AN AGENT-BASED MODEL OF OPINION FORMATION IN SOCIAL NETWORKS

by

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ABSTRACT

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In this thesis, opinion dynamics within social networks are examined by means of an agent-based simulation model. The Social Judgement Theory based model is run on random, scale-free and small-world networks, which are three widely considered topologies in the literature, and the results are comparatively analyzed to understand the effect of network topology on the opinion formation behaviors of individuals. The observations indicate that the structural changes in the social network only affect the convergence rates of opinion dynamics. An influential external actor is also introduced to the model, and its impact on opinion dynamics is studied. One critical observation is that a moderate external opinion, compared to an extreme one, has a better chance to push the majority towards extreme opinions in some populations. In addition, a population that normally results in consensus is seen to split into extremes under the effect of intense extreme propaganda. It is observed that polarization dominates the opinion dynamics in the presence of close-minded agents, when populations having heterogeneous thresholds are considered.

ÖZET

SOSYAL AĞLARDA DÜŞÜNCE OLUŞUMUNUN ETMEN TABANLI MODELLENMESİ

Bu tezde, sosyal ağlarda düşünce dinamikleri etmen tabanlı modelleme ve benzetim yöntemleri kullanılarak incelenmiştir. Ağ topolojisindeki değişimin sosyal yargı kuramını temel alarak kurulan modelin davranışına herhangi bir etkisi olup olmadığını anlamak amacıyla, yazında yaygın olarak kullanılan ve gerçek hayatta gözlemlenen ağları temsil etmede başarılı olduklarına inanılan farklı ağ yapıları kullanılarak model sınanmıştır. Elde edilen sonuçların karşılaştırmalı çözümlemesi, ağdaki yapısal değişikliklerin yalnızca benzetimin denge durumuna ulaşma hızını etkilediğini göstermektedir. Ek olarak, modele ağdaki kişilerle tek yönlü iletişim kuran etkili bir dış aktör eklenmiş ve bu durumun düşünce devinimleri üzerindeki etkisi gözlemlenmiştir. Deney sonuçları bazı toplulukların dışarıdan gelen etkili söylemlere karşı daha dirençli olduğunu ortaya koyarken bazılarındaki devinimlerin ise öncesine göre büyük değişimler gösterdiğini ortaya çıkarmıştır. Çoktürlü eşik değerlerine sahip bireylerin oluşturduğu farklı topluluklar gözlemlendiğinde kapalı görüşlü kişilerin varlığının devinimi örttüğü saptanmıştır.

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LIST OF SYMBOLS

$ex\text{-}op$	Opinion of the external actor
k	Average node degree for small-world networks
l_i	Lower threshold of agent i
min_d	Minimum degree for scale-free networks
n	Number of nodes
p_c	Connection probability for random networks
p_r	Rewiring probability for small-world networks
q	Probability of communicating with EA
r_c	Proportion of close-minded agents
r_i	Proportion of indifferent agents
r_o	Proportion of open-minded agents
u_i	Upper threshold of agent i
x_i	Opinion of agent i
y	Dispersion index
α	Mixing *or* Convergence parameter
μ	Mixing parameter for external actor

LIST OF ACRONYMS/ABBREVIATIONS

DW	Deffuant-Weisbuch Model
EA	External Actor
LL	Low Lower and Upper Thresholds
LH	Low Lower Threshold and High Upper Threshold
HH	High Lower and Upper Thresholds
HK	Hegselmann-Krause Model
RA	Relative Agreement Model
SJT	Social Judgement Theory
VV	Very Low Lower Threshold and Very High Upper Threshold

1. INTRODUCTION

Humans are social beings by nature that require communication and interaction with each other. With this necessity, we become entities of various social networks forming both face-to-face (e.g., family or work-related) and online (e.g., follower/followee on Twitter) relationships. We share our experiences, collect information, and exchange opinions through these networks. Interactions between the people within these networks create complex dynamics and result in significant social phenomena.

Presently, social networks are enormously large and produce an outrageous amount of data. The desire to benefit from this big data accumulated may be one reason why social networks field draws attention progressively. Although there has always been an interest in social networks, the concept became so attractive due to the rising popularity of internet-based ones. Figure 1.1 demonstrates the number of articles mentioning social networks with an easily noticeable increasing trend.

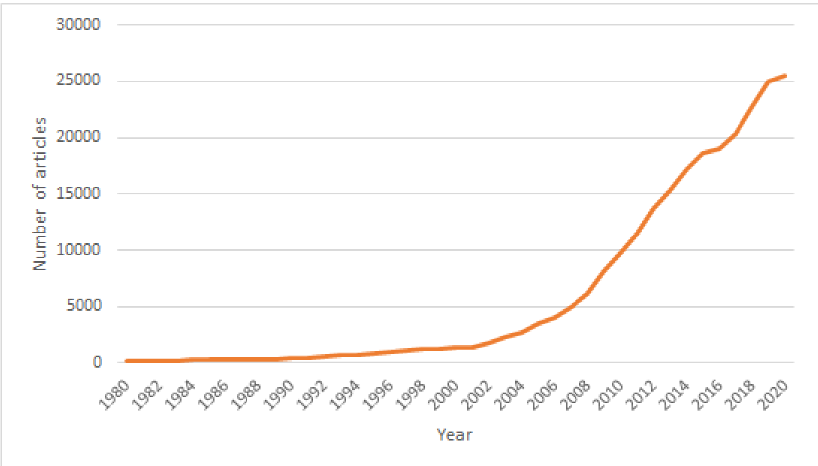

Figure 1.1. Number of articles mentioning social networks, from Scopus database.

Since people have opinions about almost everything and act based on them, understanding the evolution of opinions is vital. Numerous factors, such as the information received and the influential interactions within social networks, affect one's opinion. Therefore, opinion formation is a dynamic and complex process [1]. Opinion dynamics is an emerging sub-field of computational social science that its earliest models are mostly adapted from physics [2,3]. A great amount of studies has been provided to grasp how opinions evolve in a society, and most of them have been primarily interested in opinion clusters obtained when the dynamics become steady [4–6].

A consensus is that the network has only one opinion cluster as time progresses. The necessary and sufficient conditions to reach consensus are greatly wondered; however, it still remains as an open question of the field [7]. Even though the opportunity to communicate and the amount of communication has dramatically increased, this cannot be considered the guarantee of cooperation.

Opinion polarization, standing on the opposite side to consensus, is a natural result of this large amount of interactions and has been observed over a variety of issues and topics. As Matakos *et al.*. [8] state, polarization separates individuals into sides with almost no communication with and understanding of each other, which notably destroys societies' functionality.

A recent study conducted by TurkuazLab [9] demonstrates that political polarization in Turkish society is extremely high, and there are only a few issues, e.g., foreign policy, that the nation finds common ground. Similarly, Abramowitz and Saunders [10] mention that polarization among the American public is a fact and has significantly increased over years. Therefore, the existence of any characteristics or procedures that cause these specific opinion clusters is extremely wondered, and it is of great importance to find out strategies that may be helpful for reducing the effects of polarization.

Barabási [11] states that although networks are somewhat simple models consisting of only nodes and links, it is challenging to determine where to place the links between the nodes in order to reflect the actual system's complexity parameter. Considering the role of interactions within a social network, obtaining a complete understanding of how social networks' structure affects the formation of opinion and shapes individual behaviors is critical [12]. A question at this point is whether it is possible to relate these particular opinion clusters to the nature of network topology.

Additionally, we often observe the influence of an external actor on the people. This influence is especially valid for political issues. For instance, we sometimes see that an influential speech given by a political party leader immediately affects their followers' minds. Besides, McCombs [13] explains that mass media has an agenda-setting influence on society. Thus, understanding the effect of these actors on people's opinion is another issue that significantly concerns us.

Overall, this study aims to understand opinion formation within social networks by varying several model settings. In Chapter 2, recent literature on opinion dynamics is briefly summarized. Chapter 3 explains the methodology and introduces the simulation model. In Chapter 4, all experimentation process and essential findings are given. Finally, Chapter 5 sums up the critical observations of the study and states the possible directions for future work.

2. LITERATURE REVIEW

Opinion dynamics within social networks have been widely studied, both from the perspective of analytical modeling and experimental psychology. With the rising interest in social networks, a broad range of computational models is presented to understand opinion evolution on social networks [1,14]. Opinion representations in the models vary from one-dimensional binary opinions to multi-dimensional continuous opinions. In addition, various mechanisms are used to determine communication and opinion update rules. Thus, it is somehow possible to categorize the opinion dynamics literature regarding opinion types, opinion exchange rules, and model characteristics. Not all but the most crucial studies of the literature are encapsulated below.

One of the simplest models of opinion dynamics which involves one-dimensional and discrete opinion representation is initially proposed by Clifford and Sudbury [15] to analyze two species competing for territory and later named as the "voter model" by Holley and Liggett [2] due to reevaluation and selection procedures in their model. The voter model consists of a population laying on a network where each agent holds one of the discrete opinions ± 1. At each time step, a random agent i and one of its neighbors j is selected, and agent i changes its opinion as to the opinion of agent j. Here, the opinions of the majority only play an indirect role since the agents consider the opinion of one neighbor only.

A variant of the voter model has been developed by Takaguchi *et al.* [16], using power-law intervals between interactions (i.e., inter-event intervals) opposing the original model. Their analysis with different network topologies (i.e., complete graphs, ring, and random graphs) suggests that the convergence times of the dynamics increase to different degrees based on the network structure.

Also, Mobilia [17] extends the original model by introducing a third opinion that stands in the middle of the two extreme opinions, and a parameter q denotes the bias towards either extremism or centrism. The results indicate that when $q > 0$ polarization is more probable; whereas, consensus has a higher change to occur when $q < 0$.

Another simple opinion formation model is the majority rule model proposed by Galam [18]. The model starts with a population that holds discrete opinions ± 1 and lays on a complete graph, i.e., agents can contact all other agents. At each time step, a group of m agents is selected randomly, and the opinion that has more followers within the group, namely the majority opinion, becomes the final opinion of all agents forming the group. Depending on the group size m, either the majority opinion is distinct, or a tie exists. When m is odd, there always has to be a majority opinion, and when there is a tie, the group tends not to change their opinions. The author notes that this procedure denies the initial minority opinions and causes the whole population to follow the dominant opinion. In their subsequent studies [19–21], they extend the model by varying agent characteristics (i.e., inflexible agents that do not change their opinions at all and contrarians that take the minority opinion within the group) and observe how dynamics are driven under different circumstances.

Sznajd model [22] is an extension of the Ising spin model [23], which employs the social impact theory [24]. The social impact theory states that the strength of individuals, the number of people involved and proximity are the factors that have an effect on an individual's opinion. The Sznajd model suggests that a group of people is more influential than a single individual while persuading others; in addition, it takes into account the proximity factor with limited neighboring of the lattice network. The last factor mentioned in the theory, however, has not been considered in the Sznajd model. In the model, each agent holds a discrete opinion ± 1, and at each time step, a pair of incident agents is selected. If selected agents share the same opinion, then all their neighbors follow that opinion. If the selected pair contradicts, then their neighbors adopt the opposite opinions. It has been shown that the model always reaches a steady state with agreeing opinions, either 1 or -1.

Castellano *et al.* [25] introduces the q-voter model as a nonlinear extension of the voter model in which an individual takes into consideration the opinions of q neighbors and reacts based on them. When all neighbors share the same opinion, the neighboring agent takes that opinion. Opposingly, if the neighbors disagree, then the agent switches opinion with a probability ϵ. Analytical studies suggest that the model can be considered a generalized version of the voter and Sznajd models. For $q > 3$, the model generates a new type of transition between the two phases.

So far, we only mentioned models with discrete opinions; however, in the literature, there also exist models that use continuous opinion representation. One of them is DeGroot model [26]. The model considers a population of n agents that have initial opinions $\mathbf{p}(0) = (p_1(0), \ldots, p_n(0))$. The weights each agent put on other agents is represented by a matrix $\mathbf{T}$, where T_{ij} is the weight that agent i gives to opinion of agent j. Here, $\mathbf{T}$ is a right stochastic matrix meaning that each row of the matrix sums to 1. At each time step t, opinions are updated as follows:

$$\mathbf{p}(t) = \mathbf{T}\mathbf{p}(t-1). \tag{2.1}$$

It is proven that consensus is only reachable when $\mathbf{T}$ is irreducible and aperiodic. The linear combination coefficients are identified with the eigenvector associated with the eigenvalue 1 of $\mathbf{T}$.

Regarding the psychological idea of bounded confidence, some popular opinion dynamics models assume that people only interact when their opinions are already close enough. One of those, the Deffuant-Weisbuch model (DW) [4], considers a population with continuous opinions, in which agents only adjust their opinion if the difference between their opinions is smaller than a threshold d. The threshold can be interpreted as openness or some uncertainty in opinion. At each time step, a pair of agents with opinions x_i and x_j, are randomly chosen, and if $|x_i - x_j| < d$, they update their opinions as follows:

$$
\begin{aligned}
x_i &= x_i + \mu(x_j - x_i) \\
x_j &= x_j + \mu(x_i - x_j).
\end{aligned}
\tag{2.2}
$$

Here, μ is the convergence parameter of the dynamics. Results suggest that opinion clusters only depend on d; whereas, μ and N only affect the convergence time of the dynamics. With the presumption that people are only influenced through a pre-existing social connection, they also carry out simulations on square lattice networks and investigate the effect of social networks on the dynamics. It is noted that for larger values of d, results do not significantly differ from the fully connected case, and the most apparent difference is the importance of the extremists. For $d < 0.3$, it is observed that consensus only occurs on connected sets of agents, and it results in local clustering. Later, it is demonstrated that the number of final opinion clusters c depends on the value of the bounded confidence parameter $c \approx \left\lfloor \frac{1}{2d} \right\rfloor$ [27].

The Hegselmann-Krause model (HK) [28] is another opinion dynamics model that involves the bounded confidence principle. In the model, agents hold continuous opinions $\mathbf{x}$ and are willing to interact with their neighbors holding opinions not farther than the uncertainty ϵ. The model only differs from DW on the opinion update rule, and the opinion of an agent i at time t is:

$$
x_i(t+1) = \frac{\sum_{j:|x_i(t)-x_j(t)|<\epsilon} a_{ij} x_j(t)}{\sum_{j:|x_i(t)-x_j(t)|<\epsilon} a_{ij}}, \tag{2.3}
$$

where a_{ij} is the adjacency matrix of the network. With this update mechanism, an agent takes the average opinion of its suitable neighbors. It is shown that the number of final opinion clusters decreases when ϵ increases. Lorenz [29] reports the critical threshold values for reaching consensus as 0.27 and 0.19 for DW and HK rules, respectively. One should note that DW rule is more useful to reflect dynamics between a large group of people, in which interactions take place in pairs. On the other hand, HK mechanism is designed to describe formal group meetings, where many people interact at the same time [14].

Heterogeneous bounds of confidence with DW dynamics is examined by Weisbuch *et al.* [30]. The simulations with open- and close-minded agents holding constant thresholds demonstrate that the existence of a small number of open-minded agents is enough to reach consensus with an adequately large time for convergence. Also, it is noted that the time required for dynamics shift from two clusters to a single one is proportional to the ratio of close- and open-minded agents and the total number of agents. Furthermore, they examine the scenario that an agent's uncertainty decreases with the number of interactions take place. When the simulation results are compared to the constant thresholds, it is observed that dynamic thresholds cause a larger variety of final opinion clusters.

Multidimensional opinions are also analyzed in [30] by characterizing agents' opinions as a vector of m discrete values. Opinion update then occurs when agents disagree on $d-1$ or fewer subjects. When opinions of a pair differ on an issue, one randomly selected agent converts his opinion with probability μ. They observe that d and m are the most effective factors of dynamics. Two results are observed with multidimensional opinion settings: consensus for larger d values and extreme diversity for smaller d values.

Later, Lorenz [6] studies both DW and HK with heterogeneous bounds of confidence. The model assumes that agent i has a bound of confidence ϵ_i which determines its confidence set of agents $I_{\epsilon_i}(i, x(t)) = \{j : |x_i(t) - x_j(t)| \leq \epsilon_i\}$. Opinion update rules are changed as follows for DW and HK, respectively:

$$x_i(t+1) = \begin{cases} \frac{x_i(t)+x_i(t)}{2} & \text{if } j \in I_{\epsilon_i}(i,t) \\ x_i(t) & \text{otherwise,} \end{cases} \tag{2.4}$$

$$x_i(t+1) = \frac{1}{\#I_{\epsilon_i}(i, x(t))} \sum_{j \in I_{\epsilon_i}(i,x(t))} x_j(t). \qquad (2.5)$$

Here, it is essential to notice that opinion updates are not necessarily symmetric with heterogeneous uncertainties. The simulation results reveal that dynamics may lead to consensus in a heterogeneous society even when both open-minded and closed-minded agents are more mistrustful than in a homogeneous society. It is also mentioned that opinion shift of the whole population is often observed even with uniform initial distributions.

Weisbuch [31] examines DW model on scale-free networks and compares the results with [4]. The opinion updating rule is assumed to be asymmetric such that well-connected nodes influence others proportional to their connectivity and are influenced as often as others. The results indicate that the number of clusters does not differ much from the fully connected networks.

Kozma and Barrat [32] investigate DW dynamics on adaptive networks. They presume that connected agents having opinions more distant than the threshold may choose to cut off their links, and then it is possible for them to create a new link with other agents. Polarization is observed with large threshold values, under the effect of this coevolution of the network and the opinions, as opposed to the original DW dynamics. Contrarily, for smaller threshold values, the number of final opinion clusters is dramatically increased due to the ease of finding other agreeable agents that the link rewiring procedure creates.

Deffuant *et al.* [33] proposes the relative agreement model (RA), which is an extension of DW. A set of agents holding extreme opinions, namely extremists, is introduced in addition to the agents with initially uniform opinions. They assume that the influence of agents varies inversely as their uncertainties, which makes the extremists more influential. The influence is also assumed to vary with respect to the distance between the opinions. Here, each agent i is characterized by its opinion x_i

and its uncertainty u_i, and the agents alter both attributes during interactions. The relative agreement of agent i with agent j is the overlap of their opinion segments h_{ij} divided by the uncertainty of the influencing segment u_i. If $h_{ij} > u_i$

$$\begin{aligned} x_j^{'} &= x_j + \mu(\frac{h_{ij}}{u_i} - 1)(x_i - x_j) \\ u_j^{'} &= u_j + \mu(\frac{h_{ij}}{u_i} - 1)(u_i - u_j). \end{aligned} \tag{2.6}$$

The simulation results show that for low uncertainty among the population, local agreements suppress extremists' influence, and a central consensus occurs. On the other hand, for high general uncertainty, extremism prevails and leads to either polarization or single extreme convergence. Thus, the convergence type depends on the initial uncertainty of moderate agents.

Amblard and Deffuant [34] investigate RA dynamics with different network structures. The simulations run on the regular lattice with a Moore neighborhood (connectivity $k = 8$) demonstrate that single extreme convergence never occurs. Moreover, beyond a critical level of connectivity, a shift to a single extreme opinion is observed with small-world networks. Additionally, for higher connectivity and lower rewiring probability, which creates a more regular network, a fast local convergence influenced by initial extremists that leads to polarization at extremes is detected.

Later, Weisbuch *et al.* [35] examines RA dynamics on square lattices and scale-free networks. Their simulation results with a lower extremist density show that all three convergence types can be observed on square lattices when the uncertainty of the moderates is in [0.25, 0.5]. This observation contradicts [34] who do not detect single extreme case. Additionally, they report that two-sided extremism is not observed with scale-free networks.

Deffuant *et al.* [36] proposes the smooth bounded confidence model to represent group opinion shift towards extremes that is observed with RA. In the model, it is supposed that individuals consider themselves from the other's point of view (empathy), and a Gaussian function of the distance between the opinions is used instead of a step function. The results with asymmetric empathy indicate that the highest shifts occur when the individuals have low uncertainties.

The characterization of agents as extremists and moderates to model opinion shifts to the extremes is a little problematic since it requires an explanation of how these initial extremists appear. Additionally, the assumption that extremists have a priori lower uncertainties is not reasonable. Besides, all of the models mentioned above only depend on the agreement, meaning that agents do not interact when their opinions are too different. However, disagreement is also common in real-life situations [37–39]. Therefore, recent studies that are mostly based upon theories from social psychology, investigate the polarization dynamics by adding a rejection mechanism.

Jager and Amblard [5] proposes a model based on the Social Judgment Theory (SJT). SJT [40] is a self-persuasion theory that suggests people change their opinion based on the position of the received message on an issue, considering whether it is acceptable or objectionable for them. Their model consists of a population where each individual i is characterized by an opinion x_i and two thresholds, u_i and t_i. At each time step, a random pair of agents is selected, and during the interactions between agents i and j, the following rules are applied:

$$dx_i = \begin{cases} \mu(x_j - x_i) & \text{if} |x_i - x_j| < u_i \\ \mu(x_i - x_j) & \text{if} |x_i - x_j| > t_i. \end{cases} \tag{2.7}$$

A series of simulations are run with continuous opinions [-1, 1] and homogeneous thresholds. They characterize different populations by varying threshold levels. It is observed that dynamics lead to various final opinion clusters, including consensus and polarization, based on the threshold settings. In their subsequent study [41], they extend the model by considering two-dimensional opinions. It is supposed that an agreement or disagreement on one dimension (A) will also be effective in a similar manner on the second dimension (B). The peripheral process is described by :

$$\begin{aligned} &\text{if } |xA_i - xA_j| < u_i \qquad dxA_i = \mu(xA_j - xA_i) \text{ and } dxB_i = \mu(xB_j - xB_i) \\ &\text{if } |xA_i - xA_j| > t_i \qquad dxA_i = \mu(xA_i - xA_j) \text{ and } dxB_i = \mu(xB_i - xB_j). \end{aligned} \tag{2.8}$$

Although most agents tend to central convergence, it is also observed, especially in later time steps, some agents having an extreme opinion on dimension A also evolve extreme position on dimension B. It is seen that the correlation between the positions held on two dimensions may be strongly positive or negative.

Later, they replicate the experiments in [5] using square lattices to study the effect of social network topology on opinion dynamics [42]. They indicate that in spite of the huge decrease in the average connectivity, model behavior does not change; therefore, three convergence types, consensus, polarization and fragmentation, are robust to the introduction of social networks.

Salzarulo [43] introduces another opinion dynamics model that regards contrasting. When the self-categorization theory based model is simulated with fully connected networks, results are found to be very similar to those have been obtained in [5]. The author also tests the model with small-world networks and observes single extreme convergence for some parameter settings. A few agents having extreme opinions are found to push the majority from center to extremes.

Chau *et al.* [44] uses two different parameters for convergence and divergence dynamics, μ and λ respectively. Simulation results with $\mu = 0.2$ and $\lambda = 0.05$ indicate that $d_1 + d_2 < 1$ and $d_2 > \frac{1}{2}$ are the conditions for having extreme and moderate final opinions respectively, where d_1 and d_2 are the lower and upper thresholds such that $d_1 \leq d_2$.

Another SJT based model is proposed by Huet *et al.* [45], which considers two-dimensional opinions with equal importance. It is assumed that if an agent strongly disagrees with someone on the first dimension, and their opinions are close on the second dimension, they move away on the second dimension to remove the existing discrepancy. Based on the experiments, the number of final opinion clusters is seen to be growing linearly with the inverse of the uncertainty. In their ensuing study [46], opinion dimensions are distinguished in terms of priority. The attraction threshold u_m for the main dimension and the rejection threshold u_s for the secondary dimension are defined. If the distance between the main opinions of two agents is smaller than the attraction threshold, they get closer on both dimensions. On the other hand, when the distance on main dimension is larger than attraction threshold and the distance on the secondary dimension is smaller than rejection threshold, they move away on the second dimension to solve the dissonance. Simulation results show that lower values of u_m cause fluctuations on the secondary dimension, especially when u_s is high.

As we mentioned earlier, it is also essential to understand the impact of external information on dynamics within the network. Thus, there is a variety of studies in the literature devoted to the analysis of communication other than word-of-mouth.

In one of these studies [41], Jager and Amblard examine the effect of a meta-actor on SJT-based dynamics with two-dimensional opinion settings. By using regular grids, they allow agents to contact either one of their neighbors or the meta-actor, i.e., the meta-actor is being interacted by a 20% chance at each time step. The meta-actor is formalized by an extreme position (-1) on the first and neutral positions (0) on the second dimension. With an accepting population setting, the majority is seen to accept the opinion of the meta-actor. In contrast, with a less accepting population, it is observed that opinions polarize on dimension A and fragmentation occurs on dimension B.

In [27], Carletti *et al.* investigate under what conditions propaganda, i.e., a message that each individual receives at the same time, could be efficient and affect the opinion dynamics. They introduce an external subject with a particular constant opinion into the network and assume that all agents interact with the exterior at every T time step. They define propaganda efficiency (E_p) as the proportion of individuals convinced by the external subject. The simulation results indicate that E_p depends on the threshold of agents, the external opinion, and the propaganda period T. It is observed that, for some cases, propaganda affects only local interactions; therefore, the global opinion dynamics do not change. Moreover, imposing propaganda too early results in extremist opposition (i.e., bipolarization).

With the assumption that radical agents prefer interacting with radicals, while neutrals randomly choose whom to communicate with, Gargiulo *et al.* [47] proposes an extension of DW. Later, they include a Big-Agent (BA) that is completely connected with all other agents and holds an extreme opinion to the model. The persuasive strength of BA is represented by $\epsilon \in [0, 1]$. Their analyses show that central clusters tend to move towards the opinion of BA, for $\epsilon > 0$. However, for $\epsilon = 1$, another cluster around the opposite extreme is also observed.

Martins *et al.* [48] present another extension of DW by assigning weights, either positive $+1$ or repulsive -1, to the links between agents. Additionally, they suppose that the entire population interacts with a constant external message at every T time step. The results demonstrate that in the presence of repulsive links, a consensus around the external message can be reached even in a close-minded population.

Pineda and Buendía [49] investigate the effect of mass media and heterogeneous bounds of confidence by using both DW and HK update rules. In their model, an agent i interacts with either the external mass media (with probability m) or one of the other agents (with probability $1 - m$). Without external mass media, the chance of a population with heterogeneous bounds of confidence to reach consensus is higher for intermediate values of heterogeneity. Likewise, the ability of the mass media to convince the majority of the population is only improved when intermediate values of heterogeneity are the case. On the contrary, if interactions with the mass media are too intense, the mass media is seen to be incapable of imposing its opinion. The importance of the global communication setting of HK to oppose the influence of the external mass media is another critical finding of the study.

In this study, SJT based opinion dynamics will be further investigated. It is observed that the previous studies do not evaluate the impact of network topology and external actors; thus, we will focus on these aspects. At first, some experiments will be conducted for verification and validation. Later, we will compare the results that are obtained from different network initializations to understand whether the underlying topology affects the dynamic on top of that. Finally, an influential external actor will be introduced into the network, and its effects on opinion clusters will be observed.

3. MODEL DESCRIPTION

In this study, we aim to have a better understanding of the dynamics causing polarization and consensus in the final opinion distributions of a society. A simulation model representing people and the interactions that form their opinions is needed to examine these phenomena more deeply. Even though they are built based on many assumptions, simulation models are evidently effective in interpreting opinion evolution and opinion-based cluster formation [1].

Since the problem apparently requires an individual-level representation, an agent-based model is constructed. Agent-based models allow us to identify agent attributes in detail and specify a rule set that determines how those agents behave under different conditions. NetLogo [50], a popular general-purpose agent-based modeling language, is used for all modeling and simulation processes.

In this chapter, we explain the opinion dynamics model and the characteristics of the network topologies that have been used in detail. Additionally, the extension of the model that involves an external actor is given. All experimentation procedure (including the ones have been performed for verification and validation purposes) and the results will be discussed in the Chapter 4.

3.1. Base Model

An agent-based opinion dynamics model similar to the one that is proposed by Jager and Amblard [5], is used as the base model of this study. The model consists of a network of people, and a rule set determining communication and opinion update mechanism.

The social network is formed by a population of n people with continuous opinions $\boldsymbol{x}$, where x_i symbolizes the opinion of agent i. Although it may be valid for some cases, choosing binary opinions restricts people to stand on one side, which we believe is inappropriate for real-life communication relations. To represent the variability of people's points of view, we select continuous opinions over binary ones. For this study, opinions are decided to take values between zero and one.

Besides an initial opinion, agents have two thresholds that demonstrate their openness or narrowness to communicate. Each agent i have lower (l_i) and upper (u_i) thresholds, which determine their latitude of acceptance and latitude of rejection, respectively as illustrated in Figure 3.1. As the naming of these parameters also indicates, u_i must be greater than l_i.

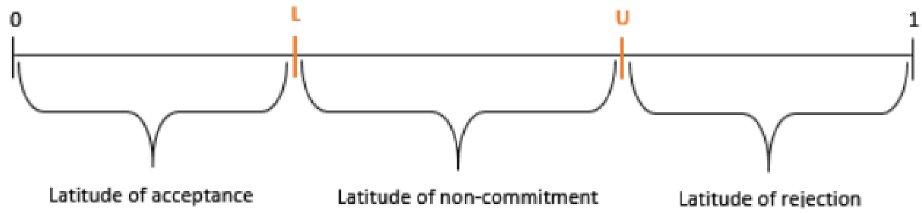

Figure 3.1. Latitudes of acceptance, non-commitment and rejection as described in [40].

The threshold levels of population are presumed to be constant and homogenous, meaning that all l_i=L and all u_i=U. A brief analysis of the model with heterogeneous thresholds will be given in Section 4.6, which is the only exception to threshold homogeneity throughout the study.

At each time step (tick), a fixed number of links are randomly selected. Both nodes incident to a selected link, i.e., nodes connected by that link, communicate. One should note that the networks used in this study are assumed to be undirected, which means the communication between individuals is symmetric.

When two agents communicate, they first calculate the distance between their opinions. Later, they control which region the distance value falls into, considering their threshold levels and decide whether they will update their opinion based on the information received and the direction of the movement. If the difference between agents' opinions is smaller than the lower threshold, agents' minds come closer, and if it is larger than the upper threshold, they move away. In short, two agents incident with a selected link update their opinions as a weighted average. When $|x_i - x_j| < L$, agents update opinions as given in [4] with

$$\begin{aligned} x_i[t+1] &= x_i[t] + \alpha(x_j[t] - x_i[t]) \\ x_j[t+1] &= x_j[t] + \alpha(x_i[t] - x_j[t]), \end{aligned} \tag{3.1}$$

where i and j are two neighbors, x_i and x_j represent their opinions and t represents the current time step. Here, α is the mixing (or convergence) parameter that demonstrates the relative value that one gives to neighbors' opinion, whereas $1 - \alpha$ is the weight of their own opinions. The mixing parameter can be assigned to a value between zero and one. Surely, opinion change is inversely signed when agents disagree, i.e., $|x_i - x_j| > U$:

$$\begin{aligned} x_i[t+1] &= x_i[t] - \alpha(x_j[t] - x_i[t]) \\ x_j[t+1] &= x_j[t] - \alpha(x_i[t] - x_j[t]). \end{aligned} \tag{3.2}$$

One should keep in mind that agents cannot exceed the limits of the opinion range, i.e., once an agent reaches one of the extremes, opinions cannot move further; however, it is always possible to move towards the center. If an agent communicates with more than one of its neighbors at a single tick, then it evaluates each communication separately and, if necessary, updates their opinions in-between the communications.

3.2. Network Topology

Three different network topologies, namely random, small-world and scale-free, is compared in this study to examine the effect of network structure on model behavior.

For an accurate comparison of the networks, one needs to familiarize with the network properties. Without understanding the networks' specific characteristics, it is impossible to know what to expect and interpret the results. The clustering coefficient, average path length, and degree distributions are main structural properties of the networks.

The local clustering coefficient of a node is a measure of how its neighbors are connected. For an undirected graph, it can be defined as

$$C_i = 2 \times \frac{NNL_i}{k_i(k_i - 1)}. \tag{3.3}$$

Here, k_i is the number of nodes in the neighborhood of node i and NNL_i represents the total number of links of those nodes. Average path length indicates the average of the shortest path lengths between any two nodes of the network and defined by

$$l_N = \frac{1}{n} \times (\frac{1}{n-1} \sum_{i \neq j} d_{ij}), \tag{3.4}$$

where d_{ij} is the shortest distance between nodes i and j. It is possible to compare the networks used in this study, in terms of these structural properties.

Random networks are characterized by low clustering coefficients and average path lengths. The degree distribution of the random networks converges to a Poisson distribution, which, for some cases, might be problematic when illustrating the real world.

Comparatively, small-world networks are known to have high clustering and moderate average path lengths. Due to nodes having almost equal degrees, small-world networks are sometimes considered to be inappropriate for representing real-life networks.

Lastly, scale-free networks have moderate clustering coefficients and very low average path lengths. Their degree distribution follows a power-law distribution, which is thought to be suitable for many real systems.

It is decided to execute Erdős-Rényi [51], Barabási–Albert [52] and Watts–Strogatz [53] algorithms to construct random, scale-free and small-world networks, respectively. Each algorithm requires additional parameters to be determined for network construction: p_c for random networks, p_r and k for small-world networks and d_{min} for scale-free networks. p_c is the probability of a link being created between any two nodes, and p_r is the rewiring probability of any link after starting from a ring lattice. k is the average degree of the network; whereas, d_{min} determines the minimum degree for each node. Figure 3.2 - Figure 3.4 illustrate the pseudo codes for each algorithm.

3.3. External Actor

Most of the time there exist some actors that are more influential than the others in social networks. Those actors may tend to use this influence to mark their own favor. We observe numerous examples of this situation in real life: a political party leader convinces the public to vote, or media channels use advertisements to persuade their viewers. Thus, it is decided to introduce such an actor into the model to inspect whether it alters the model behavior and causes outcomes different from those previously observed.

```
Input  : number of nodes n
Input  : connection probability p_c
Output: an Erdős-Rényii network G(V, E)

V ← {0, 1, ..., n − 1}
E ← ∅
for each agent i in V do
    for each agent j in V do
        p′ ← uniform(0, 1)
        if p′ < p_c then
            E ← E ∪ {i, j}
        end
    end
end
return G(V, E)
```

Figure 3.2. Pseudo code of Erdős-Rényi algorithm.

An extension of the model is given by adding a single external actor (EA) with a constant opinion to the model. It is assumed to be an influential actor who only communicates with the people in the network to convince them, i.e., the interaction is not symmetrical as it is between two people. Furthermore, the message perceived from EA is presumed not to cause contrast, meaning that the agents communicate with EA only update their opinion when the difference between opinions are smaller than L.

The communication mechanism is updated by including a probability (q), which is the probability of agents communicating with EA at any time step. A second convergence parameter (μ) is also included for the communications between EA and people.

```
Input  : number of nodes n
Input  : minimum degree d_min
Output: an Barabási–Albert network G(V, E)

V ← {0, 1, ..., n − 1}
E ← ∅
for each agent i = 0 to m − 1 do
    for each agent j = i + 1 to m do
        E ← E ∪ {i, j}
    end
end
for each agent i = m + 1 to n − 1 do
    while current degree d_i < d_min do
        Randomly select an agent j, excluding i and nodes adjacent to i
        r ← # of nodes adjacent to j/|E|
        p' ← uniform(0, 1)
        if r > p' then
            E ← E ∪ {i, j}
        end
    end
end
return G(V, E)
```

Figure 3.3. Pseudo code of Barabási–Albert algorithm.

3.4. NetLogo Implementation

The model is implemented in the NetLogo environment in a way that it can initiate all the network structures mentioned above and appropriately reflect the model description. All experimentation procedure is carried out in NetLogo as well. Figure 3.5 displays the NetLogo interface with the required parameters of the model.

```
Input   : number of nodes n
Input   : rewiring probability p_r
Input   : average degree k
Output: a Watts-Strogatz network G(V, E)

V ← {0, 1, ..., n − 1}
E ← ∅
for each agent i in V do
    for each agent j = i + 1 to i + k/2 do
        E ← ({i, j} mod n)
    end
end
for each agent i in V do
    for each link e do
        p′ ← uniform(0, 1)
        if p′ < p_r then
            Randomly select an agent j′
            while {i, j′} ∈ E do
                Randomly select an agent j′
            end
            E ← E\{e} ∪ {i, j′}
        end
    end
end
return G(V, E)
```

Figure 3.4. Pseudo code of Watts–Strogatz algorithm.

When the setup procedure is called, the model creates a network of size n. Here, n represents the number of nodes the network consists of, and each node symbolizes a person. Additionally, initial opinions of agents are uniformly distributed whenever a network is created. NetLogo's network extension, NW, is used to construct social networks [50]. This extension allows users to create most of the complex structures that are widely used to represent social networks. Figure 3.6 - Figure 3.8 exemplify all three networks with equal size and average degree.

The next step after the initialization of the network is simulating the dynamics. At each time step, $\frac{n}{2}$ links are randomly selected. Thus, one communication per node, on average, takes place in the network at every time step.

Once a simulation is started, agents keep communicating and updating their opinions based on the rules described above, until the stopping criteria are met. For this model, the stopping criterion is decided by considering the opinion variations of the agents. When the sum of opinion change of all agents is smaller than a specific value K as given by,

$$\sum_{i=1} |x_i[t] - x_i[t-1]| \leq K \tag{3.5}$$

the simulation stops and it is assumed that the network has reached a steady-state in terms of opinion clusters.

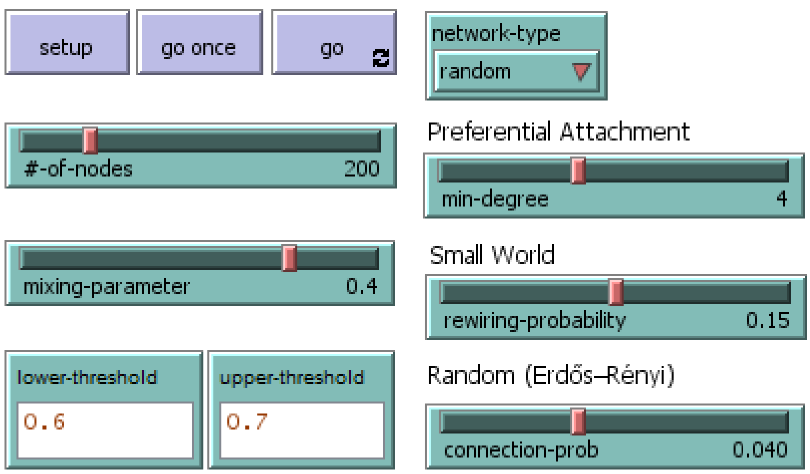

Figure 3.5. Parameter set of the model.

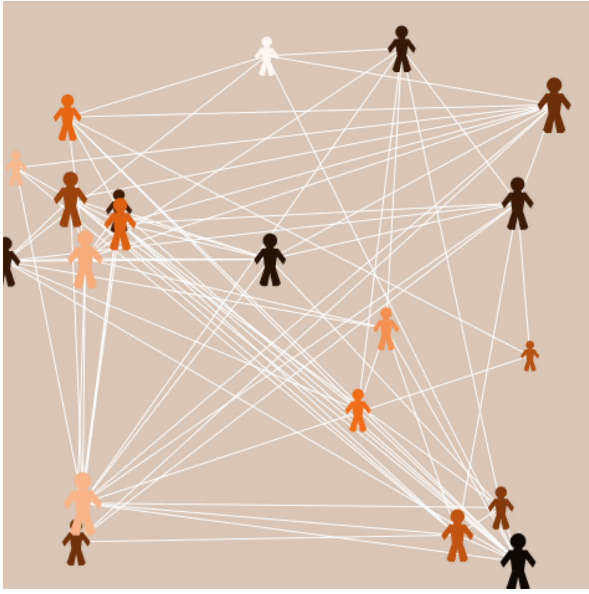

Figure 3.6. An illustration of random networks with $n = 50$.

Figure 3.7. An illustration of scale-free networks with $n = 50$.

At the end of the simulations, we need to extract valuable information from the model to interpret the results. Here, we are interested in three outcomes of the model.

Figure 3.8. An illustration of small-world networks with $n = 50$.

The time spent until reaching a steady-state, namely run length, is the first one. It is exactly the tick count of the model when stopping criterion is met. Therefore, to see under which conditions simulations quickly stop or take a longer time, we need to check the run length at the end of the simulations.

Additionally, the number of clusters at the end of simulations is another important outcome for evaluating the results. Clustering is subjective by nature, and there exist various algorithms using different approaches. Partitioning algorithms (e.g., k-means algorithm [54]) are one of the most popular clustering algorithms [55]. The biggest drawback of partitioning algorithms is that they require the number of clusters, k, to be predetermined. Some specific domain knowledge is clearly needed to determine k. Similarly, a termination condition has to be defined for hierarchical clustering algorithms, which sometimes can be problematic. In this study, we use a density-based clustering algorithm for the assessment of the opinion clusters.

DBSCAN is a density-based clustering algorithm proposed by Ester *et al.*. [56]. The algorithm relies on the basic idea that clusters have a significantly high density that is easily recognizable compared to lower density regions. Thus, the algorithm does not require k to be determined in advance. Instead, two parameters are needed for the algorithm to detect clusters: Eps and $MinPts$.

Eps-neighborhood of a point, denoted by $N_{Eps}(p)$, is the set of all points within a radius of Eps and it is defined by

$$N_{Eps}(p) = \{q \in D | dist(p, q) \leq Eps\}\cdot \tag{3.6}$$

$MinPts$ represents the minimum number of points necessary to form a cluster.

There are no specific instructions for the selection of these parameters. However, Schubert *et al.* [57] state that Eps should be chosen as small as possible considering the way the algorithm is designed. Additionally, $MinPts = 1$ is not a reasonable choice since it allows every single point to be a cluster by itself. Therefore, for our model, we select $MinPts$ as 3 and Eps as 0.05. Normally, the determination of a distance function is also needed; yet, this is not valid for our model since it contains only one agent attribute (i.e., opinion) necessary to be considered while clustering, which lies on a line segment.

In the model, NetLogo's DBSCAN extension [58] is used to identify opinion clusters. With the aforementioned settings of the algorithm parameters, the model reports the number of clusters within the network at the end of each simulation. Clearly, the number of clusters is equal to one corresponds to the population has reached a consensus. As fragmentation of agents' opinions increases, the number of clusters rises as well.

```
Input   : network G
Input   : Eps
Input   : MinPts
Input   : distance function dist
Input   : label, initially undefined
DBSCAN(G, dist, Eps, MinPts):
Cluster label C ← 0
for each agent i in G do
    if label(i) ≠ undefined then continue
    Neighbors N ← RangeQuery(G, dist, i, Eps)
    if |N| < MinPts then
        label(i) ← Noise
        continue
    end
    C ← C + 1
    label(i) ← C
    Seed set S ← N\{i}
    for each agent j in S do
        if label(j) = Noise then label(j) ← C
        if label(j) ≠ undefined then continue
        Neighbors N ← RangeQuery(G, dist, j, Eps) label(j) ← C
        if |N| < MinPts then S ← S ∪ N
    end
end
where RangeQuery(G, dist, j, Eps):
Neighbors N = null
for each agent i in G do
    if dist(x_j, x_i) ≤ Eps then
        N ← N ∪ {i}
    end
end
return N
```

Figure 3.9. Pseudo code of DBSCAN algorithm as adapted from [57].

Lastly, another outcome to investigate final opinion clusters is used in the model. That is the dispersion index y, proposed by Derrida and Flyvberg [59]:

$$y = \frac{\sum_{i=1}^{n} s_i^2}{n^2}, \tag{3.7}$$

where, s_i is the size of cluster i.

For a single cluster, $y = 1$. The value of y decreases as the number of clusters increases. It is possible to consider y as some average of the inverse number of clusters. As Weisbuch [31] states, y is an appropriate indicator of opinion dispersion for uniformly distributed opinion and large population size, when convergence is slow.

4. EXPERIMENTS AND RESULTS

After the implementation of the opinion dynamics model, some experiments are conducted using NetLogo's BehaviorSpace tool. BehaviorSpace allows users to perform various experiments with their models by changing the model's settings and records the results obtained from many runs. This way, one can explore the model's possible behaviors and determine which combinations of settings cause the behaviors of interest.

We start with a few experiments to verify that model behaves as expected. Further experimentation is done to see whether the model's behavior change when network topology differs or an external actor is introduced. In addition to those, the model is also tested with heterogeneous thresholds. The details of all these experiments and the remarkable points detected during the analyses are given in the sequel.

4.1. Verification and Validation

Before starting the experimentation we need to control whether our model demonstrates the characteristics of the conceptual model accordingly. Thus, the verification and validation of both network creation and opinion update procedures of the simulation model is required.

For the sake of comparison, networks are created in a way that they have almost equal average degrees by setting network parameters accordingly. While deciding the value of the connection probability of random networks, link density and node isolation are considered. Since social networks (e.g., citation networks, friendship networks) are known for being sparse, a value, 4%, which creates relatively sparse networks without many isolated nodes, is selected. Similarly scale-free and small-world networks are created with $d_{min} = 4$ and $k = 8$. Also, p_r is set to 0.15 by bearing in mind the tradeoff between high clustering and short path lengths. Table 4.1 summarizes network properties to verify the networks are suitably replicated in NetLogo so that they reflect their known characteristics.

Table 4.1. Average network properties for 10 replications.

	Random	**Scale-free**	**Small-world**
Clustering coefficient	0.04	0,107	0,409
Average path length	2,79	2,627	3,392

The model seems to be properly replicating network structures. Random networks have the lowest clustering coefficent, while small-world networks have the highest average path length. Moreover, scale-free networks stand in-between the others in terms of clustering coefficient and have a slightly lower average path length compared to the random networks. Overall, these results match up with the expected network characteristics. Additionally, degree distributions of sample networks are illustrated in Figure 4.1 - Figure 4.3. As expected, small-world networks have the narrowest spread and scale-free networks clearly demostrate the power-law distribution which is also expected.

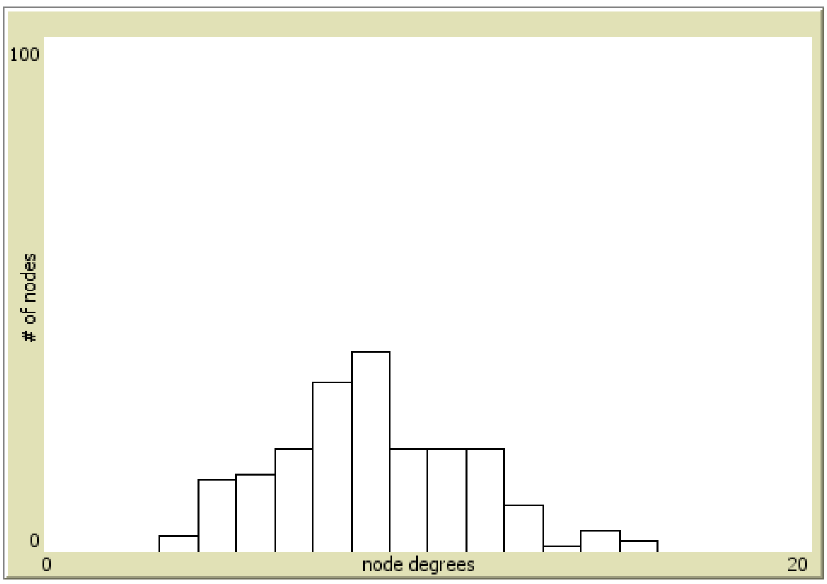

Figure 4.1. Degree distribution of a sample random network.

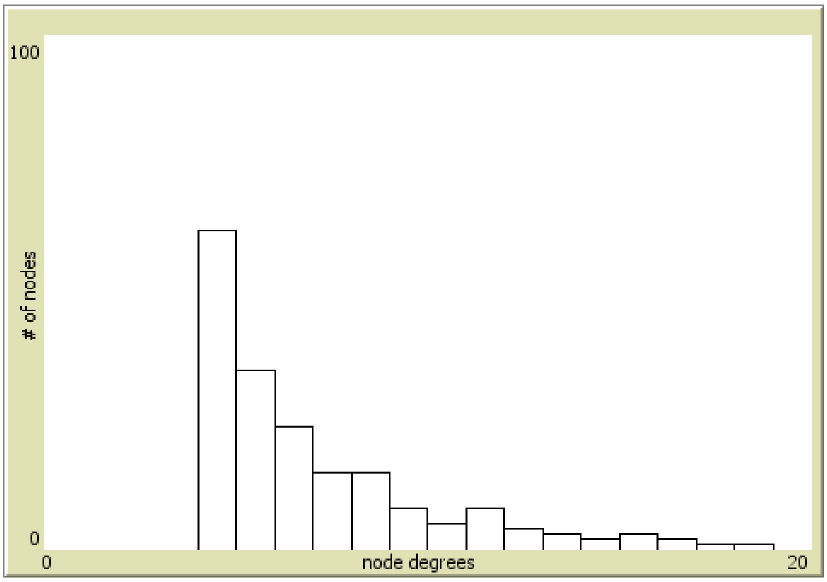

Figure 4.2. Degree distribution of a sample scale-free network.

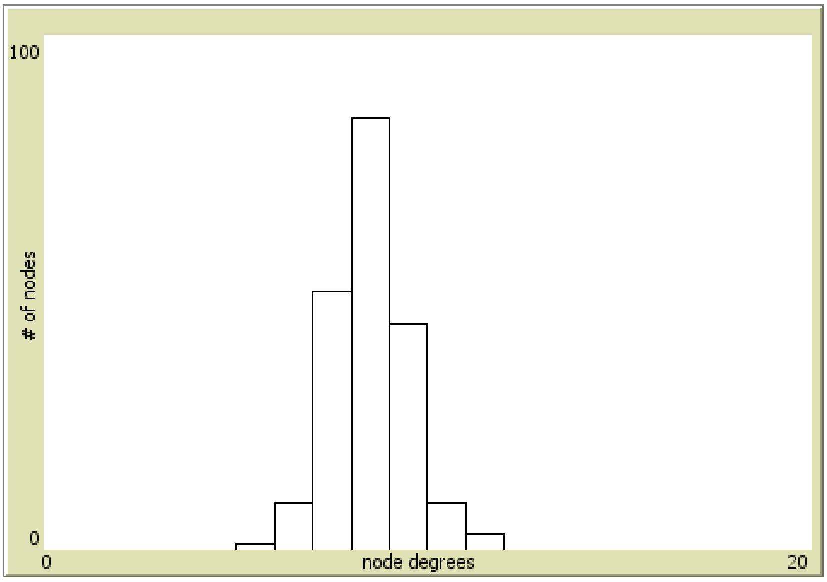

Figure 4.3. Degree distribution of a sample small-world network.

Validation of the opinion update procedure is done by the observation of individual simulation runs with small sample networks. Opinions of a randomly selected pair of agents, which are communicating at the moment, is repeatedly recorded, and the expected opinions are calculated considering their threshold levels. Then, these expected opinions are compared with the ones that is observed after one time step. The equality of the opinions is seen as a proof of appropriate implementation of the opinion update rules.

Taking everything into consideration, it is possible to say that the agent-based model coincides with the conceptual model that is expressed in the previous chapter.

4.2. Scenario Definitions and Reference Results

Since it is hard to quantify the threshold levels for any population, we come up with four threshold scenarios in a similar manner to [5]. Since the opinions are assumed to be in $[0, 1]$, thresholds cannot have values outside of this range, and threshold scenarios are generated as follows:

(i) Low-Low (LL): L=0.2, U=0.3
(ii) High-High (HH): L=0.6, U=0.7
(iii) Low-High (LH): L=0.3, U=0.6
(iv) Very low-Very high (VV): L=0.15, U=0.85

It is believed that each threshold scenario reflects populations with different characteristics. With the LL setting, agents have the largest latitude of rejection; whereas, they have the largest latitude of acceptance with the HH setting. VV scenario formalizes a massive latitude of non-commitment. One may consider a population that holds LL thresholds as a conservative one. Contrarily, populations formed by the agents that have HH thresholds can be interpreted as progressive.

Then, we collect reference results from the base model, which will be later used for comparison. Random networks are selected for this initial experiment. All four threshold scenarios are tested with $n = 200$ and $\alpha = 0.4$; the model's sensitivity to these parameters will be later examined. Since p_c is chosen as 0.04, the random networks with 200 nodes would have approximately 800 links. Table 4.2 summarizes the reference results obtained from the base model. Numerical results in the table demonstrate the averages of 50 runs.

Table 4.2. Summary table for the reference experiments.

	LL	**HH**	**LH**	**VV**
run length	59.38	291.4	807	1545.1
dispersion	0.497	0.998	0.361	0.273
# of clusters	2	1	3.4	5.42
observation	polarization	consensus	3 clusters	4 or more clusters

From Table 4.2, one can easily observe that threshold scenarios result in various opinion dynamics in the population. Final fragmentation and the time spent until reaching steady-state significantly differ based on the thresholds. It is possible to say that the LL scenario has the fastest dynamics since its average run length is relatively smaller compared to the other scenarios. It can be seen that the dispersion index converges to 1, which is the case of only one opinion cluster, with the HH scenario. Moreover, the number of final clusters varies between one and eight with different threshold levels. Details of the experiments with each scenario are explained in the subsections below.

4.2.1. Low lower and upper thresholds (LL)

Table 4.2 demonstrates that the LL threshold scenario requires the lowest time to reach the steady-state. Besides, the average dispersion index for the LL scenario, 0.497, indicates two -almost- equally sized clusters in the network. The average number of clusters crystallizes that the LL scenario results in exactly two opinion clusters.

Using low lower and upper thresholds creates a population with small latitudes of acceptance and large latitudes of rejection, which makes the possibility of contrasting between agents remarkably high. With this threshold setting, it is observed that the population splits into two groups at extreme opinions as visualized in Figure 4.4, which is the polarization case that we are interested in.

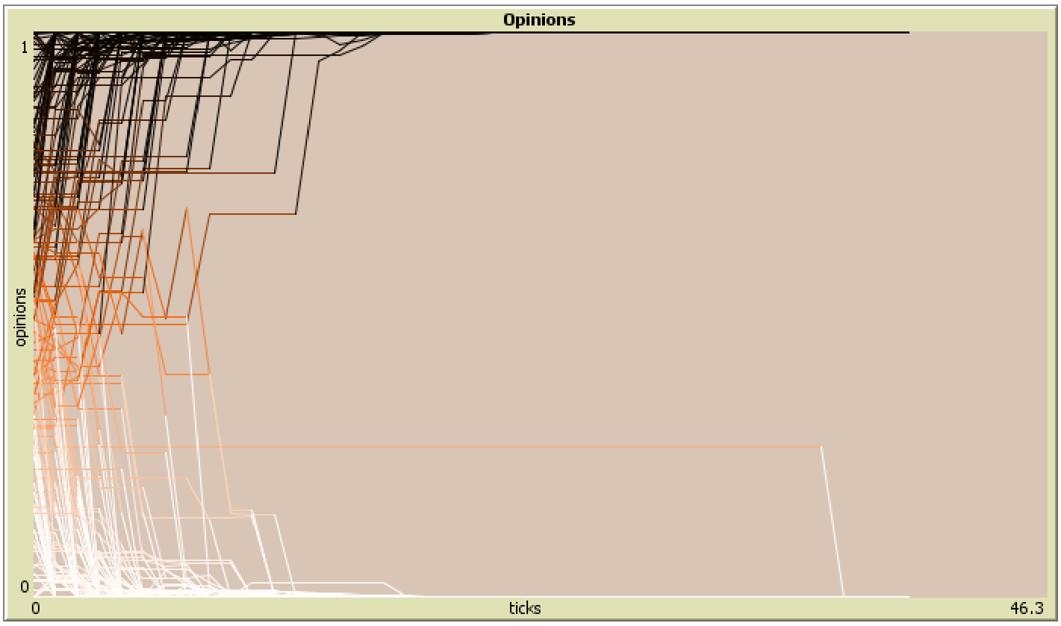

Figure 4.4. A sample simulation result of the LL scenario.

Here, it is clear that the agents with an initial opinion close (closer than $L = 0.2$) to extreme values directly pull towards the extremes. Additionally, agents with moderate opinions push each other after a communication if the distance between their opinions is greater than $U = 0.3$. These interactions together cause the emergence of a polarization.

4.2.2. High lower and upper thresholds (HH)

As the dispersion index and the number of clusters illustrate, the population forms a single cluster at the end of the simulations. In addition, dynamics result with the HH scenario are the second-fastest as can be seen from Table 4.2.

This threshold scenario allows us to model a population with large latitudes of acceptance and small latitudes of rejection. Here, latitudes of non-commitment are also small since there is almost no distance between two thresholds. Figure 4.5 displays an example of the opinion dynamics observed with the HH scenario.

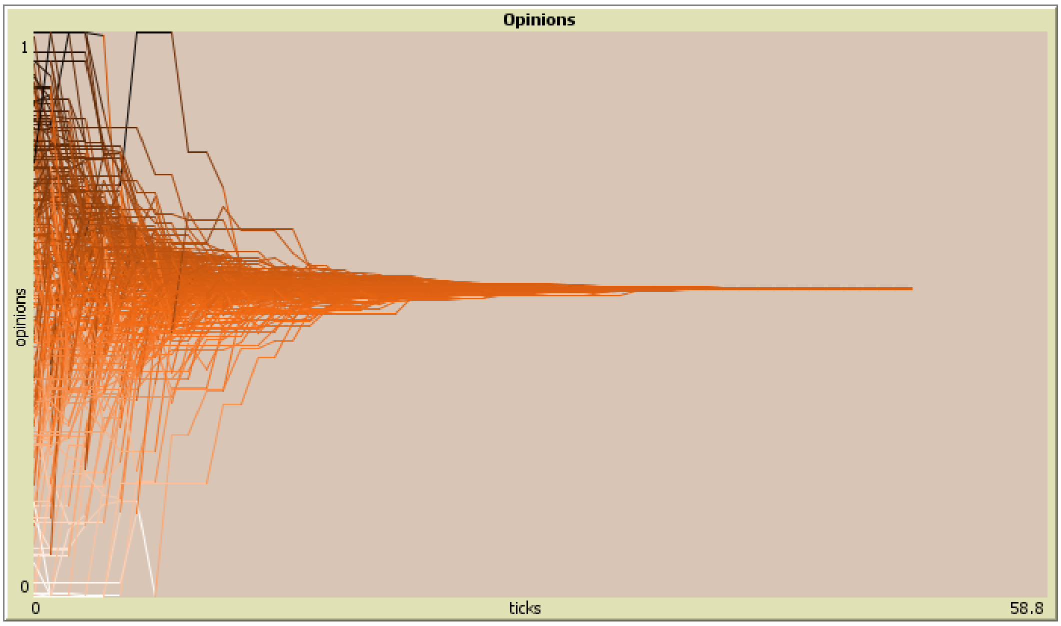

Figure 4.5. A sample simulation result of the HH scenario.

It is easily noticeable that assimilation is the dominant dynamic, and the population reaches a consensus at the average opinion (i.e., 0.5). Although contrast occurs during some communications, a more significant proportion of interactions result in convergence.

4.2.3. Low lower threshold, high upper threshold (LH)

From Table 4.2, we notice that simulations take a relatively long time with the LH scenario. The average dispersion index implies that the population is fragmented into approximately three clusters. The average number of clusters being equal to 3.4 also supports this implication.

It is somehow expected to have more opinion clusters with the LH setting since we have a population with moderate latitudes compared to the previously mentioned opposing scenarios. Figure 4.6 depicts an example of the opinion dynamics observed using the LH scenario.

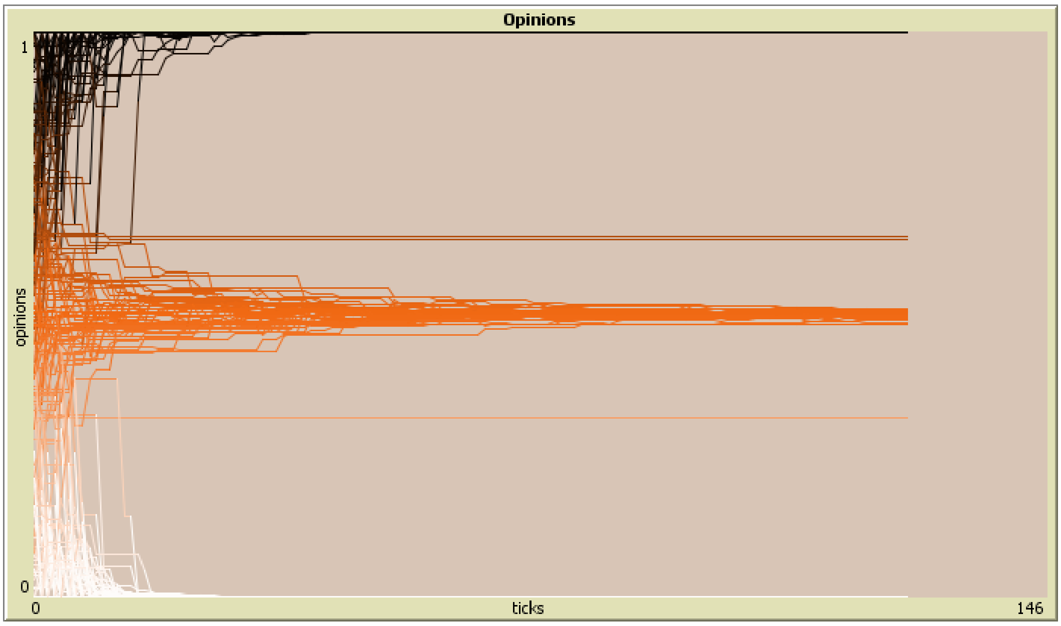

Figure 4.6. A sample simulation result of the LH scenario.

It is possible to observe that both assimilation and contrast dynamics are effective; however, none of them is able to dominate, and simulation results show a mixture of consensus and polarization with three distinct clusters positioned at the extremes and the average.

4.2.4. Very low lower, very high upper threshold (VV)

With the VV scenario, the simulation results have the longest run length, highest dispersion, and most clusters. These observations seem normal because the VV scenario forms a population with huge latitudes of non-commitment, meaning that even though they continuously communicate with others, they tend not to change their opinion. An illustration of the opinion dynamics with the VV scenario can be seen in Figure 4.7.

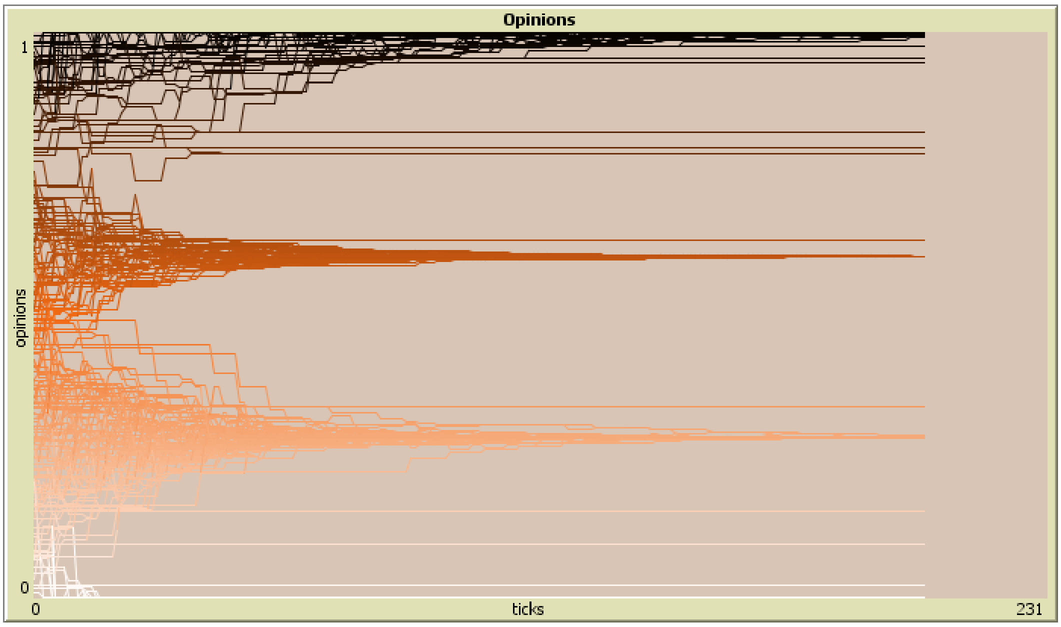

Figure 4.7. A sample simulation result of the VV scenario.

The resistance to opinion change generates some additional in-between clusters. In addition, the opinion cluster at 0.5, which was previously observed with scenarios HH and LH, is not detected in this case.

4.3. Sensitivity Analysis

A series of experiments are carried out to assess our model's robustness to uncertainties of the predetermined model parameters. The results are compared with the reference results to detect if the model behavior or the outcomes change with specific parameter combinations.

In many studies [4, 5, 31], it is stated that varying α only affects the speed of the dynamics. Opposingly, [60] indicates that it also impacts the sizes of minority clusters. Also, [61] provides evidence that varying the convergence rate has an effect on the distribution of final clusters with HK rules. Therefore, an experiment is conducted for different levels of alpha to be certain about its effect. Figure 4.8 shows the results obtained by varying α for all four threshold scenarios.

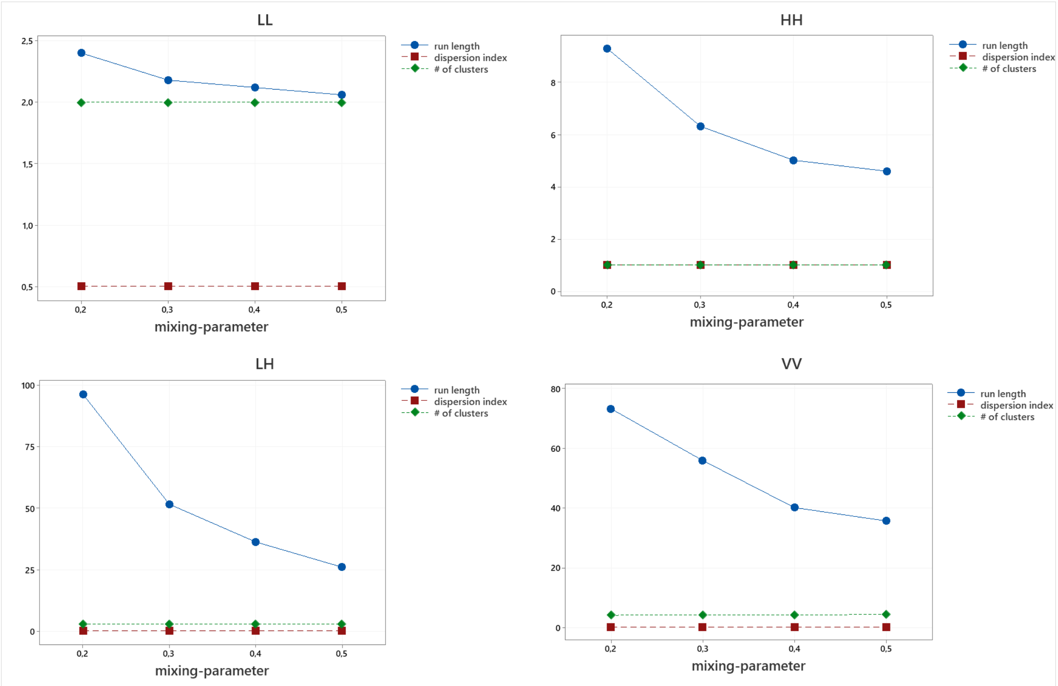

Figure 4.8. Line plots respresenting the averages of outcomes, for $\alpha = 0.2$, 0.3, 0.4 and 0.5.

Our results also indicate that the mixing parameter does not have an effect on the model's outcomes other than the run length. Hence, there is no need to change the alpha level for the rest of the analyses.

In addition, the effect of the population size is needed to be tested. The model is run by setting n to three distinct values while keeping every other parameter the same. Figure 4.9 and Figure 4.10 graphically show the effect of varying n on number of clusters and run length, respectively.

As the number of nodes increases, the simulations seem to last longer. However, for LH and VV scenarios, it is possible to say that $n = 500$ have a wider distribution compared to other values of n. When dot plots are considered one can say that n has only minor effects on the final opinion distributions.

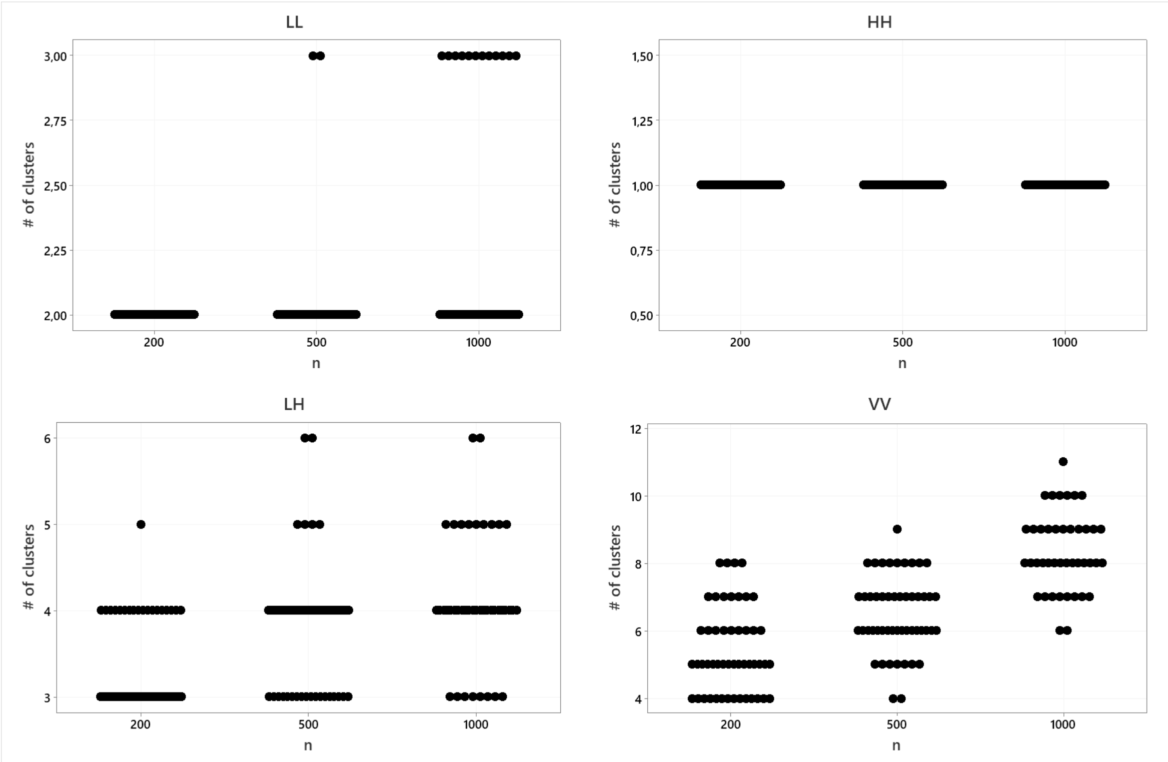

Figure 4.9. Dot plots respresenting the individual values of the number of final opinion clusters, for $n = 200$, 500 and 1000.

4.4. Impact of Network Topology

Even though a different communication regime and comparatively sparser networks are used in this study, our reference results obtained from random networks do not contradict those provided in [5]. Nonetheless, a comparison with other generic structures that are believed to represent real-life phenomena better is needed. Thus, the model is also tested with scale-free and small-world networks, and outcomes are compared with those of collected from random networks. For both networks, 50 replications are run with $n = 200$. A brief summary of the results are presented in Table 4.3.

By looking at Table 4.3, it is possible to say that different network structures do not give rise to important changes, especially in terms of the final opinion clusters.

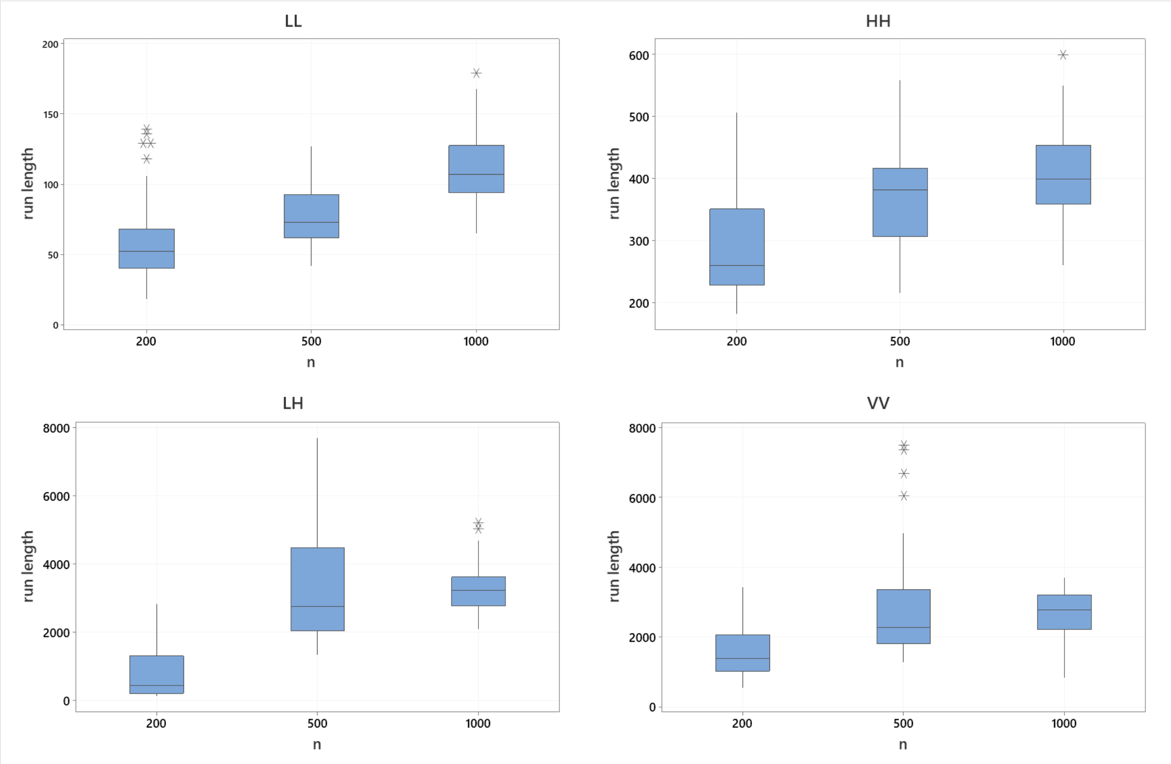

Figure 4.10. Box plots of the simulation run lengths, $n = 200$, 500 and 1000.

However, simulation times seem to differ for some cases. In general, dynamics, other than polarization, take much more time with small-world networks. Besides, the scenarios result in polarization and consensus, namely LL and HH, are more robust to the change of structures. LH and VV scenarios, on the other hand, displays a fair amount of difference in fragmentation.

Here, we must remark an important result that is observed during the analysis with different network structures. Small-world networks are seen to exhibit a different convergence type that has not been observed under any other circumstances. A single opinion cluster at one of the extremes appear on small-world networks with a lower average degree, which matches the results mentioned in [43]. Figure 4.11 depicts an example of such case with $k = 4$.

Table 4.3. Summary table for the network topology experiment.

	LL	**HH**	**LH**	**VV**
	random			
run length	59.38	291.4	807	1545.1
dispersion	0.497	0.998	0.361	0.273
# of clusters	2	1	3.4	5.42
observation	polarization	consensus	3-5 clusters	4 or more clusters
	scale-free			
run length	69.64	185.28	812	1634
dispersion	0,498	1	0.351	0.262
# of clusters	2	1	3.44	6.04
observation	polarization	consensus	3-5 clusters	4 or more clusters
	small-world			
run length	50.9	615.9	1500	4465
dispersion	0.5	0.999	0.362	0.257
# of clusters	2	1.02	3.78	6.86
observation	polarization	consensus	3-5 clusters	4 or more clusters

Other network topologies do not demonstrate the same behavior when average degree decreases. Lowering k of small-world networks may create a small group of nodes, usually less than 5% of all nodes, that are only connected each other and a few other nodes. With an initial lack of communication these nodes sometimes become like extremists, who eventually pushes all other agents in the network to the opposite extreme.

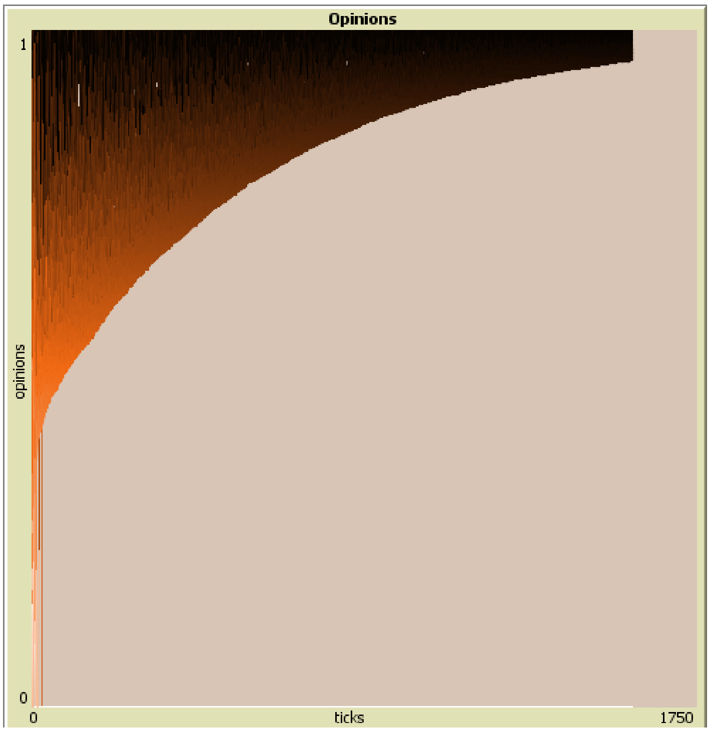

Figure 4.11. An example of single extreme convergence.

4.5. Impact of External Actor

To reflect the influence of EA, μ has to be higher than α, and it is set to 0.7. Since no significant evidence is attained from the topology experiments to suggest that the network structure has an effect on model behavior, it is preferred to use random networks for this experimentation. Experiments are conducted by using different levels for q and for the opinion of EA (ex-op):

- $q : [0.1, 0.1, 0.9]$
- ex-$op : [0, 0.1, 1]$.

Figure 4.12 demonstrates the average opinion dispersion for each parameter combination and for each threshold scenario.

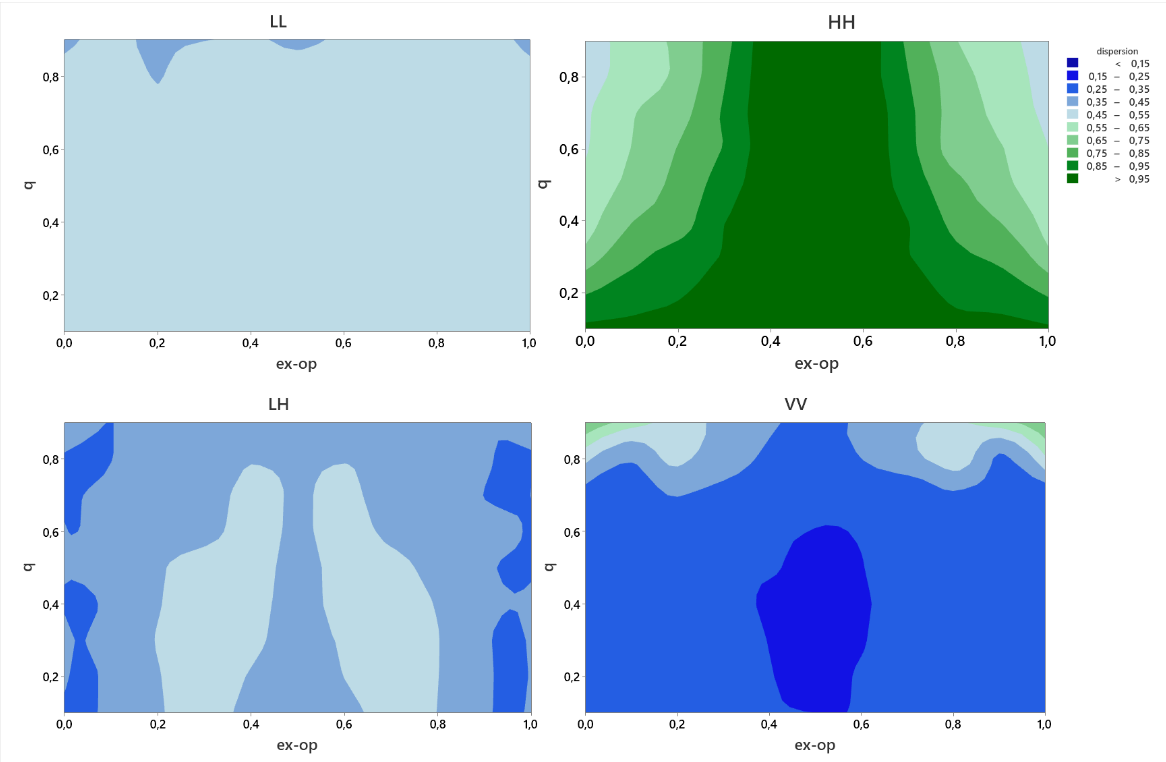

Figure 4.12. Contour plot of opinion dispersions for each threshold scenario.

At a first glance, it is possible to conlude that EA does not have an effect on the underlying dynamics when LL scenario is valid. The contour plot indicates even with the highest probability level, there is only a minor difference in dispersion of the opinions. It is expected to see the consensus result with HH scenario, since it originally results in a consensus. On the other hand, in the HH plot, we see various levels of opinion dispersion while *ex-op* converges to extreme opinions. Even polarization seems to occur for extreme values of *ex-op* and q. Other than the combinations result in polarization, LH plot could be considered unsurprising. Also, VV plot suggests the dynamics are the same and are not effected by the introduction of an influential actor.

Dynamics of HH and LH scenarios after EA is added are thought to require further investigation. Some specific combinations are examined in detail by running the simulations from NetLogo interface. Throughout the study, the only outcome observed with HH setting has been a single opinion cluster. However, it is detected that with extreme levels of *ex-op* and the highest level of q, we now see two final opinion clusters as demonstrated in Figure 4.13.

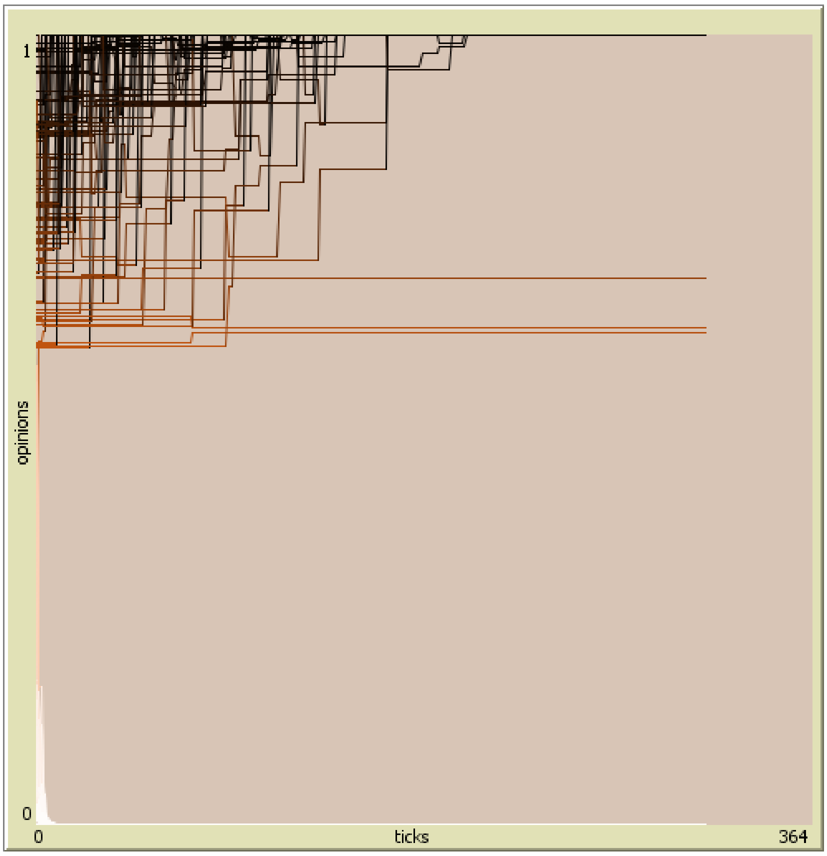

Figure 4.13. A sample trajectory of the model with *ex-op*$= 0.1$, $q = 0.9$ for HH scenario

Since the agents have a larger latitude of acceptance with HH setting, a major number of agents start to follow EA immediately after the simulation is started. This quick change in the network, eventually pushes a small group of people to the other extreme. It must be noted that this situation does not directly arise from extreme *ex-op* but its combination with q, which allows us to come up with an interpretation that too aggressive and intense propaganda creates opposing groups even if the people are open-minded.

Another interesting result is obtained with the LH setting. Normally, a population of LH scenario tends to split into three distinct groups, which are exactly located at the extremes and the center. In this case (see Figure 4.14), though, EA attractes the central group and accompanies them towards the extreme opinion. One would, intuitively, expect to see this situtation when EA have an opinion close to one of the extremes. However, it is only observed with *ex-op*= 0.4 or 0.6 and at the end EA manages to push more than 60% of the population to the extremes. Opposingly, extreme levels of *ex-op* do not effect the final opinion clusters. Figure 4.15 illustrates an example of that scenario.

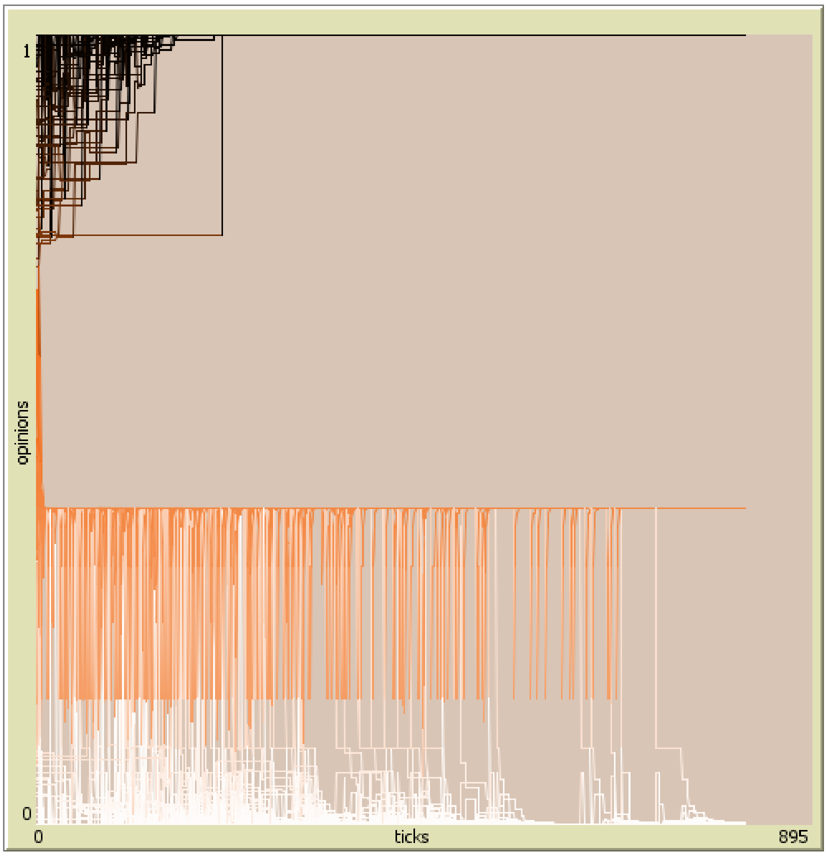

Figure 4.14. A sample trajectory of the model with *ex-op*= 0.4, $q = 0.8$ for LH scenario

Figure 4.15. A sample trajectory of the model with *ex-op*= 0, $q = 0.8$ for LH scenario

4.6. Heterogeneous Thresholds Case

We also examine the case of populations consist of people with different threhsold levels. It is possible to think that agents having LL are close-minded, having HH are open-minded and having VV are indifferent. Populations formed by agents having LL, HH or VV thresholds are considered by varying the proportions of agents having each scenario. Then, we define three parameters for each agent type:

- r_c: proportion of the close-minded agents in the population
- r_o: proportion of the open-minded agents in the population
- r_i: proportion of the indifferent agents in the population

such that $r_c + r_o + r_i = 1$.

When populations that are formed by only open- and close-minded agents considered, we see that close-minded agents dominate the dynamics. A population with $r_o = 0.7$ still demonstrates bipolarization. On the other hand, even with a small r_c, e.g., 0.05, can push the majority away from central opinion. When close-minded agents holding opposite extreme opinions exist and keep communicate for a large period of time, the polarization between them causes a single extreme convergence. Figure 4.16 exemplifies the case of single extreme convergence, where green and red colors represent open- and close-minded agents respectively.

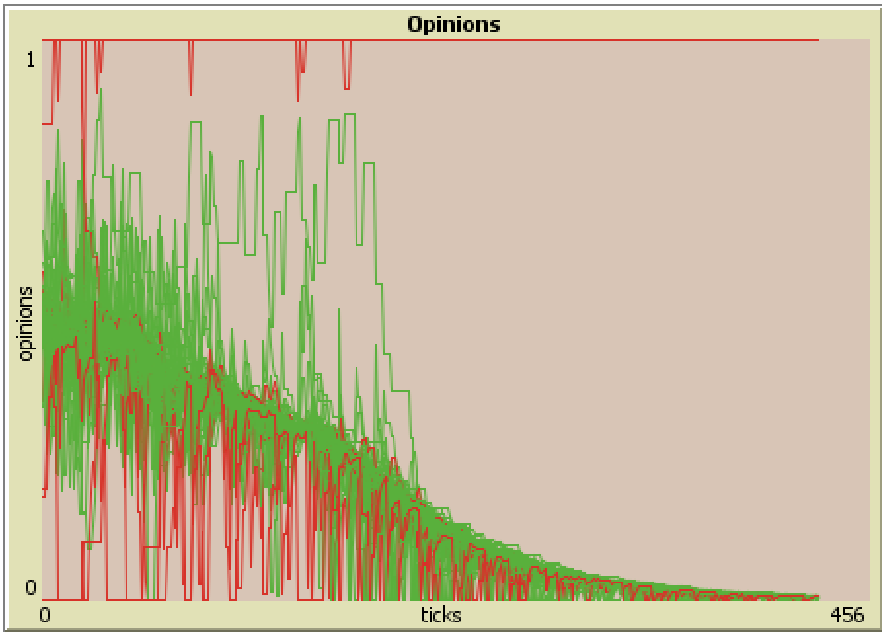

Figure 4.16. A sample trajectory of the model with $r_c = 0.09$.

Furthermore, we observe that populations with open-minded and indifferent agents do not form more than 2 clusters. This is a counter intuitive result; since homogeneous VV populations always result in 4 or more opinion clusters, we would expect to see a similar result. However, presence of indifferent agents mostly shifts consensus towards extremes as shown in Figure 4.17, where indifferent agents are portrayed by purple. When r_i increases, the majority consensus at extreme becomes more likely.

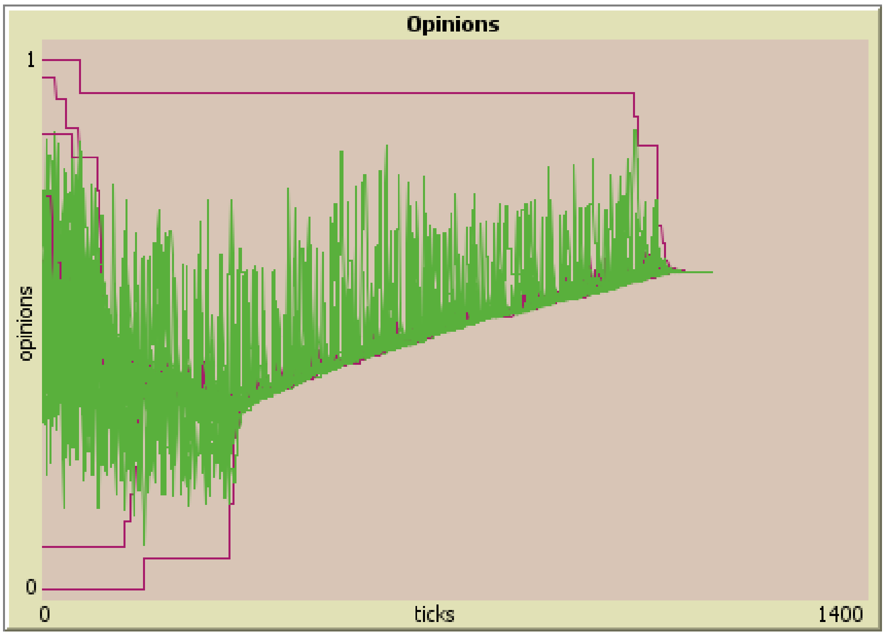

Figure 4.17. A sample trajectory of the model with $r_i = 0.2$.

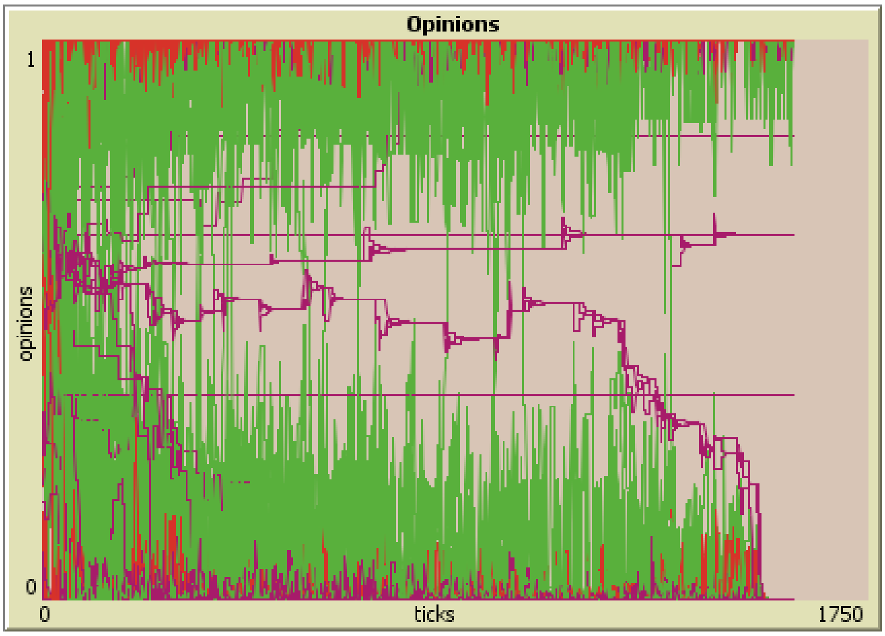

Figure 4.18. A sample trajectory of the model with $r_o = r_c = r_i$.

Even with a population in which each agent type equally represented, close-minded agents are the ones that mostly control the opinion dynamics. Figure 4.18 depicts a simulation with $r_c = r_o = r_i$, and one can see that the population reaches a state where two opposite opinion clusters exist.

5. CONCLUSION

In this study, the Social Judgment Theory-based opinion dynamics are investigated in detail. Our model consists of n people with continuous opinions and two thresholds determining individuals' latitudes of acceptance and rejection. At each time step, people connected by each of the randomly selected links communicate with each other and decide whether they agree or disagree, comparing the distance between their opinions to their thresholds. Their minds become alike if attraction occurs or more different if repulsion is the case.

An agent-based model that follows the rules described above is implemented in the NetLogo environment, and several experiments are conducted for the analysis of the model. First, we start with checking our model's robustness to parameters α and n. Then, due to the lack of real-life data necessary for model validation, four threshold scenarios are presented to illustrate different societies.

Later, the model is run with various threshold scenarios on three different networks: random, small-world, and scale-free. The simulation results are compared to determine whether the network structure alters the dynamics. It is observed that differentiating social network's topology only affects the convergence time. However, a critical result is obtained when the average degree of the small-world networks is decreased. A shift in consensus from center to one of the extremes is observed.

Furthermore, an influential external actor, which aims to convince other people in the network, is introduced. All threshold scenarios are experimented in the presence of the external actor, and the outcomes are compared to those obtained before introducing the external actor. Some populations are seen to be robust to the external actor; whereas, some others are produced surprisingly different dynamics.

Lastly, populations with heterogeneous thresholds are examined by varying the proportions of agents following different threshold scenarios. Most of the observations indicate that close-minded agents are the most influential ones on the dynamics even if they are a tiny proportion of the population. The existence of close-minded agents in a society eliminates the chance of agreement around the average. In addition, it is detected that indifferent agents may cause single extreme convergence when they are together with open-minded people.

Possible future directions exist, including, but not limited to, consideration of more than one external actor, presentation of agent attributes other than the opinion that could influence opinion formation, and differentiation of the links considering the importance of the relation.

However, there is a lack of data-oriented applications in the opinion dynamics literature; our study also does not use real-life data. Therefore, we believe that quantitative validation of models and theoretical results with real data should be the main direction of future investigations.

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