Omega 40 (2012) 445–455

Contents lists available at SciVerse ScienceDirect

Omega

journal homepage: www.elsevier.com/locate/omega

The supplier selection problem with quantity discounts and truckload shipping

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ARTICLE INFO

Article history:
Received 21 April 2011
Accepted 14 September 2011
Processed by Lodi
Available online 22 September 2011

Keywords:
Supplier selection
Quantity discounts
Truckload shipping rates
Integer programming

ABSTRACT

To minimize procurement expenditures both purchasing and transportation costs need to be considered. We study a procurement setting in which a company needs to purchase a number of products from a set of suppliers to satisfy customer demand. The suppliers offer total quantity discounts and transportation costs are based on truckload shipping rates. The goal is to select a set of suppliers so as to satisfy product demand at minimal total costs. The resulting optimization problem is strongly NP-hard. We develop integer programming based heuristics to solve the problem. Extensive computational experiments demonstrate the efficacy of the proposed heuristics and provide insight into the impact of instance characteristics on effective procurement strategies.

1. Introduction

The selection of a set of suppliers is a key procurement decision for many companies. Different considerations influence this decision, e.g., a supplier's quality and reliability, but also his pricing and discount policy (see [25] for a study on the importance of different attributes that companies consider when selecting suppliers). The continuing growth of e-commerce, with reverse auctions as a web-based counterpart to traditional procurement, has fueled an interest in procurement optimization, and in recent years the research community has made significant advances in this area.

Two distinct streams of research can be identified in procurement optimization: a first stream assuming that demand is deterministic and a second stream assuming that demand is stochastic. We highlight a few papers in each research stream. Rosenblatt et al. [22] analyze procurement policies involving both supplier selection and purchase frequency and quantity setting. Benton [7] studies a procurement problem in which suppliers offer quantity discounts. Various extensions of the problem considered by Benton are investigated and an exact method for their solution is presented in Goossens et al. [16]. Xia and Wu [26] also look at supplier selection in the presence of quantity discounts. The setting studied has multiple suppliers offering multiple products with limited availability and discounts on total business volume and considers qualitative and quantitative factors in their selection. Chauhan and Proth [10] present a procurement problem with concave purchase cost and minimal and maximal order quantities and propose different heuristics as solution methods. Awasthi et al. [6] study a supplier selection problem for a single manufacturer facing stochastic demand. The suppliers quote different prices and have restrictions on minimum and maximum order quantities. A heuristic solution procedure is proposed. Zhang and Zhang [28] extend the previous setting by including holding and shortage costs and a fixed cost incurred when a supplier is selected. A branch and bound algorithm is presented for its solution. Burke et al. [8] study single period, single product sourcing decisions under demand uncertainty. Their approach considers product prices, supplier costs, supplier capacities, supplier reliabilities and firm specific inventory costs. The same authors analyze the impact of supplier pricing schemes and supplier capacity limitations on the optimal sourcing policy for a single firm [9]. Heuristic solution methods are developed to identify a quantity allocation decision for the firm. Anupindi and Akella [4] analyze purchase quantity allocation between two uncertain suppliers and its effects on inventory policies, whereas Dada et al. [11] study an environment with multiple unreliable suppliers. Yang et al. [27] study a procurement problem, where a buyer, while facing random demand, has to decide ordering quantities from a set of suppliers with different yields and prices.

However, procurement costs are not just determined by purchasing costs. Typically, transportation costs form a substantial component of procurement cost. Therefore, we study a procurement setting that explicitly incorporates both. We consider a company that has to select a set of suppliers from which to purchase a number of products. The suppliers offer discounts based on the total quantity purchased. The transportation costs are modeled as truckload shipping costs, and thus depend on the

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doi:10.1016/j.omega.2011.09.001

total quantity purchased as well. This transportation cost structure is appropriate as it is common practice to out-source the transportation to a carrier company.

More specifically, let $K := \{1,\ldots,n\}$ be the set of products to be purchased and let $S := \{1,\ldots,m\}$ be the set of suppliers to choose from. Each product k, $k \in K$, can be purchased at a subset $S_k \subseteq S$ of suppliers at a unit price $p_{ik} > 0$, $i \in S_k$ (potentially different for each supplier). There is discrete demand d_k for product $k \in K$ and product availability q_{ik} for product $k \in K$ at supplier $i \in S_k$. We assume that $\sum_{i \in S_k} q_{ik} \geq d_k$ for all $k \in K$. Each supplier $i \in S$ defines r_i consecutive, non-overlapping discount intervals $[l_i^r, u_i^r]$, $r \in R_i := \{1,\ldots,r_i\}$, each with an associated discount rate f_i^r; discount rates satisfy $f_i^r > f_i^{r-1}$ for $r = 2,\ldots,r_i$ for $i \in S$. The interval in which the total quantity purchased falls determines the discount applied to the total purchase cost. For convenience, we convert these discounts into unit prices p_{ik}^r for product k, $k \in K$, at supplier i, $i \in S_k$, in interval r, $r \in R_i$. A fixed transportation cost c_i is charged for each visit to a supplier i, $i \in S$. The number of visits to a supplier depends on the total quantity purchased at that supplier and the truck capacity Q. The company wants to select a set of suppliers to satisfy the demand for each product in such a way that procurement costs, i.e., total purchase and transportation costs, are minimized. We refer to this problem as the *Supplier Selection Problem with Quantity Discounts and Truckload Shipping* (SSP-QDTS). The piecewise linear non-convex and non-concave purchase cost function and the step-wise transportation cost function make this a computationally challenging optimization problem.

SSP-QDTS generalizes the total quantity discount problem, which is known to be strongly NP-hard and for which exact and heuristic approaches have been proposed (see [16] and more recently [19]). SSP-QDTS is also related to the traveling purchaser problem (see [18]) which takes purchasing costs and traveling costs into account (see [20] for a variant of the problem with a budget constraint and [2,3] for a dynamic variant with supplier quantities decreasing over time). The traveling purchaser problem does not consider total quantity discounts, but allows for more general transportation options.

Given the complexity of the problem, obtaining optimal solutions to meaningful instances in a reasonable amount of time is virtually impossible. Therefore, we have developed integer programming based heuristics that are capable of producing high-quality solutions quickly. The proposed heuristics are iterative rounding schemes based on the linear programming relaxation of a natural integer programming formulation of the SSP-QDTS. The linear programming solution in combination with a quantity discount analysis guides the choice of both a supplier and the number of visits to that supplier. After fixing the number of visits to the selected supplier, the linear programming relaxation is resolved and the process is repeated. When the number of visits to each of the suppliers is fixed, the remaining integer program is solved to determine the product quantities to be bought from the selected suppliers. To enhance the effectiveness of the guidance provided by the linear programming relaxation, the formulation has been strengthened by a set of valid inequalities. Finally, simple post-processing is used to eliminate any unnecessary costs incurred.

Our research provides various contributions. First, it explicitly integrates purchasing and transportation considerations in the context of procurement, which, as far as we have been able to establish, has not been done before, even though both aspects are relevant in almost all practical procurement environments. Second, our heuristics can be seen as another example of the growing body of literature on solution approaches that incorporate the solution of small well-chosen integer programs (see for example [1,5,12,17,24]). Finally, the extensive experimental analysis on a large set of instances provides insights and rules-of-thumb that can be used by companies to help them develop effective vendor selection strategies. In fact, a major goal of our research has been to study the impact of the characteristics of the procurement environment on the structure of vendor selection strategies. Effective vendor selection strategies depend on the number of suppliers and products, on the product availabilities (both on the number of suppliers of a particular product and the quantities of product available at the suppliers), on the quantity discount parameters, the location of the suppliers and the truckload shipping costs, the relative importance of purchase and transportation costs, etc.

The paper is organized as follows. In Section 2, we describe the mathematical formulation of the SSP-QDTS, discuss properties of its continuous relaxation, and introduce valid inequalities. In Section 3, we present a number of heuristics, each representing a specific implementation of a general iterative rounding scheme. In Section 4, we discuss an extensive computational study in which we introduce various procurement settings and analyze the characteristics of effective supplier selection strategies. Finally, in Section 5, we draw some conclusions and indicate some possible future research direction.

2. An integer programming formulation

To formulate the SSP-QDTS the following sets of decision variables are introduced:

$x_i \geq 0$ the number of visits to supplier i, $\quad i \in S$

$z_{ik} \geq 0$ the number of units of product k purchased from supplier i, $k \in K$, $i \in S_k$

$$z_{ik}^r := \begin{cases} z_{ik} & \text{if } \sum_{j \in K} z_{ij} \in [l_i^r, u_i^r], \\ 0 & \text{otherwise,} \end{cases} \qquad i \in S,\ k \in K,\ r \in R_i$$

$$y_i^r := \begin{cases} 1 & \text{if } \sum_{k \in K} z_{ik} \in [l_i^r, u_i^r], \\ 0 & \text{otherwise,} \end{cases} \qquad i \in S,\ r \in R_i$$

The SSP-QDTS can be formulated as follows:

$$(SSP\text{-}QDTS)\ \min \sum_{k \in K} \sum_{i \in S_k} \sum_{r \in R_i} p_{ik}^r z_{ik}^r + \sum_{i \in S} c_i x_i \tag{1}$$

$$\sum_{i \in S} z_{ik} = d_k, \quad k \in K \tag{2}$$

$$z_{ik} \leq q_{ik}, \quad i \in S,\ k \in K \tag{3}$$

$$\sum_{k \in K} z_{ik} \leq Q x_i, \quad i \in S \tag{4}$$

$$z_{ik} = \sum_{r \in R_i} z_{ik}^r, \quad i \in S,\ k \in K \tag{5}$$

$$l_i^r y_i^r \leq \sum_{k \in K} z_{ik}^r \leq u_i^r y_i^r, \quad i \in S,\ r \in R_i \tag{6}$$

$$\sum_{r \in R_i} y_i^r \leq 1, \quad i \in S \tag{7}$$

$$x_i \geq 0 \text{ integer}, \quad i \in S \tag{8}$$

$$y_i^r \in \{0,1\}, \quad i \in S,\ r \in R_i \tag{9}$$

$$z_{ik} \geq 0 \text{ integer}, \quad i \in S,\ k \in K \tag{10}$$

$$z_{ik}^r \geq 0 \text{ integer}, \quad i \in S,\ k \in K,\ r \in R_i \tag{11}$$

Objective function (1) represents the goal of minimizing the procurement costs, i.e., the sum of purchasing and truckload shipping costs. Constraints (2) guarantee that the demand d_k of product k is satisfied and constraints (3) ensure that the amount of product k purchased from supplier i is available at supplier i. Constraints (4) establish the number of visits to supplier i. Constraints (5)–(7) determine the discount intervals. Constraints (8)–(11) provide integer, binary, and non-negativity conditions on the variables.

Before discussing solution approaches, we make a few observations regarding the above formulation. The variables z_{ik} are required to be integer as they represent the number of units of product k purchased at supplier i. However, if product demand, product availability, vehicle capacity, and the discount intervals are integral, then there is no need to require integrality of the z variables as indicated in the following proposition:

Proposition 1. *If all the input data other than costs are integral and the continuous relaxation of SSP-QDTS has an optimal solution, then there exists an optimal solution in which all z variables have integer values.*

In Goossens et al. [16] it is shown that the continuous relaxation of the total quantity discount (TQD) problem can be obtained by solving a min-cost flow problem and that there exists an optimal solution which selects the highest discount interval for each supplier, i.e., products are bought at the lowest price offered by a supplier. This property also holds for the continuous relaxation of *SSP-QDTS* since constraints (3) and (4) will never force the optimal solution of the continuous relaxation away from the cheapest intervals. This gives the following proposition:

Proposition 2. *If the continuous relaxation of SSP-QDTS has an optimal solution, then there exists an optimal solution in which all z and y variables are equal to* 0*, except those corresponding to the highest interval of each supplier.*

The rounding methods to be described in the next section use the optimal solution to the continuous relaxation of *SSP-QDTS* to guide the search. To make sure that we can extract as much useful information from the relaxation as possible, we strengthen it by the following valid inequalities:

$$z_{ik} \leq \min\{q_{ik}, Q\} x_i, \quad \forall k \in K,\ \forall i \in S_k \tag{12}$$

$$\sum_{i \in S_k} x_i \geq \left\lceil \frac{d_k}{Q} \right\rceil, \quad \forall k \in K \tag{13}$$

$$\sum_{i \in S_k \cap S_l} x_i \geq \left\lceil \frac{d_k - \sum_{i \in S_k \setminus S_k \cap S_l} q_{ik} + d_l - \sum_{i \in S_l \setminus S_k \cap S_l} q_{il}}{Q} \right\rceil, \quad \forall k,l \in K, k \neq l \tag{14}$$

$$z_{ik}^r \leq q_{ik} y_i^r, \quad \forall i \in S,\ \forall k \in K,\ \forall r \in R_i \tag{15}$$

Constraints (12) simply disaggregate constraints (4). Constraints (13) ensure that the number of visits required to satisfy the demand d_k is at least equal to $\lceil d_k/Q \rceil$. Constraints (14) generalize this idea to sets of products. In our implementation, we only consider sets of at most two products. Constraints (15) reflect that if interval r is selected for a supplier i, then y_i^r has to be at least $\max_k\{z_{ik}^r/q_{ik}\}$. As there are only polynomially many constraints, they can be added to the formulation upfront, i.e., there is no need to develop separation routines to add them on the fly.

3. An iterative rounding scheme

The idea of rounding solutions to a linear programming relaxation to obtain integer feasible solutions has been around for some time and has been embedded in several approaches to solving pure and mixed integer programs. Eppen and Martin [13] use a rounding heuristic to obtain solutions to multi-item capacitated lot-sizing problems. Franz and Miller [14] use a rounding heuristic to obtain solutions to multi-period assignment problem. The same authors have developed a binary-rounding heuristic (BRH) for solving an especially difficult class of constrained multi-period assignment problems [21]. BRH iteratively rounds up sets of fractional values in such a way that feasibility of the rounded solution is guaranteed and the value of the objective function is improved. In general, guaranteeing feasibility of rounded solutions is hard, as observed early by Glover and Sommer [15]. The above approaches are all for problems with binary variables. Far fewer rounding heuristics have been reported on that round integer variables. An exception is Saltzman and Hillier [23] who present a heuristic for general integer programs which is based, in part, on rounding integer variables.

We propose various iterative rounding schemes which start from an optimal solution to the LP relaxation of the formulation presented in Section 2. At each iteration, an x-variable with a fractional value is selected, the value is rounded and fixed, and then the LP relaxation is resolved. When all x-variables in an optimal solution to the LP relaxation have integer values, all x-variables that were not yet fixed are now fixed, the remaining integer program is solved to optimality (recall that the y- and z-variables must have integer values as well). That is, we focus on deciding the number of visits to a supplier and, once those decisions have been made, determine the product quantities bought at the selected suppliers. The iterative rounding scheme is described in Fig. 1.

Since the number of visits to the selected suppliers is fixed during construction, it might happen that this number is too high given the final product quantities purchased. Therefore, a *post-processing* step is introduced to possibly reduce the number of trips to the selected suppliers.

As it is the case with all LP-based rounding schemes, the crucial issue is preserving feasibility. It is easy to see that a feasible solution to the LP relaxation remains feasible when the value of a fractional x-variable is rounded up. On the other hand, rounding down the value of a fractional x-variable may lead to infeasibility. Fortunately, we can check efficiently whether this happens or not by solving an auxiliary flow problem. Let (z^{LP}, y^{LP}, x^{LP}) be a feasible solution to the LP relaxation. Assume that the number of visits to the suppliers in $\overline{S} \subset S$ has already been fixed, i.e., $x_j = \overline{x}_j$, $j \in \overline{S}$, with $\overline{x}_j$ an integer value.

We want to evaluate whether it is feasible to set $x_i = \lfloor x_i^{LP} \rfloor$ $(i \in S \setminus \overline{S})$. This can be done by solving an instance of a flow problem. Introduce a source node with supply equal to $\sum_{k \in K} d_k$, a supplier node for each supplier $j \in S$, and a sink node for each product $k \in K$ with demand equal to d_k. There is an arc between the source node and each supplier node j with capacity $\overline{x}_j Q$ if $j \in \overline{S}$, $\lfloor x_i^{LP} \rfloor Q$ if $j = i$, and $\sum_{k \in K} q_{jk}$ otherwise. There is an arc between a supplier node j and each product node k with capacity q_{jk} (see Fig. 2). If there exists a feasible flow, then x_i can be set to $\lfloor x_i^{LP} \rfloor$, otherwise it will have to be rounded up to preserve feasibility.

Next, we return to the iterative rounding scheme outlined in Fig. 1. Depending on the *sorting rule* used to order the suppliers, and thus to identify the x-variable to be rounded next, and the *rounding rule* used to assign an integer value to the selected x-variable, different heuristics can be defined.

Table 1 lists the sorting and rounding rules that we have explored. The sorting is done implicitly; we simply find the

INPUT:
problem instance
OUTPUT:
integer feasible solution s with value $v(s)$.

ITERATIVE ROUNDING SCHEME:
Set $S' := \emptyset$;
Solve the LP relaxation and let S^{LP} be the set of suppliers associated with non-zero x_i values;
while $(S^{LP} \setminus S' \neq \emptyset)$ **do**
    Let i be the first supplier in $S^{LP} \setminus S'$ after sorting S^{LP} according to a *sorting rule*;
    Set $S' := S' \cup \{i\}$;
    Set the value of variable x_i according to a *rounding rule*;
    Solve the LP relaxation and let S^{LP} be the set of suppliers associated with non-zero x_i values;
end while
Solve the resulting mixed integer program.
Let s^I be its solution and $v(s^I)$ its value.
$s := PostProcessing(s^I)$;
Return solution s and its value $v(s)$.

Fig. 1. Pseudo-code of the iterative rounding scheme.

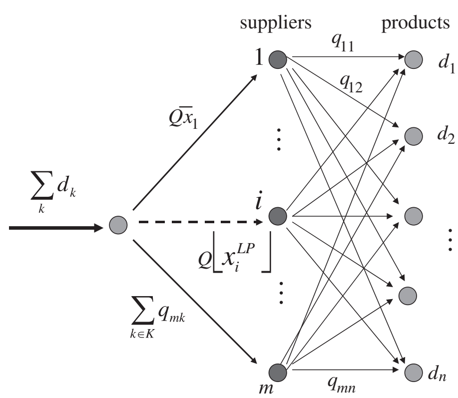

Fig. 2. The auxiliary flow problem.

Table 1
Sorting and rounding rules.

Sorting rules	
SU:	$i^* := \text{argmin}_i\{\lceil x_i^{LP} \rceil - x_i^{LP}\}$
SD:	$i^* := \text{argmin}_i\{x_i^{LP} - \lfloor x_i^{LP} \rfloor\}$
SC:	$i^* := \text{argmin}_i\{\delta_i\}$ with $\delta_i = \min\{x_i^{LP} - \lfloor x_i^{LP} \rfloor, \lceil x_i^{LP} \rceil - x_i^{LP}\}$
SH:	$i^* := \text{argmax}_i\{x_i^{LP}\}$
Rounding rules	
RU (RU-I):	$x_{i^*} := \lceil x_{i^*}^{LP} \rceil$ $(x_{i^*} \geq \lceil x_{i^*}^{LP} \rceil)$
RD(RD-I):	$x_{i^*} = \lfloor x_{i^*}^{LP} \rfloor$ if feasible otherwise round up $(x_{i^*} \leq \lfloor x_{i^*}^{LP} \rfloor)$
RC (RC-I):	$x_{i^*} =$ closest integer
RD-Iy:	As RD-I, but if $x_{i^*} \leq \lfloor x_{i^*}^{LP} \rfloor$ then compute the minimum s such that $l_{i^*}^s > Q\lfloor x_{i^*}^{LP} \rfloor$ and set $\sum_{r=s}^{r_i} y_{i^*}^r = 0$, whereas if $x_{i^*} \geq \lceil x_{i^*}^{LP} \rceil$ compute the largest s such that $u_{i^*}^s < Q\lceil x_{i^*}^{LP} \rceil$ and set $\sum_{r=1}^{s} y_{i^*}^r = 0$
Rvalue:	Let s be the interval to which $x_{i^*}^{LP}Q$ belongs. If $\lceil x_{i^*}^{LP} \rceil Q > l_{i^*}^{s+1}$ or $\lfloor x_{i^*}^{LP} \rfloor Q < u_{i^*}^{s-1}$ rounds variable up otherwise rounds it down checking feasibility.

variable that would be selected after sorting (indexed by i^*). The first letter of each acronym refers to the type of rule: S stands for sorting rule and R stands for rounding rule. The second letter indicates the type of sorting or rounding. When U is associated with a sorting rule it means that the fractional variable closest to its rounded up value is selected, and when U is associated with a rounding rule it means the value is rounded up. Similarly, when D is associated with a sorting rule it means that the fractional variable closest to its rounded down value is selected, and when D is associated with a rounding rule it means the value is rounded down. Recall that before a value is rounded down, an auxiliary flow problem is solved to ensure that doing so preserves feasibility; if feasibility cannot be preserved, the variable is rounded up. When C is associated with a sorting rule it means that the fractional variable closest to its rounded value (regardless of direction) is selected, and when C is associated with a rounding rule it means the value is rounded (regular rounding). An H with a sorting rule means that we select the supplier for which the (fractional) number of visits is the highest. The variants "I" and "Iy" associated with rounding rules indicate that instead of fixing the value of a variable, a bound inequality is added (variant I) and that the value of some y-variables is set to zero (variant Iy). Finally, the variant "Rvalue" for a rounding rule evaluates the impact of the rounding on the discount interval. If by rounding up or down the discount interval changes, we round up; otherwise, we round down provided that feasibility is preserved.

All these sorting and rounding rules can be efficiently implemented. We experimented with various other more involved rules, but the benefits in terms of quality were too small to warrant the substantial increase in computation time. The following combinations gave the best results and are used in computational study: (SU,RU), (SU,RU-I), (SD,RD), (SD,RD-I), (SD,RD-Iy), (SC,RC), and (SH,Rvalue).

In our computational study, we will also use, for comparison purposes, the simplest possible rounding heuristic (`BasicRounding`):

Solve the LP relaxation; let the optimal solution be (x^{LP},y^{LP},z^{LP}).
Let $S_0 = \{i \mid x_i^{LP} = 0\}$ and $S_1 = \{i \mid x_i^{LP} > 0\}$.
Fix all variables in S_0 to 0 and all variables in S_1 to $\lceil x_i^{LP} \rceil$.
Solve resulting IP.

Although rounding up all fractional variables at once may seem to be too aggressive, the following example shows that heuristic H(SU,RU) does not always do better.

Example. Consider an instance with two products A and B with $d_A=3$, $d_B=6$ and $Q=3$. The remaining data are shown in the following table:

	Transportation cost	Discount (%)	Interval bounds	Product A		Product B	
				Quantity	Price	Quantity	Price
Supplier 1	2	20–40	0–7–10	1	1	8	2
Supplier 2	1	20–40	0–4–9	2	1	7	3

The LP relaxation has optimal solution $x_1=2.33$, $x_2=0.67$, $z_{1A}=1$, $z_{1B}=6$, and $z_{2A}=2$. The objective value is 14.33 with transportation cost being 5.33 and purchasing cost being 9. Heuristic (SU,RU) in the first iteration will set $x_2=1$ and then, after solving the new LP relaxation, it will set $x_1=2$. Then the resulting mixed integer program gives $z_{1A}=1$, $z_{1B}=5$, $z_{2A}=2$, and $z_{2B}=1$ with objective value 21. On the other hand, `BasicRounding` sets $x_1=3$ and $x_2=1$, and then solves the resulting mixed integer program obtaining $z_{1A}=1$, $z_{1B}=6$, $z_{2A}=2$ and objective value 7 + 12.4=19.4.

3.1. Local improvement

A relatively simple improvement scheme, again relying on solving small integer programs, was found to be quite powerful and added as a post-processing step. Let the set of suppliers used in the solution be denoted by H. For each supplier in H, we solve an integer program in which the number of visits to that supplier is forced to be at least one more than in the current solution, and the number of visits to all other suppliers can be at most the same as in the current solution. The suppliers in H are evaluated in order of non-increasing unit procurement cost, defined as the ratio of the transportation cost of a visit to the supplier and the total product quantity we can purchase from the supplier, and we continue to evaluate suppliers as long as improved solutions are found, where we ensure that each supplier is evaluated at least once.

4. Computational study

The objective of our computational study is two-fold. First, we want to evaluate the performance of the proposed iterative rounding heuristics. Second, we want to gain insight into how the characteristics of the environment influence the structure of solutions.

There are a few key trade-offs that can be studied using SSP-QDTS: (1) transportation costs versus purchase costs and (2) individual item price versus total quantity discounts. Depending on the total quantity discounts offered, it may not be best to purchase a particular product from the supplier offering the lowest selling price for that product. Similarly, depending on the transportation costs, it may not necessarily be best to purchase a set of products from the supplier offering the lowest total selling price for that set of products. There are a number of factors that influence these trade-offs. For example, the distance to the supplier and the vehicle capacity clearly impacts the transportation costs, and the number of products offered and the discount scheme applied by a supplier clearly impacts the purchase costs. However, there are more subtle and intricate interactions. For example, if a supplier offers a large number of products and has a favorable discount scheme, it may be an attractive supplier even if he is located far from the purchaser.

To be able to carry out our analysis, we have created a set of randomly generated instances with various characteristics. There are two base instances and several variations derived from these base instances. The two base instances consist of a single purchaser, 49 suppliers, and 50 products. The suppliers are located in an area of size $[0, 10\,000] \times [0, 10\,000]$ with the purchaser located at the center. In the first base instance suppliers are uniformly distributed (*instance uniform*). In the second base instance suppliers are in bands around the purchaser at certain distances (*instance bands*). Fig. 3 shows the distribution of the distance of the suppliers to the purchaser in the two base instances (grouped in 10 buckets of length 500). Note that in instance bands, the suppliers are concentrated in only five buckets.

The other characteristics of the base instances are generated as follows. Each product k is offered from at least 40% of the suppliers. The set S_k of suppliers offering product k is randomly chosen. The quantity $q_{ik} \in \mathbb{Z}$ of product k available at supplier i is randomly chosen in the interval $[1+\beta_i q_{max}, q_{max}]$, where $q_{max}=500$ and $0 \le \beta_i < 1$. The value β_i depends on the distance of supplier i to the purchaser, with a supplier further away from the purchaser having a larger value β_i, which implies that such suppliers are more likely to have a larger product availability. Demand d_k for each product k is set to $\lfloor \lambda_k \sum_{i \in S_k} q_{ki} \rfloor$ with λ_k randomly generated in the interval [0.4,1). All suppliers have three discount intervals. The interval bounds are specified as a fraction of the total quantity available at a supplier, i.e., $u_i^r = \delta_r(\sum_{k \in K} q_{ik})$ for $r=1,2$, and 3 with $\delta_1=0.15, \delta_2=0.85$, and $\delta_3=1$ and $l_i^1=0$ and $l_i^r = u_i^{r-1}+1$ for $r=2$ and 3. Discount rates are the same for all products and all suppliers

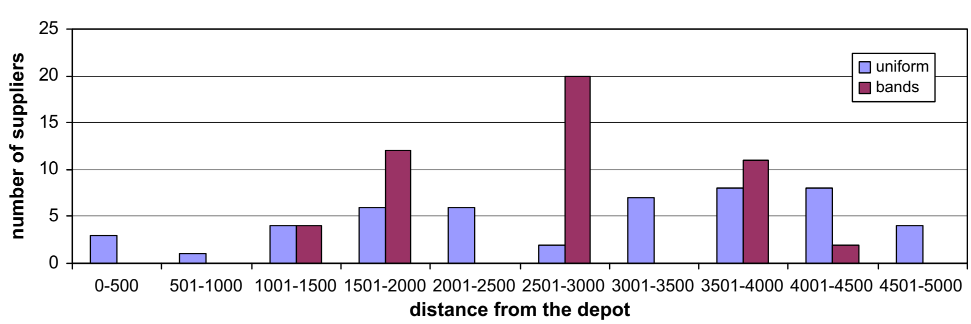

Fig. 3. Basic instances: suppliers distribution.

and are equal to 0, 5% and 35%. Prices for each interval are computed as $p_{ik}^r = (1-f^r)p_{ik}$, where f^r is the discount rate applied in interval r, and prices p_{ik} are randomly generated in the interval $[1,30]$. Finally, the truckload transportation costs c_i for supplier i are equal to the truncated Euclidean distance from the purchaser to supplier i, and the truck capacity Q is set to 500.

As mentioned above, several variations are derived from the two base instances. More specifically, for each of the base instances we create several variations:

- *Distance set* (D): The transportation costs are modified by setting $c_i^{new} = const \times c_i$ with $const$=2, 5, 7 and 10, i.e., four additional instances D1, D2, D3 and D4.
- *Capacity set* (C): The vehicle capacity is modified by setting Q^{new}=1000, 1500, 2000, and 2500, i.e., four additional instances C1, C2, C3, and C4.
- *Product set* (P): The number of products is limited to 5, 10, 15, 30, and 40, i.e., five additional instances P1, P2, P3, P4, and P5. The construction process randomly selects a subset of products and eliminates all other products from the instance. The construction process is such that the set of products offered in Pi is a subset of the products offered in Pi+1 and it ensures that each supplier offers at least one product. (Note that each supplier offers a subset of the i products in instance Pi.)
- *Interval bounds set* (I): The interval bounds are modified by setting $(\delta_1,\delta_2,\delta_3) = (0.05, 0.10, 1)$, (0.33, 0.66, 1), and (0.90, 0.95, 1), i.e., three additional instances I1, I2, and I3.
- *Discount supplier set* (S): Only a randomly chosen subset of size k of the suppliers offers total quantity discounts, with k=5, 15, 25, and 35, i.e., four additional instances S1, S2, S3, and S4.
- *Rate set* (R): The discount rates are modified by setting $(f^2,f^3) =$ (0%,25%), (0%,35%), (5%,25%), (10%,25%), and (10%,35%), i.e., five additional instances R1, R2, R3, R4, and R5. Note that setting the discount rate for the second interval f^2 to 0% effectively eliminates that interval.

4.1. Performance analysis

All the heuristics as well as the local search procedures have been coded in Visual C++ using CPLEX 10.1 and Concert Technology 2.3. Before analyzing the performance of the heuristics and the local search procedures, we investigate the benefits of the inequalities introduced to strengthen the formulation. We present results for three formulations: (a) the basic problem formulation, (b) the basic formulation plus the classes of inequalities (13) and (15), and (c) the basic formulation plus all four classes of inequalities. The second formulation was included because initial experiments indicated that these two classes of inequalities were the most effective.

More precisely, in Table 2, we present, for each formulation, the value of the linear programming relaxation and the value of the best feasible solution found in 5 h of computing. We present these values relative to the value of the linear program relaxation of the basic formulation plus all four classes of inequalities. That is, if z represents the value of the linear program relaxation of the basic formulation plus all four classes of inequalities, then the value v of a linear programming relaxation is presented as $100 \times (z-v)/z$ (LP (%)) and the value v of a feasible solution is presented as $100 \times (v-z)/z$ (IP (%)). In addition, we provide the computing time and whether or not an instance was solved to optimality (1), whether the time limit of 5 h was reached (2), or whether CPLEX ran out of memory before completing the search (3).

The results show, first and foremost, that many of these instances are difficult and cannot be solved within 5 h of computing. Furthermore, we see that instances where locations are uniformly distributed are, on average, easier to solve than instances with locations in bands around the purchaser, as more instances can be solved to optimality within 5 h of computing.

When trying to solve instances to optimality, it is not clear that a stronger formulation results in better performance as it comes at the price of many more constraints and thus an increase in linear program solve times. The situation is different when using heuristics to produce high-quality solutions quickly as there are fewer linear program solves. We found that using the strongest formulation (with all four classes of inequalities) works best when rounding heuristics are used without local search, but that when rounding heuristics are combined with local search the basic formulation plus the classes of inequalities (13) and (15) gives the best performance, and thus these settings are used in the remaining experiments.

In Tables 3 and 4, we present the results for the three best rounding heuristics (in terms of average performance) and the four best rounding heuristics combined with local search, respectively. As in Table 2, we present solution values relative to the value of the linear program relaxation of the basic formulation plus all four classes of inequalities. For each group of instances, we show the minimum, the average, and the maximum deviation as well as the minimum, average and maximum solution time (in s). Group D includes the base instance. The additional column labeled # in Table 4 gives the number of instances for which the solution value is as good or better (in parentheses) than the value of the best integer solution found by CPLEX in 5 h of computing. (None of the solutions produced by just the rounding heuristics improved upon the integer solution found by CPLEX in 5 h of computing.)

The results demonstrate unequivocally that the heuristic H(SU,RU-I) performs best as a pure iterative rounding heuristic. The results are not as clear cut when the rounding heuristics are combined with local search, but H(SU,RU-I) still performs best overall, always ending up among the best four heuristics (see Table 4). We also see that for several instances H(SU,RU-I) produces a solution with the value as the solution obtained by CPLEX in 5 h of computing (and does so an order of magnitude faster) and in some cases even a *better* one. Because H(SU,RU-I) as a pure iterative rounding heuristic provides the best trade-off in terms of quality of the solution and time required to produce a solution, we have chosen to use it in our in-depth analysis of how instance characteristics influence the structure of high-quality solutions.

4.2. Structural analysis

As mentioned in Introduction, one of the main objectives of our work is to gain insight into the structure of effective procurement strategies in varying environments. Tables 5 and 6 show various characteristics of the solutions obtained by heuristic H(SU,RU-I) for all instances with locations in bands around the purchaser and all instances where locations are uniformly distributed, respectively. More specifically, we report the per unit total cost and its breakdown into purchasing and transportation costs (columns “per unit cost”, “Purchase (%)”, and “Transport (%)”), the savings resulting for total quantity discounts (column “Saving (%)”), the number of suppliers used, the average distance to the suppliers used, and the corresponding standard deviation (columns “#”, “Avg. distance” and “Std. dev.”), the total number of trips and the percentage out of it of partially loaded trips (columns “total trips” and “non-full trips (%)”), the share of the total demand purchased from the supplier from which we decided to purchase the most, the share of the total demand purchased from the five suppliers from which we decided to purchase the most, the share of the total demand purchased from the supplier from which we decided to purchase the least, the share of the total demand purchased from the five suppliers from which

Table 2
CPLEX (within 5 h): time and percentage deviation from LP with all inequalities values.

Instances	Bands											Uniform										
	No ineq.				2-ineq				All-ineq.			No ineq.				2-ineq				All-ineq.		
	LP (%)	**IP** (%)	**Time** (s)	#	**LP** (%)	**IP** (%)	**Time** (s)	#	**IP** (%)	**Time** (s)	#	**LP** (%)	**IP** (%)	**Time** (s)	#	**LP** (%)	**IP** (%)	**Time** (s)	#	**IP** (%)	**Time** (s)	#
Base	5.236	0.909	18 001	(2)	0.005	0.909	18 004	(2)	0.909	18 004	(2)	7.794	0.761	1344	(1)	0.000	0.761	6188	(3)	0.761	880	(1)
D1	4.812	0.999	18 001	(2)	0.001	0.999	18 002	(2)	0.999	18 002	(2)	7.460	0.884	18 001	(2)	0.003	0.884	18 023	(2)	0.884	18 066	(2)
D2	3.732	0.597	18 009	(2)	0.000	0.625	18 001	(2)	0.640	13 667	(3)	5.622	0.894	18 002	(2)	0.000	0.894	18 002	(2)	0.894	16 388	(3)
D3	3.194	7.000	15 521	(3)	0.001	0.424	11 409	(3)	0.424	11 409	(3)	4.814	0.676	5013	(1)	0.003	0.676	4834	(1)	0.676	7748	(1)
D4	2.616	0.308	18 106	(2)	0.001	0.308	18 003	(2)	0.308	18 003	(2)	3.970	0.543	3299	(1)	0.007	0.543	7800	(1)	0.543	4702	(1)
C1	5.164	1.179	18 001	(2)	0.005	1.342	10 215	(3)	1.178	18 005	(2)	7.769	1.380	18 002	(2)	0.003	1.380	18 004	(2)	1.380	18 003	(2)
C2	4.887	1.680	18 001	(2)	0.040	1.680	18 003	(2)	1.688	18 002	(2)	7.538	1.699	18 001	(2)	0.022	1.698	18 002	(2)	1.738	18 003	(2)
C3	4.895	1.256	18 001	(2)	0.090	1.256	18 011	(2)	1.256	18 002	(2)	7.504	1.655	18 000	(2)	0.025	1.645	18 001	(2)	1.645	18 002	(2)
C4	4.948	1.524	18 001	(2)	0.140	1.440	18 001	(2)	1.440	18 002	(2)	7.521	1.868	18 007	(2)	0.031	1.872	18 003	(2)	1.888	18 000	(2)
P1	2.127	7.002	2459	(1)	0.251	7.002	107	(1)	7.002	93	(1)	3.930	5.345	22	(1)	0.615	5.345	5	(1)	5.345	4	(1)
P2	2.402	2.331	1216	(1)	0.000	2.331	1783	(1)	2.331	579	(1)	7.928	3.477	321	(1)	0.133	3.477	345	(1)	3.477	581	(1)
P3	2.676	2.096	18 012	(2)	0.000	2.096	5582	(1)	2.096	8545	(1)	6.535	2.204	1247	(1)	0.067	2.204	147	(1)	2.204	42	(1)
P4	5.049	1.435	18 009	(2)	0.002	1.435	12 019	(1)	1.435	18 001	(2)	6.680	1.258	411	(1)	0.007	1.258	81	(1)	1.258	215	(1)
P5	5.840	1.129	18 002	(2)	0.002	1.108	18 009	(2)	1.108	18 002	(2)	6.630	1.077	3006	(1)	0.004	1.077	1969	(1)	1.077	1242	(1)
I1	0.015	0.045	11 039	(3)	0.000	0.045	11 780	(3)	0.045	11 131	(3)	0.007	0.084	5380	(1)	0.000	0.084	18 063	(2)	0.084	1561	(1)
I2	2.017	0.380	1077	(1)	0.007	0.380	1162	(1)	0.380	1847	(1)	3.992	0.670	2544	(1)	0.004	0.670	792	(1)	0.670	639	(1)
I3	11.069	1.139	6723	(3)	0.005	1.137	4497	(3)	1.137	5588	(3)	14.653	0.989	2920	(3)	0.002	0.987	4268	(3)	0.987	10 368	(3)
S1	0.006	0.015	37	(1)	0.006	0.015	5	(1)	0.015	66	(1)	0.236	0.084	643	(1)	0.001	0.084	9	(1)	0.084	11	(1)
S2	0.990	0.122	9916	(1)	0.005	0.122	18	(1)	0.122	60	(1)	1.964	0.182	24	(1)	0.007	0.182	108	(1)	0.182	64	(1)
S3	2.751	0.448	5783	(3)	0.001	0.421	3184	(3)	0.421	207	(1)	5.172	0.452	496	(1)	0.003	0.452	875	(1)	0.487	3204	(3)
S4	4.169	0.706	16 026	(3)	0.005	0.708	18 002	(2)	0.706	18 004	(2)	6.916	0.623	2115	(1)	0.000	0.629	3998	(3)	0.623	7848	(1)
R1	3.847	0.658	18 002	(2)	0.007	0.658	18 005	(2)	0.658	18 002	(2)	5.410	0.583	18 003	(2)	0.002	0.583	7833	(3)	0.583	17 249	(3)
R2	5.216	0.886	18 001	(2)	0.006	0.886	13 694	(1)	0.886	18 002	(2)	7.767	0.713	8724	(1)	0.002	0.713	5713	(1)	0.713	12 221	(1)
R3	3.865	0.681	18 003	(2)	0.005	0.688	18 006	(2)	0.681	18 007	(2)	5.460	0.583	1244	(1)	0.000	0.583	5942	(1)	0.583	213	(1)
R4	3.850	0.695	14 460	(3)	0.000	0.683	18 006	(2)	0.683	18 003	(2)	5.486	0.583	106	(1)	0.000	0.583	603	(1)	0.583	271	(1)
R5	5.228	0.919	18 003	(2)	0.001	0.919	8131	(1)	0.919	18 002	(2)	7.795	0.790	1159	(1)	0.000	0.790	5071	(1)	0.790	639	(1)

Table 3
Heuristics without LS: minimum, average and maximum time and percentage deviation from LP with all inequalities values.

Instances Set	Heuristics	Bands Gap (%) Min	Avg	Max	Time (s) Min	Avg	Max	Heuristics	Uniform Gap (%) Min	Avg	Max	Time (s) Min	Avg	Max
D	H(SU,RU-I)	0.682	1.172	1.647	34	60.60	117	H(SU,RU-I)	0.768	1.120	1.528	30	46.00	90
	H(SH,Rvalue)	0.591	1.906	4.184	55	72.40	111	H(SH,Rvalue)	1.183	1.712	3.442	50	67.20	118
	BasicRounding	1.325	2.212	3.116	6	7.00	10	BasicRounding	1.567	2.027	2.344	5	6.80	10
C	H(SU,RU-I)	1.549	1.996	2.363	101	300.75	560	H(SU,RU-I)	2.078	2.306	2.810	145	282.75	502
	H(SH,Rvalue)	2.509	3.613	6.320	147	172.00	196	H(SH,Rvalue)	1.863	4.301	6.182	123	159.25	188
	BasicRounding	4.707	6.652	8.764	10	10.75	11	BasicRounding	6.826	7.986	9.176	7	9.00	10
P	H(SU,RU-I)	1.935	4.349	9.057	3	24.00	60	H(SU,RU-I)	1.501	3.497	6.175	1	15.80	38
	H(SH,Rvalue)	2.347	4.578	8.344	5	27.40	63	H(SH,Rvalue)	2.973	4.534	5.851	3	26.20	62
	BasicRounding	2.774	5.243	9.660	0	2.40	6	BasicRounding	3.383	5.629	8.064	0	1.60	4
I	H(SU,RU-I)	0.123	1.325	2.498	17	66.00	139	H(SU,RU-I)	0.385	0.943	1.281	16	39.33	69
	BasicRounding	0.507	3.685	9.105	3	6.67	12	BasicRounding	0.875	2.432	4.211	3	5.33	8
	H(SH,Rvalue)	0.160	3.889	10.154	33	55.00	83	H(SH,Rvalue)	0.337	2.605	4.098	32	62.67	108
S	H(SU,RU-I)	0.020	0.597	1.239	27	39.50	61	H(SU,RU-I)	0.130	0.560	1.018	21	27.75	38
	H(SH,Rvalue)	0.093	1.061	1.832	51	60.50	78	H(SU,RU)	0.130	0.927	2.124	20	23.25	27
	BasicRounding	0.304	1.212	2.153	4	5.50	7	H(SH,Rvalue)	0.206	0.974	1.982	46	52.50	66
R	H(SU,RU-I)	1.049	1.127	1.220	78	94.40	107	H(SU,RU-I)	0.750	0.866	1.036	89	101.20	109
	BasicRounding	1.927	2.801	4.045	9	10.40	12	BasicRounding	1.760	2.229	2.937	8	9.00	10
	H(SH,Rvalue)	2.458	2.944	4.236	94	106.80	119	H(SH,Rvalue)	1.740	2.451	3.423	97	114.80	140

Table 4
Heuristics with LS: minimum, average and maximum time and percentage deviation from LP with all inequalities values.

Instances Set	Heuristics	Bands Gap (%) Min	Avg	Max	Time (s) Min	Avg	Max	#	Heuristics	Uniform Gap (%) Min	Avg	Max	Time (s) Min	Avg	Max	#
D	H(SU,RU-I)	0.308	0.656	0.999	240	2102.4	5271	3(1)	H(SH,Rvalue)	0.543	0.759	0.910	254	928.2	2506	3(0)
	H(SU,RU)	0.308	0.661	1.037	243	1412.8	2816	3(1)	BasicRounding	0.573	0.762	0.910	239	679.0	1778	2(0)
	H(SD,RD)	0.308	0.662	1.031	322	2689.2	5737	1(1)	H(SD,RD)	0.586	0.777	0.941	159	534.0	1425	1(0)
	H(SH,Rvalue)	0.308	0.663	1.037	265	1510.6	3351	2(1)	H(SU,RU-I)	0.543	0.803	0.941	190	703.0	1853	2(0)
C	H(SU,RU-I)	1.253	1.555	1.947	998	1687.0	3189	0(0)	H(SU,RU-I)	1.418	1.791	2.093	422	875.2	1517	1(1)
	H(SC,RC)	1.253	1.625	1.945	940	2353.2	5015	0(0)	BasicRounding	1.826	2.110	2.569	585	1368.2	2698	0(0)
	H(SU,RU)	1.253	1.651	1.931	998	2540.2	5065	0(0)	H(SU,RU)	1.489	2.206	2.652	628	1478.7	3430	0(0)
	H(SD,RD)	1.253	1.687	2.021	992	2273.2	5131	0(0)	H(SC,RC)	1.489	2.242	2.765	480	1312.7	3087	0(0)
P	H(SH,Rvalue)	1.108	2.913	7.506	7	1181.6	3747	2(0)	H(SU,RU)	1.077	2.734	5.345	2	257.0	1005	3(0)
	BasicRounding	1.108	3.169	7.965	5	990.6	2727	1(0)	BasicRounding	1.077	2.734	5.345	2	313.2	1337	3(0)
	H(SC,RC)	1.122	3.170	7.990	6	772.4	2254	0(0)	H(SU,RU-I)	1.077	2.769	5.345	3	252.4	928	3(0)
	H(SU,RU-I)	1.122	3.196	7.990	4	821.0	2751	0(0)	H(SC,RC)	1.077	2.769	5.345	4	255.2	902	3(0)
I	H(SU,RU-I)	0.046	0.527	1.139	87	1353.6	3769	0(0)	H(SU,RU-I)	0.084	0.609	0.984	89	721.6	1769	2(1)
	BasicRounding	0.045	0.986	2.510	66	942.6	2516	1(0)	H(SH,Rvalue)	0.084	0.933	1.993	119	914.0	2173	1(0)
	H(SH,Rvalue)	0.046	1.362	3.637	65	1481.0	4130	0(0)	H(SC,RC)	0.084	1.013	1.993	90	853.3	2033	1(0)
	H(SC,RC)	0.038	1.796	4.954	82	2115.7	5713	1(0)	BasicRounding	0.084	1.014	1.993	97	659.3	1512	1(0)
S	H(SH,Rvalue)	0.016	0.321	0.706	157	605.5	1783	2(0)	H(SC,RC)	0.084	0.343	0.623	93	270.0	610	3(0)
	H(SU,RU)	0.015	0.323	0.718	127	776.0	2586	1(0)	H(SD,RD)	0.084	0.343	0.623	103	314.0	726	3(0)
	H(SC,RC)	0.015	0.323	0.718	122	865.0	2922	1(0)	BasicRounding	0.084	0.345	0.623	98	245.7	581	3(0)
	H(SU,RU-I)	0.015	0.330	0.720	128	1002.0	3347	1(0)	H(SU,RU-I)	0.086	0.348	0.623	84	224.7	415	1(0)
R	H(SU,RU-I)	0.661	0.771	0.925	1032	1645.2	2257	2(0)	H(SD,RD-ly)	0.583	0.651	0.790	904	1296.8	2496	5(0)
	H(SU,RU)	0.671	0.776	0.947	1272	1996.4	2673	3(0)	H(SC,RC)	0.583	0.651	0.790	876	1425.4	3302	5(0)
	H(SD,RD)	0.683	0.795	0.947	1068	2440.8	3867	3(0)	H(SH,Rvalue)	0.583	0.652	0.790	607	1418.0	2464	4(0)
	BasicRounding	0.683	0.799	0.965	2785	3557.0	4009	1(0)	H(SU,RU-I)	0.583	0.654	0.803	802	1371.4	2543	3(0)

we decided to purchase the least (columns "Max share", "Top 5 share", "Min share", and "Bottom 5 share"), the number of times a purchase at a supplier occurred in a particular discount interval (columns "Int. 1", "Int. 2", and "Int. 3"), and finally, the share of the total demand purchased with a discount being applied and without a discount (columns "q.ty disc. (%)" and "q.ty no disc. (%)").

Instances in Class D allow us to analyze the effect of increasing the per-unit transportation costs. As expected, the transportation costs as a fraction of the total costs increase as the per-unit transportation cost increases. However, somewhat surprisingly, the number of suppliers and their average distance do not seem to decrease. That is, an increase in per-unit transportation cost does not necessarily lead to a concentration of purchasing with fewer and closer suppliers.

A more in-depth analysis does reveal some changes: the concentration of purchases is primarily aimed at reducing the

Table 5
H(SU,RU-I) statistics: bands instances.

Prob.	Per unit cost	Purchase (%)	Transport (%)	Saving (%)	#	Avg. distance	Std. dev.	Total trips	Non-full (%)	Max share	Top 5 share	Min share	Bottom 5 share	Int. 1	Int. 2	Int. 3	q.ty disc. (%)	q.ty no disc (%)
Basic	12.73	67.66	32.34	34.24	34	2195.97	588.48	443	4.29	5.99	26.91	0.14	1.52	7	8	19	97.10	2.90
D1	16.78	52.26	47.74	33.68	34	2195.97	588.48	445	5.39	5.99	26.02	0.07	1.08	5	10	19	98.92	1.08
D2	28.30	32.96	67.04	29.82	34	2184.00	576.03	435	0.92	6.68	26.19	0.23	2.07	7	11	16	96.54	3.46
D3	36.07	25.59	74.41	30.69	34	2184.00	576.03	440	2.73	6.84	25.96	0.23	2.07	6	11	17	97.23	2.77
D4	47.22	19.89	80.11	29.68	34	2184.00	576.03	437	1.83	6.84	26.57	0.23	2.07	7	11	16	96.54	3.46
C1	10.73	77.39	22.61	35.32	33	2270.12	742.94	230	12.61	5.99	27.27	0.02	1.36	5	9	19	98.64	1.36
C2	10.00	83.18	16.82	35.16	33	2393.30	781.23	158	16.46	6.14	27.31	0.24	1.99	10	4	19	94.56	5.44
C3	9.53	86.34	13.66	36.30	32	2419.66	796.60	120	25.83	5.98	27.14	0.07	1.34	5	7	20	98.46	1.54
C4	9.34	87.74	12.26	36.38	32	2494.28	826.23	102	31.37	5.82	26.98	0.03	0.93	6	6	20	98.72	1.28
P1	10.77	59.68	40.32	37.22	19	1885.16	542.10	49	34.69	9.86	41.72	1.39	13.19	0	2	17	100.00	0.00
P2	12.41	65.20	34.80	36.98	23	2051.39	654.02	92	15.22	8.13	33.37	0.93	9.03	0	3	20	100.00	0.00
P3	12.29	65.46	34.54	36.61	26	2091.58	652.17	140	15.71	6.87	29.59	0.46	4.61	1	5	20	99.22	0.78
P4	12.77	67.98	32.02	34.03	33	2185.94	594.46	261	8.43	6.40	28.90	0.12	1.90	4	11	18	98.79	1.21
P5	12.94	68.07	31.93	33.65	34	2195.97	588.48	347	6.63	6.25	27.34	0.10	1.53	5	11	18	98.47	1.53
I1	11.94	67.79	32.21	38.27	33	2176.42	586.35	435	1.15	6.84	28.04	0.46	2.77	0	1	32	100.00	0.00
I2	12.34	66.83	33.17	36.85	33	2202.18	623.09	441	2.49	6.84	25.53	0.23	1.61	8	0	25	94.90	5.10
I3	13.75	70.11	29.89	27.21	36	2255.44	625.47	444	4.95	6.65	27.73	0.23	1.15	22	0	14	63.37	31.90
S1	15.89	74.05	25.95	9.82	36	2286.36	664.42	434	0.23	6.68	26.73	0.23	1.15	8	19	9	23.27	76.73
S2	14.69	65.25	34.75	26.26	34	2377.91	826.50	437	1.60	5.76	27.34	0.18	1.12	5	15	14	64.98	35.02
S3	13.79	64.77	35.23	31.82	33	2235.15	674.84	440	3.64	5.53	26.04	0.03	1.36	6	9	18	83.12	16.88
S4	13.39	65.59	34.41	32.40	33	2223.55	647.96	442	4.07	5.99	27.42	0.23	1.97	5	10	18	87.07	12.93
R1	14.37	71.32	28.68	21.78	36	2273.25	654.82	447	5.82	5.97	26.50	0.18	1.10	10	7	19	86.22	13.78
R2	13.18	68.76	31.24	30.83	35	2237.20	627.89	442	4.07	5.99	26.91	0.23	1.38	8	8	19	87.01	12.99
R3	13.86	70.37	29.63	25.61	35	2247.69	632.63	447	5.59	5.97	26.50	0.23	1.15	8	8	19	96.83	3.17
R4	13.34	69.31	30.69	29.55	33	2185.91	594.42	447	5.59	5.97	26.50	0.19	2.17	4	10	19	98.52	1.48
R5	12.29	66.62	33.38	37.71	33	2171.64	589.23	443	4.29	5.99	26.96	0.18	1.89	5	9	19	98.11	1.89

Table 6
H(SU,RU-I) statistics: uniform instances.

Problem	Per unit cost	Purchase (%)	Travel (%)	Saving (%)	#	Avg. distance	Std. dev.	Total trips	Non-full (%)	Max share	Top 5 share	Min share	Last 5 share	Int. 1	Int. 2	Int 3	q.ty disc. (%)	q.ty no disc (%)
Basic	13.16	72.09	27.91	32.22	31	2200.13	1118.12	374	4.81	5.72	26.65	0.27	3.54	3	11	17	98.36	1.64
D1	16.69	58.41	41.59	31.34	32	2207.28	1057.95	374	5.08	5.73	26.06	0.27	2.73	5	10	17	97.27	2.73
D2	26.90	37.09	62.91	29.96	31	2157.52	1037.36	372	4.03	5.45	26.28	0.27	4.36	2	13	16	98.91	1.09
D3	33.54	30.32	69.68	28.72	31	2157.52	1037.36	369	2.71	5.65	26.89	0.55	4.09	3	13	15	98.09	1.91
D4	43.20	24.04	75.96	27.30	32	2191.22	1038.12	369	1.08	5.88	27.74	0.27	3.00	4	14	14	98.09	1.91
C1	11.42	80.97	19.03	32.75	30	2408.93	1276.04	194	10.82	6.54	28.09	0.09	3.36	2	11	17	99.37	0.63
C2	10.68	83.98	16.02	34.68	27	2362.30	1317.53	135	16.30	6.47	28.03	0.01	4.52	2	7	18	99.45	0.55
C3	10.30	86.59	13.41	34.02	27	2443.48	1332.91	101	19.80	6.54	29.03	0.03	3.47	1	9	17	99.97	0.03
C4	10.07	87.77	12.23	34.29	28	2541.14	1391.46	85	28.24	6.82	30.13	0.00	2.15	3	8	17	99.62	0.38
P1	11.10	73.58	26.42	37.23	12	1492.08	1045.00	24	37.50	16.01	64.86	0.13	19.04	1	1	10	99.87	0.13
P2	12.74	65.48	34.52	33.88	25	2140.40	1018.72	83	28.92	7.97	34.30	0.30	4.66	1	8	16	99.70	0.30
P3	11.90	65.57	34.43	35.34	27	2154.04	991.90	121	19.01	7.12	31.00	0.33	3.35	2	6	19	98.78	1.22
P4	12.93	71.65	28.35	33.02	30	2165.00	1119.65	231	9.09	5.82	27.93	0.30	3.18	4	9	17	97.58	2.42
P5	13.20	71.58	28.42	33.43	31	2200.13	1118.12	312	7.05	5.47	26.53	0.25	2.13	6	7	18	96.67	3.33
I1	12.08	72.58	27.42	38.06	32	2207.28	1057.95	369	1.36	5.89	28.33	0.27	2.91	2	1	29	99.45	0.55
I2	12.69	70.00	30.00	36.30	33	2288.33	1138.23	371	2.96	5.73	25.82	0.10	1.06	10	0	23	94.35	5.65
I3	14.26	74.13	25.87	24.29	32	2248.56	1133.07	373	4.56	6.39	29.29	0.27	2.91	20	0	12	64.28	33.82
S1	16.28	75.64	24.36	10.57	33	2320.58	1187.81	368	1.09	6.00	29.16	0.27	1.91	6	18	9	27.92	72.08
S2	14.88	70.58	29.42	23.87	31	2209.94	1128.63	368	0.82	6.54	30.27	0.27	2.45	4	15	12	62.44	37.56
S3	13.98	70.13	29.87	29.56	32	2242.66	1125.70	373	4.83	6.00	28.23	0.27	2.98	5	11	16	80.04	19.96
S4	13.61	71.80	28.20	30.76	31	2200.13	1118.12	376	5.32	6.00	26.70	0.55	4.09	3	11	17	87.41	12.59
R1	14.86	75.48	24.52	19.66	32	2248.56	1133.07	371	3.77	5.73	27.21	0.27	2.45	5	11	16	78.71	21.29
R2	13.66	73.32	26.68	28.46	32	2248.56	1133.07	373	4.83	5.73	26.79	0.27	2.45	5	10	17	81.83	18.17
R3	14.27	74.02	25.98	24.45	31	2200.13	1118.12	375	5.60	5.72	26.65	0.27	3.69	3	11	17	98.09	1.91
R4	13.70	73.56	26.44	28.17	31	2200.13	1118.12	373	5.90	5.73	26.34	0.24	3.46	2	12	17	99.22	0.78
R5	12.67	71.11	28.89	35.67	31	2200.13	1118.12	374	4.81	5.73	26.65	0.27	3.54	3	11	17	98.36	1.64

number of partially loaded trips, i.e., the total number of trips tends to decrease whereas the percentage of fully loaded trips increases (especially clear in the uniform instances).

Instances in Class C allow us to analyze the effect of increasing vehicle capacities. As expected, the number of trips and the transportation costs as a fraction of the total costs decrease as the vehicle capacity increases. However, the decrease in the number of suppliers used may be somewhat smaller than expected. A more detailed analysis does reveal some changes in purchasing behavior. Consider Fig. 4 which shows, for a specific instance, the quantities bought from each supplier when the vehicle capacity is relatively small (C1) and when the vehicle capacity is relatively large (C4); the suppliers are listed in order of non-decreasing distance to the purchaser. We see that when the vehicle capacity is larger, suppliers further away from the purchaser are used. Most likely, these suppliers offer lower prices and/or larger total quantity discounts that have become attractive only because of the reduced transportation costs associated with larger vehicle capacities.

Instances in Class P allow us to analyze the effect of the number of products in the market. When there are few products in the market, it is relatively easy for the purchaser to reach the threshold that qualifies him for the largest discount rate at a supplier and therefore he tends to buy almost all product available from the selected suppliers. This is true even if it implies a partially loaded trip (the total quantity discount outweighs the extra transportation costs, which are relatively small since the suppliers selected are relatively close to the purchaser). When the number of products increases, the product offerings of the suppliers differ more and it becomes harder for the purchaser to reach the threshold that qualifies him for the largest discount rate. This explains the increase in per-unit cost when the number of products in the market increases. However, an increase of products in the market allows the purchaser to concentrate his purchases, thereby decreasing his transportation costs, in relative terms.

Instances in Class I allow us to analyze the effect of the discount structure offered by suppliers. The instances I1, I2, and I3 represent different attitudes towards discounting by the suppliers. In Instance I1, the suppliers discount aggressively, i.e., a discount is offered already when purchasing only 5% of the total quantity available. On the other hand, in Instance I3, the suppliers apply discounts conservatively, i.e., a discount is offered only when purchasing at least 90% of the total quantity available. As it is relatively easy to secure large discounts in Instance 1, optimizing transportation costs becomes key. This is reflected in the fact that the number of suppliers used and the average distance to these suppliers are comparatively small and that all but a handful of trips are full-load trips. On the other hand, in Instance I3, where it is relatively hard to secure large discounts, carefully selecting the suppliers where to purchase a large quantity becomes crucial as well as sacrificing transportation costs, in the form of more partially loaded trips, to qualify for the large discounts. The remaining demand can be fulfilled from nearby suppliers offering the best prices, which leads to a larger number of suppliers overall.

Instances in Class S allow us to analyze the effect of having only a limited number of suppliers offering discounts. When the number of suppliers offering discounts is small, products are purchased from suppliers that are close and offer reasonable prices. When the number of suppliers offering discounts increases, then it becomes more important to concentrate purchases at fewer suppliers so as to benefit from their total quantity discounts. To reach the threshold for qualifying for discounts, it may be necessary to accept partially loaded trips, which also explains why the total number of trips increases. Interestingly, and somewhat surprising, the market share of the five suppliers from which the largest quantity is purchased does not vary much. A more in-depth analysis of the solutions shows how important the role of discounts is: for the instances with suppliers in bands around the purchaser, the number of suppliers used that offer discounts is 5 out of 5 in S1, 14 out of 15 in S2, 19 out of 25 in S3, and 20 out of 35 in S4. The numbers are similar for instances where the suppliers are located uniformly around the purchaser. Furthermore, the quantities bought from these suppliers always correspond to the minimum number of full-load trips required to exceed the threshold for the largest discount, or if partially loaded trips are used to reach the threshold for the largest discount.

Instances in Class R allow us to analyze the effect of discount rates change. We observe that when discount rates increase, transportation costs decrease slightly as purchases are concentrated with appropriately selected suppliers. With lower discount rates, a noticeable portion of purchases is made without receiving any discount (between 15% and 20%). In the presence of large discount rates, concentration of purchases increases to ensure almost all purchases qualify for discounts.

Even though our analysis has been carried out from the perspective of the purchaser, the results indicate that suppliers can substantially increase their market share by introducing appropriately calibrated total quantity discounts.

5. Conclusions

We have introduced and studied a procurement problem that simultaneously considers transportation and purchasing costs. Linear programming based iterative rounding heuristics and local search based diversification procedures have been developed to quickly produce high-quality solutions. The approach has proven

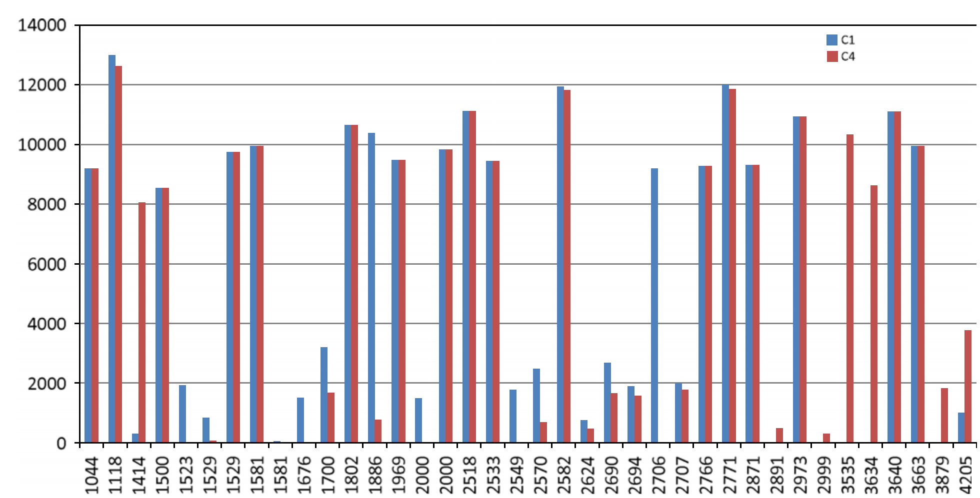

Fig. 4. Quantity bought from each supplier: instances C1 and C4, bands case.

to be efficient and effective, often producing high-quality solution an order of magnitude faster than commercial integer programming solvers. An extensive computational study was conducted to understand the sensitivity of solutions to changes in the environment. A few key insights are:

1. Instances of the problem can be quite challenging to solve. Complex trade-offs have to be made involving product availability at suppliers, discount schemes offered by suppliers, supplier locations, and transport capacity and cost structure. Model-based decision support is essential in properly evaluating these trade-offs.
2. Transport capacity and cost structures can substantially influence the structure of solutions, e.g., larger vehicle capacities may make it attractive to consider suppliers further away.
3. The number of products offered by suppliers can substantially influence the structure of solutions, e.g., when only a small number of products is offered solutions tend to concentrate on nearby suppliers, whereas when a large number of products is offered (especially when total quantity discount are offered) solutions tend to also include carefully selected suppliers further away.
4. The discount schemes offered by suppliers can substantially influence the structure of solutions, when only a small number of suppliers offer discounts solutions tend to select all of them, whereas when a large number of suppliers offer discount schemes a careful selection is warranted focusing on the trade-off between transportation costs and purchase costs savings as a result of the discount.

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