TAX POLICY AND INFORMAL ECONOMY WITHIN THE CONTEXT OF PRODUCTIVE GOVERNMENT EXPENDITURES

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Thesis Abstract

Çağrı Savcı, ‘‘Tax Policy and Informal Economy within the Context of Productive Government Expenditures’’

Using a DGE model, the thesis is an attempt to illustrate the dynamics of informal sector in relation to changes in tax policy within the context of productive government expenditures. By the help of calibration exercise, optimal tax system is tried to be explored. Moreover, introducing aggregate uncertainty into the model along technology and tax policy shocks; an attempt has been made to assess the relative ability of the model in exhibiting real business cycle dynamics. The pattern of relationship between tax rates and informality as well as optimum tax burden are observed to be determined to a large extent by government externality level. However, significant welfare gains associated with decreases in income tax rates are still observed for both US and Germany. Secondly, solutions corresponding to Ramsey equilibrium for both economies point to greater efficiency gains associated with the use of taxes on labor compared to the use of taxes on capital income. Thirdly, simulations results suggest that the use of a two-sector model can help to generate higher levels of volatility through capturing the counter-cyclical nature of informal economy. Finally, analysis of elasticities with respect to technology shock shows that higher levels of government externality tend to reinforce income effect in labor supply decision of agents. Moreover, tax policy shocks especially under initially high income tax rates are observed to be better accommodated for higher government externality levels.

Tez Özeti

Çağrı Savcı, ‘‘Üretken Kamu Harcamaları Çerçevesinde Vergi Politikası ve Kayıt Dışı Ekonomi’’

Tezin amacı, dinamik bir genel denge modeli kullanılarak kayıt dışı ekonominin dinamiklerinin kamu harcamalarının üretken olabildiği koşullarda vergi politkalarıyla ilişkilendirilmesidir. Modelin kalibrasyonu üzerinden, optimum vergi sisteminin yapısı incelenmektedir. Ayrıca, modele vergi politikası ve sektörel teknoloji şokları yoluyla genel belirsizlik eklenerek reel iş çevrim dinamiklerinin analizine yer verilmektedir. Gerek vergi oranlarıyla kayıt dışı ekonominin büyüklüğünün ilişkisinin, gerekse de optimum vergi yükünün büyük ölçüde kamu harcamalarının üretimdeki dışsallık parametresine bağlı olduğu gözlenmektedir. Ancak, hem ABD hem de Almanya ekonomileri için gelir vergilerinin indirilmesiyle yadsınamayacak kadar ciddi ölçülerde refahta artış ve kayıt dışı ekonomide küçülme gerçekleştiği görülmektedir. Buna ek olarak, Ramsey probleminin çözümü her iki ekonomi için de ücretler üzerinden vergilendirmenin sermaye üzerinden vergilendirmeye oranla daha verimli olduğuna işaret etmektedir. Simulasyon sonuçları, çift-sektör modellerinin tek-sektör modellerine oranla reel iş çevrim dinamiklerini daha iyi yakalayabileceğini göstermektedir. Bunun dışında, elastikiyet analizleri kamu harcamalarının üretimdeki dışsallığının gelir etkisini güçlendirme eğiliminde olduğuna işaret etmektedir. Son olarak, kamu harcamalarının getirdiği dışsallığın vergi şoklarının olumsuz etkilerinin yumuşatılmasında rol oynadığı gözlenmektedir.

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CHAPTER 1

INTRODUCTION AND LITERATURE REVIEW

Recent estimates indicate that around 8.8 percent of GNP produced in US is produced by individuals or firms who evade taxes. As long as the West European OECD countries concerned, 18 percent of GNP on average produced in the region belongs to informal economy. The picture is even more demanding attention for developing countries with numbers of 41 percent in Latin America and 38 percent for European Transformation Countries on averages.[1] Hence, the question of what the determinants of informality are turns out to be a demanding one.

The investigation of this issue has been grounded in the literature upon three basic conceptualizations of informal sector. The first one puts an emphasis on divergent organizational efficiency of firms which in turn is reflected upon their sizes. The most prominent of formulations following this conceptualization is the model provided by Rauch(1991). In his model, firms differ in terms of their managerial ability while workers are homogeneous. There is an exogenously given formal sector wage rate from which some firms may decide to abstain and rather operate on a smaller scale. Within this setting, the accompanying result is size dualism which refers to a difference in size between largest informal sector firm and smallest formal sector firm. Later this approach got able to incorporate taxes and labor-market regulations in the setting. Fortin et al. (1997) demonstrated that given the facts that firms differ in managerial capacity and costs of taxes and regulations increase with size; there exists positive relationship between tax rates and informal sector size. For instance, an increase in government-set wage rate leads to adjustments at both intensive and extensive margins. At the intensive margin, formal firms employ less labor

[1] See Schneider(2002), (2007)

and at the extensive one recognizing higher profitability in informal sector, some formal sector firms with lower managerial ability shift to informality. Hence, first of the conceptualizations mentioned that is the one putting emphasis on size dualism finds evidence for a positive relationship between tax rates and informal sector size.

Second conceptualization is based on the observed wage differential between formal and informal sector workers of identical abilities. Hence, the emphasis in this approach is on wage dualism. This wage differential had been most frequently explained by labor-market segmentation.[2] The points made range from entry barriers or labor-market regulations such as minimum wage laws or presence of trade unions driving a wedge between formal and informal sector salaries. As a challenge, departing from all these different versions of non-competitive labor market assumptions; Amaral and Quintin(2006) developed a model which can still account for wage differential. They take both firms and workers as heterogeneous in their abilities. labor market is competitive in the sense that both workers with same ability earn the same at both informal and formal sectors. So, labor-market is not segmented in their model. Formal sector firms pay taxes unlike informal ones, and have better access to credit markets. Finally, cost of defaulting on debt is higher for formal firms than informal ones. Their two period model ends up with the fact that skilled labor is being employed in formal sector more intensively, which means in turn a wage differential between the two sectors. They have also acknowledged the fact that in response to sequences of non-increasing default costs and non-decreasing tax rates, informal sector grows in size. Thus, second conceptualization of informal sector that is the one putting emphasis on wage dualism yields some evidence in favour of a positive relationship between tax rates and informal sector size as well.

[2] See Ray(1997)

The third basic conceptualization of informal sector in literature is concerned with evasion dualism. The setups following this approach model informal sector either by taking it as composed of firms not paying social security contributions, taxes on profits or by excluding informal sector output from taxes at self-employer/consumer side in the models. Basing the whole definition of informality upon the fact of being exempt from taxes; among all the three, this approach relates the issue to tax rates in the most direct way. Theoretically speaking, evasion dualism notion is grounded upon microeconomic evidence to distortionary effects of taxes on households' decision of labor allocation across formal and informal sectors.[3] Hence the proposed requirement to reduce tax rates in order to lower the size of informality has been emphasized even more intensively by the ones following evasion dualism notion. Notable examples are models of Busato and Chiarini(2004) and Ihrig and Moe(2004). Both of the two models make a distinction between formal and informal sectors along two main dimensions. Firstly, formal sector is modelled as more capital intensive which is reasonable assumption once the limited access of informal firms to external funds is taken into consideration.[4] Secondly, both of them associate informality with being exempt from taxes as well as facing a certain probability of being caught. The two models derive the same result that informal sector responds to a decrease in tax rates by declining in size. The intuition is that lower tax rates imply higher opportunity cost of working in informal sector, as a result people shift their labor supply to formal sector facing lower tax rates.

To sum up, all three basic conceptualizations of informality end up with similar results with regard to the relationship between tax rates and informal

[3]Schneider and Enste(1998) point out that the bigger is the difference between total cost of labour in official economy and the after tax earnings(from work), the greater is the incentive to avoid this difference and work in the shadow economy.

[4]Using a survey data, Thomas(1992) finds evidence to around twelve times more capital intensity of formal firms than informal ones in Lima, Peru. In a similar study, Carstens(1995) concludes that financial intermediation is extremely limited for informal firms.

sector size. Notable examples are the papers of Fortin et. al(1997), Busato and Chiarini(2004), Ihrig and Moe(2004), Amaral and Quintin(2006) and Galiani and Weinschelbaum(2007) and Prado(2008). All their models though differing in terms of channels resulting in a positive relationship between tax rates and informal sector size, have some commonalities. The most important commonality among them is that neither one models government expenditures as productive. They introduce heterogeneity into their models either through differing managerial capacities of firms or through taking formal sector firms more capital intensive than informal ones. What is more, none of them except for Prado(2008) takes into account the utility associated with provision of public goods.

In models of Fortin et. al(1997) and Galiani and Weinschelbaum(2007), production function is identical for both formal and informal firms and depending on its managerial ability level a firm makes a choice between formal and informal sectors. In this setting, higher tax rates drives the threshold of managerial ability which determines the separating equilibrium to a higher level below which greater number of firms are remaining. Hence, they reach naturally the conclusion that higher tax rates lead to increase in informality. Prado(2008) makes models of this class more sensible through associating government expenditures with a positive level of utility to agents. However, depending on degree of profitability across two sectors, the size of informal sector is still being determined by firm side. As a result, he reaches the same conclusion but his public good in utility assumption is at least helpful in recognition of a possibility of lower welfare level in equilibrium with lower size of informality.

Amaral and Quintin(2006) conceptualize firm heterogeneity through an additional dimension in their model. In the model, they assume continuum of managerial ability as well as differential access of informal and formal firms to credit markets. The result is that informality increases in response to sequences of non-increasing default costs and non-decreasing tax rates. Non-increasing default costs imply a stricter incentive compatibility constraint for formal sector firms. In response to this, they may choose either to lower their size or to shift to informal sector. Thus incentive compatibility constraint and higher costs of defaulting on debt for formal firms can be viewed as the main dynamics behind the positive relationship between tax rates and informality in their model. This fact could be glimpsed in the following intuitive way. Incentive compatible contracts assumption means that debt contracts must be self-enforcing. Lower default costs would make the act of defaulting on their debt preferable for firms which remain below a certain efficiency level. Hence, banks would not be willing to give credits and unable to find credits some firms would find it more profitable to produce in informal sector.[5]

Finally, models of Busato and Chiarini(2004) and Ihrig and Moe(2004) exhibit a positive relation between informality and tax rates as well. In both models, distinguishing feature is that formal sector is conceptualized as more capital intensive. Formal sector employs both capital and labor, but is also subject to taxes; whereas informal sector is able to use only labor and can pay taxes only if caught. Within this setting, the key assumption is that tax revenues are used by government to produce non-productive services.[6] What is more, agents do not derive utility from government expenditures.[7] Finally, neither of two models includes labor-leisure choice. Agents allocate their time endowments only between formal and informal sector work. Within this setup, the fact that informality increases with tax rates turns out to be a very natural result for two reasons. Firstly, if agents are obliged to divide their time between two sectors and if they do not derive any utility from government services,

[5] Amaral and Quintin(2006) also normalize the default cost for informal firms to zero.

[6] See Ihrig and Moe(2004). Similar assumption is evident also in the model of Busato and Chiarini(2004)

[7] Busato and Chiarini(2004) associate informal sector work with higher disutility than formal work, driving an extra incentive to work in formal sector compared to the model of Ihrig and Moe(2004)

facing an increase in tax rates they would necessarily shift their labor supply to informal sector for a fixed level of probability of detection. Secondly, the assumption that government expenditures are non-productive for both sectors makes this shift even more ready to occur. This assumption means that agents can have no incentive to pay taxes other than deterrent effect of audits, as government does not contribute to formal sector production. Hence, agents would have no hesitation in shifting their labor supply to informal sector once they face higher tax rates.

The objective of the thesis is to relate that choice of agents between formal and informal sector employment to tax policy within the context of productive government expenditures. The critical role played by productive government expenditures assumption lies in the fact that agents face an extra trade-off in allocating their time across formal sector, self-employment as well as leisure. Thus, agents can use leisure as well as informal sector work as a substitute to their formal sector employment in times of high tax rates. Or they may still prefer to supply as much labor to formal sector as they were supplying before tax rate increase and benefit from increased level of public expenditures. Thus, these two extensions of labor-leisure choice and more importantly productive government expenditures turn the problem into a multi-faceted one involving more than one trade-off in itself. Previously, some but only a few incorporated in their setting that captures informal economy, the role of productive government expenditures. Penalosa and Turnovsky(2005) point out, government purchases of some final goods or some government services may provide the infrastructure for formal sector production.[8] Loayza(1996) makes a similar assumption to distinguish between formal and informal firms' divergent access to services provided by government. However, his model is not able to account

[8] In their model, Penalosa and Turnovsky(2005) derives the result that maximizing social welfare, the government should tax capital at higher rate if tax revenues are used for infrastructure compared to the case in which they are unproductive. This result holds only if formal sector is more capital-intensive.

for effect of productive government expenditures on informality as he takes government externality parameter the same across formal and informal sectors. What is more, previously the relationship between tax policy and informality have been approached only through a single tax instrument of flat income taxes. Hence, the thesis is unique in its attempt to formulate the relationship between tax policy and informality in a multi-faceted manner taking into account the contribution of public services to formal sector production and probing into the dynamics behind informality within its rich framework of a variety of tax instruments that range from consumption taxes to double-taxation of capital income.

Once implications of productive government expenditures to the relationship between tax policy and informal economy are explored, an attempt will be made to assess the relative ability of the model in exhibiting real business cycle dynamics. In this part, the path that will be followed consists of mainly two phases. Firstly, a comparison will be made between relative strengths of models with and without informal economies on the basis of some well-known problems in real business cycle literature. These problems include the facts that compared to actual data, in the classical RBC models; the facts that output fluctuates too little, that consumption fluctuates too little relative to output, on the other hand investment fluctuating too much relative to output and finally the fact that labor hours fluctuate too little. Secondly, based on impulse response function analyses; implications of existence of an informal economy to functioning of aggregate economy are investigated within the times of sectoral and tax policy shocks. To make it more explicit, the problematic of informal economy as an extra insurance mechanism to smooth consumption is taken on.

The thesis is composed of following components. In Chapter 2, making use of a simple static model; the relationship between optimal public expenditures and government externality level is attempted to be grasped analytically. In

Chapter 3, the model economy is presented in detail and competitive equilibrium conditions as well as solution to Ramsey Planner's Problem are derived. In Chapter 4 and 5, the model is calibrated for US, Germany and Turkey, thereby comparative statics between informality and tax rates are derived under different tax systems and corresponding to different levels of government externality. In addition to these, at the end of Chapter 5, once recursive equilibrium law of motion is obtained, implications of the model to real business cycle facts are investigated, hence dynamics between the two sectors at times of formal sector technology and tax policy shock are explored.

CHAPTER 2

SIMPLE STATIC MODEL

In a simple model abstracted from labor-leisure choice, assuming that a representative agent allocates his time endowment between formal and informal sector employment; an agent's optimization problem can be expressed as:

$$\max_{l_f, l_i \geq 0} [(1-\tau) l_f^{\Gamma} g^{\theta} + A l_i^{\alpha}]$$

$$s.t \quad l_f + l_i = T$$

in which τ is the tax rate, g is level of public expenditures, θ is government externality level, A is informal sector technology parameter while l_f, l_i and T are levels of formal sector employment, informal sector employment and total time endowments respectively.

The problem yields the following FOC:

$$(1-\tau)\Gamma l_f^{\Gamma-1} g^{\theta} - A\alpha(T - l_f)^{\alpha-1} = 0 \quad (1)$$

Proposition 1: For the informal sector employment to be minimized in consistency with optimization problem of agent, $\tau = \theta$ must be satisfied provided that inequalities $\Gamma + \theta \leq 1$ and $\alpha \leq 1$ hold. That is the tax rate which minimizes informal sector employment subject to (1) equals the level of government externality as long as increasing returns to scale is non-existent in both formal and informal sector production.

Proof of Proposition 1 is given in Appendix A.

It is clear that as there is no labor-leisure choice in the model, maximization of formal employment is equivalent to minimization of informal sector size in the economy.

Proposition 2: If a fixed level of capital stock is entered into formal and informal sector production functions in such a way that $y_f = l_f^{\Gamma} k^{1-\Gamma} g^{\theta}$and $y_i = A l_i^{\alpha} k^{\beta}$. Then despite the increasing returns to scale existent in formal sector production, Proposition 1 still holds as long as inequalities $\Gamma + \theta \leq 1$ and $\alpha \leq 1$ are met. Moreover, if these inequalities are met and capital stock is assumed to be completely independent of labor supply; then informal sector employment minimizing condition $\tau = \theta$ directly applies to the case in which increasing returns to scale exists at informal sector as well, that is $\alpha + \beta \geq 1$.

Proof of Proposition 2 is given in Appendix A.

Hence, if the level of capital stock in the economy is exogenous; then the claim is that it is not increasing returns to scale at all that matters for $\tau = \theta$ condition not necessarily holding but it is increasing returns to scale in l_f and g as well as increasing returns in l_i that matters.

Proposition 3: If level of capital stock introduced into formal and informal sector production functions is in turn endogenous, that is to some extent dependent on level of labor supply(i.e., as it is the case in dynamic models), then even if $\Gamma + \theta \leq 1$ and $\alpha \leq 1$ are met; informal sector employment minimizing condition $\tau = \theta$ does not necessarily hold.

Proof of Proposition 3 is given in Appendix A.

As a result, not only incorporation of labor-leisure choice but also the endogeneity of capital stock in the setting necessitate the use of a dynamic general equilibrium model while investigating the amount of optimal tax burden.

CHAPTER 3

SETUP OF THE MODEL

The economy is characterized by a representative agent, who is endowed with a fixed amount of time endowments T per period and an initial capital stock, k_0. The agent decides on next period capital stock, levels of labor in formal and informal sectors as well as leisure. The distinction between formal and informal sectors is based on evasion dualism as well as differential access to capital markets and externality of government expenditures. To make it more explicit, formal sector benefits from government externality, while incurring cost of paying taxes. Whereas informal sector has no access to government's contribution to production but benefits from a lower cost by evading taxes. In addition, due to its limited access to capital markets, informal sector is taken as more labor-intensive one. There is a single homogenous consumption good in economy in terms of which every factor is paid.

The Consumer-Worker-Investor

In each period t, agents choose next period's capital stock k_{t+1}, consumption c_t and allocate their time endowments T across formal and informal sector work amounts of which denoted by l_t^f and l_t^i respectively as well as leisure that can be expressed as $T - l_t^f - l_t^i$. Hence agents' problem can be expressed as follows:

$$\max_{c_t, k_t, l_t^f, l_t^i \geq 0} \sum_{t=1}^{\infty} \beta^{t-1} \left\{ \frac{\left[c_t^v (T - l_t^f - l_t^i)^{1-v} \right]^{1-\sigma} - 1}{1-\sigma} \right\}^{9}$$

$$s.t \quad (1+\tau_c)c_t + k_t - (1-\delta)k_{t-1} = y_t^f + y_t^i - \zeta(y_t^f) \tag{2}$$

[9] For speacial case of $\sigma = 1$, the utility function resolves into log-log form of $U(c_t, T - l_t^f - l_t^i) = \log(c_t) + \gamma \log(T - l_t^f - l_t^i)$ in which $\gamma = \frac{v}{1-v}$. This special case facilitates the operations while implementing method of undetermined coefficients to derive recursive equilibrium law of motion.

$$l_t^f + l_t^i \leq T \text{ and for a given } k_0$$

$$\text{where } y_t^f = w_t l_t^f + r_t k_{t-1} \text{ is formal sector income} \tag{3}$$

while w_t, r_t are denoting wage and interest rates respectively,

$$y_t^i = A(l_t^i)^{(1-\rho)} \; for \; 0 < \rho < 1 \tag{4}$$

finally, τ_c is consumption tax rate and $\zeta(y_t^f)$ is total taxes paid out of income.

Tax Function and Government

A functional form that is derived by Gouveia and Strauss(1994) based on equal sacrifice principle will be used. The functional form is useful by its means of encompassing a wide range of tax schedules belonging to different degree of progressivities. The tax schedule is expressed in the functional form:

$$\zeta(y_t^f) = b_1 \left\{ y_t^f - [(y_t^f)^{-b_2} + b_3]^{\frac{-1}{b_2}} \right\} \tag{5}$$

in which b_1, b_2, b_3 are parameters. Thus, the effective tax function is not dependent on total income, y_t but only on formal sector income, y_t^f.The rationale behind this assumption is based on existence of mainly two ways of evading taxes. The first one is underreporting of total income through concealing the income gained from informal sector employment. The second one is the formal deductions and allowances from reported tax burden the account of which is required to derive effective income tax function.[10] $\lim_{y_t^f \to \infty} \frac{\zeta(y_t^f)}{y_t^f} = \lim_{y_t^f \to \infty} \zeta'(y_t^f) = b_1$. Thus the limiting average and marginal income tax rate equals b_1.For $b_2 = 0, \zeta(y_t^f) = b_1$ so that the tax scheme is proportional whereas for $b_2 > 0$ it is progressive which is evident from the fact that;

[10] As Gouveia and Strauss(1994) remark, their estimate for effective income tax function excludes some category of income(production for self-consumption, unrealized capital gains etc.)

$$\frac{\zeta(y_t^f)}{y_t^f} = b_1 \left\{1 - [1 + b_3 (y_t^f)^{b_2}]^{\frac{-1}{b_2}}\right\} \tag{6}$$

$$\zeta'(y_t^f) = b_1 \left\{1 - [1 + b_3 (y_t^f)^{b_2}]^{\frac{-1}{b_2}-1}\right\} \tag{7}$$

that is, average and marginal tax functions becoming strictly increasing function of formal sector income y_t^f.In the functional form above, b_3 can be used to scale the tax rate to income.

Remark 1: If income is scaled by a fixed factor $\lambda \geq 0$, in order for tax schedule to be unaffected by this scaling, b_3 needs to be adjusted such that b_3 $(y_t^f)^{b_2} = \widetilde{b_3}(\lambda y_t^f)^{b_2}, that\ is\ \widetilde{b_3} = b_3 \lambda^{-b_2}$.

Following Hall and Rabushka's(1995) proposal, under proportional or flat tax scheme; double-taxation of capital income will not be allowed unlike the case in progressive tax schemes. That is provided that government budget is balanced in each period t, total tax revenues of government can be expressed in the following ways for flat and progressive tax schemes:

$$g_t = \begin{array}{c} \tau_c c_t + b_1 y_t^f \text{ for flat tax scheme} \\ \tau_c c_t + \tau_k r_t k_{t-1} + b_1 \left\{ y_t^f - [(y_t^f)^{-b_2} + b_3]^{\frac{-1}{b_2}} \right\} \text{for progressive tax schemes} \end{array} \tag{8}$$

in which y_t^f is expressed in (3)

Formal Firm

There is a single consumption good produced by formal firms as well as informal sector. Formal sector production function is differentiated from informal sector by its means of access to capital and government services externality which is evident from the production functions:

$$y_{t,j}^f = k_{t-1,j}^{\rho} (l_{t,j}^f)^{1-\rho} G_t^{\theta} \text{ for } 0 < \rho < 1,\ 0 \leq \theta < 1 \text{ and} \tag{9}$$

$$y^i_{t,j} = A(l^i_{t,j})^{(1-\rho)} \qquad (10)$$

in which $y^f_{t,j}$ and $y^i_{t,j}$ denote formal and informal sector production owned by agent j; $l^f_{t,j}$ and $l^i_{t,j}$ are his formal and informal sector labor supplies while $k_{t-1,j}$ is his capital stock at the beginning of period t. Here, G_t is aggregate flow of public services which are assumed to be non-excludable and non-rival. There exists constant returns to scale in privately provided inputs at formal sector and decreasing returns to scale at informal sector.

Remark 2: This differential production function specification is equivalent to a more general setup in which $y^f_t = k^\rho_{t-1}(l^f_t)^{1-\rho}G^\theta_t$ and $y^i_t = Ak^\varphi_{t-1}(l^f_t)^{1-\varphi}$. Following Lucas(1988), under the condition that $\varphi < \rho$, the smaller capital share can be equalized to zero without loss of any generality. Hence, the assumption of a more labor-intensive informal sector is sufficient condition for it to be modelled as being able to use only labor as an input. In other words, none of the results of the model would change if informal sector production were taken as $y^i_t = Ak^\varphi_t(l^f_t)^{1-\varphi}$ as long as $\varphi < \rho$ holds.

Government's contribution to formal sector production can be taken either as a flow or stock variable. In the former case, government spending instantaneously increase the production through the provision of public services such as expenditures on collective goods(i.e infrastructure). If it is taken as a flow, there is no need to include congestion into model and more importantly it provides analytical simplicity.[11] Hence, it is being taken as a flow in the model.

Both capital and labor markets are assumed to be perfectly competitive. As a result, each factor is paid its marginal product in formal sector and formal firm's problem is as follows:

[11] This analytical simplicity becomes explicit especially when aggregate uncertainty is introduced into the model. Derivation of recursive equilibrium law of motion is simpler in this case.

$$\max_{k_{t-1}, l_t^f \geq 0} \pi_t = k_{t-1}^{\rho}(l_t^f)^{1-\rho} G_t^{\theta} - w_t l_t^f - r_t k_{t-1} \text{ taking } G_t \text{ as given.}^{12}$$

Then, sourcing from competitivity assumption:

$$r_t = \rho(\frac{l_t^f}{k_{t-1}})^{1-\rho} G_t^{\theta} \tag{11}$$

$$w_t = (1-\rho)(\frac{k_{t-1}}{l_t^f})^{\rho} G_t^{\theta} \tag{12}$$

Remark 3: Following Barro(1990), the number of agents can be normalized to unity as long as the government services are taken as non-excludable and non-rival, that is the model is abstracted from the externalities associated with the use of public services. As a result of this normalization, G_t is equalized to g_t given in (8).

Competitive Equilibrium

Having described each of consumers, firms and government side of the problem, competitive equilibrium can be defined in the following way.

Definition 1: For given sequences of tax instruments $\{\tau_c^t, \tau_k^t, b_1^t, b_2^t\}_{t=1}^{\infty}$ and $\{g_t\}_{t=1}^{\infty}$ and for a given initial capital stock k_o, a competitive equilibrium for this economy is a sequence of allocations for households $\left\{c_t, k_t, l_t^f, l_t^i\right\}_{t=1}^{\infty}$ and prices $\{r_t, w_t\}_{t=1}^{\infty}$ such that;

- Given prices and rates of tax instruments, the sequences of allocations solve households' optimization problem.
- Formal firms maximize their profits and each factor is paid its marginal product, that is (11) and (12) hold.
- Government's budget is in balance in each period given values of tax instruments, that is (8) is satisfied.
- Aggregate resource constraint holds.

These competitive equilibrium conditions, in turn can be expressed in the following set of equations which hold for all $t \geq 1$:

$$c_t + k_t - (1-\delta)k_{t-1} + g_t = k_{t-1}^{\rho}(l_t^f)^{1-\rho}g_t^{\theta} + A(l_t^i)^{(1-\rho)}$$

$$g_t = \left\{ \begin{array}{c} \tau_c c_t + b_1 y_t^f \text{ for flat tax scheme} \\ \tau_c c_t + \tau_k r_t k_{t-1} + b_1 \left\{ y_t^f - [(y_t^f)^{-b_2} + b_3]^{\frac{-1}{b_2}} \right\} \text{for progressive tax scheme} \end{array} \right\}$$

$$r_t = \rho(\frac{l_t^f}{k_{t-1}})^{1-\rho}g_t^{\theta}$$

$$w_t = (1-\rho)(\frac{k_{t-1}}{l_t^f})^{\rho}g_t^{\theta}$$

$$\frac{U_{c_t}w_t[1-\zeta'(y_t^f)]}{(1+\tau_c)} = -U_{l_t^f} \tag{13}$$

$$w_t[1-\zeta'(y_t^f)] = (1-\rho)A(l_t^i)^{-\rho} \tag{14}$$

$$\beta E_t \left\{ U_{c_{t+1}}[(1-\tau_k-\zeta'(y_t^f))r_t + 1 - \delta] \right\} = U_{c_t} \tag{15}$$

$$\lim_{t\to\infty} \beta^t \mu_t k_t = 0 \tag{16}$$

in which $\zeta'(y_t^f)$ is given in (7). Note that exceptionally for flat tax system $\tau_k = 0$ and $\zeta'(y_t^f) = b_1$ above.

The first equation stands for aggregate resource constraint. Second equation is equivalent to balanced government budget under flat and progressive tax schemes respectively. Third and fourth equations follow from competitivity assumptions through which each factor is paid its marginal product. Fifth equation is intratemporal Euler equation and states that in equilibrium marginal utility associated with extra consumption sourcing from an increment of increase in labor supply must be equal to the negative of the disutility associated with this increment of increase in labor supply(i.e, equivalently marginal utility of leisure). Sixth equation stands for equalised rates of return to formal and informal sector work at equilibrium and can be interpreted as production efficiency condition. Seventh equation is intertemporal Euler equation that underlines the fact that marginal utility of consumption today and tomorrow must be equal in equilibrium. Finally, eighth equation is the transversality condition in which μ_t is mutliplier of household's budget constraint.

Ramsey Equilibrium

Definition 2: The solution to Ramsey Planner's problem is to choose the sequences $\left\{c_t, k_t, l_t^f, l_t^i\right\}_{t=1}^{\infty}$ and $\{g_t, \tau_t\}_{t=1}^{\infty}$ that maximize household utility subject to competitive equilibrium conditions. Ramsey Planner takes k_0, τ_c as given.[13] In the first version of the Ramsey Problem, a single income tax instrument of τ is presumed to be existent being charged on both of capital and formal sector labor income at the flat rate. Formally, the problem can be expressed in the following way:

$$\max_{c_t,k_t,l_t^f,l_t^i,\tau_t\geq 0} \sum_{t=1}^{\infty}\beta^{t-1}\left\{ \begin{array}{c} U(c_t, T - l_t^f - l_t^i) \\ +\lambda_t[y_t^i(l_t^i) + (1-\tau_t)y_t^f(k_t, l_t^f, g_t) - c_t - k_t - (1-\delta)k_{t-1}] \\ +\mu_t[(1-\tau_t)y^f_{l_t^f,t} - y^i_{l_t^i,t}] \\ +\phi_t[\beta(U_{c,t+1}(y^f_{k_t,t}(1-\tau_t) + 1 - \delta) - U_{c,t}] \\ +\chi_t[\frac{(1-\tau_t)y^f_{l_t^f,t}U_{c_t}}{(1+\tau_c)} + U_{l_t^f}] + \xi_t(T - l_t^f - l_t^i) \end{array} \right\}$$

In the second version of the problem, relative efficiency of using wage versus capital taxes is investigated. In that case, the Ramsey Planner has two tax instruments at hand which are τ_w and τ_k, wage and capital taxes respectively. The steady-state versions of the first order conditions to both versions of the problem are given in Appendix B.

[13] Following Domeij and Klein(2005), the consumption tax is left unchanged at its benchmark rate by the Ramsey Planner.

CHAPTER 4

CALIBRATION

The model is calibrated for US, Germany and Turkey. The reason for selecting Germany sources from two concerns. Firstly, the German economy is differentiated from US economy with its amount of labor hours remaining significantly below the level in US. This fact, in turn may cause important differences in real business cycles implications of the model. Furthermore, other macro variables being close across two economies, the comparison can be made without any problem. Secondly, tax system in Germany is more dependent on consumption taxes and labor taxes than US economy which may have implications to the relationship between tax rates and informality.

US and Germany

There are mainly 3 categories of parameters which are those corresponding to utility function or preferences, the ones associated with technologies, finally the parameters describing the tax system in economy.

Tax System

There are 5 parameters. These are parameters that represent consumption and capital tax rates denoted by τ_c and τ_k respectively and parameters that describe income tax scheme, specifically the limiting income tax rate denoted by b_1 and the one that control wide variety of progressivities, denoted by b_2, finally there is a parameter in income tax function, b_3 which needs to be adjusted for the income tax rates to be in line with units of measurement of income at hand.

Values of b_1 and b_2 are directly taken from Gouveia and Strauss'(1994) estimate for current income tax code in US. The corresponding values of the same parameters b_1 and b_2 for Germany have been taken from Lim and

Hyun(2004). τ_c is taken from Mendoza et. al.(1994) and Trabandt and Uhlig(2006). τ_k for each economy has been derived following the method of Mendoza et. al.(1994)[14] Finally, b_3 is adjusted so as to match equilibrium income tax rate for benchmark models to match average effective income tax rates in US and Germany.[15]

Technology

Depreciation rate, δ is calibrated together with discount factor, β to match $\frac{K}{Y}$, $\frac{C}{Y}$ and finally $\frac{I}{Y}$ that has to remain once $\frac{G}{Y}$ is taken into account for each economy. There remains 3 free parameters describing formal and informal sector technology. Here, ρ is taken to be equal to its classical value for both US and Germany. For government externality parameter θ there are wide range of estimates from 0.05 to as high as 0.3.[16] For US economy, subtracting from the public expenditures; the components interest on debt, other capital outlay, unallocable expenditures and insurance trust expenditures; the expenditure share equaling to 0.125 is obtained. Following a similar approach, this value has been observed to be a good fit for German economy as well after calculating the share of expenditures on collective goods and services. This simple manner of estimate is also in line with many other studies.[17] Furthermore, for the sake of sensitivity; in the figures and tables the values corresponding to levels $\theta = 0.10$ and $\theta = 0.15$ will be used as well. Finally, there is the informal sector technology parameter A which affects the equilibrium informality rate to a large extent. Hence, this parameter is calibrated to match informality rate in each economy.

[14] See Data Details section for calculation of effective tax rates on capital.Indeed Mendoza et.al.(1994) has also an estimate for capital tax rates but his estimate is based on total tax rate of capital income, that is $\tau_k + \frac{\zeta(y_t^f)}{y_t^f}$.The value taken for τ_k here, match his estimate closely.

[15] See Data Details for the calculation of the target of average effective income tax rates.

[16] Once government is modelled by a flow of expenditures rather than a stock of accumulated government capital; the estimates found are usually much higher.

[17] See Munnell and Cook(1990) and Aschauer(1989)

Preferences

Once β is calibrated jointly with δ, the only parameter remaining parameter is $\gamma = \frac{v}{1-v}$ as long as $\sigma = 1$, correspondingly utility function resolving into a log-log form. $\sigma = 1$ is set for analytical simplicity. γ, which is the coefficient of log of leisure in utility function is calibrated to match average working hours in formal sector.[18]

All of the 11 parameters and 5 targets are summarized in Table 1 and 2. Although, the parameters β, δ, γ, A and b_3 are jointly calibrated to match 5 targets; in Table 1 each parameter is put in front of the target it affects to the highest extent.

Table 1. Free Parameters and Targets, US and GER

Parameter	US Value	Target	GER Value	Target
γ	1.55	$\frac{L_F}{T} = 0.30$	2.00	$\frac{L_F}{T} = 0.23$
A	4.35	$\frac{Y_i}{Y} = 0.08$	5.25	$\frac{Y_i}{Y} = 0.14$
β	0.96	$\frac{K}{Y} = 2.7$	0.985	$\frac{K}{Y} = 3.1$
δ	0.05	$\frac{I}{Y} = 0.135$	0.06	$\frac{I}{Y} = 0.185$
b_3	0.023	$\frac{\varsigma(Y_F)}{Y_F} = 0.13$	0.017	$\frac{\varsigma(Y_F)}{Y_F} = 0.15$

Table 2. Other Parameters, US and GER

Parameter	US Value	GER Value
τ_c	0.052	0.15
τ_k	0.146	0.03
b_1	0.258	0.365
b_2	0.768	0.757
ρ	0.36	0.36
θ	0.125	0.125
T	100	100

[18] The value taken for γ implies share of consumption in the composite good, $v = 0.396$ for US and $v = 0.333$ for Germany which are close to values ranging from 0.3 to 0.4 used most frequently.

Turkey

Some changes have been made while calibrating the model for Turkish economy. The necessity for these changes stems from mainly two kinds of difficulties. First one is the lack of an estimate for an effective income tax function for Turkey. Secondly, some of the targets seem to be incompatible for some parameter values presumed. As a result, while calibrating the model for Turkey; income is presumed to be taxed at the flat rate and an additional parameter is incorporated into informal sector production function.[19]

The effective income tax rate for Turkey has been derived following the procedure used to obtain a target for average effective income tax rate.[20] The factor income shares of capital in formal and informal sectors have been derived from estimates of Saracoglu(2008). Except for these two differences, there is no difference between the procedure followed in calibration exercise for Turkey and US and Germany. All the parameters and targets are given in Table 3 and 4.

Table 3. Free Parameters and Targets, TUR

Parameter	Value	Target
γ	1.122	$\frac{L_f+L_i}{T}=0.33$
A	2.4	$\frac{Y_i}{Y}=0.25$
β	0.922	$\frac{K}{Y}=3$
δ	0.05	$\frac{I}{Y}=0.15$

Table 4. Other Parameters, TUR

Parameter	τ	τ_c	τ_k	ρ	α	θ	T
Value	0.065	0.15	0.04	0.6	0.34	0.125	30

[19]The problem mainly stems from the incompatibility between $\frac{K}{Y}$ target ratio and factor income share of capital, ρ. The reported value of $\frac{K}{Y}$ is 3 which is difficult to be matched with $\rho = 0.6$, the factor income share of capital taken usually for Turkish economy. To overcome that problem, informal sector production function has been presumed in the form: $y_i = Ak^{\alpha}(l_t^i)^{1-\alpha}$ As mentioned in Remark 1, there is no difference between using this form and the one, $y_i = A(l_t^i)^{1-\alpha}$ as long as $\alpha < \rho$,that is formal sector is more capital intensive.

[20]See Data Details for the calculation of the target of average effective income tax rates.

CHAPTER 5

RESULTS

Comparative Statics

In this part, the concern is to investigate the relationship between tax rates and informality at different government externality levels and under different tax systems. The part consists of mainly two attempts. The first one is to explore the relationship between income tax rates and informal sector size and welfare without taking on the difference between taxes on different sources of income. The second one, on the other hand, is to formulate the Ramsey Problem so as to capture relative efficiency of using wage vs. capital taxes to target informality and maximize welfare. The results suggest that the worst of the tax systems would be the one without consumption taxes provided that informal sector output is subject to some portion of indirect taxes at the market. Secondly, the results point to higher efficiency gains associated with the use of wage taxes compared to capital taxes.

Before moving onto analysis, several findings from previous studies with respect to optimal tax burden need to be shared. Barro(1990), Baxter and King(1994) and Loayza(1996) mention two opposing effects of raising tax rates on welfare. On the one hand, a higher tax rate reduces the rate of return to formal sector work for a fixed level of government expenditures. On the other hand, a higher tax rate increases the level of government expenditures for a fixed level of output. The combination of these two effects are in display to determine whether it is optimal for a government to increase tax rates at any initial state. This part enriches the framework to demonstrate how the combination of these two effects force the optimal tax burden to be around government externality parameter.

The relationship between income tax rates and informality have been investigated under three basic tax systems in addition to the benchmark tax system. In the first one of these, τ_k has been set to zero, thereby double-taxation of capital income is removed while progressivity parameter b_2 is assumed to be the same with benchmark model. In the second one, τ_k has been set to zero again while both capital and formal sector labor income were assumed to be taxed at the flat rate, thus this system is the one proposed by Hall and Rabushka(1995). In the final case, only consumption tax has been eliminated, thereby both τ_k and b_2 have been preserved at their benchmark level.

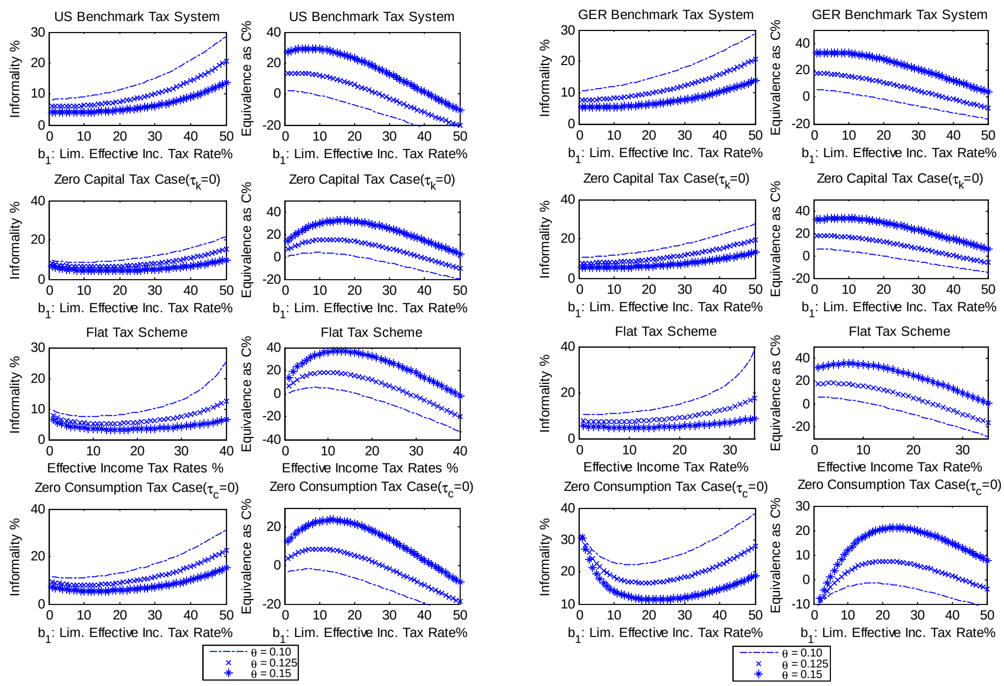

Figure 1. Informality, welfare vs. income tax rates, US and GER

Figure 1 displays the relationship between limiting income tax rates and informality, in addition demonstrates consumption equivalences of the welfare changes compared to the equilibrium in benchmark economies of US and Germany. In Figure 1, what takes notice primarily is the fact that though a positive and convex relationship between income tax rates and informality exists under benchmark tax system; once double-taxation of capital income or consumption taxes is removed, this relationship is observed to be no longer holding. This observation suggests that provided that tax revenue base is negligible already in economy, lowering of tax rates may not be a plausible policy to reduce informal sector size for significantly high levels of government externality. To put it differently, lower tax revenues and so lower level of productive government expenditures imply lesser return from formal sector employment for agents; hence the greater would be the incentives to increase informal labor supply in response to tax rate decreases at such initial points. However, significant welfare gains associated with removal of double-taxation of capital income and decreases in limiting income tax rates are still observed for both economies. In the figure, it is also evident that for higher levels of government externality, informality is lower for all income tax rates and is minimized at higher rates of taxes.

Parallel to the observations made through Figure 1, tax revenues are maximized at higher tax rates for higher levels of government externality. Figure 2 exhibits the Laffer curves associated with different tax instruments and for different government externality levels. What needs emphasis in the figure is that, for higher government externality levels; the tax rates at which the tops of the Laffer curves reached remain farther away from current tax rates. In addition, it should be noted that the tops of laffer curves lie farther away from the the tax rates which have been observed to be maximizing welfare. This is simply due to the fact that there exists a neighbourhood around optimal tax rates for which an increase in the tax rate outweigh the decreases in private inputs of capital and formal sector labor supply, thereby still increasing the tax revenue of the government within that neighbourhood.

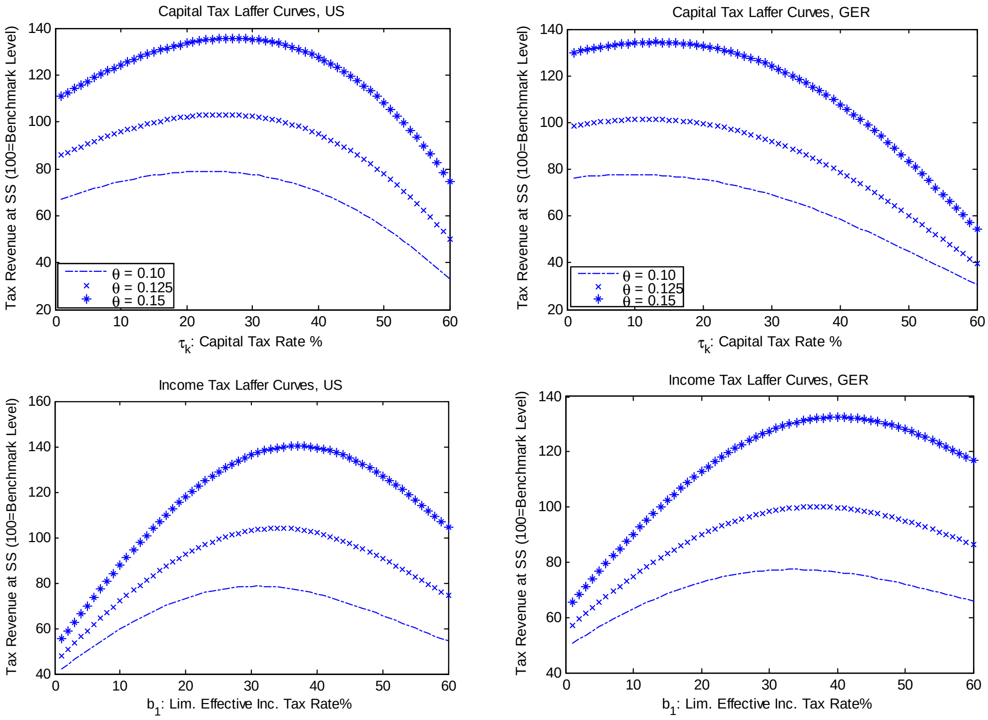

Figure 2. Capital and income tax laffer curves, US and GER

Table 5 displays the informality rates, welfare changes as well as total tax burdens at limiting effective income tax rates b_1 which minimize informality for the two artificial economies. At the table, all of the consumption equivalence values are computed against the welfare level at the benchmark tax system and benchmark government externality level of $\theta = 0.125$ in order to capture welfare levels at different degrees of government externality. Firstly, it is evident in the table the problems of informality minimization and welfare maximization almost coincide with each other. This fact can be grasped from the consumption equivalence values at informality minimizing income tax rates being almost equal to maximum level of consumption equivalences possible. Secondly, without the tax instruments of τ_k or τ_c, optimal level of public expenditures to output ratio remain significantly below the government externality level. This result points to greater inefficiencies associated with distortionary income tax. It is observed that especially without consumption tax instrument at hand, informality increases for each income tax rate compared to equilibrium in benchmark tax system while welfare remaining around same levels thanks to increases in consumption.

Table 5. Informality, Welfare and Optimal Tax Burden; US and GER

Benchmark, US	min. $\frac{Y_i}{Y}$	C% equi.	$\frac{G}{Y}$	max$C$%	Benchmark, GER	min. $\frac{Y_i}{Y}$	C% equi.	$\frac{G}{Y}$	max$C$%
$\theta = 0.10$	8.2%	2.18%	8.67%	2.18%	$\theta = 0.10$	10.62%	5.77%	10.64%	10.64%
$\theta = 0.125$	6.0%	13.57%	9.76%	13.57%	$\theta = 0.125$	7.62%	17.62%	10.55%	10.55%
$\theta = 0.15$	4.09%	29.43%	12.09%	29.43%	$\theta = 0.15$	5.27	33.08	11.35%	11.35%
$\tau_k = 0$ case	min. $\frac{Y_i}{Y}$	C% equi.	$\frac{G}{Y}$	max$C$%	$\tau_k = 0$ case	min. $\frac{Y_i}{Y}$	C% equi.	$\frac{G}{Y}$	max$C$%
$\theta = 0.10$	8.59%	3.68%	7.61%	3.68%	$\theta = 0.10$	10.56%	6.30%	9.69%	6.30%
$\theta = 0.125$	6.25%	15.41%	9.41%	15.45%	$\theta = 0.125$	7.69%	17.64%	9.59%	17.68%
$\theta = 0.15$	4.21%	32.04%	12.07%	32.04%	$\theta = 0.15$	5.34%	33.26%	11.65%	33.33%
Flat Tax Sch.	min. $\frac{Y_i}{Y}$	C% equi.	$\frac{G}{Y}$	max$C$%	Flat Tax Sch.	min. $\frac{Y_i}{Y}$	C% equi.	$\frac{G}{Y}$	max$C$%
$\theta = 0.10$	7.57%	5.05%	10.97%	5.38%	$\theta = 0.10$	10.53%	6.11%	10.48%	6.29%
$\theta = 0.125$	5.18%	18.10%	13.84%	18.52%	$\theta = 0.125$	7.33%	17.68%	13.64%	18.11%
$\theta = 0.15$	3.29%	35.90%	16.78%	36.77%	$\theta = 0.15$	4.73%	34.29%	17.01%	35.06%
$\tau_c = 0$ case	min. $\frac{Y_i}{Y}$	C% equi.	$\frac{G}{Y}$	max$C$%	$\tau_c = 0$ case	min. $\frac{Y_i}{Y}$	C% equi.	$\frac{G}{Y}$	max$C$%
$\theta = 0.10$	10.94%	−1.88%	6.89%	−1.86%	$\theta = 0.10$	22.32%	−1.63%	5.40%	−1.52%
$\theta = 0.125$	8.00%	8.72%	9.22%	8.72%	$\theta = 0.125$	16.58%	7.39%	7.45%	7.49%
$\theta = 0.15$	5.45%	23.68%	11.41%	23.68%	$\theta = 0.15$	11.31%	20.91%	9.69%	21.02%

The second problematic taken on in this part is the relative efficiency of taxing wages vs. capital. Table 6 displays the wage and capital tax rates which are solution to the Ramsey Planner's Problem taking benchmark rate of consumption taxes, τ_c as given. To facilitate the comparison, the other tax instrument is assumed to be absent at the hand of the Ramsey Planner. That is, the welfare maximizing tax rates at the table are the optimum rates given one of τ_w or τ_k is zero. It is evident in the table that for both economies, the informality rates are lower and welfare levels are higher in the case τ_w is used compared to the case τ_k is used to maximize welfare. This same fact can also be grasped from total tax burden levels at the optimum for the two case. It is observed that tax burden can readily be made close to government externality level if wage taxes are the only option. On the other hand, if capital taxes are used to maximize welfare, tax burden at the optimum lies farther from the government externality level. Hence, the results suggest that the use of wage taxes brings about far more efficiency gains than the use of capital taxes can bring in.

Table 6. Ramsey Equilibria, Wage vs. Capital Taxes

US	τ_w^*	$\frac{Y_i}{Y}$	$\frac{G}{Y}$	C% equi.	τ_k^*	$\frac{Y_i}{Y}$	$\frac{G}{Y}$	C% equi.
$\theta = 0.10$	0.159	5.42%	0.096	6.08%	0.111	7.21%	0.037	−2.81%
$\theta = 0.125$	0.193	3.22%	0.119	27.63%	0.138	4.69%	0.047	11.35%
$\theta = 0.15$	0.224	1.77%	0.147	56.33%	0.165	2.86%	0.058	30.16%
GER	τ_w^*	$\frac{Y_i}{Y}$	$\frac{G}{Y}$	C% equi.	τ_k^*	$\frac{Y_i}{Y}$	$\frac{G}{Y}$	C% equi.
$\theta = 0.10$	0.162	9.95%	0.094	6.72%	0.117	13.43%	0.036	−2.60%
$\theta = 0.125$	0.199	5.82%	0.119	26.29%	0.143	8.76%	0.047	10.68%
$\theta = 0.15$	0.232	3.16%	0.143	54.55%	0.168	5.36%	0.057	29.34%

The relationship between government externality parameter and informality rates can better be grasped through observing the rates at Ramsey Equilibria. Figure 3 exhibits that relationship for both US and Germany in case Ramsey Planner has only a single tax instrument at hand τ being charged on each factor income at the flat rate. It is observed that informality rates decrease at decreasing rate with government externality parameter.

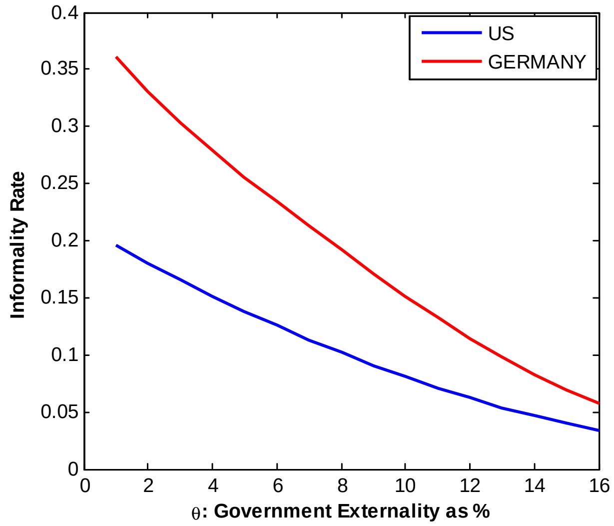

Figure 3. Informality vs. θ at ramsey equilibria, US and GER

Finally, calibration results for Turkey which are displayed at Figure 4 and 5 are in line with those for US and Germany despite the slightly different functional form presumed to be existent in informal sector production, through which Remark 2 is acknowledged.

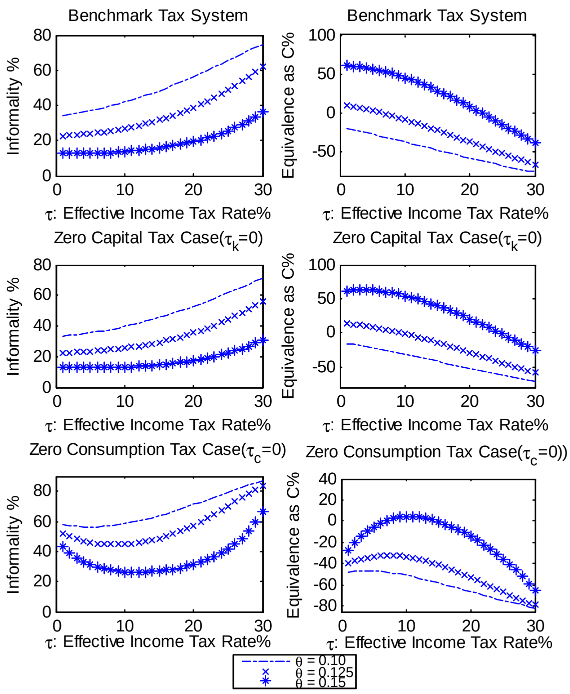

Figure 4. Informality, welfare vs. income tax rates, TUR

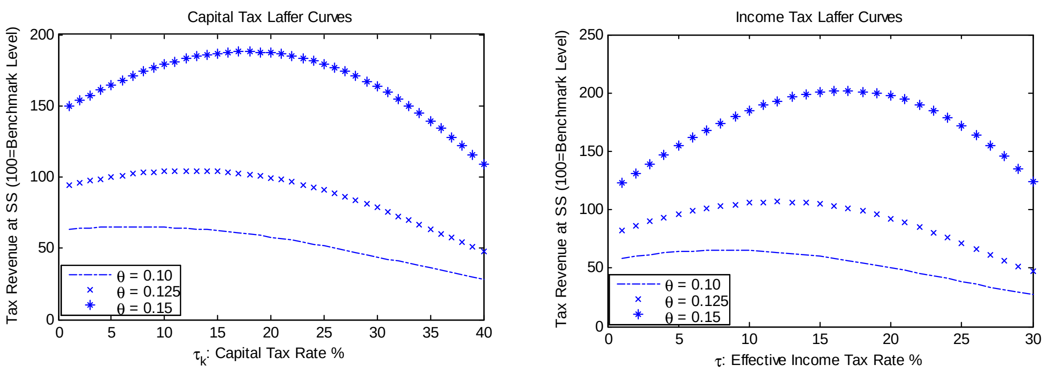

Figure 5. Capital and income tax laffer curves, TUR

Real Business Cycle Implications

Before moving on to numerical simulations and assessment of relative ability of the model with and without informal economy, it needs to be cleared why any two-sector model can be helpful to enhance the merits of real business cycle models in explaining aggregate fluctuations. In this literature; as Benhabib, Rogerson and Wright(1991) show contrary to the case in one-sector models, once agents are able to substitute between market and nonmarket production, there exists an extra dimension creating volatility that is based on the productivity differential across the two sectors changing over time. What is more, as Christiano and Eichembaum(1992) show, introduction of taxation, thereby government side into the standard RBC model is able to improve the ability of the model to match actual data. Along this line; as Mcgrattan, Rogerson and Wright(1997) have demonstrated, the impact of fiscal policies are to a large extent dependent on agents' ability to substitute in and out of market activities.

Built upon this line of research but based on different models accounting for informal economy, both Roca, Moreno and Sanchez(2001) and Busato and Chiarini(2004) have been able to demonstrate that inclusion of underground economic activities helps the models to match better the fluctuations in macroeconomics variables such as consumption, investment and employment. A similar approach is taken here through comparing the fits of models with and without an informal economy to actual data.

Numerical Simulations

Table 7 displays the standard deviations of basic macroeconomic variables and correlations of them with formal sector production under three parts which include the one representing the actual data, the one exhibited by the model without informal economy and finally the one with informal economy. The model without informal economy here is similar to the one-sector stochastic growth model with divisible labor in Hansen(1985) except for including government side. It should be noted that in order to facilitate the comparison, the models with and without the informal economy are calibrated to match the same targets.[21] The actual data statistics for US and Germany are derived from annual series involving the time period between 1970 and 2007.[22] All the series of variables have been logged and detrended using Hodrick and Prescott filter.[23] The number of simulation periods taken is equal to 38 to match the same number of observations between 1970 and 2007. In calibration process, firstly the standard deviation of the technology shock, z is calibrated at benchmark model economies to match standard deviation of output in actual data.[24] Then, the same shock level is used in the model without informal economy in order to facilitate comparison between the case with informal sector and the one without informal sector. When comparing the statistics describing the two artificial economies, one discovers that the economy with informal sector displays larger fluctuations in output than the economy without an informal sector. In fact, for the model without informal economy the standard deviation of output remains 14% and 19% below the level exhibited in the model with informal economy for US and Germany respectively. Similarly, for all of the other basic macroeconomic variables except for consumption, level of volatility exhibited is higher once informal economy is incorporated into the model.

[21] The only difference between the two models is that one of them includes an informal economy in addition to the formal one. The formal economies in the two models match the same targets of $\frac{K}{Y_F}$,$\frac{G}{Y_F}$, $\frac{I}{Y_F}$ and $\frac{L_F}{T}$. For that purpose, γ is being increased in the model without informal economy as much as to match the same $\frac{L_F}{T}$. Following the home production literature, correlations have been taken with output produced in formal sector instead of total output. See Benhabib, Rogerson and Wright(1991). Similarly, Roca, Moreno and Sanchez(2001) use registered ouput instead of total output in their two-sector model.

[22] Except for the labor hours and productivity data, the data are obtained from OECD.Stat Extracts. The labor hours and productivity series are from International Labor Comparisons(ILC) programme published by Bureau of Labor Statistics.

[23] As all the series are annually, $\lambda = 100$ has been taken in detrending procedure following the rule of thumb stated by Prescott(1983).

[24] While doing calibration, the autocorrelation coefficient, $\psi = 0.90$ has been taken which is the generally preferred value in the literature.

Table 7. Standard Deviations in %, Correlations with Output; US and GER

Actual Data, US	y^{Tot}	y^i	y^f	k	c	i	g	l^f	$\prod$	z
σ	–	–	1.88	0.59	1.79	7.23	2.14	0.94	0.76	–
$\sigma(y^f,x)$	–	–	–	0.63	0.89	0.88	0.09	0.61	0.33	–
Without Inf. Ec.	y^{Tot}	y^i	y^f	k	c	i	g	l^f	$\prod$	z
σ	–	–	1.61	0.53	0.60	5.86	1.85	0.76	0.89	0.92
$\sigma(y^f,x)$	–	–	–	0.51	0.92	0.98	0.99	0.98	0.98	0.99
With Inf. Ec	y^{Tot}	y^i	y^f	k	c	i	g	l^f	$\prod$	z
σ	1.62	1.43	1.88	0.60	0.47	6.77	2.07	1.11	0.78	0.92
$\sigma(y^f,x)$	1.00	–0.98	–	0.54	0.87	0.98	0.99	0.99	0.98	0.99

Actual Data, GER	y^{Tot}	y^i	y^f	k	c	i	g	l^f	$\prod$	z
σ	–	–	1.68	0.73	1.57	5.61	1.69	0.96	0.68	–
$\sigma(y^f,x)$	–	–	–	0.31	0.80	0.81	0.21	-0.05	0.66	–
Without Inf. Ec.	y^{Tot}	y^i	y^f	k	c	i	g	l^f	$\prod$	z
σ	–	–	1.37	0.42	0.43	3.79	1.45	0.77	0.63	0.72
$\sigma(y^f,x)$	–	–	–	0.51	0.90	0.99	0.99	0.99	0.97	1.00
With Inf. Ec.	y^{Tot}	y^i	y^f	k	c	i	g	l^f	$\prod$	z
σ	1.31	0.80	1.68	0.52	0.27	4.74	1.54	1.35	0.49	0.72
$\sigma(y^f,x)$	1.00	–0.96	–	0.55	0.68	0.98	0.99	0.99	0.96	0.99

This result originates in the fact that the productivity differential between the two sectors works as an extra mechanism to enhance volatility. To put it differently, not only absolute productivity shocks as it is the case in one-sector models but also the productivity differential between the two sectors helps to generate volatility once informal economy is incorporated. Intuitively speaking, in contrast to one-sector models; there is intratemporal substitution between formal and informal sector work in addition to intertemporal substitution which helps the formal sector production to attain even more procyclical property. To make it more explicit, in addition to the standard motive(i.e., motive of capital accumulation) for increasing labor hours when productivity is high, in the model with informal economy there is the additional motive of transferring labor hours from informal to formal sector. This extra motive in turn enhances the volatility of labor hours and improves the ability of the model to account for the fact of larger variability of hours worked compared to variability of productivity which is simply denoted by $\prod = \frac{y^f}{l^f}$. In fact, the ratio of standard deviation of labor hours to standard deviation of productivity increases from 0.85 to 1.42 and from 1.22 to 2.75 once informal economy is incorporated into the model for US and Germany respectively.

On the other hand, the model with informal economy is observed to be performing worse in terms of matching the fluctuations in consumption. However, this is an evidence to the fact that countercyclical nature of informal economy is being captured better in the two-sector model here. In other words, informal sector works as an insurance mechanism helping the agents to smooth their consumption. In times of low productivity, they are able substitute their labor from the formal sector to informal sector which in turn insures them against such adverse productivity times. This countercyclical property of informal economy can be grasped from the very strong negative correlation between formal and informal sector production. To mention another evidence supporting this argument, the correlation between consumption and output is observed to be decreasing as much as 5.4% and 24.4% for US and Germany respectively once informal economy is incorporated in the model. Finally, the model misses badly on the correlation between productivity and labor hours which is another puzzle in real business cycle literature. This correlation can even be negative for some countries as it is the case here for German economy but both models exhibit correlations as high as 0.98 and 0.99 for US and Germany respectively. This inconsistency with the data is by-product of the assumption that there is a single technology shock as a result of which labor hours follow the path of technology almost one-to-one.

To sum up, the simulation results point to the fact that two-sector models can help to generate higher levels of volatility through capturing the countercyclical nature of informal economy and the procyclical property of formal economy. However, this same ability turns out to be ineffective in accounting for volatility of consumption.

Sectoral and Policy Shocks

In this part, after deriving recursive equilibrium law of motion by the use of method of undetermined coefficients,[25] an attempt will be made to explore the behaviour of elasticities of formal and informal sector labor supplies with respect to technology and fiscal policy shocks for different levels of government externalities and limiting effective income tax rates. Analysis of elasticities can be useful to analyze which of income and substitution effects dominates over the other for which government externality levels and tax rates. In addition to elasticities analyses, for understanding how the dynamics between the two sectors work when the equilibrium is disturbed by exogenous shocks, analysis through impulse response functions will be made. There are mainly two kinds of shocks implemented which are positive shocks to formal sector technology and income tax rates.

Elasticities

As log-linearization is applied before method of undetermined coefficients used, the recursive equilibrium law of motion obtained will be represented by the log-deviations of variables. Hence, the deviation of each variable in response to a one-percent deviation of an exogenous state variable can directly be interpreted as elasticity of that variable with respect to a shock.[26]

[25] See Appendix B.

[26] See Appendix B for log-linearized conditions after formal sector technology and tax policy shocks.

Positive Tax Policy Shock

In Figure 6 and 7, elasticities of formal, informal and total labor supplies with respect to a positive tax policy shock are plotted onto limiting income tax rates, b_1 for different government externality levels. What takes notice in the figures corresponding to both US and Germany benchmark economies, is that except for very high(e.g. around 0.20) government externality parameters, the elasticity of formal sector labor supply is always negative and increasing in absolute value as income tax rates increase. Parallel to this observation, there exists a positive relationship between elasticity of informal sector labor supply and income tax rates except for the case with very high government externality level. On the other hand, at very high government externality levels, it is observed formal sector labor supply can increase in response to tax policy shock especially around the points in which informality is close to its minimum. Finally, for all income tax rates it is observed that elasticity of formal sector labor supply is lower in absolute value at higher government externality levels, which is in turn reflected on the pattern of response of total output during such tax policy shocks. Hence government externality can be said to work as an insurance mechanism for agents during positive tax policy shock times especially if initially income tax rates are high. In other words, positive tax policy shocks are observed to be better accommodated at higher government externality levels.

These results are in line with those of Baxter and King(1994) who reach the conclusion that private inputs, capital and labor have their highest response to increased government expenditures around points where government expenditures to output ratio(i.e, or tax burden alternatively) is close to government externality parameter. Indeed, substitution effects dominate but their direction varies depending on the significance of government externality level. At very high government externality levels, a positive tax policy shock implies higher rate of return to formal sector work, hence formal sector labor supply can respond by increasing.

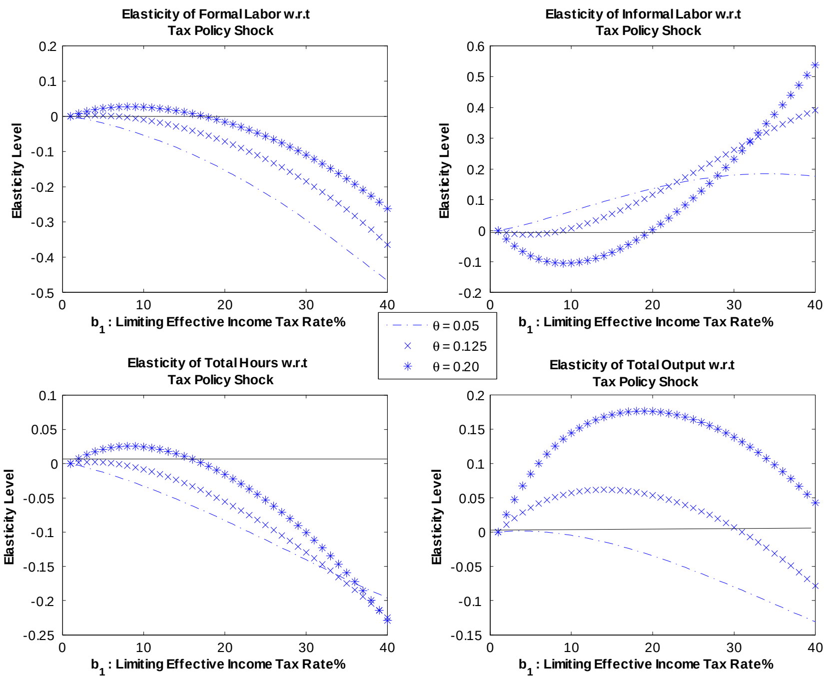

Figure 6. Elasticities and tax policy shock, US

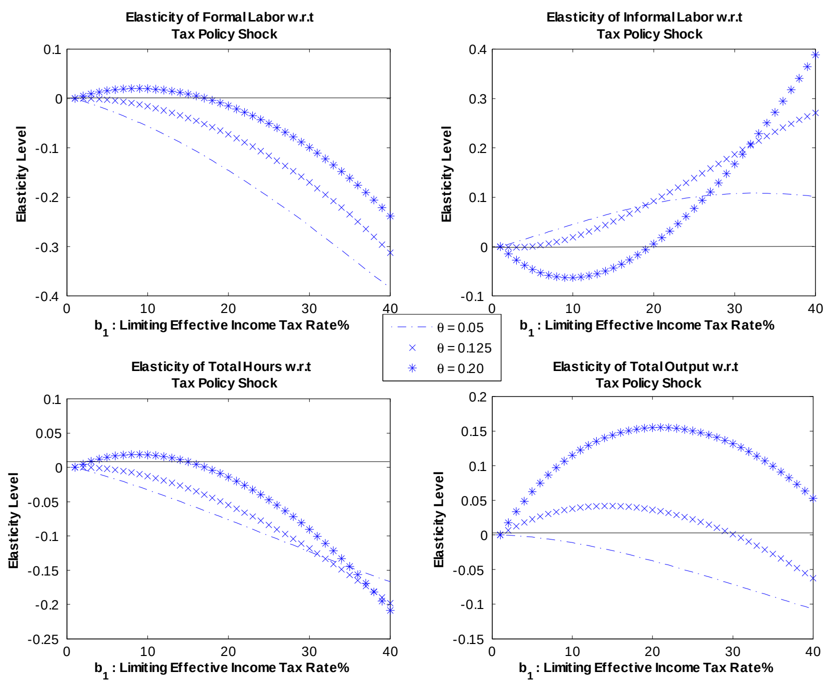

Figure 7. Elasticities and tax policy shock, GER

Positive Formal Sector Technology Shock

A positive shock on formal sector technology would imply increased rates of return to formal sector employment, hence provided that substitution effect dominates it should be expected that formal sector labor supply must temporarily increase. In other words, a positive elasticity of formal sector labor supply would indicate that substitution effect dominates. The observation of elasticities for different levels of government externality can be helpful in exploring which one of the effects is enhanced by government externality.

In Figure 8 and 9, elasticities with respect to formal sector productivity shock are plotted for different government externality levels. What takes notice is that formal sector labor supply's elasticity with respect to technology shock is lower at higher government externality levels. That is, formal sector labor supply is more responsive to positive changes in productivity at lower levels of government externality. Hence, it can be argued that government externality reinforces income effect against substitution effect. Secondly, it is observed that substitution effect tends to be more powerful for high tax rates; that is agents are more responsive to productivity shocks at high tax rates as leisure is less valued compared to consumption in such points. To sum up, at initial states corresponding to significantly high government externality levels and low rates of taxes agents tend to keep their formal sector labor supplies around same levels despite the productivity increase in the sector. Finally, it is observed that for higher government externality levels, the decrease in informal sector labor supply in response to formal sector technology shock is both higher and more stable for a wide range of income tax rates. However, informality rates corresponding to these initial points tend to determine the response of total output to formal sector technology shocks. Thanks to the lower informality rates at higher government externality levels, the response of both total working hours and total output tend to be greater in absolute value.

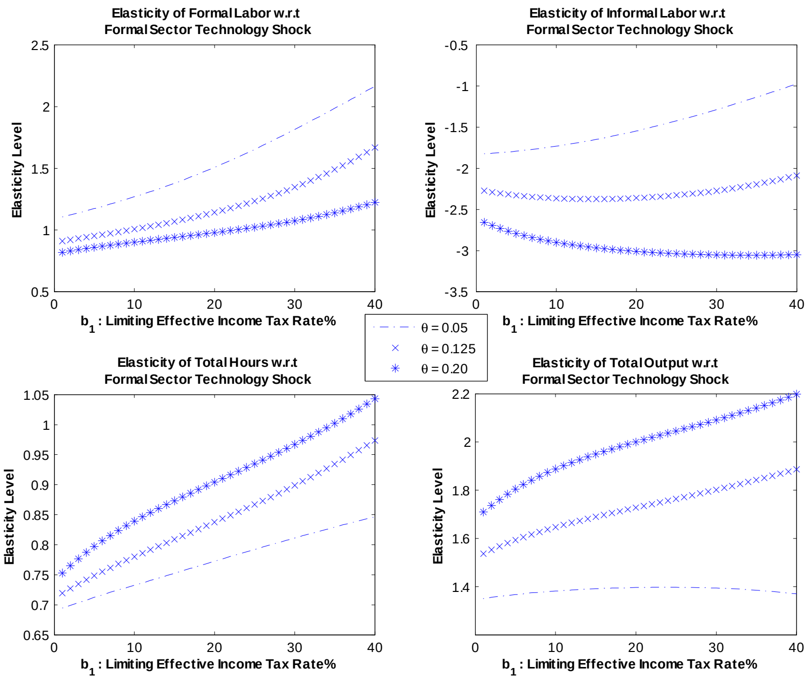

Figure 8. Elasticities and formal sector technology shock, US

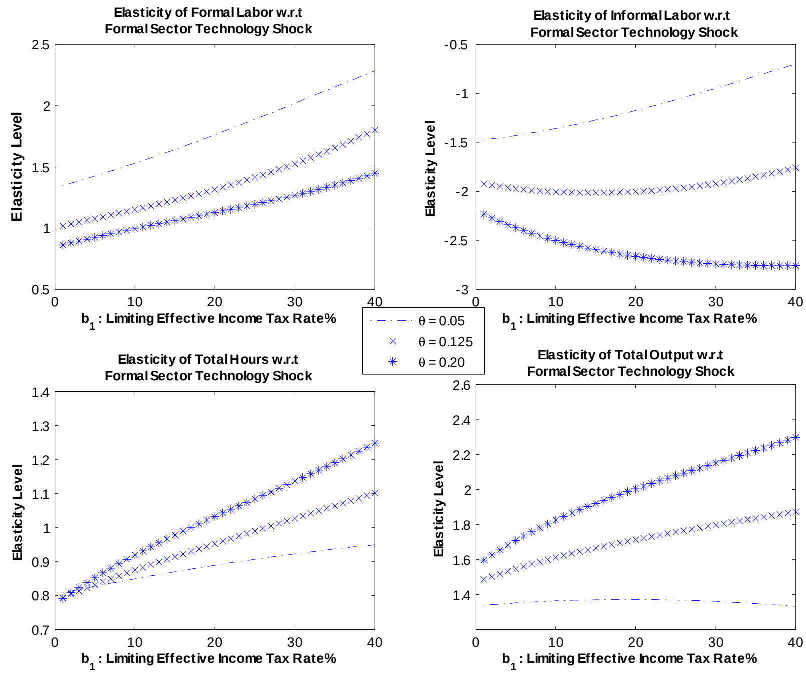

Figure 9. Elasticities and formal sector technology shock, GER

Impulse Responses

Positive Tax Policy Shock

In Figure 10 and 11, IRFs are displayed for benchmark economies corresponding to US and Germany. After a positive tax policy shock, there is initially a little jump observed in formal output owing to increased government expenditures except for cases of negligible levels of government externality. However, the increase in informality is still brought about by both the decrease in formal labor supply and increase in informal labor supply. Due to less efficient production taking part in the process, consumption and investment decrease. As the shock begins to die out, reversals in response of each of these variables begin to occur following reversals in the paths of formal and informal labor supplies.

As far as the effect of government externality on the patterns of fluctuations in variables are concerned, it needs emphasis that government externality plays insuring role in times of positive tax policy shocks. It is observed that the decrease in formal sector labor supply is lower at higher government externality levels as a result of which formal sector production is preserved and even enhanced at very high government externality levels. Indeed, formal sector production is observed to be initially jumping as much as 0.15% and 0.05% for US and Germany respectively at the government externality level of 0.20. On the other hand, it is also probable for it to be reducing at the rates around 0.15% if the level of government externality is of negligible size. These differential changes in formal sector production in turn determines the direction of change in total output as informal sector are of negligible sizes in both economies. However, what takes notice is that the changes in consumption and capital stock do not much depend on the degree of government externality.

Comparing the figures corresponding to US and Germany, it is remarkable that the responses of informal sector labor supply and hence informal sector production follow a different path. The changes in informal sector labor supply and production are observed to be independent of government externality parameter in benchmark economy of US, while they seem to be sensitive to this parameter for German artificial economy. What is more, the increase in informal sector labor supply comes about at higher rate at higher government externality levels. These discrepancies in the two economies can be attributed to the significantly lower amount of labor hours at the equilibrium in Germany compared to the case in US as a result of which substitution effects are reinforced.

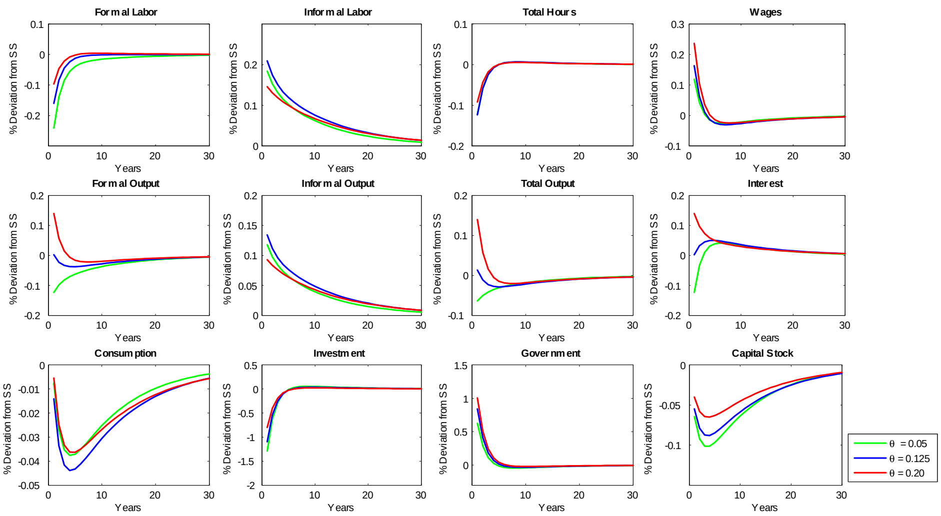

Figure 10. Impulse responses after tax policy shock, US

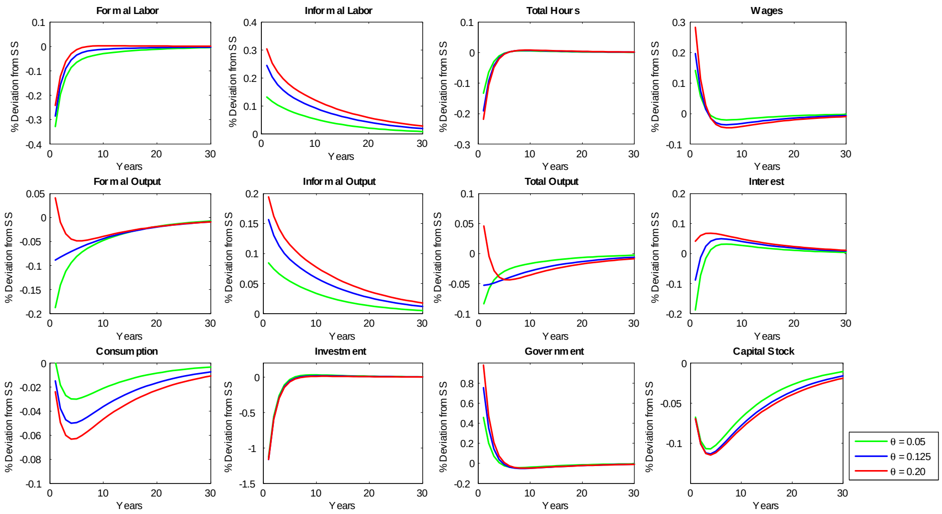

Figure 11. Impulse responses after tax policy shock, GER

Positive Formal Sector Technology Shock

Temporary change in relative sectoral productivity of a sector would be another disturbance cyclical fluctuations of which would be interesting. Introduction of positive shock to formal sector productivity would lead to a temporary wedge between rates of return to labor across two sectors in favour of formal sector. As displayed in Figure 12 and 13 for the US and Germany benchmark cases, a positive shock to formal sector productivity yields a shift of some labor from informal to formal sector as well as an increase in total output. Hence levels of savings and consumption increase until shock begins to die out.

As far as implications of different government externality levels to patterns of fluctuations observed in variables are concerned; what takes primary notice is that while the response of informal sector labor supply is quite sensitive to different levels of government externality, formal sector labor supply seems to be almost independent of them. Indeed, the initial decrease in informal sector labor supply ranges from around 1% to around 3% for both benchmark economies depending on the degree of government externality while formal sector labor supply jumps around 1.5% for all government externality levels though this jump is only a bit higher for lower government externality levels. The sharper decrease in informal labor supply for higher government externality levels point to the role of income effect as a result of which leisure becomes more preferable at higher government externality levels. Secondly, it is observed that capital stock continues its increase more at higher government externality levels as a result of which the small differentials between increases in formal sector labor supply at different government externality levels are offset. Through this way, the response of formal sector production seems to be independent of government externality levels. Indeed, the initial jump in formal sector production is observed to be around 2.5% for both economies. Though the decreases in informal sector output are lower at lower government externality levels, due to initially higher informality rates; the increase in total output come about at lower rates in case of lower levels of externality. This higher increase in total output is in turn reflected on the higher percantage increase in consumption enjoyed in economies with high government externality. Therefore, unlike the case of tax policy shock, in times of formal sector technology shocks; the changes in consumption are quite sensitive to levels of government externality.

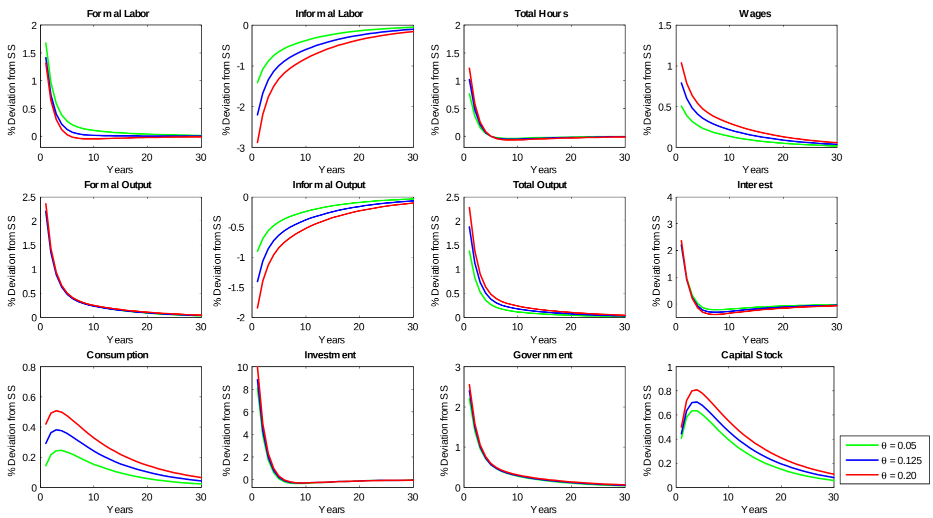

Figure 12. Impulse responses after formal sector technology shock, US

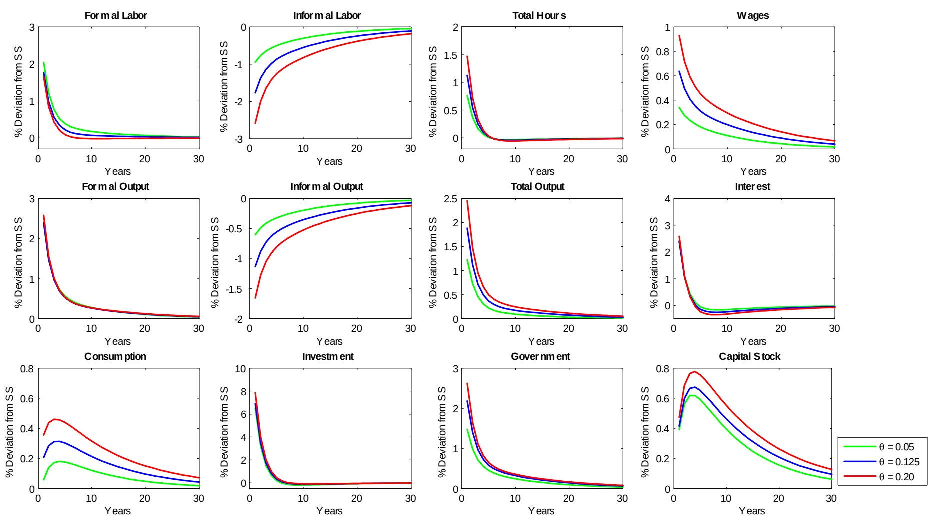

Figure 13. Impulse responses after formal sector technology shock, GER

CHAPTER 6

CONCLUSION

In this thesis, an attempt is made to probe into dynamics between tax policy and informality within a dynamic general equilibrium model which takes into account contribution of public services to formal sector production. By the help of calibration exercise, it has been shown that the pattern of relationship between tax rates and informality as well as optimum tax burden are to a large extent determined by government externality level. Hence, previously established result that point to a positive and convex relationship between tax rates and informality is observed to be holding only for negligible levels of government externality. This observation suggests that for initially low tax revenue bases in an economy and given the fact that formal sector has differential access to public services, lowering of tax rates might not be a plausible policy tool at hand to target informality.

Secondly, comparative statics explored between informality and different tax instruments for US, Germany and Turkey point to greater inefficiencies associated with income taxes compared to taxes on consumption and capital income. Thirdly, once income tax is differentiated between wage and capital taxes, thereby the problem is formulated so as to capture the relative efficiency of wage vs. capital taxes; both of the calibration results for US and Germany point to greater efficiency gains associated with the use of taxes on labor compared to the use of capital taxes. However, significant welfare gains associated with removal of double-taxation of capital income and decreases in limiting income tax rates are still observed for both economies.

In the remaining part of the thesis, an attempt has been made to assess the relative ability of the model in exhibiting real business cycle dynamics. The comparison between relative strengths of models with and without informal economies supports the rationale of using a two-sector model in order to help the model generate higher levels of volatility through capturing the countercyclical nature of informal economy and the procyclical property of formal economy. Impulse response analyses used to grasp this countercyclical nature of informal sector shows how informal economy works as an extra insurance mechanism in times of adverse shocks.

In the final part, effects of government externality and taxes on labor supply elasticities have been investigated. The results demonstrate that government externality enhances income effect in labor supply decision of agents. What is more, analysis of elasticities with respect to tax policy shocks shows that effects of such shocks are better accommodated for higher government externality levels.

Finally probing into further steps for future work, two extensions are worth emphasis. The first valuable extension would be incorporating individual level uncertainty into the model while assessing the consequences of different tax reforms. Heterogeneity introduced to the the model through the use of partially uninsurable productivity risk would be far more useful than the use of fixed productivity levels attributed to agents for two reasons. Firstly, economy wide repercussions of tax reforms would be captured in a more sensible manner thanks to an enhancement of the motive of capital accumulation with uninsurable income risk. Secondly, agents' response to tax reforms along the dimension of saving and labor supply decisions can be differentiated from each other only if their asset holding levels differ as well as their productivities. Thus, it would be possible to capture different combinations of income and substitution effects at different income quintiles. The second important step would be to supplement the analyses through the use of different production and utility functions. For instance, analyses under the conditions of positive utility associated with government expenditures or extra disutility associated with informal sector employment could be interesting.

APPENDIX A: PROOFS OF PROPOSITIONS

Proof of Proposition 1: The solution to agent's problem is expressed by the following equation which is the solution to Problem 1:

$$(1-\tau)\Gamma l_f^{\Gamma-1} g^{\theta} - A\alpha(T-l_f)^{\alpha-1} = 0$$

Firstly, SOC can be shown to unambiguously holding once $\Gamma \leq 1$ and $\alpha \leq 1$, as it is evident that;

$$(1-\tau)\Gamma(\Gamma-1)l_f^{\Gamma-2} g^{\theta} - A\alpha(\alpha-1)(T-l_f)^{\alpha-2} < 0$$

Once the government budget constraint is required to be matched, that is $g = \tau y_f$, then FOC can be shown to turn into:

$$(1-\tau)\tau l_f^{\frac{\Gamma+\theta-1}{1-\theta}} \tau^{\frac{\theta}{1-\theta}} = A\alpha(T-l_f)^{\alpha-1} \tag{17}$$

Implementing implicit differentiation onto this equation with respect to tax rate yields;

$$\begin{aligned} &\Gamma\left[\frac{(\Gamma+\theta-1)}{1-\theta} l_f^{\frac{\Gamma+\theta-1}{1-\theta}-1}(\tau^{\frac{\theta}{1-\theta}} - \tau^{\frac{1}{1-\theta}})\frac{dl_f}{d\tau} + (\frac{\theta}{1-\theta}\tau^{\frac{\theta}{1-\theta}-1} - \frac{1}{1-\theta}\tau^{\frac{1}{1-\theta}-1}) l_f^{\frac{\Gamma+\theta-1}{1-\theta}}\right] \\ = \quad & -A\alpha(\alpha-1)(T-l_f)^{\alpha-2}\frac{dl_f}{d\tau} \end{aligned}$$

which can be transformed after some algebra into:

$$\left[\frac{(\Gamma+\theta-1)}{(1-\theta)l_f}(\tau^{\frac{\theta}{1-\theta}} - \tau^{\frac{1}{1-\theta}}) + \frac{A\alpha(\alpha-1)(T-l_f)^{\alpha-2} l_f^{\frac{1-\Gamma-\theta}{1-\theta}}}{\Gamma}\right]\frac{dl_f}{d\tau} \tag{18}$$

$$= \quad (\frac{1}{1-\theta})(\tau^{\frac{\theta}{1-\theta}} - \theta\tau^{\frac{\theta}{1-\theta}-1}) \tag{19}$$

Here, the first term in the LHS is unambiguously negative only if $\Gamma + \theta \leq 1$

as $(\tau^{\frac{\theta}{1-\theta}} - \tau^{\frac{1}{1-\theta}}) \geq 0$ can easily be derived once $\tau \leq 1$ and $\theta \leq 1$ is taken into account. Similarly, the second term in the LHS is negative only if $\alpha \leq 1$ is met as $T - l_f \geq 0$ is always the case. Thus, once $\Gamma + \theta \leq 1$ and $\alpha \leq 1$ are met, LHS is unambiguously negative. On the other hand, The RHS can be shown transformed into $\frac{\tau^{\frac{\theta}{1-\theta}}}{1-\theta}(1 - \frac{\theta}{\tau})$ which is strictly negative only if $\frac{\theta}{\tau} > 1$ and strictly positive only if $\frac{\theta}{\tau} < 1$.Hence, maximum formal sector employment level is achieved at point $\tau = \theta$ which completes the proof.

Proof of Proposition 2: If a fixed level of capital stock that is completely independent of labor supply is entered into formal and informal sector production functions in such a way that $y_f = l_f^{\Gamma} k^{1-\Gamma} g^{\theta}$and $y_i = A l_i^{\alpha} k^{\beta}$, nothing changes in above equations except for LHS in the final equation getting transformed into;

$$\left[\frac{(\Gamma + \theta - 1)}{(1-\theta) l_f} (\tau^{\frac{\theta}{1-\theta}} - \tau^{\frac{1}{1-\theta}}) + \frac{A\alpha(\alpha - 1)(T - l_f)^{\alpha-2} k^{\beta - \frac{1-\Gamma}{1-\theta}} l_f^{\frac{1-\Gamma-\theta}{1-\theta}}}{\Gamma} \right] \frac{dl_f}{d\tau}$$

Then under the same conditions of the proof above, Proposition 2 is satisfied.

Proof of Proposition 3: Contrary to the case in Proposition 1 & 2, if the level of capital stock is made endogenous such that it is increasing function of amount of labor supplied like it is the case in dynamic models; that is provided that $k = f(L)$ in which $L = l_i + l_f$ and $f'(L) > 0$, the claim is that even if $\Gamma + \theta \leq 1$ and $\alpha \leq 1$ are met; informal sector employment minimizing condition $\tau = \theta$ does not necessarily hold.

Inserting $k = f(L)$ into the FOC to agents' utility maximization problem turns into:

$$(1-\tau)\tau l_f^{\frac{\Gamma+\theta-1}{1-\theta}} \tau^{\frac{\theta}{1-\theta}} (f(L))^{\frac{(1-\Gamma)(2-\theta)}{1-\theta} - \beta} = A\alpha(T - l_f)^{\alpha-1}$$

Then, following the similar path used in proofs of Proposition 1 & 2, implicitly differentiating the FOC to agents' utility maximization problem the following equality can be derived after some simple algebra:

$$\left\{\begin{array}{c}\left[\frac{(\Gamma+\theta-1)}{(1-\theta)l_f}+\left(\frac{(1-\Gamma)(2-\theta)}{1-\theta}-\beta\right)\frac{f'(L)}{f(L)}\right]\left(\tau^{\frac{\theta}{1-\theta}}-\tau^{\frac{1}{1-\theta}}\right)\\+\frac{A\alpha(\alpha-1)(T-l_f)^{\alpha-2}l_f^{\frac{1-\Gamma-\theta}{1-\theta}}}{\Gamma}f(L)^{(\beta-\frac{(1-\Gamma)(2-\theta)}{1-\theta})}\end{array}\right\}\frac{dl_f}{d\tau}=\frac{\tau^{\frac{\theta}{1-\theta}}}{1-\theta}(1-\frac{\theta}{\tau}) \quad (20)$$

Using (19), it can be shown that as long as $\left(\frac{(1-\Gamma)(2-\theta)}{1-\theta}-\beta\right)\frac{f'(L)}{f(L)}>0$ holds, the sign of LHS is ambiguous which in turn results in ambiguity of the sign of the term $\frac{dl_f}{d\tau}$ both beyond and until the point $\tau=\theta$. Thus, the tax rate which minimizes the informal sector size in consistency with agents' utility maximization problem, is not necessarily equal to government externality parameter.

APPENDIX B: SOLUTIONS TO DGE MODEL

First-order Conditions to Ramsey Problem: Substituting w_t, r_t, g_t into each condition they take place in and removing time subscripts, competitive equilibrium conditions can be reduced to 4 equations. Taking utility function and production functions of the forms $U(c_t, T - l_t^f - l_t^i) = \log(c_t) + \gamma \log(T - l_t^f - l_t^i)$, $y_t^f = k_{t-1}^{\rho}(l_t^f)^{1-\rho} g_t^{\theta}$, $y_t^i = A(l_i)^{1-\rho}$, the following 4 equations summarize the competitive equilibrium at the steady-state:

$$
\begin{aligned}
c(1+\tau_c) + \delta k &= (1-\tau) k^{\frac{\rho}{1-\theta}} l_f^{\frac{1-\rho}{1-\theta}} \tau^{\frac{\theta}{1-\theta}} + A(l_i)^{1-\rho} \\
A(l_i)^{-\rho} &= \frac{(1-\tau) k^{\frac{\rho}{1-\theta}} l_f^{\frac{\theta-\rho}{1-\theta}} \tau^{\frac{\theta}{1-\theta}}}{1-\theta} \\
\frac{1}{\beta} &= \frac{\rho(1-\tau) k^{\frac{\rho+\theta-1}{1-\theta}} l_f^{\frac{\theta-\rho}{1-\theta}} \tau^{\frac{\theta}{1-\theta}}}{1-\theta} + 1 - \delta \\
\frac{\gamma c(1+\tau_c)}{T - l_t^f - l_t^i} &= \frac{(1-\tau)(1-\rho) k^{\frac{\rho}{1-\theta}} l_f^{\frac{\theta-\rho}{1-\theta}} \tau^{\frac{\theta}{1-\theta}}}{(1-\theta)}
\end{aligned}
$$

Thus, the maximization problem can be put in the form:

$$
\begin{aligned}
&\max_{\tau,c,k,l_f,l_i \geq 0} \log c + \gamma \log(T - l_f - l_i) \\
&+\lambda[(1-\tau) k^{\frac{\rho}{1-\theta}} l_f^{\frac{1-\rho}{1-\theta}} \tau^{\frac{\theta}{1-\theta}} + A(l_i)^{1-\rho} - c(1+\tau_c) - \delta k] \\
&+\mu[\frac{(1-\tau) k^{\frac{\rho}{1-\theta}} l_f^{\frac{\theta-\rho}{1-\theta}} \tau^{\frac{\theta}{1-\theta}}}{1-\theta} - A(l_i)^{-\rho}] + \phi[\frac{1}{\beta} - \frac{\rho(1-\tau) k^{\frac{\rho+\theta-1}{1-\theta}} l_f^{\frac{\theta-\rho}{1-\theta}} \tau^{\frac{\theta}{1-\theta}}}{1-\theta} - (1-\delta)] \\
&+\chi[\frac{\gamma c(1+\tau_c)}{T - l_t^f - l_t^i} - \frac{(1-\tau)(1-\rho) k^{\frac{\rho}{1-\theta}} l_f^{\frac{\theta-\rho}{1-\theta}} \tau^{\frac{\theta}{1-\theta}}}{(1-\theta)}] + \xi(T - l_f - l_i)
\end{aligned}
$$

First order conditions can now be derived in the forms:

$$\tau \; : \; 0 = \lambda[\frac{\theta}{1-\theta}k^{\frac{\rho}{1-\theta}}l_f^{\frac{1-\rho}{1-\theta}}\tau^{\frac{\theta}{1-\theta}-1} - \frac{1}{1-\theta}k^{\frac{\rho}{1-\theta}}l_f^{\frac{1-\rho}{1-\theta}}\tau^{\frac{1}{1-\theta}-1}]$$
$$+\mu[\frac{\theta}{1-\theta}k^{\frac{\rho}{1-\theta}}l_f^{\frac{\theta-\rho}{1-\theta}}\tau^{\frac{\theta}{1-\theta}-1} - \frac{1}{1-\theta}k^{\frac{\rho}{1-\theta}}l_f^{\frac{\theta-\rho}{1-\theta}}\tau^{\frac{1}{1-\theta}-1}]$$
$$+\phi[\frac{\rho}{(1-\theta)^2}k^{\frac{\rho+\theta-1}{1-\theta}}l_f^{\frac{1-\rho}{1-\theta}}\tau^{\frac{1}{1-\theta}-1} - \frac{\rho\theta}{(1-\theta)^2}k^{\frac{\rho+\theta-1}{1-\theta}}l_f^{\frac{1-\rho}{1-\theta}}\tau^{\frac{\theta}{1-\theta}-1}]$$
$$+\chi[\frac{1-\rho}{(1-\theta)^2}k^{\frac{\rho}{1-\theta}}l_f^{\frac{\theta-\rho}{1-\theta}}\tau^{\frac{1}{1-\theta}-1} - \frac{\theta(1-\rho)}{(1-\theta)^2}k^{\frac{\rho}{1-\theta}}l_f^{\frac{\theta-\rho}{1-\theta}}\tau^{\frac{\theta}{1-\theta}-1}] \tag{21}$$

$$c : 0 = \frac{1}{c} - \lambda(1+\tau_c) + \frac{\gamma\chi}{T-l_f-l_i} \tag{22}$$

$$l_f \; : \; 0 = -\frac{\gamma}{T-l_f-l_i} + \lambda\frac{(1-\rho)(1-\tau)}{1-\theta}k^{\frac{\rho}{1-\theta}}l_f^{\frac{\theta-\rho}{1-\theta}}\tau^{\frac{\theta}{1-\theta}}$$
$$+\mu\frac{(\theta-\rho)(1-\tau)}{1-\theta}k^{\frac{\rho}{1-\theta}}l_f^{\frac{\theta-\rho}{1-\theta}-1}\tau^{\frac{\theta}{1-\theta}} - \phi\frac{\rho(1-\rho)(1-\tau)}{(1-\theta)^2}k^{\frac{\rho+\theta-1}{1-\theta}}l_f^{\frac{\theta-\rho}{1-\theta}}\tau^{\frac{\theta}{1-\theta}}$$
$$-\chi[\frac{(1-\rho)(1-\tau)(\theta-\rho)}{(1-\theta)^2}k^{\frac{\rho}{1-\theta}}l_f^{\frac{\theta-\rho}{1-\theta}-1}\tau^{\frac{\theta}{1-\theta}} + \gamma\frac{c(1+\tau_c)}{(T-l_f-l_i)^2}] - \xi \tag{23}$$

$$l_i : 0 = -\frac{\gamma}{T-l_f-l_i} + \lambda A(1-\rho)l_i^{-\rho} + \mu(1-\theta)A\rho l_i^{-\rho-1} - \chi\frac{\gamma c(1+\tau_c)}{(T-l_f-l_i)^2} - \xi \tag{24}$$

$$k \; : \; 0 = \lambda[\frac{(1-\tau)\rho}{(1-\theta)}k^{\frac{\rho+\theta-1}{1-\theta}}l_f^{\frac{1-\rho}{1-\theta}}\tau^{\frac{\theta}{1-\theta}} - \delta] + \mu[\frac{\rho(1-\tau)}{(1-\theta)}k^{\frac{\rho+\theta-1}{1-\theta}}l_f^{\frac{\theta-\rho}{1-\theta}}\tau^{\frac{\theta}{1-\theta}}]$$
$$-\phi\frac{(1-\tau)\rho(\rho+\theta-1)}{(1-\theta)^2}k^{\frac{\rho+\theta-1}{1-\theta}-1}l_f^{\frac{1-\rho}{1-\theta}}\tau^{\frac{\theta}{1-\theta}} - \chi\frac{(1-\tau)(1-\rho)\rho}{(1-\theta)^2}k^{\frac{\rho+\theta-1}{1-\theta}}l_f^{\frac{\theta-\rho}{1-\theta}}\tau^{\frac{\theta}{1-\theta}} \tag{25}$$

Then, inserting $\xi = 0$ to maintain $T - l_f - l_i > 0$ and incorporating the 4 competitive equilibrium conditions as well as (20), (21), (22), (23) and (24) the problem reduces to 9 equations and 9 unknowns.

Ramsey Problem can also be formulated on the basis of wage taxes versus capital taxes. In that case, the Ramsey Planner has two tax instruments τ_k and τ_w at hand taking τ_c as given. The Ramsey Problem can now be put in the form;

$$\max_{\tau_w,\tau_k,c,k,l_f,l_i\geq 0} \log c + \gamma \log(T - l_f - l_i)$$
$$+\lambda[\frac{k^{\frac{\rho}{1-\theta}} l_f^{\frac{1-\rho}{1-\theta}} (\rho\tau_k + \tau_w(1-\rho))^{\frac{\theta}{1-\theta}}}{1-\theta}(\rho(1-\tau_k) + (1-\rho)(1-\tau_w)) + A(l_i)^{1-\rho}$$
$$-c(1+\tau_c) - \delta k] + \mu[\frac{(1-\tau_w)k^{\frac{\rho}{1-\theta}} l_f^{\frac{1-\rho}{1-\theta}} (\rho\tau_k + \tau_w(1-\rho))^{\frac{\theta}{1-\theta}}}{1-\theta} - A(l_i)^{-\rho}]$$
$$+\phi[\frac{1}{\beta} - \frac{\rho(1-\tau_k)k^{\frac{\rho}{1-\theta}} l_f^{\frac{1-\rho}{1-\theta}} (\rho\tau_k + \tau_w(1-\rho))^{\frac{\theta}{1-\theta}}}{1-\theta} - (1-\delta)] + \chi[\frac{\gamma c(1+\tau_c)}{T - l_t^f - l_t^i}$$
$$-\frac{(1-\tau_w)(1-\rho)k^{\frac{\rho}{1-\theta}} l_f^{\frac{1-\rho}{1-\theta}} (\rho\tau_k + \tau_w(1-\rho))^{\frac{\theta}{1-\theta}}}{1-\theta}] + \xi(T - l_f - l_i)$$

through which the following first order conditions are obtained:

$$\begin{aligned}
\tau_w \quad : \quad 0 = {} & \lambda[\frac{(1-\rho)^2(1-\tau_w)\theta}{(1-\theta)^2} k^{\frac{\rho}{1-\theta}} l_f^{\frac{1-\rho}{1-\theta}} (\rho\tau_k + \tau_w(1-\rho))^{\frac{\theta}{1-\theta}} \\
& -\frac{1-\rho}{1-\theta} k^{\frac{\rho}{1-\theta}} l_f^{\frac{1-\rho}{1-\theta}} (\rho\tau_k + \tau_w(1-\rho))^{\frac{\theta}{1-\theta}}] \\
& +\mu[\frac{(1-\tau_w)\theta(1-\rho)}{1-\theta} k^{\frac{\rho}{1-\theta}} l_f^{\frac{1-\rho}{1-\theta}} (\rho\tau_k + \tau_w(1-\rho))^{\frac{\theta}{1-\theta}-1} \\
& -\frac{1}{1-\theta} k^{\frac{\rho}{1-\theta}} l_f^{\frac{\theta-\rho}{1-\theta}} (\rho\tau_k + \tau_w(1-\rho))^{\frac{\theta}{1-\theta}}] \\
& -\phi[\frac{\theta(1-\rho)\rho(1-\tau_k)}{(1-\theta)^2} k^{\frac{\rho+\theta-1}{1-\theta}} l_f^{\frac{1-\rho}{1-\theta}} (\rho\tau_k + \tau_w(1-\rho))^{\frac{\theta}{1-\theta}-1}] \\
& -\chi[\frac{\theta(1-\tau_w)(1-\rho)^2}{(1-\theta)^2} k^{\frac{\rho}{1-\theta}} l_f^{\frac{\theta-\rho}{1-\theta}} (\rho\tau_k + \tau_w(1-\rho))^{\frac{\theta}{1-\theta}-1} \\
& -\frac{1-\rho}{1-\theta} k^{\frac{\rho}{1-\theta}} l_f^{\frac{\theta-\rho}{1-\theta}} (\rho\tau_k + \tau_w(1-\rho))^{\frac{\theta}{1-\theta}}]
\end{aligned} \tag{26}$$

$$\tau_k \quad : \quad 0 = \lambda[\frac{(1-\tau_k)\theta\rho^2}{(1-\theta)^2} k^{\frac{\rho}{1-\theta}} l_f^{\frac{1-\rho}{1-\theta}} (\rho\tau_k + \tau_w(1-\rho))^{\frac{\theta}{1-\theta}-1} \tag{27}$$

$$-\frac{\rho}{1-\theta} k^{\frac{\rho}{1-\theta}} l_f^{\frac{1-\rho}{1-\theta}} (\rho\tau_k + \tau_w(1-\rho))^{\frac{\theta}{1-\theta}}]$$

$$\mu[\frac{(1-\tau_w)\rho\theta}{1-\theta} k^{\frac{\rho}{1-\theta}} l_f^{\frac{\theta-\rho}{1-\theta}} (\rho\tau_k + \tau_w(1-\rho))^{\frac{\theta}{1-\theta}-1}]$$

$$-\phi[\frac{(1-\tau_k)\theta\rho^2}{(1-\theta)^2} k^{\frac{\rho+\theta-1}{1-\theta}} l_f^{\frac{1-\rho}{1-\theta}} (\rho\tau_k + \tau_w(1-\rho))^{\frac{\theta}{1-\theta}-1}$$

$$-\frac{\rho}{1-\theta} k^{\frac{\rho+\theta-1}{1-\theta}} l_f^{\frac{1-\rho}{1-\theta}} (\rho\tau_k + \tau_w(1-\rho))^{\frac{\theta}{1-\theta}}]$$

$$-\chi[\frac{(1-\rho)\rho\theta(1-\tau_w)}{(1-\theta)^2} k^{\frac{\rho}{1-\theta}} l_f^{\frac{\theta-\rho}{1-\theta}} (\rho\tau_k + \tau_w(1-\rho))^{\frac{\theta}{1-\theta}}]$$

$$c : 0 = \frac{1}{c} - \lambda(1+\tau_c) + \frac{\gamma\chi}{T - l_f - l_i} \tag{28}$$

$$l_f \quad : \quad 0 = \frac{-\gamma}{T - l_f - l_i} + \lambda[\frac{(1-\rho)^2(1-\tau_w)}{(1-\theta)^2} k^{\frac{\rho}{1-\theta}} l_f^{\frac{\theta-\rho}{1-\theta}} (\rho\tau_k + \tau_w(1-\rho))^{\frac{\theta}{1-\theta}} \tag{29}$$

$$+\frac{\rho(1-\tau_k)(1-\rho)}{(1-\theta)^2} k^{\frac{\rho}{1-\theta}} l_f^{\frac{\theta-\rho}{1-\theta}} (\rho\tau_k + \tau_w(1-\rho))^{\frac{\theta}{1-\theta}}]$$

$$+\mu[\frac{(\theta-\rho)(1-\tau_w)}{(1-\theta)} k^{\frac{\rho}{1-\theta}} l_f^{\frac{\theta-\rho}{1-\theta}-1} (\rho\tau_k + \tau_w(1-\rho))^{\frac{\theta}{1-\theta}}]$$

$$-\phi[\frac{\rho(1-\rho)(1-\tau_k)}{(1-\theta)^2} k^{\frac{\rho+\theta-1}{1-\theta}} l_f^{\frac{\theta-\rho}{1-\theta}} (\rho\tau_k + \tau_w(1-\rho))^{\frac{\theta}{1-\theta}}]$$

$$-\chi[\frac{(1-\rho)(1-\tau_w)(\theta-\rho)}{(1-\theta)^2} k^{\frac{\rho}{1-\theta}} l_f^{\frac{\theta-\rho}{1-\theta}-1} (\rho\tau_k + \tau_w(1-\rho))^{\frac{\theta}{1-\theta}}$$

$$+\gamma\frac{c(1+\tau_c)}{(T - l_f - l_i)^2}] - \xi$$

$$l_i : 0 = -\frac{\gamma}{T - l_f - l_i} + \lambda A(1-\rho) l_i^{-\rho} + \mu(1-\theta)A\rho l_i^{-\rho-1} - \chi\frac{\gamma c(1+\tau_c)}{(T - l_f - l_i)^2} - \xi \tag{30}$$

$$\begin{aligned} k \quad : \quad 0 &= \lambda[-\delta + \frac{\rho(1-\rho)(1-\tau_w)}{(1-\theta)^2} k^{\frac{\rho+\theta-1}{1-\theta}} l_f^{\frac{1-\rho}{1-\theta}} (\rho\tau_k + \tau_w(1-\rho))^{\frac{\theta}{1-\theta}} \\ &+ \frac{\rho^2(1-\tau_k)}{(1-\theta)^2} k^{\frac{\rho+\theta-1}{1-\theta}} l_f^{\frac{1-\rho}{1-\theta}} (\rho\tau_k + \tau_w(1-\rho))^{\frac{\theta}{1-\theta}}] \\ &+ \mu[\frac{\rho(1-\tau_w)}{(1-\theta)} k^{\frac{\rho+\theta-1}{1-\theta}} l_f^{\frac{\theta-\rho}{1-\theta}} (\rho\tau_k + \tau_w(1-\rho))^{\frac{\theta}{1-\theta}}] \\ &- \phi[\frac{(1-\tau_k)\rho(\rho+\theta-1)}{(1-\theta)^2} k^{\frac{\rho+\theta-1}{1-\theta}-1} l_f^{\frac{1-\rho}{1-\theta}} (\rho\tau_k + \tau_w(1-\rho))^{\frac{\theta}{1-\theta}}] \\ &- \chi[\frac{(1-\tau_w)(1-\rho)\rho}{(1-\theta)^2} k^{\frac{\rho+\theta-1}{1-\theta}} l_f^{\frac{\theta-\rho}{1-\theta}} (\rho\tau_k + \tau_w(1-\rho))^{\frac{\theta}{1-\theta}}] \end{aligned} \tag{31}$$

After setting $\xi = 0$ to maintain $T - l_f - l_i > 0$ and taking together the 4 competitive equilibrium conditions with (25), (26), (27), (28), (29) and (30); the Ramsey Problem reduces to 10 equations and 10 unknowns. Then, one can easily compare the relative efficiency of using wage versus capital taxes through setting one of τ_w or τ_k to zero. By the way of solving the simultaneous 9 non-linear equations for each case(i.e., $\tau_w = 0$ and $\tau_k = 0$), the relative efficiency of wage taxes versus capital taxes is explored.

Log-Linearized Conditions after Formal Sector Technology Shock: Formal sector technology shock is incorporated into the formal sector production function as expressed in the following equation; $Y_t^f = Z_t K_{t-1}^{\rho} (L_t^f)^{1-\rho} G_t^{\theta}$ in which Z_t denotes the total factor productivity and evolves exogenously according to $\log Z_t = (1-\psi)\log Z + \psi \log Z_{t-1} + \varepsilon_t$,ε_t is $i.i.d.N(0;\sigma^2)$ and $0 < \psi < 1$, Z are parameters.

Letting variable any variable $x(t)$ to denote the logarithmic deviation from its steady-state value X, that is $x(t) = \log X_t - \log X$, replacing each variable in competitive equilibrium conditions in such a way that $X_t = Xe^{x(t)} \cong X[1 + x(t)]$; and then simplifying them using steady-state competitive equilibrium equations, following log-linearized conditions can be obtained:

$0 = -Ii(t) - Cc(t) - Gg(t) + Y_F y_f(t) + Y_i y_i(t)$

$0 = Ii(t) - Kk(t) + (1-\delta)Kk(t-1)$

$0 = \rho k(t-1) - y_f(t) + (1-\rho)l_f(t) + \theta g(t) + z(t)$

$0 = -y_f(t) + (1-\rho)l_i(t)$

$0 = -Gg(t) + [\tau_k + \zeta'(Y_F)]RKr(t) + [\tau_k + \zeta'(Y_F)]RKk(t-1) + \tau_c Cc(t) +$

$WL_F\zeta'(Y_F)w(t) + WL_F\zeta'(Y_F)l_f(t)$

$0 = y_i(t) - w(t) - l_i(t)$

$0 = y_f(t) - w(t) - l_f(t)$

$0 = [\frac{\gamma C(1+\tau_c)}{W(1-\zeta'(Y_F)))}]c(t) - [\frac{\gamma C(1+\tau_c)}{W(1-\zeta'(Y_F)))}]w(t) + L_F l_f(t) + L_i l_i(t)$

$0 = -r(t) + y_f(t) - k(t-1)$

$$0 = E_t \left\{ \begin{array}{c} -\beta R[1-\tau_k-\zeta'(Y_F)+(1-\delta)]c(t+1) + \beta R[1-\tau_k-\zeta'(Y_F)]r(t+1) \\ +\beta R[1-\tau_k-\zeta'(Y_F)+(1-\delta)]c(t) \end{array} \right\}$$

$z(t+1) = \psi z(t) + \varepsilon(t+1)$

Log-linearized Conditions after Tax Policy Shock: Incorporating a positive shock on effective income tax rates similar to the way described above, following set of log-linearized constraints is obtained;

$0 = -Ii(t) - Cc(t) - Gg(t) + Y_F y_f(t) + Y_i y_i(t)$

$0 = Ii(t) - Kk(t) + (1-\delta)Kk(t-1)$

$0 = \rho k(t-1) - y_f(t) + (1-\rho)l_f(t) + \theta g(t)$

$0 = y_i(t) - w(t) - l_i(t)$

$0 = -Gg(t) + [\tau_k + \zeta'(Y_F)]RKr(t) + [\tau_k + \zeta'(Y_F)]RKk(t-1) + \tau_c Cc(t) +$

$WL_F\zeta'(Y_F)w(t) + WL_F\zeta'(Y_F)l_f(t) + \zeta'(Y_F)(RK + WL_F)\ z(t)$

$0 = -W\zeta'(Y_F)z(t) + W[1-\zeta'(Y_F)]w(t) + (1-\rho)\frac{Y_i}{N_i}l_i(t) - (1-\rho)\frac{Y_i}{N_i}y_i(t)$

$0 = y_f(t) - w(t) - l_f(t)$

$0 = [\frac{\gamma C(1+\tau_c)}{W}]c(t) - [\frac{\gamma C(1+\tau_c)}{W}]w(t)) + L_F[1-\zeta'(Y_F)]l_f(t) + L_i[1-\zeta'(Y_F)]l_i(t) +$

$\zeta'(Y_F)(T - L_F - L_i)z(t)$

$0 = -r(t) + y_f(t) - k(t-1)$

$$0 = E_t \left\{ \begin{array}{c} -\beta R[1-\tau_k-\zeta'(Y_F)+(1-\delta)]c(t+1) + \beta R[1-\tau_k-\zeta'(Y_F)]r(t+1) \\ +\beta R[1-\tau_k-\zeta'(Y_F)+(1-\delta)]c(t) - \beta R\zeta'(Y_F)z(t) \end{array} \right\}$$

$z(t+1) = \psi z(t) + \varepsilon(t+1)$

Solving Recursive Equilibrium Law of Motion: Following Uhlig(1998), firstly the log-linearized conditions are written in the form:

$$0 = Ak(t) + Bk(t-1) + Cy(t) + Dz(t) \tag{32}$$

$$0 = E_t[Fk(t+1)+Gx(t)+Hk(t-1)+Jy(t+1)+Ky(t)+Lz(t+1)+Mz(t)] \tag{33}$$

$$z(t+1) = Nz(t) + \varepsilon(t+1) \tag{34}$$

in which $k(t)$ and $z(t)$ are state variables, the former one being an endogenous state variable and the latter one representing exogenous state variable while y(t) is a vector of other 9 endogenous variables.

For the special case of one endogenous and one exogenous state variables, Uhlig(1998) demonstrates recursive equilibrium law of motion of the form;

$$\begin{aligned} k(t) &= Pk(t-1) + Qz(t) \\ y(t) &= Rk(t-1) + Sz(t) \\ z(t) &= Nz(t-1) + \varepsilon(t) \end{aligned}$$

can easily be obtained once these equations expressing $k(t)$, $y(t)$ and $z(t)$ are substituted into (31) and (32); and then the coefficent vector corresponding to $k(t-1)$ and $z(t)$ are set to zero.

$$(F - JC^{-1}A)P^2 - (JC^{-1}B - G + KC^{-1}A)P + KC^{-1}B + H = 0 \tag{35}$$

$$R = -C^{-1}(AP + B) \tag{36}$$

$$Q = [N(F - JC^{-1}A) + (JR + FP + G - KC^{-1}A)]^{-1} \left\{ \begin{array}{c} [(JC^{-1}D) - L]N \\ +(KC^{-1}D) - M \end{array} \right\} \tag{37}$$

$$S = -C^{-1}(AQ + D) \tag{38}$$

among which (34) is a simple quadratic equation and N in (36) is exogenously set. In the recursive equilibrium law of motion described above, the coefficients P, Q and coefficient vectors R and S each of which contain coefficients corresponding to 9 endogenous variables can be interpreted as elasticities since all of $k(t)$, $y(t)$, $z(t)$ and $k(t-1)$ represent log-deviations.

APPENDIX C: DATA DETAILS

Macro Data: In order to compute calibration targets, the annual data between 1980 and 2007 have been used.

All the data for basic macroeconomics variables belong to annual national accounts at OECD.Stat Extracts: http://stats.oecd.org/Index.aspx

The labor hours and productivity series have been taken from International Labor Comparisons(ILC) programme published by Bureau of Labor Statistics: http://www.bls.gov/fls/tables.htm Finally, to obtain a rough estimate of government externality parameter the data belonging to US are derived from US Census Bureau: http://www.census.gov/govs/

Tax Rates Data: Effective consumption tax rates used are taken from Mendoza et. al.(1994) and Trabandt and Uhlig(2006). Similarly, the parameters b_1 and b_2 have been taken from Gouveia and Strauss(1994) and Lim and Hyun(2004).

While having an estimate for the remaining parameters, the data at Revenue Statistics, OECD.Stat Extracts have been used: http://stats.oecd.org/Index.aspx

The following components existent in the Revenue Statistics have been taken into account to derive estimates of average effective income tax rates and effective capital income tax rates:

1110: Taxes on income and profits of individuals

1120: Taxes on capital gains of individuals

1200: Income, profit and capital gains taxes of corporations

4100: Recurrent taxes on immovable property

4400: Taxes on financial and capital transactions

The parameter used for adjustment to units of measurement at hand is b_3, this parameter is calibrated to match average effective income tax rate. The target for average effective income tax rate is calculated in the following way:

$$\frac{\zeta(y_t^f)}{y_t^f} = \frac{1110}{(1 - \frac{Y_i}{Y})}$$

The effective tax rate on capital income is calculated through the equation:

$$\tau_k = \frac{1120 + 1200 + 4100 + 4400}{(1 - \frac{Y_i}{Y})\rho}$$

in which ρ is the share of capital income in formal sector production.

REFERENCES

Andreas Irmen & Johanna Kuehnel, (2009). "Productive Government Expenditure and Economic Growth",*Journal of Economic Surveys*,Blackwell Publishing, vol. 23(4), pages 692-733, 09.

Amaral, P., & Quintin, E. (2006). "A Competitive Model of the Informal Sector". *Journal of Monetary Economics*, 53(7), 1541-53.

Aschauer, David Alan, (1989). "Is public expenditure productive?," *Journal of Monetary Economics*, Elsevier, vol. 23(2), pages 177-200, March.

Barro, Robert J, (1990). "Government Spending in a Simple Model of Endogenous Growth," *Journal of Political Economy*, University of Chicago Press, vol. 98(5), pages S103-26, October.

Baxter, Marianne & King, Robert G, (1993). "Fiscal Policy in General Equilibrium," *American Economic Review*, American Economic Association, vol. 83(3), pages 315-34, June.

Busato, F., & Chiarini, B. (2004). "Market and. Underground Activities in a Two-Sector Dynamic Equilibrium Model". *Economic Theory*, 23, 831-861.

Benhabib, Jess & Rogerson, Richard & Wright, Randall, (1991). "Homework in Macroeconomics: Household Production and Aggregate Fluctuations," *Journal of Political Economy*, University of Chicago Press, vol. 99(6), pages 1166-87, December.

Benno & Schneider, Friedrich, (2009). "The impact of tax morale and institutional quality on the shadow economy," *Journal of Economic Psychology*, Elsevier, vol. 30(2), pages 228-245, April.

Carlos, J., Roca, C., & D, C. (2001). "Underground Economy and Aggregate Fluctuations", *Spanish Economic Review* vol3, 41-53.

Domeij, D., & Klein, P. (2005). "Pre-Announced Optimal Tax Reform." *Macroeconomic Dynamics*, 9(2), pp150-169.

Ernste, Dominik & Schneider, Friedrich, (1998). "Increasing Shadow Economies all over the World - Fiction or Reality?," *IZA Discussion Papers 26*, Institute for the Study of Labor (IZA).

Fortin, B., Marceau, N., & Savard, L. (1997). "Taxation, Wage Controls and the Informal Sector". *Journal of Public Economics*, 66, 239-312.

Friedrich Schneider & Dominik H. Enste, (2000). "Shadow Economies: Size, Causes, and Consequences," *Journal of Economic Literature*, American Economic Association, vol. 38(1), pages 77-114

Ihrig, J., & Moe, K. S. (2004). "Lurking in the Shadows:. The Informal Sector and Government Policy," *Journal of Development Economics*, vol73(2), 541-557.

Ihrig, J., & Moe, K. S. (2000). "The dynamics of informal employment," *International Finance Discussion Papers 664*, Board of Governors of the Federal Reserve System (U.S.).

Lim & Hyun, (2006). "Comparative Analysis of the Effective Income Tax function: Empirical Evidence using LIS data", *Applied Economic Letters*, vol13, issue2.

Loayza, N. V. (1996). "The Economics of the Informal Sector: A Simple Model and Some Empirical Evidence from Latin America". *Carnegie-Rochester Conference Series on Public Policy*, 45, 129-162.

Lucas, Robert Jr., (1988). "On the mechanics of economic development," *Journal of Monetary Economics*, Elsevier, vol. 22(1), pages 3-42, July.

McGrattan, Ellen R & Rogerson, Richard & Wright, Randall, (1997). "An Equilibrium Model of the Business Cycle with Household Production and Fiscal Policy," *International Economic Review*, Department of Economics, University of Pennsylvania and Osaka University Institute of Social and Economic Research Association, vol. 38(2), pages 267-90, May.

Mendoza, Enrique G & Razin, Assaf & Tesar, Linda, (1994). "Computing Effective Tax Rates on Factor Incomes and Consumption: An International Macroeconomic Perspective," *CEPR Discussion Papers 866*, C.E.P.R. Discussion Papers.

Munnell Alicia H. & Leah M. Cook, (1990). "How does public infrastructure affect regional economic performance?," *New England Economic Review*, Federal Reserve Bank of Boston, issue Sep, pages 11-33.

Prado, Jr., Jose Mauricio, (2007). "Government Policy in the Formal and Informal Sectors," *Seminar Papers 751*, *Stockholm University*, Institute for International Economic Studies.

Rauch, J. E. (1991). "Modeling the Informal Sector Formally". *Journal of Development Economics*, 35, 33-48.

Ray, D. (1998). *Development Economics*. Princeton University Press, Princeton, N.J. Rosenzweig ,1988. Labor Markets in Low-Income Countries. In: Chenery, H., Srinivasan, T.N. (Eds.), Handbook of Development Economics. Elsevier, Amsterdam, pp, 713-62.

Garcia Penalosa, Cecilia & Turnovsky, Stephen J., (2005). "Second-best optimal taxation of capital and labor in a developing economy," *Journal of Public Economics*, Elsevier, vol. 89(5-6), pages 1045-1074, June.

Saracoglu, D. (2008). "The Informal Sector and Tax on Employment: A Dynamic General Equilibrium Investigation", *Journal of Economic Dynamics and Control*, Elsevier, vol. 32(2), pages 529-549, February.

Schneider, Friedrich, (2002). "The Size and Development of the Shadow Economies of 22 Transition and 21 OECD Countries," *IZA Discussion Papers 514*, Institute for the Study of Labor

Schneider, F. (2007). "Shadow Economies and Corruption All Over the World: New Estimates for 145 Countries". *Economics-The Open-Access, Open-Assessment E-Journal*, 1(9), 1-66.

Sebastian Galiani & Federico Weinschelbaum, (2007). "Modeling Informality Formally: Households and Firms," *Working Papers 0047, CEDLAS*, Universidad Nacional de La Plata.

Thomas, J. J. (1992). "Informal Economic Activity". Harvester Wheatsheaf, New York, N.Y. Tybout, J., 2000. "Manufacturing Firms in Developing Countries: How Well Do They Do, and Why?" *Journal of Economic Literature*, 28, 11-44.

Trabandt Mathias & Harald Uhlig, (2006). "How Far Are We From The Slippery Slope? The Laffer Curve Revisited," *SFB 649 Discussion Papers SFB649DP2006-023*, Sonderforschungsbereich 649, Humboldt University, Berlin, Germany.

Trabandt Mathias, (2006). "Optimal Pre-Announced Tax Reforms Under Valuable And Productive Government Spending," 2006 Meeting Papers 668, *Society for Economic Dynamics*.

Uhlig Harald, (1998). "A Toolkit for Analysing Nonlinear Dynamic Stochastic Models Easily," QM&RBC Codes 123, *Quantitative Macroeconomics & Real Business Cycles*.