TORSIONAL ANALYSIS OF THE FRAMES

A THESIS SUBMITTED IN PARTIAL FULFILLMENT OF
THE REQUIREMENTS FOR THE DEGREE OF MASTER OF
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A C K N O W L E D G E M E N T

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INTRODUCTION

In the design of ribbed floor buildings there are many methods and many approaches to the problem. This thesis has been intended to expound the theory and several design methods of one-way ribbed floors. Loads are transmitted by ribs to the beams to which they are perpendicular. The ribs are one way only, and there are no beams between the columns, parallel to the ribs, which constitutes a homogeneous one way slab. The end moments of the ribs are transmitted as torsional moments to the beams and then to the columns. To design such a structure to resist side sway due to horizontal forces, one must determine the effective length of the slab which makes a frame with the columns. The torsional moments in the beams must also be determined. These two problems are analysed elastically.

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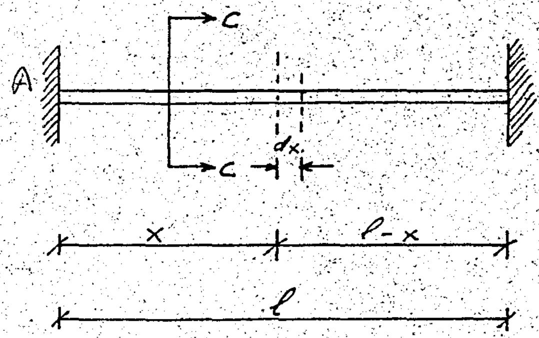
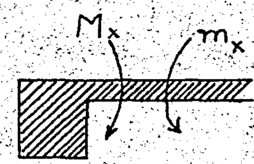
A system of edge beam and slab is taken :





(Section c-c)

The length of the beam is (l) and it is fixed supported on A and B .

?

m_x = Fixed end moment of the slab.

M_x = Torsional moment resulted on the beam due to m_x .

Then

$$dM_x = m_x \, dx$$

$$(M_x)_{x=0} = \int_0^l m_x \, dx \left(\frac{l-x}{l} \right)$$

$$M_x = (M_x)_{x=0} - \int_0^x m_x \, dx$$

φ_0 = The rotation of the slab which is hinge supported to the beam, under normal load.

φ_1 = Under ($m_x = 1$ tm) unit load, the rotation of the slab which is hinged to the beam.

$(\varphi_{pla})_x$ = The final rotation of the plate under the normal load, and plate as fixed supported to the beam.

$$\varphi_{(pl)_x} = \varphi_o + m_x \varphi_1$$

$\varphi_{(tor)_x}$ = The torsional rotation of the beam under torsion

$$\varphi_{(tor)_x} = \int_0^x \frac{M_x\, dx}{G J_t} = \int_0^x \frac{(M_x)_{x=0}\, dx}{G J_t} - \int_0^x\!\!\int_0^x \frac{m_x\, dx^2}{G J_t}$$

$$= \int_0^x\!\!\int_0^l \frac{m_x\, dx^2 (l-x)}{G J_t\, l} - \int_0^x\!\!\int_0^x \frac{m_x\, d^2x}{G J_t}$$

Keeping in mind that the rotation of the beam and the slab must be equal at every point.

$$\varphi_{(pl)_x} + \varphi_{(tor)_x} = 0$$

the general formula is obtained

$$\varphi_o + m_x \varphi_1 + \int_0^l\!\!\int_0^x \frac{m_x\, d^2x (l-x)}{G J_t\, l} - \int_0^x\!\!\int_0^x \frac{m_x\, d^2x}{G J_t} = 0$$

By assuming that (φ_1) is not a function of (x) and (φ_o) is only a linear function of (x) the above equation is differantiated twice

$$m''_x - \frac{m_x}{G J_t \varphi_1} = 0$$

taking $\varepsilon^2 = \dfrac{1}{G J_t \varphi_1}$

a second order differential equation is obtained

$$m''_x - \varepsilon^2 m_x = 0$$

taking $\varepsilon^2 > 0$, the solution is

$$m_x = c_1 \cosh \varepsilon x + C_2 \sinh \varepsilon x$$

Boundary conditions

for x = o and x = 1 $\qquad \varphi_{(pl)} = 0$

$$\varphi_{(pl)} = \varphi_o + C_1 \varphi_1 \cosh \varepsilon x + C_2 \varphi_1 \sinh \varepsilon x$$

x = o $\qquad \varphi_o + C_1 \varphi_1 = 0 \qquad C_1 = -\dfrac{\varphi_o}{\varphi_1}$

x = 1 $\qquad \varphi_o - \varphi_o \cosh \varepsilon l + C_2 \varphi_1 \sinh \varepsilon l = 0$

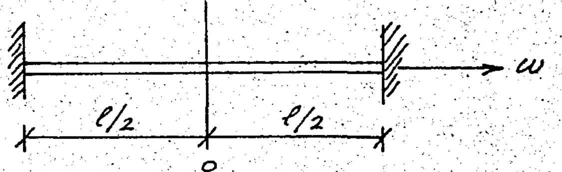
$$C_2 = \frac{\varphi_o(\cosh \varepsilon l - 1)}{\varphi_1 \sinh \varepsilon l}$$

$$m_x = -\frac{\varphi_o}{\varphi_1} \cosh \varepsilon x + \frac{\varphi_o}{\varphi_1} \frac{\cosh \varepsilon l - 1}{\sinh \varepsilon l} \sinh \varepsilon x$$

<u>**Special condition:**</u>

$\varphi_o =$ constant (for uniform slab loading)

There is symmetry around (o) axis



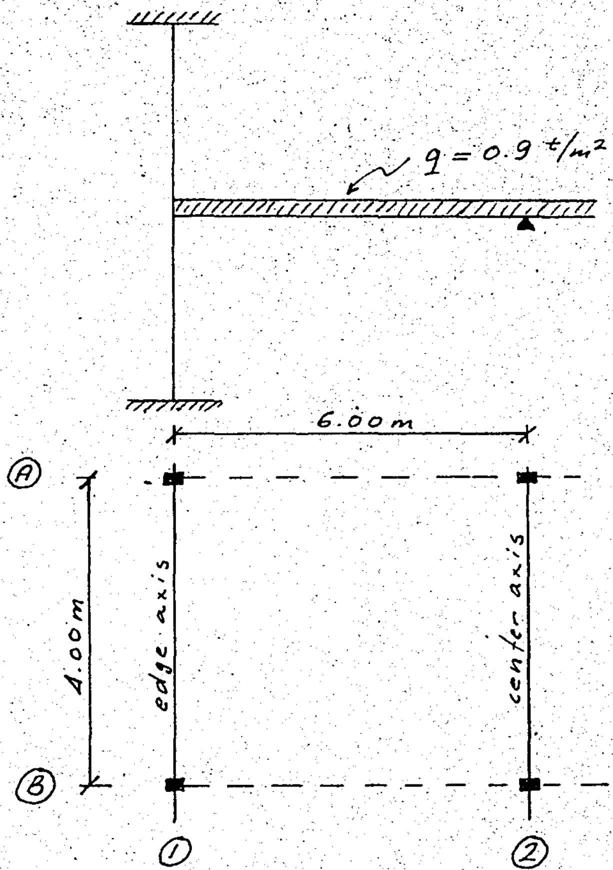
then $\qquad m_{\underset{x}{\omega}} = C \cosh \varepsilon \omega$

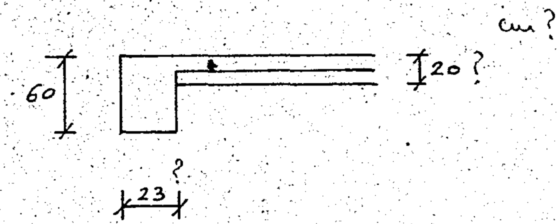
$\varphi_{(pl)} = \varphi_o + C \varphi_1 \cosh \varepsilon \omega \qquad \varphi_{(pl)} = 0 \qquad \overset{x}{\omega} = \pm \dfrac{l}{2}$

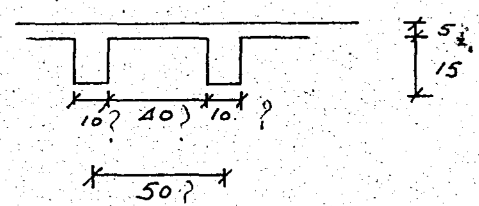
$$C = -\frac{\varphi_o}{\varphi_1} \frac{1}{\cosh \varepsilon l/2}$$

$$m_{\underset{x}{\omega}} = -\frac{\varphi_o}{\varphi_1} \frac{\cosh \varepsilon \omega}{\cosh \varepsilon l/2}$$

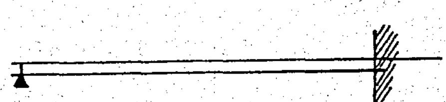
Example:

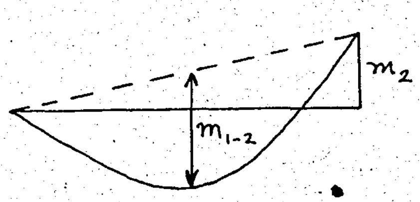






under uniform load:



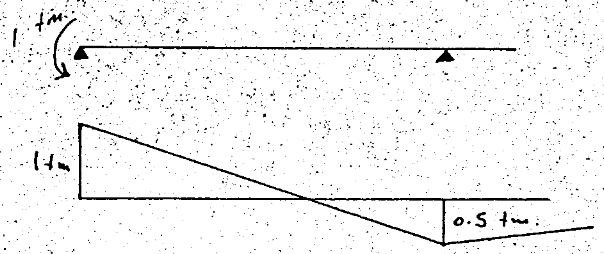


$$m_2 = -\frac{0.9\times\overline{6.00}^2}{8} = -4.05 \text{ tm/m}$$

$$m_{1-2} = +4.05 \text{ tm/m}$$

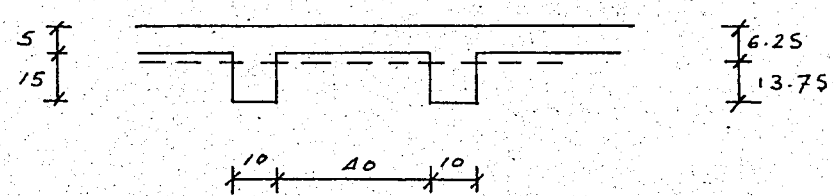
$$E J_{slab}\, \varphi_o = \frac{1}{3}\times 4.05\times 6.00 - \frac{1}{6}\times 4.05 \times 6.00 = 4.05 \text{ ? (t.m)}$$

under (1tm) load



$$E J_{slab}\, \varphi_1 = -\frac{1}{3} \times 1.00 \times 6.00 + \frac{0.5}{6} \times 1.00 \times 6.00 = -1.50 \quad ?$$

Moment of Inertia of the Slab:



$$A = 10.00 \times 0.50 + 2 \times 1.00 \times 1.50 = 8.00\ dm^2$$

$$yA = 5.00 \times 0.25 + 3.00 \times 1.25 = 5.00\ dm^3$$

$$y = \frac{5.00}{8.00} = 0.625\ dm$$

$$J = \frac{1}{12} \times 10 \times \overline{2}^3 + 2 \times 10 \times \overline{0.375}^2 - \left(\frac{4 \times \overline{1.5}^3}{12} + 4 \times 1.5 \times \overline{0.625}^2 + \frac{2}{12} \times 2 \times \overline{1.5}^3 \right.$$

$$\left. + 2 \times 2 \times 1.5 \times \overline{0.625}^2 \right) = 2.543\ dm^4/m$$

Torsional Resistence of the beam:

$$J_t = \frac{0.6 \times \overline{0.23}^3}{3} \left(1 - 0.63 \frac{0.23}{0.60} + 0.052 \frac{\overline{0.23}^5}{\overline{0.60}^5} - \dots \right.$$

$$J_t = 0.00185\ m^4$$

$$E = 2.1 \times 10^6\ t/m^2$$

$$G = 1.05 \times 10^6\ t/m^2 \quad \rightarrow (\nu = 0)$$

$$\varphi_0 = \frac{4.05}{2.1 \times 10^{6} \times 2.543 \times 10^{-4}} = 0.758 \times 10^{-2} \; ? \; \frac{1}{m}$$

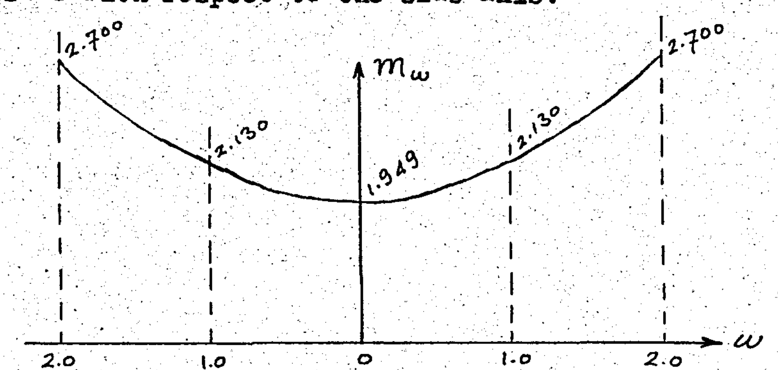
$$\varphi_1 = \frac{1.50}{2.1 \times 10^{6} \times 2.543 \times 10^{-4}} = 0.281 \times 10^{-2} \; ? \; \frac{1}{m}$$

$$\varepsilon = \sqrt{\frac{1}{1.05 \times 10^{6} \times 0.281 \times 10^{-2} \times 1.85 \times 10^{-3}}} = 0.427$$

$$-\frac{\varphi_0}{\varphi_1} = \frac{0.758 \times 10^{-2}}{0.281 \times 10^{-2}} = 2.7$$

ω	$\varepsilon\omega$	$\cosh \varepsilon\omega$	$\cosh \varepsilon\, l/2$	$\alpha = \frac{\cosh \varepsilon\omega}{\cosh \varepsilon\, l/2}$	$m_\omega = \frac{-\varphi_0}{\varphi_1}\alpha$
0	0	1.000	1.384	0.722	1.949
1.00	0.427	1.093	1.384	0.789	2.130
2.00	0.854	1.384	1.384	1.000	2.700

in this way end moments at different points are obtained. We can draw the end moment of the slab with respect to the slab axis:



$$M_t = \int_0^2 m_\omega \, d\omega$$ so the shaded part under the diagram gives the maximum torsional moment applied to the beam. ?

$$M_{t_{max}} = M_{t_a} = \int_0^l m_x \left(\frac{l-x}{l}\right) dx$$

1) Notations:

E = Young's modulus of elasticity (t/dm^2)

$G = \frac{6}{14} E$ Shear modulus (for Poisson's ratio $m = 6$)

J = Moment of Inertia of the slab for (1m) width, or moment of inertia of T beam on the edge span (dm^4)

J_o, J_u = Moment of inertia of the edge columns (dm^4)

$J_t \cong \frac{b\,d^3}{12} \, \frac{1}{0.3\left(1+\frac{d^2}{b^2}\right)}$ Torsion factor for the edge beam (dm^4) (b= width of the beam and d=depth of the beam)

$k = \sqrt{\frac{14}{(2-\gamma_1)\,l} \, \frac{J}{J_t}}$ dimensionless coefficient in hyperbolic functions

l = Span length in (x) direction (m)

s = length in (y) direction (m)

h_o, h_u = Height of the edge column (m)

q = Load on the edge slabs (t/m)

m_{oy} = The fixed end moment of the slab along the edge beam at any point (y) (tm)

m_{os} = The fixed end moment of the slab at column section at y=s (tm)

$\overline{m}_n$ = The end moment of the slab at n^{th} support, found by assuming supports to be free to rotate and as a continuous beam.

m_{ny} = The real end moment of the slab at $(n)^{th}$ support and at (y) ordinate, (tm)

$\left.\begin{matrix} M_{oa} \\ M_{ob} \\ M_{oc} \end{matrix}\right\}$ = The fixed end moment of the T beam (a,b,c) on the edge beam. (tm)

$\left.\begin{matrix} \overline{M}_{na} \\ \overline{M}_{nb} \\ \overline{M}_{nc} \end{matrix}\right\}$ = The fixed end moment of the T beam (a,b,c,) at $(n)^{th}$ support found by assuming the supports to be free to rotate and as a continuous beam (tm)

$\left.\begin{matrix} M_{na} \\ M_{nb} \\ M_{nc} \end{matrix}\right\}$ = The real end moments of the T beam (a,b,c) at $(n)^{th}$ support (tm)

γ_n = Edge moment coeficient

2) Edge beams, edge columns and slabs:

Like flat slabs, the slab makes a frame with the columns. The length of the drop panel is equol to (2s) Figure I. The maximum end and center moment of the slab occurs under the loading shown in Figure 1b .

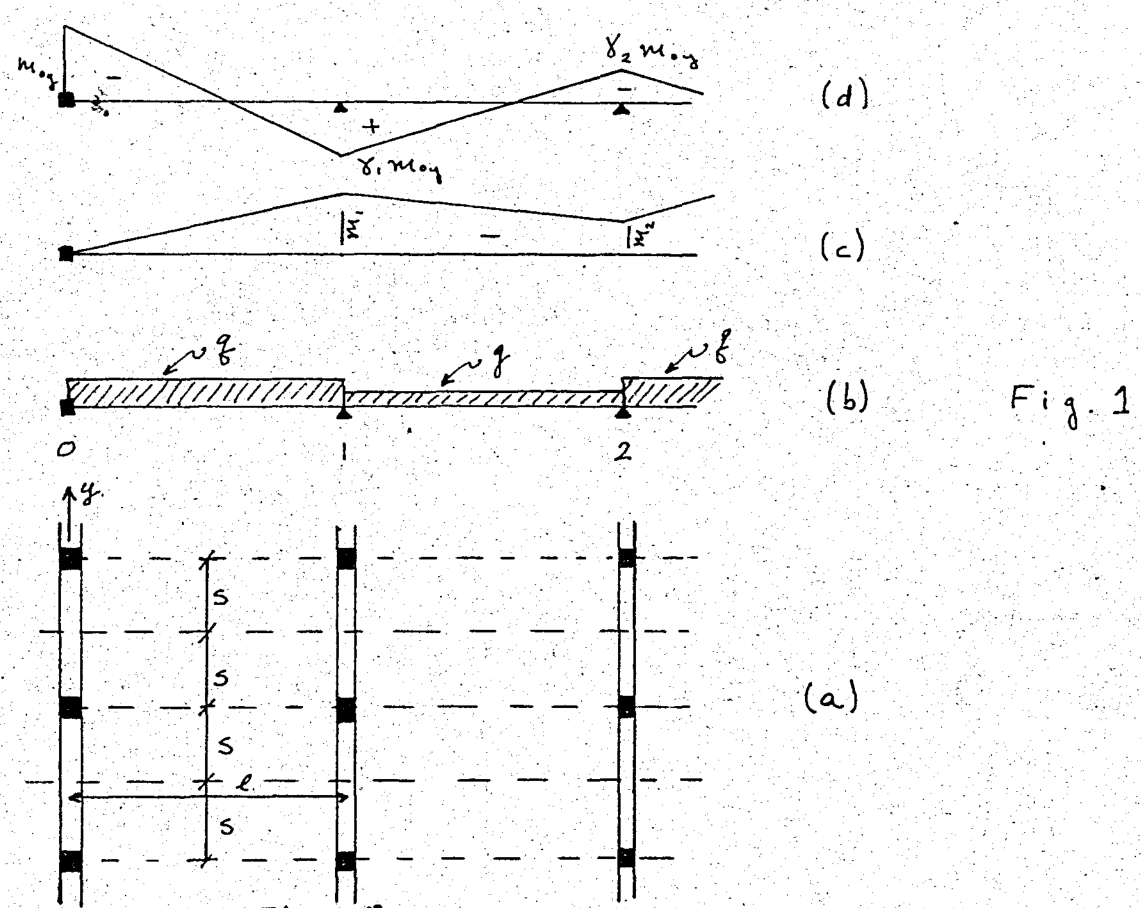


Figure 1

2a) Equation for the slab end moments.

$$m_{ny} = -\gamma_n \, m_{oy} + \overline{m}_n \qquad (1)$$

By choosing this equation interior support moments are already divided by two. Moment (m_n) is constant along (y) axis while m_{oy} and m_{ny} are variables.

In the first solution the slab is solved as a continuous beam with supports free to rotate, moments may be found using any method, coeficients are known, too.

The second solution deals with the edge slab, and is dealt more vigorously.

2b) The distribution of torsional and slab end moments.

The angle of rotation of the slab at the (y) of the edge beam

$$\varphi = m_{oy}\frac{1}{3EJ} + m_{1y}\frac{1}{6EJ} + \frac{q\ell^3}{24EJ} \qquad (I)$$

and taking (m_{1y}) from the first equation

$$\varphi = \delta m_{oy} + \overline{m}_1 \frac{1}{6EJ} + \frac{q\ell^3}{24EJ} \qquad (Ia)$$

$$\delta = \frac{(2-\gamma_1)\,l}{6EJ}$$

Three seperate conditions may help in solving the differential equation of moment distribution.

1) The angle of twist of the edge beam is equal to the tangent of the slab at that point.

$$D\rho\, dy = d\varphi + \frac{\partial\varphi}{\partial y}dy - d\varphi$$

Torsional moment $\qquad D = \frac{1}{\rho}\frac{d\varphi}{dy} \qquad (II)$

$\rho = \frac{1}{GJ_t}$ is the angle of twist for 1dm of the edge beam and D = 1tdm.

2) The differance in torsional moments of the edge beam is equal to (m_{oy}) the end moment of the slab.

$$D + \frac{\partial D}{\partial y}dy - D = m_{oy}\, dy$$

from this $$\frac{dD}{dy} = m_{oy} \qquad (III)$$

The differential of the second equation:

$$\frac{dD}{dy} = \frac{1}{\rho} \frac{d^2\varphi}{dy^2} \qquad (IV)$$

3) Differentiating (Ia) twice

$$\frac{d^2\varphi}{dy^2} = \delta \frac{d^2 m_{oy}}{dy^2} \qquad (V)$$

taking (eq.IV)into consideratian, eq. III and V result in

$$\frac{d^2 m_{oy}}{dy^2} = \frac{\rho}{\delta} m_{oy}$$

taking $k^2 = \frac{\rho}{\delta}$

$$\frac{d^2 m_{oy}}{dy^2} - k^2 m_{oy} = 0 \qquad (VI)$$

The solution of this is m_{oy} = A cosh ky + B sinh ky (VII) where A and B are constants.

From accepted symmetry

$$\frac{dm_{oy}}{dy} = 0 \qquad \text{for} \quad y = 0$$

so B = o (Fig. 2)

for y = s $\qquad m_{oy} = m_{os} \qquad$ and $A = \frac{m_{os}}{\cosh ks}$

Torsional moment is obtained by integrating (eq. III)

$$D_y = m_{os} \int_0^y \frac{\cosh ky}{\cosh ks}\, dy = \frac{m_{os}}{k} \frac{\sinh ky}{\cosh ks}$$

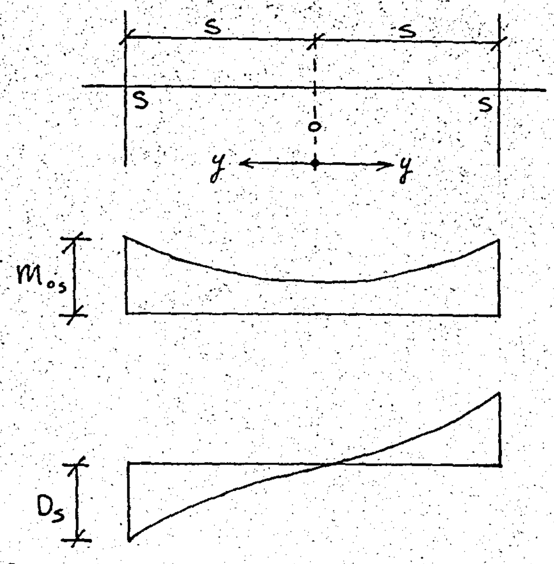


fig. 2

2.c) The condition for the equilibrium of the moment is

$$\alpha + \varphi_s = 0 \qquad (a)$$

where α = rotation of the support

φ_s = rotation of the slab

2c_1) Distribution of the end moments are according to the following equation

$$M_{oy} = M_{os} \frac{\cosh ky}{\cosh ks} \qquad (2)$$

Torsional Moment

$$D_y = \frac{M_{os}}{k} \frac{\sinh ky}{\cosh ks} \qquad (3)$$

Both ofthese are maximum near the columns (Fig.2)

The torsional moment at the edge beams is

$$D_s = M_{os} \frac{\mathrm{tgh}\, ks}{k} \qquad (4)$$

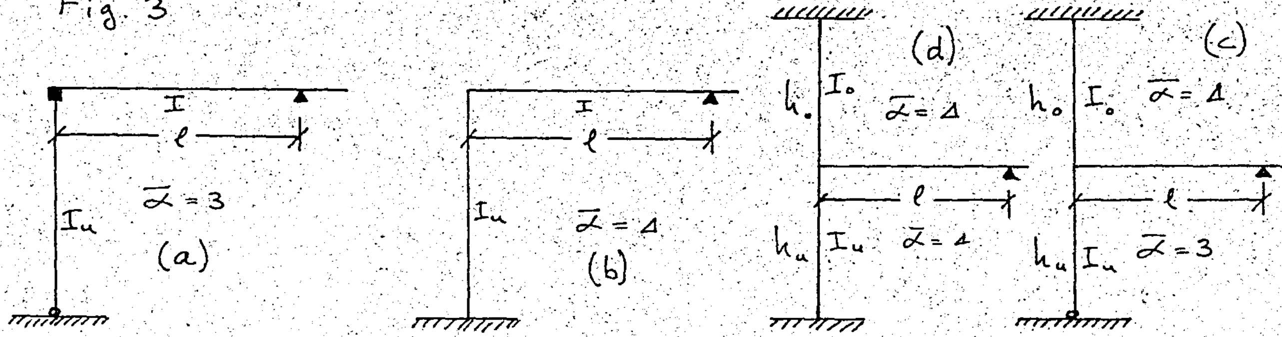
Edge columns are subjected to a moment of (2 D_s)

then

$$\alpha = \frac{2 D_s h_u}{(1+c)\,\overline{\alpha_u}\, E J_u} = M_{os} \frac{2\,\mathrm{tgh}\, ks}{(1+c)\,\overline{\alpha_u}\, k} \frac{h_u}{E J_u} \qquad (b)$$

values for different column combinations is shown in (Fig.3)

Fig. 3



(C) values are the result of the changes on column conditions

$$c = \frac{\overline{\alpha}_o}{\overline{\alpha}_u} \; \frac{J_o}{J_u} \; \frac{h_u}{h_o} \qquad (5)$$

for one floor frames c is equal to zero.

($2c_2$) Rotation of the slab

The rotation of the tangent of the slab at $y = s$

$$\varphi_s = m_{os} \frac{(2-\gamma_1)\,l}{6EJ} + \overline{m}_1 \frac{l}{3EJ} + \frac{q\,l^3}{24EJ} \qquad (6)$$

($2c_3$) In passing from (b) and (c) to (a) and multiplying by $\dfrac{6EJ}{l}$

$$V = \frac{J}{J_u} \; \frac{h_u}{l}$$ coefficient of fixity is obtained

$$m_{os}\left(\frac{12\,v}{(1+c)\,\overline{\alpha}_u} \; \frac{tgh\,ks}{k} + 2 - \gamma_1\right) = -\frac{q\,l^2}{4} - \overline{m}_1 \qquad (7)$$

$\overline{m}_1$ is known so it is on the right of the equation.

for $y = o$ end moment of the slab is maximum, so the span moment is minimum.

Calculations must be made for $y = o$ and $y = s$.

Edge columns support a moment of

$$M_s = 2\,D_s \qquad (8)$$

3) Edge beam and various T beams with edge columns.

The method of calculation is the same, instead of continuous edge support moment curve, a moment curve shaped by single moments and broken at the T beam axis, is formed. All T beams have equal J and symmetrically loaded.

3a) Case of an intermediate beam:

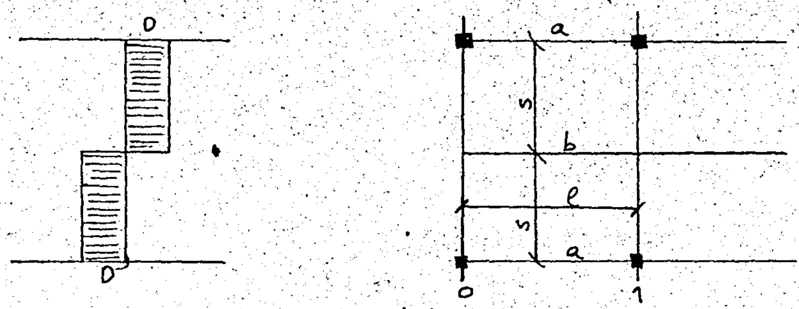


Fig. 4

Conditions at the section where (T) beams meet the edge beam

for (a) beam $\quad \alpha + \varphi_a = 0$

for (b) beam $\quad \alpha + \vartheta + \varphi_b = 0$

The angle of rotation of the column

$$\alpha = \frac{(M_{oa} + M_{ob})\, h_u}{(1+c)\, \bar{\alpha}_u\, E J_u}$$

The angle of torsion of the edge beam

$$\vartheta = \frac{7}{6}\, \frac{M_{ob}\, s}{E J_t}$$

The rotation of the (a) beam at the support

$$\varphi_a = M_{oa} \frac{(2-\gamma_1)\, l}{6EJ} + \bar{M}_{1a} \frac{l}{6EJ} + \frac{q_a\, l^3}{24\, EJ}$$

The rotation of the (b) beam at the support

$$\varphi_b = M_{ob} \frac{(2-\gamma_1)\, l}{6EJ} + \bar{M}_{1b} \frac{l}{6EJ} + \frac{q_b\, l^3}{24\, EJ}$$

These rotations are multiplied by $\frac{6EJ}{l}$ to find fixity coefficients.

$$v = \frac{6}{(1+c)\bar{\alpha}_u}\, \frac{J}{J_u}\, \frac{h_u}{l} \qquad (9)$$

$$\mu = 7\, \frac{J}{J_t}\, \frac{s}{l} \qquad (10)$$

These values are substitited into equilibrium equations to find moments

equilibrium equations for (a) and (b)

$$(2 + v + \gamma_1)\, M_{oa} + v\, M_{ob} = -\frac{q_a\, l^2}{4} - \bar{M}_{1a}$$

$$v\, M_{oa} + (2 + v + \mu - \gamma_1)\, M_{ob} = -\frac{q_b\, l^2}{4} - \bar{M}_{1b} \qquad (11)$$

$\overline{M_{1a}}$ and $\overline{M_{1b}}$ as moments of continuous beam are known from the first calculations.

The Moment (M_{ob}) is the moment which causes torsional moment on the edge beam.

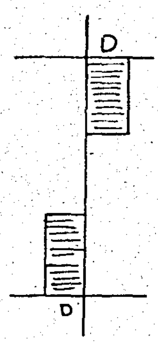
$$\text{Torsional moment} = \frac{M_{ob}}{2} = D \qquad (12)$$

Distribution occurs as shown in fig. 4

Edge columns take $M_s = M_{oa} + M_{ob}$

which is distributed between upper and lower columns.

3b) Case of two intermediate beam (Fig. 5), with conditions as (3a)



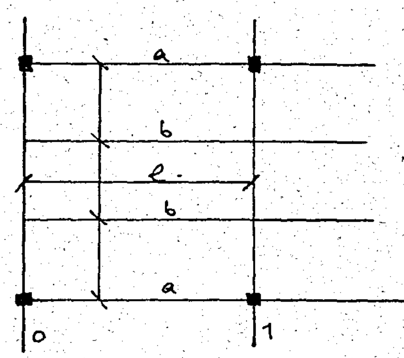


Fig. 5

The rotation of the edge column

$$\alpha = \frac{(M_{oa} + 2M_{ob})}{(1+c)\ \alpha_u} \frac{h_u}{E J_u}$$

The torsional rotation of the edge beam

$$\vartheta = \frac{14}{6} \frac{M_{ob}\, s}{E J_t}$$

The rotation of the tangent of the intermediate beam is like case (3a).

$$(2 + \nu - \gamma_1)\, M_{oa} + 2\nu\, M_{ob} = -\frac{q_a\, l^2}{4} - \overline{M_{1a}}$$

$$\nu M_{oa} + (2 + 2\nu + 2\mu - \gamma_1) M_{ob} = -\frac{q_b \ell^2}{4} - \overline{M}_{1b} \qquad (14)$$

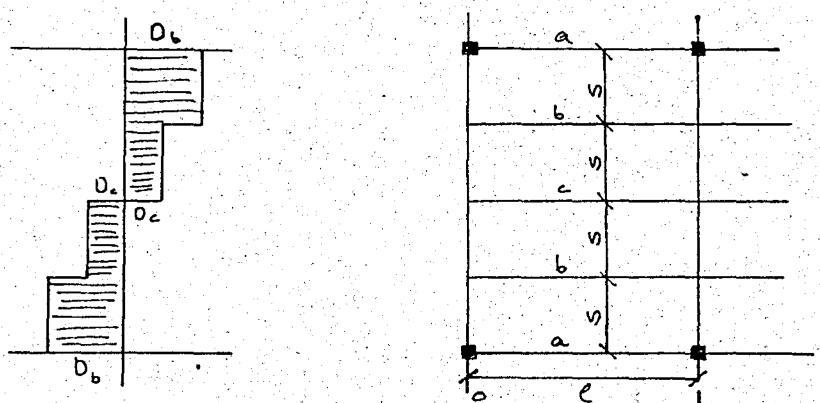
Torsional moment in the space between beams (a) and (b)

$$D = M_{ob} \qquad (15)$$

Moment on the edge beam

$$M_s = M_{oa} + 2 M_{ob} \qquad (16)$$

3c) Case of three intermediate beam (Figure 6)



The rotation of the edge column

$$\alpha = \frac{(M_{oa} + 2M_{ob} + M_{oc})\, h_u}{(1+c)\,\overline{\alpha_u}\, E J_u}$$

The torsional rotation of the edge beam at the intersection of (b)

$$\vartheta_b = \frac{7}{6}\ \frac{(2M_{ob} + M_{oc})\, s}{E J_t}$$

The torsional rotation of the edge beam at the intersection of (c)

$$\vartheta_c = \frac{14}{6}\ \frac{(M_{ob} + M_{oc})\, s}{E J_t}$$

The rotation of the tangent of the slab is like in (3a), to this rotation for beam (c) with same value is used, then conditions are

$$\alpha + \varphi_a = 0$$

$$\alpha + \vartheta_b + \varphi_b = 0$$

$$\alpha + \vartheta_c + \varphi_c = 0$$

substituting these values into equation I

$$
\begin{aligned}
(2+v-\gamma_1) M_{oa} + 2v\, M_{ob} + v\, M_{oc} &= -\frac{q_a l^2}{4} - \overline{M_{1a}} \\
v\, M_{oa} + (2+2v+2u-\gamma_1) M_{ob} + (v+u) M_{oc} &= -\frac{q_b l^2}{4} - \overline{M_{1b}} \qquad (17) \\
v\, M_{oa} + (2v+2u) M_{ob} + (2+v+2u-\gamma_1) M_{oc} &= -\frac{q_c l^2}{4} - \overline{M_{1c}}
\end{aligned}
$$

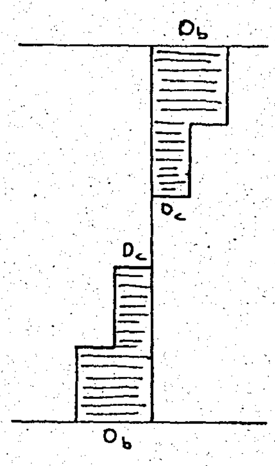
Solutions of these equations give fixed end moments of the beam.

Torsional moment between (a–b) $D_b = M_{ob} + \frac{M_{oc}}{2}$ (18)

(b–c) $D_c = \frac{M_{oc}}{2}$ (19) Fig. 6

End columns take $M_s = M_{oa} + 2M_{ob}\ M_{oc}$ (20)

3d) Case of four intermediate beams (Figure 7)



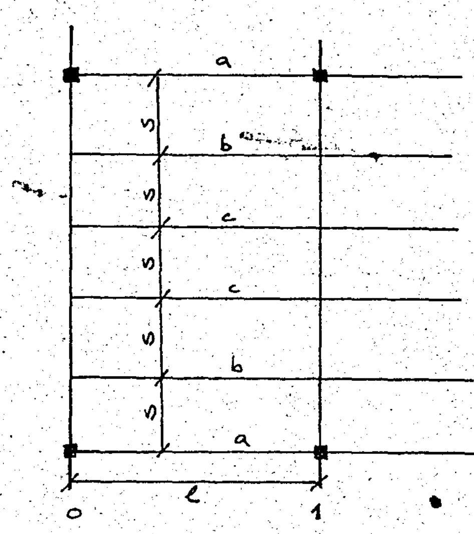


Fig. 7

Rotation of the edge column

$$\alpha = \frac{(M_{oa} + 2M_{ob} + 2M_{oc})\, h_u}{(1+c)\, \overline{\alpha_u}\, E J_u}$$

The angle of torsional rotation at (b)

$$\vartheta_b = \frac{14}{6} \cdot \frac{(M_{ob} + M_{oc})\, s}{E J_t}$$

The angle of torsional rotation at (c)

$$\vartheta_c = \frac{14}{6} \cdot \frac{(M_{ob} + 2M_{oc})\, s}{E J_t}$$

$$(2+v-\gamma_1) M_{oa} + 2v M_{ob} + 2v M_{oc} = -\frac{q_a l^2}{4} - \overline{M}_{1a}$$

$$v M_{oa} + (2+2v+2\mu-\gamma_1) M_{ob} + (2v+2\mu) M_{oc} = -\frac{q_b l^2}{4} - \overline{M}_{1b} \qquad (21)$$

$$v M_{oa} + (2v+2\mu) M_{ob} + (2+2v+4\mu-\gamma_1) M_{oc} = -\frac{q_b l^2}{4} - \overline{M}_{1c}$$

Torsional moment between beams (a) and (b)

$$D_b = M_{ob} + M_{oc} \qquad (22)$$

Torsional moment between beams (b) and (c)

$$D_c = M_{oc} \qquad (23)$$

End columns $M_s = M_{oa} + 2(M_{ob} + M_{oc})$ (24)

If there are more than four intermediate beams, the way as explained in part (2) should be followed.

4 - Examples:

4 a) Edge beam and slab (B 300)

There are many equal spans in x direction. Length in (y) direction is taken as (1 meter)

Slab : $g = 0.4\ t/m$ $p = 0.5\ t/m$ $q = 0.9\ t/m$
$d = 1.4\ dm$ $J = 2.3\ dm^4$ $l = 4.5\ m$ $s = 3.0\ m$

Edge beam:

$b = 1.8\ dm$ $d = 9.5\ dm$ $J_t = 14.9\ dm^4$

Edge columns: $b = 5.0\ dm$ $d = 2.2\ dm$ $J_o = J_u = 4.4\ dm^4$

Combination is like in figure 3d. $h_o = h_u = 3.2\ m$ $c = 1$ $\alpha_u = 4$

$$k = \sqrt{\frac{14}{(2-0.267)4.5}\ \frac{2.3}{14.9}} = 0.52 \qquad k_s = 0.52 \times 3.0 = 1.56$$

$$\overline{m}_1 = -(0.1071 \times 0.4 + 0.0536 \times 0.5)\ \overline{4.5}^2 = -1.41\ tm \qquad \gamma_1 = 0.267$$

for y = s   tgh 1.56 = 0.92   cosh 1.56 = 2.48

The coefficient of fixity

$$v = \frac{2.3}{4.4} \frac{3.2}{4.5} = 0.37 \quad (eq. 6)$$

$$m_{os}\left(\frac{12 \times 0.37}{(1+1)\,4} \frac{0.92}{0.52} + 2 - 0.267\right) = -\frac{0.9 \times \overline{4.5}^2}{4} - (-1.41) \quad (eq. 7)$$

$m_{os} = 1.16$ tm

$m_{1s} = 0.267\,(-1.16) - 1.41 = -1.10$ tm (eq.1)

$$D_s = 1.16 \frac{0.92}{0.52} = 2.05 \quad (eq. 4)$$ torsional moment

$M_s = 2 \times 2.05 = 4.10$ tm (eq.8)

Torsional moment near the column

$Y_r = (3.00 - 0.25) = 2.75$ m

$K_{yr} = 0.52 \times 2.75 = 1.43$ sinh 1.43 = 1.97

$$D_r = \frac{1.16}{0.52} \frac{1.97}{2.48} = 1.76 \text{ tm} \quad (eq. 3)$$

for y = o   cosh o = 1

$$m_{oo} = -1.16 \frac{1}{2.48} = -0.46 \text{ tm} \quad (eq. 2)$$

$$m_{1o} = -0.267\,(-0.46) - 1.41 = -1.29 \text{ tm} \quad (eq. 1)$$

4b) Equal spans in x direction, and three intermediate beams in y direction as in figure 6.

slab: $s = 1.50$ m $g = 0.60$ t/m $p = 0.75$ t/m $q = 1.35$ t/m $\ell = 4.50$ m

$d = 0.8$ dm $b_o = 2.0$ dm $d_o = 2.8$ dm $J = 6.2$ dm^4

edge beam: $b = 2.2$ dm $d = 5.0$ dm $J_t = 12.2$ dm^4

column: same as example 1.

4b$_1$) $\overline{M}_{1a} = \overline{M}_{1b} = \overline{M}_{1c} = -1.5 \times 1.41 = -2.11$ tm

$\gamma_{1a} = \gamma_{1b} = \gamma_{1c} = 0.267 \approx 0.27$

4b$_2$)

$$v = \frac{6}{(1+1)\,4}\;\frac{6.2}{4.4}\;\frac{3.2}{4.5} = 0.75 \quad (\text{eq. } 9)$$

$$\mu = 7\,\frac{6.2}{12.2}\;\frac{1.5}{4.5} = 1.18 \quad (\text{eq. } 10)$$

$$-\frac{q\ell^2}{4} - \overline{M}_1 = -\frac{1.5 \times \overline{4.5}^2}{4} - (-2.11) = -5.48 \text{ tm}$$

Moment equilibrium equation :

$$2.48\, M_{oa} + 1.50\, M_{ob} + 0.75\, M_{oc} = -5.48$$

$$0.75\, M_{oa} + 5.59\, M_{ob} + 1.93\, M_{oc} = -5.48$$

$$0.75\, M_{oa} + 3.86\, M_{ob} + 4.84\, M_{oc} = -5.48$$

$M_{oa} = -1.72$ tm $\quad M_{ob} = -0.62$ tm $\quad M_{oc} = -0.37$ tm

From eq. 1

$$M_{1a} = -0.267\,(-1.72) - 2.11 = -1.65 \text{ tm}$$

$$M_{1b} = -0.267\,(-0.62) - 2.11 = -1.94 \text{ tm}$$

$$M_{1c} = -0.267\,(-0.37) - 2.11 = -2.01 \text{ tm}$$

$$D_b = 0.62 + \frac{0.37}{2} = 0.80 \text{ tm} \quad (\text{eq. } 18)$$

$$M_s = -1.72 - 2 \times 0.62 - 0.37 = -3.33 \text{ tm}$$

General Assumptions:

1) Slabs are rigid to bending in perpendicular direction to beams and are without bending in parallel direction.

2) The vertical deformation and horizontal bending rigidity of the beam are neglected.

3) Edge beams and slab constitutes a homogeneous section. Torsion from combined forces is neglected.

4) Edge beams are assumed to be only under torsional effects.

5) Torsional rigidity of the slab is neglected.

Definitions:

E = Modulus of Elasticity (t/m^2)

$G = \dfrac{1}{2(1+u)} E$ Shear modulus (t/m^2)

J = Moment of inertia of the slab for 1 m width (edge slab) (m^4/m)

l = Length of the edge slab (m)

J_t = Torsional factor of the edge beam (m^4)

for rectangular sections

l_y = length of the edge beam (m)

y = variable distance (m)

$\eta = y/l_y$ variable without units

D = Torsional moment on the edge beam on the point n (mt)

D_s = Torsional moment on the edge beam on the s support (mt)

$\overline{D_s}$ = Torsional fixed end moment on the edge beam on support s (mt)

$\left.\begin{matrix} m_B^* \\ m_C^* \end{matrix}\right\}$ = End moment of the slab due to external loads, considering the support to rotate freely (mt/m)

m_η = end moment of the slab at point η (mt/m)

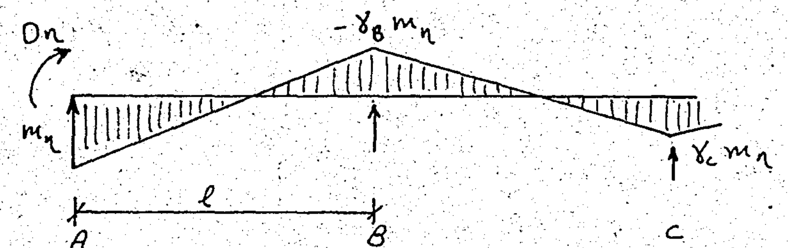
m_s = end moment of the slab at support s (mt/m)

$\overline{m_s}$ = Summation of the fixed end moments of the slab at support s (mt/m)

m_o = end moment of the slab at the middle of the span

$\left.\begin{array}{l}\varphi_\eta \\ \varphi_s\end{array}\right\}$ = The angle of rotation of the slab around A axis

γ_B, γ_c = Moment coeficients (without unit)



(Fig. 7) Distribution of m_η caused by D_η

Supports A.B.C. etc. are considered to be simple supports. The relation between torsional moment D_η and the end moment of the slab (m_η) at point η are discussed below.

The rotation of the slab around A axis

$$\varphi_\eta = \frac{(2+\gamma_B)\,\ell}{6EJ}\, m_\eta \qquad (1)$$

For the stiffness of the slab 1m width is taken, using Kani diefinition:

$$K = \frac{3}{2}\,\frac{J}{(2+\gamma_B)\,\ell} \qquad (2)$$

For fixed support $\gamma_B = -0.5$ and (eq.2) reduces to

$$K = \frac{3}{2}\,\frac{J}{(2-0.5)\,\ell} = \frac{J}{\ell}$$

For simple support $\gamma_B = 0$ (eq.2) becomes

$$K = \frac{3}{2}\,\frac{J}{2\ell} = \frac{3}{4}\,\frac{J}{\ell}$$

From (eq.1) and (eq.2) $\varphi_\eta = \frac{1}{4EK}\, m_\eta \qquad (3)$ is found

a) The torsional unit rotational angle (ϑ) is equal to the derivative of torsional rotational angle (φ_{η}), from this relation torsional moment on the edge can be found.

$$\vartheta = \frac{D_\eta}{G J_t} = \frac{d \varphi_\eta}{d y} = \frac{d \varphi_\eta}{\ell_y \, d\eta}$$

or

$$\frac{d \varphi_\eta}{d\eta} = \frac{\ell_y D_\eta}{G J_t} \qquad (4)$$

b) The derivative of torsional moment D_η is equal to the edge moment (m_η) of the slab.

$$\frac{d D_\eta}{d y} = m_\eta = \frac{1}{\ell_y} \frac{d D_\eta}{d \eta}$$

or

$$\frac{d D_\eta}{d \eta} = \ell_y m_\eta \qquad (5)$$

using

$$\lambda = \ell_y \sqrt{\frac{8(1+\mu)K}{J_t}} \qquad (6)$$

for concrete $\mu = 1/6$

$$\lambda = 3.055 \, \ell_y \sqrt{\frac{K}{J_t}} \qquad (6a)$$

and (eq.3) becomes

$$\frac{d^2 m_\eta}{d\eta^2} - \lambda^2 m_\eta = 0 \qquad (7)$$

solution of (7) does not depend on (η and x)

solution gives

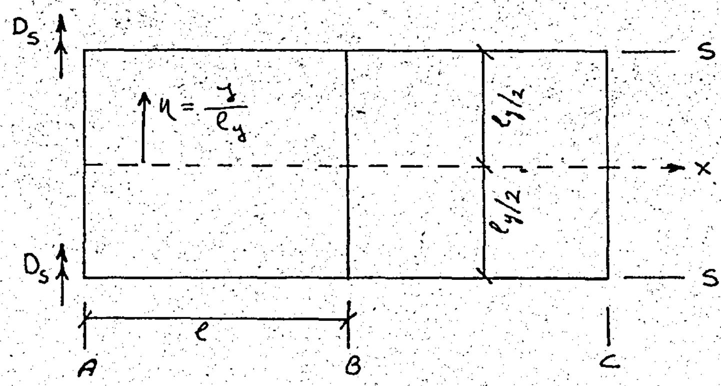
$$4EK \varphi_\eta = m_\eta = C_1 \cosh \lambda\eta + C_2 \sinh \lambda\eta \qquad (8)$$

referring to eq.3, 4, and 6

$$D_\eta = \frac{\ell_y}{\lambda} \left[C_1 \sinh \lambda\eta + C_2 \cosh \lambda\eta \right] \qquad (9)$$

is found.

2) Calculationsfor the coefficients for a one span edge beam subjected to D_s external torsional moment at supports (s) as shown at figure 3



(Fig. 2)

External torsional moment D_s on (S axis)

Under this loading there is symmetry with respect to axis = o, so must also have symmetry with respect to axis =o. antisymmetric part C_2 in equation B must equal to zero, so from boundary condition :

$$(D_\eta)_{\eta=\frac{1}{2}} = D_s$$

$$C_1 = \frac{D_s \lambda}{\ell_y \sinh \lambda/2}$$

$$D_y = D_s \frac{\sinh \lambda \eta}{\sinh \lambda/2}$$

$$4EK\varphi_\eta = m_\eta = \frac{D_s\lambda}{\ell_y} \frac{\cosh \lambda \eta}{\sinh \lambda/2}$$

And as a special condition:

$$\left| 4EK\varphi_\eta \right|_{\eta=\frac{1}{2}} = 4EK\varphi_s = m_\eta \Big|_{\eta=\frac{1}{2}} = m_s \frac{D_s}{\ell_y/2} \frac{\lambda/2}{\operatorname{tgh} \lambda/2}$$ is found.

$$\Phi_b = \frac{\tanh \lambda/2}{\lambda/2} \qquad (10)$$

together with equation 10,

$$b = \frac{1}{2} \ell_y \Phi_b \qquad (11)$$

$$K_R = bK \qquad (12)$$

The slab can be thought as a beam with width (b) and bending rigidity KR:

$$\varphi_s = \frac{1}{4EK_R} D_s$$

$$m_s = \frac{D_s}{b}$$

$$\Phi_m = \frac{\cosh \lambda \eta}{\cosh \lambda/2} \quad \text{(Fig. 4)} \qquad (15)$$

$$m_\eta = m_s \Phi_m \qquad (16)$$

Which gives the variation of end moment of the slab along η

$$\Phi_{m_o} = \frac{1}{\cosh \lambda/2} \quad \text{(see Fig. 3)} \qquad (17)$$

at the middle of the slab $m_o = m_s \Phi_{m_o}$. (18)

Torsional moment on the edge beam is found using:

$$\Phi_D = \frac{\sinh \lambda \eta}{\sinh \lambda/2} \quad \text{(see Fig. 5)} \qquad (19)$$

$$D_\eta = D_s \Phi_D \qquad (20)$$

3) Fixed end moment on the edge beam due to load (q) on the slab: (Slab moment is considered to consist of the followings)

a) m^* moments on A,B,and C axis (Fig. 6) which are considered to be simple supported.

b) $\bar{\bar{m}}_\eta$ moments (Fig. 7 and 8) due to slab being fixed to edge beam along (s) axis. In this complete fixity φ_s , the rotation of the edge beam around (s) axis is zero.

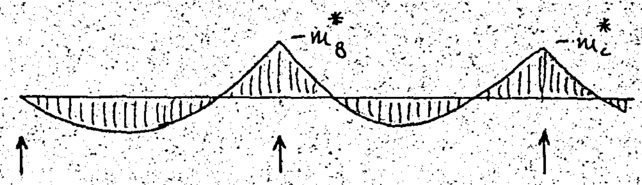


Fig. 6

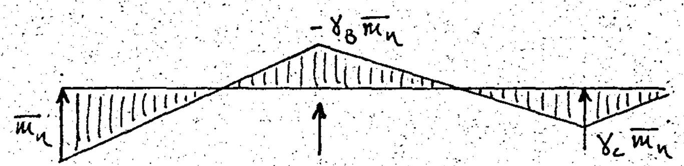


Fig. 7

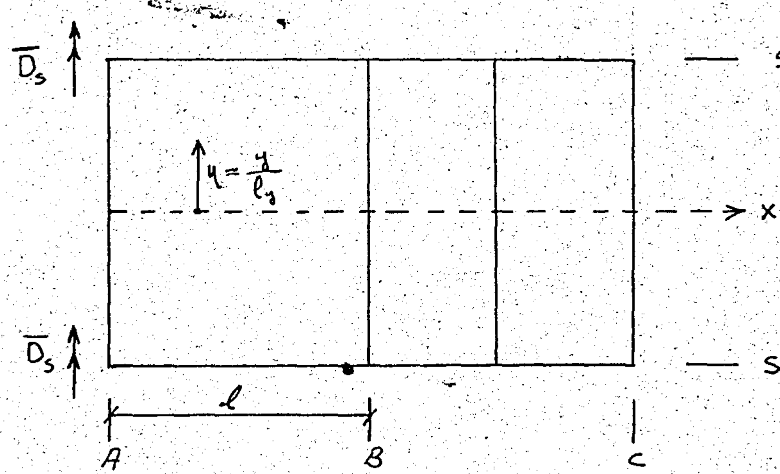


Fig. 8

$$\varphi_s = o = \frac{(2+\gamma_B)\, l}{6EJ} \overline{m}_s + \left(\frac{q l^2}{\Delta} + m_B^* \right) \frac{l}{6EJ} \qquad (21)$$

$$\overline{m}_s = - \frac{1}{(2+\gamma_B)} \left(\frac{q l^2}{\Delta} + m_B^* \right) \qquad (22)$$ is found.

If there is a moment ( ) due to an overhang at A axis:

$$\sum \overline{m}_s = - \frac{1}{(2+\gamma_B)} \left(\frac{q l^2}{\Delta} + 2\, m_A^* + m_B^* \right) \qquad (22a)$$

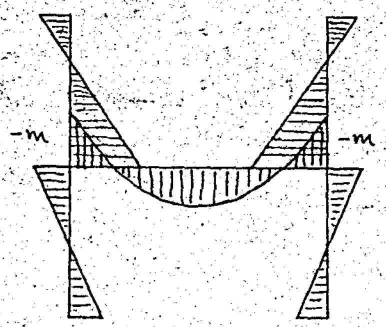
Torsional moment at (s) axis :

$$D_s = b\, \overline{m}_s \qquad (23)$$

$$\sum \overline{D}_s = b \sum \overline{m}_s \qquad (23a)$$ is found.

4) **Special cases:**

Symmetrical one span slab under symmetrical (q) loading (Fig. 9):



$$\gamma_B = +1$$

$$K = \frac{1}{2}\,\frac{J}{\ell} \qquad (24)$$

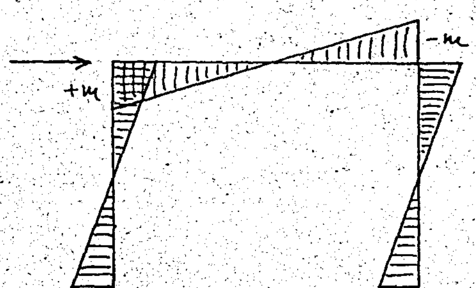
$$m_B^* = 0 \quad \text{from eq. 22}$$

$$\overline{m}_s = -q\,\frac{\ell^2}{12} \qquad (25)$$

Fig. 9

5) **Special condition:**

Symmetrical one span slab under antisymmetrical loading (Fig. 10):



$$\gamma_B = -1$$

$$K = \frac{3}{2}\,\frac{J}{\ell} \qquad (26)$$

Fig. 10

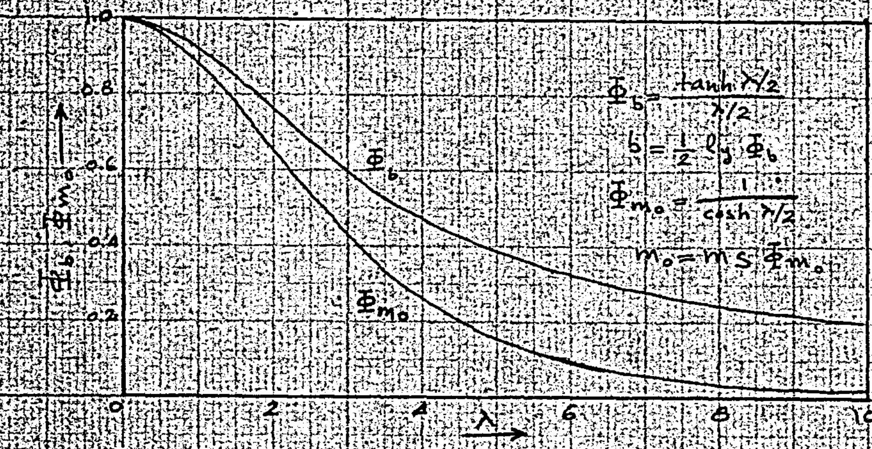


Fig. 3

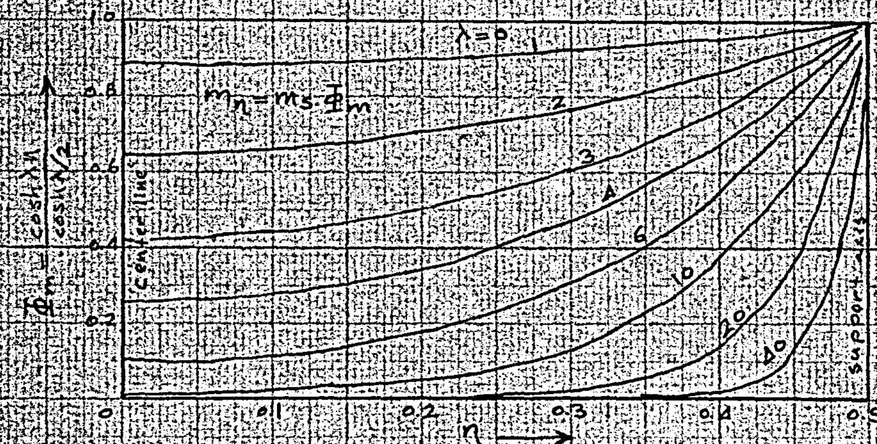


Fig. 4

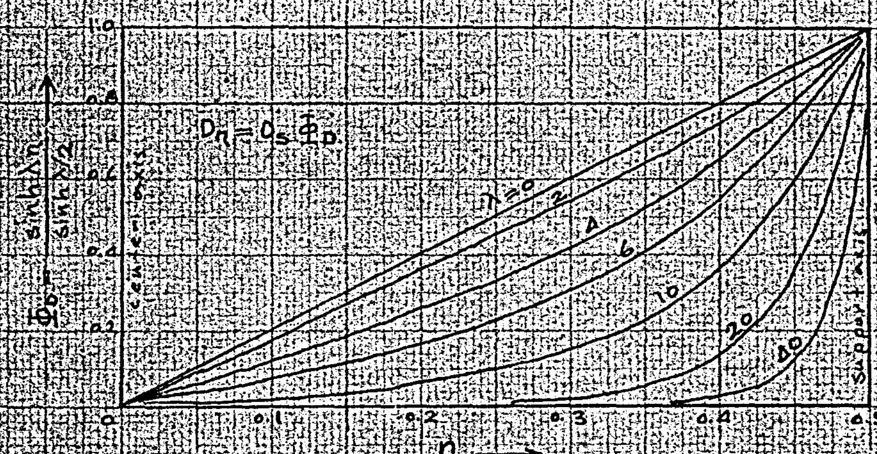


Fig. 5

Example (1)

In order to show the conformity of the method with experiments, the same dimensions used by Halbritter in his experiments are taken.

a) Edge beam : $\ell_{y_1} = \ell_{y_2} = 6.00\ m \quad b/d = 18/95$

$$J_t = 14.9 \times 10^{-4}\ m^4$$

b) Plate $\quad d = 14\ cm \quad \ell = \ell_u = 4.50\ m \quad q = 0.40\ t/m^2 \quad p = 0.50\ t/m^2$

$$J = 2.3 \times 10^{-4}\ m^4/m \qquad \gamma_B = -0.268$$

$$2 + \gamma_B = 1.732$$

$$K = \frac{3}{2}\ \frac{2.3 \times 10^{-4}}{1.732 \times 4.50} = 0.444 \times 10^{-4}\ m^3/m \qquad (eq.\ 2)$$

$$\lambda = 6.00 \times 3.055 \sqrt{\frac{0.444}{14.9}} = 3.16 \quad (eq.\ 6a)$$

$$b = \frac{1}{2}\ 6.00 \times 0.59 = 1.77\ m \qquad (fig.\ 3)$$

$$K_\ell = 1.77 \times 0.444 \times 10^{-4} = 0.79 \times 10^{-4}\ m^3 \qquad (eq.\ 12)$$

The sum of torsional stiffnesses

$$\sum K_\ell = 2.0\ 79 \times 10^{-4} = 1.58 \times 10^{-4}\ m^3,$$

$$m_B^* = -0.107 \times 0.40 \times \overline{4.50}^2 - 0.053 \times 0.50 \times \overline{4.50}^2 = -1.41\ mt/m$$

$$\overline{m_s} = -\frac{1}{1.732}\left(\frac{0.9 \times \overline{4.50}^2}{4} - 1.41\right) = -1.81\ mt/m \qquad (eq.\ 22)$$

c) Columns $\quad b/d = 50/22 \quad h = 3.20\ m \quad J_c = 4.4 \times 10^{-4}\ m^4$

$$K_o = K_u = 4.4 \times 10^{-4}\ \frac{1}{3.20} = 1.38 \times 10^{-4}\ m^3$$

d) The ratio of the column stiffnesses to column stif. plus slab stiff.

$$\varepsilon = \frac{K_o + K_u}{K_o + K_u + \sum K_\ell} = \frac{2 \times 1.38}{2 \times 1.38 + 1.58}$$

e) Resulted moments (Fig. 11)

$$m_s = \varepsilon \, \overline{m}_s = -0.64 \times 1.81 = -1.16 \text{ mt/m}$$

$$m_o = m_s \, \Phi_{m_o} = -1.16 \times 0.40 = -0.46 \text{ mt/m} \quad \text{(fig. 3)}$$

$$D_s = b \times m_s = -1.77 \times 1.16 = -2.05 \text{ mt} \quad \text{(eq. 14)}$$

Column moments

$$M_{so} = -\frac{K_o}{K_o + K_u} \, 2D_s = \frac{1}{2} \, 2 \times 2.05 = 2.05 \text{ mt}$$

$$M_{su} = \frac{K_u}{K_o + K_u} \, 2D_s = -\frac{1}{2} \, 2 \times 2.05 = -2.05 \text{ mt}$$

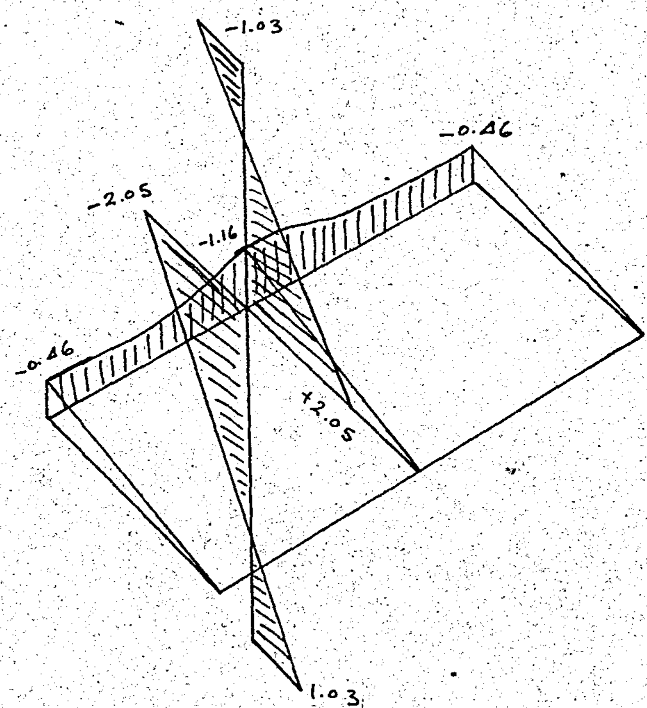


Fig. 11

Ex. 2

Two story frame, l_{y_1} and l_{y_2} chane periodically

a) $l_{y_1} = 3.00\,m$ $\quad l_{y_2} = 6.00\,m$ $\quad b/d = 60/40$ $\quad J_t = 78.3 \times 10^{-4}\,m^4$

b) Slab $\quad d_o = 20 + 8 = 28\,cm$ $\quad b_o = 20\,cm/m$

$l = l_n = 8.00\,m$ $\quad g = 0.40\,t/m^2$ $\quad p = 0.35\,t/m^2$

$J = 7.1 \times 10^{-4}\,m^3/m$ $\quad \gamma_B = -0.268$ $\quad 2 + \gamma_B = 1.732$

$$K = \frac{3}{2}\,\frac{7.1 \times 10^{-4}}{1.732 \times 8.00} = 0.769 \times 10^{-4}/m \quad (eq.\ 2)$$

$$m^* = -0.105 \times 0.40 \times \overline{8.00}^2 - 0.053 \times 0.35 \times \overline{8.00}^2 = -3.88\,mt/m$$

$$\overline{m_s} = -\frac{1}{1.732}\left(\frac{0.75 \times \overline{8.00}^2}{4} - 3.88\right) = -4.69\,mt/m \quad (eq.\ 22)$$

Slab 1 : $\lambda_1 = 3.00 \times 3.055\sqrt{\dfrac{0.769}{73.8}} = 3.00 \times 0.312 = 0.934$ (eq. 6a)

$b_1 = \frac{1}{2}\,3.00 \times 0.953 = 1.43\,m$ (eq. 11)

$K_{R_1} = 1.43 \times 0.769 \times 10^{-4} = 1.10 \times 10^{-4}\,m^3$ (eq. 12)

$\overline{D_{s_1}} = -1.43 \times 4.69 = -6.71\,mt$ (eq. 23)

Slab 2 : $\lambda_2 = 6.00 \times 0.312 = 1.868$ (eq. 6a)

$b_2 = \frac{1}{2}\,6.00 \times 0.784 = 2.35\,m$ (eq. 11)

$K_{R_2} = 2.35 \times 0.769 \times 10^{-4} = 1.81 \times 10^{-4}\,m^3$ (eq. 12)

$\overline{D_{s_2}} = -2.35 \times 4.69 = -11.03\,mt$ (eq. 23)

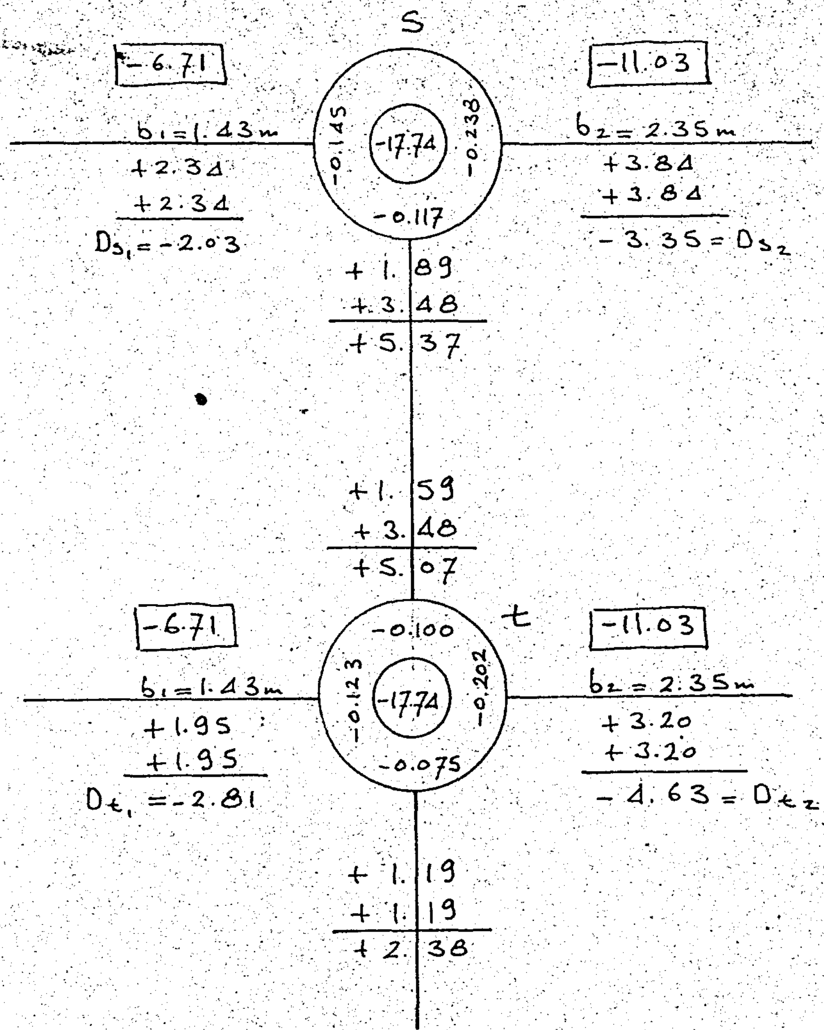
c) Columns $\quad b/d = 40/20\,cm$ $\quad h = 3.00\,m$ $\quad J_c = 2.67 \times 10^{-4}\,m^4$

$$K_o = 2.67 \times 10^{-4}\,\frac{1}{3.00} = 0.89 \times 10^{-4}\,m^3$$

$$K_u = \frac{3}{4}\,0.89 \times 10^{-4} = 0.67 \times 10^{-4}\,m^3$$

d) Distribution and carry over factors (KANI)

Joint		K_R	μ
	$Slab_1$	1.10	−0.145
	$Slab_2$	1.81	−0.238
	$Column_o$	0.89	−0.117
	$Column_u$	—	—
	Σ	3.80	−0.500
	$Slab_1$	1.10	−0.123
	$Slab_2$	1.81	−0.202
	$Column_o$	0.89	−0.100
	$Column_u$	0.67	−0.075
	Σ	4.47	−0.500



$$m_s = -2.03/1.43 = 3.35/2.35 = -1.43 \text{ mt/m}$$

$$m_{s_{10}} = -1.43 \times 0.90 = -1.29 \text{ mt/m}$$

$$m_{s_{20}} = -1.43 \times 0.68 = -0.97 \text{ mt/m}$$

$$m_t = -2.81/1.43 = -4.63/2.35 = -1.97 \text{ mt/m}$$

$$m_{t_{10}} = -1.97 \times 0.90 = -1.77 \text{ mt/m}$$

$$m_{t_{20}} = -1.97 \times 0.68 = -1.34 \text{ mt/m}$$

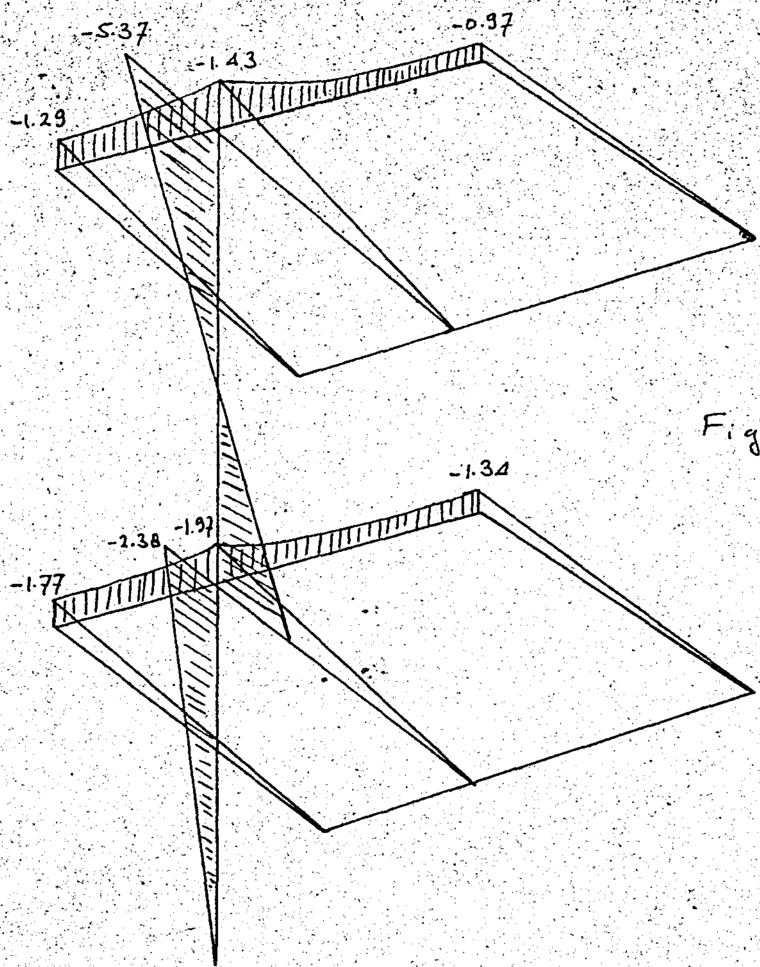


Fig. 12

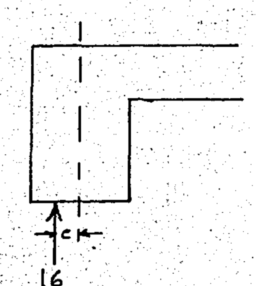
Example 3

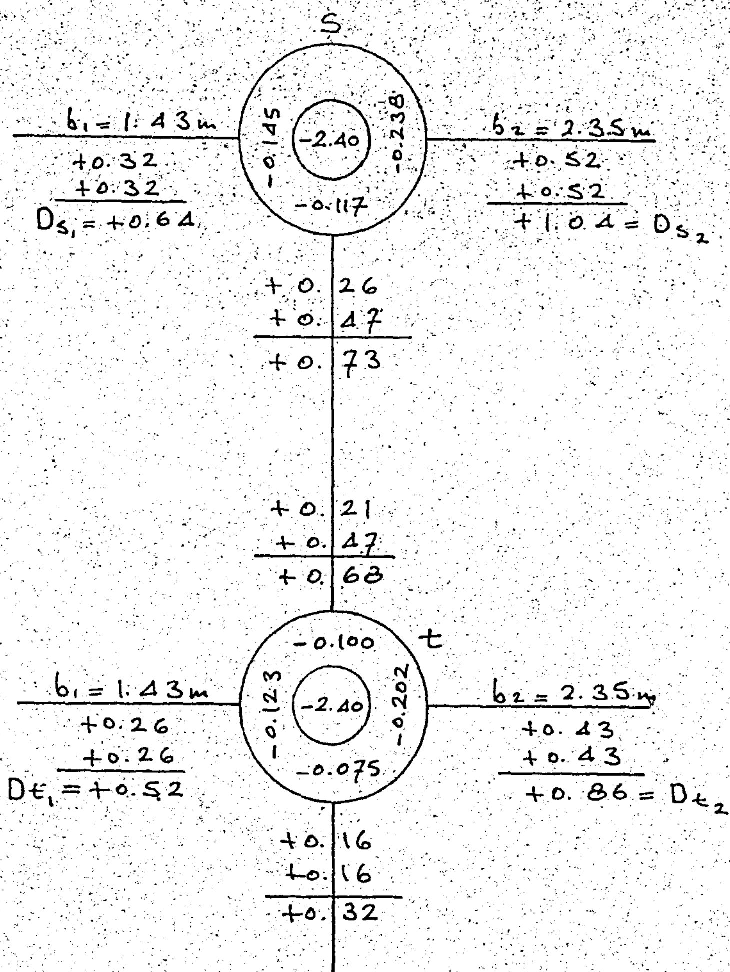
$$M_s = -Ae = -16 \times 0.15 = -2.40\,mt$$

eccentricity $e = 15$ cm

Shear on the beam is 16 t

Same dimensions as at ex. 2





$m_s = 0.64/1.43 = 1.04/2.35 = 0.44 \ mt/m$

$m_{s_{10}} = 0.45 \times 0.90 = 0.40 \ mt/m$ (fig. 3)

$m_{s_{20}} = 0.45 \times 0.68 = 0.30 \ mt/m$ (fig. 3)

$m_t = 0.52/1.43 = 0.86/2.35 = 0.36 \ mt/m$

$m_{t_{10}} = 0.36 \times 0.90 = 0.32 \ mt/m$ (fig. 3)

$m_{t_{20}} = 0.36 \times 0.68 = 0.25 \ mt/m$

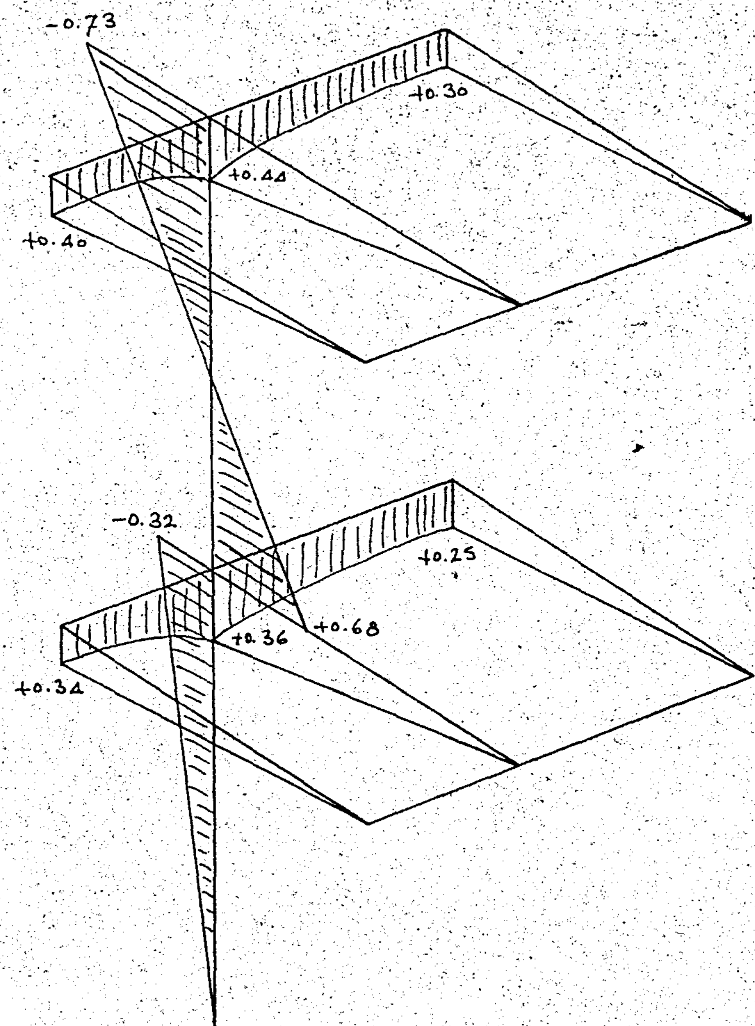
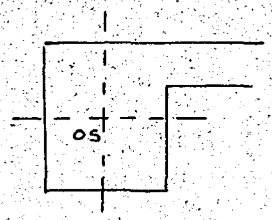
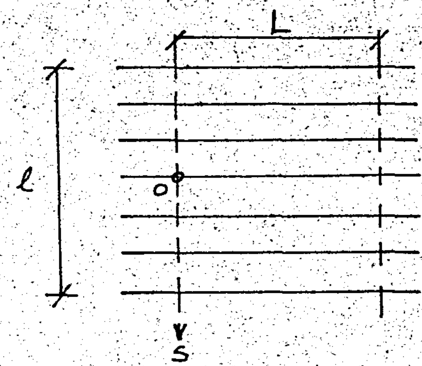


Fig. 13

Analysis of Torsional Rigidity





Notations:

l = Length of the perpendicular beam.

L = Length of the ribbed floor.

Os = Axis, which passes through the center of gravity of the perpendicular beam, middle of the beam as its starting point.

φ = Angle of rotation of the perpendicular beams, considering clockwise rotation as (+)

I = Moment of inertia of the slab for 1 m. lenght.

GJ_t = Rigidity of the perpendicular beam.

R = Summation of the bending stiffness of the upper and lower columns.

Torsional Rigidities of the end sections:

At the supports of the ribs, bending moments of M_e is created. This moment is obtained by the superposition of the two effects.

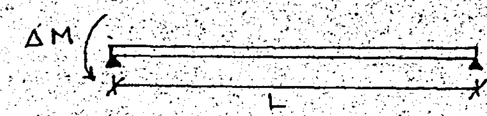
1) Fixed end moment M_o, when the rotation of the support is zero (absolute fixation).

2) $\Delta M = \frac{S\varphi}{3}$ Which is the moment induced by a rotation of magnitude φ of the support. Where (S) is the stiffness of the 1 m. wide slab.

For $I = \text{const.}$ $\quad S = \frac{6EJ}{L} \quad \Delta M = \frac{2EJ\varphi}{L} \quad \varphi = \frac{\Delta M L}{2EJ}$

Then the fixed end moment of the slab is:

$$M_s = -M_o + \frac{s\varphi}{3}$$



$$\varphi = \frac{\Delta M\, L}{2EJ}$$

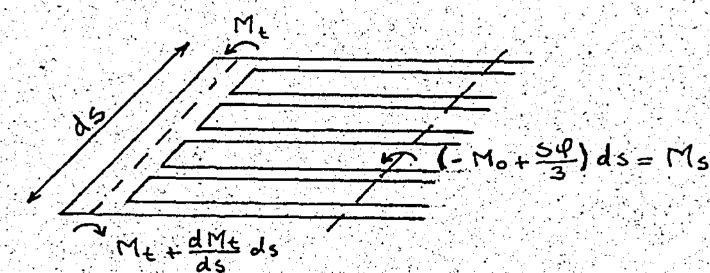
The equation of equilibrium for the perpendicular beam:

1) $$M_t + \left(-M_o + \frac{s\varphi}{3}\right) ds - \left(M_t + \frac{dM_t}{ds} ds\right) = 0$$

2) $$\frac{dM_t}{ds} = -M_o + \frac{s\varphi}{3}$$

Deformation equation of the perpendicular beam:

3) $$M_t = G J_t \frac{d\varphi}{ds}$$



Taking derivative with respect to (S)

$$\frac{dM_t}{ds} = G J_t \frac{d^2\varphi}{ds^2}$$

$$G J_t \frac{d^2\varphi}{ds^2} = -M_o + \frac{s\varphi}{3}$$

$$\frac{d^2\varphi}{ds^2} - \frac{s\varphi}{3GJ_t} = -\frac{M_o}{GJ_t}$$ taking $\alpha_1^2 = \frac{s}{3GJ_t}$

$$\frac{d^2\varphi}{ds^2} - \alpha_1^2 \varphi = -\frac{M_o}{GJ_t}$$ is obtained.

Solution of this:

4) $$\varphi = A e^{\alpha_1 s} + B e^{-\alpha_1 s} + \frac{M_o}{GJ_t \alpha_1^2}$$

Boundary conditions

for S = o from symmetry $M_t = 0$ $\frac{d\varphi}{ds} = 0$

$$\frac{d\varphi}{ds} = \alpha_1 A e^{\alpha_1 s} - \alpha_1 B e^{-\alpha_1 s} = 0 \qquad B = A$$

For $s = \ell/2$, if joint rotates an amount of φ, ribs apply a moment of $-R\varphi$ to the beam. This moment must be in equilibrium with torsional moment M_t applied on both ends.

$$M_t = -\frac{R\varphi}{2}$$

from "3"

$$G J_t \frac{d\varphi}{ds} = -\frac{R}{2}\varphi$$

This substituted into (4), A and B are solved.

$$A = B = \frac{-M_o \frac{R}{G J_t \alpha_1^2}}{4 G J_t \alpha_1 \, sh\alpha_1 \frac{\ell}{2} + 2R \, ch\alpha_1 \frac{\ell}{2}}$$

$$\varphi = \frac{-M_o \frac{R}{G J_t \alpha_1^2}}{4 G J_t \alpha_1 \, sh\alpha_1 \frac{\ell}{2} + 2R \, ch\alpha_1 \frac{\ell}{2}} \left(e^{\alpha_1 s} + e^{-\alpha_1 s}\right) + \frac{M_o}{G J_t \alpha_1^2}$$

Moment in the columns (M_s) from $M_s = \varphi R$ with $S = \frac{\ell}{2}$

5) $$M_s = \frac{M_o \ell}{\frac{2EJ}{R}\frac{\ell}{L} + \frac{\beta_1}{th\beta_1}} \qquad \beta_1^2 = \frac{EJ}{GJ_t}\frac{\ell^2}{2L}$$

from $M_s = -M_o + \frac{s\varphi}{3}$

6) $$M_s = -M_o \frac{ch\beta_1 \frac{s}{\ell/2}}{\frac{2EJ}{R}\frac{\ell}{L}\frac{sh\beta_1}{\beta_1} + ch\beta_1}$$

If equation (5) is solved for M_o and substituted into equation (6)

M_s for 1 m. width:

$$M_s = -\frac{M_s}{\ell}\beta_1 \frac{ch^2\beta_1 \, s/\ell}{sh\beta_1}$$

taking average value $\frac{M_s}{\ell} = M_{s,m}$

7) $$M_s = -M_{s,m}\,\beta_1 \frac{ch^2\beta_1 \, s/\ell}{sh\beta_1}$$ is obtained

The values of $\frac{M_s}{M_o l}$ depending on $\gamma = \frac{2EJ}{R} \frac{l}{L}$ and β, is given in diagram (1)

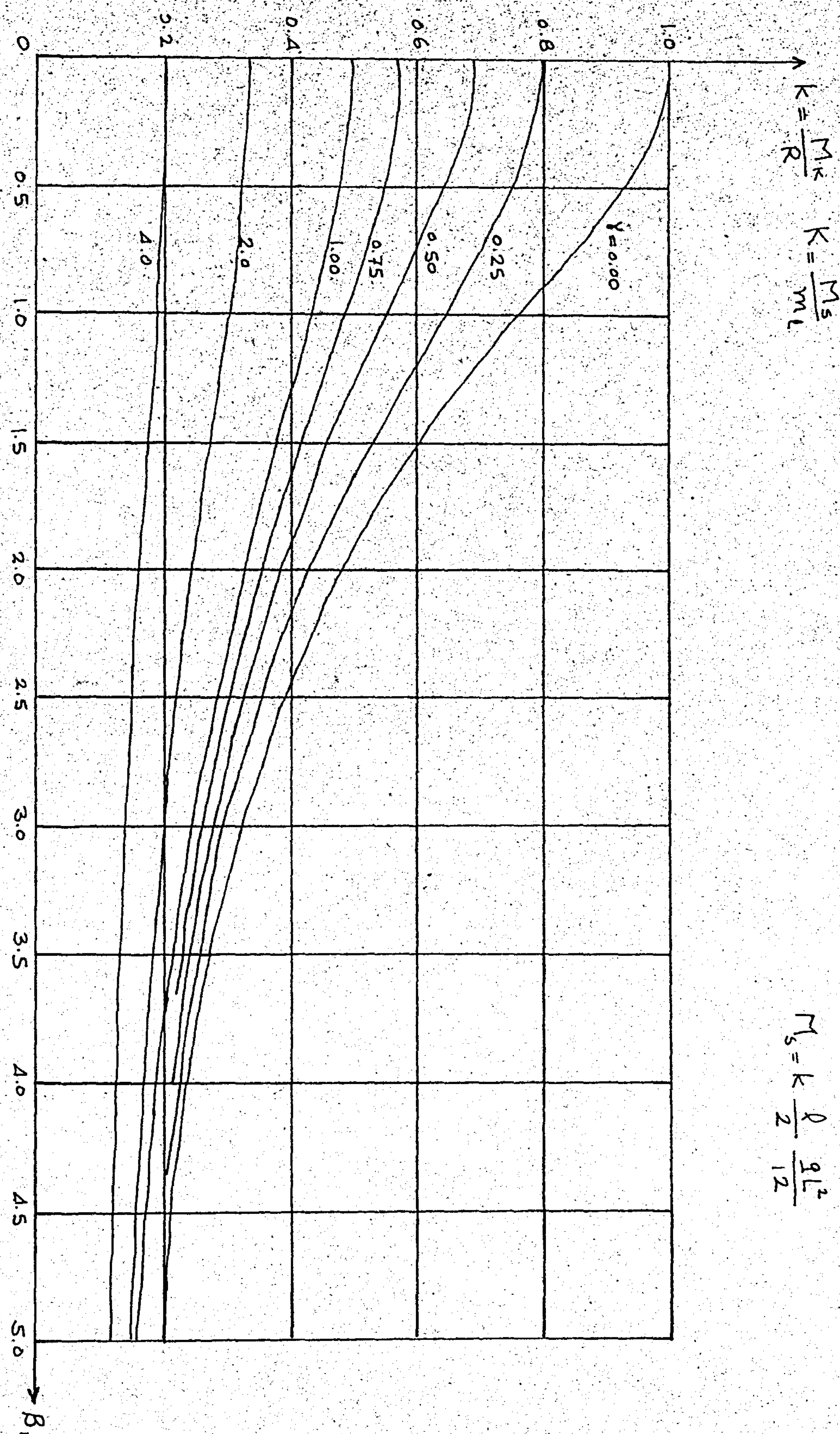
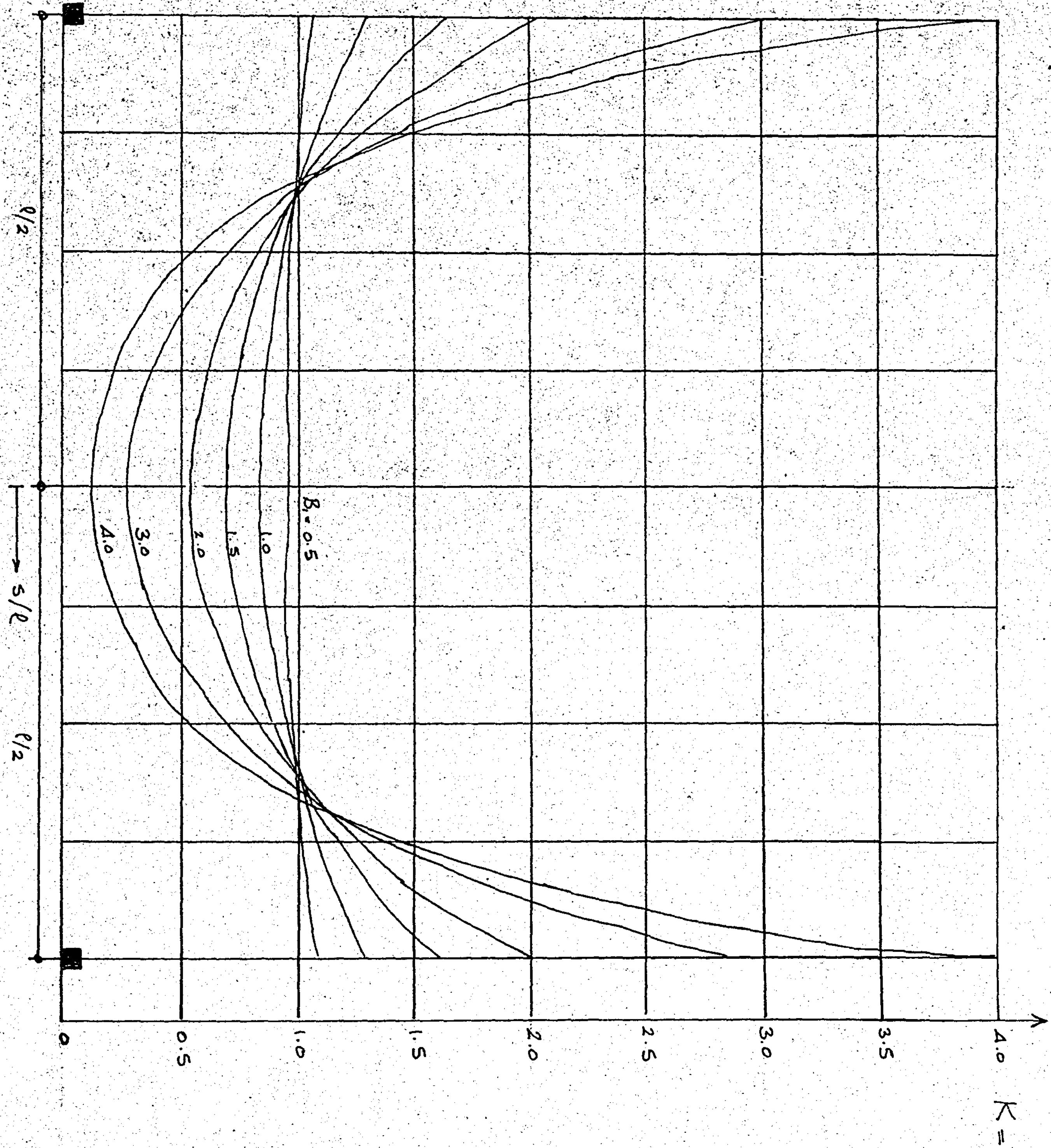


Fig. (Diagram) 1

$M_s = k \frac{l}{2} \frac{qL^2}{12}$

q = load on the ribs
L = length of the ribs
l = span distance

$\frac{M_s}{M_{s,m}}$ values depending on $\frac{s}{\ell/2}$ and β_1 are given on diagram (2)



Ib) Case of continuous ribbed floor:

If γ, β_1 and M_o values are calculated with the formulae given below, equations given in section (Ia) can be used.

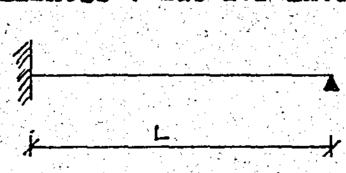
For one span $\Delta M = \frac{2EJ}{L}\varphi$ for continuous span: $\Delta M = \frac{2EJ}{\frac{L}{3}} \cdot \frac{1}{2 - \frac{1}{2+\frac{6EJ}{R_1 L}}}$

For (L) we must substitude: $\frac{L}{3}\left[2 - \frac{1}{2+\frac{6EJ}{LR_1}}\right]$

in this case:

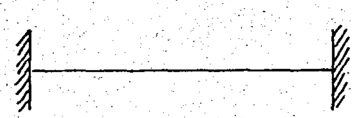
$$\beta_1^2 = \frac{l^2}{2 - \frac{1}{2+\frac{6EJ}{LR_1}}} \cdot \frac{3EJ}{2GJ_tL} \qquad \gamma = \frac{6EJ}{L\left(2 - \frac{1}{2+\frac{6EJ}{LR_1}}\right)} \cdot \frac{l}{R}$$

Here (R_1) is the stiffness value of the interior slab for 1 m., and M_o is the fixed end moment of the slab with exterior end fixed and R_1 as the stiffness value for interior.



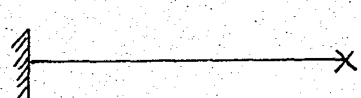
$$\gamma = \frac{3EJ}{R} \cdot \frac{l}{L} \qquad \beta_1^2 = \frac{1.5}{2} \cdot \frac{J}{J_t} \cdot \frac{l^2}{L} \cdot \frac{E}{G}$$

$$M_o = \frac{1}{8} q L^2$$



$$\gamma = \frac{4EJ}{R} \cdot \frac{l}{L} \qquad \beta_1^2 = \frac{I}{J_t} \cdot \frac{l^2}{L} \cdot \frac{E}{G}$$

$$M_o = \frac{1}{12} q L^2$$



$$\gamma \approx \frac{3.5EJ}{R} \cdot \frac{l}{L} \qquad \beta_1^2 = \frac{1.75}{2} \cdot \frac{I}{J_t} \cdot \frac{l^2}{L} \cdot \frac{E}{G}$$

$$M_o \approx \frac{1}{10} q L^2$$

Ic) Case of the Tie Beam across the Columns:

In this case the bending stiffness value of the beam in excess of the ribs for length (L) must be added to the stiffness values of the lower and upper columns.

M_o found in this way must be proportioned between excess stiffness of the beam and column stiffnesses to find the desired moments. Rib moments are found as in Ia by using: (Diagram 2)

R = For the different end conditions.

$R_K = \frac{2EI_K}{L}$ = Single span symmetrical ribbed floor.

$R_K \approx \frac{3.5EI_K}{L}$ = Continuous ribbed floor.

I_K = Moment of inertia of the beam on excess of the rib width.

R_K = The stiffness of the beam on excess of ribs.

Id) Torsional Beam With Unequal Span Length:

For this case Relaxation method is used. First torsional beam is locked to rotate around (S) axis. In this situation (diagram 1) is used to find torsional moments at the end of the torsional beam, taking γ =o. One by one the joints are unlocked. Unlocking ofthe joint means the application of the moments in opposite sense to moments which causes the joints to be locked. This applied moment also equals to the opposite of the sum of the moments caused by torsional beam in various stages, considering the equilibrium of the joint around the S axis.

In this case unlocking the joint means the summation of the moments on the joint when the other joints are locked, and moments caused on the same joint by unlocking the other joints. This moment applied on the joint is distributed among lower and upper columns and wide beam with respect to their flexural rigidity and to torsional beam with respect to their torsional rigidity.

Torsional Stiffness:

$$R_t = \frac{GJ_t}{\ell} \frac{2B_1}{th\, 2B_1}$$

Torsional carryover factor: $\mu = \dfrac{1}{ch\, 2B_1}$

After sufficient distribution final results are obtained by superposition.

Column bending moments:

It is equal to the summation of the moments caused by releasing the joint to which it belongs.

Torsional beam and torsional Moment:

It is the summation of the moments caused by external forces when its two ends are restrained against rotation around (S) axis, and moments caused by releasing the adjucent joints.

Torsional moment is obtained by the equilibrium equation depending on column and rib moments.

Ribbed Floor end moments:

It is the summation of bending moments caused by external forces when the ends of the beam are restrained against rotation around (S) axis and bending moment created by releasing towo adjucent joints. For this superposition is used. The end moment in one of the ribs is found by the Eq. 10 depending on the torsional moment created at the end of the beam when one of the joints is locked and the other is free.

(S) In this formula shows the distance from the unrotating joint:

$$M_s = \frac{n_i}{l} \frac{2B_1 \, sh \, 2B_1 \, s/l}{ch \, 2B_1} \qquad (10)$$

Ie) Cantilever Ribs:

In this case M_0 is obtained by adding cantilever moment to the fixed end moment of the fixed end beam. In case of vertical loads these two moments are in opposite sign, so that they are substracted. For determining column moments K coefficient is found from Diagram I. Column moment of M_k is distributed to the ribs using diagram 2. To these moments cantilevel moments are added to find final supports moments of the ribs.

II) Calculations for the Horizontal Forces:

IIa) For calculations of the side sway, instead of a system connecting the columns, a fictitious beam is used. In this way the system is solved by classical means and moments on the fictitious beam are found. Afterwards the moments for the various parts are found in the way shown below.

M_s = fixed end moment of the ribbed floor on (s) section.

$$M_s = S\varphi$$

$$\frac{dm_t}{ds} = s\varphi$$

$$M_t = GJ_t \frac{d\varphi}{ds}$$

Differentiated with respect to S

$$\frac{dM_t}{ds} = GJ_t \frac{d^2\varphi}{ds^2}$$

$$GJ_t \frac{d^2\varphi}{ds^2} = s\varphi$$ is obtained.

$$\varphi = Ae^{\alpha_1 s} + Be^{-\alpha_1 s}$$

$$\alpha_1 = \frac{s}{GJ_t}$$

Boundary Conditions:

a) From symmetry for $s = o$ $\quad M_t = \frac{d\varphi}{ds} = 0 \quad A = B$

b) For $s = \frac{\ell}{2}$ $\quad M = 2M_t$

$$\frac{M}{2} = GJ_t \frac{d\varphi}{ds}$$

$$\frac{d\varphi}{ds} = \alpha_1 A e^{\alpha_1 s} - \alpha_1 B e^{-\alpha_1 s}$$

$$\frac{d\varphi}{ds} = \alpha_1 A e^{\alpha_1 \frac{\ell}{2}} - \alpha_1 B e^{-\alpha_1 \ell/2}$$

$$\frac{M}{2} = GJ_t\alpha_1 (A e^{\alpha_1 \ell/2} - B e^{-\alpha_1 \ell/2})$$

$$\frac{M}{2} = GJ_t \alpha_1 A (e^{\alpha_1 \ell/2} - e^{-\alpha_1 \ell/2})$$

$$A = B = \frac{M}{2GJ_t\alpha_1 (e^{\alpha_1 \ell/2} - e^{-\alpha_1 \ell/2})}$$

$$\varphi = \frac{M\ell \, ch\, 2\beta\, s/\ell}{4\beta G J_t \, sh\, \beta} \qquad M_t = \frac{M}{2} \frac{sh\, 2\beta\, s/\ell}{sh\, \beta}$$

$$M_s = M_{s,m} \frac{\beta\, ch\, 2\beta\, s/\ell}{sh\, \beta} \qquad (11)$$

Here $M_{s,m}$ shows the end moment in case of equal distribution of end moment between the ribs. And,

$$\beta^2 = \frac{3}{2} \frac{EI}{GJ_t} \frac{\ell^2}{L}$$

Diagram 2 gives $\quad k = \frac{M_s}{M_{s,m}} \quad$ depending on β

here β is different from B_1

Moment of inertia of the fictitious beam: $\quad I_f = I\ell \frac{th\,\beta}{\beta}$

in equation (11) for $\quad s = \ell/2 \qquad M = \varphi R$

$$R = \frac{GJ_t}{\ell} 4\beta\, th\, \beta = \frac{6EI_f}{L}$$

IIb) Case of the wide rib across the columns.

Bending stiffness of the excess part than normal ribs is:

$$R_K = \frac{6EJ_K}{L}$$

$$J_f = I_K + I \ell \frac{t_k B}{B}$$

for $s = 1/2$

The stiffness of the torsional beam is $= \frac{6EJ}{L} \ell \frac{t_k B}{B}$

$$R_K = \frac{2E}{L} J_K$$ (Symmetrical rotation case)

IIc) Continuous Ribbed Floor:

Calculations are the same with part (Ib). The distribution factor for intermediate torsional beams is $B^2 = B_L^2 + B_R^2$, here

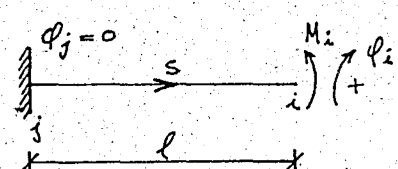
B_L = Left
B_R = Right
Are the B factors for the spans.

IId) Torsional Beam with Unequal span Length:

Relaxation method is used as explained in part (I)

III) Calculations for torsional Carryover and Distribution Factors for an Edge Beam.

Let us consider a beam with (j) edge fixed, and apply M_i torsional moment at (i)



1. $R_t = \frac{M_i}{\varphi_i}$ = torsional distribution factor.

2. $\mu = \frac{M_1}{M_i}$ = torsional carryover factor.

3 M_s = fixed end moment of the ribbed floor.

$$\frac{d^2\varphi}{ds^2} - \alpha^2\varphi = 0 \qquad \text{with integration} \qquad \text{for } m_o = 0$$

$$\varphi = A e^{\alpha s} + B e^{-\alpha s}$$

Boundary conditions:

1) $s = 0$, $\varphi = 0$, $A + B = 0$

2) $s = 1$

$$M_t = M_i = G J_t \frac{d\varphi}{ds} = \alpha G J_t \left(A e^{\alpha l} - B e^{-\alpha l} \right)$$

$$A = -B = \frac{M_i}{\alpha G J_t \left(e^{\alpha l} + e^{-\alpha l} \right)}$$

$$\varphi = \frac{M_i}{G J_t\, 2 B_1 / l} \; \frac{sh\, 2 B_1\, s/l}{ch\, 2 B_1}$$

$$M_t = M_i \frac{ch\, 2 B_1\, s/l}{ch\, 2 B_1} \qquad m_s = \varphi B_1^2 \frac{4 G J_t}{l^2}$$

is like explained in part (I)

for $s = 1$ $$\varphi = \varphi_i = \frac{M_i\, th\, 2 B_1}{G J_t\, 2 B_1 / l}$$

and torsional distribution factor is

$$R_t = \frac{M_i}{\varphi_i} = \frac{G J_t}{l} \; \frac{2 B_1}{th\, 2 B_1}$$

for $s = 0$

$$M_t = M_j = \frac{M_i}{ch\, 2 B_1}$$

carryover factor is $$\mu = \frac{M_t}{M_i} = \frac{1}{ch\, 2 B_1}$$

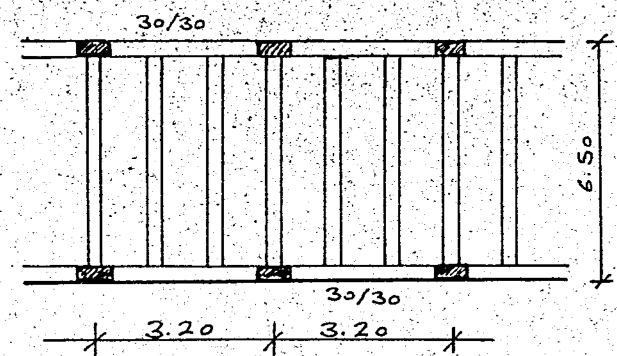
if φ is ~~distributed~~ substituted into equation 12, the ribbed floor support moments are found

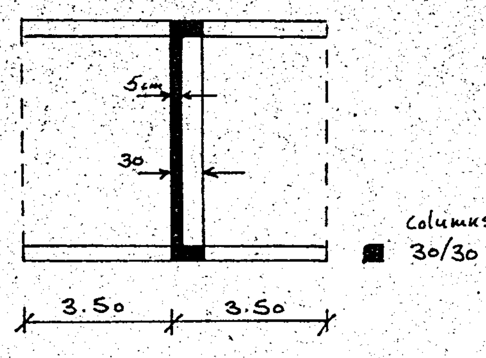
$$M_s = \frac{M_i}{l}\, 2 B_1 \frac{sh\, 2 B_1\, s/l}{ch\, 2 B_1}$$

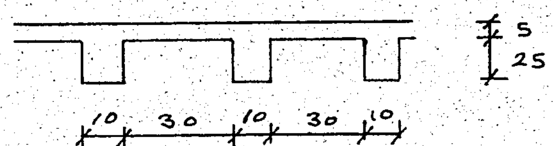
IV. Numerical Examples:

Example 1

(Loads are vertical)







Moment of Inertia of slab

$$I = \frac{1}{0.40} \times 0.435 \times \frac{4 \times 3^3}{12} = 9.60 \ dm^4$$

Sum of the column stiffnesses at the support

$$\frac{R}{E} \approx 2 \times 0.60 \times \frac{6 \times 3 \times 3^3}{12 \times 3.5} = 14.00$$

Torsional rigidity of the beams:

$$h/b = 1 \qquad \eta_3 = 0.14 \qquad J_t = 0.14 \times 3 \times 3^3 = 11.40 \ dm^4$$

Example 2

(Loads are horizontal)

Same system of example 1 is analysed for the horizontal loads.

$$\beta^2 = \frac{3}{2} \times \frac{7}{3} \times \frac{9.60}{11.40} \times \left(\frac{3.20}{6.50}\right)^2 = 4.65 \qquad \beta = 2.16$$

End moment of the rib across the columns

from chart 2:

$$2.20 \frac{M}{8} = 0.276 M$$

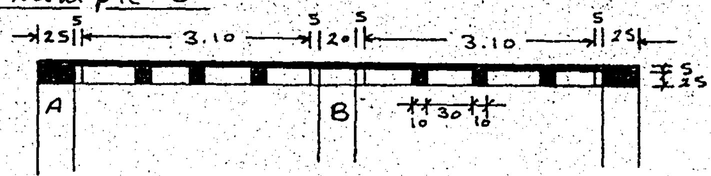
End moment of the center rib:

$$0.500 \frac{M}{8} = 0.062\,M$$

$$I_t = I \times l \frac{th\,\beta}{\beta} = 9.60 \times 3.20 \times \frac{0.973}{2.16} = 13.80 \text{ dm}^4$$

I fictitious for side sway calculations.

Example 3



Ribs are only one-span. Torsional beams are symmetrical and two-span. Upper and lower support conditions are symmetrical. Load on the slab is 0.55 t/M2 and torsional beams have additional 0.500 t/m wall load.

$$\gamma = \frac{2EJ}{R}\frac{l}{L} = \frac{2 \times 9.6 \times 3.20}{14 \times 6.50} = 0.68$$

$\frac{R}{E} = 14.00$ from example 1.

$$\beta_1^2 = \frac{7}{3}\,\frac{9.60}{2 \times 11.40}\,\frac{\overline{3.20}^2}{6.50} = 1.55 \qquad \beta_1 = 1.24 \qquad \frac{E}{G} = \frac{7}{3}$$

From chart 1. $K = 0.47$

$$M_s = \frac{3.20 \times \overline{6.50}^2}{12} \times 0.47\,q = 5.30\,q \text{ tm}$$

Bending moment for one support

$$\frac{5.30}{2}\,q = 2.65\,q$$

Ribbed floor average support moment

$$M_{s.m} = -\frac{5.30}{8}\,q = -0.662\,q \text{ tm}$$

Fixed end moment for each rib from chart 2.

$$X = -0.662\,q \times 1.465 = -0.970\,q \text{ tm}$$

Fixed end moment for the rib which is at the center of the torsional beam.

$$X = -0.662\,q \times 0.785 = -0.520\,q \text{ tm}$$

Moment at the Span of the Rib:

At column section $M = 1.14\, q$

At center of the torsional beam $M = 1.59\, q$

Torsional moment at support of the torsional beam:

$$M_t = \left(\frac{5.30}{2} - \frac{0.97}{2}\right) q = 2.1659\, q$$

If the ribs were considered to be simple supported, the relative mistakes of the moments at the span would be:

For column section % 85
For center section % 35

According to example 1.:

$$I = 9.60\ dm^4/m \qquad \frac{R}{E} = 14.00 \qquad J_t = 11.40\ dm^4$$

The differance of side torsional beams from ribs 25/30

The differance of center torsional beams from ribs 20/30

In this case the stiffnesses are:

$$R_K = \frac{2E \times 2.5 \times \bar{3}^3}{12 \times 6.50} = 1.73\, E \quad \text{for the side beam}$$

$$R_K = \frac{2E \times 2.0 \times \bar{3}^3}{12 \times 6.50} = 1.39\, E \quad \text{for the center beam}$$

Torsional Rigidities:

$$R_t = \frac{G J_t}{l} \frac{2 B_1}{th\, 2 B_1} = G \frac{11.40}{3.20} \frac{2 \times 1.24}{0.986} = 3.84\, E$$

Torsional carryover factor

$$\mu = \frac{1}{ch\, 2 B_1} = \frac{1}{6.012} = 0.166$$

The summation of the stiffnesses

At the edge : $\sum R = (3.84 + 1.73 + 14.00)\, E = 19.57\, E$

At the center: $\sum R = (3.84 + 3.84 + 14.00 + 1.39)\, E = 21.07\, E$

For schematic represantation look at figure 10a.

If 1 tm (total moment) fixed end moment acts at the side beams which are fixed at one edge:

From equation 10:

For $s = \ell \quad M_s = 0.760 \quad , \quad s = \ell/2 \quad M_s = 0.204 \quad , \quad s = 0 \quad M_s = 0$

Moments due to $0.550\ t/m^2 \quad (\gamma = 0 \ , \ \beta = 1.24)$

At the edge $\quad M_s = 0.68 \dfrac{(3.20/2)\ \overline{6.50}^2}{12} \times 0.550 = 2.11\ tm$

At the center $\quad M_s = 2 \times 2.11 = 4.22\ tm$

From wall load $\quad M_s = 0.500 \dfrac{\overline{6.50}^2}{12} = 1.76\ tm$

Bending moments at the edge supports = 1.443 tm

Bending moment at the middle supports = 1.890 tm

To find ribs support moments considering 30 cm. as average support width for the ribs:

$$X = -2\left(\frac{2.11}{8}\right) 1.465 + (1 + 0.045)\ 0.760 \times 0.20 \times 2 - 1.384 = -3.84\ tm$$

$$X = -\frac{2.11}{8}\ 1.465 + 0.795 \times 0.760 \times 0.20 - 1.395 = -1.66\ tm$$

At the rib which is at the center:

$$X = -2\ \frac{2.11}{8}\ 0.785 + 0.204\,(1.045 + 0.795)\ 0.40 = -0.264\ tm$$

Torsional moments at the edge beams:

At th edge support: $\quad M_t = -1.443 - 1.443 + 1.660 = -1.226\ tm$

At the center support: $\quad M_t = \dfrac{1.89 + 1.89}{2} - \dfrac{1.84}{2} = 0.970\ tm$

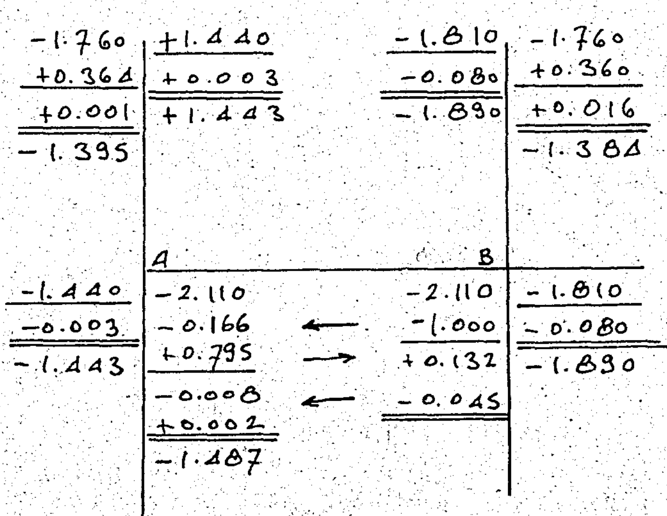


Fig. 10a

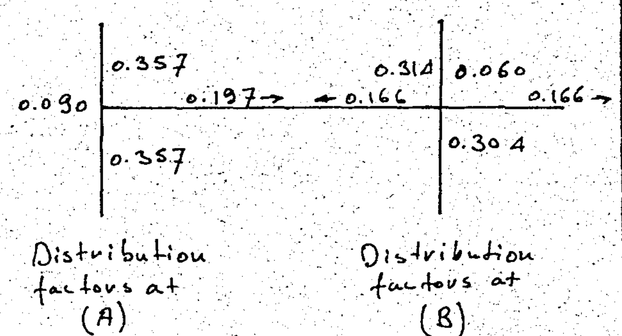


Fig. 10b

Part II. Calculations of the Rib Systems

1. Introduction:

Beams which support the rib have torsional rigidities, because of that these beams are subjected to torsional moments, and columns to bending moments, as a result of these moments support moments of the ribs aere reduced considerably. In the part (I) only side torsional beams and ribs were considered because of its importance. In this part, the application of formulaes developed in part (I) for intermediate beams are analysed. In this way rib frames are analysed with a Hardy Cross like calculation to some accuracy. At the end of the article a numerical example is gieven to compare the results thus obtained to an accurate solution; also an example is given using methods developed in part (I) for side sway analysis In this article like part (I), ribs are considered to be infinitly close and the effect of horizontal bending due to the fact that the center of gravity and the center of rotation of the beam do not cross , is not considered.

This moment is neglected when the ratio of the height of the torsional beam to the rib height is not large, and this fact is confirmed by analysing chart 7.

2) Vertical Loads:

Here the systems which are encountered usually in practice, with different rib openings, and equaled span torsional beams with infinite length are considered. The vertical loads acting on the ribs are considered to be uniform along vertical directions to the ribs, so in this direction all spans are same with each other. To analyse one of them is enought. The case of the torsional beam with different span is analysed in part (I). In practice ribs are made wide at the column section and reinforced. In this article the section which is the difference of the wide beam to ordinary rib is called the rib excess beam. In figure 4 the rib excess beam is shown by hatched lines. To make the calculations simple, the rib and column moments turning the torsional beam from left to right are taken as positive like the Hardy Cross Method. The effect of wall loads on rib excess beam and the effect of the uniform loads on the ribs are considered seperately.

2) 1. The effect of loads on the rib:

The results as shown on different steps below are superposed to find the final answer.

a) Torsional beams are restrained to rotate around their axis. In this case end moments of the ribs are found easily, becuase the ribs are like fixed supported beams. Unit width end moments M_0 are considered to be positive on left support and to be negative on right support, for loads acting downward.

b) One of the torsional beams is left free to rotate around its axis. In this case, the sum of the end moments of the upper and lower columns, and of the rib excess beam is designated as M_s its value was given in part I as:

1) $M_s = (M_{o\,left} + M_{o\,right})\, \ell \times k$

here

2) $k = \dfrac{1}{\gamma + \dfrac{B_1}{\tanh B_1}}$

γ and B are defined as

On edge torsional beam = $\gamma = \dfrac{4EI}{R}\dfrac{\ell}{L}$ $\qquad B_1^2 = \dfrac{EI}{GJ_t}\dfrac{\ell^2}{L}$ (3)

EI = Unit width bending rigidity of the ribbed floor.

R = The sum of the stiffnesses of the upper and lower columns, and of the rib excess beam. (Other end of the columns are considered to be half fixed, and the rib excess beam as fully fixed on both ends.)

L = Lenghth of the ribbed floor.

l = Length of the torsional beams. (If there are rib excess beams then l = length of the torsional beam - width of the rib excess beam.)

GJ_t = Torsional rigidity of the torsional beam.

For intermediate torsional beam : $B_1^2 = B_{1\,left}^2 + B_1^2$ right

$\gamma = \gamma_{left} + \gamma$ right

To make the calculations simpler, coefficient k versus γ and B was given in part (I)

Moment M_s found in this way is distributed among upper and lower columns, and left and right rib excess beams, with respect to their stiffnesses; for the ribbed floor end moments in this case are :

$$-(M_{o\,left} + M_{o\,right})\, \ell\, (1-k) \qquad (4)$$

This moment is divided between left and right ribbed floors with respect to 1/2 ratio. Later rib excess moments and ribbed floor end moments are carried over in half to find adjucent end moments. While releasing a torsional beam, to $(M_{o_{left}} + M_{o_{right}})$ fixed end moments at formula (1) and (4) carried over moments of the slab and rib excess beam from adjucent spans must be added if the torsional beam has been released before. Only the carried over moments from adjucents spans are not distributed uniformly, so the coefficient (k) found from (eq.2) is not exact, however this differance is of second order magnitude and it does not effect to the result considerably. Torsional beams are released till the system is inequilibrium. To find the real results values onevery step is summed. Because of the uniqueness of the problem, the systematic solution which converges rapidly, is shown in (ex. 4.1) Column end moments found in this way are the desired real moments on the ribs and on torsional beams. One must follow the steps as explained below:

The summation of the end moments of the ribbed slab on two sides of the torsional beam is equal to the summation of the applied moment on torsional beam by ribs. The ratio of this summed moment to number of ribs for one span, is multiplied by:

$$k_1 = \frac{\beta_1}{l} \frac{\cosh 2\beta_1 s/l}{\sinh \beta_1}$$

To obtain the applied moments (M_s) by each rib to torsional beam, the chart giving k_1 depending on B_1 was given in part (I).

The moment applied to the ribs by torsional beam then is $- M_s$. The ribs can be analysed by any method like Hardy Cross, since the moments acting on supports ($-M_s$) and external loads are known (Fig.3)

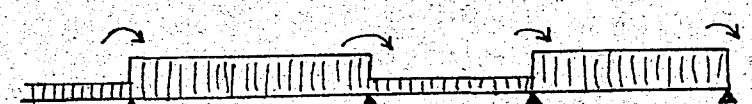


Fig. 3

Torsional moments on torsional beams are found easily since column and rib excess beam end moments, and M_s moments are known.

2.2 - The effect of the load on rib excess beam.

To find this effect the step as outlined in part (2.1) is followed exactly. Only in step (a) the ribbed excess beam fixed end moments due to the loads on it, and in step (b) for coeficient k:

$$k = \frac{1}{1 + \gamma \frac{\tanh \beta_1}{\beta_1}} \qquad (2')$$

On figure 1 the curve for $\gamma = o$ gives the value for $\frac{\tanh \beta_1}{\beta_1}$ this value can be substituted into eq. 2 to get the coeficient (K).

3.) **Horizontal Loads:**

Same method as explained in part (I) is followed.

4.) Examples:

Figure 4 shows the one span of a ribbed floor system with equal spans and infinite length. Ribs are on two spans with an additional cantilevel part, these two spans are 6.00 m. and 4.00 m respectively. Beam width across the columns parallel to ribs are 0.50 m., rib width is 0.10 m., so the width of the rib excess beam is 0.50 - 0.10 = 0.40 m. So according to definition 1 = 4.40 - 0.40 = 4.00 m. : 1 equals 4.00 m. and the stiffness of the rib excess beam is:

For (AB) $R_K = \left(4 \frac{4 \times \overline{3.3}^{3}}{12}\right) \frac{1}{6} = 7.99\ dm^4/m$

For (BC) $R_K = \left(4 \frac{4 \times \overline{3.3}^{3}}{12}\right) \frac{1}{4} = 11.98\ dm^4/m$ here $E = 1$

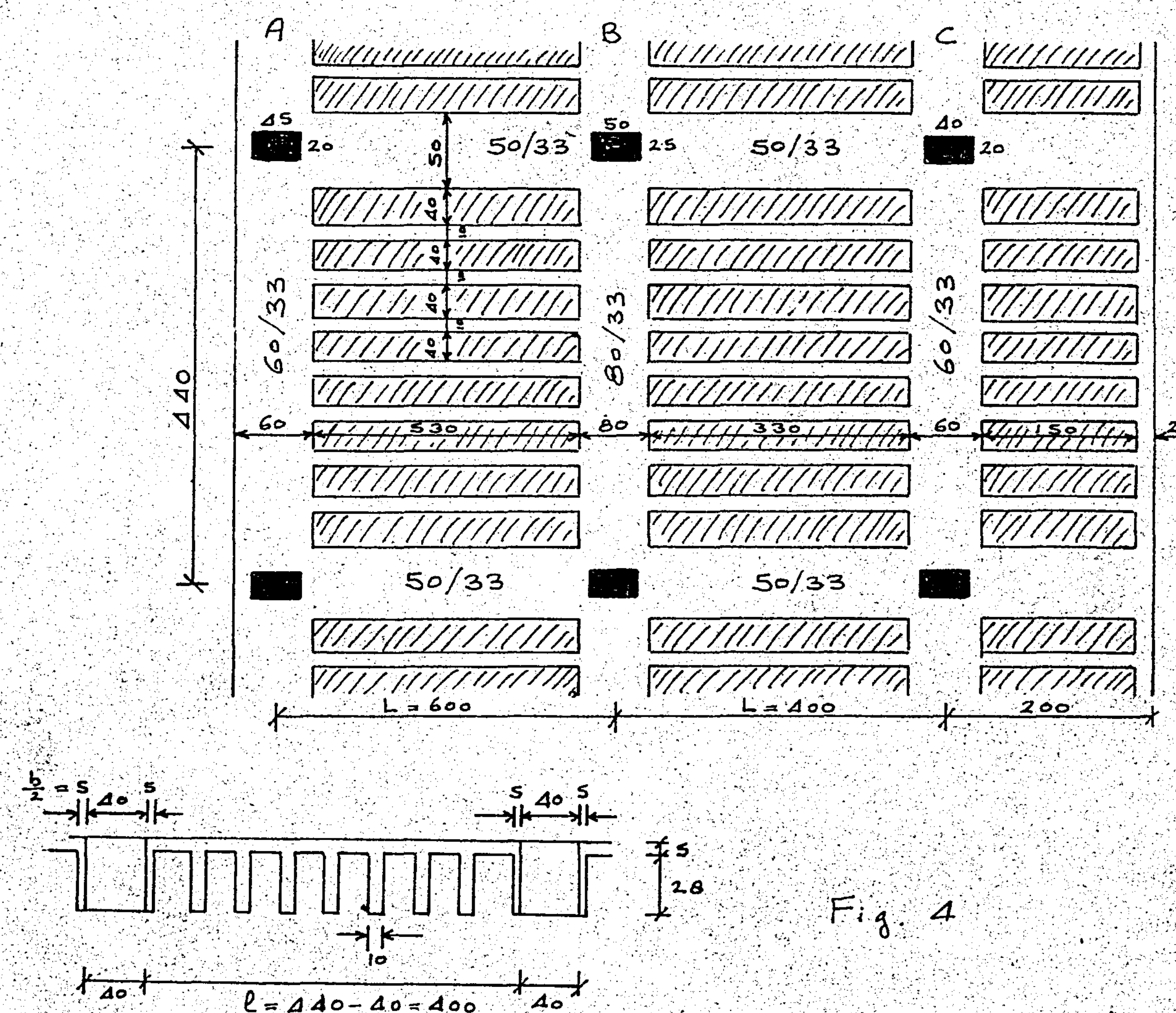


Fig. 4

Column Stiffnesses:

A axis= $h = 3.50\,m$ section 20/45

$$R_s = \left(3.5\,\frac{2 \times \overline{4.5}^3}{12}\right)\frac{1}{3.50} = 15.19\ dm^4/m$$

B axis= $h = 3.50\,m$ section 25/50

$$R_s = \left(3.5\,\frac{2.5 \times \overline{5.0}^3}{12}\right)\frac{1}{3.50} = 26.04\ dm^4/m$$

C axis = $h = 3.50\,m$ section 20/40

$$R_s = \left(3.5\,\frac{2 \times \overline{4.0}^3}{12}\right)\frac{1}{3.50} = 10.67\ dm^4/m$$

then R, by definition, is the sum of the stiffnesses found above

(A axis) $R = (2 \times 15.19) + 7.99 = 38.37$

(B axis) $R = (2 \times 26.04) + 7.99 + 11.98 = 72.05$

(C axis) $R = (2 \times 10.67) + 11.98 = 33.32$

Torsional rigidities of A and C axis, section being 60/33

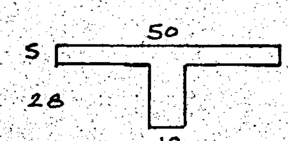
$$J_t = 0.217 \times 6 \times \overline{3.3}^3 = 46.79\ dm^4$$

for B axis, section 80/33

$$J_t = 0.246 \times 8 \times \overline{3.3}^3 = 70.72\ dm^4$$

The values 0.217 and 0.246 depend on the ration of the sides of the section they can be obtained from Beton Kalendar 1961, page 191.

Moment of Inertia of the ribbed floor for unit width (1m.)



$$I = \frac{10 \times \overline{33}^3}{32.9} = 10.92\ dm^4/m$$

32.9 From Löser

4.1) Calculations for the effect of the uniform load:

$q = 0.900\ t/m^2$

Fixed end moments due to (q)

For (AB)

$$M_{o_{left}} = 0.900\ \frac{\overline{6.00}^2}{12} \times 4.00 = +10.80\ tm$$

$$M_{o_{right}} = \qquad = -10.80\ tm$$

For (BC)

$$M_{o_{left}} = 0.900\ \frac{\overline{4.00}^2}{12} \times 4.00 = +4.80\ tm$$

$$M_{o_{right}} = \qquad = -4.80\ tm$$

For cantilevel when an uniform load of 0.320 t:m is assumed to act at the end of the ribs.

$$M_{o_c} = 0.900\ \frac{\overline{2.00}^2}{2} \times 4.00 + 0.32 \times 1.90 \times 4.00 = 9.63\ tm$$

1.90 is the distance from the center of the 0.320 t/m load to the mid section of axis (c)

γ and B_1 values using eq. 3 and 3'

For A axis $\quad \gamma = \frac{4 \times 10.92}{38.37}\ \frac{4.00}{6.00} = 0.76$

$$B_1^2 = \frac{8}{3}\ \frac{10.92}{46.79}\ \frac{\overline{4.00}^2}{6.00} = 1.66 \qquad B_1 = 1.29$$

For B axis $\quad \gamma = \frac{4 \times 10.92}{72.05}\ \frac{4.00}{6.00} + \frac{4 \times 10.92}{72.05}\ \frac{4.00}{4.00} = 1.01$

$$B_1^2 = \frac{8}{3}\ \frac{10.92}{70.72}\ \frac{\overline{4.00}^2}{6.00} + \frac{8}{3}\ \frac{10.92}{70.72}\ \frac{\overline{4.00}^2}{4.00} = 2.755 \qquad B_1 = 1.66$$

here $\frac{E}{G} = \frac{8}{3}$

$$\gamma_c = 1.31 \qquad B_{1_c} = 1.58$$

The value of the coefficient (k) for the values of and B are taken from Fig I

$k_A = 0.44 \qquad k_B = 0.36 \qquad k_c = 0.33$

Distribution factors for A axis:

Upper and lower columns = $r_s = -\frac{\Sigma R_s}{R} k = -\frac{2 \times 15.19}{38.37} \times 0.44 = -0.35$

Rib excess beam = $r_K = -\frac{R_K}{R} k = -\frac{7.99}{38.37} \times 0.44 = -0.09$

Ribs = $r_N = -(1-k) = -(1-0.44) = -0.56$

Distribution Factors for B axis:

upper and lower columns = $r_s = -\frac{2 \times 26.04}{72.05} \times 0.36 = -0.26$

AB rib excess beam = $r_K = -\frac{7.99}{72.05} \times 0.36 = -0.04$

BC rib excess beam = $r_K = -\frac{11.98}{72.05} \times 0.36 = -0.06$

Ribs = $r_N = -(1-0.36) = -0.64$

for (AB)ribs = $r_N = -0.64 \frac{4}{4+6} = -0.26$

for (BC) ribs= $r_N = -0.64 \frac{6}{4+6} = -0.38$

Distribution factors for C axis:

Upper and lower columns = $r_s = -\frac{2 \times 10.67}{33.32} \times 0.33 = -0.21$

Rib excess beam = $r_K = -\frac{11.98 \times 0.33}{33.32} = -0.12$

Ribs = $r_N = -(1-0.33) = -0.67$

In table 1. the distribution factors are on the first row. Fixed end moments are on the second row, and by sequance A,C,B, - A,C,B, - A,C,B, axis are released to obtain equilibrium condition.

Table I

A axis			B axis					C axis			
upper and lower columns	Rib excess beam AB	Ribs AB	Ribs AB	Rib excess beam AB	upper and lower Columns	Rib excess beam BC	Ribs BC	Ribs BC	Rib excess beam BC	upper and lower Columns	Ribs cantilever
-0.35	-0.09	-0.56	-0.26	-0.04	-0.26	-0.06	-0.38	-0.67	-0.12	-0.21	—
		+10.80	-10.80				+4.80	-4.80			+9.63
-3.78	-0.97	-6.05	-3.02	-0.48		-0.29	-1.62	-3.24	-0.58	-1.01	
	+0.23	+1.48	+2.97	+0.46	+2.97	+0.68	+4.33	+2.16	+0.34		
-0.60	-0.15	-0.96	-0.48	-0.08		-0.15	-0.84	-1.68	-0.30	-0.52	
	+0.03	+0.20	+0.40	+0.06	+0.41	+0.09	+0.59	+0.30	+0.04		
-0.08	-0.02	-0.13	-0.06	-0.01		-0.02	-0.12	-0.23	-0.04	-0.07	
			+0.06	+0.01	+0.05	+0.01	+0.08				
-4.46	-0.88	+5.34	-10.93	-0.04	+3.43	+0.32	+7.22	-7.49	-0.54	-1.60	+9.63

For this table end moments of the columns A,B, and C respectively:

$$-\frac{4.46}{2} = -2.23\ tm \qquad +\frac{3.43}{2} = 1.72\ tm \qquad -\frac{1.60}{2} = -0.80\ tm$$

The stiffnesses of the upper and lower columns for particular column are same, so moments found from table (1) are divided into (2).

The sum of the rib moments on two sides of the torsional beam are:

at A = $0 + 5.34 = +5.34\ tm$

at B = $-10.93 + 7.22 = -3.71\ tm$

at C = $-7.49 + 9.63 = +2.14\ tm$

There are 8 ribs in one span so we divide these moments into 8, the results are:

at A = $\dfrac{5.34}{8} = +0.668\ tm$

at B = $\dfrac{-3.71}{8} = -0.464\ tm$

at C = $\dfrac{2.14}{8} = +0.268\ tm$

In calculation only the center rib (N_1) and the rib across the columns are considered, so M_s moments applied to torsional beams by ribs are found by multiplying (k_1) factor obtained from figure 2 by the moments found above.

At A axis $B_1 = 1.29$

N_1 centre rib $s/\ell = 0$ $k_1 = 0.77$

$M_s = 0.668 \times 0.77 = 0.51$ tm

Column section $s/\ell = 0.50$ $k_1 = 1.50$

$M_s = -0.668 \times 1.50 = -1.00$ tm

At B axis $B_1 = 1.66$

N_1 centre rib $s/\ell = 0$ $k_1 = 0.65$

$M_s = -0.464 \times 0.65 = -0.30$ tm

Column section $s/\ell = 0.50$ $k_1 = 1.78$

$M_s = -0.464 \times 1.78 = -0.83$ tm

At C axis $B_1 = 1.58$

N_1 centre rib $s/\ell = 0$ $k_1 = 0.68$

$M_s = 0.268 \times 0.68 = 0.18$ tm

Column section $s/\ell = 0.50$ $k_1 = 1.72$

$M_s = 0.268 \times 1.72 = 0.46$ tm

In this case support moments acting at A, B, and C for center rib are -0.51 tm, $+0.30$ tm and -0.18 tm respectively. For column sections these moments are -1.00 tm, $+0.83$ tm and -0.46 tm. (Figure 3). In these two ribs end moments due to M_s and uniform loads are determined by Hardy Cross method (Table 2).

Moments acting on the parallel beam are found by adding rib excess beam moment (Table I) to column section rib end moment (Table 2).

A		B		C
+0.12	-1.50	+0.95	-1.28	+1.20

A) + 1.00 - 0.88 = + 0.12

B left) - 1.86 - 0.04 = - 1.50

B right) + 0.32 + 0.63 = + 0.95

C left) - 0.74 - 0.54 = - 1.28

C right + 1.20

Torsional moments, at the section where torsional beams cut parallel beams, are found by adding half of the total column moments to half of the moments on parallel beam.

A axis = $M_t = -\frac{1}{2}(-4.46 + 0.12) = +2.17$ tm

B axis = $M_t = -\frac{1}{2}(3.43 - 1.50 + 0.95) = -1.44$ tm

C axis = $M_t = -\frac{1}{2}(-1.60 - 1.28 + 1.20) = +0.84$ tm

	A	B		C	
Moments acting on the supports-Ms	-0.51	+0.30		-0.18	
Distribution factors	—	-0.40	-0.60	—	—
Fixed end Moments	—	-2.02	+0.90	—	+1.20
Distribution	+0.51 →	+0.25 +0.43	-0.51 +0.65 ←	-1.02	
Result	+0.51	-1.34	+1.04	-1.02	+1.20

N_1 Centre rib

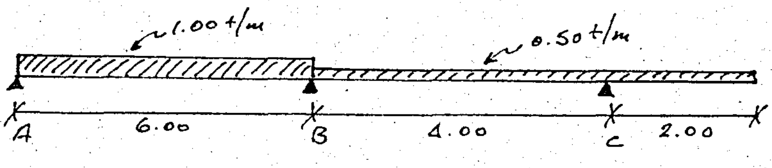
Table 2

Column strip rib

	A	B		C	
Moments acting on the supports-Ms	-1.00	+0.83		-0.46	
Distribution	—	-0.40	-0.60	—	—
Fixed end moments		-2.02	+0.90		+1.20
Distribution	+1.00	+0.50 +0.06	-0.37 +0.10	-0.74	
Result	+1.00	-1.46	+0.63	-0.74	+1.20

4.2) The effect of the wall load on the rib excess beam:

Loads acting is shown in figure 5.



Fixed end moments due to the loads shown are :

(AB) = $M_o = \frac{1}{12} \times 1.00 \times \overline{6.00}^2 = 3.00$ tm

(BC) = $M_o = \frac{1}{12} \times 0.50 \times \overline{4.00}^2 = 0.67$ tm

Overhanging part $M_o = \frac{1}{2} \times 0.50 \times \overline{2.00}^2 = 1.00$ tm

K values as found by using (2')

$K_A = 0.67 \qquad K_B = 0.64 \qquad K_C = 0.57$

Distributing factors considering K values as in part (4.1)

A axis $r_s = -0.53 \qquad r_K = -0.14 \qquad r_N = -0.33$

B axis $r_s = -0.46 \qquad r_{BA_K} = -0.07 \qquad r_{BC_K} = -0.11 \qquad r_{BA_N} = -0.14 \qquad r_{BC_N} = -0.22$

C axis $r_s = -0.36 \qquad r_K = -0.21 \qquad r_N = -0.43$

In table 3 the distribution factors are on the first row, the fixed end moments are on the second row. Axis with A,C,B - and A,C,B, sequences are released.

A axis		
Upper and lower columns	rib excess beam (AB)	Ribs (AB)
-0.53	-0.14	-0.33
	+3.00	
-1.59	-0.42	-0.99
	+0.11	+0.21
-0.17	-0.05	-0.11
-1.76	+2.64	-0.88

B axis				
Ribs (AB)	rib excess beam (AB)	Upper and lower columns	rib excess beam (BC)	Ribs (BC)
-0.14	-0.07	-0.46	-0.11	-0.22
	-3.00		+0.67	
-0.50	-0.21		-0.04	-0.07
+0.44	+0.22	+1.45	+0.35	+0.69
-0.06	-0.02		-0.05	-0.11
+0.03	+0.02	+0.12	+0.03	+0.05
-0.09	-2.99	+1.57	+0.95	+0.56

C axis			
Ribs (BC)	rib excess beam (BC)	Upper and lower columns	rib excess beam (cantilever)
-0.43	-0.21	-0.36	—
	-0.67		+1.00
-0.14	-0.07	-0.12	
+0.34	+0.18		
-0.22	-0.11	-0.19	
-0.02	-0.67	-0.31	+1.00

Column end moments:

A) $-\frac{1.76}{2} = -0.88$

B) $+\frac{1.57}{2} = +0.78$

C) $-\frac{0.31}{2} = -0.16$

The sum of the rib moments on two sides of the torsional beam.

At A - 0.88 tm

At B - 0.09 + 0.56 = + 0.47 tm

At C - 0.02tm

Dividing these moments into the number of ribs in one span:

At A -0.110 tm

At B +0.059 tm

At C -0.002 tm

For M_s (Moments applied by the ribs to torsional beam) above moments are multiplied by k_1 taken from figure 2.

A axis

N_1 center rib M_s = -0.11 x 0.77 = - 0.08

Column strip rib M_s = -0.11 x 1.50 = - 0.16

B axis

N_1 center rib M_s = 0.059 x 0.65 = 0.04

Column strip rib = M_s = 0.059 x 1.78 = 0.10

C axis

N_1 center rib M_s = - 0.002 x 0.68 = 0

Column strip rib M_s = - 0.002 x 1.72 = 0

The end moments due to these moments applied at the supports are found using Hardy Cross method. (Table 4).

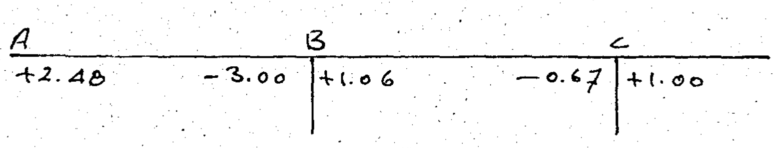
Table 4 N_1 centre rib

	A	B		C	
Moments acting on the supports -Ms	+0.08	-0.04		~0	
Distribution factors	—	-0.40	-0.60	—	—
Fixed end Moments	0	0	0	0	0
Distribution	-0.08 →	-0.04 +0.03	0 +0.05	← 0	
Result	-0.08	-0.01	+0.05	0	0

Column strip rib

	A	B		C	
Moments acting on the supports -Ms	+0.16	-0.10		~0	
Distribution	—	-0.40	-0.60	—	—
Fixed end moments		0	0	0	0
Distribution	-0.16 →	-0.08 +0.07	0 +0.11	← 0	
Result	-0.16	-0.01	+0.11	0	0

Moments acting in the parallel beam are found by adding the moments of the rib excess beam (table 3) and column strip rib (table 4).



The torsional moments at the section, where torsional beams cut into parallel beams are found by adding half of column moments to half of the parallel beam moments as found above, changing the signs:

A axis $\quad M_t = -\frac{1}{2}(-1.76+2.48) = -0.36$ tm

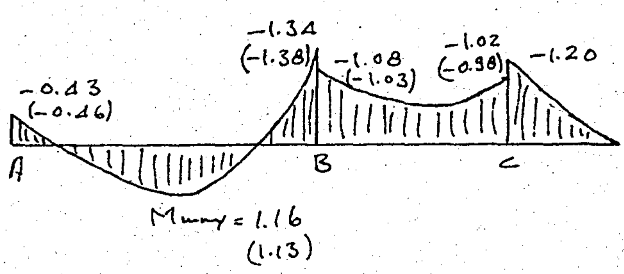
B axis $\quad M_t = -\frac{1}{2}(+1.57-3.00+1.06) = +0.18$ tm

C axis $\quad M_t = -\frac{1}{2}(-0.31-0.67+1.00) = -0.01$ tm

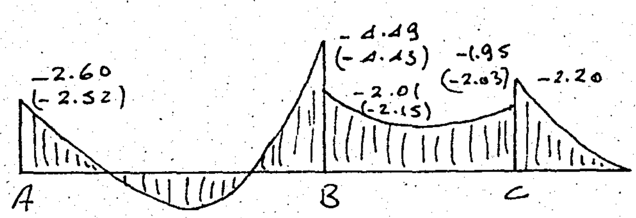
4.3-) Exact Solution:

To check the results found at (4.1) and (4.2) same system is solved by using slope-deflection method. The rotation of the ribs on torsional beams are taken as the unknowns. Because of the symmetry, the number of the unknowns is reduced to fifteen. On figure 6, the values found by slope-deflection formulaes are written in paranthesis beside the values found in (4.1) and (4.2).

Center Rib



Parallel beam
(rib excess beam)



The torsional Moments on torsional beam:

A axis = Mt = +1.81 (+ 1.86)

B axis = Mt = -1.26 (- 1.36)

C axis = Mt = +0.83 (+ 0.89)

The column moments:

A axis = M_s = -3.11 (-3.12)

B axis = M_s = +2.50 (+2.50)

C axis = M_s = -0.95 (-0.90)

By the inspection of figure 6 two results are very close to each other.

4.4-) Calculations for the Effect of the Horizontal Loads:

(Figure 7) shows the dimensions and magnitudes of the forces acting on the system. The slab on each floor is the same with ribbed floor as explained in (4) Here only the effects on the III floor are calculated.

Coefficient B from equation 7 and 7'

A axis $B^2 = 2.49$ $B = 1.58$

B axis $B^2 = 4.12$ $B = 2.03$

C axis $B^2 = 3.73$ $B = 1.93$

$\frac{\tanh B}{B}$ values are taken from figure 1.

A axis $\frac{\tanh B}{B} = 0.58$

B axis " $= 0.48$

C axis " $= 0.50$

The value of I_f is found using equation 6 :

A axis $I_f = 10.92 \times 4 \times 0.50 + 11.98 = 37.31$

$$\frac{I_f}{L} = \frac{37.31}{6} = 6.22$$

B axis $I_f = 10.92 \times 4 \times 0.48 + 11.98 = 32.95$

at left $\frac{I_f}{L} = 5.49$ at right $\frac{I_f}{L} = \frac{32.95}{4} = 8.24$

C axis : $I_f = 10.92 \times 4 \times 0.50 + 11.98 = 33.82$

$\frac{I_f}{L} = 8.46$

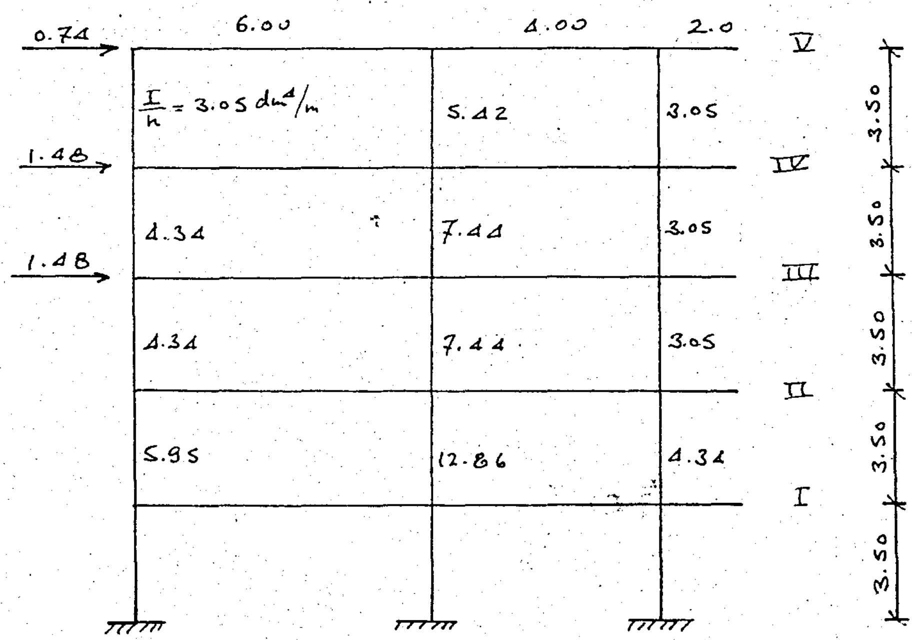


Fig. 7

Column moments are found (ref 3) as:

A axis	upper	0.07 tm	Lower 1.72 tm
B axis	upper	1.89 tm	Lower 3.38 tm
C axis	upper	0.96 tm	Lower 1.57 tm

Moments on the fictitious beam are:

$M_{AB} = -2.59$ tm $\qquad M_{BA} = -2.38$ tm

$M_{BC} = -2.89$ tm $\qquad M_{CB} = -2.53$ tm

Fictitious beam moments are distributed between rib excess beam and ribs with respect to I_k and $I\ell \frac{\tanh B}{B}$ values. These distributed moments are listed at table 5.

	Rib excess Beam		Ribbed Floor	
	Left	Right	Left	Right
A axis	0	-0.83	0	-1.76
B axis	-0.87	-1.06	-1.51	-1.83
C axis	-0.90	0	-1.63	0

End moments for one rib at A axis $-\frac{1.76}{8} = -0.220$ tm

B axis at left $-\frac{1.51}{8} = -0.189$ tm

at right $-\frac{1.83}{8} = -0.229$ tm

C axis $-\frac{1.63}{8} = -0.204$ tm

These end moments are to be multiplied by (k_1) taken from figure 2

A axis (B = 1.50)

Ni centre ribb $s/\ell = 0$ $k_1 = 0.68 \longrightarrow -0.15$ tm

Column strip rib $s/\ell = 0.50$ $k_1 = 1.72 \longrightarrow -0.38$ tm

B axis (B = 0.203)

Ni centre rib $s/\ell = 0$ $k_1 = 0.54 \longrightarrow$ at left -0.10 right -0.12

Col. rib $s/\ell = 0.50$ $k_1 = 2.10 \longrightarrow$ at left -0.40 tm right -0.48 tm

C axis (B = 1.93)

Ni $s/\ell = 0$ $k_1 = 0.57 \longrightarrow -0.12$ tm
Col. $s/\ell = 0.50$ $k_1 = 2.01 \longrightarrow -0.41$ tm

Moments on the parallel beams are found by adding rib excess beam moments (table 5) to column strip rib moments.

A		B		C
-1.21	-1.27	-1.54	-1.31	

The torsional moments on the section where torsional beams cut parallel beams is found by adding half of the column moments to half of the parallel beam moments, with changing signs.

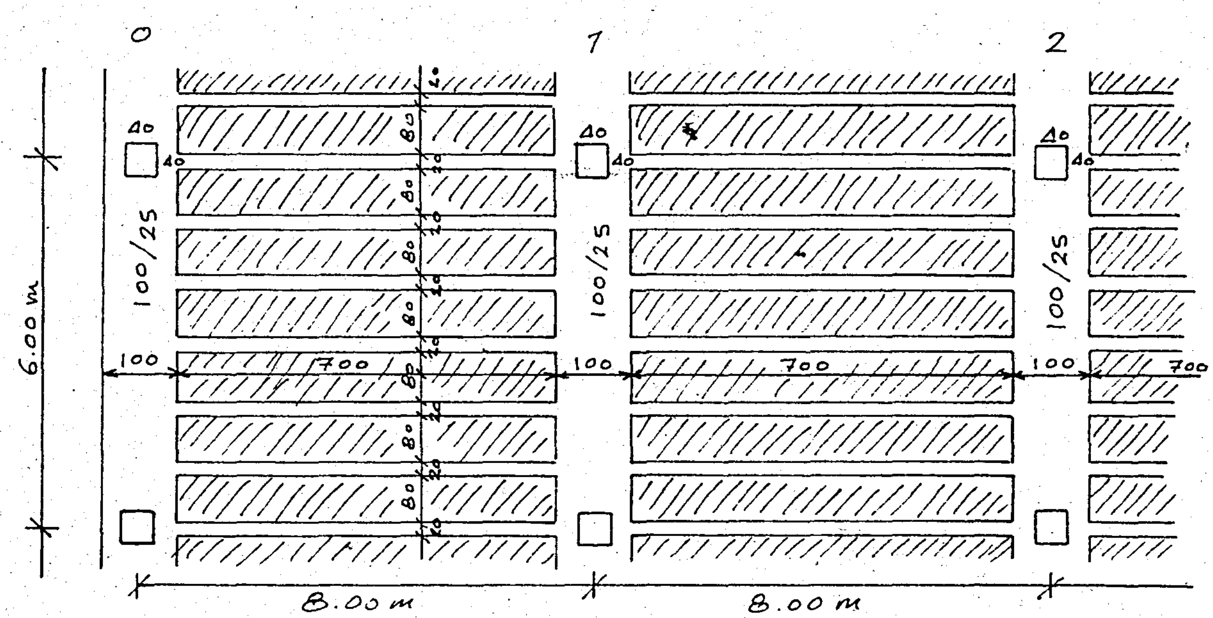
A axis = $M_t = -\frac{1}{2}(0.87 + 1.72 - 1.21) = -0.69$ tm

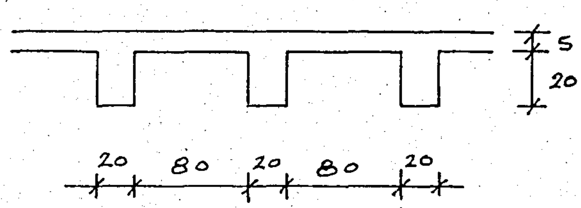
B axis = $M_t = -\frac{1}{2}(1.89 + 3.38 - 1.27 - 1.54) = -1.23$ tm

C axis = $M_t = -\frac{1}{2}(0.96 + 1.57 - 1.31) = -0.61$ tm

Comparison Between the Methods :

An example will be solved using different methods. Among these methods only Mr. Çakıroğlu considers the case of the tie beam across the columns, so our example will be without a tie beam. The dimensions of the example will be imaginery.





Load on the slab

$g = 0.400 \ t/m^2 \qquad p = 0.400 \ t/m^2$

Column height $h = 3.00 m$

$E = 2.00 \times 10^6 \ t/m^2$

$G = 0.75 \times 10^6 \ t/m^2$

Moment of Inertia of the Slab

$$A = 10.00 \times 0.5 + 2.00 \times 2.00 = 5.00 + 4.00 = 9.00\ dm^2$$

$$y \times A = 5.00 \times 0.5 \times \frac{1}{2} + 4.00 \times 1.50 = 1.25 + 6.00 = 7.25\ dm^3$$

$$y = \frac{7.25}{9.00} = 0.805\ dm$$

$$I = \frac{1}{12} \times 10 \times \overline{2.5}^3 + 10 \times 2.5\,(1.250 - 0.805)^2 - \frac{2}{12}\,4 \times \overline{2}^3 + 16\,(1.69 - 1.00)^2$$

$$I = 4.95\ dm^4/m$$

For 6 ribs $4.95 \times 6 = 29.70\ dm^4/span$

Torsional Resistance of the Beam

$$J_t = \frac{b^3 d}{3}\left(1 - 0.63\frac{b}{d} + 0.052\frac{b^5}{d^5} - \dots\right)$$

According to the Concrete Engi. Handbook, it is better to use the above formula, if the ratio of the long side (d) to short side (b) is greater or equal to 1.6, so:

$$J_t = \frac{1.0 \times \overline{0.3}^3}{3}\left(1 - 0.63\frac{0.3}{1.0} + 0.052\frac{\overline{0.3}^5}{\overline{1.0}^5}\right) = 0.007299\ m^4$$

Columns

$$I_c = \frac{1}{12} \times 4 \times \overline{4}^3 = 21.35\ dm^4$$

$$K_c = \frac{21.35}{3.00} = 7.11\ dm^4/m$$

Slab

$$K_s = \frac{29.70}{8.00} = 3.72\ dm^4/m$$

Distribution Factors

1- Axis Columns $\longrightarrow$ 0.338 Slab $\longrightarrow$ 0.133 left 0.190 right

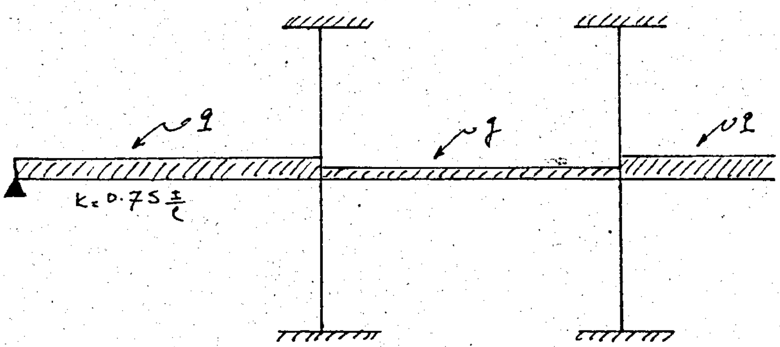
2 - Axis Columns $\longrightarrow$ 0.328 Slabs $\longrightarrow$ 0.172

Fixed End Moments

$$M_g = \frac{1}{12} \times 0.4 \times \overline{8.00}^2 \times 6.00 = 12.80\ tm \qquad M_{q_{(0\cdot1)}} = 38.40\ tm$$

$$M_q = 25.60\ tm$$

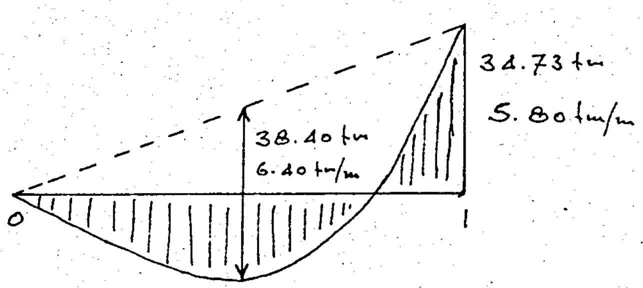
Loading conditions to give max. Torque at the edge beam



0	1		2		3	
	0.133	0.190	0.172	0.172	0.172	0.172
	-38.40	+12.80	-12.80	+25.60	-25.60	+12.80
	+3.48	+4.85	+2.43			
		-1.31	-2.63	-2.63	-1.31	
	+0.17	+0.25	+0.13	+1.22	+2.43	+2.43
		-0.13	-0.25	-0.25		
	+0.02	+0.02	-13.12	+23.94		
	-34.73	+16.48				

Center moment at span (0) and (1) considering the slab as simple supported:

$$M = \frac{1}{8} \times 4.80 \times \overline{8.00}^2 = 38.40 \text{ tm}$$



$$E J_s \varphi_0 = \frac{1}{3} \times 6.40 \times 8.00 - \frac{1}{6} \times 5.80 \times 8.00 = 9.43$$

$$E J_s \varphi_1 = -\frac{1}{3} \times 1.00 \times 8.00 + \frac{1}{6} \times 0.08 \times 8 = -2.56$$

$$\frac{\varphi_0}{\varphi_1} = 3.68 \qquad \varepsilon^2 = 0.408 \qquad \varepsilon = 0.069$$

ω	$\varepsilon\omega$	coh $\varepsilon\omega$	coh $\varepsilon l/2$	$\alpha = \frac{\cosh \varepsilon\omega}{\cosh \varepsilon l/2}$	$m_\omega = -\frac{\ell_0}{\ell_1}\alpha$
0	0	1.000	1.3286	0.75	2.76
0.5	0.13	1.0085	1.3286	0.760	2.80
1.0	0.26	1.034	1.3286	0.776	2.86
1.5	0.39	1.077	1.3286	0.808	2.97
2.0	0.52	1.1383	1.3286	0.817	3.15
2.5	0.65	1.2188	1.3286	0.915	3.37
3.0	0.79	1.32.86	1.3286	1.000	3.68

$$A = 9.19 \text{ tm} = \frac{0.5}{2}\left[2.76 + 3.68 + 2(2.80 + 2.86 + 2.97 + 3.15 + 3.37)\right]$$

so the column moments are

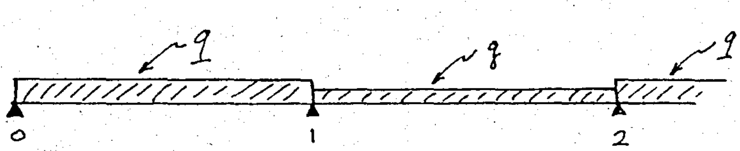
$$M_s = 2 \times 9.19 = 18.38 \text{ tm}$$

Mr. Franz's solution

Slab $g = 0.400$ t/m $p = 0.400$ t/m $J = 4.95$ dm^4/m $l = 8.00$ m

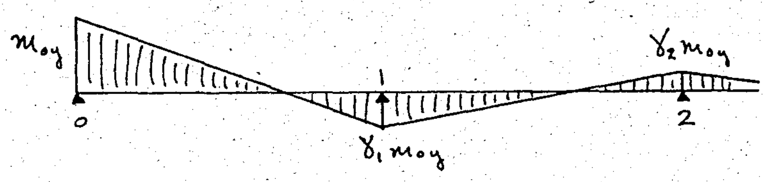
Edge beam $J_t = 72.99$ dm^4

Edge columns $J_o = J_u = 21.35$ dm^4 $\bar{\alpha} = 4$ $c = 1$ $h = 3.00$ m



$$\overline{M}_1 = -(0.1071 \times 0.4 + 0.0536 \times 0.4)\, \overline{8.00}^2 = -4.11 \text{ tm}$$

coeff. 0.1071 and 0.0536 can be found in Beton Kalender for the loading shown above. Likewise $\gamma_1 = 0.267$ is the moment induced on the far end when 1 tm load is applied at support (0), for equal spans.



$$k = \sqrt{\frac{14}{(2-\gamma_1)\,\ell}\;\frac{J}{J_t}} = \sqrt{\frac{14}{(2-0.267)\,8.00}\;\frac{4.95}{72.99}} = 0.262$$

$$k_s = 0.262 \times 3.00 = 0.786$$

for $y = s$ $\quad tgh\ 0.786 = 0.65 \qquad \cosh 0.786 = 0.65$

$$v = \frac{J}{J_u}\,\frac{h_u}{\ell} = \frac{4.95}{21.35}\,\frac{3.00}{8.00} = 0.09$$

$$m_{os}\left(\frac{12 \times 0.09}{(1+1)\,4}\,\frac{0.65}{0.262} + 2 - 0.267\right) = -\frac{0.8 \times 8.00^2}{4} + 4.11$$

$$m_{os} = -4.18\ tm$$

$$m_{1s} = -0.267(-4.18) + (-4.11) = -2.99\ tm$$

$$D_s = 4.18\,\frac{0.65}{0.262} = 10.38\ tm$$ Torsional moment on (0) axis at one span

$$M_s = 2 \times 10.38 = 20.76\ tm$$ Column moment

Torsional moment near the column

$$y_r = (3.00 - 0.20) = 2.80$$

$$k y_r = 0.262 \times 2.80 = 0.734$$

$$\sinh 0.734 = 0.80$$

$$D_r = \frac{4.18}{0.262}\,\frac{0.80}{1.33} = 9.60\ tm$$

for $y = 0$ $\quad \cosh 0 = 1$

$$m_{oo} = -4.18\,\frac{1}{1.33} = -3.15\ tm$$

$$m_{1o} = -0.267(-3.15) - 4.11 = -3.27\ tm$$

Mr. Erich's solution

Slab $g = 0.400\ t/m^2$ $p = 0.400\ t/m^2$ $J = 1.95\ dm^4/m$ $l = l_n = 8.00\ m$

Edge beam $l_{y_1} = l_{y_2} = 6.00\ m$ $b/d = 100/25$ $J_t = 72.99\ dm^4$

$\gamma_t = -0.267$

Columns $40/40$ $J = 21.35\ dm^4$ $h = 3.00\ m$ $K = 7.11\ dm^4/m$

$2 + \gamma_1 = 1.732$

$$K = \frac{3}{2}\ \frac{1.95 \times 10^{-4}}{1.732 \times 8.00} = 0.535 \times 10^{-4}\ m^3/m$$

$$\lambda = 6.00\sqrt{\frac{8(1+\frac{1}{3})\,0.535}{72.99}} = 1.68$$

$$\phi_b = \frac{\tanh \lambda/2}{\lambda/2} = \frac{\tanh 0.84}{0.84} = 0.816$$

$$b = \frac{1}{2} \times 6.00 \times 0.816 = 2.45\ m$$

$$K_R = 2.45 \times 0.535 \times 10^{-4} = 1.31 \times 10^{-4}\ m^3$$

$$\sum K_R = 2 \times 1.31 \times 10^{-4} = 2.62 \times 10^{-4}\ m^3$$

$$m_1^* = -4.11\ tm/m$$

$$\overline{m}_s = -\frac{1}{1.732}\left(\frac{0.8 \times 8.00^2}{4} - 4.11\right) = -5.02\ tm/m$$

$$\varepsilon = \frac{2 \times 7.11 \times 10^{-4}}{2 \times 7.11 \times 10^{-4} + 2.62 \times 10^{-4}} = 0.845$$

$$m_s = -0.845 \times 5.02 = -4.24\ tm/m$$

$$m_0 = -4.24\ \frac{1}{\cosh 0.84} = -3.09\ tm/m$$

$$D_s = 2.45 \times 4.24 = 10.40\ tm$$

Column moment 20.80 tm

Mr. Cakiroglu's solution

Edge beam: $l_{y_1} = l_{y_2} = 6.00\ m$ $\quad J_t = 0.0073\ m^4$

Slab: $l = l_n = 8.00\ m$ $\quad g = 0.40\ t/m^2$ $\quad p = 0.40\ t/m^2$ $\quad J = 4.95 \times 10^{-4}$

Columns: $40/40$ $\quad I = 21.35\ dm^4$

$$R_s = 3.5\ \frac{21.35}{3.00} = 24.90\ dm^4/m$$

Fixed End Moments

$$M_{o_q} = 0.8\ \frac{\overline{8.00}^2}{12} \times 6.00 = 25.60\ tm$$

$$M_{o_g} = 12.80\ tm$$

(a) axis

$$\gamma = \frac{4EI}{R}\ \frac{l}{L} = \frac{4 \times 4.95}{2 \times 24.90}\ \frac{6.00}{8.00} = 0.30$$

$$B_1^2 = \frac{E}{G}\ \frac{J}{J_t}\ \frac{l^2}{L} = \frac{2.00}{0.75}\ \frac{4.95 \times 10^{-4}}{0.0073}\ \frac{\overline{6.00}^2}{8.00} = 0.793$$

$B_1 = 0.89$ $\qquad k = 0.645$

(1) axis

$\gamma = 0.60$

$B_1^2 = 1.586$

$B_1 = 1.26$ $\qquad k = 0.480$

$$k = \frac{1}{\gamma + \frac{B_1}{\tanh B_1}}$$

Distribution factors

(o) axis Columns → 0.645 Ribs (1−k) → 0.355

(1) axis Columns → 0.600 Ribs → 0.200

0 (0.355)	1 (0.20)	1 (0.20)	2 (0.20)	2 (0.20)	3 (0.20)	3 (0.20)
+25.60	−25.60	+12.80	−12.80	+25.60	−25.60	+12.80
− 9.10	− 4.55	− 1.28	− 2.56	− 2.56	− 1.28	
+ 1.86	+3.73	+ 3.73	+ 1.86	+ 1.41	+2.82	+ 2.82
− 0.66	− 0.33	− 0.33	− 0.65	− 0.65		
+ 0.07	+0.14	+ 0.14	−14.15	+23.80		
− 0.03	−26.61	+15.06				
+17.74						

The sum of the rib end moments

(0) axis +17.74 tm

(1) axis −26.61 + 15.06 = −11.55 tm

(2) axis −14.15 + 23.80 = + 9.65 tm

Dividing these values into number of ribs in one span:

(0) axis $\frac{17.74}{6} = +2.96$ tm

(1) axis $\frac{11.55}{6} = -1.93$ tm

(2) axis $\frac{9.65}{6} = +1.61$ tm

The moment (m_s) caused by ribs on the torsional beam, considering only center and column strips:

(0) axis $B_1 = 0.89$ $\quad k_1 = \frac{0.89}{6.00} \frac{\cosh 0}{\sinh 0.89} = 0.990$ for center

alredy used

$k_1 = \frac{0.89}{6.00} \frac{\cosh 0.89}{\sinh 0.89} = 1.420$ for column strip

$m_{s\,center} = +2.96 \times 0.990 = 2.94$ tm

$m_{s\,column} = +2.96 \times 1.420 = 4.21$ tm

(1) axis $B_1 = 1.26$ $\quad k_{1\,center} = 0.62$

$k_{1\,column} = 1.17$

$m_{s\,center} = 1.93 \times 0.62 = 1.20$ tm

$m_{s\,column} = 1.93 \times 1.17 = 2.26$ tm

CONCLUSION

As it has been seen from the solutions of the example, all the methods give approximately the same result; so the amount of the calculation required to get the solution is the important factor. The first and Mr. Çakıroğlu's methods are the most laborous.

Mr. Çakıroğlu's differential equation:

$$M_t + \left(-M_o + \frac{s\varphi}{3}\right) ds - \left(M_t + \frac{dM_t}{ds} ds\right) = 0$$

without $\frac{s\varphi}{3}$, values are the same with the first equation in the first method. The solution of the all differential equations are the same. The solution involves hyperbolic functions.

The torsional moment found in the example is considerably large. Usual practice ignores torsional moments on the edge beams, on ribbed floor buildings with large spans usually the torsional moment governs the dimensions of the edge beam.

For side sway calculations, Mr Çakıroğlu gives the value of the fictitious beam to be considered in the analysis. Mr. Erich Schmid also gives the value of b which is multiplied by (K) of the slab.

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3) Muto

4) Çakıroğlu et Özmen - Caleul des partiques à étanger soumis à des charges horizontales, Technique des Trewanx Sept. Oct. 1961.

5) Adnan Çakıroğlu - Statik sistemelerin hesap metodları.

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8) Kani - Istanbul Teknik Üniversitesi Yayınlarından