European Journal of Operational Research 32 (1987) 363–379
North-Holland

An interactive, computer-aided ship scheduling system

David M. MILLER
Department of Management Science and Statistics, University of Alabama, U.S.A.

Abstract: This paper is concerned with a fleet scheduling and inventory resupply problem faced by an international chemical operation. The firm uses a fleet of small ocean-going tankers to deliver bulk fluid to warehouses all over the world. The scheduling problem centers around decisions on routes, arrival/departure times, and inventory replenishment quantities. An interactive computer system was developed and implemented at the firm, and was successfully used to address daily scheduling issues as well as longer range planning problems. The purpose of this paper is to first present how the underlying decision problem was analyzed using both a network flow model and a mixed integer programming model, and then to describe the components of the decision support system developed to generate schedules. The use of the system in various decision making applications is also described.

Keywords: Scheduling, water transportation

Introduction

The Ethyl Corporation utilizes a fleet of bulk ocean-going tankers to deliver gasoline antiknock compounds to distribution and customer terminals all over the world. Managing this fleet involves strategic planning decisions, as well as many day-to-day operational decisions. However, the central issue around which most of these management decisions evolve is that of how to schedule each ship in the fleet so that inventory needs at all terminals are satisfied in the least-cost manner.

This central problem can be viewed either as an inventory reorder decision in which resupply lead times are controllable, or as a vehicle scheduling decision in which demands (i.e., warehouse inventory reorders) are controllable. In any event the problem is a complex one, requiring an effective decision support system to allow the distribution system to be operated in an efficient manner. The purpose of this paper is to describe the decision support system successfully developed by the Ethyl Corporation to address this problem; while tailored to Ethyl's particular needs, the system nevertheless has a generic structure and should therefore be of interest to others facing similar distribution/inventory problems.

After discussing research reported by others on related problems, the paper gives a thorough description of the distribution system involved as well as the decision problem that is being addressed. Then, it gives a description of a network model that was used to formulate the primary schedule generation components of the decision support system. In the final sections of the paper, these components are described and the use of the system in various decision applications are presented.

Related work

The generic ship scheduling problem has been addressed in the literature through several different avenues. These include applications oriented papers such as vehicle routing and tanker scheduling, as well as model structure oriented papers such as the multicommodity network flow problem and the dynamic transportation problem [4].

Received March 1986; revised August 1986

The most recent developments in this general area involve interactive computer systems to provide key information to the user. Cullen, Jarvis and Ratliff [6] have developed effective software along these lines, as have Scott and Douglas [11], and Boykin and Levary [5].

Early research in the ship scheduling area was that done by Dantzig and Fulkerson [7] in which they addressed the problem of determining the fewest number of identical cargo tankers needed to satisfy several demands. Their problem was a relatively simplified version of the general ship scheduling problem in that it assumed a single source, demand requirements that are integer multiples of ship capacities, and identical vessels. Bellmore, et al. [3] considered a very similar problem, but one in which there are an insufficient number of tankers to meet the demand requirements. Delivery 'utilities' were introduced to help decide which deliveries to cancel. A more expanded problem was addressed by Applegreen [2], but it was still assumed that the size of cargoe is the same as the capacity of the ship; that is, even if two or more separate cargoes could be carried simultaneously on one ship, this possibility is not considered in the model.

Koenigsberg and Lam [8] presented a stochastic approach to a version of the ship scheduling problem. They developed a cyclic queue model of a closed loop, fixed route delivery system involving several identical vehicles or tankers, one loading port, and one or two discharge port(s). The model was analyzed to find key statistics such as the expected number of tankers in each port and their expected waiting time.

McKay and Hartley [9], and Agin and Cullen [1] considered a version of the ship scheduling problem which resembles the line addressed in this paper. The decision problem was to construct a set of routes for each ship in the fleet and to determine the amount of each type of cargo to be delivered by the ships at each stop in its route(s). The resulting schedule had to deliver cargo to each of several demand locations within specified calendar dates and at minimum cost. The generalized features of their formulations included:

(1) Multiple loading ports,

(2) Multiple deliveries of multiple cargoes at each demand location,

(3) Multiple ships with different characteristics such as capacity and speed, and

(4) Cargoes which are not preassigned to ships but rather are decision variables.

Ronen [10] provided a good overview of the various types of ship scheduling situations that have been researched. He classified them as either liner operations (analogous to a bus line), tramp operations (like a taxicab service), and industrial operations (like a private truck fleet). The situation faced by the Ethyl Corporation falls in this last category. However, because the cargo being transported is owned by Ethyl, and due to the fact that the driving force behind ship operations is meeting terminal inventory requirements, Ethyl's ship scheduling problem can also be viewed as an inventory scheduling problem. Cargoes are essential inventory resupply quantities, whose lead times are a function of how the delivery ships are scheduled. In the remainder of this paper, the scheduling problem is defined and modeled. In addition, the computer-human interactive system developed to resolve this problem is described. First, the underlying distribution network and delivery system is presented.

The distribution system

The physical distribution system supporting the international sales of Ethyl's antiknock compounds normally consists of a fleet of 4 bulk tankers delivering a mix of around 20 products to 30 or so warehouses around the world. There is one primary loading port, with additional sourcing taking place as transshipments at certain warehouses which function as distribution terminals. The primary characteristics of each of the components of this distribution system are briefly described below.

Ships. The ships in Ethyl's fleet are relatively small bulk tankers with buoyance capacities between 2100 and 6700 metric tons. Cargo is stored in one or more of the 8–10 individual compartments or tanks, each with a different volume as well as weight capacity.

Each of the ships has features that prevent the ships from being treated as identical for scheduling purposes. These features include speed, type of fuels used, draft, country in which the vessel is registered, cargo capacity, and configuration of storage tanks. Differences in these features affect a ship's transit time between stops on a voyage, its feasibility for use in delivering cargo to certain individual terminals of certain combinations of terminals, and its daily operating cost. In order to be effective, the scheduling system needs to recognize such differences and allow their effects to influence scheduling decisions.

Cargo. The material being transported is a liquid lead chemical having a relatively high specific gravity. Differences in the chemical formula and in the fluid's color as required by various customers create more than 20 separate products. Each product has its own demand pattern, inventory requirements, and cost structure. The high specific gravity of the products causes extra consideration to be given to certain ship operating issues, such as maintaining adequate draft and maintaining proper trim of the ship throughout its voyage. (trim is determined by how the 6 or 7 products on a ship are assigned to the 8–10 storage compartments built into the ship; the capacity of each compartment is a function of the products as well as the strength of the compartment itself).

Terminals. There are more than 30 ports to which cargo is delivered. At each of these, there are storage tanks which serve as bulk warehouses. The number and capacity of the short tanks differs among the terminals, as does the inventory policy with respect to safety stocks and average level requirements. Another difference arises due to the fact that some ports will not allow ships carrying 'flags of convenience' to call on that port; some of Ethyl's ships have this type registration. Another characteristic of some terminals which dictates which ships can be used to deliver cargo to those terminals is the draft or depth restrictions of the port. Even in cases in which all ships can be used at a given port, there are often time restrictions on when ships can call such as only during months in which there is no ice blockage of the port.

The scheduling decision problem

There are several decisions that must be made periodically in operating this distribution system. For instance, the basic configuration of the system is reexamined every year or so as major shifts in demands appear likely. This means adding additional ships to the fleet or removing them as overall requirements change. When a ship is added, a decision must be made as to its characteristics such as capacity, crew nationality, and registration. Other configuration issues that are periodically addressed included whether or not to add a new distribution terminal, whether or not to close a terminal, and whether to expand or contract storage capacity.

The basic decision that underlies both long range planning strategies as well as short term operating issues is a scheduling problem. Specifically, the problem is how to schedule each ship so that inventory needs of all terminals are met during a prescribed planning horizon while the combined effect of ocean transportation costs and inventory carrying costs are minimized. Each alternative solution to this problem is a possible *ship schedule*. Such a schedule consists of a set of voyages for each ship which covers the use of that ship during the planning horizon. A voyage is an itinerary for a ship which accounts for the deliveries made by the ship from the time it leaves a loading port until it returns to a loading port.

The typical application of this problem involves an 18 month planning horizon. Each of the four ships makes 4 or 5 voyages during this period, and each terminal is visited 1–10 times depending on its demand rate, storage capacity, and inventory requirements. These demand rates and inventory requirements change from one month to the next. Forecasting of these demands is currently done on a subjective basis by the marketing and sales departments. Time series analysis is not used and as a consequence intermediate and long term forecasts are not available. (Changes in customer orders occur frequently causing inaccuracies in predictions two or three months in the future.) This forces the schedule to be revised at least once a month, and a new 18-month schedule to be developed.

Decision variables

There are three types of decision variables involved in constructing a schedule: routing assignments, delivery quantities, and arrival/departure dates. The principle characteristics of these variables are as follows.

Routes. The specification of a route for a ship consists of identifying each of the terminals that ship is to visit in a specific voyage as well as the order in which it is to call on these terminals. The voyages made by a ship within the planning horizon may involve completely different routes. A stop on a particular voyage can be a dry dock, at which time the ship is taken out of service for extended maintenance.

Delivery quantities. At each stop on a route, a ship can unload an amount of each type product handled by that terminal. The sum of the amount of products delivered to terminals on a voyage constitutes the total cargo loaded on that ship at the beginning of that voyage. The amount of a given type of product delivered to a terminal during a particular stop constitutes an inventory resupply. The demand on the terminal is assumed to occur at a uniform rate during any given month (but varying from month to month).

Arrival/departure times. The arrival date at a terminal is equal to the departure date at the previous stop plus the transit time between this pair of ports. Transit times vary from one day up to 35–40 days. In the model used to solve the scheduling problem, these times are formulated as a deterministic function of the ship's speed as well as the direction of travel (due to ocean currents). However, transit times are not modeled as a function of the ship's load or of weather conditions. The departure date from a terminal is a function of the length of time to unload cargo (assumed independent of the amount of cargo) and the scheduler's desire to speed up or slow down the voyage.

Objective function

There are several criteria which are used by Ethyl's distribution management in evaluating the worth of a given ship schedule. These include tangible factors such as average and minimum terminal inventory levels imputed by the schedule, the average unit freight cost, and the average utilization of the ship's cargo capacity which is realized. Intangible criteria used include an assessment of the implications of low inventory levels at various terminals and an assessment of the flexibility inherent in the schedule—i.e., the ability for ships to be rerouted and delivery quantities to be altered during the course of the schedule as demands differ from those forecasted.

The approach used in developing the scheduling system was to utilize a single criterion as the driving force for use in constructing an initial schedule, then to have distribution management modify this solution as needed to reflect other relevant criteria. The central measure of effectiveness used was the total transportation and inventory carrying cost during the planning horizon. Transportation costs include fixed costs such as ship capital costs, crew payroll, and crew food costs. Those expenses deemed to vary as one or more decision variables change (i.e., routes, load/unload quantities, and arrival/departure times) include ship fuel costs, in-transit inventory carrying costs, port in/out charges, canal crossing fees, and terminal inventory carrying costs. Some of these variable costs are incurred as 'fixed charges' with respect to a given leg of a voyage. For instance, if a ship makes a voyage that includes a stop at port A then at port B, the full port charges at B are incurred. If port B had not been visited, no port charges would have been incurred. Fuel costs are also treated as fixed charges, as are canal fees. On the other hand, inventory costs vary directly with delivery quantities.

Constraints

Listed below are the primary restrictions on the routes, delivery quantities, and arrival/departure times which were used in developing a useable schedule for any particular planning horizon.

(1) The total metric tonnage of products loaded on a ship must not exceed that ship's carrying capacity. The products being transported are liquids with varying densities and cannot be commingled. Therefore, the total carrying capacity of a ship depends not only on the number and size of storage compartments or tanks it has, but also on the particular mix of products to be carried.

(2) The cargo loaded on a ship must not lead to instability problems at sea. Such problems occur when a weight imbalance is caused by overloading the storage tanks on one side of the ship or overloading the tanks located at the bow or the stern. In addition, the sloshing action associated with partially filled tanks can also create instability problems. As in the case of the restriction on

the overall carrying capacity of the ship, the assessment of whether or not the loading on a ship is stable depends on the amounts as well as the mix of products involved.

(3) Only one ship at a time can load at a source port. However, two ships can be stationed in a source port at the same time. Similarly, only one ship at a time can unload cargo at a given terminal, although more ships can be stationed at that terminal for a prolonged period of time. Such stationing typically occurs during a 20–30 day dry dock for maintenance.

(4) Certain ships must be stationed at a given port during a specified time period for dry dock maintenance.

(5) The amount of inventory on hand for each product at each terminal cannot exceed that terminal's storage capacity for that product. Similarly, a product's inventory level should not go below a minimum safety stock level. This safety stock restriction is a 'soft' constraint in that, in practice, it is likely to be occasionally violated to some degree. The 'hard' version of this lower bound constraint is that inventory at a terminal is never exhausted.

(6) Due to handling restrictions and customer requirements, the amount of a product delivered to or 'dropped' at certain terminals must not exceed upper and lower limits.

(7) Deliveries at certain terminals must be made within prescribed time windows. This restriction holds, for example, for terminals in the Baltic Sea area where ice conditions in the port make it necessary to schedule deliveries only between April and November.

(8) Some ships cannot call on certain terminals. This arises because of factors such as a port's restrictions on which 'flags' or country-of-registration it will accept, and draft limitations of the ships.

The decision variables, objective function, and constraints described above define the basic decision problem. The scheduling problem involves a number of complicating 'realities' and trade-offs that make it extremely difficult to develop a minimum cost schedule, or even a feasible schedule. These complexities, as well as the size of a typical application, necessitated the use of a computer-based decision support system to analyze the problem and to help generate optimal or near-optimal schedules.

The decision support system developed by Ethyl was built around a mixed integer, 0–1 programming model of the scheduling problem. This model is given in the Appendix (and is labeled Model M1). The formulation given in the Appendix uses a network flow approach and differs from other ship scheduling problems described in the literature cited earlier. The primary differences are (1) the costs included in the objective function, (2) inventory related constraints, and (3) detailed representation of routing decisions.

Due to its large size and lack of an efficient structure, a heuristic approach was used to solve the model (see later section). The heuristics themselves as well as the organization of the overall decision support system were developed only after analyzing the scheduling problem using a network model. This network model proved to be extremely useful in providing insight as to the structure of feasible schedules as well as the identification of several subproblems within the overall model. The network representation is described in the next section.

Network model

A node-arc representation of the scheduling problem can be constructed by first defining certain types of nodes. Specifically, a 'loading node' is the activity of transferring products from a production supply to a given ship. A 'loading port ship node' is an artificial node representing the recipients of the cargo from a loading node. There is one loading port ship node for each ship; this allows the total supply available at the loading course in a period to be allocated among the various ships.

A 'terminal ship node' represents the positioning of a given ship at a terminal at a given point in time. There is one such node for each ship-terminal-product combination. These nodes are used to trace the movement of ships during the schedule. The final type of node is a 'terminal inventory node' which is used to represent the flow of inventories at a terminal from one time period to another. There is one terminal inventory node for each terminal-product combination. Product inflow to one of these nodes can come from the inventory on hand in the prior period, as well as from a delivery by a ship. Two outflows are possible, one representing inventory carried over to the next time period, and the other representing product demand or withdrawal during the current period.

All types of nodes are replicated for each time period in the planning horizon. Due to the need to have a well defined, micro level schedule, the time periods in Ethyl's application are days. Obviously, this makes the size of the network extremely large for the usual 18 month planning horizon. In fact, in a typical application, the total number of nodes is (1 loading port) + (4 ships × 1 loading port) + (4 ships × 30 terminals × 20 products) + (30 terminals × 20 products) for each of the (30 days × 18 months) time periods, or 16×10^5 nodes. An illustration of the nodes involves in a one loading port, 2 ship, 2 terminal, 2 product, 4 period application is at Figure 1.

The types of arcs involved in this network are illustrated in Figure 2. A complete voyage for each ship is illustrated by the arcs between the various 'ship nodes'. The unloading or delivery of products at a terminal is represented by the flow along the arc connecting a terminal ship node to a

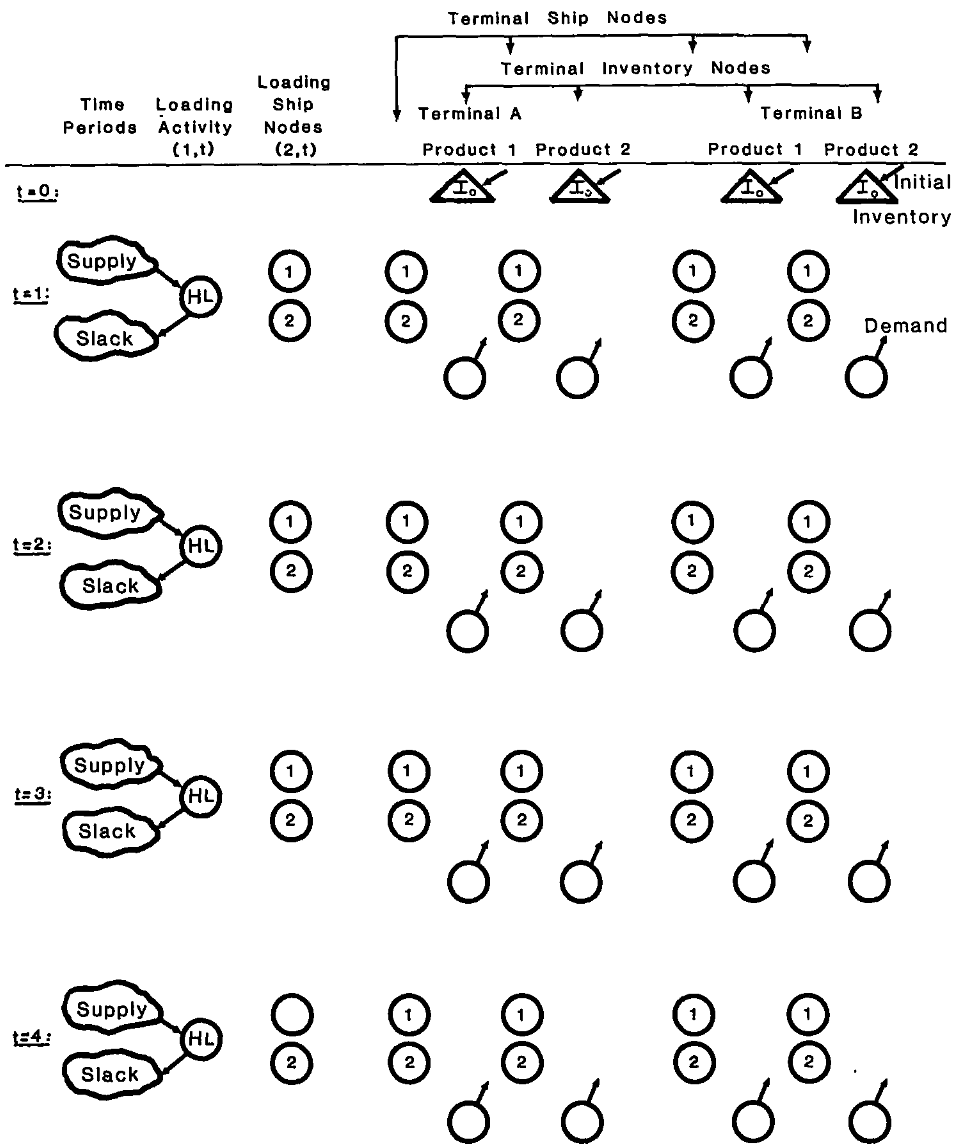

Figure 1. Illustration of the node structure in the network model.

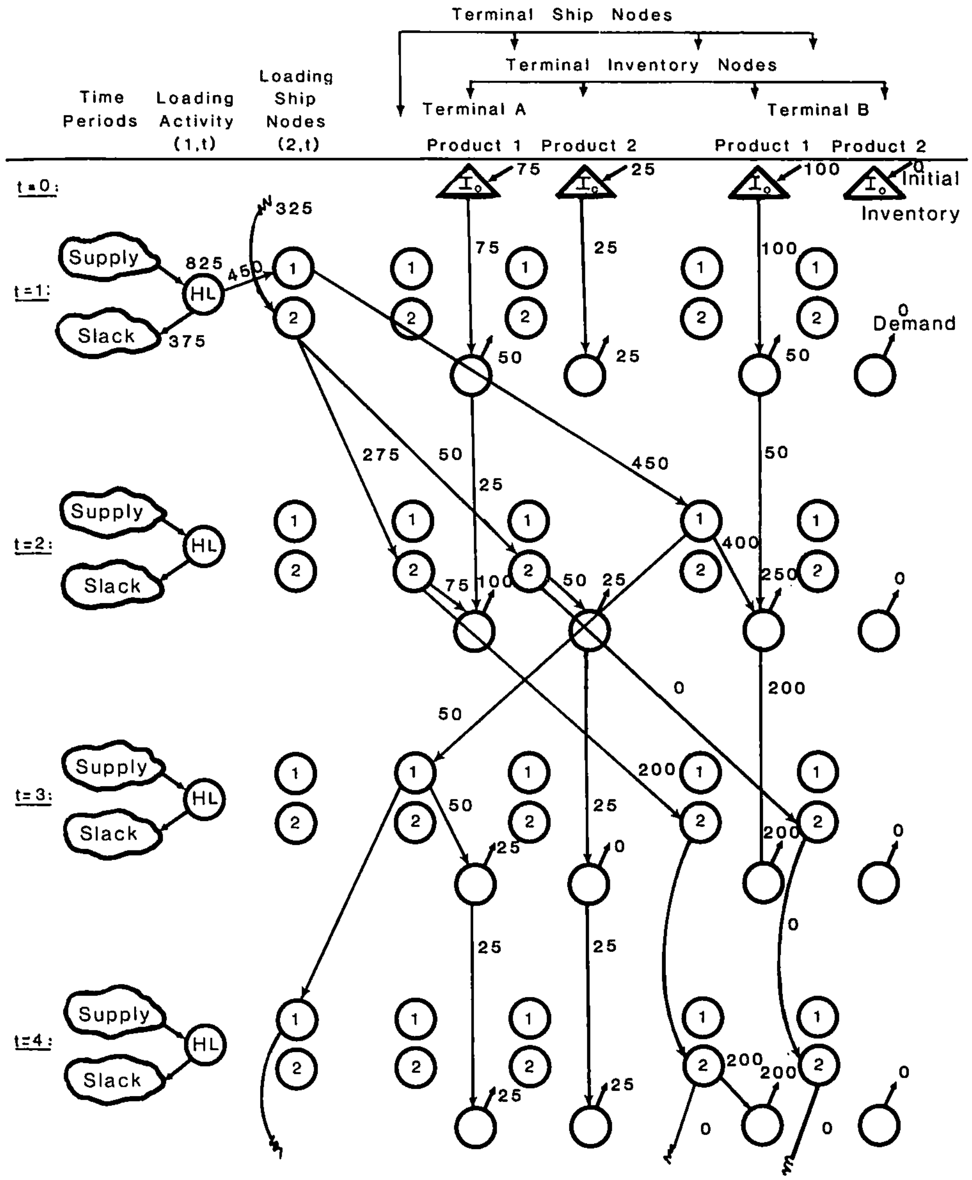

Figure 2. Example of flows in the network model.

terminal inventory node in the same period. In this example, ship 1 carries only one product, and is routed from the loading port to terminal B, then to terminal A, and finally back to the loading port. Ship 2 transports two products; therefore, its voyage is indicated by two parallel arcs throughout the network. Ship 2 was loaded during a prior period but remained in port until period 1. During its voyage, ship 2 is delayed at terminal B for an extra time period before it unloads cargo. This is often necessary in practice due to the need to wait until space is available in the terminal storage tanks before additional products can be added.

The example in Figure 2 shows only those arcs actually used in the schedule. There are many more connecting arcs that are candidates for flows. In fact, for a practical size problem there are more than 22 million arcs. The majority of these arcs connect the ship nodes throughout the network; this allows alternative ship routes to be considered.

Generating a schedule

The decision support system developed to aid the scheduler is built around an interactive, improvement type procedure designed to move toward an optimal solution to the model given in the Appendix (Model M1). Various steps in this procedure involve suboptimization problems derived from the network model cited earlier.

In the course of the procedure, the scheduler attempts to improve the optimality and (or) feasibility of a base or trial solution to Model (M1) by suggesting one or more perturbations to the schedule. The motivation for suggestions comes in part from the scheduler's experience, and in part from various computer analyses. The computer accepts each set of suggested perturbations as creating a new trial schedule, and passes back to the scheduler information on its optimality and feasibility. This interactive trial-and-error solution procedure continues until the scheduler is satisfied with the results.

The computerized system consists of four basic components. The first component is an algorithm to generate a feasible schedule or solution to Model (M1); this is used as a starting point for a later improvement phase. This algorithm is a heuristic, construction type procedure that uses a preselected set of candidate routes to build a schedule that meets the schedule constraints, including ship stability requirements.

Input data to this algorithm include information on the current configuration of the distribution system such as the number of terminals, their storage capacities, and required safety stocks, plus ship and product data. In other words, numerical values are provided for those parameters involved in Model (M1) which are feasibility (rather than cost) related. However, the primary input is a set of seven or eight optional routes for each ship. The algorithm attempts to build a feasible schedule by picking one route from each ship's set and blending this with the selected routes from the other ships. A trial-and-error improvement effort is engaged in by swapping optional routes in and out of the potential schedule. While convergence of this procedure to a feasible schedule is not guaranteed, experience has proven that using routes that worked in the past almost always leads to the production of a feasible schedule for future requirements.

The second component is a deterministic schedule simulator. Its purpose is to evaluate a trial schedule in terms of its objective function value and its feasibility. Constraint violations are called 'exceptions', and are clearly delinearized so that the scheduler can assess their severity. An illustration of the exceptions found in one particular schedule is shown in Table 1. The results of this simulator provide much of the guidance required by the scheduler for directions for schedule improvements.

The third system's component is a report and graph generator. At the scheduler's discretion, any one of 13 different reports or graphical aids can be displayed at his computer terminal. The reports summarize and display the results of submitting a given trial schedule to the simulation procedure. The reports are intended to provide a visual image to the scheduler of exactly what the schedule in question looks like, as well as provide him with evaluation statistics such as overall schedule costs, length of time a terminal is below its safety stock, ship trim conditions, etc. From this type of information, the scheduler can quickly spot areas for improvement.

This visual analysis of the schedule has several dimensions, with various reports and graphs designed to support the analysis in each of these areas. For example, an important view of the schedule is from the standpoint of the day-to-day stationing of each ship. This provides availability information, projected arrival times, future transport capacities, conflicting port assignments and so forth. A portrayal of this type of information arranged on a voyage by voyage basis for each ship is illustrated by the report in Table 2. Summary statistics concerning the quality of a given schedule can also be presented from a ship standpoint, as illustrated by the utilization report shown in Table 3 and the cost report in Table 4. Another important view of the analysis of a schedule is from the vantage point of the present and future inventory position at each terminal. This includes both quantity information as illustrated by the terminal report shown in Table 5, and cost information as shown in Table 6.

The final component of the scheduling system is a collection of interactive improvement routines. Each of these routines can be categorized as being either a *manual* or an *automatic* procedure depending on whether the scheduler makes changes

Table 1
Exceptions report

File name: Newbud80
Date and time of storage
05/06/80 08 : 42 : 58

Inventory problems

Month	Terminal	Product	Problem
January			
	Cadiz	CB	Below minimum by 27(MT)
	Cadiz	B	Below minimum by 60(MT)
	Cadiz	T	Below minimum by 11(MT)
	Dord	CB	Below minimum by 49(MT)
	Dord	B	Below minimum by 178(MT)
	Salonica	EDB	Below minimum by 146(MT)
February			
	Cadiz	T	Unable to discharge 145(MT)
	Cepsa	B	Below contract minimum by 4(MT)
	Peru	CB	Below contract minimum by 263(MT)
March			
	Cadiz	T	Unable to discharge 91(MT)
	Turku	CB	The drop on 3–14–1980 is out of its limits
	Slagen	CB	The drop on 3–10–1980 is out of its limits
	Slagen	T	The drop on 3–10–1980 is out of its limits
	Dord	T	Below minimum by 14(MT)
	Taiwan	B	Below contract minimum by 5(MT)
	Singapore	T	Below minimum by 46(MT)

Ship problems

Month	Ship	Problem
January		
February		
March		
Aoril		
May		
June		
	Joy	There is a Houston department conflict on 6-3-1980 with EE on 6-9-1980.
July		
August		
September		
October		
November		
December		

to specific, individual decision variables or whether a computerized, suboptimization procedure is used to change one or more variables. Examples of manual improvement routines are as follows:

(1) "Change a specific drop quantity",
(2) "Add a stop to a specific voyage",
(3) "Delete a stop from a specific voyage",
(4) "Change the expected arrival time of a given stop",
(5) "Rearrange the order of stops on a specific voyage", and
(6) "Change the timing of a dry dock".

These and other options to alter a schedule are presented to the scheduler in a menu format using terminology familiar to him. However, each option is directed toward changing one or more specific decision variables in Model (M1). For instance, option (1) listed above allows the sched-

Table 2
Ship activity report

11:13:56	Ship activity by voyage File name: Newbud80 From 1-1-1980 to 12-31-1980 Date and time of storage 05/06/80 08:42:58 Ship's name is Eid			02/05/81
Date	Port	Product	Amount (MT)	
01-09	Bajo Gran	CB-Y-51	31	
01-17	Houston-AR			
-				
-				
-				
-	Total load	Houston	1673	
03-21	Houston-DP	CB	573	
-		MLA	1100	
04-01	Peru	CB-W-51	573	
04-14	St. Croix	MA-W-51	1100	
04-23	Houston-AR			
-				
-				
-				
-	Total load	Houston	2050	
04-30	Houston-DP	CB	1075	
-		B	475	
-		T	500	
05-24	Dord	CB-W-51	700	
-		T-W-51	500	
-		B-W-52	200	
05-31	Cadiz	CB-W-51	375	
-		B-W-52	275	
06-22	Houston-AR			
-				

Table 3
Fleet utilization report

Fleet utilization fractions

Ship	Steam	Unload	Dry dock	Houston	Idle
Eid	0.0	0.0	0.0	0.0	1.0000
EE	0.0	0.0	0.0	0.0	1.0000
Joy	0.5464	0.0659	0.0549	0.0962	0.2366
Just	0.9505	0.0604	0.0	0.0577	−0.0687
Liby	0.7308	0.1044	0.0	0.0769	0.0879
Fleet total	0.06215	0.0695	0.0121	0.0646	0.2322

Utilization with idle time in voyages partially loaded

	Eid	Ee	Joy	Just	Liby	Fleet
Idle days	364.00	364.00	134.67	59.63	65.43	127.25
Until %	0.0%	0.0%	0.630%	0.836%	0.820%	0.650%

Table 4
Ship cost report

Ship costs (over all voyages in M$)	Eid	EE	Joy	Just	Liby
Var loading cost:	0.0	0.0	0.0	0.0	0.0
Fuel:	467.8	409.4	619.5	637.4	0.0
Port costs:	58.1	71.9	131.3	115.8	0.0
Canal costs:	7.3	0.0	18.3	18.3	0.0
Fixed loading cost:	0.0	0.0	0.0	0.0	0.0
Dry dock cost:	0.0	0.0	0.0	0.0	0.0
In trans inv:	55.2	63.2	201.2	174.1	0.0
Total var cost:	588.4	544.5	970.3	945.6	0.0
Book cost:	2907.5	2441.3	3045.9	2661.8	0.0

Total ship variable cost = 3048.8
Total terminal inventory cost = 1191.4
Total ship inventory cost = 493.8
Total fixed cost = 11056.6
Total schedule cost = 15296.8

	Eid	EE	Joy	Just	Liby
Steam time:	212	247	285	293	0
Dock time:	0	0	0	0	0
Port time:	129	86	62	45	0
Avg. util.	0.87	0.99	0.91	0.93	0.0

uler to alter the current value of one of the $x^{im}_{(j,t)(k,t')}$ variables. The scheduler specifies the product (m) whose delivery quantity at port (k) in period (t') is to be changed as well as the new quantity. The computer then changes the appropriate $x^{im}_{(j,t)(k,t')}$ variable and updates all other

Table 5
Terminal inventory report

Terminal information on Cadiz
Product–T
Tank Cap – 285
Min. days – 15

Month	Drop	Sale	Inventories			Run out
			Beg.	Max.	Min	
November		134	−44	−44	−178	0
December		126	−178	−178	−304	0
January	1–15 Just 243	85	−304	−101	−344	0
February	2–20 Liby 260	0	−146	115	−145	0
March		0	114	114	114	971
April		0	114	114	114	941
May	5– 6 Liby 340	0	114	454	114	935
June		0	454	454	454	3166
July	7– 7 Liby 340	0	454	794	454	3159
August	8–31 Liby 380	0	794	1174	794	5389
September		0	1174	1174	1174	7913
October		0	1174	1174	1174	7882
November	11–20 Liby 300	0	1174	1474	1174	7862
December		0	1474	1474	1474	9837
January		0	1474	1474	1474	9811

Table 6
Terminal cost report

11:02:13	Cost report										
Terminal inventory costs (annual cost at 11% per year carrying charge)								02/05/81			
	Cadiz			Cepsa	Turku	Turke	Slagen		Dord		
	CB	B	T	B	CB	B	CB	T	CB	T	B
Avg. inv(MT):	501	297	264	317	376	0	96	171	951	625	849
Max. inv:	960	519	446	564	653	154	107	320	2109	1115	1485
Min. inv:	96	35	61	87	161	−46	57	58	844	189	284
Cost(M$)	59.8	36.8	31.2	39.1	44.9	0.0	11.5	20.3	113.5	74.0	104.9

affected decision variables, such as those representing the amount of that product on board the appropriate ship at all previous stops on the affected voyage. The result is a new, trial schedule. No attempt is made by the computer to prevent these changes from adversely affecting either the feasibility or the optimality of the resulting new schedule. However, through the simulator mentioned earlier, the scheduler can instantly have displayed the impact of his desired individual changes and thereby know whether or not the new (trial) schedule is better than the current one. 'Better' here means either (or both) a reduction in the number and severity of constraint violations or a reduction in the overall schedule cost.

In addition to the manual improvement routines mentioned above, the scheduler can call on several automatic procedures designed to optimize some limited aspect of the overall problem. For example, one procedure can be used to minimize in-transit and shore inventory costs within the existing voyage structure. Specifically, all stops on all voyages are held fixed as they exist in the current schedule; in other words, the *arcs* in use in the existing network representation of the schedule remain as the only available arcs in the trial schedule. The *flows* along these arcs are optimized with respect to the inventory costs and inventory related constraints shown in Model (M1).

A similar automatic routine is oriented towards reducing shipping costs by minimizing transit times. Specifically, in this procedure delivery quantities are held fixed while better routes are searched for using an embedded 'traveling salesman' logic. These automatic routines are used in a similar manner to the manual ones—they are tools for generating a host of new trial schedules under the guidance of the scheduler. The obvious difference in the two sets of tools is that the automatic procedures save the scheduler a considerable amount of time and find improvements he may well miss.

While the interactive solution approach described above is not guaranteed to generate "the optimum" solution, it does have the ability to provide good, *satisficing* solutions. This feature is especially useful in view of the fact that in real world scheduling problems many of the parameters and constraints are not fixed and 'hard' as a model such as (M1) would indicate. Rather, they are *guidelines*, subject to modification if the circumstances warrant. For example, the system described here has proven useful in allowing the scheduler to modify an existing schedule to reflect a subjective tradeoff between the cost of running below the safety stock level of inventory at a particular port and the cost savings in fuel cost associated with the shortened voyage that would lead to the reduced inventory. In this case, the safety stock is not an absolute minimum, but a target. In addition, savings in inventory 'dollar' costs is not always viewed by management the same as a similar savings in the 'dollar' costs associated with a cash item such as fuel.

The solution approach described in this section is used to solve a particular numerical version of the scheduling problem represented by Model (M1). Alternative versions of the problem arise due to day-to-day changes in parameters such as terminal demands or ship speeds, as well as to different uses of the system in medium and long range planning. Several of these alternative applications are described in the next section.

Applications

One of the uses of the interactive scheduling system has been to develop a 'budget schedule' to use as a guide for next year's ship costs, inventory levels, and ship activity. Such a schedule usually covers January–December and is developed in the summer of the prior year. It is an integral part of the entire budget process. Prior to the existence of the computer system this budget schedule was developed by hand. As such, it was time consuming, not very cost-effective in its results, and often in error. In fact, a one 'hand' schedule was fed into the simulator portion of the system and over 100 constraint violations were uncovered. Due to the size and complexity of the scheduling task, it was extremely difficult to identify such violations without the assistance of a computer-aided tracking system.

The primary use of the system is in updating the current operating schedule in view of recent changes in factors such as terminal demand forecasts, ship positions, and current inventory levels (i.e., the parameters in Model M1). Typical of such changes is the following telex from a ship's operator: "The M/V Jenny is expected at the Panama Canal on April 12 which would put her four days late. She altered course and reduced speed to avoid hurricane Meli." A revised schedule is routinely generated once a month to meet such parameter changes, and on an emergency basis as needed. The interactive solution procedure uses as input the old (current) schedule and allows the scheduler to attempt to optimize ship usage in view of the latest parameter information.

The system has also proven effective as a 'what if' tool in addressing potential modifications to current operating plans. For instance, a typical use is in responding to a regional marketing manager's question. "My most important customer (i.e., terminal) needs twice as much fluid next month as he originally specified; can we change our schedule to accommodate him?" Not only can the system be used to investigate instantly the feasibility of the requested change, but also to determine the cost to the company of the change. Then, the cost of the change can be compared by management with the benefits of meeting the customer's request.

Other applications include developing five year distribution plans, calculating standard freight rates for internal cost accounting needs and pricing decisions, investigating ship capacity needs for future operations, developing customer profitability data, and in investigating alternative distribution philosophies or strategies. Alternative philosophies can easily be tested in terms of comparative feasibility and cost statistics, such as whether or not the scheduler should emphasize smaller but more frequent deliveries, or the strategy of creating 'flexible' schedules with large inventories in remote terminal areas.

In such applications the system offers significant benefits over the manual approach in areas such as the time to develop a given schedule, the annual distribution and inventory costs achieved, the 'implementability' of the schedule, and the ability to investigate proposed changes in the fleet capacity or terminal locations. For instance, a monthly updated schedule can be developed in about one-tenth of the time required by the manual scheduling approach. Further, it has been conservatively estimated that the schedule generated by the computer system saves 5%–15% of the variable schedule cost over that possible with a manual approach.

Summary

A particular version of the general ship scheduling problem has been described along with a useful network model of the problem and a computer-aided solution procedure. The problem, as much an inventory replenishment problem as a ship scheduling one, is embedded in the daily operating decisions faced by distribution management in the Ethyl Corporation, as well as in the long range planning decisions faced by company management.

The nature of the underlying distribution system is complex and can be characterized as: dynamic, multiproduct, multisource, multisink, and having a fixed charge cost structure. The system's complexity and large size–16×10^5 nodes and 22×10^6 arcs–favors the use of an interactive computer-aided procedure for schedule development. Such a solution system was developed and implemented with useful results. The system utilizes four components: a feasible schedule generator, a simulator, a report/graphics generator, and an improvement module. The improvement module has both manual, limited change options completely directed by the user, and automatic analysis procedures designed to suboptimize a portion of the overall scheduling problem. Linkage and direction of the improvement process is under the control of the scheduler. While the end schedule is not necessarily optimal, it has proven to lead to good, satisficing solutions.

The scheduling system has proven useful in several types of applications and offers the potential to assist in others. The most common use of the system has been in the monthly updating of the present ship schedule on a rolling 18 month basis. This application amounts to developing a new schedule in view of the latest forecast of demands, inventory levels, and ship availability. Other uses of the system include development of schedules for use in long range plans and budget preparation, use as a 'what if' tool in assessing changes in marketing and distribution plans, and in structuring future capital needs as ship capacity is matched to future marketing plans. In applications such as these, the computer scheduling system has proven to save time over the previous manual approach, generate more accurate and more cost-effective schedules, and to have a great deal of potential as a distribution planning tool.

Appendix. Mixed integer programming model of the scheduling problem (M1)

Indices

i → ships; $i = 1, 2, \dots, N$,
j → ports; $j = 1$ → loading source; $j = 2$ → ships at the loading source; $j = 3, 4, \dots, J$ → terminals,
t → time periods; $t = 1, 2, \dots, P$,
m → products; $m = 1, 2, \dots, M$.

Decision variables

$x^{im}_{(j,t)(k,t')}$ = the flow (in metric tons) of product m from node (j, t) to node (k, t') via ship i; i.e., flow of product m from port j originating in time period t to port k ending in period t', as transported by ship i,
$y^{i}_{(j,t)(k,t')}$ = 1 if $\sum_{m=1}^{M} x^{im}_{(j,t)(k,t')} > 0$; 0, otherwise,
$z^{m}_{(j,t)(j,t+1)}$ = the flow of product m from node (j, t) to node $(j, t+1)$; i.e., the ending inventory of product m at port j in period t,
$e_{(l,t)}$ = the amount of unused supply at the loading source in period t.

Note that:

$x^{im}_{(j,t)(j,t)}$ = the quantity dropped or delivered at terminal j in period t, and
$y^{i}_{(j,t)(j,t+1)}$ = 1 implies that ship i is in lay berth at port j in period t.

Parameters

A_t = the total supply of cargo (in metric tons) available at the loading source in period t,
d_{jmt} = the demand (in metric tons) for product m at terminal j in period t,
$U_i^{S}(x)$ = the effective capacity (in metric tons) of ship i; capacity is a function of the mix of products on board the ship,
L^{D}_{ijmt}, U^{D}_{ijmt} = the lower and upper limits, respectively, on the amount of product m that can be dropped at terminal j by ship i in period t,
L^{I}_{jm}, U^{I}_{jm} = the safety stock and storage capacity, respectively, of the inventory level of product m at terminal j,
c_{1i} = the variable cost (if any) per metric ton of loading fluid on ship i at the loading source,
c_{2jm} = the average unit production cost for fluid type m delivered to terminal j,
r_1 = the percentage carrying cost per period for each dollar of production cost tied up in in-transit inventory.

r_{2j} = the percentage carrying cost per period for each dollar of production cost tied up in inventory at terminal j,
f_{1i} = the fixed charge incurred each time ship i is loaded,
f_{2ik} = the fixed charge incurred in port charges by ship i in calling on port k,
f_{3ijk} = the canal fees incurred by ship i in going between ports j and k,
f_{4i} = the average cost of fuel per period incurred by ship i while steaming,
f_{5ik} = the fuel expense per period while ship i is in temporary lay berth at port k,
T_{ijk} = the transit time between ports j and k via ship i ($T_{ikk} = 1$ for lay berth and $T_{ikk} = 0$ for drops),
B = a large number

The objective function for this model is to find values of the decision variables (x, y, and z) that minimize the combined cost of loading ships, voyage related ship costs, lay berth costs, in-transit inventory costs, and terminal inventory costs. Using the above notation, the following objective function can be defined (only a single source is considered here):

$$\begin{aligned}\text{Min} \quad x_0 = {} & \sum_{i=1}^{N} \sum_{m=1}^{M} \sum_{t=1}^{P} c_{1i} x^{im}_{(1,t)(2,t)} + \sum_{i=1}^{N} \sum_{t=1}^{P} f_{1i} y^{i}_{(1,t)(2,t)} \\ & + \sum_{i=1}^{N} \sum_{j \geq 2}^{J} \sum_{t=1}^{P} \sum_{k \geq 1}^{J} \left(f_{2ik} + f_{3ijk} + f_{4i} \right) y^{i}_{(j,t)(k,t+T_{ijk})} \\ & + \sum_{i=1}^{N} \sum_{j \geq 2}^{J} \sum_{t=1}^{P} f_{5ij} y^{i}_{(j,t)(j,t+1)} + \sum_{i=1}^{N} \sum_{m=1}^{M} \sum_{j \geq 2}^{J} \sum_{\substack{k \geq 2 \\ k \neq j}}^{J} \sum_{t=1}^{P} r_1 c_{2km} x^{im}_{(j,t)(k,t+T_{ijk})} \\ & + \sum_{m=1}^{M} \sum_{j \geq 3}^{J} \sum_{t=1}^{P} r_{2j} c_{2jm} z^{m}_{(j,t)(j,t+1)}. \end{aligned} \tag{M1}$$

Minimization of this objective function is subject to the following constraints.

Constraint 1. Ship capacity:

$$\sum_{m=1}^{M} x^{im}_{(1,t)(2,t)} \leq U_i^{S}(x) \quad \text{for all } i,\ t.$$

Constraint 2. No simultaneous loadings:

$$\sum_{i=1}^{N} y^{i}_{(1,t)(2,t)} \leq 1 \quad \text{for all } t.$$

Constraint 3. Conservation of flow at each node:
(a) *Loading source nodes* ($j = 1$, t):

$$\sum_{i=1}^{N} \sum_{m=1}^{M} x^{im}_{(1,t)(2,t)} + e_{(1,t)} = A_t \quad \text{for all } t.$$

(b) *Loading source ship nodes* ($j = 2$, t):

$$\sum_{m=1}^{M} x^{im}_{(1,t)(2,t)} - \sum_{k=3}^{J} \sum_{m=1}^{M} x^{im}_{(2,t)(k,t+T_{i2k})} = 0 \quad \text{for all } i,\ t.$$

(c) *Terminal ship nodes* ($j > 1$, t):

$$\underbrace{\sum_{k=2}^{J} x^{im}_{(k,t-T_{ijk})(j,t)}}_{\substack{\text{Inflow}\\\text{(from previous}\\\text{terminals)}}} - \overbrace{\underbrace{\sum_{k=3}^{J} x^{im}_{(j,t)(k,t+T_{ijk})}}_{\text{To next terminal}} - \underbrace{x^{im}_{(j,t)(j,t)}}_{\text{Delivery}} - \underbrace{x^{im}_{(j,t)(j,t+1)}}_{\substack{\text{Lay berth}\\\text{at port } j}}}^{\text{Outflow}} = 0$$

for all i, y, $j > 1$.

(d) *Terminal inventory nodes* ($j > 2$, t):

$$\underbrace{\underbrace{z^{m}_{(j,t-1)(j,t)}}_{\substack{\text{Beginning}\\\text{period}\\\text{inventory}}} + \underbrace{\sum x^{im}_{(j,t)(j,t)}}_{\text{Deliveries}}}_{\text{Inflow}} - \underbrace{z^{m}_{(j,t)(j,t+1)}}_{\substack{\text{Outflow}\\\text{(End of period}\\\text{inventory)}}} = \underbrace{d_{jmt}}_{\substack{\text{Terminal}\\\text{demand}}} \quad \text{for all } m,\ t,\ j > 2.$$

Constraint 4. Arc capacity constraints:
(a) *Inventory flow arcs*:

$$L^{1}_{jm} \leqslant z^{m}_{(j,t)(j,t+1)} \leqslant U^{1}_{jm} \quad \text{for all } m,\ t,\ j > 2.$$

(b) *Delivery arcs*:

$$L^{\mathrm{D}}_{ijmt} \leqslant x^{im}_{(j,t)(j,t)} \leqslant U^{\mathrm{D}}_{ijmt} \quad \text{for all } i,\ m,\ t,\ j > 2.$$

Constraint 5. Other arc flow feasibility requirements:
(a) *No simultaneous ship routes*:

$$\sum_{k=2}^{J} y^{i}_{(j,t)(k,t+T_{ijk})} \leqslant 1 \quad \text{for all } i,\ j,\ t.$$

(b) *No simultaneous deliveries*:

$$\sum_{i=1}^{N} y^{i}_{(j,t)(j,t)} \leqslant 1 \quad \text{for all } t,\ j > 2.$$

(c) *Completion of routes*:

$$y^{i}_{(k,t)(2,t+T_{ik2})} = 1 \quad \text{if } \left[x^{im}_{(j,t-T_{ijk})(k,t)} = x^{im}_{(k,t)(k,t)} \forall m \right] \quad \text{for each } i,\ j > 1,\ k > 2.$$

(d) *Nonnegativity and x, y relation*:

$$x^{im}_{(j,t)(k,t')} \geqslant 0 \quad \text{for all } i,\ m,\ j,\ k,\ t,\ t'.$$

$$\sum_{m=1}^{M} x^{im}_{(j,t)(k,t')} - By^{i}_{(j,t)(k,t')} \leqslant 0 \quad \text{for all } i,\ j,\ k,\ t,\ t'.$$

A constraint that is not shown in the above formulation is the requirement that the loading on board a ship as it departs any ship node must not lead to ship trim or stability problems. This is a difficult requirement to represent in a useful mathematical manner (other than set notation). However, it is an important part of the scheduling problem and must be considered in the overall solution process. Also note that the port charge, f_{2ik}, for the loading port node ($k = 1$) is set at zero so that the economics of the solution process, along with constraint 5(c), will ensure completion of each voyage. (Without a nonpositive port charge it would be more economical for a product to be carried from terminal to terminal without returning to the loading port.)

References

[1] Agin, N.I. and Cullen, D.E., "An algorithm for transportation routing and vehicle loading", *Logistics* (A special issue of *Management Science*, October 1977).

[2] Applegreen, L.H., "Integer programming methods form a vessel scheduling problem", *Transportation Science* 5 (1971) 64–78.

[3] Bellmore, M., Bennington, G., and Lubore, S., "A maximum utility solution to a vehicle constrained tanker scheduling problem", *Naval Research Logistics Quarterly* 15 (1968) 404–411.

[4] Bookbinder, J.H., and Sethi, S.P., "The dynamic transportation problem: A survey", Naval Research Logistical Quarterly 27 (1980) 65–87.

[5] Boykin, R.F.m and Levary, R.R., "An interactive decision support system for analyzing ship voyage alternatives", *Interfaces* 15(2) (1985).

[6] Cullen, F.H., Jarvis, J.J., and Ratliff, H.D., "Set partitioning based heuristics for interactive routing", Working paper no. J-80-1, ISYE, Georgia Institute of Technology, August 1980.

[7] Dantzig, G.G., and Fulkerson, D.R., "Minimizing the number of tankers to meet a schedule", *Naval Research Logistics Quarterly* 1 (1954) 217–222.

[8] Koenigsberg, E., and Lam, R.C., "Cyclic queue model of fleet operations", *Operations Research* 24(3) (1976).

[9] McKay, M.D., and Hartley, H.O., "Computerized scheduling of seagoing tankers", *Naval Research Logistics Quarterly* 21 (1974) 255–264.

[10] Ronen, D., "Cargo ships routing and scheduling: Survey of models and problems", *European Journal of Operations Research* 12(2) (1983) 119–126.

[11] Scott, K.L. , and Douglas, B.W., "A model-based decision support system for planning and scheduling ocean-borne transportation", *Interfaces* (August 1981).