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Optimization in inventory-routing problem with planned transshipment: A case study in the retail industry

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A R T I C L E I N F O

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A B S T R A C T

Logistics is a science widely recognized as a value-added component for organizations and their products and services as it optimizes the use of resources, minimizes costs and maximizes level of service by coordinating activities such as transportation, inventory management and material handling. In traditional systems, transportation streams from one echelon of the supply chain to the next. Systems that are more flexible allow transshipment movements within an echelon, enabling the sharing of inventories among wholesalers and consequently reducing inventories and handling costs without changing the service level. This paper introduces a multi-period, multi-product Inventory-Routing Problem with planned Transshipment (IRPT). The novelty with regard to existing approaches is that transshipment movements are performed by the same vehicles that are distributing from the factory, meaning that they are subjected to the same capacity, time and cost restrictions. We present an exact formulation and develop a metaheuristic for the problem. The methods are applied to a real large-scale Brazilian retail industry producing a reduction of 9% in logistics costs and 70% in inventory level compared to the existing operation.

1. Introduction

Logistic systems are seen as essential tools to grant competitiveness and efficiency to companies in order to keep a sustainable business and reach global scale. Their main goal is to coordinate activities, such as transportation, warehousing, order processing, administration and maintenance of stock, also designated as inventory management. Along with service level, total logistic cost that derives from these activities has become one of the most important economic indicator for efficiency of a supply chain (Zeng and Rossetti, 2003).

In 2014, logistic costs represented 11.2% of the Brazilian companies' revenue, where, in general, inventory and transportation were the two biggest cost components (Resende and Sousa, 2014). That explains why companies and academic researchers are putting so much effort looking for efficient and economic combined management systems for transportation and inventory.

According to Andersson et al. (2010), there are no commercial systems available for supporting decisions regarding inventory management and vehicle routing problems simultaneously. In this context, a recurrent topic in recent literature is the Inventory-routing Problem (IRP), which results from the combination of inventory management and vehicle routing (Mirzapour Al-e-hashem and Rekik, 2014). The use of IRP models allows to simultaneously determining optimal inventory levels, delivery routes and vehicles' schedules based on the criterion of minimum cost (Moin et al., 2011). Guemri et al. (2016) state that the objective of IRP is to minimize distribution costs and inventory, while some restrictions are met, such as those relating to the vehicle and the storage capacity of the facility.

Coelho et al. (2012) state that scientific research on IRP is relatively recent compared to those related to optimization problems, such as Vehicle Routing Problem (VRP). The authors also refer that, although there are several literature reviews on routing problems and inventory management problems, relatively few of them explore the integration of the two subjects. The motivation to pursuit the work described in this paper arise from this gap in literature and from the interest revealed by companies, especially the one used as case study in this paper.

Traditionally, inventory systems are designed in a hierarchical fashion, with transportation streaming from one echelon of the supply chain to the next, i.e., from manufacturers to wholesalers and then from wholesalers to retailers. More flexible systems allow lateral

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transshipments within an echelon, i.e., between wholesalers or between retailers. In this case, members of the same echelon can share their inventories, which allows them to reduce inventory levels while still ensuring certain required service levels. When transshipment is included, the problem is designated as Inventory-routing Problem with Transshipment (IRPT).

In this paper, we introduce a new multi-product, multi-period and multi-vehicle IRP model where planned transshipment between distribution centers (DCs) is enabled. The problem concerns a distribution system between a set of factories and a set of DCs performed in a medium length multi-period context. The novelty regarding transshipment is that it is performed by the same vehicles responsible for distributing products from the factory to the distribution centers, instead of been outsourced as it is common in existing IRPT models. This option integrates transshipment movements within the regular routing operations, which makes it easier to be accepted and implemented by companies. Additionally, we consider a time restriction ensuring that each vehicle must return to the factory until the end of each planning period making it available to the following period. The inclusion of this restriction is especially important for companies operating with their own vehicles, as the fleet is less flexible. Finally, we actually apply the model to a real case study and provide the instances for future studies.

The new model is validated through the application to a large Brazilian industry from the retail sector. The paper tests a Proposed Scenario, which includes vehicle routing, inventory management and the possibility of transshipments between DCs, against a Base Scenario characterized by direct shipping from the factory to each DC, without any transferences between DCs and with full vehicle load from factory to DC. A metaheuristic approach is also proposed to solve the problem. It is worth noting that the industry, up to the moment, uses no helping decision software to take advantage of routing distribution, i.e., all distribution is currently done in a direct shipping basis as preconized in the Base Scenario (see Section 5). In addition, nowadays, the industry defines the quantities to deliver to each DC and the schedule based on empirical expertise and experience using at most spreadsheets.

The main contributions of this paper are: (i) a new modeling approach to incorporate transshipment performed by the same vehicles used in regular distribution; (ii) a new variant of IRP metaheuristics for solving problems with transshipment performed by the same vehicles used in regular distribution; (iii) an application of (i) and (ii) to a real case study.

The paper is organized as follows. Section 2 reviews existing literature related to IRP and, more specifically, IRPT. Section 3 describes the IRPT dealt with in detail and presents the model formulation. Section 4 describes the metaheuristic used to solve the case study presented in Section 5. The computational results are listed and analyzed in Section 6 and, finally, Section 7 presents the conclusions and proposes future research on the topic.

2. Literature review

IRP literature has its origin in a paper by Bell et al. (1983). In that paper the authors developed an advanced decision support system that integrates inventory management with vehicle scheduling and dispatching in the context of industrial gases distribution. Since then, many variations of the IRP have been published. Mirzapour Al-e-hashem and Rekik (2014) classify IRP models according to several criteria: finite or infinite planning horizons, single or multiple periods, single or multiple customers, single or multiple items, homogeneous or heterogeneous vehicles, and deterministic or stochastic demand. Two comprehensive reviews of IRP literature may be found in Andersson et al. (2010) and Coelho et al. (2013).

More recently, a new variant of IRP that makes use of lateral transshipment along with multicustomer routes has appeared providing more flexibility to the distribution systems. It is designated as Inventory-Routing Problem with Transshipment (IRPT). Paterson et al. (2011) conducted a review on inventory management models with transshipment. In their conclusions they state that planned transshipments have the potential to redistribute inventories among customers, to reduce handling costs and to be used as a method of meeting the demand that is not being met with the existing inventory in the local.

Under a transshipment policy, products can be shipped either from the factory to the distributer or from a distributer to another distributer. In other words, it happens when an element (e.g. client, distribution center, or wholesaler) is supplied by another element of the same echelon. In the retail industry, this practice is very common and useful, especially among stores belonging to the same supply chain when there are unforeseen variations in demand. Mercer and Tao (1996) provide an example of an inventory-routing system used by the supermarket Tesco in the United Kingdom, where deliveries are made from a factory to multiple warehouses and transshipment occurs between warehouses. Shen et al. (2011) describe a three-level crude oil supply chain consisting of a supplier (a single port), distribution ports and customers, where transshipment occurs between the ports. The objective is to determine in each period the number of vehicles used, how many will be used in each route and the amount of oil flowing through the pipeline in order to minimize the total logistics costs.

When transshipment is proactively planned in a deterministic context, in which there is no shortage, instead of being used as a mitigation for a certain unforeseen event (e.g. demand variation), it can produce an overall reduction in distribution and inventory holding cost (Coelho et al., 2012). This is the case, for example, when vehicle capacity and storage limits at customer locations restrict the amount that can be delivered to these customers per time period. More discussions about transshipment options can be found in the studies of Herer et al. (2002), Burton and Banerjee (2005), Nonås and Jörnsten (2007), Tiacci and Saetta (2011), Chen et al. (2012) and Hochmuth and Köchel (2012).

Two papers are closely related to the work here presented, Coelho et al. (2012) and Mirzapour Al-e-hashem and Rekik (2014). Coelho et al. (2012) were the first authors to introduce transshipments formally in the IRP context. They developed a model of IRPT that allows distribution of products from supplier to customer as well as between customers. However, transshipment movements are performed by subcontracted carriers. That option simplifies the optimization problem as cargo volumes and transportation times related to transshipments are not taken into account upon designing the routes between factory and distribution centers. The authors only acknowledge the points and periods where transshipment movements occur and manage the inventory so that transshipment can be performed, but transportation decisions are not optimized within routing design. Additionally, the authors do not consider multiple vehicles and multiple products and it is not mandatory for the vehicles to return to the depot until the end of each period. As a solution method, the authors use an Adaptive Large Neighborhood Search (ALNS) heuristic that manipulates the routes of the vehicles while determining the quantities to be delivered and the transshipment are solved by a network flow algorithm.

Mirzapour Al-e-hashem and Rekik (2014) also consider a transshipment option within the IRP. The model considers routing a rental truck that starts from one depot, pickups goods in suppliers to deliver in the assembly plant in each period. The model is characterized as multi-product, multi-period and multi-vehicle. Additionally, the model includes a green logistic approach by incorporating the reduction of greenhouse gas emission as part of the objective function. The authors assume that each assembly-plant may only be visited by a single vehicle in each period. Additionally, while their distribution is made from many-to-one, the structure we are considering in our model aims at the distribution from many-to-many (from a set of factories to a set of DCs).

3. Problem description

The new model considers the presence of multiple products, multiple vehicles, multiple periods and planned transshipments between distribution centers, seeking to reduce the total transportation and inventory

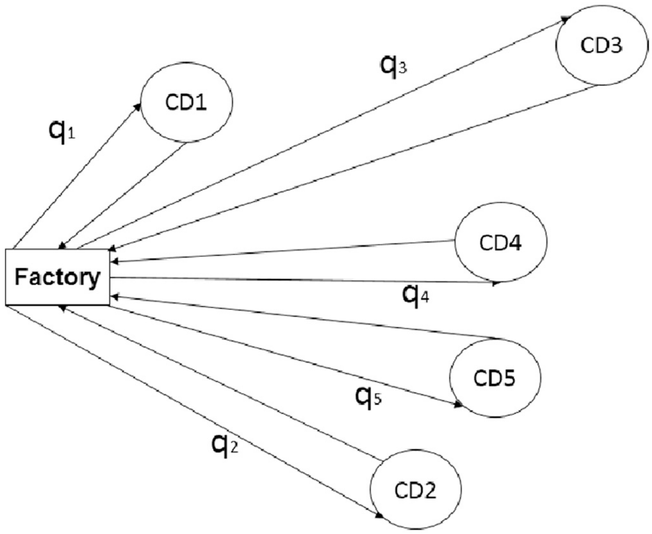

Fig. 1. Conceptual model of the original problem.

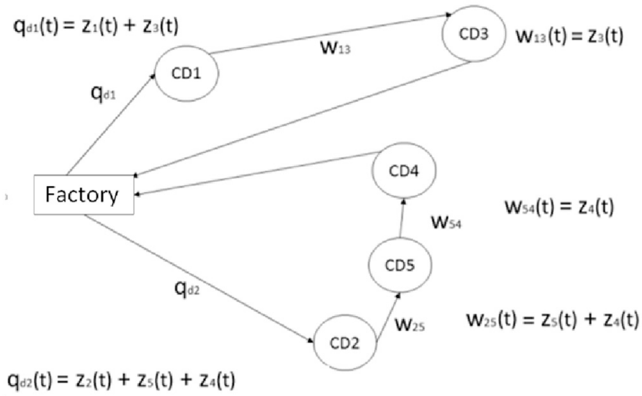

Fig. 2. Conceptual model of a classic vehicle routing problem.

cost. The formulation is generalized for the case of multiple origin factories, even though the case study presented in Section 5 only has one factory. A set of three figures is used to represent three different distribution strategies. Fig. 1 represents a traditional direct distribution strategy between factory and distribution centers (the one presently followed by the company considered in the case study under the Proposed Scenario). Fig. 2 represents a classic vehicle routing distribution strategy. Finally, Fig. 3 illustrates a strategy with routing and transshipment corresponding to the one proposed by the new model (under the Base Scenario).

By comparing Figs. 1–3, it is clear that direct distribution strategies (Fig. 1) have a large number of displacements and use more vehicles than classic routing strategies (Fig. 2) and more than the proposed model with routing and transshipment (Fig. 3). In addition, it is clear that the model with transshipment avoids these costs and also enables an improved performance when it is done in multiple periods, compared to the model with classic routing. This can be demonstrated by the example in Figs. 2 and 3. Fig. 2 shows that, since it is possible to ship products from the factory to others DCs in a single route, the vehicle can be loaded with the demand of others DCs. For example, the vehicle 2 was loaded with the demand of CD2, CD5 and CD4 and these three centers will be served in a single route, which contributes to the reduction in the number of vehicles used and to the reduction of the distance traveled, compared to the original problem (Fig. 1). But still, the solution (Fig. 2) comprehends two vehicles in each period. By comparison, Fig. 3 proposes a solution where besides classic multicustomer routes, vehicles may load products from one DC to deliver to another DC during the planned route. This allows the vehicle to carry excess products for certain DCs to be sent to other DCs in future periods. For example, in this scheme vehicle 2 is loaded not only with the current demand of CD2, CD5 and CD4, but also with the future demand of CD3 for the time period $t+1$, which is pre-shipped and temporarily stocked in CD4 during period t. So, the demand of CD3 on time period $t+1$ will be met by the stock of CD4 on time period t, occurring a transshipment from CD4 to CD3. This allows savings in distribution costs on time period $t+1$ since only one vehicle will be required to do the entire route, compared to the original necessity of two vehicles.

3.1. Mathematical formulation

The IRPT framework proposed in this study uses the following notation:

Sets	
V	Distribution centers and factories, $V = V^d \cup V^c$
V^d	Factories
V^c	Distribution centers
A	All arcs: $A = \{(i,j) \mid i,j \in V, i \neq j\}$
N	Vehicles
T	Time periods
P	Products
Parameters	
C_i	Maximum storage capacity of each node i
z_{ctp}	Demand of each product p for each DC c during each time period t
f_{tp}	Product units p produced by the factory during each time period t
k	Number of vehicles available
Q	Vehicle capacity
r_{ij}	Distance of the arc (i,j)
o_{ij}	Travel time of the arc (i,j)
h_c	Inventory cost per product unit per period at distribution center c
CT	Distribution cost per kilometer
CF	Cost of using vehicle n in the time period t
s	Upper time window
m	Maximum number of days for each vehicle n to return to the factory during each time period t
UT	Vehicle unloading and loading time
Decision variables	
x_{ijtn}	Decision to use or not vehicle n in arc (i,j) in time period t
y_{nt}	Decision to use or not vehicle n in time period t
I_{itp}	Inventory level of each product p in each node i at the end of time period t
q_{dctpn}	Amount of product p shipped from factory d to DC c by vehicle n in time period t
w_{cgtpn}	Total amount of product p shipped from DC c to g by vehicle n in time period t
u_{nt}	Total travel time of each vehicle until returning to the factory
a_{int}	Arrival time of the vehicle n at node i during each time period t

This problem is defined on a graph $G = (V, A)$, where the node set $V = V^d \cup V^c$ is composed by the factories V^d and the distribution centers V^c, and $A = \{(i,j) \mid i,j \in V, i \neq j\}$ is the arc set. The set of vehicles is N and the set of products is P. The set of time periods T denotes the planning horizon. The inventory cost of each unit product per period in each DC $c \in V^c$ is represented by h_c. The maximum storage capacity of each node $i \in$ V is defined as C_i. It is assumed that the factory has always enough capacity to meet all the distribution centers' demand during the planning horizon and that inventories are not allowed to be negative, i.e., the factory can only ship what is produced and/or what is in stock in the same time period. The demand of each product p in each DC c during each time period t is represented by z_{ctp}. The factory produces f_{tp} product units $p$during each time period t.

A number k of vehicles with a capacity Q is available. These vehicles are able to perform a route at the beginning of each time period to provide products p from the factory to all or part of the distribution centers V^c. It is assumed that the distribution cost per kilometer CT is the same for any route and the distance of the arc (i,j) is given by r_{ij}. The travel time of the arc (i,j) is given by o_{ij}.

CF represents the fixed costs for using any vehicle n in any time period t, being the same for all vehicles in all time periods. Transshipments can happen between any two DCs c and $g \in V^c$. The maximum

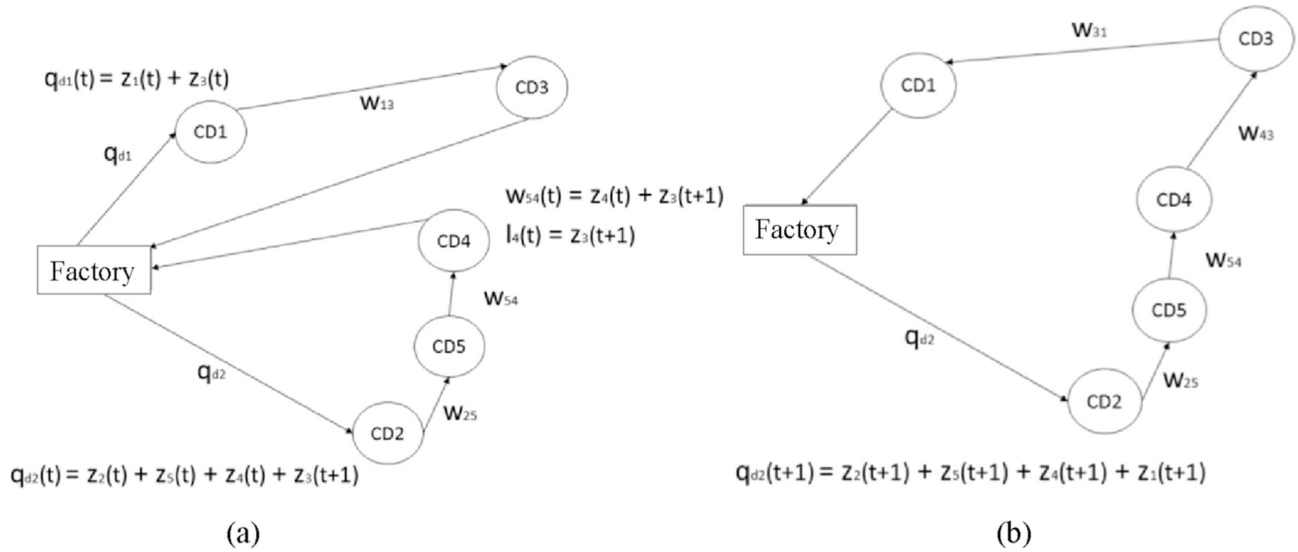

Fig. 3. Proposed model (with routing and transshipment) to time period t (a) and to time period $t+1$ (b).

deadline in days for the return to the factory of each vehicle n during each time period t is given by m.

The IRPT model developed seeks to find an optimal set of routes that meets the problem constraints and that minimizes the total cost. The problem can be modeled using the variable x_{ijtn}, where $x_{ijtn} \in \{0,1\}$ is 1 if, and only if, vehicle n uses the arc (i,j) in time period t. The variable $y_{nt} \in \{0,1\}$ is 1 if, and only if, vehicle n is used in time period t. The variable w_{cgtpn} denotes the amount of product p shipped from DC c to g by vehicle n in time period t, which represents the amount of transshipment plus the amount coming from the factory. The variable q_{dctpn} represents the amount of product p shipped from factory d to DC c by vehicle n in time period t. The inventory level of each product p in each node i at the end of time period t is measured by variable I_{itp}. The total travel time of each vehicle until returning to the factory is given by u_{nt}. The variable a_{int} represents the time when vehicle n arrives at node i during each time period t, while parameter s represents the upper time window operation bound for the DCs. The last two variables are used to ensure that DC's are supplied within their operation time window and to prevent the creation of nonrealistic sub routes. The variables w_{cgtpn}, q_{dctpn}, I_{itp}, u_{nt} e a_{int} are not negative and belong to the real numbers.

The new mathematical IRPT model is formulated as follows:

$$\text{Min} \sum_{c\in V^c}\sum_{p\in P}\sum_{t\in T} h_c I_{ctp} + \sum_{i\in V}\sum_{j\in V}\sum_{t\in T}\sum_{n\in N} CT\, r_{ij} x_{ijtn} + \sum_{n\in N}\sum_{t\in T} CF\, y_{nt} \tag{1}$$

Subject to:

$$I_{c\,t-1\,p} + \sum_{d\,\in\,V^d}\sum_{n\,\in\,N} q_{dctpn} + \sum_{g\,\in\,V^c}\sum_{n\,\in\,N} w_{gctpn} = I_{ctp} + z_{ctp} + \sum_{c\,\in\,V^c}\sum_{n\,\in\,N} w_{cgtpn},\ \forall c\in V^c, \forall t\in T, \forall p\in P \tag{2}$$

$$I_{d(t-1)p} + f_{tp} = I_{dtp} + \sum_{c\in V^c}\sum_{n\in N} q_{dctpn}, \forall d\in V^d, \forall t\in T, \forall p\in P \tag{3}$$

$$\sum_{p\in P} I_{itp} \le C_i, \forall i\in V, \forall t\in T \tag{4}$$

$$q_{dctpn} \le x_{dctn} C_c,\ \forall d\in V^d, \forall c\in V^c, \forall t\in T, \forall p\in P, \forall n\in N \tag{5}$$

$$w_{cgtpn} \le x_{cgtn}\, C_g,\ \forall c\in V^c, \forall g\in V^c, \forall t\in T, \forall p\in P, \forall n\in N \tag{6}$$

$$\sum_{p\in P} q_{dctpn} \le Q\, y_{nt}\ , \forall d\in V^d, \forall c\in V^c, \forall t\in T, \forall n\in N \tag{7}$$

$$\sum_{p\in P} w_{cgtpn} \le Q\, y_{nt}, \forall c\in V^c, \forall g\in V^c, \forall t\in T, \forall n\in N \tag{8}$$

$$\sum_{i\,\in\,V} x_{ictn} = \sum_{i\,\in\,V} x_{citn},\ \forall c\in V^c, \forall t\in T, \forall n\in N \tag{9}$$

$$\sum_{c\,\in\,V^c} x_{dctn} = y_{nt}\ , \forall d\in V^d, \forall t\in T, \forall n\in N \tag{10}$$

$$\sum_{c\,\in\,V^c} x_{cdtn} = y_{nt}\ , \forall d\in V^d, \forall t\in T, \forall n\in N \tag{11}$$

$$u_{nt} = \sum_{i\in V}\sum_{j\in V} o_{ij} x_{ijtn} + \left(\sum_{i\in V}\sum_{j\in V} x_{ijtn} - y_n\right) UT\ , \forall t\in T, \forall n\in N \tag{12}$$

$$u_{nt} \le m\, y_{nt}, \forall t\in T, \forall n\in N \tag{13}$$

$$\sum_{n\in N} y_{nt} \le k, \forall t\in T \tag{14}$$

$$a_{int} - a_{jnt} + \left(s + o_{ij} + UT\right)^{*} x_{ijtn} \le s\ ,\ \forall i\in V, \forall j\in V, \forall n\in N, \forall t\in T \tag{15}$$

$$\sum_{n\in N}\sum_{t\in T} x_{ijtn} = 0\ , \forall i\in V, \forall j\in V \ \Big|\ i=j \tag{16}$$

$$x_{ijtn},\ y_{nt} \in \{0,1\}\ , \forall i\in V, \forall j\in V,\ \forall t\in T,\ \forall n\in N \tag{17}$$

$$w_{cgtpn},\ q_{dctpn},\ I_{itp},\ u_{nt},\ a_{int} \ge 0\ , \forall i\in V, \forall j\in V,\ \forall t\in T,\ \forall n\in N,\ \forall p\in P \tag{18}$$

The objective function (1) minimizes the inventory cost and the variable and fixed distribution costs. Constraints (2) ensure conservation of stock flow at each DC, so that for each DC c, for each product p and during each time period t, the inventory level during the previous time period plus the amount received from the factory d by each vehicles n

plus transshipment amount received from each DC g is equal to the inventory level in current time period plus demand plus transshipment amount shipped to each DC g by each vehicles n. Constraints (3) ensure the conservation of stock flow at the factory, so that for each factory d, for each product p and during each time period t, the inventory level in the previous time period plus the amount produced is equal to the inventory level during current time period plus the amount shipped to each DC c by each vehicle n. Constraints (4) ensure that, during each time period t and for each node i, the inventory amount respects the storage capacity of node i. Constraints (5) ensure that products p will only be shipped from the factory d to a DC c with vehicle n if arc (d, c) is being used by vehicle n during the same time period t. Constraints (6) ensure that products p will only be transshipped from a DC c to a DC g by vehicle n if arc (c, g) is being used by vehicle n during the same time period t. Constraints (7) ensure that the sum of products p shipped from the factory d to the DC c by the vehicle n respects the capacity of vehicle n during each time period t. Constraints (8) ensure that the sum of products p carried by each vehicle n between DC c and DC g by the vehicle n during period t respects the capacity of vehicle (this includes products that are being transshipped and regular distribution from the factory. Constraints (9) ensure the conservation of the vehicles flow in each DC, so that every vehicle n arriving on a DC c coming from any node i must leave the DC c going to any node i. Constraints (10) and (11) ensure that only vehicles carrying cargo can leave the factory and that every vehicle must return to the factory. Constraints (12) and (13) ensure that the total travel time of each vehicle n respects the deadline for returning to the factory during any time period t. Total travel time includes loading and unloading time at DCs besides actual travelling time between nodes. Constraints (14) ensures that the sum of vehicles n used in time period t respects the total number of vehicles available. Constraints (15) prevent sub routes from happening and constraints (16) prevent the existence of arcs (i, j) wherein i equals j. Finally, constraints (17) and (18) define the domain of the respective variables.

Constraints (2), (3), (4), (7) and (15) were adapted from the model of Coelho et al. (2012). Constraints (10), (11) and (14) were adapted from the model of Mirzapour Al-e-hashem and Rekik (2014). The objective function (1) and constraints (5) and (9) were presented in a similar way in these two works, and were adapted for the current study. Constraints (6), (8), (12), (13) and (16) were developed specifically for the current study.

In order to simulate the current operation (represented by the Base Scenario) where the company only uses a direct shipping policy two additional sets of constraints are required. When added to the model described, constraints (19) prevent transshipment between DCs and constraints (20) forces direct shipping instead of multicustomer routes.

$$\sum_{c\in V^c}\sum_{g\in V^c}\sum_{t\in T}\sum_{p\in P}\sum_{n\in N} w_{cgtpn} = 0 \tag{19}$$

$$\sum_{c\in V^c}\sum_{g\in V^c}\sum_{t\in T}\sum_{n\in N} x_{cgtn} = 0 \tag{20}$$

4. Metaheuristic algorithm

VRP models are in general classified as NP-hard and cannot be solved using standard optimization packages in feasible time. By incorporating inventory management issues and transshipment the problem becomes significantly harder. For this reason we present in this section a metaheuristic algorithm developed to solve the case study presented in Section 5. The algorithm is a hybrid Randomized Variable Neighborhood Descent (RVND) which searches over large neighborhoods mostly based on the IRP literature (Coelho et al., 2012; Guemri et al., 2016) and allowing non-improvement moves to include diversity. The approach is different from the ones in the literature because it includes changes to accommodate transshipment movements performed by the same vehicles used in regular distribution and the maximum time window corresponding to the period extension.

The choice of using the RVND metaheuristic was made because there are several works in the literature showing successful results for several classes of VRPs (Subramanian et al., 2010; Martinelli et al., 2013). A full description of the method and the application for many VRPs variations can be found in Subramanian (2012).

4.1. Neighborhoods

The metaheuristic uses five neighborhoods as described below. Given the characteristics of the IRP, any neighborhood developed must be evaluated for each period (or pair of periods) and each vehicle (or pair of vehicles).

- Insert (v, p, n, t): This neighborhood tries to insert a customer v in any position p of route n in period t. The complete neighborhood can be tested in $O(|V^c|^2|T||N|)$.
- Remove (p, n, t): This neighborhood tries to remove the customer from position p of route n in period t. The complete neighborhood can be tested in $O(|V^c||T||N|)$.
- Shift (p, n, t): This neighborhood tries to shift the customers at position p and $(p+1)$ of route n in period t. The complete neighborhood can be tested in $O(|V^c||T||N|)$.
- Swap $(p_1, n_1, t_1, p_2, n_2, t_2)$: This neighborhood tries to swap the customer at position p_1 of route n_1 in period t_1 with the customer at position p_2 of route n_2 in period t_2. The complete neighborhood can be tested in $O(|V^c|^2|T|^2|N|^2)$.
- Relocate $(p_1, n_1, t_1, p_2, n_2, t_2)$: This neighborhood tries to remove the customer at position p_1 of route n_1 in period t_1 and insert it after the customer at position p_2 of route n_2 in period t_2. The complete neighborhood can be tested in $O(|V^c|^2|T|^2|N|^2)$.

It is clear that neighborhoods Swap and Relocate includes neighborhood Shift. However we decided to keep neighborhood Shift because Swap is less complex to code when it is not allowed to do consecutive moves.

At each neighbor evaluation, we have to recalculate the routing and inventory costs. The routing cost recalculation is easy to perform and can be done in constant time for all neighborhoods. On the other hand, the inventory cost is not trivial to recalculate. Given the characteristic of the problem we are trying to solve, it is enough to use a minimum cost flow algorithm (Ahuja et al., 1993) in an auxiliary graph to obtain the inventory cost for a given solution. Similar approaches were used by some authors in the literature (Coelho et al., 2012; Guemri et al., 2016). However, to the best of our knowledge there is no work which considers transshipment operations with the original vehicles.

4.2. Minimum cost flow algorithm

Given a solution consisting of the routes performed in each period for each vehicle, we build an auxiliary graph $G' = (V', A')$, with the vertex set V' consisting of the following vertices:

- V'_v: For each node, we create $(|T|+1)$ vertices representing its inventory in each period and at the end of the horizon. For the factory, its supply is the production and for the customers, the supplies are their negative demands.
- V'_n: For each vehicle, we create $|T|$ vertices representing the start of its route for each period. These vertices have no supply.
- e: One final vertex representing the inventory excess at the end of the horizon. Its supply is the difference between the total demand and the total production at the factory for all periods.

In order to consider the transshipment operation, we need to define

Table 1
Demand of each product family per DC in 15 days.

DC	Demand of each product family per DC (10,000 units)											
	1	2	3	4	5	6	7	8	9	10	11	12
Campinas	4.25	32.80	55.40	30.80	11.00	33.50	3.35	13.30	4.65	0.67	1.81	0.32
Contagem	4.89	37.80	63.80	35.50	12.70	38.60	3.86	15.30	5.36	0.77	2.08	0.37
Praia Grande	1.27	9.79	16.50	9.20	3.29	10.00	1.00	3.96	1.39	0.20	0.54	0.10
Ribeirão Preto	2.98	23.00	38.80	21.60	7.72	23.50	2.35	9.31	3.26	0.47	1.27	0.22
Rio de Janeiro	12.20	93.90	159.00	88.30	31.50	96.00	9.60	38.00	13.30	1.92	5.17	0.91
São José dos Campos	1.48	11.40	19.30	10.70	3.84	11.70	1.17	4.63	1.62	0.24	0.63	0.11
São Paulo	10.80	83.40	141.00	78.40	28.00	85.20	8.53	33.70	11.80	1.70	4.59	0.80
Vila Velha	1.84	14.20	24.00	13.30	4.76	14.50	1.45	5.74	2.01	0.29	0.78	0.14

the arc set A' with the following arcs:

- Vehicle arcs: For each period, we create $|N|$ arcs between the factory vertex from V_v' and the vehicles vertices V_n'. The capacity of each arc is the vehicle's capacity and its cost is zero.
- Inventory arcs: For each period and each node, we create an arc between the vertex in V_v' and the corresponding vertex in the next period. The capacity of each arc is the node maximum inventory level minus the demand (if it is a customer node), and its cost is the customer inventory cost.
- Final inventory arcs: we create $|V|$ arcs between the nodes from V_v' in period $(t+1)$ and the excess vertex e. The capacity of each arc is the same as inventory arcs, but its cost is zero.
- Routing arcs: For each period and route in the solution, we create an arc between the corresponding pair of customers in V_v'. For the first customer visited by the route, the arc is created between the corresponding vehicle vertex in V_n' and the customer vertex in V_v'. The capacity of each arc is the vehicle's capacity and its cost is zero.

The main difference between our auxiliary graph to the ones from the literature is that when there is no need to consider transshipment the routing arcs does not exist, we can create arcs from the vehicles vertices directly to the customers vertices, making the minimum cost flow problem easier to solve.

It is important to mention that we solve a minimum cost flow problem not only to obtain the inventory costs, but also the loads for each vehicle and to decide if the solution is feasible. We evaluate the feasibility of the solution checking if all demands are satisfied by the flows. If the solution is not feasible, we penalize its value adding to it a large value multiplied to the total demand not satisfied.

4.3. Randomized Variable Neighborhood Descent

As previously mentioned, we use the described neighborhoods in a Randomized Variable Neighborhood Descent. The approach is a local search-based method which starts with an initial solution (empty, full or random) and at each iteration it chooses a random ordering for the neighborhoods. Following this order, the algorithm performs a first improvement local search on the current neighborhood. If an improvement is found, the algorithm reshuffles the neighborhoods list and starts over. If no improvement is found for a given neighborhood, the algorithms goes to the next neighborhood. The method terminates when no neighborhood is able to improve the current solution.

It is important to notice that the local searches performed are also randomized. Every time a local search is called, the lists of periods, vehicles, customers and positions are shuffled. Therefore, on each call of the local search the neighborhood will be evaluated in a different ordering.

4.4. Hybridization

Following the ideas of Coelho et al. (2012), when the RVND reaches a local optima, non-improvement moves are allowed using an acceptance criterion based on simulated annealing. The advantage of this approach is to add more diversity to the search. When a better solution is found during the search, we restart the RVND.

5. Case study

The case study focus on a large industry from the Brazilian retail sector. Its supply chain is composed of three levels, one factory, a set of distribution centers and a set of customers. Presently, logistic operations are managed in two levels: (i) the distribution from the factory to the DCs (primary distribution); and (ii) the distribution from the DCs to the customers (secondary distribution). This study will focus on the primary distribution, where, presently, the company does not consider routing or transshipment between DCs. The company's objective is to find the optimal solution regarding distribution routes and inventory management for each period and for each DC.

The factory produces 12 different families of products and distributes to 32 DCs located in different regions of Brazil. The distribution in each region is managed and performed separately. In this study we will restrict the scope to the distribution on the Southeast Region of Brazil, which has eight DCs. This choice is based on the relevance of this region compared to the others as it accounts for about 60% of total product demand, which is equal to approximately 36 million of products per month. This region has 30 homogeneous trucks available for distribution with a capacity of 800,000 products each. Transportation costs for each truck have two components: a fixed amount of 514 monetary units (m.u.) per trip (going and return from and to the fabric) and a variable amount of 1.862 m.u. per kilometer. The company plans the distribution on a monthly base divided in two periods of 15 days, i.e., the length of the planning horizon for each time period t is 15 days. Consequently, all trucks leaving the factory must return empty within a maximum period of 15 days.

Demand is assumed deterministic and cyclic, that is, it is the same for each 15 days period. Table 1 shows the demand of each product family in the horizon of 15 days. The production capacity of the factory each 15 days for each product family is listed in Table 2. The study assumes that there is no inventory cost in the factory. The DCs have a limited inventory capacity of 35 million products and the inventory cost is represented by applying a rate of 0.5% to the value of the product in stock (3.75 m.u.),

Table 2
Factory production of each product family in 15 days.

Factory Production per product family (10,000 units)											
1	2	3	4	5	6	7	8	9	10	11	12
80	613	1035	576	206	626	63	248	87	13	34	6

Table 3
Distance of arc (i,j) in 1000 km.

Node i/Node j	Factory	Campinas	Contagem	Praia Grande	Ribeirão Preto	Rio de Janeiro	São José dos Campos	São Paulo	Vila Velha
Factory	0.000	0.503	0.526	0.663	0.280	1.000	0.713	0.600	1.089
Campinas	0.503	0.000	0.565	0.189	0.227	0.494	0.148	0.099	0.926
Contagem	0.526	0.565	0.000	0.642	0.500	0.451	0.593	0.569	0.537
Praia Grande	0.663	0.185	0.642	0.000	0.408	0.508	0.158	0.078	0.94
Ribeirão Preto	0.280	0.223	0.501	0.407	0.000	0.718	0.372	0.316	1.034
Rio de Janeiro	1.000	0.493	0.451	0.508	0.717	0.000	0.347	0.433	0.514
São José dos Campos	0.713	0.148	0.592	0.16	0.372	0.347	0.000	0.088	0.779
São Paulo	0.600	0.095	0.571	0.076	0.318	0.436	0.090	0.000	0.867
Vila Velha	1.089	0.926	0.537	0.941	1.034	0.515	0.780	0.866	0.000

Table 4
Route time of arc (i,j) in days.

Node i/Node j	Factory	Campinas	Contagem	Praia Grande	Ribeirão Preto	Rio de Janeiro	São José dos Campos	São Paulo	Vila Velha
Factory	0.0	0.2	0.3	1.0	0.1	1.0	1.0	0.3	1.0
Campinas	0.2	0.0	0.3	0.1	0.1	0.2	0.1	0.1	1.0
Contagem	0.3	0.3	0.0	1.0	0.3	0.2	0.3	0.3	1.0
Praia Grande	1.0	0.1	1.0	0.0	0.2	0.3	0.1	0.1	1.0
Ribeirão Preto	0.1	0.1	0.3	0.2	0.0	1.0	0.2	0.2	2.0
Rio de Janeiro	1.0	0.2	0.2	0.2	1.0	0.0	0.2	0.2	1.0
São José dos Campos	1.0	0.1	0.3	0.1	0.2	0.2	0.0	0.1	1.0
São Paulo	0.3	0.1	0.3	0.1	0.2	0.2	0.1	0.0	1.0
Vila Velha	2.0	1.0	1.0	1.0	2.0	1.0	1.0	1.0	0.0

Table 5
Comparison of objective function between the two scenarios.

Object Function	Base Scenario	Proposed Scenario
Inventory Cost	2005.31	553.12
Variable Distribution Cost	130,760.81	120,488.16
Fixed Distribution Cost	25,700.00	23,644.00
Total Cost	**158,466.12**	**144,685.28**

Table 6
Comparison of inventory level between the Base Scenario and the Proposed Scenario.

Time Period	Units in Stock	
	Base scenario	Proposed scenario
1	106,950	29,500
2	0	0
Total	**106,950**	**29,500**

Table 7
Comparison of the number of vehicles.

Time period	Number of Vehicles		
	Analytical account	Base scenario	Proposed scenario
1	22.39	26	23
2	22.39	24	23
Total	**44.78**	**50**	**46**

i.e., 0.02 m.u. per product in stock in the DC in each horizon of 15 days. Since lead times are long and measured in days, service time in the DC is not relevant and, therefore, it is not considered by the company.

Table 3 shows the distances r_{ij} corresponding to the arches connecting each pair (i,j) and Table 4 shows the travel time o_{ij}. Both distance and time were calculated using the Applications Programming Interface (APIs) "Create a locations dataset" and "Calculate distances and time matrix", provided by the Universitat Pompeu Fabra, with free access by the website vrp.upf.edu.

It's assumed that the vehicle unloading and loading time (UT) is 1.2 h, for any quantity or type of product p on all time periods t and for all DCs c. The initial stock in the DCs c and in the factory d was considered equal to zero. In order to not increase the complexity, the model assumes that there is no time window to receive products in the DCs, and such premise is also used in the current planning of the company.

We consider two scenarios for running the model. The Base Scenario, which corresponds to the present logistic operation of the company and the Proposed Scenario, which includes transshipment and is the main numerical experiment of the paper. The Base Scenario is then used for benchmark comparison with the Proposed Scenario.

6. Computational results

The mathematical model presented in Section 3 was solved through an exact method and through the metaheuristic algorithm presented in Section 4 for both the Base Scenario and the Proposed Scenario. The Base Scenario is considered for comparison purposes, while the Proposed Scenario constitutes the paper's main experiment.

The exact method was implemented using CPLEX software in AIMMS platform. The algorithm was developed and executed on an Intel Core i5 2.5 GHz with 6 GB RAM operating under Windows 7 Professional. The solution obtained for the Base Scenario was optimal and took about 6 h running. Regarding the Proposed Scenario, due to the complexity of the model, the algorithm did not reach an optimal solution for a time period of 15 h running, but a partial sub-optimal solution was obtained. While not optimal, the gap of the solution compared to the linear relaxation was 13%.

Table 5 shows the objective function value for the two scenarios studied. It can be noted that, with the inclusion of routing and transshipment (Proposed Scenario), there was a reduction in total logistics costs of approximately 9% compared to the result of the Base Scenario. In addition, it is clear that all costs were reduced, both inventory and distribution. The inventory cost has decreased by about 70%, the variable distribution cost decreased by about 8% and the fixed distribution cost decreased 8%. From Table 6, it can be seen that the average inventory level decreased 70%, which explains the reduction in inventory costs, since this is directly related to the number of units in inventory.

Table 7 shows the number of vehicles used in each scenario. The result of the analytical account presented in the table was obtained by dividing the total demand of each time period (about 18 million units) by the vehicle capacity (800,000 units). It can be noted that, despite the solution of the Proposed Scenario has been suboptimal, the number of vehicles used was the optimal integer value (23 vehicles). Thus, the

Table 8
Metaheuristic results for the Proposed Scenario.

	Empty			Full		
	Minimum	Average	Maximum	Minimum	Average	Maximum
Inventory Cost	553.12	553.12	553.12	553.12	553.12	553.12
Distribution Cost	144,128.43	144,131.23	144,139.30	144,130.30	144,133.28	144,135.88
Total Cost	144,681.56	144,684.35	144,692.73	144,683.42	144,686.40	144,689.01
Time (seconds)	471.79	510.59	607.53	652.41	748.99	820.52

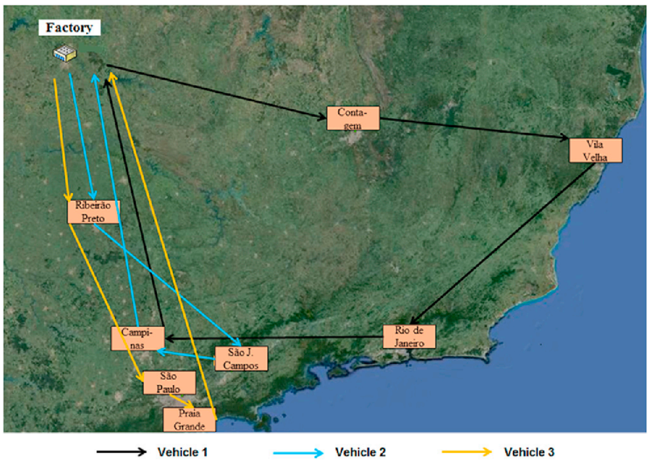

Fig. 4. Example of routes of the proposed model.

Proposed Scenario uses four vehicles less than the Base Scenario, which is 8% less of vehicles and thus 8% less of fixed distribution cost.

The metaheuristic algorithm presented in Section 4 was developed in C++ and tested using an Intel Core i7 3.3 GHz with 64 GB of RAM running Ubuntu Linux 14.04. The minimum cost flow algorithm was solved using the network simplex algorithm (Orlin, 1997) from the LEMON Graph Library (Dezső et al., 2011).

We first tested the metaheuristic algorithm for the Base Scenario. We start the algorithm with an empty solution (which will always be infeasible, generating a very high penalty cost). Given the constraints imposed by the scenario, it is rather easy to find the optimal solution. For all ten runs, the approach was able to obtain it in just one call of the local search. The average runtime was 0.13 s, with the fastest run taking 0.08 s and the longest one taking 0.27 s.

For the Proposed Scenario, we tested two types of initial solution, the empty solution and the full solution, where each vehicle visits all customers on all time periods (which will always be feasible, but with a high distribution cost). For each initial solution, we run the metaheuristic ten times and report the minimum, average and maximum costs and times in Table 8.

Table 8 shows results very close to the solution obtained by the mathematical model. The best solution found by the metaheuristic from an empty initial solution is only 0.0026% better than the one found by the model. This is an evidence that the solutions may be close to the optimal. Furthermore, it is noteworthy the small amount of time the metaheuristic algorithm took to obtain the solution and also the small variance on the solution values.

For illustration purposes, Fig. 4 represents three of the 23 routes created. It can be noted that the vehicles depart and return to the factory, and that each vehicle went through each DC only once.

7. Conclusions

As far as the authors are aware, there is no equivalent model in literature to a process of product distribution with transshipment that tackles all aspects included in the IRPT proposed: multi-vehicle, multi-product and multi-period. The most similar model to the one developed in this paper is the one of Coelho et al. (2012), which is a single vehicle, single product, multi-period model and outsourced transshipment movements and was used as the basis for building the new IRPT formulation. The characteristics included in the new formulation are not specific of the case study presented but can be found in several real world applications making it a valid contribution either for academics and practitioners.

The approaches were applied to a leading and large industry from the Brazilian retail sector. From the results analysis, it can be proved that the mathematical formulation and the metaheuristic of IRP with transshipment proposed (Proposed Scenario) in this paper are more efficient than the formulation of the original problem with direct deliveries (Base Scenario), reducing the total cost by approximately 9%. There was a reduction in all logistical costs of the company studied, i.e., reduction in transportation costs (fixed and variable) and in the inventory cost.

In terms of inventory level, the IRPT approach proposed produced a reduction of about 70% in the quantity of products in stock and therefore in the inventory cost. By allowing transferences between DCs there is a decrease in the level of inventory of the company, without changing the level of service offered.

The approach developed in this paper may be used as a logistic planning system. Specifically for the company under study, they constitute an advantage weapon to modernize their operations as up to the moment no other optimization tools were being used.

In this work, we assumed a constant unloading and loading time (UT). It might be arguable, as this parameter may affect transshipment performances. However, and given the lack of other sources, we followed the information given by the company under study. In a future work, it might be worth to test a variable parameter.

It would have also been interesting to separate the benefits of routing and the ones of transshipment in order to compare to the present direct shipment model. However, the formulation developed analyses movements in an integrated manner, making it impossible to separate the benefits. Such task would require a new formulation for the sole routing scheme which would make the formulation more computing demanding.

Finally, two research extensions related to sustainability issues are suggested. First, including greenhouse gas emission reduction in the objective function as proposed by Mirzapour Al-e-hashem and Rekik (2014). Second, introducing backhaul of raw materials consumed by the factory that may come from the distribution centers, so that the vehicles do not return empty to the factory.

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