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A STUDY ON RESOURCE ALLOCATION, JOINT COSTS AND OPTIMAL PRODUCT MIX OF A CHEMICAL COMPANY

Güngör KAVADARLI

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CHAPTER I

INTRODUCTION

A problem which is encountered very often in the product industries and which is crucial for the profitable performance of many companies is the determination of their optimal resource allocation and optimal product mix. If the resources of a company are unlimited, a resource allocation study for that firm is not necessitated and efficiency in the utilization or processing of the particular resource need not be high. This is true in the sense that although inefficiencies contribute to higher costs, and thus, result in lower profits for the firm than is attainable by it, still there is a sufficient amount of resource to keep every activity of the particular company going. In reality, however, industrial companies rarely have access to unlimited resources, and the limited nature of one or more resources impose certain restrictions on the company activities. These resources may be physical plant capacities and equipment, labor supply, availability of capable management and funds of capital. Since every limited resource is assigned to one type of activity always at the expense of another activity, the aim of such allocations is to maximize a certain objective of the company.

For an industrial firm the determination of its optimal product mix may properly be treated within the general field of production programming where the objective is to determine:

1. what products should be produced, and
2. what quantity of each selected product should be produced and when should they be produced,

such that the company activities are optimal.. In this paper, the term "optimal" is used synonymously with "maximum profits." However, for different companies the design of optimal conditions may include considerations such as market share, contribution to the general economy, community welfare, social benefits, etc.

An integral part of production programming problems is the inventory considerations. In order to be genuinely optimal, a production program for optimal product mix should take into account restrictions on and costs of inventory. In this paper, no consideration is made of inventory costs and policies, therefore, the optimal production mix determined for the company under study is truly a sub-optimal program, since inventory considerations might change the proposed mix. However, it must be added that this product mix is optimal within the scope of this study. Furthermore, the period of production for the various products is of no interest and only the quantities to be produced annually will be determined.

In a large number of industries, it is just because of the nature of the materials and activities involved that there are more than one product and/or by-product. A typical case is that of the chemical industries where it is most common to observe single raw materials or processes resulting in a number of products. These are usually joint products, i.e. they originate through a single activity or material which is common to all. There is no way of getting the desired product exclusively without also obtaining the other joint products. It follows then that the costs associated with such cases are common to all products. In other words, they are joint costs. It is believed by the writer that certain aspects of joint costs and fixed costs for decision making purposes are not properly treated in Turkey. Therefore, in this paper, one of the considerations will be to discuss the relationships between joint costs, fixed costs, variable costs, and revenues so as to provide general guidelines for optimum output decisions. The main tools in the analysis of output decisions will be revenue functions and functions of costs identifiable by individual products. The figures on these matters will constitute the "givens" of the analysis. Assumptions and simplifications will be introduced at the relevant points during the study.

Another major consideration will be formulating a general mathematical model that can be used for resource allocation problems. By transforming this model to a flow chart, it is believed that this model will be applicable to computer solution. It is thought that this model will provide the optimal solution for a given set of conditions. It will also be able to adjust its optimal solution in view of changing conditions and modified activities of the company. The main advantage of such a model is its facility in predicting the effects of changing conditions and the complementary adjustments thus necessitated in the resource allocation process. After formulating the mathematical model, a graphical solution will be proposed for the special case where the revenue and variable cost functions are linear.

In the final phase of the analysis, the optimal product mix of the company under study will be determined by illustrating the optimal allocation of its raw material to the derivatives obtained from it.

CHAPTER II

DESCRIPTION OF THE COMPANY ACTIVITIES AND PROCESSES

The company which is the subject of this paper is one of the Turkish chemical industries and will hereafter be referred to by the fictitious name of "CHEMCO". It consists of a number of integrated processing units where many products are manufactured through a series of operations and processes. The products of the company are:

1. Caustic soda ($NaOH$)
2. Sodium hypochlorite ($NaOCl$)
3. Sulfuric acid (H_2SO_4)
4. Hydrochloric acid (HCl): grade A and grade B
5. Liquid chlorine
6. Benzene hexachloride (BHC)
7. Dichlorodiphenyltrichloroethane (DDT)
8. Orthodichlorobenzene
9. Paradichlorobenzene
10. Monochlorobenzene (MCB)
11. Hydrogen gas (H_2)
12. Chlorine gas (Cl_2)

The location of the Chemco factory is very suitable for industrial purposes, and indeed, there are many other factories within its vicinity. Being situated in an industrial district, it enjoys a sufficient supply of skilled labor force. It has ready access to a main highway, railroad, and the sea. This situation provides not only a reduction in costs of transportation for materials that are received and shipped, it also enables the factory to have a more convenient and simple materials handling system. The firm owns an area of land adjacent to the factory which will make a twofold to threefold expansion possible when the need arises.

The production facilities are modern, and as far as these facilities are concerned, they do not impose a joint production constraint on the various products. In other words, since the DDT producing reactors and equipment are utilised only for that purpose and the BHC reactors and equipment are different, from the viewpoint of availability of physical manufacturing equipment, the two products are independent of one another. This is also true for other products because a different set of reactors is necessary to produce different products; for example, sulfuric acid and hydrochloric acid. The Chemco factory consists of the following self-contained and integrated manufacturing units:

1. Electrolysis plant
2. Chlorine liquefaction unit
3. Hydrochloric acid unit

4. BHC plant
5. DDT plant
6. Sodium hypochlorite unit
7. Sulfuric acid plant
8. o-Dichlorobenzene and p-dichlorobenzene separation unit

At this point, to provide some general insight for the reader on the activities and products of the company, basic information about the products and the facilities will be presented.

The Products

There are twelve products in the list presented at the beginning of this chapter. However, it must be pointed out that all twelve are products only in the general scope of the word "product". Chlorine gas, hydrogen gas, and monochlorobenzene (MCB) are intermediate products and are further processed into other final products as will be explained. Sodium hypochlorite, and the two dichlorobenzenes are by-products and are produced along with the main products just because of the nature of the chemical reactions involved. It is possible to classify only six of the twelve products as main products, and they are caustic soda, sulfuric acid and hydrochloric acids, liquid chlorine, BHC, and DDT. Some basic information about the individual products are presented in the following paragraphs.

1. Caustic Soda - It is one of the products of the electrolysis process. Metallic sodium reacts with water to liberate hydrogen gas and caustic soda liquor. This is one of the most profitable products in Chemco's product line and is sold in solution form. Besides being manufactured by other companies in Turkey, caustic soda is also imported but the imports are in solid form. The price is set by competitive considerations especially that of the international market. Since frequent ordering and fast deliveries are possible to domestic customers and this enables these customers to incur lower inventory costs, the management feels that by setting their price at the import price level the company gets a competitive edge.
2. Sodium Hypochlorite - This is an involuntarily produced product. According to industrial safety regulations, due to its toxicity only a certain amount of chlorine gas can be liberated to the atmosphere. Therefore, wasted chlorine gas from three different sources and operations cannot be freed to the air and have to be passed through caustic soda and absorbed by it. This process results in production of sodium hypochlorite. It is not desirable to produce hypochlorite, because it not only consumes caustic soda which is more valuable than hypochlorite but also necessi-

tates the incurring of additional direct costs. The amount of hypochlorite produced exceeds the domestic demand by three to four times; thus, the unsold portion is disposed of.

3. Sulfuric Acid - It is produced by the contact process which uses sulfur as the raw material, converts it first to sulfur trioxide and finally to sulfuric acid. At Chemco, it is used in the production of DDT, in the liquefaction of chlorine gas, and is also sold on the market. The Turkish companies cannot meet the internal demand for sulfuric acid which is very high; consequently, this acid is also imported.

4. Hydrochloric Acid - There are two grades of this acid produced at Chemco. Grade A of the acid is produced from two electrolysis products by burning chlorine gas in an excess of hydrogen gas to produce hydrogen chloride gas. This gas is then dissolved in water in absorption towers to yield pure, water-white acid. Hydrochloric acid is not imported and is produced by a number of companies. The price is set competitively.

5. Liquid Chlorine - Gaseous chlorine is easily converted to liquid chlorine which is the form demanded in the market due to its conveniennce in transportation and storage. The chlorine gas from the anode of the electrolysis cell is warm and moist. Therefore, it is pumped to a cooling tower where first, the moisture content of the raw gas is reduced by a countercurrent stream of cold water and then it is dried

by a countercurrent stream of concentrated sulfuric acid. After compressing the gas to about 3 atmospheres of pressure, at a liquefying unit, it is subjected to cold surface temperature of about -5 degrees centigrade which cause the gas to liquefy at the prevalent pressure of the system. The heat exchanger of the liquefying unit employs Freon for the cooling purposes. Liquid chlorine is marketed in small cylindrical tanks and is also exported to the neighboring countries such as Rumania and Lebanon.

6. Benzene Hexachloride (BHC) - This product is used as a soil insecticide. BHC is manufactured by the direct addition of chlorine gas to benzene in the presence of ultraviolet light which is a catalyser for this reaction. The raw material, benzene, is purchased from the Iron and Steel Works at Ereğli. Of the five isomers formed, only the gamma form is insecticidally active, and the commercial grade of BHC contains about 12 percent gamma isomer. The Chemco company is not free to set a price for this product. Since it is used within the scope of agricultural disease prevention as with other such drugs (Zıraî Mücadele İlaçları), its price is determined and fixed by the Ministry of Agriculture. The managers of the company feel that there is usually a time lag between the rising costs and the adjustment of prices accordingly since the Ministry attend the matter very infrequently. BHC is produced to international specifications and is exported.

7. DDT - This is a general insecticide employed both in the field and the home. It is used to control a wide variety of insects. The benzene from Ereğli is one of the starting materials of the DDT process and is converted to monochlorobenzene (MCB) by reacting with the chlorine which is an electrolysis product. Another raw material is ethanol (ethyl alcohol) which is bought from the State Monopolies (T.C. Tekel İdaresi). Ethanol is chlorinated to yield chloral which is a chlorine-substituted aldehyde. The monochlorobenzene and chloral are then reacted under the catalysis of oleum (H_2SO_4) to produce DDT. The pricing of this product for the internal market is also done by the Ministry of Agriculture based on the cost figures of Chemco. Since DDT is also exported, its export price is lower than the domestic price in order to match international competition. The firm feels that DDT is becoming a burden on the company and if a better outlet for chlorine consumption is found the DDT production should be curtailed.

A by-product of the DDT manufacturing is hydrochloric acid of grade B. For each ton of DDT produced three tons of the acid is obtained. The grade B acid does not have a market in Turkey and the great amount produced (about 7,000 tons per annum) is destroyed. The acid obtained from this process cannot be sold because of the mercury it contains as an impurity. Various attempts to sell this acid were not successful due to the impurity which is poisonous. Thus, it cannot be used in several industries such as the food processing industry. However, the management is trying hard either to be able to find sources where it can be sold as it is or to convert it to grade A acid through a purification process. If they succeed in this it will amply reward these attempts by providing substantial additional revenues.

8. o-Dichlorobenzene - This is obtained as a by-product in the chlorination of benzene to monochlorobenzene. Some portion of the raw benzene is substituted twice by chlorine to give the two dichlorobenzene isomers, i.e., the ortho and the para isomers. The dichlorobenzenes are recovered in a separation unit by fractional crystallization. Until recently, there was no market for the ortho isomer but now it is sold to tank cleaners. It has a potential use as a solvent or cleaner; it is particularly useful for metal degreasing. If purified and stabilized, it can also be used as a heat transfer fluid in the temperature range of 150-260 degrees centigrade.[1] The cost figures for the two dichlorobenzenes are not available and they are sold at rather arbitrary prices.

9. p-Dichlorobenzene - The para isomer has a pleasant odor and is used as a home deodorizer. In fact, the "Ernet" deodorizers are nothing

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[1] William H. Haberstroh, Daniel E. Collins, "Synthetic Organic Chemicals," Riegel's Industrial Chemistry, ed. James A. Kent (New York: Reinhold Publishing Corporation, 1962), p. 937.

more than this product with a little amount of perfume added and then shaped. It can also be used for protecting woolen material from moths and for sanitary purposes.

10. Monochlorobenzene - It is only an intermediary product which is used in the production of DDT as explained previously. Monochlorobenzene is produced in just the sufficient amount that is needed for DDT and of course all of it is used for that purpose. It is not manufactured in excess quantities so that it can be marketed as a raw material for certain sulfur dyes, perfumes, drugs and solvent applications.

11. Hydrogen Gas - Each year the electrolysis process produces 2,850,000 m^3 of hydrogen gas under standard temperature and pressure, i.e. at zero degree centigrade and one atmosphere of pressure. In order to manufacture 3,000 tons of hydrochloric acid about 82.5 tons of hydrogen are needed and corresponds to nearly 900,000 m^3 of the gas at standard conditions. It is seen that only one third of the hydrogen gas is used in the company processes. The engineers of the company earlier have made a study on the possibility of using the remaining wasted hydrogen gas for energy generation purposes. This project was not found feasible because mixtures of hydrogen gas and air are explosive in a wide range of relative proportions. Since many of the materials which Chemco deals with are highly inflammable, this was considered a risky process. Furthermore, the available quantity of excess hydrogen was found inadequate for meeting the energy requirements

of the firm. Therefore, even if the risks are borne, there would still be the need for a different energy generating system and having different systems would not have been practical.

12. Chlorine Gas - Chlorine gas, like the hydrogen gas, is only an intermediate product and is not marketed in gaseous form. This is the key material in Chemco's product line since its production is limited by the maximum capacity of the plant. Currently being the starting material for four products, i.e., liquid chlorine, hydrochloric acid, BHC, and DDT, and a potential raw material for other chlorine derivatives in the future, the total amount of chlorine available sets a restriction for the manufacture of those products which contain chlorine. Since the chlorine produced has alternative uses, the usage of chlorine creates a resource allocation problem. This problem, which is the main concern of this paper, will be treated extensively in Chapter IV.

The following tables present quantitative information on the various aspects of the Chemco products.

TABLE 1

MAXIMUM ANNUAL PRODUCTION CAPACITIES (IN TONS PER YEAR) FOR THE VARIOUS PRODUCTS[a]

Product	Output
Chlorine gas	9,900
Caustic soda	11,220
Hydrochloric acid, grade A (33%)	4,500
Hydrochloric acid, grade B	7,200
Sulfuric acid (100%)	9,000
DDT	2,400
BHC	5,400
Sodium hypochlorite	6,700
Hydrogen gas (m^3 at STP)	2,850,000

[a] Source: Ermukan Şengezer, Chemco optimal istihsal programı ile klor değerlendirme imkânları hakkında rapor (İstanbul: Türkiye Sınaf Kalkınma Bankası A.Ş. Dahili Muhtıra, 14 Mayıs, 1968), 2.

TABLE 2

PRICES (IN T.L. PER TON) AND SALES POTENTIALS (IN TONS) IN DOMESTIC AND FOREIGN MARKETS FOR CERTAIN PRODUCTS[a]

Product	Price Domestic	 Foreign	Sales Potential Domestic	 Foreign
BHC	2,292	1,653	1,000	....[b]
DDT	5,256	3,986	1,500	....
HCl, Grade A	766	?[c]	3,000	?
H_2SO_4	923	?	--	?
Liquid chlorine	1,430	?	460	?
NaOH	1,734	?	--	?
NaOCl	616	?	3,000	?

[a] Source: Ibid., 3.

[b] All of the quantities that can be produced can be sold.

[c] Prices are not known.

Now that the products of Chemco have been adequately described, the writer feels that one of the processes, namely, the electrolysis process, should be described because the concept of joint costs, which is the subject of the next chapter, can be more clearly explained if the reader is acquainted with this process.

The Electrolysis Process

The raw material for electrolysis is a concentrated and purified water solution of sodium chloride, i.e., the common table salt. The salt solution is called "brine" in chemical technology. At Chemco, the electrolysis cell that processes the entering concentrated brine is a mercury type cell. In this type of cell the cathode is made of mercury and the anode is of graphite.[2] By applying a voltage of 4.5-5 volts and passing a current of 60,000-66,000 amperes, the brine is decomposed into its constituents. The sodium ions form metallic sodium and chlorine gas is liberated at the anode in the electrolysing chamber. At the denuding chamber, the metallic sodium is reacted with water to produce caustic soda solution and to liberate hydrogen gas. The caustic liquor thus obtained is pure and does not contain salt. Although some companies and processes evaporate

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[2] J. A. Falcone, D. W. Duncan, D. J. Saunders, "Miscellaneous Heavy Chemicals," Riegel's Industrial Chemistry, pp. 170-172.

the resulting caustic liquor to concentrate it, at Chemco it is not concentrated further and is marketed as it is produced.

It should be observed that the three products resulting from the electrolysis process, i.e., chlorine gas, caustic soda and hydrogen gas, are produced simiiltaneously. Even if only one of these products are desired, the process produces all three of them because of the nature of the system. Along with the desired material, the other materials are necessarily obtained. Furthermore, these products which are called "joint products" because of this characteristic, are always produced in fixed relative proportions for a given state of the system.

The next chapter will treat the joint products, and the joint costs which are the costs associated with their production.

CHAPTER III

JOINT COSTS AND JOINT PRODUCTS

The concept of joint products was introduced at the end of the previous chapter. Here this phenomenon and the accompanying joint costs will be discussed comprehensively.

The Nature of Joint Costs and Joint Products

Today a majority of manufacturing activities or processes result in the production of more than one product, and these products are usually produced by making use of more than one department. In other words, a significant characteristic of today's industries as contrasted to the industries of some fifty years ago is having multi-products and multi-departments. By the nature of current technology, attempts for the production of one or more products almost invariably result in the production of additional products or by-products. Chemical industries, petroleum refining, mining, lumber, trucking, railroading, canning, meat packing, flour milling, leather making, soap making, and gas manufacturing industries are typical examples where the multi-product and multi-department characteristics are observed. The joint product that result from the processes of these industries may be produced in fixed proportions or in variable proportions. In these joint products which are produced in the former manner, the output ratio of products are fixed relative to each other. In other words, if the output ratio of two joint products is one to two, in a process producing these two, one product will always be obtained twice as much as the other. As the name implies, in the case of variable proportions, the output of one of the joint products can be changed at the expense of the others.

Joint costs are a group of costs that are intrinsic to the activity in which they originate. They arise in situations where the factor of production, whether it be the machine, the raw material or the process, brings forth two or more products. The relevance of joint costs in a given situation may be investigated qualitatively by either one or by a combination of a number of criteria. The constituents of a raw material is one such criterion. For example, the raw material in question may be crude oil. Since it is a mixture of many different compounds, it will be fractionated into its several constituents such as gasoline, benzene, liquefied petroleum gases, fuel oil, various aromatic compounds, etc. when it is processed, i.e., distilled. The cost of processing and the cost of the crude oil is common to all the resulting fractions of petroleum. When a meat packing company purchases a calf, the purchase price of one calf is joint to the steaks, chops, hamburger, fat, bones, etc. that are obtained from it. Railroad transportation and trucking are two other examples. In the former, the cost of the rails is joint to both the passengers and the freight, whereas in the latter case the round trip costs of a truck are incurred by both the outgoing and incoming freight. It should be observed that the common characteristic of each foregoing example is the association of the costs incurred with the output of two or more products or services.

Besides the nature of the raw material, another qualitative criterion to determine the presence of joint costs is the number of products that a particular company produces and the number of departments involved in the manufacturing process. If the company produces a single product by manufacturing it in a single department through a process which does not also produce by-products, then this is a case where joint costs are nonexistent. Here it is only the single type of product that uniquely incurs the costs involved. On the other hand, if the company produces a single product by using a number of departments, or if it manufactures a number of products and by-products, then joint costs are relevant.

Another characteristic of joint costs which should be made clear is the impossibility of directly identifying such costs with the resulting two or more end products, i.e., joint products. Although indirect costs are also a class of costs which cannot be traced to the products when there are more than one type of product, there is a fundamental difference between them and joint costs.[1] For instance, indirect costs will be incurred if the

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[1] Harold Bierman, Jr., Topics in Cost Accounting and Decisions, (New York: McGraw-Hill Book Company, 1963), p. 59.

same factor of production, i.e., the same equipment, is used to produce different kinds of spare parts for different types of machinery. In such a situation the indirect costs cannot be identified by the type of spare parts produced. However, here it is a decision of the management to produce different types of spare parts. If the management decision is to manufacture only one type of spare part, then any indirect costs can be traced to that product. On the other hand, if joint costs are incurred from the production of two or more products, these costs result due to the nature of the raw material or because of the method of production employed, and not due to the decision of management. As an example, considering the previous case of the meat-packing company, it is seen that since when a calf is purchased a variety of meats are already embodied in the animal, it is not possible to avoid incurring the total cost of the whole calf, which is the joint cost, by a management decision. Therefore, no single cut of meat can be economically obtained from the total animal without producing the various other cuts. The only thing over which the management has control is to decide not to apply a "finishing process" to one of the joint products if it is found uneconomical, i.e., the cost of finishing for the particular product is greater than the revenues arising from its sale. This means that decisions on the finishing processes are at the discretion of management. This is so, because in the processing of joint products, there is a definite stage of production beyond which it is possible to identify the individual products. This stage in the process is called the "split-off point."[2] Since the individual products are identifiable after the split-off point, the costs resulting from further processing are also traceable and are no longer joint costs. Any further costs that are incurred are direct to the product of interest and management will decide to finish the product if the additional costs that have to be incurred do not exceed the increase in revenues. Such decision making criteria concerning joint costs will be treated at a later section of this chapter.

A further point should be mentioned to complete the concept of joint products. In dealing with such commodities, a class of products called "by-products" are frequently encountered. If the products obtained by incurring joint costs are of relatively equal value and importance, they are called joint products. However, if they are of little value, they are properly called by-products. Thus, it is seen that in this context the term by-products does not imply those products which are auxiliary to the main products but refers to those that are of little importance. For example, in the case of the Chemco company common usage suggests that sodium hypochlorite and hydrogen gas are by-products, but within the concept of joint products, ortho and para-dichlorobenzenes are the proper by-products.

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[2] Charles T. Horngren, Cost Accounting a Managerial Emphasis (Englewood Cliffs, N.J.0 Prentice-Hall, Inc., 1967), p. 602.

Accounting Aspects of Joint Costs and Joint Products

Accounting can be considered as a tool which facilitates the control of a business and also measures its performance. In its function of measuring performance, product costing, inventory valuation, and income determination are significant topics. For product costing purposes the accountant has to find a place for every penny spent. In case he cannot trace and document what costs are directly incurred in producing one product or another, he may therefore allocate these costs. This is done by certain assumptions and making use of different bases of allocation such as floor space, number of working hours, proportion to direct costs, proportion to physical output, and proportion to sales value, etc. However, it must be pointed out that any allocation scheme is arbitrary, and therefore allocation of joint costs is not meaningful and valid for decision making purposes on the output and pricing of the products. Now, it is intended to present the accounting treatment of such costs and implications of allocating them for accounting purposes.

As an example, consider the case of a certain chemical process which takes place under a given set of operating conditions and using sodium chloride as the raw material produces two products: caustic soda and liquid chlorine. It is assumed that the following information pertains to this process.

Cost of raw material: TL. 1,900

Cost of processing : TL. 1,100

Quantity of caustic soda produced: 1,500 kg.

Quantity of chlorine gas produced: 1,000 kg.

Sales value of caustic soda at T.L. 1.8/kg : T.L.2,700

Sales value of liquid chlorine at T.L. 1.1/kg: T.L.1,100

In this process, the joint cost consists of the sum of raw material and processing costs. At the end of a business period, these costs may be allocated for income determination purposes. "Relative quantity method" and "Relative sales value method" are two commonly used allocation bases. The allocation of the joint cost of T.L.3,000 under the different methods will be illustrated.

1. Relative quantity method.

In this method the total joint cost is allocated to the products in proportion to their physical quantities.

Product	Joint Cost x Quantity Ratio	Cost Allocation
Caustic soda	$3000 \times \frac{1500}{2500}$	T.L.1,800
Liquid chlorine	$3000 \times \frac{1000}{2500}$	T.L.1,200
		T.L.3,000

Assuming that no inventories remain on hand at the end of the period the income statement will show:

	Caustic Soda	Liquid Chlorine	Total
Sales	T.L. 2,700	T.L. 1,100	T.L. 3,800
Cost of sales	1,800	1,200	3,000
Gross margin	T.L. 900	T.L. (100)	T.L. 800
Gross margin percentage	33.3%		21.1%

Thus, the relative quantity method shows a gross profit of T.L. 900 for caustic soda and a loss of T.L. 100 for liquid chlorine.

2. Relative sales value method.

This method takes into consideration the relative revenue generating power of the products, and for this reason, it is preferred to the former method.

Product	Joint Cost x Sales Value Ratio	Cost Allocation
Caustic soda	$3000 \times \frac{2700}{3800}$	T.L. 2,130
Liquid chlorine	$3000 \times \frac{1100}{3800}$	T.L. 870
		T.L. 3,000

Under the assumption of no ending inventories, the income statement for the period will be:

	Caustic Soda	Liquid Chlorine	Total
Sales	T.L. 2,700	T.L. 1,100	T.L. 3,800
Cost of sales	2,130	870	3,000
Gross margin	T.L. 570	T.L. 230	T.L. 800
Gross margin percentage	21.1%	21.1%	21.1%

The statement about the arbitrariness of joint cost allocation is evident from the foregoing examples. If two different companies are producing these two products through the same process, the company using the former method will think that liquid chlorine is a burden on the company, while the second, using the latter method, will consider both products to be profitable.

Fixed Costs and Decision Making

Before preceding with the treatment of joint costs and decision making, it will be enlightening to consider the relevance of fixed costs for decisions. It has been said by many people that a significant characteristic of output decisions is the exclusion of fixed costs from the decision process. The irrelavance of fixed costs in such cases will be illustrated by a hypothetical example.

Consider a factory whose fixed costs amount to T.L.500,000 per month. If the variable costs of producing a ton of its product is constant at T.L. 200 per ton, should it accept an order from a customer that demands for that month 10,000 tons of product at a price of T.L. 225 per ton?

This problem may be approached in two different ways. In the first method fixed costs will be included in the analysis. The average total cost is

$$ATC = \frac{500{,}000 + 200(10{,}000)}{10{,}000} = \frac{500{,}000 + 2{,}000{,}000}{10{,}000}$$

$$= \frac{2{,}500{,}000}{10{,}000} = \text{T.L. } 250 \text{ per ton}$$

Since the offered price is T.L. 225 per ton and it is less than the average total cost, it might be suggested that because the company is losing it should turn down the offer. However, such a decision will be incorrect.

A second approach is to consider and compare the variable costs and the revenue, i.e., the offered price. In the absence of other orders, the company should accept this order since the revenue per ton of the product exceeds the variables costs per ton. The excess of revenue over available costs is the contribution margin per ton and is equal to T.L. 25.

$$\text{Contribution margin} = \text{Revenue} - \text{Variable costs}$$

$$= p - C$$

$$= 225 - 200 = \text{T.L. } 25 \text{ per ton.}$$

It should be obvious that this contribution margin per ton amounts to

T.L. 25 x 10,000 = 250,000 for the entire order and reduces that month's loss of the company from T.L. 500,000 to T.L. 250,000. Thus, the difference between the revenue and the variable costs provides the opportunity to recover a portion of the fixed costs. By means of the foregoing illustration it can now be said that fixed costs are irrelevant for decision making purposes and the factors that affect output decisions are the revenue and the variable costs.

Joint Costs and Joint Products in the Chemco Product Line

Within the light of the concepts and examples presented in the previous sections of this chapter, it is now possible to analyze the Chemco product line and the related activities that were described in Chapter II. It is remembered that the electrolysis of brine results in the three products, chlorine gas, caustic soda, and hydrogen gas. It is recognized that while chlorine gas and caustic soda are joint products, hydrogen gas is a by-product. In this particular case this result is due both to the constituents of the raw material and the nature of the process. The constituents of the brine are sodium, chloride, hydrogen and hydroxyl ions. When electricity is passed through them during the process, by their intrinsic characteristics these constituents combine under the given conditions in such a manner that they always result in the mentioned joint products. Furthermore, as long as the brine is processed under the same conditions and by the same method, i.e., electrolysis, the resulting products must be the same and cannot be otherwise and will always appear in fixed relative proportions.

Some of the costs incurred before the electrolysis process and the costs that are incurred during the process are joint costs. The purchase price of raw sodium chloride, its cost of transportation from Çamaltı Tuzlası, İzmir by boat to the factory, the handling and storage costs, the cost of dissolving raw sodium chloride in water and purifying it, and the cost of the actual electrolysis process where the three joint products are obtained, constitute the joint costs incurred by the company.

Tables 3 and 4 represent the annual fixed costs of some departments of interest and the variable costs per ton of individual products, respectively.

TABLE 3

ANNUAL FIXED COSTS (IN TURKISH LIRAS) FOR SELECTED DEPARTMENTS[a]

Department	T.L./Year
Caustic soda, chlorine gas, liquid chlorine	3,406,406
Sodium hypochlorite	248,340
Hydrochloric acid	190,572
BHC	1,648,565
DDT	4,645,489
	10,139,372

[a]Source: The accounting department of Chemco.

TABLE 4

VARIABLE COSTS (IN T.L. PER TON) AND DOMESTIC AND FOREIGN PRICES (IN T.L. PER TON) FOR CHEMCO PRODUCTS[a]

Product	Variable Cost (T.L./Ton)	Revenue (T.L./Ton)
Caustic soda	614	1,734
Sodium hypochlorite	251	616
Liquid chlorine	688	1,430
Hydrochloric acid, grade A	299	766
BHC	1,115	2,292/1,653
DDT	4,252	5,256/3,986
Sulfuric acid	390	923

[a] Source: The accounting department of Chemco.

In this paper it is contended that in order to decide on the profitability and the output of an individual product the joint and fixed costs of the process should not be allocated. Since there are joint costs encountered in the system, it is not possible to identify these costs with individual products. Furthermore, because of the fact that the allocation of these costs will be arbitrary, an allocation of cost will provide misleading results. Therefore, instead of allocating, if the revenues to cover chlorine costs per ton plus fixed costs are maximized, then profits will be maximised no matter what joint costs are relevant to the individual products. By using such an approach, the necessity of allocating joint costs are eliminated. These considerations can be illustrated as follows.

The profit function is considered to be of the form

$$R_i = p_i - C_i$$

where, R_i = profit from the sale of ith product

p_i = sales price of the ith product

C_i = variable costs associated with the ith product.

In the case of Chemco, in general the total profit function for the production and sale of the chlorine derivatives can be represented as:

$$\sum_{i=1}^{n} Q_i R_i = \sum_{i=1}^{n} Q_i p_i - TFC - \sum_{i=1}^{n} Q_{ci} a_{ci} x - \sum_{i=1}^{n} Q_i C_i$$

where, $Q_{ci} a_{ci} = Q_i$

Q_i = quantity of ith product

Q_{ci} = quantity of chlorine allocated to ith product

a_{ci} = chlorine coefficient for the ith product

x = chlorine cost incident to each unit of product i. This is a constant for given operating conditions and is an unknown.

n = number of chlorine derivatives.

TFC = total fixed costs incurred by the chlorine derivatives.

Although the chlorine cost per unit of its derivative is an unknown, the method used here does not require finding its value. The chlorine coefficient for a product is the conversion factor of chlorine to that particular product. For example, since 3.05 tons of hydrochloric acid is produced from 1 ton of chlorine, the chlorine coefficient for the acid is

$$a_{c\,HCl} = 3.05$$

Rewriting the total profit function for one product and rearranging, an expression of profit (contribution margin) is obtained:

$$QR = Qp - FC - Q_c a_c x - QC, \qquad \text{since } Q_c a_c = Q$$

$$Q(p - C) = QR + Qx + FC$$

where (p - C) was already defined as the profit expression. Differentiating with respect to Q the expression becomes

$$\frac{d}{dQ}\left[Q(p - C)\right] = \frac{d}{dQ}\left[(QR + Qx + FC)\right]$$

$$(p - C) = R + x$$

$$(p - C) - x = R$$

since the constant terms are eliminated. In words, this expression shows that profit equals the contribution margin less chlorine cost per unit of the product. However, it should be noted that x, chlorine cost per unit, is a joint cost and its presence in the profit expression indicated that it has been allocated to the product under consideration. It was previously shown that allocation of joint costs is irrelevant for decision making purposes. Based on this, if joint costs are not allocated the last equation reduces to

$$p - C = R$$

which validates the contention of this paper. Thus, it is shown that in deciding on the profitability and the output of an item, the bases of decision should be the revenue from the item and the variable costs associated with it.

Joint Costs and Decision Making

The proper treatment of joint costs in decision making situations will first be considered by a hypothetical numerical example to illustrate the general principles, then these considerations will be applied to two items of the Chemco product line.

Let us assume a production situation where from a 100-kg raw material which costs T.L. 5/kg, joint products X and Y are obtained. Assume further that the following output and cost information represents this process.

	X	Y
Quantity of joint products obtained from 100-kg of raw material	70 kg	30 kg
Direct finishing costs	T.L. 1/kg	T.L.3/kg.

Indirect finishing costs T.L.140 (applies jointly to X and Y).

It should be noted that any purely fixed costs have been omitted since they should not be used in making decisions. Also, both types of finishing costs are incremental costs. The total cost of manufacturing seventy kilograms of X and thirty kilograms of Y costs T.L. 800.

Raw material (100 x 5)	T.L. 500
Joint processing costs	140
Direct costs for finishing X (70 x 1)	70
Direct costs for finishing Y (30 x 3)	90
	T.L. 800

The individual cost of producing product X and product Y cannot be determined accurately since the T.L. 640 cost for raw material and joint processing costs are joint to both. The management now faces the two options concerning production of X and Y.

a) Produce X and do not finish Y - This would cost the company T.L. 710, i.e. (500 + 140 + 70) T.L.

b) Produce Y and do not finish X - The cost of this alternative is T.L. 730, i.e. (500 + 140 + 90) T.L.

Evaluating the problem by including revenue considerations, the following conclusions can be reached concerning output decisions:

1. If the combined revenues from the sale of X and Y exceed T.L. 800, the company should produce.
2. If the revenue from the sale of X exceed T.L. 710, the company should produce X. It should not finish Y if its revenues are less than T.L. 90 which is the cost of finishing Y.

3. If the revenues from the sale of Y exceed T.L. 730, the company should produce Y. It should not finish X if its revenues are less than 70 which is the cost of finished X.
4. If the combined revenues from the sale of X and Y are less than T.L. 710, then the company should not produce X or Y.

These conclusions, as should be noticed, do not necessitate any allocation of joint costs. This is the reason why it is contended that for output decisions cost allocation is irrelevant and basing the decisions on incremental costs, as was done in the above example, will be theoretically sound.

The foregoing discussion can be applied to the production of ortho- and para-dichlorobenzenes in the case of the Chemco company. It is recalled that these two are by-products of the manufacture of monochlorobenzene (MCB) in the DDT process. The cost of these isomers are joint to the cost of MCB and the company does not have available cost figures for the isomers. After the removal of MCB from the process, the dichlorobenzenes remain as a mixture. As a first step they have to be separated, and then they have to go through finishing processes such as drying, weighing, packaging, etc. If the revenues from the sale of one of the isomers exceeds the joint costs of separation and its individual finishing costs, it should be produced. The remaining isomer should not be finished (of course provided that it cannot be sold without finishing) if its revenues are not greater than its finishing costs. If the combined revenues from the ortho and para isomers do not cover the sum of the cost of separation and the smaller of the finishing costs of the individual products, the original mixture should be disposed of without separation and any further processing.

It is now evident that although several items may be joint products, the management may decide on producing some of them and not producing the others along the theoretical guidelines discussed. Thus, if the use or profitability for one of the items in the product line has decreased, the management can stop the production of that item and continue to process the others.

For a more comprehensive treatment of decision making with joint costs--including pricing of joint procts--the reader is referred to Harold Bierman, Jr.'s book named Topics in Cost Accounting and Decisions. Indeed, the theoretical background of the discussion in this section has been facilitated by this cited work.

CHAPTER IV

RESOURCE ALLOCATION

Allocation of resources to various competing activities is a very common problem for all kinds of business firms whether they be manufacturing, merchandising, or service companies. In a manufacturing company the resource allocation problem might be investing a certain sum of available capital to the most profitable alternative among a number of investment possibilities; or it might be scheduling the time of a certain machine or machine group to the production of given types of parts; or it might deal with assigning the available time of a manager to various problems or activities. A small single-proprietorship may have to decide to utilize a limited sum of money for various different kinds of inventories or between carrying some inventories with a portion of the money and also buying a fixed asset such as a cash register with the remaining amount. A service company might face a situation where its limited number of repairmen is to service a greater number of customers complaining of various disorders.

In some industries a problem encountered often is the allocation of raw materials to the manufacturing of a number of products. Should such cases arise, the allocation is made such that an objective function is maximized subject to a set of independent and/or joint constraints. Usually the objective is to maximize profits subject to capacity, price, sales restraints. The problem in this paper is to allocate the current annual chlorine gas capacity of 9,900 tons of Chemco to its four derivatives, i.e., liquid chlorine, hydrochloric acid, BHC, and DDT, in such a manner that the profit of the firm per ton of chlorine gas allocated is maximized. However, before solving this special case a more general mathematical model for resource allocation will be proposed. Then a general graphical solution subject to certain assumptions will be developed.

Since there are four possible products for allocation of the chlorine gas, the mathematical and the general graphical solution will be developed in terms of a four-product system, because firstly, the n-product system can readily be derived from a four-product system by applying the same principles and procedures but is only more complicated, and secondly, because a four-product system facilitates a better graphical illustration of the mathematical considerations and equations involved in developing the theory.

A Logical Mathematical Model for Resource Allocation

This resource allocation model in essence is an iterative procedure which arrives at the optimum solution by a series of stepwise allocation and checks of the results of each individual allocation. These individual allocations can be taken as an appropriate constant increment whose value will be dependent on the accuracy desired, i.e., the number of steps necessary or required, the amount of raw material to be allocated, the revenue and cost functions of the products to which the allocations are to be made, etc. For example, if the raw material amounts to 1,000 tons, the allocation may be assigned in increments of 5 tons for a 200-step iteration process, or in 10-ton units for 100 steps, or 25-ton unit allocations for a 40-step allocation process.

The following symbols and variables will be used in developing the mathematical model.

A = the basic raw material (or resource) to be allocated.

Q = the amount of basic raw material A to be allocated to its derivatives.

q = quantity of raw material (a variable)

i = W, X, Y, Z are the products to which allocations are to be made.

$AR_i = p_i = f(q)$ = average revenue or price functions of the ith product which is any function of the quantity of material A allocated to it.

$C_i = g(q)$ = variable cost of ith product as any function of q.

$MC_i = h(q)$ = marginal cost of ith product as any function of q.

$MR_i = k(q)$ = marginal revenue of ith product as any function of q.

q_i' = optimum quantity of raw material A allocated to i th product where the q_i''s correspond to the quantity at the point $MR_i = MC_i$. No allocations will be made to i after q_i' since beyond this point the marginal contribution becomes negative.

w, x, y, z = allocated optimum amount of products W, X, Y, Z, respectively.

$R_i = p_i - C_i$ = contribution margin of i th product (profit per unit of i as a function of total allocation of A to product i).

$P_i = R_i \times q_i$ = total profit from the sale of q_i units of i th product.

$= (p_i - C_i) q_i$

Assumptions:

1. All functions are defined and continuous in the domain of the problem.
2. $0 \leq q_i \leq Q$, where Q = maximum quantity available for allocation.
3. $\sum q_i' > Q$. This is an economic constraint which means that the sum of the optimum quantities of the raw material A allocated for producing products W, X, Y, Z should be greater than or equal to Q. Otherwise, for optimum profits, Q should be reduced down to $\sum q_i'$.

The relationships between these variables are illustrated in Fig. 1, Fig. 2 and Fig. 3.

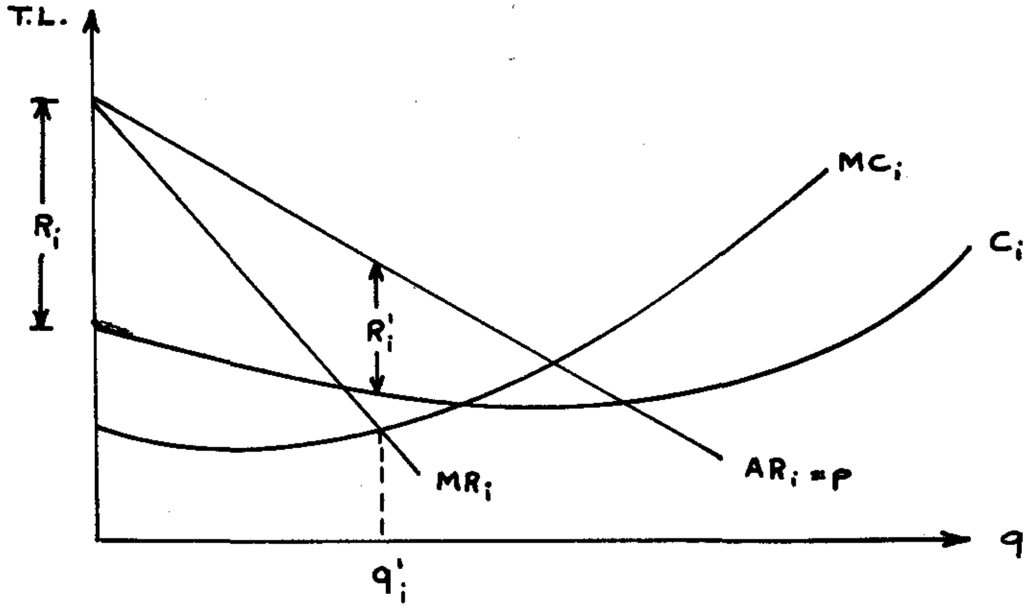


FIG. 1

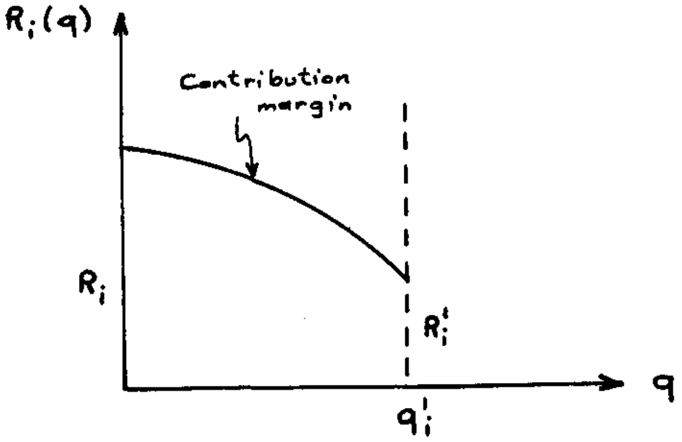


FIG. 2

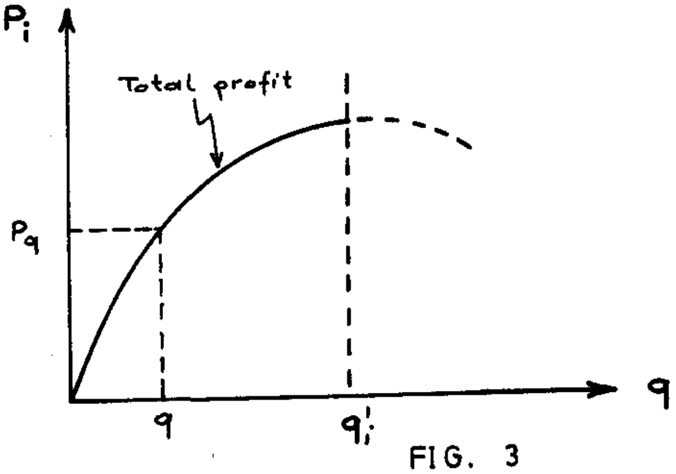


FIG. 3

The two boundary conditions of the model are:

$$w + x + y + z = Q \qquad (1)$$

$$\begin{aligned} 0 &\leq w \leq q'_w \\ 0 &\leq x \leq q'_x \\ 0 &\leq y \leq q'_y \qquad (2) \\ 0 &\leq z \leq q'_z \end{aligned}$$

The total profit function, denoted by TP, is:

$$TP = \sum_{i=W,X,Y,Z} (p_i - C_i) q_j \qquad (j = w, x, y, z)$$

Now there are four variables namely, w, x, y, z, and the objective is to determine the set values of w, x, y, z which maximize the total profit function. The procedure proposed for this is to successively assign to w the values in the range $0 \leq w \leq q'_w$ and to keep it constant and to reduce the total profit function to a function of the remaining variables, x, y, z. Thus, symbolically this amounts to

$$\begin{aligned} w &= 0 \ldots\ldots\ldots \quad TP(w=0) = f_0(x, y, z) \\ w &= 1 \ldots\ldots\ldots \quad TP(w=1) = f_1(x, y, z) \\ &\vdots \qquad\qquad\qquad\qquad\quad \vdots \\ w &= n \ldots\ldots\ldots \quad TP(w=n) = f_n(x, y, z) \\ &\vdots \qquad\qquad\qquad\qquad\quad \vdots \\ w &= q'_w \ldots\ldots\ldots \quad TP(w=q'_w) = f_{q'}(x, y, z) \end{aligned}$$

Now let's consider any one of these functions, say f_n.

$$TP(w=n) = f_n(x, y, z).$$

For $w = n$, Eq. (1) becomes

$$x + y + z = Q - n$$

$$z = Q - n - (x + y)$$

$$z = f'_n(x + y)$$

This equation shows that when the variable w is held constant at the value $w = n$, the variable z is a function of variables x and y for the given value of $w = n$.

The boundary conditions for the new version of TP are

$$TP(w=n) = f_n(x, y, z) = f_n(x, y) \text{ since } z = f'(x + y)$$

$$w + x + y + z = Q \qquad (1)$$

$$\left.\begin{array}{l} 0 \leq x \leq q'_x \\ 0 \leq y \leq q'_y \end{array}\right. \qquad (2)'$$

Since, $0 \leq z \leq q'_z$ and $z = (Q - n) - (x + y)$, the equation for z is substituted in the inequality expression and it yields a third boundary condition for the possible set of $x + y$ values:

$$(Q - n) - q'_z \leq x + y \leq (Q - n) \qquad (3)$$

Boundary conditions $(2)'$ and (3) define in the function $TP(w=n)$ a domain where the maximum value of $TP(w=n)$ is the optimum profit looked for the value of $w=n$. This domain is bounded within the planes $x=0$ and $x=q'_x$, $y=0$, and $y=q'_y$, $x + y = (Q - n) - q'_z$ and $x + y = (Q - n)$.

To find the maximum TP for a given value of w=n, the following set of equations are used which when taken simultaneously gives the general expression for equation of a family of parallel intersections on the total profit surface. In Figure 4, these intersections are shown by the parallel lines on the total profit surface. The simultaneous set is:

$$\begin{aligned} TP(w=n) &= f_n(x, y) \\ x + y &= k \quad \text{for} \quad (Q - n) - q'_x \leq k \leq (Q - n) \end{aligned} \qquad (4)$$

and it can be represented by a single equation as a function of x and k, since y can readily be determined from x and k. Thus,

$$TP'(w=n) = f'_n(x, k) \qquad (5)$$

where the prime signs are introduced to represent the equation of an intersection.

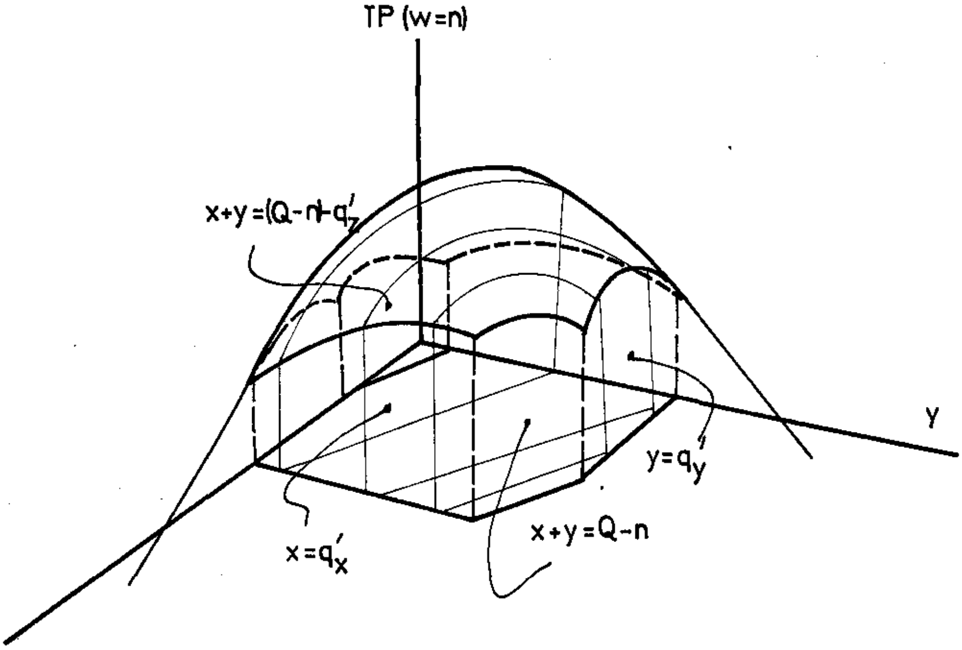
Maximizing the function in Equation (5) with respect to x for a given value of k, yields

$$x = x_o(k) \qquad (6)$$

The subscript "o" indicates the optimum value of x, i.e., the value of x which causes the TP' function to be a maximum for a given k. Therefore, by assigning different values to x within the permissible range, a set of values for maximum TP' is obtained which is represented as

$$TP'_{max} = f(k) \qquad (7)$$

The value of k which yields the highest TP'_{max} is $k = k_o$. This value of $k = k_o$, when substituted in Equation (7) gives Max TP'_{max}, i.e., the



A THREE-DIMENSIONAL PRESENTATION OF TOTAL PROFIT SURFACE AND THE BOUNDARY CONDITIONS

FIGURE 4

maximum value of TP'_{max}, which is equal to the maximum total profit attainable for the given value of w, in other words, $TP_{max}(w=n)$. Likewise, by determining the maximum values of TP for each value of w in the range $0 \leq w \leq q'_w$ and tabulating those maximum values obtained, the maximum of the maximum $TP(w=n)$ values will be chosen, and it will represent the optimum total profit figure that can be obtained by the process of allocation. The set of values of the variables w, x, y, z that give the optimum TP will represent the optimum allocation schedule of raw material A to its derivatives W, X, Y, Z. The optimum allocation values w, x, y, z correspond to the highest point of that portion of total profit surface which is bounded by the six planes in Figure 4.

The following procedures for product W can be applied to either one of the products, X, Y, Z in a similar manner. The chosen product, let us say Y, will be assigned a value $y=n$ within the range $0 \leq y \leq q'_y$ and held constant at that value. Then the maximum value of TP for each value of y will be determined and tabulated as below:

$$\begin{array}{lcl}
y = 0 & \ldots\ldots\ldots\ldots & TP(y = 0)_{max} \\
y = 1 & \ldots\ldots\ldots\ldots & TP(y = 1)_{max} \\
\vdots & & \vdots \\
y = n & \ldots\ldots\ldots\ldots & TP(y = n)_{max} \\
\vdots & & \vdots \\
y = q'_y & \ldots\ldots\ldots\ldots & TP(y = q'_y)_{max}
\end{array}$$

The maximum of the above TP_{max} values for product Y will be the optimum solution and, of course, this TP_{max} value coincides with the TP_{max} value determined for product W. Furthermore, the optimum values determined above for w, x, y, z are also identical with the optimum w, x, y, z values that were determined in the solution procedure for product W. This is true because no matter from which product the approach is made, all possible sets of values are enumerated and investigated for optimality.

It is believed that the proposed mathematical model is suitable for solution by a computer if these logical considerations are transferred to a flow diagram for a computer and programmed accordingly. All the computer would have to do is to assign different set of values to the variables, evaluate these sets and tabulate the results for the total profit values obtained from the various sets. The maximum of the tabulated values will be investigated and it will represent the solution of the problem.

Instead of tabulating the values and then choosing the maximum, the computer might alternatively enumerate different sets of values and proceed by eliminating the smaller values, retaining the largest and comparing it again with other sets, etc. As an illustration, assume that five units of a resource are to be allocated as integer values. Assuming that as a first trial product W is allocated three units and the permissible range of k is between one and two (these values being inclusive), this is represented as:

$Q = 5 \qquad w = 3 \qquad 1 \leq k \leq 2.$

For $w = 3$ and $k = 1$, $x + y = 1$ and the possible sets of values of the variables and the corresponding total profit values are:

$x = 0,\ y = 1$ .... $TP_1 (w = 3)$

$x = 1,\ y = 0$ .... $TP_2 (w = 3)$.

If $TP_2 > TP_1$, eliminate the latter.

For $w = 3$ and $k = 2$, the possible sets are:

$x = 0,\ y = 2$ .... $TP_3 (w = 3)$

$x = 1,\ y = 1$ .... $TP_4 (w = 3)$

$x = 2,\ y = 0$ .... $TP_5 (w = 3)$

If $TP_4 > TP_5 > TP_3$, eliminate TP_3 and TP_5, and $TP_4 > TP_2$, then Max $TP(w=3) = TP_4$. According to this, the optimum set of values for $w=3$ is:

$w = 3,\ x = 1,\ y = 1,\ z = 0.$

If this procedure is repeated for values of $0 \leq w \leq 4$ (provided that $q'_w = 4$), and for every permissible value of k, the maximum total profit figure will be obtained.

The summary of steps in this procedure is that firstly, w is determined (is assigned a value). Based on the given value of k, x is determined and then y is obtained from the values of k and x. Finally z is determined from the values of w, x, and y and the corresponding total profit figure is determined.

A Graphical Solution for the Resource Allocation Problem for the Case of Linear Price Function and Linear Variable Cost Function

In the previous general mathematical model the price, variable cost, marginal revenue and marginal cost functions were any function of q, the quantity of raw material A allocated. The graphical solution to be developed in this section assumes a linear function of variable costs and a linear function of price, i.e., average revenue. This case is illustrated in Figure 5.

The graphical procedure will again be illustrated for a four-product system, i.e., products W, X, Y, Z. The same set of symbols as used in the previous general model will be used in this section.

Assumptions:

1. $C_i = f(q)$ and $p_i = h(q)$ are linear functions of q and have the general form:

$p_i = a_i q + b_i$ and $C_i = a_i' q + b_i'$

where $a_i \nleq a_i'$ and $b_i \nleq b_i'$

2. $R_i = p_i - C_i$ where $R_i =$ contribution margin

$=$ profit per unit

$= A_i q + B_i = R_i(q)$

It can be shown that:

$A_i = a_i - a_i'$

$B_i = b_i - b_i'$

This function is illustrated in Figure 6.

3. $B_W > B_X > B_Y > B_Z$

Since the C_i and p_i functions are known, the y-intercepts of the R_i functions ($i = W, X, Y, Z$), i.e., the initial contribution margins of the individual products denoted by B_i, can be determined. Then, the product with the largest contribution margin is denoted product W, the next largest is denoted product X, etc. This ordering is not a requirement for solving the problem, but is devised in order to have a more systematic and orderly solution procedure. If any $B_i = B_j$, then the ordering will be based on the comparison of A_i's, i.e., the slopes of contribution margin functions.

4. The functions p_i, C_i, and R_i are linear functions of q that are defined and continuous for every value of q within the domain of the problem.

5. $P_i = R_i q$

$= (A_i q + B_i)q = A_i q^2 + B_i q$, therefore,

$P_W = A_W q^2 + B_W q$

$P_X = A_X q^2 + B_X q$

$P_Y = A_Y q^2 + B_Y q$

$P_Z = A_Z q^2 + B_Z q$

Thus, all total profit functions are quadratic (parabolic), and are illustrated in Figure 7.

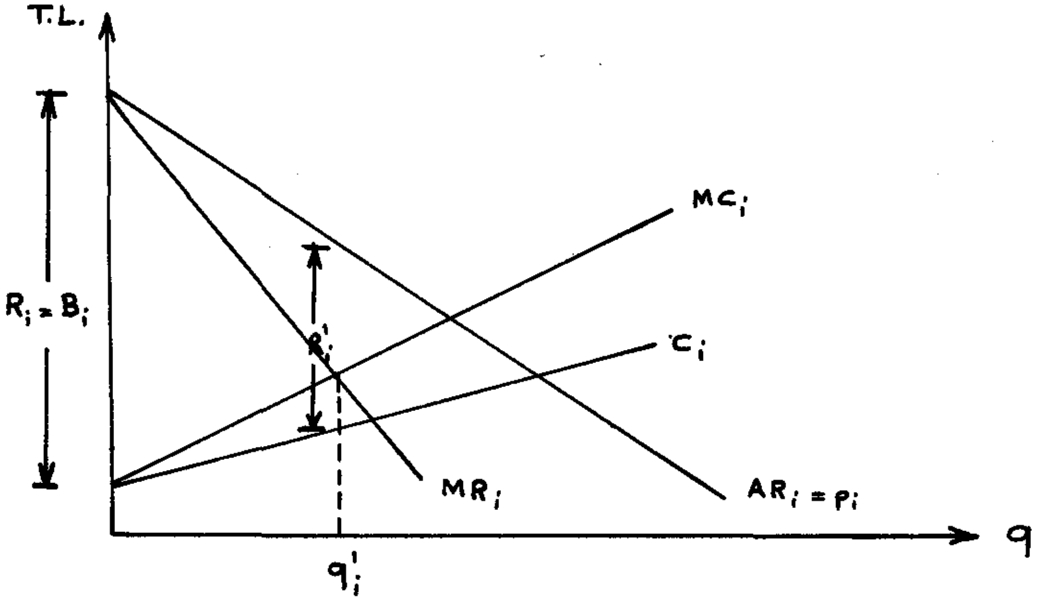


FIG. 5

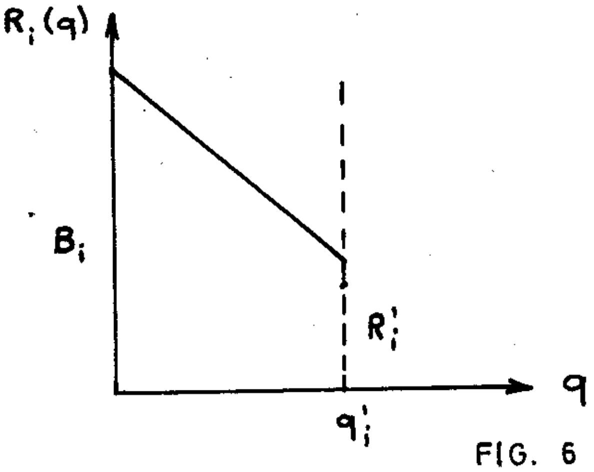


FIG. 6

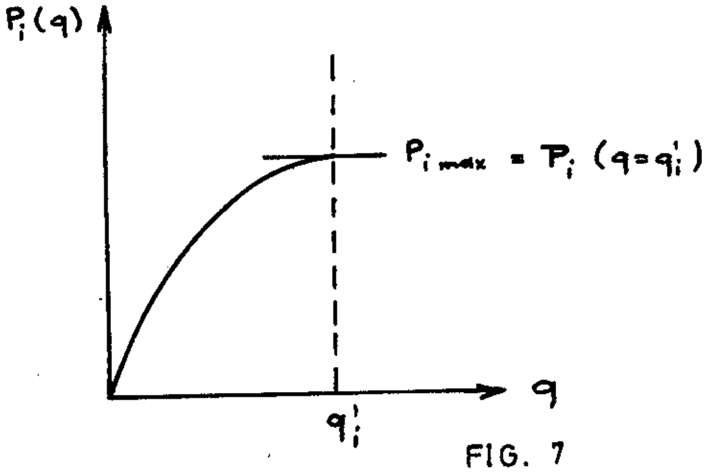


FIG. 7

6. $q'_w + q'_x + q'_y + q'_z > Q$

   There is the same economic constraint as discussed in the assumptions of the mathematical model.

7. q'_i = optimum output of i th product that is determined at the point $MC_i = MR_i$. There will be no further allocations to product i beyond the quantity q'_i.

The Marginal Profit Function

The total profit function was shown to have the form $P_i = A_i q^2 + B_i q$, and, since the marginal profit is by definition $\frac{dP_i}{dq} = MP_i$, $MP_i = \frac{dP_i}{dq} = 2A_i q + B_i$.

Since the contribution margin function (profit per unit) is

$$R_i = A_i q + B_i,$$

for $R_i = 0$, $q = -\frac{B_i}{A_i}$ and

for $q = 0$, $R_i = B_i$.

Also for $MP_i = 0$, $q = -\frac{B_i}{2A_i}$ and

for $q = 0$, $MP_i = B_i$.

The negative signs in the q expressions should not mislead the reader, because when the functions slope downward to the right, the $A_i q$ expression is negative, and therefore, the q expressions are positive.

Thus, the MP_i function crosses the horizontal axis at a point whose abcissa is equal to one-half of the abcissa of the point where the R_i function crosses the same axis and their intercepts on the vertical axis are equal. See Figure 8. Furthermore, it should be noted that unless there are market limitations, the maximum quantity q_i' which can be allocated to product i is

$$q_i' = -\frac{B_i}{2A_i}$$

since beyond that point the marginal profit is negative. The total profit is defined, for any quantity q, by

$$P_i = \int_0^{q_i} MP_i \, dq \qquad \text{for} \qquad 0 \leq q \leq q_i'.$$

This is the area under the curve MP_i, and the two vertical lines $q = 0$ and $q = q_i$. See Figure 9. For the generality of the discussion, in all further illustrations the maximum quantities q_i' will be taken smaller than $q = -\frac{B_i}{2A_i}$ where $\frac{dP}{dq} = 0$.

The Graphical Solution Procedure

The ranges of the P_i curves are restricted by

$$0 \leq q_i \leq q_i'$$

where q_i' is the upper limit for the allocation of raw material to product i.

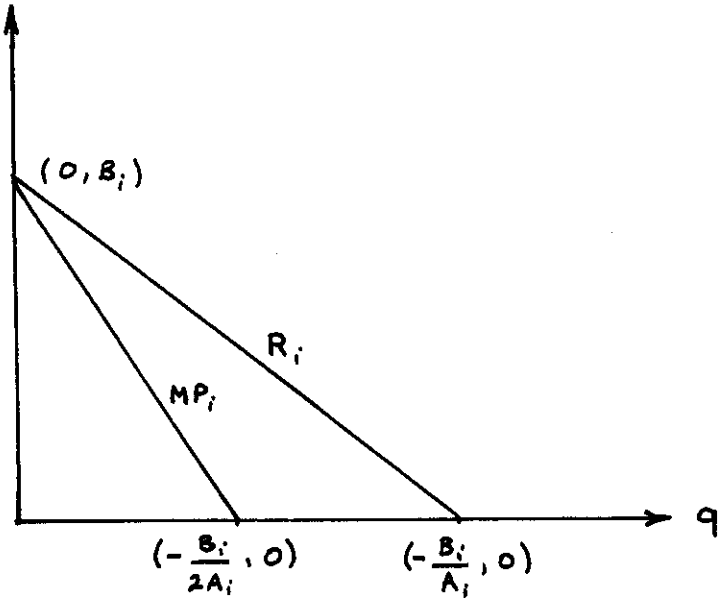


FIG. 8

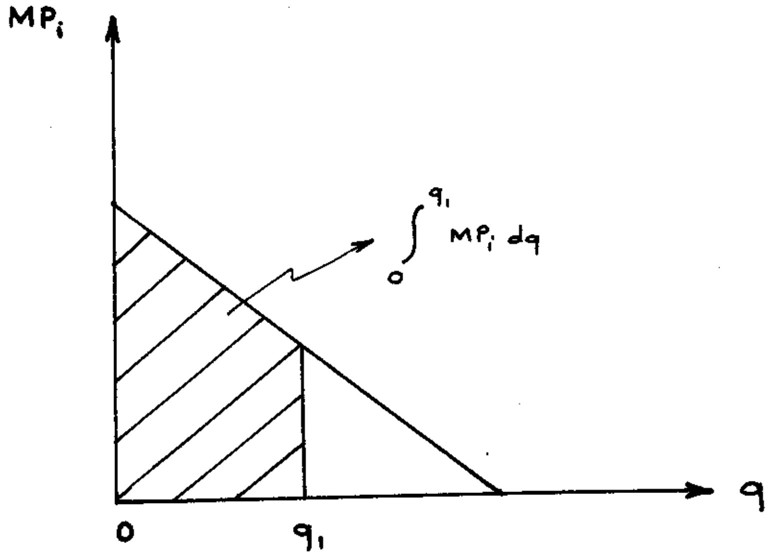


FIG. 9

q_i' is equal either to optimum quantity $(\frac{dP}{dq} = 0)$ or Q, the total raw material available for allocation, whichever is the smallest.

Case I:

$$q_w' + q_x' + q_y' + q_z' = \sum q_i'$$

If $\sum q_i' = Q$, then allocate

$$w = q_w'$$

$$x = q_x'$$

$$y = q_y'$$

$$z = q_z'$$

In this case no further computation is needed.

Case II:

$$\sum q_i' < Q$$

In that case there is an over-production of Q; reduce quantity of Q to $\sum q_i'$ and allocate the same set of solutions as in Case I.

Case III:

$$\sum q_i' > Q$$

If this is the case, then there is a problem of resource allocation and the graphical approach to solving this problem will be explained and illustrated below.

The marginal profit functions for products W, X, Y, Z are drawn on a graph paper. Then the initial marginal profit (B_i) and the smallest marginal profit ($MP_{i\,min}$) for each product is indicated. As illustrated in Figure 10, the points B_i and $MP_{i\,min}$ correspond to the extreme points in the range $0 \leq q \leq q_i'$. The x-axis in this case represents the amount of raw material or resource allocated to the individual products. The y-axis represents the value of marginal profits for the products. However, a word of caution must be said on the marginal profit figures. Since the horizontal axis represents the amount of the raw material and not the amount of its derivatives, the marginal profit figures correspond to the marginal profit of that quantity of the individual product which is obtained from allocating to it any quantity, q, of the raw material. In other words, if 1 ton of chlorine is allocated to production of BHC and the chlorine coefficient for BHC is 1.3, then, the marginal profit value read on the y-axis is the marginal profit of 1.3 tons of BHC.

Since this is a stepwise allocation process, in each step equal increments of q will be assigned to the products and the areas remaining under their individual marginal profit functions and between the values q and $q + dq$ will be compared. The allocation will be made to that product which gives the greatest area for the incremental allocation dq. The value of dq, as has been discussed in the previous model, is determined by the number of steps required and the amount of resource available. Also it might be worth noting that since total profit is the integral of marginal profit with respect to q, quantity of output, the trapezoidal areas under the MP function in the graphical solution correspond to total profit obtained from the allocation. The area of a trapezoidal segment, of course, is dq times the height of the MP function at the point $q + \frac{dq}{2}$. The objective of the graphical solution, therefore, is to maximize the area under the MP functions for a given amount of available resource, Q, for $Q = \sum q_i$ where $i = w, x, y, z$. This solution procedure will consist of the following steps:

1. Draw the MP functions on a common q axis in the order such that $B_w > B_x > B_y > B_z$. Label the extreme points B_i and $MP_{i\,min}$. Refer to Figure 10.
2. Draw the horizontal line I through the second highest extreme point, B_x. Allocate to product W the quantity q_{w1}, since it bounds all the area for MP above line I. This allocation gives area A_1 in Figure 10.
3. Draw the horizontal line II through the next highest extreme point, $MP_{x\,min}$. Products W and X are the only ones that have their MP functions above line II.
4. Allocate an amount of resource $dq_{w1} = dq_{x1}$ to W and X, respectively. Check the resulting trapezoidal area: if $dA_{x1} > dA_{w1}$, allocate to X. The resulting area is A_2 in Figure 10.
5. Allocate $dq_{w1} = dq_{x2}$. If $dA_{x2} > dA_{w1}$, allocate to X. This gives A_3.
6. Allocate $dq_{w1} = dq_{x3}$. If $dA_{w1} > dA_{x3}$, allocate to W. This is area A_4.

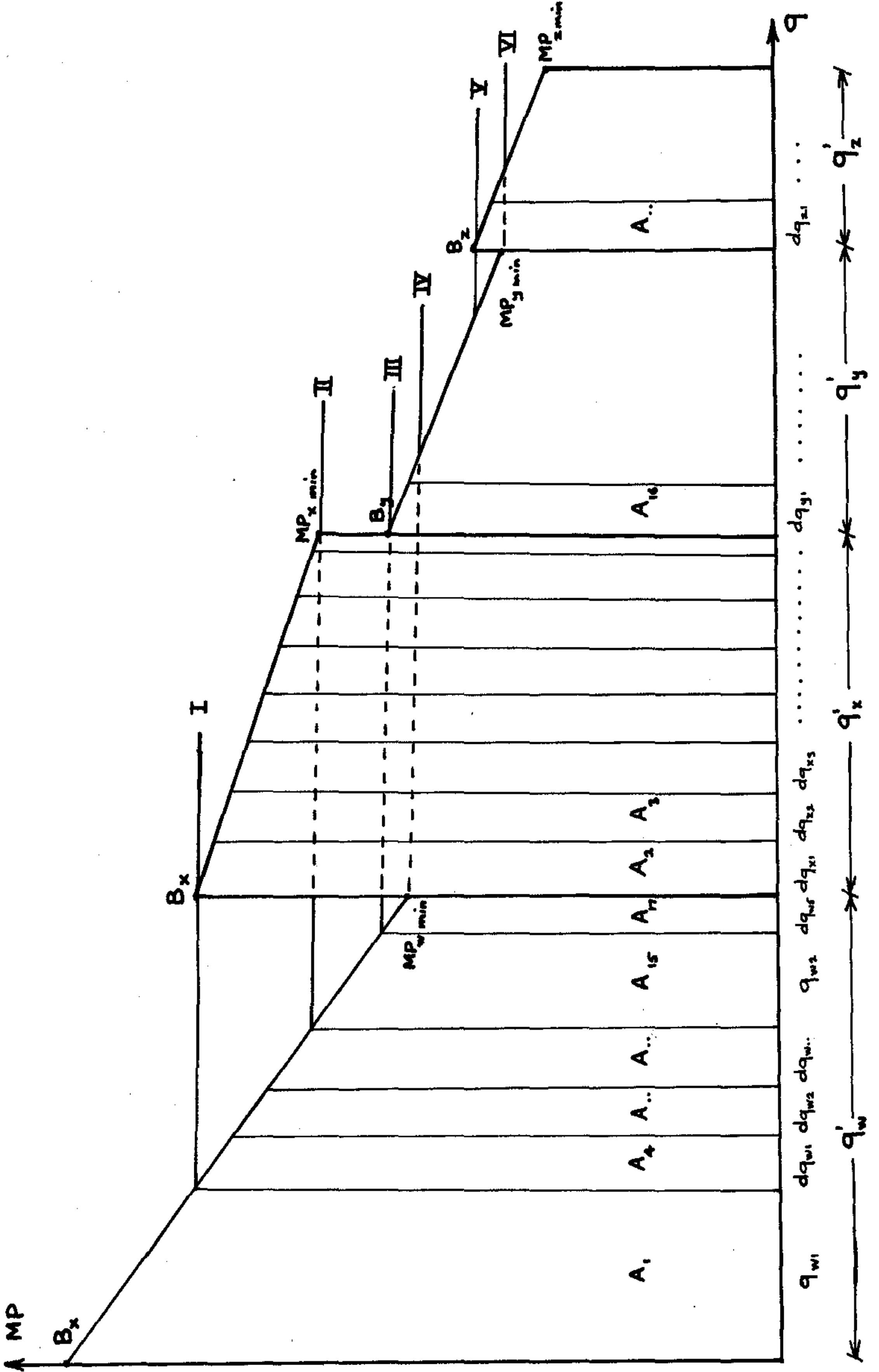

MP
q
B_x
I
II
III
IV
V
VI
MP_x min
MP_y min
MP_z min
MP_w min
B_x
B_y
B_z
A_1
A_4
A..
A..
A_15
A_17
A_2
A_3
A_16
A..
q_w1
dq_w1 dq_w2 dq_w..
q_w2
dq_w5 dq_x1 dq_x2 dq_x3
dq_y1
dq_z1
q'_w
q'_x
q'_y
q'_z

FIGURE. 10

7. Allocate $dq_{w2} = dq_{x3}$, etc. and proceed as before until all the areas that can be formed by MP values above line II; let us assume the areas to be $A_5, A_6, \ldots\ldots, A_{14}$.

8. Draw line III through the next highest point, B_y. Since all the area for product X has been completed, there will be no additional allocations to it and for product X, $q = q_x'$. Allocate q_{w2} to W which corresponds to area A_{15}.

9. Draw line IV through $MP_{w\,min}$. Only W and Y have MP functions above this line.

10. Allocate $dq_{w5} = dq_{y1}$. If $dA_{y1} > dA_{w5}$, allocate to Y. This gives area A_{16}.

11. Allocate $dq_{w5} = dq_{y2}$. If $dA_{w5} > dA_{y2}$, allocate to W. This is area A_{17} and exhausts product X.

12. Proceed in this manner until the sum of q_i's and dq_i's equals to Q, i.e. $\sum q_i + \sum dq_i = Q$.

In the above procedure to avoid repetition, the need for checking to see if the quantity Q is attained or is exceeded was not mentioned. Obviously, after each allocation, the total quantity allocated to each product up to that step must be checked and the process should stop when the allocations sum up to Q.

An Alternative Approach to the Graphical Solution

The condition for the existence of a resource allocation was mentioned to be $\sum q_i' > Q$. The graphical solution illustrated in the previous section started the allocation process from the highest marginal profit region and tried to maximize the area. However, it is equally feasible to approach the problem from the opposite region, that is, the region of minimum marginal profits. This approach will only be explained in words and will not be illustrated since the procedure will be very similar to the prevopis approach.

If $\sum q_i' > Q$, then it may be written as

$$\sum q_i' = Q + \Delta Q.$$

After representing the expression as such, the problem of allocation is transformed into a minimization problem. Now, the objective is still maximising the profit from an allocation of quantity Q by maximizing the area under the MP functions but this time instead of adding incremental areas, this approach will subtract incremental areas from the total area under the MP functions.

The area to be minimized and subtracted from the total area corresponds to the area which would be obtained from allocating the quantity ΔQ to the individual products. If the quantity to be subtracted from q_w', q_x', q_y', and q_z' represented as Δq_w, Δq_x, Δq_y, and Δq_z, respectively, then

$$\Delta Q = \sum \Delta q_i.$$

Since the areas corresponding to Δq_i's are profits foregone by not being able to allocate up to q_i' values, the reduction in profits should be as small as possible in order to maximize the profits from sales of W, X, Y, Z. Thus, the solution consists of finding quantities Δq_i so that the profit corresponding to them which is $\sum \Delta P_i$ is a minimum. In symbolic form these thoughts can be summarized as:

$$(\sum P_i)_{MAX} = \sum P_i' - (\sum \Delta P_i)_{MIN}$$

Optimal Product Mix for Chemco

In the previous chapter, tables were presented which summarised the capacity, demand, price and cost characteristics of the Chemco company. Furthermore, in the discussion of joint costs, the treatment of costs in decision making situations were analyzed. Within the light of these information and criteria, an optimal product mix for Chemco will be determined.

There are certain points that are significant in the product mix problem. Firstly, products such as caustic soda, and sulfuric acid do not utilize any of the chlorine produced by the firm. Secondly, the compulsory production of sodium hypochlorite due to air pollution regulations and which absorbs the chlorine wasted during various processes, amounts to about 5 percent of chlorine production. Thus, of the 9,900 tons chlorine produced annually, about 500 tons have to be used for manufacturing sodium hypochlorite. This leaves 9,400 tons of chlorine as a raw material which is to be allocated to its derivatives in such a manner that the contribution margin (profit) from the allocation shall be a maximum.

Before proceeding with the calculation of contribution to profits by the various chlorine derivatives, the chlorine coefficients for the individual products should be known. This information is presented in Table 5.

TABLE 5

CHLORINE COEFFICIENTS FOR THE CHLORINE DERIVATIVES[a]

Product	Chlorine Coefficient	Reciprocal of Chlorine Coefficient
Liquid chlorine	1	1
Hydrochloric acid	3.05	0.33
BHC	1.30	0.77
DDT	0.51	1.95
Sodium hypochlorite	6.57	0.15

[a] Source: Chemco production department.

The optimal product mix can now be determined under the following assumptions:

1. Joint costs and fixed costs cannot be allocated.

2. Variable costs, markets and the corresponding prices are given,

and the prices and the variable costs are fixed.

The profits derived from each ton of Chemco products are calculated and ranked in the order of decreasing contribution margin.

$$R_i = p_i - C_i$$

Caustic soda	: $R = 1734 - 614 =$	T.L. 1120	2
Sulfuric acid	: $R = 923 - 390 =$	533	6
Sodium hypochlorite	: $R = 616 - 251 =$	365	8
Liquid chlorine	: $R = 1430 - 688 =$	742	4
Hydrochloric acid - A	: $R = 766 - 299 =$	467	7
BHC	: $R = 2292 - 1115 =$	1177 (internal) ...	1
	$R = 1653 - 1115 =$	538 (export).....	5
DDT	: $R = 5256 - 4252 =$	1004 (internal) ...	3
	$R = 3986 - 4252 =$	-266 (export).....	9

The calculations indicate that since caustic soda and sulfuric acid have positive contributions, they should be produced up to capacity, and since Chemco incurs loss from exporting DDT, it should not export DDT. The 9400 tons of chlorine is allocated to its derivatives such that the demand for its highest contribution margin product is exhausted before it is allocated to the next profitable derivative. The chlorine requirements are:

a) 1000 tons of BHC (for domestic consumption)

: $1000 \times 0.77 =$ 770 tons of chlorine

b) 1500 tons of DDT (for domestic consumption)

: 1500 x 1.95 = 2925

c) 460 tons of liquid chlorine: 460 x 1 = 460

4155 tons of chlorine

d) The next highest contribution is derived from exporting BHC. Since the external market can buy as much as Chemco can produce, a portion of the chlorine remaining on hand should be allocated to BHC to be exported.

Chlorine available = 9400 - 4155 = 5245 tons

Unused BHC production capacity = 5400 - 1000 = 4400 tons

Chlorine requirement for 4400 tons of BHC = 4400 x 0.77 = 3390 tons

Therefore, chlorine available for allocation to next profitable product = 5245 - 3390 = 1855 tons of chlorine.

e) 4500 tons of hydrochloric acid - grade A

= 4500 x 0.33 = 1500 tons of chlorine

The 355 tons of chlorine (1855-1500) still available for allocation cannot be used for manufacturing BHC, hydrochloric acid, and liquid chlorine since the maximum production capacities are reacted for these products. Also, even though the capacity of DDT is yet not reached, since the domestic demand is exhausted and by exporting losses will be incurred, the remaining 355 tons will be used for production of additional sodium hypochlorite.

f) Chlorine from process wastes = 500 tons

Excess chlorine = 355 tons (carried over)

Chlorine to be used for sodium hypochlorite = 855 tons

855 x 6.57 = 5620 tons of sodium hypochlorite.

As mentioned in Chapter II, the excess of sodium hypochlorite (2620 tons) is disposed of since the demand for it is only 3000 tons.

Therefore, based on the foregoing computations, the optimal product mix for Chemco is to produce the following products in the indicated quantities:

BHC	5400 tons
Caustic soda	11220 tons
DDT	1500 tons
Liquid chlorine	460 tons
Sulfuric acid	9000 tons
Hydrochloric acid - A	3000 tons
Sodium hypochlorite	5620 tons.

CHAPTER V

CONCLUSION

In the foregoing chapters of this thesis, the optimal product mix for a chemical company was determined. The actual determination of the product mix was not complicated, because the previous comprehensive discussions on joint costs and resource allocation validated this simplified solution. These discussions, it is hoped, will provide a theoretical starting point from which some managers can develop more extensive and detailed analysis on these subjects.

The treatment of joint costs warrants exercise of great care. Methods of their allocation must be questioned critically and they must be employed only under relevant situations. As shown in this paper, joint costs are at times very misleading if they are applied blindly. Therefore, an analytical approach should be used where they are involved.

At the very best, joint costs are suitable for purposes of inventory valuation and income determination. For purposes of managerial decision making, they should not be used since they present a distorted picture. Consideration of whether selling a product, dropping a product, or processing it further will not be meaningful if joint costing methods are implemented. For such decisions joint costs are irrelevant and the decision maker must use incremental costs (which are variable costs) and incremental benefits to arrive at a sound decision.

The mathematical model which was developed in the resource allocation chapter shows that such allocations can be approached in a stepwise manner. Although in theory these problems can be solved by exhaustive enumeration and evaluation of each of them, if the number of products is large and the amount to be allocated is also large with small unit-allocations, this task becomes impossible. Therefore, in developing the model, it was primarily intended to design one which can be worked out by a computer.

A further remark should be made for the proposed graphical solution. Even though it was outlined and illustrated for the case of linear functions, since the method consists of evaluating areas under a curve between two boundaries, it can be applied to obtain an approximate solution in case the functions are non-linear.

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