

THE IMPACT OF HUMAN CAPITAL ON THE SOCIAL SECURITY SYSTEM OF  
TURKEY

by

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B.S. in Math.E., İstanbul Technical University, 2006

Submitted to the Institute for Graduate Studies in  
Science and Engineering in partial fulfillment of  
the requirements for the degree of  
Master of Science

Graduate Program in Industrial Engineering

Boğaziçi University

2010

## ACKNOWLEDGEMENTS

First of all, I would like to thank to my thesis supervisor, Assist. Prof. Aybek Korugan, for his guidance, support and motivation throughout my thesis.

I am very thankful to members of my committee who generously contributed me; namely to Prof. Dr. İlhan Or and Assoc. Prof. Aslı Erdem for suggesting me many helpful revisions.

I am also grateful to my friends who motivated me during my thesis study. Especially, I owe a debt of gratitude to Fatma Akpınar for her close interest and help.

Furthermore, special thanks go to my boyfriend, İnanç Gülfidan, for being always with me by giving endless support and motivation.

I wish to utter my special appreciation to my family for their support, encouragement and patience not only for this thesis but also for my entire life.

I am also grateful to Turkish Scientific and Technological Research Institute (Türkiye Bilimsel ve Teknik Araştırma Kurumu- TÜBİTAK) for the scholarship.

## **ABSTRACT**

### **THE IMPACT OF HUMAN CAPITAL ON THE SOCIAL SECURITY SYSTEM OF TURKEY**

The purpose of the study is to analyze the impact of human capital, which is based on the years of education and working experience, on the duration of education, working and retirement which maximize the average lifetime utility by satisfying the constraints of the social security system in Turkey.

In order to conduct the study, firstly, a general nonlinear optimization model that considers average education, working and retirement duration is constructed. After the determination of the human capital by conducting the regression analysis for determined datasets, the years of education, working and retirement period is determined within the PAYG pension plan structure of Turkey considering the current system parameters. Two models are constructed in the study. The first model examines the impact of human capital on the average income and decisions while the second model examines the impact of human capital by taking production in the economy into account. In other words, the optimization model is modified to measure only the impact of education and working experience and then the education and working experience impact with the economic growth concept.

The models are run for different levels of social security and human capital parameters in order to examine the behaviors of the models due to the changes of these parameters. By the developed models in the study, the optimal duration of education, working and retirement, which maximize the average lifetime utility gained from the consumptions during working and retirement, are explained with the contribution of economic growth theory, which is based on human capital.

## ÖZET

### BEŞERİ SERMAYENİN TÜRKİYE SOSYAL GÜVENLİK SİSTEMİ ÜZERİNDEKİ ETKİSİ

Bu çalışma eğitim süresi ve çalışma tecrübesine bağlı olarak ele alınan beşeri sermayenin, Türk sosyal güvenlik sisteminin kısıtlarını sağlayarak, ortalama yaşam yararlılığını maksimize eden eğitim, çalışma ve emeklilik süresi üzerindeki etkisini analiz etmeyi amaçlamaktadır.

Çalışmayı yürütmek için ilk olarak eğitim, çalışma ve emeklilik sürelerini ele alan genel bir nonlinear optimizasyon problemi oluşturulmuştur. Beşeri sermayenin belirlenmiş veri setlerinin regresyon analizi ile belirlenmesinden sonra, bireylerin alacakları eğitim süresi, çalışma ve emeklilik süreleri üzerindeki kararları PAYG tipi emeklilik sisteminin kapsamındaki aktüel parametreler de göz önünde bulundurularak belirlenmiştir. Çalışmada iki model oluşturulmuştur. İlk modelde beşeri sermayenin ortalama gelir üzerindeki etkisi ele alınırken, ikinci modelde beşeri sermayenin etkisi ekonomideki üretim kavramı göz önünde bulundurularak incelenmiştir. Başka bir deyişle, optimizasyon problemi sadece eğitim ve çalışma süresinin etkisini açıklayacak, daha sonra da eğitim ve çalışma süresinin etkisini ekonomik büyüme kavramı içinde açıklayacak şekilde belirlenmiştir.

Kurulan modeller farklı sosyal güvenlik ve beşeri sermaye parametreleri için çalıştırılarak, modellerin bu parametre değişimleri karşısındaki davranışı incelenmektedir. Bu çalışmada geliştirilen modellerle, bireylerin yaşamlarında çalışma ve emeklilik dönemlerindeki tüketimlerinden elde ettikleri ortalama yararlılığı maksimize eden optimal eğitim, çalışma ve emeklilik süreleri beşeri sermayeyi temel alan ekonomik büyüme teorisiyle açıklanmaktadır.

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## LIST OF SYMBOLS / ABBREVIATIONS

b	Retirement benefit
C	Consumption
E	Education expenditure per child
k	Capital per worker
K	Aggregate capital stock
<i>l</i>	Leisure time (retirement)
L	Labor force
n	Population growth rate
N	Number of children per consumer
r	Interest rate
S	Saving
W	Wage
$X_1$	Education duration
$X_2$	Working duration
$X_3$	Retirement duration
Y	Aggregate output
$\beta_1$	Share of education on wage
$\beta_2$	Share of working experience on wage
DB	Defined benefit
DC	Defined contribution
GDP	Gross domestic product
GE	General equilibrium
PAYG	Pay-as-you-go
SE	Standard error

## 1. INTRODUCTION

Social security has a very important place in the lives of individuals since it is a system which is constructed to protect individuals and their dependents when they are exposed to income loss, which occurs in the case of retiring from working because of old age or disability, or outcome increases, which occurs in the case of illnesses. The most important component of the social security systems is pension systems. The aim of pension systems is to sustain the individuals' life standards, which they had throughout their working period, in their elderly or in their retirement. Developing and developed countries have begun to attach importance to the structure of their pension plans due to the problems which threaten the sustainability of the social security systems. Since the financial problems cause important deficits in the economies of the countries, many modelling studies have been introduced to cope with the problems in social security systems for more than 30 years.

Economic growth theories aim to discover the components which determine economic growth. Economists began to develop mathematical models in order to understand the growth process. There are many growth models depending on different assumptions, and the two main models of economic growth are neoclassical and endogenous growth models. Human capital is an important concept which has drawn interest since neoclassical growth theories started to be inadequate to explain the economic growth due to the widening gap between technological progress of developing and developed countries. It began to be used for explaining the unexplained part of the neoclassical growth theories at the beginning of the 1960s. Endogenous growth theories, which have been debated since the early 1980s, determined human capital as the engine of the economic growth. After the appearance of endogenous growth theories, human capital has been used in modelling the economies as a determinant of productivity. Since productivity is an indicator of a good living standards and income, many studies have been conducted to analyze the impact of human capital on individual's earning.

While analyzing the economic growth, it is important to consider the preferences of individuals in order to examine the economy completely. General equilibrium models,

which are the measurement methods for examining the effects of fiscal policies on the individuals' consumption and welfare, are used to analyze the preferences of individuals within the production in the economy. The most important of the fiscal policies which is examined in general equilibrium models is social security systems since it has a significant effect on the decisions of individuals such as consumption and saving levels which maximize their lifetime utilities.

In this study, different from the general equilibrium studies in the literature, instead of deciding the consumption and saving levels, the model is constructed so that it determines the average years of education, working and retirement period in order to maximize the average lifetime utility. Moreover, this study differs from many studies by the definition of human capital which is used to explain the income of individuals. Since we consider human capital not only the knowledge obtained during education at school but also during the working period, we try to construct average income as a function of the years of education and working. Furthermore, defining human capital as a function of education and working naturally brings the fact that individuals need to be considered as young, working and retired in the study. Instead of using arbitrarily determined values for parameters of model as done in many studies in the literature, we use real data analysis results. Considering these differences and contributions, we aim to examine the impact of human capital on the optimal duration of education, working and retirement within the context of social security system in Turkey with two models. First model is covered by directly defining the earnings (income/wage) of individuals by Mincer earning function without considering general equilibrium while the second is covered by the income function obtained by the production side of the general equilibrium model structure.

The rest of the thesis is organized as follows. In Chapter 2, a literature review on social security systems is presented. This chapter explains the general equilibrium models and gives a quick overview of the main structure which comprises the framework of general equilibrium models such as economic growth and overlapping generations models. Moreover, the structure and parameters of the base models which form the production side of our model is covered in terms of understanding the main structure to be used. In addition, the studies related to the social security modelling with general equilibrium models are discussed briefly. Chapter 3 states the objective of the study. Chapter 4 covers

the construction of two models. Chapter 5 contains the analysis of two models which is developed in Chapter 4. Moreover, the construction of the dataset to be used in the second model is discussed comprehensively. The parameters of the base case analysis are discussed. Afterwards, the sensitivity of the models to deviations in the assumed values of parameters for the base case are investigated for the first and second model, respectively. Finally, the general conclusions derived from the analysis study are covered in Chapter 6.

## **2. BACKGROUND & LITERATURE SURVEY**

In this chapter, firstly social security and pension plans are reviewed. Secondly, social security system in Turkey is summarized. Afterwards, general equilibrium models and the main sub-structures, which all constitute the background of our models which will be constructed in Chapter 4, is covered. Finally, the modelling studies in literature are discussed briefly in order to recognize the place of our models among them.

### **2.1. Social Security Systems**

Social security is a governmental program designed to protect individuals and their families from the economic insecurity which can be caused by the income reduction due to sickness, unemployment, disability, retirement or death and is also designed to guarantee they have a healthy and minimum standard of living. In addition to retirees who receive retirement benefits, spouses and other family members can utilize from the benefits provided by the social security program. Social security is also an insurance mechanism since it provides disability and survivor's benefits and a medical program to people within the coverage of the social security program.

Since individuals sought to protect themselves against risks in any period of the history of humanity, the social security concept has been an important part of the social life. Furthermore, social security systems play a major role in preventing the poverty and the inequality of income distribution and also in providing the social welfare. There exists a social security system in the countries all around the world. Although the concept of social security is a need emerged along with the human existence, it was begun to be used for the first time in 20th century. It was first covered in "Social Security Act" approved in USA, in 1935. Afterwards, William Beveridge prepared Beveridge Report which handled social security more detailed.

In 1944, social security was discussed comprehensively in the International Working Conference organized in Philadelphia; thus, the necessity of social security in theory and practice was introduced. Moreover, the concept of social security acquired a global

qualification by the Human Rights Universal Statement approved by the United Nations General Assembly in 1948 (Bilman, 2007).

### **2.1.1. Pension Plans**

Pension is the long-term periodical benefit paid to a retired person for his contribution to the social security system while working. Pension plans are the regulations made for the individuals of social security system, who fulfill the employment duration and age conditions, to receive benefits. The aim of the pension plans is not to sustain the same welfare standard in their working period but to prevent them from being dependent on others by providing a reasonable standard for the rest of their life.

Pension systems can be classified according to several dimensions. In this study, we are interested in two of these dimensions which are benefit determination and financing methods of the pension plans.

2.1.1.1. Benefit Determination Methods: Depending on the basis of benefit determination, pension plans are classified into three categories; defined benefit pension plan, defined contribution pension plan and hybrid pension plans. Pension is provided by three major pension plan types:

- (i) Defined benefit (DB)
- (ii) Defined contribution (DC)
- (iii) Hybrid plans

Defined benefit and defined contribution plans are the most widely used types of pension plans. The main point which DB and DC plans differ is the fact that whether the amount of benefits or the annual contributions are primarily determined.

#### **(i) Defined Benefit (DB) Plan**

In defined benefit plans, the retirement benefits are determined by a formula based on the earnings and service year of social security system contributors but without

considering the contributions and the return of the contributions. Here first, the benefits to be paid during retirement are defined and then required contributions are evaluated by the plan formula. Generally, social security systems in the world are financed by defined benefit pay-as-you-go system. Most of the DB plans, pay death and disability benefits to the contributors beside the old-age benefit.

#### (ii) Defined Contribution (DC) Plan

Defined contribution plan is another type of pension plan which is based on the amount of the contributions made by contributors of the system and the investment return of these contributions. In DC plans, the contributions of the employee are primarily defined and afterwards the benefits are calculated depending on the defined contributions. The contributions of individuals are kept in the funds belong to them and are invested; moreover, the proceeds from contributions and investments are used to buy annuities at retirement. In this plan the individuals have investment risk, since they are given the options of where to invest the account. The individual pension system that was introduced by the 1999 social security reform in Turkey is an application of DC plans.

#### (iii) Hybrid Plans

The other type of pension plans is hybrid plans which are constructed by combining the characteristics of DB and DC plans. In hybrid plans, benefits are determined by a formula similar to DB plans and the funds are directed by employers. Moreover, the formula which is used to calculate benefits is based on the earnings and interest return instead of service year in the system. As DC plans, contributors have individual accounts which consist of the contributions collected during working period and retirement benefits are paid as lump sum or paid by annuities.

2.1.1.2. Financing Methods: The pension plans can be classified into two categories depending on the basis of financing; pay-as-you-go (unfunded) and funded pension plan.

#### (i) PAYG (Unfunded) Pension Plans

PAYG system can be defined as a social contract between working and retiree generations. The current social security expenditures are financed by the income provided from the current working population. In other words, the payroll taxes collected from the current working population are used to finance the retirement benefits paid to the current retirees.

(ii) Funded Pension Plans

In funded pension plans the premiums paid by workers are accumulated in funds. Participants pay premiums to these funds in order to avoid financial embarrassment at retirement and secure their future. Payments accumulated in funds and earning on investments are paid to workers when they retire.

### **2.1.2. The Social Security System in Turkey**

When we look at the evolution of social security in Turkey, we can see that the strong public spirit which fundamentally takes part in the Turkish traditions, constituted the basis of Turkish social security. The Turkish social security, afterwards, institutionalized during the Republic period. After 1923, it was tried to construct a government and society respectful to the rights by the Ataturk's revolutions. Beginning from the 1930s, the effort of government for strengthening the weak economy provided the rise of the public finance institutions. In addition, Public Health Protection Law, Municipal Corporations Law, Labor Law were enacted and these laws underlay the current social security institutions in Turkey.

The social security system in Turkey is constructed by three different major organizations:

- Social Insurance Institution (SSK)
- Pension Fund for Civil Servants (ES)
- Social Security Institution for the Self-Employed (BK)

2.1.2.1. Social Insurance Institution: SSK is the social security organization which covers the highest number of participants. Private sector and blue-collar public sector workers are insured for industrial accidents and occupational diseases, sickness (health), maternity, disability, old-age and death. Old-age insurance comprises the insured persons who have provided the requirements of contribution payments and have fulfilled the required insurance time and pays them retirement benefits. In the case of the insured person's death, the spouse and orphan or the parents of him are entitled for the survivor pension benefits. The disability insurance entitles the disabled people who have lost their working ability or who have been determined not to be in convenient condition for working.

The main financing resource of SSK is the premiums which are paid by employers and employees. SSK works on the basis of PAYG and DB structure.

2.1.2.2. Pension Fund for Civil Servants: ES is a social security organization which covers all civil servants, the personnels of the special provincial administration and municipality. The fund of ES is constructed by the subsidies cut from the wages of civil servants, the payments of the institutions for which they work and the proceeds provided from the management of the fund. ES provides retirement, disability, occupational disability, spouse and orphan benefits and retirement bonuses for participants.

2.1.2.3. Social Security Institution for the Self-Employed: BK is the organization with the highest number of participants after SSK. This organization covers the self-employed people which are not in the coverage of the SSK and provides old-age, disability, death and health insurance for its participants.

BK is financed by the premiums paid by insured people in its coverage, grants given by real and legal people, the proceeds gained from the portable and not portable goods of the institution, the subsidies from the public budget and other incomes.

The participants of these three social security organizations, SSK, ES, Bag-Kur, were unified under one roof by the Act. No. 5510 which was enacted on October 1<sup>st</sup> 2008.

## **2.2. Modelling the Pension Plans**

The models of pension systems are generally based on the general equilibrium models. General equilibrium models have turned out to be very important measurement methods for examining the effects of all sorts of fiscal policies on the individual's consumption and welfare. Moreover, reform schemes, which lead government bills, have been introduced and the macroeconomic effects of these pension schemes have been searched out especially by using overlapping generations models.

The main purpose of this chapter is to present a brief summary on the general framework of these types of models and the modelling studies in the literature.

### **2.2.1. General Equilibrium Models**

The general equilibrium models consist of goods, services, prices and public policies and satisfy the conditions stated below:

(i) Given the policy and prices, computed amount of the goods and services solve the optimization problem of the individuals which is choosing the optimal consumption, saving, investment and labor supply which maximize their lifetime utility.

(ii) Given the policy and prices, production units maximize their profits by equating factor prices, which are interest rate and wage, to the marginal product of the production inputs, which are capital and labor, respectively.

(iii) The labor supply of individuals has to be equal to labor demand of the firms.

(iv) The budget of government is balanced by the consumptions and taxes.

Hence, individuals maximize their utilities and firms maximize their profits satisfying a defined budget constraint.

GE models generally prefer individuals or sometimes households as decision maker. The aim of this decision maker is to choose the optimal consumption, saving, investment and labor supply in the current economic environment. The time span which the decision maker considers while deciding is his own lifetime duration. Individual likes consumption; however, requires working for earning and for providing consumption and saving. Moreover, the objective function which the individual tries to maximize is a function of the consumption, saving and labor supply which are chosen by the decision maker. On the other hand, individual prefers current consumption to future consumption for his lifetime optimization which means that a lower weight is given to the future consumption during optimization.

The production technology is represented by a production function which uses labor and capital as inputs and produces Gross Domestic Product (GDP). The elements required for the production of goods and services are named as factors of production which are capital, labor, natural resources and entrepreneurship. Moreover, the factor price is the price paid for and received by the services of the factors of production.

The production agents (firms) demand labor and hire the capital owned by individuals and they pay for labor and capital by the factor prices wage and interest rate determined by the general equilibrium conditions. The production technology of the GE models is based on the growth theory (Tusiad, 2004).

### **2.2.2. Economic Growth Theory**

Economic growth can be defined as the change in the amount of the production of goods and services over time and it often refers to the change rate of gross domestic product (GDP).

The economic growth rate is affected by natural, human and capital resources and technological progress. The growth theory can be covered in two approaches: neoclassical (exogenous) growth and endogenous growth.

2.2.2.1. Neoclassical (Exogenous) Growth: In neoclassical growth theory, which is based on the fact that growth is exogenous, population and technological progress are determined exogenously. This type of growth models emphasizes the importance of physical capital and labor force during the process of growth. Neoclassical economists suppose that increasing the labor supply and improving the productivity of labor and capital raise the long-term growth rate of economy.

The Harrod-Domar model which was developed by Harrod (1939) and Domar (1946) was the pioneering study of neoclassical growth models. The model defined production as a function of capital stock. Solow (1956) made an important contribution to the growth theory by extending the Harrod-Domar model. In addition to physical capital, Solow added labor as a factor in the production function. His model assumed diminishing returns to physical capital and labor and constant returns to scale for physical capital stock and labor together. Solow also included a technology variable as a function of time. Solow model can be stated briefly as follows:

2.2.2.2. Solow Model: Solow model takes population growth rate, saving rate and technological progress as exogenous variables. The production function can be denoted as:

$$Y(t) = K(t)^\alpha (A(t)L(t))^{1-\alpha} \quad 0 < \alpha < 1 \quad (2.1)$$

where Y is aggregated production output, K is aggregated physical capital, L is labor and A is the level of technology. L and A are assumed to grow exogenously.

2.2.2.3. Endogenous Growth: Since technological progress was determined as exogenous variable in neoclassical growth models, it remained unexplained in the analysis of the growth theories. Hence, in 1980s, the endogenous growth theory was introduced as an alternative to neoclassical growth models. The fundamental of endogenous growth theory was improved Romer (1986) and Lucas (1988) by taking population and technological progress as endogenous variables.

Endogenous growth models can be classified into three groups: human capital due to education investments, technological progress due to Research&Development (R&D) investments and technological progress due to learning-by-doing externalities.

The approach of endogenous growth models with human capital due to education investments is based on the study of Uzawa (1965) proposing that human capital, which is accumulated through households' education investments, drives the productivity of the economy. Lucas (1988) extended the Uzawa's model into the endogenous growth literature. The R&D model, which proposes that the investments in R&D incite the productivity of economy, is based on the study of Nelson and Phelps (1966) and was extended by Romer (1990), and Aghion and Howitt (1992). Learning-by-doing model, which proposes that knowledge gained during the production process increases the productivity of the economy, is based on the study of Arrow (1962) and it was improved by Romer (1986) in the endogenous growth concept.

### **2.2.3. Human Capital**

Human capital is an economic term which can be defined in many ways. In OECD (1998), human capital is defined as the knowledge, skills, competences and other attributes formed in individuals who are related to economic activity. The forerunner of human capital study was published in 1964 by Professor Gary S. Becker who is a Nobel Laureate. According to him, the macroeconomic aspect of education and human capital investment make contribution to economic growth and the value of education and returns of educational investment have raised (Becker, 1993). In macroeconomic theory, economic growth can be explained by factors of physical capital, natural resources, technology and human capital. Since organizations and nations has been increasingly recognizing that high level of knowledge, skills and competence are fundamental to their future security and welfare, human capital has begun to draw attention among other factors. Therefore, as stated in OECD (1998), political and social expectations for reaching social economic goals through human capital investment have increased. However, the investments in human capital need to be well designed to meet desired objectives and therefore these expectations. This requires a good understanding of the features of human capital and its role in individual, social and economic welfare. Thus, there are many studies in order to

understand the measurement of human capital and its return. Income differences across workers are based on the differences in the market values of their human capital inputs. Moreover, these market values are highly determined by the differences in human capital inputs of workers. Samuelson (1995) states that highly educated and skilled people have an advantage of earning more in the labor market as a result of the return of their investment in education.

There are many approaches to measure human capital. The most common approaches are human capital investment and years of schooling. However, as Jeong (2001) states that the method which uses years of schooling has some drawbacks since it does not measure the human capital obtained outside the school such as in job learning. It cannot be disregarded that workers continue to acquire human capital after school when they work. Depending on this fact, we defined human capital not only as a function of the years of schooling but also as a function of the years of working.

Lucas (1988, 1993) suggested that human capital was more important than physical capital for the determination of the long-term growth process. According to his approach, investment on education provides increasing return to scale by creating positive externalities. The model which is introduced by Lucas is as shown below:

The production function of Lucas was defined in per-capita terms:

$$y = Ak^\beta [uh]^{1-\beta} \quad (2.2)$$

where  $u$  is the fraction of non-leisure time agents spend working, so producing the output  $y$  and  $h$  is the measure of the average quality of workers and  $L$  is the number of working agents. The term  $uh$  is called human capital. This production function indicates constant returns to scale in  $k$  and  $uh$ . In addition, Lucas assumed that people get more productive when they are with intelligent people and insert the average human capital term,  $h_a^\varphi$ , into the model.

$$y = Ak^\beta [uh]^{1-\beta} h_a^\varphi \quad (2.3)$$

This term was included to analyze the results of the migration across countries.

Moreover, if the economy is in steady state, growth rate per capita is equal to the human capital growth rate per capita. Mankiw *et al.* (1992) introduced the augmented Solow model by adding human capital accumulation to the Solow model. Furthermore, they took human capital investment in the form of education by ignoring investments in health. The model structure which we adopted is augmented Solow model and it can be stated as follows:

The augmented Solow model defines production function as:

$$Y(t) = K(t)^\alpha H(t)^\beta (A(t)L(t))^{1-\alpha-\beta} \quad (2.4)$$

where the added variable H is aggregated human capital. Mankiw *et al.* assume that  $\alpha + \beta < 1$ , implying that there are decreasing returns to physical and human capital.

One category of the GE model is overlapping generations (OLG) models which are used to analyze the effects of pension reforms. OLG model, which was first introduced by Samuelson (1958) and Diamond (1965), is a general structure used in GE models for analyzing the economic studies, such as macroeconomic effects of public pension systems. In OLG model, agents are considered as consumers who live for a finite length of time. Agents consume in all periods and their lifetime utility is defined over their consumption in all periods.

In the literature, there are many methods of OLG modelling. In the most basic OLG model, which suppose that agents live for two periods, there are two cohorts; young generation which works and old generation which has retired.

At any time t, there are young agents and old agents alive. If  $N_t$  is defined as the number of agents born in period t, it is obvious that there are  $N_t$  young agents and  $N_{t-1}$  old agents alive in period t.

		Time		
		1	2	3
Generations	G1			
	G2			
	G3			
	G4			

Figure 2.1. The demographic structure of OLG models

### 2.3. Modelling Studies in Literature

Auerbach and Kotlikoff (1987) developed a 55-period overlapping generations model, meaning 55 generations of agents, in order to analyze the social security system and fiscal policies effects. They assumed agents to enter the system by starting to work at age 21 and die at age 75, and hereby each period corresponded to one year.

Pak (2002) developed a model which involves death and disability old age benefits procured under SSK Act. No. 506. In the model, participation age, contribution period, retirement age, contribution rate, real interest rate was used as parameters. In this study, it was aimed to examine the effects of parameter changes on a defined benefit pay-as-you-go pension system that used the reformed parameters introduced by social security reform in September 1999, in Turkey, and to determine the same pension system participants which gain and lose benefits. The deterministic model constructed by Pak was analyzed by Individual Actuarial Balance (IAB) Analysis. The analysis yielded the results that the redesigned parameters are more promising for the pension system sustainability in contrast to the old parameters.

Yew and Zhang (2008) examine pay-as-you-go social security by adding human capital investments and endogenous fertility in a dynastic family model. The model that was developed consists of agents who live for 3 periods which are education, working and retirement. This study differs from the neoclassical models introduced before since it includes the human capital investment. They formulize the education of a child such that it depends on final good investment per child, human capital of his or her parents and the average human capital in the economy.

Forni (2004) worked on a standard overlapping generations model in which the agents live for 2 periods, that is, agents work in the first period and retire in the second period of their life. While developing the model, Forni concentrated on Markov subgame perfect equilibrium of median voting mechanism with punishment. The model includes a game among generations in which a median voter is determined among the members of young generation. The median voter has to decide the payroll tax the young generation has to pay to the current old generation whose members receive retirement benefits. This game is also supported by a punishment mechanism. If a generation does not transfer the required resource amount, it will be punished and the future old age transfer for it will be lowered. Moreover, since the model is Markov subgame perfect equilibrium, the government strategies depend only on capital stock which is state variable at time  $t$  and the strategies of short-lived governments try to maximize the lifetime utility of young generation.

Imrohoroglu (1995) *et al.* developed an overlapping generations model consists of agents who live for 65 periods which are each of 1 year. The model took into consideration the concept of accidental bequest. Moreover, stochastic survival and employment opportunities for each agent were included in the model. It was aimed to analyze the welfare benefits resulted from the arrangements on the optimal social security replacement rate. Since the human capital investment was not used in the construction of the model, it is an neoclassical (exogenous) growth model.

Deger (2008) presented an overlapping generations model whose agents live for 6 periods; 4 periods were allocated for working while 2 periods for retirement duration of households. Deger studied on the analysis of multiple social security systems with different tax payments and retirement benefits by using the 6-period OLG model. Furthermore, the model was constructed and analyzed on the basis of pay-as-you-go social security structure. However, human capital; education, investment on education and working experience, was not included in the consumption and production structure of the study.

Echevarria (2004) discussed the effects of life expectancy on education span, retirement age and economic growth in his study which covers a continuous time, overlapping generations model with endogenous growth. He obtained the optimal duration

of education and retirement age so as to maximize lifetime utility by employing different life expectancy values as the result of the analysis. In order to represent economic growth better, physical and human capital were used in the definition of production technology. Moreover, human capital was formulated by using the schooling time. The model was solved numerically and the analysis gave the result that increases in life expectancy lengthen the duration of education and raised the retirement age. Although the retirement age was included, the retirement benefit and payroll taxes were not considered in the construction of the model.

Auerbach and Kotlikoff (1987), Forni (2004), Imrohoroglu (1995) and Deger (2008) covered the general equilibrium model for two types of consumers, working and retiree and without considering human capital. On the other hand, Yew and Zhang (2008) and Echevarria (2004) dealt with the models by considering three types of consumers, young, working and retiree also by including human capital to their models. However, they do not use human capital as a function of the years of education and working experience. Since the aim of this thesis is to analyze the optimal duration of schooling, working and retirement, we defined human capital as the knowledge and ability obtained through education and experience.

### 3. PROBLEM DEFINITION

In the literature, there are various models studying social security systems with general equilibrium structure. In the models in which the human capital is not included for explaining the economic growth, only working and retired individuals are taken into account. In this study, we aim to analyze the impact of human capital on the individual's lifetime allocation, depending on the models which determine human capital as an engine of the economic growth. Therefore, it is additionally required to consider the young individual which gathers human capital by taking education before starting to work and entering the social security system.

In general equilibrium models, individuals choose the optimal consumption, saving, investment and labor supply in order to maximize their lifetime utility. However, in our study, different from general equilibrium models used in the analysis of social security systems, rather than considering individual's preferences we are interested in an average steady state condition and we try to obtain optimal duration of schooling, working and retirement which maximize the average lifetime utility. Different from the studies which include young individuals, we do not break down lifetime into fixed periods to let the model determines the required years of education, working and retirement satisfying the lifetime restriction of the study during the average lifetime.

In addition, instead of using human capital proxies such as human capital investment, secondary school enrollment, human development index, health investment, number of patents in a country, we define human capital as a function of the years of education and working since we want to obtain average income based on the years of education and working experience.

At the first step of the study, we aim to model the system by assuming an income function of the Mincer earning function form, without covering production side of the economy. At the second step, a general equilibrium model structure which uses human capital will be used based on the augmented Solow model structure. In order to analyze the optimal education, working and retirement periods which maximize the lifetime utility,

both of the models will be formed on three types of individuals; young, working and retired; in other words, the model considers that the lifetime consists of three parts; youth, adult and old. Afterwards, a sensitivity analysis of the model parameters will be carried out to examine how the model determines the duration of education, working and retirement as a result of the changes in human capital by satisfying the current social security system constraints in Turkey.

## 4. MODEL

In the study, a model with human capital is developed to analyze the PAYG social security system in Turkey. It is aimed to find the optimal duration of education, occupation and retirement by studying this model.

### 4.1 First Model

In this model an average lifetime utility is considered within the social security system. Here, a static social security model which includes the average lifetime utility of consumers constrained by the required balance of the PAYG social security budget is introduced. The model is analyzed by observing the effects of the parameters used in the construction of the model. The basic parameters used in this model are as shown in Table 4.1.

Table 4.1. Basic parameters of the models

Parameter	Description	Parameter	Description
$X_1$	education time	$E$	education expenditure per child (annual)
$X_2$	working time	$N$	average number of children per consumer
$X_3$	retirement time	$\tau$	payroll tax for retirement benefit
$w$	wage (hourly)	$b$	retirement benefit (annual)
$W$	wage (annual)	$r$	interest rate (annual)
$S$	saving (annual)	$l$	leisure in retirement

In the model, two agents are covered; namely consumers and government. Consumers are classified as young, working and old, depending on their position in the social security system. Young consumers receive education, while working consumers contribute to the system by working and old consumers receive retirement benefits. The problem of the social security system comprises of the optimal duration of education, occupational and retirement time in order to maximize the lifetime utility. Different from GE models, consumers do not choose their consumption and saving. Instead, the duration of education, working and retirement periods which maximize the lifetime utility through

consumption, are determined. The consumers are assumed to receive no inheritance and leave no bequest, they are born without wealth.

In the model, the constant relative risk aversion (CRRA) utility function is used to represent the working and old consumer. The utility function used in the model is known as CRRA utility since the relative risk aversion coefficient is constant. Moreover, it is defined over the consumption of working and old consumer. The consumption of young individuals consists of the investment on their education. However, since their parents undertake the educational consumption, a utility function is not defined for young individuals. The lifetime utility is defined as:

$$U(C_1, C_2) = U(C_1) + \sigma U(C_2)$$

where  $U(\cdot)$  is a utility function, with  $U'(\cdot) > 0$  and  $U''(\cdot) < 0$ . Since CRRA utility function is assumed, utility functions can be denoted as below:

$$U(C_1) = \frac{C_1^{1-\alpha} - 1}{1-\alpha} \quad (4.1)$$

$$U(C_2) = \frac{(lC_2)^{1-\alpha} - 1}{1-\alpha} \quad (4.2)$$

which yields

$$U(C_1, C_2) = \frac{C_1^{1-\alpha} - 1}{1-\alpha} + \sigma \frac{(lC_2)^{1-\alpha} - 1}{1-\alpha}. \quad (4.3)$$

Here  $C_1$  is the consumption of the working consumer, while  $C_2$  is the consumption of the old consumer during the retirement period.  $\alpha$  is denoted as the relative risk aversion coefficient in the models with uncertainty.  $1/\alpha$  is the intertemporal elasticity of substitution which measures the tendency to substitute consumption of different periods with the value greater than zero. The smaller  $1/\alpha$  the less willing is the consumer to

substitute the consumption between periods. Moreover, here  $l$  is the leisure time in retirement and as Shesinki (1978) uses, it is taken as the ratio of retirement duration to the lifetime,  $l = X_3 / (X_1 + X_2 + X_3 + 6)$ .

In multi-period models, consumers have different utility functions for different time periods. Since the impact of the future events is not the same as the impact of current events, consumers place less weight on utility gained during the retirement than the utility gained during working. This is not because the future events are not valued, but because they may not occur. In this model, the weight of future events, that is the utility gained from the consumption during retirement, is weighted with  $\sigma$ ,  $0 < \sigma < 1$ .

Utility discount factor and relative risk aversion parameters are assumed to be 0.8 and 0.1, respectively. They are chosen arbitrarily conforming the allowed range value. Therefore, the working consumer is assumed to value his retirement period at a rate of 0.8 in comparison with the working period.

The working consumer's income is  $(1 - \tau) * W$ , since  $\tau$  is the payroll tax rate for social security contribution. Furthermore, consumer spends his income on the consumption of working period  $C_1$ , savings  $S$ , which are accumulated during working period, and education of each child  $E$ . After retiring, the individual consumes the retirement benefit  $b$ , received from the social security system and what he saved before. Thus, the consumption of working and retiree consumers can be defined as shown below:

$$C_1 = X_2 * (1 - \tau) * W - X_2 * S - E * X_1 * N \quad (4.4)$$

$$C_2 = X_2 * S * (1 + r)^{X_2 - 1} + b * X_3 \quad (4.5)$$

In this model, the wage of the working individual is defined on the basis of the Mincer earning function. Although the standard Mincer function also includes a quadratic experience term, in the first model, the (log) wage is assumed to be a linear function of education and working experience as specified below:

$$\ln(W) = \beta_0 + \beta_1 X_1 + \beta_2 X_2 \quad (4.6)$$

where  $\ln W$  is the log wage of individual, and as mentioned before  $X_1$  is years of schooling and  $X_2$  is years of working experience.

Government side of the model represents the social security which is a pay as you go (PAYG) system. Since the PAYG system is based on the unfunded structure, the contribution of the working consumer is not invested after collection but transferred to the current retired by the government. The government does not have any financial contribution to the social security system. Thus the problem can be modeled as a constrained problem that balances the behavior of consumers with respect to PAYG system where the objective is to maximize the lifetime utility by determining the optimal values of education time, working time and retirement time:

$$\underset{X_1, X_2, X_3}{Max} U(C_1, C_2) = \frac{C_1^{1-\alpha} - 1}{1-\alpha} + \sigma \frac{[X_3 / (X_1 + X_2 + X_3 + 6) C_2]^{1-\alpha} - 1}{1-\alpha} \quad (4.7)$$

The models do not consider the unemployment, disability and survivors benefit. Therefore, the PAYG balance is given as:

$$b \times X_3 \times retiree \leq W \times \tau \times X_2 \times worker \quad (4.8)$$

where retiree is number of retirees who receive retirement benefit and worker is number of workers who contribute to the social security system by paying payroll tax. Here, it is assumed that there is no government budget in order to help the benefit payment of the social security system. The ratio of the working population to the retired population is taken as  $worker = retiree \times 2$  depending on the data of the Social Security Institution. Therefore, we set these variables as  $worker = 2$  and  $retiree = 1$ . Furthermore, although we set a static model, considering the sustainability of social security in the future, number of children,  $N$ , is assumed to be 4. Therefore, the model can satisfy the constraints, when it is thought to be an overlapping generations model.

As stated by the Social Security Institution, the retirement benefit is calculated by multiplying the average wage of an individual, replacement rate and working years. Since we construct the model over an average individual and we use an average wage to analyze the individual's preferences, in this calculation  $W$  is used instead of average wage. Moreover, the social security reform, which was approved by the Turkish Parliament in 2008, offers replacement rate as 2% for each year of working. Thus, the formula which explains the annual retirement benefit can be stated as:

$$b = W \times 0.02 \times X_2 \quad (4.9)$$

Since compulsory education is 8 years in Turkey, the lower bound of the education time is denoted to be 8. Depending on the new reform, minimum premium payment duration is 9000 days. Therefore, 25 is the minimum working years required to receive retirement benefit due to the new reform. Hence, these constraints for decision variables can be stated as:

$$\begin{aligned} 8 &\leq X_1 \leq 20 \\ 25 &\leq X_2 \leq 40 \\ 0 &\leq X_3 \leq 30 \end{aligned} \quad (4.10)$$

The Turkish Statistical Institute denotes life expectancy at birth as around 71 years for 2009 and considering the individuals begin to receive education at age 6. Therefore, the sum of education, working and retirement periods are assumed to be approximately 65. Therefore, maximum lifetime constraint can be stated as below:

$$X_1 + X_2 + X_3 = 65 \quad (4.11)$$

In Tusiad (2004), as a result of the calibration of general equilibrium model constructed for Turkey, interest rate is obtained to be around 8%. Considering this calibration result, the interest rate is also adopted as 8%.

## 4.2 Second Model

The first model is not a general equilibrium model, since it does not cover the production side of the economy. Here, the first model is expanded to a kind of general equilibrium structure which considers all of the agents; consumers, government and the firm. The main point in this general equilibrium model is satisfying the requirements of consumers and firm without creating surpluses or shortages. In addition to the consumer and government, components used in the previous model, the behaviour of the firm is also taken into consideration by including the production side of the economy. Hence, the wage of the working individual is defined by using the production side of the model.

Here, the augmented Solow model is adopted without considering the level of technology. A representative firm is assumed to produce a single good.  $Y$  is the output produced in each period according to the production function, given as:

$$Y = K^\alpha H^\theta L^{1-\alpha-\theta} \quad (4.12)$$

The production technology of the firm is represented by Cobb-Douglas production function. Mankiw *et al.* (1992) assume that  $\alpha + \theta < 1$ , which implies that there are decreasing returns to physical and human capital. Decreasing return means that when a factor which is used in production is increased, the increase in production output does not remain at the same amount, since a point will eventually be reached at which that factor yields smaller increase in production output.

The output  $Y$  is the total production of the inputs,  $K$ ,  $H$  and  $L$ . GDP (Gross Domestic Product) is defined as the market value of all final goods and services made within the borders of a country and is used as a measure of overall economic output of countries. Therefore, like many economic studies in literature  $Y$  is interpreted as real GDP. Furthermore, the inputs  $K$ ,  $H$  denote the aggregate physical capital and the aggregate human capital, respectively.  $\alpha$  is the physical capital's share of GDP,  $\theta$  is the human capital's share of GDP. Mankiw *et al.* (1992) assume that  $L$  grows exogenously at rate  $n$ . Since the technology level is not included in the model, the growth rate of technology,  $g$ , which is mentioned before is also omitted. Moreover, they also define  $s_k$  as the fraction of

income invested in physical capital and  $s_h$  as the fraction invested in human capital. The evolution of the economy is determined by

$$dk/dt = s_k y - (n + \delta)k \quad (4.13)$$

$$dh/dt = s_h y - (n + \delta)h \quad (4.14)$$

where  $y = Y/L$ ,  $k = K/L$  and  $h = H/L$  are output per worker, physical capital per worker and human capital per worker, respectively. In addition,  $\delta$  is the rate of depreciation where human capital depreciates at the same rate as physical capital. If we divide right and left-hand-side of Equation 4.12 by  $L$ , it follows that

$$y = k^\alpha h^\theta. \quad (4.15)$$

Substituting Equation 4.15 into Equation 4.13 and 4.14, yields

$$dk/dt = s_k k^\alpha h^\theta - (n + \delta)k \quad (4.16)$$

$$dh/dt = s_h k^\alpha h^\theta - (n + \delta)h. \quad (4.17)$$

Mankiw *et al.* (1992) show that Equations 4.16 and 4.17 imply that  $k$  and  $h$  converge to steady state values  $k^*$  and  $h^*$  where  $dk/dt = 0$  and  $dh/dt = 0$ :

$$k^* = \left( \frac{s_k h^\theta}{n + \delta} \right)^{1/1-\alpha} \quad (4.18)$$

$$h^* = \left( \frac{s_h k^\alpha}{n + \delta} \right)^{1/1-\theta} \quad (4.19)$$

If we substitute Equation 4.19 into 4.18, we obtain  $k^*$  as shown below:

$$k^* = \left[ \left( \frac{s_k}{n + \delta} \right)^{1/1-\alpha} \left( \frac{s_h}{n + \delta} \right)^{\theta/(1-\theta)(1-\alpha)} \right]^{\frac{(1-\theta)(1-\alpha)}{1-\alpha-\theta}} \quad (4.20)$$

In our model, we define aggregate human capital  $H$  as:

$$H = Lh \quad (4.21)$$

where  $L$  is the number of working consumers and  $h$  is the human capital per consumer. Human capital is introduced as a function of education and occupation duration, since the model assumes the human capital is the combination of education and working experience. Hence,

$$h = Ze^{(\beta_1 X_1 + \beta_2 X_2)}, \quad (4.22)$$

In production side of the model, the firm is assumed to maximize its profit by choosing the optimal physical capital  $K$  and labor  $L$ . It hires labor and capital with factor prices  $r$  and  $W$  and chooses capital input,  $K$ , and labor input,  $L$ , in order to maximize its profit as given below:

$$P = K^\alpha H^\theta L^{1-\alpha-\theta} - rK - WL. \quad (4.23)$$

Hence, the firm's maximization problem can be denoted as:

$$\underset{K,L}{Max} \quad K^\alpha H^\theta L^{1-\alpha-\theta} - rK - WL$$

The factor price equations can be achieved from the first order necessary conditions:

$$L : \frac{\partial P}{\partial L} = (1 - \alpha - \theta)K^\alpha H^\theta L^{-\alpha-\theta} - W = 0 \quad (4.24)$$

$$K : \frac{\partial P}{\partial K} = \alpha K^{\alpha-1} H^\theta L^{1-\alpha-\theta} - r = 0 \quad (4.25)$$

Thus,  $r$  and  $W$  become:

$$r = \alpha k^{\alpha-1} h^\theta \quad (4.26)$$

$$W = (1 - \alpha - \theta)k^\alpha h^\theta \quad (4.27)$$

where  $k = \frac{K}{L}$  and  $h = \frac{H}{L}$ .

In the model, as mentioned before it is aimed to define income per worker as a function of years of schooling and working; therefore, we will be interested in the estimation of  $W$ . Taking natural logarithm of  $W$  gives

$$\ln W = \ln(1 - \alpha - \theta) + \alpha \ln k + \theta \ln h \quad (4.28)$$

Moreover, substituting Equation 4.22 into the Equation 4.26 yields

$$\ln W = \text{const} + \alpha \ln k + \theta(\beta_1 X_1 + \beta_2 X_2) \quad (4.29)$$

where  $\text{const} = \ln(1 - \alpha - \theta) + \theta \ln Z$  is a constant. After substituting Equation 4.20 into Equation 4.29, it becomes

$$\ln W = \text{const} + \frac{\alpha(1-\theta)}{1-\alpha-\theta} \ln\left(\frac{s_k}{n+\delta}\right) + \frac{\alpha\theta}{1-\alpha-\theta} \ln\left(\frac{s_h}{n+\delta}\right) + \theta(\beta_1 X_1 + \beta_2 X_2) \quad (4.30)$$

Equation 4.30 will be used as income of an average worker after a regression analysis is conducted. Thus, the coefficients which constitutes  $W$  will be found in order to use in the analysis of the model. Moreover, if we take natural logarithm of Equation 4.15, we obtain

$$\ln y = \alpha \ln k + \theta \ln h, \quad (4.31)$$

and substituting Equation 4.22 into Equation 4.31 yields:

$$\ln y = \theta \ln Z + \alpha \ln k + \theta(\beta_1 X_1 + \beta_2 X_2). \quad (4.32)$$

Again, when we substitute Equation 4.20 into Equation 4.32, we obtain an equation very similar to Equation 4.30 as can be seen below:

$$\ln y = \frac{\alpha(1-\theta)}{1-\alpha-\theta} \ln\left(\frac{s_k}{n+\delta}\right) + \frac{\alpha\theta}{1-\alpha-\theta} \ln\left(\frac{s_h}{n+\delta}\right) + \theta(\beta_1 X_1 + \beta_2 X_2) \quad (4.33)$$

In the economic growth literature, since it is not possible to find cross-country data for wages depending on human capital data and Equation 4.30 and Equation 4.33 have same nature in terms of variables in their structure, the income analysis is conducted by using Equation 4.33. Thus, the Equation 4.33 will be the base of our analysis study.

The production side of the second model is the main property that differs it from the first model. This gives two different income formulas which will be analyzed. The individual's optimization problem is same as stated in Section 4.1.

Therefore, we can state the mathematical structure of the optimization problem as:

$$\underset{X_1, X_2, X_3}{Max} U = \frac{C_1^{1-\alpha} - 1}{1-\alpha} + \sigma \frac{[X_3 / (X_1 + X_2 + X_3 + 6) C_2]^{1-\alpha} - 1}{1-\alpha}$$

*s.t.*

$$b \times X_3 \times \text{retiree} \leq W \times \tau \times X_2 \times \text{worker}$$

$$8 \leq X_1 \leq 20$$

$$25 \leq X_2 \leq 40$$

$$0 \leq X_3 \leq 30$$

$$X_1 + X_2 + X_3 = 65$$

where

$$C_1 = X_2 * (1 - \tau) * W - X_2 * S - E * X_1 * N$$

$$C_2 = X_2 * S * (1 + r)^{X_2} + b * X_3$$

$$b = W \times 0.02 \times X_2$$

## 5. ANALYSIS OF THE MODELS

The results of the optimization problem vary depending on the changes in model parameters. This chapter contains the analysis of the parameter change and the results of analysis. The optimization problem of individuals is solved using Mathematica 6.0 and the analysis is conducted by Excel.

### 5.1. Analysis of the First Model

As mentioned before, in the first model, the income of individual is in the form of Mincer earning function and is a function of the duration of education and working (experience). Here, we need the income data with the years of education and working experience. In order to make a regression analysis to estimate the share of schooling and experience on wage, a dataset which contains wage, years of schooling and years of working experience is required. However, because of the lack of data for analyzing the Mincer earning function in Turkey, the results of a monograph prepared by Stukel and Engelhardt are used for the construction of log wage regression model. This monograph obtains the statistics of a random sample of 534 people from the 1995 Current Population Survey (CPS) which is a statistical survey which is performed by the United States Census Bureau for the Bureau of Labor Statistics. The dataset consists of hourly wages and the characteristics of the workers such as gender, years of education, years of work experience, occupational status, region of residence and marital status. This dataset was used to understand the effects of these characteristics on the hourly wages and it was analyzed by a macro written in STATA.

By using the results of the regression analysis which takes log wage as response variable and education time and experience as predictor variables, the Mincer earning function (hourly) can be stated as:

$$w = e^{0.8628728+0.1034977X_1+0.0128666X_2} \quad (5.1)$$

However, since the regression analysis is run by using hourly wage data by Stukel and Engelhardt in order to obtain monthly wage, we multiply the earning by 180 ( $=9 \times 5 \times 4$ ), considering 9 working hours per day, 5 working days per week and 4 weeks per month. Afterwards, we can state the annual wage by multiplying monthly wage by 12. Thus, Equation 4.7 becomes:

$$W = 12 \times 180 \times e^{0.8628728 + 0.1034977X_1 + 0.0128666X_2} \quad (5.2)$$

### 5.1.1. Base Case

In this section, in order to analyze the optimization problem which has restrictions in the coverage of social security system, a base case is implemented with the basic parameters of the model. The base case assumes the system parameters as given in Table 5.1.

Table 5.1. Values of the base case parameters

Parameters	Base Values
Payroll Tax ( $\tau$ )	0.15
Share of Education on Wage ( $\beta_1$ )	0.10
Share of Working on Wage ( $\beta_2$ )	0.012
Maximum Lifetime	65

Payroll tax rate is taken as the premium rate for the pension system. Although the stated amount for Turkey is 20%, Tusiad (2004) calculates payroll tax as 15% due to the problems in collecting the premiums and since about 3% of GDP is transferred to the social security system as treasury aid. Therefore, payroll tax is assumed to be 15%. As mentioned before, the Turkish Statistical Institute denotes life expectancy at birth about 71 years for 2009 and considering the individuals begin to receive education at age 6. Therefore, the sum of education, working and retirement period is assumed to be 65.

### 5.1.2. Single Parameter Analysis

In this section, the individual effect of parameters, payroll tax ( $\tau$ ), share of education on wage ( $\beta_l$ ) and maximum lifetime level on the education, working and retirement duration, which are required for the sustainability of the social security system, is examined given that the other parameters are held constant.

5.1.2.1. Changes in Payroll Tax: Payroll tax is one of the most important parameters in the social security systems. Therefore, the effect of changes from the base payroll tax value is analyzed. Payroll tax is varied between 5% and 75% as the other parameters are held constant. In Table 5.2, the results of payroll tax changes are displayed.

Table 5.2. Analysis results for changes in  $\tau$

	$\tau$ (%)							
	5	15	25	35	45	55	65	75
$X_1$	20	20	20	20	20	20	20	20
$X_2$	40	39.2541	39.1294	38.942	38.6281	37.9899	35.9137	25
$X_3$	5	5.74592	5.87058	6.05796	6.37194	7.0101	9.08626	20
Utility	536966	477616	416337	354116	290814	226339	161373	94248.8

As it can be observed from the Figure 5.1, decreasing the payroll tax causes an increase in the working period, since the decrease in the payroll tax means increase in the after-tax income of the consumers, there would be a tendency for working as much as possible. The working period,  $X_2$ , reaches its upper bound, 40 years. Besides, the decrease in payroll tax from 15% to 5% causes a decrease in the duration of retirement,  $X_3$ . Decreasing payroll tax means the increase of average after-tax income. Moreover, with a higher after-tax income it is more probable to allocate more income for the education of young individuals, namely children of consumers. Although an increase is expected in  $X_2$ , it remains the same since it is already at its maximum value while  $\tau = 15\%$ .

When  $\tau$  is increased from 15% to 75%, the working period decreases gradually due to the decrease of after-tax income of individual. Moreover, the duration of education remains unchanged. As a result of this, the duration of retirement increases.

The payroll tax between 65% and 75% is the range where the model reacts significantly. As a result of this increase, the after-tax income of the consumers decreases by an important amount and the years of working decreases; therefore, longer retirement periods begin to be obtained to increase the lifetime utility. At  $\tau = 75\%$ , only the duration of retirement increases, since the upper bound of education stops  $X_1$  from being larger than 20. In Table 5.2 the utility values are represented as lifetime utility, while in Figure 5.2 average utility is displayed. During the analysis, utility will be examined in this way.

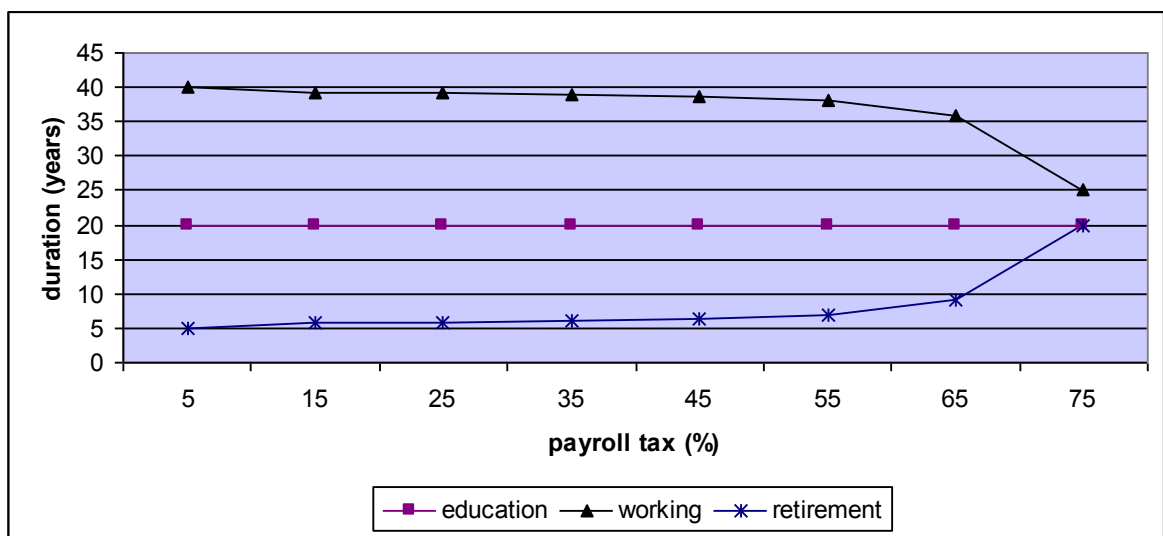


Figure 5.1. Effect of payroll tax ( $\tau$ ) on  $X_1$ ,  $X_2$  and  $X_3$

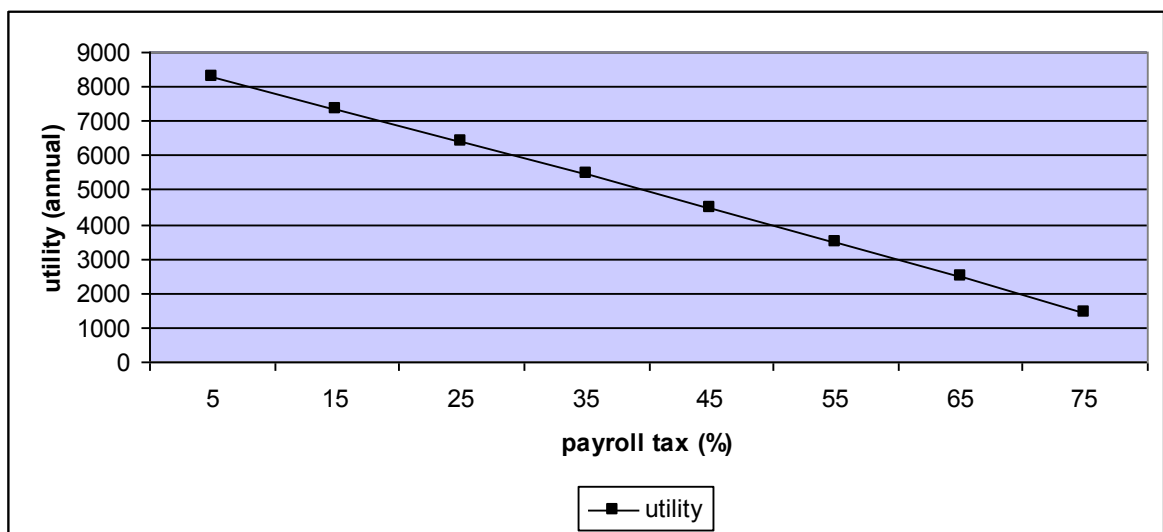


Figure 5.2. Effect of payroll tax ( $\tau$ ) on average utility

In addition, as it is seen in the Figure 5.2, when the payroll tax is decreased, the lifetime utility increases despite of the decrease in the duration of retirement, since the duration of working increases. Therefore, this indicates that the duration of working has a more important effect than the duration of retirement in the lifetime utility of individual. On the other hand, the increase in payroll tax between 15% and 75% causes decrease in the after-tax income of individual; therefore, the lifetime utility decreases.

5.1.2.2. Changes in the Share of Education on Wage: Since it is aimed to observe the impact of human capital on  $X_1$ ,  $X_2$  and  $X_3$  values in the coverage of social security system, the effect which the share of education on wage causes is investigated by 2% variation from the base value of  $\beta_1$ . The analysis results for this parameter are given in Table 5.3.

Table 5.3. Analysis results for changes in  $\beta_1$

	$\beta_1$ (%)			
	6	8	10	12
$X_1$	13.426	17.8546	20	20
$X_2$	40	40	39.2541	39.2541
$X_3$	11.574	7.14536	5.74592	5.74592
Utility	253448	337238	477616	684582

When the share of education on wage is decreased, the duration of schooling, working and retirement and the lifetime utility are all changed. Here, the schooling time decreases, the working time remains the same and the retirement period increases. Shorter studying period is obtained since the effect of the education on the wage is decreased. Moreover, the lifetime utility decreases due to the decrease in the wage of individuals. As a result of this decrease in the duration of education, the duration of working reaches its maximum value 40. Moreover, the duration of retirement increases such that the lifetime constraint is satisfied.

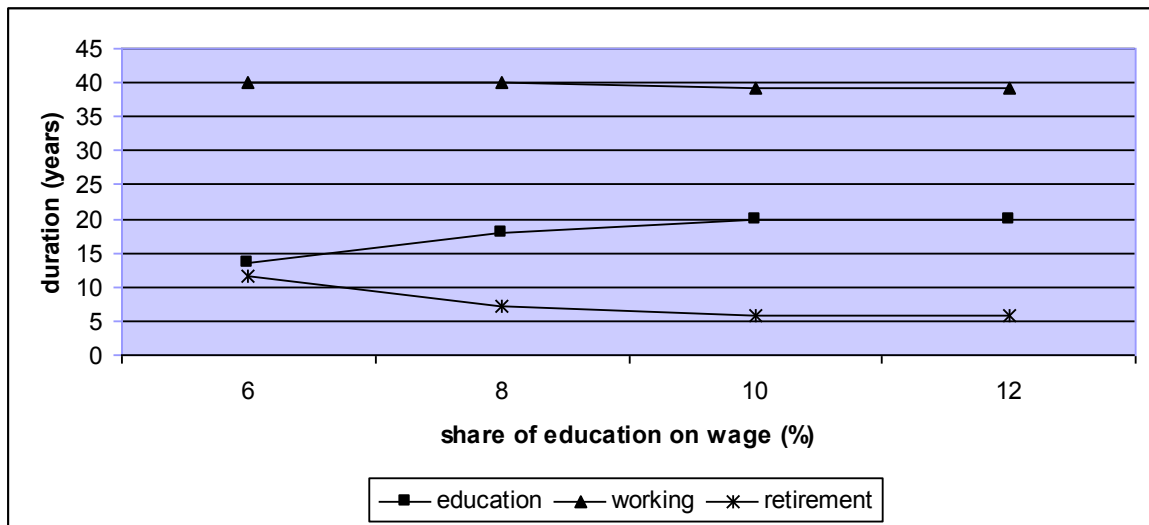


Figure 5.3. Effect of  $\beta_1$  on  $X_1$ ,  $X_2$  and  $X_3$

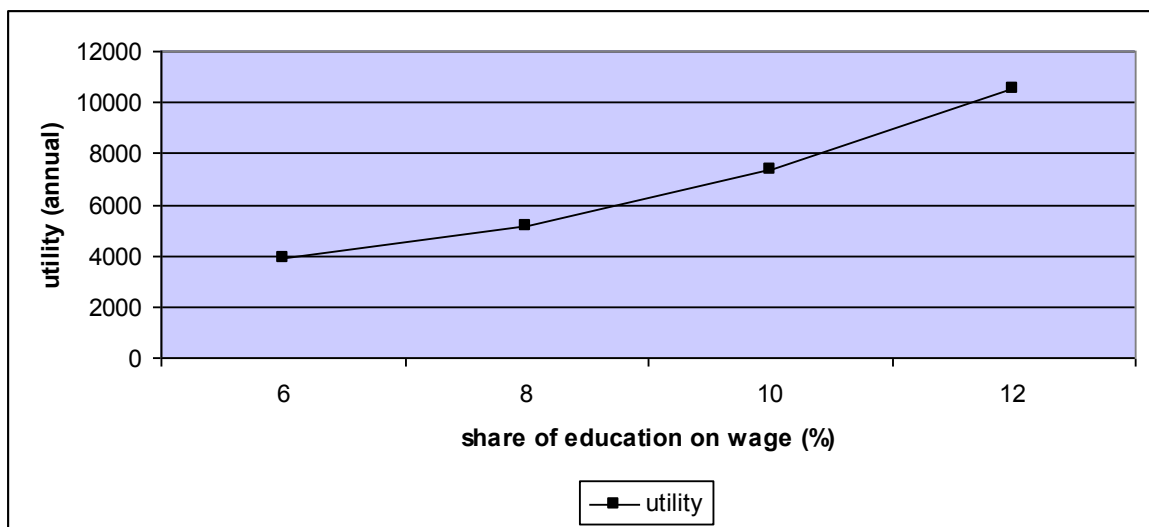


Figure 5.4. Effect of  $\beta_1$  on average utility

When the share of education on wage is increased, the duration of education is expected to increase; however, since it's at its upper bound value, it remains the same. Furthermore, the duration of working and retirement remains unchanged. Since the income of the individual depends considerably on the duration of education, the increase on the share of education on wage makes the income increase. As a result of the increase in income, the lifetime utility increases as can be seen in the Figure 5.4.

5.1.2.3. Changes in the Share of Working Experience on Wage: Share of working experience on wage is an important parameter since the income of individual is a function of it. In this section the results of its change is discussed. Figure 5.5 demonstrates the changes on  $X_1$ ,  $X_2$  and  $X_3$ , while Table 5.4 shows some result points. In the model, the lifetime utility which is a function of individual's consumption, is tried to be maximized. Since the consumption is defined based on the share of education and working experience on wage, when  $\beta_2$  is decreased,  $X_2$  shows a decrease.

Table 5.4. Analysis results for changes in  $\beta_2$

	$\beta_2$		
	0.8	1.2	1.6
$X_1$	20	20	20
$X_2$	38.8268	39.2541	39.6444
$X_3$	6.17324	5.74592	5.3556
Utility	414988	477616	550505

On the other hand, if  $\beta_2$  is increased, it is expected to obtain longer working period for increasing the lifetime utility. As it can be seen from Table 5.4, as  $\beta_2$  is decreased,  $X_1$  is expected to increase due to the decrease in  $X_2$ . However, since it's already 20 and cannot increase,  $X_3$  increases in order to increase the lifetime utility, by satisfying the lifetime constraint. Increasing  $\beta_2$  causes increase in  $X_2$ . Since  $X_1$  remains the same,  $X_3$  decreases due to the increase in  $X_2$ . As Figure 5.6 demonstrates, lifetime utility is an increasing function of  $\beta_2$ .

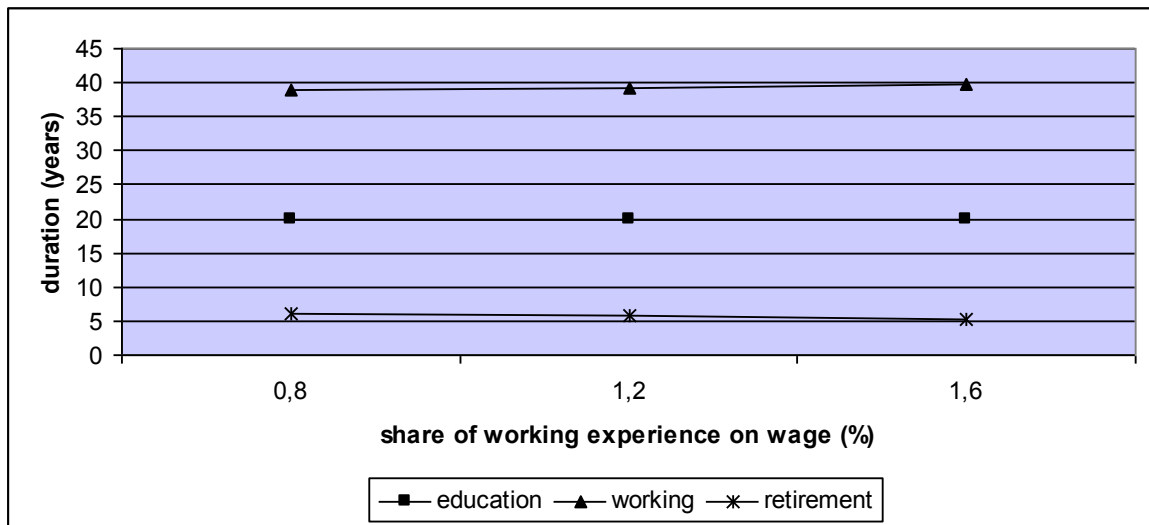


Figure 5.5. Effect of  $\beta_2$  on  $X_1$ ,  $X_2$  and  $X_3$

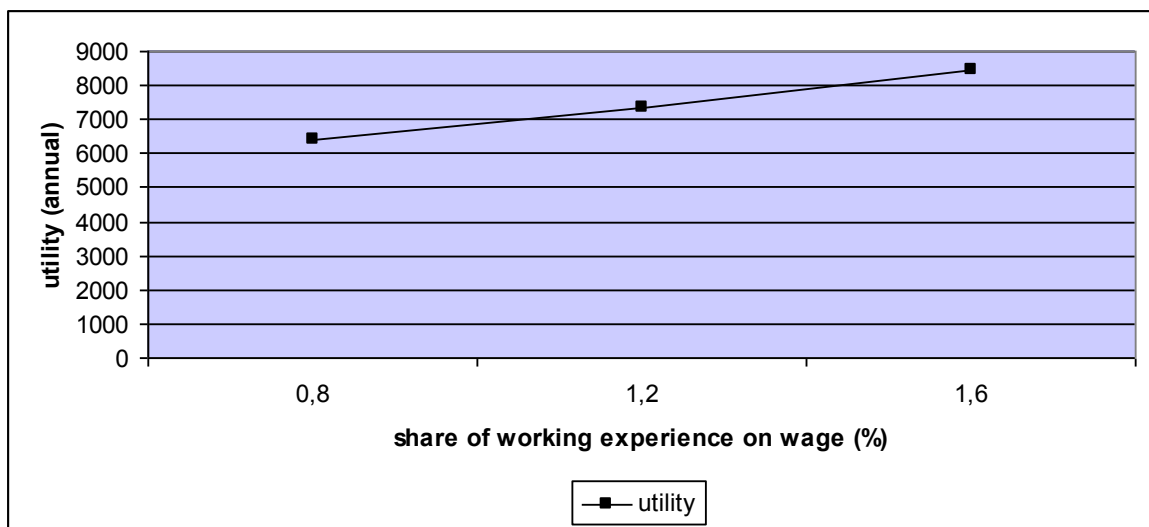


Figure 5.6. Effect of  $\beta_2$  on lifetime utility of individual

5.1.2.4. Changes in the Maximum Lifetime: This section investigates the effect of maximum lifetime on the years of education, working and retirement which is determined by the analysis of the model. Table 5.5 summarizes the results of changes on the upper bound of lifetime constraint.

Table 5.5. Analysis results for changes in maximum lifetime

	Maximum Lifetime				
	55	60	65	70	75
$X_1$	18.3665	20	20	20	20
$X_2$	32.8258	35.3778	39.2541	40	40
$X_3$	3.80771	4.62223	5.74592	10	15
Utility	247953	345485	477616	633331	776719

As Figure 5.7 illustrates, since the upper bound value of education period is 20 years, it remains at its maximum when the maximum lifetime is increased. The working period increases to 40 years when the lifetime is increased to 70 and then remains at its maximum; besides, the duration of the retirement increases.

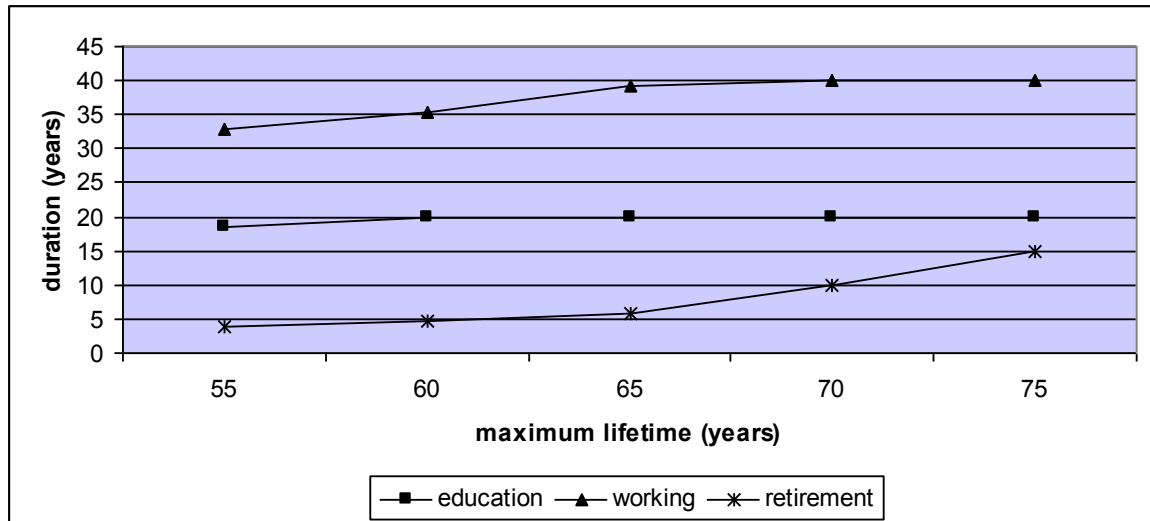
Figure 5.7. Effect of maximum lifetime on  $X_1$ ,  $X_2$  and  $X_3$ 

Figure 5.8 demonstrates the behavior of individual's lifetime utility as a function of the changes on maximum lifetime. As a natural result, the lifetime utility decreases due to the decrease in the upper bound of lifetime constraint. Since the average lifetime is shorter in comparison with the base case, he studies and works less and that causes lower income and therefore a lower lifetime utility. Moreover, since the utility of leisure is involved in

the consumption function of a retired individual, the duration of retirement does not change a lot while schooling and working duration decrease. As the maximum lifetime decreases to 60,  $X_2$  and  $X_3$  decrease and  $X_1$  remains unchanged due to its important effect on the average income.

When the maximum lifetime is 55, the model has to decrease all three decision variables due to the restricted lifetime. The decrease in retirement period is less than the decrease in working period because of the effect of the utility gained from retirement (leisure).

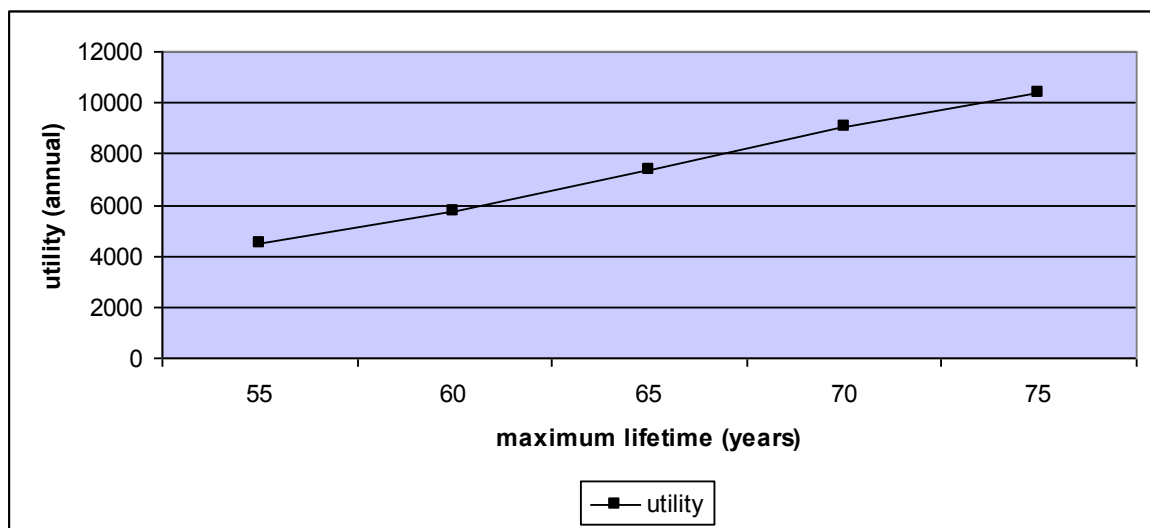


Figure 5.8. Effect of maximum lifetime on the lifetime utility of individual

### 5.1.3. Multi-parameter Analysis

In addition to the single parameter analysis in the previous section, we aim to examine the model's behavior on the duration of education, working and retirement period as a result of aggregate effect of parameters.

**5.1.3.1. Changes in Payroll Tax and the Share of Education on Wage:** In this section, since payroll tax and share of education on wage are important parameters for the decision variables, it is aimed to investigate the aggregate effect of payroll tax and share of education on wage. Payroll tax is varied between 5% and 75%, while at the same time share of education on wage is varied between 6% and 12%. Figure 5.9 displays the effect

of these parameters on  $X_1$ , while some of the points are presented in Table 5.6. The behavior of  $X_2$  and  $X_3$  due to the aggregated effect of  $\tau$  and  $\beta_1$  changes is also illustrated in the Figure 5.10 and Figure 5.11.

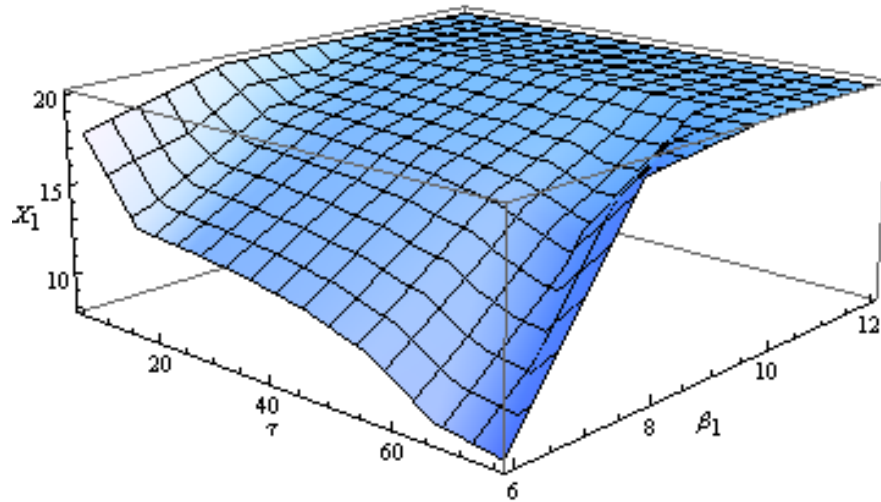


Figure 5.9. Aggregate effect of  $\tau$  and  $\beta_1$  on  $X_1$

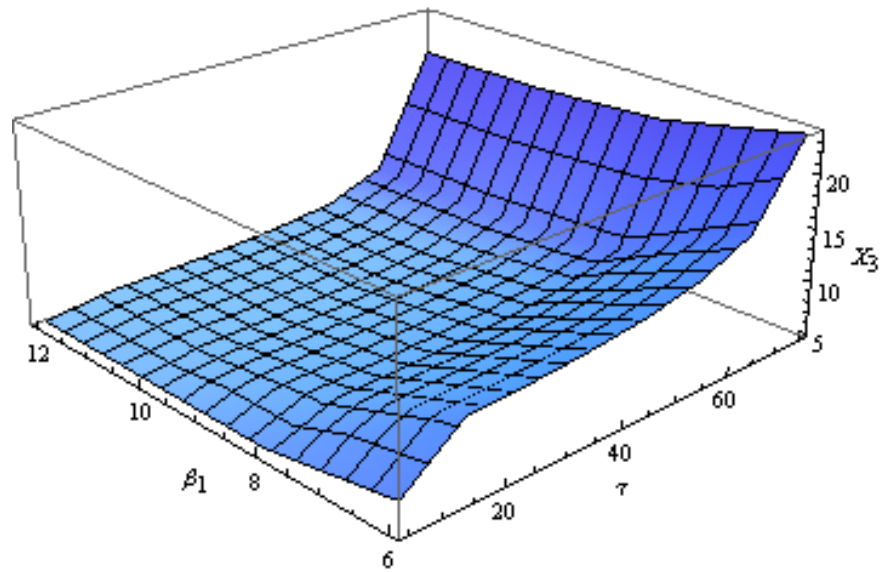
It is seen that whatever the payroll tax is, while  $\beta_1$  increases  $X_1$  also increases until  $X_1$  reaches its upper bound at 10% , since our model is constructed on the base of human capital, the share of an additional year of education is very important for the duration of three parts in an average lifetime; studying, working and retirement.

Considering the base case where  $\beta_1$  is 10%; as expected, the decrease of  $\beta_1$  makes the model yield lower  $X_1$  values for any  $\tau$  values 5%. When  $\beta_1$  is increased to 12%,  $X_1$  remains to be 20 despite of the changes of any  $\tau$  values, since it is already at its upper bound value at the base case value of  $\beta_1$ . If we think the cases where  $\beta_1 = 6\%$  and  $8\%$ , when  $\tau$  is increased,  $X_1$  decreases. However, at the case where  $\beta_1 = 8\%$  and  $\tau = 75\%$ , an increase is observed due to the high decrease in  $X_2$  value. Again we can see the importance of the share of education on wage for the optimal solution of the model by this result.

Table 5.6.  $X_1$  values for different  $\tau$  and  $\beta_1$ 

		$\beta_1$ (%)			
		6	8	10	12
$\tau$ (%)	5	17.8542	20	20	20
	15	13.426	17.8546	20	20
	25	13.1194	17.6959	20	20
	35	12.6805	17.4665	20	20
	45	12.0054	17.1073	20	20
	55	10.8542	16.4717	20	20
	65	8.58624	15.0992	20	20
	75	8	19.1047	20	20

Figure 5.10. Aggregate effect of  $\tau$  and  $\beta_1$  on  $X_2$

Figure 5.11. Aggregate effect of  $\tau$  and  $\beta_1$  on  $X_3$ Table 5.7.  $X_2$  values for different  $\tau$  and  $\beta_1$ 

		$\beta_1$ (%)			
		6	8	10	12
$\tau$ (%)	5	40	40	40	40
	15	40	40	39.2541	39.2541
	25	40	40	39.1294	39.1294
	35	40	40	38.942	38.942
	45	40	40	38.6281	38.6281
	55	40	40	37.9899	37.9899
	65	40	40	35.9137	35.9137
	75	33.2662	25	25	25

Since the income of an individual is defined by the share of education and working experience on wage, when  $\beta_1$  is decreased, the income of individual decreases. In order to compensate this loss, it is required to study less and work more. Therefore,  $X_2$  is observed to increase its upper bound, 40, as expected. Considering the base case where  $\beta_1 = 10\%$  it

can be seen that  $X_2$  decreases as a result of increases in  $\tau$ . Higher  $\tau$  values decreases after-tax income of individual; therefore, there is a tendency of working less. The change of  $\beta_1$  from 10% to 12% does not change  $X_2$  values. Since  $X_1$  reaches its maximum value,  $X_2$  and  $X_3$  remains the same as it is represented in Table 5.7 and Table 5.8. Moreover, for the lowest value of  $\tau$ ,  $X_3$  remains to be 5, because  $X_1$  and  $X_2$  take their upper bound values at this point.

For very low  $\tau$  values, it is preferred to work more because of the increase in the average after-tax income. Moreover, individual is responsible for his children; therefore, higher after-tax income provides longer period of education to his children. Thus,  $X_2$  increases as a result of this situation. As expected  $X_3$  decreases in order to satisfy related lifetime constraint of our problem.

Table 5.8.  $X_3$  values for different  $\tau$  and  $\beta_1$

		$\beta_1$ (%)			
		6	8	10	12
$\tau$ (%)	5	7.14578	5	5	5
	15	11.574	7.14536	5.74592	5.74592
	25	11.8806	7.30413	5.87058	5.87058
	35	12.3195	7.53353	6.05796	6.05796
	45	12.9946	7.89271	6.37194	6.37194
	55	14.1458	8.52829	7.0101	7.0101
	65	16.4138	9.90085	9.08629	9.08629
	75	23.7338	20.8953	20	20

As represented in Table 5.6, Table 5.7 and Table 5.8, when we look for  $X_1$ ,  $X_2$  and  $X_3$  for  $\beta_1$  is 6% and 8%, the increase in  $\tau$  causes a decrease in  $X_1$  considering the low effect of  $\beta_1$  here. Therefore, the model tries to increase the lifetime utility by increasing the duration of the retirement period,  $X_3$ .  $X_3$  reaches the longest retirement duration of this analysis where  $\beta_1 = 6\%$  and  $\tau = 75\%$ , since  $X_1$  is at its lower bound 8 and  $X_2$  decreases.  $X_1$  takes the value of 8 years since the share of education on wage is at its minimum and the payroll tax

is at its maximum. This causes lower after-tax income; therefore, shorter studying and working period. Hence,  $X_3$  increases by a great amount.

5.1.3.2. Changes in Payroll Tax and the Share of Working Experience on Wage: In this section it is aimed to investigate the combined effect of payroll tax and share of working experience on the optimal values of the decision variables,  $X_1$ ,  $X_2$  and  $X_3$ . The analysis is conducted by three points for  $\beta_2$  and eight points for  $\tau$  including base case values for each. Table 5.9, Table 5.10 and Table 5.11 shows some points of  $X_1$ ,  $X_2$  and  $X_3$  due to the changes in  $\beta_2$  and  $\tau$ .

Since the income of individual is defined as a function of working years, when we increase the share of an additional working year on wage from 1.2% to 1.6% , the system wants individuals to work as more as possible providing the constraints of the optimization problem. Moreover, decreasing  $\beta_2$  causes decreases on the working period results of the model. When we look at each  $\beta_2$  value, increasing the payroll tax decreases the after-tax income; therefore,  $X_2$  values decrease.  $\tau = 75\%$  is the point where  $X_2$  drops to its lower bound, 25.

Table 5.9.  $X_1$  values for different  $\tau$  and  $\beta_2$

		$\beta_2$ (%)		
		0.8	1.2	1.6
$\tau$ (%)	5	20	20	20
	15	20	20	20
	25	20	20	20
	35	20	20	20
	45	20	20	20
	55	20	20	20
	65	20	20	20
	75	20	20	20

Table 5.9 and Table 5.10 shows some points of  $X_1$  and  $X_3$  due to the changes in  $\beta_2$  and  $\tau$  while Figure 5.13 demonstrates the results of analysis for  $X_3$ . If we look at the case where  $\tau = 5\%$ ,  $X_2$  values are all 40. Since a payroll tax of 5% is very low, individuals have higher after-tax income and it's natural that the system prefers that the individuals work more at  $\tau = 5\%$ .

Table 5.10.  $X_3$  values for different  $\tau$  and  $\beta_2$

		$\beta_2$ (%)		
		0.8	1.2	1.6
$\tau$ (%)	5	5	5	5
	15	6.17324	5.74592	5.3556
	25	6.31852	5.87058	5.46326
	35	6.53851	6.05796	5.62413
	45	6.91166	6.37194	5.89095
	55	7.68991	7.0101	6.42215
	65	10.4637	9.08629	8.03728
	75	20	20	20

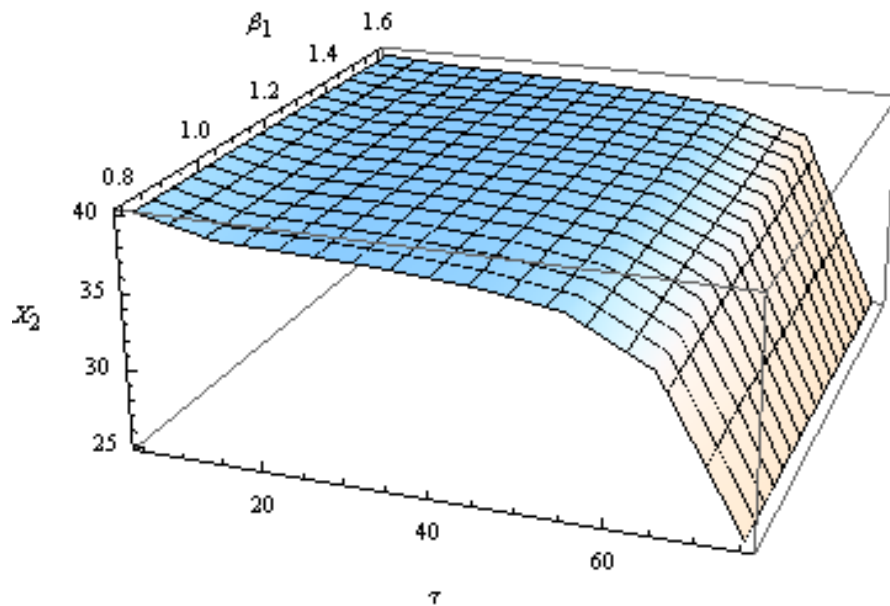


Figure 5.12. Aggregate effect of  $\tau$  and  $\beta_2$  on  $X_2$

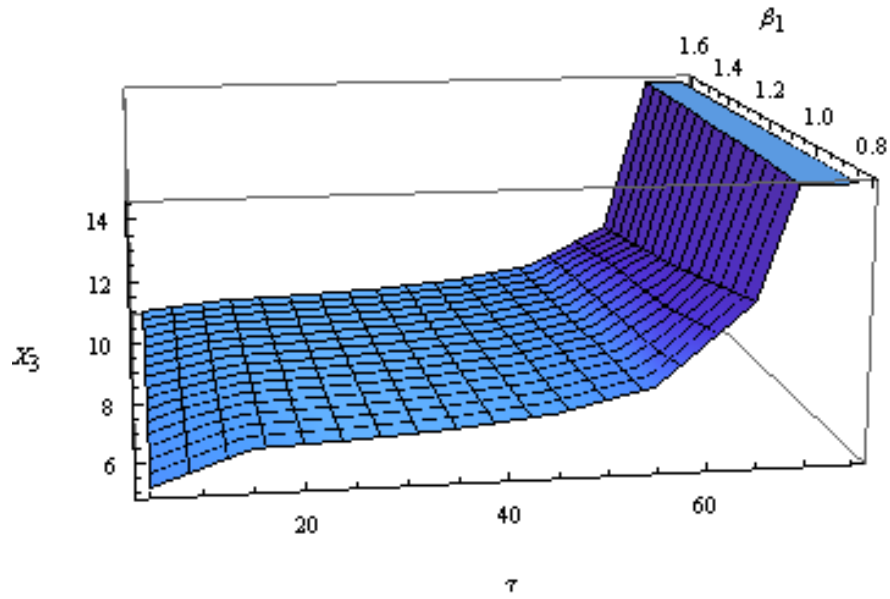


Figure 5.13. Aggregate effect of  $\tau$  and  $\beta_2$  on  $X_3$

Table 5.11.  $X_2$  values for different  $\tau$  and  $\beta_2$

		$\beta_2$ (%)		
		0.8	1.2	1.6
$\tau$ (%)	5	40	40	40
	15	38.8268	39.2541	39.6444
	25	38.6815	39.1294	39.5367
	35	38.4615	38.942	39.3759
	45	38.0883	38.6281	39.109
	55	37.3101	37.9899	38.5779
	65	34.5363	35.9137	36.9627
	75	25	25	25

In the model, individual is assumed to save his after-tax income for his retirement period and his income is based on the duration of education and working. Moreover, the retirement benefit is also defined as a function of his average income during his working period. While  $\tau$  increases, since after-tax income decreases, it is required to work less;

hence,  $X_2$  decreases. Furthermore, this causes a lower lifetime utility. In order to increase the lifetime utility  $X_3$  increases by satisfying the lifetime constraint. At  $\tau = 75\%$ ,  $X_3$  reaches its maximum due to the fact that  $X_2$  drops to its lower bound. As can be seen from Table 5.9, the changes in  $X_2$  due to the  $\tau$  variation do not affect  $X_1$  and it remains unchanged for all  $\tau$  and  $\beta_2$  values.

## 5.2. Analysis of the Second Model

Regression analysis is conducted for describing and evaluating the relationship between a dependent variable and one or more other variables which have an effect on this dependent variable. Regression model which describes this relationship is stated as:

$$y = \beta_0 + \beta_1 X_1 + \beta_2 X_2 + \dots + \beta_k X_k + \varepsilon \quad (5.3)$$

where the parameters  $\beta_j$ ,  $j = 0, 1, \dots, k$ , are called the regression coefficients. The parameter  $\beta_j$  is the expected change in  $y$  per unit change in  $X_j$ , holding all the other independent variables constant.

The significance of regression model is determined based on the fact that whether a linear relationship exists between  $y$  and  $X_j$ s. The hypotheses for checking the significance of the model are,

$$H_0 : \beta_1 = \beta_2 = \dots = \beta_n = 0 \quad (5.4)$$

$$H_1 : \beta_j \neq 0 \text{ for at least one } j. \quad (5.5)$$

Here, rejection of  $H_0$  means that at least one of the independent variables  $X_j$ s has a significant contribution to the model. For hypothesis testing  $F_0$  and P-value, which are calculated by statistical software tools.  $H_0$  is rejected if  $F_0$  exceeds  $F_{\alpha, k, n-k-1}$  where  $\alpha$  is the significance level,  $k$  is the number of independent variables and  $n$  is the number of observations used in the regression analysis. Alternatively, if the P-value is less than  $\alpha$ , the null hypothesis  $H_0$  is rejected.

Moreover,  $R^2$  value is examined to see the goodness of fit of the regression model.  $R^2$  value is an indicator of how well the model fits the data. An  $R^2$  value close to 100% indicates that we have accounted for almost all of the variability with the variables specified in the model.

In addition to the significance test of the regression model, the significance of individual regression coefficients is also tested. The hypotheses testing for testing the significance of any individual coefficient,  $\beta_j$ , are

$$H_0 : \beta_j = 0 \quad (5.6)$$

$$H_1 : \beta_j \neq 0 \quad (5.7)$$

If  $H_0 : \beta_j = 0$  is not rejected, this implies that  $X_j$  can be deleted from the model. The hypothesis is tested by using  $t_0$  value. The null hypothesis  $H_0 : \beta_j = 0$  is rejected if  $|t_0| > t_{\alpha/2, n-k-1}$ . P-value of the coefficients is also used for testing the significance of the independent variables (Montgomery, 2005).

### 5.2.1. Data for Regression Analysis

5.2.1.1. Dataset 1: In the second model, we can estimate the share of average years of schooling and average years of working experience on wage by using Equation 4.30. We carry out our empirical analysis with a sample of 60 countries. The variables which constitute our model can be stated as GDP per worker (capita) for  $y$ , investment share on GDP per capita for  $s_k$ , human capital investment share on GDP per capita for  $s_h$ , labor force growth for  $n$ , average years of schooling and average years of experience for  $X_1$  and  $X_2$ , respectively. Since in Turkey, there is a lack of data required to conduct the analysis, we use international data of different studies on economic growth. In order to obtain data, we use the database of United Nations, Penn World Table 6.2 and the dataset of Barro-Lee (2000).

The Penn World Table is a rich macroeconomic panel dataset which is also known as Summers-Heston dataset. The table comprises about 30 variables for about 188 countries

over some or all the years 1950-2004. It is a predecessor of international database and is used for making comparisons between countries and over time in the analysis of economic growth. Beside it provides purchasing power parity and national accounts changed into international prices and it also displays demographic data.

Barro-Lee dataset is another fundamental study which is used for economic growth studies. This dataset includes the estimates of educational attainment for the male and female populations of 138 countries. In addition the data are presented between 1960-2000 quinquennially.

The variable of  $X_1$  is obtained from the dataset of Barro-Lee (2000). We use years of schooling attained by the population aged 15 and over. Our measure of the average years of experience of the working population is adopted from Cook (2004) and is based on the data taken from Barro-Lee (2000) and United Nations Population Division (2008). Population by five year age group data were obtained for 60 countries between the years 1960-2000 quinquennially. Because of the missing laborforce data of International Labor Organization, the population between 15-64 is assumed to be working age population as in the study of Easterly and Levine (2001). For each age group, we calculate the working experience as age less schooling years less 6. For the variable of age in this calculation, we use the midpoint age for each 5-year age group between 15-64. After calculating experience for each group, we take the working population weighted average of these experience levels. In addition, we use population growth rate for the variable of  $n$  and it is calculated as the average population growth rate between 1960-2000 for each country. The data for GDP per capita ( $y$ ), investment share on GDP per capita ( $s_k$ ) and human capital investment share on GDP per capita ( $s_h$ ) are obtained from Penn World Table 6.2; in addition, population growth rate ( $n$ ) is obtained from United Nations Population Division (2008) for 60 countries used in the analysis. These countries are represented by the datasets in Appendix A.

The regression analysis is carried out with the significance level ( $\alpha$ ) of 0.05 by using the dataset obtained from mentioned sources. The results of analysis are represented in Table 5.12 and Table 5.13.

$$(R^2 = 68.1\%; \text{Adj. } R^2 = 67\%)$$

Table 5.12 Regression results for dataset 1

Predictors	Coefficient	SE Coefficient	T-value	P-value
Constant	3.5221	0.9144	3.85	0.000
$X_1$	0.2776	0.02574	10.79	0.000
$X_2$	0.13784	0.03922	3.51	0.001

Table 5.13 ANOVA for the regression model for dataset 1

Source	DF	Sum of Squares	Mean Square	F-value	P-value
Regression	2	31.085	15.542	60.96	0.00
Residual	57	14.533	0.255		
Total	59	45.618			

The coefficients which are obtained from the regression analysis are used in the definition of individual's income stated by Equation 4.30. However, the model does not react significantly during the sensitivity analysis of the optimization problem. When we look at the  $R^2$  value, it can be seen that 61.8% of the model can be explained by the independent variables,  $X_1$  and  $X_2$ .

5.2.1.2. Dataset 2: Because of the dissatisfaction of these results and inefficiency of the model in the sensitivity analysis, we constructed a different dataset based on the Equation 4.32. However, this time, the variable  $k$  is not taken as the definition of augmented Solow model which depends on  $s_k$  and  $s_h$ . Here,  $k$  is assumed to be physical capital per worker,  $K/L$ . Therefore, the dataset is constructed in such a way that it includes total GDP ( $Y$ ), aggregated physical capital ( $K$ ), labor force ( $L$ ),  $X_1$  and  $X_2$ .  $K$ ,  $L$  and  $X_1$  data is obtained from the study of Benhabib and Spiegel (1994), while  $X_2$  is calculated as mentioned before. All of these variables are taken for 104 countries for 1985. These countries can be found with corresponding data in Appendix A.

The regression analysis is conducted for new dataset and the results are demonstrated in Table 5.14 and Table 5.15.

$$(R^2 = 91.6\%; \text{Adj. } R^2 = 91.4\%)$$

Table 5.14 Regression results of dataset 2

Predictors	Coefficient	SE Coefficient	T-value	P-value
<b>Constant</b>	6.8247	0.1479	46.15	0.000
<i>lnk</i>	0.60351	0.03785	15.94	0.000
$X_1$	0.06656	0.019822	3.36	0.000
$X_2$	0.005986	0.004498	1.33	0.186

Table 5.15 ANOVA for the regression model of dataset 2

Source	DF	Sum of Squares	Mean Square	F-value	P-value
<b>Regression</b>	3	103.101	34.367	365.70	0.000
<b>Residual</b>	100	9.398	0.094		
<b>Total</b>	103	112.499			

As it can be observed from Table 5.14, P-value of  $X_2$  is greater than  $\alpha = 0.05$ ; therefore, we cannot reject the null hypothesis  $H_0: \beta_j = 0$  for  $X_2$ . This implies that  $X_2$  can be deleted from the model. The regression analysis is carried out after excluding  $X_2$  from the model. Table 5.16 and Table 5.17 demonstrate the regression results of the model without  $X_2$ .

$$(R^2 = 91.5\%; \text{Adj. } R^2 = 91.3\%)$$

Table 5.16. Regression results of dataset 2 without experience variable

Predictors	Coefficient	SE Coefficient	T-value	P-value
<b>Constant</b>	6.99804	0.07025	99.62	0.000
<i>lnk</i>	0.60253	0.03799	15.86	0.000
$X_1$	0.06265	0.01967	0.18	0.002

Since  $R^2$  value is 91.5%, it can be stated that the independent variables can explain 91.5% of the variation in the dependent variable and this shows the goodness of fit of the model. As shown in Table 5.17, since P-value of the model is less than the significance

level 0.05, it is clear that the model is significant. Moreover, P-values for the variables are less than the 0.05 and this implies that independent variables are also significant.

Table 5.17. ANOVA for the regression model of dataset 2 without experience variable

Source	DF	Sum of Squares	Mean Square	F-value	P-value
<b>Regression</b>	2	102.935	51.467	543.52	0.00
<b>Residual</b>	101	9.564	0.095		
<b>Total</b>	103	112.499			

However, when we look for the correlation between  $X_I$  and  $lnk$ , we observe that the Pearson coefficient, which is a measure of correlation, is about 0.8. Since the correlation between  $X_I$  and  $lnk$  is considerably high, in order to eliminate this problem, we use a different independent variable in the regression. The new independent variable is formed by multiplying  $X_I$  and  $lnk$ ; therefore, it is  $ln k \times X_1$ . After running the regression again we obtain the results displayed in Table 5.19.

$$(R^2 = 81.8\%; \text{Adj. } R^2 = 81.6\%)$$

Table 5.18. Regression results of dataset 2 with new independent variable

Predictors	Coefficient	SE Coefficient	T-value	P-value
<b>Constant</b>	7.65657	0.07054	108.54	0.000
$ln k \times X_1$	0.068541	0.003201	21.42	0.000

Table 5.19. ANOVA for the regression model of dataset 2 with new independent variable

Source	DF	Sum of Squares	Mean Square	F-value	P-value
<b>Regression</b>	1	92.030	92.030	458.61	0.000
<b>Residual</b>	102	20.469	0.201		
<b>Total</b>	103	112.499			

The model is significant, since the P-value of the model is 0 and less than the significance level 0.05.

Hence, as a result of the regression analysis, Equation 4.32 can be stated as

$$\ln y = 7.66 + 0.0685 \ln k \times X_1 \quad (5.8)$$

Therefore, the average income is calculated as shown below:

$$y = e^{7.66+0.0685 \ln k \times X_1} \quad (5.9)$$

Here, the variable  $k$  is set as the average of the physical capital per worker,  $k$ , values for Turkey between 1973-2003 (Saygili, 2005) and it is 3.935648. Thus, the income of individual which will be used in the analysis of the optimization problem becomes

$$y = e^{7.66+0.269592X_1} \quad (5.10)$$

### 5.2.2. Base Case

In this section, the base case parameters for the second model are presented. As mentioned before, payroll tax is assumed to be 15% depending on the calculation of Tusiad (2004). In addition, like the base case of the first model, maximum lifetime is assumed to be 65. Here, a regression analysis is conducted for the datasets mentioned before. Therefore, the share of education parameter  $\beta_1$  is assumed to be 26% depending on the regression analysis results obtained in Table 5.20 and stated in Equation 5.10. The base case assumes the system parameters as given in Table 5.20.

Table 5.20. Values of the base case parameters

Parameters	Base Values
Payroll Tax ( $\tau$ )	0.15
Share of Education on Wage ( $\beta_1$ )	0.26
Maximum Lifetime	65

### 5.2.3. Single Parameter Analysis

In this section, the individual effect of parameters, payroll tax ( $\tau$ ), share of education ( $\beta_1$ ) and maximum lifetime level on the preferences of individuals is discussed by holding other parameters constant.

5.2.3.1. Changes in Payroll Tax: This section discusses the effect of payroll tax changes on the analysis results of the model and the related results are displayed in Table 5.21. As done in the first model, payroll tax ( $\tau$ ) is varied between 5% and 75%. Similar to the results obtained in Section 5.1.2.1, as demonstrated in Figure 5.14, the duration of education does not change if  $\tau$  is decreased to 5%, because it is already 20. Since lower  $\tau$  yields higher after-tax income, the system requires that working consumers work as much as possible. Therefore,  $X_2$  increases to 40 years due to the decrease of  $\tau$  to 5%. As obtained in the analysis of the first model,  $X_1$  is 20 years for all  $\tau$  values.

Table 5.21. Analysis results for changes in  $\tau$

	$\tau$ (%)							
	5	15	25	35	45	55	65	75
$X_1$	20	20	20	20	20	20	20	20
$X_2$	40	37.7189	37.509	37.1843	36.6121	35.3123	29.3222	25
$X_3$	5	7.28114	7.49102	7.81575	8.38792	9.68768	15.6778	20
Utility	3040490	2750980	2402530	2049350	1691320	1330340	989034	752708

When  $\tau$  is increased from its base case value 15%, after-tax income decreases. Since the impact of  $X_1$  is really important in the income of individual, it does not change. Therefore, working less is preferred while  $\tau$  is increasing. Hence,  $X_3$  increases gradually depending on the decrease of  $X_2$ . Unlike the analysis of the first model, the model reacts significantly when  $\tau$  is between 55% and 65%. As expected the duration of working decreases because of very low after-tax income due to the high payroll tax. Moreover, since  $X_1$  cannot increase, a high  $X_3$  value is obtained. In addition, as mentioned before, between  $\tau = 55\%$  and  $\tau = 65\%$  working period decreases by a great amount and reaches its lower bound and this reaction is observed earlier in this model due to the fact that individual's income does not depend on the duration of working, namely experience.

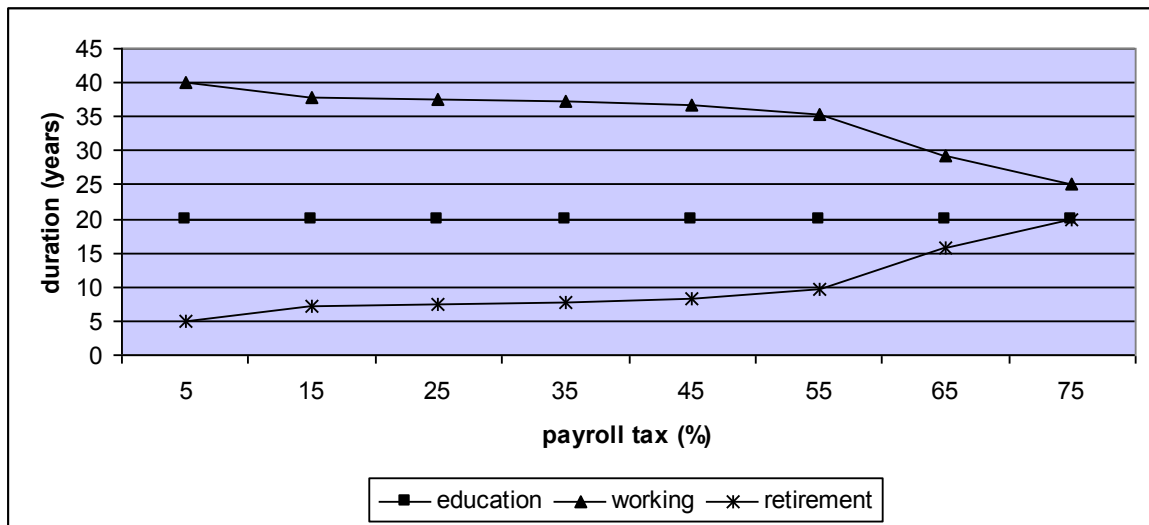


Figure 5.14. Effect of payroll tax ( $\tau$ ) on  $X_1$ ,  $X_2$  and  $X_3$

Furthermore, as it is demonstrated in Figure 5.15, the reaction of the lifetime utility is same as the results of the first model analysis for payroll tax changes. When  $\tau$  is decreased, since the after-tax income increases due to the increase in  $X_2$ , the lifetime utility increases. On the other hand, as seen previously, the increase of  $\tau$  from the base case value causes a decrease in lifetime utility due to the low after-tax income of individual and the decreasing working period.

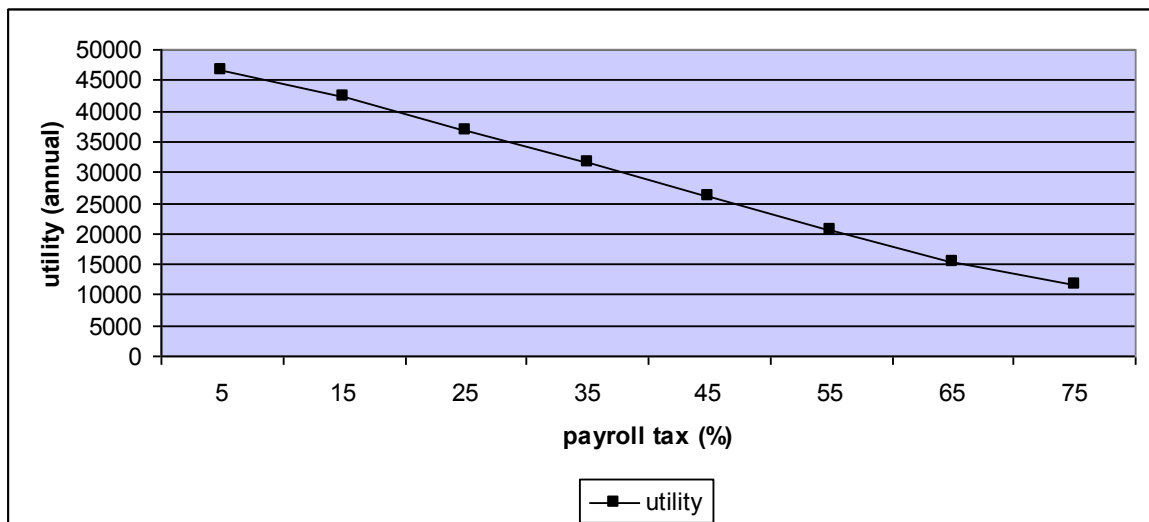


Figure 5.15. Effect of maximum lifetime on the lifetime utility of individual

5.2.3.2. Changes in the Share of Education on Wage: This section covers the effect of the share of education changes on the optimal solutions of education, working and retirement period. The analysis is conducted by 10% variation from the base value of  $\beta_1$  and the results are given in Table 5.22.

Table 5.22. Analysis results for changes in  $\beta_1$

	$\beta_1$ (%)		
	6	16	26
$X_1$	15.0813	20	20
$X_2$	40	37.7189	37.7189
$X_3$	9.91866	7.28114	7.28114
Utility	78070.9	454732	2750980

When the share of education is decreased to 16%, the duration of education, working and retirement do not change. Besides, if  $\beta_1$  is decreased to 6%,  $X_1$  decreases; therefore, the average years of working experience and retirement are obtained to be higher as a natural result. The reaction of the model results to the changes in  $\beta_1$  can be observed from Figure 5.16.

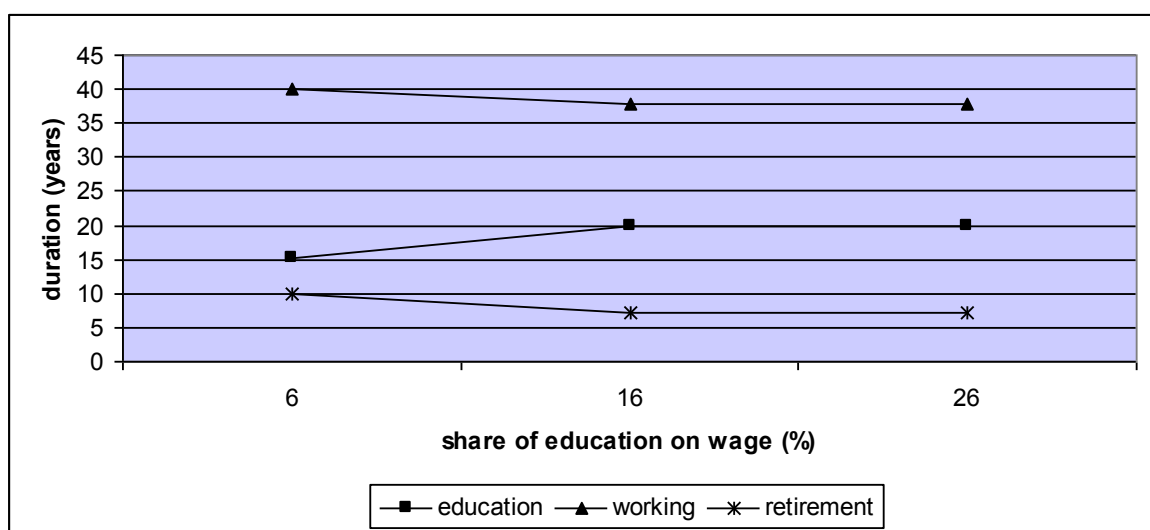


Figure 5.16. Effect of  $\beta_1$  on  $X_1$ ,  $X_2$  and  $X_3$

On the other hand, as a result of the increase in  $\beta_1$ , individual is expected to study more, since the income of individual increases due to the increase in  $\beta_1$ , and human capital is enhanced by increasing the duration of education. However,  $X_1$  does not change since it cannot exceed its upper bound value 20.

In addition, it can be seen from Figure 5.17. that the lifetime utility of individual decreases when  $\beta_1$  is decreased and increases when  $\beta_1$  is increased. The changes in the income of individual are the reason for these changes in lifetime utility, since the increase of income causes increase in lifetime utility while decrease of income causes decrease in lifetime utility.

In this model, the share of education is really high in comparison with the first model. As it can be observed,  $X_1$  has its maximum at  $\beta_1 = 16\%$ ; therefore, any increase in  $\beta_1$  does not make any change in  $X_1$  values. Moreover,  $\beta_1$  does not directly affect  $X_2$  and  $X_3$ , since it is the impact of individual's income. Hence,  $X_2$  and  $X_3$  remain the same.

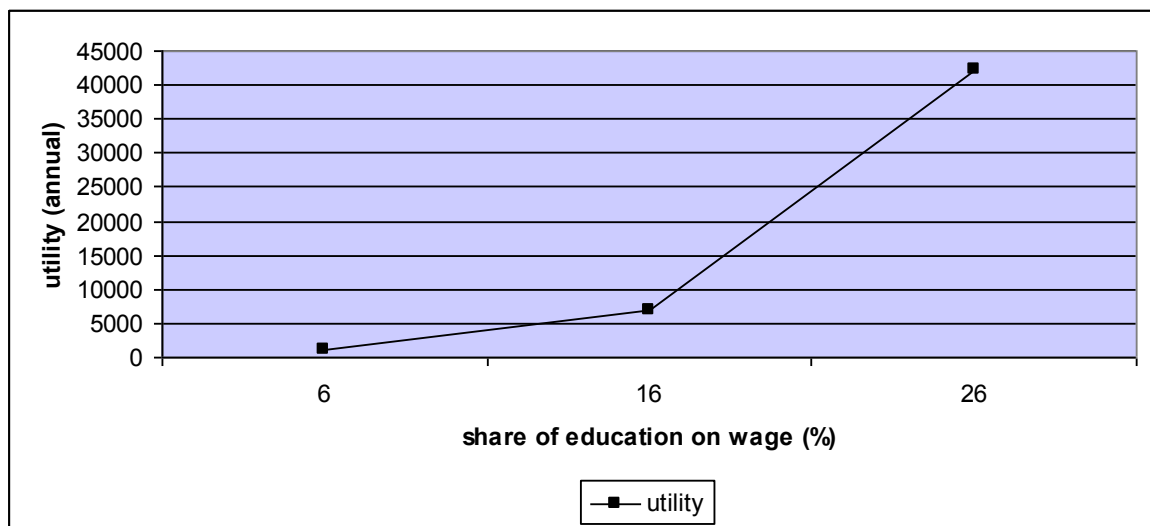


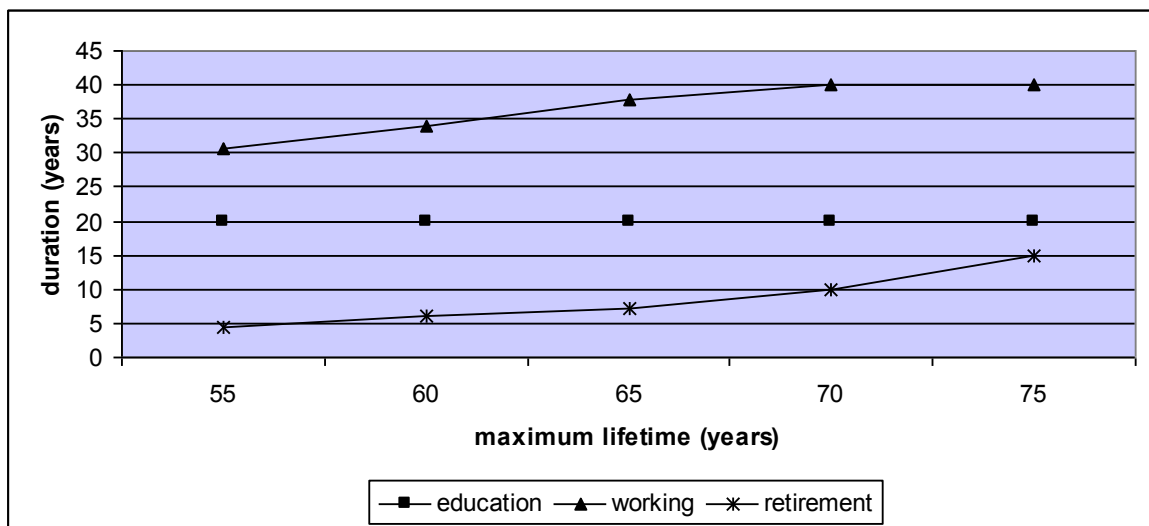
Figure 5.17. Effect of  $\beta_1$  on lifetime utility

**5.2.3.3. Changes in Maximum Lifetime:** In this section, the effect of maximum lifetime on the determination of education, working and retirement duration is discussed. Table 5.21 summarizes the results of the changes in maximum lifetime. Figure 5.18 demonstrates the lifetime allocation which the model makes due to the changes in maximum lifetime.

Table 5.23. Analysis results for changes in maximum lifetime

	Maximum Lifetime				
	55	60	65	70	75
$X_1$	20	20	20	20	20
$X_2$	30.5742	33.9932	37.7189	40	40
$X_3$	4.42584	6.00681	7.28114	10	15
Utility	1546280	2079630	2750980	3586140	4398020

Similar to the analysis in Section 5.1.2.4, when the lifetime is increased from the base value, it is preferred to work and enjoy the retirement period more. Since the income of individual is a function of education and working period, as a natural result, individual is expected to study more. However, since  $X_1$  is at its maximum value at the base case, no increase is observed.

Figure 5.18. Effect of maximum lifetime on  $X_1$ ,  $X_2$  and  $X_3$ 

This analysis differs from the analysis in Section 5.1.2.4 by the fact that the retirement period is generally more than the values obtained in the first model. In the second model the individual's income is only a function of the duration of education;

therefore, we obtain lower  $X_2$  values comparing the analysis of the first model. Hence,  $X_3$  has higher values here.

When maximum lifetime is increased,  $X_2$  and  $X_3$  increase as much as possible. Moreover, the average income and therefore the average consumption, which forms the lifetime utility, increase. As a result of this, the lifetime utility increases as illustrated in Figure 5.19. Furthermore, the decrease of maximum lifetime causes the decrease of lifetime utility as a result of working and enjoying the retirement less.

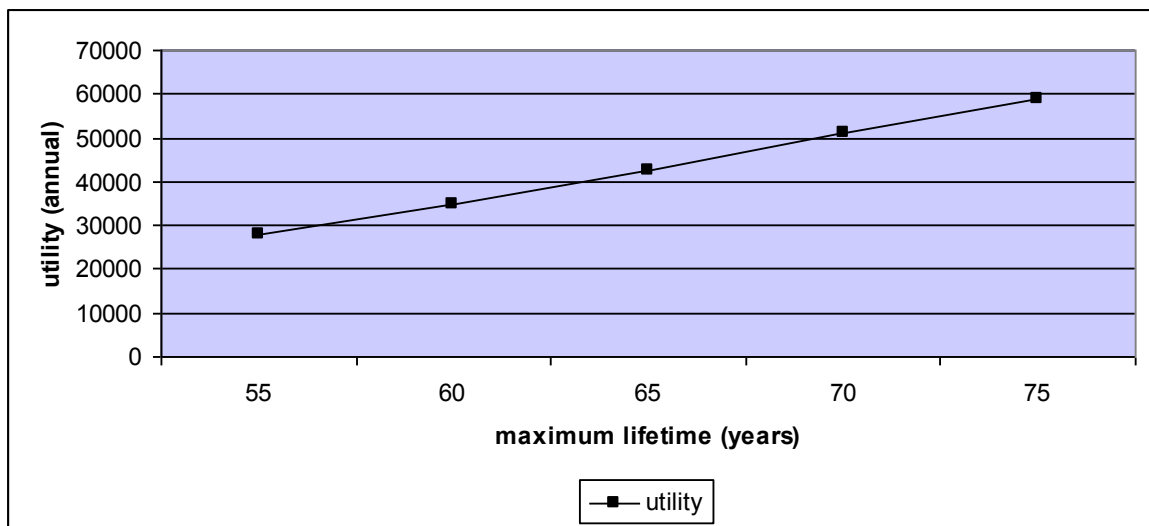


Figure 5.19. Effect of maximum lifetime on the lifetime utility of individual

#### 5.2.4. Multi-parameter Analysis

In this section, we aim to examine the results of the aggregate effect of payroll tax and share of education on the behavior of the model on the education, working and retirement period. Different from the first model analysis, the effect of the share of education on wage is not covered since here individual's income is only a function of education period. The behavior of  $X_1$ ,  $X_2$ ,  $X_3$  due to the changes of  $\tau$  and  $\beta_1$  is displayed in Figure 5.20, Figure 5.21 and Figure 5.22, respectively while results points are shown in Table 5.24, Table 5.25 and Table 5.26.

$\tau$  is varied between 5% and 75% while  $\beta_1$  is varied between 6% and 26%. When  $\beta_1$  is decreased to 16%, no change is observed in  $X_1$  values. As Table 5.24, Table 5.25, Table

5.26 show, after  $\beta_1 = 16\%$  the reaction of the model is fixed and  $X_1$ ,  $X_2$  and  $X_3$  remains unchanged. Therefore, we do not include the case where  $\beta_1$  is increased. Moreover, at  $\beta_1 = 16\%$  and  $\beta_1 = 26\%$ , increasing the payroll tax does not cause any change since the share of  $X_1$  on individual's income is high. However, at  $\beta_1 = 6\%$  this resistance does not continue and while  $\tau$  is increased, there is a decreasing manner in  $X_1$ , since  $\tau$  reaches to 65%. Until  $\tau$  reaches to 65% ,  $X_2$  remains the same. As mentioned in the single parameter analysis,  $\tau = 65\%$  is the point where  $X_2$  changes by a great amount. At  $\beta_1 = 6\%$ , this amount is lower than the amount of decrease at  $\beta_1 = 26\%$ . Here, payroll tax reaches a point where shorter working period is obtained because of low after-tax income. Therefore, here  $X_1$  and  $X_3$  values increase in order to compensate the decreasing lifetime utility by satisfying the lifetime constraint.

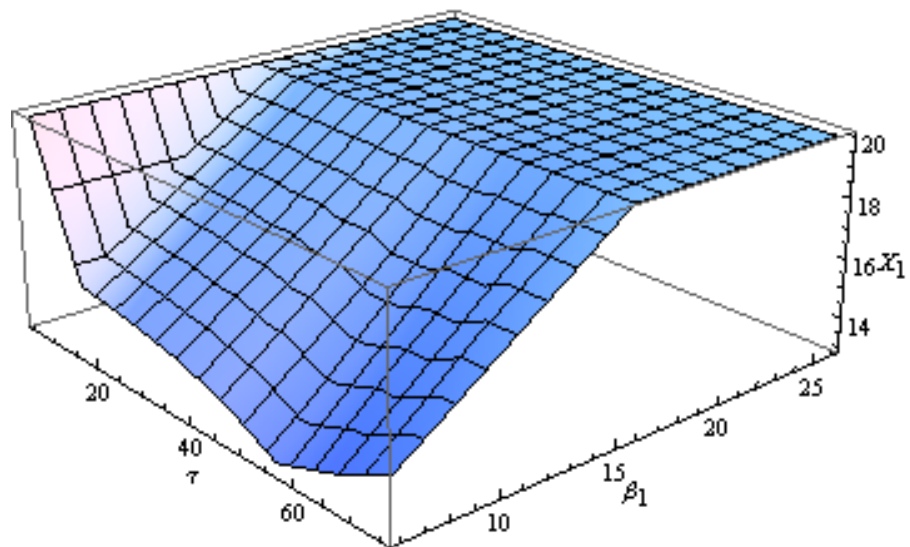
Table 5.24.  $X_1$  values for different  $\tau$  and  $\beta_1$

		$\beta_1(\%)$		
		6	16	26
$\tau(\%)$	5	20	20	20
	15	15.0813	20	20
	25	14.8329	20	20
	35	14.476	20	20
	45	13.9232	20	20
	55	12.9669	20	20
	65	13.6762	20	20
	75	14.6334	20	20

When we look the changes in  $X_2$  due to the changes in  $\beta_1$  and  $\tau$ , at  $\beta_1 = 16\%$  and 26%, increasing  $\tau$  causes decrease in  $X_2$  values. Moreover,  $X_2$  drops to its lower bound when  $\tau = 75\%$  which is a very high payroll tax value. In addition, at  $\beta_1 = 16\%$  and 26%,  $X_2$  does not change; therefore,  $X_3$  increases while  $\tau$  is increased gradually.

Table 5.25.  $X_2$  values for different  $\tau$  and  $\beta_1$ 

		$\beta_1$ (%)		
		6	16	26
$\tau$ (%)	5	40	40	40
	15	40	37.7189	37.7189
	25	40	37.509	37.509
	35	40	37.1843	37.1843
	45	40	36.6121	36.6121
	55	40	35.3123	35.3123
	65	35.5261	29.3222	29.3222
	75	25	25	25

Figure 5.20. Aggregate effect of  $\tau$  and  $\beta_2$  on  $X_1$

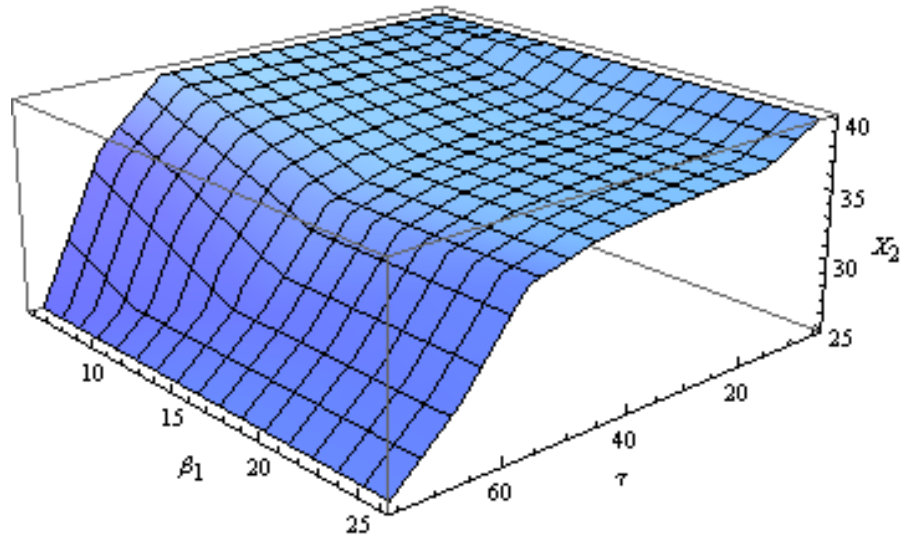


Figure 5.21. Aggregate effect of  $\tau$  and  $\beta_1$  on  $X_2$

As Table 5.26 shows,  $X_3$  takes its maximum value in the analysis of the second model when  $\tau = 75\%$  and  $\beta_1 = 6\%$ . Due to the high  $\tau$ ,  $X_2$  decreases; moreover,  $X_2$  cannot increase since  $\beta_1$  is very low. In order to increase lifetime utility and satisfying the lifetime constraint,  $X_3$  reaches a high value here.

Table 5.26.  $X_3$  values for different  $\tau$  and  $\beta_1$

		$\beta_1$ (%)		
		6	16	26
$\tau$ (%)	5	5	5	5
	15	9.91866	7.28114	7.28114
	25	10.1671	7.49102	7.49102
	35	10.524	7.81575	7.81575
	45	11.0768	8.38792	8.38792
	55	12.9669	9.68768	9.68768
	65	15.7977	15.6778	15.6778
	75	25.3666	20	20

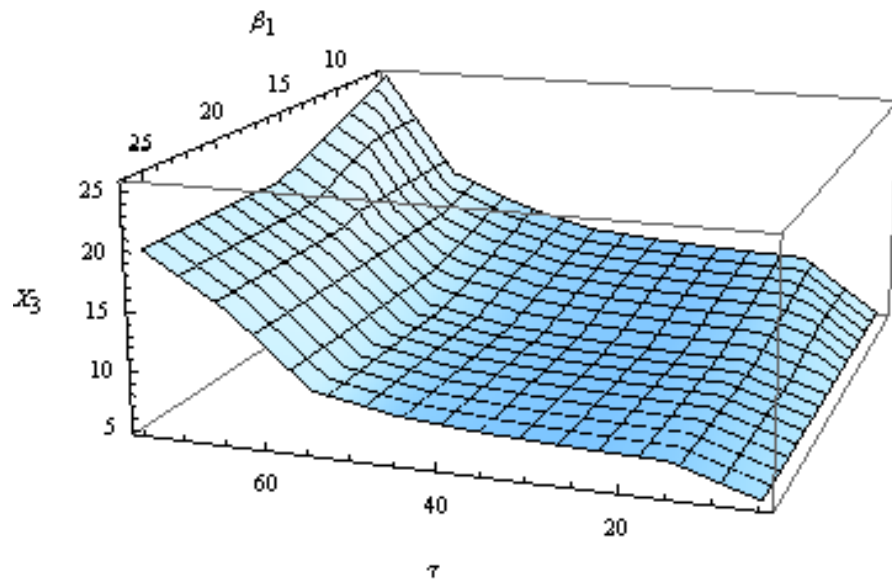


Figure 5.22. Aggregate effect of  $\tau$  and  $\beta_1$  on  $X_3$

## 6. CONCLUSIONS

In this study, an important issue which has taken the attraction of the economists, the human capital, is discussed. There have been many studies on the economic growth which takes human capital as the engine of the growth. Moreover, since there are financial problems of social security systems in many developing and developed countries, economists also study on modelling the social security systems, especially the pension plans, considering the individual's consumption and saving decisions. Different from these studies, this thesis covers two models which consider the average duration of education, working and retirement. One of the models examines the average lifetime allocation for education, working and retirement with human capital but without considering the economic growth while the other covers the economic growth with human capital to examine the average lifetime allocation within the social security system in Turkey.

In our study human capital which is a function of the years of education and working experience is used. In order to examine the impact of human capital on the average education, working and retirement duration for the sustainability of the social security system, two datasets which include the average years of education and working experience, gross domestic product, physical capital and laborforce information are used as shown in Appendix A. The regression analysis of the Dataset 1 indicates that the years of education and working experience have significant effect on the gross domestic product per worker, which is an indicator of the income level of individuals. However, in the regression analysis of Dataset 2, it is observed that the factor of average education years is significant for explaining the gross domestic product per worker. The regression results of Dataset 2 constitute the production side of the second model. By using the share of human capital on the average income obtained from the regression analysis, the optimization problem is solved with the current parameters of the social security system in Turkey.

An important conclusion arising from this study is that, human capital is an important determinant of the average income. More human capital brings more production as a nature of the production structure used in the study. Average lifetime utility which is a function of average consumption during working and average consumption during

retirement is tried to be maximized. Moreover, the consumptions are the functions of the income of individual formed by human capital. Satisfying the constraints of social security system, the optimal duration of education, working and retirement are determined.

It is observed from the analysis that the share of human capital on the average wage (income) affects the results for how much to study, to work and to enjoy retirement. Higher share of human capital on the average income yields higher lifetime utility. As a natural result, when the share of education on wage increases, models yield longer education period and when the share of working experience on wage increases, longer working experience is obtained. However, the retirement period is not ignored since the model is constructed in such a way that leisure time in retirement also has an important effect on the lifetime utility. Therefore, the results are achieved considering the leisure time which will be enjoyed during retirement.

Since the model is analyzed by using the social security parameters of Turkey, it is very essential to say that payroll taxes, which are collected for financing the social security system, also have an effect on the average lifetime allocation of education, working and retirement. Very high payroll taxes make the models yield shorter working period, longer education and longer retirement period. When the average lifetime increases, primarily longer studying and working periods are obtained. The duration of retirement also increases but slightly comparing the increase of education. This indicates that the return of human capital is more effective than the utility gained from retirement.

If we compare two models which are analyzed in this study, the behaviours of the models are in similar nature, although the working experience is not included in the second model. However, when we look at the payroll tax changes effect, the payroll tax values where the models react significantly and decrease the working duration are different. In the first model, 75% is the point which has the most significant effect on the duration of working while it is 65% in the second model. The reason is that in the second model the average wage is not a function of the working experience. Therefore, the resistance of the duration of working across the payroll tax changes is less than the first model. Hence, the significant reaction is observed earlier in the analysis of the second model.

When we observe the results of the retirement period for two models, the highest duration of retirement value is observed in the case where the share of education is 6% with the payroll tax 75%, in the second model. The highest retirement duration value in the first model is about 23 years while it is about 25 years in the second model. Moreover, since the share of education period on wage is higher in the second model, the duration of education is higher than the first model. In addition, the average wage does not depend on the working experience in the second model; therefore, the working duration is obtained to be less than the first model. Hence, in the case of the first model, the analysis yields shorter education period, and longer working and retirement period. We can see that despite of the similarities in the models, different results are also observed due to the base case values of the model parameters.

In the current situation of Turkey, depending on the educational attainment data of TurkStat, average years of education is obtained to be about 7 years. This value is very low in comparison with the results obtained in our study. Especially, in the second model, the average income is only a function of the education period; therefore, 7 years of education yields very low income. In addition, the average duration of retirement is about 30 years which is very high because of early retirement issue happened formerly. As a result, considering the lifetime constraint in our models, average working experience is 28 years. This value is really close to the lower bound of working duration in our study. It is clear that a pension system with these values cannot keep the sustainability. Working less means, lower total tax collected from the working consumers. Furthermore, the population is getting older, this means that less working consumers will participate in the system. Thus, it will cause important problems for financing the retirement benefits; in other words, more deficit in the budget of the social security system.

This study can be further extended by including the other two institutions, ES and BK, in order to analyze the models in the full coverage of the pension system in Turkey. The models analyzed in this study are static and deterministic in nature. It would be beneficial to construct a dynamic and stochastic model which also considers the mortality probabilities of individuals, growth rate of wages and unemployment rates.

The production side of the second model can be analyzed by using a specific sector in the economy. As analyzing the impact of human capital, different proxies can be used for the construction of individual's human capital, such as investment in health and number of patents taken in a certain time interval.

In addition, a questionnaire which includes the information about the years of education, working experience, income level and gender can be applied to the employees of different companies in Turkey. Moreover, the data obtained from this survey can yield a more specific analysis for Turkey which gives the share of education and working experience on the average income.

We can conclude that since human capital has a significant effect on the production of goods and services beside physical capital, the workers' knowledge and skills should be increased by education at school and training at work. Thus, the growth of the economy can come true by increasing the quality of education and making more active and innovative workers. According to the results obtained in the analysis of two models, it can be observed that the duration of education is obtained to be considerably high, usually at its upper bound. Therefore, the government can increase the standard of living by supporting the schools, by investing on the education in order to improve the quality and productivity at work. Therefore, this will provide a more competitive and knowledge-based economy with a sustainable economic growth.

## APPENDIX A: DATASETS

Table A.1. Dataset 1

Country	$n$	$s_k$	$s_h$	$Y$	$X_1$	$X_2$
Argentina	1,45	17,36	0,89	11331,958	8,83	20,86
Austria	0,32	26,50	3,75	26999,767	8,35	24,48
Barbados	0,22	5,63	5,78	16086,000	8,73	21,67
Belgium	0,27	25,25	6,59	24661,914	9,34	23,56
Benin	2,69	10,41	4,38	1251,474	2,34	23,64
Bolivia	2,27	11,08	4,31	2929,186	5,58	21,31
Brazil	2,18	21,52	1,09	7193,597	4,88	23,06
Cameroon	2,69	6,01	1,70	2471,727	3,54	22,28
Central African Republic	2,28	10,23	3,36	945,102	2,53	23,88
Chile	1,75	21,67	4,62	11430,188	7,55	22,06
Congo	2,75	28,75	4,47	1286,190	5,14	20,63
Costa Rica	2,70	9,14	4,49	8341,470	6,05	21,70
Cyprus	0,79	21,82	4,42	20456,780	9,15	22,09
Denmark	0,38	25,02	4,26	27827,281	9,66	23,74
El Salvador	1,90	7,89	2,33	4732,128	5,15	22,47
Finland	0,39	33,82	3,48	22740,686	9,99	23,42
France	0,65	25,42	3,05	25044,538	7,86	24,45
Gambia	3,53	6,05	3,03	953,859	2,31	24,87
Germany	0,30	27,33	0,20	25061,340	10,20	23,63
Greece	0,68	28,47	3,34	13982,385	8,67	23,44
Guyana	0,71	16,63	6,55	3733,182	6,25	20,27
Haiti	2,01	4,73	1,32	2069,288	2,77	23,52
Honduras	2,84	13,88	3,26	2239,656	4,80	20,67
Hungary	0,06	20,07	3,78	11382,950	9,12	23,40
Iceland	1,17	28,46	2,92	25794,627	8,83	22,06
India	2,11	12,01	0,41	2643,851	5,06	22,54
Indonesia	1,98	16,57	1,22	3771,861	4,99	22,68
Iran (Islamic Republic)	2,82	29,26	4,81	6045,526	5,31	20,28
Ireland	0,74	22,45	4,88	24947,554	9,35	21,16
Israel	2,64	28,71	6,35	22236,900	9,60	19,93
Italy	0,36	27,24	3,67	22487,212	7,18	26,00
Jamaica	1,14	19,90	6,29	4520,838	5,26	22,31
Japan	0,77	32,88	1,38	23970,563	9,47	24,22
Mali	1,83	8,94	2,93	1046,719	0,88	24,49
Mexico	2,41	20,22	3,16	8082,091	7,23	19,77
Netherlands	0,82	26,84	5,87	26293,085	9,35	23,47
Nicaragua	2,64	9,43	4,65	3437,851	4,58	20,69
Niger	3,06	6,58	2,88	807,454	1,02	25,07
Panama	2,41	19,52	5,03	7934,799	8,55	19,33
Paraguay	2,58	11,97	1,82	4965,414	6,18	20,63
Peru	2,41	21,18	3,32	4204,500	7,58	19,60
Poland	0,65	22,98	2,39	8611,004	9,84	21,37
Portugal	0,36	22,81	3,89	17323,140	5,87	26,18
Rwanda	2,53	2,70	2,24	1018,070	2,56	22,04

Table A.1. Continued

<b>Country</b>	<b><i>n</i></b>	<b><i>s<sub>k</sub></i></b>	<b><i>s<sub>h</sub></i></b>	<b><i>Y</i></b>	<b><i>X<sub>1</sub></i></b>	<b><i>X<sub>2</sub></i></b>
Senegal	2,92	4,91	5,18	1571,367	2,55	22,63
Singapore	2,25	51,83	3,57	29433,771	7,05	24,72
South Africa	2,37	11,14	1,84	8226,063	6,14	21,10
Spain	0,70	25,56	1,50	19536,381	7,28	24,26
Sri Lanka	1,57	14,78	2,62	4046,630	6,87	22,16
Sweden	0,43	25,32	2,85	25231,770	11,41	22,02
Switzerland	0,75	31,64	0,69	28831,251	10,48	22,68
Syrian Arab Republic	3,19	8,71	2,45	2000,888	5,77	18,82
Thailand	2,03	30,07	4,21	6473,595	6,50	23,19
Togo	3,02	11,11	5,39	823,166	3,33	22,35
Trinidad	1,08	20,91	3,81	14770,034	7,76	20,40
Turkey	2,14	14,53	3,76	5714,591	5,29	22,11
UK	0,30	19,27	1,47	24666,411	9,42	23,32
Uruguay	0,67	14,49	2,15	10739,737	7,56	23,11
Venezuela	2,92	21,36	4,35	7322,970	6,64	20,93
Zambia	3,09	22,72	4,02	865,649	5,46	20,03

Table A.2. Dataset 2

Country	$K$	$L$	$Y$	$X_1$	$X_2$
Algeria	180215	4834,1	69693064	4,6570	36,5834
Angola	13180	3718,2	7228200	3,6898	22,1507
Argentina	178450	10884	120778040	8,0239	22,3001
Australia	534246	7364,1	197762896	8,7223	36,4957
Austria	221013	3503,9	77748504	8,5742	22,7138
Bangladesh	47998	28846	70413000	3,4821	22,5736
Barbados	3515	127,00	1556456	8,0090	19,2332
Belgium	257377	4092,0	103094960	9,3513	22,3466
Benin	3510	1963,6	4459429	2,3348	24,0586
Bolivia	18120	1987,1	9976986	5,3649	21,2947
Botswana	4363	381,02	2733850	3,5326	21,7652
Brazil	980791	49641	541578176	5,5357	21,1208
Burkina Faso	7301	3763,4	4019310	0,7332	24,9130
Burundi	2752	2521,1	2531144	1,7341	23,4901
Cameroon	17873	3958,0	18262272	5,4293	21,2868
Canada	868868	12723	381014912	9,9838	19,8254
Central African Rep	2028	1282,3	1805517	3,5607	23,5772
Chad	4974	1789,9	2604342	1,8253	24,9392
Chile	65426	4275,9	45611324	6,9627	30,7768
Colombia	152093	9195,0	93779400	6,5211	40,8826
Costa Rica	12778	904,02	8987779	8,2263	17,5698
Denmark	160007	2784,0	61265720	6,9074	24,4750
Dominican Rep	22423	1861,9	13480016	6,6521	19,2583
Ecuador	62199	2838,9	26023950	8,7616	17,2695
Egypt	48557	12836	91920696	5,6956	21,0834
El Salvador	7416	1832,1	8420288	4,2209	22,4221
Ethiopia	7500	19187	13738075	1,1401	51,6964
Fiji	5929	230,99	2461900	6,6368	19,4114
Finland	175072	2488,0	55092300	10,8279	20,5389
France	1689367	24639	627613952	9,5391	21,7501
Gabon	14693	518,00	4197370	8,0129	19,7594
Gambia	290	307,08	542300	1,5525	26,3947
Germany	1954396	29403	710813632	10,3321	21,4380
Ghana	9490	4964,1	10752240	3,8589	29,8266
Greece	138324	3779,9	56657328	8,4000	23,6372

Table A.2. Continued

Country	<i>K</i>	<i>L</i>	<i>Y</i>	<i>X<sub>1</sub></i>	<i>X<sub>2</sub></i>
Guatemala	16467	2261,0	17518600	3,6760	22,6577
Guyana	4597	337,05	999350	6,2113	19,0125
Haiti	6215	2822,0	5471928	2,6625	24,0027
Honduras	7150	1303,0	5434920	5,6431	19,6243
Hong Kong	88232	2866,1	55558448	7,8005	20,9646
Iceland	6885	127,00	2867900	8,5581	21,1407
India	926307	293250	532541632	4,7514	22,3613
Indonesia	578202	63425	280529536	4,4695	22,3468
Iran	295697	13023	163685792	5,7493	20,6981
Iraq	244547	4258,9	55370272	4,5519	36,5218
Ireland	64542	1367,0	21154168	8,8303	20,4675
Israel	84087	1610,0	39337268	10,0337	18,8184
Italy	1575287	22763	604780352	9,1318	22,8279
Jamaica	13365	1095,1	5562016	5,8989	19,9018
Japan	3608437	59773	1301848832	9,4693	22,6748
Jordan	18375	798,97	9574886	7,4699	17,6013
Kenya	24453	8389,4	17198284	3,4454	37,374601
Kuwait	65983	677,01	21715008	6,9188	19,288533
Lesotho	2221	729,98	1909620	4,8838	21,613522
Liberia	5591	807,97	2053854	3,2279	22,703649
Luxembourg	12331	155,00	4531812	6,9022	24,817642
Madagascar	8815	4509,8	6913524	4,3069	21,622661
Malawi	4772	3074,9	4237175	1,9665	23,846399
Malaysia	207538	6171,2	74505184	5,7288	34,769025
Mali	3171	2598,4	3591054	1,4435	24,871438
Malta	4634	139,00	1989270	6,8231	23,957573
Mauritania	3560	589,94	1635316	1,0287	24,976793
Mauritius	4441	390,02	3831120	6,3028	20,429059
Mexico	828662	26080	418689984	7,0618	18,930738
Morocco	39320	6675,8	44527560	3,4852	40,98812
Mozambique	25068	7670,9	11460321	2,1021	31,858719
Nepal	16653	6868,9	12164823	2,0246	38,838783
Netherlands	368831	5861,0	158488064	9,4748	35,75137
New Zealand	79636	1458,0	32918086	9,2745	20,151696
Nicaragua	14689	992,95	6184080	6,0230	19,233053

Table A.2. Continued

Country	<i>K</i>	<i>L</i>	<i>Y</i>	<i>X</i> <sub>1</sub>	<i>X</i> <sub>2</sub>
Niger	4910	3203,2	3994375	0,8309	25,292733
Nigeria	189261	36574	106247152	2,0057	24,520547
Norway	169468	2039,0	56044736	9,2365	21,74098
Pakistan	140879	29802	139653360	2,5400	24,141342
Panama	16707	760,01	7967900	7,9892	18,22228
Papua New Guinea	16549	1684,8	5859859	2,7990	22,446096
Paraguay	10356	1223,0	8660085	6,1659	19,933343
Peru	97838	6204,2	52915592	7,9436	36,111369
Philippines	211298	19875	97627432	8,8683	46,202019
Portugal	109656	4562,9	46061996	6,5156	24,530039
Rwanda	2604	3063,3	4405006	3,2368	22,15665
Saudi Arabia	223459	3405,0	109797824	2,9513	36,3139
Senegal	6795	2897,5	7591452	2,4804	24,119109
Sierra Leone	1672	1351,9	3719169	1,9828	25,511677
Singapore	75127	1226,0	26646686	6,8895	20,647167
Somalia	6581	1999,4	4538712	0,8255	24,99443
Spain	667000	13725	247818464	9,6992	21,430861
Sri Lanka	67085	5919,6	31072194	6,0320	38,359385
Sudan	6105	6991,8	20639828	2,0888	44,943454
Suriname	2828	117,00	1621452	6,0845	20,404368
Swaziland	3511	273,00	1405145	5,4018	20,364963
Sweden	231996	4236,9	103389696	9,6351	22,513373
Syria	84321	2596,0	52457328	6,6225	18,439697
Thailand	160879	26657	130034432	5,5127	21,194647
Trinidad&Tobago	23897	450,00	8861430	5,9157	20,71021
Tunisia	29123	2224,0	22538144	5,6554	20,808771
Turkey	335542	21384	161193248	6,3271	20,763534
Uganda	2153	7053,0	6312400	2,9334	34,160445
United Kingdom	1180783	27432	604623616	8,4984	22,874877
Uruguay	32888	1171,0	13291740	7,6588	23,602552
USA	7223147	116801	4014929408	12,0864	17,939423
Venezuela	185689	5870,9	98014224	6,8983	37,025342
Zambia	17313	2241,5	5108448	3,8305	21,957244
Zimbabwe	16974	3410,0	12054204	4,8537	20,230283

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