STATIC AND DYNAMIC ANALYSIS OF SUPER-ELLIPTICAL PLATES

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B.S. in Yıldız Technical University, 1995

M.S. in Yıldız Technical University, 1998

Submitted to the Institute for Graduate Studies in
Science and Engineering in partial fulfillment of
the requirements for the degree of
Doctor of Philosophy

Graduate Program in Civil Engineering

Boğaziçi University

2005

ACKNOWLEDGEMENTS

Very special thanks to my advisor Prof. Gülay Altay to whom I owe so much for all those I have learnt, and for all those I have experienced. It is a great pleasure for me to complete a dissertation supervised by her. The years I spent in BU were fantastic. My performance and capabilities improved, and my skills developed. Thanks so much.

Next, I'd like to thank to Prof. M. Cengiz Dökmeci (ITU, Department of Naval Architecture) not only for his great contribution to this study but for his didactic and friendly guidance to my academic career as well. He always encouraged me to work harder. Studying with him was splendid. Thanks again.

I also wish to thank to Can Lütfüoğlu (ITU, Department of Physics), and Seyit Çeribaşı (BU, Civil Engineering Department) who have helped me in many different ways. I won't forget their support.

My sincere gratitude is also due to the members of the jury for their useful suggestions and comments.

Finally, I am thankful to my mother and my father for their support and understanding during the period devoted to my graduate studies. This dissertation is dedicated to my dear parents and to my lovely sister.

ABSTRACT

STATIC AND DYNAMIC ANALYSIS OF SUPER-ELLIPTICAL PLATES

This study reports the static and dynamic analysis of super-elliptical plates of uniform thickness. There are mainly three distinct boundary conditions (clamped/simply-supported along the contour and point supported on the contour) to be examined in the dissertation. In static analysis the plate is subjected to constant lateral load. On the other hand, in dynamic analysis the undamped free vibration is concerned. The plate perimeter is defined by a super-elliptic function with a power corresponding to the shape ranging from an ellipse to a rectangle. The Kirchhoff fourth-order plate theory is employed for the isotropic and the functionally graded elastic plate. Numerical examples for plates with different shapes are solved by the Ritz method and the methods of weighted residuals. The results are compared with known ones where possible.

ÖZET

SÜPER-ELİPTİK PLAKLARIN STATİK VE DİNAMİK ANALİZİ

Bu çalışmada sabit kalınlıklı süper-eliptik plakların statik ve dinamik analizi yapılmıştır. Plak çevresi boyunca ankastre, sabit mesnetli ve dört adet nokta mesnet olmak üzere, üç farklı sınır şartı gözönüne alınmıştır. Statik analizde plak, düşey olarak etkiyen üniform yüke maruz bırakılmış, dinamik analizde ise serbest sönümsüz titreşim durumu incelenmiştir. Plağın sınırları, süper-elips denklemi ile tanımlanmıştır. Bu denklem, plak geometrisinin elipsten dikdörtgene olan dönüşümünü göstermektedir. Plak modeli olarak dördüncü mertebeden Kirchhoff plak teorisi kullanılmıştır. Statik analizde çözümler hem izotrop ve homojen malzeme hem de pozisyon bağımlı tabakalı malzeme için yapılmıştır. Ritz yöntemi ve ağırlıklı artıklar metoduyla farklı geometrilerdeki plaklar için sayısal çözümler yapılmış ve bulunan sonuçlar özel durumlara karşı gelen az sayıdaki bilinen değerlerle karşılaştırılmıştır.

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LIST OF SYMBOLS / ABBREVIATIONS

a	The semi-major axis (parallel to x-axis)
b	The semi-minor axis (parallel to y-axis)
c	Aspect ratio
D	Flexural rigidity
D_1	Effective material property
E	Modulus of elasticity
g_i	The i th constraint which denotes zero deflection at the i th point-support
h	Plate thickness
h_m	A complete set of independent functions
k	The super-elliptical power
m	The mass of the plate per unit area
n	Order of the polynomial h_m
n_1	The outward normal of the contour
P	Uniformly distributed lateral load acting vertically downward
r	Total number of α_m
R	Subgrade reaction modulus
R_e, R_b	Residual
S	Contour equation
T	The kinetic energy of the plate
U_b	The strain energy of the plate
V	The potential energy of the lateral load
w	Deflection
w_{max}	Maximum deflection
x,y	Rectangular Cartesian coordinates
(x_i,y_i)	The location of the i th point support
α_m	Unknown coefficients
β_m	The lever arm in method of moments
θ_γ	Basic function including weier-strass polynomial where γ=0,1,2,3,..13
χ	Total number of point-supports

λ_i	The i th Lagrange multiplier
ν	Poisson's ratio
Π	The total potential energy of the plate
Π^*	The extended (modified) total potential energy of the plate including constraints
ϕ_m	An arbitrary function satisfying the boundary conditions
ω	The natural circular frequency of the plate

BVP	Boundary value problems
CS	Case study
FGM	Functionally graded material
FS	Fixed support
GM	Galerkin method
LM	Lagrange multiplier
MOLS	Method of least squares
MOM	Method of moments
MWR	Methods of weighted residuals
PS	Point-supported
RVM	Ritz variational method
SE	Super-ellipse
SS	Simply supported

1. INTRODUCTION

Plates which are of different shapes and sizes, depending on the design requirements, have always attracted the attention of researchers [1]. There are tens of books and hundreds of published papers, each of which presents an in depth study of a specific area of plate analysis. Apart from a small number of exceptions, most of the studies deal with rectangular/circular plates which are relatively simple to handle in comparison with those having irregular shapes or curved boundaries. This is partly because these geometric figures offer numerous advantages in the solution stage among various alternatives, and partly because they are easily adoptable for distinct plate models like Kirchhoff, Mindlin, Reissner, and Reddy.

Due to material properties, the studies on isotropic/orthotropic plates still constitute the majority of the existing literature. With the help of the advance in computer science in the last decade, the analyses on composites and functionally graded materials have been getting more in number.

Real materials and structural components are often non-homogeneous, either by design or because of the physical composition and imperfections in the underlying material. Thus, analytical solutions for non-homogeneous materials under mechanical loads are of considerable interest to engineers and have widespread applications, given the prevalence of these materials in fields as diverse as aerospace, construction, electronics, etc. More precisely, those are essentially composites with carefully manufactured properties that yield desirable mechanical characteristics and properties, such as optimal arrangement of the material, minimum weight, etc [2].

The concept of functionally graded materials (FGMs) was first introduced in 1984 by a group of material scientists in Japan, as ultrahigh temperature resistant materials for aircraft, space vehicles and other engineering applications. Since then, FGMs have attracted much interest as heat-shielding materials. FGMs are composite materials that are microscopically nonhomogeneous but at macro level, the material properties vary continuously from one interface to the other. This is achieved by gradually varying the volume fraction of the constituent materials. The advantage of using these materials is that they can survive environments with high temperature gradients, while maintaining their structural integrity.

FGMs are also known to occur in nature. The structures of bamboos have been found to have macroscopic gradient structure leading to a constant surface stress at every height as well as microscopic gradient structure providing a strength distribution in the radial direction for adapting to bending stresses due to wind loading [3, 4, 5].

Independent of the material, the 2-dimensional structural action of plates results in lighter structures and therefore offers numerous economic advantages. In recent years, lightweight plate structures have been widely used in many engineering applications and vibration analyses of plates of different shapes have been carried out extensively [6].

A good understanding of the free vibration behavior of plate components is crucial to the design and performance evaluation of a mechanical system. The plate resonant frequencies and vibration mode shapes, for instance, are often used to establish the dynamic response of complex engineering systems. Free vibration data are used to derive the resonant behaviors and fatigue stresses of critical machine parts. In the design of aerodynamic lifting surfaces (such as gas turbine blades, helicopter rotors and aircraft wings), free vibration analyses are often examined prior to the detailed forced response and flutter boundary calculations [7].

There are many applications in engineering design where the study of vibration of plates plays an important role [8]. The free vibrations of plates have been of practical and academic interest for over a century [9]. Closed form solutions are possible only for a limited set of simple boundary conditions and geometries [10]. Extensive data on eigenvalues and mode shapes are available when the plate is rectangular or circular.

A rather limited amount of information is available for elliptical plates, while the number of papers dealing with the super-elliptical plates is even smaller.

The current work aims to present an acceptable accuracy for design purposes in the static and dynamic analysis of plates having super-elliptical boundaries. What makes the work original/different in the numerical case studies is “the choice of the shape functions” which is crucial in overcoming/eliminating the corner effect whish arises as the super-elliptical power “k” increases. Since only a little in dynamic analysis and almost none in static analysis are available in the existing literature, this study attempts to present a consistent precision in the analysis of super-elliptical plates as high as possible.

1.1. Literature Review

Plates in general are three-dimensional in nature, but most of the studies on the analyses of plates in the literature are quite often approximated by two-dimensional theories.

The first significant analyses of plates began in the early 1800s, with much of the work attributed to Cauchy, Poisson, Navier, Lagrange, and Kirchhoff. The research done by these early “engineers” was extremely significant. Many of the techniques which they developed are still used in engineering analysis today.

There has been a great deal of research on the vibration analysis of thin elastic plates since the pioneering work of Chladni in 1802. Comprehensive literature surveys and reviews by Leissa present studies of a large variety of plate shapes and different combinations of boundary conditions.

In view of the difficulty of studying the irregular-shaped plates, the preliminary studies on super-elliptical plates have been quite poor in number. The present literature (e.g.,[11, 12, 13, 7], and the references cited therein) is basically focused on the vibration and buckling analysis of super-elliptical plates of different boundary conditions and distinct plate models. As far as the author knows, there is absolutely no work done about the static analysis. Furthermore, the dynamic analysis of point-supported super-elliptical plates has not been carried out yet. In addition to these, the super-elliptical plates made up of functionally graded material have not been investigated either.

The following table lists the preliminary studies on super-elliptical plates:

Table 1.1. The preliminary studies

Reference no	Dynamic analysis	Buckling analysis	Static analysis	Model	Material, thickness
11	X	X		Kirchhoff plate theory	Isotropic, thin plate
12	X			Reddy third-order plate theory	Symmetrically laminated thick plate
13	X			A higher-order shear deformation theory	Thick plate
7	X			3-D elasticity theory	Perforated plate

There are four studies each of which presents a different approach due to plate model or material properties. What these papers have in common in dynamic analysis is that, they all investigate the free undamped vibration analysis of super-elliptical plates as the present dissertation does. Among these papers, there is only one research matching with my study. Therefore, since the buckling analysis is beyond the scope of the thesis, a comparison can only be made with the study which has used Kirchhoff plate theory in case of dynamic analysis.

1.2. Objective

The probable behavior of the solution method can not be well estimated in the very beginning of the stage when the problem is identified if the exact solutions are not known. And the work turns out to be harder unless the problem can be solved analytically.

The current work is motivated primarily by the lack of contributions on the deflection analysis of plates with rounded corners. The rounded corners are advantageous in helping to diffuse and dilute stress concentrations at sharp corners [12].

Rectangular plates with rounded corners are used extensively as structural and machine elements in engineering structures. Although elliptical plates which are commonly used as cover plates for cut-outs in engineering systems, have attracted the attention of researchers over the years [14], there is comparatively insufficient data presented especially for the analysis of super-elliptical plates.

Since the literature devoted to the analysis of point-supported super-elliptical plates is definitely none, both the static and dynamic analyses are carried out. Finally, the functionally graded super-elliptical plates are examined.

The present work provides static analysis and free vibration analysis of plates of rounded corners.

The objectives of the dissertation may be outlined as follows:

- A suitable shape function which can characterize the deformed form of the plate is determined.
- The deflections at the centroid of the plate are computed and are checked with the known values where possible.
- The lowest natural frequencies of transverse free vibration of super-elliptical plates are obtained and are checked with the known values where possible.

2. BASIC EQUATIONS

2.1. The Plate Model

In the analysis of structures, we frequently encounter structural components for which one dimension, referred to as the thickness, is much smaller than the other dimensions. This type of structural component is called "plate". Plates are straight, plane (flat, non-curved) surface structures which have free, simply supported, and fixed boundary conditions, including elastic supports/restraints, or, in some cases, point supports. The shape of a plate is adequately defined by describing the geometry of its middle surface, which is a surface that bisects the plate thickness "h" at each point.

A plate is typically considered to be thin when the ratio of its thickness to a smaller representative lateral dimension is less than 1/20 (a commonly accepted ratio). Using the Kirchhoff (or classical thin) plate theory, the bending solutions may be determined with good accuracy [15]. The small deflection plate theory, generally attributed to Kirchhoff and Love, is based on the following assumptions [16, 17]:

- The material of the plate is elastic, homogeneous, and isotropic.
- The plate is initially flat.
- The thickness of the plate is small compared to its other dimensions. The smallest lateral dimension of the plate is at least ten times larger than its thickness.
- The deflections are small compared to the plate thickness. A maximum deflection from one tenth to one fifth of the thickness is considered as the limit for small-deflection theory. This limitation can also be stated in terms of length; i.e., the maximum deflection is less than one fiftieth of the smaller span length.
- The slopes of the deflected middle surface are small compared to unity.
- The deformations are such that straight lines, initially normal to the middle surface, remain straight lines and normal to the middle surface (deformations due to transverse shear will be neglected).

- The deflection of the plate is produced by displacement of points of the middle surface normal to its initial plane.
- The stresses normal to the middle surface are of a negligible order of magnitude.
- The strains in the middle surface produced by inplane forces can usually be neglected in comparison with strains due to bending (inextensional plate theory).

Unless otherwise stated, the validity of the "Kirchhoff-Love plate theory" is assumed throughout this thesis.

2.2. The Governing Equation

The governing plate equation in the Cartesian coordinate system is given by

$$\nabla^4 w - \frac{P}{D} = 0 \tag{2.1}$$

where w, P and D denote the deflection function, the lateral load and the flexural rigidity, respectively [17].

$$\mathrm{D} = \frac{\mathrm{Eh}^3}{12(1-\nu^2)} \tag{2.2}$$

2.2.1. Rigorous Solution of The Governing Differential Equation

$$\nabla^4 w = \frac{P}{D} \tag{2.3}$$

Mathematically, the differential equation of plates is classified as a linear partial differential equation of the fourth order having constant coefficients. Its homogeneous form,

$$\nabla^4 \mathrm{w} = 0 \tag{2.4}$$

is called the biharmonic equation.

In general, there are four types of mathematically "exact" solutions available for plate problems [16]:

- Closed-form solution.
- Solution of the biharmonic equation upon which a "particular solution" of the governing differential equation of plate is superimposed.
- Double trigonometric series solution.
- Single series solution.

2.2.2. Numerical and Approximate Methods

The solution of plate problems via the classical route, is limited to relatively simple plate geometry, load, and boundary conditions. If these conditions are more complex, the analysis becomes increasingly tedious and even impossible. In such cases numerical and approximate methods are the only approaches that can be employed.

In the engineering application of the theory of plates, we are usually dealing with inaccurate input data. In the first place, the external loads are known only with a certain degree of accuracy. In addition, the material properties, such as the modulus of elasticity, and the Poisson's ratio can contain considerable inaccuracies. Furthermore, the actual boundary conditions are merely approximations of the theoretical ones. Consequently, even an "exact" solution only approximates the real behavior of a plate.

Employing approximate methods, an additional inaccuracy is introduced in our computation, which is called the "error of calculation". Naturally, this must be smaller than the error of data. As a rule, it is desirable that the error of calculation have less than ±5% discrepancy in comparison with the exact solution. In some practical applications, however, a ±10% error of calculation might be permissible. If the approximate method is used for checking or estimating purposes, even larger errors of computation can be tolerated [18].

2.3. The Plate Geometry

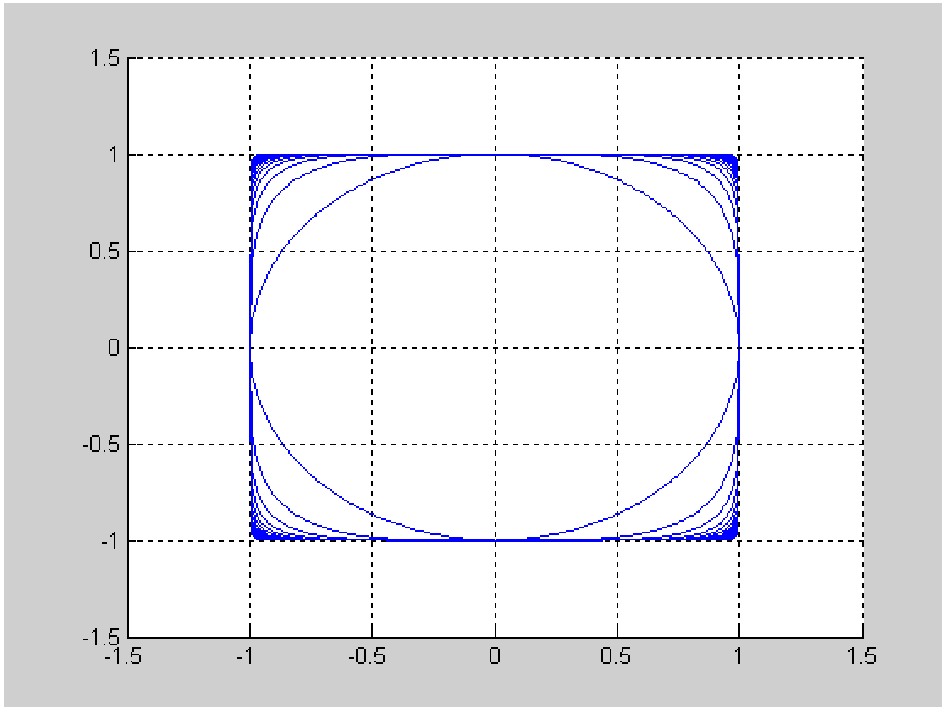

Figure 2.1. The shape of a super-ellipse (a=1, c=1, k=1, 2, ...20)

The plate has the form of a super-ellipse (SE) of uniform thickness "h". The periphery shape of the plate can be defined by the equation

$$\left(\frac{x}{a}\right)^{2k} + \left(\frac{y}{b}\right)^{2k} = 1 \tag{2.5}$$

where a, b and k are the semi-major axis, the semi-minor axis and the positive integer super-elliptical power, respectively.

The aspect ratio can be defined as

$$c = \frac{a}{b} \tag{2.6}$$

The shape becomes an ellipse (a circle if c=1) when k=2. Interestingly, if k is continually increased, the plate turns out to be a rectangle (a square if c=1) with four rounded corners. The form becomes a rectangle as k approaches infinity. The coordinate axes are located such that the origin of the xy-frame and the centroid of the super-ellipse coincide. Therefore, the plate possesses two-fold symmetry.

Table 2.1. The area of a super-ellipse (a=1, c=1, k=1, 2, ..20)

k	area	k	area	k	area	k	area
1	3.1416	6	3.9579	11	3.9852	16	3.9916
2	3.7077	7	3.9680	12	3.9871	17	3.9923
3	3.8544	8	3.9747	13	3.9886	18	3.9928
4	3.9127	9	3.9793	14	3.9898	19	3.9933
5	3.9415	10	3.9827	15	3.9908	20	3.9937

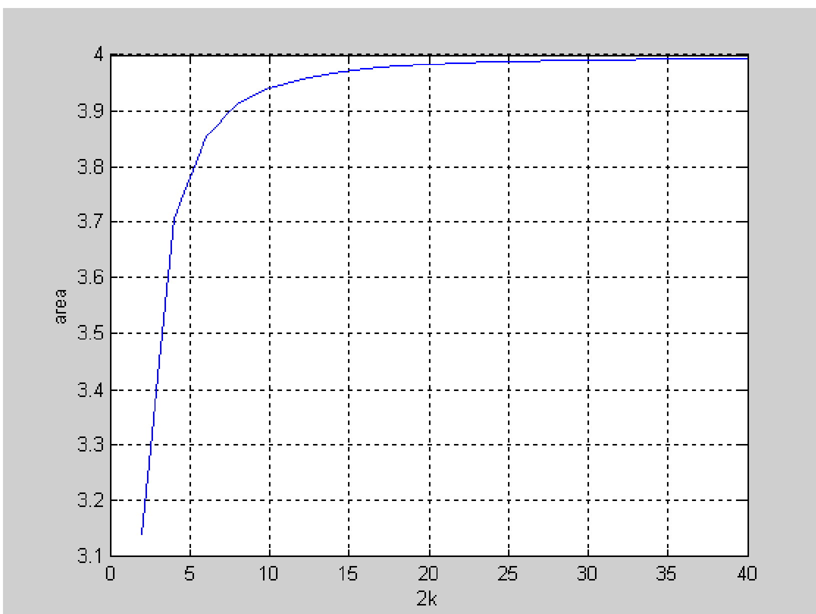

Figure 2.2. The area of a super-ellipse (a=1, c=1, k=1, 2, ..20)

2.4. The Loading

In static analysis, the plate is subjected to static uniform transverse load “P”. Since only the free undamped vibration of super-elliptical plates is examined, no load is considered in dynamic analysis.

2.5. The Boundary Conditions

This dissertation presents mainly 3 distinct boundary conditions for the static and dynamic analysis of super-elliptical plates.

2.5.1. Fixed along the Contour

The built-in edge has no degree of freedom against deflection and rotation. Therefore, the boundary conditions along the plate contour are zero deflection and zero slope, namely

$$\mathrm{w} = 0 \tag{2.7}$$

$$\frac{\partial \mathrm{w}}{\partial \mathrm{n}_1} = 0 \tag{2.8}$$

where n_1 is the outward normal of the contour

2.5.2. Point-Supported on the Contour

The plate is assumed to be placed on 4 point supports which are symmetrical with respect to x and y axes. The exact locations of the supports are the points at which the line “ $y = \pm \frac{x}{c}$ ” and the contour intersect.

The boundary conditions to be considered are zero deflection at the point-supports, namely

$$w(x_i, y_i) = 0 \tag{2.9}$$

The zero deflections at the supports are characterized by the constraints "$g_i(x_i, y_i)$".

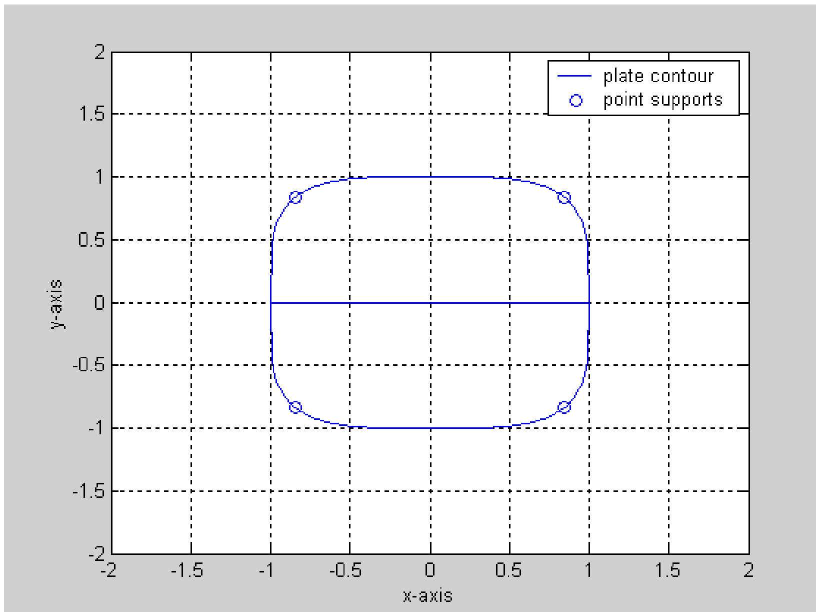

Figure 2.3. Location of the point-supports (c=1, k=2)

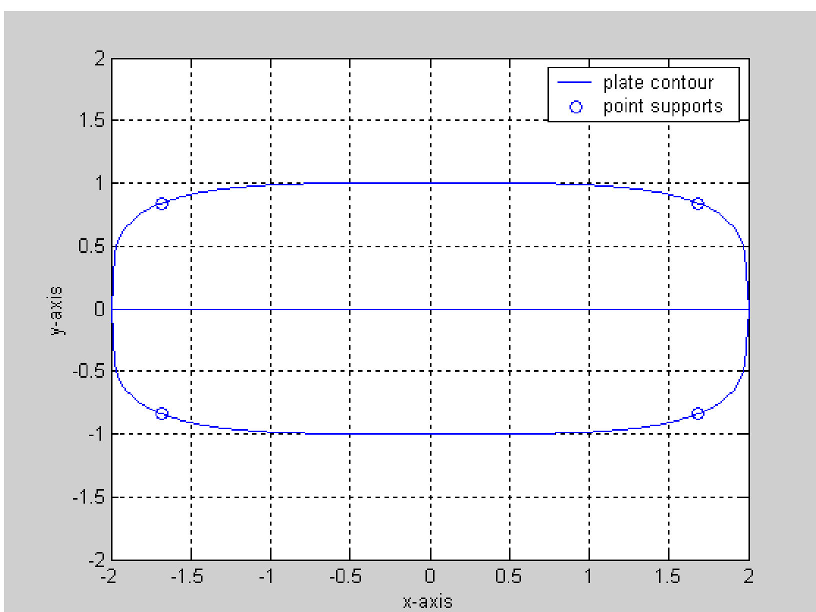

Figure 2.4. Location of the point-supports (c=2, k=2)

2.5.3. Simply Supported along the Contour

The geometrical boundary condition to be satisfied is zero deflection along the contour, namely

$$w = 0 \tag{2.10}$$

2.6. The Material and the Environment Conditions

The first five case studies deal with isotropic and homogeneous material. The other ones investigate the functionally graded material. The humidity is neglected, and no temperature change is considered.

2.7. The Potential Energy of the Plate

The strain energy of the plate in bending may be written as [16]

$$U_b = \frac{D}{2}\left[\int_{x_1}^{x_2}\int_{y_1}^{y_2}\left[\left(\nabla^2 w\right)^2 - 2(1-\nu)\left\{\frac{\partial^2 w}{\partial x^2}\frac{\partial^2 w}{\partial y^2} - \left(\frac{\partial^2 w}{\partial x \partial y}\right)^2\right\}\right]\right]dxdy \tag{2.11}$$

where

$$x_1 = -a \tag{2.12}$$

$$x_2 = a \tag{2.13}$$

$$y_1 = -\left(\frac{b}{a}\right)\sqrt[2k]{a^{2k} - x^{2k}} \tag{2.14}$$

$$y_2 = \left(\frac{b}{a}\right)\sqrt[2k]{a^{2k} - x^{2k}} \tag{2.15}$$

If all edges of the plate are fixed, equation (2.11) becomes

$$U_b = \frac{D}{2}\left[\int_{x_1}^{x_2}\int_{y_1}^{y_2}\left[\left(\nabla^2 w\right)^2\right]\right]dxdy \tag{2.16}$$

The potential energy of the lateral load is

$$V = -\int_{x_1}^{x_2}\int_{y_1}^{y_2}(Pw)dxdy \tag{2.17}$$

The total potential energy of the plate is given by [16, 18]

$$\Pi = U_b + V \tag{2.18}$$

2.8. The Selection of the Trial Functions

2.8.1. Static Analysis

$$w = \sum_{q=0}^{n}\sum_{i=0}^{q} \alpha_m \phi_m \tag{2.19}$$

$$m = \frac{\left(\frac{q-2}{2}+1\right)\left(\frac{q-2}{2}+2\right)}{2} + \left(\frac{q-i}{2}+1\right) \tag{2.20}$$

$$\phi_m = h_m \theta_\gamma \tag{2.21}$$

$$h_m = x^i y^{q-i} \tag{2.22}$$

Since the origin of the coordinate system is in the center of the plate, “q” and “i” have to be even numbers only.

$$r = \frac{\left(\frac{n}{2}+1\right)\left(\frac{n}{2}+2\right)}{2} \tag{2.23}$$

2.8.2. Dynamic Analysis

$$w = \sum_{q=0}^{n}\sum_{i=0}^{q} \alpha_m \phi_m \tag{2.24}$$

$$m = \frac{q(q+1)}{2} + q - i + 1 \tag{2.25}$$

$$\phi_m = h_m \theta_\gamma \tag{2.26}$$

$$h_m = x^i y^{q-i} \tag{2.27}$$

In order not to skip the frequencies corresponding to unsymmetrical modes, a complete weier-strass polynomial including both “odd” and “even” values of “q” and “i” are considered.

$$r = \frac{(n+1)(n+2)}{2} \tag{2.28}$$

2.8.3. The Trial Functions

$$S = \left[1 - \left(\frac{x}{a}\right)^{2k} - \left(\frac{y}{b}\right)^{2k}\right] \tag{2.29}$$

$$R_1 = (a^2 - x^2) \tag{2.30}$$

$$R_2 = (b^2 - y^2) \tag{2.31}$$

$$R_3 = (a^2b^2 - x^2y^2) \tag{2.32}$$

$$S_4 = \left[1 - \left[\left(\frac{x}{a}\right)^{2k} + \left(\frac{y}{b}\right)^{2k}\right]^2\right] \tag{2.33}$$

$$S_5 = \left[1 - \left[\left(\frac{x}{a}\right)^{2k} + \left(\frac{y}{b}\right)^{2k}\right]^3\right] \tag{2.34}$$

$$\theta_0 = 1 \tag{2.35}$$

$$\theta_1 = (S)^2 \tag{2.36}$$

$$\theta_2 = (R_1)^2(R_2)^2(S)^2 \tag{2.37}$$

$$\theta_3 = (R_1)(R_2)(S)^2 \tag{2.38}$$

$$\theta_4 = (R_3)(S)^2 \tag{2.39}$$

$$\theta_5 = (R_3)^2(S)^2 \tag{2.40}$$

$$\theta_6 = (S_4)^2 \tag{2.41}$$

$$\theta_7 = (S_5)^2 \tag{2.42}$$

$$\theta_8 = (R_1)(R_2)^2(S)^2 \tag{2.43}$$

$$\theta_9 = (R_1)^2(R_2)(S)^2 \tag{2.44}$$

$$\theta_{10} = (R_1)^2(R_2)^2(S) \tag{2.45}$$

$$\theta_{11} = (R_1)(R_2)(S) \tag{2.46}$$

$$\theta_{12} = (R_1)^2(R_2)(S) \tag{2.47}$$

$$\theta_{13} = (R_1)(R_2)^2(S) \tag{2.48}$$

3. METHOD OF SOLUTION

3.1. Methods of Weighted Residuals

Methods of weighted residuals (MWR) are applicable when the governing equations are differential equations.

The MWR has been used in solving a variety of boundary value problems (BVP), ranging from fluid flow to heat and mass transfer problems. It is popular because of the interactive nature of the first step, that is, the user provides a first guess at the solution and this is then forced to satisfy the governing equations along with the conditions imposed at the boundaries. The left-over terms, called “residuals”, arise because the chosen form of solution does not exactly satisfy either the equation or the boundary conditions. How these residual terms are minimized provides the basis for parameter or function selection. Of course, the optimum solution depends on the intelligent selection of a proposed solution [19].

The essential idea of the MWR is to construct an approximate solution. Because of the approximate solution of the estimated solution, it may not, in general, satisfy the equation and the boundary conditions; that is: the residuals (R_e and R_b) are not identically zero. If the approximate solution is constructed such that the differential equation is satisfied exactly (i.e., R_e=0) the method is called the “boundary method”. However, if it is constructed such that the boundary conditions are satisfied exactly (i.e., R_b=0) the method is called the “interior or domain method”. If neither the differential equation nor the boundary conditions are satisfied exactly, it is referred to as the “mixed method” [20, 19].

MWR criteria seek to minimize an expression of error in the differential equation (not the unknown function itself). There are many different MWR criteria four of which are [21, 22]:

3.1.1. The Galerkin Method

The Galerkin method is one of the well-known weighted residual methods. Although the mathematical theory behind it is quite complex, its physical interpretation is relatively simple. A trial function "w" such that it is a complete set of linearly independent functions capable of representing the lateral deflections is selected. Each term of this expression must satisfy all (both geometrical and static) boundary conditions of the problem but not necessarily the governing differential equation of the plate. Accuracy of the method depends considerably on the selected shape function, which is the case for all energy approaches [16].

3.1.2. The Method of Moments

The moment method, one of the well known –although rarely applied in mechanical engineering problems- weighted residual methods, is used to solve the problem. The first application of the moment method appears to be due to Yamada (1947). As was already mentioned, the weight function "w" is a complete set of linearly independent functions. The reason for choosing even values of "i" and "q" for β_m is that the method does not yield any result if odd values are included [14, 20, 23, 24].

3.1.3. The Collocation Method

Instead of trying to satisfy the differential equations in an "average" form, the collocation method tries to satisfy them at only a series of chosen points called the collocation points at which the residual vanishes. These points are usually but not necessarily, evenly distributed in the domain. In principle the number of terms in the trial function has to be the same as the number of collocation points [24].

3.1.4. The Method of Least Squares

The method of least squares is applied directly to the residual. The trial function is chosen to satisfy the boundary conditions and the unknown coefficients are determined by bringing the residual-the error towards zero, as much as possible. This procedure, as in the case of Ritz method, leads to a set of linear equations in terms of unknown parameters. This approach, however, often leads to an entirely incorrect solution, in which the deflections even have wrong signs throughout [16].

3.2. Ritz Variational Method

Ritz variational method (RVM) is applicable when the governing equations are variational (integral) equations.

Variational principles, sometimes referred to as extremum or minimum principles, seeks to minimize, or find an extremum in, some physical quantity, such as energy. While RVM is the most popular one among energy methods, the Lagrange multipliers method is another effective technique used in optimization.

3.2.1. The Ritz Method

The Ritz method is widely used to solve problems in structural mechanics, particularly ones dealing with vibrations, buckling, and deflection. The first two types are eigenvalue problems, whereas the last is an equilibrium problem [25].

An energy method developed by Ritz applies the principle of minimum potential energy. According to this theorem, of all displacements that satisfy the boundary conditions, those making the total potential energy of the structure a minimum are the sought deflections pertinent to the stable equilibrium conditions. Satisfaction of the governing equation of the plate by the assumed expression "w" is not required. The deflected plate surface is represented by "w" which is a complete set of linearly independent functions ϕ_m that satisfy individually at least the geometrical boundary conditions. The unknown constants α_1, α_2, α_3, ..., α_r are determined from the minimum potential energy principle. This minimization procedure yields r simultaneous algebraic equations from which the unknown parameters α_m can be computed. It should be noted that during the partial differentiation all coefficients, except the spesific α_m under consideration, are taken constant. The accuracy of the Ritz method depends considerably on how well the assumed shape functions are capable of describing the actual deflection surface [16].

As in the case of static analysis of plates, the energy methods occupy an important position among the approximate methods for solving problems on dynamic analysis. But, the free vibration analysis of plates by energy methods is not as sensitive to the proper choice of the shape functions as its static counterpart. Unlike Rayleigh's method which gives remarkably accurate lowest frequency values, which are slightly higher than the actual ones, the Ritz method is capable of yielding information on the higher frequencies. On the other hand, the accuracy of these frequencies usually deteriorates progressively for the higher modes.

3.2.2. The Lagrange Multiplier (LM) Method

An extension of the Ritz method is the Lagrange multiplier method, which follows the general procedure outlined above. In many problems involving the deflection of plates for which the Ritz method is used, it is indeed difficult to construct a series of assumed functions for the deflection that satisfies all the boundary conditions, especially if these functions are to possess the properties of orthogonality. It is possible to avoid this difficulty by the Lagrange multipliers to enforce boundary conditions or constraints not satisfied by the assumed series of functions [18]. Consequently, this approach is less restrictive. By using certain multipliers during the minimization process, we can arrange that the series expression "w" as a whole will satisfy the boundary conditions, although the individual functions do not fulfill them.

The Lagrange Multipliers method can be outlined as follows:

All relative extrema of the function z=f(x, y), subject to a constraint g(x, y)=0, will be found among those points (x, y) for which there exists a value of λ such that

$$F_x(x, y, \lambda)=0 \tag{3.1}$$

$$F_y(x, y, \lambda)=0 \tag{3.2}$$

$$F_{\lambda}(x, y, \lambda)=0 \tag{3.3}$$

where

$$F(x, y, \lambda) = f(x, y) + \lambda g(x, y) \tag{3.4}$$

and all indicated partial derivatives exist.

In the theorem, the function $F(x, y, \lambda)= f(x, y)+ \lambda g(x, y)$ is called the Lagrange function, and "λ" is called the Lagrange multiplier.

The process can be described step by step as shown below [26]:

- Write the constraint in the form

$$g(x, y)=0 \tag{3.5}$$

- Form the Lagrange function

$$F(x, y, \lambda) = f(x, y) + \lambda g(x, y) \tag{3.6}$$

- Find F_x, F_y, F_{λ}.

- Form the system of equations

$$F_x=0 \tag{3.7}$$

$$F_y=0 \tag{3.8}$$

$$F_{\lambda}=0 \tag{3.9}$$

- Solve the system in the previous step; the relative extrema for "f" are among the solutions of the system.

The numerical methods have the advantages and disadvantages reversed. The trial functions are easy to construct but the choice of the shape functions is crucial in getting satisfactory results. Sometimes, it is quite a hard task to determine the suitable trial function among a large number of alternatives. The crux of the solution of plate problems by numerical methods is the selection of the proper shape (trial) functions, on which the obtainable accuracy depends.

4. CASE STUDIES

In the dissertation, the so-called pb-2 function is employed for different methods like RVM, and MWR. The function is taken as the product of a complete 2-dimensional polynomial function (p-2) and a basic function (b). The reason why the basic function in static analysis is even is that both the plate geometry and the loading is symmetrical with respect to the xy-frame. In other words, there is two-fold symmetry. However, both even and odd powers of "x" and "y" are used in computing the natural frequencies in dynamic analysis. If only the even powers were used, just the natural frequencies corresponding to symmetrical modes would be obtained.

The definite integration over the plate area is a difficult task to carry out analytically. Therefore, the 12-point Gaussian quadrature technique [27] is employed in evaluating the integration with respect to "x" in the range $[x_1 \; x_2]$. The computations are carried out by MATLAB and FORTRAN.

The entire work consists of nine case studies including two different aspect ratios provided that the semi-minor axis and the Poisson's ratio remain constant.

$$b=1 \tag{4.1}$$

$$\nu=0.3 \tag{4.2}$$

The numerical results correspond to plates of 20 different shapes with a super-elliptical power varying from 1 to 20. The maximum deflection and the lowest natural frequency for each super-elliptical power are listed in the tables and the results are compared with known ones where possible. The more detailed tabulated results are shown in Appendix A.

Table 4.1. The content of the case studies

Case study no	FS	PS	SS	Elastic foundation	Static analysis	Dynamic analysis	Isotropic, homogeneous material	FGM
1	X				X		X	
2	X			X	X		X	
3	X					X	X	
4		X			X		X	
5		X				X	X	
6	X				X			X
7			X		X			X
8		X			X			X
9		X				X		X

4.1. Static Analysis of Clamped Super-Elliptical Plates

4.1.1. Abstract

This paper presents the static analysis of clamped super-elliptical plates subject to uniformly distributed lateral load acting vertically downward. The plate thickness is constant, and the material is isotropic and homogeneous. The exact deflection at the centroid of circular/square and elliptical/rectangular clamped plates are available. Hence, the upper and lower bounds of the interval are known and are expressed in terms of "$\frac{P}{D}$" in the tables/graphs.

4.1.2. The Governing Equation and The Potential Energy of The Plate

$$\nabla^4 w - \frac{P}{D} = 0 \tag{4.3}$$

$$U_b = \frac{D}{2}\left[\int_{x_1}^{x_2}\int_{y_1}^{y_2}\left[\left(\nabla^2 w\right)^2 - 2(1-\nu)\left\{\frac{\partial^2 w}{\partial x^2}\frac{\partial^2 w}{\partial y^2} - \left(\frac{\partial^2 w}{\partial x \partial y}\right)^2\right\}\right]\right]dxdy \tag{4.4}$$

$$V = -\int_{x_1}^{x_2}\int_{y_1}^{y_2}\left(Pw\right)dxdy \tag{4.5}$$

$$\Pi = U_b + V \tag{4.6}$$

4.1.3. Methods of Solution

- Ritz method

$$\frac{\partial \Pi}{\partial \alpha_m} = 0 \tag{4.7}$$

- Galerkin method

$$\int_{x_1}^{x_2}\int_{y_1}^{y_2}\left(\nabla^4 w - \frac{P}{D}\right)\phi_m dxdy = 0 \tag{4.8}$$

- Method of moments

$$\int_{x_1}^{x_2}\int_{y_1}^{y_2}\left(\nabla^4 w - \frac{P}{D}\right)\beta_m dxdy = 0 \tag{4.9}$$

$$\beta_m = h_m = x^i . y^{q-i} \tag{4.10}$$

where “i”, and “q” are even natural numbers.

- Method of least squares

$$\int_{x_1}^{x_2}\int_{y_1}^{y_2}\left(\nabla^4 w-\frac{P}{D}\right)^2 dxdy=0 \qquad (4.11)$$

- Collocation method

$$\nabla^4 w-\frac{P}{D}=0 \qquad (4.12)$$

4.1.4. Numerical Results

The tables shown below list "the maximum deflection (denoted by w_{max})" as "k" increases. Each table not only corresponds to a specific shape function, aspect ratio and method of solution, but also lists the values of "w_{max}" for distinct number of terms of weier-strass polynomial. The known exact values of "w_{max}" for circular/square and elliptic/rectangular plates are shown in the following to make a comparison.

- "0.0156" corresponds to the lower bound of interval for " w_{max}" when "a=1" and "b=1"
- "0.0202" corresponds to the upper bound of interval for " w_{max}" when "a=1" and "b=1"
- "0.0339" corresponds to the lower bound of interval for " w_{max}" when "a=2" and "b=1"
- "0.0406" corresponds to the upper bound of interval for " w_{max}" when "a=2" and "b=1"

Table 4.2. Static analysis (CS=1, RVM, γ=11, c=1)

c=1	w_{max}	w_{max}	w_{max}	w_{max}	w_{max}
γ=11	r=1	r=3	r=6	r=10	r=15
2k	n=0	n=2	n=4	n=6	n=8
2	0.0147	0.0154	0.0182	0.0188	0.0209
4	0.0175	0.0164	0.0189	0.0188	0.0194
6	0.0186	0.0175	0.0191	0.0191	0.0186
8	0.0192	0.0183	0.0192	0.0193	0.0201
10	0.0196	0.0187	0.0193	0.0194	0.0196
12	0.0199	0.0190	0.0194	0.0193	0.0203
14	0.0202	0.0193	0.0196	0.0195	0.0196
16	0.0204	0.0194	0.0197	0.0205	0.0197
18	0.0206	0.0196	0.0198	0.0198	0.0180
20	0.0207	0.0197	0.0198	0.0243	0.0197
22	0.0208	0.0198	0.0199	0.0199	0.0198
24	0.0209	0.0199	0.0200	0.0197	0.0197
26	0.0209	0.0199	0.0199	0.0199	0.0199
28	0.0209	0.0199	0.0200	0.0200	0.0198
30	0.0209	0.0200	0.0200	0.0197	0.0200
32	0.0209	0.0199	0.0198	0.0199	0.0198
34	0.0209	0.0199	0.0198	0.0163	0.0199
36	0.0208	0.0199	0.0198	0.0198	0.0199
38	0.0207	0.0198	0.0196	0.0198	0.0195
40	0.0207	0.0198	0.0196	0.0197	0.0188

Table 4.3. Static analysis (CS=1, RVM, γ=11, c=2)

c=2	w_{max}	w_{max}	w_{max}	w_{max}	w_{max}
γ=11	r=1	r=3	r=6	r=10	r=15
2k	n=0	n=2	n=4	n=6	n=8
2	0.0319	0.0312	0.0367	0.0370	0.0413
4	0.0376	0.0325	0.0384	0.0375	0.0396
6	0.0397	0.0346	0.0386	0.0380	0.0395
8	0.0410	0.0361	0.0387	0.0381	0.0396
10	0.0418	0.0370	0.0388	0.0382	0.0396
12	0.0424	0.0376	0.0390	0.0384	0.0396
14	0.0428	0.0380	0.0392	0.0385	0.0396
16	0.0431	0.0382	0.0393	0.0387	0.0396
18	0.0434	0.0384	0.0394	0.0389	0.0396
20	0.0436	0.0386	0.0395	0.0391	0.0396
22	0.0438	0.0388	0.0395	0.0392	0.0396
24	0.0439	0.0389	0.0395	0.0394	0.0396
26	0.0440	0.0390	0.0395	0.0395	0.0396
28	0.0441	0.0391	0.0396	0.0396	0.0396
30	0.0442	0.0392	0.0396	0.0397	0.0396
32	0.0443	0.0393	0.0396	0.0398	0.0397
34	0.0443	0.0393	0.0396	0.0398	0.0397
36	0.0444	0.0394	0.0396	0.0399	0.0397
38	0.0444	0.0395	0.0396	0.0399	0.0397
40	0.0444	0.0395	0.0396	0.0400	0.0397

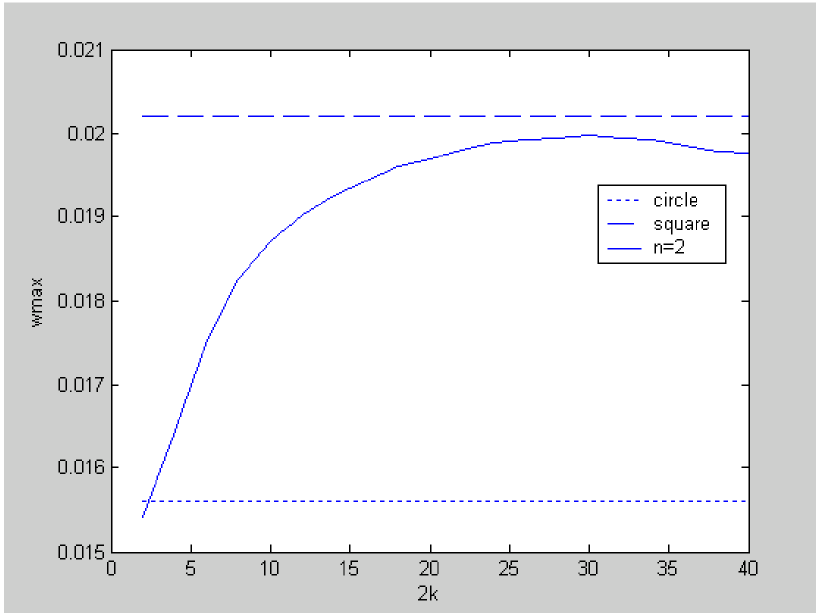

Figure 4.1. w_{max}-2k graph (CS=1, c=1, γ=11)

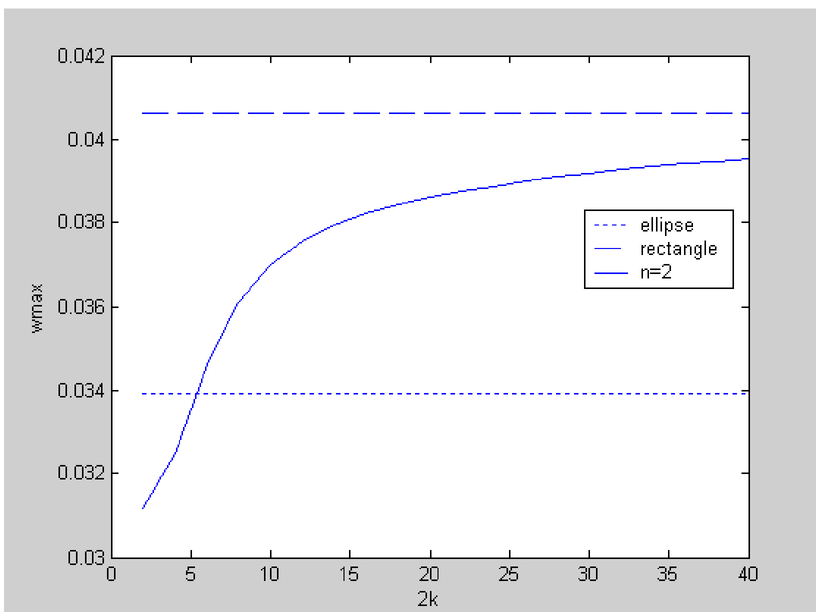

Figure 4.2. w_{max}-2k graph (CS=1, c=2, γ=11)

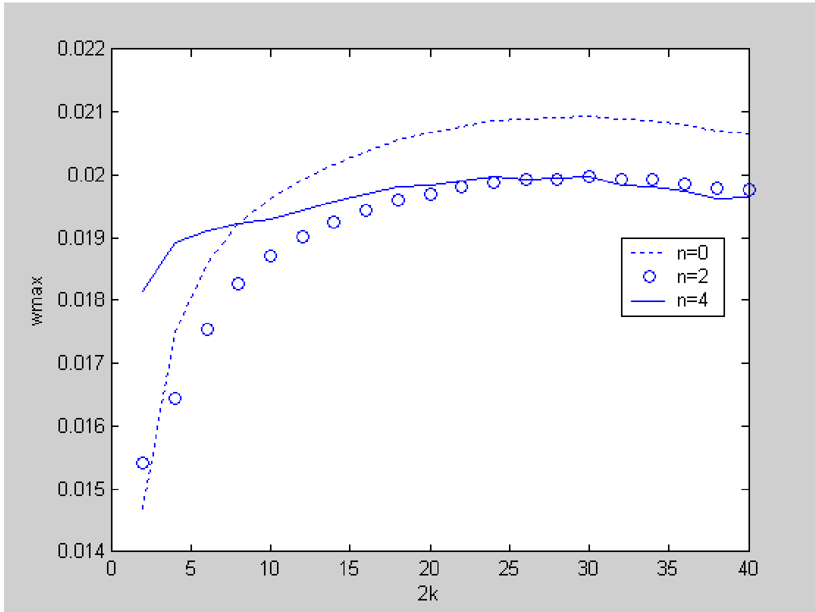

Figure 4.3. Convergence of "w_{max}" (CS=1, c=1, γ=11)

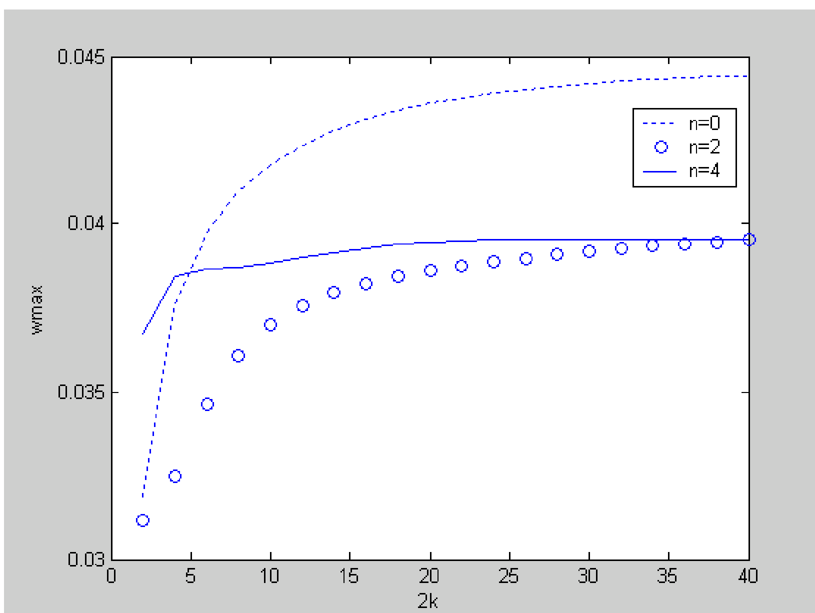

Figure 4.4. Convergence of "w_{max}" (CS=1, c=2, γ=11)

4.1.5. Conclusions

It can easily be seen from the tables that the Ritz method gives the best and the most sensitive results while Galerkin method and the method of moments yield significant values for some certain interval of k as shown in the appendix A. As a result of the solution method, there are infinitely many alternatives in choosing the shape functions. Among the ones tried, there are at least 2 different shape functions which can define the deformed surface approximately. Since approximate methods are used, some of the values are slightly out of the expected range. As far as the author knows, this is partly because of the corner effect, and partly because of the inevitable truncation and round-off errors.

4.2. Static Analysis of Clamped Super-Elliptical Plates Resting on Elastic Foundation

4.2.1. Abstract

This work deals with the deflection analysis of clamped super-elliptical plates of uniform thickness subjected to constant lateral load and placed on elastic foundation of Winkler type. The material is isotropic and homogeneous. The results listed in tables below are in terms of “P”.

4.2.2. The Governing Equation and The Potential Energy of The Plate

$$\nabla^4 w - \frac{P - Rw}{D} = 0 \qquad (4.13)$$

$$U_b = \frac{D}{2}\left[\int_{x_1}^{x_2}\int_{y_1}^{y_2}\left[\left(\nabla^2 w\right)^2 - 2(1-\nu)\left\{\frac{\partial^2 w}{\partial x^2}\frac{\partial^2 w}{\partial y^2} - \left(\frac{\partial^2 w}{\partial x \partial y}\right)^2\right\}\right]\right] dxdy + \frac{1}{2}\int_{x1}^{x2}\int_{y1}^{y2} Rw^2 dxdy \qquad (4.14)$$

$$V = -\int_{x_1}^{x_2}\int_{y_1}^{y_2} (Pw) dxdy \qquad (4.15)$$

$$\Pi = U_b + V \qquad (4.16)$$

4.2.3. Methods of Solution

- Ritz method

$$\frac{\partial \Pi}{\partial \alpha_m} = 0 \tag{4.17}$$

- Galerkin method

$$\int_{x_1}^{x_2}\int_{y_1}^{y_2}\left(\nabla^4 w - \frac{P - Rw}{D}\right)\phi_m dxdy = 0 \tag{4.18}$$

- Method of moments

$$\int_{x_1}^{x_2}\int_{y_1}^{y_2}\left(\nabla^4 w - \frac{P - Rw}{D}\right)\beta_m dxdy = 0 \tag{4.19}$$

$$\beta_m = h_m = x^i y^{q-i} \tag{4.20}$$

where “i”, and “q” are even natural numbers.

4.2.4. Numerical Results

The parameters used in the computations are shown in the following:

$$h=0.2 \tag{4.21}$$

$$E=25x10^6 \tag{4.22}$$

$$R=300x10^3 \tag{4.23}$$

The tables shown below depict “the maximum deflection (denoted by w_{max})” as “k” increases. Each table not only corresponds to a specific shape function, aspect ratio and method of solution, but also lists the values of “w_{max}” for distinct number of terms of weier-strass polynomial.

Table 4.4. Static analysis (CS=2, RVM, γ=1, c=1)

c=1	w_{max}	w_{max}	w_{max}	w_{max}	w_{max}
γ=1	r=1	r=3	r=6	r=10	r=15
2k	n=0	n=2	n=4	n=6	n=8
2	7.3901e-7	7.3274e-7	7.3213e-7	7.3239e-7	7.3219e-7
4	2.9751e-7	6.4841e-7	8.2562e-7	8.7927e-7	8.8892e-7
6	1.1881e-7	4.8855e-7	8.1722e-7	9.0349e-7	9.0829e-7
8	5.6924e-8	3.5008e-7	7.7802e-7	9.0647e-7	9.1144e-7
10	3.1275e-8	2.4561e-7	7.2289e-7	9.0458e-7	9.1168e-7
12	1.9124e-8	1.7241e-7	6.6538e-7	9.0125e-7	9.1207e-7
14	1.2775e-8	1.2317e-7	6.1263e-7	8.9924e-7	9.1502e-7
16	9.1761e-9	9.0700e-8	5.6604e-7	8.9981e-7	9.2348e-7
18	6.9805e-9	6.9269e-8	5.2449e-7	9.0305e-7	9.4046e-7
20	5.5486e-9	5.4964e-8	4.8616e-7	9.0876e-7	9.6657e-7
22	4.5556e-9	4.5334e-8	4.4931e-7	9.1665e-7	9.9822e-7
24	3.8268e-9	3.8946e-8	4.1242e-7	9.2606e-7	1.0302e-6
26	3.2638e-9	3.5049e-8	3.7404e-7	9.3560e-7	1.0594e-6
28	2.8093e-9	3.3403e-8	3.3311e-7	9.4311e-7	1.0853e-6
30	2.4293e-9	3.4270e-8	2.9007e-7	9.4647e-7	1.1084e-6
32	2.1040e-9	3.8531e-8	2.4939e-7	9.4509e-7	1.1292e-6
34	1.8215e-9	4.7781e-8	2.2187e-7	9.4154e-7	1.1476e-6
36	1.5749e-9	6.3002e-8	2.2296e-7	9.4236e-7	1.1638e-6
38	1.3599e-9	7.7650e-8	2.6283e-7	9.5568e-7	1.1779e-6
40	1.1731e-9	7.4751e-8	3.3142e-7	9.8485e-7	1.1900e-6

Table 4.5. Static analysis (CS=2, RVM, γ=1, c=2)

c=2	w_{max}	w_{max}	w_{max}	w_{max}	w_{max}
γ=1	r=1	r=3	r=6	r=10	r=15
2k	n=0	n=2	n=4	n=6	n=8
2	1.3882e-6	1.3415e-6	1.3388e-6	1.3393e-6	1.3389e-6
4	5.6052e-7	1.1355e-6	1.3963e-6	1.4667e-6	1.4764e-6
6	2.2796e-7	9.0261e-7	1.4057e-6	1.4998e-6	1.4937e-6
8	1.1015e-7	7.0576e-7	1.3854e-6	1.5137e-6	1.4976e-6
10	6.0687e-8	5.4895e-7	1.3464e-6	1.5234e-6	1.4990e-6
12	3.6798e-8	4.2817e-7	1.2975e-6	1.5296e-6	1.4985e-6
14	2.3977e-8	3.3863e-7	1.2427e-6	1.5339e-6	1.4957e-6
16	1.6510e-8	2.7473e-7	1.1848e-6	1.5384e-6	1.4901e-6
18	1.1870e-8	2.3059e-7	1.1264e-6	1.5445e-6	1.4824e-6
20	8.8300e-9	2.0113e-7	1.0694e-6	1.5524e-6	1.4744e-6
22	6.7519e-9	1.8256e-7	1.0146e-6	1.5619e-6	1.4683e-6
24	5.2803e-9	1.7227e-7	9.6196e-7	1.5722e-6	1.4660e-6
26	4.2075e-9	1.6854e-7	9.1076e-7	1.5819e-6	1.4680e-6
28	3.4061e-9	1.7020e-7	8.6091e-7	1.5890e-6	1.4736e-6
30	2.7950e-9	1.7622e-7	8.1489e-7	1.5915e-6	1.4814e-6
32	2.3207e-9	1.8526e-7	7.8151e-7	1.5888e-6	1.4901e-6
34	1.9470e-9	1.9501e-7	7.7960e-7	1.5821e-6	1.4983e-6
36	1.6486e-9	2.0187e-7	8.2899e-7	1.5741e-6	1.5052e-6
38	1.4076e-9	2.0145e-7	9.1668e-7	1.5678e-6	1.5105e-6
40	1.2108e-9	1.9080e-7	9.8974e-7	1.5648e-6	1.5140e-6

Table 4.6. Static analysis (CS=2, RVM, γ=1, c=2)

c=2	w_{max}	w_{max}	w_{max}	w_{max}	w_{max}
γ=1	r=21	r=28	r=36	r=45	r=55
2k	n=10	n=12	n=14	n=16	n=18
2	1.3415e-6	1.3358e-6	1.3446e-6	1.3212e-6	1.3573e-6
4	1.4869e-6	1.4711e-6	1.4833e-6	1.5414e-6	1.3836e-6
6	1.4949e-6	1.4945e-6	1.4515e-6	1.5696e-6	1.4755e-6
8	1.4930e-6	1.5103e-6	1.4449e-6	1.5308e-6	1.5148e-6
10	1.4921e-6	1.5190e-6	1.4689e-6	1.5025e-6	1.5219e-6
12	1.4943e-6	1.5189e-6	1.4964e-6	1.4916e-6	1.5193e-6
14	1.4999e-6	1.5141e-6	1.5131e-6	1.4905e-6	1.5154e-6
16	1.5075e-6	1.5094e-6	1.5197e-6	1.4932e-6	1.5122e-6
18	1.5141e-6	1.5066e-6	1.5203e-6	1.4967e-6	1.5097e-6
20	1.5175e-6	1.5053e-6	1.5182e-6	1.4999e-6	1.5081e-6
22	1.5178e-6	1.5049e-6	1.5153e-6	1.5025e-6	1.5071e-6
24	1.5161e-6	1.5050e-6	1.5124e-6	1.5046e-6	1.5063e-6
26	1.5136e-6	1.5054e-6	1.5098e-6	1.5061e-6	1.5058e-6
28	1.5109e-6	1.5058e-6	1.5078e-6	1.5072e-6	1.5056e-6
30	1.5086e-6	1.5062e-6	1.5062e-6	1.5080e-6	1.5054e-6
32	1.5066e-6	1.5065e-6	1.5049e-6	1.5085e-6	1.5052e-6
34	1.5050e-6	1.5068e-6	1.5040e-6	1.5088e-6	1.5050e-6
36	1.5038e-6	1.5069e-6	1.5034e-6	1.5090e-6	1.5048e-6
38	1.5028e-6	1.5070e-6	1.5030e-6	1.5091e-6	1.5049e-6
40	1.5020e-6	1.5071e-6	1.5027e-6	1.5090e-6	1.5048e-6

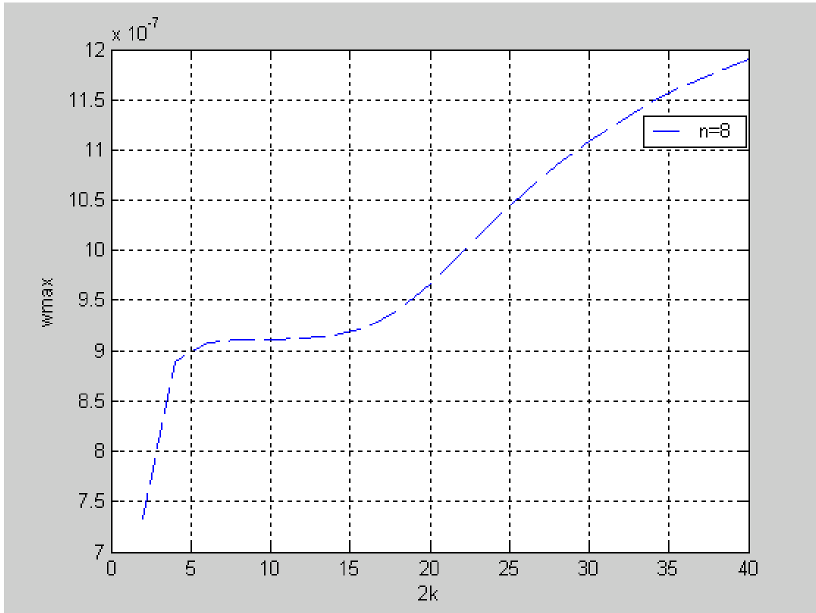

Figure 4.5. w_{max}-2k graph (CS=2, c=1, γ=1)

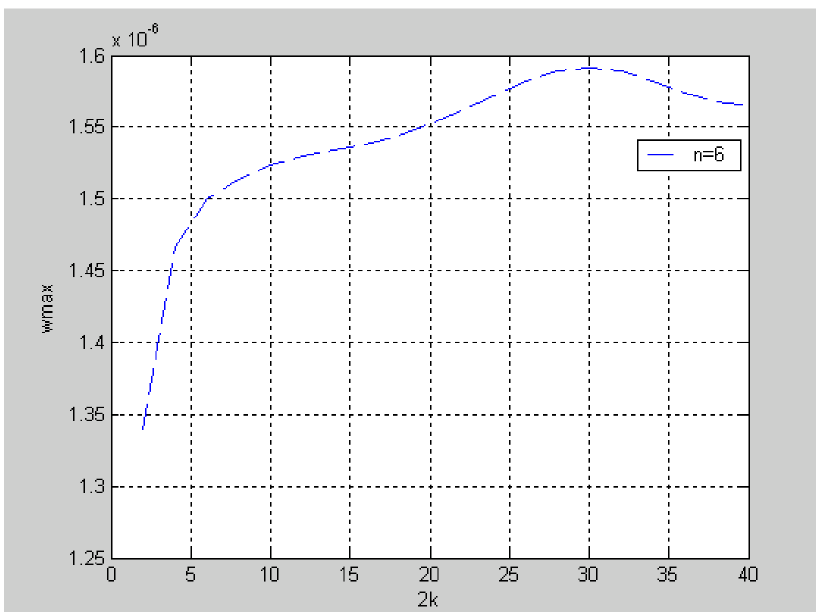

Figure 4.6. w_{max}-2k graph (CS=2, c=2, γ=1)

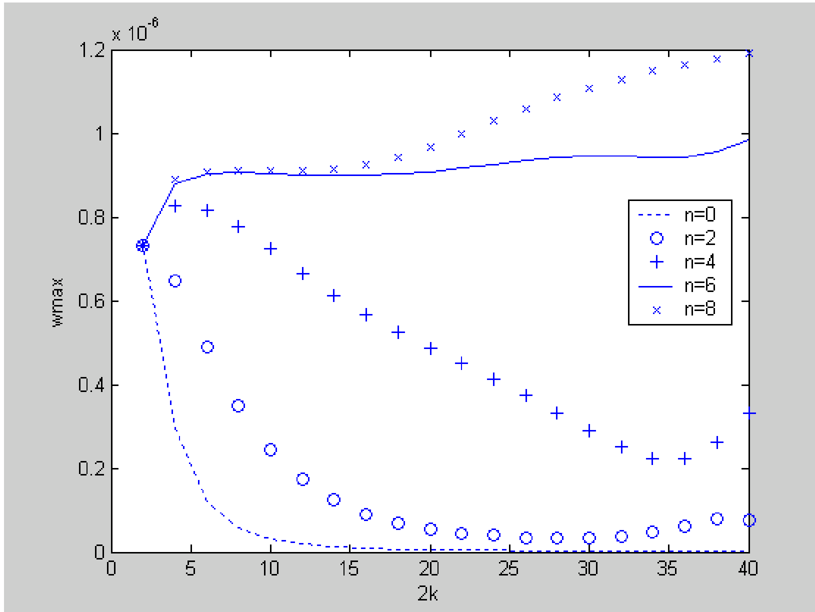

Figure 4.7. Convergence of "w_{max}" (CS=2, c=1, γ=1)

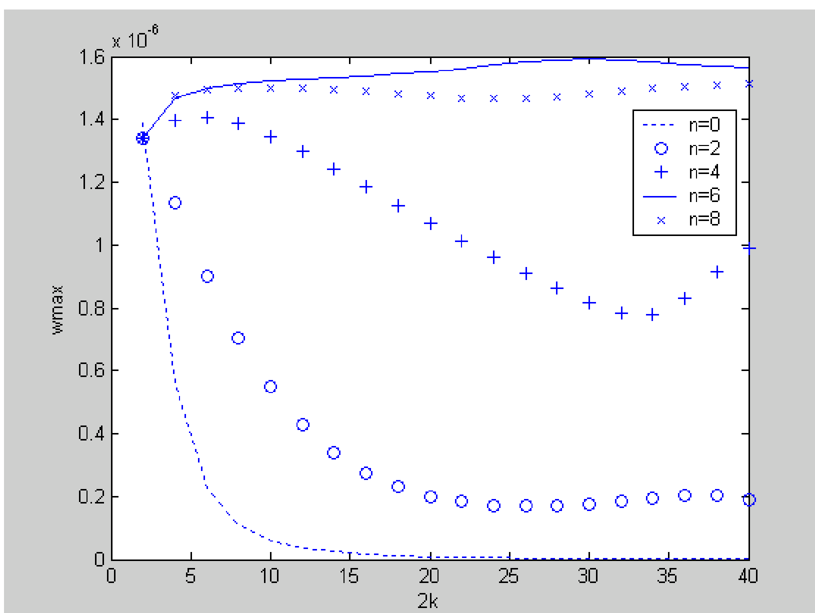

Figure 4.8. Convergence of "wmax" (CS=2, c=2, γ=1)

4.2.5. Conclusions

The tables show that the Ritz method again yields the best results. The deflections tend to get bigger as the area of the plate or in other words "k" increases. Apparently, the results are satisfactory for c=1, and quite reasonable except a few values for c=2.

4.3. Dynamic Analysis of Clamped Super-Elliptical Plates

4.3.1. Abstract

This study provides the undamped free vibration analysis of clamped super-elliptical plates of uniform thickness. The material is isotropic and homogeneous. The exact lowest natural frequencies of circular/square and elliptical/rectangular clamped plates are available. Hence, the upper and lower bounds of the interval are known and are expressed in terms of "$\sqrt{\frac{D}{m}}$" in the tables/graphs.

4.3.2. The Governing Equation, the Potential and the Kinetic Energy of the Plate

$$D\nabla^4 w - m\omega^2 w = 0 \tag{4.24}$$

$$U_b = \frac{D}{2}\left[\int_{x_1}^{x_2}\int_{y_1}^{y_2}\left[\left(\nabla^2 w\right)^2 - 2(1-\nu)\left\{\frac{\partial^2 w}{\partial x^2}\frac{\partial^2 w}{\partial y^2} - \left(\frac{\partial^2 w}{\partial x \partial y}\right)^2\right\}\right]\right]dxdy \tag{4.25}$$

$$T = \tfrac{1}{2}\omega^2 m \int_{x_1}^{x_2}\int_{y_1}^{y_2} w^2 dxdy \tag{4.26}$$

$$\Pi = U_b - T \tag{4.27}$$

4.3.3. Methods of Solution

- Ritz method

$$\frac{\partial \Pi}{\partial \alpha_m} = 0 \tag{4.28}$$

- Galerkin method

$$\int_{x_1}^{x_2}\int_{y_1}^{y_2} [D\nabla^4 w - \omega^2 m w]\phi_m \, dxdy = 0 \tag{4.29}$$

4.3.4. Numerical Results

The tables shown below demonstrate “the natural frequencies (denoted by ω1, ω2, ..)” as “k” increases. Each table corresponds to a specific shape function, aspect ratio, number of terms of weier-strass polynomial and method of solution, and lists the values of “ω1, ω2, ...”. The known exact values of the lowest natural frequency “ω1” for circular/square plates are shown in the following to make a comparison.

- “8.9965” corresponds to the lower bound of interval for "ω1" when “a=1” and “b=1”
- “10.216” corresponds to the upper bound of interval for "ω1" when “a=1” and “b=1”

Table 4.7. Dynamic analysis (CS=3, RVM, γ=8, n=4, c=1)

γ=8	Frequencies					
2k	ω1	ω2	ω3	ω4	ω5	ω6
2	10.7030	22.1814	23.3187	36.0907	37.2667	42.0256
4	9.6646	20.4804	21.2178	30.9378	36.2158	37.7880
6	9.5394	20.2146	20.5040	29.9566	36.3110	36.8105
8	9.4742	20.0381	20.1133	29.4724	36.1074	36.8188
10	9.4165	19.7209	20.1024	29.1771	35.6186	37.1140
12	9.3632	19.4846	19.9892	28.8846	35.2204	36.8571
14	9.3124	19.2937	19.7212	28.5512	34.8976	36.2395
16	9.2639	19.1417	19.3927	28.2297	34.6417	35.5350
18	9.2187	19.0270	19.0842	27.9589	34.4349	34.8837
20	9.1766	18.8243	18.9431	27.7449	34.1788	34.4154
22	9.1376	18.6149	18.8817	27.5795	33.7545	34.2454
24	9.1019	18.4495	18.8363	27.4526	33.3711	34.1514
26	9.0703	18.3213	18.8016	27.3558	33.0647	34.0822
28	9.0427	18.2242	18.7747	27.2828	32.8311	34.0281
30	9.0200	18.1536	18.7535	27.2293	32.6620	33.9844
32	9.0019	18.1058	18.7367	27.1919	32.5490	33.9483
34	8.9892	18.0773	18.7232	27.1682	32.4841	33.9178
36	8.9808	18.0654	18.7124	27.1559	32.4601	33.8918
38	8.9766	18.0679	18.7039	27.1535	32.4712	33.8693
40	8.9766	18.0826	18.6972	27.1596	32.5121	33.8494

Table 4.8. Dynamic analysis (CS=3, RVM, γ=8, n=5, c=1)

γ=8	Frequencies					
2k	ω1	ω2	ω3	ω4	ω5	ω6
2	10.7030	21.6852	22.4009	36.0907	37.2667	42.0256
4	9.6646	19.3106	20.0004	30.9378	36.2158	37.7880
6	9.5394	18.9193	19.6517	29.9566	36.3110	36.8105
8	9.4742	18.7707	19.5005	29.4724	36.1074	36.8188
10	9.4165	18.7075	19.3795	29.1771	35.6186	37.1140
12	9.3632	18.6732	19.2669	28.8846	35.2204	36.8571
14	9.3124	18.6434	19.1621	28.5512	34.8976	36.2395
16	9.2639	18.6050	19.0672	28.2297	34.6417	35.5350
18	9.2187	18.5528	18.9835	27.9589	34.4349	34.8837
20	9.1766	18.4873	18.9104	27.7449	34.1788	34.4154
22	9.1376	18.4122	18.8470	27.5795	33.7545	34.2454
24	9.1019	18.3338	18.7923	27.4526	33.3711	34.1514
26	9.0703	18.2581	18.7453	27.3558	33.0647	34.0822
28	9.0427	18.1905	18.7053	27.2828	32.8311	34.0281
30	9.0200	18.1345	18.6719	27.2293	32.6620	33.9844
32	9.0019	18.0927	18.6443	27.1919	32.5490	33.9483
34	8.9892	18.0656	18.6219	27.1682	32.4841	33.9178
36	8.9808	18.0531	18.6043	27.1559	32.4601	33.8918
38	8.9766	18.0544	18.5907	27.1535	32.4712	33.8693
40	8.9766	18.0679	18.5809	27.1596	32.5121	33.8494

Table 4.9. Dynamic analysis (CS=3, RVM, γ=8, n=4, c=2)

γ=8	Frequencies					
2k	ω1	ω2	ω3	ω4	ω5	ω6
2	7.4505	10.6513	14.6472	19.9432	20.9573	23.8995
4	6.7716	9.3040	12.8002	18.3753	18.6287	20.8309
6	6.6772	8.8983	12.4352	18.0119	18.9046	19.9677
8	6.6155	8.6951	12.3355	17.5812	19.4304	19.4732
10	6.5555	8.5773	12.2768	17.2760	19.1426	20.1551
12	6.5015	8.4817	12.1534	17.0552	18.9008	21.5853
14	6.4552	8.3887	11.9879	16.8926	18.7167	23.8093
16	6.4160	8.3045	11.8220	16.7717	18.5774	24.9424
18	6.3840	8.2347	11.6750	16.6807	18.4731	25.9217
20	6.3569	8.1795	11.5514	16.6111	18.3946	27.0425
22	6.3344	8.1363	11.4505	16.5566	18.3345	28.2161
24	6.3155	8.1028	11.3699	16.5129	18.2880	29.3733
26	6.2996	8.0768	11.3073	16.4771	18.2509	30.4665
28	6.2861	8.0567	11.2597	16.4470	18.2210	31.4663
30	6.2750	8.0415	11.2254	16.4215	18.1966	31.7115
32	6.2654	8.0303	11.2020	16.3995	18.1764	31.6613
34	6.2574	8.0222	11.1879	16.3803	18.1596	31.6175
36	6.2506	8.0172	11.1817	16.3634	18.1454	31.5791
38	6.2450	8.0147	11.1824	16.3484	18.1335	31.5450
40	6.2406	8.0141	11.1888	16.3349	18.1235	31.5145

Table 4.10. Dynamic analysis (CS=3, RVM, γ=8, n=5, c=2)

γ=8	Frequencies					
2k	ω1	ω2	ω3	ω4	ω5	ω6
2	7.4505	10.2296	14.6472	18.9625	19.9223	23.8995
4	6.7716	8.7023	12.8002	17.5371	17.8240	20.8309
6	6.6772	8.5056	12.4352	17.2446	17.7035	19.9677
8	6.6155	8.4155	12.3355	17.1042	17.8461	19.4732
10	6.5555	8.3501	12.2768	16.9903	17.7719	19.1426
12	6.5015	8.2979	12.1534	16.8864	17.5234	18.9008
14	6.4552	8.2544	11.9879	16.7929	17.2259	18.7167
16	6.4160	8.2164	11.8220	16.7109	16.9359	18.5774
18	6.3840	8.1820	11.6750	16.6403	16.6759	18.4731
20	6.3569	8.1508	11.5514	16.4540	16.5800	18.3946
22	6.3344	8.1219	11.4505	16.2721	16.5288	18.3345
24	6.3155	8.0960	11.3699	16.1282	16.4850	18.2880
26	6.2996	8.0731	11.3073	16.0192	16.4475	18.2509
28	6.2861	8.0539	11.2597	15.9404	16.4154	18.2210
30	6.2750	8.0380	11.2254	15.8873	16.3876	18.1966
32	6.2654	8.0256	11.2020	15.8558	16.3637	18.1764
34	6.2574	8.0165	11.1879	15.8422	16.3427	18.1596
36	6.2506	8.0103	11.1817	15.8438	16.3245	18.1454
38	6.2450	8.0069	11.1824	15.8583	16.3084	18.1335
40	6.2406	8.0062	11.1888	15.8838	16.2943	18.1235

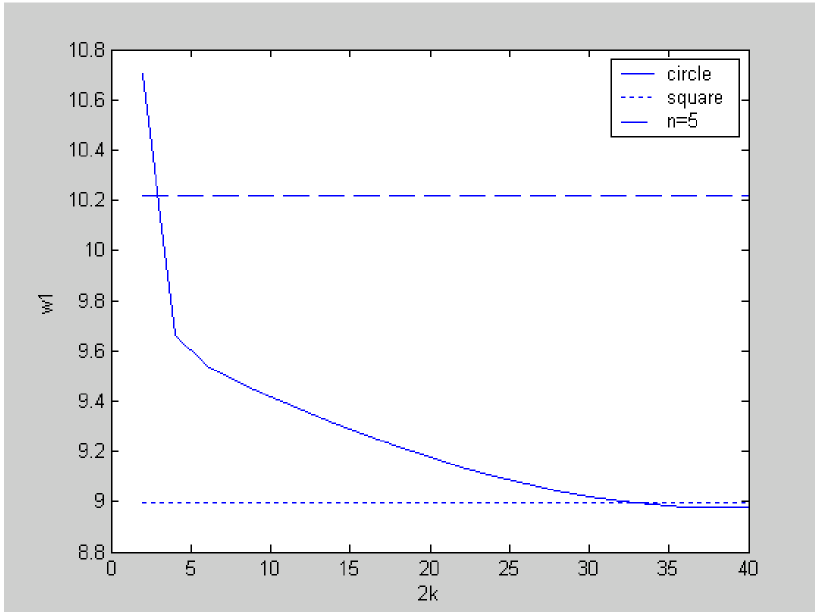

Figure 4.9. ω1-2k graph (CS=3, c=1, γ=8)

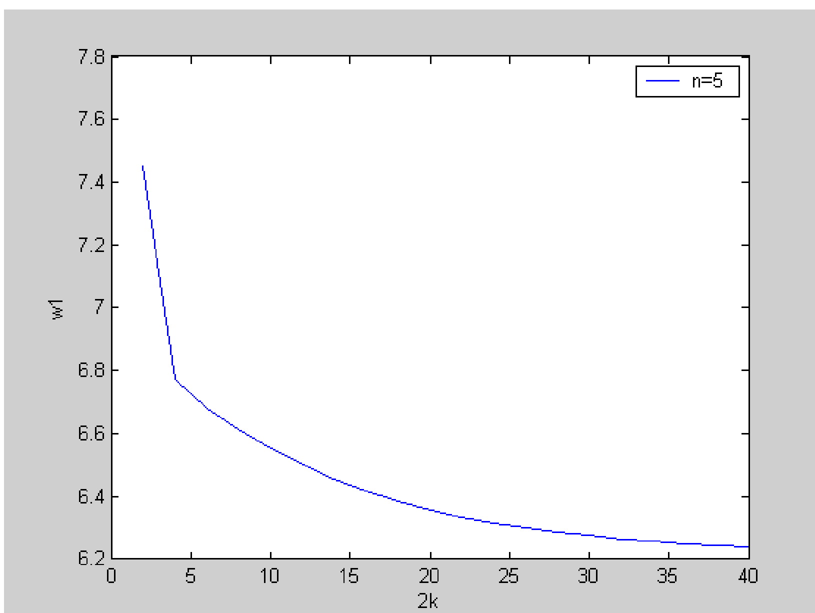

Figure 4.10. ω1-2k graph (CS=3, c=2, γ=8)

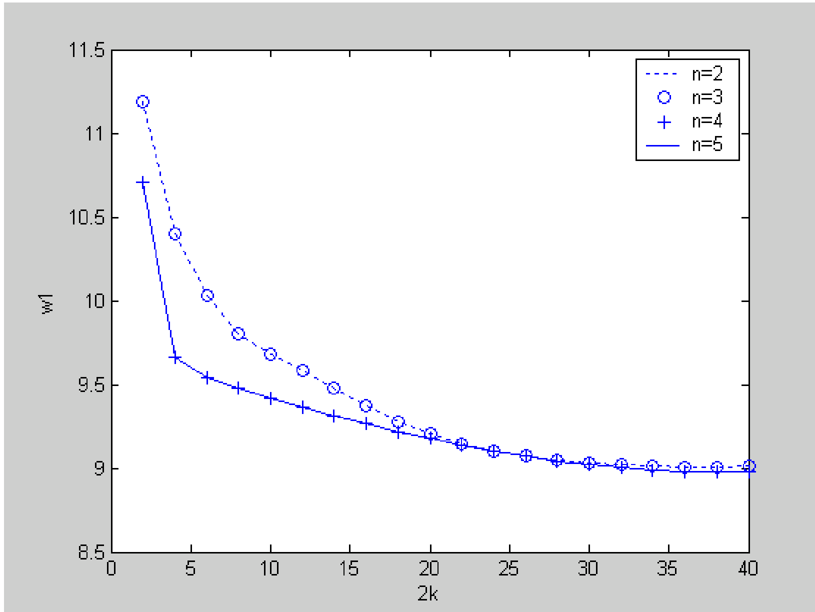

Figure 4.11. Convergence of “ω1” (CS=3, c=1, γ=8)

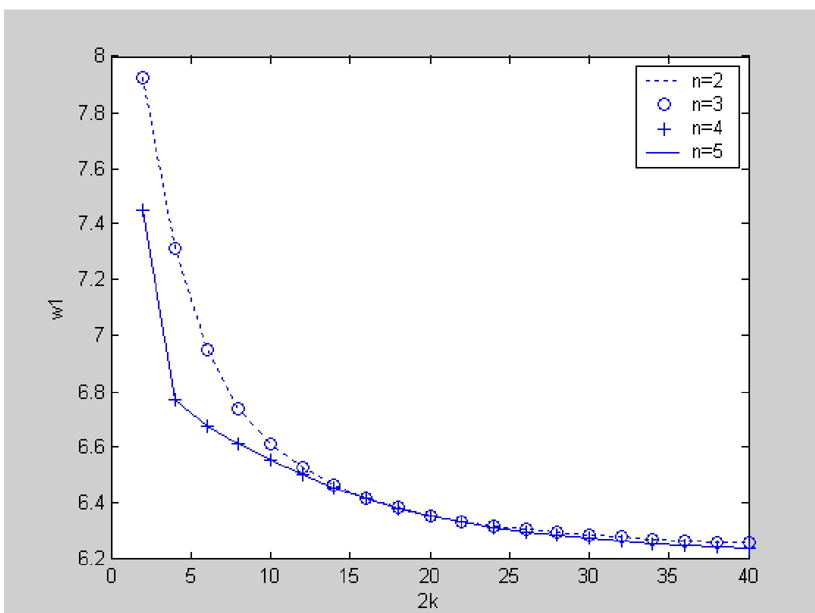

Figure 4.12. Convergence of “ω1” (CS=3, c=2, γ=8)

4.3.5. Conclusions

Since the results have to be in descending order, the lowest frequencies as shown in the tables represent an expected curve with perfect convergence as demonstrated in the figures. Almost all of the lowest natural frequencies corresponding to c=1 are in agreement with the exact ones. Small deviations are not surprising because an approximate method is employed and in general the Ritz method yields upper bounds to the exact values of free vibration frequencies [18].

4.4. Static Analysis of Super-Elliptical Plates Resting on 4 Point-Supports

4.4.1. Abstract

This paper investigates the static analysis of point-supported super-elliptical plates subject to uniformly distributed lateral load acting vertically downward. The plate thickness is constant, and the material is isotropic and homogeneous. The exact deflection at the centroid of circular/square and elliptical/rectangular clamped plates are not available. Hence, the upper and lower bounds of the interval are not known. The results representing the maximum deflections are expressed in terms of "$\frac{P}{D}$" in the tables/graphs.

4.4.2. The Potential Energy of The Plate

$$U_b = \frac{D}{2}\left[\int_{x_1}^{x_2}\int_{y_1}^{y_2}\left[\left(\nabla^2 w\right)^2 - 2(1-\nu)\left\{\frac{\partial^2 w}{\partial x^2}\frac{\partial^2 w}{\partial y^2} - \left(\frac{\partial^2 w}{\partial x \partial y}\right)^2\right\}\right]\right]dxdy \tag{4.30}$$

$$V = -\int_{x_1}^{x_2}\int_{y_1}^{y_2}\left(Pw\right)dxdy \tag{4.31}$$

$$\Pi = U_b + V \tag{4.32}$$

4.4.3. Methods of Solution

- Ritz + LM method

$$\Pi^* = \Pi + \lambda_i \sum_{i=1}^{\kappa} g_i(x_i, y_i) \tag{4.33}$$

$$\frac{\partial \Pi^*}{\partial \alpha_m} = 0 \tag{4.34}$$

$$\frac{\partial \Pi^*}{\partial \lambda_i} = 0 \tag{4.35}$$

4.4.4. Numerical Results

The tables shown below denote "the maximum deflection (denoted by w_{max})" as "k" increases. Each table not only corresponds to a specific shape function, aspect ratio and method of solution, but also lists the values of "w_{max}" for distinct number of terms of weier-strass polynomial.

Table 4.11. Static analysis (CS=4, RVM+LM, γ=0, c=1)

c=1	w_{max}	w_{max}	w_{max}	w_{max}	w_{max}
γ=0	r=3	r=6	r=10	r=15	r=21
2k	n=2	n=4	n=6	n=8	n=10
2	0.0481	0.0786	0.0805	0.0821	0.0823
4	0.1108	0.1789	0.1794	0.1802	0.1802
6	0.1461	0.2347	0.2345	0.2351	0.2351
8	0.1678	0.2689	0.2683	0.2689	0.2688
10	0.1825	0.2918	0.2910	0.2917	0.2915
12	0.1930	0.3082	0.3073	0.3080	0.3079
14	0.2009	0.3206	0.3196	0.3203	0.3201
16	0.2070	0.3302	0.3292	0.3299	0.3297
18	0.2119	0.3379	0.3369	0.3376	0.3374
20	0.2160	0.3443	0.3432	0.3439	0.3437
22	0.2193	0.3496	0.3484	0.3491	0.3489
24	0.2222	0.3540	0.3529	0.3536	0.3534
26	0.2246	0.3579	0.3568	0.3574	0.3572
28	0.2267	0.3612	0.3601	0.3607	0.3605
30	0.2286	0.3641	0.3630	0.3637	0.3635
32	0.2302	0.3667	0.3656	0.3662	0.3660
34	0.2317	0.3690	0.3679	0.3685	0.3683
36	0.2330	0.3711	0.3700	0.3706	0.3704
38	0.2342	0.3729	0.3718	0.3724	0.3722
40	0.2352	0.3746	0.3735	0.3741	0.3739

Table 4.12. Static analysis (CS=4, RVM+LM, γ=0, c=2)

c=2	w_{max}	w_{max}	w_{max}	w_{max}	w_{max}
γ=0	r=3	r=6	r=10	r=15	r=21
2k	n=2	n=4	n=6	n=8	n=10
2	0.5009	0.7608	0.7852	0.7919	0.7934
4	1.1547	1.7049	1.7192	1.7234	1.7239
6	1.5225	2.2144	2.2220	2.2255	2.2260
8	1.7494	2.5198	2.5235	2.5269	2.5271
10	1.9022	2.7216	2.7230	2.7264	2.7264
12	2.0118	2.8645	2.8645	2.8679	2.8677
14	2.0943	2.9710	2.9701	2.9734	2.9730
16	2.1585	3.0533	3.0518	3.0551	3.0546
18	2.2100	3.1189	3.1169	3.1202	3.1196
20	2.2521	3.1723	3.1701	3.1733	3.1727
22	2.2872	3.2167	3.2143	3.2174	3.2168
24	2.3169	3.2542	3.2517	3.2547	3.2540
26	2.3424	3.2862	3.2837	3.2866	3.2859
28	2.3645	3.3139	3.3114	3.3142	3.3135
30	2.3838	3.3382	3.3356	3.3384	3.3376
32	2.4009	3.3595	3.3569	3.3596	3.3589
34	2.4160	3.3785	3.3759	3.3785	3.3778
36	2.4296	3.3954	3.3929	3.3954	3.3947
38	2.4418	3.4106	3.4081	3.4106	3.4099
40	2.4529	3.4244	3.4219	3.4244	3.4237

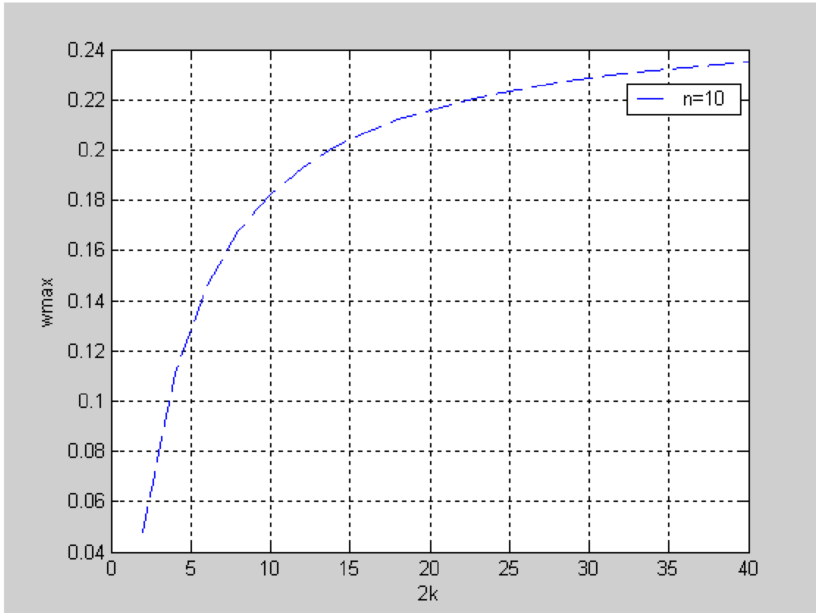

Figure 4.13. w_{max}-2k graph (CS=4, c=1, γ=0)

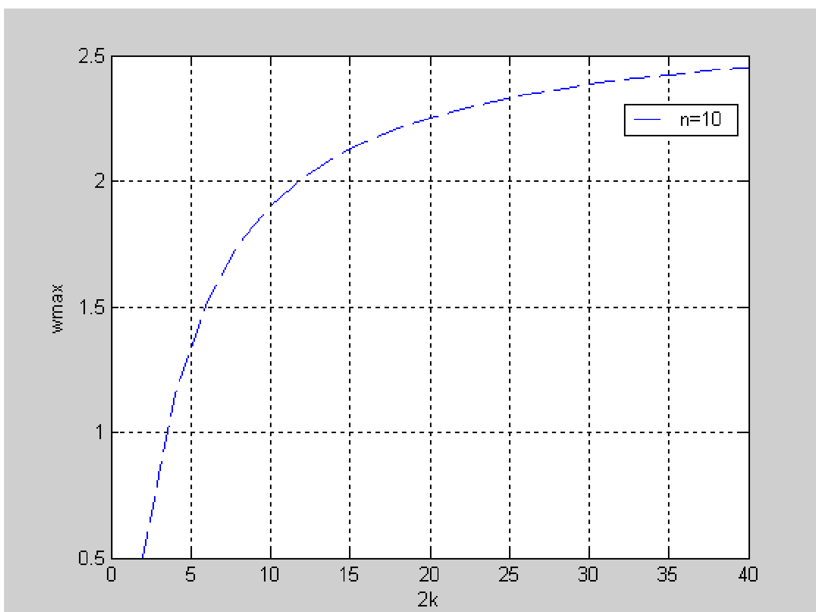

Figure 4.14. w_{max}-2k graph (CS=4, c=2, γ=0)

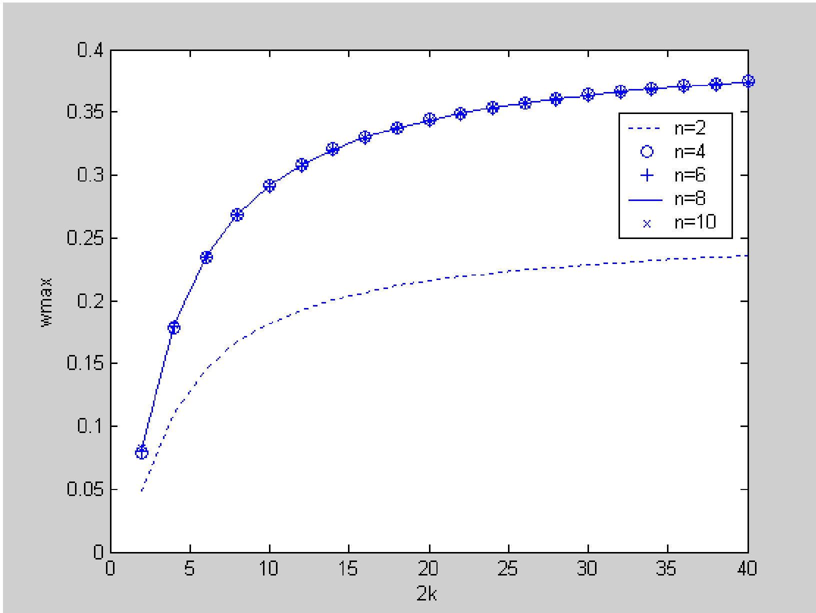

Figure 4.15. Convergence of "w_{max}" (CS=4, c=1, γ=0)

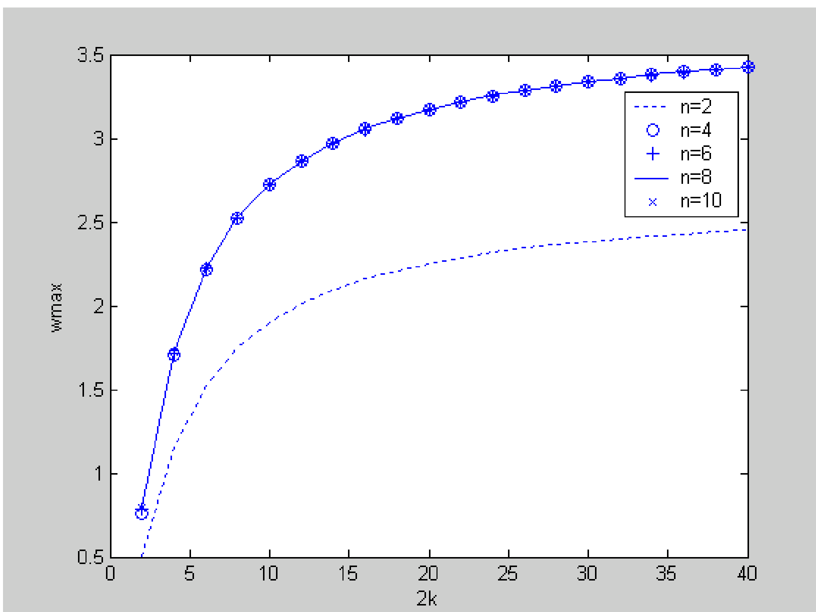

Figure 4.16. Convergence of "w_{max}" (CS=4, c=2, γ=0)

4.4.5. Conclusions

In spite of the fact that there is no existing data available for checking the "w_{max}" values, the results show that high convergence is obtained as the number of terms in the shape function increases. The "w_{max}" values are in ascending order as it is expected. Design purposes are met with acceptable accuracy.

4.5. Dynamic Analysis of Super-Elliptical Plates Resting on 4 Point-Supports

4.5.1. Abstract

This paper reports the undamped free vibration analysis of point-supported super-elliptical plates of uniform thickness. The material is isotropic and homogeneous. The exact lowest natural frequencies of circular/square and elliptical/rectangular clamped plates are not available. Hence, the upper and lower bounds of the interval are not known. The results representing the natural frequencies are expressed in terms of "$\sqrt{\frac{D}{m}}$" in the tables/graphs.

4.5.2. The Potential And The Kinetic Energy Of The Plate

$$U_b = \frac{D}{2}\left[\int_{x_1}^{x_2}\int_{y_1}^{y_2}\left[\left(\nabla^2 w\right)^2 - 2(1-\nu)\left\{\frac{\partial^2 w}{\partial x^2}\frac{\partial^2 w}{\partial y^2} - \left(\frac{\partial^2 w}{\partial x \partial y}\right)^2\right\}\right]\right]dxdy \qquad (4.36)$$

$$T = \tfrac{1}{2}\omega^2 m \int_{x_1}^{x_2}\int_{y_1}^{y_2} w^2 dxdy \qquad (4.37)$$

$$\Pi = U_b - T \qquad (4.38)$$

4.5.3. Methods of Solution

- Ritz + LM method

$$\Pi^{*} = \Pi + \lambda_i \sum_{i=1}^{\kappa} g_i(x_i, y_i) \tag{4.39}$$

$$\frac{\partial \Pi^{*}}{\partial \alpha_m} = 0 \tag{4.40}$$

$$\frac{\partial \Pi^{*}}{\partial \lambda_i} = 0 \tag{4.41}$$

4.5.4. Numerical Results

The tables shown below indicate the first six natural frequencies (denoted by ω1, ω2, ...ω6) as “k” increases. Each table corresponds to a specific shape function, aspect ratio, number of terms of weier-strass polynomial and method of solution, and lists the values of “ω1, ω2, ...”.

Table 4.13. Dynamic analysis (CS=5, RVM+LM, γ=0, n=4, c=1)

γ=0	Frequencies					
2k	ω1	ω2	ω3	ω4	ω5	ω6
2	4.3501	5.3773	7.7886	7.7992	14.5836	25.0814
4	2.8097	5.0135	5.9767	5.9886	14.0782	21.1484
6	2.4243	4.9628	5.4076	5.4144	13.6841	17.9494
8	2.2498	4.9506	5.1237	5.1274	13.3908	16.4069
10	2.1502	4.9467	4.9517	4.9538	13.1728	15.5229
12	2.0856	4.8359	4.8372	4.9451	13.0067	14.9462
14	2.0402	4.7524	4.7533	4.9444	12.8767	14.5389
16	2.0067	4.6893	4.6899	4.9441	12.7725	14.2350
18	1.9808	4.6400	4.6404	4.9439	12.6871	13.9993
20	1.9602	4.6003	4.6006	4.9438	12.6160	13.8110
22	1.9435	4.5676	4.5679	4.9437	12.5559	13.6569
24	1.9296	4.5404	4.5405	4.9437	12.5044	13.5285
26	1.9178	4.5172	4.5173	4.9437	12.4598	13.4198
28	1.9078	4.4972	4.4974	4.9437	12.4209	13.3265
30	1.8991	4.4800	4.4800	4.9436	12.3865	13.2456
32	1.8915	4.4648	4.4648	4.9436	12.3560	13.1748
34	1.8848	4.4514	4.4514	4.9436	12.3288	13.1122
36	1.8789	4.4394	4.4394	4.9436	12.3043	13.0565
38	1.8736	4.4287	4.4287	4.9436	12.2821	13.0067
40	1.8688	4.4190	4.4191	4.9436	12.2620	12.9618

Table 4.14. Dynamic analysis (CS=5, RVM+LM, γ=0, n=6, c=1)

γ=0	Frequencies					
2k	ω1	ω2	ω3	ω4	ω5	ω6
2	4.2781	5.3550	6.9310	6.9365	13.4048	18.0914
4	2.7908	4.9784	5.3881	5.3946	12.7766	14.7663
6	2.4128	4.9114	4.9147	4.9220	12.4030	12.8809
8	2.2414	4.6764	4.6780	4.9078	12.0220	12.1617
10	2.1435	4.5352	4.5360	4.9031	11.5275	11.9959
12	2.0800	4.4404	4.4408	4.9012	11.2049	11.8758
14	2.0355	4.3723	4.3725	4.9003	10.9773	11.7852
16	2.0025	4.3208	4.3209	4.8999	10.8079	11.7145
18	1.9771	4.2805	4.2806	4.8997	10.6767	11.6578
20	1.9568	4.2481	4.2482	4.8996	10.5721	11.6113
22	1.9404	4.2215	4.2215	4.8995	10.4865	11.5726
24	1.9267	4.1992	4.1992	4.8994	10.4153	11.5398
26	1.9152	4.1802	4.1803	4.8994	10.3551	11.5117
28	1.9053	4.1639	4.1640	4.8993	10.3034	11.4873
30	1.8968	4.1498	4.1498	4.8993	10.2586	11.4659
32	1.8893	4.1373	4.1374	4.8993	10.2194	11.4471
34	1.8828	4.1263	4.1264	4.8993	10.1847	11.4303
36	1.8769	4.1165	4.1166	4.8993	10.1539	11.4154
38	1.8717	4.1077	4.1078	4.8993	10.1263	11.4019
40	1.8670	4.0998	4.0999	4.8993	10.1015	11.3896

Table 4.15. Dynamic analysis (CS=5, RVM+LM, γ=0, n=8, c=1)

γ=0	Frequencies					
2k	ω1	ω2	ω3	ω4	ω5	ω6
2	4.2262	5.3550	6.8166	6.8221	13.2147	17.4883
4	2.7846	4.9783	5.3713	5.3781	12.7132	14.5420
6	2.4110	4.9044	4.9083	4.9219	12.3615	12.8218
8	2.2405	4.6722	4.6741	4.9078	11.9814	12.1275
10	2.1428	4.5321	4.5331	4.9030	11.4962	11.9647
12	2.0795	4.4379	4.4384	4.9011	11.1793	11.8459
14	2.0350	4.3702	4.3704	4.9003	10.9554	11.7557
16	2.0021	4.3190	4.3191	4.8998	10.7886	11.6851
18	1.9767	4.2789	4.2789	4.8996	10.6593	11.6283
20	1.9565	4.2466	4.2466	4.8995	10.5560	11.5816
22	1.9400	4.2201	4.2201	4.8994	10.4716	11.5425
24	1.9264	4.1978	4.1978	4.8993	10.4012	11.5093
26	1.9149	4.1789	4.1789	4.8993	10.3416	11.4807
28	1.9050	4.1626	4.1627	4.8993	10.2905	11.4559
30	1.8965	4.1485	4.1485	4.8993	10.2461	11.4341
32	1.8890	4.1361	4.1361	4.8993	10.2072	11.4148
34	1.8825	4.1251	4.1251	4.8992	10.1729	11.3976
36	1.8766	4.1153	4.1153	4.8992	10.1423	11.3822
38	1.8714	4.1065	4.1065	4.8992	10.1149	11.3683
40	1.8667	4.0985	4.0985	4.8992	10.0902	11.3557

Table 4.16. Dynamic analysis (CS=5, RVM+LM, γ=0, n=10, c=1)

γ=0	Frequencies					
2k	ω1	ω2	ω3	ω4	ω5	ω6
2	4.2178	5.3554	6.7709	6.7779	13.1860	17.4151
4	2.7848	4.9785	5.3619	5.3689	12.7055	14.5365
6	2.4114	4.9015	4.9046	4.9219	12.3560	12.8027
8	2.2405	4.6707	4.6723	4.9076	11.9743	12.1223
10	2.1436	4.5310	4.5321	4.9031	11.4926	11.9591
12	2.0794	4.4368	4.4372	4.9010	11.1764	11.8399
14	2.0349	4.3690	4.3692	4.9001	10.9527	11.7496
16	2.0020	4.3178	4.3179	4.8997	10.7859	11.6787
18	1.9766	4.2777	4.2778	4.8994	10.6566	11.6218
20	1.9563	4.2455	4.2455	4.8993	10.5533	11.5751
22	1.9399	4.2190	4.2190	4.8992	10.4688	11.5360
24	1.9262	4.1968	4.1968	4.8992	10.3985	11.5029
26	1.9147	4.1779	4.1779	4.8991	10.3389	11.4744
28	1.9049	4.1617	4.1617	4.8991	10.2878	11.4496
30	1.8963	4.1475	4.1476	4.8991	10.2435	11.4280
32	1.8889	4.1352	4.1352	4.8991	10.2048	11.4088
34	1.8824	4.1242	4.1243	4.8991	10.1705	11.3917
36	1.8766	4.1144	4.1145	4.8991	10.1400	11.3764
38	1.8714	4.1057	4.1057	4.8991	10.1126	11.3626
40	1.8666	4.0977	4.0978	4.8991	10.0880	11.3501

Table 4.17. Dynamic analysis (CS=5, RVM+LM, γ=0, n=4, c=2)

γ=0	Frequencies					
2k	ω1	ω2	ω3	ω4	ω5	ω6
2	1.1790	3.3287	4.6276	4.7043	7.6688	7.9850
4	0.8426	2.4890	4.0435	4.4911	5.1986	7.3014
6	0.7483	2.2439	3.8451	3.8839	5.2716	6.3510
8	0.7036	2.1249	3.5277	3.8007	5.3568	5.9013
10	0.6782	2.0530	3.3451	3.7463	5.4134	5.6365
12	0.6599	2.0050	3.2262	3.7072	5.4512	5.4608
14	0.6485	1.9707	3.1431	3.6778	5.3354	5.4781
16	0.6399	1.9449	3.0816	3.6547	5.2412	5.4976
18	0.6332	1.9246	3.0343	3.6361	5.1677	5.5123
20	0.6277	1.9084	2.9967	3.6207	5.1088	5.5237
22	0.6233	1.8950	2.9661	3.6078	5.0603	5.5326
24	0.6201	1.8839	2.9407	3.5968	5.0199	5.5399
26	0.6168	1.8744	2.9194	3.5873	4.9855	5.5458
28	0.6144	1.8663	2.9011	3.5791	4.9560	5.5508
30	0.6120	1.8593	2.8853	3.5719	4.9303	5.5550
32	0.6099	1.8531	2.8716	3.5655	4.9078	5.5585
34	0.6083	1.8477	2.8595	3.5597	4.8879	5.5616
36	0.6070	1.8428	2.8487	3.5546	4.8702	5.5642
38	0.6054	1.8385	2.8391	3.5499	4.8543	5.5666
40	0.6042	1.8345	2.8305	3.5457	4.8399	5.5686

Table 4.18. Dynamic analysis (CS=5, RVM+LM, γ=0, n=6, c=2)

γ=0	Frequencies					
2k	ω1	ω2	ω3	ω4	ω5	ω6
2	1.1683	2.7505	4.2273	4.4142	5.4544	7.3007
4	0.8367	2.2305	3.2924	3.9680	4.4649	5.4818
6	0.7450	2.0700	2.8240	3.7716	4.5128	4.9634
8	0.7036	1.9887	2.6153	3.6613	4.5420	4.7196
10	0.6782	1.9391	2.4960	3.5903	4.5591	4.5760
12	0.6592	1.9037	2.4192	3.5406	4.4812	4.5697
14	0.6477	1.8790	2.3656	3.5036	4.4136	4.5766
16	0.6391	1.8600	2.3258	3.4751	4.3628	4.5812
18	0.6325	1.8451	2.2953	3.4525	4.3232	4.5845
20	0.6273	1.8332	2.2710	3.4339	4.2914	4.5869
22	0.6229	1.8232	2.2512	3.4185	4.2654	4.5886
24	0.6193	1.8149	2.2347	3.4054	4.2436	4.5899
26	0.6164	1.8079	2.2209	3.3943	4.2252	4.5910
28	0.6140	1.8017	2.2091	3.3846	4.2093	4.5917
30	0.6116	1.7964	2.1987	3.3761	4.1955	4.5924
32	0.6099	1.7918	2.1897	3.3686	4.1835	4.5929
34	0.6079	1.7876	2.1819	3.3620	4.1728	4.5933
36	0.6066	1.7840	2.1749	3.3560	4.1632	4.5936
38	0.6050	1.7806	2.1685	3.3507	4.1547	4.5939
40	0.6042	1.7776	2.1629	3.3458	4.1470	4.5941

Table 4.19. Dynamic analysis (CS=5, RVM+LM, γ=0, n=8, c=2)

γ=0	Frequencies					
2k	ω1	ω2	ω3	ω4	ω5	ω6
2	1.1640	2.7350	4.1815	4.3180	4.8775	7.1934
4	0.8367	2.2293	3.1812	3.9262	4.4244	5.4083
6	0.7450	2.0700	2.7686	3.7457	4.4475	4.9204
8	0.7036	1.9887	2.5788	3.6428	4.4587	4.6883
10	0.6782	1.9378	2.4688	3.5756	4.4649	4.5514
12	0.6592	1.9034	2.3975	3.5280	4.4603	4.4678
14	0.6477	1.8786	2.3473	3.4926	4.3953	4.4694
16	0.6391	1.8596	2.3100	3.4651	4.3464	4.4701
18	0.6325	1.8449	2.2812	3.4432	4.3082	4.4704
20	0.6273	1.8328	2.2582	3.4253	4.2776	4.4704
22	0.6229	1.8229	2.2395	3.4103	4.2523	4.4702
24	0.6193	1.8147	2.2240	3.3976	4.2312	4.4700
26	0.6164	1.8076	2.2108	3.3868	4.2133	4.4697
28	0.6140	1.8015	2.1994	3.3774	4.1978	4.4693
30	0.6116	1.7961	2.1896	3.3691	4.1844	4.4689
32	0.6095	1.7915	2.1812	3.3618	4.1726	4.4686
34	0.6079	1.7873	2.1736	3.3554	4.1622	4.4683
36	0.6066	1.7837	2.1669	3.3496	4.1528	4.4679
38	0.6050	1.7804	2.1609	3.3443	4.1445	4.4676
40	0.6037	1.7774	2.1555	3.3395	4.1369	4.4673

Table 4.20. Dynamic analysis (CS=5, RVM+LM, γ=0, n=10, c=2)

γ=0	Frequencies					
2k	ω1	ω2	ω3	ω4	ω5	ω6
2	1.1640	2.7102	4.1797	4.2924	4.8425	7.1516
4	0.8367	2.2237	3.1765	3.9256	4.4113	5.3921
6	0.7450	2.0676	2.7668	3.7450	4.4379	4.9122
8	0.7036	1.9862	2.5768	3.6421	4.4520	4.6829
10	0.6782	1.9365	2.4678	3.5749	4.4593	4.5475
12	0.6592	1.9024	2.3967	3.5273	4.4572	4.4627
14	0.6477	1.8776	2.3466	3.4918	4.3926	4.4649
16	0.6391	1.8589	2.3093	3.4644	4.3440	4.4659
18	0.6325	1.8442	2.2806	3.4425	4.3059	4.4664
20	0.6273	1.8322	2.2577	3.4245	4.2753	4.4665
22	0.6229	1.8224	2.2390	3.4096	4.2502	4.4665
24	0.6193	1.8141	2.2234	3.3970	4.2292	4.4663
26	0.6164	1.8071	2.2103	3.3861	4.2114	4.4660
28	0.6140	1.8010	2.1990	3.3768	4.1960	4.4658
30	0.6116	1.7958	2.1893	3.3685	4.1826	4.4654
32	0.6095	1.7911	2.1807	3.3613	4.1708	4.4651
34	0.6079	1.7870	2.1732	3.3548	4.1604	4.4648
36	0.6066	1.7834	2.1666	3.3490	4.1511	4.4645
38	0.6050	1.7800	2.1606	3.3438	4.1428	4.4641
40	0.6037	1.7771	2.1551	3.3391	4.1353	4.4639

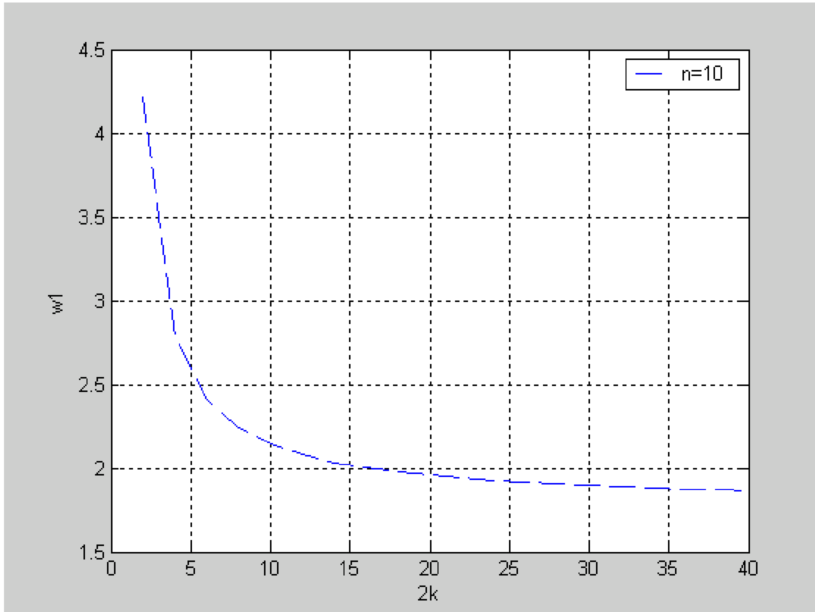

Figure 4.17. ω1-2k graph (CS=5, c=1, γ=0)

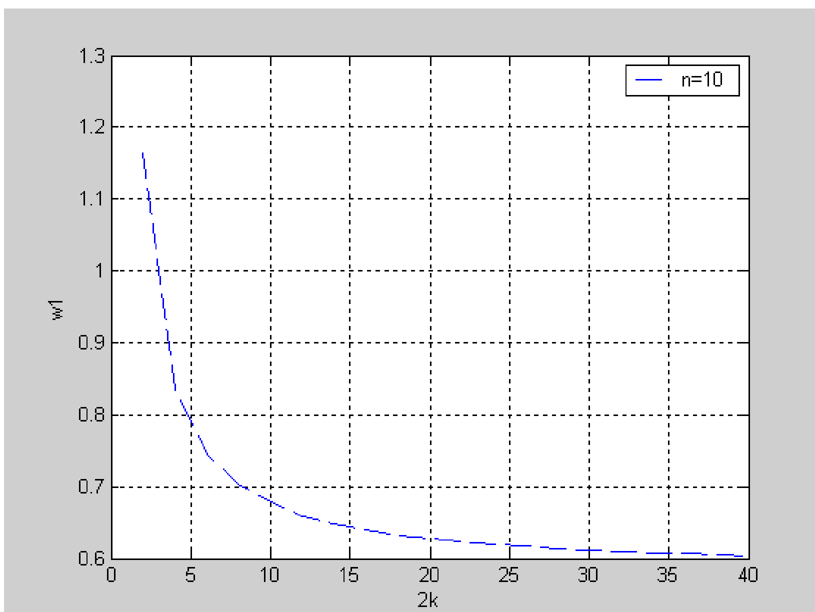

Figure 4.18. ω1-2k graph (CS=5, c=2, γ=0)

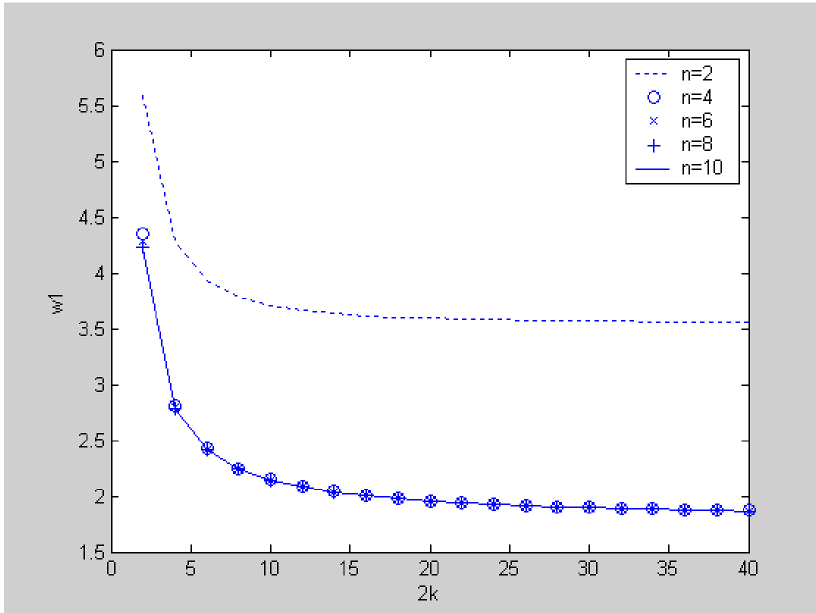

Figure 4.19. Convergence of “ω1” (CS=5, c=1, γ=0)

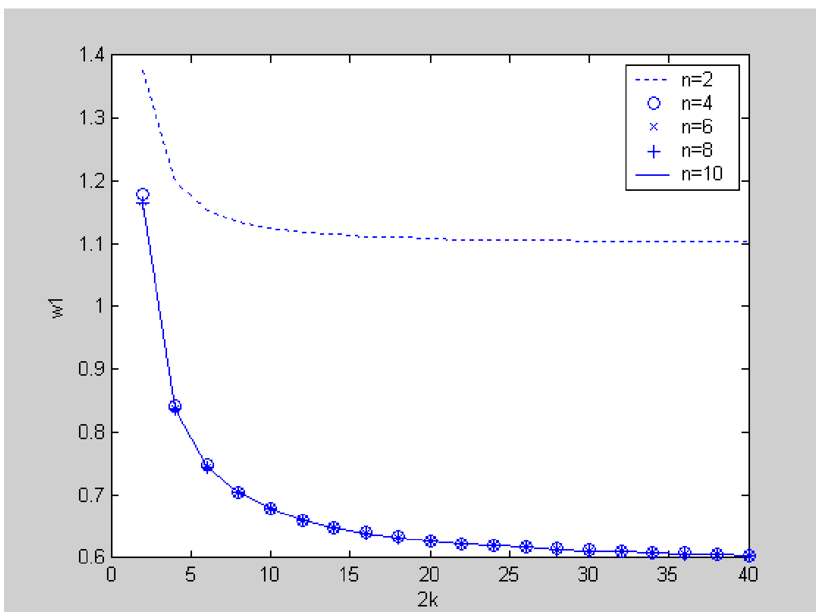

Figure 4.20. Convergence of “ω1” (CS=5, c=2, γ=0)

4.5.5. Conclusions

Despite the non-existing literature on the analysis of super-elliptical plates resting on point-supports, the tabulated results listed above seem to meet the design purposes with acceptable accuracy. The natural frequencies tend to decrease as “k” increases and high convergence is achieved as shown in the figures.

4.6. Static Analysis of Functionally Graded Clamped Super-Elliptical Plates

4.6.1. Abstract

In the last decade, a number of studies on FGMs have been available but no prior work has been done on the analysis of functionally graded super-elliptical plates. Therefore there is no way to check/compare the results which are written in terms of “P/D” in the tables.

4.6.2. The Potential Energy of The Plate

$$D_1 = De^{x^2} \tag{4.42}$$

$$U_b = \frac{1}{2}\left[\int_{x_1}^{x_2}\int_{y_1}^{y_2} D_1\left[\left(\nabla^2 w\right)^2 - 2(1-\nu)\left\{\frac{\partial^2 w}{\partial x^2}\frac{\partial^2 w}{\partial y^2} - \left(\frac{\partial^2 w}{\partial x \partial y}\right)^2\right\}\right]\right]dxdy \tag{4.43}$$

$$V = -\int_{x_1}^{x_2}\int_{y_1}^{y_2}\left(Pw\right)dxdy \tag{4.44}$$

$$\Pi = U_b + V \tag{4.45}$$

D_1 denotes the “position dependent” generic material property corresponding to the flexural rigidity of the plate.

4.6.3. Methods of Solution

- Ritz method

$$\frac{\partial \Pi}{\partial \alpha_m} = 0 \qquad (4.46)$$

4.6.4. Numerical Results

The tables shown below depict the maximum deflection (denoted by w_{max}) as "k" increases. Each table not only corresponds to a specific shape function, aspect ratio and method of solution, but also lists the values of "w_{max}" for distinct number of terms of weier-strass polynomial.

Table 4.21. Static analysis (CS=6, RVM, γ=1, c=1)

c=1	w_{max}	w_{max}	w_{max}	w_{max}	w_{max}
γ=1	r=1	r=3	r=6	r=10	r=15
2k	n=0	n=2	n=4	n=6	n=8
2	0.0111	0.0121	0.0121	0.0121	0.0121
4	0.0034	0.0093	0.0131	0.0145	0.0147
6	0.0012	0.0062	0.0124	0.0148	0.0151
8	0.0006	0.0041	0.0113	0.0148	0.0151
10	0.0003	0.0027	0.0100	0.0147	0.0152
12	0.0002	0.0018	0.0089	0.0145	0.0152
14	0.0001	0.0013	0.0079	0.0144	0.0152
16	0.0001	0.0009	0.0071	0.0144	0.0154
18	0.0001	0.0007	0.0065	0.0145	0.0157
20	0.0001	0.0006	0.0060	0.0146	0.0164
22	0.0000	0.0004	0.0055	0.0149	0.0172
24	0.0000	0.0004	0.0049	0.0152	0.0181
26	0.0000	0.0003	0.0043	0.0155	0.0190
28	0.0000	0.0003	0.0037	0.0157	0.0198
30	0.0000	0.0003	0.0031	0.0158	0.0205
32	0.0000	0.0003	0.0026	0.0157	0.0211
34	0.0000	0.0004	0.0022	0.0155	0.0217
36	0.0000	0.0006	0.0022	0.0154	0.0221
38	0.0000	0.0007	0.0026	0.0155	0.0225
40	0.0000	0.0007	0.0033	0.0161	0.0228

Table 4.22. Static analysis (CS=6, RVM, γ=1, c=1)

c=1	w_{max}	w_{max}	w_{max}	w_{max}	w_{max}
γ=1	r=21	r=28	r=36	r=45	r=55
2k	n=10	n=12	n=14	n=16	n=18
2	0.0122	0.0121	0.0123	0.0120	0.0168
4	0.0150	0.0151	0.0156	0.0158	0.0155
6	0.0152	0.0152	0.0150	0.0181	0.0216
8	0.0152	0.0153	0.0156	0.0213	0.0236
10	0.0151	0.0159	0.0182	0.0227	0.0242
12	0.0153	0.0173	0.0209	0.0234	0.0244
14	0.0158	0.0191	0.0225	0.0238	0.0245
16	0.0168	0.0206	0.0235	0.0241	0.0246
18	0.0184	0.0217	0.0240	0.0243	0.0246
20	0.0200	0.0226	0.0243	0.0245	0.0246
22	0.0213	0.0232	0.0245	0.0246	0.0247
24	0.0223	0.0237	0.0246	0.0246	0.0247
26	0.0230	0.0240	0.0247	0.0247	0.0247
28	0.0234	0.0242	0.0247	0.0247	0.0247
30	0.0238	0.0244	0.0247	0.0247	0.0247
32	0.0240	0.0245	0.0248	0.0248	0.0247
34	0.0241	0.0245	0.0248	0.0248	0.0248
36	0.0242	0.0246	0.0248	0.0248	0.0248
38	0.0243	0.0246	0.0248	0.0248	0.0248
40	0.0244	0.0246	0.0248	0.0248	0.0248

Table 4.23. Static analysis (CS=6, RVM, γ=1, c=2)

c=2	w_{max}	w_{max}	w_{max}	w_{max}	w_{max}	w_{max}	w_{max}
γ=1	r=1	r=3	r=6	r=10	r=15	r=21	r=28
2k	n=0	n=2	n=4	n=6	n=8	n=10	n=12
2	0.0108	0.0192	0.0223	0.0230	0.0231	0.0231	0.0231
4	0.0025	0.0101	0.0187	0.0237	0.0253	0.0258	0.0258
6	0.0008	0.0054	0.0149	0.0226	0.0253	0.0260	0.0261
8	0.0003	0.0031	0.0117	0.0211	0.0252	0.0260	0.0262
10	0.0002	0.0019	0.0093	0.0199	0.0249	0.0260	0.0262
12	0.0001	0.0012	0.0076	0.0189	0.0247	0.0259	0.0263
14	0.0001	0.0008	0.0064	0.0181	0.0244	0.0260	0.0263
16	0.0000	0.0006	0.0057	0.0175	0.0241	0.0260	0.0264
18	0.0000	0.0004	0.0051	0.0171	0.0238	0.0261	0.0265
20	0.0000	0.0003	0.0046	0.0168	0.0236	0.0261	0.0265
22	0.0000	0.0003	0.0041	0.0166	0.0234	0.0262	0.0266
24	0.0000	0.0002	0.0036	0.0164	0.0234	0.0262	0.0267
26	0.0000	0.0002	0.0032	0.0161	0.0233	0.0263	0.0267
28	0.0000	0.0002	0.0026	0.0158	0.0233	0.0263	0.0268
30	0.0000	0.0002	0.0021	0.0152	0.0232	0.0263	0.0268
32	0.0000	0.0002	0.0016	0.0146	0.0232	0.0263	0.0268
34	0.0000	0.0003	0.0014	0.0140	0.0231	0.0263	0.0269
36	0.0000	0.0003	0.0013	0.0133	0.0231	0.0263	0.0269
38	0.0000	0.0004	0.0015	0.0128	0.0230	0.0263	0.0269
40	0.0000	0.0003	0.0019	0.0125	0.0230	0.0263	0.0269

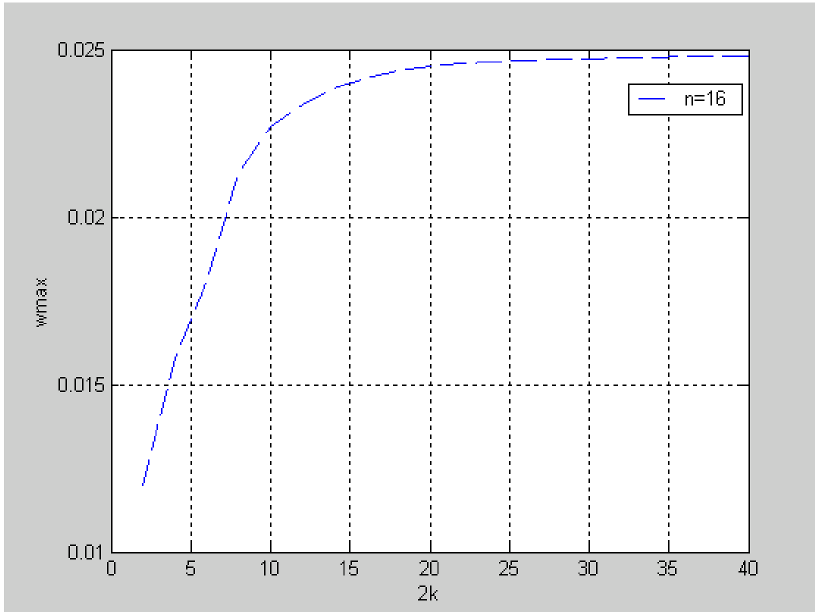

Figure 4.21. w_{max}-2k graph (CS=6, c=1, γ=1)

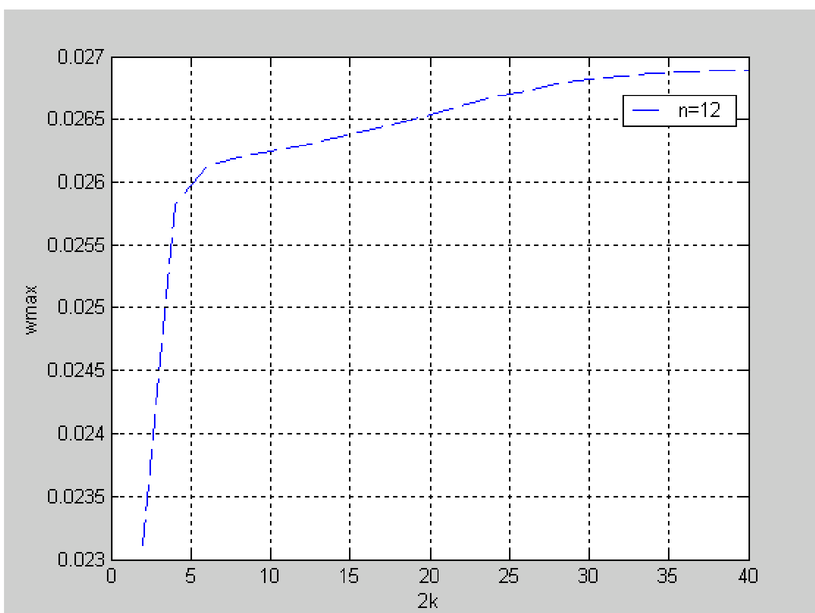

Figure 4.22. w_{max}-2k graph (CS=6, c=2, γ=1)

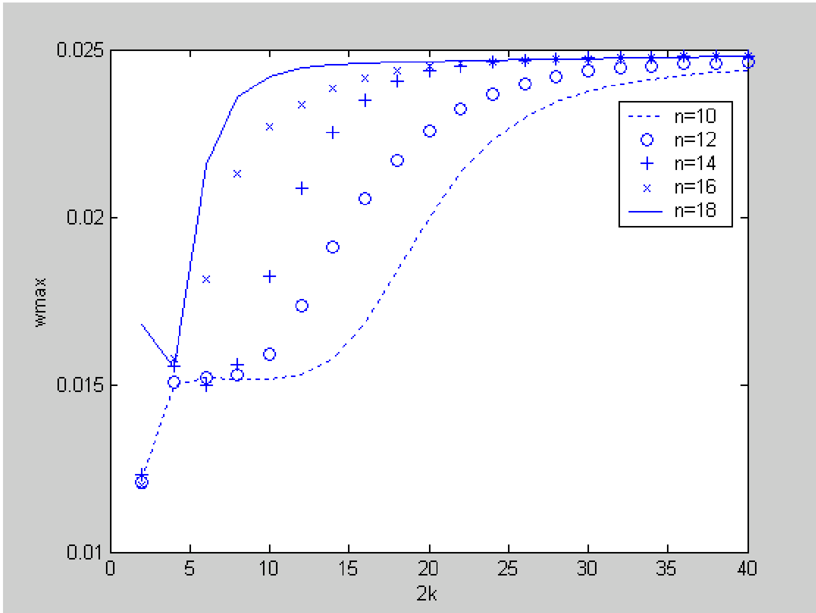

Figure 4.23. Convergence of "w_{max}" (CS=6, c=1, γ=1)

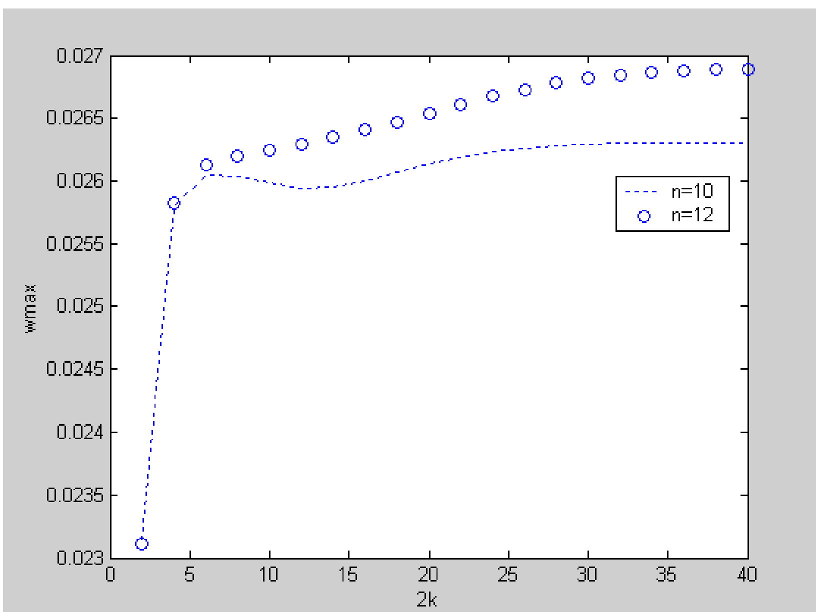

Figure 4.24. Convergence of "w_{max}" (CS=6, c=2, γ=1)

4.6.5. Conclusions

Unsurprisingly, the tables show that the Ritz method again yields satisfactory results. Although a few of the "w_{max}" values seem to be inconsistent, because of the convergence, the results which are in ascending order can be considered to be sufficiently accurate.

4.7. Static Analysis of Functionally Graded Simply-Supported Super-Elliptical Plates

4.7.1. Abstract

In the last decade, a number of studies on FGMs have been available but no prior work has been done on the analysis of functionally graded super-elliptical plates. Therefore there is no way to check/compare the results which are written in terms of "P/D" in the tables.

4.7.2. The Potential Energy of The Plate

$$D_1 = De^{x^2} \tag{4.47}$$

$$U_b = \frac{1}{2}\left[\int_{x_1}^{x_2}\int_{y_1}^{y_2} D_1\left[\left(\nabla^2 w\right)^2 - 2(1-\nu)\left\{\frac{\partial^2 w}{\partial x^2}\frac{\partial^2 w}{\partial y^2} - \left(\frac{\partial^2 w}{\partial x \partial y}\right)^2\right\}\right]\right]dxdy \tag{4.48}$$

$$V = -\int_{x_1}^{x_2}\int_{y_1}^{y_2} (Pw)dxdy \tag{4.49}$$

$$\Pi = U_b + V \tag{4.50}$$

D_1 denotes the "position dependent" generic material property corresponding to the flexural rigidity of the plate.

4.7.3. Methods of Solution

- Ritz method

$$\frac{\partial \Pi}{\partial \alpha_m} = 0 \tag{4.51}$$

4.7.4. Numerical Results

The tables shown below list the maximum deflection (denoted by w_{max}) as "k" increases. Each table not only corresponds to a specific shape function, aspect ratio and method of solution, but also lists the values of "w_{max}" for distinct number of terms of weier-strass polynomial.

Table 4.24. Static analysis (CS=7, RVM, γ=11, c=1)

c=1	w_{max}	w_{max}	w_{max}	w_{max}	w_{max}
γ=11	r=1	r=3	r=6	r=10	r=15
2k	n=0	n=2	n=4	n=6	n=8
2	0.0121	0.0121	0.0141	0.0144	0.0164
4	0.0132	0.0126	0.0144	0.0142	0.0150
6	0.0136	0.0133	0.0146	0.0145	0.0150
8	0.0138	0.0137	0.0146	0.0145	0.0150
10	0.0139	0.0141	0.0147	0.0146	0.0150
12	0.0141	0.0143	0.0147	0.0146	0.0150
14	0.0142	0.0144	0.0148	0.0147	0.0150
16	0.0144	0.0146	0.0149	0.0147	0.0150
18	0.0145	0.0147	0.0149	0.0148	0.0150
20	0.0146	0.0147	0.0150	0.0148	0.0150
22	0.0147	0.0148	0.0150	0.0149	0.0150
24	0.0147	0.0149	0.0150	0.0149	0.0150
26	0.0147	0.0149	0.0150	0.0149	0.0150
28	0.0147	0.0149	0.0150	0.0150	0.0150
30	0.0147	0.0150	0.0150	0.0150	0.0150
32	0.0147	0.0150	0.0149	0.0150	0.0150
34	0.0146	0.0150	0.0149	0.0150	0.0150
36	0.0146	0.0150	0.0149	0.0150	0.0150
38	0.0145	0.0150	0.0149	0.0150	0.0150
40	0.0144	0.0150	0.0148	0.0150	0.0150

Table 4.25. Static analysis (CS=7, RVM, γ=11, c=2)

c=2	w_{max}	w_{max}	w_{max}	w_{max}	w_{max}
γ=11	r=1	r=3	r=6	r=10	r=15
2k	n=0	n=2	n=4	n=6	n=8
2	0.0178	0.0210	0.0245	0.0247	0.0266
4	0.0171	0.0204	0.0245	0.0245	0.0255
6	0.0164	0.0210	0.0247	0.0248	0.0255
8	0.0160	0.0215	0.0249	0.0250	0.0255
10	0.0159	0.0219	0.0250	0.0250	0.0256
12	0.0158	0.0221	0.0250	0.0251	0.0256
14	0.0158	0.0223	0.0251	0.0252	0.0256
16	0.0159	0.0224	0.0251	0.0252	0.0256
18	0.0159	0.0225	0.0251	0.0253	0.0256
20	0.0159	0.0225	0.0251	0.0254	0.0256
22	0.0159	0.0226	0.0251	0.0255	0.0256
24	0.0159	0.0226	0.0251	0.0255	0.0256
26	0.0159	0.0227	0.0251	0.0256	0.0256
28	0.0158	0.0227	0.0251	0.0256	0.0256
30	0.0157	0.0228	0.0251	0.0257	0.0256
32	0.0157	0.0228	0.0251	0.0257	0.0256
34	0.0156	0.0229	0.0251	0.0258	0.0257
36	0.0155	0.0229	0.0251	0.0258	0.0257
38	0.0154	0.0229	0.0251	0.0258	0.0257
40	0.0153	0.0230	0.0251	0.0258	0.0257

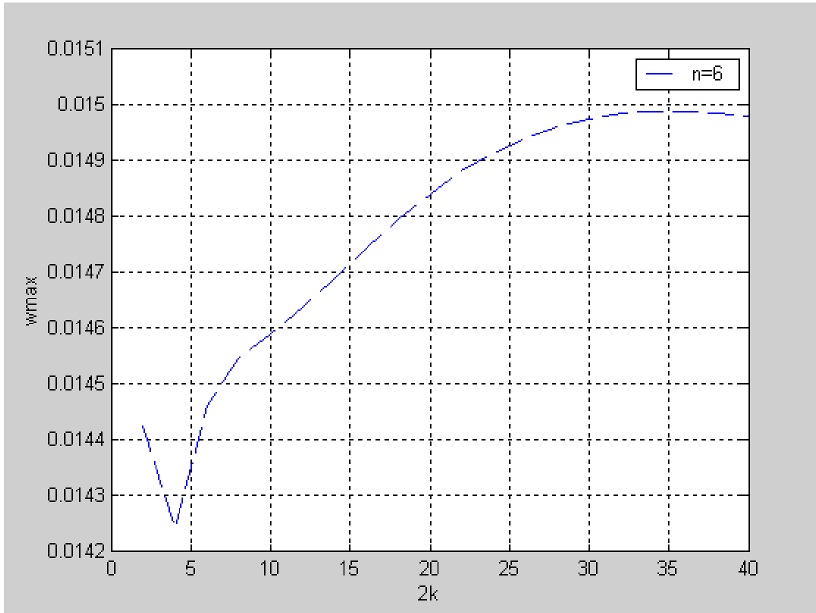

Figure 4.25. w_{max}-2k graph (CS=7, c=1, γ=11)

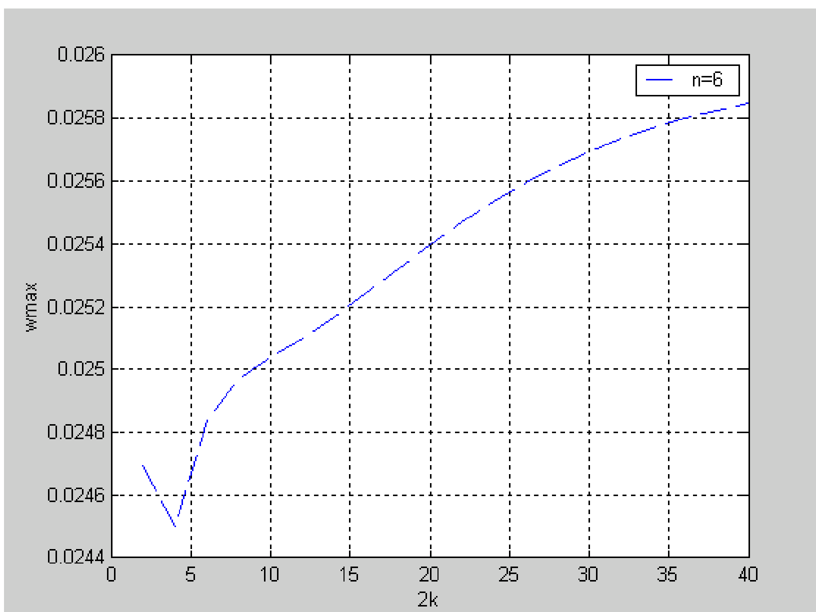

Figure 4.26. w_{max}-2k graph (CS=7, c=2, γ=11)

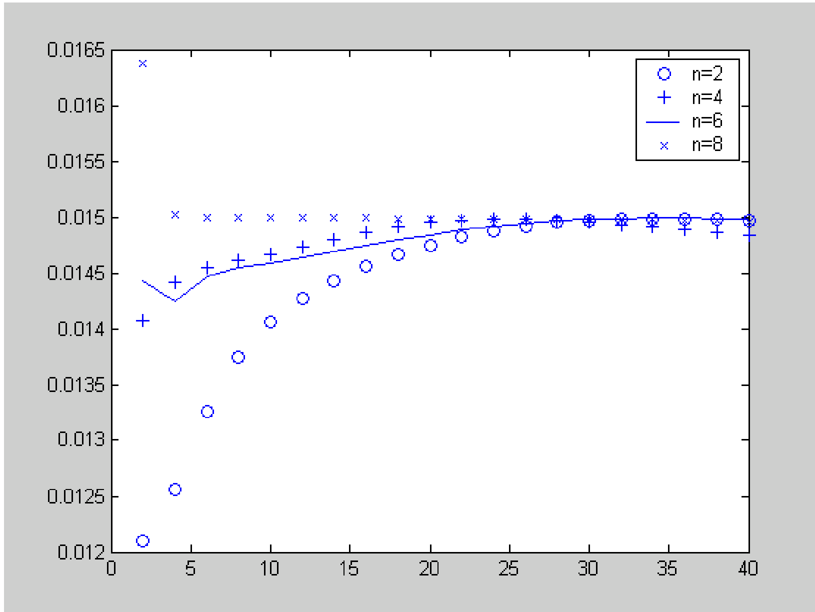

Figure 4.27. Convergence of "w_{max}" (CS=7, c=1, γ=11)

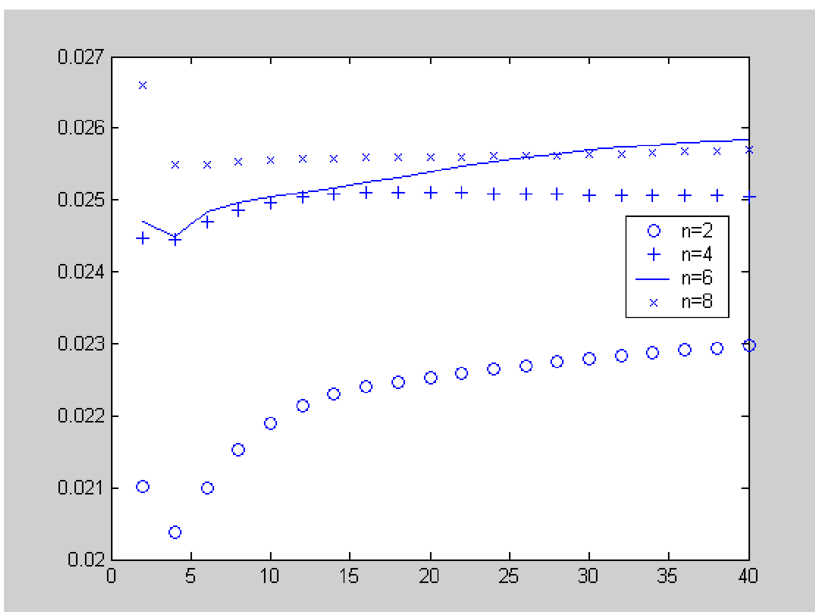

Figure 4.28. Convergence of "w_{max}" (CS=7, c=2, γ=11)

4.7.5. Conclusions

Unsurprisingly, the tables show that the Ritz method again produces satisfactory results. Because of the convergence, the results can be considered to be sufficiently accurate. Apart from the first two "w_{max}" values corresponding to both of the aspect ratios, no inconsistency about the form of the curves is detected.

4.8. Static Analysis of Functionally Graded Super-Elliptical Plates Resting on 4 Point-Supports

4.8.1. Abstract

In the last decade, a number of studies on FGMs have been available but no prior work has been done on the analysis of functionally graded super-elliptical plates. Therefore there is no way to check/compare the results which are written in terms of "P/D" in the tables.

4.8.2. The Potential Energy of The Plate

$$D_1 = De^{x^2} \tag{4.52}$$

$$U_b = \frac{1}{2}\left[\int_{x_1}^{x_2}\int_{y_1}^{y_2} D_1\left[\left(\nabla^2 w\right)^2 - 2(1-\nu)\left\{\frac{\partial^2 w}{\partial x^2}\frac{\partial^2 w}{\partial y^2} - \left(\frac{\partial^2 w}{\partial x \partial y}\right)^2\right\}\right]\right]dxdy \tag{4.53}$$

$$V = -\int_{x_1}^{x_2}\int_{y_1}^{y_2}\left(Pw\right)dxdy \tag{4.54}$$

$$\Pi = U_b + V \tag{4.55}$$

D_1 denotes the "position dependent" generic material property corresponding to the flexural rigidity of the plate.

4.8.3. Methods of Solution

- Ritz + LM method

$$\Pi^* = \Pi + \lambda_i \sum_{i=1}^{\kappa} g_i(x_i, y_i) \tag{4.56}$$

$$\frac{\partial \Pi^*}{\partial \alpha_m} = 0 \tag{4.57}$$

$$\frac{\partial \Pi^*}{\partial \lambda_i} = 0 \tag{4.58}$$

4.8.4. Numerical Results

The tables shown below denote "the maximum deflection (denoted by w_{max})" as "k" increases. Each table not only corresponds to a specific shape function, aspect ratio and method of solution, but also lists the values of "w_{max}" for distinct number of terms of weier-strass polynomial.

Table 4.26. Static analysis (CS=8, RVM, γ=0, c=1)

c=1	w_{max}	w_{max}	w_{max}	w_{max}	w_{max}	w_{max}
γ=0	r=3	r=6	r=10	r=15	r=21	r=28
2k	n=2	n=4	n=6	n=8	n=10	n=12
2	0.0362	0.0637	0.0669	0.0680	0.0682	0.0684
4	0.0788	0.1396	0.1430	0.1433	0.1433	0.1436
6	0.1020	0.1803	0.1839	0.1841	0.1841	0.1842
8	0.1163	0.2048	0.2084	0.2087	0.2086	0.2087
10	0.1259	0.2210	0.2246	0.2250	0.2249	0.2250
12	0.1328	0.2325	0.2361	0.2366	0.2365	0.2366
14	0.1380	0.2411	0.2448	0.2453	0.2452	0.2452
16	0.1421	0.2478	0.2514	0.2520	0.2519	0.2519
18	0.1454	0.2531	0.2568	0.2573	0.2572	0.2573
20	0.1480	0.2574	0.2611	0.2617	0.2616	0.2617
22	0.1503	0.2610	0.2648	0.2653	0.2652	0.2653
24	0.1522	0.2641	0.2678	0.2684	0.2683	0.2684
26	0.1538	0.2667	0.2705	0.2711	0.2709	0.2710
28	0.1552	0.2690	0.2727	0.2733	0.2732	0.2733
30	0.1565	0.2710	0.2747	0.2753	0.2752	0.2753
32	0.1576	0.2727	0.2765	0.2771	0.2770	0.2770
34	0.1585	0.2743	0.2781	0.2786	0.2785	0.2786
36	0.1594	0.2757	0.2795	0.2800	0.2799	0.2800
38	0.1602	0.2769	0.2807	0.2813	0.2812	0.2813
40	0.1609	0.2780	0.2819	0.2824	0.2823	0.2824

Table 4.27. Static analysis (CS=8, RVM, γ=0, c=2)

c=2	w_{max}	w_{max}	w_{max}	w_{max}	w_{max}	w_{max}
γ=0	r=3	r=6	r=10	r=15	r=21	r=28
2k	n=2	n=4	n=6	n=8	n=10	n=12
2	0.0981	0.4084	0.5850	0.6241	0.6287	0.6292
4	0.1708	0.8292	1.1859	1.2606	1.2704	1.2723
6	0.2064	1.0417	1.4836	1.5777	1.5908	1.5928
8	0.2281	1.1665	1.6553	1.7605	1.7753	1.7772
10	0.2428	1.2482	1.7664	1.8785	1.8941	1.8961
12	0.2535	1.3058	1.8438	1.9607	1.9769	1.9788
14	0.2617	1.3485	1.9009	2.0212	2.0377	2.0396
16	0.2682	1.3814	1.9447	2.0675	2.0843	2.0863
18	0.2734	1.4075	1.9794	2.1041	2.1211	2.1231
20	0.2778	1.4288	2.0075	2.1338	2.1510	2.1529
22	0.2814	1.4464	2.0308	2.1583	2.1756	2.1776
24	0.2846	1.4613	2.0503	2.1789	2.1963	2.1983
26	0.2873	1.4740	2.0670	2.1964	2.2140	2.2160
28	0.2896	1.4850	2.0814	2.2116	2.2292	2.2312
30	0.2917	1.4945	2.0940	2.2248	2.2425	2.2445
32	0.2936	1.5030	2.1051	2.2363	2.2541	2.2561
34	0.2952	1.5104	2.1148	2.2466	2.2644	2.2665
36	0.2967	1.5171	2.1236	2.2558	2.2737	2.2757
38	0.2981	1.5231	2.1314	2.2640	2.2819	2.2839
40	0.2993	1.5286	2.1385	2.2714	2.2894	2.2914

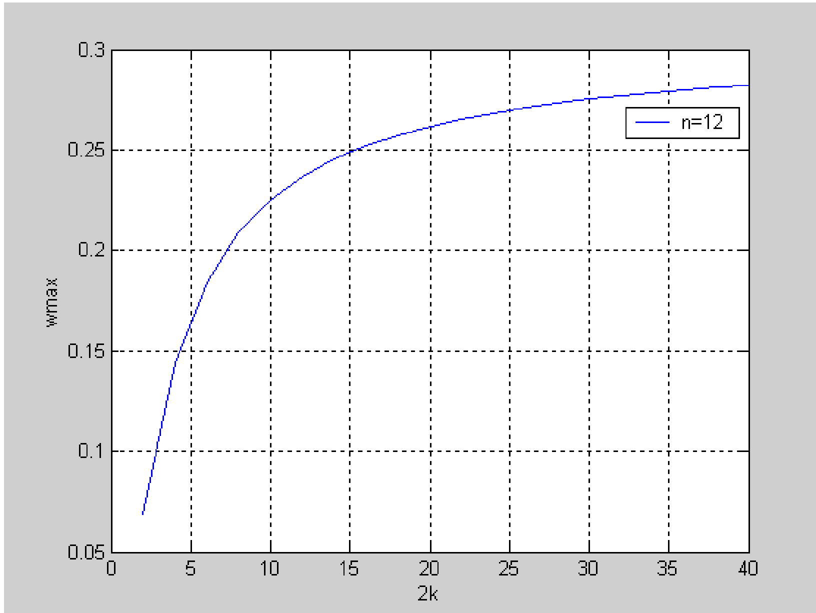

Figure 4.29. w_{max}-2k graph (CS=8, c=1, γ=0)

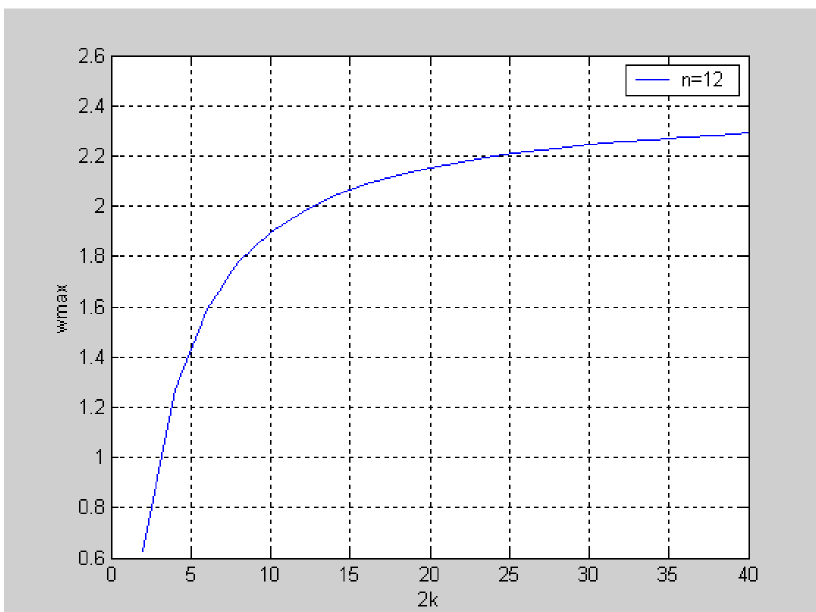

Figure 4.30. w_{max}-2k graph (CS=8, c=2, γ=0)

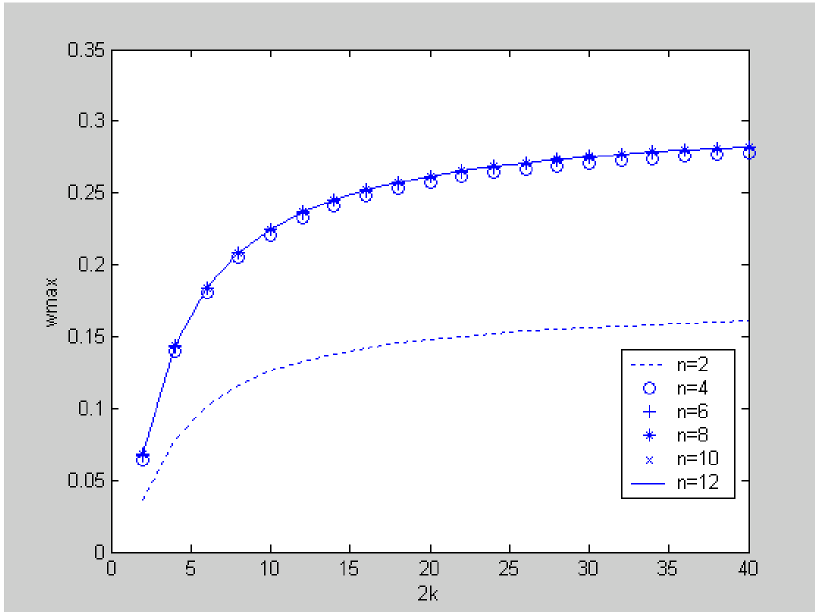

Figure 4.31. Convergence of "w_{max}" (CS=8, c=1, γ=0)

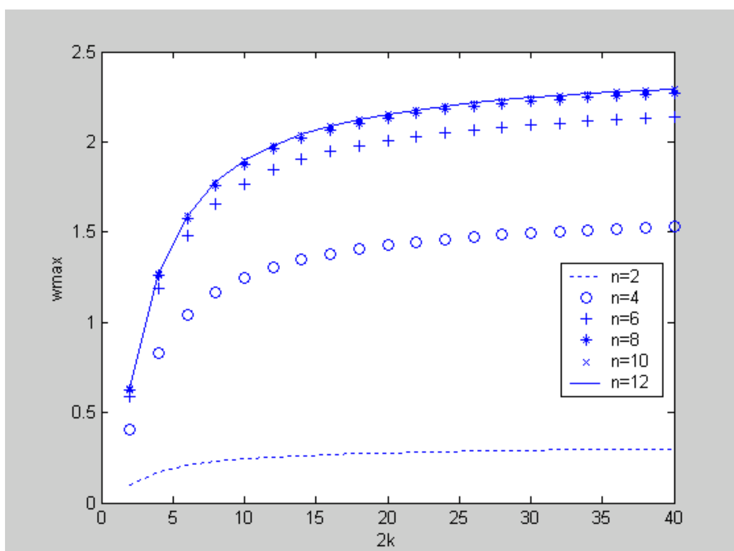

Figure 4.32. Convergence of "w_{max}" (CS=8, c=2, γ=0)

4.8.5. Conclusions

In spite of the fact that there is no existing data available for checking the "w_{max}" values, the results show that high convergence is obtained as the number of terms in the shape function increases. Apparently, there is no discrepancy. The design purposes are met with acceptable accuracy.

4.9. Dynamic Analysis of Functionally Graded Super-Elliptical Plates Resting on 4 Point-Supports

4.9.1. Abstract

This paper reports the undamped free vibration analysis of functionally graded point-supported super-elliptical plates of uniform thickness. The results representing the natural frequencies are expressed in terms of "$\sqrt{\frac{D}{m}}$" in the tables/graphs.

4.9.2. The Potential Energy of The Plate

$$U_b = \frac{1}{2}\left[\int_{x_1}^{x_2}\int_{y_1}^{y_2} D_1\left[\left(\nabla^2 w\right)^2 - 2(1-\nu)\left\{\frac{\partial^2 w}{\partial x^2}\frac{\partial^2 w}{\partial y^2} - \left(\frac{\partial^2 w}{\partial x \partial y}\right)^2\right\}\right]\right]dxdy \qquad (4.59)$$

$$T = \tfrac{1}{2}\omega^2 m \int_{x_1}^{x_2}\int_{y_1}^{y_2} w^2 dxdy \qquad (4.60)$$

$$\Pi = U_b - T \qquad (4.61)$$

D_1 denotes the "position dependent" generic material property corresponding to the flexural rigidity of the plate.

4.9.3. Methods of Solution

- Ritz + LM method

$$\Pi^* = \Pi + \lambda_i \sum_{i=1}^{\kappa} g_i(x_i, y_i) \tag{4.62}$$

$$\frac{\partial \Pi^*}{\partial \alpha_m} = 0 \tag{4.63}$$

$$\frac{\partial \Pi^*}{\partial \lambda_i} = 0 \tag{4.64}$$

4.9.4. Numerical Results

The tables shown below depict the first six natural frequencies (denoted by ω1, ω2, …ω6) as "k" increases. Each table corresponds to a specific shape function, aspect ratio, number of terms of weier-strass polynomial and method of solution, and lists the values of "ω1, ω2,..".

Table 4.28. Dynamic analysis (CS=9, RVM+LM, γ=0, n=4, c=1)

γ=0	Frequencies					
2k	ω1	ω2	ω3	ω4	ω5	ω6
2	4.8043	6.1561	9.2248	9.5584	17.3480	27.5862
4	3.2026	5.7927	7.3683	7.6102	17.4772	24.4553
6	2.7910	5.7458	6.7731	6.9580	17.2731	23.6203
8	2.6043	5.7341	6.4673	6.6233	17.0357	21.8402
10	2.4977	5.7301	6.2777	6.4173	16.8284	20.7912
12	2.4288	5.7282	6.1476	6.2769	16.6562	20.0936
14	2.3804	5.7273	6.0524	6.1748	16.5138	19.5932
16	2.3447	5.7267	5.9797	6.0971	16.3951	19.2155
18	2.3171	5.7263	5.9222	6.0359	16.2950	18.9194
20	2.2952	5.7261	5.8756	5.9864	16.2098	18.6809
22	2.2774	5.7259	5.8371	5.9456	16.1365	18.4843
24	2.2626	5.7258	5.8047	5.9113	16.0727	18.3194
26	2.2501	5.7256	5.7770	5.8821	16.0168	18.1790
28	2.2395	5.7256	5.7532	5.8569	15.9675	18.0580
30	2.2302	5.7255	5.7323	5.8349	15.9236	17.9526
32	2.2222	5.7140	5.7254	5.8157	15.8843	17.8600
34	2.2151	5.6978	5.7253	5.7986	15.8489	17.7779
36	2.2088	5.6833	5.7253	5.7833	15.8169	17.7046
38	2.2032	5.6702	5.7253	5.7696	15.7878	17.6388
40	2.1982	5.6585	5.7252	5.7573	15.7612	17.5794

Table 4.29. Dynamic analysis (CS=9, RVM+LM, γ=0, n=6, c=1)

γ=0	Frequencies					
2k	ω1	ω2	ω3	ω4	ω5	ω6
2	4.6699	6.1233	7.8812	8.3747	15.3853	20.2165
4	3.1526	5.7179	6.3887	6.6486	15.0235	17.4945
6	2.7564	5.6531	5.9048	6.1086	14.7061	15.8347
8	2.5758	5.6348	5.6613	5.8435	14.4800	14.8874
10	2.4728	5.5135	5.6278	5.6847	14.3172	14.3381
12	2.4059	5.4139	5.5784	5.6244	13.9785	14.1961
14	2.3590	5.3420	5.5021	5.6225	13.7243	14.1029
16	2.3244	5.2875	5.4446	5.6214	13.5348	14.0292
18	2.2976	5.2449	5.3996	5.6206	13.3880	13.9695
20	2.2764	5.2105	5.3635	5.6200	13.2708	13.9202
22	2.2591	5.1821	5.3338	5.6196	13.1749	13.8789
24	2.2448	5.1584	5.3089	5.6192	13.0951	13.8436
26	2.2327	5.1382	5.2878	5.6189	13.0275	13.8133
28	2.2223	5.1208	5.2696	5.6187	12.9695	13.7869
30	2.2134	5.1057	5.2539	5.6185	12.9192	13.7637
32	2.2056	5.0924	5.2400	5.6183	12.8752	13.7432
34	2.1986	5.0806	5.2278	5.6181	12.8363	13.7248
36	2.1926	5.0702	5.2169	5.6180	12.8017	13.7084
38	2.1871	5.0608	5.2071	5.6179	12.7707	13.6936
40	2.1822	5.0523	5.1983	5.6177	12.7427	13.6801

Table 4.30. Dynamic analysis (CS=9, RVM+LM, γ=0, n=8, c=1)

γ=0	Frequencies					
2k	ω1	ω2	ω3	ω4	ω5	ω6
2	4.6301	6.1165	7.7187	8.2217	15.1116	19.1304
4	3.1504	5.7141	6.3142	6.6266	14.8352	16.1154
6	2.7557	5.6492	5.8385	6.0927	14.4960	15.4157
8	2.5750	5.5981	5.6306	5.8273	14.2546	14.5415
10	2.4714	5.4527	5.6234	5.6678	14.0272	14.0831
12	2.4044	5.3549	5.5611	5.6199	13.6898	13.9571
14	2.3573	5.2845	5.4845	5.6180	13.4510	13.8614
16	2.3226	5.2314	5.4267	5.6168	13.2728	13.7864
18	2.2958	5.1898	5.3817	5.6159	13.1345	13.7261
20	2.2745	5.1564	5.3455	5.6153	13.0240	13.6766
22	2.2572	5.1289	5.3157	5.6148	12.9336	13.6352
24	2.2429	5.1059	5.2909	5.6145	12.8581	13.6002
26	2.2307	5.0864	5.2698	5.6142	12.7943	13.5700
28	2.2204	5.0696	5.2516	5.6139	12.7394	13.5439
30	2.2114	5.0550	5.2358	5.6137	12.6918	13.5209
32	2.2036	5.0422	5.2220	5.6135	12.6501	13.5007
34	2.1968	5.0308	5.2098	5.6133	12.6133	13.4826
36	2.1907	5.0207	5.1988	5.6132	12.5804	13.4664
38	2.1853	5.0116	5.1891	5.6131	12.5510	13.4518
40	2.1804	5.0034	5.1803	5.6130	12.5245	13.4386

Table 4.31. Dynamic analysis (CS=9, RVM+LM, γ=0, n=10, c=1)

γ=0	Frequencies					
2k	ω1	ω2	ω3	ω4	ω5	ω6
2	4.6220	6.1151	7.6660	8.1817	15.0704	18.9906
4	3.1499	5.7131	6.3067	6.6215	14.8200	16.0504
6	2.7555	5.6481	5.8364	6.0908	14.4797	15.3460
8	2.5749	5.5964	5.6297	5.8253	14.2360	14.5265
10	2.4713	5.4502	5.6225	5.6654	14.0093	14.0627
12	2.4043	5.3516	5.5583	5.6191	13.6690	13.9354
14	2.3572	5.2807	5.4814	5.6172	13.4279	13.8388
16	2.3225	5.2271	5.4235	5.6159	13.2480	13.7631
18	2.2956	5.1852	5.3783	5.6151	13.1085	13.7023
20	2.2744	5.1515	5.3420	5.6145	12.9972	13.6525
22	2.2571	5.1239	5.3122	5.6140	12.9063	13.6110
24	2.2428	5.1008	5.2874	5.6136	12.8305	13.5758
26	2.2306	5.0811	5.2663	5.6133	12.7664	13.5456
28	2.2202	5.0643	5.2481	5.6131	12.7115	13.5194
30	2.2113	5.0497	5.2324	5.6129	12.6639	13.4966
32	2.2035	5.0368	5.2186	5.6127	12.6222	13.4763
34	2.1967	5.0255	5.2064	5.6125	12.5855	13.4584
36	2.1905	5.0154	5.1955	5.6124	12.5527	13.4423
38	2.1852	5.0063	5.1857	5.6123	12.5234	13.4278
40	2.1803	4.9981	5.1770	5.6121	12.4971	13.4147

Table 4.32. Dynamic analysis (CS=9, RVM+LM, γ=0, n=4, c=2)

γ=0	Frequencies					
2k	ω1	ω2	ω3	ω4	ω5	ω6
2	1.5956	10.2341	10.7979	14.9330	22.8603	25.3495
4	1.2056	9.0824	11.1870	15.7382	19.7292	24.2994
6	1.0902	8.5025	11.5258	14.4384	20.1950	24.5603
8	1.0351	8.1940	11.6610	13.6019	20.7415	24.4619
10	1.0030	7.9976	11.7276	13.0761	21.1114	24.0588
12	0.9818	7.8599	11.7655	12.7180	21.3618	23.3955
14	0.9670	7.7574	11.7893	12.4584	21.5370	22.9104
16	0.9558	7.6778	11.8053	12.2614	21.6640	22.5381
18	0.9471	7.6141	11.8167	12.1067	21.7588	22.2421
20	0.9402	7.5619	11.8250	11.9818	21.8314	22.0006
22	0.9346	7.5183	11.8313	11.8789	21.7995	21.8884
24	0.9301	7.4812	11.7925	11.8363	21.6292	21.9338
26	0.9263	7.4494	11.7190	11.8402	21.4830	21.9707
28	0.9228	7.4218	11.6556	11.8434	21.3560	22.0011
30	0.9201	7.3975	11.6004	11.8460	21.2446	22.0264
32	0.9173	7.3760	11.5519	11.8481	21.1461	22.0478
34	0.9152	7.3569	11.5089	11.8500	21.0584	22.0659
36	0.9132	7.3397	11.4706	11.8516	20.9797	22.0816
38	0.9116	7.3242	11.4361	11.8529	20.9087	22.0951
40	0.9099	7.3102	11.4051	11.8541	20.8443	22.1069

Table 4.33. Dynamic analysis (CS=9, RVM+LM, γ=0, n=6, c=2)

γ=0	Frequencies					
2k	ω1	ω2	ω3	ω4	ω5	ω6
2	1.3603	5.5303	8.4022	9.4419	13.5706	14.0425
4	1.0156	5.6895	7.1453	10.0570	14.0319	14.4855
6	0.9192	5.7562	6.4210	10.3200	13.8178	14.5844
8	0.8741	5.7557	6.0677	10.4234	13.4256	14.8691
10	0.8482	5.7381	5.8619	10.4659	13.1641	14.7431
12	0.8313	5.7176	5.7275	10.4826	12.9771	14.4961
14	0.8191	5.6330	5.6982	10.4875	12.8367	14.3254
16	0.8106	5.5630	5.6809	10.4868	12.7273	14.2006
18	0.8037	5.5089	5.6657	10.4834	12.6396	14.1053
20	0.7981	5.4660	5.6523	10.4788	12.5677	14.0300
22	0.7937	5.4312	5.6406	10.4736	12.5076	13.9689
24	0.7903	5.4022	5.6302	10.4684	12.4566	13.9184
26	0.7871	5.3778	5.6211	10.4632	12.4129	13.8757
28	0.7845	5.3570	5.6129	10.4583	12.3748	13.8393
30	0.7820	5.3390	5.6055	10.4536	12.3415	13.8078
32	0.7801	5.3233	5.5989	10.4491	12.3119	13.7802
34	0.7785	5.3094	5.5929	10.4450	12.2856	13.7559
36	0.7769	5.2971	5.5875	10.4410	12.2621	13.7342
38	0.7756	5.2862	5.5826	10.4374	12.2408	13.7148
40	0.7743	5.2763	5.5780	10.4339	12.2215	13.6973

Table 4.34. Dynamic analysis (CS=9, RVM+LM, γ=0, n=8, c=2)

γ=0	Frequencies					
2k	ω1	ω2	ω3	ω4	ω5	ω6
2	1.3259	4.6315	6.4210	9.1970	11.6707	12.1727
4	0.9877	4.6787	5.1935	9.4542	10.5613	13.2675
6	0.8936	4.6967	4.7649	9.5444	10.3177	11.9723
8	0.8497	4.5603	4.6848	9.5586	10.1666	11.4319
10	0.8243	4.4412	4.6674	9.5466	10.0564	11.1506
12	0.8081	4.3635	4.6508	9.5260	9.9724	10.9834
14	0.7962	4.3087	4.6362	9.5037	9.9063	10.8746
16	0.7877	4.2681	4.6238	9.4822	9.8532	10.7991
18	0.7813	4.2367	4.6133	9.4624	9.8096	10.7439
20	0.7759	4.2117	4.6043	9.4446	9.7733	10.7019
22	0.7717	4.1914	4.5966	9.4287	9.7426	10.6690
24	0.7681	4.1744	4.5900	9.4145	9.7164	10.6425
26	0.7652	4.1602	4.5842	9.4017	9.6937	10.6207
28	0.7629	4.1480	4.5791	9.3903	9.6738	10.6025
30	0.7606	4.1375	4.5746	9.3801	9.6564	10.5870
32	0.7586	4.1282	4.5706	9.3708	9.6409	10.5738
34	0.7570	4.1201	4.5671	9.3624	9.6271	10.5622
36	0.7556	4.1128	4.5638	9.3548	9.6148	10.5521
38	0.7543	4.1064	4.5609	9.3478	9.6036	10.5431
40	0.7530	4.1006	4.5583	9.3414	9.5934	10.5352

Table 4.35. Dynamic analysis (CS=9, RVM+LM, γ=0, n=10, c=2)

γ=0	Frequencies					
2k	ω1	ω2	ω3	ω4	ω5	ω6
2	1.3233	4.5352	5.9522	9.1096	10.8011	11.8624
4	0.9849	4.5147	4.8154	9.2757	9.7993	11.6048
6	0.8905	4.4431	4.4903	9.3016	9.5342	10.7411
8	0.8468	4.2645	4.4578	9.2675	9.3724	10.3693
10	0.8216	4.1602	4.4301	9.2159	9.2595	10.1822
12	0.8053	4.0920	4.4082	9.1636	9.1766	10.0775
14	0.7937	4.0439	4.3910	9.1135	9.1167	10.0135
16	0.7852	4.0082	4.3772	9.0642	9.0762	9.9713
18	0.7788	3.9808	4.3661	9.0247	9.0418	9.9417
20	0.7733	3.9589	4.3570	8.9924	9.0125	9.9201
22	0.7691	3.9411	4.3494	8.9657	8.9874	9.9036
24	0.7658	3.9263	4.3429	8.9431	8.9658	9.8907
26	0.7629	3.9138	4.3375	8.9238	8.9470	9.8803
28	0.7603	3.9031	4.3327	8.9072	8.9306	9.8718
30	0.7580	3.8939	4.3285	8.8927	8.9161	9.8646
32	0.7563	3.8859	4.3249	8.8799	8.9033	9.8586
34	0.7543	3.8789	4.3216	8.8686	8.8918	9.8534
36	0.7530	3.8725	4.3187	8.8586	8.8815	9.8489
38	0.7517	3.8669	4.3162	8.8495	8.8722	9.8450
40	0.7503	3.8619	4.3138	8.8414	8.8638	9.8415

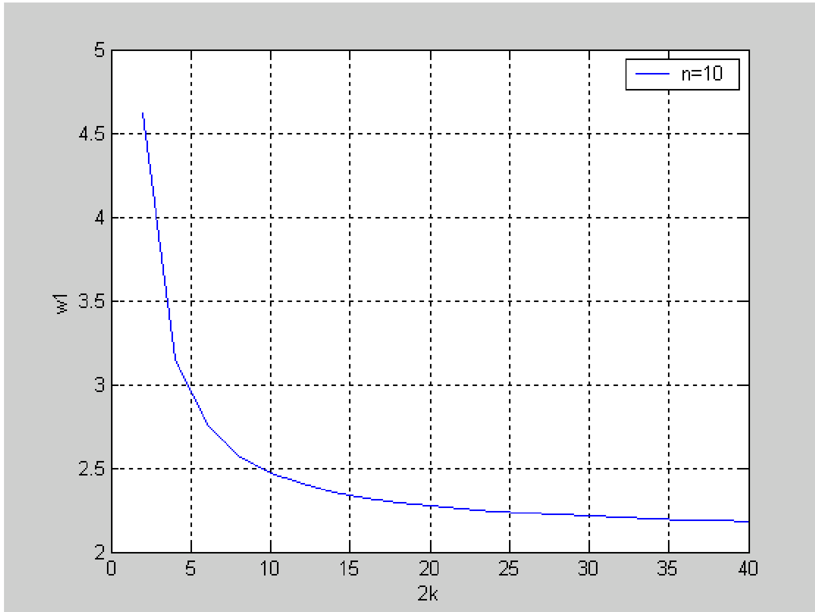

Figure 4.33. ω1-2k graph (CS=9, c=1, γ=0)

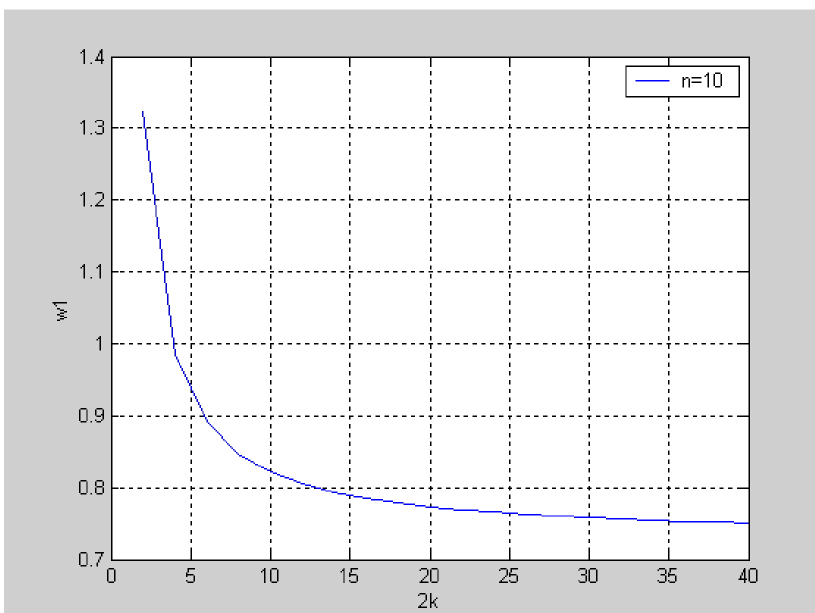

Figure 4.34. ω1-2k graph (CS=9, c=2, γ=0)

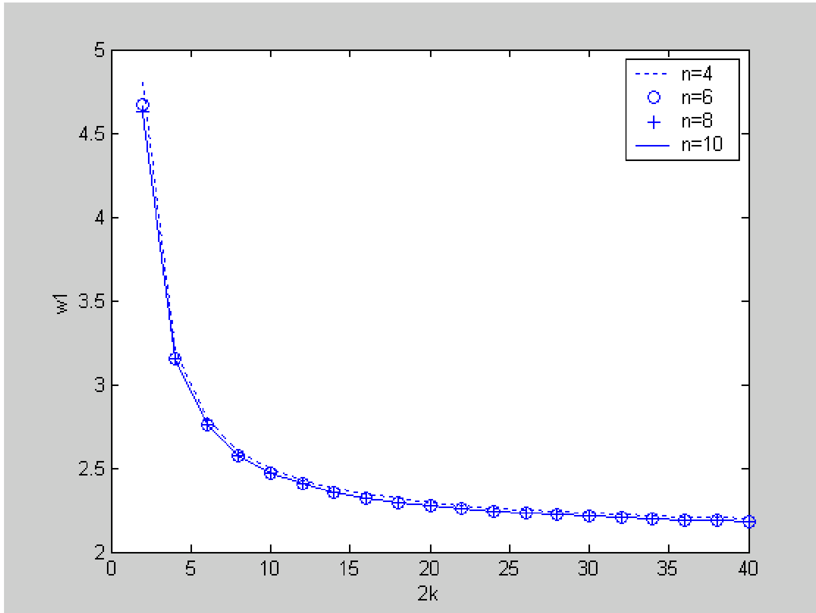

Figure 4.35. Convergence of “ω1” (CS=9, c=1, γ=0)

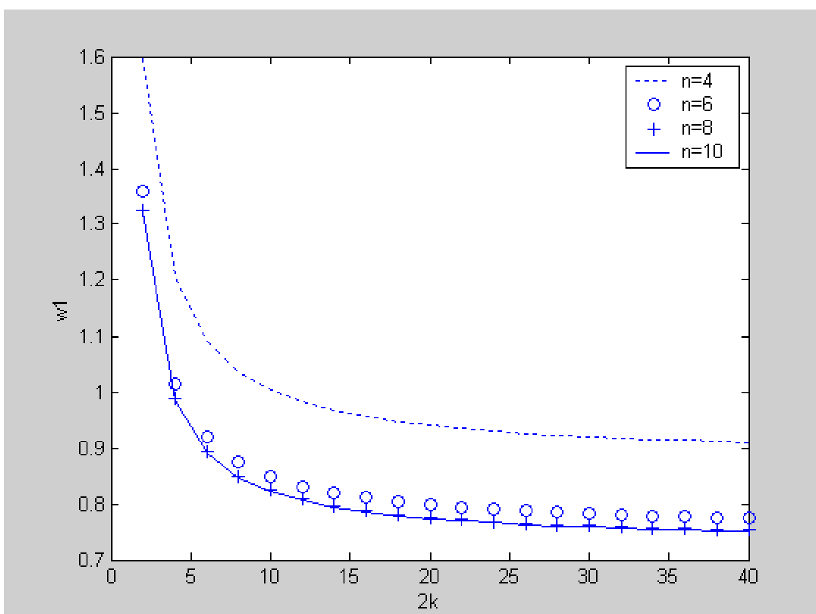

Figure 4.36. Convergence of “ω1” (CS=9, c=2, γ=0)

4.9.5. Conclusions

Despite the non-existing literature on the analysis of functionally graded super-elliptical plates resting on point-supports, the tabulated results listed above seem to meet the design purposes with acceptable accuracy. The natural frequencies tend to decrease as "k" increases and high convergence is obtained as illustrated in the figures.

5. CONCLUSIONS / FINAL REMARKS

As it was mentioned by the researchers, although the pb-2 Ritz method works for a large class of plate shapes, it encounters problems when

- the boundary equations of simple and clamped supports pass through the plate domain,
- the boundary conditions are mixed along the same edge,
- the internal curved supports do not span the plate [28].

It can be seen from the tabulated results that although for clamped circular plate case (c=1, a=1), all of the methods are in agreement with the exact ones –totally independent of the polynomial order denoted by “n”-, as “k” increases distortion occurs beyond the control of the researcher. Therefore, in our case, -as a result of the corner effect-, the approximate methods employed in the dissertation need modification in the selection of shape functions.

It is quite hard to find an expression representing the deformed plate surface among infinitely many alternatives. Apart from the boundary conditions, there are two more conditions to be satisfied in order to guarantee that the results are sufficiently close to the exact ones. The “w_{max}” values have to be in the interval bounded by those known ones and they have to be in ascending order as the super-elliptical power “k” increases. The latter statement is not available in the literature as far as the author knows.

On the contrary, it is stated that the results do not necessarily lie in the interval bounded by circular/elliptical and square/rectangular plates [11]. This statement can not be considered to be wrong because it supports the idea that the results may or may not lie between the bounds. It is not surprising to verify it when only the shape function (γ=1) is employed.

First it was shown that relatively many terms in number could not yield satisfactory results. At most 120 terms were used for the static analysis of simply supported super-elliptical plates (γ=10) as shown in appendix A. The task was harder than it seemed because the interval was too narrow. The “w_{max}” values had to be in the range [0.0637 0.0667] in ascending order. Finally, almost all of the results were in the desired interval but not in ascending order at all. This was the break point to think about something else/new. So the author decided to build another way.

The trial functions were chosen mostly by deep-thinking and partly by inspiration/guess. The motive in the selection of the convenient shape functions lies in the detection of distortion in the straightforward approach which corresponds to $\gamma=1$. Although it seems to be the solution to the problem at first glance, surprisingly, distortion was detected after solving a set of polynomials. The theoretical background states that the solution gets too close to the exact solution if infinitely many terms are considered. The long runtime of the codes and the memory problems with the computer were the inevitable effects which slowed down the work.

During the time spent for the dissertation, a large number of computations have been carried out with great care. The results were checked over and over.

Finally, the modifications were accomplished and the expected results were computed faster and faster day by day. In the end, satisfactory results were found and considerably high accuracy was obtained. Especially in static analysis, for the first half of the interval, convergence was achieved even for the shape functions which produced reasonable results but not shown in Table 5.1.

In some cases in static analysis, two methods produced close results to each other in a small interval of “k”. But it was the Ritz method which stepped forward among the others. Table 5.1 points the (assumed to be) the best results. The details of the computations can be seen in Appendix A.

Table 5.1. The list of the results (probably sufficiently close to the exact values)

Case study no	Shape function no (c=1)	Method	n (c=1)	Shape function no (c=2)	Method	n (c=2)
1	11	Ritz	2	11	Ritz	2
2	1	Ritz	8	1	Ritz	6
3	8	Ritz	5	8	Ritz	5
4	0	Ritz+LM	10	0	Ritz+LM	10
5	0	Ritz+LM	10	0	Ritz+LM	10
6	1	Ritz	16	1	Ritz	12
7	11	Ritz	6	11	Ritz	6
8	0	Ritz+LM	12	0	Ritz+LM	12
9	0	Ritz+LM	10	0	Ritz+LM	10

If the computations are examined carefully, the following comments/statements can be made:

- For the third, fourth, fifth, eighth, and the ninth case studies, extremely high convergence is achieved. Design purposes are met with accuracy.
- There are at least two possibilities in determining the shape function which can be considered to identify the deformed plate surface as well as possible in the first case study. The one which fits the interval better, is chosen to be the shape function which is assumed to be sufficiently close to the exact ones.
- The convergence for the second case study can be considered to be sufficient for practical purposes.
- The sixth case study presents significant convergence as the order of the polynomial increases. There is nothing unexpected in the trend of the incrementation of the results.
- Finally, we have the case study number seven where apart from a few exceptions for the smallest and the greatest values of “k”, quite satisfactory results are obtained with convergence.

APPENDIX A: DETAILED RESULTS OF THE CASE STUDIES

One of the significant points is that the shape functions number 2, 3, 8, and 9 produce acceptable satisfactory results in both static and dynamic analysis except a few number of exceptions with sufficient convergence as shown in the following tables. Interestingly, the results obtained by the above mentioned shape functions are close to the ones demonstrated in Table 5.1.

Table A.1. Static analysis (CS=1, RVM, γ=1, c=1)

c=1	w_{max}	w_{max}	w_{max}	w_{max}	w_{max}	w_{max}	w_{max}	w_{max}	w_{max}
γ=1	r=3	r=6	r=10	r=15	r=21	r=28	r=36	r=45	r=55
2k	n=2	n=4	n=6	n=8	n=10	n=12	n=14	n=16	n=18
2	0.0156	0.0156	0.0156	0.0156	0.0157	0.0156	0.0158	0.0154	0.0202
4	0.0137	0.0181	0.0194	0.0197	0.0200	0.0200	0.0206	0.0210	0.0206
6	0.0100	0.0178	0.0200	0.0202	0.0203	0.0203	0.0200	0.0238	0.0280
8	0.0069	0.0168	0.0201	0.0203	0.0202	0.0204	0.0208	0.0276	0.0302
10	0.0047	0.0154	0.0200	0.0203	0.0202	0.0212	0.0242	0.0291	0.0306
12	0.0033	0.0140	0.0199	0.0203	0.0204	0.0231	0.0274	0.0297	0.0306
14	0.0023	0.0127	0.0199	0.0203	0.0210	0.0253	0.0292	0.0301	0.0306
16	0.0017	0.0116	0.0198	0.0204	0.0224	0.0270	0.0300	0.0303	0.0305
18	0.0013	0.0107	0.0201	0.0214	0.0244	0.0282	0.0304	0.0304	0.0304
20	0.0010	0.0098	0.0201	0.0218	0.0262	0.0291	0.0306	0.0305	0.0304
22	0.0008	0.0090	0.0202	0.0225	0.0276	0.0296	0.0306	0.0305	0.0304
24	0.0007	0.0082	0.0207	0.0240	0.0286	0.0300	0.0306	0.0305	0.0304
26	0.0006	0.0074	0.0205	0.0242	0.0292	0.0302	0.0305	0.0305	0.0303
28	0.0006	0.0065	0.0206	0.0248	0.0296	0.0303	0.0305	0.0305	0.0303
30	0.0006	0.0056	0.0209	0.0259	0.0298	0.0304	0.0305	0.0305	0.0303
32	0.0007	0.0048	0.0205	0.0262	0.0300	0.0304	0.0304	0.0305	0.0303
34	0.0009	0.0043	0.0213	0.0297	0.0301	0.0304	0.0304	0.0305	0.0303
36	0.0012	0.0043	0.0207	0.0281	0.0302	0.0304	0.0304	0.0305	0.0303
38	0.0014	0.0051	0.0210	0.0276	0.0303	0.0305	0.0304	0.0305	0.0303
40	0.0014	0.0065	0.0208	0.0268	0.0303	0.0305	0.0304	0.0305	0.0303

Table A.2. Static analysis (CS=1, RVM, γ=1, c=2)

c=2	w_{max}	w_{max}	w_{max}	w_{max}	w_{max}
γ=1	r=1	r=3	r=6	r=10	r=15
2k	n=0	n=2	n=4	n=6	n=8
2	0.0339	0.0339	0.0339	0.0339	0.0339
4	0.0118	0.0278	0.0368	0.0393	0.0397
6	0.0044	0.0207	0.0367	0.0404	0.0404
8	0.0021	0.0153	0.0356	0.0408	0.0406
10	0.0011	0.0114	0.0340	0.0409	0.0407
12	0.0007	0.0087	0.0321	0.0410	0.0406
14	0.0004	0.0067	0.0302	0.0410	0.0406
16	0.0003	0.0054	0.0283	0.0411	0.0405
18	0.0002	0.0045	0.0265	0.0412	0.0403
20	0.0002	0.0039	0.0248	0.0413	0.0402
22	0.0001	0.0035	0.0232	0.0414	0.0402
24	0.0001	0.0033	0.0217	0.0415	0.0402
26	0.0001	0.0032	0.0203	0.0416	0.0403
28	0.0001	0.0032	0.0190	0.0416	0.0405
30	0.0001	0.0034	0.0178	0.0416	0.0408
32	0.0000	0.0035	0.0170	0.0415	0.0411
34	0.0000	0.0037	0.0170	0.0414	0.0413
36	0.0000	0.0039	0.0183	0.0413	0.0415
38	0.0000	0.0039	0.0207	0.0412	0.0417
40	0.0000	0.0037	0.0227	0.0414	0.0418

Table A.3. Static analysis (CS=1, RVM, γ=2, c=1)

c=1	w_{max}	w_{max}	w_{max}	w_{max}	w_{max}
γ=2	r=1	r=3	r=6	r=10	r=15
2k	n=0	n=2	n=4	n=6	n=8
2	0.0097	0.0112	0.0137	0.0141	0.0154
4	0.0141	0.0132	0.0168	0.0168	0.0171
6	0.0160	0.0151	0.0175	0.0176	0.0175
8	0.0171	0.0165	0.0176	0.0177	0.0183
10	0.0178	0.0173	0.0178	0.0178	0.0178
12	0.0181	0.0177	0.0179	0.0181	0.0183
14	0.0185	0.0180	0.0182	0.0186	0.0193
16	0.0188	0.0182	0.0183	0.0183	0.0188
18	0.0194	0.0188	0.0199	0.0184	-0.0020
20	0.0198	0.0190	0.0194	0.0206	0.0192
22	0.0201	0.0191	0.0204	0.0199	0.0212
24	0.0207	0.0198	0.0107	0.0195	0.0207
26	0.0206	0.0197	0.0199	0.0202	0.0211
28	0.0206	0.0195	0.0198	0.0198	0.0202
30	0.0207	0.0198	0.0200	0.0200	0.0197
32	0.0206	0.0196	0.0197	0.0170	0.0209
34	0.0220	0.0218	0.0202	0.0106	0.0187
36	0.0213	0.0205	0.0196	0.0175	0.0188
38	0.0210	0.0202	0.0279	0.0153	0.0214
40	0.0196	0.0188	0.0209	0.0189	0.0164

Table A.4. Static analysis (CS=1, RVM, γ=2, c=2)

c=2	w_{max}	w_{max}	w_{max}	w_{max}	w_{max}
γ=2	r=1	r=3	r=6	r=10	r=15
2k	n=0	n=2	n=4	n=6	n=8
2	0.0211	0.0234	0.0289	0.0296	0.0316
4	0.0303	0.0261	0.0344	0.0340	0.0367
6	0.0340	0.0298	0.0352	0.0353	0.0379
8	0.0363	0.0325	0.0355	0.0357	0.0384
10	0.0378	0.0343	0.0359	0.0358	0.0386
12	0.0388	0.0354	0.0364	0.0359	0.0387
14	0.0396	0.0361	0.0369	0.0361	0.0387
16	0.0403	0.0366	0.0374	0.0364	0.0387
18	0.0409	0.0369	0.0379	0.0368	0.0387
20	0.0414	0.0371	0.0382	0.0372	0.0387
22	0.0418	0.0373	0.0385	0.0376	0.0387
24	0.0421	0.0375	0.0387	0.0379	0.0387
26	0.0424	0.0377	0.0388	0.0382	0.0388
28	0.0426	0.0379	0.0389	0.0384	0.0388
30	0.0428	0.0380	0.0389	0.0387	0.0388
32	0.0430	0.0382	0.0389	0.0388	0.0389
34	0.0431	0.0383	0.0389	0.0390	0.0389
36	0.0432	0.0385	0.0389	0.0391	0.0390
38	0.0433	0.0386	0.0389	0.0392	0.0390
40	0.0434	0.0387	0.0389	0.0393	0.0390

Table A.5. Static analysis (CS=1, RVM, γ=3, c=1)

c=1	w_{max}	w_{max}	w_{max}	w_{max}	w_{max}
γ=3	r=1	r=3	r=6	r=10	r=15
2k	n=0	n=2	n=4	n=6	n=8
2	0.0137	0.0138	0.0152	0.0152	0.0156
4	0.0144	0.0151	0.0189	0.0186	0.0209
6	0.0120	0.0161	0.0196	0.0195	0.0197
8	0.0100	0.0168	0.0197	0.0198	0.0205
10	0.0087	0.0171	0.0198	0.0198	0.0199
12	0.0078	0.0174	0.0198	0.0198	0.0199
14	0.0070	0.0177	0.0198	0.0197	0.0200
16	0.0062	0.0179	0.0198	0.0197	0.0197
18	0.0053	0.0182	0.0202	0.0201	0.0197
20	0.0046	0.0182	0.0200	0.0202	0.0189
22	0.0040	0.0180	0.0199	0.0201	0.0205
24	0.0034	0.0179	0.0205	0.0207	0.0206
26	0.0030	0.0175	0.0200	0.0200	0.0200
28	0.0027	0.0171	0.0198	0.0216	0.0203
30	0.0025	0.0168	0.0199	0.0200	0.0198
32	0.0023	0.0164	0.0197	0.0201	0.0204
34	0.0021	0.0167	0.0216	0.0201	-0.0466
36	0.0020	0.0162	0.0207	0.0189	0.0178
38	0.0019	0.0158	0.0204	0.0233	0.0128
40	0.0018	0.0153	0.0189	0.0265	0.0242

Table A.6. Static analysis (CS=1, RVM, γ=3, c=2)

c=2	w_{max}	w_{max}	w_{max}	w_{max}	w_{max}
γ=3	r=1	r=3	r=6	r=10	r=15
2k	n=0	n=2	n=4	n=6	n=8
2	0.0297	0.0293	0.0326	0.0326	0.0335
4	0.0300	0.0299	0.0384	0.0373	0.0389
6	0.0242	0.0321	0.0396	0.0384	0.0394
8	0.0199	0.0337	0.0398	0.0390	0.0395
10	0.0168	0.0349	0.0399	0.0392	0.0396
12	0.0146	0.0358	0.0398	0.0393	0.0396
14	0.0128	0.0365	0.0397	0.0393	0.0396
16	0.0114	0.0371	0.0396	0.0394	0.0396
18	0.0102	0.0374	0.0396	0.0395	0.0395
20	0.0092	0.0375	0.0396	0.0395	0.0396
22	0.0084	0.0374	0.0397	0.0396	0.0396
24	0.0077	0.0372	0.0398	0.0396	0.0396
26	0.0071	0.0370	0.0399	0.0397	0.0396
28	0.0065	0.0367	0.0400	0.0397	0.0397
30	0.0061	0.0363	0.0401	0.0398	0.0397
32	0.0057	0.0360	0.0402	0.0398	0.0398
34	0.0054	0.0356	0.0403	0.0398	0.0398
36	0.0051	0.0353	0.0403	0.0398	0.0398
38	0.0048	0.0350	0.0404	0.0398	0.0399
40	0.0046	0.0347	0.0405	0.0398	0.0399

Table A.7. Static analysis (CS=1, RVM, γ=4, c=1)

c=1	w_{max}	w_{max}	w_{max}	w_{max}	w_{max}
γ=4	r=1	r=3	r=6	r=10	r=15
2k	n=0	n=2	n=4	n=6	n=8
2	0.0157	0.0154	0.0156	0.0156	0.0156
4	0.0069	0.0140	0.0182	0.0194	0.0197
6	0.0029	0.0109	0.0180	0.0200	0.0202
8	0.0015	0.0084	0.0174	0.0200	0.0202
10	0.0008	0.0064	0.0165	0.0199	0.0202
12	0.0005	0.0050	0.0157	0.0198	0.0202
14	0.0004	0.0039	0.0149	0.0197	0.0202
16	0.0003	0.0031	0.0142	0.0195	0.0204
18	0.0002	0.0025	0.0136	0.0198	0.0224
20	0.0002	0.0021	0.0128	0.0198	0.0236
22	0.0001	0.0017	0.0121	0.0201	0.0240
24	0.0001	0.0015	0.0114	0.0205	0.0257
26	0.0001	0.0013	0.0106	0.0208	0.0269
28	0.0001	0.0012	0.0098	0.0214	0.0382
30	0.0001	0.0011	0.0091	0.0223	0.0276
32	0.0001	0.0011	0.0084	0.0223	0.0280
34	0.0001	0.0012	0.0080	0.0231	0.0190
36	0.0000	0.0015	0.0081	0.0226	0.0281
38	0.0000	0.0019	0.0087	0.0222	0.0274
40	0.0000	0.0020	0.0100	0.0232	0.0281

Table A.8. Static analysis (CS=1, RVM, γ=4, c=2)

c=2	w_{max}	w_{max}	w_{max}	w_{max}	w_{max}
γ=4	r=1	r=3	r=6	r=10	r=15
2k	n=0	n=2	n=4	n=6	n=8
2	0.0341	0.0336	0.0339	0.0339	0.0339
4	0.0138	0.0282	0.0369	0.0393	0.0397
6	0.0057	0.0224	0.0370	0.0402	0.0404
8	0.0028	0.0178	0.0362	0.0404	0.0405
10	0.0016	0.0142	0.0351	0.0404	0.0405
12	0.0010	0.0113	0.0340	0.0404	0.0406
14	0.0007	0.0092	0.0329	0.0403	0.0406
16	0.0005	0.0075	0.0318	0.0403	0.0406
18	0.0003	0.0063	0.0308	0.0403	0.0405
20	0.0002	0.0055	0.0297	0.0404	0.0404
22	0.0002	0.0049	0.0286	0.0405	0.0404
24	0.0002	0.0045	0.0275	0.0408	0.0403
26	0.0001	0.0044	0.0264	0.0412	0.0403
28	0.0001	0.0044	0.0252	0.0416	0.0404
30	0.0001	0.0045	0.0238	0.0420	0.0405
32	0.0001	0.0048	0.0225	0.0423	0.0406
34	0.0001	0.0052	0.0216	0.0425	0.0407
36	0.0000	0.0055	0.0219	0.0425	0.0409
38	0.0000	0.0056	0.0235	0.0425	0.0410
40	0.0000	0.0054	0.0255	0.0423	0.0412

Table A.9. Static analysis (CS=1, RVM, γ=5, c=1)

c=1	w_{max}	w_{max}	w_{max}	w_{max}	w_{max}
γ=5	r=1	r=3	r=6	r=10	r=15
2k	n=0	n=2	n=4	n=6	n=8
2	0.0156	0.0152	0.0156	0.0156	0.0156
4	0.0076	0.0140	0.0183	0.0194	0.0197
6	0.0034	0.0114	0.0182	0.0199	0.0202
8	0.0018	0.0093	0.0177	0.0199	0.0202
10	0.0010	0.0077	0.0171	0.0198	0.0202
12	0.0006	0.0065	0.0165	0.0196	0.0202
14	0.0004	0.0055	0.0160	0.0195	0.0202
16	0.0003	0.0046	0.0157	0.0192	0.0202
18	0.0002	0.0040	0.0154	0.0195	0.0226
20	0.0002	0.0034	0.0148	0.0191	0.0229
22	0.0002	0.0029	0.0145	0.0198	0.0245
24	0.0001	0.0025	0.0140	0.0202	0.0369
26	0.0001	0.0022	0.0134	0.0202	0.0268
28	0.0001	0.0019	0.0131	0.0213	0.0130
30	0.0001	0.0017	0.0128	0.0222	0.0281
32	0.0001	0.0016	0.0125	0.0228	0.0360
34	0.0001	0.0016	0.0124	0.0236	0.0315
36	0.0001	0.0018	0.0127	0.0233	0.0270
38	0.0000	0.0023	0.0131	0.0224	0.0267
40	0.0000	0.0026	0.0141	0.0235	0.0352

Table A.10. Static analysis (CS=1, RVM, γ=5, c=2)

c=2	w_{max}	w_{max}	w_{max}	w_{max}	w_{max}
γ=5	r=1	r=3	r=6	r=10	r=15
2k	n=0	n=2	n=4	n=6	n=8
2	0.0339	0.0331	0.0339	0.0339	0.0339
4	0.0153	0.0283	0.0371	0.0392	0.0397
6	0.0067	0.0233	0.0372	0.0401	0.0404
8	0.0034	0.0193	0.0365	0.0401	0.0405
10	0.0019	0.0161	0.0355	0.0400	0.0405
12	0.0012	0.0135	0.0347	0.0399	0.0406
14	0.0008	0.0113	0.0339	0.0397	0.0407
16	0.0006	0.0096	0.0332	0.0396	0.0409
18	0.0004	0.0082	0.0325	0.0394	0.0411
20	0.0003	0.0071	0.0318	0.0393	0.0413
22	0.0002	0.0063	0.0311	0.0393	0.0414
24	0.0002	0.0057	0.0304	0.0393	0.0414
26	0.0001	0.0054	0.0295	0.0396	0.0414
28	0.0001	0.0053	0.0286	0.0399	0.0414
30	0.0001	0.0053	0.0275	0.0403	0.0413
32	0.0001	0.0056	0.0264	0.0408	0.0412
34	0.0001	0.0059	0.0256	0.0411	0.0412
36	0.0001	0.0063	0.0255	0.0413	0.0412
38	0.0001	0.0065	0.0263	0.0414	0.0412
40	0.0000	0.0062	0.0274	0.0415	0.0412

Table A.11. Static analysis (CS=1, RVM, γ=6, c=1)

c=1	w_{max}	w_{max}	w_{max}	w_{max}	w_{max}
γ=6	r=1	r=3	r=6	r=10	r=15
2k	n=0	n=2	n=4	n=6	n=8
2	0.0050	0.0112	0.0145	0.0154	0.0156
4	0.0011	0.0078	0.0175	0.0198	0.0198
6	0.0004	0.0040	0.0155	0.0200	0.0201
8	0.0002	0.0020	0.0131	0.0199	0.0203
10	0.0001	0.0012	0.0109	0.0200	0.0214
12	0.0001	0.0008	0.0089	0.0200	0.0232
14	0.0001	0.0007	0.0071	0.0207	0.0257
16	0.0000	0.0007	0.0052	0.0202	0.0255
18	0.0000	0.0012	0.0046	0.0209	0.0273
20	0.0000	0.0014	0.0069	0.0231	0.0316
22	0.0000	0.0007	0.0089	0.0233	0.0278
24	0.0000	0.0004	0.0081	0.0245	0.0278
26	0.0000	0.0002	0.0068	0.0287	0.0305
28	0.0000	0.0001	0.0057	0.0328	0.2850
30	0.0000	0.0001	0.0049	0.0267	0.0267
32	0.0000	0.0001	0.0043	0.0290	0.0300
34	0.0000	0.0001	0.0039	0.0281	0.0418
36	0.0000	0.0000	0.0036	0.0569	0.0275
38	0.0000	0.0000	0.0033	0.0255	0.0343
40	0.0000	0.0000	0.0031	0.0263	0.0275

Table A.12. Static analysis (CS=1, RVM, γ=7, c=1)

c=1	w_{max}	w_{max}	w_{max}	w_{max}	w_{max}
γ=7	r=1	r=3	r=6	r=10	r=15
2k	n=0	n=2	n=4	n=6	n=8
2	0.0019	0.0086	0.0145	0.0157	0.0157
4	0.0004	0.0048	0.0166	0.0197	0.0197
6	0.0001	0.0018	0.0133	0.0199	0.0202
8	0.0001	0.0009	0.0098	0.0199	0.0226
10	0.0000	0.0007	0.0067	0.0199	0.0253
12	0.0000	0.0011	0.0052	0.0214	0.0278
14	0.0000	0.0011	0.0088	0.0234	0.0287
16	0.0000	0.0004	0.0089	0.0293	0.0315
18	0.0000	0.0002	0.0065	0.0275	0.0301
20	0.0000	0.0001	0.0050	0.0291	0.0307
22	0.0000	0.0001	0.0041	0.0299	0.0373
24	0.0000	0.0000	0.0037	0.0713	0.0228
26	0.0000	0.0000	0.0033	0.0306	0.0268
28	0.0000	0.0000	0.0029	0.0281	0.0175
30	0.0000	0.0000	0.0027	0.0339	0.0212
32	0.0000	0.0000	0.0025	0.0214	0.0106
34	0.0000	0.0000	0.0024	0.0268	0.0389
36	0.0000	0.0000	0.0022	0.0217	0.0305
38	0.0000	0.0000	0.0021	0.0279	0.0348
40	0.0000	0.0000	0.0019	0.0136	-0.0319

Table A.13. Static analysis (CS=1, RVM, γ=8, c=1)

c=1	w_{max}	w_{max}	w_{max}	w_{max}	w_{max}
γ=8	r=1	r=3	r=6	r=10	r=15
2k	n=0	n=2	n=4	n=6	n=8
2	0.0114	0.0124	0.0145	0.0146	0.0152
4	0.0144	0.0141	0.0177	0.0177	0.0188
6	0.0138	0.0157	0.0185	0.0183	0.0238
8	0.0129	0.0168	0.0188	0.0188	0.0188
10	0.0122	0.0174	0.0190	0.0190	0.0189
12	0.0117	0.0179	0.0191	0.0191	0.0191
14	0.0110	0.0184	0.0192	0.0193	0.0208
16	0.0100	0.0188	0.0192	0.0192	0.0195
18	0.0087	0.0195	0.0197	0.0132	0.0173
20	0.0074	0.0198	0.0196	0.0201	0.0199
22	0.0063	0.0200	0.0195	0.0186	0.0198
24	0.0054	0.0205	0.0200	0.0195	0.0203
26	0.0047	0.0203	0.0197	0.0196	0.0206
28	0.0041	0.0203	0.0196	0.0195	0.0212
30	0.0037	0.0204	0.0197	0.0203	0.0240
32	0.0034	0.0202	0.0197	0.0201	0.0208
34	0.0032	0.0217	0.0178	0.0231	0.0312
36	0.0030	0.0212	0.0204	0.0177	0.0190
38	0.0029	0.0209	0.0191	0.0108	0.0202
40	0.0027	0.0194	0.0195	0.0181	0.0232

Table A.14. Static analysis (CS=1, RVM, γ=8, c=2)

c=2	w_{max}	w_{max}	w_{max}	w_{max}	w_{max}
γ=8	r=1	r=3	r=6	r=10	r=15
2k	n=0	n=2	n=4	n=6	n=8
2	0.0213	0.0239	0.0289	0.0299	0.0316
4	0.0297	0.0265	0.0343	0.0339	0.0363
6	0.0323	0.0305	0.0354	0.0353	0.0370
8	0.0336	0.0334	0.0362	0.0359	0.0373
10	0.0344	0.0353	0.0367	0.0363	0.0375
12	0.0349	0.0367	0.0371	0.0366	0.0377
14	0.0351	0.0380	0.0374	0.0369	0.0377
16	0.0348	0.0390	0.0375	0.0372	0.0377
18	0.0341	0.0399	0.0376	0.0376	0.0377
20	0.0329	0.0406	0.0377	0.0379	0.0378
22	0.0316	0.0412	0.0377	0.0382	0.0379
24	0.0301	0.0416	0.0378	0.0385	0.0380
26	0.0288	0.0419	0.0378	0.0388	0.0381
28	0.0275	0.0422	0.0379	0.0390	0.0382
30	0.0264	0.0424	0.0380	0.0392	0.0383
32	0.0254	0.0426	0.0380	0.0394	0.0384
34	0.0246	0.0427	0.0381	0.0395	0.0385
36	0.0239	0.0428	0.0381	0.0396	0.0386
38	0.0234	0.0429	0.0382	0.0397	0.0387
40	0.0230	0.0430	0.0382	0.0398	0.0387

Table A.15. Static analysis (CS=1, RVM, γ=9, c=1)

c=1	w_{max}	w_{max}	w_{max}	w_{max}	w_{max}
γ=9	r=1	r=3	r=6	r=10	r=15
2k	n=0	n=2	n=4	n=6	n=8
2	0.0114	0.0124	0.0145	0.0147	0.0148
4	0.0144	0.0141	0.0179	0.0178	0.0160
6	0.0138	0.0157	0.0184	0.0181	0.0181
8	0.0128	0.0168	0.0185	0.0183	0.0182
10	0.0117	0.0175	0.0186	0.0191	0.0186
12	0.0107	0.0179	0.0187	0.0188	0.0189
14	0.0099	0.0183	0.0188	0.0186	0.0197
16	0.0091	0.0185	0.0189	0.0191	0.0187
18	0.0085	0.0190	0.0198	0.0202	0.0167
20	0.0080	0.0193	0.0197	0.0169	0.0192
22	0.0075	0.0196	0.0198	0.0205	0.0202
24	0.0071	0.0201	0.0206	0.0206	0.0215
26	0.0067	0.0201	0.0203	0.0201	0.0201
28	0.0063	0.0200	0.0200	0.0228	0.0204
30	0.0060	0.0202	0.0200	0.0201	0.0203
32	0.0057	0.0201	0.0197	0.0207	0.0199
34	0.0054	0.0215	0.0235	0.0242	0.0153
36	0.0052	0.0206	0.0208	0.0214	0.0194
38	0.0049	0.0207	0.0210	0.0195	0.0252
40	0.0047	0.0190	0.0184	0.0193	0.0193

Table A.16. Static analysis (CS=1, RVM, γ=9, c=2)

c=2	w_{max}	w_{max}	w_{max}	w_{max}	w_{max}
γ=9	r=1	r=3	r=6	r=10	r=15
2k	n=0	n=2	n=4	n=6	n=8
2	0.0294	0.0288	0.0326	0.0323	0.0335
4	0.0309	0.0295	0.0385	0.0372	0.0393
6	0.0257	0.0317	0.0393	0.0383	0.0402
8	0.0214	0.0337	0.0392	0.0386	0.0405
10	0.0182	0.0352	0.0391	0.0386	0.0405
12	0.0158	0.0362	0.0391	0.0385	0.0404
14	0.0139	0.0371	0.0391	0.0385	0.0404
16	0.0124	0.0378	0.0392	0.0386	0.0404
18	0.0112	0.0384	0.0393	0.0388	0.0403
20	0.0102	0.0390	0.0394	0.0389	0.0403
22	0.0094	0.0394	0.0394	0.0391	0.0403
24	0.0086	0.0399	0.0395	0.0392	0.0402
26	0.0080	0.0402	0.0394	0.0393	0.0402
28	0.0075	0.0406	0.0394	0.0394	0.0402
30	0.0070	0.0409	0.0394	0.0395	0.0401
32	0.0066	0.0411	0.0394	0.0395	0.0401
34	0.0062	0.0413	0.0394	0.0396	0.0401
36	0.0059	0.0415	0.0393	0.0396	0.0400
38	0.0056	0.0417	0.0393	0.0396	0.0400
40	0.0054	0.0418	0.0393	0.0396	0.0400

Table A.17. Static analysis (RVM, γ=10, c=1)

c=1	w_{max}	w_{max}	w_{max}	w_{max}	w_{max}	w_{max}	w_{max}	w_{max}
γ=10	r=1	r=3	r=6	r=10	r=15	r=21	r=28	r=36
2k	n=0	n=2	n=4	n=6	n=8	n=10	n=12	n=14
2	0.0481	0.0637	0.0637	0.0637	0.0637	0.0637	0.0637	0.0637
4	0.0122	0.0496	0.0682	0.0722	0.0728	0.0729	0.0729	0.0729
6	0.0041	0.0333	0.0650	0.0700	0.0703	0.0702	0.0702	0.0701
8	0.0018	0.0214	0.0609	0.0680	0.0684	0.0684	0.0684	0.0685
10	0.0009	0.0135	0.0563	0.0665	0.0673	0.0673	0.0673	0.0677
12	0.0006	0.0086	0.0516	0.0652	0.0665	0.0666	0.0667	0.0672
14	0.0003	0.0056	0.0468	0.0640	0.0660	0.0662	0.0663	0.0669
16	0.0002	0.0038	0.0423	0.0627	0.0655	0.0660	0.0661	0.0667
18	0.0002	0.0026	0.0382	0.0614	0.0652	0.0657	0.0659	0.0665
20	0.0001	0.0019	0.0348	0.0601	0.0648	0.0656	0.0657	0.0663
22	0.0001	0.0014	0.0320	0.0590	0.0644	0.0654	0.0656	0.0661
24	0.0001	0.0011	0.0298	0.0581	0.0640	0.0653	0.0655	0.0659
26	0.0001	0.0008	0.0280	0.0573	0.0636	0.0652	0.0654	0.0658
28	0.0000	0.0007	0.0267	0.0564	0.0629	0.0651	0.0653	0.0656
30	0.0000	0.0005	0.0256	0.0560	0.0624	0.0650	0.0652	0.0655
32	0.0000	0.0004	0.0248	0.0556	0.0621	0.0650	0.0651	0.0654
34	0.0000	0.0004	0.0241	0.0556	0.0620	0.0649	0.0651	0.0653
36	0.0000	0.0003	0.0235	0.0553	0.0616	0.0648	0.0651	0.0652
38	0.0000	0.0003	0.0229	0.0549	0.0610	0.0648	0.0650	0.0651
40	0.0000	0.0002	0.0225	0.0550	0.0609	0.0647	0.0650	0.0651

Table A.18. Static analysis (RVM, γ=10, c=1)

c=1	w_{max}	w_{max}	w_{max}	w_{max}	w_{max}	w_{max}	w_{max}
γ=10	r=45	r=55	r=66	r=78	r=91	r=105	r=120
2k	n=16	n=18	n=20	n=22	n=24	n=26	n=28
2	0.0638	0.0632	0.0727	0.1835	0.1344	0.1426	0.1606
4	0.0732	0.0735	0.0765	0.1572	0.1714	0.1781	0.1863
6	0.0701	0.0755	0.0819	0.1180	0.1186	0.1594	0.1905
8	0.0684	0.0758	0.0789	0.1059	0.0786	0.3823	0.1703
10	0.0678	0.0746	0.0752	0.0925	0.0930	0.1758	0.1130
12	0.0676	0.0728	0.0724	0.0804	0.0763	0.0938	0.1505
14	0.0674	0.0712	0.0703	0.0812	0.0615	0.1007	0.1758
16	0.0672	0.0698	0.0689	0.0720	0.0405	0.9237	0.2136
18	0.0670	0.0688	0.0680	0.0699	0.0823	0.1278	0.3479
20	0.0667	0.0680	0.0672	0.0678	0.0896	-0.0765	-0.2343
22	0.0665	0.0673	0.0667	0.0669	0.0683	-0.6933	0.1254
24	0.0663	0.0668	0.0663	0.0661	0.0726	0.1611	-0.0236
26	0.0661	0.0664	0.0660	0.0639	0.0653	0.1445	0.3765
28	0.0660	0.0661	0.0659	0.0643	0.0606	0.1580	0.1397
30	0.0659	0.0658	0.0657	0.0628	0.0719	1.2679	0.1412
32	0.0657	0.0656	0.0657	0.0644	0.0739	0.1526	-26.2794
34	0.0656	0.0655	0.0656	0.0638	0.0944	-0.2447	0.1761
36	0.0655	0.0654	0.0655	0.0633	0.0791	0.1322	-0.1171
38	0.0655	0.0653	0.0655	0.0644	0.0687	0.0974	-0.1392
40	0.0654	0.0652	0.0654	0.0649	0.0712	0.8501	-0.1471

Table A.19. Static analysis (RVM, γ=10, c=2)

c=2	w_{max}	w_{max}	w_{max}	w_{max}	w_{max}
γ=10	r=1	r=3	r=6	r=10	r=15
2k	n=0	n=2	n=4	n=6	n=8
2	0.1031	0.1421	0.1423	0.1423	0.1423
4	0.0246	0.1114	0.1577	0.1673	0.1687
6	0.0080	0.0779	0.1569	0.1679	0.1681
8	0.0035	0.0535	0.1507	0.1662	0.1663
10	0.0018	0.0364	0.1419	0.1648	0.1649
12	0.0011	0.0249	0.1320	0.1635	0.1637
14	0.0007	0.0174	0.1220	0.1618	0.1627
16	0.0005	0.0124	0.1123	0.1596	0.1619
18	0.0003	0.0091	0.1033	0.1568	0.1612
20	0.0002	0.0068	0.0951	0.1536	0.1606
22	0.0002	0.0052	0.0877	0.1502	0.1599
24	0.0001	0.0041	0.0810	0.1467	0.1592
26	0.0001	0.0032	0.0750	0.1431	0.1584
28	0.0001	0.0026	0.0697	0.1397	0.1575
30	0.0001	0.0021	0.0649	0.1363	0.1565
32	0.0001	0.0018	0.0605	0.1331	0.1552
34	0.0000	0.0015	0.0567	0.1301	0.1538
36	0.0000	0.0013	0.0532	0.1271	0.1522
38	0.0000	0.0011	0.0500	0.1243	0.1505
40	0.0000	0.0010	0.0472	0.1216	0.1486

Table A.20. Static analysis (CS=1, RVM, γ=12, c=1)

c=1	w_{max}	w_{max}	w_{max}	w_{max}	w_{max}
γ=12	r=1	r=3	r=6	r=10	r=15
2k	n=0	n=2	n=4	n=6	n=8
2	0.0214	0.0204	0.0230	0.0233	0.0247
4	0.0151	0.0188	0.0204	0.0202	0.0207
6	0.0112	0.0190	0.0204	0.0202	0.0203
8	0.0089	0.0191	0.0204	0.0203	0.0203
10	0.0073	0.0191	0.0204	0.0202	0.0202
12	0.0062	0.0190	0.0204	0.0202	0.0202
14	0.0054	0.0189	0.0204	0.0202	0.0202
16	0.0048	0.0187	0.0204	0.0202	0.0203
18	0.0044	0.0185	0.0204	0.0202	0.0203
20	0.0040	0.0184	0.0203	0.0201	0.0202
22	0.0037	0.0182	0.0203	0.0202	0.0203
24	0.0034	0.0180	0.0204	0.0202	0.0205
26	0.0033	0.0178	0.0203	0.0201	0.0206
28	0.0031	0.0176	0.0203	0.0201	0.0203
30	0.0030	0.0175	0.0203	0.0201	0.0204
32	0.0029	0.0173	0.0202	0.0200	0.0201
34	0.0028	0.0172	0.0202	0.0199	0.0209
36	0.0027	0.0171	0.0201	0.0199	0.0198
38	0.0027	0.0169	0.0200	0.0197	0.0202
40	0.0026	0.0168	0.0201	0.0198	0.0210

Table A.21. Static analysis (CS=1, RVM, γ=12, c=2)

c=2	w_{max}	w_{max}	w_{max}	w_{max}	w_{max}
γ=12	r=1	r=3	r=6	r=10	r=15
2k	n=0	n=2	n=4	n=6	n=8
2	0.0452	0.0398	0.0449	0.0452	0.0484
4	0.0309	0.0375	0.0411	0.0403	0.0413
6	0.0224	0.0384	0.0411	0.0404	0.0407
8	0.0175	0.0391	0.0411	0.0405	0.0406
10	0.0143	0.0394	0.0411	0.0405	0.0405
12	0.0121	0.0395	0.0411	0.0404	0.0405
14	0.0105	0.0395	0.0410	0.0404	0.0405
16	0.0093	0.0394	0.0410	0.0403	0.0406
18	0.0083	0.0392	0.0411	0.0403	0.0406
20	0.0075	0.0390	0.0411	0.0403	0.0407
22	0.0069	0.0387	0.0411	0.0402	0.0407
24	0.0064	0.0384	0.0412	0.0402	0.0408
26	0.0059	0.0382	0.0412	0.0402	0.0408
28	0.0055	0.0380	0.0413	0.0401	0.0409
30	0.0052	0.0378	0.0413	0.0401	0.0410
32	0.0049	0.0376	0.0414	0.0400	0.0411
34	0.0046	0.0375	0.0414	0.0400	0.0411
36	0.0044	0.0374	0.0415	0.0399	0.0412
38	0.0041	0.0373	0.0415	0.0399	0.0413
40	0.0040	0.0373	0.0416	0.0398	0.0414

Table A.22. Static analysis (CS=1, GM, γ=1, c=1)

c=1	w_{max}	w_{max}	w_{max}	w_{max}	w_{max}
γ=1	r=1	r=3	r=6	r=10	r=15
2k	n=0	n=2	n=4	n=6	n=8
2	0.0156	0.0156	0.0156	0.0156	0.0156
4	0.0059	0.0138	0.0182	0.0194	0.0197
6	0.0023	0.0100	0.0179	0.0200	0.0202
8	0.0011	0.0070	0.0168	0.0201	0.0203
10	0.0006	0.0048	0.0155	0.0200	0.0202
12	0.0004	0.0034	0.0142	0.0200	0.0202
14	0.0002	0.0024	0.0128	0.0199	0.0203
16	0.0002	0.0018	0.0113	0.0198	0.0204
18	0.0001	0.0013	0.0098	0.0196	0.0204
20	0.0001	0.0010	0.0086	0.0192	0.0204
22	0.0001	0.0008	0.0075	0.0188	0.0201
24	0.0001	0.0006	0.0068	0.0184	0.0197
26	0.0000	0.0005	0.0063	0.0180	0.0192
28	0.0000	0.0004	0.0059	0.0177	0.0186
30	0.0000	0.0003	0.0058	0.0173	0.0178
32	0.0000	0.0002	0.0058	0.0168	0.0166
34	0.0000	0.0002	0.0060	0.0159	0.0145
36	0.0000	0.0002	0.0063	0.0127	0.0082
38	0.0000	0.0001	0.0066	0.0292	0.1111
40	0.0000	0.0001	0.0070	0.0205	0.0288

Table A.23. Static analysis (CS=1, GM, γ=1, c=2)

c=2	w_{max}	w_{max}	w_{max}	w_{max}	w_{max}
γ=1	r=1	r=3	r=6	r=10	r=15
2k	n=0	n=2	n=4	n=6	n=8
2	0.0339	0.0339	0.0339	0.0339	0.0339
4	0.0118	0.0278	0.0368	0.0393	0.0397
6	0.0044	0.0207	0.0368	0.0404	0.0405
8	0.0021	0.0153	0.0356	0.0407	0.0406
10	0.0011	0.0114	0.0340	0.0409	0.0406
12	0.0007	0.0087	0.0320	0.0410	0.0406
14	0.0004	0.0067	0.0299	0.0411	0.0407
16	0.0003	0.0053	0.0276	0.0411	0.0407
18	0.0002	0.0042	0.0254	0.0411	0.0406
20	0.0002	0.0034	0.0234	0.0409	0.0406
22	0.0001	0.0028	0.0217	0.0406	0.0406
24	0.0001	0.0022	0.0202	0.0403	0.0406
26	0.0001	0.0017	0.0189	0.0402	0.0405
28	0.0001	0.0014	0.0176	0.0401	0.0405
30	0.0000	0.0011	0.0164	0.0402	0.0404
32	0.0000	0.0009	0.0151	0.0407	0.0402
34	0.0000	0.0007	0.0139	0.0415	0.0399
36	0.0000	0.0006	0.0128	0.0429	0.0370
38	0.0000	0.0005	0.0117	0.0453	0.0416
40	0.0000	0.0004	0.0108	0.0492	0.0410

Table A.24. Static analysis (CS=1, GM, γ=2, c=1)

c=1	w_{max}	w_{max}	w_{max}	w_{max}	w_{max}
γ=2	r=1	r=3	r=6	r=10	r=15
2k	n=0	n=2	n=4	n=6	n=8
2	0.0097	0.0112	0.0137	0.0142	-0.0107
4	0.0141	0.0132	0.0168	0.0169	0.0139
6	0.0160	0.0151	0.0173	0.0174	0.0540
8	0.0171	0.0164	0.0176	0.0177	0.0246
10	0.0177	0.0172	0.0178	0.0182	0.0220
12	0.0182	0.0177	0.0182	0.0189	0.0207
14	0.0189	0.0182	0.0187	0.0192	0.0200
16	0.0195	0.0188	0.0192	0.0193	0.0197
18	0.0200	0.0192	0.0195	0.0194	0.0195
20	0.0202	0.0195	0.0195	0.0194	0.0195
22	0.0202	0.0195	0.0194	0.0194	0.0195
24	0.0200	0.0193	0.0192	0.0193	0.0195
26	0.0197	0.0192	0.0190	0.0192	0.0194
28	0.0193	0.0190	0.0189	0.0191	0.0193
30	0.0190	0.0189	0.0187	0.0190	0.0191
32	0.0188	0.0188	0.0186	0.0189	0.0190
34	0.0187	0.0187	0.0185	0.0189	0.0189
36	0.0187	0.0187	0.0185	0.0188	0.0188
38	0.0188	0.0187	0.0185	0.0188	0.0187
40	0.0190	0.0188	0.0186	0.0188	0.0187

Table A.25. Static analysis (CS=1, GM, γ=2, c=2)

c=2	w_{max}	w_{max}	w_{max}	w_{max}	w_{max}
γ=2	r=1	r=3	r=6	r=10	r=15
2k	n=0	n=2	n=4	n=6	n=8
2	0.0211	0.0234	0.0289	0.0296	-0.2548
4	0.0303	0.0261	0.0344	0.0339	-0.0244
6	0.0340	0.0298	0.0352	0.0350	0.0440
8	0.0363	0.0325	0.0356	0.0354	0.0388
10	0.0378	0.0342	0.0361	0.0357	0.0384
12	0.0389	0.0352	0.0367	0.0361	0.0383
14	0.0397	0.0359	0.0372	0.0366	0.0383
16	0.0405	0.0364	0.0376	0.0370	0.0383
18	0.0411	0.0369	0.0378	0.0374	0.0383
20	0.0415	0.0373	0.0378	0.0377	0.0383
22	0.0418	0.0377	0.0378	0.0379	0.0384
24	0.0420	0.0381	0.0378	0.0381	0.0385
26	0.0421	0.0384	0.0379	0.0383	0.0385
28	0.0422	0.0386	0.0379	0.0384	0.0386
30	0.0423	0.0387	0.0379	0.0385	0.0386
32	0.0424	0.0389	0.0380	0.0386	0.0386
34	0.0424	0.0389	0.0381	0.0387	0.0387
36	0.0426	0.0390	0.0382	0.0388	0.0387
38	0.0427	0.0390	0.0383	0.0389	0.0388
40	0.0428	0.0390	0.0384	0.0389	0.0389

Table A.26. Static analysis (CS=1, GM, γ=3, c=1)

c=1	w_{max}	w_{max}	w_{max}	w_{max}	w_{max}
γ=3	r=1	r=3	r=6	r=10	r=15
2k	n=0	n=2	n=4	n=6	n=8
2	0.0137	0.0138	0.0152	0.0152	0.0157
4	0.0144	0.0151	0.0189	0.0187	0.0194
6	0.0120	0.0161	0.0195	0.0194	0.0197
8	0.0100	0.0168	0.0196	0.0196	0.0197
10	0.0086	0.0172	0.0196	0.0195	0.0196
12	0.0075	0.0175	0.0197	0.0194	0.0193
14	0.0064	0.0178	0.0198	0.0194	0.0190
16	0.0054	0.0180	0.0199	0.0194	0.0184
18	0.0046	0.0179	0.0199	0.0195	0.0237
20	0.0040	0.0177	0.0199	0.0196	0.0202
22	0.0037	0.0175	0.0198	0.0197	0.0199
24	0.0035	0.0172	0.0196	0.0197	0.0198
26	0.0035	0.0171	0.0193	0.0194	0.0197
28	0.0036	0.0169	0.0186	0.0193	0.0195
30	0.0039	0.0168	0.0127	0.0190	0.0194
32	0.0043	0.0166	0.0209	0.0186	0.0192
34	0.0049	0.0115	0.0198	0.0190	0.0191
36	0.0058	0.0176	0.0194	0.0187	0.0189
38	0.0069	0.0174	0.0190	0.0185	0.0188
40	0.0082	0.0173	0.0187	0.0184	0.0186

Table A.27. Static analysis (CS=1, GM, γ=3, c=2)

c=2	w_{max}	w_{max}	w_{max}	w_{max}	w_{max}
γ=3	r=1	r=3	r=6	r=10	r=15
2k	n=0	n=2	n=4	n=6	n=8
2	0.0297	0.0293	0.0326	0.0326	0.0336
4	0.0300	0.0299	0.0385	0.0373	0.0388
6	0.0242	0.0321	0.0395	0.0387	0.0393
8	0.0199	0.0338	0.0397	0.0391	0.0395
10	0.0168	0.0350	0.0396	0.0392	0.0396
12	0.0145	0.0359	0.0396	0.0392	0.0397
14	0.0127	0.0366	0.0396	0.0392	0.0398
16	0.0112	0.0369	0.0396	0.0392	0.0400
18	0.0100	0.0369	0.0396	0.0392	0.0450
20	0.0090	0.0369	0.0396	0.0392	0.0398
22	0.0083	0.0367	0.0397	0.0393	0.0399
24	0.0077	0.0365	0.0397	0.0395	0.0400
26	0.0072	0.0365	0.0397	0.0368	0.0400
28	0.0068	0.0365	0.0397	0.0386	0.0399
30	0.0065	0.0369	0.0395	0.0387	0.0399
32	0.0062	0.0378	0.0418	0.0388	0.0399
34	0.0059	0.0398	0.0403	0.0388	0.0389
36	0.0057	0.0445	0.0402	0.0387	0.0397
38	0.0055	0.0586	0.0401	0.0387	0.0396
40	0.0053	0.1771	0.0401	0.0387	0.0396

Table A.28. Static analysis (CS=1, GM, γ=4, c=1)

c=1	w_{max}	w_{max}	w_{max}	w_{max}	w_{max}
γ=4	r=1	r=3	r=6	r=10	r=15
2k	n=0	n=2	n=4	n=6	n=8
2	0.0157	0.0154	0.0156	0.0156	0.0156
4	0.0069	0.0140	0.0182	0.0194	0.0197
6	0.0029	0.0109	0.0181	0.0200	0.0202
8	0.0015	0.0084	0.0174	0.0200	0.0202
10	0.0008	0.0065	0.0166	0.0199	0.0202
12	0.0005	0.0050	0.0158	0.0198	0.0201
14	0.0004	0.0039	0.0150	0.0198	0.0202
16	0.0003	0.0030	0.0140	0.0197	0.0203
18	0.0002	0.0024	0.0130	0.0195	0.0203
20	0.0001	0.0020	0.0120	0.0192	0.0202
22	0.0001	0.0016	0.0111	0.0189	0.0200
24	0.0001	0.0014	0.0103	0.0185	0.0195
26	0.0001	0.0011	0.0097	0.0181	0.0190
28	0.0000	0.0009	0.0093	0.0178	0.0183
30	0.0000	0.0008	0.0090	0.0173	0.0175
32	0.0000	0.0006	0.0088	0.0168	0.0164
34	0.0000	0.0005	0.0089	0.0159	0.0144
36	0.0000	0.0004	0.0091	0.0128	0.0091
38	0.0000	0.0004	0.0094	0.0340	-1.4908
40	0.0000	0.0003	0.0097	0.0209	0.0293

Table A.29. Static analysis (CS=1, GM, γ=4, c=2)

c=2	w_{max}	w_{max}	w_{max}	w_{max}	w_{max}
γ=4	r=1	r=3	r=6	r=10	r=15
2k	n=0	n=2	n=4	n=6	n=8
2	0.0341	0.0336	0.0339	0.0339	0.0339
4	0.0138	0.0282	0.0370	0.0392	0.0397
6	0.0057	0.0224	0.0370	0.0402	0.0405
8	0.0028	0.0178	0.0362	0.0404	0.0405
10	0.0016	0.0142	0.0352	0.0404	0.0405
12	0.0010	0.0113	0.0340	0.0404	0.0405
14	0.0007	0.0092	0.0327	0.0404	0.0405
16	0.0005	0.0075	0.0313	0.0404	0.0405
18	0.0003	0.0062	0.0299	0.0403	0.0404
20	0.0002	0.0052	0.0285	0.0403	0.0404
22	0.0002	0.0043	0.0273	0.0401	0.0403
24	0.0001	0.0036	0.0262	0.0400	0.0402
26	0.0001	0.0030	0.0253	0.0398	0.0401
28	0.0001	0.0025	0.0244	0.0397	0.0400
30	0.0001	0.0020	0.0234	0.0397	0.0399
32	0.0001	0.0017	0.0224	0.0399	0.0396
34	0.0001	0.0014	0.0212	0.0404	0.0390
36	0.0000	0.0012	0.0200	0.0414	0.0289
38	0.0000	0.0010	0.0188	0.0435	0.0415
40	0.0000	0.0009	0.0176	0.0483	0.0407

Table A.30. Static analysis (CS=1, GM, γ=5, c=1)

c=1	w_{max}	w_{max}	w_{max}	w_{max}	w_{max}
γ=5	r=1	r=3	r=6	r=10	r=15
2k	n=0	n=2	n=4	n=6	n=8
2	0.0156	0.0152	0.0156	0.0156	0.0156
4	0.0076	0.0140	0.0183	0.0194	0.0197
6	0.0034	0.0114	0.0183	0.0199	0.0202
8	0.0018	0.0093	0.0177	0.0199	0.0202
10	0.0010	0.0077	0.0171	0.0198	0.0201
12	0.0006	0.0064	0.0166	0.0197	0.0201
14	0.0004	0.0052	0.0161	0.0196	0.0201
16	0.0003	0.0043	0.0155	0.0196	0.0202
18	0.0002	0.0036	0.0149	0.0194	0.0202
20	0.0002	0.0030	0.0142	0.0191	0.0201
22	0.0001	0.0026	0.0136	0.0187	0.0199
24	0.0001	0.0023	0.0130	0.0183	0.0194
26	0.0001	0.0020	0.0126	0.0179	0.0189
28	0.0001	0.0017	0.0122	0.0175	0.0182
30	0.0000	0.0015	0.0120	0.0170	0.0174
32	0.0000	0.0013	0.0119	0.0163	0.0162
34	0.0000	0.0011	0.0120	0.0152	0.0143
36	0.0000	0.0009	0.0122	0.0114	0.0092
38	0.0000	0.0008	0.0125	0.0359	-0.2252
40	0.0000	0.0007	0.0130	0.0211	0.0295

Table A.31. Static analysis (CS=1, GM, γ=5, c=2)

c=2	w_{max}	w_{max}	w_{max}	w_{max}	w_{max}
γ=5	r=1	r=3	r=6	r=10	r=15
2k	n=0	n=2	n=4	n=6	n=8
2	0.0339	0.0331	0.0340	0.0339	0.0339
4	0.0154	0.0283	0.0372	0.0392	0.0397
6	0.0067	0.0233	0.0373	0.0400	0.0404
8	0.0034	0.0193	0.0365	0.0401	0.0405
10	0.0019	0.0161	0.0356	0.0400	0.0404
12	0.0012	0.0135	0.0347	0.0399	0.0404
14	0.0008	0.0113	0.0339	0.0398	0.0404
16	0.0006	0.0096	0.0331	0.0397	0.0404
18	0.0004	0.0082	0.0322	0.0396	0.0403
20	0.0003	0.0071	0.0313	0.0395	0.0403
22	0.0002	0.0061	0.0305	0.0394	0.0402
24	0.0002	0.0053	0.0298	0.0392	0.0401
26	0.0001	0.0045	0.0292	0.0391	0.0400
28	0.0001	0.0039	0.0286	0.0390	0.0399
30	0.0001	0.0033	0.0281	0.0390	0.0397
32	0.0001	0.0028	0.0276	0.0391	0.0393
34	0.0001	0.0024	0.0269	0.0394	0.0384
36	0.0001	0.0021	0.0261	0.0401	-2.0308
38	0.0000	0.0018	0.0252	0.0420	0.0418
40	0.0000	0.0016	0.0242	0.0491	0.0409

Table A.32. Static analysis (CS=1, GM, γ=8, c=1)

c=1	w_{max}	w_{max}	w_{max}	w_{max}	w_{max}
γ=8	r=1	r=3	r=6	r=10	r=15
2k	n=0	n=2	n=4	n=6	n=8
2	0.0114	0.0124	0.0145	0.0147	0.0153
4	0.0144	0.0141	0.0178	0.0177	0.0187
6	0.0138	0.0157	0.0184	0.0185	0.0190
8	0.0129	0.0168	0.0187	0.0187	0.0191
10	0.0120	0.0175	0.0188	0.0187	0.0190
12	0.0110	0.0181	0.0190	0.0187	0.0188
14	0.0097	0.0187	0.0192	0.0187	0.0185
16	0.0082	0.0192	0.0193	0.0188	0.0180
18	0.0070	0.0195	0.0194	0.0190	0.0231
20	0.0061	0.0197	0.0195	0.0192	0.0197
22	0.0057	0.0198	0.0194	0.0193	0.0195
24	0.0056	0.0198	0.0192	0.0194	0.0194
26	0.0058	0.0196	0.0189	0.0192	0.0194
28	0.0065	0.0195	0.0181	0.0191	0.0192
30	0.0079	0.0192	0.0107	0.0190	0.0191
32	0.0106	0.0189	0.0208	0.0187	0.0189
34	0.0173	0.0243	0.0197	0.0186	0.0188
36	0.0482	0.0192	0.0192	0.0185	0.0187
38	-0.0683	0.0188	0.0189	0.0184	0.0186
40	-0.0216	0.0186	0.0186	0.0182	0.0184

Table A.33. Static analysis (CS=1, GM, γ=8, c=2)

c=2	w_{max}	w_{max}	w_{max}	w_{max}	w_{max}
γ=8	r=1	r=3	r=6	r=10	r=15
2k	n=0	n=2	n=4	n=6	n=8
2	0.0213	0.0239	0.0289	0.0299	0.0317
4	0.0297	0.0265	0.0343	0.0340	0.0361
6	0.0323	0.0305	0.0354	0.0355	0.0369
8	0.0336	0.0335	0.0360	0.0361	0.0373
10	0.0343	0.0355	0.0365	0.0363	0.0376
12	0.0345	0.0371	0.0369	0.0365	0.0378
14	0.0342	0.0383	0.0372	0.0367	0.0380
16	0.0333	0.0393	0.0374	0.0370	0.0383
18	0.0322	0.0400	0.0375	0.0373	0.0304
20	0.0312	0.0405	0.0376	0.0376	0.0381
22	0.0306	0.0408	0.0377	0.0379	0.0384
24	0.0305	0.0410	0.0377	0.0385	0.0385
26	0.0311	0.0411	0.0376	0.0363	0.0386
28	0.0324	0.0412	0.0373	0.0376	0.0386
30	0.0343	0.0410	0.0351	0.0379	0.0389
32	0.0370	0.0404	0.0409	0.0380	0.0387
34	0.0405	0.0375	0.0394	0.0381	0.0388
36	0.0449	0.0494	0.0391	0.0382	0.0388
38	0.0500	0.0445	0.0389	0.0382	0.0388
40	0.0557	0.0437	0.0389	0.0382	0.0388

Table A.34. Static analysis (CS=1, GM, γ=9, c=1)

c=1	w_{max}	w_{max}	w_{max}	w_{max}	w_{max}
γ=9	r=1	r=3	r=6	r=10	r=15
2k	n=0	n=2	n=4	n=6	n=8
2	0.0114	0.0124	0.0145	0.0147	-0.0077
4	0.0144	0.0141	0.0178	0.0177	0.0164
6	0.0138	0.0157	0.0183	0.0182	-0.0335
8	0.0128	0.0168	0.0185	0.0185	0.0257
10	0.0117	0.0175	0.0186	0.0190	0.0226
12	0.0108	0.0180	0.0189	0.0196	0.0212
14	0.0100	0.0185	0.0193	0.0199	0.0205
16	0.0093	0.0190	0.0198	0.0199	0.0201
18	0.0087	0.0195	0.0200	0.0198	0.0200
20	0.0081	0.0198	0.0200	0.0198	0.0200
22	0.0075	0.0198	0.0198	0.0197	0.0200
24	0.0070	0.0197	0.0196	0.0196	0.0199
26	0.0066	0.0195	0.0194	0.0194	0.0198
28	0.0062	0.0193	0.0192	0.0193	0.0196
30	0.0058	0.0191	0.0190	0.0192	0.0195
32	0.0055	0.0190	0.0189	0.0190	0.0193
34	0.0052	0.0190	0.0188	0.0189	0.0192
36	0.0050	0.0190	0.0188	0.0188	0.0191
38	0.0048	0.0190	0.0188	0.0188	0.0190
40	0.0046	0.0191	0.0188	0.0188	0.0190

Table A.35. Static analysis (CS=1, GM, γ=9, c=2)

c=2	w_{max}	w_{max}	w_{max}	w_{max}	w_{max}
γ=9	r=1	r=3	r=6	r=10	r=15
2k	n=0	n=2	n=4	n=6	n=8
2	0.0294	0.0288	0.0326	0.0323	-0.1778
4	0.0309	0.0295	0.0385	0.0372	0.0126
6	0.0257	0.0318	0.0393	0.0381	0.0468
8	0.0214	0.0337	0.0393	0.0384	0.0409
10	0.0182	0.0351	0.0392	0.0385	0.0403
12	0.0158	0.0362	0.0393	0.0388	0.0402
14	0.0139	0.0371	0.0394	0.0390	0.0401
16	0.0124	0.0379	0.0394	0.0392	0.0400
18	0.0112	0.0385	0.0393	0.0393	0.0400
20	0.0102	0.0391	0.0392	0.0393	0.0400
22	0.0094	0.0395	0.0391	0.0393	0.0400
24	0.0086	0.0399	0.0390	0.0393	0.0400
26	0.0080	0.0402	0.0390	0.0393	0.0400
28	0.0075	0.0404	0.0389	0.0392	0.0399
30	0.0070	0.0405	0.0389	0.0392	0.0399
32	0.0066	0.0407	0.0390	0.0392	0.0398
34	0.0062	0.0409	0.0390	0.0392	0.0398
36	0.0059	0.0410	0.0390	0.0392	0.0398
38	0.0056	0.0412	0.0391	0.0392	0.0398
40	0.0053	0.0414	0.0391	0.0392	0.0398

Table A.36. Static analysis (CS=1, MOM, γ=1, c=1)

c=1	w_{max}	w_{max}	w_{max}	w_{max}	w_{max}
γ=1	r=1	r=3	r=6	r=10	r=15
2k	n=0	n=2	n=4	n=6	n=8
2	0.0156	0.0156	0.0156	0.0156	0.0156
4	0.0019	0.0096	0.0175	0.0193	0.0197
6	0.0005	0.0047	0.0156	0.0198	0.0202
8	0.0002	0.0023	0.0130	0.0198	0.0203
10	0.0001	0.0012	0.0103	0.0195	0.0204
12	0.0001	0.0007	0.0081	0.0192	0.0204
14	0.0000	0.0004	0.0064	0.0189	0.0205
16	0.0000	0.0003	0.0052	0.0185	0.0206
18	0.0000	0.0002	0.0043	0.0181	0.0206
20	0.0000	0.0001	0.0037	0.0177	0.0205
22	0.0000	0.0001	0.0033	0.0173	0.0203
24	0.0000	0.0001	0.0031	0.0169	0.0199
26	0.0000	0.0001	0.0030	0.0166	0.0194
28	0.0000	0.0000	0.0029	0.0164	0.0188
30	0.0000	0.0000	0.0029	0.0162	0.0180
32	0.0000	0.0000	0.0029	0.0160	0.0168
34	0.0000	0.0000	0.0029	0.0159	0.0145
36	0.0000	0.0000	0.0029	0.0162	0.0069
38	0.0000	0.0000	0.0029	0.0099	0.0615
40	0.0000	0.0000	0.0029	0.0141	0.0277

Table A.37. Static analysis (CS=1, MOM, γ=1, c=2)

c=2	w_{max}	w_{max}	w_{max}	w_{max}	w_{max}
γ=1	r=1	r=3	r=6	r=10	r=15
2k	n=0	n=2	n=4	n=6	n=8
2	0.0339	0.0339	0.0339	0.0339	0.0339
4	0.0038	0.0204	0.0365	0.0390	0.0397
6	0.0010	0.0113	0.0342	0.0403	0.0406
8	0.0004	0.0064	0.0303	0.0408	0.0409
10	0.0002	0.0038	0.0260	0.0409	0.0411
12	0.0001	0.0024	0.0219	0.0407	0.0412
14	0.0001	0.0016	0.0184	0.0404	0.0412
16	0.0000	0.0011	0.0155	0.0400	0.0413
18	0.0000	0.0008	0.0131	0.0395	0.0413
20	0.0000	0.0006	0.0112	0.0391	0.0413
22	0.0000	0.0004	0.0098	0.0387	0.0413
24	0.0000	0.0003	0.0086	0.0385	0.0413
26	0.0000	0.0003	0.0076	0.0384	0.0413
28	0.0000	0.0002	0.0068	0.0386	0.0413
30	0.0000	0.0002	0.0062	0.0391	0.0413
32	0.0000	0.0001	0.0056	0.0399	0.0412
34	0.0000	0.0001	0.0052	0.0413	0.0412
36	0.0000	0.0001	0.0048	0.0432	0.0408
38	0.0000	0.0001	0.0044	0.0457	0.0418
40	0.0000	0.0001	0.0041	0.0489	0.0415

Table A.38. Static analysis (CS=1, MOM, γ=4, c=1)

c=1	w_{max}	w_{max}	w_{max}	w_{max}	w_{max}
γ=4	r=1	r=3	r=6	r=10	r=15
2k	n=0	n=2	n=4	n=6	n=8
2	0.0312	0.0128	0.0159	0.0156	0.0156
4	0.0031	0.0080	0.0176	0.0191	0.0197
6	0.0008	0.0049	0.0158	0.0196	0.0202
8	0.0003	0.0030	0.0138	0.0197	0.0203
10	0.0002	0.0018	0.0118	0.0195	0.0203
12	0.0001	0.0012	0.0100	0.0194	0.0203
14	0.0001	0.0008	0.0084	0.0192	0.0204
16	0.0000	0.0005	0.0071	0.0189	0.0205
18	0.0000	0.0004	0.0060	0.0187	0.0205
20	0.0000	0.0003	0.0052	0.0184	0.0204
22	0.0000	0.0002	0.0047	0.0181	0.0201
24	0.0000	0.0002	0.0042	0.0179	0.0197
26	0.0000	0.0001	0.0040	0.0176	0.0192
28	0.0000	0.0001	0.0038	0.0174	0.0186
30	0.0000	0.0001	0.0037	0.0172	0.0178
32	0.0000	0.0001	0.0036	0.0171	0.0167
34	0.0000	0.0001	0.0036	0.0171	0.0146
36	0.0000	0.0000	0.0035	0.0177	0.0078
38	0.0000	0.0000	0.0035	0.0011	0.0656
40	0.0000	0.0000	0.0035	0.0145	0.0271

Table A.39. Static analysis (CS=1, MOM, γ=4, c=2)

c=2	w_{max}	w_{max}	w_{max}	w_{max}	w_{max}
γ=4	r=1	r=3	r=6	r=10	r=15
2k	n=0	n=2	n=4	n=6	n=8
2	0.0608	0.0255	0.0346	0.0338	0.0339
4	0.0058	0.0160	0.0371	0.0387	0.0397
6	0.0015	0.0107	0.0343	0.0398	0.0406
8	0.0006	0.0070	0.0310	0.0402	0.0409
10	0.0003	0.0046	0.0277	0.0403	0.0409
12	0.0002	0.0031	0.0246	0.0403	0.0410
14	0.0001	0.0021	0.0217	0.0402	0.0410
16	0.0001	0.0015	0.0191	0.0399	0.0410
18	0.0000	0.0011	0.0167	0.0397	0.0410
20	0.0000	0.0008	0.0147	0.0394	0.0410
22	0.0000	0.0006	0.0130	0.0391	0.0409
24	0.0000	0.0005	0.0115	0.0389	0.0409
26	0.0000	0.0004	0.0103	0.0389	0.0408
28	0.0000	0.0003	0.0093	0.0390	0.0408
30	0.0000	0.0003	0.0084	0.0393	0.0407
32	0.0000	0.0002	0.0076	0.0400	0.0406
34	0.0000	0.0002	0.0070	0.0411	0.0405
36	0.0000	0.0002	0.0064	0.0427	0.0395
38	0.0000	0.0001	0.0059	0.0451	0.0413
40	0.0000	0.0001	0.0055	0.0482	0.0409

Table A.40. Static analysis (CS=1, MOM, γ=5, c=1)

c=1	w_{max}	w_{max}	w_{max}	w_{max}	w_{max}
γ=5	r=1	r=3	r=6	r=10	r=15
2k	n=0	n=2	n=4	n=6	n=8
2	0.0952	0.0096	0.0165	0.0155	0.0156
4	0.0044	0.0069	0.0192	0.0188	0.0197
6	0.0010	0.0053	0.0167	0.0193	0.0202
8	0.0004	0.0038	0.0148	0.0195	0.0203
10	0.0002	0.0026	0.0134	0.0196	0.0203
12	0.0001	0.0018	0.0122	0.0195	0.0203
14	0.0001	0.0012	0.0109	0.0194	0.0204
16	0.0000	0.0009	0.0097	0.0193	0.0205
18	0.0000	0.0006	0.0085	0.0191	0.0205
20	0.0000	0.0005	0.0075	0.0188	0.0203
22	0.0000	0.0003	0.0067	0.0186	0.0201
24	0.0000	0.0003	0.0060	0.0183	0.0196
26	0.0000	0.0002	0.0055	0.0181	0.0191
28	0.0000	0.0002	0.0051	0.0178	0.0184
30	0.0000	0.0001	0.0048	0.0177	0.0176
32	0.0000	0.0001	0.0047	0.0176	0.0165
34	0.0000	0.0001	0.0045	0.0176	0.0144
36	0.0000	0.0001	0.0044	0.0183	0.0078
38	0.0000	0.0001	0.0044	-0.0002	0.0727
40	0.0000	0.0001	0.0043	0.0150	0.0271

Table A.41. Static analysis (CS=1, MOM, γ=5, c=2)

c=2	w_{max}	w_{max}	w_{max}	w_{max}	w_{max}
γ=5	r=1	r=3	r=6	r=10	r=15
2k	n=0	n=2	n=4	n=6	n=8
2	0.1318	0.0111	0.0362	0.0336	0.0339
4	0.0080	0.0097	0.0406	0.0370	0.0399
6	0.0019	0.0097	0.0360	0.0379	0.0409
8	0.0007	0.0076	0.0321	0.0387	0.0411
10	0.0004	0.0055	0.0292	0.0393	0.0411
12	0.0002	0.0040	0.0269	0.0396	0.0410
14	0.0001	0.0028	0.0247	0.0398	0.0409
16	0.0001	0.0021	0.0225	0.0398	0.0408
18	0.0001	0.0015	0.0205	0.0396	0.0408
20	0.0000	0.0012	0.0185	0.0394	0.0408
22	0.0000	0.0009	0.0167	0.0392	0.0407
24	0.0000	0.0007	0.0151	0.0390	0.0407
26	0.0000	0.0006	0.0137	0.0389	0.0406
28	0.0000	0.0005	0.0124	0.0389	0.0405
30	0.0000	0.0004	0.0112	0.0391	0.0404
32	0.0000	0.0003	0.0102	0.0396	0.0403
34	0.0000	0.0003	0.0094	0.0404	0.0400
36	0.0000	0.0002	0.0086	0.0418	0.0360
38	0.0000	0.0002	0.0079	0.0438	0.0410
40	0.0000	0.0002	0.0073	0.0470	0.0406

Table A.42. Static analysis (CS=1, MOLS, γ=1, c=1)

c=1	w_{max}	w_{max}	w_{max}	w_{max}	w_{max}
γ=1	r=1	r=3	r=6	r=10	r=15
2k	n=0	n=2	n=4	n=6	n=8
2	0.0156	0.0156	0.0156	0.0156	0.0156
4	0.0009	0.0030	0.0061	0.0107	0.0160
6	0.0002	0.0007	0.0028	0.0083	0.0151
8	0.0000	0.0002	0.0009	0.0036	0.0092
10	0.0000	0.0001	0.0003	0.0015	0.0044
12	0.0000	0.0000	0.0002	0.0006	0.0025
14	0.0000	0.0000	0.0001	0.0003	0.0015
16	0.0000	0.0000	0.0001	0.0002	0.0009
18	0.0000	0.0000	0.0001	0.0001	0.0006
20	0.0000	0.0000	0.0000	0.0001	0.0003
22	0.0000	0.0000	0.0000	0.0001	0.0002
24	0.0000	0.0000	0.0000	0.0001	0.0001
26	0.0000	0.0000	0.0000	0.0001	0.0000
28	0.0000	0.0000	0.0000	0.0002	-0.0000
30	0.0000	0.0000	0.0000	0.0002	0.0000
32	0.0000	0.0000	0.0000	0.0002	0.0002
34	0.0000	0.0000	0.0000	0.0002	0.0004
36	0.0000	0.0000	0.0000	0.0002	0.0007
38	0.0000	0.0000	0.0000	0.0001	-0.0012
40	0.0000	0.0000	0.0000	0.0001	-0.0015

Table A.43. Static analysis (CS=1, MOLS, γ=1, c=1)

c=1	w_{max}	w_{max}	w_{max}	w_{max}	w_{max}
γ=1	r=21	r=28	r=36	r=45	r=55
2k	n=10	n=12	n=14	n=16	n=18
2	0.0156	0.0156	0.0156	0.0156	0.0156
4	0.0192	0.0197	0.0198	0.0197	0.0196
6	0.0183	0.0199	0.0202	0.0245	0.0196
8	0.0163	0.0197	0.0197	0.0201	0.0185
10	0.0121	0.0181	0.0196	0.0183	0.0147
12	0.0081	0.0158	0.0187	0.0080	0.0116
14	0.0065	0.0129	0.0172	0.0133	-0.0038
16	0.0061	0.0098	0.0148	0.0157	0.0196
18	0.0057	0.0073	0.0118	-0.0091	0.0102
20	0.0051	0.0056	0.0069	0.0088	-0.0013
22	0.0043	0.0044	0.0042	0.0083	0.0015
24	0.0036	0.0036	0.0031	0.0060	0.0003
26	0.0031	0.0030	0.0024	-0.0001	-0.0025
28	0.0027	0.0025	0.0018	-0.0007	0.0006
30	0.0024	0.0022	0.0013	-0.0013	-0.0022
32	0.0022	0.0020	0.0009	-0.0021	-0.0036
34	0.0021	0.0019	0.0005	-0.0029	-0.0057
36	0.0013	0.0015	-0.0011	-0.0051	-0.0061
38	-0.0025	-0.0024	-0.0021	-0.0033	-0.0044
40	-0.0011	-0.0011	-0.0011	-0.0028	-0.0039

Table A.44. Static analysis (CS=1, MOLS, γ=1, c=2)

c=2	w_{max}	w_{max}	w_{max}	w_{max}	w_{max}
γ=1	r=1	r=3	r=6	r=10	r=15
2k	n=0	n=2	n=4	n=6	n=8
2	0.0339	0.0339	0.0339	0.0339	0.0339
4	0.0012	0.0037	0.0073	0.0141	0.0259
6	0.0002	0.0012	0.0043	0.0154	0.0278
8	0.0001	0.0005	0.0022	0.0098	0.0167
10	0.0000	0.0002	0.0010	0.0048	0.0099
12	0.0000	0.0001	0.0005	0.0024	0.0051
14	0.0000	0.0001	0.0002	0.0013	0.0021
16	0.0000	0.0000	0.0001	0.0009	0.0007
18	0.0000	0.0000	0.0001	0.0007	0.0003
20	0.0000	0.0000	0.0000	0.0006	0.0003
22	0.0000	0.0000	0.0000	0.0005	0.0005
24	0.0000	0.0000	0.0000	0.0004	0.0007
26	0.0000	0.0000	0.0000	0.0003	0.0010
28	0.0000	0.0000	0.0000	0.0003	0.0012
30	0.0000	0.0000	0.0000	0.0002	0.0013
32	0.0000	0.0000	0.0000	0.0002	0.0012
34	0.0000	0.0000	0.0000	0.0002	0.0008
36	0.0000	0.0000	0.0000	0.0001	0.0001
38	0.0000	0.0000	0.0000	0.0001	-0.0008
40	0.0000	0.0000	0.0000	0.0001	-0.0015

Table A.45. Static analysis (CS=1, MOLS, γ=1, c=2)

c=2	w_{max}	w_{max}	w_{max}	w_{max}	w_{max}
γ=1	r=21	r=28	r=36	r=45	r=55
2k	n=10	n=12	n=14	n=16	n=18
2	0.0339	0.0339	0.0339	0.0339	0.0339
4	0.0360	0.0392	0.0433	0.0428	0.0401
6	0.0312	0.0382	0.0392	0.0420	0.0969
8	0.0311	0.0370	0.0397	0.0423	0.0396
10	0.0255	0.0308	0.0393	0.0390	0.0442
12	0.0161	0.0272	0.0352	0.0470	0.0395
14	0.0093	0.0227	0.0300	0.0424	0.0633
16	0.0052	0.0147	0.0256	0.0299	0.0406
18	0.0029	0.0076	0.0232	0.0208	0.0399
20	0.0016	0.0035	0.0204	0.0220	0.0328
22	0.0008	0.0012	0.0140	0.0137	0.0290
24	0.0002	-0.0001	0.0066	0.0208	0.0214
26	-0.0002	-0.0008	0.0018	0.0127	0.0157
28	-0.0006	-0.0013	-0.0005	0.0101	0.0152
30	-0.0009	-0.0017	-0.0016	0.0086	0.0102
32	-0.0010	-0.0020	-0.0020	0.0018	0.0092
34	-0.0009	-0.0023	-0.0022	0.0004	0.0074
36	-0.0003	-0.0025	-0.0023	-0.0012	0.0056
38	0.0001	-0.0026	-0.0024	-0.0019	0.0038
40	-0.0004	-0.0024	-0.0026	-0.0024	-0.0076

Table A.46. Static analysis (CS=2, RVM, γ=2, c=1)

c=1	w_{max}	w_{max}	w_{max}	w_{max}	w_{max}
γ=2	r=1	r=3	r=6	r=10	r=15
2k	n=0	n=2	n=4	n=6	n=8
2	4.8626e-7	5.4326e-7	6.5721e-7	6.7256e-7	7.0744e-7
4	6.7751e-7	6.2821e-7	7.8624e-7	7.8301e-7	8.4485e-7
6	7.5524e-7	7.0868e-7	8.0544e-7	8.0351e-7	8.8182e-7
8	8.0216e-7	7.6359e-7	8.1295e-7	8.1102e-7	9.0004e-7
10	8.2820e-7	7.9655e-7	8.1957e-7	8.1610e-7	9.1248e-7
12	8.4362e-7	8.1541e-7	8.2643e-7	8.2247e-7	9.2029e-7
14	8.5761e-7	8.2830e-7	8.3518e-7	8.3291e-7	9.2407e-7
16	8.7335e-7	8.3963e-7	8.4679e-7	8.4758e-7	9.2497e-7
18	8.9035e-7	8.5105e-7	8.6096e-7	8.6396e-7	9.2418e-7
20	9.0711e-7	8.6270e-7	8.7613e-7	8.7897e-7	9.2259e-7
22	9.2228e-7	8.7403e-7	8.9011e-7	8.9084e-7	9.2071e-7
24	9.3501e-7	8.8427e-7	9.0108e-7	8.9923e-7	9.1877e-7
26	9.4496e-7	8.9278e-7	9.0825e-7	9.0461e-7	9.1687e-7
28	9.5210e-7	8.9921e-7	9.1179e-7	9.0767e-7	9.1498e-7
30	9.5667e-7	9.0354e-7	9.1244e-7	9.0906e-7	9.1310e-7
32	9.5900e-7	9.0597e-7	9.1104e-7	9.0926e-7	9.1119e-7
34	9.5947e-7	9.0683e-7	9.0833e-7	9.0866e-7	9.0923e-7
36	9.5844e-7	9.0648e-7	9.0488e-7	9.0751e-7	9.0724e-7
38	9.5625e-7	9.0524e-7	9.0108e-7	9.0602e-7	9.0521e-7
40	9.5321e-7	9.0341e-7	8.9718e-7	9.0430e-7	9.0318e-7

Table A.47. Static analysis (CS=2, RVM, γ=2, c=2)

c=2	w_{max}	w_{max}	w_{max}	w_{max}	w_{max}
γ=2	r=1	r=3	r=6	r=10	r=15
2k	n=0	n=2	n=4	n=6	n=8
2	9.6673e-7	9.9342e-7	1.1990e-6	1.2013e-6	1.2753e-6
4	1.2762e-6	1.0614e-6	1.3603e-6	1.3170e-6	1.4173e-6
6	1.3938e-6	1.1813e-6	1.3767e-6	1.3527e-6	1.4537e-6
8	1.4650e-6	1.2648e-6	1.3795e-6	1.3611e-6	1.4638e-6
10	1.5115e-6	1.3194e-6	1.3872e-6	1.3616e-6	1.4623e-6
12	1.5438e-6	1.3527e-6	1.3990e-6	1.3613e-6	1.4587e-6
14	1.5684e-6	1.3720e-6	1.4124e-6	1.3642e-6	1.4554e-6
16	1.5883e-6	1.3827e-6	1.4256e-6	1.3715e-6	1.4528e-6
18	1.6049e-6	1.3888e-6	1.4369e-6	1.3822e-6	1.4508e-6
20	1.6189e-6	1.3927e-6	1.4454e-6	1.3946e-6	1.4495e-6
22	1.6307e-6	1.3959e-6	1.4510e-6	1.4068e-6	1.4487e-6
24	1.6406e-6	1.3993e-6	1.4540e-6	1.4180e-6	1.4483e-6
26	1.6487e-6	1.4030e-6	1.4551e-6	1.4278e-6	1.4484e-6
28	1.6555e-6	1.4072e-6	1.4549e-6	1.4363e-6	1.4486e-6
30	1.6611e-6	1.4118e-6	1.4540e-6	1.4434e-6	1.4491e-6
32	1.6657e-6	1.4165e-6	1.4528e-6	1.4495e-6	1.4498e-6
34	1.6694e-6	1.4213e-6	1.4515e-6	1.4547e-6	1.4505e-6
36	1.6725e-6	1.4260e-6	1.4502e-6	1.4591e-6	1.4512e-6
38	1.6750e-6	1.4306e-6	1.4490e-6	1.4628e-6	1.4520e-6
40	1.6771e-6	1.4350e-6	1.4479e-6	1.4660e-6	1.4528e-6

Table A.48. Static analysis (CS=2, RVM, γ=3, c=1)

c=1	w_{max}	w_{max}	w_{max}	w_{max}	w_{max}
γ=3	r=1	r=3	r=6	r=10	r=15
2k	n=0	n=2	n=4	n=6	n=8
2	6.6359e-7	6.5532e-7	7.1860e-7	7.1430e-7	7.3764e-7
4	6.8339e-7	7.0366e-7	8.6111e-7	8.4821e-7	8.8882e-7
6	5.7855e-7	7.4787e-7	8.8868e-7	8.8230e-7	9.0064e-7
8	4.9445e-7	7.7760e-7	8.9715e-7	8.9267e-7	9.0151e-7
10	4.3644e-7	7.9353e-7	9.0031e-7	8.9409e-7	9.0116e-7
12	3.9440e-7	8.0533e-7	9.0129e-7	8.9328e-7	8.9971e-7
14	3.5600e-7	8.1856e-7	9.0197e-7	8.9256e-7	8.9811e-7
16	3.1592e-7	8.3065e-7	9.0301e-7	8.9297e-7	8.9788e-7
18	2.7563e-7	8.3762e-7	9.0443e-7	8.9491e-7	9.0029e-7
20	2.3860e-7	8.3834e-7	9.0614e-7	8.9841e-7	9.0587e-7
22	2.0699e-7	8.3387e-7	9.0803e-7	9.0320e-7	9.1427e-7
24	1.8118e-7	8.2585e-7	9.0997e-7	9.0884e-7	9.2457e-7
26	1.6057e-7	8.1573e-7	9.1186e-7	9.1478e-7	9.3555e-7
28	1.4425e-7	8.0459e-7	9.1355e-7	9.2051e-7	9.4607e-7
30	1.3133e-7	7.9314e-7	9.1495e-7	9.2561e-7	9.5527e-7
32	1.2107e-7	7.8185e-7	9.1597e-7	9.2980e-7	9.6261e-7
34	1.1289e-7	7.7103e-7	9.1657e-7	9.3290e-7	9.6787e-7
36	1.0634e-7	7.6086e-7	9.1672e-7	9.3486e-7	9.7102e-7
38	1.0110e-7	7.5144e-7	9.1645e-7	9.3573e-7	9.7218e-7
40	9.6922e-8	7.4284e-7	9.1579e-7	9.3557e-7	9.7153e-7

Table A.49. Static analysis (CS=2, RVM, γ=3, c=2)

c=2	w_{max}	w_{max}	w_{max}	w_{max}	w_{max}
γ=2	r=1	r=3	r=6	r=10	r=15
2k	n=0	n=2	n=4	n=6	n=8
2	1.2720e-6	1.1834e-6	1.3070e-6	1.2970e-6	1.3326e-6
4	1.2428e-6	1.1803e-6	1.4613e-6	1.4032e-6	1.4626e-6
6	1.0482e-6	1.2594e-6	1.4931e-6	1.4315e-6	1.4689e-6
8	8.9460e-7	1.3205e-6	1.5009e-6	1.4470e-6	1.4693e-6
10	7.7810e-7	1.3624e-6	1.5007e-6	1.4540e-6	1.4692e-6
12	6.8780e-7	1.3953e-6	1.4957e-6	1.4575e-6	1.4682e-6
14	6.1488e-7	1.4237e-6	1.4896e-6	1.4601e-6	1.4664e-6
16	5.5395e-7	1.4467e-6	1.4848e-6	1.4625e-6	1.4644e-6
18	5.0211e-7	1.4623e-6	1.4820e-6	1.4650e-6	1.4626e-6
20	4.5768e-7	1.4705e-6	1.4812e-6	1.4673e-6	1.4613e-6
22	4.1953e-7	1.4727e-6	1.4820e-6	1.4694e-6	1.4608e-6
24	3.8675e-7	1.4705e-6	1.4840e-6	1.4712e-6	1.4609e-6
26	3.5851e-7	1.4652e-6	1.4866e-6	1.4727e-6	1.4615e-6
28	3.3411e-7	1.4579e-6	1.4898e-6	1.4738e-6	1.4624e-6
30	3.1293e-7	1.4495e-6	1.4932e-6	1.4747e-6	1.4634e-6
32	2.9446e-7	1.4404e-6	1.4967e-6	1.4753e-6	1.4644e-6
34	2.7826e-7	1.4310e-6	1.5002e-6	1.4756e-6	1.4654e-6
36	2.6397e-7	1.4215e-6	1.5036e-6	1.4758e-6	1.4664e-6
38	2.5130e-7	1.4123e-6	1.5069e-6	1.4758e-6	1.4672e-6
40	2.3999e-7	1.4033e-6	1.5101e-6	1.4756e-6	1.4680e-6

Table A.50. Static analysis (CS=2, RVM, γ=4, c=1)

c=1	w_{max}	w_{max}	w_{max}	w_{max}	w_{max}
γ=4	r=1	r=3	r=6	r=10	r=15
2k	n=0	n=2	n=4	n=6	n=8
2	7.4255e-7	7.2427e-7	7.3261e-7	7.3227e-7	7.3220e-7
4	3.4480e-7	6.5746e-7	8.2836e-7	8.7882e-7	8.8882e-7
6	1.5250e-7	5.2892e-7	8.2514e-7	9.0081e-7	9.0736e-7
8	7.8174e-8	4.1687e-7	8.0015e-7	9.0201e-7	9.0952e-7
10	4.4987e-8	3.2784e-7	7.6686e-7	8.9859e-7	9.0913e-7
12	2.8381e-8	2.5882e-7	7.3355e-7	8.9365e-7	9.0981e-7
14	1.9357e-8	2.0545e-7	7.0325e-7	8.8981e-7	9.1526e-7
16	1.4098e-8	1.6469e-7	6.7504e-7	8.8825e-7	9.3112e-7
18	1.0831e-8	1.3395e-7	6.4717e-7	8.8918e-7	9.6366e-7
20	8.6730e-9	1.1081e-7	6.1837e-7	8.9308e-7	1.0125e-6
22	7.1639e-9	9.3275e-8	5.8794e-7	9.0122e-7	1.0659e-6
24	6.0492e-9	7.9900e-8	5.5561e-7	9.1491e-7	1.1101e-6
26	5.1834e-9	6.9778e-8	5.2137e-7	9.3409e-7	1.1401e-6
28	4.4807e-9	6.2528e-8	4.8575e-7	9.5630e-7	1.1585e-6
30	3.8902e-9	5.8375e-8	4.5052e-7	9.7738e-7	1.1697e-6
32	3.3816e-9	5.8395e-8	4.2004e-7	9.9355e-7	1.1777e-6
34	2.9375e-9	6.4917e-8	4.0211e-7	1.0036e-6	1.1845e-6
36	2.5475e-9	8.0814e-8	4.0629e-7	1.0105e-6	1.1909e-6
38	2.2055e-9	1.0222e-7	4.3841e-7	1.0198e-6	1.1970e-6
40	1.9069e-9	1.0881e-7	4.9342e-7	1.0352e-6	1.2026e-6

Table A.51. Static analysis (CS=2, RVM, γ=4, c=2)

c=2	w_{max}	w_{max}	w_{max}	w_{max}	w_{max}
γ=4	r=1	r=3	r=6	r=10	r=15
2k	n=0	n=2	n=4	n=6	n=8
2	1.3943e-6	1.3279e-6	1.3401e-6	1.3391e-6	1.3389e-6
4	6.4422e-7	1.1395e-6	1.3990e-6	1.4649e-6	1.4764e-6
6	2.8892e-7	9.5703e-7	1.4086e-6	1.4925e-6	1.4924e-6
8	1.4859e-7	7.9792e-7	1.3962e-6	1.4993e-6	1.4948e-6
10	8.5311e-8	6.6087e-7	1.3733e-6	1.5025e-6	1.4959e-6
12	5.3228e-8	5.4529e-7	1.3473e-6	1.5036e-6	1.4971e-6
14	3.5405e-8	4.5105e-7	1.3200e-6	1.5036e-6	1.4977e-6
16	2.4760e-8	3.7715e-7	1.2913e-6	1.5038e-6	1.4961e-6
18	1.8018e-8	3.2131e-7	1.2608e-6	1.5059e-6	1.4916e-6
20	1.3536e-8	2.8070e-7	1.2289e-6	1.5110e-6	1.4848e-6
22	1.0435e-8	2.5267e-7	1.1958e-6	1.5201e-6	1.4777e-6
24	8.2160e-9	2.3519e-7	1.1612e-6	1.5333e-6	1.4719e-6
26	6.5845e-9	2.2684e-7	1.1238e-6	1.5497e-6	1.4686e-6
28	5.3564e-9	2.2672e-7	1.0819e-6	1.5672e-6	1.4679e-6
30	4.4137e-9	2.3412e-7	1.0349e-6	1.5824e-6	1.4694e-6
32	3.6779e-9	2.4784e-7	9.8697e-7	1.5924e-6	1.4728e-6
34	3.0950e-9	2.6518e-7	9.5446e-7	1.5959e-6	1.4774e-6
36	2.6276e-9	2.8067e-7	9.6344e-7	1.5935e-6	1.4827e-6
38	2.2484e-9	2.8626e-7	1.0188e-6	1.5868e-6	1.4880e-6
40	1.9378e-9	2.7522e-7	1.0871e-6	1.5780e-6	1.4930e-6

Table A.52. Static analysis (CS=2, RVM, γ=5, c=1)

c=1	w_{max}	w_{max}	w_{max}	w_{max}	w_{max}
γ=1	r=1	r=3	r=6	r=10	r=15
2k	n=0	n=2	n=4	n=6	n=8
2	7.3848e-7	7.1254e-7	7.3329e-7	7.3198e-7	7.3224e-7
4	3.7795e-7	6.5843e-7	8.3209e-7	8.7794e-7	8.8894e-7
6	1.7743e-7	5.5093e-7	8.3087e-7	8.9857e-7	9.0688e-7
8	9.3559e-8	4.5939e-7	8.1165e-7	8.9842e-7	9.0844e-7
10	5.4524e-8	3.8698e-7	7.8831e-7	8.9365e-7	9.0774e-7
12	3.4585e-8	3.2936e-7	7.6718e-7	8.8724e-7	9.0847e-7
14	2.3638e-8	2.8145e-7	7.5001e-7	8.8166e-7	9.1460e-7
16	1.7226e-8	2.4083e-7	7.3521e-7	8.7802e-7	9.3250e-7
18	1.3232e-8	2.0657e-7	7.2064e-7	8.7655e-7	9.6947e-7
20	1.0592e-8	1.7787e-7	7.0488e-7	8.7791e-7	1.0251e-6
22	8.7441e-9	1.5386e-7	6.8742e-7	8.8389e-7	1.0859e-6
24	7.3797e-9	1.3365e-7	6.6845e-7	8.9669e-7	1.1365e-6
26	6.3204e-9	1.1657e-7	6.4877e-7	9.1717e-7	1.1712e-6
28	5.4612e-9	1.0232e-7	6.2975e-7	9.4339e-7	1.1922e-6
30	4.7396e-9	9.1170e-8	6.1352e-7	9.7117e-7	1.2034e-6
32	4.1185e-9	8.4440e-8	6.0304e-7	9.9575e-7	1.2085e-6
34	3.5765e-9	8.5165e-8	6.0167e-7	1.0138e-6	1.2104e-6
36	3.1008e-9	9.7902e-8	6.1237e-7	1.0249e-6	1.2112e-6
38	2.6838e-9	1.2158e-7	6.3635e-7	1.0326e-6	1.2118e-6
40	2.3198e-9	1.3712e-7	6.7117e-7	1.0411e-6	1.2125e-6

Table A.53. Static analysis (CS=2, RVM, γ=5, c=2)

c=2	w_{max}	w_{max}	w_{max}	w_{max}	w_{max}
γ=5	r=1	r=3	r=6	r=10	r=15
2k	n=0	n=2	n=4	n=6	n=8
2	1.3886e-6	1.3135e-6	1.3420e-6	1.3386e-6	1.3390e-6
4	7.0474e-7	1.1355e-6	1.4060e-6	1.4623e-6	1.4768e-6
6	3.3448e-7	9.8540e-7	1.4141e-6	1.4864e-6	1.4924e-6
8	1.7680e-7	8.5405e-7	1.4013e-6	1.4893e-6	1.4941e-6
10	1.0285e-7	7.3714e-7	1.3816e-6	1.4886e-6	1.4953e-6
12	6.4584e-8	6.3398e-7	1.3621e-6	1.4864e-6	1.4987e-6
14	4.3093e-8	5.4457e-7	1.3445e-6	1.4826e-6	1.5048e-6
16	3.0180e-8	4.6907e-7	1.3280e-6	1.4778e-6	1.5121e-6
18	2.1975e-8	4.0715e-7	1.3112e-6	1.4729e-6	1.5182e-6
20	1.6510e-8	3.5795e-7	1.2932e-6	1.4692e-6	1.5212e-6
22	1.2726e-8	3.2042e-7	1.2734e-6	1.4680e-6	1.5207e-6
24	1.0017e-8	2.9360e-7	1.2513e-6	1.4708e-6	1.5179e-6
26	8.0257e-9	2.7684e-7	1.2259e-6	1.4783e-6	1.5138e-6
28	6.5268e-9	2.6980e-7	1.1959e-6	1.4899e-6	1.5094e-6
30	5.3763e-9	2.7225e-7	1.1610e-6	1.5035e-6	1.5052e-6
32	4.4785e-9	2.8343e-7	1.1242e-6	1.5165e-6	1.5019e-6
34	3.7677e-9	3.0075e-7	1.0964e-6	1.5270e-6	1.4997e-6
36	3.1977e-9	3.1811e-7	1.0924e-6	1.5337e-6	1.4988e-6
38	2.7354e-9	3.2585e-7	1.1159e-6	1.5365e-6	1.4990e-6
40	2.3569e-9	3.1559e-7	1.1525e-6	1.5358e-6	1.5001e-6

Table A.54. Static analysis (CS=2, RVM, γ=8, c=1)

c=1	w_{max}	w_{max}	w_{max}	w_{max}	w_{max}
γ=8	r=1	r=3	r=6	r=10	r=15
2k	n=0	n=2	n=4	n=6	n=8
2	5.6296e-7	5.9687e-7	6.8810e-7	6.9260e-7	7.2506e-7
4	6.8335e-7	6.6558e-7	8.2045e-7	8.1303e-7	8.6212e-7
6	6.6104e-7	7.3183e-7	8.4787e-7	8.4676e-7	8.7499e-7
8	6.2182e-7	7.7696e-7	8.6045e-7	8.5887e-7	8.7846e-7
10	5.9198e-7	8.0377e-7	8.6872e-7	8.6278e-7	8.8010e-7
12	5.7078e-7	8.2338e-7	8.7421e-7	8.6499e-7	8.7993e-7
14	5.4210e-7	8.4406e-7	8.7816e-7	8.6765e-7	8.7911e-7
16	4.9579e-7	8.6561e-7	8.8135e-7	8.7149e-7	8.7939e-7
18	4.3656e-7	8.8490e-7	8.8420e-7	8.7672e-7	8.8216e-7
20	3.7595e-7	9.0046e-7	8.8692e-7	8.8326e-7	8.8806e-7
22	3.2227e-7	9.1247e-7	8.8960e-7	8.9083e-7	8.9680e-7
24	2.7839e-7	9.2164e-7	8.9223e-7	8.9897e-7	9.0749e-7
26	2.4386e-7	9.2863e-7	8.9474e-7	9.0714e-7	9.1896e-7
28	2.1710e-7	9.3396e-7	8.9702e-7	9.1485e-7	9.3005e-7
30	1.9645e-7	9.3795e-7	8.9897e-7	9.2169e-7	9.3989e-7
32	1.8050e-7	9.4085e-7	9.0049e-7	9.2737e-7	9.4794e-7
34	1.6818e-7	9.4282e-7	9.0155e-7	9.3175e-7	9.5393e-7
36	1.5871e-7	9.4399e-7	9.0214e-7	9.3478e-7	9.5782e-7
38	1.5150e-7	9.4449e-7	9.0226e-7	9.3653e-7	9.5971e-7
40	1.4614e-7	9.4439e-7	9.0194e-7	9.3708e-7	9.5977e-7

Table A.55. Static analysis (CS=2, RVM, γ=8, c=2)

c=2	w_{max}	w_{max}	w_{max}	w_{max}	w_{max}
γ =8	r=1	r=3	r=6	r=10	r=15
2k	n=0	n=2	n=4	n=6	n=8
2	9.7230e-7	1.0103e-6	1.1941e-6	1.2143e-6	1.2765e-6
4	1.2437e-6	1.0750e-6	1.3499e-6	1.3105e-6	1.3935e-6
6	1.3231e-6	1.2098e-6	1.3814e-6	1.3446e-6	1.4056e-6
8	1.3638e-6	1.3048e-6	1.3999e-6	1.3638e-6	1.4128e-6
10	1.3892e-6	1.3667e-6	1.4130e-6	1.3752e-6	1.4172e-6
12	1.4068e-6	1.4129e-6	1.4189e-6	1.3849e-6	1.4185e-6
14	1.4141e-6	1.4541e-6	1.4199e-6	1.3952e-6	1.4179e-6
16	1.4071e-6	1.4921e-6	1.4185e-6	1.4061e-6	1.4163e-6
18	1.3849e-6	1.5248e-6	1.4164e-6	1.4172e-6	1.4148e-6
20	1.3504e-6	1.5512e-6	1.4144e-6	1.4278e-6	1.4140e-6
22	1.3082e-6	1.5717e-6	1.4127e-6	1.4377e-6	1.4141e-6
24	1.2631e-6	1.5875e-6	1.4115e-6	1.4466e-6	1.4152e-6
26	1.2185e-6	1.5998e-6	1.4107e-6	1.4545e-6	1.4170e-6
28	1.1767e-6	1.6093e-6	1.4104e-6	1.4613e-6	1.4192e-6
30	1.1389e-6	1.6168e-6	1.4105e-6	1.4670e-6	1.4217e-6
32	1.1059e-6	1.6228e-6	1.4110e-6	1.4719e-6	1.4244e-6
34	1.0777e-6	1.6276e-6	1.4117e-6	1.4758e-6	1.4269e-6
36	1.0541e-6	1.6314e-6	1.4126e-6	1.4791e-6	1.4294e-6
38	1.0351e-6	1.6344e-6	1.4137e-6	1.4816e-6	1.4318e-6
40	1.0204e-6	1.6367e-6	1.4151e-6	1.4836e-6	1.4341e-6

Table A.56. Static analysis (CS=2, RVM, γ=9, c=1)

c=1	w_{max}	w_{max}	w_{max}	w_{max}	w_{max}
γ =9	r=1	r=3	r=6	r=10	r=15
2k	n=0	n=2	n=4	n=6	n=8
2	5.6297e-7	5.9688e-7	6.8803e-7	6.9543e-7	7.2108e-7
4	6.8341e-7	6.6555e-7	8.2446e-7	8.1663e-7	8.6958e-7
6	6.6151e-7	7.3182e-7	8.4438e-7	8.3552e-7	9.0507e-7
8	6.1933e-7	7.7963e-7	8.4972e-7	8.4110e-7	9.2025e-7
10	5.7204e-7	8.0940e-7	8.5281e-7	8.4391e-7	9.3012e-7
12	5.2769e-7	8.2725e-7	8.5581e-7	8.4771e-7	9.3638e-7
14	4.8962e-7	8.4052e-7	8.6093e-7	8.5551e-7	9.3956e-7
16	4.5733e-7	8.5311e-7	8.6933e-7	8.6762e-7	9.4047e-7
18	4.2934e-7	8.6629e-7	8.8072e-7	8.8156e-7	9.3990e-7
20	4.0446e-7	8.7987e-7	8.9353e-7	8.9426e-7	9.3844e-7
22	3.8194e-7	8.9306e-7	9.0556e-7	9.0397e-7	9.3648e-7
24	3.6135e-7	9.0494e-7	9.1500e-7	9.1036e-7	9.3427e-7
26	3.4245e-7	9.1482e-7	9.2101e-7	9.1388e-7	9.3191e-7
28	3.2507e-7	9.2235e-7	9.2370e-7	9.1526e-7	9.2946e-7
30	3.0908e-7	9.2755e-7	9.2371e-7	9.1513e-7	9.2694e-7
32	2.9436e-7	9.3064e-7	9.2182e-7	9.1398e-7	9.2436e-7
34	2.8082e-7	9.3199e-7	9.1872e-7	9.1219e-7	9.2173e-7
36	2.6834e-7	9.3198e-7	9.1493e-7	9.1000e-7	9.1908e-7
38	2.5684e-7	9.3095e-7	9.1082e-7	9.0760e-7	9.1641e-7
40	2.4622e-7	9.2921e-7	9.0664e-7	9.0511e-7	9.1378e-7

Table A.57. Static analysis (CS=2, RVM, γ=9, c=2)

c=2	w_{max}	w_{max}	w_{max}	w_{max}	w_{max}
γ=9	r=1	r=3	r=6	r=10	r=15
2k	n=0	n=2	n=4	n=6	n=8
2	1.2630e-6	1.1653e-6	1.3136e-6	1.2844e-6	1.3316e-6
4	1.2828e-6	1.1674e-6	1.4720e-6	1.4091e-6	1.4868e-6
6	1.1088e-6	1.2448e-6	1.4886e-6	1.4376e-6	1.5137e-6
8	9.5874e-7	1.3123e-6	1.4816e-6	1.4415e-6	1.5138e-6
10	8.3834e-7	1.3638e-6	1.4757e-6	1.4382e-6	1.5067e-6
12	7.4228e-7	1.4016e-6	1.4736e-6	1.4333e-6	1.5008e-6
14	6.6494e-7	1.4303e-6	1.4739e-6	1.4308e-6	1.4968e-6
16	6.0170e-7	1.4536e-6	1.4750e-6	1.4319e-6	1.4941e-6
18	5.4915e-7	1.4737e-6	1.4757e-6	1.4361e-6	1.4920e-6
20	5.0484e-7	1.4916e-6	1.4753e-6	1.4417e-6	1.4902e-6
22	4.6699e-7	1.5077e-6	1.4739e-6	1.4474e-6	1.4886e-6
24	4.3430e-7	1.5222e-6	1.4718e-6	1.4525e-6	1.4872e-6
26	4.0579e-7	1.5352e-6	1.4694e-6	1.4568e-6	1.4859e-6
28	3.8073e-7	1.5469e-6	1.4670e-6	1.4603e-6	1.4848e-6
30	3.5853e-7	1.5572e-6	1.4649e-6	1.4632e-6	1.4837e-6
32	3.3874e-7	1.5663e-6	1.4632e-6	1.4654e-6	1.4828e-6
34	3.2099e-7	1.5743e-6	1.4618e-6	1.4673e-6	1.4820e-6
36	3.0499e-7	1.5814e-6	1.4607e-6	1.4688e-6	1.4812e-6
38	2.9049e-7	1.5877e-6	1.4599e-6	1.4700e-6	1.4806e-6
40	2.7730e-7	1.5933e-6	1.4594e-6	1.4709e-6	1.4800e-6

Table A.58. Static analysis (CS=2, RVM, γ=10, c=1, SS)

c=1	w_{max}	w_{max}	w_{max}	w_{max}	w_{max}
γ=10	r=1	r=3	r=6	r=10	r=15
2k	n=0	n=2	n=4	n=6	n=8
2	1.7210e-6	2.0724e-6	2.0580e-6	2.0578e-6	2.0578e-6
4	5.7401e-7	1.7177e-6	2.1219e-6	2.2089e-6	2.2237e-6
6	2.1061e-7	1.3068e-6	2.0779e-6	2.1835e-6	2.1852e-6
8	9.5669e-8	9.3439e-7	2.0105e-6	2.1540e-6	2.1533e-6
10	5.0644e-8	6.3579e-7	1.9240e-6	2.1291e-6	2.1311e-6
12	2.9835e-8	4.2505e-7	1.8229e-6	2.1067e-6	2.1161e-6
14	1.8989e-8	2.8646e-7	1.7120e-6	2.0841e-6	2.1060e-6
16	1.2809e-8	1.9727e-7	1.5982e-6	2.0601e-6	2.0981e-6
18	9.0398e-9	1.3949e-7	1.4894e-6	2.0355e-6	2.0905e-6
20	6.6152e-9	1.0130e-7	1.3917e-6	2.0119e-6	2.0826e-6
22	4.9875e-9	7.5435e-8	1.3080e-6	1.9910e-6	2.0743e-6
24	3.8559e-9	5.7479e-8	1.2389e-6	1.9735e-6	2.0658e-6
26	3.0459e-9	4.4715e-8	1.1831e-6	1.9594e-6	2.0574e-6
28	2.4514e-9	3.5442e-8	1.1385e-6	1.9484e-6	2.0492e-6
30	2.0054e-9	2.8568e-8	1.1027e-6	1.9399e-6	2.0412e-6
32	1.6645e-9	2.3380e-8	1.0736e-6	1.9332e-6	2.0333e-6
34	1.3994e-9	1.9398e-8	1.0493e-6	1.9279e-6	2.0255e-6
36	1.1901e-9	1.6294e-8	1.0283e-6	1.9232e-6	2.0175e-6
38	1.0226e-9	1.3842e-8	1.0093e-6	1.9189e-6	2.0093e-6
40	8.8686e-10	1.1879e-8	9.9169e-6	1.9146e-6	2.0009e-6

Table A.59. Static analysis (CS=2, RVM, γ=10, c=2, SS)

c=2	w_{max}	w_{max}	w_{max}	w_{max}	w_{max}
γ=10	r=1	r=3	r=6	r=10	r=15
2k	n=0	n=2	n=4	n=6	n=8
2	2.6476e-6	3.0503e-6	2.9712e-6	2.9705e-6	2.9707e-6
4	1.0143e-6	2.6136e-6	3.0013e-6	3.0775e-6	3.0907e-6
6	3.9289e-7	2.1960e-6	3.0466e-6	3.1007e-6	3.0877e-6
8	1.8140e-7	1.7767e-6	3.0519e-6	3.1077e-6	3.0760e-6
10	9.6540e-8	1.3819e-6	3.0198e-6	3.1138e-6	3.0655e-6
12	5.6976e-8	1.0473e-6	2.9597e-6	3.1189e-6	3.0566e-6
14	3.6285e-8	7.8550e-7	2.8800e-6	3.1206e-6	3.0488e-6
16	2.4479e-8	5.9001e-7	2.7879e-6	3.1168e-6	3.0420e-6
18	1.7273e-8	4.4712e-7	2.6891e-6	3.1066e-6	3.0359e-6
20	1.2634e-8	3.4319e-7	2.5878e-6	3.0901e-6	3.0305e-6
22	9.5165e-9	2.6726e-7	2.4865e-6	3.0686e-6	3.0254e-6
24	7.3453e-9	2.1128e-7	2.3871e-6	3.0432e-6	3.0205e-6
26	5.7877e-9	1.6952e-7	2.2904e-6	3.0152e-6	3.0155e-6
28	4.6417e-9	1.3797e-7	2.1971e-6	2.9857e-6	3.0099e-6
30	3.7799e-9	1.1382e-7	2.1078e-6	2.9553e-6	3.0035e-6
32	3.1195e-9	9.5091e-8	2.0226e-6	2.9248e-6	2.9960e-6
34	2.6050e-9	8.0394e-8	1.9418e-6	2.8944e-6	2.9873e-6
36	2.1982e-9	6.8723e-8	1.8655e-6	2.8646e-6	2.9772e-6
38	1.8722e-9	5.9352e-8	1.7936e-6	2.8354e-6	2.9658e-6
40	1.6080e-9	5.1748e-8	1.7262e-6	2.8070e-6	2.9529e-6

Table A.60. Static analysis (CS=2, RVM, γ=11, c=1)

c=1	w_{max}	w_{max}	w_{max}	w_{max}	w_{max}
γ=11	r=1	r=3	r=6	r=10	r=15
2k	n=0	n=2	n=4	n=6	n=8
2	7.0604e-7	7.1815e-7	8.3558e-7	8.5366e-7	9.6208e-7
4	8.1681e-7	7.5881e-7	8.6509e-7	8.5564e-7	9.0450e-7
6	8.6043e-7	8.0403e-7	8.7242e-7	8.6732e-7	9.0269e-7
8	8.8579e-7	8.3337e-7	8.7546e-7	8.7166e-7	9.0238e-7
10	9.0212e-7	8.5176e-7	8.7894e-7	8.7449e-7	9.0265e-7
12	9.1390e-7	8.6396e-7	8.8303e-7	8.7766e-7	9.0270e-7
14	9.2342e-7	8.7291e-7	8.8748e-7	8.8138e-7	9.0243e-7
16	9.3142e-7	8.7999e-7	8.9173e-7	8.8523e-7	9.0200e-7
18	9.3805e-7	8.8573e-7	8.9528e-7	8.8880e-7	9.0157e-7
20	9.4331e-7	8.9033e-7	8.9786e-7	8.9190e-7	9.0123e-7
22	9.4726e-7	8.9390e-7	8.9944e-7	8.9444e-7	9.0103e-7
24	9.5002e-7	8.9656e-7	9.0013e-7	8.9645e-7	9.0091e-7
26	9.5174e-7	8.9840e-7	9.0011e-7	8.9796e-7	9.0082e-7
28	9.5257e-7	8.9956e-7	8.9957e-7	8.9903e-7	9.0072e-7
30	9.5268e-7	9.0016e-7	8.9866e-7	8.9973e-7	9.0054e-7
32	9.5222e-7	9.0030e-7	8.9750e-7	9.0010e-7	9.0027e-7
34	9.5132e-7	9.0009e-7	8.9619e-7	9.0019e-7	8.9989e-7
36	9.5010e-7	8.9962e-7	8.9481e-7	9.0007e-7	8.9941e-7
38	9.4866e-7	8.9896e-7	8.9340e-7	8.9977e-7	8.9884e-7
40	9.4709e-7	8.9818e-7	8.9201e-7	8.9934e-7	8.9819e-7

Table A.61. Static analysis (CS=2, RVM, γ=11, c=2)

c=2	w_{max}	w_{max}	w_{max}	w_{max}	w_{max}
γ=11	r=1	r=3	r=6	r=10	r=15
2k	n=0	n=2	n=4	n=6	n=8
2	1.3444e-6	1.2240e-6	1.4227e-6	1.3945e-6	1.5354e-6
4	1.5052e-6	1.2502e-6	1.4635e-6	1.4086e-6	1.4845e-6
6	1.5673e-6	1.3161e-6	1.4657e-6	1.4205e-6	1.4769e-6
8	1.6045e-6	1.3597e-6	1.4638e-6	1.4229e-6	1.4754e-6
10	1.6289e-6	1.3864e-6	1.4644e-6	1.4251e-6	1.4753e-6
12	1.6459e-6	1.4027e-6	1.4665e-6	1.4290e-6	1.4749e-6
14	1.6584e-6	1.4130e-6	1.4688e-6	1.4344e-6	1.4740e-6
16	1.6681e-6	1.4199e-6	1.4705e-6	1.4405e-6	1.4730e-6
18	1.6757e-6	1.4250e-6	1.4715e-6	1.4468e-6	1.4721e-6
20	1.6818e-6	1.4291e-6	1.4718e-6	1.4528e-6	1.4714e-6
22	1.6867e-6	1.4328e-6	1.4716e-6	1.4582e-6	1.4709e-6
24	1.6907e-6	1.4362e-6	1.4710e-6	1.4630e-6	1.4706e-6
26	1.6939e-6	1.4395e-6	1.4703e-6	1.4672e-6	1.4705e-6
28	1.6965e-6	1.4426e-6	1.4696e-6	1.4709e-6	1.4705e-6
30	1.6987e-6	1.4455e-6	1.4688e-6	1.4740e-6	1.4707e-6
32	1.7004e-6	1.4482e-6	1.4682e-6	1.4767e-6	1.4709e-6
34	1.7019e-6	1.4507e-6	1.4676e-6	1.4790e-6	1.4712e-6
36	1.7031e-6	1.4531e-6	1.4671e-6	1.4809e-6	1.4715e-6
38	1.7041e-6	1.4552e-6	1.4668e-6	1.4826e-6	1.4719e-6
40	1.7049e-6	1.4571e-6	1.4665e-6	1.4840e-6	1.4723e-6

Table A.62. Static analysis (CS=2, RVM, γ=12, c=1)

c=1	w_{max}	w_{max}	w_{max}	w_{max}	w_{max}
γ=12	r=1	r=3	r=6	r=10	r=15
2k	n=0	n=2	n=4	n=6	n=8
2	9.6695e-7	9.1033e-7	1.0148e-6	1.0163e-6	1.0713e-6
4	7.1287e-7	8.5395e-7	9.1648e-7	9.0891e-7	9.2759e-7
6	5.4576e-7	8.6530e-7	9.1711e-7	9.1055e-7	9.1399e-7
8	4.4184e-7	8.7213e-7	9.1941e-7	9.1139e-7	9.1124e-7
10	3.7114e-7	8.7297e-7	9.1982e-7	9.1053e-7	9.1052e-7
12	3.1990e-7	8.7045e-7	9.1931e-7	9.0929e-7	9.1041e-7
14	2.8113e-7	8.6614e-7	9.1866e-7	9.0832e-7	9.1064e-7
16	2.5106e-7	8.6063e-7	9.1807e-7	9.0769e-7	9.1108e-7
18	2.2740e-7	8.5426e-7	9.1753e-7	9.0729e-7	9.1170e-7
20	2.0865e-7	8.4734e-7	9.1699e-7	9.0696e-7	9.1246e-7
22	1.9370e-7	8.4019e-7	9.1642e-7	9.0656e-7	9.1335e-7
24	1.8171e-7	8.3306e-7	9.1579e-7	9.0598e-7	9.1439e-7
26	1.7205e-7	8.2612e-7	9.1508e-7	9.0515e-7	9.1564e-7
28	1.6421e-7	8.1951e-7	9.1427e-7	9.0404e-7	9.1724e-7
30	1.5780e-7	8.1328e-7	9.1337e-7	9.0265e-7	9.1940e-7
32	1.5253e-7	8.0748e-7	9.1238e-7	9.0105e-7	9.2236e-7
34	1.4815e-7	8.0211e-7	9.1134e-7	8.9931e-7	9.2638e-7
36	1.4448e-7	7.9720e-7	9.1028e-7	8.9754e-7	9.3174e-7
38	1.4137e-7	7.9276e-7	9.0925e-7	8.9588e-7	9.3870e-7
40	1.3870e-7	7.8880e-7	9.0832e-7	8.9446e-7	9.4747e-7

Table A.63. Static analysis (CS=2, RVM, γ=12, c=2)

c=2	w_{max}	w_{max}	w_{max}	w_{max}	w_{max}
γ=12	r=1	r=3	r=6	r=10	r=15
2k	n=0	n=2	n=4	n=6	n=8
2	1.7130e-6	1.4522e-6	1.6123e-6	1.5878e-6	1.6730e-6
4	1.2725e-6	1.4025e-6	1.5172e-6	1.4768e-6	1.5163e-6
6	9.8643e-7	1.4513e-6	1.5163e-6	1.4888e-6	1.5006e-6
8	8.0468e-7	1.4849e-6	1.5175e-6	1.4910e-6	1.4957e-6
10	6.7923e-7	1.5047e-6	1.5174e-6	1.4899e-6	1.4942e-6
12	5.8745e-7	1.5156e-6	1.5169e-6	1.4883e-6	1.4942e-6
14	5.1743e-7	1.5203e-6	1.5166e-6	1.4870e-6	1.4950e-6
16	4.6231e-7	1.5210e-6	1.5168e-6	1.4859e-6	1.4965e-6
18	4.1789e-7	1.5187e-6	1.5176e-6	1.4848e-6	1.4983e-6
20	3.8139e-7	1.5145e-6	1.5190e-6	1.4837e-6	1.5005e-6
22	3.5092e-7	1.5091e-6	1.5209e-6	1.4825e-6	1.5030e-6
24	3.2514e-7	1.5032e-6	1.5231e-6	1.4811e-6	1.5057e-6
26	3.0305e-7	1.4973e-6	1.5255e-6	1.4796e-6	1.5085e-6
28	2.8392e-7	1.4916e-6	1.5281e-6	1.4779e-6	1.5115e-6
30	2.6718e-7	1.4865e-6	1.5307e-6	1.4761e-6	1.5146e-6
32	2.5242e-7	1.4821e-6	1.5333e-6	1.4742e-6	1.5177e-6
34	2.3929e-7	1.4786e-6	1.5359e-6	1.4723e-6	1.5208e-6
36	2.2752e-7	1.4760e-6	1.5384e-6	1.4703e-6	1.5237e-6
38	2.1691e-7	1.4746e-6	1.5407e-6	1.4682e-6	1.5264e-6
40	2.0729e-7	1.4743e-6	1.5428e-6	1.4662e-6	1.5288e-6

Table A.64. Dynamic analysis (CS=3, RVM, γ=1, n=3, c=1)

γ=1	Frequencies					
2k	ω1	ω2	ω3	ω4	ω5	ω6
2	10.2198	21.2750	21.2852	36.6590	36.7188	43.1002
4	10.3182	20.4761	20.4808	40.7485	47.9339	53.3496
6	11.9994	23.0580	23.0605	55.0747	65.1545	72.1371
8	14.3720	26.6095	26.6394	73.3238	84.6160	94.5314
10	17.3504	30.7598	30.8995	94.1753	105.3880	119.0907
12	20.8724	35.4121	35.8688	116.3030	126.1822	144.4346
14	24.8240	40.6832	41.3319	138.4103	145.1763	170.0752
16	29.0250	46.5372	46.9638	159.6252	160.6587	196.1808
18	33.2782	52.3874	52.8358	171.8380	179.5760	208.3544
20	37.3940	57.2299	59.3918	178.8231	198.2866	212.2284
22	41.1735	61.1423	65.9614	182.2295	209.5949	216.0404
24	44.3685	63.8144	72.1365	182.9768	202.8243	233.2665
26	46.6325	65.0095	77.0959	182.2553	194.5669	250.4594
28	47.5016	64.6246	79.2872	181.5323	188.2568	268.1215
30	46.4844	62.7636	76.7590	182.4766	187.9648	286.7235
32	43.3400	59.8016	69.1033	186.7071	196.5587	306.6772
34	38.4756	56.4234	58.8519	195.4416	214.3651	328.3167
36	33.2312	50.2164	53.6093	209.3094	240.4431	351.8903
38	29.8515	47.2456	52.5015	228.4334	273.7987	377.5587
40	30.5432	51.4047	54.0601	252.6120	313.5203	405.4012

Table A.65. Dynamic analysis (CS=3, RVM, γ=1, n=4, c=1)

γ=1	Frequencies					
2k	ω1	ω2	ω3	ω4	ω5	ω6
2	10.2189	21.2750	21.2852	34.9404	34.9561	39.9286
4	9.3086	20.4761	20.4808	30.5670	35.7300	37.9941
6	9.4074	23.0580	23.0605	33.4825	39.9941	41.8008
8	9.7188	26.6095	26.6394	38.3699	45.3720	46.2010
10	10.1605	30.7598	30.8995	44.6549	49.6606	52.0582
12	10.6637	35.4121	35.8688	52.1960	53.5483	58.9890
14	11.1786	40.6832	41.3319	58.0882	60.7048	66.3677
16	11.6867	46.5372	46.9638	63.6535	69.6990	73.8796
18	12.1918	52.3874	52.8358	70.2293	78.6011	81.3047
20	12.7098	57.2299	59.3918	77.3368	86.7673	88.7726
22	13.2646	61.1423	65.9614	83.8302	93.4599	97.0279
24	13.8879	63.8144	72.1365	88.4827	97.8450	106.5466
26	14.6257	65.0095	77.0959	91.0937	99.1190	115.0263
28	15.5432	64.6246	79.2872	92.0215	96.8188	115.3423
30	16.7082	62.7636	76.7590	91.1864	91.4546	104.1061
32	18.0912	59.8016	69.1033	83.3515	85.4288	93.5536
34	19.2697	56.4234	58.8519	73.5885	75.2980	91.7686
36	19.2221	50.2164	53.6093	67.9895	69.6869	91.4641
38	17.5502	47.2456	52.5015	69.2021	70.7306	92.2895
40	15.4790	51.4047	54.0601	75.1414	79.7379	94.6263

Table A.66. Dynamic analysis (CS=3, RVM, γ=1, n=5, c=1)

γ=1	Frequencies					
2k	ω1	ω2	ω3	ω4	ω5	ω6
2	10.2189	21.2604	21.2725	34.9404	34.9561	39.9286
4	9.3086	18.9646	19.0124	30.5670	35.7300	37.9941
6	9.4074	19.0986	19.1279	33.4825	39.9941	41.8008
8	9.7188	19.6147	19.6410	38.3699	45.3720	46.2010
10	10.1605	20.3262	20.3874	44.6549	49.6606	52.0582
12	10.6637	21.1658	21.1933	52.1960	53.5483	58.9890
14	11.1786	21.9413	22.1011	58.0882	60.7048	66.3677
16	11.6867	22.6182	23.1014	63.6535	69.6990	73.8796
18	12.1918	23.3044	24.1281	70.2293	78.6011	81.3047
20	12.7098	24.1267	25.1453	77.3368	86.7673	88.7726
22	13.2646	25.2186	26.1296	83.8302	93.4599	97.0279
24	13.8879	26.7051	27.0762	88.4827	97.8450	106.5466
26	14.6257	28.0035	28.6909	91.0937	99.1190	115.0263
28	15.5432	28.9499	31.2091	92.0215	96.8188	115.3423
30	16.7082	29.9402	34.0599	91.1864	91.4546	104.1061
32	18.0912	30.8513	36.4693	83.3515	85.4288	93.5536
34	19.2697	31.1306	36.9343	73.5885	75.2980	90.2010
36	19.2221	29.9852	34.7109	67.9895	69.6869	88.4865
38	17.5502	27.8735	31.2227	69.2021	70.7306	92.2895
40	15.4790	26.2961	28.1012	75.1414	79.7379	94.6263

Table A.67. Dynamic analysis (CS=3, RVM, γ=1, n=3, c=2)

γ=1	Frequencies					
2k	ω1	ω2	ω3	ω4	ω5	ω6
2	6.8491	9.8987	15.3794	17.5057	22.2032	23.9216
4	7.1428	9.1905	17.8478	17.8711	25.4801	27.6916
6	8.2219	10.3930	19.9330	24.5816	33.7316	38.0698
8	9.5208	12.1932	22.2691	32.7443	43.2690	50.8263
10	10.9900	14.4430	24.6615	41.7818	53.0370	65.2522
12	12.6020	17.0737	27.0528	51.2247	62.3262	80.9923
14	14.2843	19.9281	29.3876	60.7743	70.9311	97.7993
16	15.9236	22.7597	31.5980	70.3338	79.0110	115.5171
18	17.3986	25.2886	33.6211	79.9522	86.9063	134.0646
20	18.6058	27.2582	35.4135	89.7527	95.0037	153.4104
22	19.4746	28.4812	36.9575	99.8792	103.6497	173.5499
24	19.9735	28.8699	38.2606	110.4644	113.1034	194.4894
26	20.1098	28.4426	39.3511	121.6162	123.5272	216.2372
28	19.9280	27.3141	40.2731	133.4142	135.0031	238.7994
30	19.5069	25.6855	41.0816	145.9123	147.5580	262.1782
32	18.9582	23.8477	41.8387	159.1428	161.1856	286.3714
34	18.4257	22.1966	42.6095	173.1213	175.8606	311.3728
36	18.0786	21.2295	43.4581	187.8496	191.5463	337.1728
38	18.0932	21.4474	44.4446	203.3187	208.1980	363.7590
40	18.6202	23.1423	45.6211	219.5111	225.7649	391.1164

Table A.68. Dynamic analysis (CS=3, RVM, γ=1, n=4, c=2)

γ=1	Frequencies					
2k	ω1	ω2	ω3	ω4	ω5	ω6
2	6.8447	9.8987	14.0823	17.5057	22.1661	22.2032
4	6.3738	9.1905	12.8351	17.8478	20.3155	25.4801
6	6.4055	10.3930	14.2892	19.9330	22.2507	33.7316
8	6.5445	12.1932	16.5582	22.2691	24.9703	41.8848
10	6.7316	14.4430	19.4813	24.6615	28.0519	45.8061
12	6.9444	17.0737	23.0227	27.0528	31.3311	49.6422
14	7.1760	19.9281	26.9793	29.3876	34.6138	53.3507
16	7.4229	22.7597	30.9760	31.5980	37.6749	56.9077
18	7.6798	25.2886	33.6211	34.5169	40.3205	60.3045
20	7.9420	27.2582	35.4135	37.0740	42.4285	63.5374
22	8.2073	28.4812	36.9575	38.2451	43.9580	66.5907
24	8.4773	28.8699	37.8984	38.2606	44.9390	69.4424
26	8.7561	28.4426	36.1892	39.3511	45.4567	72.0897
28	9.0454	27.3141	33.4772	40.2731	45.6375	74.5594
30	9.3285	25.6855	30.2696	41.0816	45.6389	76.8947
32	9.5302	23.8477	27.2567	41.8387	45.6404	79.1418
34	9.4744	22.1966	25.3743	42.6095	45.8345	81.3435
36	9.0322	21.2295	25.5766	43.4581	46.4113	83.5375
38	8.4525	21.4474	28.1962	44.4446	47.5400	85.7563
40	8.0988	23.1423	32.8762	45.6211	49.3485	88.0270

Table A.69. Dynamic analysis (CS=3, RVM, γ=1, n=5, c=2)

γ=1	Frequencies					
2k	ω1	ω2	ω3	ω4	ω5	ω6
2	6.8447	9.8752	14.0823	17.4657	19.5062	22.1661
4	6.3738	8.3675	12.8351	16.4833	18.2062	20.3155
6	6.4055	8.3678	14.2892	16.5676	20.1599	22.2507
8	6.5445	8.6533	16.5582	16.7775	22.8681	24.9703
10	6.7316	9.0424	17.0498	19.4813	26.3891	28.0519
12	6.9444	9.4515	17.3809	23.0227	30.8109	31.3311
14	7.1760	9.8610	17.7633	26.9793	34.6138	35.9269
16	7.4229	10.2808	18.1797	30.9760	37.6749	41.2231
18	7.6798	10.7266	18.6063	34.5169	40.3205	45.7948
20	7.9420	11.2165	19.0192	37.0740	42.4285	48.4418
22	8.2073	11.7752	19.3937	38.2451	43.9580	48.3937
24	8.4773	12.4386	19.7047	37.8984	44.9390	45.9071
26	8.7561	13.2518	19.9223	36.1892	41.8319	45.4567
28	9.0454	14.2408	20.0122	33.4772	37.1201	45.6375
30	9.3285	15.2887	19.9402	30.2696	32.8358	45.6389
32	9.5302	15.8247	19.6938	27.2567	30.4694	45.6404
34	9.4744	15.0969	19.3177	25.3743	31.5434	45.8345
36	9.0322	13.6702	18.9326	25.5766	35.8717	46.4113
38	8.4525	12.4848	18.6961	28.1962	42.3867	47.5400
40	8.0988	11.8127	18.7210	32.8762	49.3485	50.3594

Table A.70. Dynamic analysis (CS=3, RVM, γ=2, n=2, c=1)

γ=2	Frequencies					
2k	ω1	ω2	ω3	ω4	ω5	ω6
2	11.7549	26.2850	26.2851	40.0176	41.7333	45.7452
4	10.7658	21.7209	21.7210	32.2063	37.7378	38.5376
6	10.2274	20.7171	20.7557	30.5123	36.9134	37.3068
8	9.8995	20.3322	20.3372	29.8094	36.8483	37.2398
10	9.7263	20.0998	20.2830	29.5161	36.7804	37.6537
12	9.6333	19.9399	20.2485	29.3087	36.7178	37.6617
14	9.5603	19.8141	20.0699	29.0551	36.6306	37.1510
16	9.4868	19.6926	19.7636	28.7402	36.1994	36.6470
18	9.4021	19.3959	19.5755	28.4077	35.2959	36.5294
20	9.3059	19.0316	19.4731	28.0963	34.4587	36.4362
22	9.2195	18.7243	19.3804	27.8316	33.8172	36.3456
24	9.1542	18.4878	19.3028	27.6261	33.4006	36.2629
26	9.0995	18.3248	19.2406	27.4809	33.2024	36.1884
28	9.0554	18.2373	19.1885	27.3861	33.1904	36.1165
30	9.0333	18.2099	19.1416	27.3386	33.3257	36.0527
32	9.0222	18.2373	19.1102	27.3313	33.5708	35.9944
34	9.0222	18.3030	19.0840	27.3496	33.8940	35.9444
36	9.0333	18.3957	19.0631	27.3898	34.2695	35.8999
38	9.0443	18.5095	19.0473	27.4481	34.6699	35.8608
40	9.0664	18.6387	19.0368	27.5209	35.0799	35.8329

Table A.71. Dynamic analysis (CS=3, RVM, γ=2, n=2, c=2)

γ=2	Frequencies					
2k	ω1	ω2	ω3	ω4	ω5	ω6
2	7.9875	12.2229	16.3401	22.1088	26.0806	39.6333
4	7.3756	9.5917	13.2514	18.7723	21.1092	35.3582
6	7.0000	9.0222	12.6174	18.1163	20.1345	34.7390
8	6.7823	8.7864	12.4499	17.8382	19.7282	34.6266
10	6.6483	8.6603	12.4177	17.6692	19.5038	34.6092
12	6.5574	8.5790	12.3450	17.5499	19.3391	34.5977
14	6.5115	8.4971	12.1902	17.4471	19.2042	34.5746
16	6.4498	8.4143	11.9833	17.3609	19.0840	34.5340
18	6.4187	8.3307	11.7813	17.2858	18.9789	34.4848
20	6.3875	8.2583	11.6017	17.2163	18.8839	34.4267
22	6.3561	8.1854	11.4630	17.1581	18.8043	34.3628
24	6.3246	8.1363	11.3754	17.1056	18.7403	34.2987
26	6.3087	8.0994	11.3225	17.0587	18.6815	34.2374
28	6.2929	8.0747	11.3137	17.0176	18.6333	34.1760
30	6.2929	8.0623	11.3314	16.9765	18.5957	34.1145
32	6.2769	8.0623	11.3666	16.9470	18.5634	34.0588
34	6.2769	8.0623	11.4193	16.9115	18.5365	34.0029
36	6.2610	8.0623	11.4804	16.8879	18.5149	33.9529
38	6.2610	8.0747	11.5499	16.8642	18.4986	33.9028
40	6.2610	8.0870	11.6276	16.8404	18.4824	33.8556

Table A.72. Dynamic analysis (CS=3, RVM, γ=3, n=2, c=1)

γ=3	Frequencies					
2k	ω1	ω2	ω3	ω4	ω5	ω6
2	10.6501	22.5862	22.5866	35.7400	36.6603	40.9104
4	10.0404	20.5662	20.5705	30.4990	36.4174	37.5749
6	9.8509	22.1028	22.1058	31.8544	39.4193	40.2768
8	9.7455	23.8513	24.0545	33.9282	42.2415	43.6037
10	9.7023	25.2043	25.9780	35.8426	44.3510	47.1141
12	9.6688	26.4569	27.8126	37.7254	47.0420	50.6279
14	9.6177	28.2012	29.6241	39.9673	51.3186	54.1363
16	9.5695	30.7331	31.4737	42.7656	56.7242	58.1280
18	9.5491	33.3757	33.9541	46.0607	60.5893	64.6690
20	9.5637	35.3089	37.5742	49.6577	63.9453	72.0339
22	9.6078	37.2384	41.3048	53.3472	67.1818	79.3065
24	9.6724	39.1314	44.9343	56.9633	70.3162	86.1417
26	9.7499	40.9622	48.3318	60.3950	73.3456	92.3661
28	9.8339	42.7143	51.4257	63.5769	76.2676	97.9044
30	9.9204	44.3785	54.1835	66.4762	79.0821	102.7374
32	10.0065	45.9507	56.5958	69.0809	81.7911	106.8752
34	10.0901	47.4308	58.6658	71.3912	84.3983	110.3422
36	10.1698	48.8209	60.4038	73.4146	86.9082	113.1685
38	10.2445	50.1245	61.8231	75.1625	89.3256	115.3860
40	10.3138	51.3459	62.9386	76.6484	91.6555	117.0265

Table A.73. Dynamic analysis (CS=3, RVM, γ=3, n=2, c=2)

γ=3	Frequencies					
2k	ω1	ω2	ω3	ω4	ω5	ω6
2	7.1997	10.5620	14.6258	18.9762	23.3197	35.0941
4	6.9018	9.1564	12.7814	17.9814	20.2738	34.5868
6	6.7823	9.7801	13.4770	19.6450	21.5361	37.7338
8	6.7052	10.5785	14.3197	21.6132	23.3276	41.3829
10	6.6539	11.3329	15.0929	23.5791	25.1713	45.0799
12	6.6151	12.0584	15.9922	25.4754	26.9849	48.7017
14	6.5860	12.8289	17.1873	27.2925	28.7737	52.2113
16	6.5711	13.6854	18.6882	29.0347	30.5468	55.5995
18	6.5715	14.6216	20.3955	30.7080	32.3009	58.8671
20	6.5856	15.6045	22.1851	32.3170	34.0242	62.0182
22	6.6095	16.5967	23.9553	33.8657	35.7041	65.0589
24	6.6393	17.5669	25.6375	35.3575	37.3302	67.9954
26	6.6735	18.4930	27.1922	36.7957	38.8960	70.8343
28	6.7093	19.3619	28.5999	38.1835	40.3983	73.5823
30	6.7461	20.1671	29.8539	39.5244	41.8366	76.2454
32	6.7831	20.9056	30.9547	40.8215	43.2123	78.8294
34	6.8198	21.5779	31.9067	42.0778	44.5275	81.3398
36	6.8549	22.1857	32.7161	43.2960	45.7853	83.7815
38	6.8891	22.7316	33.3902	44.4788	46.9890	86.1592
40	6.9213	23.2190	33.9362	45.6285	48.1419	88.4770

Table A.74. Dynamic analysis (CS=3, RVM, γ=3, n=4, c=1)

γ=3	Frequencies					
2k	ω1	ω2	ω3	ω4	ω5	ω6
2	10.4159	21.9132	21.9146	35.3184	36.1358	40.6589
4	9.3434	20.1863	20.1872	30.0435	35.5831	36.5140
6	9.2469	20.0229	20.0292	29.4334	35.8651	36.4376
8	9.2326	19.9800	20.0225	29.3205	36.2332	36.8361
10	9.2247	19.9529	20.1021	29.3356	36.4710	37.1681
12	9.2122	19.9166	20.0726	29.3035	36.5631	37.0164
14	9.1918	19.8776	19.9015	29.2389	36.3826	36.7582
16	9.1657	19.6999	19.8657	29.2422	35.8638	36.7813
18	9.1376	19.5512	19.8985	29.3656	35.3605	36.8802
20	9.1091	19.4769	19.9754	29.6041	34.9380	37.0271
22	9.0819	19.4682	20.0862	29.9282	34.6030	37.2099
24	9.0562	19.5092	20.2195	30.3068	34.3500	37.4169
26	9.0333	19.5858	20.3658	30.7135	34.1701	37.6381
28	9.0136	19.6872	20.5178	31.1297	34.0539	37.8657
30	8.9975	19.8050	20.6701	31.5420	33.9915	38.0942
32	8.9853	19.9336	20.8193	31.9417	33.9743	38.3194
34	8.9769	20.0685	20.9627	32.3226	33.9943	38.5382
36	8.9725	20.2058	21.0985	32.6810	34.0443	38.7482
38	8.9713	20.3431	21.2255	33.0142	34.1185	38.9474
40	8.9736	20.4777	21.3428	33.3207	34.2121	39.1342

Table A.75. Dynamic analysis (CS=3, RVM, γ=3, n=4, c=2)

γ=3	Frequencies					
2k	ω1	ω2	ω3	ω4	ω5	ω6
2	7.0228	10.1111	14.2415	18.2714	19.9729	22.5560
4	6.3851	8.9224	12.4579	17.5691	18.1583	19.8312
6	6.3186	8.7115	12.2407	17.5730	19.1374	19.5127
8	6.3127	8.6400	12.2607	17.5645	19.4312	20.0052
10	6.3111	8.6171	12.2959	17.5257	19.3861	21.0286
12	6.3067	8.5945	12.2623	17.4703	19.3454	22.7028
14	6.2992	8.5735	12.2078	17.4134	19.3268	25.0989
16	6.2905	8.5735	12.1815	17.3663	19.3499	27.9762
18	6.2817	8.6038	12.1992	17.3339	19.4194	31.0378
20	6.2730	8.6605	12.2577	17.3163	19.5281	34.0573
22	6.2654	8.7370	12.3479	17.3111	19.6649	34.4337
24	6.2586	8.8264	12.4609	17.3152	19.8196	34.4031
26	6.2526	8.9238	12.5893	17.3256	19.9834	34.3731
28	6.2474	9.0252	12.7271	17.3400	20.1499	34.3439
30	6.2430	9.1277	12.8701	17.3566	20.3144	34.3154
32	6.2390	9.2298	13.0152	17.3737	20.4735	34.2872
34	6.2358	9.3303	13.1598	17.3902	20.6246	34.2592
36	6.2330	9.4279	13.3026	17.4053	20.7658	34.2308
38	6.2310	9.5221	13.4427	17.4181	20.8957	34.2017
40	6.2290	9.6128	13.5788	17.4281	21.0129	34.1717

Table A.76. Dynamic analysis (CS=3, RVM, γ=8, n=4, c=1)

γ=8	Frequencies					
2k	ω1	ω2	ω3	ω4	ω5	ω6
2	10.7030	22.1814	23.3187	36.0907	37.2667	42.0256
4	9.6646	20.4804	21.2178	30.9378	36.2158	37.7880
6	9.5394	20.2146	20.5040	29.9566	36.3110	36.8105
8	9.4742	20.0381	20.1133	29.4724	36.1074	36.8188
10	9.4165	19.7209	20.1024	29.1771	35.6186	37.1140
12	9.3632	19.4846	19.9892	28.8846	35.2204	36.8571
14	9.3124	19.2937	19.7212	28.5512	34.8976	36.2395
16	9.2639	19.1417	19.3927	28.2297	34.6417	35.5350
18	9.2187	19.0270	19.0842	27.9589	34.4349	34.8837
20	9.1766	18.8243	18.9431	27.7449	34.1788	34.4154
22	9.1376	18.6149	18.8817	27.5795	33.7545	34.2454
24	9.1019	18.4495	18.8363	27.4526	33.3711	34.1514
26	9.0703	18.3213	18.8016	27.3558	33.0647	34.0822
28	9.0427	18.2242	18.7747	27.2828	32.8311	34.0281
30	9.0200	18.1536	18.7535	27.2293	32.6620	33.9844
32	9.0019	18.1058	18.7367	27.1919	32.5490	33.9483
34	8.9892	18.0773	18.7232	27.1682	32.4841	33.9178
36	8.9808	18.0654	18.7124	27.1559	32.4601	33.8918
38	8.9766	18.0679	18.7039	27.1535	32.4712	33.8693
40	8.9766	18.0826	18.6972	27.1596	32.5121	33.8494

Table A.77. Dynamic analysis (CS=3, RVM, γ=8, n=5, c=1)

γ=8	Frequencies					
2k	ω1	ω2	ω3	ω4	ω5	ω6
2	10.7030	21.6852	22.4009	36.0907	37.2667	42.0256
4	9.6646	19.3106	20.0004	30.9378	36.2158	37.7880
6	9.5394	18.9193	19.6517	29.9566	36.3110	36.8105
8	9.4742	18.7707	19.5005	29.4724	36.1074	36.8188
10	9.4165	18.7075	19.3795	29.1771	35.6186	37.1140
12	9.3632	18.6732	19.2669	28.8846	35.2204	36.8571
14	9.3124	18.6434	19.1621	28.5512	34.8976	36.2395
16	9.2639	18.6050	19.0672	28.2297	34.6417	35.5350
18	9.2187	18.5528	18.9835	27.9589	34.4349	34.8837
20	9.1766	18.4873	18.9104	27.7449	34.1788	34.4154
22	9.1376	18.4122	18.8470	27.5795	33.7545	34.2454
24	9.1019	18.3338	18.7923	27.4526	33.3711	34.1514
26	9.0703	18.2581	18.7453	27.3558	33.0647	34.0822
28	9.0427	18.1905	18.7053	27.2828	32.8311	34.0281
30	9.0200	18.1345	18.6719	27.2293	32.6620	33.9844
32	9.0019	18.0927	18.6443	27.1919	32.5490	33.9483
34	8.9892	18.0656	18.6219	27.1682	32.4841	33.9178
36	8.9808	18.0531	18.6043	27.1559	32.4601	33.8918
38	8.9766	18.0544	18.5907	27.1535	32.4712	33.8693
40	8.9766	18.0679	18.5809	27.1596	32.5121	33.8494

Table A.78. Dynamic analysis (CS=3, RVM, γ=8, n=4, c=2)

γ=8	Frequencies					
2k	ω1	ω2	ω3	ω4	ω5	ω6
2	7.4505	10.6513	14.6472	19.9432	20.9573	23.8995
4	6.7716	9.3040	12.8002	18.3753	18.6287	20.8309
6	6.6772	8.8983	12.4352	18.0119	18.9046	19.9677
8	6.6155	8.6951	12.3355	17.5812	19.4304	19.4732
10	6.5555	8.5773	12.2768	17.2760	19.1426	20.1551
12	6.5015	8.4817	12.1534	17.0552	18.9008	21.5853
14	6.4552	8.3887	11.9879	16.8926	18.7167	23.8093
16	6.4160	8.3045	11.8220	16.7717	18.5774	24.9424
18	6.3840	8.2347	11.6750	16.6807	18.4731	25.9217
20	6.3569	8.1795	11.5514	16.6111	18.3946	27.0425
22	6.3344	8.1363	11.4505	16.5566	18.3345	28.2161
24	6.3155	8.1028	11.3699	16.5129	18.2880	29.3733
26	6.2996	8.0768	11.3073	16.4771	18.2509	30.4665
28	6.2861	8.0567	11.2597	16.4470	18.2210	31.4663
30	6.2750	8.0415	11.2254	16.4215	18.1966	31.7115
32	6.2654	8.0303	11.2020	16.3995	18.1764	31.6613
34	6.2574	8.0222	11.1879	16.3803	18.1596	31.6175
36	6.2506	8.0172	11.1817	16.3634	18.1454	31.5791
38	6.2450	8.0147	11.1824	16.3484	18.1335	31.5450
40	6.2406	8.0141	11.1888	16.3349	18.1235	31.5145

Table A.79. Dynamic analysis (CS=3, RVM, γ=8, n=5, c=2)

γ=8	Frequencies					
2k	ω1	ω2	ω3	ω4	ω5	ω6
2	7.4505	10.2296	14.6472	18.9625	19.9223	23.8995
4	6.7716	8.7023	12.8002	17.5371	17.8240	20.8309
6	6.6772	8.5056	12.4352	17.2446	17.7035	19.9677
8	6.6155	8.4155	12.3355	17.1042	17.8461	19.4732
10	6.5555	8.3501	12.2768	16.9903	17.7719	19.1426
12	6.5015	8.2979	12.1534	16.8864	17.5234	18.9008
14	6.4552	8.2544	11.9879	16.7929	17.2259	18.7167
16	6.4160	8.2164	11.8220	16.7109	16.9359	18.5774
18	6.3840	8.1820	11.6750	16.6403	16.6759	18.4731
20	6.3569	8.1508	11.5514	16.4540	16.5800	18.3946
22	6.3344	8.1219	11.4505	16.2721	16.5288	18.3345
24	6.3155	8.0960	11.3699	16.1282	16.4850	18.2880
26	6.2996	8.0731	11.3073	16.0192	16.4475	18.2509
28	6.2861	8.0539	11.2597	15.9404	16.4154	18.2210
30	6.2750	8.0380	11.2254	15.8873	16.3876	18.1966
32	6.2654	8.0256	11.2020	15.8558	16.3637	18.1764
34	6.2574	8.0165	11.1879	15.8422	16.3427	18.1596
36	6.2506	8.0103	11.1817	15.8438	16.3245	18.1454
38	6.2450	8.0069	11.1824	15.8583	16.3084	18.1335
40	6.2406	8.0062	11.1888	15.8838	16.2943	18.1235

Table A.80. Dynamic analysis (CS=3, RVM, γ=9, n=4, c=1)

γ=9	Frequencies					
2k	ω1	ω2	ω3	ω4	ω5	ω6
2	10.7037	22.1799	23.3197	36.0895	37.2671	42.0343
4	9.6447	20.4789	21.2150	30.9323	36.2119	37.7499
6	9.5467	20.2157	20.4710	29.9245	36.2846	36.6739
8	9.5087	20.0031	20.0587	29.4185	36.0314	36.5068
10	9.4789	19.7708	19.9453	29.1500	35.7898	36.5982
12	9.4538	19.6410	19.8494	28.9658	35.6386	36.6374
14	9.4199	19.5097	19.7525	28.7740	35.4093	36.6238
16	9.3678	19.3308	19.6502	28.5436	35.0149	36.5715
18	9.2960	19.1010	19.5461	28.2826	34.4730	36.4962
20	9.2133	18.8450	19.4463	28.0183	33.8818	36.4108
22	9.1321	18.5988	19.3560	27.7798	33.3545	36.3239
24	9.0645	18.3936	19.2780	27.5868	32.9607	36.2403
26	9.0161	18.2465	19.2131	27.4469	32.7151	36.1624
28	8.9878	18.1596	19.1612	27.3578	32.5983	36.0910
30	8.9761	18.1253	19.1204	27.3123	32.5786	36.0261
32	8.9769	18.1320	19.0891	27.3010	32.6260	35.9672
34	8.9861	18.1685	19.0657	27.3156	32.7165	35.9139
36	9.0011	18.2248	19.0485	27.3486	32.8330	35.8654
38	9.0191	18.2933	19.0361	27.3942	32.9632	35.8213
40	9.0385	18.3683	19.0276	27.4479	33.0993	35.7810

Table A.81. Dynamic analysis (CS=3, RVM, γ=9, n=5, c=1)

γ=9	Frequencies					
2k	ω1	ω2	ω3	ω4	ω5	ω6
2	10.7037	21.6797	22.4214	36.0895	37.2671	42.0343
4	9.6447	19.3136	19.9324	30.9323	36.2119	37.7499
6	9.5467	19.0436	19.7257	29.9245	36.2846	36.6739
8	9.5087	18.9552	19.6465	29.4185	36.0314	36.5068
10	9.4789	18.9095	19.5818	29.1500	35.7898	36.5982
12	9.4538	18.8780	19.4967	28.9658	35.6386	36.6374
14	9.4199	18.8428	19.3508	28.7740	35.4093	36.6238
16	9.3678	18.7953	19.1242	28.5436	35.0149	36.5715
18	9.2960	18.7367	18.8416	28.2826	34.4730	36.4962
20	9.2133	18.5592	18.6738	28.0183	33.8818	36.4108
22	9.1321	18.3280	18.6150	27.7798	33.3545	36.3239
24	9.0645	18.1709	18.5670	27.5868	32.9607	36.2403
26	9.0161	18.0849	18.5325	27.4469	32.7151	36.1624
28	8.9878	18.0555	18.5109	27.3578	32.5983	36.0910
30	8.9761	18.0660	18.5001	27.3123	32.5786	36.0261
32	8.9769	18.1022	18.4976	27.3010	32.6260	35.9672
34	8.9861	18.1542	18.5008	27.3156	32.7165	35.9139
36	9.0011	18.2147	18.5080	27.3486	32.8330	35.8654
38	9.0191	18.2791	18.5174	27.3942	32.9632	35.8213
40	9.0385	18.3442	18.5282	27.4479	33.0993	35.7810

Table A.82. Dynamic analysis (CS=3, RVM, γ=9, n=4, c=2)

γ=9	Frequencies					
2k	ω1	ω2	ω3	ω4	ω5	ω6
2	7.0367	10.2467	14.5215	18.2738	20.6126	22.5563
4	6.4292	9.1132	12.8018	17.6047	17.9855	19.9484
6	6.3608	8.8153	12.3734	17.6111	18.0419	19.6124
8	6.3498	8.6449	12.1659	17.5895	18.6870	19.4621
10	6.3451	8.5469	12.0650	17.5370	19.1779	19.3507
12	6.3392	8.4800	11.9950	17.4703	19.2486	19.3730
14	6.3305	8.4190	11.9080	17.3989	19.1471	19.3896
16	6.3182	8.3540	11.7875	17.3284	19.0467	19.3683
18	6.3032	8.2858	11.6447	17.2618	18.9515	19.4027
20	6.2869	8.2195	11.5037	17.2004	18.8650	19.5352
22	6.2722	8.1612	11.3855	17.1445	18.7894	19.7712
24	6.2590	8.1142	11.3004	17.0942	18.7248	20.0964
26	6.2490	8.0799	11.2490	17.0489	18.6707	20.4895
28	6.2414	8.0570	11.2263	17.0082	18.6259	20.9290
30	6.2366	8.0439	11.2256	16.9716	18.5888	21.3973
32	6.2330	8.0390	11.2408	16.9386	18.5582	21.8802
34	6.2310	8.0399	11.2670	16.9089	18.5329	22.3679
36	6.2298	8.0455	11.3004	16.8818	18.5122	22.8534
38	6.2294	8.0539	11.3382	16.8572	18.4949	23.3321
40	6.2294	8.0647	11.3785	16.8348	18.4804	23.8005

Table A.83. Dynamic analysis (CS=3, RVM, γ=9, n=5, c=2)

γ=9	Frequencies					
2k	ω1	ω2	ω3	ω4	ω5	ω6
2	7.0367	10.0260	14.5215	17.9040	20.0386	22.5563
4	6.4292	8.5302	12.8018	16.6205	17.9208	19.9484
6	6.3608	8.3803	12.3734	16.4142	17.4494	19.6124
8	6.3498	8.3469	12.1659	16.3501	17.2926	19.4621
10	6.3451	8.3274	12.0650	16.3244	17.2683	19.3507
12	6.3392	8.3042	11.9950	16.3106	17.2274	19.2486
14	6.3305	8.2689	11.9080	16.2992	17.0753	19.1471
16	6.3182	8.2198	11.7875	16.2874	16.8103	19.0467
18	6.3032	8.1630	11.6447	16.2748	16.5071	18.9515
20	6.2869	8.1080	11.5037	16.2450	16.2624	18.8650
22	6.2722	8.0632	11.3855	16.0599	16.2505	18.7894
24	6.2590	8.0315	11.3004	15.9521	16.2398	18.7248
26	6.2490	8.0122	11.2490	15.9058	16.2304	18.6707
28	6.2414	8.0025	11.2263	15.9038	16.2224	18.6259
30	6.2366	8.0006	11.2256	15.9313	16.2154	18.5888
32	6.2330	8.0034	11.2408	15.9781	16.2096	18.5582
34	6.2310	8.0094	11.2670	16.0368	16.2045	18.5329
36	6.2298	8.0172	11.3004	16.1025	16.2000	18.5122
38	6.2294	8.0262	11.3382	16.1717	16.1961	18.4949
40	6.2294	8.0359	11.3785	16.1926	16.2425	18.4804

Table A.84. Static analysis (CS=6, RVM, γ=3, c=1)

c=1	w_{max}	w_{max}	w_{max}	w_{max}	w_{max}
γ=3	r=1	r=3	r=6	r=10	r=15
2k	n=0	n=2	n=4	n=6	n=8
2	0.0113	0.0110	0.0119	0.0118	0.0122
4	0.0100	0.0112	0.0142	0.0141	0.0148
6	0.0076	0.0116	0.0148	0.0147	0.0150
8	0.0061	0.0118	0.0149	0.0149	0.0150
10	0.0052	0.0118	0.0150	0.0149	0.0150
12	0.0046	0.0118	0.0149	0.0149	0.0150
14	0.0041	0.0119	0.0149	0.0149	0.0149
16	0.0035	0.0120	0.0149	0.0149	0.0149
18	0.0029	0.0121	0.0149	0.0149	0.0150
20	0.0024	0.0121	0.0149	0.0150	0.0151
22	0.0020	0.0121	0.0149	0.0151	0.0154
24	0.0017	0.0119	0.0150	0.0152	0.0156
26	0.0015	0.0117	0.0150	0.0153	0.0159
28	0.0013	0.0115	0.0151	0.0155	0.0162
30	0.0012	0.0113	0.0151	0.0156	0.0165
32	0.0011	0.0111	0.0152	0.0157	0.0167
34	0.0010	0.0109	0.0152	0.0158	0.0169
36	0.0010	0.0107	0.0152	0.0158	0.0170
38	0.0009	0.0105	0.0152	0.0158	0.0170
40	0.0009	0.0103	0.0152	0.0159	0.0170

Table A.85. Static analysis (CS=6, RVM, γ=3, c=2)

c=2	w_{max}	w_{max}	w_{max}	w_{max}
γ=3	r=1	r=3	r=6	r=10
2k	n=0	n=2	n=4	n=6
2	0.0159	0.0198	0.0221	0.0224
4	0.0108	0.0168	0.0232	0.0241
6	0.0072	0.0155	0.0230	0.0247
8	0.0054	0.0145	0.0226	0.0250
10	0.0044	0.0136	0.0223	0.0250
12	0.0036	0.0129	0.0221	0.0250
14	0.0030	0.0125	0.0220	0.0251
16	0.0025	0.0122	0.0218	0.0251
18	0.0021	0.0120	0.0216	0.0252
20	0.0018	0.0118	0.0215	0.0252
22	0.0015	0.0115	0.0213	0.0253
24	0.0013	0.0112	0.0212	0.0253
26	0.0011	0.0109	0.0211	0.0254
28	0.0010	0.0106	0.0211	0.0254
30	0.0009	0.0103	0.0210	0.0254
32	0.0008	0.0100	0.0210	0.0255
34	0.0008	0.0097	0.0209	0.0255
36	0.0007	0.0095	0.0209	0.0255
38	0.0007	0.0092	0.0209	0.0255
40	0.0007	0.0090	0.0208	0.0254

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