THE IMPACT OF ORBIT DEPENDENT RETURN RATE ON THE CONTROL POLICIES OF A HYBRID PRODUCTION SYSTEM

by
Seval Ata
B.S., Industrial Engineering, Boğaziçi University, 2015

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ABSTRACT

THE IMPACT OF ORBIT DEPENDENT RETURN RATE ON THE CONTROL POLICIES OF A HYBRID PRODUCTION SYSTEM

Reprocessing of products at the end of their useful lives has become a profitable business option for many markets. Both original equipment manufacturers and third party remanufacturers collect products back from the end users and reprocess them to satisfy the demand. Demand flows for new and reprocessed products are distinct in some cases, while in others the demand is only interested in the functionality and the warranty of the product. In both cases the amount of reprocessed products are bounded by the amount of new products sold. Furthermore, the uncertainty in quality, quantity and timing of product returns generates a yield of less than one. The literature on the analysis and control of return flows does not consider the correlation between demand flow and return flow except for a few studies. None of these studies regard the satisfied demand for new products as potential returns that can satisfy the second hand demand. On the other hand, because remanufacturing is usually less costly and faster than manufacturing by its definition, remanufactured products are expected to bring more profit than a new product. Therefore, in this study it is aimed to consider the production decision of new products based on the correlation among demand and return processes. To this end, the supply chain structure will be modeled as a queueing network and optimal stationary decision policies are going to be sought for production control by using Markov decision process models.

ÖZET

KULLANIMDAKİ ÜRÜN SAYISINA BAĞLI GERİ DÖNÜŞ HIZININ HİBRİT ÜRETİM MODELİ KONTROL KARARLARINA ETKİSİ

Kullanım ömürlerinin sonuna gelmiş ürünleri yeniden işlemden geçirmek birçok pazar için karlı bir iş imkanı haline gelmiştir. Hem orijinal ekipman üreticileri hem de yeniden imalat şirketleri kullanıcılardan ürünleri toplayıp yeniden işlemden geçirdikten sonra ürün talebini karşılamak üzere satmaktadırlar. Kimi durumlarda yeni ürüne olan talep ile yeniden işlemden geçmiş ürüne talep ayrı iken, diğer durumlarda talep yalnızca ürünün işlevi ve garantisine bağlıdır. Her iki durumda da satılmış olan yeni ürün miktarı yeniden işlemden geçen ürün miktarı için bir üst sınır oluşturmaktadır. Ayrıca, geri dönen ürünlerin miktar, kalite ve geri dönme zamanlarındaki belirsizlik bu miktarın talep edilmiş ürün miktarının altında olmasına yol açmaktadır. Geri dönen ürün akışını inceleyen ve kontrol sistemleri öneren çalışmalarda, talep ve geri dönüşler arasındaki ilişki bir kaç yeni çalışma dışında göz önüne alınmamaktadır. Bu çalışmaların ise hiçbiri, karşılanmış yeni ürün taleplerinin, yeniden işlemden geçen ürün arzını oluşturma potansiyeline sahip olduğunu göz önüne almamaktadır. Halbuki, yeniden işlemenin genellikle tanımı gereği daha ucuz ve daha hızlı olması nedeniyle yeniden işlenmiş ürünlerin yeni ürünlerden daha çok kar getirmesi beklenmektedir. Bu nedenle bu çalışmada, yeni ürün imal etme kararını talep ve geri dönen ürün akışları arasındaki ilişkiye bağlı olarak irdeleyeceğiz. Bu amaçla, problemdeki tedarik zinciri yapısını bir kuyruk ağı olarak modelledikten sonra, Markov karar süreci modelleri ile uzun vade için en iyi karar politikalarını belirleyeceğiz.

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LIST OF SYMBOLS

c_L	Cost of lost sales
c_M	Cost of manufacturing
c_R	Cost of remanufacturing
d	Decision
d_M	Manufacturing decision
d_R	Remanufacturing decision
h	Holding cost
M	Maximum number of products in orbit
N	Maximum number of products in inventory
p	Return probability
x_0	Number of products in orbit
x_I	Number of products in inventory
γ	Loss rate
θ	Production rate
λ	Demand rate
Λ	Total rate of all events
μ	Usage rate
ν_n	Value function at iteration n

LIST OF ACRONYMS/ABBREVIATIONS

LP	Linear Programming

1. INTRODUCTION

Remanufacturing of products has become a beneficial business option for many markets. While some companies like Kodak and Fuji which produce single-use cameras, or Xerox, Canon and Hewlett-Packard which produce toner cartridges or IBM voluntarily collect used products to recover their residual value, some other companies collect products such as automobiles, electronic goods or packaging because of take back legislation [1]. In both cases remanufacturing offers a sustainable way of dealing with waste accumulation and raw material shortage, and opportunity to increase competitiveness of a company. By definition of remanufacturing, the aim is returning a used product to its as good as new performance by using minimum resources and effort. Therefore, even though economical feasibility of remanufacturing depends on the type of products, it generates more value for environment than recycling.

While many third party companies remanufacture certain type of products, a great amount of original equipment manufacturers collect and remanufacture their own products [2]. In the latter case which engages in both manufacturing of new products and remanufacturing of used products, the system is considered as hybrid production system. In some cases, demand flows for newly manufactured and remanufactured products are distinct on the contrary of others where demand is only interested in the functionality of the product. The profitability of remanufacturing originates from the relatively low effort required to restore used products in comparison with manufacturing. On the other hand, the uncertainty in quality, quantity and timing of collected products causes further complexity for production planning.

In this thesis, a hybrid production system in which the optimal control policy is sought for the production decision of new products based on the correlation among demand and return processes is considered. Additionally, an admission decision for returned products is introduced to system. To this end, the supply chain structure is modeled as a queueing network and optimal stationary decision policies are studied for both production and admission control by using Markov decision process models.

This study contributes to expanding literature on remanufacturing in terms of generating a framework based on Markov decision processes which can measure the value of return flows and admission control, and the consequences of misjudging the correlation between number of products in use and return flows in a more realistic system where product return flows are dependent on the amount of products in use.

The rest of this thesis is organized as follows: Chapter 2 presents a literature review on studies considering closed loop supply chain models. In Chapter 3, the model of hybrid production system is explained in detail and parameters used in this system are introduced. Chapter 4 covers the numerical experiments and sensitivity analysis on main parameters. In the last chapter, Chapter 5, concluding remarks are provided.

2. LITERATURE REVIEW

The first studies published in the field of closed loop supply chains (CLSC) considered problems of packaging returns *viz.* deposit returns of beverage bottles, or maintenance and remanufacturing of expensive equipment *viz.* military equipment. While in the first case an uncertainty in quantity and timing is present, in the second case a failed part is immediately replaced by a spare part and then remanufactured before being sent to the spare parts inventory. Thus, return flows for maintenance are perfectly correlated with the demand of spare parts [3]. On the other hand, in the first case, returns are not perfectly correlated with demands. Yet the increase in demand generates an influx in returns with a random delay causing both uncertainty and dependence in the timing, and quantity of core returns.

The majority of studies in this field are based on the idea of uncorrelated and stationary product return flows. This assumption is initially stated in one of the first studies conducted by Heyman [4] and is inherited subsequently. Here, Heyman [4] considers a single echelon continuous review inventory system operating as an $M/M/1/N$ queue where returns are assumed to arrive following a Poisson process. Demands occur one by one following an independent exponentially distributed time and are satisfied as long as there is a returned product in the queue. Returns are accepted in the system if the inventory of returns does not exceed N and are disposed otherwise. The objective is to determine the optimal inventory level N that minimizes the total inventory cost by using a push production control. The model introduces an admission control for returns but does not consider the production and remanufacturing lead times. Muckstadt and Isaac [5] considered constant lead times as an extension but ignore the admission control while developing a continuous review (Q, r) policy inventory model. The results obtained from the single-echelon model are applied to a two-echelon model. Van der Laan *et al.* [6] considered various periodic review inventory control mechanisms by including the disposal option to the single-echelon model in [5]. They conducted a comparative study between inventory policies with and without disposal that shows disposal is necessary for cost minimization. In [7], they analyzed three different inventory control policies with disposal option, compared the performance of each alternative and showed that a four-parameter control policy is optimal. In [8], they evaluated the effects of lead-time duration and lead-time variability on total expected costs numerically and showed that the pull control strategy is more cost effective than the push control strategy for hybrid production systems without disposal option. Fleischmann *et al.* [9] considered a basic uncapacitated (s, Q) continuous review inventory system with independent Poisson demands and returns which is similar to the case without returns. They allowed returns by modelling positive and negative demand. In this model, they considered a manufacturing lead time, a fixed order cost, and linear holding and backlogging costs while disregarding lead times and cost for core remanufacturing. They showed that conventional (s, Q) policies remain optimal when introducing Poisson item return flows. For the same system, they propose an optimization algorithm to compute the control parameters for an (s, S) policy in [10].

Van der Laan *et al.* [11] numerically showed that disposal option is essential for achieving maximum system efficiency in both PUSH and PULL controlled systems as it reduces variability. In [12], even though their control strategies rely on [5], they allowed non-zero fixed remanufacturing costs and separate holding costs for remanufacturables and serviceables. To investigate the effect of lead time, they assumed deterministic lead times for both manufacturing and remanufacturing activities, rather than a deterministic manufacturing lead time and stochastic lead time resulting from limited remanufacturing capacity. Yuan and Cheung [13] proposed an (s, S) inventory control model with correlated demand and returns on par with maintenance systems, where a return generates a demand. They considered a Poisson demand process, exponential return lead times and instantaneous procurement lead times. At the end of return lead time, items are either returned or kept by customer with a given probability. Toktay *et al.* [14] considered an industry problem in the Kodak single use camera remanufacturing case. They constructed a closed queueing network model to study the effects of various system parameters such as information flow, procurement delay, demand rate, and the length of the product's life cycle. Bayındır *et al.* [15] considered a remanufacturing system as a queueing network with different system parameters, including capacity of the production facility to find the ratio of returned products. In their next study, [16] they relaxed the as good as new assumption of remanufactured products by considering two customer classes and two product classes as new and remanufactured with a common capacity constraint. They modeled the system for an $(S-1, S)$ inventory control policy and downward product substitution, where a remanufactured demand may be satisfied by a new product. The main aim was investigating the system conditions, under which utilization of remanufacturing option provides profit improvement. Behret and Korugan [17] provided a detailed hybrid manufacturing/remanufacturing model using queueing networks that considers the quality grades of returns and remanufacturing processing and lead times dependent on quality grades along with independent demand rates and manufacturing lead times. They explored optimal values of a push type control for returns and finished goods inventories. Korugan and Gupta [2] considered an adaptive kanban control procedure for a hybrid remanufacturing/manufacturing system, where the number of kanbans increase and decrease based on the intensity of arrivals. Here kanbans balance the manufacturing and remanufacturing capacities.

In periodic review models, a few studies model a remanufacturing system for deterministic demand and return processes. Cohen *et al.* [18] assumed that a fixed fraction of the products issued in a given period is returned after a fixed sojourn time in the market and may then be reused. Optimality of an order up-to policy is shown when disregarding fixed costs and procurement lead times. Kelle and Silver [19] determined an optimal purchasing policy for reusable containers with a finite horizon model subject to random demands and returns. The associated stochastic model was reduced to a deterministic, dynamic lot sizing problem by allowing negative net demand with non-zero order cost and zero lead time. Teunter *et al.* [20] considered two lot sizing models with deterministic demand rates and return probabilities which differs in taking joint or separate costs for manufacturing and remanufacturing setups. They showed the lot sizing approach for new unit production and used unit remanufacturing is optimal for both capacitated and uncapacitated models.

Periodic review models commonly consider the stochasticity in remanufacturing systems. Simpson [21] studied a system where products arrive at a separate remanufacturing facility and can be disposed of or held for a while before they are remanufactured for considering the tradeoff between material savings due to reuse versus additional inventory holding costs. He optimized a three-parameter control policy which controls order, repair and disposal activities without fixed costs and lead times. Inderfurth [22] showed this policy is optimal also in the case of non-stochastic and identical non-zero lead times for manufacturing and remanufacturing. He compared the lead times for the two processes and concluded that the deviation between the two lead times is a critical factor for the simplicity or complexity of the optimal policy because different lead times cause a higher dimensionality of the underlying Markov model. Buchanan and Abad [23] assumed that returns are a stochastic fraction of the number of items in use for each period and the time before return is exponentially distributed. For a finite horizon, they derived an optimal production policy depending on two state variables which are on-hand inventory and the number of items in use. Kiesmüller and van der Laan [24] considered returns depending on demand in a periodic review model with a finite horizon. In this study, an order up-to policy is considered and both demand and return quantities follow a Poisson distribution with returns depending on previous demands. All lead times are constant and returned items are either remanufactured or disposed of with a constant probability. They compared the dependent returns with the independent returns and numerically showed that the average cost is smaller in the dependent case. In a series of multi-echelon systems considering independent demand and returns, De Croix *et al.* [25] allowed negative demand and proved that a base-stock echelon is still optimal. As an alternative to the echelon base-stock policy, they also discussed a policy which uses local information only. They presented exact and approximate methods for a finite horizon problem and explained how to extend the model when returns occur at different stages. De Croix [1] extended Simpson [21] and Inderfurth [22] by combining the multi-echelon structure of [25] to a series system without disposal. [26] and [27] considered assemble-to-order systems with returns of components or finished products for single-item and multi-item respectively by using a base stock policy. An extensive and detailed comparison of production-inventory control models for production systems in CLSC can be found in the review papers of [28] and [29].

All studies in the literature considered modifications of well-known production-inventory control mechanisms and optimization of related control parameters. However, none of them investigated how to optimally control a hybrid production system. In their review paper, Akçalı and Çetinkaya [28] emphasized the importance of studying the characteristics of optimal control for systems in CLSC. To the best of our knowledge there are no other studies than Zerhouni *et al.* [30] that considered operational level optimal control models for production systems with return flows and Flapper *et al.* [31] that considered product returns that are announced in advance by the customers. In [31], a two-dimensional state space is considered to keep information of both finished goods inventory and announced returns with continuous review and random lead times. Their focus was the potential value of advance return information. Zerhouni *et al.* [30] relaxed the instantaneous procurement lead-time assumption of [32] by considering capacitated production and analyzed the impact of ignoring dependency between demands and returns. However, they assumed instantaneous lead times for product returns for the sake of tractability.

The correlation between products in use and return rate of products is an important issue in controlling of hybrid production systems. The literature on the analysis and control of return flows does not consider this correlation except for a few studies as mentioned above. Those studies usually make generalizing assumptions of zero usage time for products. Therefore, production control policies proposed so far usually use an estimate of constant return rate. However, the return rate is a function of the number of sold products, the time each product is in use and the probability of a product being returned to the manufacturer at the end of its use. Thus, satisfying a demand has both a primary and a secondary effect in a CLSC. An analysis for the optimal control mechanisms of hybrid systems cannot avoid this correlation. In our study we will explore the impact of the correlation among demand and return processes on the production decision of new products by considering the information of sold products. To this end, we will model the supply chain structure as a queueing network and we will generate framework based on Markov decision processes that can measure the value added of return flows to the supply chain, the cost of misjudging the correlation between products in use and return flows, and the value of admission control on product returns.

3. DESCRIPTION OF THE MODEL

In this study, we consider a hybrid make-to-stock manufacturing/remanufacturing system that satisfies a single type of product demand either by manufacturing new products or by processing cores. Thus, similar to [30], we assume that the demand does not distinguish between new and reprocessed products. Here the system produces new products with an exponentially distributed rate θ and places them in the finished goods inventory to meet the demand. Demands arrive one by one following independent exponential interarrival times with rate λ. After a product is sold, it is used for an exponential lead-time with rate μ before it is returned to the hybrid system. We consider two different control mechanisms one of which has admission control for returns. For both models, if a returned product is accepted it is assumed that it is added to the finished goods inventory as soon as it is returned similar to Heyman [4]. The considered products do not require reprocessing or require only minor reprocessing to be considered as good as new. Also while being used a product may be disposed off without being returned to the system after an independent exponential time with rate γ. The considered initial queueing network is given in Figure 3.1.

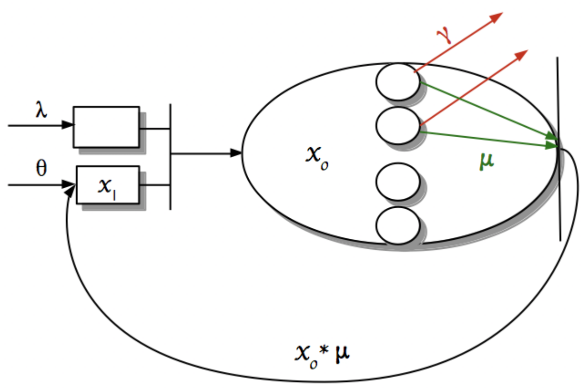

Figure 3.1. The production system with return flows.

When nonzero lead times are introduced, the number of products in use (orbit) are to be considered along with the finished goods inventory level. Zerhouni *et al.* [30] consider that the number of products in orbit is unobservable. In reality, this quantity is partially observable. In this study, in order to understand the impact of the orbit information, it is first assumed that the orbit is fully observable and the difference between this case and the one that completely ignores the orbit information is examined. Therefore, the system state is denoted as $x(t) = (x_0(t), x_I(t))$ where $x_0(t)$ represents the number of products in orbit at time t, while $x_I(t)$ gives the number of products in the finished goods inventory. Using this state definition, two models are considered where in the first one only the production decision is given, while in the second one also the admission of product returns are controlled. Each event causes a shift in the system state as given in the Figure 3.2.

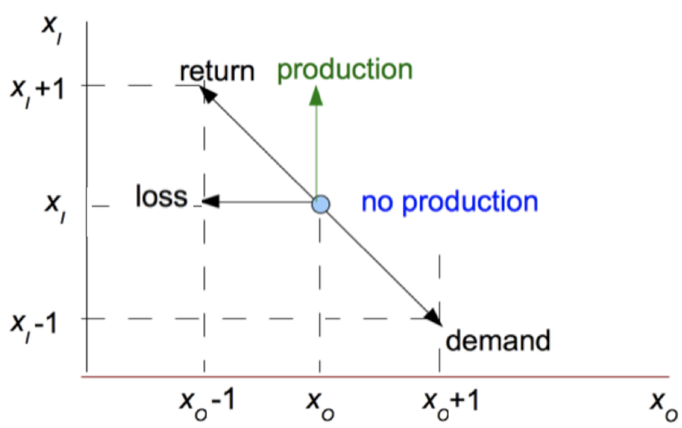

Figure 3.2. State transitions.

3.1. The Model with Production Control

In this model, there is only one control point in the system which allows to decide on producing or not in the relevant state. To find the optimal production policy for the first model, value iteration algorithm of a Markov decision process model is constructed with the following Bellman equation:

$$\nu_{n+1}(x) = \Lambda^{-1}[hx_I^+ + \mu x_0(\nu_n(x_0-1, x_I+1) + c_R) + \gamma x_0 \nu_n(x_0-1, x_I)$$
$$+ \lambda(\nu_n(x_0+1, x_I-1) + c_L\alpha(x_I)) + \theta min\{\nu_n(x_0, x_I+1) + c_M, \nu_n(x_0, x_I)\}$$
$$+ (\Lambda - \mu x_0 - \gamma x_0 - \lambda - \theta)(\nu_n(x_0, x_I))] \tag{3.1}$$

where,

$$\Lambda = \lambda + \theta + M(\mu + \gamma)$$

and

$$\alpha(x_I) = \begin{cases} 1 & x_I = 0 \\ 0 & x_I > 0 \end{cases}$$

Here, h is the holding cost per period per product, c_L is cost of lost sales per product, c_M is cost of manufacturing per product, c_R is cost of remanufacturing per product and M is the maximum number of products in orbit. Note that, although the remanufacturing is assumed to be instantaneous, considering the cost associated for a product to be redirected to the value stream gives an added flexibility for the cost analysis.

After defining the Bellman equations in order to get the stationary distribution of being in state x and giving decision d, $y_{x,d} = P(x_0, x_I, d)$, of the optimal decision the LP equivalent of the value iteration algorithm is constructed as:

$$
\begin{aligned}
minZ = \sum_{x \in S} (c_{x,0} \times y_{x,0} + c_{x,1} \times y_{x,1}) \\
s.t. \sum_{x \in S} (y_{x,0} + y_{x,1}) = 1 \\
y_{x,0} + y_{x,1} - \Lambda^{-1}\{\theta(y_{x,0} + y_{x-e_I,1} + \lambda(y_{x-e_0+e_I,0} + y_{x-e_0+e_I,1}) \\
+ x_0\mu(y_{x+e_0-e_I,0} + y_{x+e_0-e_I,1}) + x_0\gamma(y_{x+e_0,0} + y_{x+e_0,1}) \\
+ (M - x_0)(\gamma + \mu)(y_{x,0} + y_{x,1}\} = 0 \qquad (3.2)
\end{aligned}
$$

$$
\begin{aligned}
& y_{x,0} \geq 0, y_{x,1} \geq 0 \qquad \forall \quad x \in S \\
& x = (x_0, x_I), e_0 = (1, 0), e_I = (0, 1) \\
& c_{x,d} = hx_I + (1 - x_I)^+ \lambda c_L + \theta d c_M + x_0 \mu c_R \\
& d = \{0, 1\}
\end{aligned}
$$

where $c_{x,d}$ is the cost of making decision d in state x, $d = 0$ and $d = 1$ are do not produce and produce decisions, respectively.

The given LP equivalent algorithm is constructed based on balance equations which consider the relationships of states with each other. An example state transition diagram for an inner state is given in Figure 3.3. When system state is $x = (x_0, x_I)$ where $0 < x_0 < M$ and $0 < x_I < N$ there are several possible transitions which are from $x = (x_0, x_I)$ to $x = (x_0 + 1, x_I - 1)$ in case of a demand arrival with rate λ, to $x = (x_0 - 1, x_I + 1)$ in case of a product return with rate $x_0\mu$, to $x = (x_0 - 1, x_I)$ in case of a loss of a product with rate $x_0\gamma$, to $x = (x_0, x_I + 1)$ in case of a produce decision is given with rate θ and to itself in case there is any demand arrival, production, or return or loss of a product with rate $\theta + (M - x_0)(\mu + \gamma)$. Here, production rate, θ, is multiplied with production decision variable, d, or with $(1 - d)$. In this way if produce decision is given in relevant state, $d = 1$, inventory size increases one or stays same otherwise. Also, the transitions that do not involve state $x = (x_0, x_I)$ are not represented in this figure for simplicity.

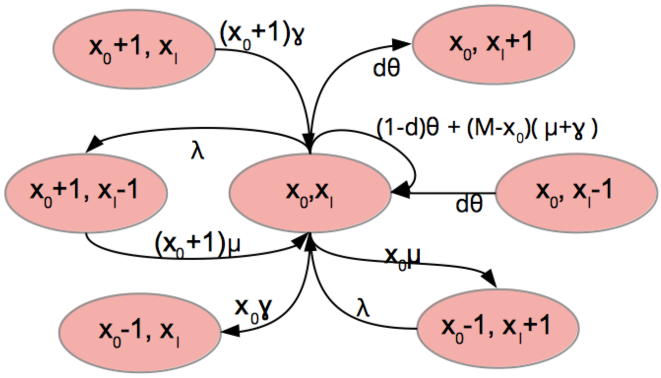

Figure 3.3. State transition diagram of state $x = (x_0, x_I)$, $0 < x_0 < M$ and $0 < x_I < N$

All state transition diagrams of the model with production control are included in the Appendix A.

3.2. The Model with Production and Admission Controls

In addition to production control, the decision of accepting or rejecting a returned product is introduced to the model by extending the equation 3.1 of previous model is given as:

$$\begin{aligned} \nu_{n+1}(x) = \Lambda^{-1}[hx_I^+ &+ \mu x_0 min\{\nu_n(x_0-1, x_I+1) + c_R, \nu_n(x_0-1, x_I)\} \\ &+ \gamma x_0 \nu_n(x_0-1, x_I) + \lambda(\nu_n(x_0+1, x_I-1) + c_L \alpha(x_I)) \\ &+ \theta min\{\nu_n(x_0, x_I+1) + c_M, \nu_n(x_0, x_I)\} \\ &+ (\Lambda - \mu x_0 - \gamma x_0 - \lambda - \theta)(\nu_n(x_0, x_I))] \end{aligned} \tag{3.3}$$

where,

$$\Lambda = \lambda + \theta + M(\mu + \gamma)$$

and

$$\alpha(x_I) = \begin{cases} 1 & x_I = 0 \\ 0 & x_I > 0 \end{cases}$$

The LP equivalent of this value iteration approach which calculates the stationary distribution of being in state x and giving decision β, $y_{x,\beta} = P(x_0, x_I, \beta)$ is given as:

$$\begin{aligned}
minZ = \sum_{x \in S} & (c_{x,0} \times y_{x,0} + c_{x,1} \times y_{x,1} + c_{x,2} \times y_{x,2} + c_{x,3} \times y_{x,3}) \\
& s.t. \sum_{x \in S} (y_{x,0} + y_{x,1} + y_{x,2} + y_{x,3}) = 1 \\
& y_{x,0} + y_{x,1} + y_{x,2} + y_{x,3} - \Lambda^{-1}\{\theta(y_{x,0} + y_{x,1} + y_{x-e_I,2} + y_{x-e_I,3}) \\
& + \lambda(y_{x-e_0+e_I,0} + y_{x-e_0+e_I,1} + y_{x-e_0+e_I,2} + y_{x-e_0+e_I,3}) \\
& + x_0\mu(y_{x+e_0,0} + y_{x+e_0-e_I,1} + y_{x+e_0,2} + y_{x+e_0-e_I,3}) \\
& + x_0\gamma(y_{x+e_0,0} + y_{x+e_0,1} + y_{x+e_0,2} + y_{x+e_0,3}) \\
& + (M - x_0)(\gamma + \mu)(y_{x,0} + y_{x,1} + y_{x,2} + y_{x,3}\} = 0
\end{aligned} \tag{3.4}$$

$$\begin{aligned}
& y_{x,0} \geq 0,\ y_{x,1} \geq 0,\ y_{x,2} \geq 0,\ y_{x,3} \geq 0 \qquad \forall x \in S \\
& x = (x_0, x_I),\ e_0 = (1, 0),\ e_I = (0, 1) \\
& c_{x,\beta} = hx_I + (1 - x_I)^+\lambda c_L + \theta d_M c_M + x_0\mu d_R c_R \\
& \beta = \{0, 1, 2, 3\}
\end{aligned}$$

where,

$$\beta(d_M, d_R) = \begin{cases} 0 & d_M = 0, \quad d_R = 0 \\ 1 & d_M = 0, \quad d_R = 1 \\ 2 & d_M = 1, \quad d_R = 0 \\ 3 & d_M = 1, \quad d_R = 1 \end{cases}$$

Here, $d_M = 0$ and $d_M = 1$ are do not produce and produce decisions, respectively and $d_R = 0$ and $d_R = 1$ are do not accept and, accept and remanufacture the returned product decisions, respectively. Besides, $c_{x,\beta}$ is the cost of making decision β in state x while $\beta = 0$ represents do not produce and do not accept a returned product decision, $\beta = 1$ represents do not produce but accept and remanufacture the returned product decision, $\beta = 2$ represents produce but do not accept a returned product decision and $\beta = 3$ represents produce and, accept and remanufacture the returned product decision.

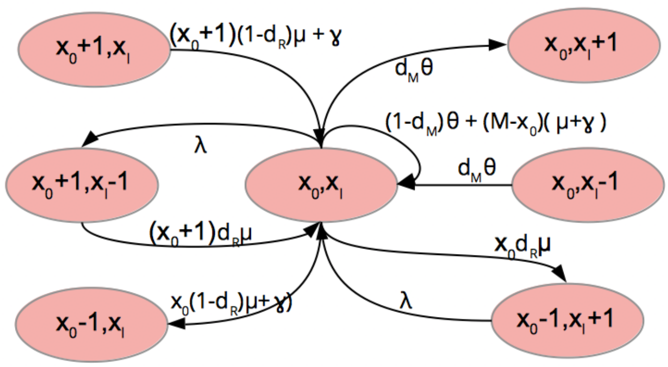

Figure 3.4. State transition diagram of state $x = (x_0, x_I)$, $0 < x_0 < M$ and $0 < x_I < N$

A state transition diagram for an inner state is given in Figure 3.4 to present the differences from the first model and to clarify the LP equivalent algorithm. When system state is $x = (x_0, x_I)$ where $0 < x_0 < M$ and $0 < x_I < N$ there are several possible transitions which are from $x = (x_0, x_I)$ to $x = (x_0 + 1, x_I - 1)$ in case of a demand arrival with rate λ, to $x = (x_0 - 1, x_I + 1)$ in case of a returned product is accepted to the inventory with rate $x_0\mu$, to $x = (x_0 - 1, x_I)$ in case of a loss of a product or not accepting the returned product to the inventory with rate $x_0\gamma + x_0\mu$, to $x = (x_0, x_I + 1)$ in case of a produce decision is given with rate θ and to itself in case there is any demand arrival, production, or return or loss of a product with rate $\theta + (M - x_0)(\mu + \gamma)$. Here, production rate, θ, is multiplied with production decision variable, d_M, or with $(1 - d_M)$ and usage rate, μ, is multiplied with remanufacturing decision variable, d_R, or with $(1 - d_R)$. In this way if production decision is given in the relevant state, $d_M = 1$, inventory size increases one or stays same otherwise. Similarly, if remanufacturing decision is given in relevant state, $d_R = 1$, inventory size increases one and orbit size decreases one or orbit size decreases one without an increase in the inventory otherwise. Also, the transitions that do not involve state $x = (x_0, x_I)$ are not represented in this figure for simplicity.

In both models the initial decision is when to produce a new product based on the inventory of finished goods and the number of products in orbit. In the second model the decision of remanufacturing and transferring a return to the finished goods inventory is added. The effect and interaction of these decisions are studied in the Numerical Analysis section.

4. NUMERICAL ANALYSIS

The main aim of this thesis is to analyze the change of production decision under return flows dependent on the size of orbit. Also the value of using return flows is studied by comparing production systems with and without return flows. To extend the insights on return flows the value of being able to control the admission of returned product in addition to production control is analyzed. As mentioned earlier, the number of products is at best partially observable in reality and therefore, return rate estimates are inaccurate. In the last part of numerical analysis, the costs of underestimating or overestimating the return rate are examined for both the single and two control parameter cases.

In order to carry out the analysis we first determine the state space size that would minimize the upper boundary effects of the inventory level and orbit size on the expected reward. We assume that when the size of orbit reaches its maximum value, a new demand is satisfied if there is at least one product in inventory but it does not increase the size of orbit. Similarly, when the inventory reaches its maximum value a returned product does not increase the amount of products in inventory. For the experiments to determine the state space, we fixed the production rate, θ, and arrival rate, λ, to 1 and studied the sizes of orbit and inventory by changing return rate, μ, and loss rate, γ, as 0.0005, 0.0015, 0.0025, 0.005, 0.001, 0.003, 0.01. Our objective in setting $\theta = \lambda$ was to observe the cases with maximum variance. Since the system generates a semi- closed queueing network with an M/M/1 server that shuts off when the queue is $\geq S$ and a $./M/\infty$ server the system will never diverge. One can easily argue that as x_0 increases the probability of a return before a demand satisfaction converges to one.

The results of experiments showed that the effect of setting orbit size to more than 100 and the inventory size to more than 40 on the expected cost per unit time is insignificant for all experiments. Thus, we assumed that setting the maximum orbit size M as 100 and the maximum inventory size N as 40 is equivalent to having infinite orbit and inventory sizes. After determining the state space size, we constructed an experiment set to study the behavior of this system in detail. To this end, the estimated rate of leaving the orbit which is calculated as $M/2$ times the sum of μ and γ is made equal to the initial demand rate λ which was taken as 1. From this calculation, the sum of μ and γ is taken as 0.02. To observe the effect of return probability p which is calculated as $\mu/(\mu+\gamma)$ is decided to have five different values as given in Table 4.1.

Table 4.1. Parameters: μ, γ and p.

	1	**2**	**3**	**4**	**5**
μ	0.002	0.006	0.01	0.014	0.018
γ	0.018	0.014	0.01	0.006	0.002
p	0.1	0.3	0.5	0.7	0.9

While specifying the cost parameters, the holding cost h is taken as a reference point by setting its value to 1. To examine its relationship with cost of lost sales, c_L, two different h/c_L ratios, 0.1 and 0.02, are determined. Furthermore, it is assumed that $c_M \leq c_L$ to have an incentive to produce and three different c_M/c_L ratios are determined as 0.1, 0.5 and 1 to examine the relationship between manufacturing and lost sales costs. Finally, the costs of manufacturing and remanufacturing, c_M and c_R, are determined as given in Table 4.2 to satisfy five different ratios as c_R/c_M. It is also assumed that $c_R \leq c_M$ to let remanufacturing preferable. Since the interpretation on cost parameters are made based on ratios, the real monetary values of the costs were not necessary to obtain in this study. Finally, the demand rate λ is decided to have two more values as 0.8 and 1.2 to examine the system behavior when $\lambda < \theta$, $\lambda = \theta$ and $\lambda > \theta$. In this way, an experiment set with 360 different experiments is obtained.

Table 4.2. Parameters: c_M and c_R.

c_M	c_R			
1	0.1	0.3	0.6	0.9
5	0.5	1.5	3	4.5
10	1	3	6	9
25	2.5	7.5	15	22.5
50	5	15	30	45
c_R/c_M	0.1	0.3	0.6	0.9

4.1. The Effect of Orbit Size Dependent Return Flows on Production Decision

It was shown in [30] that the optimal production policy is a base stock policy when the return flow is independent of the orbit size. However, the return rate is a function of the number of products in use, the usage time and the probability of a product to be returned to the manufacturer. Therefore, the aim is analyzing how orbit dependent return rate affects the optimal production policy here. To this end, decision spaces of the MDP of model with production decision is obtained by using value iteration algorithm. The following Figure 4.1 gives the optimal orbit state dependent decision space for products with high return probability and an average usage length of 50 unit. Here, x-axis represents the size of orbit and y-axis represents the size of inventory. Also, 1 represents the decision to produce a new unit for the stock and 0 represents otherwise.

It can be observed that when there are 0 to 6 products in the orbit, the system prefers to produce until the number of products in the inventory reaches to 6 which is called as base stock level. However, there is a decline in the base stock level as the number of products in use increases. The reason behind that is the increase in return rate which is calculated as μ times x_0. When the return rate gets higher, a higher percentage of the demands is satisfied from the returned products and the need for

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5	1111111111111111000000000000000000000000000000000000000000000000000000000000000000000000000000000000
4	1111111111111111111111111110000000000000000000000000000000000000000000000000000000000000000000000000
3	1111111111111111111111111111111111111100000000000000000000000000000000000000000000000000000000000000
2	1111111111111111111111111111111111111111111111111000000000000000000000000000000000000000000000000000
1	1111111111111111111111111111111111111111111111111111111110000000000000000000000000000000000000000000
0	1111111111111111111111111111111111111111111111111111111111111111100000000000000000000000000000000000

Figure 4.1. Decision space where return probability $p = 0.9$, $\mu = 0.018$, $\theta = 1$, $\lambda = 1$, $h/c_L = 0.02$, $c_M/c_L = 0.1$, $c_M = 5$, $c_R = 3$ and $h = 1$

production decreases which allows the base stock level to decrease. This behavior of the switching curve is also observed by Flapper *et al.* [31] where they have modeled for a partial knowledge for products to be returned.

4.1.1. Sensitivity Analysis on the Switching Curve Behavior

In this section, the changes in the behavior of switching curve with different return probabilities, h/c_L, c_M/c_L and c_R/c_M ratios and different demand rates are examined, respectively. In the following Figure 4.2, the decision spaces of three different experiments with increasing return probabilities are given. It is observed that when the return probability is low, 0.1, the control policy is very similar to the base stock type policy. A base stock level S is decided by control policy and whenever the number of finished goods in inventory is less than the S system produces new products. On the other hand as the return probability increases, the impact of orbit information becomes more noticeable. The increase in the impact of orbit can be observed with the change of the switching curve. Number of states where system decides to produce decreases and base stock level is updated more frequently by decreasing it one by one as the orbit size increases. After reaching a certain orbit size which depends on the magnitude of return probability, control policy may decide not to produce since the return rate is big enough to satisfy demand by itself. These results can be interpreted as the increase in probability of return increases the value of orbit information.

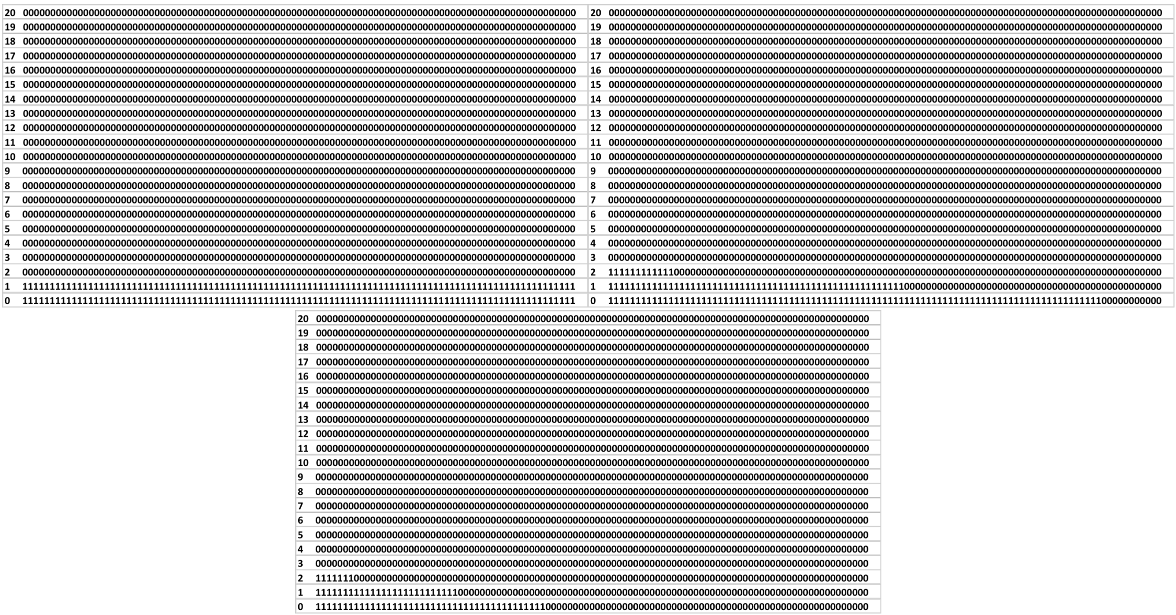

Figure 4.2. Decision spaces where $p = (0.1, 0.5, 0.9)$, (top left clockwise) $\mu = (0.02, 0.1, 0.18)$, $\theta = 1$, $\lambda = 1$, $h/c_L = 0.1$, $c_M/c_L = 0.5$, $c_R/c_M = 0.6$ and $h = 1$

The same effect in Figure 4.2 can also be observed in Figure 4.3. However, in this figure the control policy decides to hold a higher finished goods inventory and the area of production decisions is increased since the cost of lost sales is increased significantly. In this case, it can also be observed by comparing Figure 4.2 and Figure 4.3 that base stock levels are updated more frequently while orbit size increases.

When the cost of manufacturing gets closer to the cost of lost sales, the benefit of production decreases. In the Figure 4.4 and 4.5 it is seen that the increase in c_M/c_L ratio results in a decrease in the area of production decisions. Base stock levels are pulled down and updated less frequently. Furthermore, by comparing Figure 4.4 and 4.5 with each other, it can be observed that a higher c_R/c_M ratio makes the effect of c_M/c_L ratio more powerful since the value of remanufacturing also decreases because of its increasing cost. Therefore, it is expected that the probability of incurring a cost of lost sale increases with an increase in both c_M/c_L and c_R/c_M ratios.

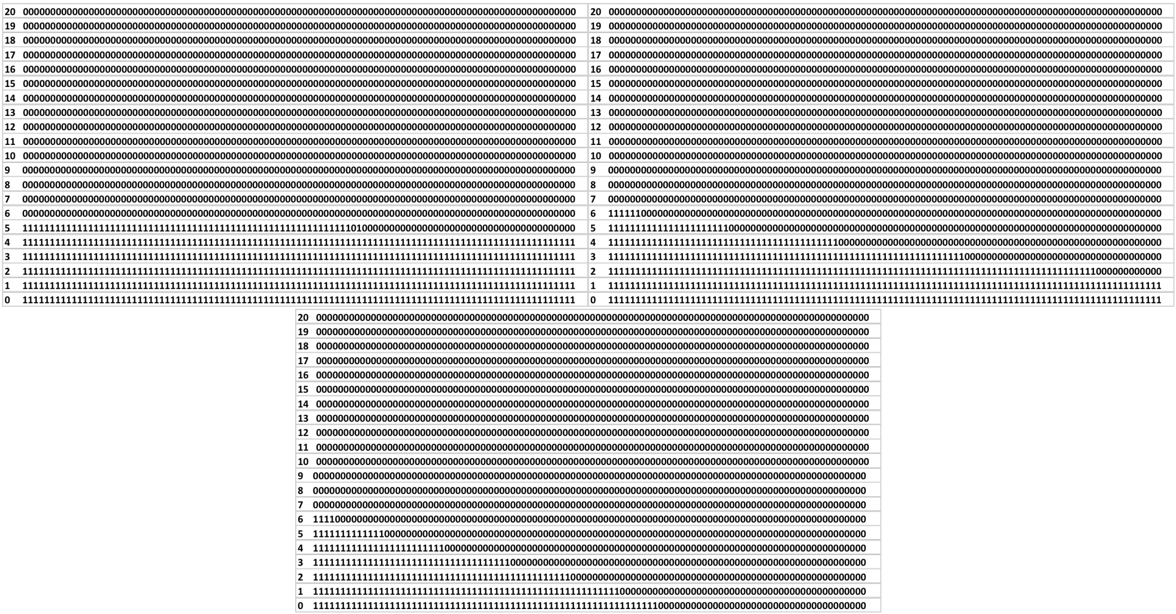

Figure 4.3. Decision spaces where $p = (0.1, 0.5, 0.9)$, (top left clockwise) $\mu = (0.02, 0.1, 0.18)$, $\theta = 1$, $\lambda = 1$, $h/c_L = 0.02$, $c_M/c_L = 0.5$, $c_R/c_M = 0.6$ and $h = 1$

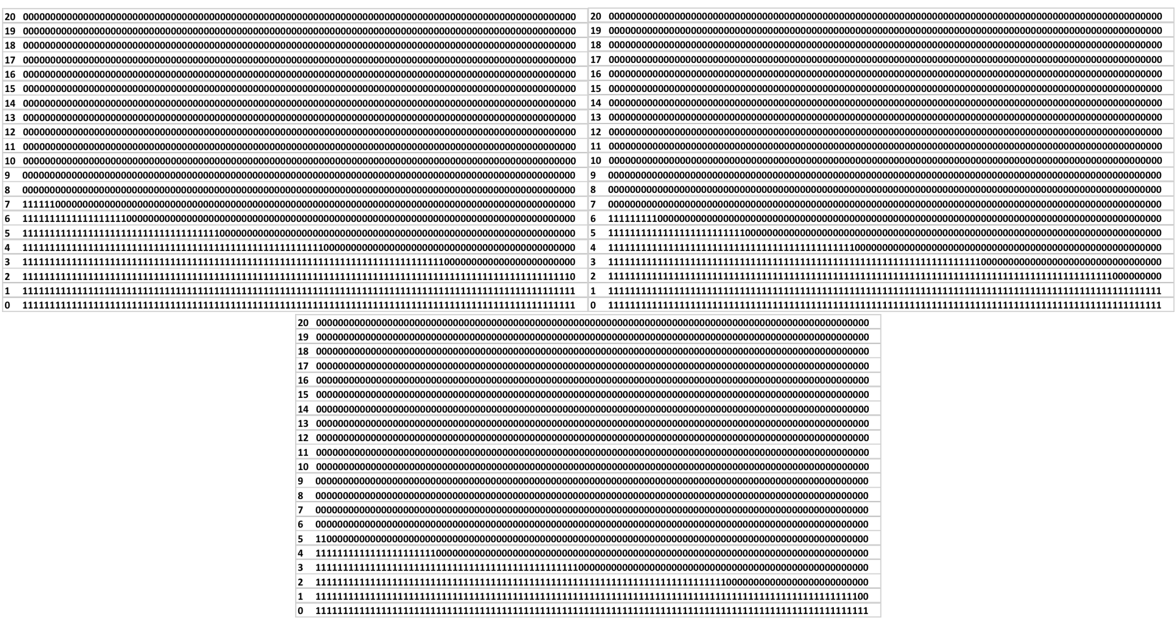

Figure 4.4. Decision spaces where $p = 0.5$, $\mu = 0.1$, $\theta = 1$, $\lambda = 1$, $h/c_L = 0.02$, $c_M/c_L = (0.1, 0.5, 1)$ (top left clockwise), $c_R/c_M = 0.1$ and $h = 1$

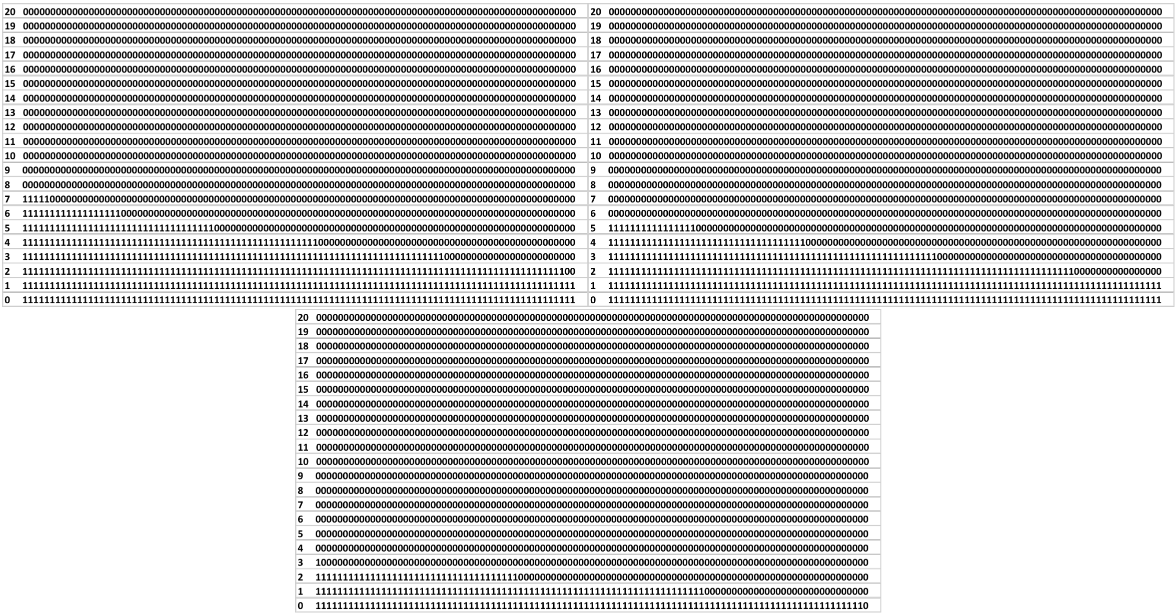

Figure 4.5. Decision spaces where $p = 0.5$, $\mu = 0.1$, $\theta = 1$, $\lambda = 1$, $h/c_L = 0.02$, $c_M/c_L = (0.1, 0.5, 1)$ (top left clockwise), $c_R/c_M = 0.6$ and $h = 1$

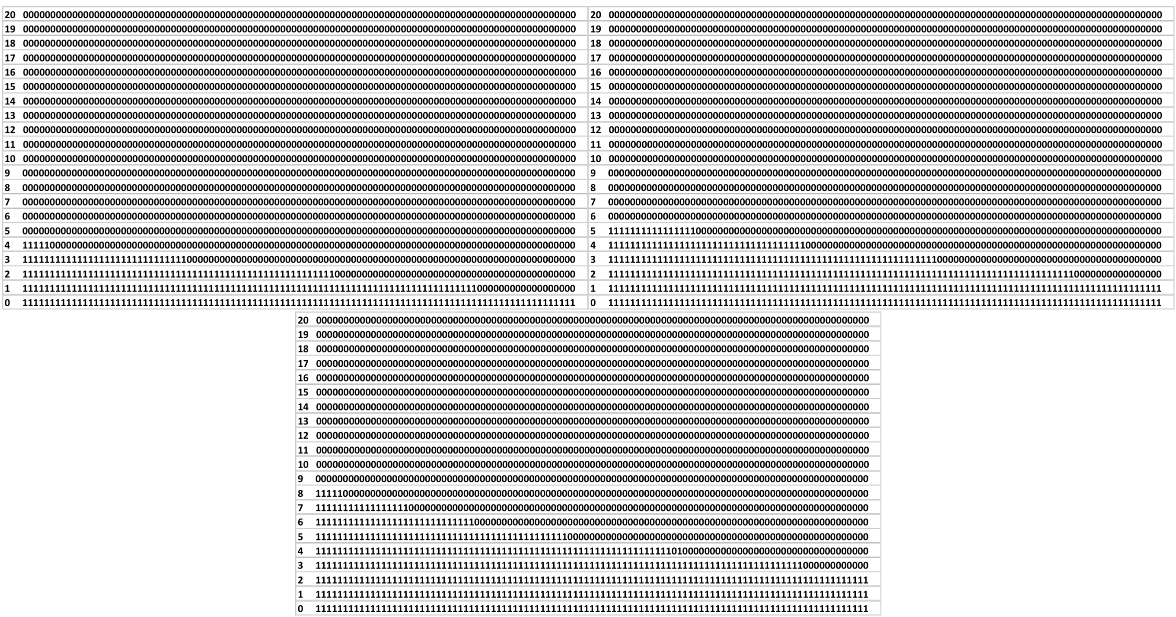

Figure 4.6. Decision spaces where $p = 0.5$, $\mu = 0.1$, $\theta = 1$, $\lambda = (0.8, 1, 1.2)$ (top left clockwise), $h/c_L = 0.02$, $c_M/c_L = 0.5$, $c_R/c_M = 0.6$ and $h = 1$

Figure 4.6 indicates that when demand rate increases, area of production decision increases because control policy increases the amount of finished goods in the inventory to be able to cope with increased demand. The behavior of switching curve also changes similar to Figure 4.3 which indicates the increase in cost of lost sales and increase in demand affect the switching curve similarly.

4.1.2. Sensitivity Analysis on the Cost of Model with Production Decision

After analysing the effect of return flow on the production decision space, the cost behavior of the relevant model is discussed here by also considering the probability distributions. Costs and probability distributions are obtained from the LP equivalent of the model.

In the Figure 4.7, 6 graphs in which the costs of the chosen experiments are plotted according to their c_R/c_M ratio with the increasing return probability in the x-axis are represented in two rows and three columns. The graphs in the first row belong to the experiments with $c_M/c_L = 0.1$ and the ones in the second row belong to the experiments with $c_M/c_L = 0.5$. Each column includes the experiments with same λ parameters.

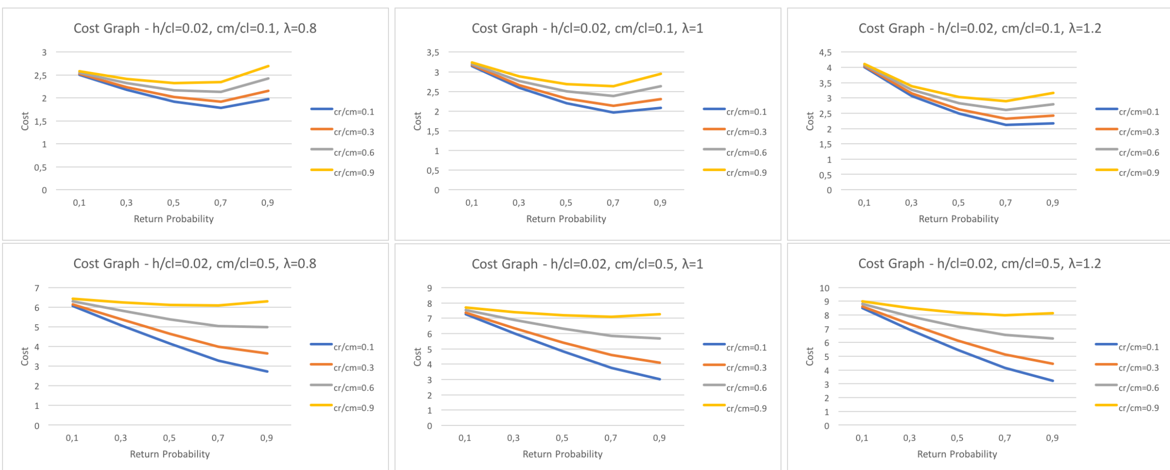

Figure 4.7. Cost graphs of model with production control, $h/c_L = 0.02$, $p = (0.1, 0.3, 0.5, 0.7, 0.9)$, $\mu = (0.002, ..0.018)$, $\lambda = (0.8, 1, 1.2)$, $\theta = 1$, and $h = 1$

From Figure 4.7, it is observed that average costs increase with increasing λ rate. As it is discussed in the Figure 4.6, the increase in demand causes both an increase in the number of finished products in the inventory and the number of states where production decision is given. Consequently, there are more products both in use and in inventory with increasing λ as seen in the marginal probability distributions of orbit and inventory in Figures 4.8 and 4.9, respectively. The increase in the expected number of products in use also increases the expected number of returned products. Therefore, control policy updates the base stock level frequently with increasing λ as in Figure 4.6. However, having more products in the inventory causes a higher holding cost and higher level of manufacturing and remanufacturing activities also increases the cost. However, the magnitude of cost decrease when λ is 20% smaller than θ is a little more than the magnitude of cost increase when λ is 20% more than θ as can be seen in the Table 4.3. Therefore, it might be interpreted that the effect of the increase in λ on the cost is more powerful when $\lambda < \theta$. Also, it is observed that cost differences caused by λ increases when c_M/c_L and c_L increases.

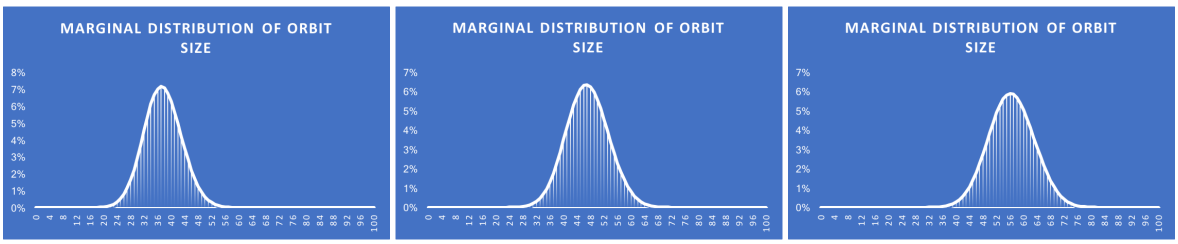

Figure 4.8. Marginal probability distributions of orbit size of model with production control, $h/c_L = 0.02$, $c_M/c_L = 0.5$, $c_R/c_M = 0.6$, $p = 0.5$, $\mu = 0.01$, $\lambda = (0.8, 1, 1.2)$, $\theta = 1$, and $h = 1$

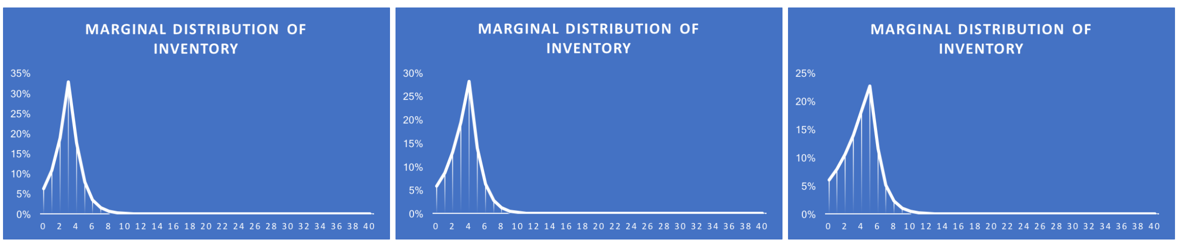

Figure 4.9. Marginal probability distributions of inventory of model with production control, $h/c_L = 0.02$, $c_M/c_L = 0.5$, $c_R/c_M = 0.6$, $p = 0.5$, $\mu = 0.01$, $\lambda = (0.8, 1, 1.2)$, $\theta = 1$, and $h = 1$

Table 4.3. Comparison of cost difference where $\lambda = (0.8, 1, 1.2)$, $p = 0.5$ $c_R/c_M = 0.6$, $c_M/c_L = (0.1, 0.5)$, $c_L = (10, 50)$, $\theta = 1$ and $h = 1$.

λ	c_R/c_M	c_M/c_L	c_L	Optimal Cost	Cost Difference
0.8	0.6	0.1	10	0.942	-9.81%
1	0.6	0.1	10	1.044	0%
1.2	0.6	0.1	10	1.146	9.70%
0.8	0.6	0.5	10	1.490	-12.69%
1	0.6	0.5	10	1.707	0%
1.2	0.6	0.5	10	1.897	11.13%
0.8	0.6	0.1	50	2.172	-13.20%
1	0.6	0.1	50	2.502	0%
1.2	0.6	0.1	50	2.828	13.03%
0.8	0.6	0.5	50	5.379	-14.74%
1	0.6	0.5	50	6.310	0%
1.2	0.6	0.5	50	7.171	13.65%

In all graphs within the Figure 4.7, cost lines start almost from the same point which indicates that there is not much cost difference in experiments with different c_R/c_M ratios when return probability is as low as 0.1. When return probability increases, cost difference becomes more noticeable between experiments with different c_R/c_M ratios. The experiments with higher c_R/c_M ratio have higher costs because of both of an increase in c_R and in probability of lost sale as it can be seen in Table 4.4 by comparing the experiments with same c_L value and c_M/c_L ratio. The probability of lost sale is calculated as probability of having zero finished goods in inventory, $P(x_I = 0)$. It was mentioned that the change in behavior of switching curve is more noticeable when both c_M/c_L and c_R/c_M increases as in Figures 4.4 and 4.5. It was also mentioned that it is expected that the probability of incurring a lost sale cost increases with an increase in both c_M/c_L and c_R/c_M ratios. As expected, the increase in probability of lost sale with increasing c_R/c_M ratio is small where c_M/c_L is low but when c_M/c_L ratio is higher, the probability of lost sale increases more with increasing c_R/c_M ratio because of both decrease in the value of manufacturing and remanufacturing.

Table 4.4. Probability of lost sales where $\lambda = 1$, $p = 0.5$ $c_R/c_M = (0.1, 0.6)$, $c_M/c_L = (0.1, 0.5)$, $c_L = (10, 50)$, $\theta = 1$ and $h = 1$.

c_M	c_L	c_M/c_L	c_R/c_M	Probability
1	10	0.1	0.1	0.165
5	10	0.5	0.1	0.171
1	10	0.1	0.6	0.167
5	10	0.5	0.6	0.185
5	50	0.1	0.1	0.038
25	50	0.5	0.1	0.051
5	50	0.1	0.6	0.039
25	50	0.5	0.6	0.058

It can also be observed from Figure 4.7 that, the cost keeps decreasing as return probability increases. However, after a while it starts to increase. It can be explained with the increasing number of returned products that is caused by increasing return probability. Since all returned products are accepted, remanufacturing of more than necessary returns causes an increase in the cost. Also, the amount of products in finished goods inventory increases while return probability increases as can be seen in the Figure 4.10 which causes an increase in the total holding cost. The cost of a system with high return probability as 0.9 may be even higher than a system with low return probability as 0.1, as it is seen in the graph on first row and first column of Figure 4.7. However, when c_M/c_L ratio increases while keeping λ same, cost curves start to turn into monotonically decreasing lines even though average costs increases because of increase in manufacturing and remanufacturing costs and the increase in probability of lost sale as can be seen in Table 4.4. In other words, increasing amount of returned products increases the total cost where manufacturing cost is low compared to cost of lost sale but decreases the total cost when manufacturing cost increases as long as remanufacturing cost, c_R, is not very close to c_M. Therefore, it can be interpreted that the importance of remanufactured products increases while c_M/c_L increases. It is also observed in graphs in the second row of Figure 4.7 that as λ increases where c_M/c_L is higher, even higher c_R/c_M ratio does not cause an increase in the cost at higher return probability. The reason is that when demand is increased, the higher amount of returned products are more beneficial and used to satisfy increased demand. While discussing switching curves it has been discussed that the production area decreases as c_M/c_L increases because of the decreasing benefit of production. Because of less production, the average orbit and inventory sizes decreases as can be observed in marginal distribution of orbit size and finished goods inventory in the Figure 4.11 and Figure 4.12. This also confirms the increase in probability of lost sales with increasing c_M/c_L ratio.

In Figure 4.13, 6 graphs in which the costs of the chosen experiments are plotted according to their c_R/c_M ratio with the increasing return probability in the x-axis are represented in two rows and three columns. The graphs in the first row belong to the experiments with $h/c_L = 0.1$ and the ones in the second row belong to the experiments with $h/c_L = 0.02$. Each column includes the experiments with same λ parameters.

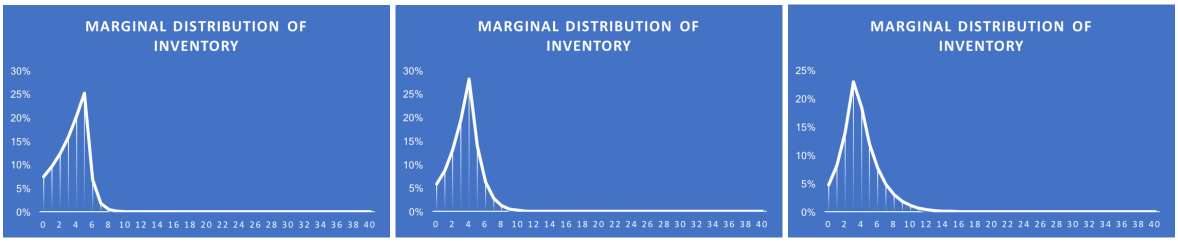

Figure 4.10. Marginal probability distributions of inventory of model with production control, $h/c_L = 0.02$, $c_M/c_L = 0.5$, $c_R/c_M = 0.6$, $p = (0.3, 0.5, 0.7)$, $\mu = 0.01$, $\lambda = 1$, $\theta = 1$, and $h = 1$

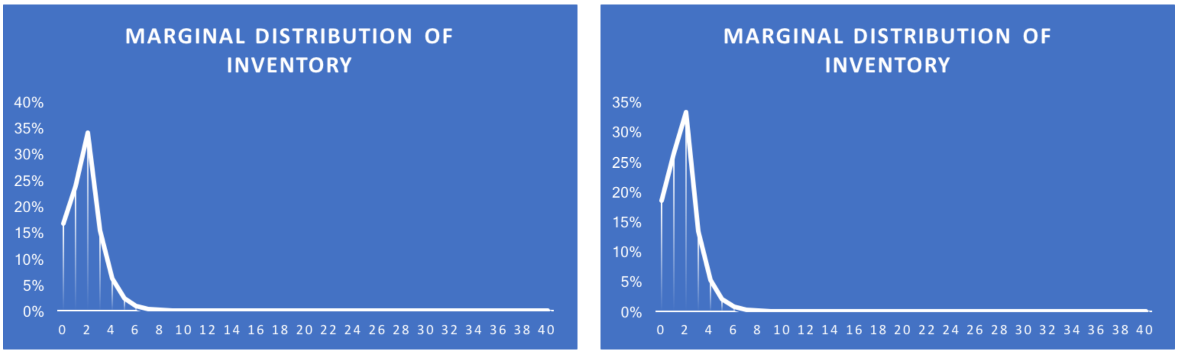

Figure 4.11. Marginal probability distributions of inventory of model with production control, $h/c_L = 0.1$, $c_M/c_L = (0.1, 0.5)$, $c_R/c_M = 0.6$, $p = 0.5$, $\mu = 0.01$, $\lambda = 1$, $\theta = 1$, and $h = 1$

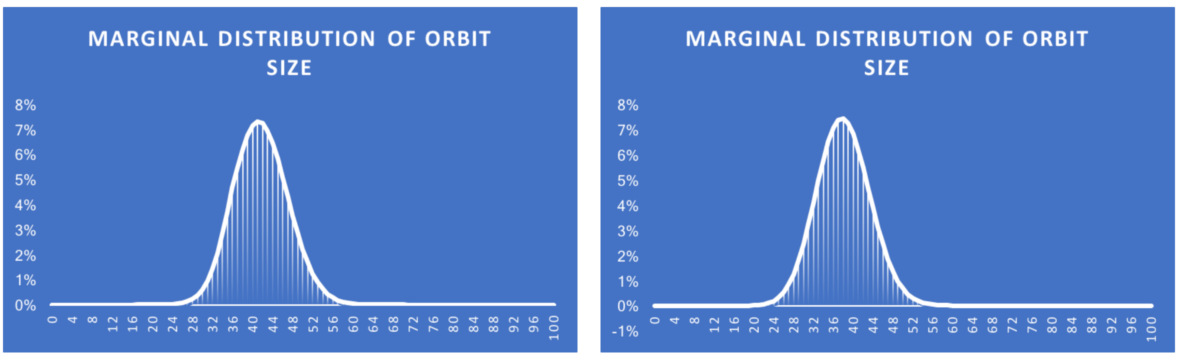

Figure 4.12. Marginal probability distributions of orbit size of model with production control, $h/c_L = 0.1$, $c_M/c_L = (0.1, 0.5)$, $c_R/c_M = 0.6$, $p = 0.5$, $\mu = 0.01$, $\lambda = 1$, $\theta = 1$, and $h = 1$

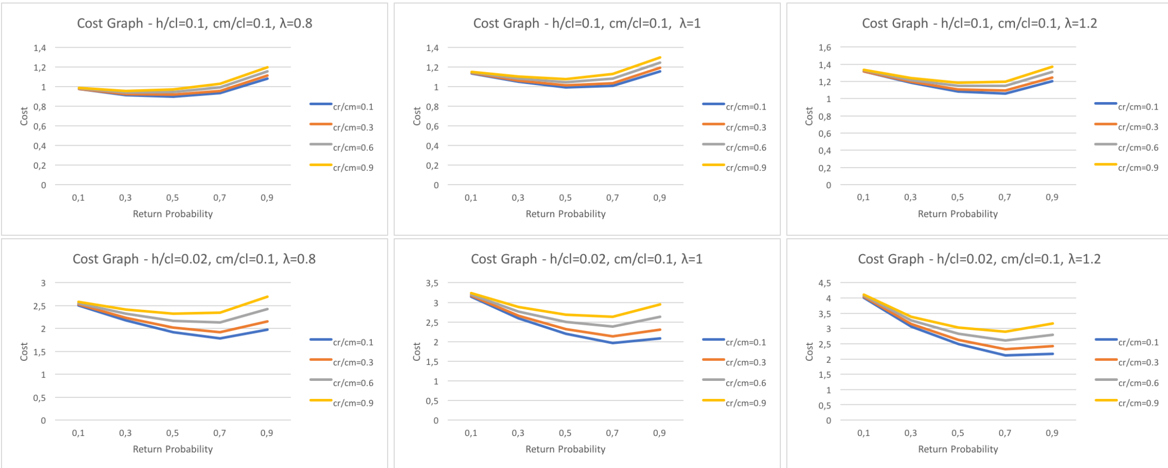

Figure 4.13. Cost graphs of model with production control, $h/c_L = (0.1, 0.02)$, $c_M/c_L = 0.1$, $p = (0.1, 0.3, 0.5, 0.7, 0.9)$, $\mu = (0.002, ..0.018)$, $\lambda = (0.8, 1, 1.2)$, $\theta = 1$, and $h = 1$

From Figure 4.13 it can be observed that when c_L is low, increasing return probability may not allow cost to decrease significantly. However, when c_L is high, there might be more improvement in cost with increasing return probability even though average costs increase because of both an increase in manufacturing and remanufacturing costs and holding more finished goods in the inventory as it is discussed in Figure 4.6 and can be observed in marginal distribution of inventory in Figure 4.14. Also, when the cost of a lost sale is high relative to its holding cost, probability of a lost sale decreases as can be seen in Table 4.4 and therefore high percentage of demand is satisfied. This causes orbit size to expand as shown in Figure 4.15 where marginal orbit size distribution moves to the right. The reason of being able to have more room for improving the cost is the increase in numerical value of difference between cost of lost sales and the manufacturing/remanufacturing costs.

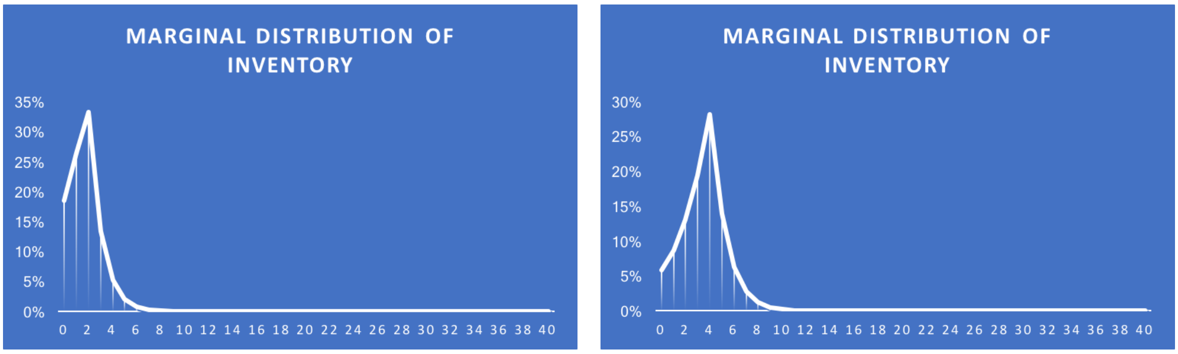

Figure 4.14. Marginal probability distributions of inventory of model with production control, $h/c_L = (0.1, 0.02)$, $c_M/c_L = 0.5$, $c_R/c_M = 0.6$, $p = 0.5$, $\mu = 0.01$, $\lambda = 1$, $\theta = 1$, and $h = 1$

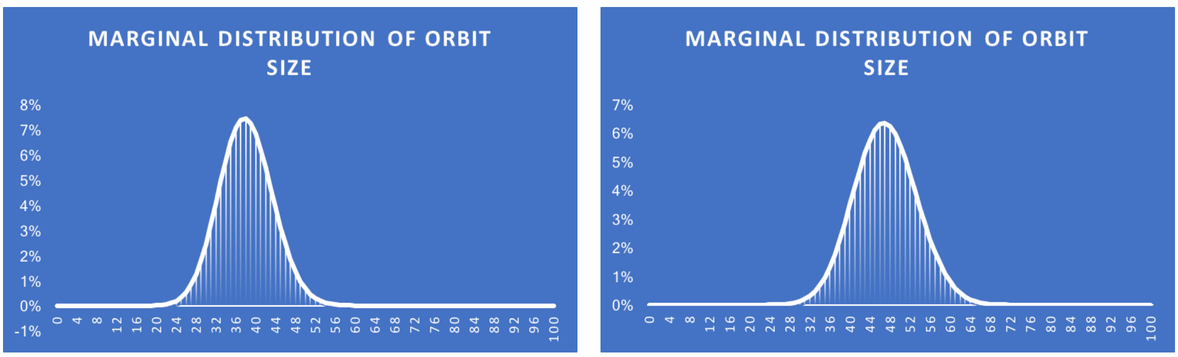

Figure 4.15. Marginal probability distributions of orbit size of model with production control, $h/c_L = (0.1, 0.02)$, $c_M/c_L = 0.5$, $c_R/c_M = 0.6$, $p = 0.5$, $\mu = 0.01$, $\lambda = 1$, $\theta = 1$, and $h = 1$

4.1.3. Value of the Return Flow

To understand the value of using return flows, a pure production system with an optimal production decision policy is compared to the optimal control policy systems with return flows. Here pure production model is obtained by modifying our current model to reject all the returned products. In Figure 4.16, 6 graphs in which the percentage difference of costs between the pure production model and the model with orbit dependent return flow are plotted according to their λ rates with the increasing return probability in the x-axis are represented in two rows and three columns. The graphs in the first row belong to the experiments with $c_R/c_M = 0.1$ and the ones in the second row belong to the experiments with $c_R/c_M = 0.6$. Each column includes the experiments with same c_M/c_L parameters.

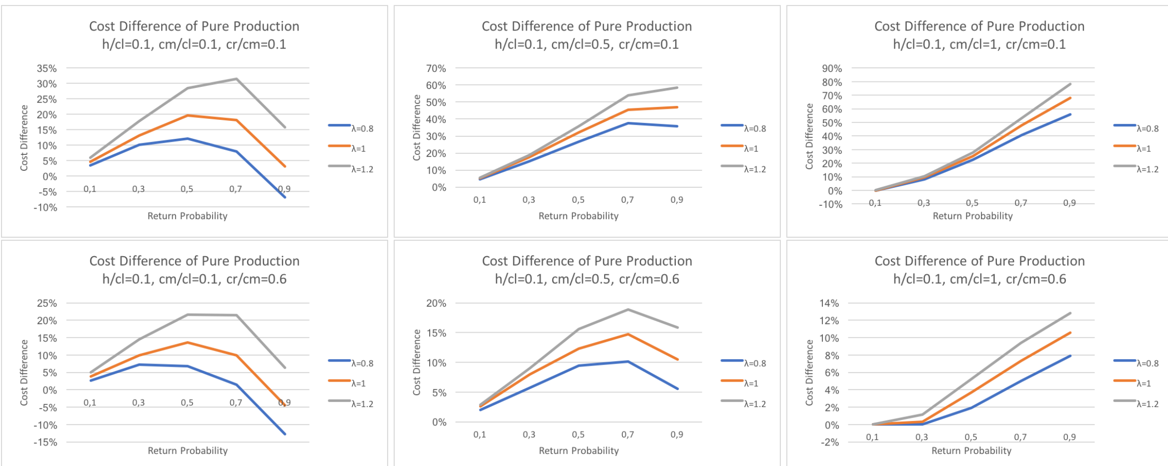

Figure 4.16. Cost comparison of model with production control and pure production model , $h/c_L = 0.1$, $c_M/c_L = (0.1, 0.5, 1)$, $c_R/c_M = (0.1, 0.6)$, $p = (0.1, 0.3, 0.5, 0.7, 0.9)$, $\mu = (0.002, ..0.018)$, $\lambda = (0.8, 1, 1.2)$, $\theta = 1$, and $h = 1$

Figure 4.16 shows that the cost difference is around 5% when return probability is low, 0.1, yet increases significantly with increasing return probability, since the cost of model with return flow decreases as seen in Figure 4.7 while the cost of pure production model is independent from return probability. On the other hand, as it is also mentioned in Figure 4.7, cost of model with return flow increases when return probability increases too much and therefore the cost difference with pure production model decreases at high return probabilities. There are even some experiments which has a negative cost difference indicating the cost of model with return flow is higher than the cost of pure production. In our experiment set, negative cost differences are observed in only experiments with $c_L = 10$, $c_M/c_L \leq 0.5$, $p \geq 0.7$ and $\lambda \leq \theta$. By comparing these experiments it is possible to observe that increase in c_M/c_L ratio and λ decreases the absolute value of negative cost difference, but increase in p and c_M/c_R increases the absolute value of negative cost difference as given in Table 4.5. As it is observed, high return probability causes return flow to worsen the total cost where the value of remanufacturing decreases. In this case, it can be concluded that admission control on return flow is necessary to improve the cost of system. This can also be interpreted as using return flows especially when the demand is higher than the supply or when h/c_L is as low as 0.02 or as a strategic decision of reducing production rate to incorporate return flows.

Table 4.5. Parameter sets where the cost of model with return flow is higher than the cost of pure production model, $\theta = 1$ and $h = 1$

c_L	λ	c_M/c_L	c_R/c_M	p	Cost Difference
10	0.8	0.1	0.9	0.7	-2%
10	0.8	0.1	0.1	0.9	-7%
10	0.8	0.1	0.3	0.9	-9%
10	0.8	0.1	0.6	0.9	-13%
10	1	0.1	0.6	0.9	-4%
10	0.8	0.1	0.9	0.9	-16%
10	1	0.1	0.9	0.9	-8%
10	0.8	0.5	0.9	0.7	-1%
10	0.8	0.5	0.9	0.9	-1%
10	1	0.5	0.9	0.9	-3%

It is also observed in Figure 4.16 that even though the increase in λ causes an increase the cost of model with return flow as discussed earlier, the cost of pure production model increases more with increasing λ. For example, pure production systems are 5% up to 45% more costly when demand and production rates are equal where $c_M/c_L = 0.5$. The cost difference increases significantly when $\lambda > \theta$ and still 5% up to 35% more costly when $\lambda < \theta$ in the same setting. On the other hand, as the c_R/c_M ratio increases, the cost difference decreases on average which means the benefit of using return flow decreases as remanufacturing gets more costly.

It can also be observed from Figure 4.16 that with the increasing c_M/c_L ratio, cost difference keeps increasing until a higher return probability. The reason behind that is remanufacturing becomes a more valuable option when manufacturing cost c_M is close to c_L as discussed earlier. As the amount of returned products increases with increasing return probability, demand is satisfied more economically compare to pure production and cost difference increases therefore. Finally, when the $c_M/c_L = 1$ the advantage of using return flow seems more significant since the manufacturing decision is not a valuable decision in this case. However, $c_M/c_L = 1$ is an extreme case and usually cost of manufacturing is less than cost of lost sales. Yet it is important to see what happens when the lost sale cost is not significant.

In the Figure 4.17, 6 graphs in which the percentage difference of costs between the pure production model and the model with orbit dependent return flow are plotted according to their λ rates with the increasing return probability in the x-axis are represented in two rows and three columns. The graphs in the first row belong to the experiments with $h/c_L = 0.1$ and the ones in the second row belong to the experiments with $h/c_L = 0.02$ where c_L is 10 and 50, respectively. Each column includes the experiments with same c_M/c_L parameters. Here, the effect of c_L can be observed by comparing the figures in first and second rows with each other. As it is seen, an increase in c_L causes cost difference to increase significantly. It can be interpreted as the value of return flow increases when c_L is higher compared to holding cost, h, since it is more economical to keep inventory high with returned products.

As a summary, using return flows decreases the cost of the system significantly. The value of using return flows is higher especially when c_L, c_M/c_L and λ are higher and c_R/c_M is lower. Even though it is mostly beneficial, return flows might cause

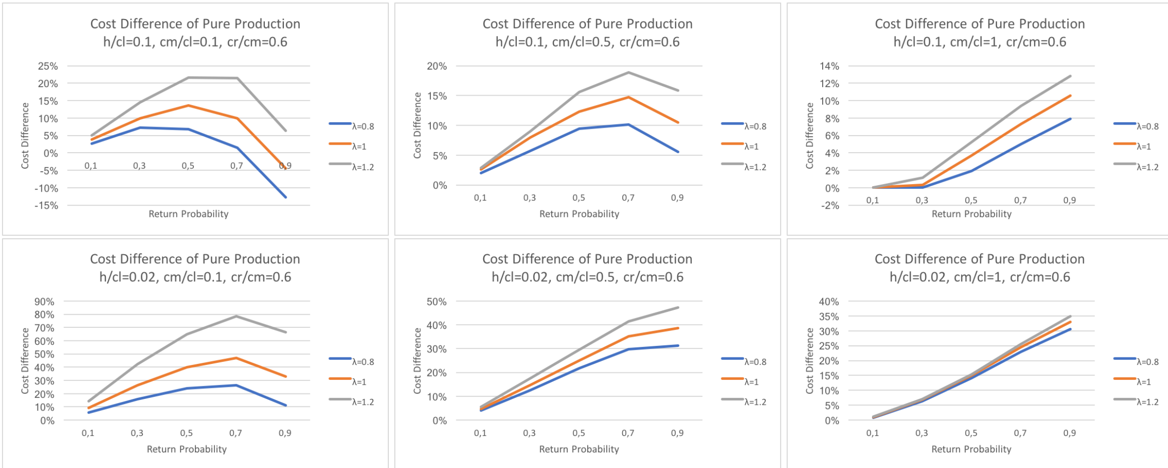

Figure 4.17. Cost comparison of model with production control and pure production model , $h/c_L = (0.1, 0.02)$, $c_M/c_L = (0.1, 0.5, 1)$, $c_R/c_M = 0.6$, $p = (0.1, 0.3, 0.5, 0.7, 0.9)$, $\mu = (0.002, ..0.018)$, $\lambda = (0.8, 1, 1.2)$, $\theta = 1$, and $h = 1$

system cost to increase especially where the cost of remanufacturing is very close to the cost of manufacturing and the amount of returned products becomes more than necessary because of not having an admission control on the returned products.

4.2. Value of Admission Control

In the previous analysis it has been observed that even though return flow has a significant effect on decreasing the cost of the system, after the return probability increases too much its benefits diminish because of uncontrolled return of products in use. Therefore, an admission control becomes necessary at this point.

In the left of Figure 4.18 the optimal costs of model with only production is given. The costs are convex in return probabilities when only the production decision is controlled. The improvement in cost decreases with increasing return probability and after a certain probability cost starts to increase. On the other hand, the effect of being able to reject the unnecessary returns generates a monotonically decreasing cost structure as can be seen in the right of Figure 4.18. Admission control helps to control inventory level which prevents system to incur unnecessary holding and remanufacturing costs. van der Laan [11] also discussed that the increase in return rate decreases the total cost at first but then cost increases due to the higher variability of output of remanufacturing as we show here. However, unlike our study, [11] allow for correlation between demands and returns with Coxian-2 distributed demand and return inter-arrival time distributions where a product return generates a demand with a specified probability. Also manufacturing and remanufacturing are in batches where they optimize 4-parameter PUSH-disposal and 5-parameter PULL-disposal strategies as a modification of a classical inventory control policy.

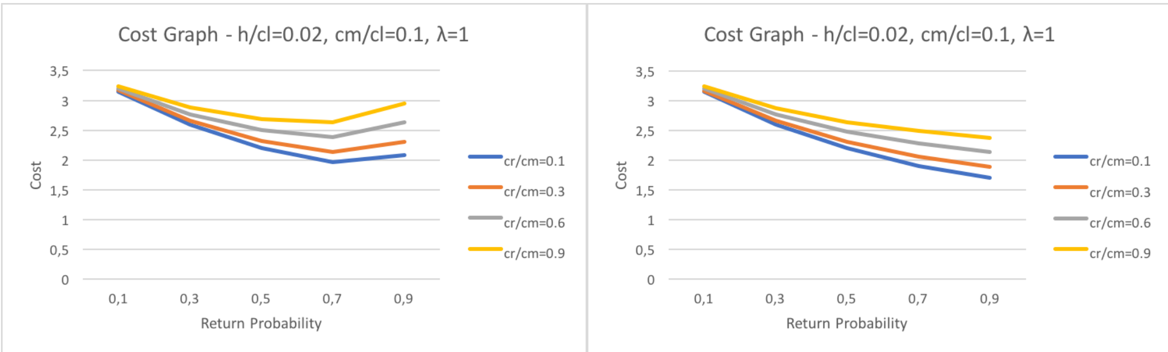

Figure 4.18. Optimal cost graphs of models with only production control (left) and with both production and admission controls (right) $h/c_L = 0.02$, $p = (0.1, ..0.9)$, $\mu = (0.002, ..0.018)$, $\lambda = 1$, $\theta = 1$, and $h = 1$

The effect of admission control can also be observed in Figure 4.19 for different λ values while comparing the costs of both models with only production control and with both production and admission control against to cost of pure production model, respectively. Since the admission control allows cost to decrease with increasing return probability, cost difference of model with both production and admission control with pure production model keeps increasing as in the graph on the right.

The marginal inventory distribution graphs in Figure 4.20 reveal that the control policy keeps the same inventory level in both one control model and two-control model. However, it is seen that the probability of having 10 or more products in the inventory is equal to zero for the model with admission control since holding more than that amount of product in the inventory is unnecessary. For the experiments used in Figure 4.20 probability of lost sales, $P(x_I = 0)$, are 0.04725 and 0.04766 in one control and two-control models, respectively. It is seen that there is no significant change in lost sales probability since admission control both ensures to keep enough inventory and to keep holding and remanufacturing costs as low as possible by rejecting unnecessary returned products.

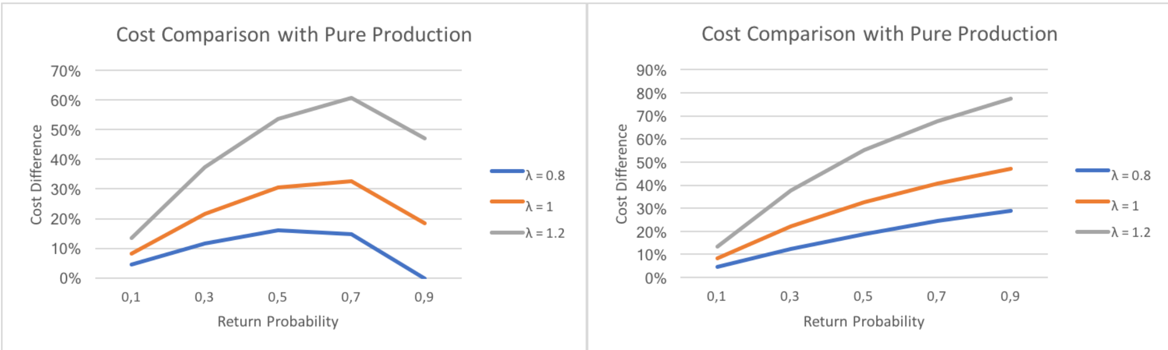

Figure 4.19. Cost comparisons of pure production model both with optimal one-control model (left), and two-control model (right), $h/c_L = 0.02$, $p = (0.1, ..0.9)$, $\mu = (0.002, ..0.018)$, $c_R/c_M = 0.9$, $\lambda = (0.8, 1, 1.2)$, $\theta = 1$, and $h = 1$

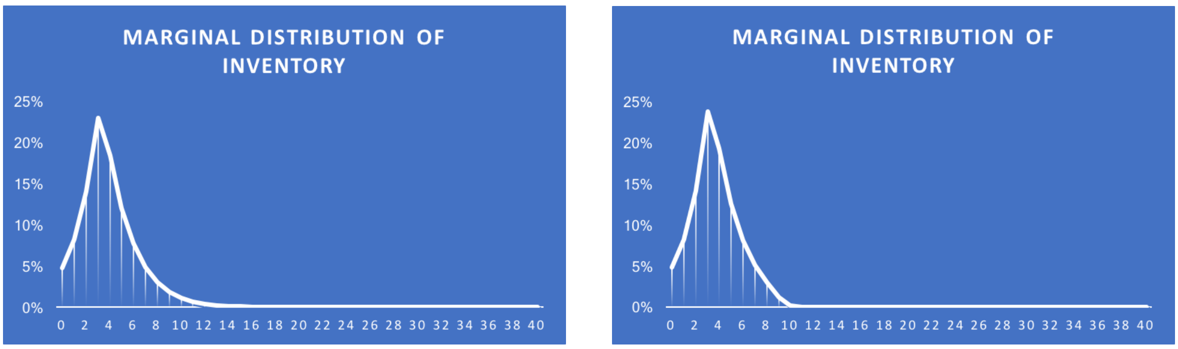

Figure 4.20. Marginal inventory distributions of one and two control models, $c_L = 50$, return probability $p = 0.7$, $c_M/c_L = 0.5$, $c_R/c_M = 0.6$, $\mu = 0.014$, $\gamma = 0.006$, $\lambda = 1$, $\theta = 1$ and $h = 1$

It is also important to examine how the magnitude of the effect of admission control changes with different parameter sets. In the Figure 4.21, 6 graphs in which the percentage cost differences of model with production control from model with both production and admission control are plotted according to their c_M/c_R ratio with the increasing return probability in the x-axis. The graphs in the first row belong to the experiments with $c_L/c_M = 0.1$ and the ones in the second row belong to the experiments with $c_L/c_M = 0.5$. Each column includes the experiments with same λ parameters. It is observed that when c_M/c_L is low where both manufacturing and remanufacturing costs are also low comparing to cost of lost sales, the cost difference between two models are not sensitive to c_R/c_M ratios. On the other hand, when c_M/c_L is higher, the effect of c_R/c_M ratio becomes more noticeable. Especially the experiments with $c_R/c_M = 0.9$ have a little higher cost difference and slope of the cost difference curve increases at a smaller return probability since unnecessary returned products cause a higher cost of remanufacturing with higher c_R in the model with only production decision. Furthermore, the average cost difference of two models decreases significantly with a higher c_M/c_L ratio and a higher λ. As it is mentioned earlier, the increase in c_M/c_L ratio makes remanufacturing more valuable and increase in returned products helps in satisfying demand more economically in that case. Since the main reason of cost difference between these two models is the costs of unnecessary returns, this cost difference decreases when the amount of returned products which are considered unnecessary decreases as c_M/c_L and λ increases.

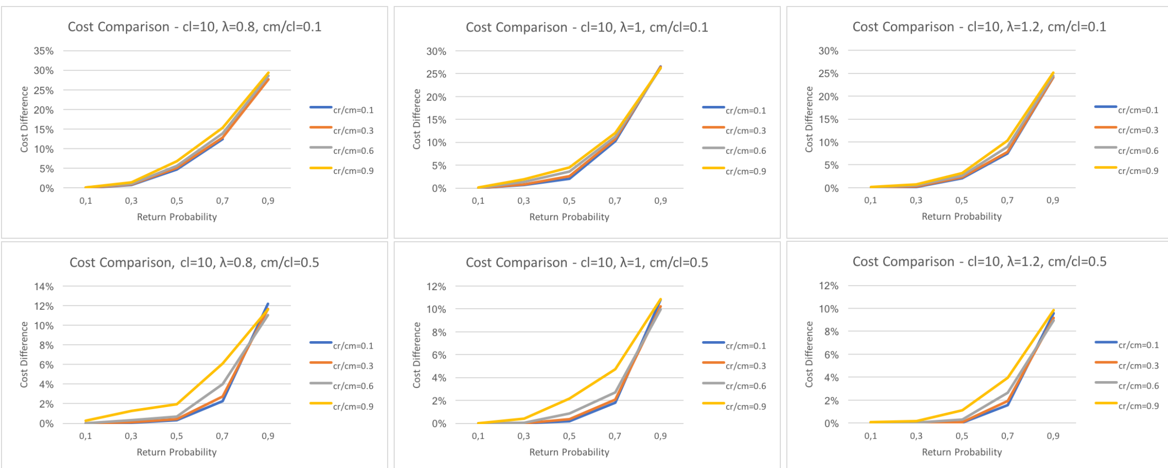

Figure 4.21. Cost difference of model with production control from model with both production and admission controls, $c_L = 10$, $c_M/c_L = (0.1, 0.5)$, $c_R/c_M = (0.1, 0.3, 0.6, 0.9)$, $p = (0.1, ..0.9)$, $\mu = (0.002, ..0.018)$, $\lambda = (0.8, 1, 1.2)$, $\theta = 1$, and $h = 1$

In the Figure 4.22, the effect of c_L on cost differences can be observed by comparing graphs within different rows and the effect of λ can be observed by comparing graphs in different columns. It is seen that cost difference of two models decreases when c_L increases. As it is mentioned earlier, when c_L increases, control policy decides to hold more finished goods in the inventory and satisfies a higher percentage of demand because cost of a lost sale is high. Therefore, the value of remanufacturing increases as in the previous case and cost difference of two models decreases.

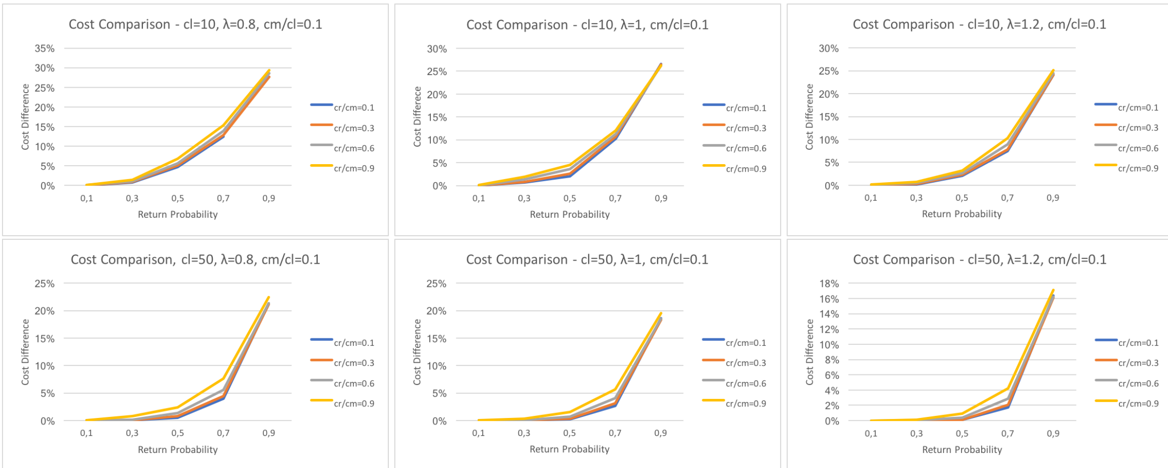

Figure 4.22. Cost comparison of models with production control and with both production and admission controls, $c_L = (10, 50)$, $c_M/c_L = 0.1$, $c_R/c_M = (0.1, 0.3, 0.6, 0.9)$, $p = (0.1, ..0.9)$, $\mu = (0.002, ..0.018)$, $\lambda = (0.8, 1, 1.2)$, $\theta = 1$, and $h = 1$

In both of the Figures 4.21 and 4.22, the cost difference increases significantly with the increase of return probability, as mentioned before. As a summary, where the c_L, c_M/c_L and λ are low, and the return probability and c_R/c_M is high, the benefit of using admission control is higher. Especially when the return probability gets higher, admission control becomes more necessary. On the contrary, in the previous section it was observed that with the increase in return probability, the higher amount of returned products allowed higher amount of cost improvement when c_L, c_M/c_L and λ are high and c_R/c_M is low. Therefore it can be concluded that value of admission control decreases with the increasing importance of returned products and increases otherwise.

4.3. Cost of Misjudging the Return Rate

As it is discussed in the previous sections, using return flows are beneficial to decrease the cost of the system significantly. However, the return flow is usually considered by taking a fixed return estimate since the orbit is at best partially observable in real life. In this case, a fixed base stock control policy would be applied to the system where the actual return flow is dependent to orbit size. Therefore, it is expected to create a cost difference with optimal control policy and it is important to evaluate this cost difference especially when estimated return rate is misjudged. Here, the return rate estimate is obtained from the optimal LP model with orbit dependent return rate as $E[\mu] = \bar{\mu} = E[x_0]\mu$. This is considered as best estimate of return rate for a base stock policy. Then, three different cases for overestimating and underestimating are also considered by multiplying return estimate $\bar{\mu}$ by 0.5, 0.8 and 1.2. The base stock levels which are obtained by using these four return estimates in a fixed return estimate model are forced as a base stock control policy in the model with orbit dependent return flow which leads to a suboptimal solution.

In Figure 4.23, 6 graphs in which the percentage cost differences of model which assumes fixed return rate where returns are actually orbit dependent from the optimal model with production control are plotted according to different return estimates as $0.5\bar{\mu}$, $0.8\bar{\mu}$, $\bar{\mu}$ and $1.2\bar{\mu}$ with the increasing return probability in the x-axis. The graphs in the first row belong to the experiments with $c_R/c_M = 0.1$ and the ones in the second row belong to the experiments with $c_R/c_M = 0.6$. Each column includes the experiments with same λ parameters.

Figure 4.23. Cost difference of misjudging the return rate where $c_L = 50$, $p = (0.1, ..0.9)$, $\hat{\mu} = \bar{\mu}(0.5, 0.8, 1, 1.2)$, $\lambda = (0.8, 1, 1.2)$, $c_M/c_L = 0.1$, $c_R/c_M = (0.1, 0.6)$, $\theta = 1$, and $h = 1$

It can be observed from Figure 4.23 that using a fixed return estimate as $\bar{\mu}$ for a base stock policy causes at most 5% cost difference with optimal control model which has only production control. However, overestimating and underestimating the return rate increases the cost significantly with increasing return probability. By comparing the cost differences in case of taking return rate estimate as $0.5\bar{\mu}$ and $0.8\bar{\mu}$ it can be observed that the cost difference increases with the increasing absolute difference of average return rate obtained from LP model considering orbit size and return rate relationship, and estimated return rate. In the Table 4.6, the base stock levels which are obtained by using four return estimates which are $\bar{\mu}$, $0.5\bar{\mu}$, $0.8\bar{\mu}$ and $1.2\bar{\mu}$ in a fixed return estimate model are given with increasing p and c_R/c_M values. Here, it is seen that the specified base stock levels of model with $0.5\bar{\mu}$ as return estimate are always higher than the base stock levels of model with $\bar{\mu}$ as return estimate unlike the base stock levels of models with $0.8\bar{\mu}$ and $1.2\bar{\mu}$ as return estimates. It also helps to explain the higher cost difference of the case in which return rate is misjudged as $0.5\bar{\mu}$. Furthermore, the cost differences of the experiments with return probability as 0.7 and 0.9 among the experiments in Table 4.6 are given in Table 4.7 that represents the highest cost differences belong to model with $0.5\bar{\mu}$ as return estimate.

It is also observed from Figure 4.23 that cost difference is negligible when return probability is as low as 0.1 but increases with increasing return probability. Since the returns are actually orbit dependent and the orbit information becomes more valuable as return probability increases, the cost difference of model which assumes fixed return rate from the optimal model increases with increasing return probability as it is observed in Table 4.7. Cost difference of model which assumes fixed return rate as $\bar{\mu}$ increases up to 21% as return probability increases. The biggest cost difference is obtained where return probability, p, is 0.9, $c_L = 10$, $c_M/c_L = 0.1$ and $\lambda \leq \theta$.

In the Figure 4.23 it is also observed that when return probability is higher, especially after 0.5, the cost differences decreases as c_R/c_M ratio increases. Since the base stock levels obtained from fixed return rate model are not sensitive to the increase in c_R/c_M ratio as can be seen in Table 4.6, cost of model with orbit dependent return rate in which these base stock levels are forced as a base stock control policy does not

Table 4.6. Base stock levels obtained from fixed return estimate model, $c_L = 50$, $c_M/c_L = 0.1$, $\lambda = 1$, $\theta = 1$ and $h = 1$

c_R/c_M	p	$\bar{\mu}$	$0.5\bar{\mu}$	$0.8\bar{\mu}$	$1.2\bar{\mu}$
0.1	0.1	6	7	7	6
0.3	0.1	6	7	7	6
0.6	0.1	6	7	7	6
0.9	0.1	6	7	7	6
0.1	0.3	5	6	5	4
0.3	0.3	5	6	5	4
0.6	0.3	5	6	5	4
0.9	0.3	5	6	5	4
0.1	0.5	4	5	4	3
0.3	0.5	4	5	4	3
0.6	0.5	4	5	4	3
0.9	0.5	4	5	4	3
0.1	0.7	3	4	3	2
0.3	0.7	3	4	3	2
0.6	0.7	3	4	3	2
0.9	0.7	3	4	3	2
0.1	0.9	1	4	3	0
0.3	0.9	1	4	3	0
0.6	0.9	1	4	3	0
0.9	0.9	1	4	3	0

Table 4.7. Percentage cost difference of model which assumes fixed return rate where returns are actually orbit dependent from the optimal model with production control, $c_L = 50$, $c_M/c_L = 0.1$, $\lambda = 1$, $\theta = 1$ and $h = 1$

p	c_R/c_M	$\bar{\mu}$	$0.5\bar{\mu}$	$0.8\bar{\mu}$	$1.2\bar{\mu}$
0.7	0.1	0.70%	5.68%	0.70%	2.77%
0.7	0.3	0.69%	5.37%	0.68%	2.40%
0.7	0.6	0.68%	5.00%	0.68%	1.94%
0.7	0.9	0.68%	4.70%	0.68%	1.58%
0.9	0.1	1.66%	27.47%	16.60%	13.19%
0.9	0.3	1.53%	25.20%	15.31%	11.41%
0.9	0.6	1.38%	22.50%	13.75%	9.28%
0.9	0.9	1.28%	20.42%	12.63%	7.65%

change with c_R/c_M ratio. On the other hand, the cost of model with optimal control policy increases with increasing c_R/c_M ratio and therefore, the cost differences of these two models decrease as c_R/c_M increases. Even though the decrease in the model with return estimated as $\bar{\mu}$ is not significant, the cost of overestimating and underestimating decreases more noticeable as in the Table 4.7. Moreover, it has been discussed that the behavior of switching curve of optimal control model becomes increasingly similar to a base stock control policy as c_R/c_M increases. This might be interpreted as when the behavior of switching curve in an optimal control model gets similar with a base stock control policy, the cost differences decrease.

In the Figure 4.24, 2 graphs in which the percentage cost differences of model which assumes fixed return rate where returns are actually orbit dependent from the optimal model with production control are plotted according to different return rate estimates as $0.5\bar{\mu}$, $0.8\bar{\mu}$, $\bar{\mu}$ and $1.2\bar{\mu}$ with the increasing return probability in the x-axis. In the graph on the left $c_M/c_L = 0.1$ and graph on the right $c_M/c_L = 0.5$. It is observed that, when c_M/c_L ratio increases the cost of overestimating becomes slightly more than underestimating even though average cost differences decrease. The reason of the decrease in cost differences at every return probability is that when c_M/c_L increases, switching curve behavior gets similar to the behavior in a fixed return rate model as in previous case. However, since the value of return flow increases as c_M/c_L increases as it is discussed earlier, having less return than the expected return increases total cost significantly in case of overestimating.

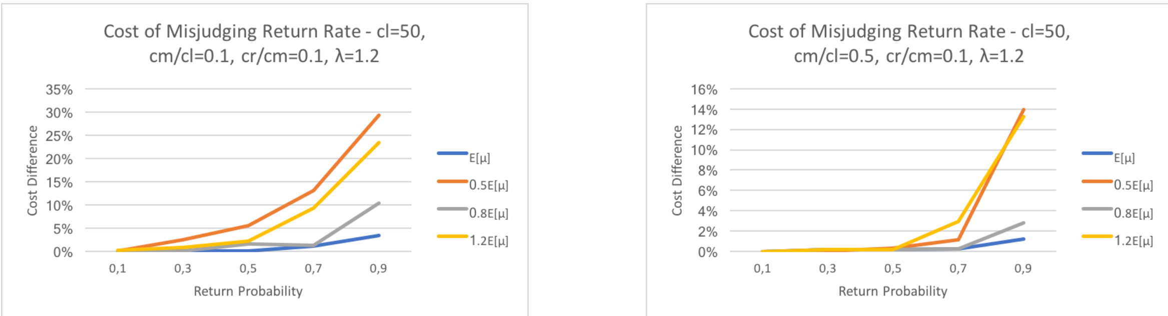

Figure 4.24. Cost difference of misjudging return rate where $c_L = 50$, $p = (0.1, ..0.9)$, $\hat{\mu} = \bar{\mu}(0.5, 0.8, 1, 1.2)$, $\lambda = 1.2$, $c_M/c_L = (0.1, 0.5)$, $c_R/c_M = 0.1$, $\theta = 1$, and $h = 1$

In the Figure 4.25, 4 graphs in which the percentage cost differences of model which assumes fixed return rate where returns are actually orbit dependent from the optimal model with production control are plotted according to different return rate estimates as $0.5\bar{\mu}$, $0.8\bar{\mu}$, $\bar{\mu}$ and $1.2\bar{\mu}$ with the increasing return probability in the x-axis. The graphs in the first row belong to the experiments with $c_R/c_M = 0.1$ and the ones in the second row belong to the experiments with $c_R/c_M = 0.3$. Each column includes the experiments with same λ parameters where λ is equal to 1 and 1.2, respectively. Here, it is aimed to show that since the production level decreases when c_M/c_L is high, overestimating of return rate causes to decrease the production level to even zero when return probability is as high as 0.9, $\lambda \leq \theta$ and c_R/c_M is small. Therefore, the cost difference of overestimating grows very much as can be seen in the graphs on the first column. It is an extreme case but it is important to evaluate the impact of c_M/c_L ratio. However, when $\lambda > \theta$ as in the graphs on the right, control policy decides to produce to satisfy increased demand. Therefore, cost difference of overestimating decreases and it even becomes less than the cost difference caused by taking a fixed estimate return rate as $0.5\bar{\mu}$ while c_R/c_M increases.

Figure 4.25. Cost difference of misjudging return rate, $c_L = 50$, $p = (0.1, ..0.9)$, $\hat{\mu} = \bar{\mu}(0.5, 0.8, 1, 1.2)$, $\lambda = (1, 1.2)$, $c_M/c_L = (0.1, 0.5)$, $c_R/c_M = (0.1, 0.3)$, $\theta = 1$, and $h = 1$

Comparing the optimal control policy with both production and admission controls exhibits the same behaviors. In the Figure 4.26, cost differences with optimal decision policy with both production and admission control is given, respectively. It is observed that overestimating and underestimating the return rate increases the cost significantly with increasing return probability and also average cost difference decreases when c_R/c_M ratio increases as in the Figure 4.23. However, comparing the Figure 4.23, the cost differences are higher because of the additional benefits of admission control in cost reduction.

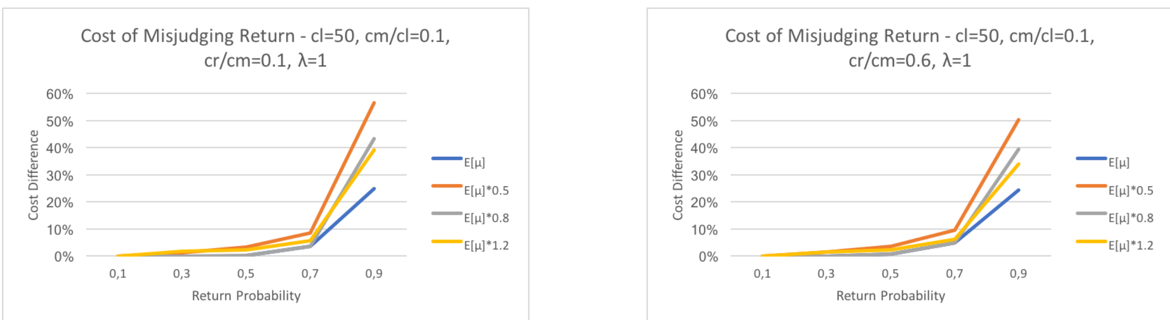

Figure 4.26. Cost difference of misjudging return rate, $c_L = 50$, $p = (0.1, ..0.9)$, $\hat{\mu} = \bar{\mu} * (0.5, 0.8, 1, 1.2)$, $\lambda = 1$, $cm/cl = 0.1$, $cr/cm = (0.1, 0.6)$, $\theta = 1$, and $h = 1$

5. CONCLUSION

In this study, a testbed for measuring the impact of the number of products in use (orbit) on the production control of hybrid production systems is introduced. It is shown that base stock levels change with the number of products in use and there is a step-wise production policy. Results indicate that the control policy is dependent on the orbit size and return probabilities. When the expected return rate increases because of increase in the expected number of products in use or in return probability, control policy updates base stock levels more frequently. The increase in λ, c_L and p results in an increase in the expected return rate and a step-wise production policy is observed where base stock levels are updated frequently. On the other hand, increase in c_M/c_L and c_R/c_M causes return rate to decrease because of decreasing the orbit size which leads a less frequent base stock level updating and production decision to converge to a policy similar to a base stock type policy.

The numerical study indicates that using return flows to supply the demand generates up to 80 % cost reduction. The value of using return flows gets higher as c_L, c_M/c_L and λ get higher and as c_R/c_M gets lower. Even though it is mostly beneficial, return flows might cause system cost to increase especially where the cost of remanufacturing is very close to the cost of manufacturing and where the amount of returned products is more than necessary like in the cases with high return probability p, $\lambda \leq \theta$ and a small c_M/c_L ratio. Therefore, having an admission control on returned products is useful to take advantage of the return flow in all circumstances.

The numerical study also indicates that introducing an additional control for admissions of returned products generates an additional 30% cost reduction. However, unlike the value of using return flows, the value of using admission control decreases as c_L, c_M/c_L and λ get higher, and as c_R/c_M gets lower because of the increase in benefits of additional returned products. Yet it is usually necessary to lower the cost of the system especially when return probability, p, is high.

Finally, it is shown that both overestimating and underestimating the return rate increase the cost significantly where orbit information has a higher importance as in cases with higher return probability. Similarly, the cost of misjudging the return rate decreases when the switching curve behavior gets similar to fixed return rate model as in cases with higher c_M/c_L and c_R/c_M ratios.

This project enabled us to thoroughly describe the behavior of the optimal decision for hybrid production systems. As future study, we aim to expand the proposed model to include remanufacturing decision and the remanufacturing lead time, and to develop new control mechanisms customized for hybrid systems. Also we have generated a testbed based on the LP model of a stochastic problem. Using the new computational capabilities developed for LP based optimization we aim to expand both our state space definition and decision parameters both to test previously proposed control mechanisms and to advance our understanding on the problem.

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APPENDIX A: STATE TRANSITION DIAGRAMS-PRODUCTION CONTROL

Here, the state transition diagrams of the system which allows only production control are given.

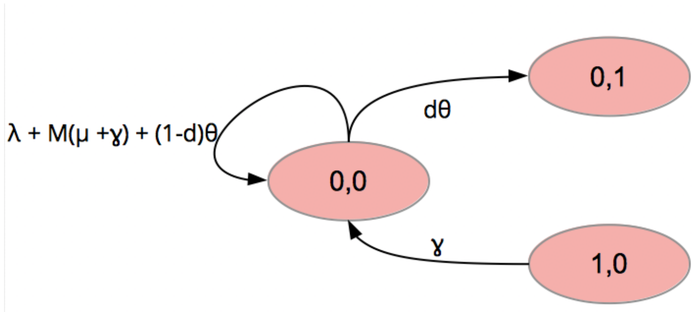

Figure A.1. State transition diagram of state $x = (0, 0)$

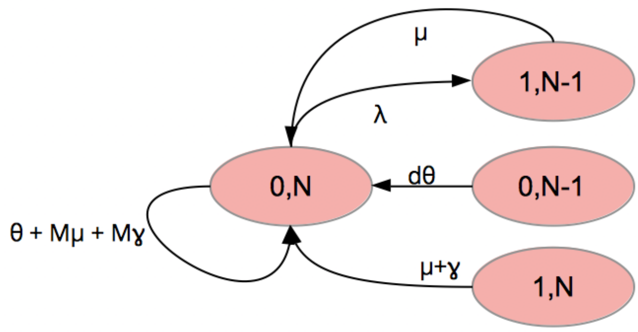

Figure A.2. State transition diagram of state $x = (0, N)$

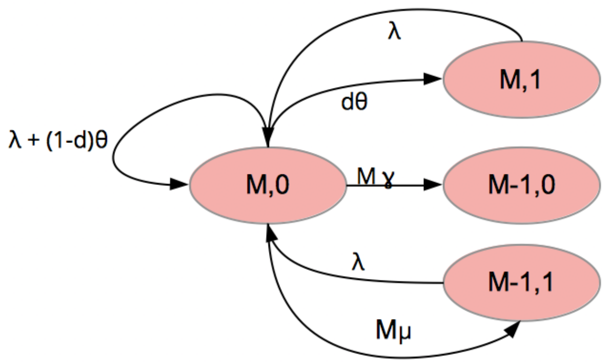

Figure A.3. State transition diagram of state $x = (M, 0)$

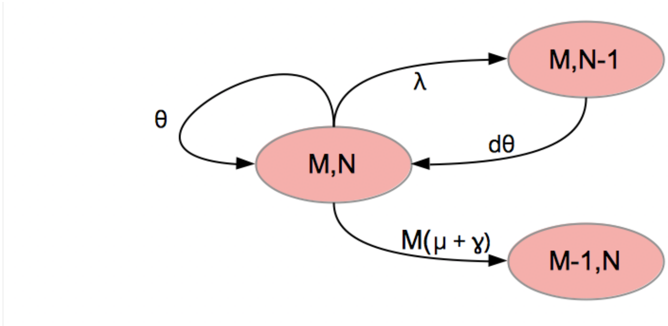

Figure A.4. State transition diagram of state $x = (M, N)$

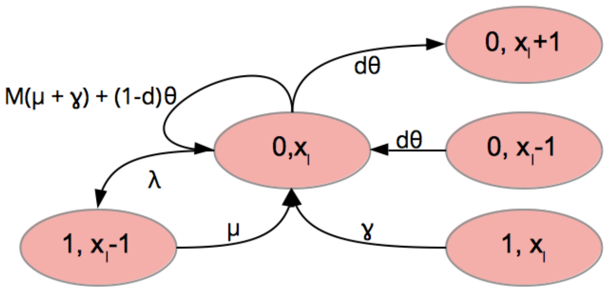

Figure A.5. State transition diagram of state $x = (0, x_I)$, $0 < x_I < N$

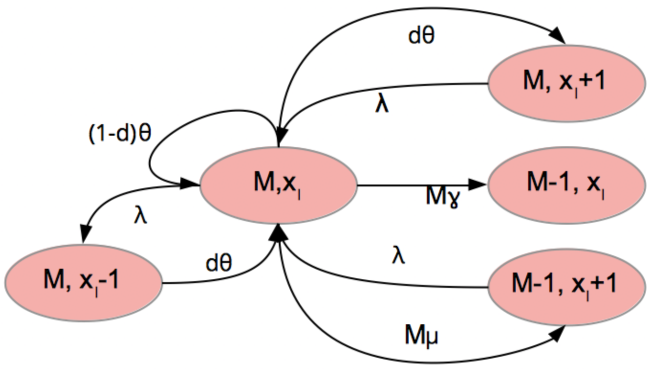

Figure A.6. State transition diagram of state $x = (M, x_I)$, $0 < x_I < N$

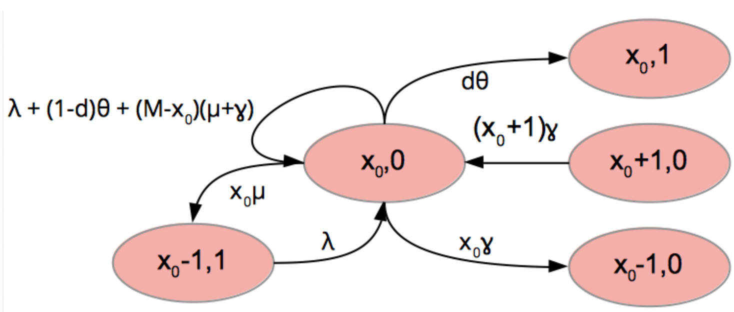

Figure A.7. State transition diagram of state $x = (x_0, 0)$, $0 < x_0 < M$

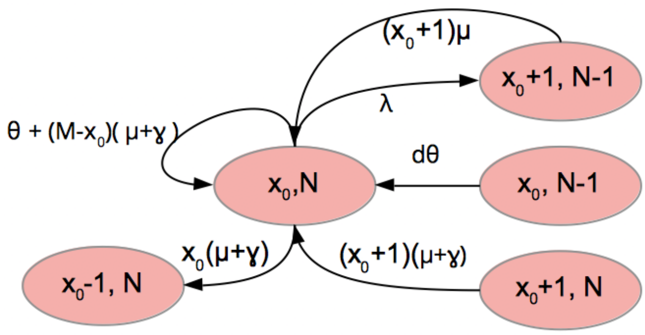

Figure A.8. State transition diagram of state $x = (x_0, N)$, $0 < x_0 < M$

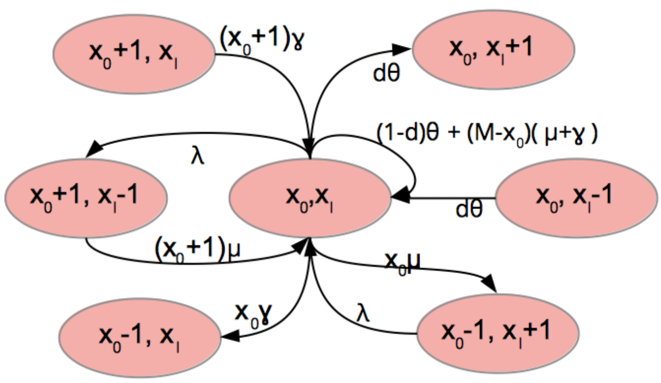

Figure A.9. State transition diagram of state $x = (x_0, x_I)$, $0 < x_0 < M$ and $0 < x_I < N$

APPENDIX B: STATE TRANSITION DIAGRAMS-PRODUCTION AND ADMISSION CONTROL

Here, the state transition diagrams of the system which allows both production and admission control are given.

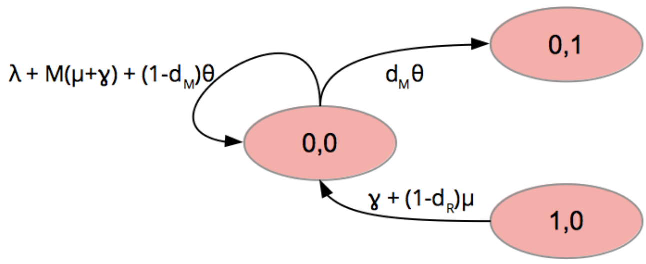

Figure B.1. State transition diagram of state $x = (0, 0)$

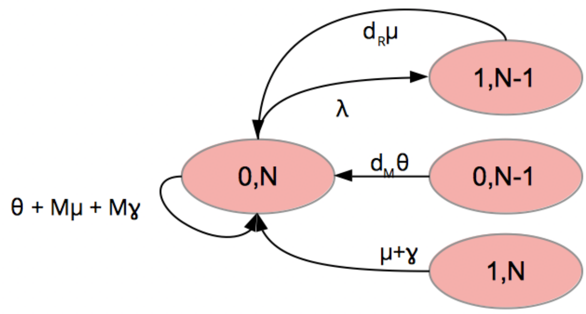

Figure B.2. State transition diagram of state $x = (0, N)$

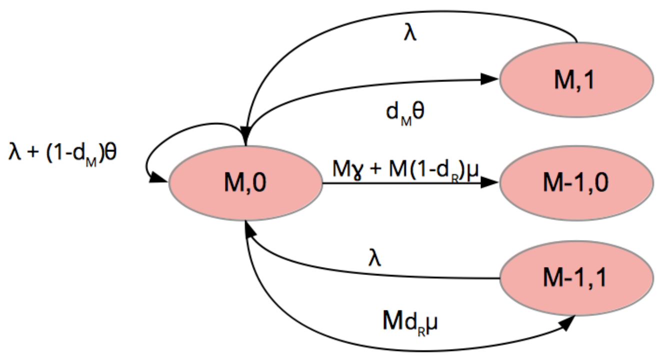

Figure B.3. State transition diagram of state $x = (M, 0)$

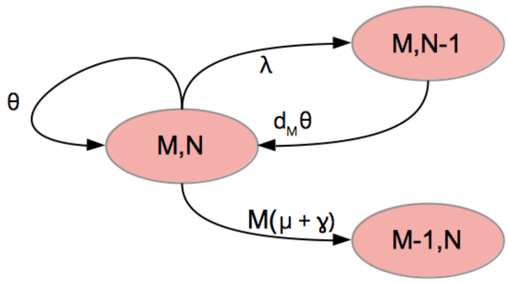

Figure B.4. State transition diagram of state $x = (M, N)$

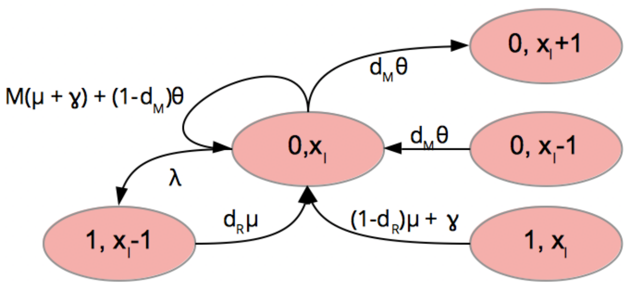

Figure B.5. State transition diagram of state $x = (0, x_I)$, $0 < x_I < N$

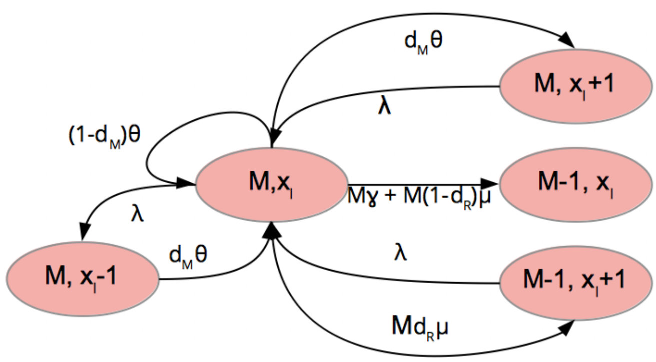

Figure B.6. State transition diagram of state $x = (M, x_I)$, $0 < x_I < N$

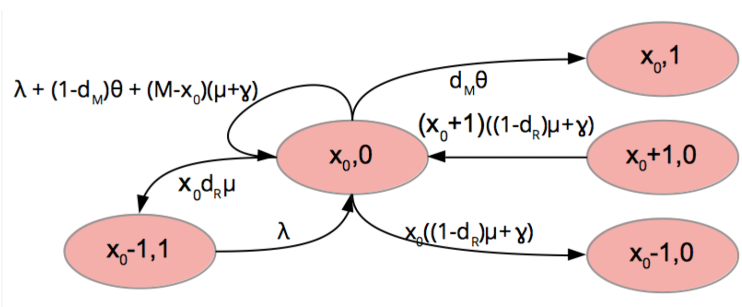

Figure B.7. State transition diagram of state $x = (x_0, 0)$, $0 < x_0 < M$

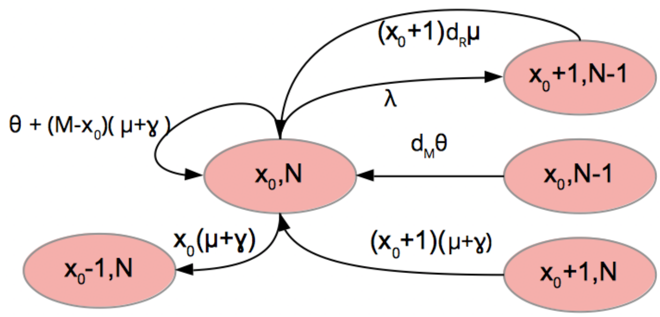

Figure B.8. State transition diagram of state $x = (x_0, N)$, $0 < x_0 < M$

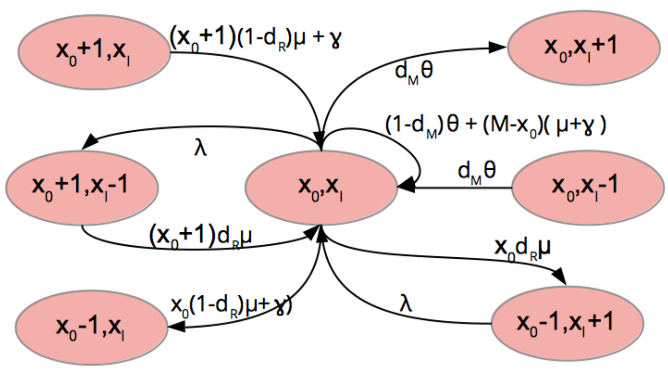

Figure B.9. State transition diagram of state $x = (x_0, x_I)$, $0 < x_0 < M$ and $0 < x_I < N$