THE EFFECT OF SOCIOMATHEMATICAL NORMS AND TECHNOLOGY INTEGRATED INSTRUCTION ON 6^{TH} GRADE STUDENTS' UNDERSTANDING OF ALTITUDE

by

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ABSTRACT

THE EFFECT OF SOCIOMATHEMATICAL NORMS AND TECHNOLOGY INTEGRATED INSTRUCTION ON 6^{TH} GRADE STUDENTS' UNDERSTANDING OF ALTITUDE

This action research study was designed to improve students' conceptual understanding of the concept of the altitude. For this purpose, an instruction was designed in the environment enriched with sociomathematical norms which are sharing solutions, working collaboratively acceptable mathematical explanations and being free to make mistakes. To improve the learning environment, instruction enhanced with technology by using various simulations and GeoGebra as dynamic tool. Before and after the 5-week long instruction with forty-eight 6^{th} grade students, The Altitude Test was implemented to assess students' conceptual understanding of the concept of the altitude and misconceptions related to the concept of the altitude. Teacher's field notes and classroom audio and board recordings were used to detect sociomathematical norms and use of technology in the classroom. Students' views about sociomathematical norms and technology were determined by students' journals. The results of Altitude test showed that students improved their understanding of the concept of the altitude. The result of the pre and post-tests showed that students had misconceptions related to the concept of altitude. Sociomathematically enriched, technology incorporated instruction led to decreasing or eliminating many misconceptions related to the concept of altitude and improved students' conceptual understanding. Students' journals showed that students' views about norms and technology in the classroom reflected their learning positively. The result of altitude tests and the frequency of the misconceptions were also parallel with students' views. It can be stated from the findings that the instruction enriched with sociomathematical norms helped students improve their conceptual understanding.

ÖZET

SOSYOMATEMATİKSEL NORMLAR VE TEKNOLOJİ İLE ZENGİNLEŞTİRİLMİŞ ÖĞRETİMİN 6. SINIF ÖĞRENCİLERİNİN YÜKSEKLİK KAVRAMINI ANLAMASINA ETKİSİ

Bu çalışmanın temel amacı, sosyomatematiksel normlar ve teknoloji kullanılarak tasarlanmış öğrenme ortamında yükseklik kavramının öğrenimini ve öğretimini geliştirmektir. Bu amacı gerçekleştirmek için, 4 tane sosyomatematiksel norm belirlemiş ve öğrenciler ile birlikte geliştirilmiştir. Bu normlar: birlikte paylaşarak öğrenme, sınıfta yapılan çözümleri sınıfla paylaşma, hata yapmaktan çekinmeden düşüncelerini ifade edebilme ve matematiksel açıklamalar yapabilmedir. Bu normlarla birlikte kavramsal anlamayı geliştirmek amacıyla çeşitli simülasyonlar ve GeoGebra program kullanılmıştır. Yükseklik öğretimi 5 hafta boyunca 48 altıncı sınıf öğrencisiyle yapılmıştır. Öğrencilerin ön bilgilerini kontrol etmek amacıyla öntest ve çalışma bitiminde öğrencilerin kavramsal anlamalarını ölçmek amacıyla son test uygulanmıştır. Ayrıca, sınıf içindeki sosyomatematiksel normlar ve teknoloji kullanımı öğretmen notları, akıllı tahta ve ses kayıtları alınarak belirlenmiştir. Öğrenciler her hafta sosyomatematiksel normlar ve teknoloji kullanımı hakkındaki düşüncelerini günlük tutarak düşüncelerini belirtmiştir. Yapılan analizler sonucunda, sosyomatematiksel normlar ve teknoloji kullanımıyla zenginleştirilmiş ögretim, öğrencilerin yükseklik kavramını anlamalarını olumlu yönde katkısı olmuştur. Bununla birlikte, uygulanan ön test ve son testte öğrencilerin bazı kavramsal yanılgılara sahip olduğu bulunmuştur. Uygulanan öğretim sonucunda, kavram yanılgılarının pek çoğunu azaldığı ya da tamamen yok olduğu tespit edilmiştir. Öğrenciler günlüklerinde, sınıf içindeki sosyomatematiksel normlar ve teknoloji kullanımının öğrenme süreçlerini olumlu etkilediğini belirtmişlerdir.

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LIST OF SYMBOLS

AsympSig	Asymptotic Significance
df	Degree of Freedom
f	Frequency
N	Number
Std	Standard

LIST OF ACRONYMS/ABBREVIATIONS

MEB/MoNE	Milli Eğitim Bakanlığı / Ministry of National Education
NCTM	National Council of Teachers of Mathematics

1. INTRODUCTION

1.1. General

The concept of altitude has significant place for students to understand and learn the geometry topics of area and volume (Van de Walle, 1998). In Turkish curriculum, the students learn area and volume of geometric shapes in all levels of middle school and high school. Altitude is one of basic concepts to understand these topics and students starts to learn at the 6^{th} grade. It is significant that students at 6^{th} grade understand the concept of altitude in a deep way.

I have been teaching for five years in both middle school and high school. The concept of altitude is placed in curriculum in different levels from primary school to high school. The 6^{th} grade mathematics course book, which is accepted by National Educational Ministry, the altitude of triangle defined as "a perpendicular line segment which is drawn from any vertex to the opposite side" (Güven, 2014 p. 264). I experienced that students with all levels had difficulty in drawing altitudes. Some students cannot understand the definition of the concept properly, some students cannot construct the visual image of the concept in their minds or some students cannot build the relationship between the image and the definition of the altitude exactly. Thus, students cannot understand the concept and they might have misconceptions related to the concept. They might maintain their misconceptions with them as they learn different geometry concepts such as area of the triangle. An instruction design needs to be created for better understanding of the concept of altitude.

Sociomathematical norms can be defined as a way of acting and interacting that become routine in mathematics classrooms (Cobb and Yackel, 1996). Kazemi and Stipek (2001) found that sociomathematical norms such as sharing students' solutions to classroom, working collaboratively or being free to making mistakes improve students' conceptual understanding of the mathematics topic. The classroom enriched with sociomathematical norms enable students to talk on mathematics concepts, establish communication on mathematics problems and share their ideas on problems and solution strategies. When such interactive learning environment becomes routine, it helps students' conceptual understanding improve. Researchers reached that the sociomathematical norms such as working collaboratively, sharing solution strategies or solving problems with a different method improved students' conceptual understanding of geometry concepts (Cobb and Yackel, 1996; Kazemi and Stipek, 2001; Lopez and Allal, 2007).

Researchers reached that there has been limited understanding on the geometry concept of altitude among students from primary school to college level (Cunningham and Roberts, 2010; Vinner and Hershkowitz, 1983; Gutierrez and Jaime, 1999). Students try to memorize geometric concepts without that they can understand the relationship and properties of the geometric concepts and shapes (Clements, Sarama and Battista, 1998). To teach a geometry concept better, Clements (2003) suggested that using multiple drawings and creating discussion on these drawings facilitate students' understanding of geometry.

The effective use of technology is one of the important factors that improve learning mathematics (Roschelle, Pea, Hoadley, Gordin, & Means, 2000). Researchers expressed that students communicate and reflect on the concept effectively while visualizing and modeling the concepts, designing solution strategies and verifying solutions by using the properties of technological tools. Laborde (2003) stated that technology has important role on representing, exploring and constructing abstract geometric objects and working with these objects such as rotating and reflecting. Herskowitz and Schwarz (1999) found that creating rich environment supported with technology had positive effect on understanding geometry. Researchers (Herskowitz and Schwarz, 1999) reached such result in environment that sociomathematical norms are rooted. The environment enhanced with sociomathematical norms and technology improved students' understanding of geometry.

There have been so many studies that analyze sociomathematical norms and technology integration with geometry separately. However, the number of studies which investigate teaching geometry enriched with sociomathematical norms in technology supported classrooms is very limited (Akyüz, 2014; Herskowitz and Schwarz, 1999; Kozaklı and Akkoç, 2015). I wanted to analyze the effects of the instruction enriched with sociomathematical norms and supported with technology on 6^{th} grade students' understanding of altitude.

2. REVIEW OF LITERATURE

2.1. Social Norms

Students build knowledge both individually and collaboratively while they are learning mathematics. Interactions between students and teachers help learners construct mathematical knowledge. It can be stated that doing mathematics involves social activities (Hershkowitz and Schwarz, 1999). These social activities occur in the classrooms according to some norms.

Yackel, Rasmussen and King (2000) defined the social norms as "a way of acting and interacting that become routine through ongoing participation". These norms can be the way of explaining, justifying, solving questions in different methods and interpreting the others' results or reasoning. These norms can occur in any course.

Social norms are established by teacher and students through the interactions between students and teacher. Social norms can be formed very differently, even though same teachers teach the same content. It can be stated that these norms make each classroom different (Cobb and Yackel, 1996). Every classroom constitutes their own norms according to the quality and the quantity of the interactions.

Social norms are not composed of static rules or are not prepared and presented to the classrooms only by teachers (Cobb *et al.*, 1991). Students and teacher construct and develop social norms in a mutual way. They can establish the norms by discussing and negotiating the roles and expectations of students and teachers during the learning process. Teachers have central role on establishing and developing norms. Cobb and Yackel (1996) illustrated that social norms can be built or revised again and again, as teacher and students share common understanding and interact with each other.

Social norms in the classroom are directly related to the roles of teachers and students that are casted a role by themselves or each other (Özmantar *et al.*, 2009).

These roles influence the way of behaviors, interaction and communication. In the study (Cheval, 2009), the teacher wanted to support collaborative work. Teacher emphasized on the work that group did as providing opportunities for working as a group and sharing ideas in the group. Students improved the norms of working and sharing together. Students developed and maintained the norms in the way that teacher supported students for development of the norms.

2.2. Sociomathematical Norms

Cobb and Yackel (1996) wanted to explore how learning happens in mathematics classes in the aspect of social perspectives. They focused on interactions among students and the teacher. The patterns of social interactions become routine in the mathematics classroom. They called these patterns of social norms as "sociomathematical norms". There is a significant difference between social norms and sociomathematical norms. Students demonstrate sociomathematical norms during mathematical activity. For example, giving explanation to one's thinking or reasoning is a social norm whereas giving mathematical explanation or reasoning is a sociomathematical norm (Cobb and Yackel, 1996; Kazemi and Stipek, 2001).

Mathematics classrooms can be characterized by analyzing mathematical activities. These activities include problems, solutions, justifications and explanations (Cobb *et al.*, 1992). Students interact with teacher and other students while solving problems, explaining and justifying. In a time, a common understanding constitutes related to how explanations and justifications enact. Such common shared interactions become norms.

Yackel, Rasmussen, & King (2000) explained that social norms are "normative interactions" in the classroom while sociomathematical norms are "normative understanding" related to mathematics. However, Mottier Lopez (2005) broadened the definition of sociomathematical norms as "if a norm of social interaction is negotiated and interpreted in terms of mathematical meaning, it should be considered as a sociomathematical norm".

Social norms help sociomathematical norms to construct and develop in mathematics classrooms (Cheval, 2009). Researcher revealed that teacher developed a social norm related to students' solutions, explanations and strategies. This norm transformed into a sociomathematical norm while students were explaining the way of thinking. Their dialogues included mathematical reasoning, justification and argumentation. Teacher started with a social norm, but students formed sociomathematical norms at the end of the learning process.

Social and sociomathematical norms cannot be predicted unless they are observed inside of the classroom. Researchers can understand the norms after they observed in a specific amount of time (Cobb and Yackel, 1996). Researchers need to analyze the process of learning and teaching and then they can decide which one is a sociomathematical norm or how these norms are developing. The existence or the quality of explanation, justifications and solutions cannot be determined at once. Determination is needed to study for a while. Most of students need to perform the same behaviors (Güven and Dede, 2017). For example, if most of students try to explain their solutions or solution strategies to class, it can be accepted as a norm. This norm is also needed to occur more than once. It means that students share their solutions in more than one lesson.

It is significant how sociomathematical norms introduced, constructed, developed and sustained between students and teacher in the classroom. Dixon, Andreasen and Stephan (2009) studied with elementary school teachers and designed their study as three parts: planning for negotiatian, negotiating new norms and sustaining norms. In the study (Dixon, Andreasen and Stephan, 2009), teachers firstly determined which norms were constructed. Teacher also established criteria for the norms to implement it. Researchers (Dixon, Andreasen and Stephan, 2009) stated that while establishing norms, teacher and students needed to negotiate on the norms. The norms could not be imposed by the teacher. Teacher may create opportunuties for students to negotiate on the norms. After establishing the norms, teacher maintains the norm by supporting and promoting students on taking responsibility for sustaining the norm. Similarly, Roy, Tobias, Safi and Dixon (2014) studied with undergraduate students but investigated on the reestablished and renegotiated norms. Roy *at al.* (2014) stated that teachers might need to reestablish the sociomathematical norms when the content or the topic changed. Teachers in this study sustained the sociomathematical norms of explaining, justifying and making sense of others. However, Roy *at al.* (2014) found that teachers re-established the norm of finding acceptable solutions containing justification and explanation when the content changed from whole number unit to rational number unit, Teachers promoted to reestablish the sociomathematical norm of making acceptable explanations by asking questions about students' solutions and ideas and directing students to explain the reasons of their thinking (Roy *at al.*, 2014). For example, while students were expected to make acceptable explanations for their thinking, students stated that they knew from previous knowledge without giving any other explanations (Roy, Tobias, Safi and Dixon, 2014). Teacher reestablish the norm so that previous knowledge could not be acceptable without making explanations and justifications. Teacher promoted students to give reasons for their explanations. Roy *at al.* (2014) explained the reason of reestablishment of the norms so that teachers used more procedural methods in the whole number unit and they were unfamiliar with the whole number units. When the topic of rational numbers started, teachers had more experience on the topic of rational numbers and they tried to construct conceptual understanding. The demand of the course or the experiences of the teachers might be changed and the norms might be needed to reestablish when the mathematic content changed.

Tatsis and Koleza (2008) analyzed how sociomathematical norms established and how these norms could be identified during the process of problem solving. Researchers described the norms by observing students' interactions and dialogues while solving problems cooperatively in the classroom. They found that the establishment of norms influenced understanding of concept. Cobb and Yackel (1996) were not deal with how these norms were constructed and they concentrated on the process of norms. Researchers (Cobb and Yackel, 1996) studied on second grade students' process of explanation, justification and argumentation during one year. They interviewed each student in the beginning, middle and at the end of the study. During process, they took video-recording and also assessed the notes of students and researchers. Teacher in the study listened to students' solutions and explanations and promoted their discussions. Teacher followed their developments and showed students that they improved their conceptual understanding. They reached the conclusion that students started to present better explanations, justifications and argumentations at the end of the study.

Social and sociomathematical norms can be formed with the teacher and the students together. Some researchers prefer to focus on the relationship between the norms and teacher while some of them prefer to concentrate on the relationship between the norms and the students. In both conditions, researchers analyzed the construction of norms or improvement of norms by analyzing the interactions among the students or between the teacher and the students.

2.2.1. Perspective of Teacher

There are many studies which examined sociomathematical norms in the perspective of teacher. Teachers create differences on the construction or development of the norms. There were plenty of factors that influence the sociomathematical norms such as teachers' expectations, beliefs, actions, selection and design of the tasks (Cheval, 2009; Cobb and Yackel, 1996; Kang and Kim, 2016; Kazemi and Stipek 2001). For example, Kazemi and Stipek (2001) compared different teachers' teaching strategies and examined the differences among norms in the mathematics classrooms. One teacher emphasized and promoted the norms in the mathematics classroom more than the other teacher did. Researchers found that the improvement of the sociomathematical norms showed differences on two different classrooms.

Most of the research included in comparison between classes or teachers. Güven and Dede (2017) studied with same teacher but in different courses. They compared the quality of norms in two different courses. Levenson, Tirosh and Tsamir (2009) and Lopez and Allal (2007) worked on two different classes with different teacher who gave the same courses. However, Tatsis and Koleza (2008) preferred to study on a single group. They observed how students in the same group established norms. Yackel and Cobb (1996) and Kang and Kim (2016) also studied norms with one classroom since they investigated the role of teacher in the classroom.

Cheval (2009) were looking for the answer of which actions of the teachers can help developing sociomathematical norms. Researcher found the selection of high cognitive demanding task and promotion of working collaboratively had significant effect on forming sociomathematical norms for fifth grade students. When students had difficulty in doing task, they tried to solve the problem together. They explained their strategies and try to find an efficient solution by discussing each other. Researcher also emphasized that creating supportive and safe environment played crucial role on developing norms. Students can think different solutions, find efficient solutions and explain their strategies to the classroom if classroom environment is positive and supportive. Students have the courage of sharing their ideas, even though they are not sure that it is true.

Researchers (Cobb and Yackel, 1996) revealed that teachers' roles influence formation and development of norms. Teachers' beliefs and values have effective role for forming norms. (Cobb and Yackel, 1996; Cheval, 2009). In the study (Cheval, 2009), the teacher chose high level of cognitive tasks so that students can work on and discuss on them. Teacher gave more importance to share students' ideas and listen to their explanations. Researchers found that students improve the norm of working collaboratively.

Kazemi and Stipek (2001) emphasized that sociomathematical norms are related to the expectations of teachers. Teachers create a classroom environment according to their expectation. For example, if a teacher expects students to find a different solution, teacher will also show and promote finding different solutions in the classroom. Students will spend more effort to find different solutions. Teachers play central role on formation and development of the norms as Cobb and Yackel (1996) stated. Teachers directed students according to their expectations, beliefs or values and the norms were formed according to teachers? expectations, beliefs or values. Kang and Kim (2016) investigated the relationship between sociomathematical norms and teachers' belief in elementary mathematics classrooms. Researchers (Kang and Kim, 2016) reached two important conclusions. First result was that students establish sociomathematical norms according to teacher's belief. Teachers have different beliefs about teaching and learning. This variation leads students to form different norms. The second result was that teachers' belief had important effect on decision making during learning process. Since decisions are also related to forming social norms in the classrooms, there is close relationship between teachers' belief and sociomathematical norms.

2.2.2. Perspective of Student

Researchers analyzed sociomathematical norms with students at different ages. Bowers, Cobb, and McClain (1999), Yackel and Cobb (1996), Lopez and Allal (2007), Dixon, Egendoerfer and Clements (2009) analyzed sociomathematical norms with elementary school students. Levenson, Tirosh and Tsamir (2009) and Cheval (2009) investigated sociomathematical norms for 5^{th} graders while Kazemi and Stipek (2001) studied with 4^{th} and 5^{th} graders. Hershkowitz and Schwarz (1999) analyzed sociomathematical norms with high school students. Dixon, Andreasen and Stephan (2009), Yackel, Rasmussen and King (2000), Roy, Tobias, Safi and Dixon (2014) studied on sociomathematical norms with college level students. It can be stated that researchers mostly preferred to study sociomathematical norms with students from elementary and college schools.

Dixon, Egendoerfer and Clements (2009) studied with second grade students and constructed the norms of making explanations and justifications in the student-centered environment. Students became more motivated and engaged in classroom activities while discussing their ideas, making explanations and sharing their solutions. The communication and interaction among students help students improve their understanding. Researchers (Dixon, Egendoerfer and Clements, 2009) stated that students felt free more in the mathematics classroom and more comfortable in sharing their ideas.

Levenson, Tirosh and Tsamir (2009) focused on how students perceive sociomathematical norms. Although teacher decided to build norms and endorsed the norms in the classroom, students understood that norms very differently. Students could think and imagine in a different way. For example, teacher directs students to their explanation on their practical knowledge, but students may insist on explaining their ideas in a more abstract or mathematical way. Researchers reached the result that it is difficult to assure for teachers that all students perceive the norms in a same way. As Cobb and Yackel (1996) did, teacher needed to provide negotiation on the sociomathematical norms between students. By promoting and supporting students' discourse and discussions in the classroom, students might construct the norms in the same way that teacher tried to establish.

2.2.3. Observing Sociomathematical Norms

There were many ways of observing sociomathematical norms that the researchers preferred. The norms such as making mathematical explanations could be understood from discourses in the classroom so that majority of the researchers analyzed the sociomathematical norms as videotaping the lessons or monitoring students by an observer (Kang and Kim, 2016; Kazemi and Stipek, 2001; Yackel and Cobb 1996). Sekiguchi (2005) and Kang and Kim (2016) also used students' activity sheets beside observing students. Sekiguchi (2005) found that using students' works played significant role on forming and developing norms. Teachers and researchers could follow the ways of solving problems, explaining their ideas and interpreting the answers. The students' performances on the norms were observed more easily.

There are many types of sociomathematical norms that students and teachers can construct and improve during the learning process, but researchers select and examine different types of norms according to their studies. However, some norms are analyzed more often in the literature. For example, Yackel and Cobb (1996) who introduced the term of sociomathematical norm focused on the norms of mathematically different, sophisticated and efficient. After Yackel and Cobb, many researchers started to study on one of these norms or combinations of them. Researchers (Kang and Kim, 2016; McClain and Cobb, 2001; Sekiguchi, 2005; Tatsis and Koleza, 2008) who studied with sociomathematical norms develops the norms of efficiency. It is related to solving problems in easiest and meaningful way. Lopez and Allal (2007) stated that it is really important that teachers encourage students to find effective solutions, strategies and procedures. Students try to solve the problems more efficiently. The norm of effectiveness is influenced directly by the interaction of teacher and students (Lopez and Allal, 2007). When teachers expect and value that students find easiest and meaningful solutions in the classroom, students deal with finding solutions efficiently.

Another significant norm that researchers investigated on is mathematical difference (Kang and Kim, 2016; Lopez and Allal, 2007; McClain and Cobb, 2001; Yackel and Cobb, 1996). Cobb and Yackel (1996) defined "mathematical difference so that if students solve the problem in a different way or explain and justify their answers differently, then it creates a difference mathematically. Students can explain a mathematical idea, solve a problem or inquire the process in a different way that the other students did not solve the problem in that way before. Teachers can use the term "difference" in the class such as "who wants to solve the problem differently?". When students know the meaning of it, students try to make different explanations and argumentations to the class. Lopez and Allal (2007) also reached similar results so that solving problems from several ways, working on alternative solutions and trying new strategies helped elementary school students improve conceptual understanding.

Cobb and Yackel (1996) found that students started to take more sophisticated and efficient actions when teacher explained the terms of sophistication and efficiency and applied these norms in his/her actions. Teachers also started to select more different and sophisticated tasks and tried to do sophisticated interpretations to students' answers. Teachers need to explain which solutions are accepted as sophisticated or efficient and the students understand how problems could be solved in sophisticated or efficient ways. The results of this study showed both students and also teachers seemed to have a common understanding of the norms and students presented sophisticated and efficiency solutions in the classroom (Cobb and Yackel, 1996).

Sekiguchi (2005) developed a norm that inefficient solutions might also include significant ideas. When students start to solve a problem, they may solve it in an inefficient way or may even fail to solve. Students need to feel that their way of solutions is also valuable. Teacher need to provide supporting environment such that every students' solutions in the classroom is significant. In the time, teacher directs them to find the efficient solutions. The other norm was built by Sekiguchi (2005) that accuracy is more valuable than speed. To reach the correct result, it is important for students to check their answers. Teacher may emphasize that writing each step during solution process are helpful for students to check their answers. Sekiguchi (2005) reached that while students are solving problems, it was crucial to develop this norm in the mathematics classrooms.

Sometimes norms may seem as contradicting with each other. For example, the norms of “solving problems efficiently” and “inefficient solutions also have significant ideas” can be seen as contradiction. The second norm is crucial to encourage students to try to solve the problems. While second norm is important for starting point, solving efficiently has crucial role on reaching correct answer in a short time. Students need both of the norms to improve their understanding of mathematics.

Explanations are also important parts of the sociomathematical norms. Cobb and Yackel (1996) found that students improved their mathematical explanations as they participated inquiry process in the mathematics classroom. Mathematical explanations can be constituted by mathematical reasoning, solution strategies, conceptual or procedural ideas. It is crucial to determine what type of the explanation is acceptable in the classroom. Cobb and Yackel (1996) set acceptable explanation which students can construct the explanation how they understand in their own meaningful way. Teacher needs to determine on what acceptable explanations are for mathematics classrooms and needs to negotiate on this norm with students. Teachers and students could construct such explanations interactively in the process of learning. Teachers need to encourage students from participating explanation to generating explanation. Cobb and Yackel (1996) emphasized that when students take the place of explanation, they have deeper conceptual understanding. Students improve their understanding as making explanations since they think deeply and find a logical reason to make acceptable explanations.

Kazemi and Stipek (2001) established four norms: elementary school students solve the problems with different methods; they explain their solutions to the class; they work collaboratively; they are free to making mistakes. Researchers constructed these norms since they expected that these norms supported conceptual understanding. In the study, students were promoted to think and find different solutions. To create the norm of working collaboratively, students were supported to construct mutual understanding among group members and have individual responsibility for understanding group work. Students were provided opportunities to share their solutions with the classrooms. The environment was also supportive that students feel free to share and discuss students' ideas without being afraid of making mistakes. At the end of the study, students learned that they need to justify their solutions, establish connections among mathematical ideas and use mistakes as an opportunity to rethink the problem. They found that when teachers increasingly encourage students to develop these norms, students also understand the mathematical subject conceptually better. Kazemi and Stipek (2001) stated that students present better mathematical reasoning, argumentations and solution strategies after constructing and developing these sociomathematical norms.

2.2.4. Classroom Settings

Researchers designed different types of classroom settings while investigating sociomathematical norms. Lopez and Allal (2007), Tatsis and Koleza (2008) established and developed norms in problem solving activities. They found that problem solving process helps sociomathematical norms improve. Cobb and Yackel (1996) created inquiry-based environment such that teachers and students construct sociomathematical norms together. In such different environments, sociomathematical norms improved students' understanding.

Lopez and Allal (2007) stated that whole class, small group (Tatsis and Koleza, 2008) or both of them were used to investigate norms in problem solving activities. During these discussions, students have opportunity to explain and justify their reasoning and interpretation (Yackel and Cobb, 1996). Lopez and Allal (2007) and Tatsis and

Koleza (2008) examined how sociomathematical norms are constructed and developed during problem solving activities. Lopez and Allal (2007) emerged two sociomathematical norms from problem-solving environment: explanation of problem-solving strategies to the class and mathematical differentiation on problem solving. Tatsis and Koleza (2008) found that collaborative problem-solving environment helps sociomathematical norms establish.

Kazemi and Stipek (2001) studied on the establishment of sociomathematical norms in inquiry-based environment. They reached the conclusion that when teachers had a framework of sociomathematical norms, they had good opportunity to think on which norms should be emphasized. If teachers wanted to create an inquiry-based environment, they promoted students to investigate the unknown facts and challenging tasks. Students were promoted to think on the facts, solve problems, share and discuss their ideas. Kazemi and Stipek (2001) developed sociomathematical norms which support the conceptual understanding such finding different solution methods and explaining their solutions to the classroom. They observed that students improved their conceptual understanding about a specific math topic.

Cheval (2009) designed a research to explore which aspects teachers can promote $5th$ grade students to develop sociomathematical norms in a reform-based classroom. Researcher described reform-based classroom such that students play active role during the learning process. High level of cognitive demand task was used since this kind of activities includes challenging questions and encourage students think deeply and talk about it. One of important results was that the teacher' choices play crucial role on shaping sociomathematical norms. These choices included selection of high demand task, creating collaborative environment, encouraging students to think by inquiry method. Researcher emphasized that first three weeks are good opportunity for teacher to establish sociomathematical norms. While students were starting to build interactions among each other and with the teacher, the teacher promoted students to express their mathematical thinking and share their ideas with the classroom. Researcher also reached that creating safe and supportive environment helped students construct sociomathematical norms like working collaboratively and sharing their solutions.

Dixon, Egendoerfer and Clements (2009) examined the effects of the student-centered dialogues on the sociomathematical norms of making mathematical explanations and justifications during whole class discussions for second grade students. Researchers (Dixon, Egendoerfer and Clements, 2009) found that whole class discussions helped students make exploration and their understanding became more conceptual. The norms of making explanations and justifications increased students' thinking skills and improved their understanding.

There are limited studies which investigated the sociomathematical norms in technology enriched classrooms (Akyüz, 2014; Herskowitz and Schwarz, 1999; Kozaklı Akkoç 2015). Akyüz (2014) designed a research to investigate the sociomathematical norms in technology and inquiry-based classroom for teaching the properties of circle. Researcher established three sociomathematical norms: inquiring the effects of changing a variable in the question or in the answer, draw a conclusion by using the properties of the software and justifying the solution or result dynamically. Researcher called these technology related sociomathematical norms as "techno-sociomathematical norms" since these sociomathematical norms were integrated with technology. To determine these norms, researcher set three criteria; norms should be repeated sufficient number of times, these norms should occur only in mathematics classrooms, a mathematics subject should be taught by using technology which needed to improve a norm. If a norm included three criteria at the same time, then it was defined as "techno-sociomathematical norm". According to the study, it can be stated that technology can be used to facilitate and develop effective sociomathematical norms.

Herskowitz and Schwarz (1999) designed rich learning environment including open-ended problem situations, small group working, use of technological tools and multi-phases such as problem solving, reflection and reporting. They examined how social norms and sociomathematical norms were developed in a such rich environment. They reached significant conclusion that norms did not only emerged from verbal interactions between teacher and student or among students but also from interaction between students and technological tools.

Kozaklı and Akkoç (2015) analyzed the social and sociomathematical norms in the technology enhanced mathematics classroom where pre-service mathematics teachers chose and developed the norms in high school. Teachers prepared their lesson plans for geometry topics and integrated some lessons with technology. All teachers in the study selected and constructed various sociomathematical norms for their classrooms such as making acceptable explanations for students' own solutions and being free for making mistakes. However, all of the teachers built one norm as common which was the sociomathematical norms of "software is used for justification, proving and visualization". (Kozaklı and Akkoç, 2015 p. 92). Teachers promoted students to justify and prove their findings with the technological tools. After teachers used technology in their lessons, they also gave importance to sociomathematical norm of justifying their solutions in the environment with no technology. Another result was that the property of dragging in GeoGebra might direct students to examine the reasons behind the mathematical phenomena and justify mathematical theorems and properties. Moreover, researchers also found that the sociomathematical norms of sharing students' ideas and working collaboratively were encountered more in the environment of technology. As Akyüz, (2014) found, technology might help some sociomathematical norms develop.

The sociomathematical norms that students experience in the mathematics classroom help them improve their learning and understanding process (Yackel and Cobb, 1996). The environment supported with sociomathematical norms provides both individual and cooperative construction. When students learn mathematics, they construct the mathematics knowledge with individually and cooperatively. Such environment enhanced with sociomathematical norms improves students' understanding.

2.3. Geometry

Geometry helps students analyze properties and relations of geometric shapes to develop students' justification, visualization, modeling and reasoning skills (NCTM, 2000). Geometry lessons provide rich environment for students such that students can draw, describe and explore the geometric shapes and the properties of geometric shapes. Turkish Elementary Mathematics Curriculum (MoNE, 2017) expected that students have general knowledge about points, angles, geometric shapes in the primary school, but in middle grades, students learn the properties of the geometric shapes and explore geometric shapes more deeply. Similarly, NCTM (2000) stated that students in the middle school can define, classify, draw, compare, visualize and transform geometric shapes to explore relationship among geometric concepts. However, students may have difficulty in visualizing the geometric shapes, understanding the properties of shapes or building connections among concepts. Many researchers reached that students had limited understanding on geometry concepts among students from primary school to college level (Cunningham and Roberts, 2010; Vinner and Hershkowitz, 1983; Gutierrez and Jaime, 1999). To teach and learn geometry better, there have been presented so many techniques, strategies, models and tools.

Vinner and Hershkowitz (1983) defined a model that students can form a concept in their minds via the definition of the concept and the image of the concept. They stated that students can learn a concept better when they have concept image and definition together. It is also significant how students establish connections between concept image and definition. Students can form concept image and definition when teachers create an environment that students can examine the examples of the concept (Gutierrez and Jaime, 1999). Students need to build relationship between the concept image and concept definition to understand the concept better.

To learn geometry better, visualization has important effect on it (NCTM, 2000). There is a close relationship between geometry and visualization since geometry includes visual elements in it. Students can perceive any geometric concept better with visualizing it (Hershkowitz, 1989). Students form a visual image of a concept with the help of the definition of the concept. When they face with different examples, they take the concept image as representative and make interpretation for the concept. Concept image plays significant role on learning a geometric concept and it is formed with visualization.

Students with different levels deal with geometric constructions while learning different attributes of geometry. Geometric constructions help students explore the properties of the shapes with hands-on experience (Cheung, 2011). They enable students represent the properties through drawings and proving geometrical explanations (Chan, 2006). When students are experiencing the construction, they can build connections among abstract concepts and deepen geometric thinking.

Clements (2003) expressed that interactive and dynamic computer programs are also helpful for understanding the geometry topics better. These programs enable students to manipulate geometry objects, their elements and features. Students can learn abstract geometry concepts better with the help of these programs (Clements, 2003). These programs help students to visualize the geometry objects better, explore the features of objects and increase the interaction between students and geometry concepts (King and Schattschneider, 1997).

Wilson (1986) expressed that a geometry concept can be thought in three ways; teaching and practicing with all types of examples, relevant features of the concept and related rules defining the concept. Teaching with all types of examples includes negative and positive examples related to the concept. Wilson (1986) found that students learned the geometry concepts when they experienced with both positive and negative examples of the concepts. Students could identify relevant and irrelevant characteristics of the geometry concepts.

In addition to Wilson (1986), Clements (2003) presented four significant suggestions to help teaching geometry. Firstly, geometry teaching is more effective when teacher give important clues or feedback to avoid misconceptions. Secondly, examples and non-examples, prototype and non-prototype examples are needed be practiced very well. Thirdly, students need to examine and discuss every attributes of given concept. Fourthly, more than one concept related to concept are needed be taught at the same time. Teachers can create environment that students discuss on examples and non-examples, prototype and non-prototype examples with each other. While students are sharing their ideas and solutions, their misconceptions can be revealed and eliminated during the discussions. When students learn more than one concept together, students can perceive and realize the attributes of the geometric concepts better.

There are many concepts in the geometry and one of them is the concept of altitude. The concept of altitude was mostly misunderstood by students. These topics are placed in the topics of auxiliary elements of triangle, area and volume in the Turkish Mathematics Curriculum (MoNE, 2017). Students started to learn this concept in the middle school and continue their learning during the high school. If they can learn the concept properly and eliminate misconceptions in the beginning of middle school, they can understand many geometric concepts easily.

2.3.1. Altitude

Altitude defined as "an altitude of a triangle is the perpendicular segment from a vertex to the line that contains the opposite side" (Cunningham and Roberts, 2010). The mathematics curriculum in Turkey expected that 6th grade students understand the concept of altitude and calculate the area of parallelogram and triangle (MoNE, 2018). In Turkish settings, students continue to learn this concept with the concepts of the properties of polygons, area and volume in middle and high school. There were many researches related to the concept of altitude. According to previous studies (Fischbein and Nachlieli 1998; Hershkowitz 1989; Vinner and Hershkowitz 1983) drawing altitude is crucial factor that affects the understanding the area of triangles.

Orhan (2013) found that students had strong procedural knowledge related to the concept of area. They could calculate the questions of area easily when the edges and heights were given directly. However, they had difficulty in finding the area of parallelograms and obtuse-angled triangle. The reason behind it is that students could not determine the height of the geometric shapes. The researcher realized that most students had problems with determining the altitudes of these geometric shapes.

Orhan (2013) studied with 6^{th}, 7^{th} and 8^{th} grade students and researcher wanted them to calculate the areas of parallelograms and triangles. Results of the study stated that 8^{th} graders showed more success on determining the base and height of the triangle than 6^{th} and 7^{th} graders. 6^{th} graders had higher procedural performance on the finding area of triangle and parallelogram than 7^{th} and 8^{th} graders. However, 8^{th} graders were the most successful grade on the conceptual test. Researcher explained the reason such that students learned the concept of area of triangle and parallelogram in 6^{th} grade and they had all the procedural knowledge in their minds, but they started to understand the concept more deeply in later grades.

Researchers (Vinner and Hershkowitz, 1983) reached a generalization such that students who could succeed in drawing altitude in right angled and obtuse triangle also showed achievement on drawing altitude in all types of triangle. Most of students could draw the altitude to the triangle which had altitude inside of it. Students showed higher success on triangles which had equal sides. The reason behind it was that altitude was also bisector and median at the same time. Majority of students had difficulty in drawing altitude for right angled and obtuse triangle. There were so many unanswered questions including obtuse and right-angled triangles for 5^{th} and 6^{th} graders. Only 30% of 8^{th} graders could have the concept of altitude. As the grade increased, the misconceptions related to the altitude inside of triangle were decreasing. However, the misconceptions related to the outside of triangle did not change across the grades.

Vinner and Hershkowitz (1983) found that 20 in-service mathematics teachers experienced similar difficulties as students did. Only 8 in-service mathematics teachers could draw the altitudes correctly. Teachers could not also understand the concept of altitude. Gutierrez and Jaime (1999) investigated similar topic and researchers examined 190 pre-service primary teachers' understanding of the concept of altitude of a triangle. They concluded that the concept is not easily grasped by pre-service primary teachers.

Wilson (1986) defined relevant and irrelevant features related to the concept of altitude. Relevant features were expressed as "originating at a vertex and terminating perpendicular to a line containing the side of the vertex" while irrelevant features were described as orientation of triangle, slope of the altitude (vertical/not vertical), location of the proposed altitude (inside/outside) (Wilson, 1986 p. 132). It can be beneficial that students understand and differentiate relevant and irrelevant features of the concept. Students also need to practice drawing altitude to the triangles which vary relevant and irrelevant features.

The researchers (Gürefe, Yarar, Pazarbasi, & Es, 2014) designed a research to investigate the effects of Conceptual Change Text on 5^{th} grade students' understanding of the concept of altitude. Conceptual change text included some texts related to explanations related to misconceptions of the concept of altitude, detailed explanations related to the definition of altitude and problems related to altitude. The students who take the instruction of Conceptual Change Text had more success on the altitude test. Researchers expressed that Conceptual Change Text is very helpful for understanding of the concept of altitude since it improved conceptual understanding.

Yıldız, Güven and Koparan (2010) worked with 25 students at 8^{th} grade and found that Cabri as a dynamic software affected students' understanding on the concept of altitude, diagonal and perpendicular bisector positively. Researchers reached that all of the students could draw the altitude to acute angled triangle, 16 students could be successful at drawing altitude to right angled triangle and 19 students could draw altitude to the obtuse angled triangles after using Cabri. When learning environment enhance with dynamic software, Cabri, most of students could understand the auxiliary elements of triangle. However, there might be still some students who could not draw altitude to the all types of triangles successfully.

According to the studies, altitude is significant concept to understand for almost all grades from primary school to college school. It is significant that how students learn the concept of altitude. Teaching strategies and methods need to be designed in a way that students can eliminate misconceptions related to the topic and they can understand the concept very deeply.

2.3.1.1. Concept of Altitude. The 6^{th} grade mathematics course book, which is accepted by National Educational Ministry in Turkey, the altitude of triangle defined as "a perpendicular line segment which is drawn from any vertex to the opposite side" (Güven, 2014 p. 264). Researchers (Cunningham and Roberts, 2010) stated that the definition is inadequate for drawing altitude to the obtuse angled triangle. This definition could be accepted as "adequate" if the extension of the opposite side is mentioned (Cunningham and Roberts, 2010). When students learn the concept of altitude form such inadequate definitions, they may have difficulty in understanding the concept properly and drawing altitude to the obtuse angled triangles.

Guo and Pang (2011) determined six critical aspects for the concept of altitude. They were vertex, perpendicularity, opposition, orientation, location and altitude-base-correspondence. It is significant for students to examine in their drawings that if altitude passes through or not, it is perpendicular to the side, it is correspondence with the right bases, it is inside or outside the triangle, it could be any orientations and the vertex does not have to be opposite to the side (Guo and Pang, 2011). Students need to understand these aspects and practice drawing altitudes to the triangles. These aspects are so crucial to understand the concept of altitude completely (Guo and Pang, 2011).

Guo and Pang (2011) explained how conceptual knowledge can be measured for the concept of the altitude. The conceptual knowledge on altitude can be measured via the ability to recognize and to explain (a) whether or not a segment was an altitude of a triangle, (b) a specified side of a triangle to which an altitude was perpendicular, and (c) three specified sides of a triangle to which three altitudes were perpendicular (Guo and Pang, 2011).

Students can inquire if the drawn line segment is an altitude or not by using six critical aspects of the altitude. Students can also share and discuss these three points with each other in the classroom after drawing altitudes. It can be stated that students have conceptual knowledge about the topic of altitude if they can understand these three points completely.

Vinner and Hershkowitz (1983) developed a model that students can learn a concept via the definition of the concept and the image of the concept. The concept image is that students remember an image or a representation belongs to the concept in their mind (Gutierrez and Jaime, 1999). It may contain an example related to the concept or properties of the related to concept. The concept definition includes all necessary properties related to the concept. With the definition of the concept, students identify the critical features of the concept. Concept image facilitates recognition of visual characteristics of the concept. Vinner and Hershkowitz (1983) expressed that students can learn a concept better when they have concept image and definition together. It is also significant how students establish connections between concept image and definition. Students can form concept image and definition when teachers create an environment that students can examine the examples of the concept (Gutierrez and Jaime, 1999). Students need to build relationship between the concept image and concept definition to understand the concept better.

Blanco (2001) examined the activities of students and found that students were successful at defining the altitude of triangle. However, they could not draw the altitude of the triangle in a correct way. The researcher reached that there is a difference between the definition and the representation of altitude. Azcarate (1997, p 29) explained the reason behind it that "memorization of definition is no guarantee of understanding its meaning". Student may memorize the definition of the concept, but they cannot draw altitudes to all types of triangles correctly without understanding the definition. The researcher stated that students needed to understand the mental image, properties, procedures and needed to experience some critical examples related to the concept.

Vinner and Hershkowitz (1983) studied with 189 students in grades 6'8 to assess their understanding of basic geometric concepts. The results of the study showed that only about 30% of the participants could correctly construct the required altitude of a right-angled or obtuse triangle, even though they were provided with the definition of the concept. About 20% of the students could construct the altitude correctly without being provided with the definition. The results showed that students did not understand the concept of altitude properly.

Gürefe and Gültekin (2016) examined 8^{th} grade students' knowledge on the concept of altitude. They reached that most of the students had difficulty in defining the altitude. Some students could define altitude pictorially or symbolically. Although there were no students who defined the altitude as perpendicular line segment, students who defined the altitude pictorially drew the altitude as perpendicular and line segment. Researchers explained the reason such that students could memorize the altitude as an image and they could not understand the topic of altitude as conceptual. This result showed that the study Vinner and Hershkowitz (1983) conducted had different results from the study Blanco (2001) conducted.

Fischbein and Nachlieli (1998) found that when the instruction level related to drawing altitude was increased, the definition of altitude showed improvement. Students from 9^{th} to 11^{th} grades who took the high level of instruction could mention about extension of the base and altitude outside of the triangle. Even though there was close relationship between defining correctly and drawing properly, the percentage of students who drew altitude properly was quite lower than the students who defined it correctly. As Blanco (2001) and Vinner and Hershkowitz (1983) explained, defining altitude correctly could not guarantee that students could draw the altitude correctly.

2.3.1.2. Misconceptions. Students learn the attributes of concepts and related to examples while learning the mathematics concepts. However, students may have misunderstandings related to the concept. Misconceptions can be defined that which students' understanding of the concept is different from its actual meaning accepted as scientific (Yag-basan and Gülçiçek, 2003). Misconceptions may seem sensible to students and may stay students' minds. It is significant for students to understand the concept and eliminate the misconceptions with a proper instruction. (Smith, DiSessa and Roschelle, 1994).

Learning consists of both learning new concept and eliminating misconceptions (Smith, DiSessa and Roschelle, 1994). The misconceptions need to be revealed and eliminated so that students can construct understanding of the concept completely.

Smith *et al.* (1994) stated that misconceptions mostly originated from previous knowledge or arise in the mathematics classroom. Misconceptions stays permanently if it is not replaced by expert concept. Sometimes the misconceptions were changed with a designed instruction or sometimes they were replaced with expert concept accidentally (Smith, DiSessa and Roschelle, 1994). The misconceptions from elementary schools may last during high school or college level if it is not changed (Bütüner and Filiz, 2017).

Students may make errors or have misconceptions while they are learning mathematics. Borasi (1994) investigated how secondary school students took advantages of making error as learning opportunity. The researcher found that making error helped students reexamine the solutions, discuss and reflect on their findings. Making errors or having misconceptions created an opportunity for reexploring the problem and creating communication with peers (Borasi, 1994). While students discussed the reasons of the misconceptions or errors with peers, they had better understanding. Researcher emphasized on the role of the teacher in the classroom. Teacher needed to be aware of potential errors or misconceptions, support the inquiry in the classroom and promote students to discuss on them. If teachers can create an environment that students reflect on their understanding, sharing their solutions and discussing on their errors, students improve their understanding.

There are some reasons which students have misconceptions related to geometry. Bütüner and Filiz (2017) explained the reasons so that students may overgeneralize the rules or they cannot understand the geometry concepts conceptually. For example, while students are finding the area of parallelogram, they tend to multiply length and width. Students generalize the area of square and rectangle to the area of parallelogram. There are many geometric concepts such as area that students have misconceptions and one of them is the concept of altitude. Many students from middle school to high school have various misconceptions or partial conceptions related to the concept of altitude (Fischbein and Nachlieli, 1998; Gutierrez and Jaime, 1999; Vinner and Hershkowitz, 1983).

Guo and Pang (2011) studied with 4th grade students for the concept of altitude. They expressed that students had difficulty in drawing altitude to the right angled triangle and obtuse triangle. Since students have the concept image of altitude only as "internal altitude", students gave no answer for right angled triangles or gave wrong answer. It showed that students had limited understanding related to the definition and they also could not establish the relationship between concept image and definition.

Hızarcı, Ada and Elmas (2006) studied with 230 preservice mathematics teachers to analyzed participants' definitions and drawings of the concept of angle, diagonal and altitude. As Guo and Pang (2011) found, researchers (Hızarcı, Ada and Elmas, 2006) found that most of the preservice mathematics teachers drew all the altitudes to obtuse triangle inside of the triangles and showed the center of the altitudes (orthocenter) inside of the obtuse angled triangle. The image of the all altitudes to the triangles in students' minds is as internal. Similarly, Bütüner (2017) analyzed how 52 preservice mathematics teachers defined the concept of altitude and drew altitudes to the triangles. Researcher found that participants had misconceptions while drawing altitude to obtuse angled triangle. The most common misconceptions in the study (Bütüner, 2017) were drawing altitude inside, drawing median or perpendicular bisector and drawing altitude to the wrong side. Students even at college level had misconception related to the concept of altitude especially for obtuse angled triangle.

Fischbein and Nachlieli (1998) found that students did rarely make mistake while drawing altitude to isosceles triangle. They reached a generalization from their studies such that isosceles triangle is prototype example for drawing altitude. The reason behind it was that altitude was a median and a perpendicular bisector and it was also inside of the triangle. Although they confused the concepts of median, perpendicular bisector and altitude, they can draw altitude properly to the isosceles triangle.

There is significant impact of the position of triangle to draw the altitude. Students have difficulty in drawing altitude while the position of triangles is different from the original one. Fischbein and Nachlieli (1998) found that 93% of the students from 9^{th} to 11^{th} graders could draw the altitude of acute triangle standing on its base correctly. However, they could not draw the altitude of the triangle when it was changing the position upside down. Researchers reached the conclusion that the position of the triangle had strong effect on students. Similarly, Vinner and Hershkowitz (1983) found that the position of the acute angled triangle affected middle school students' drawing altitudes negatively. However, the rotation of obtuse and right angled triangles did not create any differences on students' drawing (Vinner and Hershkowitz, 1983). The reason is that majority of the students already could not draw altitude to obtuse and right-angled triangles.

Gutierrez and Jaime (1999) mainly determined six misconceptions in their study. These misconceptions were the most common ones who were encountered in students' drawings of altitude to the triangle. These misconceptions were altitude vs. median, altitude vs. perpendicular bisector, limitation to internal altitudes, disregard of length, fixation on side and marked base as distracter.

2.3.1.3. Altitude vs. Median. Students may confuse the concept images of altitude and median and draw median instead of altitude. Gutierrez and Jaime (1999) found this misconception the most frequent error in his study. Researchers stated that there are two reasons behind it. One of them is that students keep the image of the concept in their mind instead of understanding the definition of the concept. This image is mostly composed of an altitude which is drawn inside of the acute angled triangle. The other reason is that when students draw the altitude of the obtuse triangle, they could not imagine drawing the external altitude and they draw a median inside of the triangle.

Fischbein and Nachlieli (1998) found that 26% of the sample had the mistake of drawing median instead of altitude. Researchers pointed out that the most frequent error was drawing median to required side or drawing an altitude from different vertex. However, Vinner and Hershkowitz (1983) observed this misconception only once in their sample. One of their sample always drew median to right-angled, acute or obtuse triangle instead of drawing altitude.

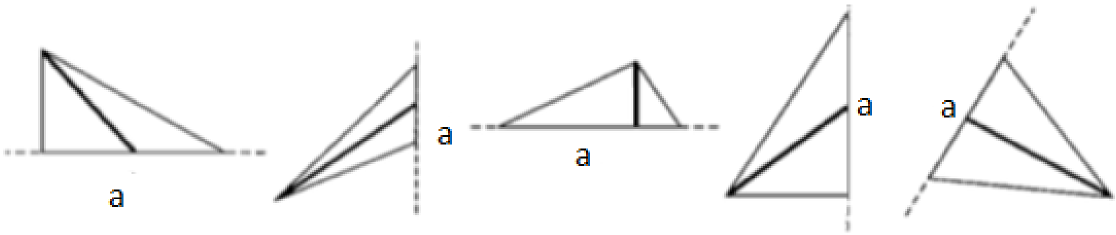


Figure 2.1. The misconception of altitude and median (Gutierrez and Jaime, 1999 p. 269).

<u>2.3.1.4. Altitude vs. Perpendicular Bisector.</u> Students may confuse the altitude and the perpendicular bisector of triangle. Gutierrez and Jaime (1999) encountered with this misconception in their study very rarely. Similarly, Vinner and Hershkowitz (1983) found that only one of their participants from middle school systematically draw perpendicular bisector, instead of altitude, to all types of triangle. Fischbein and Nachlieli (1998) reached similar result such that 8% of the sample from high school drew perpendicular bisector.

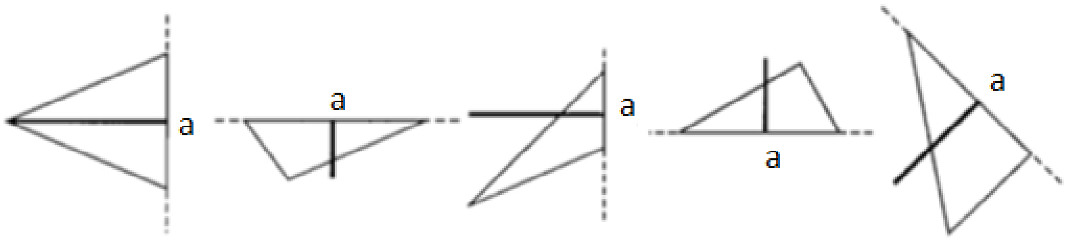


Figure 2.2. The misconception of altitude and perpendicular bisector (Gutierrez and Jaime, 1999 p. 269).

<u>2.3.1.5. Limitation to Internal Altitudes.</u> Student may draw the internal altitudes of the different side. Students understand the concept of altitude partially. They could not draw an altitude to the specified side in the acute triangle. Vinner and Hershkowitz (1983) encountered this misconception with such students who could success at drawing altitude correctly to all types of triangles except from obtuse triangle. Researchers (Vinner and Hershkowitz, 1983) explained the reason behind it such that students had a concept image of altitude as "being inside". Students drew altitudes in a way of being inside of the triangle.

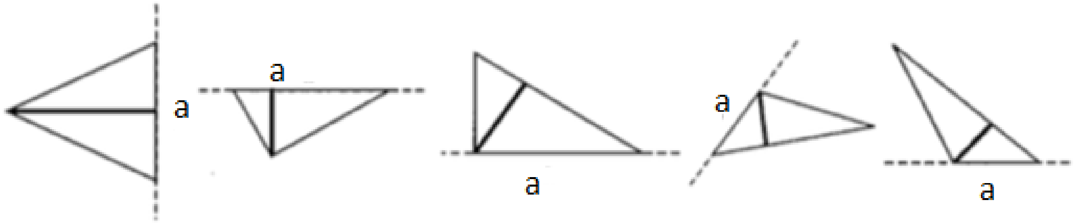


Figure 2.3. The misconception of internal altitude (Gutierrez and Jaime, 1999 p. 270).

2.3.1.6. Disregard of Length. Students draw the altitude to right side, but the length of the altitude is inappropriate. They draw the altitude such as ray or segment of undetermined length. Except from the length of altitude, all characteristics of altitude are proper. Gutierrez and Jaime (1999) found this misconception the second frequent one in the study. Gürefe and Gültekin (2016) found that there were no 8^{th} grade students who define the altitude as perpendicular line segment. Students had partial image of the concept of altitude. They disregarded the length of the altitude.

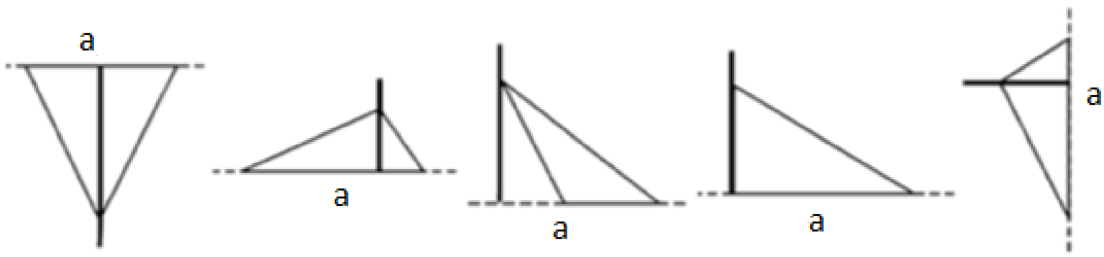


Figure 2.4. The misconception of disregard of the length (Gutierrez and Jaime, 1999 p. 270).

2.3.1.7. Fixation on Side. Some students may understand the concept of the altitude, but they can draw the altitude to the only specific type of triangle. For example, Gutierrez and Jaime (1999) stated that one of the samples can draw the altitude of right triangle and isosceles triangle. The participants could not draw the altitude for the other type of triangles. Fischbein and Nachlieli (1998) found that 93% of students in grades 9-11 were successful at drawing the altitude to the base of a triangle. When the triangles were rotated, they could not determine the altitude because of change of base.

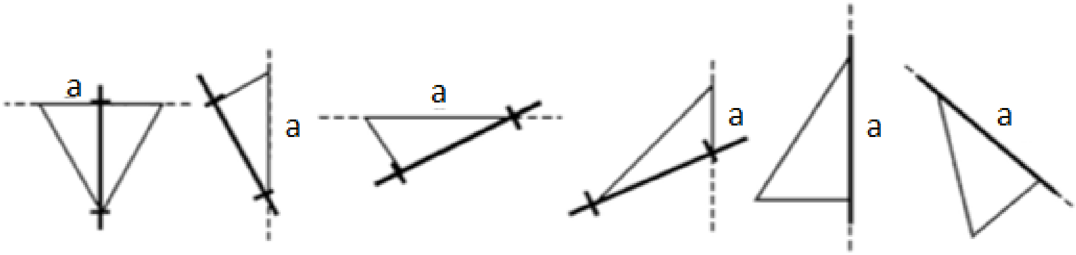


Figure 2.5. The misconception of fixation on side (Gutierrez and Jaime, 1999 p. 271).

2.3.1.8. Marked Base as Distracter. Students may learn critical characteristics of altitude as starting from the corresponding vertex, passing through the specified base and being perpendicular. When they were drawing altitude to triangle, they may miss the information of being perpendicular. Gutierrez and Jaime (1999) presented students' responses as the Figure 2.6. Students can draw the altitudes to right triangles even they have different rotations. However, students cannot draw the altitude as being perpendicular for the right and obtuse angled triangles.

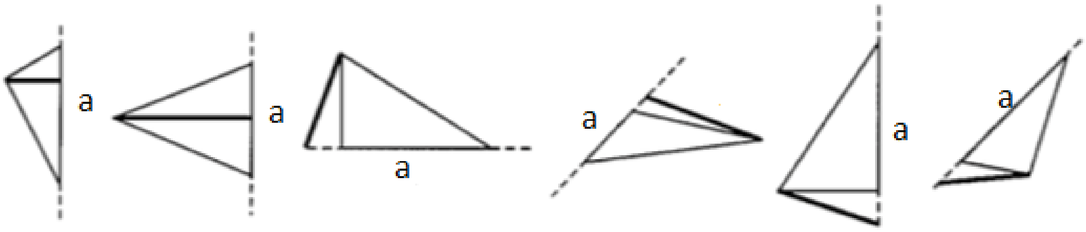


Figure 2.6. Marked base as distracter (Gutierrez and Jaime, 1999 p. 271).

Fischbein and Nachlieli (1998) expressed that effective instruction improved their learning and help eliminating misconceptions. Gutierrez and Jaime (1999) had some suggestions to eliminate the misconceptions and improve the conceptual understanding of altitude. First suggestion is that an instructional design which addresses possible misconceptions and eliminates the misconceptions related to the concept. Such an instruction secondly needs to enhance with various concept images related to topic. Another suggestion is thirdly that class discussion to emerge misconceptions and reflect what students understand. Last suggestion is that students are given opportunity to explain, defend and present their ideas during the process of learning.

2.3.2. Teaching Altitude for Understanding

In literature, it was found that there is crucial effect of teachers' definitions, concept images, difficulties and errors about altitudes of triangles on students' understanding (Gutierrez and Jaime, 1999). Knowledge of teachers is so significant that students learn the topic correctly. When teachers have information that students may have some misconception about the concept, they can put emphasize on what might be the possible errors so that students have the awareness of it.

Vinner and Hershkowitz (1983) emphasized on the relationship between concept definition and the concept image for conceptual understanding. Teachers need to build strong connection among them so that students could understand the concept deeply (Gutierrez and Jaime, 1999). While students are learning critical features of the concept, students should form concept image correspond to concept definition together. Gutierrez and Jaime (1999) suggested that teachers could want their students to draw and reflect their concept image related to the altitude. Researchers also stated that students could discuss and reflect on the others' concept images. This process could be helpful for students to realize the differentiation among other students' concept images. By reflecting and discussing on the images, they could gain better conceptual understanding.

Hershkowitz (1989) suggested that students need to experience different examples apart from prototype example. Students could take prototype example as a reference or concept image in their mind to improve understanding the concept, but they need to experience different characteristics of the concepts. Prototype examples include common visual characteristics and they are popular in the textbooks. Students need to experience non-prototype examples adequately such that they could form concept image for them.

Smith, DiSessa and Roschelle (1994) pointed out that the first step to overcome misconceptions should be identify the misconception. The learning should be designed for removing misconception and then inserting the correct one. To do this, researchers stated that students needed to confront the misconception. After the confrontation, students started to accept the new concept with appropriate instruction. Olkun and Toluk (2004) suggested that teachers promote students to talk on their misconceptions and errors. They could realize their errors during the discussion. If students have misconceptions related to topic, these misconceptions can be revealed in that discussions.

Guo and Pang (2011) described six critical aspects for the concept of altitude; vertex, perpendicularity, opposition, orientation, location and altitude-based-correspondence. These aspects are so significant for understanding the concept of altitude. They are needed to be varied among questions. Guo and Pang (2011) indicated that use of multiple examples including these critical aspects helped students understand the concept of altitude. Students need to experience these critical aspects with adequate examples such that they could understand the difference among the aspects “vertex, perpendicularity, opposition, orientation, location”. Acute, right and obtuse angled triangles with these critical aspects also created a variation in this study. According to the findings, students learned altitude better when they identified the aspects and compared the examples within each other.

The design of questions is significant for teaching the concept of altitude (Guo and Pang, 2011). The examples are needed to be designed starting from the easiest one to difficult one. Guo and Pang (2011) investigated the effect of prior knowledge with 4th graders who did not learn the altitude of the triangle before.

Researchers (Guo and Pang, 2011) found that students who had prior knowledge about the topic were not affected by the design of the questions. However, the design of the questions was so important for learners who did not have previous knowledge. It might be more beneficial for the students to determine the previous knowledge about the topic and to design proper questions for a better instruction.

Laborde (2003) stated that technology has important role on representing, exploring and constructing abstract geometric objects. Working with these objects such as rotating and observing the results of these changes on technological tools facilitate students visualize and understand the geometry (Laborde, 2003). The concept of altitude can be taught with the help of technological tools. There are limited studies could be reached such that technology integrates in teaching the concept of altitude. Yıldız, Güven and Koparan (2010) analyzed the 8th grade students' conceptual understanding on the concept of altitude by using Cabri as a dynamic software. Researchers found that students improved their conceptual understanding on the concept of altitude after using Cabri.

Clements (2003) expressed that interactive and dynamic computer programs facilitate understanding the geometry concepts better. These programs give opportunity to manipulate geometry objects, visualize them from different view and construct concepts in students' minds. Students can learn the concept of altitude while making sense of six critical aspects of altitude which are essential for definition of altitude (Guo and Pang, 2011). Students can experience these critical aspects better with technological tools. Determining and drawing altitude are better when the triangle is rotated or changed the position by using the dynamic tools such as GeoGebra. As Yıldız, Güven and Koparan (2010) also reached, use of technology can be helpful for understanding the concept of altitude.

Hershkowitz, Bruckheimer and Vinner (1987) suggested that teachers needed to construct rich strategies for teaching altitude. Students needed to learn trial and error method by experiencing all types of examples. Students needed to experience critical attributes of the examples. Teacher can encourage students to share and discuss their ideas in the classroom. The misconceptions may be revealed and can be eliminated by the teacher. Teacher can also use technology to improve students' conceptual understanding.

2.3.2.1. Multiple Examples. Guo and Pang (2011) found that multiple examples help students improve their learning. There are two significant criteria while determining and forming examples which are critical aspects and variance of patterns (Guo and Pang, 2011). The aspects of an example can be critical if students had difficulty in understanding. For example, if students can draw the altitude to the acute triangle but not to obtuse or right angled triangles, then the type of triangles according to the angles has critical aspects. They also emphasized on variance of patterns as much as critical aspects of example since students need to experience to understand a subject. Variance of patterns were classified as contrast, separation, fusion and generalization (Guo and Pang, 2011). Learners need to make comparison to understand all types of examples. They also needed to change a specific aspect by keeping other aspects constant. When learners encountered with an example, they needed to perceive whole aspects simultaneously to distinguish it from others. Lastly, students need to experience all types of examples but at the same time they should reach a generalization. Guo and Pang (2011) found that the examples varied according to the aspects of contrast and separation has stronger effect on students' success.

Guo and Pang (2011) studied the concept of altitude with 4th grade students by comparing multiple examples in the classrooms. Students did not have previous knowledge about the altitude of triangle. Students experienced separately and simultaneously varied examples during the treatment. Using multiple examples in this way enabled students to compare examples and understand the difference and similarities within them. In this study Guo and Pang (2011), students had opportunities to compare and experience the altitudes to the different types of triangles, the altitudes to different sides in the same triangle and the altitudes to triangles in the different rotations. Guo and Pang (2011) found that students showed improvement on their conceptual understanding after such treatment.

Each concept has so many types of examples but some of them are much more popular than others in textbooks, classrooms (Hershkowitz, 1989). These popular examples are called prototype examples. For example, acute triangle is a prototype example for drawing altitude. When altitude of triangle is asked, learners have the concept image of acute triangle and the altitude places inside of it. Altitudes for right and obtuse angled triangles are non-prototype examples for learners. They tend to not remember or understand the altitudes to right and obtuse angled triangles when non-prototype examples are not emphasized and understood conceptually very well.

Fischbein and Nachlieli (1998) found that 93% of students in grades 9-11 were successful at drawing the altitude to the base of a triangle. However, they could not show success on drawing the altitude in right or obtuse angled triangles. Researchers stated that isosceles or acute triangle are prototype to draw altitude, but obtuse triangle is nonprototype for them. They pointed out common misconceptions related to the concept of altitude. The most significant one was that students had difficulty in drawing altitude to the obtuse triangle with different orientations. The most frequent errors were drawing median to required side or drawing an altitude from different vertex.

Fischbein and Nachlieli (1998) explained the difficulty of drawing altitude to obtuse triangle so that the auxiliary elements of triangles are inside of the triangle, but the altitude of the obtuse triangle placed outside of triangle. Students from 9^{th} to 11^{th} grades found this situation extraordinary. Another reason for being non-prototype of altitude of obtuse triangle might be that questions whose altitudes are inside of triangle mostly occur in the examples that are solved in the classroom or textbook. The number of nonprototype examples that students experienced might be increased so that students can also show more success on drawing altitudes to nonprototype examples.

Hershkowitz (1989) reached significant conclusion such that if students did not understand the prototype example related concept, they could not also understand non-prototype ones. For example, if students had difficulty in drawing altitude to right angled triangle in the horizontal-vertical position which was the most common position for right angled triangle, they also could not draw altitude to right angled triangle with other orientations. Students had difficulty in constructing visual concept of altitude to the triangles with different orientations in their minds. Students need to understand the prototype examples of the concept very well and then students also need to practice on nonprototype examples with different rotations.

Cunningham and Roberts (2010) studied geometric concepts which were misunderstood according to results of pre-test with preservice elementary teachers. Participants showed improvement on drawing altitude of acute triangle as prototypical examples and altitude of right-angled triangles as nonprototype examples at the end of applying strategy of "concept attainment". Students created their own definition by examining the examples and non-examples related to the concept during the treatment. However, their success on drawing altitude to obtuse angled triangle as prototypical examples was decreasing after the treatment. Cunningham and Roberts (2010) explained the reason so that the conceptual understanding level of questions in post-test was higher than the pre-test's. The instruction the researcher applied was not sufficient for that level of conceptual understanding. Only one of third of students could realize if the given line segment of obtuse triangle was altitude or not. However, these students could not draw the altitude to the outside of the obtuse triangle in a correct way.

Fischbein and Nachlieli (1998) studied with high school students and found that drawing altitude did not have relationship with the age but the instruction had positive effect on drawing altitude and also defining the concept of altitude. Researchers found that the students who had higher level of instruction were more successful. The achievement did not show differences across the grades. Vinner and Hershkowitz (1983) reached that 7^{th} and 8^{th} graders showed higher achievement on drawing altitude than 5^{th} and 6^{th} graders. The results of the studies may seem contradictory. However, researchers explained the reason that students started to learn the concept of altitude in 5^{th} and 6^{th} grades and continue to learn the concept deeply in 7^{th} and 8^{th} grades. Students could make sense of the concepts and understand it conceptually in later grades.

2.3.2.2. Geometric Constructions. Clements (2003) expressed that using manipulatives and tools help students' conceptual understanding. Concrete manipulatives or computer tools are helpful for students to construct their geometric drawings. Students can manipulate the tools and then experience their results on their drawings. Interaction between students and tools increase geometric thinking level and improve geometric reasoning (Clements, 2003).

The construction tasks need to be designed very well that students get motivated for the construction. Cheung (2011) stated that students mostly do not pay attention to do construction tasks since construction is not an assessment type in the school. Challenging questions may help students increase the engagement and motivation. Cheung (2011) designed construction tasks such that students took their own responsibility. Students firstly planned what the steps of construction are. They remembered and used previous knowledge on building new one. They reflected what they know on the tasks. After they revised their plan, relationships among concepts and the properties, they reconstructed their concept. The researcher stated that geometric construction task presented very good learning environment and students get motivated and engaged to the tasks.

Uygun and Akyüz (2017) studied on the construction of auxiliary elements of triangle with manipulatives to get rid of the misconceptions. Participants constructed the auxiliary elements with using compass and protractor. They reached that students showed more success on defining auxiliary elements of the triangle while they were constructing. They could form the auxiliary elements of the triangle and justify their mathematical ideas with the tools. Using manipulatives facilitates students construct the concept and discover the relationship among mathematical concept (Olkun and Toluk, 2004).

<u>2.3.2.3. Use of Technology.</u> Forsythe (2007) stated that geometric shapes and Figures on the paper are static. However, dynamic geometry provides an opportunity so that students can construct and manipulate Figures. Students can also predict, observe, drag, rotate and record with the help of dynamic geometry. Ruthven (2009) stated that dynamic software has the properties of "accuracy, speed and manipulative ease" (p. 5). These properties help students do geometric exploration.

Pierce and Stacey (2001) studied with undergraduate students and analyzed the students' discussion while they were learning the concept of functions on the computer. Researchers found that students shared their ideas, worked with peers and discussed their solutions with peers and teachers more. Use of technology increased the interactions among students and the teacher. Students stated that they learned better with the use of technology since they discussed the mathematics and explore the ideas. As Ruthven (2009) stated, students also expressed that the technologic tools were better at the properties of speed and accuracy. When the concept is difficult for students to visualize in their minds or to draw on the paper, technological tools facilitate students' understanding and constructing the concept.

Ruthven *et al.* (2008) found that students understood geometric concepts better with help of the "dragging" property of dynamic software. Forsythe (2007) expressed that students can learn geometry actively with the dynamic software. Students can drag the vertices, sides, lines and shapes and can observe the result of their change. Forsythe (2007) found that the students who learned the geometry with dynamic geometry did not show more success than the students who learn the geometry without dynamic geometry. However, the researcher applied second test after a while and found that there was statistically significant difference among students. Students who learned the geometry with dynamic geometry were more successful in the second test. It can be reached that students learned the geometry concepts conceptually by using dynamic software.

Yıldız, Güven and Koparan (2010) analyzed 8^{th} grade students' conceptual understanding after the instruction enhanced with technology. Cabri as a dynamic software was used in the study and it had positive effect on students' understanding on the concept of altitude, diagonal and perpendicular bisector positively. Researchers reached that students improved their understanding of the concept of the altitude after using Cabri. Researchers also found that Cabri made geometry concepts more meaningful for students since auxiliary elements might be abstract for middle school students. However, when students worked geometry concepts on software by dragging them, concepts might become more concrete and meaningful.

Ruthven (2009) reached that working with dynamic software is more interesting and attractive than drawing with compass or ruler for students when students dealt with geometric constructions and bisecting triangles. Shadaan and Leong (2013) found that GeoGebra is a motivational tool for students. While students were exploring and visualizing the geometry on GeoGebra, students were interacting within each other and they became more motivated (Shadaan and Leong, 2013). Içel (2011) reached that students become more motivated and learn the topic better with help of the dragging" property.

Hohenwarter (2004) defines GeoGebra as an interactive geometry software for the students between 12 and 18 years. GeoGebra help students discover and construct mathematics and teachers can use GeoGebra for visualization, demonstration and preparation of teaching materials (Hohenwarter, 2004). Students can construct geometry and can examine how geometric Figures behave by dragging. Shadaan and Leong (2013) found that learning the subject of GeoGebra improved 9th grade students' conceptual understanding. Içel (2011) studied the effect of use of GeoGebra on the topic of triangle and Pythagorean theorem. Researcher (Içel, 2011) found that use of GeoGebra on teaching triangles had positive effect on 8th graders. As a result, according to literature review, GeoGebra helps students visualize, construct and understand geometry better. Students increase their interaction among peers and become more motivated by using GeoGebra.

2.3.2.4. Class Discussion. Researchers who studied with preservice middle school mathematics teachers found that class discussion helps students understand, analyze and criticize their ideas (Olkun and Toluk, 2004). Uygun and Akyüz (2017) designed an environment such that instructor encouraged students talk on misconceptions and errors. Students firstly studied the auxiliary elements within the peers and then discussed the concept with whole class. They had opportunity to realize their misunderstandings during the discussion. They reconstructed the geometrical concept correctly with justifications. Similarly, Gutierrez and Jaime (1999) created an environment such that students discussed on different solutions, findings and difficulties while learning the concept of altitude. Researchers expressed that discussion helped students reflect on their understanding. Moreover, students have opportunity to explain, defend and present their ideas during group or class discussion (Gutierrez and Jaime, 1999).

Blanco (2001) found that students have difficulty in understanding the auxiliary elements of triangle; bisector, median, orthocenter and altitude of triangles. Gutierrez and Jaime (1999) revealed that whole class discussions improved understanding of the concept of the altitude which is one of the auxiliary elements of triangles since they analyzed and criticized the concepts and their ideas about the solutions, formation of the concept, and justification of the ideas. This result is parallel to the study of Olkun and Toluk (2004). Researchers (Olkun and Toluk, 2004) stated that class discussions facilitated students' geometric thinking and understanding of geometrical concepts. Therefore, it is important to provide an environment where they can have class discussions to understand geometric constructions.

Olkun and Toluk (2004) reached the conclusion that questioning was a beneficial strategy to promote class discussion. Researchers emphasized on the content of the questions. Questions firstly need to reveal students' previous knowledge and students' thinking level. Questions also need to facilitate exploration and improvement of the mathematical idea for students. Open ended and challenging questions helped students initiate sharing and discussing ideas.

The concept of altitude is crucial to understand deeply for students since students continue to use the concept of altitude while learning in other geometry topics such as area and volume. Many students from middle school to college have difficulty in defining the concept and drawing altitude (Cunningham and Roberts, 2010; Vinner and Hershkowitz, 1983; Gutierrez and Jaime, 1999). Students could understand the concept of altitude better with an effective instruction. As Gutierrez and Jaime (1999) suggested, instruction needs to address possible misconceptions and eliminate them by enhancing with various concept images, practicing on prototype and non-prototype examples. Students can share and discuss on the different type of examples as a class or group. Students need to have opportunity to explain, defend and analyze their ideas.

It could be possible in a class environment such that students can work collaboratively, feel free to make mistake and share solutions with that class. Such instruction also needs to enrich with technology since technological tools creates an environment to manipulate geometry objects, visualize them from different view and construct concepts. As Clements (2003) expressed, technological tools facilitate understanding the geometry concepts better. From the literature review, guiding principles were determined for teaching the concept of the altitude as below.

2.3.3. Guiding Principles for Teaching Altitude with Technology Integration and Sociomathematical Norms

2.3.3.1. Geometric Constructions. Cheung (2011) found that geometric construction helps students reflect what they know and explore mathematical idea. Students can construct geometric knowledge by using manipulatives such as protractor or compass. Students can also improve their knowledge by using some technological tools such as cabri or sketchpad. Uygun and Akyüz (2017) found that the construction of auxiliary elements of triangle with manipulatives help students eliminate the misconceptions and improve their understanding. Students can construct the concept in their minds better with the help of tools.

2.3.3.2. Emphasis on Non-prototype Examples. Vinner and Hershkowitz (1983) reached conclusion from their studies such that students should establish strong connections between the definition of the concept and the concept image. Researchers reached that the definition of concept is limited by practicing prototype examples only. As Fischbein and Nachlieli (1998) and Gutierrez and Jaime (1999) found, students had difficulty in drawing altitude to the nonprototype triangles. Hershkowitz (1989) revealed that students have only the concept image of prototype examples and so their understanding was limited with prototype examples. Hershkowitz (1989) suggested that teachers need to emphasize on non-prototype examples. Students need to experience drawing altitude to the nonprototype examples as much as prototype examples.

2.3.3.3. Emphasis on Misconceptions. Gutierrez and Jaime (1999) determined basic misconceptions while students were learning the concept of altitude. These misconceptions were altitude vs. median, altitude vs. perpendicular bisector, limitation to the internal altitudes, disregard of the lengths, fixation on side and marked base as distracter. Teachers need to be aware of these misconceptions for the concept of the altitude. Researchers stated that the concept of altitude needs to be taught by addressing possible misconceptions and errors. Researchers also suggested that teachers need to create an environment so that students can share and discuss their ideas. Students can eliminate the misconceptions for the concept of the altitude with the help of the discussion. Teachers should also design their instructions to reveal and eliminate misconceptions related to the mathematical concept.

2.3.3.4. Emphasis on Six Critical Aspects. Guo and Pang (2011) set six critical aspects for the concept of altitude, namely vertex, perpendicularity, opposition, orientation, location and altitude-base-correspondence. Researchers expressed that these aspects were "necessary and sufficient for the definition of altitude" (Guo and Pang, 2011, p. 500). They found that the variation of these critical aspects on the examples helps students understand altitude better. These aspects need to be varied carefully while teaching the concept of altitude so that students experience all types of examples adequately. Students need to explain these critical aspects and use these aspects in their explanations. Understanding these critical aspects is significant for defining and drawing altitudes properly (Guo and Pang, 2011).

2.3.3.5. Technology. Clements (2003) stated that technological tools develop geometric reasoning and thinking. The concept of altitude can be taught with the help of technological tools. Yıldız, Güven and Koparan (2010) found that an environment enhanced with technology improve students' understanding of the concept of altitude. Researchers (Yıldız, Güven and Koparan, 2010) reached that dynamic software made the concepts of altitude more concrete and meaningful. Students could visualize the geometric shapes and manipulate them easily. They could observe the results of changes and justify their ideas easily on the computer.

Technological tools can be used to establish and improve sociomathematical norms. Students and teacher build interaction with technological tools. Akyüz (2014) determined three sociomathematical norms in a classroom which supported with technology. These norms are inquiring about the effects of a change made in a question or a solution; reaching conclusions by using the properties of the tools in the dynamic software; and dynamically verifying a solution or a hypothesis. As these norms, technology can be used a tool to develop sociomathematical norms.

2.3.3.6. Class Discussions. Gutierrez and Jaime (1999) found that class discussion improve students' understanding of altitude. Students shared their solutions with the classroom and misconceptions were revealed during the discussions. These discussions help students and teachers eliminate misconceptions and reconstruct the concept. Similarly, Olkun and Toluk (2004) suggested that teachers promote students to talk on their misconceptions and errors. They could realize their errors during the discussion. Olkun and Toluk (2004) stated that teachers could reveal students? thinking level by using the method of questioning during the discussions. During class discussions, following four sociomathematical norms can be developed to create rich and effective learning environment.

(i) Sharing solutions to the class: Cobb and Yackel (1996) designed a research such that students explain their solutions to their small groups or to the rest of the classrooms. They reached that students had opportunity to reflect what they learned and so improved their understanding on concept. While students are sharing solutions, they explain their solution methods, make interpretation about the problem or solution or reach a conclusion related to the problem (Özmantar, 2009). Dixon, Egendoerfer and Clements (2009) found that students were more involved in mathematics activities and more comfortable in the classroom as sharing their ideas and solutions with their peers. Researchers also reached that being a routine of sharing their solutions with classroom members helped students build conceptual understanding. According to the literature review, students improved their thinking skills, their conceptual understanding and feel more comfortable in the classroom when students share their ideas and solutions in the mathematics classroom.

(ii) Working collaboratively: Cheval (2009) found that working collaboratively provides supportive environment. Students can find efficient or different solutions by sharing and discussing them with their group members. Kazemi and Stipek (2001) constructed the norm of working collaboratively to build mutual understanding among students. Researchers also constructed the norm of working collaboratively so that each student was responsible for understanding group solutions and students came to mutual understanding after discussing on different ideas of group members. Researchers found that students developed their thinking and understanding after the norm become routine in the classroom. Similarly, Tatsis and Koleza (2008) expressed that students need to reach mutual agreement to work collaboratively. Tatsis and Koleza (2008) set two criterias for their research; using plural verbs while expressing their ideas or solutions and asking questions to understand a group member' thinking way.

(iii) Acceptable explanation: Researchers (Cobb and Yackel, 1996; Kazemi and Stipek, 2001) found that students improved their understanding through mathematical explanations. Mathematical explanations include mathematical reasoning, solution strategies, conceptual or procedural ideas (Cobb and Yackel, 1996). Dixon, Egendoerfer and Clements (2009)) found that students improved their thinking skills by making acceptable explanations. Cobb and Yackel (1996) expressed that an explanation counted as acceptable explanation mathematically when students explain their thoughts in their own mathematical thoughts. It is crucial to determine what type of the explanation is acceptable. Students can use mathematical explanations when they describe their solutions or how they understand a concept (Cobb and Yackel, 1996). It is significant how students and the teacher establish and improve the norms in the classroom.

(iv) Being free to make mistakes: Making errors provide opportunity for students to reconceptualize the problem. Students need to feel that they are free to make mistake. To build such an environment, teacher and other students in the classroom need to accept and support so that students may make mistakes (Kazemi and Stipek, 2001). Students could not learn without trial and error method. Kazemi and Stipek (2001) found that students learn to accept the mistakes as normal part of learning process. Kazemi and Stipek (2001) found that the mistakes in group or class discussion encourage students explain their ideas freely. When students feel free for making mistakes, they can explore contradiction in the solutions and can try different solution methods. Researchers (Kazemi and Stipek, 2001) also emphasized that making errors also help students and teachers identify the misconceptions and provide an opportunity to build conceptual understanding.

3. SIGNIFICANCE OF STUDY

In the literature review section, the numbers of research related to sociomathematical norms and improving the teaching of the concept of altitude were discussed separately. There are limited studies about the concept of altitude enriched with sociomathematical norms. Cobb and Yackel (1996) found that when students take the place of explanation as a norm, they have deeper conceptual understanding. Kazemi and Stipek (2001) established the norms: explaining their solutions to the class; working collaboratively, being free to making mistakes. The authors reached that these norms supported conceptual understanding. These norms were expected to improve conceptual understanding of “altitude”.

There have been also many studies related to the concept of altitude starting from elementary school to university. Students from all levels have some specific misconceptions related to the concept. Gutierrez and Jaime (1999) found that class discussion help students eliminate misconceptions. Hershkowitz (1989) found that the emphasis on non-prototype examples which are significant to understand the concept altitude better while Guo and Pang (2011) stressed on the critical attributes of the concepts for better understanding. There are limited studies on teaching altitudes with supported with technology, even though there are many studies on geometry supported with technology. Yıldız, Güven and Koparan (2010) studied that an environment supported with technology improved students’ understanding of altitude.

In Turkish Elementary Mathematics Curriculum, the concept of altitude is firstly placed in 6^{th} grade. It is so significant that students in 6^{th} grade learn the concept conceptually with reducing misconceptions since the concept of altitude continues to be learnt in many other geometry topics till college level. For this study, I wanted to improve my teaching practices by designing an instruction so that my students had better understanding on the concept of altitudes in an environment enriched with specific sociomathematical norms. The instruction supported with technology were expected to have positive effects on students’ conceptual understanding. In this study, it was investigated the effects of the instruction supported with technology and enriched with sociomathematical norms on 6th grade students' conceptual understanding of altitude.

4. STATEMENT OF THE PROBLEM

I have been teaching five years in both middle school and high school. I experienced that students had difficulty in drawing altitude and they had misconceptions related to the concept. I wanted to design an instruction for the concept of altitude so that I would improve my teaching practices. For this study, the instruction was supported technology and enriched with sociomathematical norms. The instruction both helped me reflect my teaching practices and also students to understand the concept better by applying this study.

The purpose of this study is to analyze the effectiveness of instruction enriched with sociomathematical norms and supported with technology for improving 6^{th} grade students' conceptual understanding of the concept of altitude.

4.1. Variables

The variables of this study are sociomathematical norms and conceptual understanding. These terms can be explained as:

Conceptual understanding can be defined as the "comprehension of mathematical concepts, operations, and relations" (NCTM, 2001). Guo and Pang (2011) determined six critical aspects for the concept of altitude. They were vertex, perpendicularity, opposition, orientation, location and altitude - based - correspondence. When students can explain these six critical aspects and draw the altitude of given geometric shapes in the different position and orientation, students have conceptually understanding of the concept of the altitude.

Sociomathematical norms are defined as "normative aspects of mathematics discussions specific to students' mathematical activity" (Yackel and Cobb 1996, p.461). In this study, four sociomathematical norms are constructed: sharing solutions to the class, working collaboratively, acceptable explanation and being free to make mistakes.

These norms were constructed in the classroom to promote students explaining, sharing and discussing their mathematical ideas and solutions. It is expected that these norms help students understand the concept of altitude better.

4.2. Research Questions

The study was seeking the answers of these research questions:

(i) Is there a statistically significant difference between 6^{th} grade participants' conceptual understanding of altitude before and after instruction enriched with sociomathematical norms and integrated with technology?

- What are the 6^{th} grade participating students' concept definitions and images for the conceptual understanding of altitude before and after instruction enriched with sociomathematical norms and integrated with technology?

(ii) What misconceptions related to the concept of altitude do participating 6^{th} grade students have in the pre-test and post-test as measured by the Altitude Test?

(iii) What are ththe 6th grade participating students' views on experience of sociomathematical norms and technology while learning the concept of altitude?

(iv) What are the sociomathematical norms seen during an instruction for 6th grade altitude enriched with sociomathematical norms and integrated with technology based on students' journals, teacher field notes and classroom audio recordings?

(v) What might be the effective instruction for 6th grade to improve students' conceptual understanding of the concept of the altitude?

5. METHOD

5.1. Research Design

This study was grounded in action research methodology. Mills (2011) defined action research as “systematic procedures done by teachers (or other individuals in an educational setting) to gather information about, and subsequently improve, the ways their particular educational setting operates, their teaching, and their student learning” (p.5). In this research type, teachers use data collection and analysis to solve problems by collecting and analyzing data. Teachers reflected on their problems, searched for solving problems, collected and analyzed data and implemented their findings. Mills (2011) prepared a model for teacher-researchers such that they can conduct an action research in their classrooms easily. Mills (2011) described the basic steps of action research as identifying an area of focus, collecting data, analyzing and interpreting data and developing an action plan. Firstly, a teacher-researcher reflects on teaching practices, reviews the literature and determining the action plan in the step of identifying an area of focus. Secondly, teacher-researcher collects quantitative and qualitative data with various tools. Thirdly, teacher-researcher analyzes data quantitavely and qualitatively and interprets the data according to the findings from literature and own teaching experiences. Lastly, teacher-researcher summarizes the findings and gives recommendation according to the findings. Even though the steps of actions research are described separately, Kagan, Burton and Siddiquee (2017) explained that action research is a dynamic process so that researcher cycles the process back and forth among the steps of the action research. Thus, a teacher-researcher reflects on practices based on action-research findings and revises them as she contniues to teaching.

In the literature, it is expressed that students from primary school to college have difficulty in understanding the concept of altitude properly. As a teacher, my experiences in teaching altitude as a geometry concept are in parallel with the literature. I encountered many difficulties in the classroom, while students were both learning the concept of altitude and solving the problem related to the concept of area. I could not overcome the misconceptions and misunderstandings for the concept of altitude. I decided to approach this issue as a teacher-researcher to examine middle school students' learning of altitude and to develop an instructional sequence (action plan). From the literature review, there were many studies related to improving students' understanding and reducing misconceptions for the concept of altitude. In most of these studies, students improved their understanding in an environment so that students could explain, share and discuss their ideas freely. With the help of these studies, I would like to solve the problem which students had difficulty in understanding the concept of altitude in an environment enriched with sociomathematical norms including working collaboratively, making explanations and sharing their solutions freely. I reconsidered my teaching practices related to the concept of altitude. I wanted to design effective instruction based on literature review and implement an action research. This research was an opportunity to reflect and improve my teaching practices based on results of a research.

5.2. Participants and Setting

The participants in this study were 6^{th} graders (12-13 years) attending a private school in Istanbul. The school consists of primary and middle school, including 427 students totally. The statement of the school-board is preparing students who can learn independently in enriched environment and sustain their learning throughout their lives.

Participants were heterogeneous-ability 6^{th} grade students. Participants consisted of 30 female and 18 male students. Participants in the school come from upper middle socioeconomic backgrounds. This action research was implemented with two 6^{th} grade classes.

5.2.1. Role of the Researcher

As a teacher researcher, I used the stages of action research that Mills (2011) described. The stages of action research were dynamic and cyclic. The stages were identifying an area of focus, collecting data, analyzing and interpreting data and developing action plan. Firstly, as a teacher researcher, I observed that students from each level of the middle and high school had difficulty in drawing altitude. I described my practical problem and decided to design an effective instruction to solve this problem. I reached from the examination of literature that an environment enriched with sociomathematical norms and technology improve students' conceptual understanding. I prepared an action plan to design an effective instruction. To create such an environment, I constructed sociomathematical norms with the students during one semester. I encouraged students share, explain and discuss their ideas and solutions. I tried to create supportive environment so that students feel comfortable in the mathematics classrooms. While students were working collaboratively, I emphasized that sharing ideas freely in the group, reaching mutual agreement and taking responsibility of explaining group' solution are significant in the group working. I kept emphasizing these points and giving feedback about group working before and during the instruction. I explained what the acceptable explanation is with students. While they were sharing their solutions, students learned making acceptable explanations for their solutions. I also supported my lesson plans with technology since the visualization is significant for the geometry concepts and use of technology helps students visualize the geometry concepts better. Before the implementation of the study, students learned one subject in the 6th grade curriculum by using technology. I also considered that students continued their learning with same norms in the technology integrated classrooms. I also designed my lesson plans and activities based on guiding principles for teaching altitude with technology integration and sociomathematical norm that was determined in literature.

Before starting the instruction, I applied pre-tests and revised my lesson plans and activities according to the results of pre-tests. For instance, I revised my plans as including more practices related to the misconceptions occurred in the pre-test. During the implementation of the study, I collected data from students' journals, teacher's field notes, board and audio recordings from the classroom. I wrote my notes and transcribed recordings on the day of data collection or the following day. During the instruction, I evaluated and reflected my teaching practices if students could have better instruction for the concept of altitude or not. I continuously revised lesson plans and activities according to students' needs and the teacher field notes. For example, if students still had difficulty in drawing altitude outside of the triangle after designed lesson plans, I revised my plans as adding more parts related to practicing nonprototype examples more, working on drawing altitude with dynamic tools more or creating more class discussions on the misunderstandings. During the instruction, I encouraged students to discuss on the misconceptions which revealed in the pre-tests or during the instruction. I encouraged students to explain six critical aspects of the altitude to eliminate their misconceptions. While students were sharing their solutions, I wanted them to use six critical aspects of the altitude or definition of the altitude. I also used nonprototype examples as much as prototype examples during the instruction. Students constructed the concept of the altitude on the different examples with concrete and technology tools. After implementation of instruction, I applied post-test to analyze students' conceptual understanding. I investigated if the instruction enriched with sociomathematical norms and supported with technology improve students' conceptual understanding of the concept of the altitude. To reach results, pre and post-tests, teacher field notes, classroom recordings and students journals were analyzed. After data analysis and the interpretation of the results, I evaluated if the systematic approach for reflecting my teaching practices were helpful or not.

5.3. Procedures

The concept of altitude is introduced in 6^{th} grade and it is expected that 6^{th} grade students understand the concept of altitude (MoNE, 2017). 6^{th} grade mathematics curriculum (MoNE, 2017) expressed that students start to learn the area of the square and rectangle in 4^{th} grade and they continue to calculate their areas in 5^{th} grade.

Even though students do not have the information about the concept of the altitude before starting the study, they know how to draw right angle and how to calculate an area of a square and rectangle. Students also might have information about the concept of altitude from different sources such as their elementary school teachers.

I determined the sociomathematical norms in beginning of the year. Since sociomathematical norms can form through interaction of students and teachers for a while (Cobb and Yackel, 1996), I improved these norms with the students in my classroom throughout the year. I promoted four sociomathematical norms: sharing solutions collaboratively, working cooperatively, acceptable mathematical explanations and free to make mistakes. I encouraged students explain their actions, solution strategies and their thinking way with their reasons. Students had common understanding on what kind of explanation is acceptable for mathematics lessons. Students in the classroom become individuals who demand explanations from the speakers. They practiced how they can work together and share their ideas with each other and within the classroom. Each member in the group had responsibility to think individually and contributed their group by sharing their ideas. They experienced that they could make mistakes and all students know that they feel comfortable in the lessons from their experiences. They practiced that making mistakes is a normal part of learning process. These norms were formed and developed with different mathematics subjects.

Another factor that enriches the learning environment is the use of technology for my study. I designed some lesson plans enhanced with technology such that they interact with technology in the mathematics classrooms before the implementation of the study. Since students need to study the concept of altitude on computer in computer lab during the study, students worked in computer lab while learning a different mathematics subject.

5.3.1. Plan of Teaching Altitude

The lesson plans and the lesson activities were designed according to the objectives which were determined by Turkish Elementary Mathematics Curriculum. The sequences of activities during the instruction are represented as in the following table;

Table 5.1. Activities during the instruction.

Topics	**Activities**	**Time**
Perpendicularity to a line from a point outside of the line.	• Understanding the perpendicularity. • Construction with ruler and miter on paper. • Construction with ruler and miter on smart board. • Construction using GeoGebra. • Watching videos related to drawing perpendicular to line. • Solving activity sheets including prototype and nonprototype examples. • Discussion about common understanding and misunderstanding among peers and then as a class.	2 class hours (1stweek)
Altitude to the parallelogram.	• Understanding the altitude of parallelogram. • Construction with ruler and miter on paper. • Construction with ruler and miter on smart board. • Construction using GeoGebra. • Drawing altitude to the parallelogram with different rotations on activity sheets. • Discussion about common understanding and misunderstanding among peers and then as a class.	2 class hours (1st week)
Area of parallelogram.	• Understanding the area of parallelogram. • Understanding the area of parallelogram on simulations. • Solving problems related to the area of the parallelogram. • Discussion about common understanding and misunderstanding among peers and then as a class.	2 class hours (1st week)
Altitude to the triangle.	• Understanding the critical aspects of the altitude with various examples. • Construction with ruler and miter on paper. • Construction with ruler and miter on smart board. • Construction using GeoGebra. • Drawing altitudes to different types of triangles according to their sides. • Establishing relationship between concept image and concept definitions on different examples. • Discussion about common understanding and misunderstanding among peers and then as a class.	6 class hours (2nd week)

Table 5.1. Activities during the instruction (Cont.).

Topics	Activities	Time
Area of triangle.	• Understanding the area of triangle. • Understanding the area of triangle on simulations. • Solving problems related to the area of the triangle. • Discussion about common understanding and misunderstanding among peers and then as a class.	2 class hours (3rd week)
Altitude to the triangle.	• Construction altitudes with concrete tools and GeoGebra. • Discussion about common misconceptions while drawing altitudes to the triangles. • Manipulations on simulations via GeoGebra. • Drawing altitudes to triangles with different rotations. • Drawing altitudes to the triangles including prototype and nonprototype examples on activity sheets. • Discussion about common understanding and misunderstanding among peers and then as a class.	4 class hours (3^{rd} week)
Altitude to the triangle.	• Drawing altitudes to triangles with concrete tools and GeoGebra. • Practicing drawing altitude on nonprototype examples. • Manipulations on simulations via GeoGebra. • Determining the place of altitude via GeoGebra. • Discussing six critical aspects on both examples and nonexamples of the concept of altitude. • Discussion about common understanding and misunderstanding among peers and then as a class.	3 class hours (4^{th} week)
Problems related to the area.	• Solving area problems drawing altitude to triangle or parallelogram on activity sheets. • Solving daily problems related to the area. • Creating their own problems related to area and solving them as a group. • Discussion about common understanding and misunderstanding among peers and then as a class.	3 class hours (4^{th} week)
Altitude to the triangle.	• Practicing drawing altitude on both prototype and nonprototype examples. • Drawing altitudes to all types of triangles with different rotations. • Sharing and discussing the misconceptions • Manipulations on simulations via GeoGebra. • Determining the place of altitude via GeoGebra. • Discussion about common understanding and misunderstanding among peers and then as a class.	6 class hours (5^{th} week)

During the instruction, students constructed altitude to line, triangle and parallelogram by using manipulatives such as miter. Students also constructed the altitude by using GeoGebra as dynamic tools. They worked on determining the place of altitude with the help of GeoGebra. As a teacher-researcher, I supported students' conceptual understanding with animations, simulations and videos during the instruction. Students attended series of task including prototype and non-prototype examples. I gave opportunity to students to practice their knowledge with multiple examples. I encouraged students to collaborate with their peers and then discuss their ideas as class discuss. I promoted students discussing non-prototype examples to reduce their misconceptions. During the discussion, I analyzed how sociomathematical norms support the students' conceptual understanding.

The study lasted during 5 weeks since Turkish Mathematics Curriculum, MoNE, (2017) allocates that duration for the topic. There are 6 mathematics class hours in each week and one class hour is 40 minutes. The researcher took permission from the school manager to apply pre-test and post-test. Pre-test and post-test were administrated to both of the classes two days before instruction and two days after the instruction. Same instruction was applied to the both of the classroom. Students took the series set of instruction related to the concept of altitude. The content of the instruction is drawing perpendicular to the line, drawing altitude triangle and parallelogram. Lessons were designed the focus on building the relationship between concept image and concept definition. Teacher emphasized on the critical attributes of the altitude. Teacher helped students experience the prototype and non-prototype examples of the concept.

5.3.2. Sociomathematical Norms for Teaching Altitude

It is difficult to predict the sociomathematical norms unless they are observed inside of the classroom (Cobb and Yackel, 1996). I took notes during and mostly right after the lessons about the sociomathematical norms, interaction among the students and students' misconceptions. Besides the notes, I took audio recording and record of screen of smart board during the lessons. With help of these notes and recordings, I realized that students had difficulty in drawing altitude to right angled and obtuse angled triangles. I revised my lesson plans as including more activities related to determining the place of altitudes and discussing on more nonprototype examples. I put together my notes and brought out the most common misconceptions in the classroom. I added new activities related to nonprototype examples. These activities consisted of drawing altitude to nonprototype examples with concrete tools and dynamic tools, discussing the critical aspects of altitude on nonprototype examples and working on drawing altitude via GeoGebra to improve students' concept images. I tried to give opportunity to explain their thinking on nonprototype examples. I tried to promote students discuss on their misconceptions after sharing their drawings. Students shared, explained and defended their drawings in the classroom. Students reached correct drawings by using ciritical aspects of the altitude with help of their peers.

For the instruction, I designed the lesson plans starting from the topic of perpendicularity. Students learned the definition of the perpendicularity and practiced drawing perpedicular altitude to a line from a point outside of the line. Students continued their learning with defining and drawing altitude to triangle and parallelogram. I asked students to draw perpendicular to a point and altitude to triangles and parallelogram with concrete and dynamic tools. I promoted students to share and discuss their drawings in the classrooms. They practiced the concept of altitude by finding and calculating the area of triangle and parallelogram. While practicing altitude with the concept of area, I followed and noted students' misconceptions and errors. In order to increase students' awareness about misconceptions, students did activities related to drawing altitudes to various types of triangles and parallelograms in different rotations. In the last lesson, I directed students to review what they learned about the concept of altitude and shared their ideas about sociomathematical norms and use of technology.

5.3.3. Use of Technology for Teaching Altitude

During the preparation of lesson plans, it was significant how the technology was integrated into lesson plans. Sometimes students used the technology actively like drawing with GeoGebra, sometimes they used technology as a demonstration like simulations and videos or sometimes they did not use technology while drawing with concrete tools, sharing and discussing their solutions. I planned to allocate time equally, as much as possible as following the national mathematics curriculum, for use of active technology, use of technology as a demonstration and not using technology as in the Table 5.2 below.

Table 5.2. Use of technology by topics and weeks.

Weeks	**Topics**	**No Technology**	**Technology Demonstration**	**Active Technology User**
1^{st}	Perpendicularity, Altitude to paralellogram, Area of parallelogram	2 class hours	2 class hours	2 class hours
2^{nd}	Altitude to triangle	1 class hour	3 class hours	2 class hours
3^{rd}	Altitude to triangle, Area of triangle	2 class hours	2 class hours	2 class hours
4^{th}	Altitude to triangle, Problems related to area	2 class hours	2 class hours	2 class hours
5^{th}	Altitude to triangle	3 class hours	1 class hour	2 class hours

For each category according to use of technology, 10 hours were equally allocated during the instruction. While students were using the technology actively or as a demonstration, they also shared and discussed their ideas and solutions without using technology. In the environment of no technology, students worked on the tasks collaboratively, drew altitudes with miter, shared and discussed their ideas and solutions within the group and with the classroom. Students worked on tasks collaboratively, constructed drawings with their hands, observed and experienced the changes in the simulations, explained and discussed the reasons about changes in the simulations with the property of dragging while students were using technology as a demonstration. In the environment that students used technology actively, students constructed the drawings with the help of technology, experienced the changes individually with the property of dragging, worked on the given tasks collaboratively, shared and discussed their solutions within the group and with the classroom.

To support the concept of altitude with technology, I used GeoGebra in my lessons both for demonstrating simulations and drawing altitude actively. There are some screenshots related to use of technology in the classroom as in the Figure 5.1.

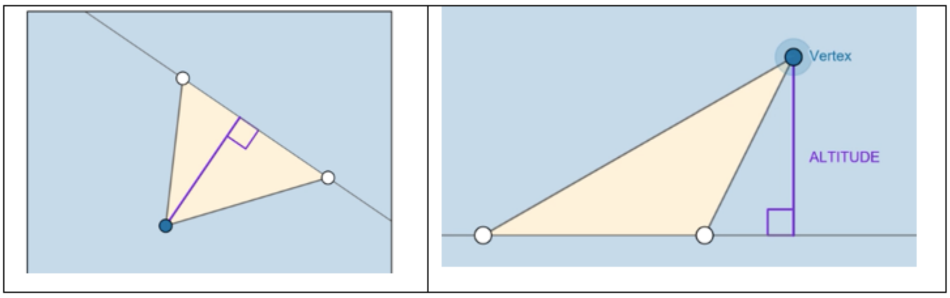


Figure 5.1. Screenshots from classroom while using teachnogy as a demostration.

The screenshots in the Figure 5.1 are simulations related to understanding of the place of the altitudes. I asked students to work on these simulations on GeoGebra. Students experienced the place of altitudes in triangle by dragging the vertex. They discussed the six critical aspects of the altitude on the simulation. Students also used technology actively by drawing perpendicularities or altitudes. The Figure 5.2 shows drawings of altitudes to the different types of altitude on GeoGebra. Students firstly constructed a triangle on GeoGebra. They drew three altitudes by constructing perpendicular lines on GeoGebra. Students experienced the changes of the place of the altitude according to the type of triangles.

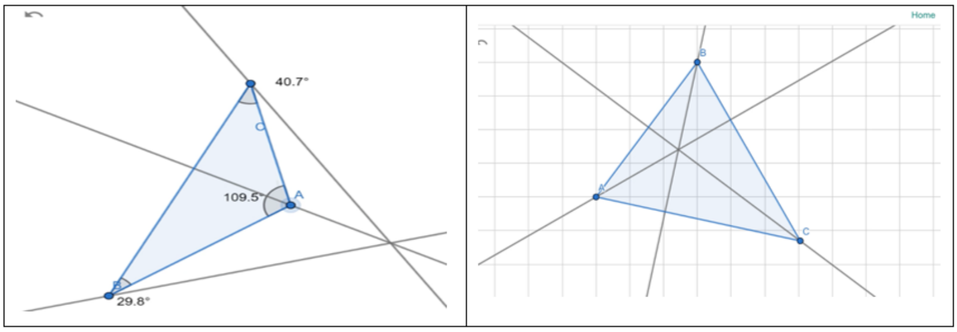


Figure 5.2. Drawing altitudes to the triangles on the GeoGebra.

Before starting instruction, students needed to learn how to use GeoGebra and I took support from information technology teacher in the school. Right before the implementation of the study, we firstly taught GeoGebra menu and made students practice on drawing triangle, perpendicular to a point and a triangle, measuring the angle of triangles during two class hours in computer lab. I explain how I used technology in the lessons with a sample lesson plan through the end of these part.

5.3.4. Examples of Misconceptions in the Classroom

During the instruction, students worked on plenty of activity sheets related to series of sequences of the tasks. In the classroom, most of the students were successful at drawing altitude to the acute triangle as in the Figure 5.3. It showed that students constructed the concept image of altitude to the acute triangle. Even though triangles were rotated, most of students have correct concept image of acute angled triangles as in the Figure 5.3.

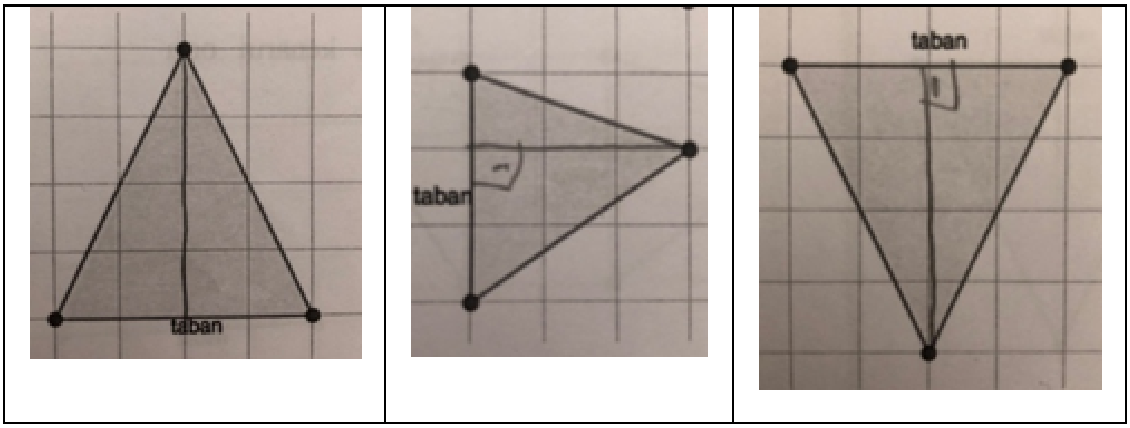


Figure 5.3. The examples related to drawing altitude to the acute angled triangle.

However, some of students drew altitude to the triangle as in the Figure 5.4. The altitude of acute triangle was drawn in a wrong way as in the Figure 5.4 since students did not start the altitude from the corresponding vertex. These students could not construct the concept definition of the altitude properly and they neglected one of six critical aspects which is vertex. I directed students to discuss why the drawn altitude was not an altitude in the classroom. They reached the correct result by using the six critical aspects of the triangle.

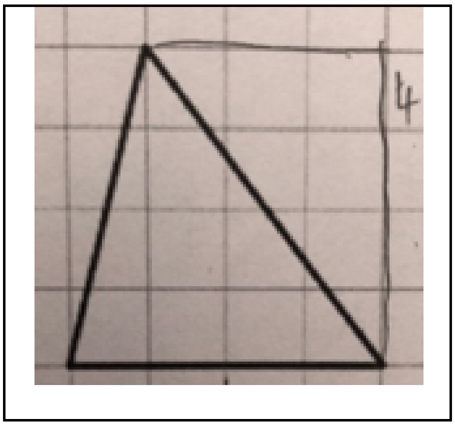


Figure 5.4. The misconception related to acute angled triangle.

In the classroom, most of misconceptions were related to right angled and obtuse angled triangles which were nonprototype examples as Figure 5.5 and Figure 5.6. Students mostly had misconceptions about the right angled and obtuse angled triangles which were rotated or not. The Figure 5.5 includes the misconceptions for the right angled triangle. The first of two images in the Figure 5.5 were misconceptions of median vs altitude and the other images were the misconceptions of the perpendicular bisector vs altitude.

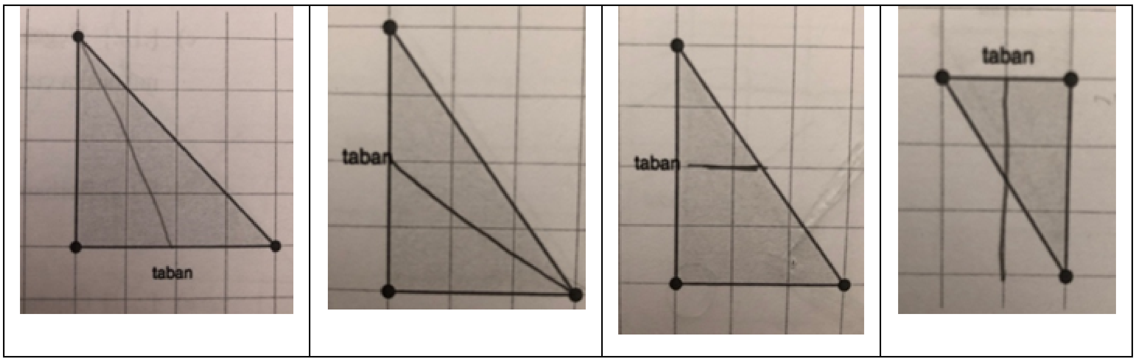


Figure 5.5. The misconceptions related to right angled triangle.

These drawings in the Figure 5.5 and Figure 5.6 belongs to students' activity sheets. Since students could not improve concept image and definition together, they might have such misconceptions. Students shared these drawings with the classroom and students discussed on these drawings. Students used six critical aspects of the altitude while reaching the correct drawings. Students also used dynamic tools, simulations and concrete tools to convince their peers. To reduce misconceptions, students worked on both concept image and concept definition as they were working on the definition and improving their concept image with various tools.

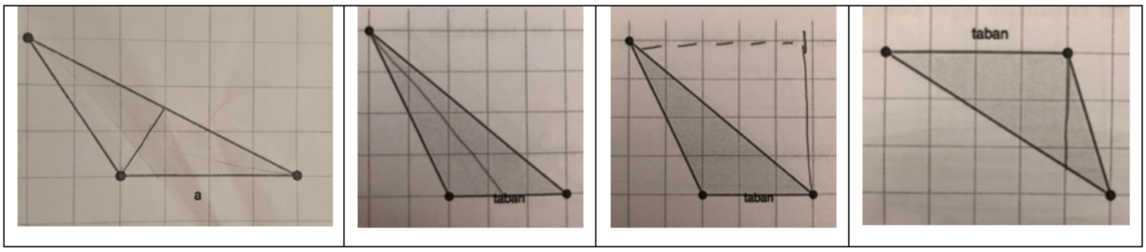


Figure 5.6. The misconception related to obtuse angled triangle.

The tasks and activity sheets were designed these misconceptions and improve students' understanding of the concept of altitude. To eliminate such misconceptions, students practiced on nonprototype examples by drawing altitude to the triangles with hand or with GeoGebra. Students discussed on the examples within the groups or with the classroom. They were promoted reaching the correct solutions and drawings by using and explaining six critical aspects of the concept of the altitude.

I designed my lesson plans based on guiding principles for teaching altitude with technology integration and sociomathematical norm that was determined in literature. The principles are geometric constructions, emphasis on nonprototype examples, misconceptions, six critical aspects, use of technology and class discussion. These lesson plans depended on the objectives that Turkish Mathematics Curriculum, MoNE, (2017) determined for 6^{th} grade mathematics lessons. Sample lesson plans was provided in Appendix A and Appendix B.

5.3.5. A Sample Lesson Plan

Series of lesson plans and activities for the instruction were implemented in the enviroment enriched with sociomathematical norms and use of technology. One of lesson plans was explained how the concept of altitude were taught in the enriched enviroment. The lesson plan were related to the altitude to the triangles. Before this lesson, students started to explore the definition of the altitude with different tasks in the previous lessons.

I started the lesson by dividing students into six groups. Students worked as four students in their groups. Before starting the activity, I remembered that every member in the group had responsibility to think and answer all the questions. The activity sheets, as in the appendix F, were given to the students. I wanted them to answer the questions, as in the below, in their activity sheets. I explained what they did for the questions to the groups briefly. Students started to answer the first three questions by drawing the altitudes according to the types of the triangles. Then, students defined the concept of altitude in 4th question.

(i) Draw an altitude to lie inside the triangle. Explain what kind of a triangle this will occur.

(ii) Draw an altitude to lie on the one side of the triangle. Explain what kind of a triangle this will occur.

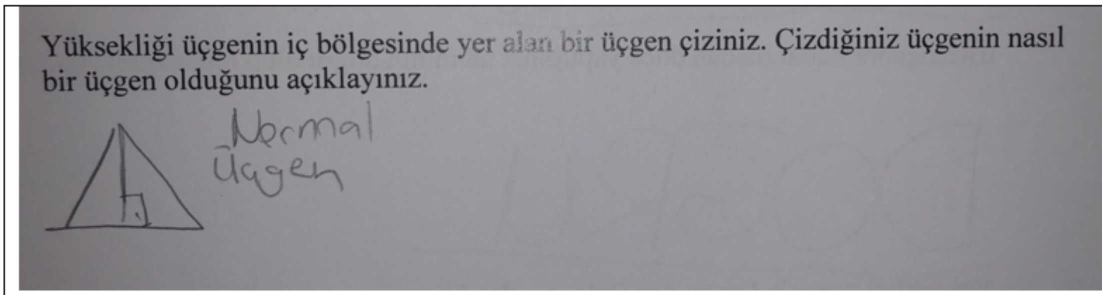
(iii) Draw an altitude to lie entirely outside the triangle. Explain what kind of a triangle this will occur.

(iv) Given your responses to this question and what you've observed, complete the following sentence definition: An altitude of a triangle is...

While students were answering the questions, I observed students' answers and actions. They had difficulty in defining the concept of altitude for the first question. Many groups asked how they could define the altitude. I gave clues that they could remember the critical aspects of the altitude or they could write definition by drawing an altitude to a triangle. Most of the group drew altitude to an acute angled triangle standing on the base and then wrote the definition. Students also asked questions related to drawing altitude to right angled and obtuse angled triangle. I suggested that they could draw the altitudes by examining the definition which they wrote for first question. Almost all of the groups had difficulty in drawing altitude for third and fourth questions.

After answering questions, students with their groups shared their drawings and definitions to the classroom. Before the process of sharing, I reminded that each group needed to explain and support their answers to the class clearly. Even if students give some wrong answers, students needed to listen to the group to the end of sharing.

Students shared their drawings to the classroom. The following Figure 5.7, Figure 5.8, Figure 5.9 and Figure 5.10 were examples of students' drawings that shared with the classroom. Through the process of sharing, all of the groups drew altitude to the acute triangle correctly. Some groups wrote the type of triangles as "acute triangle", "equilateral triangle" or "Isosceles triangle". However, one group called the acute angled triangle as "Normal Triangle" as the below Figure 5.7. When I asked the reasons of givining name as "Normal Triangle", they explained as "since it was an acute triangle".

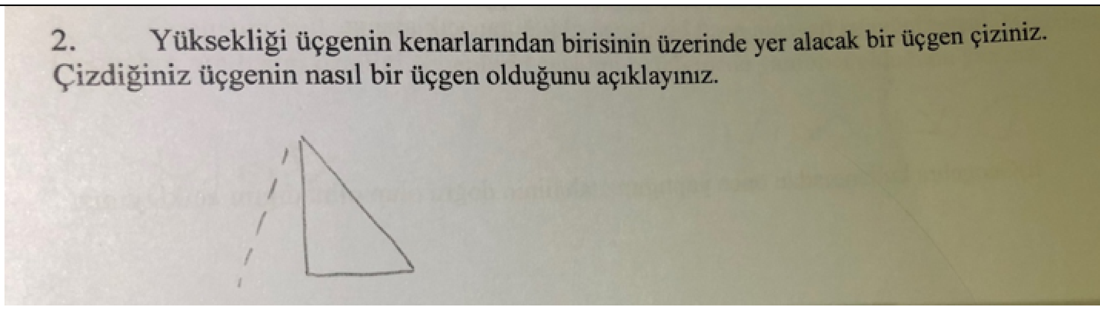


Question: Draw an *altitude* to lie *inside of the triangle*. Explain what kind of a triangle this will occur.

Student'answer: Normal triangle.

Figure 5.7. Students' drawings for the altitude being inside of the triangle.

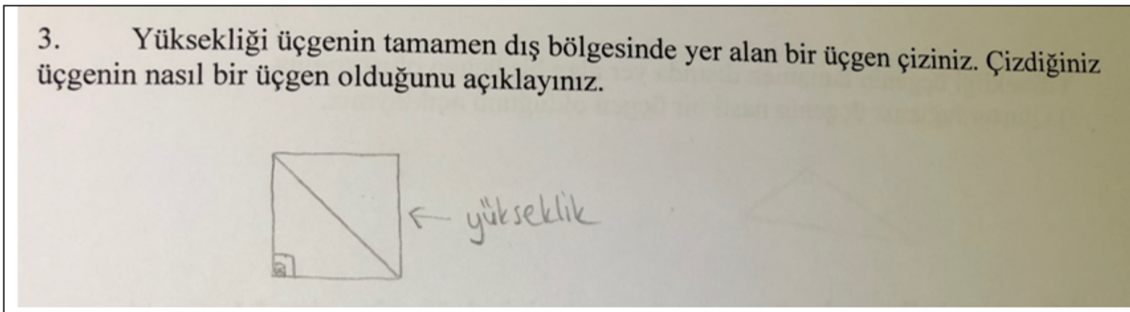
Students had difficulty more in answering the questions of altitude being on the side and outside the triangle. As the below Figure 5.8, student could draw the right angled triangle, but they drew the altitude outside of the triangle without drawing as being perpendicular. The student did not consider the one of the six critical aspects of the altitude which is perpendicularity. After sharing this below drawing, students discussed on it and find correct results with the help of making acceptable explanations.



Question: Draw an *altitude* to lie *on the one side of the triangle*. Explain what kind of a triangle this will occur.

Figure 5.8. Students' drawings for the altitude being on the side.

Students mostly did not prefer to give detailed explanations for their drawings. As the below Figure 5.9, students drew the altitude as being outside of the triangle. Some students stated that this altitude is wrong because the altitude did not start from the vertex while some of them stated that the altitude was correct because the length of the drawn altitude has same length. After students discussed for a while, they reached that altitude need to start from correspponding vertex according to the definiton of the altitude.

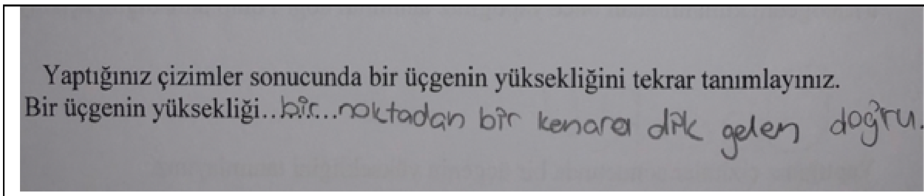


Question: Draw an *altitude* to lie *entirely outside the triangle*. Explain what kind of a triangle this will occur.

Students' Answer : Altitude (the pointed out part)

Figure 5.9. Students' drawings for the altitude being outside triangle.

Another group shared their definition as a perpendicular line passing from one point to a side as the below Figure 5.10. Some groups wrote "line" for altitude even though they drew the altitude as "line segment" in their drawings. In the classroom, students expressed that altitude is a line segment and the written definition is not totally true. Students reached this result with the help of using six critical aspects of altitude.


Yaptığınız çizimler sonucunda bir üçgenin yüksekliğini tekrar tanımlayınız.
Bir üçgenin yüksekliği..bir..noktadan bir kenara dik gelen doğru.

Question: Define the altitude of triangle after drawing the altitudes.

Student'answer: An altiude of a triangle is a perpendicular line from one point to one side.

Figure 5.10. Students' definition of the altitude.

After the part of sharing, I asked students to open the given link including simulations on GeoGebra. I gave some time to examine and manipulate the triangle from the vertex. I wanted them to answer the following questions as in the below. Students answered the below questions as a group by using GeoGebra.

(v) How can you change the vertex so that the altitude will lie inside the triangle?
- Show what kind of a triangle this will occur on the GeoGebra.
- Express if your prediction was correct before using GeoGebra.

(vi) How can you change the vertex so that the altitude will lie on the one side of the triangle?
- Show what kind of a triangle this will occur on the GeoGebra.
- Express if your prediction was correct before using GeoGebra.

(vii) How can you change the vertex so that the altitude will lie entirely outside the triangle?
- Show what kind of a triangle this will occur on the GeoGebra.
- Express if your prediction was correct before using GeoGebra.

(viii) Write the definition of altitude again according to your observation.
- An altitude of a triangle is...

Students verified their answer by using GeoGebra and they compared their answer with and without using GeoGebra. I reminded students to support their answers by presenting reasons and observations. I also directed them to write their explanations with their reasons.

While working on GeoGebra, some students suprised and stated that the altitude is outside of the triangle. Before this lesson, students drew altitude to the obtuse angled triangle many times. However, they reflected their feelings as if they saw the altitude outside of the triangle at the first time. Visulazing and manipulating the altitude on the GeoGebra was a different experience for the students. At the end of answering questions, I asked students to share solutions to the class. They explained the place of altitudes for each type of triangles. All students found correct results by drawing the triangles and dragging them on GeoGebra. They also shared their definitons with the classroom. While students were sharing their definitions of the concept of altiude for the question 8, students were more comfortable. For the closure part, students answered following questions as groups.

(ix) Is there any differences on the definitions before and after using GeoGebra? If so, explain what the reason would be.
(x) Explain what kind of differences do you observe before and after using GeoGebra. Please write the differences for each question from 1 to 3.
(xi) Do you think that GeoGebra are beneficial for you to make decision on the type of the triangle. If so, please explain which property of GeoGebra help you to decide?

After sharing and coming to a mutual understanding, they wrote the answers. Students shared their drawings and the explanations with the classroom. Most of students stated that they could draw the altiude to acute triangle properly before using GeoGebra. However, they expressed that their altitudes to right and obtuse triangle were wrong before using GeoGebra. Many students stated that GeoGebra presented a better visualization for the concept of altitude. They especially emphasized the dragging property of the GeoGebra. One of the students explained the property as they could gain different types of triangles and observed the change of place of altitude by dragging the vertex of triangle on GeoGebra. Students expressed that they understood the concept of altitude better after using GeoGebra.

For the last question, students made comments for the use of GeoGebra in the mathematics lessons as in the following statements:
"GeoGebra made the concept of altitude easier for me to understand it".
"GeoGebra helped me to draw the altitude".
"GeoGebra was helpful for visualizing altitudes and triangles"
"GeoGebra was very beneficial"
"GeoGebra was very entertaining"
"Dragging (the vertex of) triangles (on the GeoGebra) helped me understand altitude better".

Students understood the place of altitude better with the above lesson plan. Students reflected on what they learned related the concept of altitude. They shared their understandings as they were sharing and discussing their solutions. Then, they experienced how the place of altitude was changing with the dragging property of GeoGebra. After the experiences with GeoGebra, students solved the rest of questions and drew the asked altitudes better on their activity sheets. Students' views were also positive about their experience with GeoGebra.

5.4. Instruments

5.4.1. Altitude Test

The Altitude Test was provided by the researchers Gutierrez and Jaime (1999) and Gürefe *et al.* (2014) with their permission. I revised the test according to the opinions of experts. The aim of Altitude Test as pre-test is to detect students' previous knowledge about the concept of altitude. Even though students did not learn the concept before 6th grade in the curriculum, some elementary teachers might give extra information related to altitude in 5th grade or students might learn the concept from different resources. The aim of Altitude Test as post-test is to assess students' conceptual understanding of the concept of the altitude after the instruction. All questions were correspondence with the learning objectives in 6th grade mathematics curriculum (MoNE, 2017). Questions were matched with the topics for the Altitude Test (see in Appendix C) in following ways:

Table 5.3. Topics in the Altitude Test.

Items	**Topics**
1	The definition and the properties of altitudes
2	The definition and the properties of altitudes
3	Perpendicularity to a line from a point outside of the line
4	Altitude to a parallelogram
5	Altitude to the triangle
6	Area of the parallelogram
7	Area of the triangle

The question 1 consisted of open-ended questions to measure students' conceptual understanding of the definition and the properties of altitudes. The question 2 related to the definition and the properties of altitudes was placed in the test of "Height Achievement Test" which was prepared by Gürefe *et al.* (2014). This test was designed for 5th grade students to measure students' achievement on the concept of altitude. The question 5 related to drawing altitudes to triangles was originally designed by Hershkowitz and Vinner (1982) and Vinner and Hershkowitz (1983) and developed by Gutierrez and Jaime (1999). This test was designed for the students from 6th grade to 8th grades. The questions 3 and 4 related to drawing perpendicular to line and altitudes to parallelogram were designed by researcher based on the question 5.

The Altitude Test was administered two days before and two days after the instruction. The test consisted of altitude questions in different rotation and location, in different type of triangle (acute, obtuse and right-angled triangles). All questions were arranged from easy to the difficult ones. Questions were prepared by considering six critical aspects namely vertex, perpendicularity, opposition, orientation, location and altitude-based-correspondence. Both prototype and non-prototype examples placed in the test. After the test prepared, researcher took experts' opinions.

The test was applied as pilot test in a different school. For the pilot testing, 7^{th} grade students were chosen since they learned the concept of altitude before and had previous knowledge about it. According to the results of the pilot test, all misconceptions in the literature review (altitude vs. median, altitude vs. perpendicular bisector, limitation to internal altitudes, discard of length, fixation on side) were observed. According to results, I revised the questions before applying it.

Gürefe *et al.* (2014) developed a rubric to assess students' understanding of the concept of altitude. I used the same rubric (see in Appendix D) but I revised the scores of the items since the number of items were different from the test Gürefe *et al.* (2014) developed. For the questions 3, 4 and 5 related to drawing altitude, small error in the perpendicularity was allowed (Gutierrez and Jaime, 1999) since students did not use drawing tools during the administration of the test. There is a summary of rubric in the Table 5.4, but the detailed version is placed in Appendix D. I asked the rubric to the one experienced mathematics teacher and one mathematics educator. 12 pre-tests and 12 post-tests were selected and evaluated with same rubric by the experienced mathematics teacher. Inter-scorer reliability coefficient was found as .994.

Table 5.4. Scoring the items in the Altitude Test.

Questions	**Items**	**Answers**	**Scores**
1	6	Correct	2
		Partially True	1
		False	0
		Empty	0
2	8	Right Answer - True Explanation	2
		Right Answer - False(No) Explanation	1
		Wrong Answer	0
		Empty	0
3	5	True	2
		False	0
		Empty	0
4	4	True	2
		False	0
		Empty	0
5	14	True	2
		False	0
		Empty	0
6	2	True	2
		Partially True	1
		False	0
		Empty	0
7	4	True	2
		Partially True	1
		False	0
		Empty	0
Total			86

The Altitude Test is evaluated out of 86 points. There are 6 items in the first question and 8 items in the second question. These questions are related to the definition of the concept and place of the altitude. For the questions 1 and 2 in the Altitude Test, students have 2 points for each item if students'? answer and explanation are correct. However, if students have right answer without explanations, they have 1 point for each item in the questions 1 and 2. There are 8, 5 and 4 items respectively for the questions 3, 4 and 5 and these questions are related to drawing perpendicularity or altitude. If students can draw correctly, they get 2 points. Since the questions were related to drawing, there are no partial scores awarded. In the last questions 6 and 7, there are 2 and 4 items respectively. Students find the area of parallelograms and triangles in these questions. If students draw the altitudes but can not find the area, they have 1 point for each item. If students both draw altitude and find area, then they have 2 points for each item.

5.4.2. Student Journals

Students reflected what they experienced during the learning process on their journals. Students wrote journals according to the given directions and guidelines. Students answered the given questions (see in Appendix E) while writing journal once in a week. The questions were related to their views about sociomathematical norms students experienced: sharing solutions to the class, working collaboratively, acceptable explanation and being free to make mistakes. Each question in the students' journal were assigned to one sociomathematical norm or use of technology. It is significant to understand how students experience sociomathematical norms during learning process. The questions were also included learning the concept of altitude such as the difficulties, students' experience or interactions with the technology. Students totally kept four journals during the instruction process.

There are three questions in the students' journals. The first question in the students' journals are related to four sociomathematical norms and use of technology. Students select one scale from "not observed, partially observed, mostly observed, always observed". Students assessed the sociomathematical norms and use of technology by selecting one scale. The second question is composed of five items. Students reflect the sociomathematical norms and use of technology with open-ended questions. Four items in the second question correspond with the four sociomathematical norms of "working collaboratively, sharing solutions with the classroom, being free to making mistakes and making acceptable explanations" respectively. The last item in the second question corresponds with use of technology. For the third question, students can write their ideas related to mathematics lessons apart from sociomathematical norms and use of technology.

5.4.3. Teacher's Field Notes

I took notes about the instruction process during or right after the instructions related to the sociomathematical norms, common misconceptions, students' understanding way, use of technology. It helped me to determine the misconceptions related to the concept of altitude and reflect on sociomathematical norms and use of technology. I had an opportunity to reflect my teaching in the classroom as taking notes during the instruction. As a result of reflections, I continuously revised my lesson plans and my actions how I could improve students'? conceptual understanding better.

5.4.4. Classroom Audio and Board Recordings

During the instruction, I took voice and smart board recordings to improve my instruction and observe and follow the interactions between sociomathematical norms and students. Audio records was needed for detecting sociomathematical norms during class discussions, the process of sharing solutions and interactions among students or between students and teacher. With the help of board records, the flow of the instruction could be followed easily after the lessons. According to board recordings, common misconceptions could be detected on students' drawings in the classroom. I listened the audio recordings in that day or the following day. I took notes about students' interactions, actions and understanding. I continuously reflected how I could improve my teaching after listening recordings.

5.5. Data Collection

There were four main sources of data collection; the altitude test and student journals, teacher's field notes of classroom observations and classroom audio and board recordings. The data coming from pre-test and post-test was compared and analyzed. While the Altitude Test as a pre-test and post-test were assessing, each question was scoring. Students' conceptual understanding was also assessed by analyzing which six critical aspects of the altitude (vertex, perpendicularity, opposition, orientation, location and altitude-based-correspondence) are understood, what type of misconceptions exist, what kind of questions students have difficulty in answering and explaining. Teacher's field notes and classroom audio and board recordings were analyzed to determine sociomathematical norms and use of technology in the classroom. Journals were analyzed on how students experience sociomathematical norms during learning process. The following table shows the process of data collection.

Table 5.5. Process of data collection.

Before Instruction	**During Instruction**	**After Instruction**
Altitude Test	Student Journals + Teacher's Field Notes + Classroom Audio and Board Recordings	Altitude Test

5.6. Data Analysis

The data were collected with quantitative and qualitative methods to assess students' conceptual understanding of the concept of altitude. Achievement on Altitude Test, types of explanations, types of misconceptions, types of misconceptions according to prototype and nonprototype examples and achievement on prototype and nonprototype examples were analyzed for pre and post-test. The frequencies of sociomathemematical norms and students' views about norms and technology were also analyzed. The below table showed how data was analyzed.

Table 5.6. Process of data analysis.

Achievement on Altitude Test	Comparisons between pre and post-test scores with Wilcoxon signed-rank test
Concept image - definition	Frequencies of the types of explanations (pre and post-tests)
Misconceptions	Frequencies of the types of misconception (pre and post-tests)
Misconceptions for prototype and nonprototype examples	Frequencies for prototype and nonprototype examples (pre and post-tests)
Achievement on prototype and nonprototype examples	Frequencies of correct drawings according to prototype and nonprototype examples
Sociomathematical norms	Frequencies of sociomathematical norms according to the types of environment
Views about norms and technology	Frequencies of views about norms and technology
Instructions	Revision and reflection of teacher's field notes

5.6.1. Altitude Test

Altitude Test was administrated to 6^{th} grade students to assess students' conceptual understanding of the concept of altitude before and after the instruction. After both pre-test and post-test questions were scored, their scores were analyzed. Since the sample is small, Shapiro-Wilk Test was used for the normality of the data (Shapiro, Wilk and Chen, 1968). The p-values of both pre and post-test is 0.005 and 0.016 respectively. Since the p-values are less than 0.05, non-parametric test was used.

For the data analysis, Wilcoxon signed-rank test was used since the sample size was more than 30 but the data was not normally distributed (Privitera, 2012). Wilcoxon signed-rank test was preferred to investigate whether there is a significant difference between pre-test and post-test scores of participants.

Table 5.7. Tests of Normality.

Tests of Normality						
	Kolmogorov-Smirnov[a]			**Shapiro-Wilk**		
	Statistic	df	Sig.	Statistic	df	Sig.
Post-test	.128	48	.048	.940	48	.016
Pre-test	.132	48	.036	.927	48	.005
a. Lilliefors Significance Correction						

Six misconceptions that Gutierrez and Jaime (1999) determined in their study as mentioned in the literature, were detected in students' pre-tests or post-tests. The occurrence of misconceptions in pre and post-tests were compared. Questions were coded according to the misconceptions that Gutierrez and Jaime (1999) determined. Their frequencies of misconceptions in each question were determined for both pre-test and post-test.

5.6.2. Classroom Recordings and Teacher Notes

Sociomathematical norms were analyzed with the help of classroom recordings, students' journals and teacher notes. I took notes about the interactions between students, sociomathematical norms, misconceptions and the reflections on use of technology while students were working collaboratively among themselves or after the lesson in the same day. I listened to all audio and board recordings in that day or the following day. While I was listening to the records, I transcribed all records.

The sociomathematical norms in the dialogues, interactions and discussions were coded as "acceptable explanation", "being free to making mistakes", "working collaboratively" and "sharing solutions to the class". Before the process of coding, I constructed the below table for determination of sociomathematical norms. The description of the norms was prepared by summarizing the literature (Cobb and Yackel, 1996; Kazemi and Stipek, 2001; Özmantar, 2009; Tatsis and Koleza, 2008).

Table 5.8. The determination of sociomathematical norms.

The types of Sociomathematical Norms	**The Description of Norms**
Acceptable explanation	• Mathematical actions. • Consist of mathematical reasoning, solution strategies, conceptual or procedural ideas. • Being understood and interpreted the explanation by other students.
Being free to making mistakes	• Reconceptualizing a problem with the help of mistakes. • Exploring contradiction in different solutions after making mistakes. • Trying different solution methods. • Revealing misconceptions • Reaching a mutual agreement on the solution process and understanding the concepts and procedures.
Working collaboratively	• Individual accountability for thinking on problem, strategies or solutions in the group. • Using plural verbs while expressing their ideas in group working. • Asking questions to understand a group member' thinking way.
Sharing solutions to the class	• Explaining students' solution methods to the class. • Making interpretation about the problem or solution. • Reaching a conclusion related to the problem.

5.6.3. Student Journals

In order to assess students' views related to sociomathematical norms and use of technology, students wrote journals for each week. Students' journals consisted of a scale about norms and use of technology and writing their ideas about norms and use of technology for each week. Students reflected their ideas about norms and technology in the lessons at end of the week for five weeks. Student journals were comprised of four-point Likert Scale items ranging as "not observed, partially observed, always observed". There were five items which were composed of sharing solutions collaboratively, working cooperatively, acceptable mathematical explanation, being free to make mistakes and use of technology. Scores of 0 to 3 were assigned to "not observed, partially observed, mostly observed, always observed" respectively for each item.

6. RESULTS

The aims of the action research were to solve the problem which students had difficulty in understanding the concept of altitude and to improve my teaching practices to design an effective instruction enriched with technology and sociomathematical norms. The development of research could be analyzed according to the data from teacher's fields notes, students' journals, classroom audio and board recordings. Firstly, the results of pre and post Altitude Tests, students' journals, teacher's fields notes, classroom audio and board recordings were presented respectively. Then, the results for development of instruction were mentioned in the following section.

6.1. Altitude Test

The pre-test and post-test scores of Altitude Test were quantitatively analyzed to assess students' conceptual understanding. Shapiro-Wilk Test was used for the normality of the data as in the Table 5.7. The p-values of both pre and post-test were found as 0.005 and 0.016 respectively as in the Table 5.7. Since the p-values are less than 0.05, non-parametric test was used. Since the distribution of the data was not normal and the sample is more than 30 (Privitera, 2012 p.137), a nonparametric Wilcoxon signed-rank test was performed to investigate the difference between pre-test and post-test scores. The following table shows descriptive statistics related to pre-test and post-test scores of Altitude Test.

Table 6.1. Descriptive statistics related to pre-test and post-test scores of Altitude Test.

	N	Mean	Std. Deviation	Minimum	Maximum
Pre-test	48	14.729	11.988	2	41
Post-test	48	62.687	13.432	37	83

There are 48 participants in both of pre-test and post-test. The mean of pre-test scores is 14.729 out of 86, while the mean of post-test scores is 62,687 out of 86. The mean of post-test scores is higher than the mean of pre-test scores. It can be seen that there is also increase in the minimum and maximum scores of post-test scores according to the scores of pre-tests.

Table 6.2. Ranks of Wilcoxon signed-rank test.

Ranks				
		N	Mean Rank	Sum of Ranks
Post-test-Pre-test	Negative Ranks	0[a]	.00	.00
	Positive Ranks	48[b]	24.50	1176.00
	Ties	0[c]		
	Total	48		
a. Post-test¡ Pre-test				
b. Post-test¿Pre-test				
c. Post-test = Pre-test				

According to results of Wilcoxon signed-rank test, there is a statistically significant difference between pre-test and post-test scores (Z=-6.032, $p < 0.001$). The Table 6.2 and Table 6.3 demonstrate test statistics.

Table 6.3. Test statistics of Wilcoxon signed-rank test.

Test Statistics[a]	
	Post-test-Pre-test
Z	-6.032[a]
Asymp. Sig. (2-tailed)	0
a. Based on negative ranks.	
b. Wilcoxon Signed Ranks Test	

6.1.1. Concept Image and Concept Definition

To analyze students' concept image and concept definition together, students both drew the altitude to the given triangles and made explanations about being altitude in the open-ended questions of the Altitude Test. If students' explanations were composed of only words, the answers were accepted as "explanations with only words". Students mostly used six critical aspects in their explanations. If students' explanations included only drawings, then the answers were accepted as "explanations with only image". Students mostly drew altitudes to triangles in their explanations with images. If students both explained the question with words and image, then their answers were accepted as "explanations with both words and image". The questions 1 and 2 in the test were open ended. It is asked that students explain their thinking. There were totally 672 answers in the question 1 and 2 coming from all participants. Some students preferred to explain the concept of altitude with words or image, while some students did not prefer to write an explanation of their answers.

Table 6.4. The frequency of the types of explanation in question 1 and 2.

	Explanation with only image	**Explanation with only words**	**Explanation with both words and image**	**Answers without explanation**	**No answer**
Pre-test	37	60	5	172	398
Post-test	218	220	12	184	38

In the Table 6.4, all answers in the question 1 and 2 coming from pre and post-test were analyzed. In the pre-test, the students who gave no answer were at the highest level. In both of pre and post-test, students who explained the questions with words were more than the students who explained the questions with image.

Table 6.5. The frequency of the types of explanation in the pre-test of the question 1 and 2.

Concepts in pre-test	**Explanation with only image**		**Explanation with Only words**		**Explanation with both words and image**	
	correct	**wrong**	**correct**	**wrong**	**correct**	**wrong**
Defining Altitude	6	9	12	18	0	3
Placing Altitude	8	14	8	22	0	2

The Table 6.5 and Table 6.6 showed how students gave the explanation for the question of defining and placing altitude. In post-test, while students were explaining the question of defining altitude, most of students preferred to give explanation with only words. While students were explaining the question of placing altitude, students were giving explanations with only image in the post-test. However, in the pre-test, most of the students preferred to explain the question with only words, while they were both defining and placing altitude. In the pre-test, students who explained the question with both words and image gave wrong answers. However, students who explained with both words and image gave correct answers in the post-test.

Table 6.6. The frequency of the types of explanation in the post-test of the question 1 and 2.

Concepts in post-test	**Explanation with only image**		**Explanation with only words**		**Explanation with both words and image**	
	correct	**wrong**	**correct**	**wrong**	**correct**	**wrong**
Defining Altitude	53	16	118	25	12	0
Placing Altitude	132	17	79	25	0	0

6.1.2. Misconceptions

Students' understanding of the concept of the altitude in both pre-test and post-test were analyzed to detect possible misconceptions. Gutierrez and Jaime (1999) composed codes which include the common misconceptions. All of the misconceptions were detected in students' papers. I used these codes to detect misconceptions in the questions 3, 4 and 5 including drawing perpendicular line and altitude to triangle and parallelogram.

Table 6.7. The codes of misconception.

Codes	The Types of Misconceptions
M1	Altitude vs. Median Misconception
M2	Altitude vs. Perpendicular Bisector Misconception
M3	Limitation to Internal Altitudes Misconception
M4	Disregard of Length Misconception
M5	Fixation on Side Misconception
M6	Marked base as distracter Misconception
M7	Disregard of vertex misconception
M8	The error of perpendicularity
M9	The tendency of completing triangle

The misconceptions in the pre and post-test were analyzed in the question 3, 4 and 5 from 48 participants. These questions are composed of drawing perpendicularity, altitudes to parallelograms and triangles. The misconceptions of M1, M2, M3, M4, M5, M6, M7 are related to drawing altitudes to triangles. The misconception of M8 belongs to both drawing perpendicularity and altitude to parallelogram while M9 is only related to drawing perpendicularity.

Table 6.8. The misconceptions in the pre and post-test.

	The code of M1	The code of M2	The code of M3	The code of M4	The code of M5	The code of M6	The code of M7	The code of M8	The code of M9
The Misconceptions in pre-test	4	7	11	7	4	0	6	17	8
The Misconceptions in post-test	3	5	40	0	6	9	19	5	0

- Altitude vs. median misconception (The code M1): In this type of misconception, students mixed the concepts of median and altitude. 4 students had the misconception in pre-test while the number of students who had this type of concept was decreasing in the post-test. This type of misconception was the least frequent for the post-test.
- Altitude vs. perpendicular bisector misconception (The code M2): The concept of altitude and perpendicular bisector were mixed by students since both of the

concept had the characteristic of perpendicularity. Students had the knowledge of perpendicularity but the starting or ending point of the altitude was not understood. The number of students who had misconception was decreasing from 7 to 5 after the instruction.

- Limitation to internal altitudes misconception (The code M3): Students who had this misconception had tendency to draw the altitude inside of the triangle. Students had difficulty in extending the corresponding side and drawing altitude outside of obtuse angled triangle during the instruction. Students called the extended side as "imaginary side" in the classroom. When they encountered with obtuse angle triangle, they drew the altitude to the inside of the triangle. This misconception was the most common error for post-test while in pre-test this misconception was the second most frequent.
- Disregard of length misconception (The code M4): Students did not consider or mix line or line segment for the concept of altitude. 7 students had the misconception in the pre-test. However, none of students had this misconception in the post-test. After students learnt the concept of altitude, there was no error related to the starting and ending point of the altitude.
- Fixation on side misconception (The code M5): In this type of misconceptions, students only considered the side indicated for drawing altitude. They had tendency to show the side next to the indicated side or show the side itself as an altitude. There were 4 students who had this misconception in pre-test while the number was increasing to 6 for post-test after the instruction.
- Marked base as distracter misconception (The code M6): Students have the knowledge that the altitude needed to start from the vertex and pass through the indicated side. However, they missed the aspect of the perpendicularity. It showed that students had the partial image of the concept of altitude. In the pre-test there were no students who had the misconceptions, while 9 students had the misconception in the post-test included this misconception.
- Disregard of vertex misconception (The code M7) Students drew an imaginary line passing through the corresponding vertex and drew the altitude starting from any point on that imaginary line. They drew the altitude to specified base and as being perpendicular. However, the altitude did not start form the corresponding

vertex even if students found the corresponding vertex. Such students had the concept partially. This type of misconception was not found in the literature. I called it as “disregard of vertex misconception“ since students did not consider the corresponding vertex. This misconception was discussed many times with the classroom. Students stated that the altitude which was started from the corresponding vertex had same length with the one which was drawn from the extension of the vertex. Even though the disregarding of the vertex was not corresponded with definition of the altitude and was discussed with the classroom, the number of students who had this misconception increased from 6 to 19 after the instruction. It showed that these students could not understand the six critical aspects of altitude and learn the concept of altitude properly.

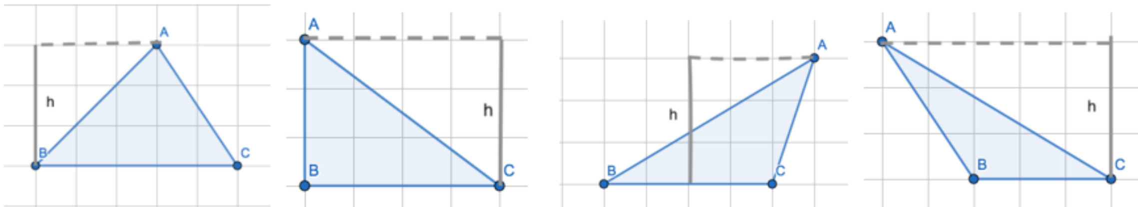


Figure 6.1. The misconception of disregard of vertex.

- The error of perpendicularity (The code M8): This misconception included the error of perpendicularity while drawing altitude. The small error of perpendicularity did not count as misconception. However, if it created a huge difference in perpendicularity, it is called as “the error of perpendicularity”. All misconceptions related to the error of perpendicularity in the pre and post-test belonged to the question of “drawing perpendicular to a line”. The answers including this misconception decreased from 17 to 5 after students took the instruction.

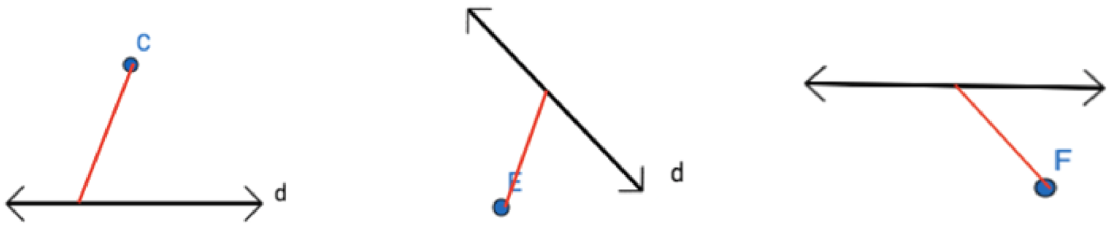


Figure 6.2. The misconception of the error of perpendicularity.

- The tendency of completing triangle (The code M9): This type of misconception was only detected in pre-test. In the question 3, it was asked for drawing a perpendicular to the given outside of the line. Students complete the given line to the triangle with dotted line segments. There were 8 students who had this misconception in the pre-test. After students took the instruction, the misconception of completing triangle disappeared.

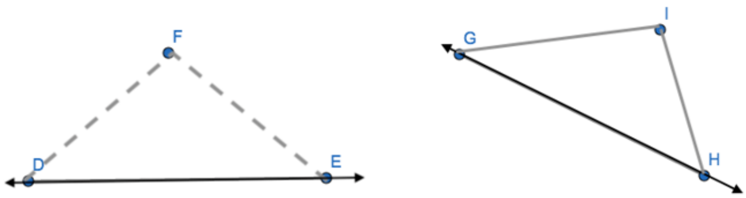


Figure 6.3. The misconception of the tendency of completing triangle.

Most of the misconceptions detected in the question 5 of pre and post-test. This question was asked to draw altitudes to given triangles. The prototype and non-prototype items were determined. The misconceptions of the error of perpendicularity (M8) and the tendency of completing triangle (M9) was not analyzed for prototype and nonprototype examples since these misconceptions (M8 and M9) related to the perpendicularity to a line from a point outside of the line, not altitude to a triangle. The acute angled triangles counted as prototype examples which were the items 1, 2, 3, 4, 6, 10 and 11 in the Altitude Test. The right and obtuse angled triangles were counted as non-prototype examples which were items 5, 7, 8, 9, 12, 13 and 14 in the Altitude Test.

Table 6.9. The misconceptions in prototype and non-prototype items.

The Codes of Misconception	**Prototype Examples**		**Non-prototype Examples**	
	Pre-test	**Post-test**	**Pre-test**	**Post-test**
M1	3	1	4	3
M2	5	4	6	4
M3	6	2	9	37*
M4	7	0*	4	0*
M5	2	0	2	6
M6	0	3	0	7
M7	4	9	4	15

- Prototype vs nonprototype:

When the Table 6.7 were analyzed, students had misconceptions mostly on non-prototype examples in the both of pre and post-test. After instruction, there were no students who had the misconception of disregard of length misconception (M4) for both prototype and nonprototype examples. All types of misconceptions for prototype examples, except from M6 and M7, were decreasing from pre-test to post-test. Students did not have any misconceptions of M4, M5, M8 and M9 for prototype examples after the instruction.

While the misconceptions of M1, M2, M4 were decreasing from pre-test to post-test, the misconceptions of M3, M5, M6 and M7 were increasing for non-prototype examples. There were no students who had the misconceptions of disregard of length for non-prototype examples.

In the Table 6.10 below, students showed more success on prototype examples than nonprototype examples. The Table 6.10 is parallel with the Table 6.9 as it is expected. However, the Table 6.10 gave detailed information about each item of "triangle" questions. Although the items 6 and 10 are prototype examples, the percentage of correct drawings had lower values according to other prototype examples. For non-prototype examples, the items 8 and 13 had lowest percentages. The triangles in the items 6, 8, 10 and 13 did not rotated vertically or horizontally. The triangles are leaning to one side.

Table 6.10. The percentage of correct drawings in prototype and non-prototype items.

	correct drawings	
	f	**%**
Item-1 (prototype)	46	95.833%
Item-2 (prototype)	41	85.416%
Item-3 (prototype)	46	95.833%
Item-4 (prototype)	41	85.416%
Item-5 (nonprototype)	34	70.833%
Item-6 (prototype)	33	68.75 %
Item-7 (nonprototype)	30	62.5 %
Item-8 (nonprototype)	5	10.416%
Item-9 (nonprototype)	23	47.916%
Item-10 (prototype)	35	72.916%
Item-11 (prototype)	42	87.5 %
Item-12 (nonprototype)	31	64.583 %
Item-13(nonprototype)	2	4.166%
Item-14 (nonprototype)	27	5.25 %
f: frequency of correct drawings, %: percentage of correct drawings		

6.2. Classroom Recordings and Teacher Notes

The action research were planned to solve the problem which students had difficulty in understanding the concept of altitude. It was planned to improve my teaching practices to design an effective instruction enriched with technology and sociomathematical norms. The results of development of research could be explained based on quantitative and qualitative data.

From beginning of the year, I was establishing four sociomathematical norms with my 6^{th} grade students. For this research, I focused on examining effect of using sociomathematical norms with technology on students learning of altitude. Detailed plan of instruction is discussed in section 5.3.1, namely “Plan of Teaching Altitude”. In this section, I will report how sociomathematical norms with use of technology is implemented. During the instruction on altitude, the transcribing data from audio and board recordings in that day or the following day contributed me observe and follow students' interactions in the classroom.

The sociomathematical norms during instruction were analyzed qualitatively. To understand the classroom environment, the frequencies of norms for each lesson were determined as constructing a table. The frequencies in the table reorganized based on the topic of the lessons and the use of technology in the following table. Four sociomathematical norms namely acceptable explanation, being free to making mistakes, working collaboratively, sharing solutions to the class were coded as N_1, N_2, N_3, and N_4 respectively. The norms in Table 6.11 were constructed with the types of topics. As the sequence of the instruction was also explained in the Table 5.3.1, students continued to learn the topic of altitude to to the triangle in the 2^{nd}, 3^{rd}, 4^{th} and 5^{th} weeks while they were also learning the topics of area of triangle and the problems related to the area in the 3^{rd} and 4^{th} weeks.

The main purpose for the Table 6.11 was to show how sociomathematical norms were according to the type of use of technology. While students were learning the topic of perpendicularity, the frequencies of acceptable explanation and working collaboratively did not show differences across the type of environment. However, students shared their ideas and solutions more in the environment of technology demonstration. Students shared their solutions freely with environment of no technology.

The frequencies of making explanations, being free to making mistakes and sharing solutions had highest values in the environment of no technology during the topic of altitude to the parallelogram. Students did not need to work collaboratively. After learning the topic of perpendicularity, students could draw the altitude to parallelogram easily and they wanted to work individually. On the other hand, students made acceptable explanations and shared their solutions to the classroom at the most in environment of technology demonstration. While students tried to understand the area of parallelogram, they took advantage of simulations and discussed the area of formula on the simulations.

Table 6.11. Sociomathematical norms and use of technology during the instruction in class-1 and 2.

Class-1 and 2	No Technology		Technology Demonstration		Active Technology User	
	Class 1	Class 2	Class 1	Class 2	Class 1	Class 2
Perpendicularity to a line from a point outside of the line. (2 class hours in the 1^{st} week)	N_1: 5 N_2: 4 N_3: 1 N_4: 5	N_1: 5 N_2: 4 N_3: 1 N_4: 5	N_1: 5 N_2: 1 N_3: 2 N_4: 9	N_1: 5 N_2: 2 N_3: 2 N_4: 9	N_1: 4 N_2: 1 N_3: 1 N_4: 4	N_1: 4 N_2: 1 N_3: 1 N_4: 4
Altitude to the parallelogram. (2 class hours in the 1^{st} week)	N_1: 13 N_2: 5 N_3: 0 N_4: 17	N_1: 13 N_2: 4 N_3: 0 N_4: 18	N_1: 4 N_2: 3 N_3: 0 N_4: 6	N_1: 4 N_2: 3 N_3: 0 N_4: 7	N_1: 2 N_2: 1 N_3: 0 N_4: 3	N_1: 2 N_2: 1 N_3: 0 N_4: 3
Area of parallelogram. (2 class hours in the 1^{st} week)	N_1: 2 N_2: 3 N_3: 2 N_4: 3	N_1: 2 N_2: 2 N_3: 2 N_4: 2	N_1: 12 N_2: 4 N_3: 1 N_4: 14	N_1: 12 N_2: 3 N_3: 1 N_4: 15	N_1: 2 N_2: 0 N_3: 0 N_4: 2	N_1: 2 N_2: 0 N_3: 0 N_4: 1
Altitude to the triangle. (19 class hours during 2^{nd}, 3^{rd} 4^{th} and 5^{th} weeks)	N_1: 39 N_2:30 N_3: 2 N_4: 44	N_1: 39 N_2:31 N_3: 2 N_4: 43	N_1: 46 N_2: 20 N_3: 6 N_4: 59	N_1: 48 N_2: 22 N_3: 6 N_4: 58	N_1: 32 N_2: 12 N_3: 5 N_4:32	N_1: 33 N_2: 11 N_3: 5 N_4:33
Area of triangle. (3 class hours in the 3^{rd} week)	N_1: 49 N_2:15 N_3: 2 N_4: 57	N_1: 48 N_2:14 N_3: 1 N_4: 61	N_1: 0 N_2: 0 N_3:0 N_4: 0	N_1: 0 N_2: 0 N_3:0 N_4: 0	N_1: 0 N_2: 0 N_3: 0 N_4: 0	N_1: 0 N_2: 0 N_3: 0 N_4: 0
Problems related to the area. (2 class hours in the 4^{th} week)	N_1: 17 N_2: 7 N_3: 4 N_4: 11	N_1: 17 N_2: 6 N_3: 4 N_4: 21	N_1: 0 N_2: 0 N_3: 0 N_4: 0	N_1: 0 N_2: 0 N_3: 0 $N_4$4: 0	N1_1: 0 N_2: 0 N_3: 0 N_4: 0	N_1: 0 N_2: 0 N_3: 0 N_4: 0
N_1: Acceptable explanation, N_2: Being free to making mistakes, N_3: Working collaboratively, N_4: Sharing solutions to the class						

Learning the concept of altitude lasted during 19 hours since students had difficulty in drawing altitude to triangles more. The frequencies of making explanations and sharing solutions had highest values in the environment of technology demonstrations. While students were learning the altitude to right and obtuse triangles, students made their explanations, worked collaboratively and shared their solutions on the simulations. Students mostly had difficulty in determining the place of altitude and simulations showed the place of altitude according to the type of triangle very clearly. However, students felt free to make mistakes in the environment of no technology.

Students made applications related to the concept of altitude in the topics of area of triangle and problems related to the area while they continued to learn the concept of altitude. Since students mostly solve area problems, students did their drawings with their hands. They did not use technology during solving problem. Students made their explanations, worked collaboratively and shared their solutions freely in the environment of no technology.

The Figure 6.4 and Figure 6.5 are presented the frequencies of sociomathematical norms visually for class1 and 2. Total values of frequencies of the norms are shown in the below tables. The total frequencies can be compared according to the types of environment.

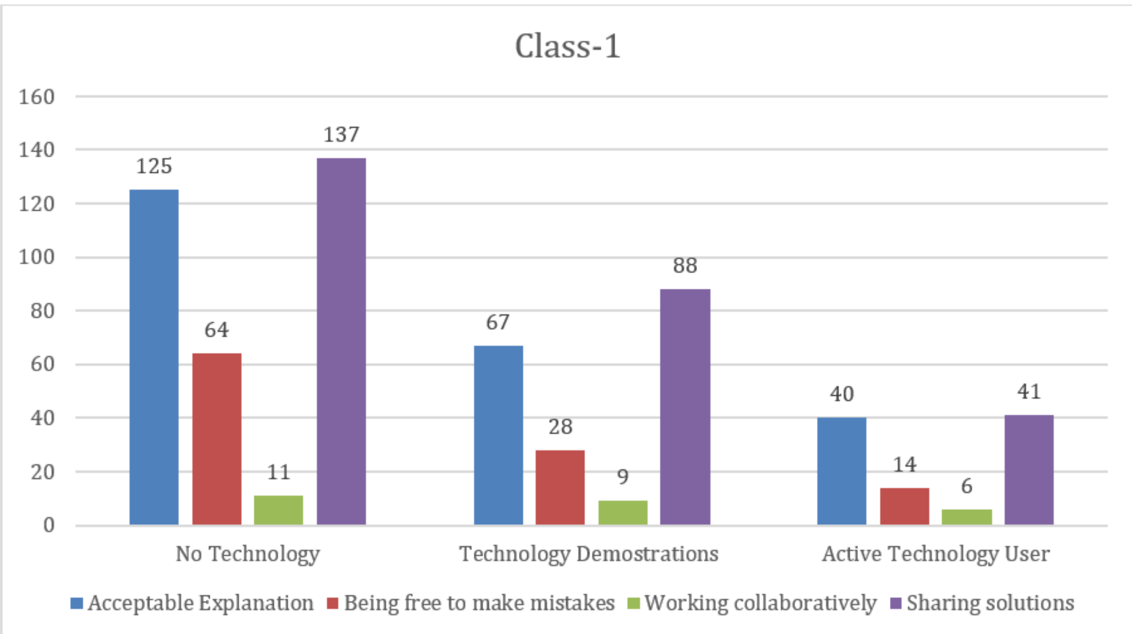


Figure 6.4. The frequencies of sociomathematical norms in class 1.

According to the Figure 6.4 and Figure 6.5, students made acceptable explanations, worked collaboratively, felt free to make mistakes and shared their solutions with the classroom at most in the environment of no technology. The frequencies of four sociomathematical norms had lowest values in the environment that students use technology actively. While the frequencies of four sociomathematical norms shows differences among the type of environment as in the Table 6.11, total frequencies of all four norms had highest values in the environment of no technology.

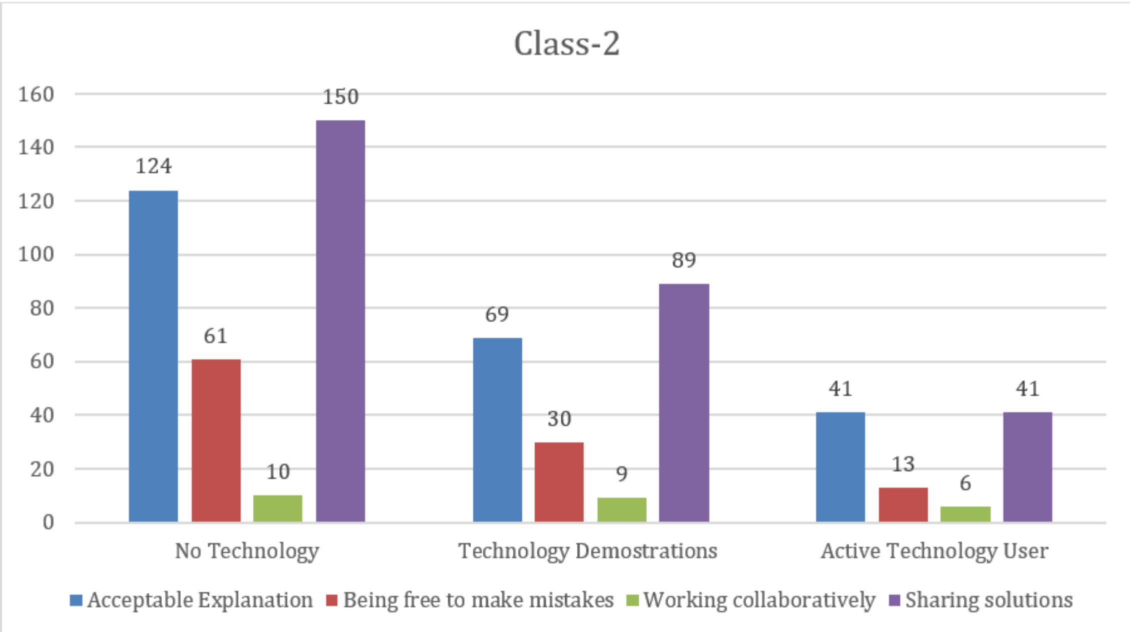


Figure 6.5. The frequencies of sociomathematical norms in class 2.

The frequencies of four sociomathematical norms were analyzed and explained by the above tables and graphs according to classroom recordings and teacher field notes. How these four norms were actualized in the classroom were explained with one sample dialogue for each sociomathematical norms.

- Acceptable explanation:

The norm of acceptable explanations showed similarity among two classes according to the Figure 6.4 and Figure 6.5. The frequency of acceptable explanations in the lessons which students used the technology actively was the lowest one. In an environment without technology, students made acceptable explanations at the most. Students explained their actions, solutions and ideas in a way that other students could also understand. When acceptable explanations in the classrooms were examined, explanations mostly based on six critical aspects of the altitude and the definition of the altitude.

Students had difficulty in providing acceptable explanations while they started to learn the concept of altitude. I supported students how they could make acceptable explanations. As I was listening to classroom recordings, I thought how I could direct students on making acceptable explanations. By revising my notes and reviewing literature, I decided to promoted students use the critical aspects of the altitude for the concept of altitude. During the class discussion, I tried to ask more challenging questions for explaning their thinking and making accepptable explanations. Some of these questions were like “What is the reason of being altitude inside of the triangle?” and “How do you know?”. These questions directed students to give acceptable explanations. While students were sharing their ideas, I also encouraged students to ask questions about their solutions. Through the end of instruction, they became learners who wanted acceptable explanations to be convinced on drawings or solutions. When the results of pre-test and post-test were compared as in the Table 6.4, the number of students who gave correct answer increased, at the same time the number of students who made explanations verbally and pictorially increased also.

Many students had difficulty in drawing altitude to right and obtuse angled triangles in the classroom. Students learned to draw altitude correctly with help of their peers? acceptable explanations. In the following example, I asked students to draw altitudes to three types of triangles according to their angles. Students made their groups and drew altitudes to acute, right and obtuse triangles in their activity sheets. After groups finished their drawings, I asked them firstly to share the drawing of altitude to acute triangle and explain their thinking with their explanations. I asked students to share their solutions of right angled triangle. The following example illustrated how students gave acceptable explanation for drawing altitude to right angled triangle in the classroom.

Example: It is asked to draw altitude to the BC side of the given triangle.

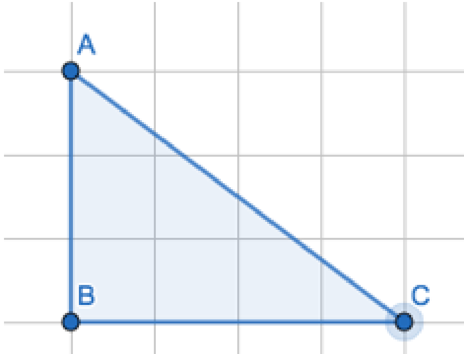


Figure 6.6. The right angled triangle.

- Teacher: Who can draw altitude to the BC side?
- Student A: Me.

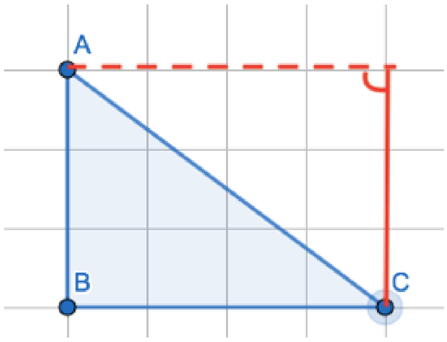


Figure 6.7. The student A's drawing altitude to the right angled triangle.

- Class: It is not passing from the vertex.
- Teacher: Is there anyone who think in a different way?
- Student B: Yes me.

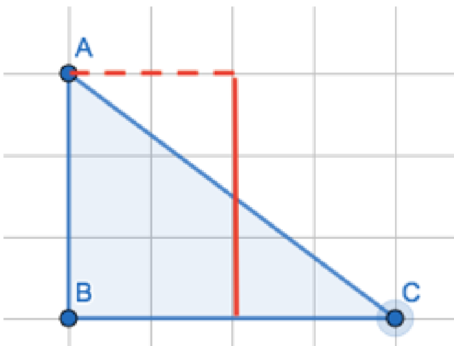


Figure 6.8. The student B's drawing altitude to the right angled triangle.

- Class: Again, it is not passing from the vertex.
- Teacher: Okay.
- Student C: I can.

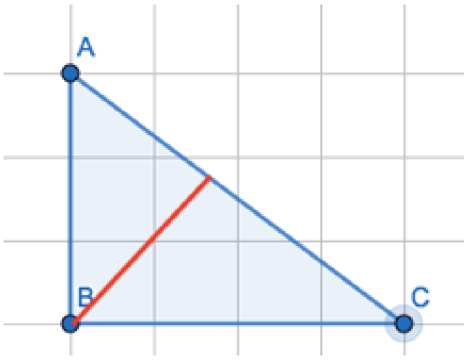


Figure 6.9. The student C's drawing altitude to the right angled triangle.

- Class: Not that side.
- Class: It was not asked to draw altitude the side AC.
- Teacher: Who else want to draw?
- Student D: Yes, me.

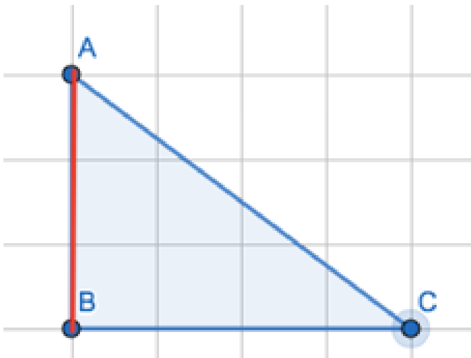


Figure 6.10. The student C's drawing altitude to the right angled triangle.

- Teacher: Is it correct?
- Student D: Yes.
- Teacher: How? Can you explain it?
- Student D: It is perpendicular.
- Class: It is starting from the opposite vertex.

In the dialogue, students shared their solutions with class. Students tried to explain their drawings with help of definition of the altitude. Especially, when students had misconceptions, they explained their ideas by using six critical aspects. With help of making acceptable explanations, students realized their misconceptions in the classroom. Students got an acceptable explanation of their errors and misconceptions. During the instruction, I promoted students to make acceptable explanations. I directed students to make explanations for their solutions by asking questions. Students were used to make acceptable explanations with using six critical aspects of the altitude as in the above dialogue.

- Being free to making mistakes:

Students in both classes mostly felt comfortable on making mistakes in the lessons. They could discuss on their mistakes and seek for different solutions after their mistakes. Students wanted to share his or her idea or solution with the class, reconceptualized the problem and produced different solutions at most in the environment with no technology, according to the Figure 6.4 and Figure 6.5. In the part of technology demonstration, there were less situations that students felt free for making mistakes. However, students rarely wanted to share their different solutions while students were using technology actively.

From the recordings of classroom and my teacher field notes, students shared their ideas without feeling fear of making mistakes during the instruction. However, sometimes, some students did not want to share their solutions with classroom when they had difficulty in drawing altitude to right and obtuse angled triangle. Students encouraged their friends as "Everyone can make a mistake, it's normal". The environmet of the classroom and the interactions among students were supportive. Students promoted their peers to share their ideas without being afraid of making mistakes. With this promotion, students shared their solutions freely. If they had mistakes, they learned the reason of incorrectness and their peers explained the correct solutions with acceptable explanations. Such an supportive enviroment helped students improve their understanding.

In the following example, students were learning to draw altitude to obtuse angled triangle with their miters. An obtuse angled triangle standing on the base was given in their activity sheets. I asked student to draw altitude to the given triangle individually. Some students asked questions related to their drawings. I tried to direct them reminding the six critical aspects of the concept of the altitude. I wanted them to share their solutions within their groups and check their solutions according to six critical aspects of altitude. I asked groups to share their solutions with the classroom. Many groups did not sure that their solutions were correct. They shared their solutions freely even though they learned to draw altitude to obtuse angled triangle firstly. The following dialogue was an example for the norms of being free to making mistakes.

Example: It is asked to draw altitude to given triangle.

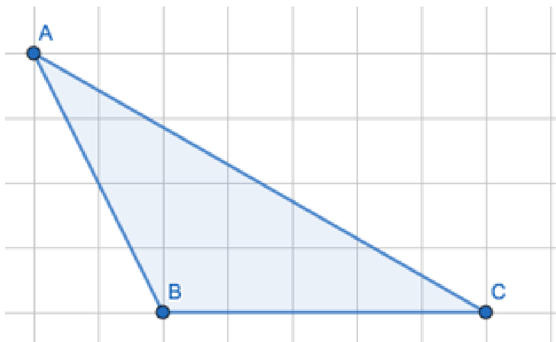


Figure 6.11. The student A's drawing altitude to the obtuse angled triangle.

- Teacher: How can we draw the altitude to given triangle?
- Student A: Yes, I can. (He is drawing the altitude as below)

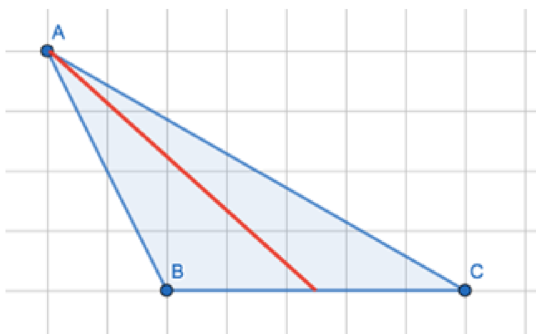


Figure 6.12. The student B's drawing altitude to the obtuse angled triangle.

- Student B: No teacher. It is not.
- Teacher: Why?
- Student B: It is not perpendicular.
- Teacher: Yes. The altitude should be perpendicular to the side. So, we cannot accept that it is altitude. Is there anyone who think in a different way?
- Student C: I have. (He is drawing the altitude as below)

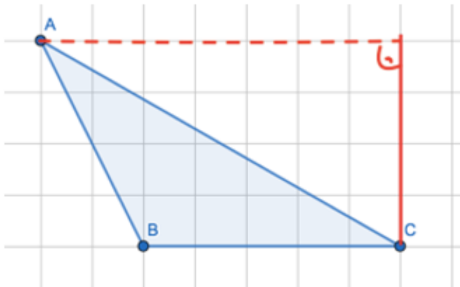


Figure 6.13. The student C's drawing altitude to the obtuse angled triangle.

- Teacher: Do you think that it is altitude?
- Class: No.
- Teacher: Why?
- Student D: It is not starting from the vertex.
- Teacher: Okay. Is there anyone who has a different idea?
- Student E: Can I draw?
- Teacher: Of course. (He is drawing the altitude as below)

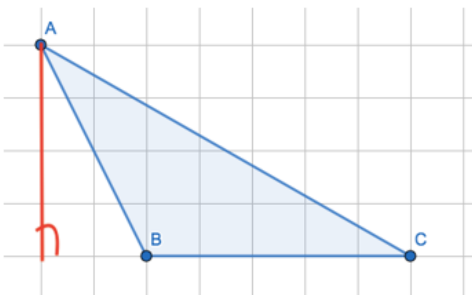


Figure 6.14. The student E's drawing altitude to the obtuse angled triangle.

- Teacher: Is it true?
- Student E: Yes, it is perpendicular and it is passing from vertex to the side.

In the dialogue, students reached the solution by error and trial method while they were drawing the altitude to obtuse angled triangle firstly. They gave acceptable explanations for wrong solutions. They tried to reconceptualize the problem by remembering the definition of the altitude if their drawings were not worked. Even though students did not draw the altitude correctly, they retried to draw the altitude without hesitation and tried different solutions to reach the correct one.

- Working collaboratively:

Students generally worked collaboratively in almost all lessons. They worked on their drawings and discussed on different solutions as a group. They tried to reach a result that every member of the group had mutual agreement. According to the Figure 6.4 and Figure 6.5, the frequency of working collaboratively in lessons with no technology is the highest. When students use technology actively during the learning process, working collaboratively is the lowest one. While students use technology actively, they were more willing to work individually.

From the recordings of classroom and my teacher field notes, students easily understood drawing the altitude to parallelogram. In the following example, I asked students to draw altitude to given parallelogram in their activiy sheets. They made their groups and most of them firstly drew altitudes individually. After they finishing their drawings, they shared their solutions within the group. Since there are many correct solutions of the questions, some groups could not decide which altitude were drawn correctly. Students wanted to reach mutual understanding and asked me to select the correct one. I tried to direct them to reach corect solutions by reminding the definition of the altitude. Students gave decision that both of the solution was correct at end of discussion. The following dialogue is an example of the norm of working collaboratively.

- Example: It is asked to draw altitude to given parallelogram. They worked on the given parallelogram as group. They cannot reach mutual agreement on their drawings.

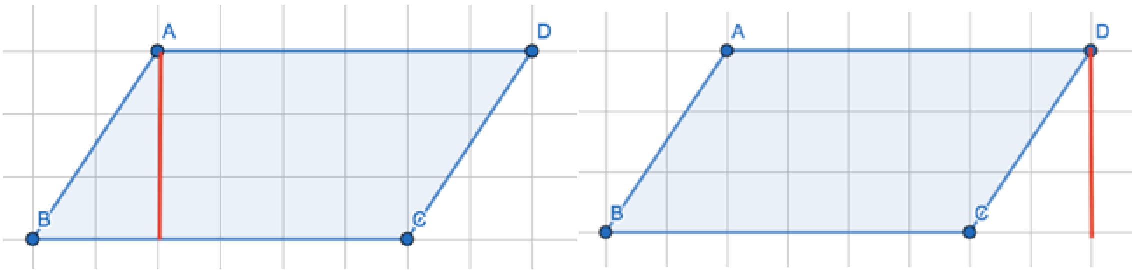


Figure 6.15. The group's drawing altitudes to parallelogram.

- Student A: I drew the altitude inside, but he drew the altitude outside.
- Student B: I did not understand it.
- Student A: Which one is correct? (or) both of them?
- Teacher: Let's draw.
- Student A: Both of them is 4 units.
- Student B: Both of them is perpendicular.
- Teacher: Perpendicular and same length.
- Student A: Both of them may be correct.
- Teacher: What is the definition of the altitude?
- Student A: From the vertices. It is a straight line.
- Teacher: Straight line?
- Student A: Moving down from the points in the parallelogram.
- Teacher: What is straight line? What might be saying when they said "straight line"?
- Class: Perpendicular.
- Teacher: Okay well. Which one might be "line, line segment, ray"?
- Class: Line segment.

In the example, students worked on the problem individually and as a group. They could not reach a mutual agreement on the solution process and they asked to reach the agreement. Students were directed to work on the problem again and try to understand the concept and procedure. By remembering the definition of altitude, they understood that both of them found correct solution even though their drawings were different from each other. During the process of learning, students took the responsibility within the group. They discussed on the different ideas and reached the agreement. They could work collaboratively during the instruction.

- Sharing solutions to the class:

According to the Figure 6.4 and Figure 6.5, similarly, students shared their solutions and ideas with the classroom at the most in the environment with no technology. Students solved questions or drew altitudes on the computer, but they shared their results with classmates more rarely than the environments with technology demonstration or with no technology.

I designed many tasks that student could draw altitudes on the paper with their hands. Students learned the concept of altitude by drawing altitudes to many types of triangles. After students were drawing individually or as a group, they shared their solutions with the classroom. During the process of sharing, students needed to explain their thinking. If students did not explain their solutions with an acceptable explanation, I asked questions to give explanations to the classroom or students demanded for acceptable explanations. Sharing of solutions were helpful for creating a discussion environment. Each student had opportunity to explain their ideas and asked questions to classroom.

In the following example, I asked students to draw altitude to side BC for a right angled triangle. Before this question, students worked on drawing altitude to the triangle with different rotation. Students made their groups and drew altitude to the triangle. When triangles were rotated, many students had difficulty in drawing altitude. Some students asked me if their altitudes were correct or not. I directed them to remember the definition of the altitude and to check their drawings with six critical aspects of the altitude.

After they completed their group working, students started to share their solutions with the classroom as in the below example.

- Example: It is asked to draw altitude to the BC side of the given triangle.

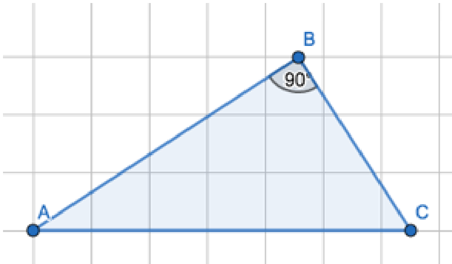


Figure 6.16. The acute angled triangle.

- Teacher: Who wants to draw altitude to the side BC?
- Student A : Yes, teacher. We drew like this. (They were drawing)

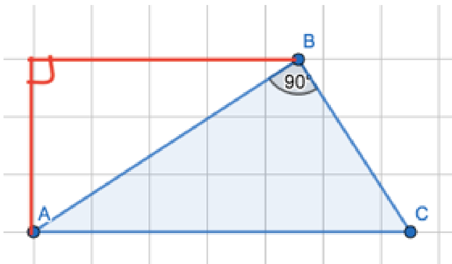


Figure 6.17. The student A's drawing altitude to the acute angled triangle.

- Teacher: Which side did you take as base?
- Student A: The side BC
- Teacher: Is it perpendicular to the side BC?
- Student A: Teacher actually we did not think like that. However, it was not dividing into two parts.
- Teacher: Do the altitude have to divide the side into two parts?
- Student A: ....
- Teacher: What was the definition of the altitude?
- Student A: From the vertex to the corresponding side (and) perpendicular.
- Teacher: Is it saying that the altitude divides the side into two parts?
- Student A: No.
- Teacher: It is so normal to think like way but the line segment you drew is not true. It is not passing through the side BC as being perpendicular. We can

measure the angle between the line segment (Student A drew) and the side BC. It is not 900. (They were measuring the angle and drawing the altitude to the side BC correctly.)

In the dialogue, students shared their idea and solutions in the classroom with no technology. Students were enthusiastic to share their own idea or the idea of the group. During the part of sharing, students could realize misconceptions related to altitude. Students who had similar misconceptions could discuss them with their peers. They could reach the correct solution by remembering the definition and critical aspects of the altitude.

From the literature review, there were studies which most of the students had misconceptions related to the concept of altitude. To reduce misconceptions and improve students' understanding of the concept of the altitude, I enriched my instruction with four sociomathematical norms and use of technology. As I planned the process of instruction with my lesson plan, students explained their ideas and solutions by using acceptable explanations. Students improved their concept image with dynamic tools and simulations during the instruction. I observed and took notes how students' understanding was improving. I focused on interactions and actions among students more. I was able to see the changes on the students' actions and interactions. I followed and observed how students were working collaboratively and explaining their ideas. They could express their understanding with the reasons. They could worked collaboratively and helped each other. Students shared their ideas without feeling fear of making mistakes.

After examining the students' answers in the pre-test and following students' learning, I reflected on my teaching more. I decided to allocate most of the instruction on drawing right and obtuse angled triangle. Since students needed to practice on drawing altitude to prototype and nonprototype examples more, I did revision on my lesson plans. With help of classroom recordings and teacher field notes, I observed and followed students? explanations in a detailed way. Some students knew the definition of altitude, however they could not draw altitude properly. Some students could draw altitude to triangles but they could support their thinking by using of six critical aspects of the altitude or the definition of the altitude. I directed and supported for improving both their drawings and explanations.

After I decided to implement this instruction, I became more systematic. I reflected on my teaching daily. After listening my lessons, I tried to select and design my questions more carefully. I thought on how I could ask right question. I tried to direct students by asking questions. I focused on interactions and actions among students more. I was able to see the changes on the students? actions and interactions. I followed and observed how students were working collaboratively and explaining their ideas. I considered the role of students in the group and class discussion more. With the help of recordings, I could followed more easily that every student could share their ideas and solutions freely in the classroom. Observing students' interactions and taking field notes on students understanding helped me improve my teaching.

6.3. Students Journals

The students' views about sociomathematical norms and technology was determined by students' journals. Students select one choice from each of four category "not done, partially done, done, always done". Scores of 0 to 3 were assigned to "not observed, partially observed, mostly observed, always observed" respectively.

There were totally 163 students' journals entries for four weeks. The frequencies and percentages related to each four categories for four journals were counted and examined in the below table. Except from one sociomathematical norm namely being free to making mistakes, it was found that the students who chose "always observed" was higher than the students who chose other scales. For the norm of being free to making mistakes, the number of students who chose "mostly observed" were higher.

Table 6.12. The frequencies of the views about norms and technology.

Norms and technology	N	Not observed		Partially observed		Mostly observed		Always observed	
		f	%	f	%	f	%	f	%
Working Collaboratively	163	21	12.88 %	15	9.20%	56	34.35 %	70	42.94 %
Sharing solutions with the classroom	163	12	7.36 %	20	12.26 %	43	26.38 %	87	53.37 %
Being free to making mistakes	163	19	11.65 %	23	14.11 %	66	40.49 %	54	33.12 %
Mathematical explanations	163	9	5.52 %	13	7.97 %	49	30.06 %	91	55.82 %
Use of technology	163	8	4.90 %	19	11.65 %	48	29.44 %	87	53.37 %
N: Number of journal entries, f: frequency of students' views, %: percentage of students' views									

Students expressed their ideas about sociomathematical norms and use of technology with open ended questions in the journals. However, most of them did not give detailed information on norms and use of technology and they mostly wrote one sentence related to each norm or technology. Most of the students assessed the use of GeoGebra in the mathematics lessons as in the following statements:

- "GeoGebra made the concept of altitude easier for me to understand it".
- "GeoGebra helped me to draw the altitude".
- "GeoGebra was very beneficial"
- "Dragging property of GeoGebra helped me understand altitude better".

Students wrote their reflection on use of technology. Students' experiences on use of technology was positive and they found use of technology beneficial for the concept of altitude. Students also reflected their ideas on four sociomathematical norms. They mostly made comments for the norms as in the following statements:

- "We shared our solutions with the class"
- "We worked as a group"
- "I worked with my friend"
- "Working together was really beneficial"

- “We explained our thinking”
- “I did not afraid of making mistake”
- “Everyone can make mistake. It is normal”.

Students preferred to express their ideas with short sentences. They tried to describe what they experienced about the sociomathematical norms in the classroom. Students’ experiences about the sociomathematical norms and use of technology positive. It can be said that reflections on the norms and use of technology was parallel with the results of the scale.

7. CONCLUSION AND DISCUSSION

For having five years of teaching experience in the middle and high school, I determined to study on a concept which students had difficulty in understanding and had misconception related to the concept. My purpose was to provide better conceptual understanding of the concept of the altitude with enriched tasks by sociomathematical norms and technology while they were learning the concept firstly. I collected and analyzed the data to gain the results about my study.

In this study, the results of Altitude Test showed that students improved their understanding of the concept of the altitude. The instruction was enriched with sociomathematical norms and supported with technology. In this enriched learning environment, students could share their solutions, discuss on the misconceptions and defend their ideas with acceptable explanations. Such an environment might help students understand the concept of the altitude better. These results were also parallel with students' journals. According to students' journals, students also realized the enriched environment and they experienced sociomathematical norms and use of technology positively for their learning environment. I expressed and discussed the results in this part.

7.1. Development of Instruction

Through the action research, I became more reflective teacher. I used the all quantitative and qualitative data that I collected to improve my teaching. I prepared the instructional sequences before implementing the instruction. I revised my lesson plans after implementing the pre-test. I gathered information about the prior knowledge or misconceptions related to the concept of altitude before starting the instruction. While I was collecting data during the instruction, I transcribed the classroom recordings and took notes about the lessons daily. I continuously studied on these recordings and notes to improve my teaching during the study. While I was transcribing the classroom recordings and writing the field notes, I reflected on asking more challenging questions to students. I realized that students engaged in sharing, explaining and discussing when I asked more challenging questions.

During the process of the study, I tried to become good listener in the classroom. I monitored students' actions and understandings. Students worked collaboratively with their peers. Each student in the group learned taking responsibility of explaining and sharing ideas. Students shared and discussed their ideas with the classroom. I guided students to reach mutual understanding and helped students create supportive environment. It could be stated that my classroom become more students centered during the instruction. I focused more interactions of the students in the classroom more. Students improved their understanding while sharing and discussing. The misconceptions and misunderstandings were discussed and students made acceptable explanations to defend their ideas. As students tried to make explanations on their drawings and solutions, they understood experienced how these interactions among peers helped students improve their understanding. I supported each student in the classroom to participate in working collaboratively. At the beginning of the study, some students did have unwilling to share their ideas. I tried to encourage students to think, explain and share their ideas and solutions during the instruction.

I implemented systematic way to solve the problem that students had difficulty in understanding the concept of altitude. I decided to design an effective instruction enriched with sociomathematical norms and supported technology to improve my teaching. This enriched instruction improved students' understanding and it was given me an opportunity to improve my teaching methods in the classroom. I will continue to implement such instruction to the other mathematics subjects and to the different grades.

7.2. Sociomathematical Norms and Technology

7.2.1. Sociomathematical Norms in the 6th Grade Geometry Class

One of the aims of the study was to analyze the students' conceptual understanding of the concept of the altitude before and after the instruction enriched with sociomathematical norms. Sociomathematical norms were established with students from the beginning of the academic year because it requires some time for a classroom (students and the teacher) to adapt sociomathematical norms. At the beginning of the academic year, I established four sociomathematical norms with students. The plan and design of the tasks were selected to promote these norms in the classroom.

In this study, I examined sociomathematical norms during the instruction on the concept of the altitude. The sociomathematical norms were analyzed from the classroom recordings and teacher field notes. Students interactions and dialogues showed how the sociomathematical norms were implemented in the classroom.

- Acceptable explanation

When the classroom recordings and teacher field notes were analyzed as in the Table 6.11, it was found that students used acceptable explanations for their solutions and actions. Students made acceptable explanations with six critical aspects of altitude in the classroom. They discussed if the shared solutions were drawn correctly or not and students reached this result by using the necessary part of the critical aspects. As Kazemi and Stipek (2001) found, students improved their understanding through mathematical explanations.

Cobb and Yackel (1996) analyzed sociomathematical norms in the second grade mathematics classroom. They found that students altered from participating explanations to making explanations. Students' explanations became more conceptual. Researchers reached that students started to judge what the acceptable explanation is. Similarly, in the beginning of the study, students could made interpretations about the problems and solutions without giving explanations with the reasons. However, they became learners who could explain their solutions and strategies by using six critical aspects of the altitude during the process of learning. Students started to need acceptable explanations to be convinced of the correctness of the solutions to the end of the study. As Cobb and Yackel (1996) found, students, in this current study, became learners who made acceptable explanations and were asked for acceptable explanations from their peers. The findings of Cobb and Yackel (1996) were parallel with the results of the current study.

- Being free to make mistakes

The classroom recordings and teacher notes (also the Table 6.11) showed that students felt comfortable in the classroom and felt free to making mistakes in the classroom. Students tried so many times to draw an altitude especially to obtuse and right angled triangles in the classroom. Although many students made mistake on drawing altitude correctly, they kept trying to reach solution. They discussed on their drawings and tried to explain their solutions with their reasons. Kazemi and Stipek (2001) found that the mistakes in group or class discussion encouraged students explain their ideas freely. During the process of learning, students had many misconceptions while students were drawing altitude to the obtuse and right angled triangle. Students understood why their drawings were wrong when acceptable explanations were made. Students had an opportunity to reconceptualize the concept with the help of mistakes. The results were parallel with Kazemi and Stipek (2001) who found that making errors also help students and teachers identify the misconceptions and provide an opportunity to build conceptual understanding.

- Working collaboratively

It could be said that students worked individually or collaboratively during the learning process from the Table 6.11. In the group working, every member in the group had the responsibility of thinking on the problem and searching for the answer individually. In the classroom activities, group members sometimes had different ideas for the solution and they discussed to reach the mutual agreement. Group members tried to convince each other by making explanations. Sometimes group members might find different solutions. They realized that both of the solutions were correct and there could be different correct solutions. As Cheval (2009) stated, working collaboratively provides supportive environment such that students find efficient or different solutions by discussing with their group members.

- Sharing solutions to the class

The Table 6.11 showed students shared their solutions with their group members and the classroom in all classes. Students explained their ideas and solutions freely with the classroom and other students in the classroom made interpretation about the solution. Students shared and discussed on the different solutions. Students found right solution by using the definition of the altitude. As Cobb and Yackel (1996) found, students had opportunity to reflect what they learned and so improved their understanding on the concept while they were sharing their solutions.

As the sociomathematical norms were analyzed from classroom recordings and teacher field notes, students' views about the experiences of the sociomathematical norms and technology were also analyzed by students' journals. As Levenson, Tirosh and Tsamir (2009) stated, students might understand that norms very differently although teacher and students build and develop norms together. Students might think and imagine in a different way. In this current study, their views about sociomathematical norms could be learned from students' journals. The results of journals showed that the students who chose "always observed" was higher than the students who chose other scales except from the norm of being free to make mistakes. Hopko at al. (2003) stated that students with mathematics anxiety make more mistakes in the mathematics classroom and students had higher anxiety when they made mistakes. Researchers (Hopko *et al.*, 2003) stated that this situation among failure and anxiety become a cycle. This process might cause that students feel fear of making mistakes. The students in this current study who selected the scale of "mostly observed" could not express their ideas freely in the classroom and had fear of making mistakes because of their mathematics anxiety. However, as a result, students had the realization of the sociomathematical norms and use of technology during the instruction. Their views about norms and technology reflected their learning positively as considering the teacher' field notes and classroom recordings. Teacher' field notes and classroom recordings corresponded with students' experiences about sociomathematical norms and technology.

Kozaklı and Akkoç (2015) reached the result that the property of dragging in GeoGebra might direct students to examine the reasons behind the mathematical phenomena and justify mathematical theorems and properties. Akyüz (2014) found that students needed to understand the conceptual ideas behind simply dragging. Students need to explain the ideas with reasons while working with technology. Akyüz (2014) stated that teachers need to promote students understand the mathematical ideas. In this current study, students explain, share and discuss their ideas and solutions while or after using technology. I directed students to think the mathematical deeply by asking questions. Students had an environment that they could understand the concept during the class discussion.

Kozaklı and Akkoç (2015) analyzed the social and sociomathematical norms in the technology integrated mathematics classroom. Researchers studied with pre-service mathematics teachers and the lesson mostly composed of geometry topics such as area of triangle, basic elements of triangle and circle. The participants selected sociomathematical norms according to their own lesson plans. Participants selected and constructed one norm as common which was the sociomathematical norms of "software is used for justification, proving and visualization.“ (Kozaklı and Akkoç, 2015 p. 92). After teachers used technology in their lessons, they even gave more importance to the sociomathematical norm of justifying their solutions in the environment with no technology. Kozaklı and Akkoç (2015) found that the sociomathematical norms of working collaboratively and sharing students' ideas improved in the environment of technology. Compared to the findings of Kozaklı and Akkoç (2015), for their study, students share their ideas and work collaboratively in the technology enhanced environment less than in no-technology environment. Students verified their solutions on GeoGebra and they did not need to make written or verbal explanations in addition to work on GeoGebra. Considering that Kozaklı and Akkoç (2015) studied in high school (9, 10 and 11th grades), students are required to produce proofs in written and verbal forms in the mathematics class. On the other hand, in the current study, it was expected that students worked collaboratively, shared their solutions to the classroom, discussed on their errors and make explanations on their solutions and strategies. The interactions were built between students and teacher or among students because of the selection of the sociomathematical norms. Students could also establish the interactions with the technological tool, but these interactions might cause the sociomathematical norms in the current study not to construct in the environment with technology as much as in the environment with no technology. It can be stated that medium affected the form of building interaction in the learning process.

Hershkowitz and Schwarz (1999) studied with middle and high school students to show the formation of sociomathematical norms while students were learning geometry, algebra, statistics and functions. Researchers enriched the environment by using open-ended problem-solving situations, collaborative working and computerized tools. Researchers found that sociomathematical norms did not only establish with the interaction between teachers and students, but norms could be also constructed in the results of manipulations on the dynamic geometry software, graphic calculators and spreadsheet. In this current study, the sociomathematical norm of working collaboratively and sharing solutions occurred more in the environment of no technology than the environment with technology. Students might not establish interactions with other class members in the environment with technology since students built interaction with technological tools as Hershkowitz and Schwarz (1999) found.

Kazemi and Stipek (2001) studied to analyze how 4^{th} and 5^{th} grade students understand the concept of addition of fractions conceptually in the environment enriched with sociomathematical norms. Researchers reached that sociomathematical norms created high press for conceptual understanding. These norms were making explanations, working collaboratively, making mistakes and understanding relations among strategies. Researchers found that students showed improvement on mathematical explanations including mathematical argumentations and reasoning when teachers promoted conceptual understanding. Students shared their strategies and discuss the similarities and differences among multiple examples in an environment that teacher put high press on conceptual understanding. Students were discussed inadequate and wrong strategies and solutions. They supported their actions with mathematical explanations. When they made mistakes on their solutions and explanations, students reconceptualized the problem and tried to find alternative solutions. While students were working collaboratively, each student was responsible for understanding group's solutions and was ready to explain and discuss the solution within the classroom.

Kazemi and Stipek (2001) studied on the concept of fraction, but sociomathematical norms showed similarities with this current study on teaching concept of altitude. In this current study, students made mathematical explanations consisted of six critical aspects of the concept while they were sharing their solutions or discussing on the problem. Students improved their understanding of the concept through the explanations. During the instruction, students worked collaboratively and shared their solutions with the classroom. They shared and discussed on the errors freely in the group or in the classroom. Making errors were an opportunity to reveal and eliminate the misconceptions. As Tatsis and Koleza (2008) found that norms affected the process of learning a mathematical concept, these sociomathematical norms help students improve their conceptual understanding. Also, in this current study, the pre and post-tests difference in the Altitude Test results showed improvement in students understanding of the concept. Thus, in the next section, the results of students' understanding of the concept as measured by Altitude Test were discussed.

7.3. Students' Understanding of Concept of Altitude

The aim of the study was to analyze the difference between the pre and post-tests. The results of Altitude Test showed that the instruction enriched with sociomathematical norms and use of technology improved students' understanding of the concept of altitude. In the National Turkish Elementary Mathematics Curriculum, the concept of altitude is introduced first time in 6^{th} grade (MoNE, 2017). However, students start to learn the concept of area in 4^{th} grade. Some students might have idea of the concept of altitude from the previous classes or they might gain the concept from different resources outside school. Thus, the pre-test was administered to analyze their previous knowledge about the concept of the altitude. After the instruction the post-test was also administered to compare the students' conceptual understanding of the altitude. The results of the pre-test showed that some students had previous knowledge about the altitude. The results of the post-test showed that students improved their understanding of the concept of the altitude. Indeed, there was a statistically significant difference between pre-test and post-test scores. It can be stated that the instruction created difference between pre and post-test. The results also were expected for 6^{th} graders after they learned the subject.

However, in the study of Fischbein and Nachlieli (1998), 93% of students in grades 9-11 were successful at drawing the altitude to the base of a triangle which includes inside of the triangle. In this regard, conceptual understanding of altitude is not limited to drawing altitude for some cases of triangles. As it was discussed by Guo and Pang (2011), conceptual understanding of altitude requires students to experience comparing varied multiple examples. Fischbein and Nachlieli (1998) found that 61% of students were not successful on drawing the altitude in a right-angled or obtuse triangle. In this current study, students improved their concept image and concept definition and try to eliminate the misconceptions related to the concept with the implemented instruction included practicing varied multiple examples. Thus, in the following sections, the results of concept definition-image and student misconceptions were discussed.

7.3.1. Concept Image and Concept Definition

Another aim of the instruction was to improve students' concept image and definition together to for the concept of the altitude. Vinner and Hershkowitz (1983) expressed that students can learn a concept better when they have concept image and definition together. Students may memorize and repeat the definition of the concept. Instruction was designed for students to construct the concept definition of the altitude with the help of six critical aspects of altitude. The concept definition was also developed with making acceptable explanations for their drawings and solutions. In the instruction, students were asked to practice on multiple examples including both nonprototype and prototype examples in order to improve concept image. Students identified and formed the concept of the altitude with the help of simulations.

The development of concept image and concept definition was analyzed by examining the drawings and explanations in the pre and post-tests. The Altitude Test included open ended questions to examine students' thinking. There were three types of answers for such open-ended questions. Some answers consist of only verbal explanations, while some had only images and there were also some answers with both words and images. The number of answers were analyzed descriptively and there were some differences between pre and post test results. Before students took the instruction for the concept of the altitude, the students mostly preferred to make verbal explanations in the pre-test. The results in the pre-test were parallel with Fischbein and Nachlieli (1998) who found that the percentage of students who drew altitude properly was quite lower than the students who defined verbally it correctly. However, in this current study, the number of students who explained with image increased from 37 to 218 in the post-test. After the instruction, the number of students (220) who explained with words were close to the number of students (218) who explained with image. By the elements of instruction, both concept image and definition were emphasized during the instruction. Students drew altitudes on the paper and on the GeoGebra. They manipulated the objects on the GeoGebra and the property of dragging helped students visualize and understand the concept of altitude. The students were also promoted to explain and discuss their solutions by using six critical aspects of the altitude during the instruction. Thus, students might show improvement on their explanations with words and image together.

Blanco (2001) studied with prospective primary teachers to analyze the difficulties and errors for the concept of altitudes on the series of the mathematical tasks. Researchers found that participants were successful at defining the altitude of triangle verbally while they were not drawing the altitude of the triangle in a correct way. On the other hand, Gürefe and Gültekin (2016) analyzed Turkish 8th grade students' knowledge about the concept of the altitude. Researchers found that most of the 8^{th}

grade students had difficulty in defining the altitude with words. Some students defined altitude visually. Even though there were no students defining the altitude as perpendicular line segment in verbal definition, visual definitions given by students were perpendicular line segment. Compared to Gürefe and Gültekin (2016), results of the post-test showed that many students defined the concept of altitude as perpendicular line segment. This difference between results of the studies can be explained by the instruction. During the instruction, the knowledge of "altitude is a line segment" were emphasized while students were making interpretations on other students' solutions and strategies. Six critical aspects of the altitude were emphasized in the classroom and students used six critical aspects on their explanations. The definition of the altitude and six critical aspects of the altitude also helped students reveal and eliminate their misconceptions. This process might help students improve their concept image and definition.

7.3.2. Misconceptions

Students from all levels from middle and high school had difficulty in understanding the concept of altitude and had misconceptions related to the concept (Gutierrez and Jaime, 1999). During the instruction, I promoted to share their ideas and solutions to reveal students' misconceptions and errors, as Olkun and Toluk (2004) suggested. I emphasized the possible misconceptions or errors. Nonprototype examples which most of the students had the misconceptions were emphasized and practiced more to avoid those misconceptions. Students discussed on nonprototype examples and explain the misconceptions with reasons based on the definition of the altitude.

There were six of misconceptions described by Gutierrez and Jaime (1999); altitude vs. median misconception, altitude vs. perpendicular bisector misconception, limitation to internal altitudes misconception, disregard of length misconception, fixation on side misconception and marked base as distracter misconception, which were used to analyze students' answers. There were also three more type of misconceptions which were emerged from the data, in other words not found in literature but found as a result of this study.

Misconceptions of altitude vs. median, altitude vs. perpendicular bisector and disregard of length decreased for the post-test. The least occurring misconception was altitude vs. perpendicular bisector, indeed Gutierrez and Jaime (1999) also found that the misconception of altitude vs. perpendicular bisector was the least frequent for preservice teachers. And, Gutierrez and Jaime (1999) reached that the misconception of altitude vs. median was the most frequent one. However, for current study, altitude vs. median misconception was the least frequent for one the post-test. The reason of the least occurrence might be that students did not learn the concepts of median and perpendicular bisector until 8th grade in the Turkish curriculum.

The misconception of disregard of length was one of the most frequent misconceptions in the pre-test of the current study while Gutierrez and Jaime (1999) found that this misconception was a frequent source of errors for their study with preservice teachers. After the instruction, in this current study, there were no students who had this misconception in the post-test. Before starting instruction, the instruction was revised according to the results of pre-test. To eliminate the misconception of disregard of length, "altitude is line segment" was emphasized during the instruction. Being line segment is one aspect of the concept definition of the altitude. While students were discussing on the altitude examples if the drawn is an altitude or not, students made their decision according to the definition of the concept. Students also made acceptable explanations based on the concept definition as they were explaining their ideas and sharing their solutions. The emphasis of the critical aspects of the altitude during explaining, sharing and discussing processes might help students eliminate the misconception.

On the other hand, some misconceptions occurrences were increased such as limitation to the internal altitude, fixation on the side and marked base as distracter. The occurrence of the misconception of limitation to the internal side had increase from pre to post-test at most. Guo and Pang (2011) expressed that students had difficulty in drawing altitude to the right and obtuse angled triangles. They expressed that students had tendency to draw the altitude to the inside of the triangle, since majority of students had the concept image of altitude only as "internal altitude" (Hershkowitz, 1989). Cunningham and Roberts (2010) studied with preservice elementary teachers to assess the conceptual understanding of geometric concepts. They found that participants did not success at drawing altitude to the obtuse angled triangle after 14-weeks-instruction. Similarly, in the current study, limitation to internal altitudes misconception was found as the most common error although the misconception was emphasized and practiced in the classroom. The reason might be the rotation of the triangles. As in the Table 6.8, students were more unsuccessful at drawing altitude of the rotated triangles such as the items 7, 9 and 14 in Altitude Test which are the rotation of the right angled and obtuse triangles. Fischbein and Nachlieli (1998) found that 93% of students in grades 9-11 were successful at drawing the altitude to the base of a triangle. When the triangles were rotated, they could not determine the altitude because of change of base. Determining the base and drawing altitude for the rotated triangles might be abstract for students.

It was found that students had three more misconception related to concept of the altitude: disregard of vertex misconception, the error of perpendicularity and the tendency of completing triangle. These misconceptions were not found from the literature but detected in the Altitude Test. For the disregard of vertex misconception, students could not draw altitude to the corresponding vertex. The number of students who had this misconception increased from 6 to 19 after they learned the concept of altitude. During the instruction, it was emphasized and practiced that the altitude is drawn to the outside of the obtuse angled triangles. Students who could not understand this concept properly did not consider the vertex and had wrong concept image. After the concept of the altitude, students continued to learn area of the triangle. During the instruction, some students stated that the length of the altitude was correct for finding the area of triangle even though they drew the altitude from extension of the vertex. It was explained to students that disregard of the vertex is a misconception because the altitude does not start with the extension of vertex. This explanation was discussed many times with the students during the instruction. However, it seemed that students did not consider the definition of altitude and six critical aspects of the altitude while drawing altitude from extending vertex.

There were small errors of perpendicularity while drawing altitude in both pre and post-test. These slightly tilted perpendicularities did not count as misconception in the Altitude Test. However, if it created a huge difference in perpendicularity, it called as “the error of perpendicularity. The answers including this misconception decreased after students took the instruction. Being perpendicular which was the one of six critical aspects was emphasized during the instruction. Students used it on their explanations and practiced drawing altitude on the paper or with technological tools. They understood and experienced the perpendicularity with different examples. The practices with sociomathematical norms in the classroom might help students understand the concept of perpendicularity. After the instruction, students could draw perpendicularities more precise.

In the pretest, students had the tendency of completing to a triangle while drawing perpendicular. There were no students who had this misconception in the post-test. This can be explained that if they do not know anything about it, they try to make it similar to what they know, triangles. They had tendency to completing a geometric shape from one point and a line. Due to the occurrence of this misconception in pre-test, I was able to be aware of it during the instruction. As classroom norms, students needed to share their solutions in the classroom and also provide acceptable solutions. When a student made a similar mistake, I asked him to explain why he drew triangle instead of just drawing the perpendicular line segment. He explained that completing triangle helped me draw perpendicular. The reason behind it might be that students had the information “altitude is related to the triangles” and “if in the question of drawing perpendicular to a line did not consisted of triangle, then a triangle must be created”. After the instruction, there were no students who had this misconception and the misconception was eliminated.

As a result, three more misconceptions were identified in student’s Altitude Test results: the misconceptions of the disregard of vertex, the error of perpendicularity and the tendency of completing triangle. As I explained before, the misconceptions were not found from the literature. It is significant that mathematics teachers are aware of that students also may have these misconceptions related to the concept of the altitude. These identified misconceptions might create an awareness among teachers so that teachers can design their lesson plan and organize the learning environment accordingly.

7.3.2.1. Misconceptions for prototype and nonprototype examples. The misconceptions in pre and post-test were also analyzed for prototype and nonprototype examples separately. According to the literature (Cunningham and Roberts, 2010; Fischbein and Nachlieli, 1998; Hershkowitz, 1989), researchers classified acute angled triangles as prototype examples and right angled and obtuse triangles as nonprototype examples. Cunningham and Roberts (2010) found that elementary mathematics pre-service teachers showed improvement on drawing altitude to prototypical triangles after the instruction. In this current study, all misconceptions except from marked base as distracter misconception and disregard of vertex misconception decreased for the prototype triangles. Moreover, the misconceptions of disregard of length and fixation on side were eliminated completely.

Fischbein and Nachlieli (1998) studied with students from high school and analyzed the concept of altitude after the instruction. Researchers (Fischbein and Nachlieli, 1998) reached for prototype examples that 26% of the students drew the median instead of altitude. 27% of the students drew altitude starting from different vertex to be inside of triangle. These students had the misconceptions of limitation to internal altitudes. 8% of the students had the misconception of altitude vs. perpendicular bisector. In this current study for the prototype examples, only one student from 46 students had the altitude vs median misconception. Two students could not eliminate the altitude vs. perpendicular bisector misconceptions. Six students had altitude vs. perpendicular bisector misconceptions. Students in this current study, showed more improvement on eliminating these misconceptions. The reason might be the difference on the instruction. In this current study, while students were sharing their solutions, the misconceptions revealed. Students had the opportunity to discuss on misconceptions and reach the correct results by using six critical aspects of the altitude and the definition of the concept. Their concept image constructed and improved by drawing altitudes both on the paper and on the GeoGebra. The instruction enriched with sociomathematical norms and supported with technology help students eliminate the misconceptions on the prototype examples.

Only three misconceptions for nonprototype examples were decreasing from pre-test to post-test, even though the majority of the misconceptions for prototype examples decreased after the instruction. Vinner and Hershkowitz (1983) found for nonprototype examples that only about 30% of the students in grades 6-8 drew the altitude of a right-angled or obtuse triangle correctly, even though they were provided with the definition of the concept. About 20% of the students could construct the altitude correctly without being provided with the definition. In this current study, the definition of the altitude was not given in pre and post-tests and students defined it by themselves. According to the Table 6.10, more than 5%0 of the students could draw the altitudes to most of the nonprototype triangles correctly. Similar to this current study, Yıldız, Güven and Koparan (2010) also found that all of the 8th grade students could draw the altitudes to the acute angled triangle, but only more than half of students could be successful at drawing altitude to right and obtuse angled triangles after using dynamic geometry. According to the findings, it could be reached that some students might still have difficulty in drawing altitude to nonprototype triangles after the instruction.

Orhan (2013) studied with 6^{th}, 7^{th} and 8^{th} grade students and analyzed their conceptual and procedural knowledge on the concept of the area. He found that 6^{th} graders had strongest procedural knowledge related to the concept of area while their conceptual knowledge related to concept of area was lowest. Orhan (2013) stated that the 6th grade students had difficulty in drawing altitude to obtuse triangle. However, 8th graders were the most successful grade on the conceptual test. Researcher explained the reason such that 6^{th} grade students learned the concept of area of triangle and parallelogram procedurally. Students had all the procedural knowledge in their minds, but they started to understand the concept more deeply in later grades. The results that Orhan (2013) might explain why some misconceptions for nonprototype examples did not decrease from pre-test to post-test in the current study. Since students in

6^{th} grade start to learn the concept of altitude based on the definition of the concept of the perpendicularity, the improvement on eliminating the misconceptions might be limited. In the further grades, as students learn the concept of area better, they might understand the concept of the altitude better. It might be expected that the students eliminate misconceptions completely after students increase their experiences on the concept of altitude in different geometric subjects such as auxiliary elements of triangle and area of triangle.

Hızarcı, Ada and Elmas (2006) studied with 230 preservice mathematics teachers and analyzed their definitions and drawings of the concept of angle, diagonal and altitude. Researchers found that most of the preservice mathematics teachers could not define the concept of altitude and could not draw the altitude to the right angled and obtuse angled triangles. Researchers explained the reason that most of the students had the image of an acute angled triangle standing on the base in their minds. Researchers also stated that students could not visualize the concept of altitude so that the side can be extended and the altitude is placed outside of triangle. Students might not have abstract thinking and they cannot draw the altitude to obtuse angled triangle outside of the triangle. In this current study, while students were learning to draw altitude to obtuse angled triangle, they called the extended side as “imaginary side”. They might have difficulty in imagining and visualizing altitude except from being inside. Actually, the labeling as “imaginary side” might explain that extending the corresponding side and drawing altitude outside of the shape might be abstract for 6th grade students.

8. LIMITATIONS AND SUGGESTIONS

In this action research, I aimed to analyze the effectiveness of instruction enriched with sociomathematical norms and supported with technology for improving 6^{th} grade students' conceptual understanding of the concept of altitude. Sociomathematical norms were examined with classroom audio and board recordings. I also kept field notes about my experiences related to sociomathematical norms in the classroom. Students expressed their views about sociomathematical norms in the journals. Students' conceptual understanding of the concept of the altitude was compared with the Altitude Test as pre and post-tests.

In this study, majority of the students showed improvement to eliminate their misconceptions from pre-test to post-test. However, some students could not eliminate all of the misconceptions completely. In the Turkish Mathematics Curriculum for grades 1-8 and 9-12, students continue to learn the concept of altitude with the geometry topics of auxiliary elements of triangle, area and volume. Students can improve their understanding for the concept of the altitude, as they keep learning the further concepts better in this enriched environment and eliminate the misconceptions completely in the further grades. Further studies could be designed so that students' learning of the understanding for the concept of the altitude. It can be extended in different geometric subjects and different grades.

According to the results of the study, it could be stated that the teaching based on tasks enriched with sociomathematical norms and technology is promising approach to eliminate the misconceptions. However, the misconceptions related to the concept of the altitude can be examined in more detailed way. The duration of the study can be extended such that students have more opportunity to reveal and eliminate their misconceptions related to the concept of altitude. Beside to time limitation, our findings are still limited to a sample of 48 students and for only the concept altitude. The results may show differences on other 6th grade students while learning other geometry subjects. More longitudinal studies could be designed with more than one subject so that understanding of the concept of the altitude could be analyzed more deeply in the environment enriched with sociomathematical norms.

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APPENDIX A: SAMPLE LESSON PLAN-1

Lesson Plan (40+40 minutes)

Objective: Students will be able to draw an altitude to the triangle.

At the end of the task, students will understand the following ideas;

(i) There are six critical aspects to draw altitude, namely vertex, perpendicularity, opposition, orientation, location and altitude-based-correspondence.

(ii) Altitude can be located inside or outside of triangle.

The principles of teaching altitudes are used in the following ways:

- Emphasis on six critical aspects
- Class discussions
- Technology

Guiding principles for teaching environment are used in the following ways:

- Sharing solutions to the class
- Working collaboratively
- Acceptable explanation
- Free to make mistakes
- Technology

Instructions to teacher

A.1. Introduction part: (20 minutes)

Teacher will want students to draw the altitude, which is asked in the questions, on the paper. Students will make predictions about the questions below:

(i) Write the definition of altitude.
(ii) Draw an altitude to lie inside the triangle. Explain what kind of a triangle this will occur.
(iii) Draw an altitude to lie on the one side of the triangle. Explain what kind of a triangle this will occur.
(iv) Draw an altitude to lie entirely outside the triangle. Explain what kind of a triangle this will occur.
(v) Given your responses to this question and what you've observed, complete the following sentence definition: An altitude of a triangle is...

After students complete the questions, teacher will create a short discussion on students' answers with students.

A.2. Main Body: (40 minutes)

Teacher will want students to open https://www.GeoGebra.org/m/q9d2yqyw
Teacher will divide the class into groups with two.
Teacher will give some time to examine and manipulate the triangle from the vertex.

Teacher will want them to answer questions below by letting students manipulate the triangle. Students will verify their answer by using GeoGebra and they will compare their answer with and without using GeoGebra. The questions are below;

(i) How can you get an altitude to lie inside the triangle by changing the place of the vertex?
  - Show what kind of a triangle this will occur on the GeoGebra.
  - Express if your prediction was correct before using GeoGebra.
(ii) How can you get an altitude to lie on the one side of the triangle by changing the place of the vertex?
  - Show what kind of a triangle this will occur on the GeoGebra.
  - Express if your prediction was correct before using GeoGebra.

(iii) How can you get an altitude to lie entirely outside the triangle by changing the place of the vertex?

- Show what kind of a triangle this will occur on the GeoGebra.
- Express if your prediction was correct before using GeoGebra.

(iv) Write the definition of altitude again according to your observation.

- An altitude of a triangle is...

(v) Are there any differences on the definitions before and after using GeoGebra? If so, explain what the reason would be.

(vi) Explain what kind of differences you observe before and after using GeoGebra. Please write the differences for each question from 1 to 3.

(vii) Do you think that GeoGebra are beneficial for you to make decision on the type of the triangle? If so, please explain which property of GeoGebra help you to decide?

Teacher will need to be careful if students explain their answers as it is asked. Teacher will direct them writing their explanations with their reasons. Teacher will remind them to support their answers by presenting reasons and observations.

During the lesson, teacher will express that students are free to make mistakes. Teacher will emphasize that they can answer the questions without hesitation. Since the teacher will create an environment such that students feel comfortable and free to make mistakes from the beginning of the year, students will answer the questions comfortably.

At the end of the task, teacher will want students to share solutions to the class. During process of sharing, teacher will stress that students should be respectful for each other. Each group can explain their answers to the class clearly. Even if students give some wrong answers, students can listen to the group to the end of sharing. After the sharing part, teacher wants them to explain the reasons of their thinking ways. Teacher will direct them to the right answer with some clues or with showing on GeoGebra.

A.3. Closure Part: (20 minutes)

Teacher will want them to reflect what they learn by answering the questions below;

(i) Write three things you found out.
(ii) Write two interesting for you.
(iii) Write one thing you still want to learn.

Teacher will create a discussion environment and promote students express their ideas on the reflections. Teacher will want students to explain their ideas on what they think on the activity.

APPENDIX B: SAMPLE LESSON PLAN-2

Lesson Plan (40+40 minutes)

Objective: Students will be able to draw an altitude to the triangle.

At the end of the task, students will understand the following ideas;

(i) There are three altitudes belong to each side in the triangle.

(ii) There are six critical aspects to draw altitude, namely vertex, perpendicularity, opposition, orientation, location and altitude-based-correspondence.

The principles of teaching altitudes are used in the following ways:

- Geometric constructions
- Emphasis on misconceptions
- Emphasis on six critical aspects
- Class discussions
- Technology

Guiding principles for teaching environment are used in the following ways:

- Sharing solutions to the class
- Working collaboratively
- Acceptable explanation
- Free to make mistakes
- Technology

Instructions to teacher

B.1. Introduction part: (20 minutes)

Teacher will open https://www.GeoGebra.org/m/NUgpgMMp and form the altitudes on the board. Teacher will manipulate one vertex such that students observe how altitudes change.

Teacher will want them to remember the definition of the altitude.

Teacher will ask "How many altitudes can be placed in the triangle?", "What happens when we change the place of the vertex?" and "How many altitudes are there even if you change the place of the vertex?"

Teacher will create a short discussion environment by asking such questions. While students are answering questions, teacher will want them to explain their ideas clearly and support their answers.

Teacher will divide the class into groups with two.

B.2. Main Body: (40 minutes)

Teacher will open https://www.GeoGebra.org/m/eMRYWGwm on the smartboard. Teacher will show how to draw the altitude for each side by using miter. Teacher will give information about how to use miter while drawing altitude for triangle.

Teacher will give the printed version of the triangle to students.

Teacher will want them to answer the questions below; (Students will have the questions on the activity sheet)

(i) Draw altitude from the vertex C to the given triangle ABC with using miter.
(ii) Draw altitude from the vertex B to the given triangle ABC with using miter.
(iii) Draw altitude from the vertex A to the given triangle ABC with using miter.

Students will answer these questions for the following each of the triangle as below.

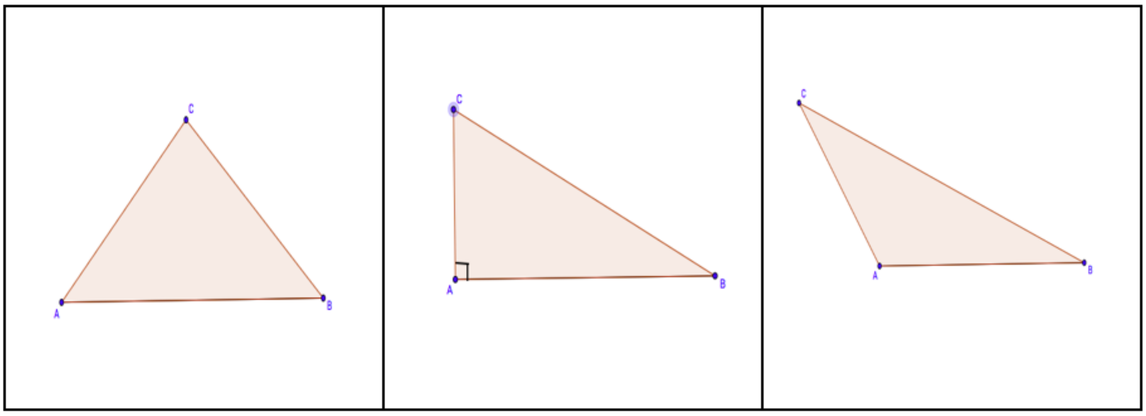


Figure B.1. The triangle ABC.

Teacher will want them to share their drawings to class and make comment to the other groups. Each group will explain how they locate the altitudes and what their reasons are. Teacher will promote students to give acceptable explanations related to the altitude they draw. During the process of learning, teacher will remind that students are free to make mistakes and so they can share their solutions without hesitation.

In the previous lesson, students do their construction by using GeoGebra. In this lesson, students can construct altitudes geometrically by using miter. Students will have opportunity to compare hand drawings and computer drawings.

B.3. Closure Part: (20 minutes)

Teacher will try to create a discussion environment and promote students talking about misconceptions on the drawings. Teacher will make students reveal their misconceptions by reminding the definition of altitude.

Teacher will give feedback to students' drawings at the end of the lesson.

APPENDIX C: ALTITUDE TEST YÜKSEKLİK TESTİ

1. Aşağıda verilen ifadeleri açıklayarak cevaplayınız.

(i) Bir üçgen ya da dörtgenin yüksekliğini nasıl tanımlarsınız?

(ii) Bir üçgenin toplam kaç tane yüksekliği vardır?

(iii) Bir üçgene yükseklik çizilmek isteniyor. Hangi üçgen çeşitlerinde yükseklik üçgenin içinde yer alacak şekilde çizilebilir? Açıklayınız.

(iv) Hangi üçgen çeşitlerinde yükseklik üçgenin dışında yer alacak şekilde çizilebilir? Açıklayınız.

(v) Bir üçgende yükseklik herhangi bir kenarının üzerinde olabilir mi? Açıklayınız.

(vi) Bir çokgende bir köşeden kenara doğru çizilen doğru parçasının yükseklik olup olmadığı nasıl anlaşılabilir? Açıklayınız.

2. Aşağıda verilen ifadelerden "Doğru" olduğunu düşündüklerinize D harfini, "Yanlış" olduğunu düşündüklerinize Y harfini yazınız. Doğru ya da yanlış bulduğunuz tüm ifadeler için neden doğru ya da yanlış olduğuna ilişkin alt kısımlarında verilen açıklama kısmına yazınız.

(i) [...] Üçgende yükseklik, üçgenin bir kenarından diğer kenara çizilen dik doğru parçasıdır.

Açıklama:

(ii) [...] Geniş açılı bir üçgenin yüksekliklerinden sadece bir tanesi üçgenin iç bölgesinde yer alır.

Açıklama:

(iii) [...] Üçgende yükseklik, üçgenin bir köşesinden karşısındaki kenarın orta noktasına çizilen dikmedir. Açıklama:

(iv) [...] Geometrik şekillerde yükseklik her zaman şekli iki eş parçaya ayırır. Açıklama:

(v) [...] Geometrik şekillerde herhangi bir kenar ile o kenara ait yükseklik arasında 90^olik bir açı oluşur.

Açıklama:

(vi) [...] Yükseklik bir doğru parçasıdır.

Açıklama:

(vii) [...] Üçgenlerde sadece 1 tane, dörtgenlerde ise sadece 2 tane yükseklik çizilebilir.

Açıklama:

(viii) [...] Herhangi bir üçgende 3 tane yükseklik çizilebilir.

Açıklama:

3. Aşağıdaki doğrulara, verilen noktalardan yükseklik çiziniz.

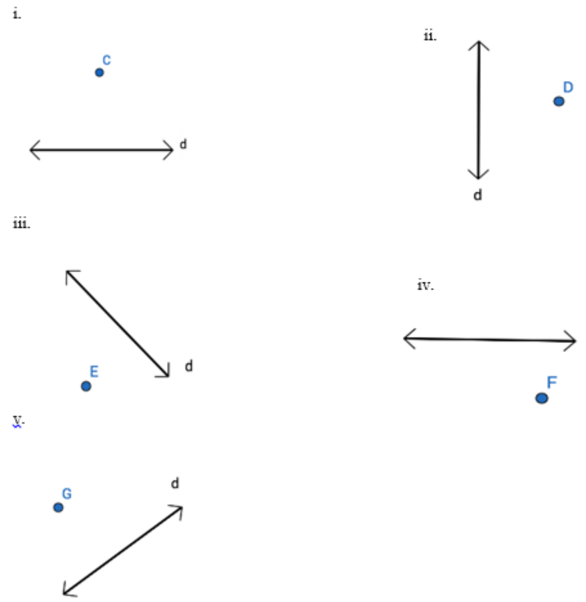


Figure C.1. Doğruya dışında verilen bir noktadan dikme çizebilme - Soru 3.

4. Aşağıdaki paralelkenarların belirtilen a kenarına ait yüksekliklerini çiziniz.

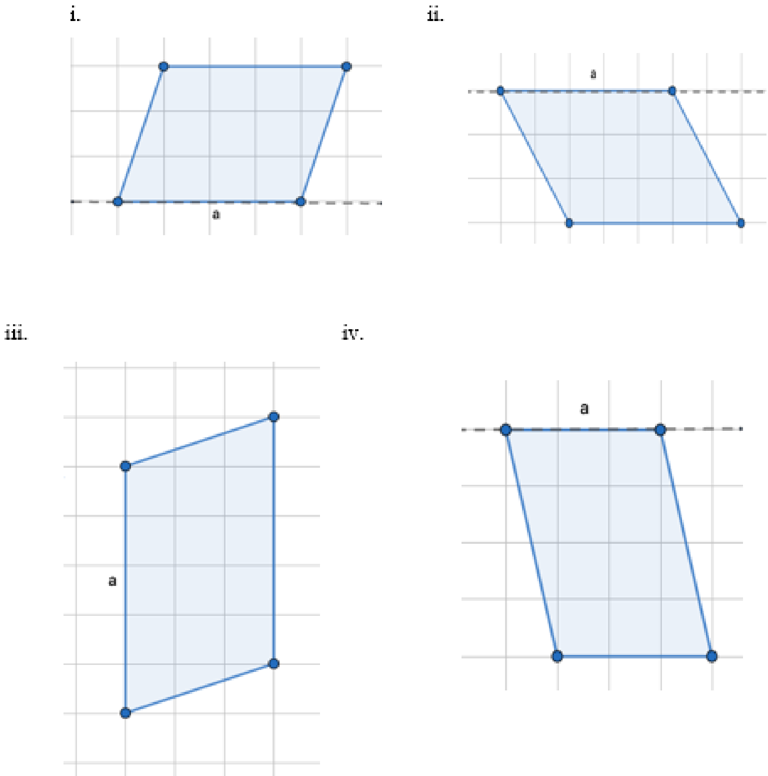


Figure C.2. Verilen paralelkenara yükseklik çizebilme ? Soru 4.

5. Aşağıdaki üçgenlerin belirtilen a kenarına ait üçgenin yüksekliğini çiziniz.

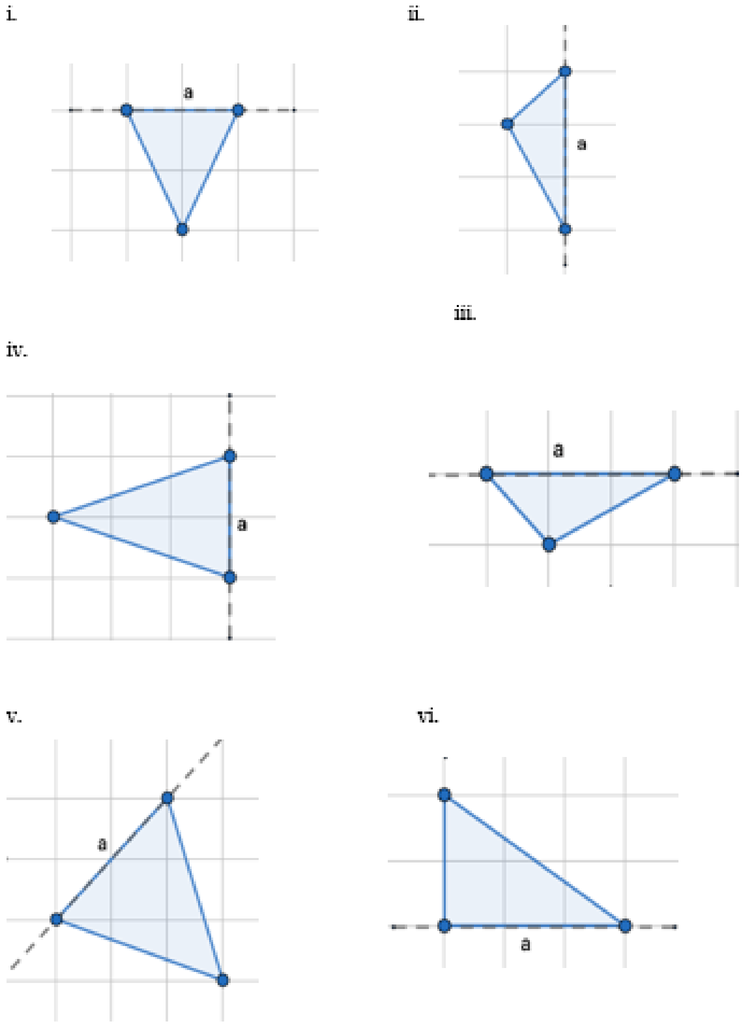


Figure C.3. Verilen üçgene yükseklik çizebilme (i, ii, iii, iv, v ve vi) - Soru 5.

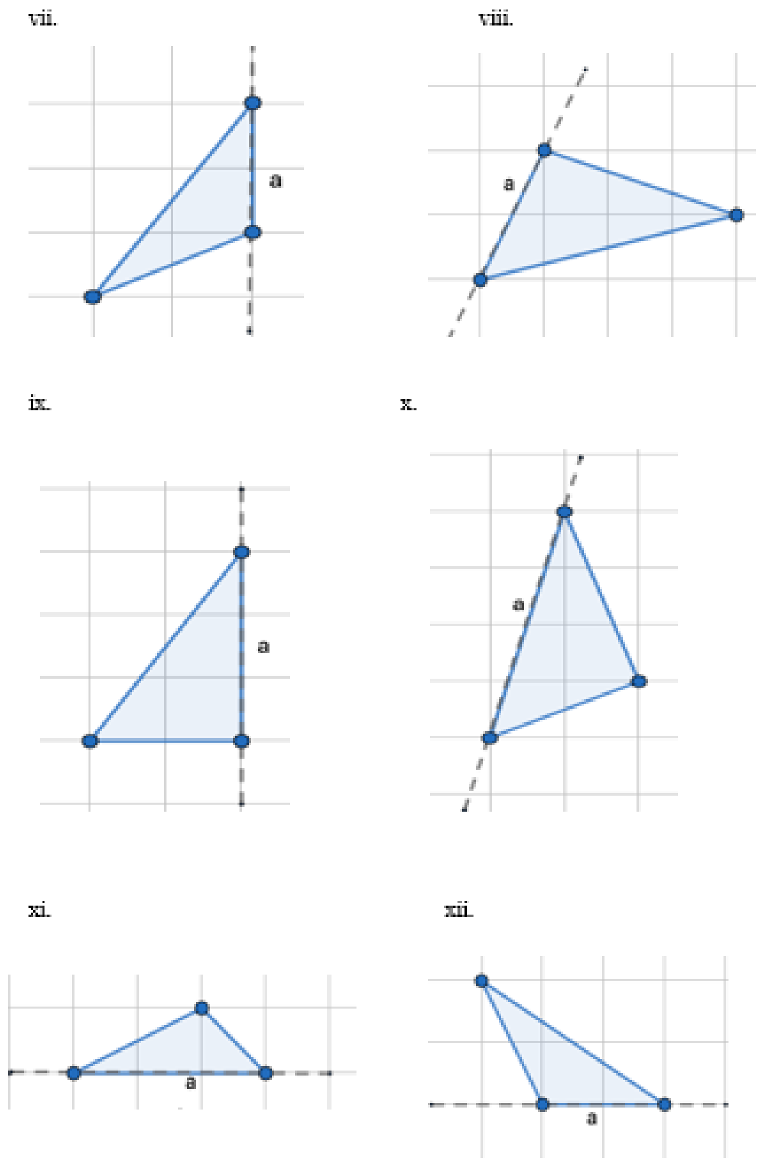


Figure C.4. Verilen üçgene yükseklik çizebilme (i, ii, iii, iv, v ve vi) - Soru 5a.

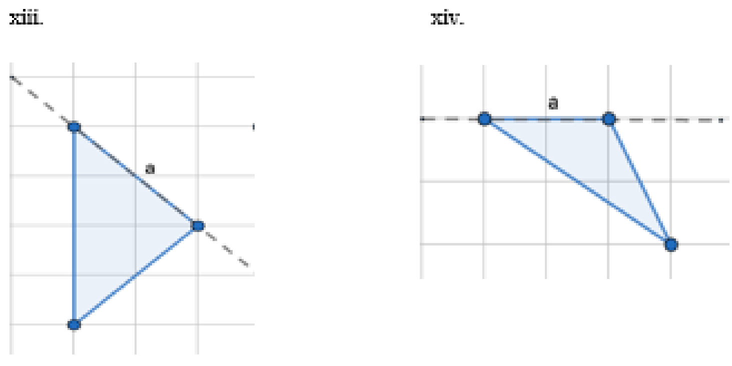


Figure C.5. Verilen üçgene yükseklik çizebilme (i, ii, iii, iv, v ve vi) - Soru 5b.

6. Aşağıda verilen paralelkenarların yüksekliklerini çizerek alanlarını hesaplayınız.

i. Alan: ....

ii. Alan: ....

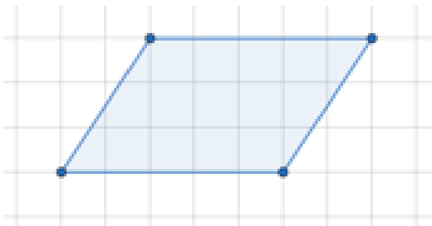
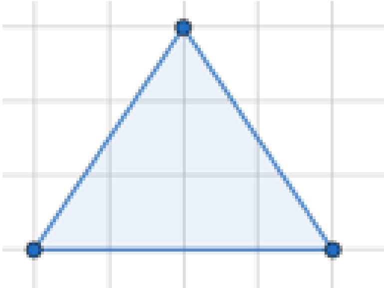


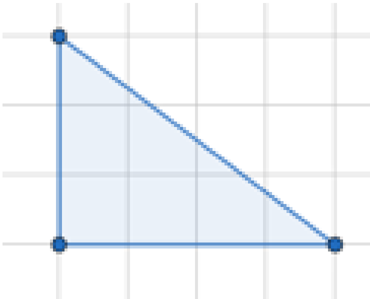
Figure C.6. Verilen paralelkenarın yüksekliğini çizerek alanını bulabilme - Soru 6.

7. Aşağıda verilen üçgenlerin yüksekliklerini çizerek alanlarını hesaplayınız. (– aralığını 1 birim olarak alınız.)

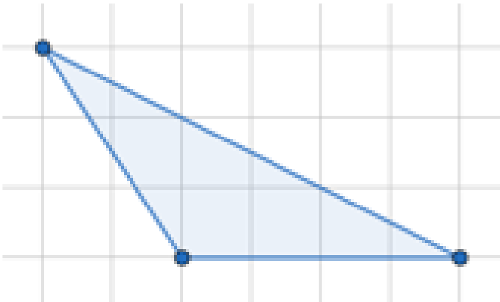
i. Alan: ....



ii. Alan: ....



i. Alan: ....



ii. Alan: ....

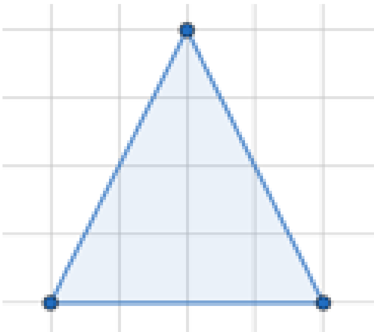


Figure C.7. Verilen üçgenin yüksekliğini çizerek alanını bulabilme - Soru 7.

APPENDIX D: RUBRIC OF ALTITUDE TEST

D.1. Yükseklik Testinin Puanlama Anahtari Yükseklik Testi

1. Aşağıda verilen ifadeleri açıklayarak cevaplayınız.

(i) Bir üçgen ya da dörtgenin yüksekliğini nasıl tanımlarsınız?

Cevap: Bir üçgenin herhangi bir köşesinden karşısındaki kenara veya uzantısına çizilen dikmenin, kenarı ya da uzantısını kestiği nokta ile bu köşeyi birleştiren doğru parçasına, o kenara ait yükseklik denir (MEB, 2016. s.80) (2 tam puan) Kenardan köşeye, diklik , doğru parçası (uzaklık ifadesi de kullanılabilir) ifadelerinin hepsi yer alıyorsa 2 tam puan, bir veya iki tanesi yer alıyorsa 1 puan alacaktır.

Bu ifadelerden eksik olan varsa ama şekil üstünde doğru gösterdiyse yine tam puan alacaktır.

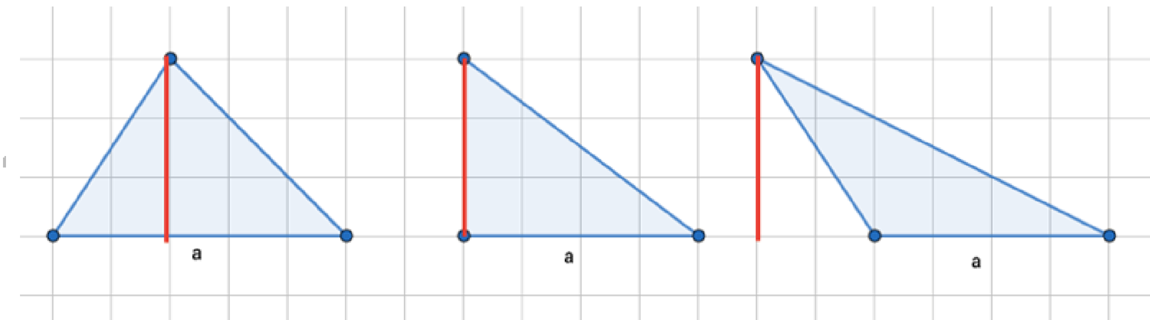


Figure D.1. Üçgen yüksekliği tanımı - Soru 1 (i).

(ii) Bir üçgenin toplam kaç tane yüksekliği vardır?

Cevap: 3 tane (1)
Açıklama: Bir üçgenin farklı tabanlarına ait 3 tane farklı yükseklik çizilebilir. Üçgenin 3 farkli kenarı olduğu için her kenara ait bir tane yükseklik çizilebilir. (1)
Ya da çizimle gösterebilir. (1)

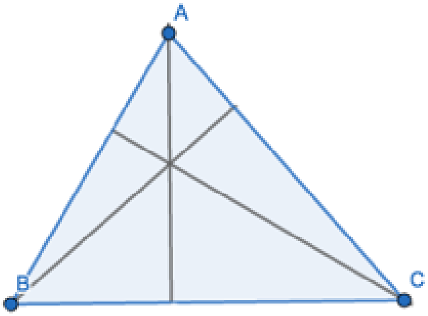


Figure D.2. Bir üçgende farklı kenarlara ait yükseklikler - Soru 1 (ii).

(iii) Bir üçgene yükseklik çizilmek isteniyor. Hangi üçgen çeşitlerinde yükseklik üçgenin içinde yer alacak şekilde çizilebilir? Açıklayınız.

Cevap: Dar Açılı Üçgen (1)

Açıklama: Üçgenin tüm açıları dar olduğu için kenarlara ait yükseklikler üçgenin içinde yer alır. (1)

Ya da çizimle gösterebilir.(1)

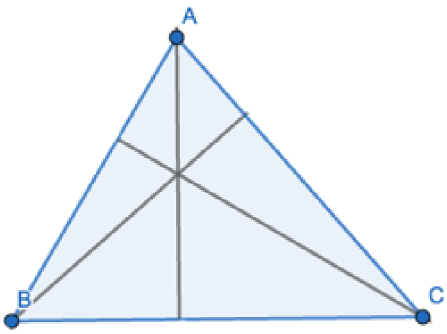


Figure D.3. Dar açılı üçgen - Soru 1 (iii).

(iv) Hangi üçgen çeşitlerinde yükseklik üçgenin dışında yer alacak şekilde çizilebilir? Açıklayınız.

Cevap: Geniş Açılı Üçgen (1)

Açıklama: Geniş açılı üçgende açılardan bir tanesi 90 dereceden büyük olduğu için, geniş açı kolları olan kenarlara ait yükseklikler üçgenin dışında yer alır. (1) Ya da çizimle gösterebilir.(1)

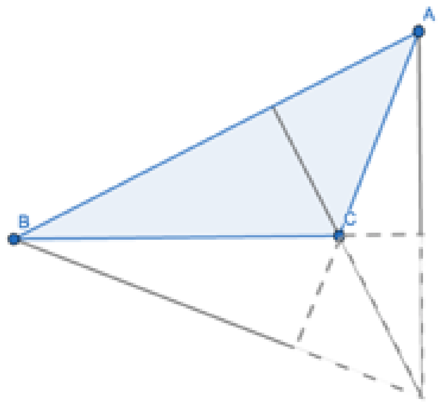


Figure D.4. Geniş açılı üçgen - Soru 1 (vi).

(v) Bir üçgende yükseklik herhangi bir kenarının üzerinde olabilir mi? Açıklayınız.

Cevap: Dik Açılı Üçgen (1)

Açıklama: Dik açılı üçgende açılardan bir tanesi 90 derece olduğu için, dik açının kolları olan kenarlara ait yükseklikler üçgenin kenarının üzerinde yer alır. (1) Ya da çizimle gösterebilir.(1)

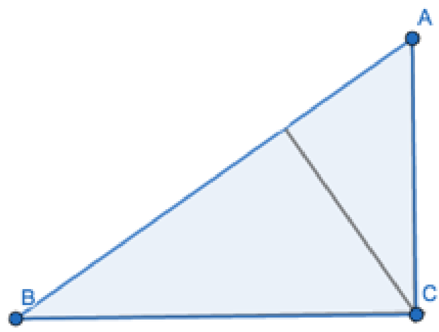


Figure D.5. Dik açılı üçgen - Soru 1 (v).

(vi) Bir çokgende bir köşeden kenara doğru çizilen doğru parçasının yükseklik olup olmadığı nasıl anlaşılabilir? Açıklayınız.

Cevap: Yükseklik ile kenar arasındaki açının 90 derece olup olmadığına bakılır. (2)

Ya da çizimle gösterebilir.(2)

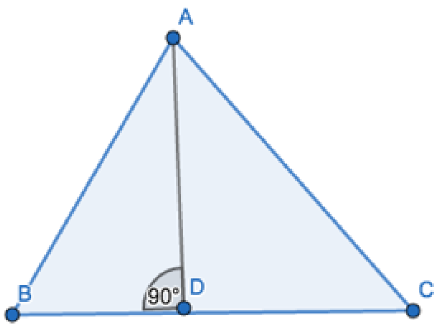


Figure D.6. Üçgende yükseklik çizimi - Soru 1 (vi).

2. Aşağıda verilen ifadelerden "Doğru" olduğunu düşündüklerinize D harfini, "Yanlış" olduğunu düşündüklerinize Y harfini yazınız. Doğru ya da yanlış bulduğunuz tüm ifadeler için neden doğru ya da yanlış olduğuna ilişkin alt kısımlarında verilen açıklama kısmına yazınız.

Doğrı yada yanlış cevabı doğru ise öğrenci 1 puan, açıklama da doğru ya da çizim ile gösterim doğru ise 1 puan alacaktır.

(i) [..Y..] Üçgende yükseklik, üçgenin bir kenarından diğer kenara çizilen dik doğru parçasıdır. (1)

Açıklama: Bir kenarından karşı köşesine çizilen dik doğru parçasıdır. (1)

(ii) [..D..] Geniş açılı bir üçgenin yüksekliklerinden sadece bir tanesi üçgenin iç bölgesinde yer alır. (1)

Açıklama: Geniş açılı üçgende açılardan bir tanesi 90 dereceden büyük olduğu için, geniş açı kolları olan kenarlara ait yükseklikler üçgenin dışında yer alır. (1)

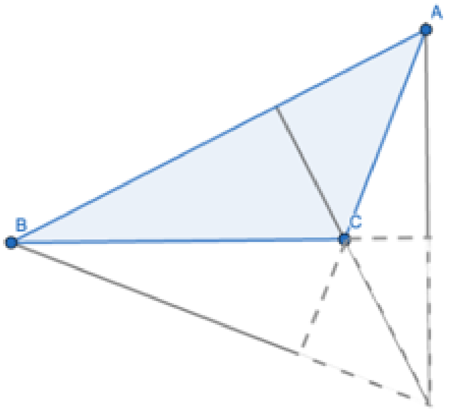


Figure D.7. Geniş açılı üçgende yüksekliklerin yeri - Soru 2 (ii).

(iii) [..Y..] Üçgende yükseklik, üçgenin bir köşesinden karşısındaki kenarın orta noktasına çizilen dikmedir. (1)

Açıklama: Yükseklik kenarı ortadan iki eşit parçaya bölmek zorunda değidir. (1)

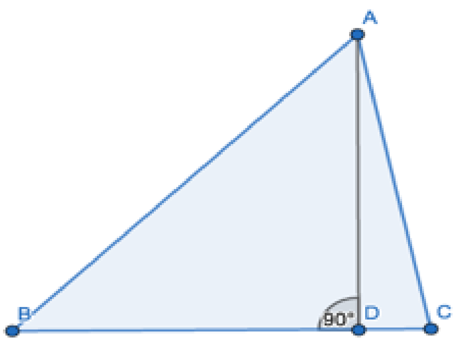


Figure D.8. Üçgenin bir kenarına ait yüksekliğin yeri - Soru 2 (iii).

(iv) [..Y..] Geometrik şekillerde yükseklik her zaman şekli iki eş parçaya ayırır. (1)

Açıklama: Yükseklik kenarı ortadan iki eşit parçaya bölmek zorunda değidir. (1)

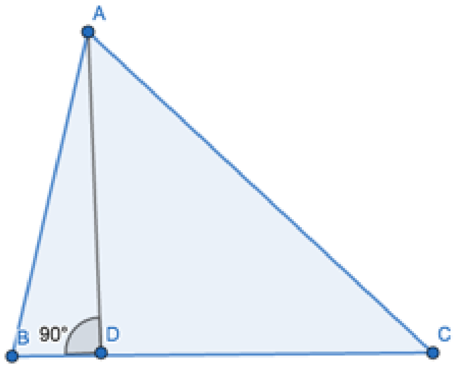


Figure D.9. Üçgenin bir kenarına ait yüksekliğinin kenar ile ilişkisi - Soru 2 (iv).

(v) [..D..] Geometrik şekillerde herhangi bir kenar ile o kenara ait yükseklik arasında 90^{o}lik bir açı oluşur. (1)

Açıklama: Yükseklik tanımı bir köşeden karşı kenarına inen dik doğru parçasıdır. Bu nedenle açı 90 derece olmalıdır. (1)

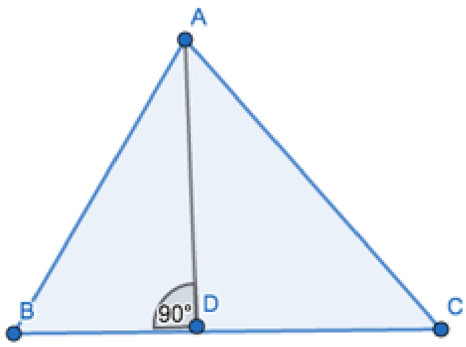


Figure D.10. Yüksekliğin bir dik doğru parçası olma durumu - Soru 2 (v).

(vi) [..D..] Yükseklik bir doğru parçasıdır. (1)

Açıklama: Yükseklik tanımı bir köşeden karşı kenarına inen dik doğru parçasıdır. Yüksekliğin başlandığı yer ve bittiği yer bellidir. (1)

(vii) [..Y..] Üçgenlerde sadece 1 tane, dörtgenlerde ise sadece 2 tane yükseklik çizilebilir. (1)

Açıklama: Üçgenlerde 3 farklı kenar olduğu için 3 farklı yükseklik, dörtgenlerde 4 farklı kenar olduğu için 4 farklı yükseklik çizilebilir. (1)

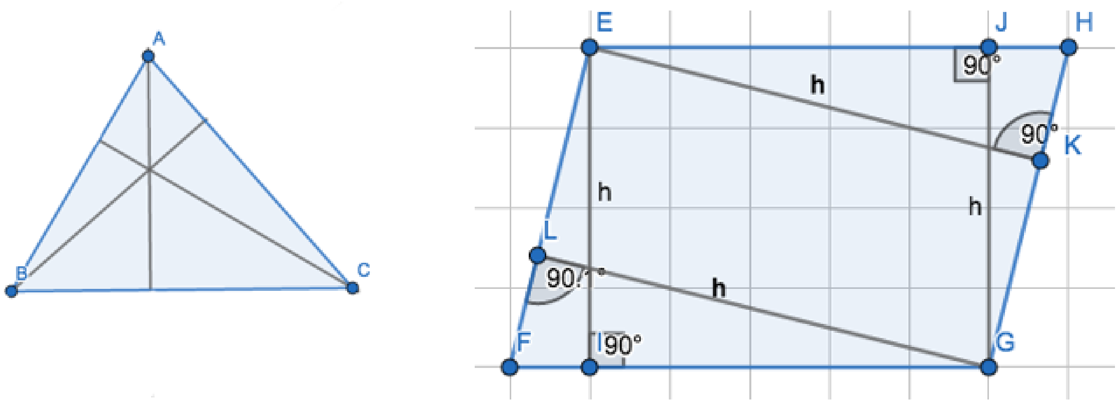


Figure D.11. Üçgen ve dörtgende farklı kenarlara ait yükseklikler - Soru 2 (vii).

(viii) [..D..] Herhangi bir üçgende 3 tane yükseklik çizilebilir. (1)

Açıklama: Üçgenlerde 3 farklı kenar olduğu için 3 farklı yükseklik çizilebilir. (1)

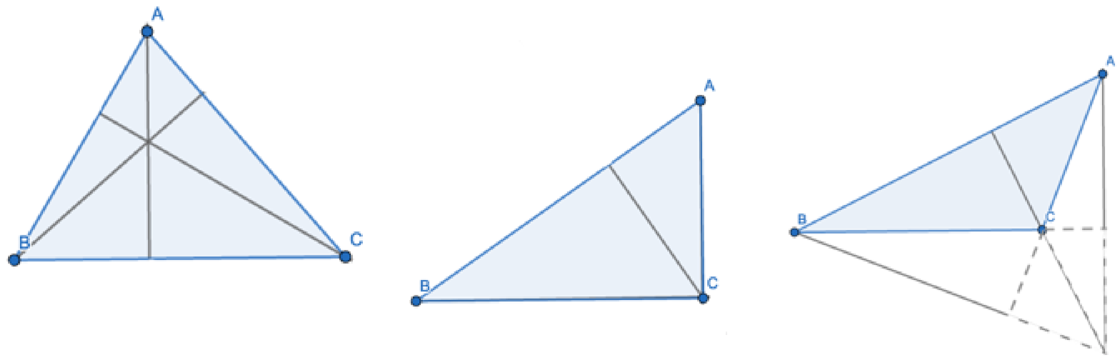


Figure D.12. Üçgende farklı kenarlara ait yükseklikler - Soru 2 (viii).

3. Aşağıdaki doğrulara, verilen noktalardan yükseklik çiziniz.

Diklik çiziminde öğrenciler geometrik araçlar kullanmadıkları için diklikteki küçük sapmalar dikkate alınmayacaktır. Her şık 2 tam puandır.

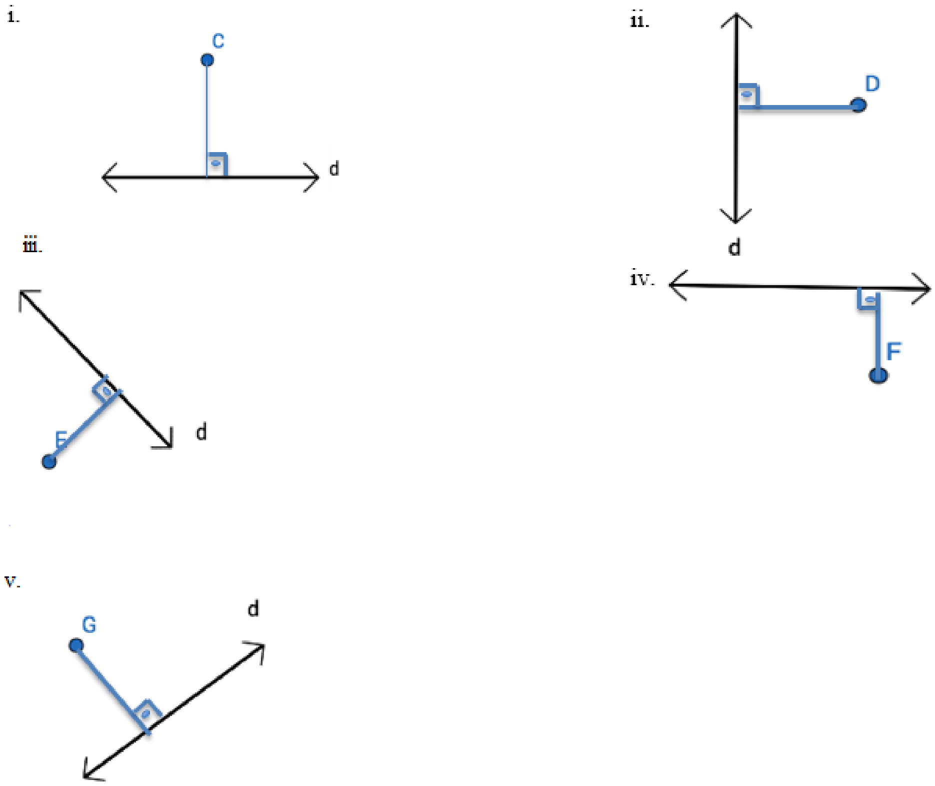


Figure D.13. Doğruya dışında verilen bir noktadan dikme çizimleri - Soru 3.

4. Aşağıdaki paralelkenarların belirtilen a kenarına ait yüksekliklerini çiziniz. Diklik çiziminde öğrenciler geometrik araçlar kullanmadıkları için diklikteki küçük sapmalar dikkate alınmayacaktır. Her şık 2 tam puandır.

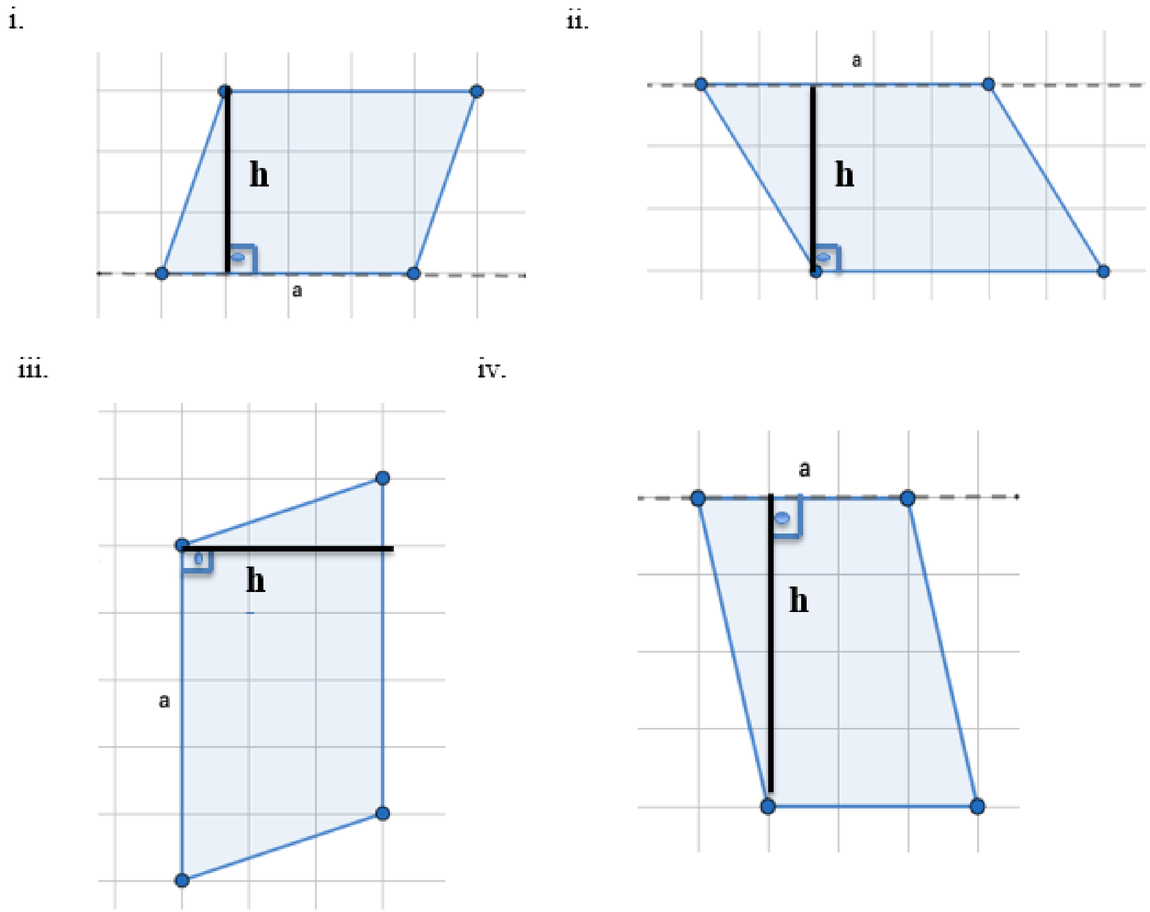


Figure D.14. Paralelkenara ait yükseklik çizimleri - Soru 4.

5. Aşağıdaki üçgenlerin belirtilen a kenarına ait üçgenin yüksekliğini çiziniz. Diklik çiziminde öğrenciler geometrik araçlar kullanmadıkları için diklikteki küçük sapmalar dikkate alınmayacaktır. Her şık 2 tam puandır

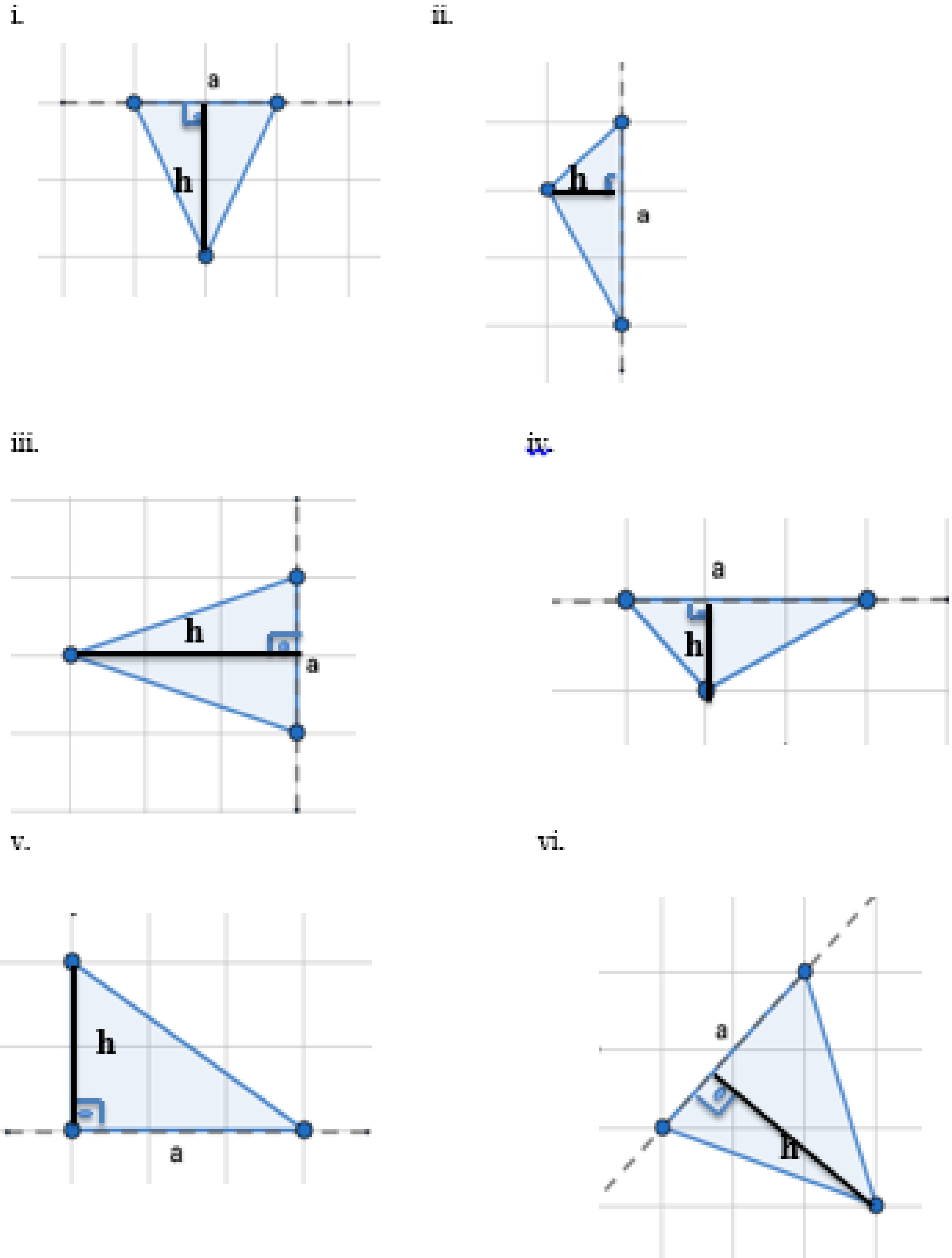


Figure D.15. Üçgenlere ait yükseklik çizimleri - Soru 5 (i, ii, iii, iv, v ve vi).

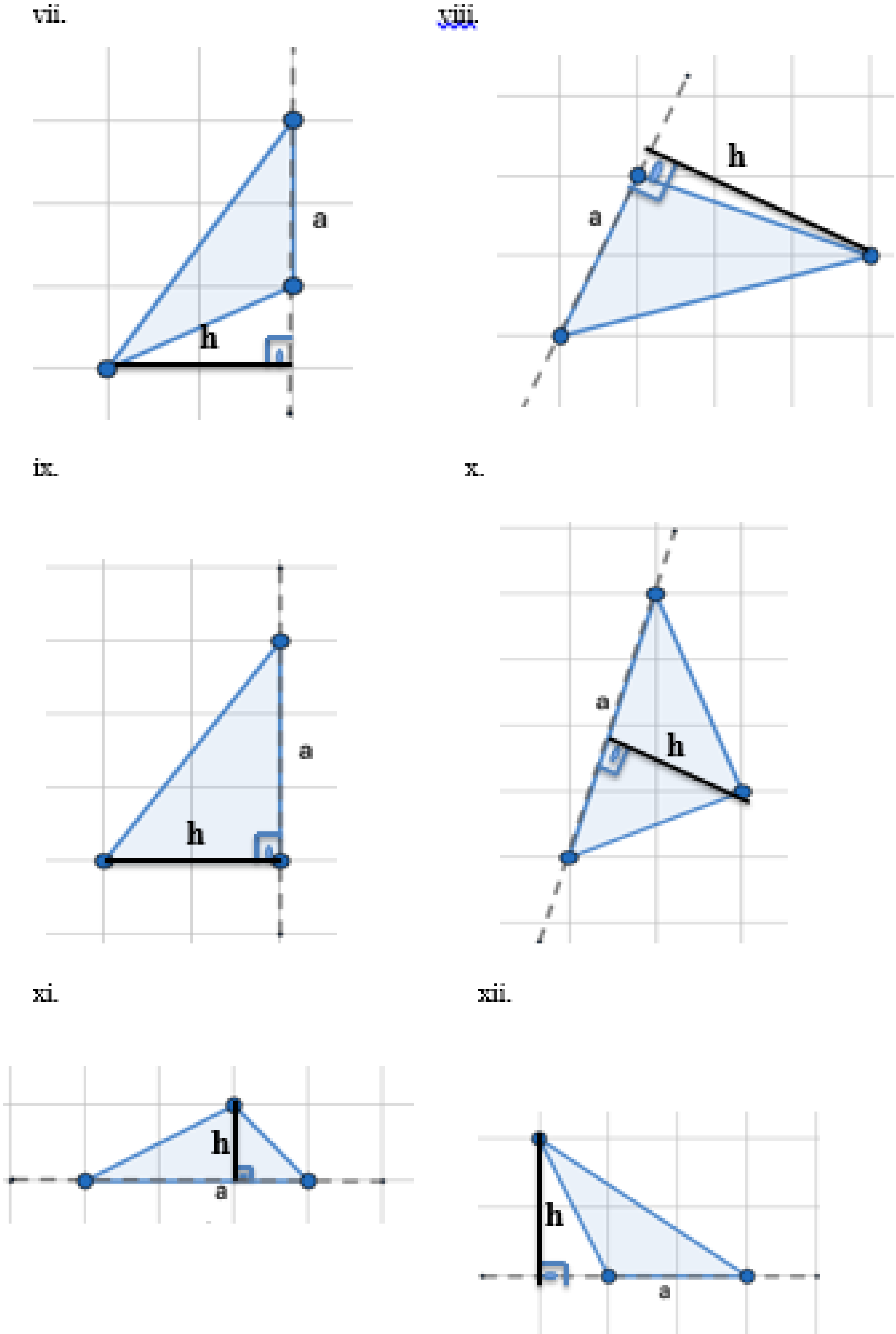
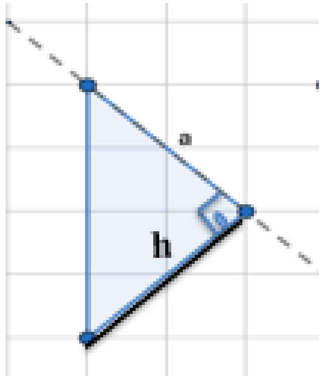
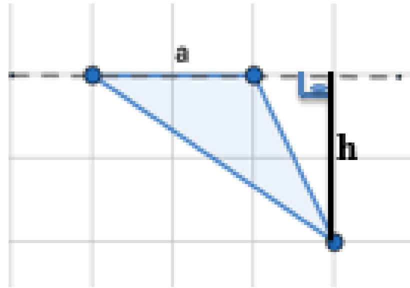


Figure D.16. Üçgenlere ait yükseklik çizimleri - Soru 5 (vii, viii, ix, x, xi ve xii).

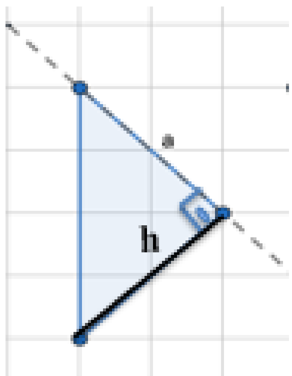
xiii.



xiv.



xv.



xiv.

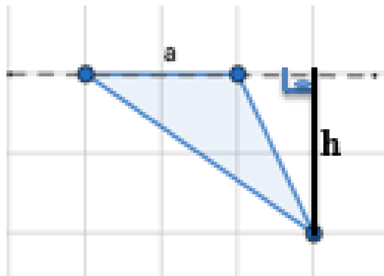
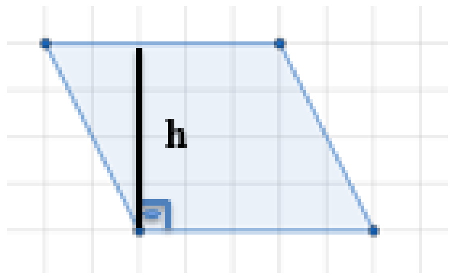


Figure D.17. Üçgenlere ait yükseklik çizimleri - Soru 5 (xiii ve xiv).

6. Aşağıda verilen paralelkenarların yüksekliklerini çizerek alanlarını hesaplayınız. (– aralığını 1 birim olarak alınız.) Yükseklik uzunluğunu doğru olarak bulunmuş ise 1 puan, paralelkenarın alanı doğru olarak bulunmuş ise 1 puan daha verilecektir.

i.



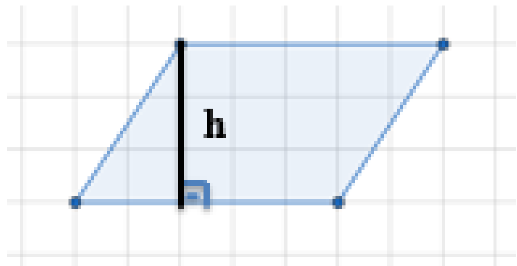
Alan: ....

h = 4 birim (1)

taban = 5 birim

Alan = 4 x 5 = 20 birim^2 (1)

ii.



Alan: ....

h = 3 birim (1)

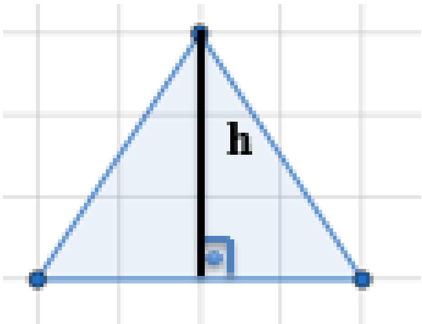
taban = 5 birim

Alan = 5x 3 = 15 birim^2 (1)

Figure D.18. Verilen paralelkenarların yüksekliklerinin çizimleri ve alanlarının çözümleri - Soru 6.

7. Aşağıda verilen üçgenlerin yüksekliklerini çizerek alanlarını hesaplayınız. (− aralığını 1 birim olarak alınız.) Yükseklik uzunluğunu doğru olarak bulunmuş ise 1 puan, üçgenin alanı doğru olarak bulunmuş ise 1 puan daha verilecektir.

i.



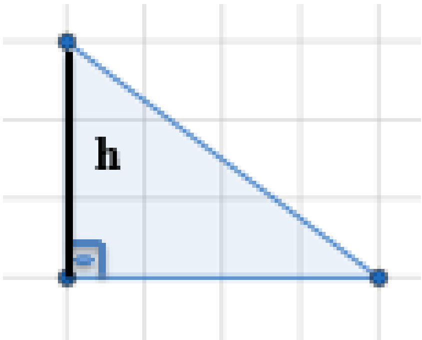
Alan: ....

h = 3 birim (1)

taban = 4 birim

$$\text{Alan} = \frac{4 \ x \ 3}{2} = 6 \text{ birim}^2 \ (1)$$

iii.



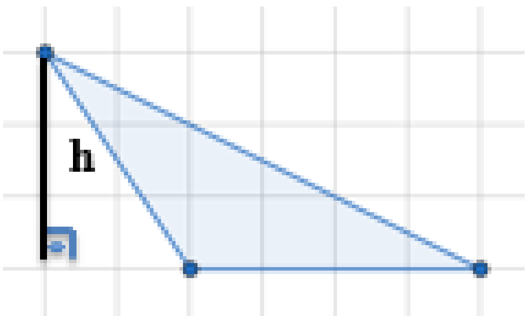
Alan: ....

h = 3 birim (1)

taban = 4 birim

$$\text{Alan} = \frac{4 \ x \ 3}{2} = 6 \text{ birim}^2 \ (1)$$

iv.



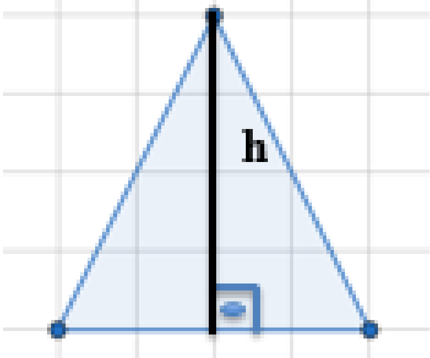
Alan: ....

h = 3 birim (1)

taban = 4 birim

$$\text{Alan} = \frac{4 \ x \ 3}{2} = 6 \text{ birim}^2 \ (1)$$

v.



Alan: ....

h = 4 birim (1)

taban = 4 birim

$$\text{Alan} = \frac{4 \ x \ 4}{2} = 8 \text{ birim}^2 \ (1)$$

Figure D.19. Verilen üçgenlerin yüksekliklerinin çizimleri ve alanlarının çözümleri - Soru 7.

APPENDIX E: JOURNAL QUESTIONS DEĞERLENDİRME

1. Aşağıdaki faktörler sınıfta ne derece uygulandığını (x) koyarak belirtiniz.

Table E.1. Faktörlerin sınıflardırılması.

	yapılmadı	az yapıldı	yapıldı	her zaman yapıldı
Birlikte paylaşarak öğrenme				
Sınıfta yapılan çözümlerin sınıfla paylaşımı				
Hata yapmaktan çekinmeden düşüncelerini ifade edebilme				
Sınıfta yapılan matematiksel açıklamalar				
Derste teknoloji kullanımı				

2. Bu haftaki matematik derslerinde yukarıda derecelendirdiğiniz faktörlerin sınıf içinde nasıl uygulandığını ve konuları öğrenmende nasıl bir etkisi olduğunu açılayınız.

(i) Grupça birlikte paylaşarak öğrenme:

(ii) Sınıfta yapılan çözümlerin sınıfla paylaşımı:

(iii) Yanlış açıklamalar veya yanlış çözümler:

(iv) Matematiksel açıklamalar:

(v) Konuyu öğrenirken teknoloji kullanımı:

3. Bu konuların dışında matematik dersi ile ilgili eklemek istediğin bir şey var mı?

APPENDIX F: SAMPLE OF AN ACTIVITY SHEET

1. Bölüm:

(i) Yüksekliği üçgenin iç bölgesinde yer alan bir üçgen çiziniz. Çizdiğiniz üçgenin nasıl bir üçgen olduğunu açıklayınız.
(ii) Yüksekliği üçgenin kenarlarından birisinin üzerinde yer alacak bir üçgen çiziniz. Çizdiğiniz üçgenin nasıl bir üçgen olduğunu açıklayınız.
(iii) Yüksekliği üçgenin tamamen dış bölgesinde yer alan bir üçgen çiziniz. Çizdiğiniz üçgenin nasıl bir üçgen olduğunu açıklayınız.
(iv) Yaptığınız çizimler sonucunda bir üçgenin yüksekliğini tekrar tanımlayınız. Bir üçgenin yüksekliği...

2. Bölüm: https://www.GeoGebra.org/m/q9d2yqyw linki ile verilen GeoGebra etkinlik sayfasını açınız. Verilen GeoGebra etkinlik sayfasını kullanarak aşağıdaki soruları cevaplayınız.

(i) Tüm yükseklikleri üçgenin içinde yer alan bir üçgen oluşturunuz.
  - Oluşturduğunuz üçgenin nasıl bir üçgen olduğunu açıklayınız.
  - GeoGebra kullanmadan önce yaptığınız tahminin doğru olup olmadığını açıklayınız.

(ii) Yüksekliği üçgenin kenarı üzerinde yer alan bir üçgen oluşturunuz.
  - Oluşturduğunuz üçgenin nasıl bir üçgen olduğunu açıklayınız.
  - GeoGebra kullanmadan önce yaptığınız tahminin doğru olup olmadığını açıklayınız.

(iii) Yüksekliği üçgenin tamamen dışında yer alan bir üçgen oluşturunuz.
  - Oluşturduğunuz üçgenin nasıl bir üçgen olduğunu açıklayınız.
  - GeoGebra kullanmadan önce yaptığınız tahminin doğru olup olmadığını açıklayınız.

(iv) Yaptığınız çizimler sonucunda bir üçgenin yüksekliğini tanımlayınız. Bir üçgenin yüksekliği...

(v) GeoGebra kullanmadan önce ve GeoGebra kullandıktan sonra oluşturduğunuz yükseklik tanımları arasında farklılık var mı? Eğer varsa, nedeniyle birlikte açıklayınız.

(vi) GeoGebra kullanmadan önce ve GeoGebra kullandıktan sonra 1. 2. ve 3. sorulara verdiğiniz cevapları karşılaştırınız. Gözlemlediğiniz farklılıkları her bir soru için ayrı ayrı açıklayınız.

(vii) GeoGebra kullanımının üçgenlerin çeşidini belirlemede nasıl etkisi olduğunu belirtiniz. GeoGebra'nın hangi özelliği üçgenlerin çeşidini belirlemede yardımcı olduğunu yazınız.