

PRICING EUROPEAN FX OPTIONS BY HESTON MODEL  
CALIBRATED WITH HISTORICAL SPOT PRICES

by

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## **ABSTRACT**

### **PRICING EUROPEAN FX OPTIONS BY HESTON MODEL CALIBRATED WITH HISTORICAL SPOT PRICES**

In this thesis, European type of FX call and put options are priced by two methods, Black and Scholes Model and Heston Model. Despite the fact that Black and Scholes is widely used by finance environment, the volatility of the underlying asset is ignored and assumed to be constant. However, since this assumption does not reflect the real market behavior, European options are priced by Heston model which consider that the volatility is stochastic, not constant. Heston model parameters are calibrated by using historical EUR/TL exchange rates with the Maximum Likelihood Estimation (MLE) method. The calibrated parameters are used in a simulation algorithm in order to forecast the future EUR/TL exchange rates. This is done to observe whether estimated parameters are usable in the option pricing procedure. Then, European FX call and put options are priced for given spot and strike prices, domestic and foreign interest rates and maturities. Black and Scholes model is also used for option pricing in order to compare the performances of the two models. Furthermore, model prices are compared to the real market prices. The deviations between the model prices and real prices are tried to be explained by a Root Mean Square Error and Average Relative Percentage Error analysis. The computational results show that the Heston Model outperforms the Black and Scholes model.

## ÖZET

### AVRUPA TİPİ FX OPSİYONLARININ GEÇMİŞ SPOT VERİLERİYLE KALİBRE EDİLEN HESTON MODELİ İLE FİYATLANDIRILMASI

Bu tez çalışmasında, Avrupa tipi FX alım ve satım opsiyonları Black-Scholes ve Heston modeli ile fiyatlandırıldı. Finans çevreleri tarafından Black-Scholes modeli yaygın bir şekilde kullanılmasına rağmen, dayanak varlığın oynaklığı bu modelde ihmal edilir ve oynaklığın sabit olduğu kabul edilir. Ancak, bu varsayım gerçek piyasa hareketini yansıtmadığı için Avrupa tipi opsiyonlar, oynaklığın sabit değil de stokastik olduğunu dikkate alan Heston modeli ile fiyatlandırıldı. Heston modele ait parametreler, Maksimum Olabilirlik Yöntemi ile geçmiş yıl EUR/TL kurları kullanılarak kalibre edilmiştir. Kalibre edilen parametreler gelecek yıllara ait EUR/TL kurlarını tahmin etmek için bir simülasyon algoritmasında kullanıldı. Bu, tahmin edilen parametrelerin opsiyon fiyatlama sürecinde kullanılabilir olup olmadığını gözlemlemek için yapılmıştır. Sonrasında, Avrupa tipi FX alım ve satım opsiyonları verilen spot ve kullanım fiyatları, iç ve dış faiz oranları ve vade günleri kullanılarak fiyatlandırılmıştır. Black-Scholes modeli de her iki modeli karşılaştırmak için opsiyon fiyatlandırılmasında kullanılmıştır. Buna ek olarak, model fiyatları ile gerçek fiyatlar da karşılaştırılmıştır. Model fiyatları ile gerçek fiyatlar arasındaki sapmalar Karekök Ortalama Hata ve Ortalama Göreceli Yüzde Hata analizleri ile açıklanmaya çalışılmıştır. Sayısal sonuçlar Heston modelin Black-Scholes modelinden üstün olduğunu göstermiştir.

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## LIST OF SYMBOLS

$C(S,t)$	Call Option Price of Asset S at time t
$K$	Strike Price
$N(\cdot)$	Cumulative Normal Distribution Function
$N(t)$	Poisson Process
$P(S,t)$	Put Option Price of Asset S at time t
$p$	Upward Jump Probability
$q$	Downward Jump Probability
$Q_t$	Asset Returns at time t
$r_d$	Domestic Interest Rate
$r_f$	Foreign Interest Rate
$S$	Spot Price
$T-t$	Time to Maturity
$V_t$	Mean Reverting Square Process
$W_t$	Brownian Motion
$Z_s$ and $Z_v$	Two standard Normal Random Variables
$\eta(x)$	Jump Size
$\theta$	Long Term Level of Variance
$\kappa$	The Rate of Mean Reversion
$\lambda$	Risk Premium
$\mu$	Percentage Drift
$\rho$	Correlation Coefficient
$\sigma$	Volatility of Volatility

**LIST OF ACRONYMS/ABBREVIATIONS**

ARPE	Average Relative Percentage Error
ATM	At the Money
CIR	Cox-Ingersoll-Ross
EUR	Euro
FX	Foreign Exchange
ITM	In the Money
MLE	Maximum Likelihood Estimation
OTM	Out of the Money
PDE	Partial Differential Equation
RMSE	Root Mean Square Error
TL	Turkish Lira

## 1. INTRODUCTION

In today's economy financial derivatives are widely used by investors for the protection from the risky scenarios which are generally unexpected changes in the foreign exchange rates. There are many products in order to hedge the risks in investments. Among the plenty of financial instruments, the option is the most crucial one, because of its flexibility in contrast to futures and forwards.

In recent years, foreign exchange (FX) option market has been the largest and the most liquid sector in financial world. Institutional and individual investors, banks, central banks, corporations, speculators are the FX market participants. Their average daily transactions are estimated around 5.3 trillion US dollar in a day.

The main purpose of the use of FX options is to utilize the ability of combating risk for the investors and businessman working with outside of their countries' borders in their investments. Determining the price of a currency pair in a long time horizon is absolutely difficult, due to the volatility of the exchange rate. Daily exchange rate of the Euro against the Turkish Liras fluctuates significantly over the last five years as shown in Figure 1.1.

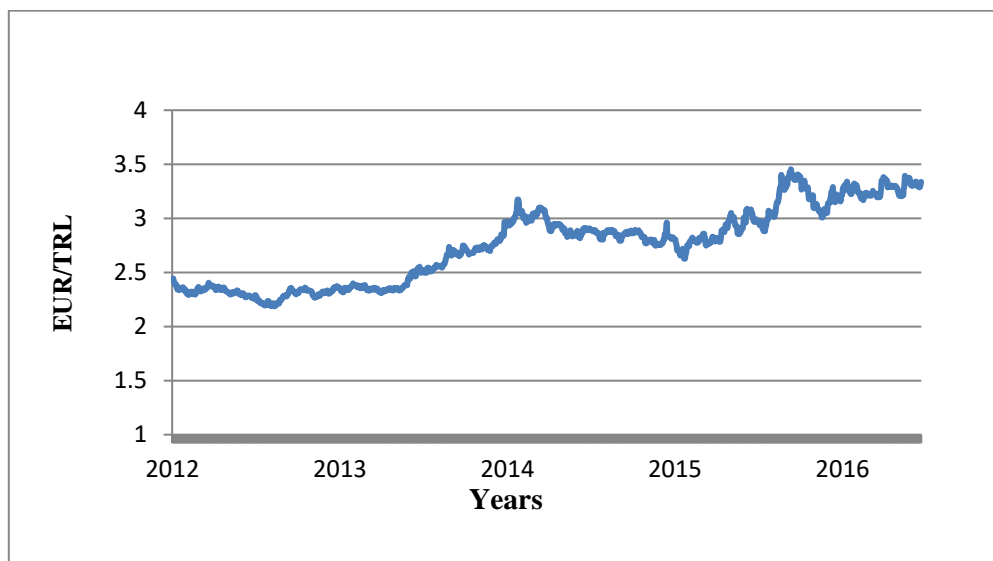


Figure 1.1. Daily Exchange Rate of the Euro against the Turkish Lira

If there is no volatility, there will be no requirement for the option (Hicks, 2000). However, FX market is very volatile and using FX options for the investors is very reasonable. Hedging feature of the option can be used not only by the investors but also by governments, banks and corporations to protect from the adverse market movements (Chisholm, 2004). Other reasons of the use of the options are to make additional income and to trade the exchange rate at a favorable price in the future (Dejanovski, 2014).

An option is defined as a right, but not the obligation, to buy (call option) or sell (put option) a specified underlying asset at a fixed price on or before a fixed maturity date. Underlying asset refers to the security on which a derivative contract is based on and can be commodities (e.g. gold, silver, wheat, coffee and other agricultural products), bonds, equities, stocks, indices, futures and foreign exchange rates (in this thesis underlying asset will be EUR/TL FX rates). Spot price is the value of the underlying asset at written day of option contract. Maturity date (expiration date) is about expiration of the option contract on the given date. At the end of the maturity date, underlying asset is bought or sold at a predetermined rate which is defined as the strike price (or exercise price). In order to buy the underlying asset at a fixed price, the buyer pays to seller a fee. This amount of money is known as the premium of the option. In other words, the premium is the price of the option. Premium represents the maximum profit of the seller whereas maximum loss of the buyer.

There are some definitions associated with the relationship between the spot and strike price of the option. Option is in the money (ITM) if the spot price is higher than the strike price. Conversely, if the spot price is less than the strike price, the option is said to be out of the money (OTM). The option is at the money (ATM) when spot price is exactly equal to the strike price. The difference between the spot and strike price is known as the intrinsic value. Above definition is about the intrinsic value of the option.

There are mainly two types of options: European option and American option. European option is exercised only at the expiration date whereas American option is exercised at any time prior to expiry. In this thesis, we focus on European call and put options written on foreign currencies.

This thesis presents pricing European FX call and put option written on EUR/TL currency pair by using both Black and Scholes model and Heston model. Parameters in Heston model are estimated by conducting *Maximum Likelihood Estimation* method, in short MLE, with the historical exchange rates of the EUR/TL. The estimated parameters are used in simulation algorithm in order to measure the usability of the parameters in option pricing. The errors between real and the estimated exchange rates are tried to be explained by a root mean square error analysis. European type of FX call and put options are priced by using calibrated parameters with Black and Scholes and Heston model. For the comparison the real market FX option prices are used and to draw a conclusion about the performance of the models root mean square error and average relative percentage error analysis are made. In Heston model estimating the parameters is the most critical part of the option pricing process, because a small change in the parameters are very sensitive to the premium of the option. In order to observe how Heston parameter affects the price of the option, sensitivity analysis is conducted.

The main objective of the thesis is to analyze the performance of the Black and Scholes model and Heston model. Furthermore, to decide which model gives better results and which one is applicable to the FX option market.

There are similar academic dissertations when literature is reviewed. This thesis is different from the others with regard to the data which is used for calibration of the Heston model parameters. In many theses, real option prices are used in order to calibrate the parameters. However, in this study, EUR/TL parity is employed for the calibration.

## 2. LITERATURE REVIEW

In the past decades, a plenty of research has been conducted in order to find foreign currency option models that fit best to the market data and to reach an option price that is in line with determined market figures. The literature is designed according to option pricing models' volatility assumption. Firstly, we define volatility and introduce the different types of the volatility. Next, volatility models, most significantly constant and stochastic models, are provided. Finally, according to volatility models, their related option pricing models are reviewed.

### 2.1. Introduction

In financial mathematics, the study of volatility becomes a significant research area, because it helps to explain the dynamics of the underlying asset price. Volatility is defined as the standard deviation of the return of an underlying asset. Wilmott (2001) distinguishes the volatility at four different categories. Those are explained as follows:

- **Actual volatility:** In underlying asset return, it is a measure of randomness at any particular time. Hence, there is no timescale associated with it.
- **Historical (or realized) volatility:** It is a measure of randomness over a specific time period. Historical volatility is calculated by using the past empirical data and steps for the calculation are shown in the section 4.2.
- **Implied volatility:** It is the market's estimate of volatility. Implied volatility is not directly derived as in the historical volatility. However, it can be calculated by the existence of other parameters in Black and Scholes equation and premium of the option. Implied volatility is important for the derivative market participant, because it is an indicator about the price of an option. The higher the implied volatility means the higher option price.
- **Forward volatility:** It is a measure of the volatility, either actual or implied, over some period in the future.

Option pricing models has been developed according to the constant volatility assumption past decades. However, this assumption has become inconsistent with the market behavior and does not reflect real life. This situation gives rise to the development of the volatility models. In literature, there are four different volatility models whose complexity increases as go downwards:

- Constant volatility  $\sigma$ ;
- Time dependent volatility  $\sigma(t)$ ;
- Time and state dependent volatility  $\sigma(t, S_t)$ ;
- Stochastic volatility  $\sigma(w)$

In the subsequent section, these models are explained in detail. However, constant volatility models and stochastic volatility models will be the main focus of the thesis.

## 2.2. Constant Volatility Models

Various models have been developed to price options but most well-known option pricing model belongs to the Nobel Prize winners Black and Scholes. This model is still implemented in the financial market to obtain the price of the option although it has some limitations (Ray, 2012). Rubinstein (1985) claims that the constant volatility assumption is the most critical bias of the Black and Scholes model, due to the fact that market behavior is not generally in line with this assumption. October 1987 stock market crash is an example that disproves the assumption. In a very short term, the volatility can be relatively constant, but it is never constant in the longer time (Yalincak, 2005).

According to the Black and Scholes model, the stock price of the underlying asset,  $S_t$ , is said to follow a Geometric Brownian Motion if it satisfies the stochastic differential equation

$$dS_t = \mu S_t dt + \sigma S_t dW_t \quad (2.1)$$

where  $W_t$  is the standard Brownian motion (or Wiener process),  $\mu$  is the percentage drift,  $\sigma$  is the volatility of the stock return. By using the separation of variables and Ito calculus for the next steps, we can obtain solution for the process  $S_t$ :

$$S_t = S_0 \exp \left\{ \left( \mu - \frac{1}{2} \sigma^2 \right) t + \sigma W_t \right\} \quad (2.2)$$

The payoff for a European put and call option is obtained by

$$F = [\emptyset(S_T - K)]^+ \quad (2.3)$$

where the  $\emptyset$  takes the value +1 for the call and -1 for the put option and the symbol  $x^+$  denotes the positive part of  $x$  (Wystup, 2006).

We can compute European call and put option prices as the solution of Black and Scholes partial differential equation or as the discounted expected value of the payoff function (Wystup, 2006). The end result of the both ways gives the following Black-Scholes formula (Black and Scholes, 1973) for pricing European call and put option

$$C(S, t) = SN(d_1) - Ke^{-r(T-t)}N(d_2) \quad (2.4)$$

$$P(S, t) = Ke^{-r(T-t)}N(-d_2) - SN(-d_1) \quad (2.5)$$

where

$$d_1 = \frac{\ln\left(\frac{S}{K}\right) + \left(r + \frac{1}{2}\sigma^2\right)(T-t)}{\sigma\sqrt{(T-t)}}$$

$$d_2 = \frac{\ln\left(\frac{S}{K}\right) + \left(r - \frac{1}{2}\sigma^2\right)(T-t)}{\sigma\sqrt{(T-t)}}$$

S: stock price of the underlying asset

K: strike or exercise price

T-t: time to maturity

$\sigma$ : volatility of the stock return

r: risk free interest rate

$N(\cdot)$ : cumulative normal distribution function

In the derivation of the formula process, Black and Scholes (1973) assume some ideal conditions in the market for the stock and for the option:

- (i) The short term interest rate is known and is constant through time.
- (ii) The stock price follows a random walk in continuous time with a variance rate proportional to the square of the stock price. The variance rate of return on the stock is constant.
- (iii) The stock pays no dividends or other distributions.
- (iv) The option is European, that is, it can only be exercised at maturity.
- (v) There are no transaction costs in buying or selling the stock or the option.
- (vi) It is possible to borrow any fraction of the price of a security to buy it or to hold it, at the short term interest rate.
- (vii) There are no penalties to short selling.

Black and Scholes model cannot be directly implemented to the foreign currency options since the assumption of that “the stock pays no dividends” during the life of the option (Biger and Hull, 1983). Therefore, some other models are developed to value foreign currency options. One of them is the Garman-Kohlhagen (1983) which is the simple extension of the Black and Scholes model. The following assumptions are made for Garman-Kohlhagen model (1983),

- (i) No transaction costs, no differential taxes, no borrowing or lending restrictions, and trading takes place continuously.
- (ii) The term-structure of interest rates in both the domestic and foreign country are flat and non-stochastic.
- (iii) The underlying state variable is the spot exchange rate.
- (iv) The exchange rate can be described by the stochastic process

Differently from Black and Scholes model, suppose that  $r_d$  is the domestic currency risk free interest rate and  $r_f$  is the foreign currency risk free interest rate. Also, in Garman

Kohlhagen,  $S$  represents the exchange rate parity. Other parameters are same as in the Black and Scholes model. Then, prices of European call and put option written on foreign currency are derived by the following formulas:

$$C(S, t) = Se^{-r_f(T-t)}N(d_1) - Ke^{-r_d(T-t)}N(d_2) \quad (2.6)$$

$$P(S, t) = Ke^{-r_d(T-t)}N(-d_2) - Se^{-r_f(T-t)}N(-d_1) \quad (2.7)$$

where

$$d_1 = \frac{\ln\left(\frac{S}{K}\right) + \left(r_d - r_f + \frac{1}{2}\sigma^2\right)(T-t)}{\sigma\sqrt{(T-t)}}$$

$$d_2 = \frac{\ln\left(\frac{S}{K}\right) + \left(r_d - r_f - \frac{1}{2}\sigma^2\right)(T-t)}{\sigma\sqrt{(T-t)}}$$

Garman-Kohlhagen model assumes that the volatility of the exchange rate is constant over the life of the option. However, this claim does not reflect the real market behavior. Therefore, it is accepted that this assumption is the major drawback of the Garman-Kohlhagen model (Kung, 2013).

### 2.3. Time Dependent Volatility Models

Time dependent volatility means that fluctuations in volatility evolve with time. Even though volatility is not constant any more, stock returns are still distributed normally.

Various efforts have been made in order to express the volatility dependent on the time. Ho and Lee (1986), Hull and White (1990), Black, Derman and Toy (1990) and Black and Karasinski (1991) have developed time dependent models. They assume the underlying asset follows the below forms:

- Ho and Lee

$$dS_t = \mu(t)dt + \sigma(t)dW_t \quad (2.8)$$

- Hull and White

$$dS_t = \{\alpha_0(t) + \alpha_1(t)S_t\}dt + \sigma(t)S_t^i dW_t \quad i = 0 \text{ or } 0.5 \quad (2.9)$$

- Black, Derman and Toy

$$dS_t = \{\alpha_1(t)S_t + \alpha_2(t)S_t \log S_t\}dt + \beta_0(t)S_t dW_t \quad (2.10)$$

- Black and Karasinski

$$dS_t = \{\alpha_1(t)S_t + \alpha_2(t)S_t \log S_t\}dt + \beta_0(t)S_t dW_t \quad (2.11)$$

where

$$\alpha_2(t) = \frac{d \log\{\beta_0(t)\}}{dt}$$

#### 2.4. Time and State Dependent Volatility Models

Since constant volatility assumption in option pricing models does not satisfy the real market, it may be reasonable to consider the volatility which depends on the both stock price and time. The underlying stock price is now represented by

$$dS_t = \mu(t)S_t dt + \sigma(t, S_t)S_t dW_t \quad (2.12)$$

where  $\sigma(t, S_t)$  is a deterministic function of time and the stock price and called the *Local Volatility Function* of the model. Black and Scholes model can be extended by inserting time and stock price into the volatility function.

Dupire (1997) has developed the idea of local volatility in order for the models to fit the actual option prices for all maturities and all strike prices. As in the implied volatility,

local volatility can be analytically extracted from the European call option prices by using the Dupire's famous equation (1997):

$$\frac{\partial C}{\partial t} = \frac{\sigma(t, S_t)^2 S_t^2}{2} \frac{\delta^2 C}{\delta S_t^2} + (r_t - D_t)(C - S_t \frac{\delta C}{\delta S_t}) \quad (2.13)$$

where  $C$  is the European call option price,  $r$  is the risk free interest rate and  $D$  is the dividend yield. The local volatility assumption provides a risk neutral probability distribution of the underlying asset.

## 2.5. Stochastic Volatility Models

Melino and Turnbull (1990) develop another model to price the currency option by relaxing the assumption about the stochastic process described the evolution of the exchange rate. Stochastic process for the exchange rate is the form of

$$dS = (a + bS)dt + \vartheta S^{\beta/2} dW_t \quad (2.14)$$

where  $W_t$  is the standard Brownian motion whose increments have instantaneous correlation  $\rho$ ;  $\alpha$ ,  $b$  and  $\beta$  are parameters and  $0 \leq \beta \leq 2$ . Different from the Black and Scholes and Garman and Kohlhagen models, Melino and Turnbull assume the volatility is not constant and follow a stochastic process described by the following differential equation:

$$d \ln \sigma = (\alpha + \delta \ln \sigma)dt + \gamma dW_\sigma \quad (2.15)$$

where  $W_\sigma$  is the standard Brownian motion and  $\sigma$  is the volatility. In order to obtain price of the call option ( $C$ ), following partial differential equation should be satisfied:

$$\begin{aligned} & \frac{1}{2}(\sigma^2 S^\beta C_{SS} + 2\rho\sigma^2 S^{\beta/2} C_{S\sigma} + \sigma^2 C_{\sigma\sigma}) \\ & + (r_d - r_f)SC_S + (\mu(\sigma) - \lambda\sigma)C_\sigma - r_d C + C_t = 0 \end{aligned} \quad (2.16)$$

where  $\mu(\sigma)$  and  $\sigma^2$  are the instantaneous mean and variance of the volatility process, respectively and  $\lambda$  is a risk premium that arises because volatility is not a traded asset

(Melino and Turnbull, 1990). Explicit solution for the above PDE is not available; however, it can be solved by employing the numerical method.

Melino and Turnbull (1990) reveal that taking volatility as a stochastic process gives a much closer fit to empirical distribution of the exchange rate changes. On the other hand, they consider that the term structure of interest rates in both domestic and foreign country is flat and non-stochastic.

The most widely used stochastic volatility model is, no doubt, Heston's model, since it provides a fast calibration to the market data (Mikhailov and Nögel, 2003). In contrast to Black and Scholes (1973), Heston (1993) proposes that stock returns are not normally distributed and also, spot asset is correlated with the volatility. In Heston model, there are several parameters different than the other models mentioned above. These are mean reversion rate ( $\kappa$ ), correlation coefficient ( $\rho$ ), volatility of volatility ( $\sigma$ ) and long run variance ( $\theta$ ). Correlation between the spot price and volatility is needed in order to build skewness in the distribution of spot returns (Heston, 1993). Long run variance is related to the implied volatility. Increasing the long run variance gives rise to implied volatility smile to move upwards. Curvature of the implied volatility smile is controlled by the mean reversion parameter. Lastly, volatility of volatility parameter has an impact on the kurtosis of the stock return distribution (Moyaert and Petitjean, 2011). Details of the model will be examined subsequent chapter of the thesis.

Observations on the market reveal that some jumps can occur in the exchange rate behavior because of the economic or political uncertainty. For this reason, although Heston model provides a good fit to the real market data and gives close estimate of the FX option prices, it does not incorporate sudden movements in the exchange rates.

Jump diffusion models contain a jump part and a diffusion part. The jump part, which provides unexpected and sudden price changes of the underlying asset, is determined by an impulse function and a distribution function whereas the diffusion part is determined by a standard Brownian motion (Runggaldier, 2002). A general formula for the underlying asset process is given by

$$dS_t = \mu S_t dt + \sigma S_t dW_t + \eta S_t dN_t \quad (2.17)$$

where  $N(t)$  is a Poisson process with rate  $\lambda$  and  $\eta$  is an impulse function which causes a jump of  $S$  to  $S(1 + \eta)$ .

Merton (1975) suggests that underlying stock returns should be governed by both continuous and jump process. In Merton's normal jump diffusion model, component of stock price change follows a Brownian motion and jump component follows a 'Poisson-driven' process (Merton, 1975). Hence, stock price process for the Merton model is given as:

$$\frac{dS_t}{S_t} = \mu dt + \sigma dW_t + d(\sum_{i=1}^{N(t)} (V_i - 1)) \quad (2.18)$$

where  $\mu$  is the instantaneous expected appreciation rate of the foreign currency,  $\sigma^2$  is the instantaneous variance of the return,  $W_t$  is the standard Brownian motion,  $N(t)$  is a Poisson process with rate  $\lambda$ ,  $\{V_i\}$  is a sequence of independent identically distributed nonnegative random variables such that  $Y = \log V$ . The logarithm of jump size is normally distributed in this model. Namely,

$$\eta(x) = N(\mu, \sigma)$$

The option prices are calculated as a weighted average of Black-Scholes prices where each weight is the probability of having  $n$  jumps (Daal and Madan, 2005). This model can give better results when the options have short time to maturity.

Like Merton, Kou develops a model which includes jump processes. It follows the same process for the exchange rate (Burger and Kiliaras, 2013). However, in Kou's model, jump size has an asymmetric double exponential distribution given by

$$\eta(x) = p\eta_1 e^{-\eta_1 x} 1_{\{x \geq 0\}} + q\eta_2 e^{\eta_2 x} 1_{\{x < 0\}} \quad (2.19)$$

$$\eta_1 > 1, \eta_2 > 0$$

where  $p$  and  $q$  are the upward and downward jump probabilities respectively. Kou (2002) assumes that  $N(t)$ ,  $W_t$  and  $Y_s$  are independent and  $\mu$  and  $\sigma$  are constant. In his evaluation part of the model, he asserts that leptokurtic feature of the return distribution and volatility smile can be reproduced, because of the feature of the double exponential distribution (Kou and Wang, 2003). However, in Black-Scholes model we cannot reach such properties since spot returns are distributed normally. According to the empirical tests, double exponential jump diffusion model fits market data better than Merton's jump diffusion model (Ramezani & Zeng, 1999). On the other hand, closed form solutions for path dependent options are obtained in Kou's model, due to memoryless property of the double exponential distribution while in Merton's model are not (Kou, 2002). For the calculation of the option prices, Kou introduces an  $Hh$  function represented by

$$Hh_{-1}(x) = e^{-\frac{x^2}{2}} \quad (2.20)$$

$$Hh_0(x) = \sqrt{2\pi}N(-x) \quad (2.21)$$

$$Hh_n(x) = \int_x^\infty Hh_{n-1}(y) dy = \frac{1}{n!} \int_x^\infty (t-x)^n e^{-\frac{t^2}{2}} dt \quad \forall n \geq 0. \quad (2.22)$$

and computes the rational expectations equilibrium option pricing formula for the European call and put options.

Bates (1996) proposes an option pricing model which combines the jump process and stochastic volatility. The exchange rate can be described by the geometric jump diffusion and the variance follows a mean reverting square root process as in the Heston model,

$$\frac{dS_t}{S_t} = (\mu - \lambda \bar{k})dt + \sqrt{V_t}dW_t + kdq_t \quad (2.23)$$

$$dV_t = \kappa(\theta - V_t)dt + \sigma_v \sqrt{V_t}dW_v \quad (2.24)$$

$$dW_t dW_v = \rho dt \quad (2.25)$$

$$prob(dq = 1) = \lambda dt \quad (2.26)$$

where  $k$  is the random percentage jump conditional on a jump occurring and  $q$  is a Poisson counter with intensity  $\lambda$ .  $k$  and  $q$  is assumed to be independent. Furthermore, they are independent with  $W_t$  and  $W_v$ . Under the risk neutral probability measure, European options on foreign currency can be evaluated as the expected value of terminal payoffs (Bates, 1996).

Madan et al. (1998) develop Variance-Gamma model which has a high activity, multiple amplitude pure jump process in order to value foreign currency options. Behavior of the exchange rate is discontinuous. In the model, drift parameter of the Brownian motion controls the skewness and the variance rate of the gamma process controls the excess kurtosis of the distribution (Madan and Daal, 2005). Madan and Daal (2005) consider the foreign interest rates as the dividend yield on the foreign currency in order to obtain price of the option. In a highly active financial market, it is very useful to use this model because the variance gamma process allows discontinuous movements in any time interval.

## 2.6. Heston Model

One of the most popular stochastic volatility models is the Heston model due to the existence of the semi-closed form solution to the European options and thus, this provides the calibration process fast and practical. On the other hand, considering the volatility as non-constant makes the model more realistic and more appropriate to apply market data than the Black and Scholes. In following sections, the model and its parameters will be introduced and explained in detail.

### 2.6.1. Description of the Model

The model is developed Steven Heston in 1993. He assumes that the stock price process ( $S_t$ ) follows a log normal distribution and the process volatility ( $V_t$ ) follows a Cox-Ingersoll-Ross (CIR) process (Cox *et al.*, 1985). The model is given as:

$$dS_t = (r_d - r_f)S_t dt + \sqrt{V_t}S_t dZ_{1t} \quad (2.27)$$

$$dV_t = \kappa(\theta - V_t)dt + \sigma\sqrt{V_t}dZ_{2t} \quad (2.28)$$

$$dZ_{1t}dZ_{2t} = \rho dt \quad (2.29)$$

where the variables:

- $\theta$  is the long term level of variance
- $\kappa$  is the rate of mean reversion
- $\sigma$  is the volatility of volatility
- $\rho$  is the correlation coefficient between the two standard Brownian motions  $dZ_{1t}$  and  $dZ_{2t}$
- $r_d$  and  $r_f$  are the domestic and foreign interest rates respectively

It is important to note that mean reverting square process,  $V_t$ , is always positive if  $2\kappa\theta > \sigma^2$  (Feller condition) and it is asymptotically stable if  $\kappa > 0$  (Mikhailov and Nogel, 2003).

### 2.6.2. Semi-closed Form Solution for European Options

According to ‘no arbitrage’ argument, the value of any asset  $U$  at time  $t$ ,  $U(S, v, t)$ , should satisfy the following partial differential equation:

$$\begin{aligned} \frac{1}{2}vS^2 \frac{d^2U}{dS^2} + \rho\sigma vS \frac{d^2U}{dSdv} + \frac{1}{2}\sigma^2v \frac{d^2U}{dv^2} + rS \frac{dU}{dS} \\ + \{\kappa(\theta - v) - \lambda(S, v, t)\} \frac{dU}{dv} - rU + \frac{dU}{dt} = 0 \end{aligned} \quad (2.30)$$

where the term  $\lambda(S, v, t)$  is the market price of the volatility risk (Heston, 1993). A European option with the strike price  $K$  and time to maturity  $T$  satisfies the above PDE subject to following boundary conditions:

$$U(S, v, T) = \text{Max}(0, S - K) \quad (2.31)$$

$$U(S, v, t) = 0 \quad (2.32)$$

$$\frac{dU}{dS}(\infty, v, t) = 1 \quad (2.33)$$

$$rS \frac{dU}{dS}(S, 0, t) + \kappa\theta \frac{dU}{dv}(S, 0, t) - rU(S, 0, t) + U_t(S, 0, t) = 0 \quad (2.34)$$

$$U(S, \infty, t) = S \quad (2.35)$$

Heston guessed a solution, analogous to Black and Scholes formula, of a European call option on an FX pair as

$$C(S, v, t) = SP_1 - Ke^{(r_d - r_f)(T-t)}P_2 \quad (2.36)$$

where

$$P_j = \frac{1}{2} + \frac{1}{\pi} \int_0^\infty \operatorname{Re} \left[ \frac{e^{-i\phi \ln(K)} f_j(x, v_t, \tau, \phi)}{i\phi} \right] d\phi$$

$$f_j = e^{C(\tau, \phi) + D(\tau, \phi)v_t + i\phi x}$$

$$C(\tau, \phi) = (r_d - r_f)\phi i \tau + \frac{a}{\sigma^2} \left[ (b_j - \rho\sigma\phi i + d)\tau - 2 \ln \left( \frac{1 - ge^{d\tau}}{1 - g} \right) \right]$$

$$D(\tau, \phi) = \frac{b_j - \rho\sigma\phi i + d}{\sigma^2} \left( \frac{1 - e^{d\tau}}{1 - ge^{d\tau}} \right)$$

$$g = \frac{b_j - \rho\sigma\phi i + d}{b_j - \rho\sigma\phi i - d}$$

$$d = \sqrt{(\rho\sigma\phi i - b_j)^2 - \sigma^2(2u_j\phi i - \phi^2)}$$

$$u_1 = \frac{1}{2}, \quad u_2 = -\frac{1}{2}, \quad a = \kappa\theta, \quad b_1 = \kappa\lambda - \rho\sigma, \quad b_2 = \kappa + \lambda$$

for  $j = \{1, 2\}$  and  $x = \ln S_0$ . In order to find the value of a put option, put-call parity can be solved

$$C(K, T) = P(K, T) + S_t e^{r_f(T-t)} P_1 - Ke^{r_d(T-t)} P_2 \quad (2.37)$$

where  $P(K, T)$  is the put option price and  $C(K, T)$  is the call option price.

### 2.6.3. Model Parameters

The volatility of volatility ( $\sigma$ ) has an impact on the kurtosis (peak) of the stock return distribution (Moyaert and Petitjean, 2011). When  $\sigma$  is zero, the volatility is deterministic. Therefore, the distribution of the stock price follows the normal distribution. Increasing  $\sigma$  increases the kurtosis of the distribution, i.e., increases the convexity of the fit (Rouah, 2013). Volatility is more volatile with the high  $\sigma$  which means that the market has a greater chance of extreme movements (Bhar, 2010).

The correlation coefficient  $\rho$  is the correlation between log returns of the stock price and volatility of the underlying asset. Correlation is needed in order to build skewness in the distribution of stock returns (Heston, 1993). If  $\rho > 0$ , then volatility will increase as the asset price increases. This will spread the right tail of the probability density. Hence, the distribution becomes negatively skewed. Conversely, if  $\rho < 0$ , then the volatility will increase as the asset price decreases. This causes the left tail of the probability density to spread. Therefore, the distribution becomes positively skewed. If  $\rho = 0$ , there will be no effect to the skewness of the distribution. Changing the parameter  $\rho$  has impact on the implied volatility surface. Positive correlation results in more expensive call options whereas negative correlation results in more expensive put options.

The mean reversion rate ( $\kappa$ ) assumes that stock prices will tend to move the average price over time. In particular, it explains how fast the variance process is reverting to its long term mean. Curvature of the implied volatility smile is controlled by the mean reversion parameter. The volatility surface becomes flat when the mean reversion parameter increases. However, the long term level of variance ( $\theta$ ) affects the height of the smile curve rather than its shape. Increasing the long run variance moves the implied volatility smile upwards. In the stock price process, drift term provides that the variance is mean reverting towards the long term mean,  $\theta$ . The high value of  $\kappa$  implies that the model reaches the long term mean of variance quickly. Furthermore, when mean reversion rate is positive, the variance has a

steady state distribution with mean  $\theta$  (Cox *et al.*, 1985). For the mean reversion property of the Heston model, these two parameters are very crucial.

#### 2.6.4. Advantages and Disadvantages of the Model

Even though Heston model is popular among the finance world, it has some flaws. In this section, we briefly summarize the advantages and disadvantages of the Heston model.

Advantages:

- For the European options provides a semi-closed form solution which allows easy and fast calibration to the market data.
- The volatility is mean reverting which implies that the volatility does not go to infinity or zero, returns a normal level (Engle and Patton, 2001).
- Contrary to the Black and Scholes, the model allows for non-Gaussian distribution which is characterized by high peak and fat tails (Cont, 2001).
- The Heston model fits the implied volatility surface well, if the maturity is not too small.
- Leverage effect can be observed in the model meaning that underlying asset returns are negatively correlated with the implied volatility of that asset.

Disadvantages:

- Since the volatility is not observable, the parameter values can be hard to estimate
- Heston model prices are sensitive to the parameter values. Therefore, calibration of the parameters bears great importance for the fitness of the model (Mikhailov and Nögel, 2003)
- For the short maturities, the model fails to fit implied volatility surface. In order to handle this, jumps can be added.

### 3. CALIBRATION OF HESTON MODEL USING HISTORICAL VALUES

In the Heston model, there are five parameters,  $\kappa, \theta, \sigma, \rho, \vartheta_0$ , which should be estimated before the pricing procedure. Since choices of the parameters have a big effect on the correctness of the option prices, calibration process becomes as crucial as the model itself (Cont, 2005).

Parameters calibration can be conducted in two different ways.

First is to estimate the Heston model parameters by using the current option prices observed in the financial market. To do so, the most useful method is to implement the non-linear least squares optimization problem. This focuses on the error minimization between the Heston model prices and real market prices.

Second way is to look at the historical data of the underlying asset returns. For this case, one need only the spot prices of the asset (for this thesis, we need the EUR/TL parity). In parameter calibration process, maximum likelihood estimation method developed by R.A. Fisher is employed. It is applied by writing the likelihood of the observed values and then choosing the parameters which maximizes that likelihood function. Details will be given in following section.

In practice, the accuracy and speed of the calibration are two important criteria since it is used in pricing huge option contracts which are traded in large market.

#### 3.1. Parameter Estimation

Heston model parameters, in this thesis, are estimated by the method of Maximum Likelihood Estimation (MLE). However, before the estimation procedure, the model will be discretized by employing Euler's discretization method (Haastrecht and Pelsser, 2010). Then, the data set of asset returns will be used to find the proper parameters.

### 3.1.1. Discretization of the Heston Model

In the Heston model, stochastic process for the stock price  $S_t$  is given by

$$dS_t = (r_d - r_f)S_t dt + \sqrt{V_t}S_t dZ_{1t} \quad (3.1)$$

and the Euler discretization of that is

$$S_{t+1} = S_t + (r_d - r_f)S_t \Delta t + \sqrt{V_t}S_t \Delta t dZ_s \quad (3.2)$$

For the simplicity, represent the model as the change in asset returns;

$$Q_{t+1} = \frac{S_{t+1}}{S_t} \quad (3.3)$$

Now, let take  $\Delta t = 1$  in order to represent the one trading day between each of asset price and divide both sides of the equation (4.2) by  $S_t$ . Obtained equation is;

$$Q_{t+1} = 1 + (r_d - r_f) + \sqrt{V_t}Z_s \quad (3.4)$$

where  $Z_s \sim N(0,1)$ .

The stochastic process for the variance of the stock price  $V_t$  is as follows,

$$dV_t = \kappa(\theta - V_t)dt + \sigma\sqrt{V_t}dZ_{2t} \quad (3.5)$$

and the Euler discretization of that process is given by

$$V_{t+1} = V_t + \kappa(\theta - V_t) + \sigma\sqrt{V_t}Z_v \quad (3.6)$$

where  $Z_v \sim N(0,1)$  and  $dt = 1$  is taken similarly as in the first discretization.

Before the discretization of the Heston model,  $Z_{1t}$  and  $Z_{2t}$  are the two standard Brownian motions which have correlation coefficient  $\rho$ . This is the continuous form of the Heston model. After the usage of Euler's discretization, we obtain the discrete form of the Heston model and now,  $Z_s$  and  $Z_v$  are two standard normal random variables with the same correlation coefficient  $\rho$ . In order to finalize the discretiation process, let set

$$Z_v = Z_1 \quad (3.7)$$

$$Z_s = \rho Z_1 + \sqrt{1 - \rho^2} Z_2 \quad (3.8)$$

where  $Z_1$  and  $Z_2 \sim N(0,1)$ . The discretization yields the following Euler scheme,

$$Q_{t+1} = 1 + (r_d - r_f) + \sqrt{V_t}(\rho Z_1 + \sqrt{1 - \rho^2} Z_2) \quad (3.9)$$

$$V_{t+1} = V_t + \kappa(\theta - V_t) + \sigma\sqrt{V_t}Z_1 \quad (3.10)$$

Above system will be utilized in the likelihood function of the data.

### 3.1.2. Maximum Likelihood Estimation

In this thesis, we employ maximum likelihood estimation for the calibration. Each step for obtaining log likelihood function is described below.

First, writing the likelihood function of data set is must. The likelihood function  $L(r_d, r_f, \kappa, \theta, \sigma, \rho)$  is the product of the joint probability density function,  $f(Q_{t+1}, V_{t+1})$ , of  $Q_{t+1}$  and  $V_{t+1}$ ,

$$L(r_d, r_f, \kappa, \theta, \sigma, \rho) = \prod_{t=1}^n f(Q_{t+1}, V_{t+1} | r_d, r_f, \kappa, \theta, \sigma, \rho) \quad (3.11)$$

The natural logarithm of the likelihood function, denoted by  $l(r_d, r_f, \kappa, \theta, \sigma, \rho)$ , is equivalent to,

$$l(r_d, r_f, \kappa, \theta, \sigma, \rho) = \sum_{t=1}^n \log(f(Q_{t+1}, V_{t+1} | r_d, r_f, \kappa, \theta, \sigma, \rho)) \quad (3.12)$$

Partial derivatives of  $l(\cdot)$  are taken with respect to each parameter by setting them to zero. Instead of getting model parameters by hand, we use a software tool R which is capable to optimize the log likelihood function under some constraints. The famous one is the Feller condition and the other constraints are as follows,

$$2\kappa\theta - \sigma^2 > 0$$

$$\kappa > 0$$

$$\theta > 0$$

$$\sigma > 0$$

$$-1 < \rho < 1$$

$Z_s$  and  $Z_v$  are the standard normal variables and hence,

$$Q_{t+1} \sim N(1 + (r_d - r_f), V_t)$$

$$V_{t+1} \sim N(V_t + \kappa(\theta - V_t), \sigma^2 V_t)$$

Moreover,  $Q_{t+1}$  and  $V_{t+1}$  have the correlation  $\rho$  since  $Z_s$  and  $Z_v$  have correlation coefficient  $\rho$ .

Definition: Two random variables  $(X, Y)$  have a bivariate normal distribution  $N(\mu_1, \mu_2, \sigma_1^2, \sigma_2^2, \rho)$  if their joint pdf is

$$f_{X,Y}(x, y) = \frac{1}{2\pi\sigma_1\sigma_2\sqrt{1-\rho^2}} \exp \left\{ -\frac{1}{2(1-\rho^2)} \left[ \left( \frac{x-\mu_1}{\sigma_1} \right)^2 - 2\rho \left( \frac{x-\mu_1}{\sigma_1} \right) \left( \frac{y-\mu_2}{\sigma_2} \right) + \left( \frac{y-\mu_2}{\sigma_2} \right)^2 \right] \right\}$$

for all  $x, y$ . The parameters  $\mu_1, \mu_2$  may be any real numbers,  $\sigma_1 > 0, \sigma_2 > 0$  and  $-1 \leq \rho \leq 1$ .

The joint probability density function of two random variables  $Q_{t+1}$  and  $V_{t+1}$  can be written by definition as,

$$f(Q_{t+1}, V_{t+1}) = \frac{1}{2\pi\sigma V_t \sqrt{1-\rho^2}} \exp \left\{ \frac{(Q_{t+1}-1-(r_d-r_f))^2}{2V_t(1-\rho^2)} + \frac{\rho(Q_{t+1}-1-(r_d-r_f))(V_{t+1}-V_t-\theta\kappa+\kappa V_t)}{V_t\sigma(1-\rho^2)} - \frac{(V_{t+1}-V_t-\theta\kappa+\kappa V_t)^2}{2\sigma^2 V_t(1-\rho^2)} \right\} \quad (3.13)$$

and the log likelihood function is,

$$l(r_d, r_f, \kappa, \theta, \sigma, \rho) = \sum_{t=1}^n \left( -\log(2\pi) - \log(\sigma) - \log(V_t) - \frac{1}{2} \log(1-\rho^2) - \frac{(Q_{t+1}-1-(r_d-r_f))^2}{2V_t(1-\rho^2)} + \frac{\rho(Q_{t+1}-1-(r_d-r_f))(V_{t+1}-V_t-\theta\kappa+\kappa V_t)}{V_t\sigma(1-\rho^2)} - \frac{(V_{t+1}-V_t-\theta\kappa+\kappa V_t)^2}{2\sigma^2 V_t(1-\rho^2)} \right) \quad (3.14)$$

Then, stock return ( $Q_{t+1}$ ) and asset variance ( $V_{t+1}$ ) are plugged into the log likelihood function. Stock returns are obtained by the historical spot prices. A vector of variance ( $V_t$ ) can be estimated from the data by determining the variance of the values  $Q_{t+1}$  up to and including its value at the given time  $t$ . For instance,  $V_2$  is found by calculating the variance of  $Q_1, Q_2, Q_3, V_3$  is found by calculating the variance of  $Q_1, Q_2, Q_3, Q_4$  and in general  $V_n$  is found by calculating the variance of  $Q_1, Q_2, Q_3, Q_4, \dots, Q_{n+1}$ .

Since we have a data set with known values of  $Q_{t+1}$ ,  $V_{t+1}$  and  $V_t$  and also know the log likelihood function, parameters can be estimated by `constroptim.nl` function in R.

### 3.2. Estimation Results

Heston model parameters are calibrated in R using historical data set of asset prices by the method of maximum likelihood estimation. EUR/TL exchange rates of year 2012, 2013, 2014 and 2015 are used for the calibration. The estimated values for the parameters are obtained as shown in Table 3.1.

Heston model parameters are calibrated by using different historical data sets. To illustrate, in order to price 2016 European FX call and put options by Heston model, we use one year EUR/TL parity (2015), two years parities (2014 – 2015), three years parities (2013 – 2014 – 2015) and four years parities (2012 – 2013 – 2014 – 2015) for the calibration. Hence, we obtain four different model parameters for 2016.

European FX option will be calculated by using those parameters with Heston model and in order to reach which model gives better results, Black and Scholes model will be employed and compared both models with real market prices in the following chapter.

Table 3.1. Calibration Results of Heston Parameters.

<b>Historical Parity Years (used for calibration)</b>	<b>Rate of Mean Reversion (<math>\kappa</math>)</b>	<b>Long Term Level of Mean Reversion (<math>\Theta</math>)</b>	<b>Volatility of Volatility (<math>\sigma</math>)</b>	<b>Correlation Coefficient (<math>\rho</math>)</b>
2012	0,00156000	0,01741000	0,00000001	0,37699000
2013	0,01299965	0,01248850	0,00060563	0,79530229
2014	1,45228526	0,00048426	0,00033420	0,00196706
2015	0,03374777	0,28726650	0,00000497	0,37813080
2012-2013	0,10800000	0,00170000	0,00001294	0,01080100
2012-2014	1,29409500	0,26195390	0,00000087	0,41767830
2012-2015	0,27655540	0,50967250	0,00000216	0,40871780
2013-2014	0,40156760	0,50429820	0,00000049	0,31789270
2014-2015	0,36371460	0,48298050	0,00000101	0,30747010
2013-2015	0,91743790	0,19894590	0,00000120	0,25326510

### 3.3. Parameter Simulation

The efficiency of maximum likelihood estimation used in the calibration of Heston model parameters is examined by some simulation experiment. In order to measure the goodness of the parameters, Monte Carlo simulation is performed. The expectation is to observe insignificant error between the original asset price and the estimated one. According to error, the proper parameters will be chosen. It is critical for pricing options correctly.

The purpose of the simulation is to determine the FX rates of the following year by considering the last year/years. If estimated FX rates close to the original rate, our estimated Heston parameters are convenient to use in pricing.

In the simulation of EUR/TL exchange rates, the years 2012, 2013, 2014 and 2015 are used. To illustrate the procedure, 2014 EUR/TL rates are estimated by using one year data (2013 full year data is used) firstly. In order to decide whether bigger data set provides better results, 2014 EUR/TL exchange rates are also simulated by considering two years data (2012 and 2013 data are used). Simulation procedure begins with choosing an initial value. Based on this, each simulation is conducted by two different ways. First, it is done by choosing the initial parity from the first day of the year and that year parities are simulated by using that parity. For instance, 2014 EUR/TL simulation results are obtained by choosing the initial value as 01 January 2014 EUR/TL parity. This value is employed to estimate the 2014 full year parity values. In other word, 01 January 2014 parity is used, for example, to find the parity of 10 January 2014, 10 February 2014 and 15 March 2014 etc. Second way is implementing the simulation weekly. Namely, choosing the initial parity from the first day of a week and simulate the rest of the parities of that week. For instance, 2014 EUR/TL parities are found by selecting the initial values from the first day's parity of each week. 01 January 2014 parity is taken as an initial value to estimate the 02 January and 03 January 2014 parities whereas 06 January 2014 parity is taken as an initial value to simulate the parities for 07 – 08 – 09 - 10 January 2014. Steps of the Monte Carlo simulation applied to Heston model are explained in following paragraph.

Monte Carlo simulation is utilized from Strong Law of Large Number Theorem and Central Limit Theorem. It is applied to make prediction for future event relying on repeated random sampling. The fundamental steps of the simulation are listed below.

- Step 1: Create the parametric model  $S = f(X_1, X_2, \dots, X_N)$ .
- Step 2: Generate random numbers  $X_1, X_2, \dots, X_N$ .
- Step 3: Evaluate the model and obtain the results as  $S$ .
- Step 4: Calculate the sample mean  $\bar{S} = \frac{1}{N} \sum_{i=1}^N S_i$

Simulation results of the EUR/TL exchange rates are revealed in Table 3.2 and Table 3.3. Table 3.2 includes yearly simulation results whereas Table 3.3 shows weekly ones. Simulation experiment is conducted for  $N = 100000$  times.

Table 3.2. Mean Values of Yearly Simulated EUR/TL Rates and Root Mean Square Errors (RMSE).

Historical Parity (used for simulation)	Simulated Year	EURTRL True Mean Value	EURTRL Simulated Mean Value	RMSE
				yearly
2012	2013	2,5068	2,3327	0,2525
2013	2014	2,9034	2,9670	0,0955
2014	2015	3,0098	2,7087	0,3530
2015	2016	3,2556	3,1852	0,0820
2012-2013	2014	2,9034	2,8777	0,0836
2012-2014	2015	3,0098	2,8237	0,2947
2012-2015	2016	3,2556	3,1645	0,1004
2013-2014	2015	3,0098	2,8441	0,2643
2014-2015	2016	3,2556	3,1795	0,0836
2013-2015	2016	3,2556	3,1757	0,0871

Table 3.3. Mean Values of Weekly Simulated EUR/TL Rates and Root Mean Square Errors (RMSE).

Historical Parity (used for simulation)	Simulated Year	EURTRL True Mean Value	EURTRL Simulated Mean Value	RMSE
				weekly
2012	2013	2,5068	2,5014	0,0240
2013	2014	2,9034	2,9371	0,0401
2014	2015	3,0098	2,9320	0,0466
2015	2016	3,2556	3,2552	0,0233
2012-2013	2014	2,9034	2,9029	0,0268
2012-2014	2015	3,0098	3,0132	0,0463
2012-2015	2016	3,2556	3,2536	0,0239
2013-2014	2015	3,0098	3,0042	0,0523
2014-2015	2016	3,2556	3,2501	0,0247
2013-2015	2016	3,2556	3,2495	0,0253

Based on the simulation results, taking bigger data set for the estimation procedure is, to some extent, reasonable. Root mean square errors for the simulation of the 2014 EUR/TL parities by using only one year data (2013) is 0,0955 and two years data (2012 – 2013) is 0,0836. For the estimation of 2016 parities, we use one year, two years, three years and four years data. Expectation is in the direction of using four years data should give better estimation results. However, choosing bigger data set can sometimes mislead. The reason of that misleading result is that values of the 2012 EUR/TL parities are far from the 2013 – 2014 – 2015 parities. In 2012, yearly average of EUR/TL parity is 2,3090, while in 2013 is 2,5285, in 2014 is 2,8976 and in 2015 is 3,0137. Therefore, it can be said that 2012 exchange rates are the outlier of our simulation.

Another result observed in above tables is that choosing initial value as in the first day of the each week and conducting simulation result in better outcomes, because RMSEs are small compared to yearly simulation. For instance, root mean square error for the estimation of 2014 EUR/TL parities by using one year data set (2013) is 0,0955 when 01 January 2014 parity is used as the initial value for the full year. However, root mean square error is 0,0401 when the initial values are taken at the first day of each week.

Implementing Monte Carlo simulation gives idea about whether calibrated Heston model parameters should be used in option pricing or not. If RMSEs were too big, probably the estimation method of parameters would be changed.

## 4. MODEL IMPLEMENTATION

In this section of the thesis, pricing of the European FX options will be carried with two methods: Black and Scholes model and Heston model. The purpose is to determine which method demonstrates more accurate results compared to real market data.

### 4.1. Market Data

The European FX option prices used in this thesis are procured from *Turkish Economic Bank*. The data consists of spot price, strike price, domestic and foreign interest rates and European call and put option prices for the maturity 90 days. Prices are given according to 1 million EUR notional amounts. A representative form of the option data is given in Table 4.1. All option prices are shown in Appendix A.1

Table 4.1. A Sample European FX Option Prices.

Price Date	Expiration Date	Days to Maturity	Spot Price	Strike Price	Foreign Interest Rate	Domestic Interest Rate	European Call Option Price	European Put Option Price
02.01.13	02.04.13	90	2.354	2.406	0.11%	5.36%	4,938	25,500
15.01.13	15.04.13	90	2.359	2.409	0.10%	5.21%	5,800	25,300
01.02.13	02.05.13	90	2.384	2.435	0.05%	5.07%	5,250	26,850
15.02.13	16.05.13	90	2.354	2.402	0.06%	4.84%	5,280	25,250
04.03.13	02.06.13	90	2.340	2.399	0.05%	5.23%	4,560	32,333
02.01.14	02.04.14	90	2.966	3.033	0.31%	7.22%	20,987	35,971
16.01.14	16.04.14	90	3.001	3.068	0.31%	7.85%	24,134	34,109
30.01.14	30.04.14	90	3.046	3.113	0.23%	9.66%	37,255	31,123
13.02.14	14.05.14	90	3.002	3.069	0.23%	11.53%	45,568	27,362
27.02.14	28.05.14	90	3.044	3.111	0.25%	11.56%	48,002	27,999

Price date is the date which option is written on that day. Expiration date implies the last day of the contract. Days to maturity gives the remaining time until the maturity of options. Spot price is the EUR/TL exchange rate on the price date. Strike price is the price at which the holder of the option can buy or sell the EUR/TL exchange rates at that price

when the option is exercised. European call and put options are the option prices on the specific trading day.

Initial starting date for the analysis is 2<sup>nd</sup> of January, 2013. The time horizon extends to 13<sup>th</sup> August, 2016. 20 different European FX option prices for each year (2013, 2014, 2015, 2016) will be used. Each option has 90 day maturity with different spot and strike prices. Moreover, daily spot FX rates, foreign and domestic interest rates are used between the specified dates.

#### 4.2. Pricing European FX Option with Black and Scholes Model

In Black and Scholes model, there is no need for the calibration process as in the Heston model. Only one parameter needed to be estimated is volatility. Based on the underlying asset price series, historical volatility is found and used in the pricing formula. The steps for computation of historical volatility are briefly described below.

First, log returns of asset prices  $S_0, S_1, \dots, S_N$  are created

$$r_i = \ln\left(\frac{S_i}{S_{i-1}}\right), \quad i = 1, \dots, N. \quad (4.1)$$

Then, average log returns are computed

$$\bar{r} = \frac{1}{N} \sum_{i=1}^N r_i \quad (4.2)$$

their variance

$$\hat{\sigma}^2 = \frac{1}{N-1} \sum_{i=1}^N (r_i - \bar{r})^2 \quad (4.3)$$

and their standard deviation

$$\hat{\sigma} = \sqrt{\frac{1}{N-1} \sum_{i=1}^N (r_i - \bar{r})^2} \quad (4.4)$$

The annualized standard deviation, volatility, is the form of

$$\hat{\sigma}_a = \sqrt{\frac{B}{N-1} \sum_{i=1}^N (r_i - \bar{r})^2} \quad (4.5)$$

where the annualization factor  $B$  is given by

$$B = \frac{N}{k} d \quad (4.6)$$

$k$  denotes the number of calendar days within the time series and  $d$  denotes the number of calendar days per year.

All the parameters which are spot price, strike price, foreign and domestic interest rate and volatility will be plugged into the Black and Scholes formula are known. Summary of European FX option prices and Black and Scholes prices are found as depicted in the Table 4.2. Appendix A.2 contains more detailed study.

Table 4.2. European FX Option Prices with Black and Scholes Model.

Date	Expiration Date	Spot Price	Strike Price	Historical Volatility	Real Price		Black Scholes Price	
					Call Option	Put Option	Call Option	Put Option
02.01.2013	02.04.2013	2,354	2,406	0,027	4.938	25.500	4.912	25.534
19.03.2013	17.06.2013	2,344	2,406	0,031	4.000	34.590	4.073	34.580
03.05.2013	01.08.2013	2,344	2,412	0,030	1.350	49.800	1.328	49.885
17.05.2013	15.08.2013	2,357	2,425	0,030	2.000	44.450	2.066	44.455
03.09.2013	02.12.2013	2,685	2,752	0,048	16.150	38.101	16.156	38.098
13.02.2014	14.05.2014	3,002	3,069	0,059	45.568	27.362	44.977	26.571
13.03.2014	11.06.2014	3,093	3,160	0,061	52.011	26.997	51.258	26.346
03.07.2014	01.10.2014	2,900	2,967	0,042	23.791	27.019	22.512	25.705
14.08.2014	12.11.2014	2,877	2,944	0,038	21.100	24.992	19.612	23.659
25.09.2014	24.12.2014	2,870	2,937	0,037	19.100	25.161	18.036	24.296
16.02.2015	17.05.2015	2,787	2,937	0,060	7.961	77.147	9.718	78.891
02.03.2015	31.05.2015	2,809	2,959	0,059	7.549	79.901	9.201	81.537
13.04.2015	12.07.2015	2,798	2,948	0,049	4.413	75.488	5.636	76.471
11.05.2015	09.08.2015	2,987	3,137	0,049	6.582	70.509	7.918	71.530
31.08.2015	29.11.2015	3,264	3,414	0,077	24.123	77.992	28.245	80.792
21.03.2016	19.06.2016	3,225	3,245	0,056	76.114	12.681	75.992	12.625
05.04.2016	04.07.2016	3,211	3,231	0,050	78.593	7.990	78.532	7.971
08.07.2016	06.10.2016	3,210	3,230	0,041	71.390	5.252	71.377	5.219
04.08.2016	02.11.2016	3,375	3,395	0,050	87.736	7.639	87.805	7.607
09.09.2016	08.12.2016	3,315	3,335	0,045	80.412	6.020	80.404	6.003

### 4.3. Pricing European FX Option with Heston Model

Using calibrated Heston model parameters and the other input variables, European FX options are priced with Heston model. Other than Black and Scholes model, due to use of different data set for the calibration, we have more than one price for the specified date. To illustrate, for 2014 FX option prices, only 2013 data set is used and also 2012 and 2013, namely 2 years data set, are used for the calibration. Hence 2 different calibrated parameters will be plugged into the Heston model and obtained 2 different option prices for the same spot price, strike price, foreign and domestic interest rate. Summary of European FX option prices and Heston prices are found as revealed in following tables. Appendix A.2 contains more detailed study.

Table 4.3. European FX Option Prices (2013) with Heston Model Calibrated with 2012 Spot Prices.

Date	Expiration Date	Spot Price	Strike Price	Initial Variance	$\kappa$	$\theta$	$\sigma$	$\rho$	Real Price		Heston Price	
									Call Option	Put Option	Call Option	Put Option
02.01.2013	02.04.2013	2,354	2,406	0,001	0,00156000	0,01741000	0,00000001	0,37699000	4.938	25.500	4.940	25.562
19.03.2013	17.06.2013	2,344	2,406	0,001	0,00156000	0,01741000	0,00000001	0,37699000	4.000	34.590	4.100	34.607
03.05.2013	01.08.2013	2,344	2,412	0,001	0,00156000	0,01741000	0,00000001	0,37699000	1.350	49.800	1.338	49.896
17.05.2013	15.08.2013	2,357	2,425	0,001	0,00156000	0,01741000	0,00000001	0,37699000	2.000	44.450	2.091	44.481
03.09.2013	02.12.2013	2,685	2,752	0,002	0,00156000	0,01741000	0,00000001	0,37699000	16.150	38.101	16.161	38.103

Above table incorporates Heston model parameters which are calibrated by using just 2012 historical spot prices. However, following tables show that two different option prices since two distinct historical data sets are used. In order to predict FX option prices expired at year 2014, Heston model parameters are calibrated both 2012-2013 (two full years) and 2013 (just one year) spot prices.

The end results of Heston model option prices are revealed in the Table 4.4 and Table 4.5.

Table 4.4. European FX Option Prices (2014) with Heston Model Calibrated with 2012 and 2013 Spot Prices.

Date	Expiration Date	Spot Price	Strike Price	Initial Variance	$\kappa$	$\theta$	$\sigma$	$\rho$	Real Price		Heston Price	
									Call Option	Put Option	Call Option	Put Option
13.02.2014	14.05.2014	3,002	3,069	0,003	0,10800000	0,00170000	0,00001294	0,01080100	45.568	27.362	45.868	27.462
13.03.2014	11.06.2014	3,093	3,160	0,004	0,10800000	0,00170000	0,00001294	0,01080100	52.011	26.997	52.111	27.197
03.07.2014	01.10.2014	2,900	2,967	0,002	0,10800000	0,00170000	0,00001294	0,01080100	23.791	27.019	23.891	27.085
14.08.2014	12.11.2014	2,877	2,944	0,001	0,10800000	0,00170000	0,00001294	0,01080100	21.100	24.992	21.146	25.192
25.09.2014	24.12.2014	2,870	2,937	0,001	0,10800000	0,00170000	0,00001294	0,01080100	19.100	25.161	19.600	25.860

Table 4.5. European FX Option Prices (2014) with Heston Model Calibrated with 2013 Spot Prices.

Date	Expiration Date	Spot Price	Strike Price	Initial Variance	$\kappa$	$\theta$	$\sigma$	$\rho$	Real Price		Heston Price	
									Call Option	Put Option	Call Option	Put Option
13.02.2014	14.05.2014	3,002	3,069	0,003	0,01299965	0,01248850	0,00060563	0,79530229	45.568	27.362	45.044	26.637
13.03.2014	11.06.2014	3,093	3,160	0,004	0,01299965	0,01248850	0,00060563	0,79530229	52.011	26.997	51.318	26.406
03.07.2014	01.10.2014	2,900	2,967	0,002	0,01299965	0,01248850	0,00060563	0,79530229	23.791	27.019	22.634	25.828
14.08.2014	12.11.2014	2,877	2,944	0,001	0,01299965	0,01248850	0,00060563	0,79530229	21.100	24.992	19.748	23.795
25.09.2014	24.12.2014	2,870	2,937	0,001	0,01299965	0,01248850	0,00060563	0,79530229	19.100	25.161	18.176	24.436

European FX option prices written on year 2015 and 2016 are found in Appendix A.3. Next section provides a comparison between two methods in order to explain the differences and give a solid idea as to whether Heston or Black and Scholes model is better.

#### 4.4. Comparison of the Black and Scholes and Heston Model

One of the main purposes of this thesis is to determine which model gives more accurate results in predicting FX option prices. For the comparison, 2016 European FX option prices which are taken from the Turkish Economic Bank's Treasury department and both two models prices are used.

Table 4.6 reveals four different Heston model prices due to using distinct data set for the calibration, Black and Scholes model prices and real market prices.

Table 4.6. A Comparison between Black and Scholes, Heston Model and Real Market Prices.

Date	Time to Maturity	Expiration Date	Spot Price	Strike Price	Real Price		Black Scholes Price		Heston Price*	
					Call Option	Put Option	Call Option	Put Option	Call Option	Put Option
04.01.2016	90	03.04.2016	3,190	3,210	86.143	20.388	86.511	20.259	86.544	20.292
05.04.2016	90	04.07.2016	3,211	3,231	78.593	7.990	78.532	7.971	78.580	8.019
20.05.2016	90	18.08.2016	3,362	3,382	82.879	4.989	82.865	4.978	82.915	5.028
09.09.2016	90	08.12.2016	3,315	3,335	80.412	6.020	80.404	6.003	80.455	6.054

\*2015 spot prices are used for Heston model parameter calibration

Date	Time to Maturity	Expiration Date	Spot Price	Strike Price	Real Price		Black Scholes Price		Heston Price*	
					Call Option	Put Option	Call Option	Put Option	Call Option	Put Option
04.01.2016	90	03.04.2016	3,190	3,210	86.143	20.388	86.511	20.259	86.697	20.446
05.04.2016	90	04.07.2016	3,211	3,231	78.593	7.990	78.532	7.971	78.757	8.196
20.05.2016	90	18.08.2016	3,362	3,382	82.879	4.989	82.865	4.978	83.090	5.203
09.09.2016	90	08.12.2016	3,315	3,335	80.412	6.020	80.404	6.003	80.635	6.234

\*2014 & 2015 spot prices are used for Heston model parameter calibration

Date	Time to Maturity	Expiration Date	Spot Price	Strike Price	Real Price		Black Scholes Price		Heston Price*	
					Call Option	Put Option	Call Option	Put Option	Call Option	Put Option
04.01.2016	90	03.04.2016	3,190	3,210	86.143	20.388	86.511	20.259	86.263	20.401
05.04.2016	90	04.07.2016	3,211	3,231	78.593	7.990	78.532	7.971	78.565	8.004
20.05.2016	90	18.08.2016	3,362	3,382	82.879	4.989	82.865	4.978	82.899	5.012
09.09.2016	90	08.12.2016	3,315	3,335	80.412	6.020	80.404	6.003	80.439	6.038

\*2013, 2014, 2015 spot prices are used for Heston model parameter calibration

Date	Time to Maturity	Expiration Date	Spot Price	Strike Price	Real Price		Black Scholes Price		Heston Price*	
					Call Option	Put Option	Call Option	Put Option	Call Option	Put Option
04.01.2016	90	03.04.2016	3,190	3,210	86.143	20.388	86.511	20.259	86.253	20.390
05.04.2016	90	04.07.2016	3,211	3,231	78.593	7.990	78.532	7.971	78.553	7.992
20.05.2016	90	18.08.2016	3,362	3,382	82.879	4.989	82.865	4.978	82.886	4.999
09.09.2016	90	08.12.2016	3,315	3,335	80.412	6.020	80.404	6.003	80.426	6.025

\*2012, 2013, 2014, 2015 spot prices are used for Heston model parameter calibration

As observed in Table 4.6, Heston model option prices whose parameters calibrated by using four years spot prices have better results than the other Heston prices. Next section examines the error between the model prices and the real market prices.

#### 4.5. Error Analysis

In order to check the accuracy of the models and to compare the performance of the pricing methods, a variety of statistical instruments can be chosen. In this thesis, widely used measures, Root Mean Squared Error (RMSE) and Average Relative Percentage Error

(ARPE), are employed. Before presenting the errors, a brief summary of the statistical measures are explained below.

#### 4.5.1. Root Mean Squared Error Analysis

The RMSE is defined as the square root of variance of the residuals. This statistical metric indicates that how close the observed data points are to the model's predicted values. It is calculated in the following way:

$$RMSE = \sqrt{\frac{\sum_{i=1}^N (\text{Market price} - \text{Model price})_i^2}{N}} \quad (4.7)$$

Lower values of RMSE shows better fit. Following tables reveal the comparison between Black and Scholes model, Heston model and real option prices for years 2013, 2014, 2015 and 2016 as a summary. Details can be found in Appendix A.3

Table 4.7. RMSE of Black and Scholes Model and Heston Model for 2013 European FX Options.

Date	Expiration Date	Spot Price	Strike Price	Real Price		Black Scholes Price		Heston Price*	
				Call Option	Put Option	Call Option	Put Option	Call Option	Put Option
02.01.2013	02.04.2013	2.354	2.406	4,938	25,500	4,912	25,534	4,940	25,562
19.03.2013	17.06.2013	2.344	2.406	4,000	34,590	4,073	34,580	4,100	34,607
03.05.2013	01.08.2013	2.344	2.412	1,350	49,800	1,328	49,885	1,338	49,896
17.05.2013	15.08.2013	2.357	2.425	2,000	44,450	2,066	44,455	2,091	44,481
03.09.2013	02.12.2013	2.685	2.752	16,150	38,101	16,156	38,098	16,161	38,103
				<b>RMSE</b>		49.5	44.7	52.0	50.0

Table 4.8. RMSE of Black and Scholes Model and Heston Model for 2014 European FX Options.

Date	Expiration Date	Spot Price	Strike Price	Real Price		Black Scholes Price		Heston Price*	
				Call Option	Put Option	Call Option	Put Option	Call Option	Put Option
13.02.2014	14.05.2014	3,002	3,069	45.568	27.362	44.977	26.571	45.044	26.637
13.03.2014	11.06.2014	3,093	3,160	52.011	26.997	51.258	26.346	51.318	26.406
03.07.2014	01.10.2014	2,900	2,967	23.791	27.019	22.512	25.705	22.634	25.828
14.08.2014	12.11.2014	2,877	2,944	21.100	24.992	19.612	23.659	19.748	23.795
25.09.2014	24.12.2014	2,870	2,937	19.100	25.161	18.036	24.296	18.176	24.436
				<b>RMSE</b>		1.029,4	982,7	924,1	877,2

\*2013 spot prices are used for Heston model parameter calibration

Date	Expiration Date	Spot Price	Strike Price	Real Price		Black Scholes Price		Heston Price*	
				Call Option	Put Option	Call Option	Put Option	Call Option	Put Option
13.02.2014	14.05.2014	3,002	3,069	45.568	27.362	44.977	26.571	45.868	27.462
13.03.2014	11.06.2014	3,093	3,160	52.011	26.997	51.258	26.346	52.111	27.197
03.07.2014	01.10.2014	2,900	2,967	23.791	27.019	22.512	25.705	23.891	27.085
14.08.2014	12.11.2014	2,877	2,944	21.100	24.992	19.612	23.659	21.146	25.192
25.09.2014	24.12.2014	2,870	2,937	19.100	25.161	18.036	24.296	19.600	25.860
				<b>RMSE</b>		1.029,4	982,7	361,8	386,1

\*2012 & 2013 spot prices are used for Heston model parameter calibration

Table 4.9. RMSE of Black and Scholes Model and Heston Model for 2015 European FX Options.

Date	Expiration Date	Spot Price	Strike Price	Real Price		Black Scholes Price		Heston Price*	
				Call Option	Put Option	Call Option	Put Option	Call Option	Put Option
16.02.2015	17.05.2015	2,787	2,937	7.961	77.147	9.718	78.891	8.032	77.205
02.03.2015	31.05.2015	2,809	2,959	7.549	79.901	9.201	81.537	7.558	79.894
13.04.2015	12.07.2015	2,798	2,948	4.413	75.488	5.636	76.471	4.566	75.401
11.05.2015	09.08.2015	2,987	3,137	6.582	70.509	7.918	71.530	6.605	70.217
31.08.2015	29.11.2015	3,264	3,414	24.123	77.992	28.245	80.792	24.728	77.275
<b>RMSE</b>						2.224,1	1.938,4	193,1	329,7

\*2014 spot prices are used for Heston model parameter calibration

Date	Expiration Date	Spot Price	Strike Price	Real Price		Black Scholes Price		Heston Price*	
				Call Option	Put Option	Call Option	Put Option	Call Option	Put Option
16.02.2015	17.05.2015	2,787	2,937	7.961	77.147	9.718	78.891	7.959	77.133
02.03.2015	31.05.2015	2,809	2,959	7.549	79.901	9.201	81.537	7.487	79.823
13.04.2015	12.07.2015	2,798	2,948	4.413	75.488	5.636	76.471	4.505	75.340
11.05.2015	09.08.2015	2,987	3,137	6.582	70.509	7.918	71.530	6.528	70.140
31.08.2015	29.11.2015	3,264	3,414	24.123	77.992	28.245	80.792	24.616	77.162
<b>RMSE</b>						2.224,1	1.938,4	170,2	380,2

\*2013 & 2014 spot prices are used for Heston model parameter calibration

Date	Expiration Date	Spot Price	Strike Price	Real Price		Black Scholes Price		Heston Price*	
				Call Option	Put Option	Call Option	Put Option	Call Option	Put Option
16.02.2015	17.05.2015	2,787	2,937	7.961	77.147	9.718	78.891	7.984	77.158
02.03.2015	31.05.2015	2,809	2,959	7.549	79.901	9.201	81.537	7.512	79.848
13.04.2015	12.07.2015	2,798	2,948	4.413	75.488	5.636	76.471	4.543	75.378
11.05.2015	09.08.2015	2,987	3,137	6.582	70.509	7.918	71.530	6.577	70.189
31.08.2015	29.11.2015	3,264	3,414	24.123	77.992	28.245	80.792	24.615	77.162
<b>RMSE</b>						2.224,1	1.938,4	166,5	371,4

\*2012, 2013, 2014 spot prices are used for Heston model parameter calibration

Table 4.10. RMSE of Black and Scholes Model and Heston Model for 2016 European FX Options.

Date	Expiration Date	Spot Price	Strike Price	Real Price		Black Scholes Price		Heston Price*	
				Call Option	Put Option	Call Option	Put Option	Call Option	Put Option
21.03.2016	19.06.2016	3,225	3,245	76.114	12.681	75.992	12.625	76.040	12.672
05.04.2016	04.07.2016	3,211	3,231	78.593	7.990	78.532	7.971	78.580	8.019
08.07.2016	06.10.2016	3,210	3,230	71.390	5.252	71.377	5.219	71.432	5.274
04.08.2016	02.11.2016	3,375	3,395	87.736	7.639	87.805	7.607	87.852	7.654
09.09.2016	08.12.2016	3,315	3,335	80.412	6.020	80.404	6.003	80.455	6.054
				<b>RMSE</b>		644,2	260,3	636,5	266,9

\*2015 spot prices are used for Heston model parameter calibration

Date	Expiration Date	Spot Price	Strike Price	Real Price		Black Scholes Price		Heston Price*	
				Call Option	Put Option	Call Option	Put Option	Call Option	Put Option
21.03.2016	19.06.2016	3,225	3,245	76.114	12.681	75.992	12.625	76.224	12.857
05.04.2016	04.07.2016	3,211	3,231	78.593	7.990	78.532	7.971	78.757	8.196
08.07.2016	06.10.2016	3,210	3,230	71.390	5.252	71.377	5.219	71.623	5.466
04.08.2016	02.11.2016	3,375	3,395	87.736	7.639	87.805	7.607	88.024	7.826
09.09.2016	08.12.2016	3,315	3,335	80.412	6.020	80.404	6.003	80.635	6.234
				<b>RMSE</b>		644,2	260,3	633,5	360,1

\*2014 & 2015 spot prices are used for Heston model parameter calibration

Date	Expiration Date	Spot Price	Strike Price	Real Price		Black Scholes Price		Heston Price*	
				Call Option	Put Option	Call Option	Put Option	Call Option	Put Option
21.03.2016	19.06.2016	3,225	3,245	76.114	12.681	75.992	12.625	76.027	12.659
05.04.2016	04.07.2016	3,211	3,231	78.593	7.990	78.532	7.971	78.565	8.004
08.07.2016	06.10.2016	3,210	3,230	71.390	5.252	71.377	5.219	71.414	5.256
04.08.2016	02.11.2016	3,375	3,395	87.736	7.639	87.805	7.607	87.838	7.640
09.09.2016	08.12.2016	3,315	3,335	80.412	6.020	80.404	6.003	80.439	6.038
				<b>RMSE</b>		644,2	260,3	51,7	13,7

\*2013, 2014, 2015 spot prices are used for Heston model parameter calibration

Date	Expiration Date	Spot Price	Strike Price	Real Price		Black Scholes Price		Heston Price*	
				Call Option	Put Option	Call Option	Put Option	Call Option	Put Option
21.03.2016	19.06.2016	3,225	3,245	76.114	12.681	75.992	12.625	76.014	12.646
05.04.2016	04.07.2016	3,211	3,231	78.593	7.990	78.532	7.971	78.553	7.992
08.07.2016	06.10.2016	3,210	3,230	71.390	5.252	71.377	5.219	71.400	5.243
04.08.2016	02.11.2016	3,375	3,395	87.736	7.639	87.805	7.607	87.826	7.628
09.09.2016	08.12.2016	3,315	3,335	80.412	6.020	80.404	6.003	80.426	6.025
				<b>RMSE</b>		644,2	260,3	51,4	15,1

\*2012, 2013, 2014, 2015 spot prices are used for Heston model parameter calibration

#### 4.5.2. Average Relative Percentage Error Analysis

Another widely used statistical measure is Average Relative Percentage Error (ARPE). It shows how large the error is in relation to the real value. Since it is expressed as percentage, interpretation is easier. Mathematical expression for ARPE is shown below:

$$ARPE = \frac{1}{N} \sum_{i=1}^N \frac{|Market\ price - Model\ price|_i}{Market\ price_i} \times 100\% \quad (4.8)$$

As in the RMSE, the model which has small percent error is the better one. Following tables indicate the comparison between Black and Scholes model, Heston model and real option prices for years 2013, 2014, 2015 and 2016 as a summary. Details can be found in Appendix A.3

Table 4.11. ARPE of Black and Scholes Model and Heston Model for 2013 European FX Options.

Date	Expiration Date	Spot Price	Strike Price	Real Price		Black Scholes Price		Heston Price*	
				Call Option	Put Option	Call Option	Put Option	Call Option	Put Option
02.01.2013	02.04.2013	2,354	2,406	4.938	25.500	4.912	25.534	4.940	25.562
19.03.2013	17.06.2013	2,344	2,406	4.000	34.590	4.073	34.580	4.100	34.607
03.05.2013	01.08.2013	2,344	2,412	1.350	49.800	1.328	49.885	1.338	49.896
17.05.2013	15.08.2013	2,357	2,425	2.000	44.450	2.066	44.455	2.091	44.481
03.09.2013	02.12.2013	2,685	2,752	16.150	38.101	16.156	38.098	16.161	38.103
				<b>ARPE</b>		1,14%	0,09%	1,13%	0,10%

\*2012 spot prices are used for Heston model parameter calibration

Table 4.12. ARPE of Black & Scholes Model and Heston Model for 2014 European FX Options.

Date	Expiration Date	Spot Price	Strike Price	Real Price		Black Scholes Price		Heston Price*	
				Call Option	Put Option	Call Option	Put Option	Call Option	Put Option
13.02.2014	14.05.2014	3,002	3,069	45.568	27.362	44.977	26.571	45.044	26.637
13.03.2014	11.06.2014	3,093	3,160	52.011	26.997	51.258	26.346	51.318	26.406
03.07.2014	01.10.2014	2,900	2,967	23.791	27.019	22.512	25.705	22.634	25.828
14.08.2014	12.11.2014	2,877	2,944	21.100	24.992	19.612	23.659	19.748	23.795
25.09.2014	24.12.2014	2,870	2,937	19.100	25.161	18.036	24.296	18.176	24.436
<b>ARPE</b>						4,02%	3,30%	3,58%	2,92%

\*2013 spot prices are used for Heston model parameter calibration

Date	Expiration Date	Spot Price	Strike Price	Real Price		Black Scholes Price		Heston Price*	
				Call Option	Put Option	Call Option	Put Option	Call Option	Put Option
13.02.2014	14.05.2014	3,002	3,069	45.568	27.362	44.977	26.571	45.868	27.462
13.03.2014	11.06.2014	3,093	3,160	52.011	26.997	51.258	26.346	52.111	27.197
03.07.2014	01.10.2014	2,900	2,967	23.791	27.019	22.512	25.705	23.891	27.085
14.08.2014	12.11.2014	2,877	2,944	21.100	24.992	19.612	23.659	21.146	25.192
25.09.2014	24.12.2014	2,870	2,937	19.100	25.161	18.036	24.296	19.600	25.860
<b>ARPE</b>						4,02%	3,30%	0,97%	1,14%

\*2012 & 2013 spot prices are used for Heston model parameter calibration

Table 4.13. ARPE of Black and Scholes Model and Heston Model for 2015 European FX Options.

Date	Expiration Date	Spot Price	Strike Price	Real Price		Black Scholes Price		Heston Price*	
				Call Option	Put Option	Call Option	Put Option	Call Option	Put Option
16.02.2015	17.05.2015	2,787	2,937	7.961	77.147	9.718	78.891	8.032	77.205
02.03.2015	31.05.2015	2,809	2,959	7.549	79.901	9.201	81.537	7.558	79.894
13.04.2015	12.07.2015	2,798	2,948	4.413	75.488	5.636	76.471	4.566	75.401
11.05.2015	09.08.2015	2,987	3,137	6.582	70.509	7.918	71.530	6.605	70.217
31.08.2015	29.11.2015	3,264	3,414	24.123	77.992	28.245	80.792	24.728	77.275
<b>ARPE</b>						20,45%	2,41%	1,26%	0,31%

\*2014 spot prices are used for Heston model parameter calibration

Date	Expiration Date	Spot Price	Strike Price	Real Price		Black Scholes Price		Heston Price*	
				Call Option	Put Option	Call Option	Put Option	Call Option	Put Option
16.02.2015	17.05.2015	2,787	2,937	7.961	77.147	9.718	78.891	7.959	77.133
02.03.2015	31.05.2015	2,809	2,959	7.549	79.901	9.201	81.537	7.487	79.823
13.04.2015	12.07.2015	2,798	2,948	4.413	75.488	5.636	76.471	4.505	75.340
11.05.2015	09.08.2015	2,987	3,137	6.582	70.509	7.918	71.530	6.528	70.140
31.08.2015	29.11.2015	3,264	3,414	24.123	77.992	28.245	80.792	24.616	77.162
<b>ARPE</b>						20,45%	2,41%	1,27%	0,36%

\*2013 & 2014 spot prices are used for Heston model parameter calibration

Date	Expiration Date	Spot Price	Strike Price	Real Price		Black Scholes Price		Heston Price*	
				Call Option	Put Option	Call Option	Put Option	Call Option	Put Option
16.02.2015	17.05.2015	2,787	2,937	7.961	77.147	9.718	78.891	7.984	77.158
02.03.2015	31.05.2015	2,809	2,959	7.549	79.901	9.201	81.537	7.512	79.848
13.04.2015	12.07.2015	2,798	2,948	4.413	75.488	5.636	76.471	4.543	75.378
11.05.2015	09.08.2015	2,987	3,137	6.582	70.509	7.918	71.530	6.577	70.189
31.08.2015	29.11.2015	3,264	3,414	24.123	77.992	28.245	80.792	24.615	77.162
<b>ARPE</b>						20,45%	2,41%	1,17%	0,34%

\*2012, 2013, 2014 spot prices are used for Heston model parameter calibration

Table 4.14. ARPE of Black & Scholes Model and Heston Model for 2016 European FX Options.

Date	Expiration Date	Spot Price	Strike Price	Real Price		Black Scholes Price		Heston Price*	
				Call Option	Put Option	Call Option	Put Option	Call Option	Put Option
21.03.2016	19.06.2016	3,225	3,245	76.114	12.681	75.992	12.625	76.040	12.672
05.04.2016	04.07.2016	3,211	3,231	78.593	7.990	78.532	7.971	78.580	8.019
08.07.2016	06.10.2016	3,210	3,230	71.390	5.252	71.377	5.219	71.432	5.274
04.08.2016	02.11.2016	3,375	3,395	87.736	7.639	87.805	7.607	87.852	7.654
09.09.2016	08.12.2016	3,315	3,335	80.412	6.020	80.404	6.003	80.455	6.054
<b>ARPE</b>						0,24%	0,68%	0,24%	0,63%

\*2015 spot prices are used for Heston model parameter calibration

Date	Expiration Date	Spot Price	Strike Price	Real Price		Black Scholes Price		Heston Price*	
				Call Option	Put Option	Call Option	Put Option	Call Option	Put Option
21.03.2016	19.06.2016	3,225	3,245	76.114	12.681	75.992	12.625	76.224	12.857
05.04.2016	04.07.2016	3,211	3,231	78.593	7.990	78.532	7.971	78.757	8.196
08.07.2016	06.10.2016	3,210	3,230	71.390	5.252	71.377	5.219	71.623	5.466
04.08.2016	02.11.2016	3,375	3,395	87.736	7.639	87.805	7.607	88.024	7.826
09.09.2016	08.12.2016	3,315	3,335	80.412	6.020	80.404	6.003	80.635	6.234
<b>ARPE</b>						0,24%	0,68%	0,42%	2,89%

\*2014 & 2015 spot prices are used for Heston model parameter calibration

Date	Expiration Date	Spot Price	Strike Price	Real Price		Black Scholes Price		Heston Price*	
				Call Option	Put Option	Call Option	Put Option	Call Option	Put Option
21.03.2016	19.06.2016	3,225	3,245	76.114	12.681	75.992	12.625	76.027	12.659
05.04.2016	04.07.2016	3,211	3,231	78.593	7.990	78.532	7.971	78.565	8.004
08.07.2016	06.10.2016	3,210	3,230	71.390	5.252	71.377	5.219	71.414	5.256
04.08.2016	02.11.2016	3,375	3,395	87.736	7.639	87.805	7.607	87.838	7.640
09.09.2016	08.12.2016	3,315	3,335	80.412	6.020	80.404	6.003	80.439	6.038
<b>ARPE</b>						0,24%	0,68%	0,05%	0,14%

\*2013, 2014, 2015 spot prices are used for Heston model parameter calibration

Date	Expiration Date	Spot Price	Strike Price	Real Price		Black Scholes Price		Heston Price*	
				Call Option	Put Option	Call Option	Put Option	Call Option	Put Option
21.03.2016	19.06.2016	3,225	3,245	76.114	12.681	75.992	12.625	76.014	12.646
05.04.2016	04.07.2016	3,211	3,231	78.593	7.990	78.532	7.971	78.553	7.992
08.07.2016	06.10.2016	3,210	3,230	71.390	5.252	71.377	5.219	71.400	5.243
04.08.2016	02.11.2016	3,375	3,395	87.736	7.639	87.805	7.607	87.826	7.628
09.09.2016	08.12.2016	3,315	3,335	80.412	6.020	80.404	6.003	80.426	6.025
<b>ARPE</b>						0,24%	0,68%	0,05%	0,13%

\*2012, 2013, 2014, 2015 spot prices are used for Heston model parameter calibration

### 4.5.3. Interpretation

From the RMSE and ARPE tables, it can be concluded that Heston model gives better results than Black and Scholes model when compared to actual option prices. The reason for this outcome is that Heston model takes the volatility into consideration but Black and Scholes not.

Heston model parameters are calibrated by using different historical spot prices as explained in previous sections. Calibration of the parameters by using large data sample leads to closer results to the real market price. In Table 4.10, calibration is conducted by using only 2015 spot prices, 2014 – 2015 spot prices, 2013 – 2014 – 2015 spot prices and lastly 2012 – 2013 – 2014 – 2015 spot prices in order to estimate the 2016 FX call and put options. RMSE is 636,5 when only 2015 spot prices used for the calibration whereas 51,4 when 2012 – 2013 – 2014 – 2015 spot prices used for the calibration in order to price the call option. Similarly, for the put option RMSE is 266,9 when only one year historical data set is used, while 15,1 when four years data are employed.

Based on the results, in order to decrease the error between the model and market price, one should increase the amount of data to calibrate the Heston parameters. However, regardless of the data set for the calibration, Heston model outperform the Black and Scholes model.

## 5. SENSITIVITY ANALYSIS OF THE HESTON MODEL PARAMETERS

This section will examine the effect of the change in the Heston model parameters which are looked into correlation coefficient  $\rho$  and volatility of volatility  $\sigma$ . The analysis will be made for the both in the money ( $S > K$ ) and out of the money ( $S < K$ ) options.

Sensitivity analysis will be made by choosing two different  $\rho$  values and then calibrating the other Heston parameters. Similarly, according to two distinct  $\sigma$  values, we will perform a calibration procedure except correlation coefficient  $\rho$ . We set  $\rho = 0$  in order not to have an impact on the skewness. After finding the parameters, we will price the European FX options by two models, Heston model and Black Scholes model. Finally, effects of changing model parameters on the option prices will be observed clearly.

### 5.1. Effects of Changing Correlation Coefficient Parameter $\rho$

Correlation coefficient  $\rho$  is set to both -0,5 and 0,5 and then the other Heston model parameters are estimated according to calibration procedure explained in Chapter 3. The calibrated parameters are found in Table 5.1.

Table 5.1. Calibration Results when  $\rho$  is fixed at -0,5 and 0,5.

Heston Model Parameters	$\rho=0.5$	$\rho=-0.5$
Volatility of Volatility $\sigma$	0,0002	0,000153937
Mean Reversion $\kappa$	0,0005	0,000538218
Long Term Level of Variance $\Theta$	0,1	0,097399176

For the two different correlation coefficient parameters, European FX options (call options used in this analysis) are priced with both Heston model and Black and Scholes model. 14 different spot prices varying from 3,18 Euro to 3,36 Euro and a strike price of 3,18 Euro to 3,33 Euro are used. The volatility of the Black and Scholes model is equal to

the square root of the volatility in the Heston model (Heston, 1993). By using those input variables, option prices are determined. In the following table details are shown.

Table 5.2. Option Prices When Correlation Coefficient Parameter is Fixed at  $-0,5$  and  $0,5$ .

Spot Price	Strike Price	$\rho=-0.5$			$\rho=0.5$		
		Black & Scholes Price	Heston Price	Difference	Black & Scholes Price	Heston Price	Difference
3,190	3,210	86.082	86.082	-2,95E-12	81.509	81.509	0,00000
3,314	3,334	90.414	90.414	-1,05E-9	85.719	85.719	0,00000
3,228	3,248	87.482	87.482	-0,00001	83.078	83.078	0,00009
3,310	3,330	88.995	88.995	-0,00030	84.512	84.512	0,00031
3,184	3,204	90.018	90.018	-0,00150	85.909	85.909	0,00200
3,225	3,245	76.224	76.224	-0,00700	71.656	71.656	0,00370
3,211	3,231	78.696	78.695	-0,01900	75.258	75.258	0,00800
3,216	3,186	104.149	104.148	-0,02500	102.802	102.802	-0,00350
3,291	3,261	113.254	113.254	-0,01500	112.234	112.234	-0,02200
3,362	3,332	127.373	127.373	-0,00080	126.664	126.664	-0,02800
3,290	3,260	118.794	118.794	0,00520	117.770	117.770	-0,01600
3,303	3,273	120.523	120.523	0,00510	119.958	119.958	-0,00550
3,210	3,180	115.748	115.748	0,00320	114.924	114.924	-0,00020
3,297	3,267	118.628	118.628	0,00140	117.209	117.209	-0,00001

As explained in the Section 2.6.3, correlation is related with the skewness in the distribution of stock returns. A positive  $\rho$  value means that the volatility will increase as the spot price increases and this will spread the right tail and the distribution becomes negatively skewed. A negative  $\rho$  has opposite outcomes compared to positive correlation. Stock price distribution will become positively skewed now.

In Figure 5.1 correlation coefficient effect on the distribution can be observed clearly. When  $\rho = 0,5$ , volatility increases as the spot prices increase and this causes the distribution negatively skewed. However, when  $\rho = -0,5$ , volatility decreases as the spot prices increase. Hence, the stock price distribution is now positively skewed.



Figure 5.1. Differences between Black and Scholes Model and Heston Model as a Result of Distinct Correlation Coefficient Parameters.

## 5.2. Effects of Changing Volatility of Volatility Parameter $\sigma$

After observing the impact of the correlation coefficient, we can continue sensitivity analysis with another Heston model parameter, volatility of volatility  $\sigma$ . In order to see effect of changing volatility of volatility, correlation coefficient parameter will set to zero. Otherwise, skewness of the distribution will be affected. As in the previous section, two different  $\sigma$  values are chosen and for the other parameters calibration procedure will be held. After calibration the following results are obtained as shown by Table 5.3.

Table 5.3. Calibration Results when  $\sigma$  is fixed at 0,1 and 0,2.

Heston Model Parameters	$\sigma=0,1$	$\sigma=0,2$
Correlation coefficient	0	0
Mean Reversion $\kappa$	0,61452777	0,5282002
Long Term Level of Variance $\Theta$	0,00813633	0,0378644

Using the calibration results, European FX options are priced by the two models. Same spot and strike prices are used as in the sensitivity analysis for the correlation coefficient. Below table incorporates the detailed option prices.

Table 5.4. Option Prices When Volatility of Volatility Parameter is fixed at 0,1 and 0,2.

Spot Price	Strike Price	$\sigma=0,1$			$\sigma=0,2$		
		Black & Scholes Price	Heston Price	Difference	Black & Scholes Price	Heston Price	Difference
3,190	3,210	86.082	86.082	0,000003	81.509	81.509	0,000007
3,314	3,334	90.414	90.414	0,000003	85.719	85.719	0,000007
3,228	3,248	87.482	87.482	0,000002	83.078	83.078	0,000000
3,310	3,330	88.995	88.995	-0,000037	84.512	84.512	-0,000099
3,184	3,204	90.018	90.018	-0,000361	85.909	85.909	-0,000779
3,225	3,245	76.224	76.224	-0,002282	71.656	71.656	-0,004222
3,211	3,231	78.696	78.695	-0,008851	75.258	75.258	-0,014796
3,216	3,186	104.149	104.148	-0,021600	102.802	102.802	-0,034344
3,291	3,261	113.254	113.254	-0,034554	112.234	112.234	-0,053955
3,362	3,332	127.373	127.373	-0,037799	126.664	126.664	-0,058892
3,290	3,260	118.794	118.794	-0,029541	117.770	117.770	-0,046464
3,303	3,273	120.523	120.523	-0,017238	119.958	119.958	-0,027846
3,210	3,180	115.748	115.748	-0,007833	114.924	114.924	-0,013325
3,297	3,267	118.628	118.628	-0,002880	117.209	117.209	-0,005300

A higher volatility of volatility parameter increases the kurtosis of the stock return distribution, i.e., increases the convexity of the fit, while a lower volatility of volatility decreases the kurtosis.

By using two different volatility of volatility parameters, the differences between option values priced by both Black and Scholes and Heston Model are presented in Figure 5.2. When the volatility of volatility is 0,2, the kurtosis of the stock return distribution is higher compared to the value of 0,1.

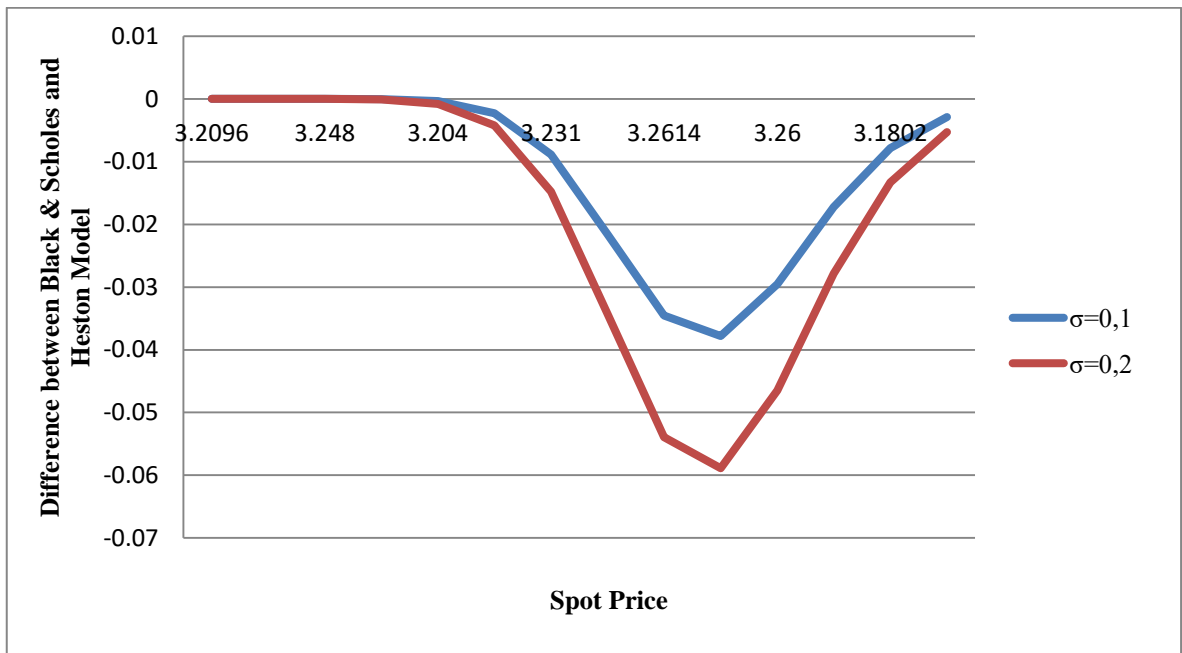


Figure 5.2. Differences between Black and Scholes Model and Heston Model as a Result of Distinct Volatility of Volatility Parameters.

Heston model gives the higher call option prices than Black and Scholes model at the cases of in the money and out of the money. Higher kurtosis results in thicker tails at the both ends of the distribution. Consequently, Heston model has ability to determine the change in the thickness whereas Black and Scholes does not detect the thicker tails.

In this section of the thesis, sensitivity analysis of the both Black and Scholes and Heston models is performed in order to observe the effect of the correlation coefficient and volatility of volatility parameters on the call option prices.

## 6. CONCLUSION

Fluctuation in exchange rates is a great threat and unavoidable risk for the investors who invest in overseas markets. However, currency options are the effective alternative so as to hedge the risk. Prices of the European type of the call and put FX options are tried to be determined by two different models in this thesis. Different from the literature, EUR/TL exchange rate parity is used for the calibration of the models' parameters.

In the thesis, two foreign currency option pricing models are conducted, Heston model and Black and Scholes model which is widely used in financial world, because of its simplicity and practicality. Even though Black and Scholes is useful for the practical approach, constant volatility assumption makes the model unrealistic. Therefore, stochastic models which rely on more realistic assumptions give close estimation results compared to deterministic model. Heston model assumes that volatility is not constant which reflects the real market behavior. Heston model parameters are calibrated by using the Maximum Likelihood Estimation method with different historical spot prices of EUR/TL. Before using the calibrated parameters, those are simulated in order to decide that whether or not they are logical estimations. Based on the Root Mean Square Error and the Average Relative Percentage Error analysis, the Heston model outstrips the performance of the Black and Scholes model. However, calibrating Heston parameters takes substantially more time compared to Black and Scholes model. Therefore, Black and Scholes model is more preferable in option pricing compared to Heston model although it has slightly higher error. In error analysis section, Heston model errors are smaller due to the consideration of the volatility. However, the root mean square and average relative percentage errors in the Heston model are still quite high. In order to obtain small errors, increasing the data set for the calibration is one of the well known alternatives.

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## APPENDIX A: MODEL DATA

### A.1. Real Market Option Prices

Table A.1. European Call and Put Option Prices for 2013.

Price Date	Expiration Date	Days to Maturity	Spot Price	Strike Price	Foreign Interest Rate	Domestic Interest Rate	European Call Option Price	European Put Option Price
02.01.13	02.04.13	90	2.354	2.406	0.11%	5.36%	4,938	25,500
15.01.13	15.04.13	90	2.359	2.409	0.10%	5.21%	5,800	25,300
01.02.13	02.05.13	90	2.384	2.435	0.05%	5.07%	5,250	26,850
15.02.13	16.05.13	90	2.354	2.402	0.06%	4.84%	5,280	25,250
04.03.13	02.06.13	90	2.340	2.399	0.05%	5.23%	4,560	32,333
19.03.13	17.06.13	90	2.344	2.406	0.05%	5.23%	4,000	34,590
03.04.13	02.07.13	90	2.309	2.388	0.07%	4.67%	1,270	52,500
18.04.13	17.07.13	90	2.342	2.401	0.07%	5.10%	3,750	33,516
03.05.13	01.08.13	90	2.344	2.412	0.08%	3.38%	1,350	49,800
17.05.13	15.08.13	90	2.357	2.425	0.09%	4.32%	2,000	44,450
03.06.13	01.09.13	90	2.460	2.535	0.13%	4.84%	2,400	47,600
18.06.13	16.09.13	90	2.465	2.512	0.13%	5.02%	11,200	28,283
03.07.13	01.10.13	90	2.502	2.590	0.15%	4.37%	3,000	63,199
18.07.13	16.10.13	90	2.517	2.602	0.17%	5.23%	5,000	57,526
02.08.13	31.10.13	90	2.572	2.635	0.15%	6.85%	14,130	33,351
19.08.13	17.11.13	90	2.602	2.688	0.12%	6.53%	7,400	50,420
03.09.13	02.12.13	90	2.685	2.752	0.16%	6.80%	16,150	38,101
18.09.13	17.12.13	90	2.684	2.750	0.14%	6.96%	16,100	35,712
03.10.13	01.01.14	90	2.747	2.821	0.17%	6.50%	13,000	43,200
21.10.13	19.01.14	90	2.663	2.746	0.08%	7.48%	10,700	43,398

Table A.2. European Call and Put Option Prices for 2014.

Price Date	Expiration Date	Days to Maturity	Spot Price	Strike Price	Foreign Interest Rate	Domestic Interest Rate	European Call Option Price	European Put Option Price
02.01.14	02.04.14	90	2.966	3.033	0.31%	7.22%	20,987	35,971
16.01.14	16.04.14	90	3.001	3.068	0.31%	7.85%	24,134	34,109
30.01.14	30.04.14	90	3.046	3.113	0.23%	9.66%	37,255	31,123
13.02.14	14.05.14	90	3.002	3.069	0.23%	11.53%	45,568	27,362
27.02.14	28.05.14	90	3.044	3.111	0.25%	11.56%	48,002	27,999
13.03.14	11.06.14	90	3.093	3.160	0.27%	12.07%	52,011	26,997
27.03.14	25.06.14	90	3.004	3.071	0.55%	12.30%	47,997	25,998
10.04.14	09.07.14	90	2.908	2.975	0.38%	10.06%	35,867	31,402
24.04.14	23.07.14	90	2.948	3.015	0.58%	10.07%	34,134	30,494
08.05.14	06.08.14	90	2.862	2.929	0.31%	10.50%	31,001	24,198
22.05.14	20.08.14	90	2.842	2.909	0.58%	9.58%	25,545	27,952
05.06.14	03.09.14	90	2.859	2.926	0.20%	9.96%	27,169	24,027
19.06.14	17.09.14	90	2.891	2.958	0.01%	8.98%	26,198	28,141
03.07.14	01.10.14	90	2.900	2.967	0.07%	8.76%	23,791	27,019
17.07.14	15.10.14	90	2.867	2.934	0.02%	8.98%	20,979	22,987
31.07.14	29.10.14	90	2.867	2.934	0.03%	7.28%	15,989	30,426
14.08.14	12.11.14	90	2.877	2.944	0.03%	8.68%	21,100	24,992
28.08.14	26.11.14	90	2.845	2.912	0.03%	8.05%	17,997	26,928
11.09.14	10.12.14	90	2.838	2.905	0.01%	7.87%	17,971	28,048
25.09.14	24.12.14	90	2.870	2.937	0.19%	8.54%	19,100	25,161

Table A.3. European Call and Put Option Prices for 2015.

Price Date	Expiration Date	Days to Maturity	Spot Price	Strike Price	Foreign Interest Rate	Domestic Interest Rate	European Call Option Price	European Put Option Price
05.01.15	05.04.15	90	2.777	2.927	0.10%	10.60%	4,536	78,600
19.01.15	19.04.15	90	2.707	2.857	0.09%	9.55%	3,890	87,009
02.02.15	03.05.15	90	2.751	2.901	0.04%	10.49%	6,310	81,472
16.02.15	17.05.15	90	2.787	2.937	0.13%	11.29%	7,961	77,147
02.03.15	31.05.15	90	2.809	2.959	0.13%	10.76%	7,549	79,901
16.03.15	14.06.15	90	2.777	2.927	0.09%	10.74%	7,089	80,251
30.03.15	28.06.15	90	2.819	2.969	0.18%	10.89%	4,817	76,460
13.04.15	12.07.15	90	2.798	2.948	0.14%	11.03%	4,413	75,488
27.04.15	26.07.15	90	2.921	3.071	0.06%	11.01%	5,549	72,125
11.05.15	09.08.15	90	2.987	3.137	0.14%	11.31%	6,582	70,509
25.05.15	23.08.15	90	2.863	3.013	0.14%	11.18%	7,070	74,817
08.06.15	06.09.15	90	3.076	3.226	0.13%	11.17%	14,109	76,371
22.06.15	20.09.15	90	3.029	3.179	0.14%	11.14%	13,258	77,931
06.07.15	04.10.15	90	2.961	3.111	0.14%	11.28%	12,784	77,787
20.07.15	18.10.15	90	2.919	3.069	0.15%	11.04%	12,264	79,472
03.08.15	01.11.15	90	3.043	3.193	0.12%	11.11%	14,521	78,530
17.08.15	15.11.15	90	3.160	3.310	0.11%	11.43%	17,314	74,993
31.08.15	29.11.15	90	3.264	3.414	0.16%	11.74%	24,123	77,992
14.09.15	13.12.15	90	3.454	3.604	0.15%	12.02%	24,801	70,000
28.09.15	27.12.15	90	3.405	3.555	0.28%	11.82%	22,488	71,984

Table A.4. European Call and Put Option Prices for 2016.

Price Date	Expiration Date	Days to Maturity	Spot Price	Strike Price	Foreign Interest Rate	Domestic Interest Rate	European Call Option Price	European Put Option Price
04.01.2016	03.04.2016	90	3.190	3.210	0.43%	11.34%	86,143	20,388
19.01.2016	18.04.2016	90	3.314	3.334	0.56%	10.98%	90,545	21,213
03.02.2016	03.05.2016	90	3.228	3.248	0.46%	12.31%	87,440	14,660
18.02.2016	18.05.2016	90	3.310	3.330	0.38%	11.76%	89,210	15,959
04.03.2016	02.06.2016	90	3.184	3.204	0.49%	12.45%	89,927	11,756
21.03.2016	19.06.2016	90	3.225	3.245	0.43%	11.43%	76,114	12,681
05.04.2016	04.07.2016	90	3.211	3.231	0.41%	11.67%	78,593	7,990
20.04.2016	19.07.2016	90	3.216	3.236	0.51%	10.21%	61,873	8,378
05.05.2016	03.08.2016	90	3.291	3.311	0.53%	10.56%	69,511	6,128
20.05.2016	18.08.2016	90	3.362	3.382	0.50%	12.01%	82,879	4,989
06.06.2016	04.09.2016	90	3.290	3.310	0.50%	11.39%	74,911	6,120
20.06.2016	18.09.2016	90	3.303	3.323	0.52%	11.37%	75,482	4,255
08.07.2016	06.10.2016	90	3.210	3.230	0.58%	11.38%	71,390	5,252
20.07.2016	18.10.2016	90	3.297	3.317	0.44%	11.58%	75,922	7,876
04.08.2016	02.11.2016	90	3.375	3.395	0.48%	11.92%	87,736	7,639
19.08.2016	17.11.2016	90	3.315	3.335	0.40%	11.19%	80,319	7,073
09.09.2016	08.12.2016	90	3.315	3.335	0.56%	12.65%	80,412	6,020

## A.2. Estimated Option Prices

Table A.5. European FX Option Prices for 2013 with Black and Scholes Model.

Date	Expiration date	Spot Price	Strike Price	Historic Volatility	Real Price		Black Sholes Price	
					Call Option	Put Option	Call Option	Put Option
02.01.2013	02.04.2013	2,354	2,406	0,027	4.938	25.500	4.912	25.534
15.01.2013	15.04.2013	2,359	2,409	0,028	5.800	25.300	5.791	25.340
01.02.2013	02.05.2013	2,384	2,435	0,028	5.250	26.850	5.232	26.792
15.02.2013	16.05.2013	2,354	2,402	0,027	5.280	25.250	5.268	25.359
04.03.2013	02.06.2013	2,340	2,399	0,031	4.560	32.333	4.535	32.381
19.03.2013	17.06.2013	2,344	2,406	0,031	4.000	34.590	4.073	34.580
03.04.2013	02.07.2013	2,309	2,388	0,031	1.270	52.500	1.316	52.486
18.04.2013	17.07.2013	2,342	2,401	0,029	3.750	33.516	3.770	33.451
03.05.2013	01.08.2013	2,344	2,412	0,030	1.350	49.800	1.328	49.885
17.05.2013	15.08.2013	2,357	2,425	0,030	2.000	44.450	2.066	44.455
03.06.2013	01.09.2013	2,460	2,535	0,032	2.400	47.600	2.474	47.591
18.06.2013	16.09.2013	2,465	2,512	0,037	11.200	28.283	11.137	28.274
03.07.2013	01.10.2013	2,502	2,590	0,041	3.000	63.199	3.081	63.220
18.07.2013	16.10.2013	2,517	2,602	0,044	5.000	57.526	5.083	57.541
02.08.2013	31.10.2013	2,572	2,635	0,044	14.130	33.351	14.127	33.351
19.08.2013	17.11.2013	2,602	2,688	0,044	7.400	50.420	7.470	50.429
03.09.2013	02.12.2013	2,685	2,752	0,048	16.150	38.101	16.156	38.098
18.09.2013	17.12.2013	2,684	2,750	0,046	16.100	35.712	16.161	35.704
03.10.2013	01.01.2014	2,747	2,821	0,046	13.000	43.200	13.014	43.282
21.10.2013	19.01.2014	2,663	2,746	0,044	10.700	43.398	10.753	43.412

Table A.6. European FX Option Prices for 2014 with Black and Scholes Model.

Date	Expiration date	Spot Price	Strike Price	Historic Volatility	Real Price		Black Sholes Price	
					Call Option	Put Option	Call Option	Put Option
02.01.2014	02.04.2014	2,966	3,033	0,045	20.987	35.971	19.815	34.812
16.01.2014	16.04.2014	3,001	3,068	0,046	24.134	34.109	23.191	32.934
30.01.2014	30.04.2014	3,046	3,113	0,055	37.255	31.123	36.346	30.764
13.02.2014	14.05.2014	3,002	3,069	0,059	45.568	27.362	44.977	26.571
27.02.2014	28.05.2014	3,044	3,111	0,060	48.002	27.999	47.154	27.404
13.03.2014	11.06.2014	3,093	3,160	0,061	52.011	26.997	51.258	26.346
27.03.2014	25.06.2014	3,004	3,071	0,060	47.997	25.998	47.752	25.878
10.04.2014	09.07.2014	2,908	2,975	0,057	35.867	31.402	35.043	30.883
24.04.2014	23.07.2014	2,948	3,015	0,054	34.134	30.494	33.630	29.929
08.05.2014	06.08.2014	2,862	2,929	0,046	31.001	24.198	29.899	23.279
22.05.2014	20.08.2014	2,842	2,909	0,046	25.545	27.952	24.632	26.939
05.06.2014	03.09.2014	2,859	2,926	0,044	27.169	24.027	26.879	23.358
19.06.2014	17.09.2014	2,891	2,958	0,046	26.198	28.141	25.578	27.007
03.07.2014	01.10.2014	2,900	2,967	0,042	23.791	27.019	22.512	25.705
17.07.2014	15.10.2014	2,867	2,934	0,036	20.979	22.987	19.403	21.400
31.07.2014	29.10.2014	2,867	2,934	0,037	15.989	30.426	14.646	28.994
14.08.2014	12.11.2014	2,877	2,944	0,038	21.100	24.992	19.612	23.659
28.08.2014	26.11.2014	2,845	2,912	0,037	17.997	26.928	16.666	25.897
11.09.2014	10.12.2014	2,838	2,905	0,038	17.971	28.048	16.587	27.065
25.09.2014	24.12.2014	2,870	2,937	0,037	19.100	25.161	18.036	24.296

Table A.7. European FX Option Prices for 2015 with Black and Scholes Model.

Date	Expiration date	Spot Price	Strike Price	Historic Volatility	Real Price		Black Sholes Price	
					Call Option	Put Option	Call Option	Put Option
05.01.2015	05.04.2015	2,777	2,927	0,051	4.536	78.600	5.683	79.827
19.01.2015	19.04.2015	2,707	2,857	0,054	3.890	87.009	4.879	88.096
02.02.2015	03.05.2015	2,751	2,901	0,058	6.310	81.472	7.753	82.969
16.02.2015	17.05.2015	2,787	2,937	0,060	7.961	77.147	9.718	78.891
02.03.2015	31.05.2015	2,809	2,959	0,059	7.549	79.901	9.201	81.537
16.03.2015	14.06.2015	2,777	2,927	0,059	7.089	80.251	8.723	81.809
30.03.2015	28.06.2015	2,819	2,969	0,051	4.817	76.460	6.128	77.650
13.04.2015	12.07.2015	2,798	2,948	0,049	4.413	75.488	5.636	76.471
27.04.2015	26.07.2015	2,921	3,071	0,048	5.549	72.125	6.514	73.570
11.05.2015	09.08.2015	2,987	3,137	0,049	6.582	70.509	7.918	71.530
25.05.2015	23.08.2015	2,863	3,013	0,055	7.070	74.817	8.536	76.489
08.06.2015	06.09.2015	3,076	3,226	0,065	14.109	76.371	16.594	78.799
22.06.2015	20.09.2015	3,029	3,179	0,065	13.258	77.931	15.815	79.577
06.07.2015	04.10.2015	2,961	3,111	0,066	12.784	77.787	15.302	79.854
20.07.2015	18.10.2015	2,919	3,069	0,067	12.264	79.472	14.524	82.041
03.08.2015	01.11.2015	3,043	3,193	0,068	14.521	78.530	17.342	80.762
17.08.2015	15.11.2015	3,160	3,310	0,067	17.314	74.993	20.040	77.650
31.08.2015	29.11.2015	3,264	3,414	0,077	24.123	77.992	28.245	80.792
14.09.2015	13.12.2015	3,454	3,604	0,068	24.801	70.000	27.853	72.522
28.09.2015	27.12.2015	3,405	3,555	0,068	22.488	71.984	25.974	74.820

Table A.8. European FX Option Prices for 2016 with Black and Scholes Model.

Date	Expiration date	Spot Price	Strike Price	Historic Volatility	Real Price		Black Sholes Price	
					Call Option	Put Option	Call Option	Put Option
04.01.2016	03.04.2016	3,190	3,210	0,073	86.143	20.388	86.511	20.259
19.01.2016	18.04.2016	3,314	3,334	0,074	90.545	21.213	87.924	22.271
03.02.2016	03.05.2016	3,228	3,248	0,065	87.440	14.660	87.415	14.611
18.02.2016	18.05.2016	3,310	3,330	0,066	89.210	15.959	89.190	15.943
04.03.2016	02.06.2016	3,184	3,204	0,062	89.927	11.756	89.889	11.727
21.03.2016	19.06.2016	3,225	3,245	0,056	76.114	12.681	75.992	12.625
05.04.2016	04.07.2016	3,211	3,231	0,050	78.593	7.990	78.532	7.971
20.04.2016	19.07.2016	3,216	3,236	0,043	61.873	8.378	61.871	8.321
05.05.2016	03.08.2016	3,291	3,311	0,041	69.511	6.128	69.429	6.109
20.05.2016	18.08.2016	3,362	3,382	0,043	82.879	4.989	82.865	4.978
06.06.2016	04.09.2016	3,290	3,310	0,043	74.911	6.120	74.897	6.093
20.06.2016	18.09.2016	3,303	3,323	0,039	75.482	4.255	75.447	4.221
08.07.2016	06.10.2016	3,210	3,230	0,041	71.390	5.252	71.377	5.219
20.07.2016	18.10.2016	3,297	3,317	0,047	75.922	7.876	75.887	7.848
04.08.2016	02.11.2016	3,375	3,395	0,050	87.736	7.639	87.805	7.607
19.08.2016	17.11.2016	3,315	3,335	0,047	80.319	7.073	80.207	7.044
09.09.2016	08.12.2016	3,315	3,335	0,045	80.412	6.020	80.404	6.003

Table A.9. European FX Option Prices (2013) with Heston Model Calibrated with 2012 Spot Prices.

Date	Expiration Date	Spot Price	Strike Price	Initial Variance	$\kappa$	$\theta$	$\sigma$	$\rho$	Real Price		Heston Price	
									Call Option	Put Option	Call Option	Put Option
02.01.2013	02.04.2013	2,354	2,406	0,0007	0,00156	0,01741	0,00000001	0,37699	4.938	25.500	4.940	25.562
15.01.2013	15.04.2013	2,359	2,409	0,0008	0,00156	0,01741	0,00000001	0,37699	5.800	25.300	5.808	25.358
01.02.2013	02.05.2013	2,384	2,435	0,0008	0,00156	0,01741	0,00000001	0,37699	5.250	26.850	5.260	26.819
15.02.2013	16.05.2013	2,354	2,402	0,0007	0,00156	0,01741	0,00000001	0,37699	5.280	25.250	5.290	25.382
04.03.2013	02.06.2013	2,340	2,399	0,0009	0,00156	0,01741	0,00000001	0,37699	4.560	32.333	4.562	32.407
19.03.2013	17.06.2013	2,344	2,406	0,0009	0,00156	0,01741	0,00000001	0,37699	4.000	34.590	4.100	34.607
03.04.2013	02.07.2013	2,309	2,388	0,0010	0,00156	0,01741	0,00000001	0,37699	1.270	52.500	1.308	52.478
18.04.2013	17.07.2013	2,342	2,401	0,0009	0,00156	0,01741	0,00000001	0,37699	3.750	33.516	3.809	33.490
03.05.2013	01.08.2013	2,344	2,412	0,0009	0,00156	0,01741	0,00000001	0,37699	1.350	49.800	1.338	49.896
17.05.2013	15.08.2013	2,357	2,425	0,0009	0,00156	0,01741	0,00000001	0,37699	2.000	44.450	2.091	44.481
03.06.2013	01.09.2013	2,460	2,535	0,0010	0,00156	0,01741	0,00000001	0,37699	2.400	47.600	2.485	47.601
18.06.2013	16.09.2013	2,465	2,512	0,0014	0,00156	0,01741	0,00000001	0,37699	11.200	28.283	11.147	28.284
03.07.2013	01.10.2013	2,502	2,590	0,0017	0,00156	0,01741	0,00000001	0,37699	3.000	63.199	3.062	63.201
18.07.2013	16.10.2013	2,517	2,602	0,0019	0,00156	0,01741	0,00000001	0,37699	5.000	57.526	5.070	57.528
02.08.2013	31.10.2013	2,572	2,635	0,0019	0,00156	0,01741	0,00000001	0,37699	14.130	33.351	14.134	33.358
19.08.2013	17.11.2013	2,602	2,688	0,0019	0,00156	0,01741	0,00000001	0,37699	7.400	50.420	7.463	50.423
03.09.2013	02.12.2013	2,685	2,752	0,0023	0,00156	0,01741	0,00000001	0,37699	16.150	38.101	16.161	38.103
18.09.2013	17.12.2013	2,684	2,750	0,0021	0,00156	0,01741	0,00000001	0,37699	16.100	35.712	16.168	35.711
03.10.2013	01.01.2014	2,747	2,821	0,0021	0,00156	0,01741	0,00000001	0,37699	13.000	43.200	13.015	43.284
21.10.2013	19.01.2014	2,663	2,746	0,0019	0,00156	0,01741	0,00000001	0,37699	10.700	43.398	10.753	43.412

Table A.10. European FX Option Prices (2014) with Heston Model Calibrated with 2013 Spot Prices.

Date	Expiration Date	Spot Price	Strike Price	Initial Variance	$\kappa$	$\theta$	$\sigma$	$\rho$	Real Price		Heston Price	
									Call Option	Put Option	Call Option	Put Option
02.01.2014	02.04.2014	2,966	3,033	0,0020	0,012999652	0,012488501	0,000605631	0,795302286	20.987	35.971	19.932	34.929
16.01.2014	16.04.2014	3,001	3,068	0,0022	0,012999652	0,012488501	0,000605631	0,795302286	24.134	34.109	23.303	33.046
30.01.2014	30.04.2014	3,046	3,113	0,0030	0,012999652	0,012488501	0,000605631	0,795302286	37.255	31.123	36.428	30.846
13.02.2014	14.05.2014	3,002	3,069	0,0034	0,012999652	0,012488501	0,000605631	0,795302286	45.568	27.362	45.044	26.637
27.02.2014	28.05.2014	3,044	3,111	0,0036	0,012999652	0,012488501	0,000605631	0,795302286	48.002	27.999	47.217	27.468
13.03.2014	11.06.2014	3,093	3,160	0,0037	0,012999652	0,012488501	0,000605631	0,795302286	52.011	26.997	51.318	26.406
27.03.2014	25.06.2014	3,004	3,071	0,0036	0,012999652	0,012488501	0,000605631	0,795302286	47.997	25.998	47.814	25.939
10.04.2014	09.07.2014	2,908	2,975	0,0032	0,012999652	0,012488501	0,000605631	0,795302286	35.867	31.402	35.118	30.958
24.04.2014	23.07.2014	2,948	3,015	0,0029	0,012999652	0,012488501	0,000605631	0,795302286	34.134	30.494	33.713	30.012
08.05.2014	06.08.2014	2,862	2,929	0,0022	0,012999652	0,012488501	0,000605631	0,795302286	31.001	24.198	29.998	23.378
22.05.2014	20.08.2014	2,842	2,909	0,0021	0,012999652	0,012488501	0,000605631	0,795302286	25.545	27.952	24.738	27.045
05.06.2014	03.09.2014	2,859	2,926	0,0019	0,012999652	0,012488501	0,000605631	0,795302286	27.169	24.027	26.987	23.466
19.06.2014	17.09.2014	2,891	2,958	0,0021	0,012999652	0,012488501	0,000605631	0,795302286	26.198	28.141	25.685	27.114
03.07.2014	01.10.2014	2,900	2,967	0,0017	0,012999652	0,012488501	0,000605631	0,795302286	23.791	27.019	22.634	25.828
17.07.2014	15.10.2014	2,867	2,934	0,0013	0,012999652	0,012488501	0,000605631	0,795302286	20.979	22.987	19.544	21.540
31.07.2014	29.10.2014	2,867	2,934	0,0013	0,012999652	0,012488501	0,000605631	0,795302286	15.989	30.426	14.789	29.137
14.08.2014	12.11.2014	2,877	2,944	0,0014	0,012999652	0,012488501	0,000605631	0,795302286	21.100	24.992	19.748	23.795
28.08.2014	26.11.2014	2,845	2,912	0,0014	0,012999652	0,012488501	0,000605631	0,795302286	17.997	26.928	16.805	26.037
11.09.2014	10.12.2014	2,838	2,905	0,0014	0,012999652	0,012488501	0,000605631	0,795302286	17.971	28.048	16.724	27.201
25.09.2014	24.12.2014	2,870	2,937	0,0013	0,012999652	0,012488501	0,000605631	0,795302286	19.100	25.161	18.176	24.436

Table A.11. European FX Option Prices (2014) with Heston Model Calibrated with 2012 and 2013 Spot Prices.

Date	Expiration Date	Spot Price	Strike Price	Initial Variance	$\kappa$	$\theta$	$\sigma$	$\rho$	Real Price		Heston Price	
									Call Option	Put Option	Call Option	Put Option
02.01.2014	02.04.2014	2,966	3,033	0,0020	0,108000000	0,001700000	0,000012945	0,010801000	20.987	35.971	19.932	34.929
16.01.2014	16.04.2014	3,001	3,068	0,0022	0,108000000	0,001700000	0,000012945	0,010801000	24.134	34.109	23.303	33.046
30.01.2014	30.04.2014	3,046	3,113	0,0030	0,108000000	0,001700000	0,000012945	0,010801000	37.255	31.123	36.428	30.846
13.02.2014	14.05.2014	3,002	3,069	0,0034	0,108000000	0,001700000	0,000012945	0,010801000	45.568	27.362	45.044	26.637
27.02.2014	28.05.2014	3,044	3,111	0,0036	0,108000000	0,001700000	0,000012945	0,010801000	48.002	27.999	47.217	27.468
13.03.2014	11.06.2014	3,093	3,160	0,0037	0,108000000	0,001700000	0,000012945	0,010801000	52.011	26.997	51.318	26.406
27.03.2014	25.06.2014	3,004	3,071	0,0036	0,108000000	0,001700000	0,000012945	0,010801000	47.997	25.998	47.814	25.939
10.04.2014	09.07.2014	2,908	2,975	0,0032	0,108000000	0,001700000	0,000012945	0,010801000	35.867	31.402	35.118	30.958
24.04.2014	23.07.2014	2,948	3,015	0,0029	0,108000000	0,001700000	0,000012945	0,010801000	34.134	30.494	33.713	30.012
08.05.2014	06.08.2014	2,862	2,929	0,0022	0,108000000	0,001700000	0,000012945	0,010801000	31.001	24.198	29.998	23.378
22.05.2014	20.08.2014	2,842	2,909	0,0021	0,108000000	0,001700000	0,000012945	0,010801000	25.545	27.952	24.738	27.045
05.06.2014	03.09.2014	2,859	2,926	0,0019	0,108000000	0,001700000	0,000012945	0,010801000	27.169	24.027	26.987	23.466
19.06.2014	17.09.2014	2,891	2,958	0,0021	0,108000000	0,001700000	0,000012945	0,010801000	26.198	28.141	25.685	27.114
03.07.2014	01.10.2014	2,900	2,967	0,0017	0,108000000	0,001700000	0,000012945	0,010801000	23.791	27.019	22.634	25.828
17.07.2014	15.10.2014	2,867	2,934	0,0013	0,108000000	0,001700000	0,000012945	0,010801000	20.979	22.987	19.544	21.540
31.07.2014	29.10.2014	2,867	2,934	0,0013	0,108000000	0,001700000	0,000012945	0,010801000	15.989	30.426	14.789	29.137
14.08.2014	12.11.2014	2,877	2,944	0,0014	0,108000000	0,001700000	0,000012945	0,010801000	21.100	24.992	19.748	23.795
28.08.2014	26.11.2014	2,845	2,912	0,0014	0,108000000	0,001700000	0,000012945	0,010801000	17.997	26.928	16.805	26.037
11.09.2014	10.12.2014	2,838	2,905	0,0014	0,108000000	0,001700000	0,000012945	0,010801000	17.971	28.048	16.724	27.201
25.09.2014	24.12.2014	2,870	2,937	0,0013	0,108000000	0,001700000	0,000012945	0,010801000	19.100	25.161	18.176	24.436

Table A.12. European FX Option Prices (2015) with Heston Model Calibrated with 2014 Spot Prices.

Date	Expiration Date	Spot Price	Strike Price	Initial Variance	$\kappa$	$\theta$	$\sigma$	$\rho$	Real Price		Heston Price	
									Call Option	Put Option	Call Option	Put Option
05.01.2015	05.04.2015	2,777	2,927	0,0026	1,452285256	0,000484257	0,000334203	0,001967059	4.536	78.600	4.572	78.717
19.01.2015	19.04.2015	2,707	2,857	0,0029	1,452285256	0,000484257	0,000334203	0,001967059	3.890	87.009	3.828	87.045
02.02.2015	03.05.2015	2,751	2,901	0,0034	1,452285256	0,000484257	0,000334203	0,001967059	6.310	81.472	6.291	81.507
16.02.2015	17.05.2015	2,787	2,937	0,0036	1,452285256	0,000484257	0,000334203	0,001967059	7.961	77.147	8.032	77.205
02.03.2015	31.05.2015	2,809	2,959	0,0035	1,452285256	0,000484257	0,000334203	0,001967059	7.549	79.901	7.558	79.894
16.03.2015	14.06.2015	2,777	2,927	0,0035	1,452285256	0,000484257	0,000334203	0,001967059	7.089	80.251	7.138	80.224
30.03.2015	28.06.2015	2,819	2,969	0,0026	1,452285256	0,000484257	0,000334203	0,001967059	4.817	76.460	4.974	76.496
13.04.2015	12.07.2015	2,798	2,948	0,0024	1,452285256	0,000484257	0,000334203	0,001967059	4.413	75.488	4.566	75.401
27.04.2015	26.07.2015	2,921	3,071	0,0023	1,452285256	0,000484257	0,000334203	0,001967059	5.549	72.125	5.363	72.419
11.05.2015	09.08.2015	2,987	3,137	0,0024	1,452285256	0,000484257	0,000334203	0,001967059	6.582	70.509	6.605	70.217
25.05.2015	23.08.2015	2,863	3,013	0,0030	1,452285256	0,000484257	0,000334203	0,001967059	7.070	74.817	7.052	75.005
08.06.2015	06.09.2015	3,076	3,226	0,0042	1,452285256	0,000484257	0,000334203	0,001967059	14.109	76.371	14.158	76.363
22.06.2015	20.09.2015	3,029	3,179	0,0043	1,452285256	0,000484257	0,000334203	0,001967059	13.258	77.931	13.440	77.202
06.07.2015	04.10.2015	2,961	3,111	0,0044	1,452285256	0,000484257	0,000334203	0,001967059	12.784	77.787	12.965	77.517
20.07.2015	18.10.2015	2,919	3,069	0,0045	1,452285256	0,000484257	0,000334203	0,001967059	12.264	79.472	12.233	79.750
03.08.2015	01.11.2015	3,043	3,193	0,0046	1,452285256	0,000484257	0,000334203	0,001967059	14.521	78.530	14.787	78.207
17.08.2015	15.11.2015	3,160	3,310	0,0045	1,452285256	0,000484257	0,000334203	0,001967059	17.314	74.993	17.295	74.905
31.08.2015	29.11.2015	3,264	3,414	0,0059	1,452285256	0,000484257	0,000334203	0,001967059	24.123	77.992	24.728	77.275
14.09.2015	13.12.2015	3,454	3,604	0,0046	1,452285256	0,000484257	0,000334203	0,001967059	24.801	70.000	24.607	69.276
28.09.2015	27.12.2015	3,405	3,555	0,0046	1,452285256	0,000484257	0,000334203	0,001967059	22.488	71.984	22.813	71.658

Table A.13. European FX Option Prices (2015) with Heston Model Calibrated with 2013 and 2014 Spot Prices.

Date	Expiration Date	Spot Price	Strike Price	Initial Variance	$\kappa$	$\theta$	$\sigma$	$\rho$	Real Price		Heston Price	
									Call Option	Put Option	Call Option	Put Option
05.01.2015	05.04.2015	2,777	2,927	0,0026	1,479318170	0,000412792	0,000342144	-0,313743939	4.536	78.600	4.512	78.657
19.01.2015	19.04.2015	2,707	2,857	0,0029	1,479318170	0,000412792	0,000342144	-0,313743939	3.890	87.009	3.776	86.993
02.02.2015	03.05.2015	2,751	2,901	0,0034	1,479318170	0,000412792	0,000342144	-0,313743939	6.310	81.472	6.225	81.442
16.02.2015	17.05.2015	2,787	2,937	0,0036	1,479318170	0,000412792	0,000342144	-0,313743939	7.961	77.147	7.959	77.133
02.03.2015	31.05.2015	2,809	2,959	0,0035	1,479318170	0,000412792	0,000342144	-0,313743939	7.549	79.901	7.487	79.823
16.03.2015	14.06.2015	2,777	2,927	0,0035	1,479318170	0,000412792	0,000342144	-0,313743939	7.089	80.251	7.069	80.155
30.03.2015	28.06.2015	2,819	2,969	0,0026	1,479318170	0,000412792	0,000342144	-0,313743939	4.817	76.460	4.911	76.433
13.04.2015	12.07.2015	2,798	2,948	0,0024	1,479318170	0,000412792	0,000342144	-0,313743939	4.413	75.488	4.505	75.340
27.04.2015	26.07.2015	2,921	3,071	0,0023	1,479318170	0,000412792	0,000342144	-0,313743939	5.549	72.125	5.293	72.349
11.05.2015	09.08.2015	2,987	3,137	0,0024	1,479318170	0,000412792	0,000342144	-0,313743939	6.582	70.509	6.528	70.140
25.05.2015	23.08.2015	2,863	3,013	0,0030	1,479318170	0,000412792	0,000342144	-0,313743939	7.070	74.817	6.979	74.933
08.06.2015	06.09.2015	3,076	3,226	0,0042	1,479318170	0,000412792	0,000342144	-0,313743939	14.109	76.371	14.065	76.269
22.06.2015	20.09.2015	3,029	3,179	0,0043	1,479318170	0,000412792	0,000342144	-0,313743939	13.258	77.931	13.349	77.111
06.07.2015	04.10.2015	2,961	3,111	0,0044	1,479318170	0,000412792	0,000342144	-0,313743939	12.784	77.787	12.877	77.429
20.07.2015	18.10.2015	2,919	3,069	0,0045	1,479318170	0,000412792	0,000342144	-0,313743939	12.264	79.472	12.148	79.665
03.08.2015	01.11.2015	3,043	3,193	0,0046	1,479318170	0,000412792	0,000342144	-0,313743939	14.521	78.530	14.694	78.114
17.08.2015	15.11.2015	3,160	3,310	0,0045	1,479318170	0,000412792	0,000342144	-0,313743939	17.314	74.993	17.194	74.804
31.08.2015	29.11.2015	3,264	3,414	0,0059	1,479318170	0,000412792	0,000342144	-0,313743939	24.123	77.992	24.616	77.162
14.09.2015	13.12.2015	3,454	3,604	0,0046	1,479318170	0,000412792	0,000342144	-0,313743939	24.801	70.000	24.488	69.157
28.09.2015	27.12.2015	3,405	3,555	0,0046	1,479318170	0,000412792	0,000342144	-0,313743939	22.488	71.984	22.697	71.543

Table A.14. European FX Option Prices (2015) with Heston Model Calibrated with 2012, 2013 and 2014 Spot Prices.

Date	Expiration Date	Spot Price	Strike Price	Initial Variance	$\kappa$	$\theta$	$\sigma$	$\rho$	Real Price		Heston Price	
									Call Option	Put Option	Call Option	Put Option
05.01.2015	05.04.2015	2,777	2,927	0,0026	1,514126317	0,000527414	0,000508276	-0,414139464	4.536	78.600	4.546	78.690
19.01.2015	19.04.2015	2,707	2,857	0,0029	1,514126317	0,000527414	0,000508276	-0,414139464	3.890	87.009	3.800	87.017
02.02.2015	03.05.2015	2,751	2,901	0,0034	1,514126317	0,000527414	0,000508276	-0,414139464	6.310	81.472	6.250	81.466
16.02.2015	17.05.2015	2,787	2,937	0,0036	1,514126317	0,000527414	0,000508276	-0,414139464	7.961	77.147	7.984	77.158
02.03.2015	31.05.2015	2,809	2,959	0,0035	1,514126317	0,000527414	0,000508276	-0,414139464	7.549	79.901	7.512	79.848
16.03.2015	14.06.2015	2,777	2,927	0,0035	1,514126317	0,000527414	0,000508276	-0,414139464	7.089	80.251	7.093	80.180
30.03.2015	28.06.2015	2,819	2,969	0,0026	1,514126317	0,000527414	0,000508276	-0,414139464	4.817	76.460	4.947	76.470
13.04.2015	12.07.2015	2,798	2,948	0,0024	1,514126317	0,000527414	0,000508276	-0,414139464	4.413	75.488	4.543	75.378
27.04.2015	26.07.2015	2,921	3,071	0,0023	1,514126317	0,000527414	0,000508276	-0,414139464	5.549	72.125	5.339	72.395
11.05.2015	09.08.2015	2,987	3,137	0,0024	1,514126317	0,000527414	0,000508276	-0,414139464	6.582	70.509	6.577	70.189
25.05.2015	23.08.2015	2,863	3,013	0,0030	1,514126317	0,000527414	0,000508276	-0,414139464	7.070	74.817	7.014	74.967
08.06.2015	06.09.2015	3,076	3,226	0,0042	1,514126317	0,000527414	0,000508276	-0,414139464	14.109	76.371	14.086	76.291
22.06.2015	20.09.2015	3,029	3,179	0,0043	1,514126317	0,000527414	0,000508276	-0,414139464	13.258	77.931	13.369	77.131
06.07.2015	04.10.2015	2,961	3,111	0,0044	1,514126317	0,000527414	0,000508276	-0,414139464	12.784	77.787	12.894	77.447
20.07.2015	18.10.2015	2,919	3,069	0,0045	1,514126317	0,000527414	0,000508276	-0,414139464	12.264	79.472	12.163	79.680
03.08.2015	01.11.2015	3,043	3,193	0,0046	1,514126317	0,000527414	0,000508276	-0,414139464	14.521	78.530	14.709	78.130
17.08.2015	15.11.2015	3,160	3,310	0,0045	1,514126317	0,000527414	0,000508276	-0,414139464	17.314	74.993	17.212	74.823
31.08.2015	29.11.2015	3,264	3,414	0,0059	1,514126317	0,000527414	0,000508276	-0,414139464	24.123	77.992	24.615	77.162
14.09.2015	13.12.2015	3,454	3,604	0,0046	1,514126317	0,000527414	0,000508276	-0,414139464	24.801	70.000	24.511	69.180
28.09.2015	27.12.2015	3,405	3,555	0,0046	1,514126317	0,000527414	0,000508276	-0,414139464	22.488	71.984	22.718	71.564

Table A.15. European FX Option Prices (2016) with Heston Model Calibrated with 2015 Spot Prices.

Date	Expiration Date	Spot Price	Strike Price	Initial Variance	$\kappa$	$\theta$	$\sigma$	$\rho$	Real Price		Heston Price	
									Call Option	Put Option	Call Option	Put Option
04.01.2016	03.04.2016	3,190	3,210	0,0053	0,006079277	0,017399749	0,000375020	0,011200030	86.143	20.388	86.544	20.292
19.01.2016	18.04.2016	3,314	3,334	0,0054	0,006079277	0,017399749	0,000375020	0,011200030	90.545	21.213	87.959	22.306
03.02.2016	03.05.2016	3,228	3,248	0,0042	0,006079277	0,017399749	0,000375020	0,011200030	87.440	14.660	87.453	14.649
18.02.2016	18.05.2016	3,310	3,330	0,0043	0,006079277	0,017399749	0,000375020	0,011200030	89.210	15.959	89.228	15.982
04.03.2016	02.06.2016	3,184	3,204	0,0039	0,006079277	0,017399749	0,000375020	0,011200030	89.927	11.756	89.926	11.764
21.03.2016	19.06.2016	3,225	3,245	0,0032	0,006079277	0,017399749	0,000375020	0,011200030	76.114	12.681	76.040	12.672
05.04.2016	04.07.2016	3,211	3,231	0,0025	0,006079277	0,017399749	0,000375020	0,011200030	78.593	7.990	78.580	8.019
20.04.2016	19.07.2016	3,216	3,236	0,0018	0,006079277	0,017399749	0,000375020	0,011200030	61.873	8.378	61.936	8.386
05.05.2016	03.08.2016	3,291	3,311	0,0017	0,006079277	0,017399749	0,000375020	0,011200030	69.511	6.128	69.488	6.169
20.05.2016	18.08.2016	3,362	3,382	0,0018	0,006079277	0,017399749	0,000375020	0,011200030	82.879	4.989	82.915	5.028
06.06.2016	04.09.2016	3,290	3,310	0,0019	0,006079277	0,017399749	0,000375020	0,011200030	74.911	6.120	74.951	6.148
20.06.2016	18.09.2016	3,303	3,323	0,0015	0,006079277	0,017399749	0,000375020	0,011200030	75.482	4.255	75.502	4.275
08.07.2016	06.10.2016	3,210	3,230	0,0017	0,006079277	0,017399749	0,000375020	0,011200030	71.390	5.252	71.432	5.274
20.07.2016	18.10.2016	3,297	3,317	0,0022	0,006079277	0,017399749	0,000375020	0,011200030	75.922	7.876	75.940	7.902
04.08.2016	02.11.2016	3,375	3,395	0,0025	0,006079277	0,017399749	0,000375020	0,011200030	87.736	7.639	87.852	7.654
19.08.2016	17.11.2016	3,315	3,335	0,0022	0,006079277	0,017399749	0,000375020	0,011200030	80.319	7.073	80.258	7.095
09.09.2016	08.12.2016	3,315	3,335	0,0020	0,006079277	0,017399749	0,000375020	0,011200030	80.412	6.020	80.455	6.054

Table A.16. European FX Option Prices (2016) with Heston Model Calibrated with 2014 and 2015 Spot Prices.

Date	Expiration Date	Spot Price	Strike Price	Initial Variance	$\kappa$	$\theta$	$\sigma$	$\rho$	Real Price		Heston Price	
									Call Option	Put Option	Call Option	Put Option
04.01.2016	03.04.2016	3,190	3,210	0,0053	0,005032987	0,092397415	0,000513313	0,672001269	86.143	20.388	86.697	20.446
19.01.2016	18.04.2016	3,314	3,334	0,0054	0,005032987	0,092397415	0,000513313	0,672001269	90.545	21.213	88.120	22.467
03.02.2016	03.05.2016	3,228	3,248	0,0042	0,005032987	0,092397415	0,000513313	0,672001269	87.440	14.660	87.612	14.808
18.02.2016	18.05.2016	3,310	3,330	0,0043	0,005032987	0,092397415	0,000513313	0,672001269	89.210	15.959	89.392	16.146
04.03.2016	02.06.2016	3,184	3,204	0,0039	0,005032987	0,092397415	0,000513313	0,672001269	89.927	11.756	90.077	11.915
21.03.2016	19.06.2016	3,225	3,245	0,0032	0,005032987	0,092397415	0,000513313	0,672001269	76.114	12.681	76.224	12.857
05.04.2016	04.07.2016	3,211	3,231	0,0025	0,005032987	0,092397415	0,000513313	0,672001269	78.593	7.990	78.757	8.196
20.04.2016	19.07.2016	3,216	3,236	0,0018	0,005032987	0,092397415	0,000513313	0,672001269	61.873	8.378	62.166	8.616
05.05.2016	03.08.2016	3,291	3,311	0,0017	0,005032987	0,092397415	0,000513313	0,672001269	69.511	6.128	69.699	6.380
20.05.2016	18.08.2016	3,362	3,382	0,0018	0,005032987	0,092397415	0,000513313	0,672001269	82.879	4.989	83.090	5.203
06.06.2016	04.09.2016	3,290	3,310	0,0019	0,005032987	0,092397415	0,000513313	0,672001269	74.911	6.120	75.145	6.341
20.06.2016	18.09.2016	3,303	3,323	0,0015	0,005032987	0,092397415	0,000513313	0,672001269	75.482	4.255	75.687	4.461
08.07.2016	06.10.2016	3,210	3,230	0,0017	0,005032987	0,092397415	0,000513313	0,672001269	71.390	5.252	71.623	5.466
20.07.2016	18.10.2016	3,297	3,317	0,0022	0,005032987	0,092397415	0,000513313	0,672001269	75.922	7.876	76.134	8.096
04.08.2016	02.11.2016	3,375	3,395	0,0025	0,005032987	0,092397415	0,000513313	0,672001269	87.736	7.639	88.024	7.826
19.08.2016	17.11.2016	3,315	3,335	0,0022	0,005032987	0,092397415	0,000513313	0,672001269	80.319	7.073	80.441	7.278
09.09.2016	08.12.2016	3,315	3,335	0,0020	0,005032987	0,092397415	0,000513313	0,672001269	80.412	6.020	80.635	6.234

Table A.17. European FX Option Prices (2016) with Heston Model Calibrated with 2013, 2014 and 2015 Spot Prices.

Date	Expiration Date	Spot Price	Strike Price	Initial Variance	$\kappa$	$\theta$	$\sigma$	$\rho$	Real Price		Heston Price	
									Call Option	Put Option	Call Option	Put Option
04.01.2016	03.04.2016	3,190	3,210	0,0053	0,000669283	0,097399943	0,000136097	0,051203129	86.143	20.388	86.263	20.401
19.01.2016	18.04.2016	3,314	3,334	0,0054	0,000669283	0,097399943	0,000136097	0,051203129	90.545	21.213	90.563	21.213
03.02.2016	03.05.2016	3,228	3,248	0,0042	0,000669283	0,097399943	0,000136097	0,051203129	87.440	14.660	87.444	14.640
18.02.2016	18.05.2016	3,310	3,330	0,0043	0,000669283	0,097399943	0,000136097	0,051203129	89.210	15.959	89.220	15.973
04.03.2016	02.06.2016	3,184	3,204	0,0039	0,000669283	0,097399943	0,000136097	0,051203129	89.927	11.756	89.917	11.755
21.03.2016	19.06.2016	3,225	3,245	0,0032	0,000669283	0,097399943	0,000136097	0,051203129	76.114	12.681	76.027	12.659
05.04.2016	04.07.2016	3,211	3,231	0,0025	0,000669283	0,097399943	0,000136097	0,051203129	78.593	7.990	78.565	8.004
20.04.2016	19.07.2016	3,216	3,236	0,0018	0,000669283	0,097399943	0,000136097	0,051203129	61.873	8.378	61.915	8.365
05.05.2016	03.08.2016	3,291	3,311	0,0017	0,000669283	0,097399943	0,000136097	0,051203129	69.511	6.128	69.470	6.150
20.05.2016	18.08.2016	3,362	3,382	0,0018	0,000669283	0,097399943	0,000136097	0,051203129	82.879	4.989	82.899	5.012
06.06.2016	04.09.2016	3,290	3,310	0,0019	0,000669283	0,097399943	0,000136097	0,051203129	74.911	6.120	74.934	6.130
20.06.2016	18.09.2016	3,303	3,323	0,0015	0,000669283	0,097399943	0,000136097	0,051203129	75.482	4.255	75.484	4.257
08.07.2016	06.10.2016	3,210	3,230	0,0017	0,000669283	0,097399943	0,000136097	0,051203129	71.390	5.252	71.414	5.256
20.07.2016	18.10.2016	3,297	3,317	0,0022	0,000669283	0,097399943	0,000136097	0,051203129	75.922	7.876	75.923	7.885
04.08.2016	02.11.2016	3,375	3,395	0,0025	0,000669283	0,097399943	0,000136097	0,051203129	87.736	7.639	87.838	7.640
19.08.2016	17.11.2016	3,315	3,335	0,0022	0,000669283	0,097399943	0,000136097	0,051203129	80.319	7.073	80.242	7.079
09.09.2016	08.12.2016	3,315	3,335	0,0020	0,000669283	0,097399943	0,000136097	0,051203129	80.412	6.020	80.439	6.038

Table A.18. European FX Option Prices (2016) with Heston Model Calibrated with 2012, 2013, 2014 and 2015 Spot Prices.

Date	Expiration Date	Spot Price	Strike Price	Initial Variance	$\kappa$	$\theta$	$\sigma$	$\rho$	Real Price		Heston Price	
									Call Option	Put Option	Call Option	Put Option
04.01.2016	03.04.2016	3,190	3,210	0,0053	0,000479513	0,087400010	0,000285063	0,051200154	86.143	20.388	86.253	20.390
19.01.2016	18.04.2016	3,314	3,334	0,0054	0,000479513	0,087400010	0,000285063	0,051200154	90.545	21.213	90.552	21.202
03.02.2016	03.05.2016	3,228	3,248	0,0042	0,000479513	0,087400010	0,000285063	0,051200154	87.440	14.660	87.433	14.629
18.02.2016	18.05.2016	3,310	3,330	0,0043	0,000479513	0,087400010	0,000285063	0,051200154	89.210	15.959	89.208	15.962
04.03.2016	02.06.2016	3,184	3,204	0,0039	0,000479513	0,087400010	0,000285063	0,051200154	89.927	11.756	89.907	11.744
21.03.2016	19.06.2016	3,225	3,245	0,0032	0,000479513	0,087400010	0,000285063	0,051200154	76.114	12.681	76.014	12.646
05.04.2016	04.07.2016	3,211	3,231	0,0025	0,000479513	0,087400010	0,000285063	0,051200154	78.593	7.990	78.553	7.992
20.04.2016	19.07.2016	3,216	3,236	0,0018	0,000479513	0,087400010	0,000285063	0,051200154	61.873	8.378	61.899	8.349
05.05.2016	03.08.2016	3,291	3,311	0,0017	0,000479513	0,087400010	0,000285063	0,051200154	69.511	6.128	69.455	6.135
20.05.2016	18.08.2016	3,362	3,382	0,0018	0,000479513	0,087400010	0,000285063	0,051200154	82.879	4.989	82.886	4.999
06.06.2016	04.09.2016	3,290	3,310	0,0019	0,000479513	0,087400010	0,000285063	0,051200154	74.911	6.120	74.920	6.117
20.06.2016	18.09.2016	3,303	3,323	0,0015	0,000479513	0,087400010	0,000285063	0,051200154	75.482	4.255	75.470	4.244
08.07.2016	06.10.2016	3,210	3,230	0,0017	0,000479513	0,087400010	0,000285063	0,051200154	71.390	5.252	71.400	5.243
20.07.2016	18.10.2016	3,297	3,317	0,0022	0,000479513	0,087400010	0,000285063	0,051200154	75.922	7.876	75.910	7.871
04.08.2016	02.11.2016	3,375	3,395	0,0025	0,000479513	0,087400010	0,000285063	0,051200154	87.736	7.639	87.826	7.628
19.08.2016	17.11.2016	3,315	3,335	0,0022	0,000479513	0,087400010	0,000285063	0,051200154	80.319	7.073	80.229	7.066
09.09.2016	08.12.2016	3,315	3,335	0,0020	0,000479513	0,087400010	0,000285063	0,051200154	80.412	6.020	80.426	6.025

### A.3. Error Analysis

Table A.19. RMSE of Black and Scholes Model and Heston Model for 2013 European FX Options.

Date	Expiration Date	Spot Price	Strike Price	Real Price		Black Scholes Price		Heston Price*	
				Call Option	Put Option	Call Option	Put Option	Call Option	Put Option
02.01.2013	02.04.2013	2,354	2,406	4.938	25.500	4.912	25.534	4.940	25.562
15.01.2013	15.04.2013	2,359	2,409	5.800	25.300	5.791	25.340	5.808	25.358
01.02.2013	02.05.2013	2,384	2,435	5.250	26.850	5.232	26.792	5.260	26.819
15.02.2013	16.05.2013	2,354	2,402	5.280	25.250	5.268	25.359	5.290	25.382
04.03.2013	02.06.2013	2,340	2,399	4.560	32.333	4.535	32.381	4.562	32.407
19.03.2013	17.06.2013	2,344	2,406	4.000	34.590	4.073	34.580	4.100	34.607
03.04.2013	02.07.2013	2,309	2,388	1.270	52.500	1.316	52.486	1.308	52.478
18.04.2013	17.07.2013	2,342	2,401	3.750	33.516	3.770	33.451	3.809	33.490
03.05.2013	01.08.2013	2,344	2,412	1.350	49.800	1.328	49.885	1.338	49.896
17.05.2013	15.08.2013	2,357	2,425	2.000	44.450	2.066	44.455	2.091	44.481
03.06.2013	01.09.2013	2,460	2,535	2.400	47.600	2.474	47.591	2.485	47.601
18.06.2013	16.09.2013	2,465	2,512	11.200	28.283	11.137	28.274	11.147	28.284
03.07.2013	01.10.2013	2,502	2,590	3.000	63.199	3.081	63.220	3.062	63.201
18.07.2013	16.10.2013	2,517	2,602	5.000	57.526	5.083	57.541	5.070	57.528
02.08.2013	31.10.2013	2,572	2,635	14.130	33.351	14.127	33.351	14.134	33.358
19.08.2013	17.11.2013	2,602	2,688	7.400	50.420	7.470	50.429	7.463	50.423
03.09.2013	02.12.2013	2,685	2,752	16.150	38.101	16.156	38.098	16.161	38.103
18.09.2013	17.12.2013	2,684	2,750	16.100	35.712	16.161	35.704	16.168	35.711
03.10.2013	01.01.2014	2,747	2,821	13.000	43.200	13.014	43.282	13.015	43.284
21.10.2013	19.01.2014	2,663	2,746	10.700	43.398	10.753	43.412	10.753	43.412
				<b>RMSE</b>		49,5	44,7	52,0	50,0

\*2012 spot prices are used for Heston model parameter calibration

Table A.20. RMSE of Black and Scholes Model and Heston Model for 2014 European FX Options.

Date	Expiration Date	Spot Price	Strike Price	Real Price		Black Scholes Price		Heston Price*	
				Call Option	Put Option	Call Option	Put Option	Call Option	Put Option
02.01.2014	02.04.2014	2,966	3,033	20.987	35.971	19.815	34.812	19.932	34.929
16.01.2014	16.04.2014	3,001	3,068	24.134	34.109	23.191	32.934	23.303	33.046
30.01.2014	30.04.2014	3,046	3,113	37.255	31.123	36.346	30.764	36.428	30.846
13.02.2014	14.05.2014	3,002	3,069	45.568	27.362	44.977	26.571	45.044	26.637
27.02.2014	28.05.2014	3,044	3,111	48.002	27.999	47.154	27.404	47.217	27.468
13.03.2014	11.06.2014	3,093	3,160	52.011	26.997	51.258	26.346	51.318	26.406
27.03.2014	25.06.2014	3,004	3,071	47.997	25.998	47.752	25.878	47.814	25.939
10.04.2014	09.07.2014	2,908	2,975	35.867	31.402	35.043	30.883	35.118	30.958
24.04.2014	23.07.2014	2,948	3,015	34.134	30.494	33.630	29.929	33.713	30.012
08.05.2014	06.08.2014	2,862	2,929	31.001	24.198	29.899	23.279	29.998	23.378
22.05.2014	20.08.2014	2,842	2,909	25.545	27.952	24.632	26.939	24.738	27.045
05.06.2014	03.09.2014	2,859	2,926	27.169	24.027	26.879	23.358	26.987	23.466
19.06.2014	17.09.2014	2,891	2,958	26.198	28.141	25.578	27.007	25.685	27.114
03.07.2014	01.10.2014	2,900	2,967	23.791	27.019	22.512	25.705	22.634	25.828
17.07.2014	15.10.2014	2,867	2,934	20.979	22.987	19.403	21.400	19.544	21.540
31.07.2014	29.10.2014	2,867	2,934	15.989	30.426	14.646	28.994	14.789	29.137
14.08.2014	12.11.2014	2,877	2,944	21.100	24.992	19.612	23.659	19.748	23.795
28.08.2014	26.11.2014	2,845	2,912	17.997	26.928	16.666	25.897	16.805	26.037
11.09.2014	10.12.2014	2,838	2,905	17.971	28.048	16.587	27.065	16.724	27.201
25.09.2014	24.12.2014	2,870	2,937	19.100	25.161	18.036	24.296	18.176	24.436
				<b>RMSE</b>		1.029,4	982,7	924,1	877,2

\*2013 spot prices are used for Heston model parameter calibration

Table A.21. RMSE of Black and Scholes Model and Heston Model for 2014 European FX Options.

Date	Expiration Date	Spot Price	Strike Price	Real Price		Black Scholes Price		Heston Price*	
				Call Option	Put Option	Call Option	Put Option	Call Option	Put Option
02.01.2014	02.04.2014	2,966	3,033	20.987	35.971	19.815	34.812	21.074	36.071
16.01.2014	16.04.2014	3,001	3,068	24.134	34.109	23.191	32.934	24.434	34.177
30.01.2014	30.04.2014	3,046	3,113	37.255	31.123	36.346	30.764	37.355	31.773
13.02.2014	14.05.2014	3,002	3,069	45.568	27.362	44.977	26.571	45.868	27.462
27.02.2014	28.05.2014	3,044	3,111	48.002	27.999	47.154	27.404	48.022	28.272
13.03.2014	11.06.2014	3,093	3,160	52.011	26.997	51.258	26.346	52.111	27.197
27.03.2014	25.06.2014	3,004	3,071	47.997	25.998	47.752	25.878	48.607	26.733
10.04.2014	09.07.2014	2,908	2,975	35.867	31.402	35.043	30.883	35.967	31.806
24.04.2014	23.07.2014	2,948	3,015	34.134	30.494	33.630	29.929	34.634	30.934
08.05.2014	06.08.2014	2,862	2,929	31.001	24.198	29.899	23.279	31.084	24.465
22.05.2014	20.08.2014	2,842	2,909	25.545	27.952	24.632	26.939	25.845	28.152
05.06.2014	03.09.2014	2,859	2,926	27.169	24.027	26.879	23.358	28.148	24.627
19.06.2014	17.09.2014	2,891	2,958	26.198	28.141	25.578	27.007	26.812	28.241
03.07.2014	01.10.2014	2,900	2,967	23.791	27.019	22.512	25.705	23.891	27.085
17.07.2014	15.10.2014	2,867	2,934	20.979	22.987	19.403	21.400	21.024	23.021
31.07.2014	29.10.2014	2,867	2,934	15.989	30.426	14.646	28.994	16.168	30.516
14.08.2014	12.11.2014	2,877	2,944	21.100	24.992	19.612	23.659	21.146	25.192
28.08.2014	26.11.2014	2,845	2,912	17.997	26.928	16.666	25.897	18.197	27.428
11.09.2014	10.12.2014	2,838	2,905	17.971	28.048	16.587	27.065	18.071	28.548
25.09.2014	24.12.2014	2,870	2,937	19.100	25.161	18.036	24.296	19.600	25.860
				<b>RMSE</b>		1.029,4	982,7	361,8	386,1

\*2012 & 2013 spot prices are used for Heston model parameter calibration

Table A.22. RMSE of Black and Scholes Model and Heston Model for 2015 European FX Options.

Date	Expiration Date	Spot Price	Strike Price	Real Price		Black Scholes Price		Heston Price*	
				Call Option	Put Option	Call Option	Put Option	Call Option	Put Option
05.01.2015	05.04.2015	2,777	2,927	4.536	78.600	5.683	79.827	4.572	78.717
19.01.2015	19.04.2015	2,707	2,857	3.890	87.009	4.879	88.096	3.828	87.045
02.02.2015	03.05.2015	2,751	2,901	6.310	81.472	7.753	82.969	6.291	81.507
16.02.2015	17.05.2015	2,787	2,937	7.961	77.147	9.718	78.891	8.032	77.205
02.03.2015	31.05.2015	2,809	2,959	7.549	79.901	9.201	81.537	7.558	79.894
16.03.2015	14.06.2015	2,777	2,927	7.089	80.251	8.723	81.809	7.138	80.224
30.03.2015	28.06.2015	2,819	2,969	4.817	76.460	6.128	77.650	4.974	76.496
13.04.2015	12.07.2015	2,798	2,948	4.413	75.488	5.636	76.471	4.566	75.401
27.04.2015	26.07.2015	2,921	3,071	5.549	72.125	6.514	73.570	5.363	72.419
11.05.2015	09.08.2015	2,987	3,137	6.582	70.509	7.918	71.530	6.605	70.217
25.05.2015	23.08.2015	2,863	3,013	7.070	74.817	8.536	76.489	7.052	75.005
08.06.2015	06.09.2015	3,076	3,226	14.109	76.371	16.594	78.799	14.158	76.363
22.06.2015	20.09.2015	3,029	3,179	13.258	77.931	15.815	79.577	13.440	77.202
06.07.2015	04.10.2015	2,961	3,111	12.784	77.787	15.302	79.854	12.965	77.517
20.07.2015	18.10.2015	2,919	3,069	12.264	79.472	14.524	82.041	12.233	79.750
03.08.2015	01.11.2015	3,043	3,193	14.521	78.530	17.342	80.762	14.787	78.207
17.08.2015	15.11.2015	3,160	3,310	17.314	74.993	20.040	77.650	17.295	74.905
31.08.2015	29.11.2015	3,264	3,414	24.123	77.992	28.245	80.792	24.728	77.275
14.09.2015	13.12.2015	3,454	3,604	24.801	70.000	27.853	72.522	24.607	69.276
28.09.2015	27.12.2015	3,405	3,555	22.488	71.984	25.974	74.820	22.813	71.658
				<b>RMSE</b>		2.224,1	1.938,4	193,1	329,7

\*2014 spot prices are used for Heston model parameter calibration

Table A.23. RMSE of Black and Scholes Model and Heston Model for 2015 European FX Options.

Date	Expiration Date	Spot Price	Strike Price	Real Price		Black Scholes Price		Heston Price*	
				Call Option	Put Option	Call Option	Put Option	Call Option	Put Option
05.01.2015	05.04.2015	2,777	2,927	4.536	78.600	5.683	79.827	4.512	78.657
19.01.2015	19.04.2015	2,707	2,857	3.890	87.009	4.879	88.096	3.776	86.993
02.02.2015	03.05.2015	2,751	2,901	6.310	81.472	7.753	82.969	6.225	81.442
16.02.2015	17.05.2015	2,787	2,937	7.961	77.147	9.718	78.891	7.959	77.133
02.03.2015	31.05.2015	2,809	2,959	7.549	79.901	9.201	81.537	7.487	79.823
16.03.2015	14.06.2015	2,777	2,927	7.089	80.251	8.723	81.809	7.069	80.155
30.03.2015	28.06.2015	2,819	2,969	4.817	76.460	6.128	77.650	4.911	76.433
13.04.2015	12.07.2015	2,798	2,948	4.413	75.488	5.636	76.471	4.505	75.340
27.04.2015	26.07.2015	2,921	3,071	5.549	72.125	6.514	73.570	5.293	72.349
11.05.2015	09.08.2015	2,987	3,137	6.582	70.509	7.918	71.530	6.528	70.140
25.05.2015	23.08.2015	2,863	3,013	7.070	74.817	8.536	76.489	6.979	74.933
08.06.2015	06.09.2015	3,076	3,226	14.109	76.371	16.594	78.799	14.065	76.269
22.06.2015	20.09.2015	3,029	3,179	13.258	77.931	15.815	79.577	13.349	77.111
06.07.2015	04.10.2015	2,961	3,111	12.784	77.787	15.302	79.854	12.877	77.429
20.07.2015	18.10.2015	2,919	3,069	12.264	79.472	14.524	82.041	12.148	79.665
03.08.2015	01.11.2015	3,043	3,193	14.521	78.530	17.342	80.762	14.694	78.114
17.08.2015	15.11.2015	3,160	3,310	17.314	74.993	20.040	77.650	17.194	74.804
31.08.2015	29.11.2015	3,264	3,414	24.123	77.992	28.245	80.792	24.616	77.162
14.09.2015	13.12.2015	3,454	3,604	24.801	70.000	27.853	72.522	24.488	69.157
28.09.2015	27.12.2015	3,405	3,555	22.488	71.984	25.974	74.820	22.697	71.543
				<b>RMSE</b>		2.224,1	1.938,4	170,2	380,2

\*2013 & 2014 spot prices are used for Heston model parameter calibration

Table A.24. RMSE of Black and Scholes Model and Heston Model for 2015 European FX Options.

Date	Expiration Date	Spot Price	Strike Price	Real Price		Black Scholes Price		Heston Price*	
				Call Option	Put Option	Call Option	Put Option	Call Option	Put Option
05.01.2015	05.04.2015	2,777	2,927	4.536	78.600	5.683	79.827	4.546	78.690
19.01.2015	19.04.2015	2,707	2,857	3.890	87.009	4.879	88.096	3.800	87.017
02.02.2015	03.05.2015	2,751	2,901	6.310	81.472	7.753	82.969	6.250	81.466
16.02.2015	17.05.2015	2,787	2,937	7.961	77.147	9.718	78.891	7.984	77.158
02.03.2015	31.05.2015	2,809	2,959	7.549	79.901	9.201	81.537	7.512	79.848
16.03.2015	14.06.2015	2,777	2,927	7.089	80.251	8.723	81.809	7.093	80.180
30.03.2015	28.06.2015	2,819	2,969	4.817	76.460	6.128	77.650	4.947	76.470
13.04.2015	12.07.2015	2,798	2,948	4.413	75.488	5.636	76.471	4.543	75.378
27.04.2015	26.07.2015	2,921	3,071	5.549	72.125	6.514	73.570	5.339	72.395
11.05.2015	09.08.2015	2,987	3,137	6.582	70.509	7.918	71.530	6.577	70.189
25.05.2015	23.08.2015	2,863	3,013	7.070	74.817	8.536	76.489	7.014	74.967
08.06.2015	06.09.2015	3,076	3,226	14.109	76.371	16.594	78.799	14.086	76.291
22.06.2015	20.09.2015	3,029	3,179	13.258	77.931	15.815	79.577	13.369	77.131
06.07.2015	04.10.2015	2,961	3,111	12.784	77.787	15.302	79.854	12.894	77.447
20.07.2015	18.10.2015	2,919	3,069	12.264	79.472	14.524	82.041	12.163	79.680
03.08.2015	01.11.2015	3,043	3,193	14.521	78.530	17.342	80.762	14.709	78.130
17.08.2015	15.11.2015	3,160	3,310	17.314	74.993	20.040	77.650	17.212	74.823
31.08.2015	29.11.2015	3,264	3,414	24.123	77.992	28.245	80.792	24.615	77.162
14.09.2015	13.12.2015	3,454	3,604	24.801	70.000	27.853	72.522	24.511	69.180
28.09.2015	27.12.2015	3,405	3,555	22.488	71.984	25.974	74.820	22.718	71.564
				<b>RMSE</b>		2.224,1	1.938,4	166,5	371,4

\*2012, 2013, 2014 spot prices are used for Heston model parameter calibration

Table A.25. RMSE of Black and Scholes Model and Heston Model for 2016 European FX Options.

Date	Expiration Date	Spot Price	Strike Price	Real Price		Black Scholes Price		Heston Price*	
				Call Option	Put Option	Call Option	Put Option	Call Option	Put Option
04.01.2016	03.04.2016	3,190	3,210	86.143	20.388	86.511	20.259	86.544	20.292
19.01.2016	18.04.2016	3,314	3,334	90.545	21.213	87.924	22.271	87.959	22.306
03.02.2016	03.05.2016	3,228	3,248	87.440	14.660	87.415	14.611	87.453	14.649
18.02.2016	18.05.2016	3,310	3,330	89.210	15.959	89.190	15.943	89.228	15.982
04.03.2016	02.06.2016	3,184	3,204	89.927	11.756	89.889	11.727	89.926	11.764
21.03.2016	19.06.2016	3,225	3,245	76.114	12.681	75.992	12.625	76.040	12.672
05.04.2016	04.07.2016	3,211	3,231	78.593	7.990	78.532	7.971	78.580	8.019
20.04.2016	19.07.2016	3,216	3,236	61.873	8.378	61.871	8.321	61.936	8.386
05.05.2016	03.08.2016	3,291	3,311	69.511	6.128	69.429	6.109	69.488	6.169
20.05.2016	18.08.2016	3,362	3,382	82.879	4.989	82.865	4.978	82.915	5.028
06.06.2016	04.09.2016	3,290	3,310	74.911	6.120	74.897	6.093	74.951	6.148
20.06.2016	18.09.2016	3,303	3,323	75.482	4.255	75.447	4.221	75.502	4.275
08.07.2016	06.10.2016	3,210	3,230	71.390	5.252	71.377	5.219	71.432	5.274
20.07.2016	18.10.2016	3,297	3,317	75.922	7.876	75.887	7.848	75.940	7.902
04.08.2016	02.11.2016	3,375	3,395	87.736	7.639	87.805	7.607	87.852	7.654
19.08.2016	17.11.2016	3,315	3,335	80.319	7.073	80.207	7.044	80.258	7.095
09.09.2016	08.12.2016	3,315	3,335	80.412	6.020	80.404	6.003	80.455	6.054
				<b>RMSE</b>		644,2	260,3	636,5	266,9

\*2015 spot prices are used for Heston model parameter calibration

Table A.26. RMSE of Black and Scholes Model and Heston Model for 2016 European FX Options.

Date	Expiration Date	Spot Price	Strike Price	Real Price		Black Scholes Price		Heston Price*	
				Call Option	Put Option	Call Option	Put Option	Call Option	Put Option
04.01.2016	03.04.2016	3,190	3,210	86.143	20.388	86.511	20.259	86.697	20.446
19.01.2016	18.04.2016	3,314	3,334	90.545	21.213	87.924	22.271	88.120	22.467
03.02.2016	03.05.2016	3,228	3,248	87.440	14.660	87.415	14.611	87.612	14.808
18.02.2016	18.05.2016	3,310	3,330	89.210	15.959	89.190	15.943	89.392	16.146
04.03.2016	02.06.2016	3,184	3,204	89.927	11.756	89.889	11.727	90.077	11.915
21.03.2016	19.06.2016	3,225	3,245	76.114	12.681	75.992	12.625	76.224	12.857
05.04.2016	04.07.2016	3,211	3,231	78.593	7.990	78.532	7.971	78.757	8.196
20.04.2016	19.07.2016	3,216	3,236	61.873	8.378	61.871	8.321	62.166	8.616
05.05.2016	03.08.2016	3,291	3,311	69.511	6.128	69.429	6.109	69.699	6.380
20.05.2016	18.08.2016	3,362	3,382	82.879	4.989	82.865	4.978	83.090	5.203
06.06.2016	04.09.2016	3,290	3,310	74.911	6.120	74.897	6.093	75.145	6.341
20.06.2016	18.09.2016	3,303	3,323	75.482	4.255	75.447	4.221	75.687	4.461
08.07.2016	06.10.2016	3,210	3,230	71.390	5.252	71.377	5.219	71.623	5.466
20.07.2016	18.10.2016	3,297	3,317	75.922	7.876	75.887	7.848	76.134	8.096
04.08.2016	02.11.2016	3,375	3,395	87.736	7.639	87.805	7.607	88.024	7.826
19.08.2016	17.11.2016	3,315	3,335	80.319	7.073	80.207	7.044	80.441	7.278
09.09.2016	08.12.2016	3,315	3,335	80.412	6.020	80.404	6.003	80.635	6.234
				<b>RMSE</b>		644,2	260,3	633,5	360,1

\*2014 & 2015 spot prices are used for Heston model parameter calibration

Table A.27. RMSE of Black and Scholes Model and Heston Model for 2016 European FX Options.

Date	Expiration Date	Spot Price	Strike Price	Real Price		Black Scholes Price		Heston Price*	
				Call Option	Put Option	Call Option	Put Option	Call Option	Put Option
04.01.2016	03.04.2016	3,190	3,210	86.143	20.388	86.511	20.259	86.263	20.401
19.01.2016	18.04.2016	3,314	3,334	90.545	21.213	87.924	22.271	90.563	21.213
03.02.2016	03.05.2016	3,228	3,248	87.440	14.660	87.415	14.611	87.444	14.640
18.02.2016	18.05.2016	3,310	3,330	89.210	15.959	89.190	15.943	89.220	15.973
04.03.2016	02.06.2016	3,184	3,204	89.927	11.756	89.889	11.727	89.917	11.755
21.03.2016	19.06.2016	3,225	3,245	76.114	12.681	75.992	12.625	76.027	12.659
05.04.2016	04.07.2016	3,211	3,231	78.593	7.990	78.532	7.971	78.565	8.004
20.04.2016	19.07.2016	3,216	3,236	61.873	8.378	61.871	8.321	61.915	8.365
05.05.2016	03.08.2016	3,291	3,311	69.511	6.128	69.429	6.109	69.470	6.150
20.05.2016	18.08.2016	3,362	3,382	82.879	4.989	82.865	4.978	82.899	5.012
06.06.2016	04.09.2016	3,290	3,310	74.911	6.120	74.897	6.093	74.934	6.130
20.06.2016	18.09.2016	3,303	3,323	75.482	4.255	75.447	4.221	75.484	4.257
08.07.2016	06.10.2016	3,210	3,230	71.390	5.252	71.377	5.219	71.414	5.256
20.07.2016	18.10.2016	3,297	3,317	75.922	7.876	75.887	7.848	75.923	7.885
04.08.2016	02.11.2016	3,375	3,395	87.736	7.639	87.805	7.607	87.838	7.640
19.08.2016	17.11.2016	3,315	3,335	80.319	7.073	80.207	7.044	80.242	7.079
09.09.2016	08.12.2016	3,315	3,335	80.412	6.020	80.404	6.003	80.439	6.038
				<b>RMSE</b>		644,2	260,3	51,7	13,7

\*2013, 2014, 2015 spot prices are used for Heston model parameter calibration

Table A.28. RMSE of Black and Scholes Model and Heston Model for 2016 European FX Options.

Date	Expiration Date	Spot Price	Strike Price	Real Price		Black Scholes Price		Heston Price*	
				Call Option	Put Option	Call Option	Put Option	Call Option	Put Option
04.01.2016	03.04.2016	3,190	3,210	86.143	20.388	86.511	20.259	86.253	20.390
19.01.2016	18.04.2016	3,314	3,334	90.545	21.213	87.924	22.271	90.552	21.202
03.02.2016	03.05.2016	3,228	3,248	87.440	14.660	87.415	14.611	87.433	14.629
18.02.2016	18.05.2016	3,310	3,330	89.210	15.959	89.190	15.943	89.208	15.962
04.03.2016	02.06.2016	3,184	3,204	89.927	11.756	89.889	11.727	89.907	11.744
21.03.2016	19.06.2016	3,225	3,245	76.114	12.681	75.992	12.625	76.014	12.646
05.04.2016	04.07.2016	3,211	3,231	78.593	7.990	78.532	7.971	78.553	7.992
20.04.2016	19.07.2016	3,216	3,236	61.873	8.378	61.871	8.321	61.899	8.349
05.05.2016	03.08.2016	3,291	3,311	69.511	6.128	69.429	6.109	69.455	6.135
20.05.2016	18.08.2016	3,362	3,382	82.879	4.989	82.865	4.978	82.886	4.999
06.06.2016	04.09.2016	3,290	3,310	74.911	6.120	74.897	6.093	74.920	6.117
20.06.2016	18.09.2016	3,303	3,323	75.482	4.255	75.447	4.221	75.470	4.244
08.07.2016	06.10.2016	3,210	3,230	71.390	5.252	71.377	5.219	71.400	5.243
20.07.2016	18.10.2016	3,297	3,317	75.922	7.876	75.887	7.848	75.910	7.871
04.08.2016	02.11.2016	3,375	3,395	87.736	7.639	87.805	7.607	87.826	7.628
19.08.2016	17.11.2016	3,315	3,335	80.319	7.073	80.207	7.044	80.229	7.066
09.09.2016	08.12.2016	3,315	3,335	80.412	6.020	80.404	6.003	80.426	6.025
<b>RMSE</b>						644,2	260,3	51,4	15,1

\*2012, 2013, 2014, 2015 spot prices are used for Heston model parameter calibration

Table A.29. ARPE of Black and Scholes Model and Heston Model for 2013 European FX Options.

Date	Expiration Date	Spot Price	Strike Price	Real Price		Black Scholes Price		Heston Price*	
				Call Option	Put Option	Call Option	Put Option	Call Option	Put Option
02.01.2013	02.04.2013	2,354	2,406	4.938	25.500	4.912	25.534	4.940	25.562
15.01.2013	15.04.2013	2,359	2,409	5.800	25.300	5.791	25.340	5.808	25.358
01.02.2013	02.05.2013	2,384	2,435	5.250	26.850	5.232	26.792	5.260	26.819
15.02.2013	16.05.2013	2,354	2,402	5.280	25.250	5.268	25.359	5.290	25.382
04.03.2013	02.06.2013	2,340	2,399	4.560	32.333	4.535	32.381	4.562	32.407
19.03.2013	17.06.2013	2,344	2,406	4.000	34.590	4.073	34.580	4.100	34.607
03.04.2013	02.07.2013	2,309	2,388	1.270	52.500	1.316	52.486	1.308	52.478
18.04.2013	17.07.2013	2,342	2,401	3.750	33.516	3.770	33.451	3.809	33.490
03.05.2013	01.08.2013	2,344	2,412	1.350	49.800	1.328	49.885	1.338	49.896
17.05.2013	15.08.2013	2,357	2,425	2.000	44.450	2.066	44.455	2.091	44.481
03.06.2013	01.09.2013	2,460	2,535	2.400	47.600	2.474	47.591	2.485	47.601
18.06.2013	16.09.2013	2,465	2,512	11.200	28.283	11.137	28.274	11.147	28.284
03.07.2013	01.10.2013	2,502	2,590	3.000	63.199	3.081	63.220	3.062	63.201
18.07.2013	16.10.2013	2,517	2,602	5.000	57.526	5.083	57.541	5.070	57.528
02.08.2013	31.10.2013	2,572	2,635	14.130	33.351	14.127	33.351	14.134	33.358
19.08.2013	17.11.2013	2,602	2,688	7.400	50.420	7.470	50.429	7.463	50.423
03.09.2013	02.12.2013	2,685	2,752	16.150	38.101	16.156	38.098	16.161	38.103
18.09.2013	17.12.2013	2,684	2,750	16.100	35.712	16.161	35.704	16.168	35.711
03.10.2013	01.01.2014	2,747	2,821	13.000	43.200	13.014	43.282	13.015	43.284
21.10.2013	19.01.2014	2,663	2,746	10.700	43.398	10.753	43.412	10.753	43.412
				<b>ARPE</b>		1,14%	0,09%	1,13%	0,10%

\*2012 spot prices are used for Heston model parameter calibration

Table A.30. ARPE of Black and Scholes Model and Heston Model for 2014 European FX Options.

Date	Expiration Date	Spot Price	Strike Price	Real Price		Black Scholes Price		Heston Price*	
				Call Option	Put Option	Call Option	Put Option	Call Option	Put Option
02.01.2014	02.04.2014	2,966	3,033	20.987	35.971	19.815	34.812	19.932	34.929
16.01.2014	16.04.2014	3,001	3,068	24.134	34.109	23.191	32.934	23.303	33.046
30.01.2014	30.04.2014	3,046	3,113	37.255	31.123	36.346	30.764	36.428	30.846
13.02.2014	14.05.2014	3,002	3,069	45.568	27.362	44.977	26.571	45.044	26.637
27.02.2014	28.05.2014	3,044	3,111	48.002	27.999	47.154	27.404	47.217	27.468
13.03.2014	11.06.2014	3,093	3,160	52.011	26.997	51.258	26.346	51.318	26.406
27.03.2014	25.06.2014	3,004	3,071	47.997	25.998	47.752	25.878	47.814	25.939
10.04.2014	09.07.2014	2,908	2,975	35.867	31.402	35.043	30.883	35.118	30.958
24.04.2014	23.07.2014	2,948	3,015	34.134	30.494	33.630	29.929	33.713	30.012
08.05.2014	06.08.2014	2,862	2,929	31.001	24.198	29.899	23.279	29.998	23.378
22.05.2014	20.08.2014	2,842	2,909	25.545	27.952	24.632	26.939	24.738	27.045
05.06.2014	03.09.2014	2,859	2,926	27.169	24.027	26.879	23.358	26.987	23.466
19.06.2014	17.09.2014	2,891	2,958	26.198	28.141	25.578	27.007	25.685	27.114
03.07.2014	01.10.2014	2,900	2,967	23.791	27.019	22.512	25.705	22.634	25.828
17.07.2014	15.10.2014	2,867	2,934	20.979	22.987	19.403	21.400	19.544	21.540
31.07.2014	29.10.2014	2,867	2,934	15.989	30.426	14.646	28.994	14.789	29.137
14.08.2014	12.11.2014	2,877	2,944	21.100	24.992	19.612	23.659	19.748	23.795
28.08.2014	26.11.2014	2,845	2,912	17.997	26.928	16.666	25.897	16.805	26.037
11.09.2014	10.12.2014	2,838	2,905	17.971	28.048	16.587	27.065	16.724	27.201
25.09.2014	24.12.2014	2,870	2,937	19.100	25.161	18.036	24.296	18.176	24.436
				<b>ARPE</b>		4,02%	3,30%	3,58%	2,92%

\*2013 spot prices are used for Heston model parameter calibration

Table A.31. ARPE of Black and Scholes Model and Heston Model for 2014 European FX Options.

Date	Expiration Date	Spot Price	Strike Price	Real Price		Black Scholes Price		Heston Price*	
				Call Option	Put Option	Call Option	Put Option	Call Option	Put Option
02.01.2014	02.04.2014	2,966	3,033	20.987	35.971	19.815	34.812	21.074	36.071
16.01.2014	16.04.2014	3,001	3,068	24.134	34.109	23.191	32.934	24.434	34.177
30.01.2014	30.04.2014	3,046	3,113	37.255	31.123	36.346	30.764	37.355	31.773
13.02.2014	14.05.2014	3,002	3,069	45.568	27.362	44.977	26.571	45.868	27.462
27.02.2014	28.05.2014	3,044	3,111	48.002	27.999	47.154	27.404	48.022	28.272
13.03.2014	11.06.2014	3,093	3,160	52.011	26.997	51.258	26.346	52.111	27.197
27.03.2014	25.06.2014	3,004	3,071	47.997	25.998	47.752	25.878	48.607	26.733
10.04.2014	09.07.2014	2,908	2,975	35.867	31.402	35.043	30.883	35.967	31.806
24.04.2014	23.07.2014	2,948	3,015	34.134	30.494	33.630	29.929	34.634	30.934
08.05.2014	06.08.2014	2,862	2,929	31.001	24.198	29.899	23.279	31.084	24.465
22.05.2014	20.08.2014	2,842	2,909	25.545	27.952	24.632	26.939	25.845	28.152
05.06.2014	03.09.2014	2,859	2,926	27.169	24.027	26.879	23.358	28.148	24.627
19.06.2014	17.09.2014	2,891	2,958	26.198	28.141	25.578	27.007	26.812	28.241
03.07.2014	01.10.2014	2,900	2,967	23.791	27.019	22.512	25.705	23.891	27.085
17.07.2014	15.10.2014	2,867	2,934	20.979	22.987	19.403	21.400	21.024	23.021
31.07.2014	29.10.2014	2,867	2,934	15.989	30.426	14.646	28.994	16.168	30.516
14.08.2014	12.11.2014	2,877	2,944	21.100	24.992	19.612	23.659	21.146	25.192
28.08.2014	26.11.2014	2,845	2,912	17.997	26.928	16.666	25.897	18.197	27.428
11.09.2014	10.12.2014	2,838	2,905	17.971	28.048	16.587	27.065	18.071	28.548
25.09.2014	24.12.2014	2,870	2,937	19.100	25.161	18.036	24.296	19.600	25.860
				<b>ARPE</b>		4,02%	3,30%	0,97%	1,14%

\*2012 & 2013 spot prices are used for Heston model parameter calibration

Table A.32. ARPE of Black and Scholes Model and Heston Model for 2015 European FX Options.

Date	Expiration Date	Spot Price	Strike Price	Real Price		Black Scholes Price		Heston Price*	
				Call Option	Put Option	Call Option	Put Option	Call Option	Put Option
05.01.2015	05.04.2015	2,777	2,927	4.536	78.600	5.683	79.827	4.572	78.717
19.01.2015	19.04.2015	2,707	2,857	3.890	87.009	4.879	88.096	3.828	87.045
02.02.2015	03.05.2015	2,751	2,901	6.310	81.472	7.753	82.969	6.291	81.507
16.02.2015	17.05.2015	2,787	2,937	7.961	77.147	9.718	78.891	8.032	77.205
02.03.2015	31.05.2015	2,809	2,959	7.549	79.901	9.201	81.537	7.558	79.894
16.03.2015	14.06.2015	2,777	2,927	7.089	80.251	8.723	81.809	7.138	80.224
30.03.2015	28.06.2015	2,819	2,969	4.817	76.460	6.128	77.650	4.974	76.496
13.04.2015	12.07.2015	2,798	2,948	4.413	75.488	5.636	76.471	4.566	75.401
27.04.2015	26.07.2015	2,921	3,071	5.549	72.125	6.514	73.570	5.363	72.419
11.05.2015	09.08.2015	2,987	3,137	6.582	70.509	7.918	71.530	6.605	70.217
25.05.2015	23.08.2015	2,863	3,013	7.070	74.817	8.536	76.489	7.052	75.005
08.06.2015	06.09.2015	3,076	3,226	14.109	76.371	16.594	78.799	14.158	76.363
22.06.2015	20.09.2015	3,029	3,179	13.258	77.931	15.815	79.577	13.440	77.202
06.07.2015	04.10.2015	2,961	3,111	12.784	77.787	15.302	79.854	12.965	77.517
20.07.2015	18.10.2015	2,919	3,069	12.264	79.472	14.524	82.041	12.233	79.750
03.08.2015	01.11.2015	3,043	3,193	14.521	78.530	17.342	80.762	14.787	78.207
17.08.2015	15.11.2015	3,160	3,310	17.314	74.993	20.040	77.650	17.295	74.905
31.08.2015	29.11.2015	3,264	3,414	24.123	77.992	28.245	80.792	24.728	77.275
14.09.2015	13.12.2015	3,454	3,604	24.801	70.000	27.853	72.522	24.607	69.276
28.09.2015	27.12.2015	3,405	3,555	22.488	71.984	25.974	74.820	22.813	71.658
				<b>ARPE</b>		20,45%	2,41%	1,26%	0,31%

\*2014 spot prices are used for Heston model parameter calibration

Table A.33. ARPE of Black and Scholes Model and Heston Model for 2015 European FX Options.

Date	Expiration Date	Spot Price	Strike Price	Real Price		Black Scholes Price		Heston Price*	
				Call Option	Put Option	Call Option	Put Option	Call Option	Put Option
05.01.2015	05.04.2015	2,777	2,927	4.536	78.600	5.683	79.827	4.512	78.657
19.01.2015	19.04.2015	2,707	2,857	3.890	87.009	4.879	88.096	3.776	86.993
02.02.2015	03.05.2015	2,751	2,901	6.310	81.472	7.753	82.969	6.225	81.442
16.02.2015	17.05.2015	2,787	2,937	7.961	77.147	9.718	78.891	7.959	77.133
02.03.2015	31.05.2015	2,809	2,959	7.549	79.901	9.201	81.537	7.487	79.823
16.03.2015	14.06.2015	2,777	2,927	7.089	80.251	8.723	81.809	7.069	80.155
30.03.2015	28.06.2015	2,819	2,969	4.817	76.460	6.128	77.650	4.911	76.433
13.04.2015	12.07.2015	2,798	2,948	4.413	75.488	5.636	76.471	4.505	75.340
27.04.2015	26.07.2015	2,921	3,071	5.549	72.125	6.514	73.570	5.293	72.349
11.05.2015	09.08.2015	2,987	3,137	6.582	70.509	7.918	71.530	6.528	70.140
25.05.2015	23.08.2015	2,863	3,013	7.070	74.817	8.536	76.489	6.979	74.933
08.06.2015	06.09.2015	3,076	3,226	14.109	76.371	16.594	78.799	14.065	76.269
22.06.2015	20.09.2015	3,029	3,179	13.258	77.931	15.815	79.577	13.349	77.111
06.07.2015	04.10.2015	2,961	3,111	12.784	77.787	15.302	79.854	12.877	77.429
20.07.2015	18.10.2015	2,919	3,069	12.264	79.472	14.524	82.041	12.148	79.665
03.08.2015	01.11.2015	3,043	3,193	14.521	78.530	17.342	80.762	14.694	78.114
17.08.2015	15.11.2015	3,160	3,310	17.314	74.993	20.040	77.650	17.194	74.804
31.08.2015	29.11.2015	3,264	3,414	24.123	77.992	28.245	80.792	24.616	77.162
14.09.2015	13.12.2015	3,454	3,604	24.801	70.000	27.853	72.522	24.488	69.157
28.09.2015	27.12.2015	3,405	3,555	22.488	71.984	25.974	74.820	22.697	71.543
				<b>ARPE</b>		20,45%	2,41%	1,27%	0,36%

\*2013 & 2014 spot prices are used for Heston model parameter calibration

Table A.34. ARPE of Black and Scholes Model and Heston Model for 2015 European FX Options.

Date	Expiration Date	Spot Price	Strike Price	Real Price		Black Scholes Price		Heston Price*	
				Call Option	Put Option	Call Option	Put Option	Call Option	Put Option
05.01.2015	05.04.2015	2,777	2,927	4.536	78.600	5.683	79.827	4.546	78.690
19.01.2015	19.04.2015	2,707	2,857	3.890	87.009	4.879	88.096	3.800	87.017
02.02.2015	03.05.2015	2,751	2,901	6.310	81.472	7.753	82.969	6.250	81.466
16.02.2015	17.05.2015	2,787	2,937	7.961	77.147	9.718	78.891	7.984	77.158
02.03.2015	31.05.2015	2,809	2,959	7.549	79.901	9.201	81.537	7.512	79.848
16.03.2015	14.06.2015	2,777	2,927	7.089	80.251	8.723	81.809	7.093	80.180
30.03.2015	28.06.2015	2,819	2,969	4.817	76.460	6.128	77.650	4.947	76.470
13.04.2015	12.07.2015	2,798	2,948	4.413	75.488	5.636	76.471	4.543	75.378
27.04.2015	26.07.2015	2,921	3,071	5.549	72.125	6.514	73.570	5.339	72.395
11.05.2015	09.08.2015	2,987	3,137	6.582	70.509	7.918	71.530	6.577	70.189
25.05.2015	23.08.2015	2,863	3,013	7.070	74.817	8.536	76.489	7.014	74.967
08.06.2015	06.09.2015	3,076	3,226	14.109	76.371	16.594	78.799	14.086	76.291
22.06.2015	20.09.2015	3,029	3,179	13.258	77.931	15.815	79.577	13.369	77.131
06.07.2015	04.10.2015	2,961	3,111	12.784	77.787	15.302	79.854	12.894	77.447
20.07.2015	18.10.2015	2,919	3,069	12.264	79.472	14.524	82.041	12.163	79.680
03.08.2015	01.11.2015	3,043	3,193	14.521	78.530	17.342	80.762	14.709	78.130
17.08.2015	15.11.2015	3,160	3,310	17.314	74.993	20.040	77.650	17.212	74.823
31.08.2015	29.11.2015	3,264	3,414	24.123	77.992	28.245	80.792	24.615	77.162
14.09.2015	13.12.2015	3,454	3,604	24.801	70.000	27.853	72.522	24.511	69.180
28.09.2015	27.12.2015	3,405	3,555	22.488	71.984	25.974	74.820	22.718	71.564
				<b>ARPE</b>		20,45%	2,41%	1,17%	0,34%

\*2012, 2013, 2014 spot prices are used for Heston model parameter calibration

Table A.35. ARPE of Black and Scholes Model and Heston Model for 2016 European FX Options.

Date	Expiration Date	Spot Price	Strike Price	Real Price		Black Scholes Price		Heston Price*	
				Call Option	Put Option	Call Option	Put Option	Call Option	Put Option
04.01.2016	03.04.2016	3,190	3,210	86.143	20.388	86.511	20.259	86.544	20.292
19.01.2016	18.04.2016	3,314	3,334	90.545	21.213	87.924	22.271	87.959	22.306
03.02.2016	03.05.2016	3,228	3,248	87.440	14.660	87.415	14.611	87.453	14.649
18.02.2016	18.05.2016	3,310	3,330	89.210	15.959	89.190	15.943	89.228	15.982
04.03.2016	02.06.2016	3,184	3,204	89.927	11.756	89.889	11.727	89.926	11.764
21.03.2016	19.06.2016	3,225	3,245	76.114	12.681	75.992	12.625	76.040	12.672
05.04.2016	04.07.2016	3,211	3,231	78.593	7.990	78.532	7.971	78.580	8.019
20.04.2016	19.07.2016	3,216	3,236	61.873	8.378	61.871	8.321	61.936	8.386
05.05.2016	03.08.2016	3,291	3,311	69.511	6.128	69.429	6.109	69.488	6.169
20.05.2016	18.08.2016	3,362	3,382	82.879	4.989	82.865	4.978	82.915	5.028
06.06.2016	04.09.2016	3,290	3,310	74.911	6.120	74.897	6.093	74.951	6.148
20.06.2016	18.09.2016	3,303	3,323	75.482	4.255	75.447	4.221	75.502	4.275
08.07.2016	06.10.2016	3,210	3,230	71.390	5.252	71.377	5.219	71.432	5.274
20.07.2016	18.10.2016	3,297	3,317	75.922	7.876	75.887	7.848	75.940	7.902
04.08.2016	02.11.2016	3,375	3,395	87.736	7.639	87.805	7.607	87.852	7.654
19.08.2016	17.11.2016	3,315	3,335	80.319	7.073	80.207	7.044	80.258	7.095
09.09.2016	08.12.2016	3,315	3,335	80.412	6.020	80.404	6.003	80.455	6.054
				<b>ARPE</b>		0,24%	0,68%	0,24%	0,63%

\*2015 spot prices are used for Heston model parameter calibration

Table A.36. ARPE of Black and Scholes Model and Heston Model for 2016 European FX Options.

Date	Expiration Date	Spot Price	Strike Price	Real Price		Black Scholes Price		Heston Price*	
				Call Option	Put Option	Call Option	Put Option	Call Option	Put Option
04.01.2016	03.04.2016	3,190	3,210	86.143	20.388	86.511	20.259	86.697	20.446
19.01.2016	18.04.2016	3,314	3,334	90.545	21.213	87.924	22.271	88.120	22.467
03.02.2016	03.05.2016	3,228	3,248	87.440	14.660	87.415	14.611	87.612	14.808
18.02.2016	18.05.2016	3,310	3,330	89.210	15.959	89.190	15.943	89.392	16.146
04.03.2016	02.06.2016	3,184	3,204	89.927	11.756	89.889	11.727	90.077	11.915
21.03.2016	19.06.2016	3,225	3,245	76.114	12.681	75.992	12.625	76.224	12.857
05.04.2016	04.07.2016	3,211	3,231	78.593	7.990	78.532	7.971	78.757	8.196
20.04.2016	19.07.2016	3,216	3,236	61.873	8.378	61.871	8.321	62.166	8.616
05.05.2016	03.08.2016	3,291	3,311	69.511	6.128	69.429	6.109	69.699	6.380
20.05.2016	18.08.2016	3,362	3,382	82.879	4.989	82.865	4.978	83.090	5.203
06.06.2016	04.09.2016	3,290	3,310	74.911	6.120	74.897	6.093	75.145	6.341
20.06.2016	18.09.2016	3,303	3,323	75.482	4.255	75.447	4.221	75.687	4.461
08.07.2016	06.10.2016	3,210	3,230	71.390	5.252	71.377	5.219	71.623	5.466
20.07.2016	18.10.2016	3,297	3,317	75.922	7.876	75.887	7.848	76.134	8.096
04.08.2016	02.11.2016	3,375	3,395	87.736	7.639	87.805	7.607	88.024	7.826
19.08.2016	17.11.2016	3,315	3,335	80.319	7.073	80.207	7.044	80.441	7.278
09.09.2016	08.12.2016	3,315	3,335	80.412	6.020	80.404	6.003	80.635	6.234
				<b>ARPE</b>		0,24%	0,68%	0,42%	2,89%

\*2014 & 2015 spot prices are used for Heston model parameter calibration

Table A.37. ARPE of Black and Scholes Model and Heston Model for 2016 European FX Options.

Date	Expiration Date	Spot Price	Strike Price	Real Price		Black Scholes Price		Heston Price*	
				Call Option	Put Option	Call Option	Put Option	Call Option	Put Option
04.01.2016	03.04.2016	3,190	3,210	86.143	20.388	86.511	20.259	86.263	20.401
19.01.2016	18.04.2016	3,314	3,334	90.545	21.213	87.924	22.271	90.563	21.213
03.02.2016	03.05.2016	3,228	3,248	87.440	14.660	87.415	14.611	87.444	14.640
18.02.2016	18.05.2016	3,310	3,330	89.210	15.959	89.190	15.943	89.220	15.973
04.03.2016	02.06.2016	3,184	3,204	89.927	11.756	89.889	11.727	89.917	11.755
21.03.2016	19.06.2016	3,225	3,245	76.114	12.681	75.992	12.625	76.027	12.659
05.04.2016	04.07.2016	3,211	3,231	78.593	7.990	78.532	7.971	78.565	8.004
20.04.2016	19.07.2016	3,216	3,236	61.873	8.378	61.871	8.321	61.915	8.365
05.05.2016	03.08.2016	3,291	3,311	69.511	6.128	69.429	6.109	69.470	6.150
20.05.2016	18.08.2016	3,362	3,382	82.879	4.989	82.865	4.978	82.899	5.012
06.06.2016	04.09.2016	3,290	3,310	74.911	6.120	74.897	6.093	74.934	6.130
20.06.2016	18.09.2016	3,303	3,323	75.482	4.255	75.447	4.221	75.484	4.257
08.07.2016	06.10.2016	3,210	3,230	71.390	5.252	71.377	5.219	71.414	5.256
20.07.2016	18.10.2016	3,297	3,317	75.922	7.876	75.887	7.848	75.923	7.885
04.08.2016	02.11.2016	3,375	3,395	87.736	7.639	87.805	7.607	87.838	7.640
19.08.2016	17.11.2016	3,315	3,335	80.319	7.073	80.207	7.044	80.242	7.079
09.09.2016	08.12.2016	3,315	3,335	80.412	6.020	80.404	6.003	80.439	6.038
				<b>ARPE</b>		0,24%	0,68%	0,05%	0,14%

\*2013, 2014, 2015 spot prices are used for Heston model parameter calibration

Table A.38. ARPE of Black and Scholes Model and Heston Model for 2016 European FX Options.

Date	Expiration Date	Spot Price	Strike Price	Real Price		Black Scholes Price		Heston Price*	
				Call Option	Put Option	Call Option	Put Option	Call Option	Put Option
04.01.2016	03.04.2016	3,190	3,210	86.143	20.388	86.511	20.259	86.253	20.390
19.01.2016	18.04.2016	3,314	3,334	90.545	21.213	87.924	22.271	90.552	21.202
03.02.2016	03.05.2016	3,228	3,248	87.440	14.660	87.415	14.611	87.433	14.629
18.02.2016	18.05.2016	3,310	3,330	89.210	15.959	89.190	15.943	89.208	15.962
04.03.2016	02.06.2016	3,184	3,204	89.927	11.756	89.889	11.727	89.907	11.744
21.03.2016	19.06.2016	3,225	3,245	76.114	12.681	75.992	12.625	76.014	12.646
05.04.2016	04.07.2016	3,211	3,231	78.593	7.990	78.532	7.971	78.553	7.992
20.04.2016	19.07.2016	3,216	3,236	61.873	8.378	61.871	8.321	61.899	8.349
05.05.2016	03.08.2016	3,291	3,311	69.511	6.128	69.429	6.109	69.455	6.135
20.05.2016	18.08.2016	3,362	3,382	82.879	4.989	82.865	4.978	82.886	4.999
06.06.2016	04.09.2016	3,290	3,310	74.911	6.120	74.897	6.093	74.920	6.117
20.06.2016	18.09.2016	3,303	3,323	75.482	4.255	75.447	4.221	75.470	4.244
08.07.2016	06.10.2016	3,210	3,230	71.390	5.252	71.377	5.219	71.400	5.243
20.07.2016	18.10.2016	3,297	3,317	75.922	7.876	75.887	7.848	75.910	7.871
04.08.2016	02.11.2016	3,375	3,395	87.736	7.639	87.805	7.607	87.826	7.628
19.08.2016	17.11.2016	3,315	3,335	80.319	7.073	80.207	7.044	80.229	7.066
09.09.2016	08.12.2016	3,315	3,335	80.412	6.020	80.404	6.003	80.426	6.025
				<b>ARPE</b>		0,24%	0,68%	0,05%	0,13%

\*2012, 2013, 2014, 2015 spot prices are used for Heston model parameter calibration

## APPENDIX B: R CODES

### B.1. R Codes for Heston Model Parameter Calibration

```

data <- read.table(file.choose(), header=F)

data <- sapply(data, gsub, pattern = ",", replacement= ".")

mydata <- as.numeric(data)

Qvec <- c(mydata)

data <- read.table(file.choose(), header=F)

data <- sapply(data, gsub, pattern = ",", replacement= ".")

mydata <- as.numeric(data)

vvec <- c(mydata)

N <- length(vvec)

library(alabama)

library(numDeriv)

for(i in 1:N){

fn <- function(theta) {sum(0.5*log(abs(2*pi*theta[3]^2*(1-theta[4]^2))))
+0.5*theta[3]^2*(1-theta[4]^2)*sum(theta[3]^2*(Qvec[i]-(rd-rf-
0.5*vvec[i]))^2/sqrt(vvec[i]^2-2*theta[4]^2*theta[3]*(Qvec[i]-(rd-rf-
0.5*vvec[i]))/sqrt(vvec[i])*(vvec[i]-theta[1]*(theta[2]-vvec[i]))/sqrt(vvec[i])+(vvec[i]-
theta[1]*(theta[2]-vvec[i]))^2/sqrt(vvec[i]^2))}

}

hin <- function(theta){

h <- rep(NA, 2)

```

```

h[1] <- theta[1]

h[2] <- theta[2]

h[3] <- theta[3]

h[4] <- theta[4]+1

h[5] <- 2*theta[1]*theta[2]-theta[3]^2

h

}

constrOptim.nl(c(*,*,*,*), fn, gr=NULL, hin=hin, hin.jac=NULL)

```

## B.2. R Codes for Model Simulation

```

N <- 10000

delta <- 1/365

S[1] <- S0

v[1] <- nu0

SumS <- 0

for(j in 1:N){

  X1 <- rnorm(T, 0, 1)

  X2 <- rnorm(T, 0, 1)

  W1 <- X1

  W2 <- rho*X1 + sqrt(1-rho^2)*X2

for( i in 2:T){

S[i] <- S[i-1] + (rd-rf-0.5*v[i-1])*S[i-1]*delta + sqrt(abs(v[i-1]))*S[i-1]*sqrt(delta)*W1[i]

```

```

v[i] <- v[i-1] + (kappa*(theta-v[i-1])-0.5*vol)*delta + vol*(1/sqrt(abs(v[i-
1]))) * sqrt(delta) * W2[i]
}

SumS <- SumS + S[T]

}

avS <- SumS/N

avS

```

### B.3. R Codes for the Estimation of Option Prices

```

callHestoncf <- function(S,X,tau,rd,rf,q,v0,vT,rho,k,sigma,
                        implVol = FALSE) {

  # S    = spot
  # X    = strike
  # tau  = time to mat
  # rd   = domestic interest rate
  # rf   = foreign interest rate
  # q    = dividend yield
  # v0   = initial variance
  # vT   = long run variance
  # rho  = correlation
  # k    = speed of mean reversion
  # sigma = vol of vol

```

```

# implVol = compute equivalent BSM volatility

# -- functions --

P1 <- function(om,S,X,tau,rd,rf,q,v0,vT,rho,k,sigma) {

  i <- 1i

  p <- Re(exp(-i*log(X)*om) *

    cfHeston(om-i,S,tau,rd,rf,q,v0,vT,rho,k,sigma) / (i * om * S * exp((rd-rf-q) *
tau)))

  return(p)

}

P2 <- function(om,S,X,tau,rd,rf,q,v0,vT,rho,k,sigma) {

  i <- 1i

  p <- Re(exp(-i*log(X)*om) *

    cfHeston(om ,S,tau,rd,rf,q,v0,vT,rho,k,sigma) / (i * om))

  return(p)

}

cfHeston <- function(om,S,tau,rd,rf,q,v0,vT,rho,k,sigma) {

  d <- sqrt((rho * sigma * 1i * om - k)^2 + sigma^2 * (1i * om + om ^ 2))

  g2 <- (k - rho * sigma * 1i * om - d) / (k - rho * sigma * 1i * om + d)

  cf1 <- 1i * om * (log(S) + (rd-rf- q) * tau)

  cf2 <- vT*k/(sigma^2)*((k - rho * sigma * 1i * om - d) *

    tau - 2 * log((1 - g2 * exp(-d * tau)) / (1 - g2)))

  cf3 <- v0 / sigma^2 * (k - rho * sigma * 1i * om - d) *

    (1 - exp(-d * tau)) / (1 - g2 * exp(-d * tau))

```

```

cf <- exp(cf1 + cf2 + cf3)

return(cf)

}

# -- pricing --

vP1 <- 0.5 + 1/pi * integrate(P1,lower = 0,upper = 200,
                             S,X,tau,rd,rf,q,v0,vT,rho,k,sigma)$value

vP2 <- 0.5 + 1/pi * integrate(P2,lower = 0,upper = 200,
                             S,X,tau,rd,rf,q,v0,vT,rho,k,sigma)$value

result <- exp(-q * tau)*S *exp(-rf*tau)*vP1 - exp(-rd * tau) * X * vP2

# -- implied BSM vol --

if (implVol) {

diffPrice <- function(vol,call,S,X,tau,rd,rf,q){

d1 <- (log(S/X)+(rd -rf- q + vol^2/2)*tau)/(vol*sqrt(tau))

d2 <- d1 - vol*sqrt(tau)

callBSM <- S * exp(-q * tau) * pnorm(d1) -

X * exp(-(rd-rf) * tau) * pnorm(d2)

return(call - callBSM)

}

impliedVol <- uniroot(diffPrice, interval = c(0,2),

                     call = result, S = S, X = X,

                     tau = tau, rd = rd,rf=rf, q = q)[[1]]

result <- list(callPrice = result,impliedVol = impliedVol)

```

```

}

return(result)

}

# -- pricing with Heston Model--

callHestoncf(S,X,tau,rd,rf,q,v0,vT,rho,k,sigma,

implVol = TRUE)

C <- callHestoncf(S,X,tau,rd,rf,q,v0,vT,rho,k,sigma,

                implVol = TRUE)

P <- C + X*exp(-rd*tau)-S * exp(-rf*tau)

# -- pricing with Black and Scholes Model--

vol <- sqrt(v0)

d1 <- (log(S/X) + (rd -rf- q + vol^2 / 2)*tau) / (vol*sqrt(tau))

d2 <- d1 - vol*sqrt(tau)

callBSM <- S * exp(-q * tau) * exp(-rf * tau )*pnorm(d1) -

        X * exp(-rd * tau) * pnorm(d2)

putBSM <- callBSM+X*exp(-rd*tau)-S*exp(-rf*tau)

```