

CREDIT RISK MODELLING AND QUANTIFICATION

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To my grandpa

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ABSTRACT

CREDIT RISK MODELLING AND QUANTIFICATION

Credit risk modelling and quantification is a very crucial issue in bank management and has become more popular among practitioners and academicians in recent years because of the changes and developments in banking and financial systems. CreditMetrics of J.P. Morgan, KMV Portfolio Manager, CreditRisk+ of Credit Suisse First Boston, and McKinsey's CreditPortfolioView are widely used frameworks in practice. Thus, this thesis focuses on these models rather than statistical modelling most academic publications are based on. Nevertheless, we find that there are several links between the models used in practice, the statistical models, and the regulatory frameworks such as Basel II. Moreover, we explore the basics of the Basel II capital accord, the principles of four frameworks and the calibration methods available in the literature. As a result of this study, it seems possible to apply the credit risk frameworks used in practice to estimate the parameters of the Basel II framework. Also, we develop a regression method to calibrate the multifactor model of CreditMetrics and determine the necessary steps for this implementation. Although, due to incomplete information provided in the literature on the calibration of these models and due to lack of data, it may not be easy to implement the models used in practice, in this thesis we calibrate the multifactor model of CreditMetrics to accessible real data by our regression based calibration method. The small credit portfolio formed by real-world data taken from *Bloomberg Data Services* is also presented within this thesis. Next, we artificially generate a large multifactor model for a large credit portfolio taking our real-world portfolio as a reference in order to inspect the loss and value distributions of a realistically large credit portfolio. Finally, by analyzing the results of multi-year Monte Carlo simulations on different portfolios of different rating concentrations, we deduce the significance of the effects of transition risk and portfolio concentration on risk-return profile, and the strength of simulation in assessing the credit spread policy. Throughout this thesis *R-Software* environment has been used for all kind of computations.

ÖZET

KREDİ RİSKİ MODELLEMESİ VE ÖLÇÜMÜ

Kredi modellemesi ve ölçümü banka yönetiminde oldukça önemli bir konudur ve son yıllarda bankacılık ve finansal sistemlerdeki değişimler ve gelişmelerden ötürü uygulamada ve akademisyenler arasında daha da popüler bir hale gelmiştir. J.P. Morgan'ın CreditMetrics, KMV Portfolio Manager, Credit Suisse First Boston'ın CreditRisk+ ve McKinsey'nin CreditPortfolioView modelleri uygulamada en çok kullanılan yaklaşımlardır. Bundan ötürü, bu çalışma, birçok akademik yayının baz aldığı istatistiksel yaklaşımlardan daha çok uygulamada kullanılan bu modeller kredi riski modellemesi üzerine üzerine yoğunlaşmaktadır. Fakat, istatistiksel modeller, uygulamada kullanılan yaklaşımlar ve Basel II gibi düzenleyici çerçeveler arasında birçok bağ olduğu sonucuna varılmıştır. Bu tezde Basel II sermaye düzenlemesinin temelleri ve bahsedilen dört uygulanabilir modelin ana ilkeleri ve bu modeller için literatürde mevcut olan kalibrasyon yöntemleri araştırılmıştır. Bu çalışmanın bir sonucu olarak, görülmeyen uygulamada kullanılan bu modeller, Basel II düzenlemesinin parametrelerini hesaplamada kullanılabilirler. Ayrıca, bu tezde, CreditMetrics çok faktörlü modelinin kalibrasyonu için, sezgisel olarak bir regresyon metodu geliştirmekteyiz ve bu metodun uygulamasındaki gerekli adımları belirlemekteyiz. Her ne kadar bu modellerin kalibrasyonu üzerine literatürde eksik sunulan bilgi ve veri yeterizliği nedeniyle, bu modelleri uygulamak çok kolay olmasa da, bu tezde sunulan regresyon tabanlı metod ile, CreditMetrics çok faktörlü modelini, elde edilebilir gerçek veriye kalibre etmekteyiz. Tez içerisinde *Bloomberg Veri Hizmetleri*'nden sağlanan gerçek veriyle oluşturulan küçük ölçekli kredi portföyü de sunulmaktadır. Ayrıca gerçeğe uygun geniş bir kredi portföyünün kayıp ve değer dağılımlarını inceleyebilmek için, bu portföyü baz alarak, geniş bir kredi portföyüne yönelik, yapay olarak geniş bir çok faktörlü model ürettik. Son olarak, farklı rating konsantrasyonlarındaki farklı portföyler üzerinde gerçekleştirdiğimiz çok seneli Monte Carlo simülasyonlarının sonuçlarını analiz ederek, rating geçiş riskinin ve portföy konsantrasyonunun risk-değer profili üzerinde önemli derecede etkili olduğu ve kredi faiz

uygulamalarını deęerlendirmede simulasyonun gçlü bir araç olduęu sonucuna varmaktayız. Bu tez süresince tüm hesaplamalarda *R*-Programlama dilinden yararlanılmıştır.

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LIST OF SYMBOLS/ABBREVIATIONS

α	Dependent systematic factor loading
$\bar{\alpha}$	Independent systematic factor loading
$\hat{\alpha}$	Normalized factor loading
Δ^E	Equity Delta
Σ	Variance-covariance matrix of systematic factors
Σ_e	Variance-covariance matrix of disturbances
$\Sigma_{\epsilon, e}, \Sigma_{e, \epsilon}$	Cross-correlation matrices for error terms
Φ	Cumulative standard normal distribution function
Φ^{-1}	Inverse cdf of standard normal distribution
ϕDP	Long-term default probability average for a speculative grade firm
ϕM	Rating-transition matrix to be adjusted periodically
ϵ_k	Idiosyncratic risk of the k^{th} obligor
α	Shape parameter of a beta or gamma distribution
β	Shape parameter of a beta distribution / scale parameter of a gamma distribution / regression coefficient
θ^E	Equity Theta
Γ	Gamma function
Γ^E	Equity Gamma
I_n	Industry return of the n^{th} industry
C_m	Country return of the m^{th} country
\mathbb{L}	Cholesky decomposition of a variance-covariance matrix
ℓ	Likelihood function
l_j	Expected loss of the j^{th} portfolio segment
ϵ	Idiosyncratic risk factor
ξ_{t+1}	Error term vector
$e_{k,t+1}$	Error terms
θ	Sector allocation vector
θ_{Ak}	Percent allocation for obligor A to sector k

$\hat{\mu}_A$	Average default intensity of obligor A
μ_A	Asset drift
μ_k	Mean default rate of portfolio segment k
μ_M	Mean return of the market portfolio
π	Market risk premium
ρ	Asset correlation
$\rho_{A,M}$	Correlation between asset value and the market portfolio
ρ_{ij}	Correlation between the i^{th} and the j^{th} obligor
ρ_{jl}	Correlation between systematic factors j and l
σ	Deviation of the total portfolio loss
σ_M	Volatility of the market portfolio
σ_A	Asset volatility
$\hat{\sigma}_A$	Standard deviation of the default event indicator for obligor A
$\acute{\sigma}_A$	Standard deviation of the default intensity of obligor A
σ_E	Volatility of equity
σ_j	Standard deviation of time series of the j^{th} factor / Standard deviation of the j^{th} portfolio segment's default rate
σ_k	Standard deviation of the k^{th} obligor's return
r_t	Log-return of asset value after t units of time
\bar{r}_t	Normalized log-return of asset value after t units of time
Z	Systematic factor / Standard normal variate
z_{kl}	Threshold of a k -rated obligor for a transition to rating l
A-IRB	Advanced IRB
ARIMA	Auto Regressive Integrated Moving Average
ARMA	Auto Regressive Moving Average
BCBS	Basel Committee on Banking Supervision
BIS	Bank for International Settlements
CAPM	Capital Asset Pricing Model
CEL	Conditional expected loss
CPV	CreditPortfolioView
CSFB	Credit Suisse First Boston

CF	Composite factor
DD	Distance-to-default
DL	Default loss
DP (PD)	Default probability
EAD	Exposure-at-default
EDF	Expected Default Frequency
ES	Expected Shortfall
F-IRB	Foundation IRB
IRB	Internal ratings-based
LGD	Loss given default
LR	Loss rate
MtM	Mark-to-market
RWA	Risk-weighted assets
S&P	Standard & Poor
SR	Sharpe ratio
UL	Unexpected loss
VaR	Value-at-risk

1. INTRODUCTION

Financial crises, such as mortgage crisis that popped up in recent years, unexpected or foretold company defaults, increasing number of new markets, developments in credit markets during the last few years resulting in more complex risk structures, and emerging regulations based on standardized approaches like Basel II Accord developed in Bank for International Settlements (BIS) force the financial institutions to evaluate and thus manage their credit risk more adequately and cautiously. For instance, banks try to diversify or reduce the risk of their credit portfolios by trying to distribute their concentration over several different regions, rating groups, and industries, and by developing and utilizing various credit products such as credit risk derivatives. Also, in Turkey, we have been witnessing the same picture. This year, in 2009, several international banks in Turkey requested extra capital from their central reserves in Europe in order to be able to give credit to firms or individuals because their liquidity would not be enough to cover their risk in case they decided to lend money as a credit. The banks' representatives that we interviewed during the completion of this thesis affirmed that they would start to implement Basel II Framework in 2009 or at the very latest 2010. This means the risk structure of the banking and financial industry is changing, and the banks try to keep up with these changes in order to manage their risk more adequately and efficiently.

Credit risk is the risk of unexpected losses due to defaults of obligors or downturns in market or internal conditions that decrease the credit worthiness of an obligor. For example, when an obligor's rating is downgraded because of specific conditions, the risk a bank bears by lending money to that obligor increases. In other words, in that case, on the average the bank will lose more than the previous situation. Default occurs when an obligor fails to pay its debt or a part of its exposure. In fact, it need not be a debt obligation. It can be a situation where a party fails to meet its obligation to any other counterparty. Furthermore, there are several different models used in credit risk, and numerous methods used in measuring credit risk within these models. Besides these models explained in literature, a frequently used tool by practitioners to evaluate risk of a credit instrument is Monte Carlo Simulation. Yet, modelling credit risk is not an easy task

since data used by the credit risk modellers are usually confidential and thus not available. Therefore, it is difficult to compare a model to another due to lack of data. In addition, it is hard to implement an algorithm or test a model for validation purposes on real data while most of the published studies on credit risk do not explain in detail how to estimate the required parameters from data. In other words, the calibration of these models is not discussed.

There are two fundamental models used in modelling credit risk; reduced-form models and structural models. These models are also used in pricing defaultable bonds (or risky bonds), such as sovereign or corporate bonds, and pricing credit derivatives, such as credit swaps, credit spread options, and collateralized debt obligations (Schmid, 2004). Furthermore, each model group has various types. Even though, there are models for credit risk, the challenging issue is how to define and integrate default correlations into a model. For example, if the economy is in recession period, then it is more likely that there will be greater number of defaults than usual. Likewise, when a political condition changes affecting a particular industry, then all default probabilities of firms within that industry will change in a similar fashion. Default correlation is essential to capture simultaneous or successive defaults. In defining default correlations, several methods are applied. These methods exploit the fact that credit risk is mainly based on a variety of systematic risk factors as well as the idiosyncratic risk (firm-specific risk) of the individual parties. Systematic risk can be highly related to region, industry, country, business cycle (growth, peak, recession, trough, and recovery), other economic conditions, credit ratings, or to a number of additional risk factors. On the other hand, idiosyncratic risk is a risk arising from the factors such as operational risk, specific to that individual. Besides, reduced-form models try to model default intensity by examining all past information and try to capture correlation between intensities through additional external variables. On the other hand, structural models try to model default and/or rating transition probabilities by defining default and transition boundaries around market value of a corporate, so that it defaults in case its value hits some barrier (Schmid, 2004). Moreover, the credit risk valuation methodologies can be used to measure different statistics. Value-at-risk (VaR), Expected Shortfall (ES), probability of shortfall, and distance-to-default (DD) are among the commonly used measures. However, the effectiveness of each of these measures in characterizing risk is discussed by some practitioners and academicians. For instance, VaR

alone is considered an insufficient measure for understanding credit risk by Duffie and Singleton (2003). Within this thesis, several of these measures will be referred.

The literature on credit risk modelling is very wide and mainly concentrated around three different views. One view is the view of Basel II. Studies based on this view focus on the implications, drawbacks or deficiencies, modifications and implementation of the capital accord, Basel II. However, there is no such study that explicitly gives the statistical model that lies behind Basel II calculations and the mathematical background of its assumptions and constants used within the risk calculations. Yet, there are several references, such as Engelmann and Rauhmeier (2006) and Ong (2005), on how to estimate the Basel II parameters. Another view is the view of the academic side. The academic studies, such as Glasserman and Li (2005) and Kalkbrener *et al.* (2007), examine the statistical modelling of, generally, default loss without any discussion of on what basis they choose to adopt these models or how widely these models are used. Academic studies, which highly concentrate on risk quantification and simulation, randomly select the factors and the other model parameters, base their calculations on an artificial structure instead of a real-world structure achieved from real data, and leave it to the reader to infer or guess how realistic and sensible these models and factors are and to find calibration methods for these models. The final view is the view of practitioners. The questions of interest are; what kind of models banks and financial institutions use to measure and manage their credit risk in the financial world and how they calibrate these models to real data. Practitioners utilize several standard frameworks for credit risk modelling but the calibration of these models is not provided in detail. Moreover, these three views seem to hang in the air without any solid link between them. In other words, it is not precise whether there is a connection between these views. However, for instance, there are studies such as Gürtler *et al.* (2008) and Jacobs (2004) that try to link the credit risk models used in practice to the Basel II framework in several ways. This thesis focuses on the methods and methodologies used by practitioners.

Consequently, one aim of this thesis is to identify the standard frameworks that are used widely in practice for the purpose of credit risk measurement and management, to find methods or tips for the calibration of these models from the literature, and to explore the link between these frameworks and the Basel II capital accord. Another aim is to find a

relevant calibration methodology in order to calibrate and then apply one important credit risk framework among those used widely in practice, and to determine all necessary steps of the implementation for real-world credit portfolios. In this thesis, we also try to comprehend the true nature of a large credit portfolio and assess our interest rate policy by utilizing Monte Carlo simulations. The structure of this thesis is as follows. First, we briefly give the results of several surveys performed among the banks and financial institutions in Turkey, Europe and the United States to find out what the most commonly used frameworks are in practice, and then explain the details of the four most popular frameworks, and the methods we have found in the literature for the calibration of these models. Next, we give a summary of the Basel II framework and the links between this framework and the models used in practice. After Basel II, we explain our calibration method we have employed for the multifactor model of CreditMetrics and give the necessary steps for the implementation. Following this section, we present a small credit portfolio formed with real data obtained from *Bloomberg Data Services* and give the required *R* codes (*R* is a statistical software environment used wide-spread by statisticians) to implement the CreditMetrics' multifactor model for this small portfolio. Then, we give the *R* codes and results of our default loss and mark-to-market simulations¹ of this real-world portfolio together with the inputs we used and our assumptions. In the last chapter, it is explained how we created an artificial portfolio out of our real credit portfolio, and our simulation results on this large credit portfolio are presented hoping these results will reveal the risk profile of a realistic large credit portfolio or at least help us to realize a few benchmarks and assess our rates. Finally, we give a brief discussion of this study and draw our conclusions.

¹ These concepts are explained in the next chapters of this thesis in detail.

2. CREDIT RISK FRAMEWORKS IN PRACTICE

The survey results provided in the literature can be explored to determine the most commonly used frameworks among banks and financial institutions. One survey we could find in the literature is the survey of Fatemi and Fooladi (2006) performed among the 21 top banking firms in the United States. The results of this survey show that most of these firms use or are planning to use the CreditMetrics framework of J. P. Morgan or the Portfolio Manager framework of KMV, and some use the CreditRisk+ model of Credit Suisse First Boston. Another survey whose results are presented by Smithson *et al.* (2002) and which was carried out by Rutter Associates in 2002 among 41 financial institutions around the world reveals that 20 per cent of the institutions that use a credit risk model (85 per cent) use CreditManager, which is the application service based upon CreditMetrics, 69 per cent use Portfolio Manager, and the remaining use their own internal models. Moreover, ECB (2007) states that most central banks use a model based on the CreditMetrics framework. Besides these surveys, there are also surveys performed among Turkish banks. One of these surveys is the survey of Anbar (2006) that was carried out among 20 banks in Turkey in 2005. The results show that only 30 per cent of these banks use a credit risk model or software but 33 per cent of those who use a credit risk model use RiskMetrics, which was developed by J. P. Morgan and is philosophically very similar to CreditMetrics, while the remaining banks use their own models. Another survey, which was carried out by Oktay and Temel (2007) in 2006 among 34 commercial banks in Turkey, shows that most of the banks participated in the survey use Portfolio Manager, CreditMetrics and/or CreditRisk+. Besides these models, the CreditPortfolioView framework developed by McKinsey & Company is another model we frequently run into in the literature. The following subsections explain the methodologies of these models.

2.1. CreditMetrics

In this section, we first give a brief review of the literature over CreditMetrics methodology. Then, we discuss the statistical basis of the model and look into the calibration of the necessary parameters. Finally, the drawbacks of the methodology mentioned in the literature are stated.

2.1.1. The Framework

CreditMetrics framework proposed by J.P. Morgan is one of the portfolio credit value-at-risk models. Gupton *et al.* (1997) give details of CreditMetrics as follows. CreditMetrics essentially utilizes the fact that if asset returns (percent changes in assets value) of a firm, namely an obligor, fall below a certain threshold, then that firm defaults. In fact, CreditMetrics is not a pure-default or default-mode model, meaning a model which accepts loss only in case of a default. It combines the default process with credit migrations which correspond to rating transitions. These kinds of models are called Mark-to-Market Models. So, by applying forward yield curves for each rating group, CreditMetrics is able to estimate credit portfolio value and the unexpected loss of a credit portfolio. In that manner, as Crouhy *et al.* (2000) state that CreditMetrics is an extended version of Merton's option pricing approach to the valuation of a firm's assets since Merton only considers default².

Although CreditMetrics offers the estimation of joint credit quality migration likelihoods as a way of observing the correlation structure, it is not very practical to do so when dealing with extremely large credit portfolios. In addition, it will require a huge data set. For these reasons, it handles the correlations between obligors by introducing a multifactor model. In a multifactor model, a latent variable triggers a change (default or rating transition) in the credit worthiness of a firm. Furthermore, in CreditMetrics framework, the asset return of a firm is used as a latent variable driven both by systematic risk factors, such as country index, industry index and regional index, and by a firm-specific factor (nonsystematic or idiosyncratic risk factor). However, CreditMetrics proposes the use of equity returns to reveal the correlation structure of a credit portfolio instead of asset returns while asset returns are not always observable in the market. Yet, equity correlations are not equal to asset correlations. This assumption does not take into account the firm leverage effect on asset values (Jacobs, 2004). By examining the effect of different systematic factors (for instance, country-industry index) over the changes in

² Merton (1974) assumes that a firm's assets value follow a standard Geometric Brownian Motion (BM) explaining the percent change in assets value by a constant drift parameter and volatility influenced by a standard BM.

equity of a firm and time series data belonging to those systematic factors, one can determine the correlation structure as explained in Section 2.1.3.

Correlations between firms are explained through systematic factors in Creditmetrics. These factors are more like global parameters that may affect the firms, which have direct or indirect connections or relations, concurrently. Non-systematic factors do not contribute any correlation between firms while they are firm specific. Moreover, CreditMetrics framework assumes that normalized asset returns are distributed standard normally and so are systematic and non-systematic risk factors. In Equation (2.1), X_k is the latent variable, normalized asset return of a firm, whereas Z_1, \dots, Z_d are systematic risk factors, and ϵ_k is the idiosyncratic risk of the k^{th} firm. a_{k1}, \dots, a_{kd} and b_k are the factor loadings. It is substantial to affirm that systematic risks cannot be diversified away by portfolio diversification whereas non-systematic risk can. In case of independently distributed systematic risk factors, the factor loading of idiosyncratic risk is chosen as in Equation (2.2), so that X_k can still have a standard normal distribution (sum of independently distributed standard normal variates has still a normal distribution).

$$X_k = a_{k1}Z_1 + a_{k2}Z_2 + \dots + a_{kd}Z_d + b_k\epsilon_k \quad (2.1)$$

$$b_k = \sqrt{1 - (a_{k1}^2 + a_{k2}^2 + \dots + a_{kd}^2)} \quad (2.2)$$

In order to monitor credit migrations, the framework defines the rating thresholds as well as the default threshold for each rating group (Figure 2.1) in such a way that they match the marginal transition probabilities achieved from a transition matrix, which is usually estimated over a long horizon. These transition matrices are assumed to be Markovian and stationary over time. Then, CreditMetrics requires the generation of random values of systematic factors from standard normal distribution (by using Cholesky decomposition in case of dependent factors) to obtain the asset returns of the next simulation period. Therefore, regarding the simulated asset return of a firm from a particular rating group, the rating category of that firm for the next time period can be derived by finding the interval it hits. If we chose to simulate normalized asset returns by calculating joint transition likelihoods, for instance, for a 100- obligor credit portfolio, we

would need to estimate 4950 ($N \times (N-1)/2$) correlations. On the other hand, with an asset model with five factors, we need to estimate only 510 ($d \times (d-1)/2 + d \times N$) correlations.

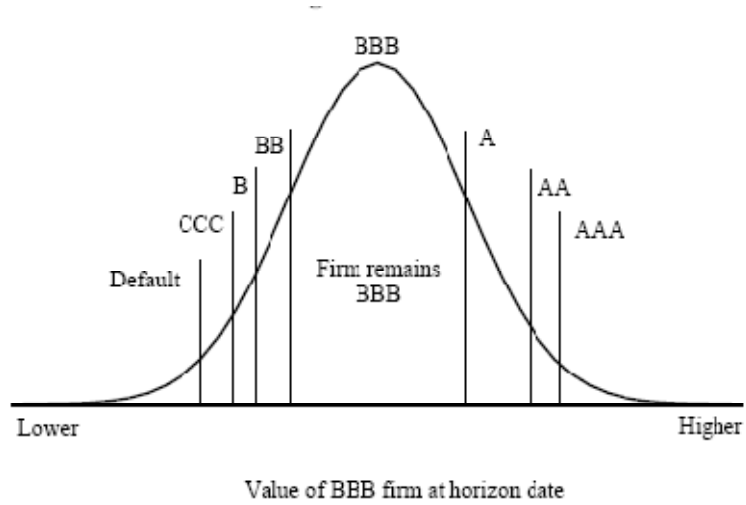


Figure 2.1. Model of firm value with rating transition thresholds (Gupton *et al.*, 1997)

After the simulation of asset returns and thus rating changes, at the beginning of each period CreditMetrics reevaluates the future cash flows (monthly or yearly payments of a debt or coupon payments of a bond) in order to find the current portfolio value. In revaluation of future cash flows (Equation (2.3)), CreditMetrics uses spot rates, which are also called forward zero-coupon rates. Spot rates or forward zero-coupon rates are nothing but the geometric mean of the forward rates, which are agreed interest rates between two parties for the upcoming years of the payment horizon (Choudhry, 2003). Equation (2.4) shows how to calculate the spot rate of the j^{th} year from forward rates where f_k and s_j represent the forward rate of the k^{th} and spot rate of the j^{th} year, respectively.

$$PV_i = \sum_{k=1}^{T-i} \frac{C_{i+k}}{s_{i+k}} \quad (2.3)$$

$$\prod_{k=1}^j (1 + f_k) = (1 + s_j)^j \quad (2.4)$$

C_j is the cash flow of the j^{th} year whereas PV_i is the present value of the future cash flows at the i^{th} year, and T is the length of the horizon.

2.1.2. Link to Statistical Models

At first glance, CreditMetrics methodology looks like a qualitative-response model, such as an ordered probit model. Duffie and Singleton (2003) explains that these models link the ratings and thus the rating changes to an underlying variable worthy to explain the credit-worthiness of an obligor (or a bond). They further explain that this underlying variable depends on a vector of factors, Z_t , which depends on time, as seen in Equation (2.5). Again, the boundaries around this underlying variable are determined in order to be able to trigger a rating change. However, these boundaries are specific to each obligor not to each rating group as in CreditMetrics. In addition, these factors are far from being external factors and thus different from the CreditMetrics' correlation structure, which triggers correlated transitions. For instance, Duffie and Singleton (2003) gives asset return coefficient beta of the obligor, and its balance sheet information as examples to possible factors of an ordered probit model. Moreover, a probit model uses the cdf of the standard normal distribution to map the value of the latent variable, X_t , to an event probability (Rachev *et al.*, 2007). In other words, a probit model finds default probabilities without the use of a default threshold. In fact, X_t , itself, acts as a default threshold in probit models.

$$X_t = \alpha + \beta Z_t + \epsilon_t \quad (2.5)$$

Glasserman and Li (2005) call the multifactor model of CreditMetrics as the normal copula model and use this model in their simulation studies whereas Kalkbrener *et al.* (2007) call this group of models as Gaussian multifactor models. Both ideas are based on the same assumption that systematic factors, which affect the asset value of a firm, have a multivariate normal distribution and so do the asset values of different firms or obligors. The statistical models that Glasserman and Li (2005) and Kalkbrener *et al.* (2007) use and the CreditMetrics framework in default mode are very similar models. However, the latent variable in the statistical model used by Glasserman and Li (2005) triggers a default if it crosses a barrier, which is again calculated from the long-term average default probabilities as done in CreditMetrics but these probabilities are assumed to be firm specific (but

obviously these probabilities can be chosen again with respect to the ratings of obligors to obtain a CreditMetrics-wise model). Yet, all these models are asset based models.

Next, CreditMetrics essentially uses a rating transition matrix, which is nothing but the transition matrix of a Markov Chain Process. Therefore, in a sense, CreditMetrics tries to model the intensities to default and to other states, namely ratings. Nonetheless, we cannot call it simply as a default-intensity (also called reduced-form) model while it does not use these intensities in valuing the bond or the credit exposure. Besides, it does not use a tractable stochastic model for the intensities. Instead, CreditMetrics merely makes use of rating-specific historical averages of the transitions in order to explain a steady-state Markov Chain. Although CreditMetrics has an inspiration from asset-value models, or in other words structural models, it does not utilize the log-normality assumption of asset returns in any estimation procedure within the credit portfolio simulation. As a result, we choose to call CreditMetrics merely as a historical method explained by Schmid (2004).

2.1.3. Calibration

Calibration of the model to the real data is a trivial task after the correlations between systematic factors are determined. However, it is critical to detect how much of the equity movements can be explained by which factors. Then, calibration consists of assessment of the rating transition thresholds and the factor loadings.

Let z_D , z_{CCC} , z_B , z_{BB} , z_A , z_{AA} , and z_{AAA} be transition thresholds for a BBB rated obligor. So, for instance, if the normalized asset return of this obligor falls below the threshold, z_D , then it is an indication to default while it shows that assets return of this obligor falls drastically. Besides, probabilities of default and transition to other states (ratings) resulting from these thresholds should match the estimated probabilities of a BBB rated firm shown by a transition matrix. Hence,

$$p_{default} = Pr\{X < z_D\} = \Phi(z_D)$$

$$p_{CCC} = Pr\{z_D < X < z_{CCC}\} = \Phi(z_{CCC}) - \Phi(z_D)$$

$$p_B = Pr\{z_{CCC} < X < z_B\} = \Phi(z_B) - \Phi(z_{CCC})$$

$$p_{BB} = Pr\{z_B < X < z_{BB}\} = \Phi(z_{BB}) - \Phi(z_B)$$

$$p_{BBB} = Pr\{z_{BB} < X < z_A\} = \Phi(z_A) - \Phi(z_{BB})$$

$$p_A = Pr\{z_A < X < z_{AA}\} = \Phi(z_{AA}) - \Phi(z_A)$$

$$p_{AA} = Pr\{z_{AA} < X < z_{AAA}\} = \Phi(z_{AAA}) - \Phi(z_{AA})$$

$$p_{AAA} = Pr\{z_{AAA} < X\} = \Phi(-z_{AAA}) \quad (2.6)$$

where X is the latent variable or normalized assets return, and p_a is the probability of a transition to rating a . CreditMetrics was developed over Merton's (1974) asset valuation model, which assumes that asset value movements mimic a Geometric Brownian Motion. Hence, it is easy to define these transition thresholds since CreditMetrics utilizes the assumption that the normalized asset returns follow standard normal distribution.

$$\bar{r}_t = \frac{\ln(A_t/A_0) - (\mu_A - (\sigma_A^2/2))t}{\sigma_A\sqrt{t}} \quad (2.7)$$

As a result, since we assume that asset returns are standard normal, then Φ in Equation (2.6) is the cumulative density function (cdf) of standard normal distribution. Next, we can calculate the transition thresholds by the following equation (thresholds appear around zero as in Figure 2.1).

$$z_{ik} = \Phi^{-1} \left(\sum_{\substack{j \geq k \text{ for } k > i \\ j > k \text{ for } k < i}} p_{ij} \right) \quad (2.8)$$

Here, p_{ij} and z_{ik} are the transition probability from rating i to j and threshold between rating i and k , respectively, where numbers from one to eight are assigned for ratings from

AAA to Default, correspondingly. In addition, Φ^{-1} is the inverse cdf of standard normal distribution.

In fact, as also mentioned in the previous section, CreditMetrics do not use the normality assumption and Equation (2.7) in any valuation step. It seeks default and transition correlations through this assumption. Thus, it might be stated that any multifactor model, which is thought to be able to explain the transition dynamics, can be used within this framework by assuring that the distribution of its latent variables is known. For instance, Löffler (2004) again uses a multi-factor model of asset returns to illustrate the correlation between obligors; however, he affirms that implied asset returns have heavy-tailed distribution such as t -distribution rather than a symmetric distribution like normal. He further explains the choice of degree of freedom parameter of t -distribution, adequate for asset returns and also the way of transforming normal asset returns to t -distributed asset returns via *Chi-square* distribution.

Next, what remains is to determine the factor loadings in such a way that they comprise the real correlation structure while leaving the standard normal assumption of normalized asset returns still valid. First of all, it is necessary to obtain the variance-covariance matrix of systematic factors. This is easily done via inspection of time series data of the factors. Then, we need to determine how accurately each factor can explain the asset return of each obligor. This step is done via regression analysis. Therefore, if we have N obligors, we need to fit N many regressions using d many factors as regressors. We use R^2 statistics $\left(R^2 = \frac{\text{Sum of Squares of regressors}}{\text{Sum of Squares of residuals}}\right)$ of these regressions for the percent of explained variance. Let's say R_k is the R^2 statistics obtained for the k^{th} obligor, and w_{kj} is the allocation of assets of the k^{th} obligor to the j^{th} factor. Assume, for example, one of the obligors use a 40 per cent of its assets in metal industry and the remaining assets in chemical industry. Then, $w_{11}= 0.4$ and $w_{12}= 0.6$. The CreditMetrics framework defines the factor loadings as follows:

$$a_{kj} = R_k w_{kj} \frac{\sigma_j}{\sigma_k} \quad (2.9)$$

where σ_j is the standard deviation of the j^{th} factor and

$$\sigma_k^2 = \sum_j \sum_l w_{kj} w_{kl} \text{Cov}(Z_j, Z_l) \quad (2.10)$$

is the variance of the k^{th} obligor. Factor loading a_{kj} is, in other words, real level of the variation explained by that factor, regarding that the factors together can explain R_k portion of the asset return variation and that because of the correlation between these factors and the asset allocations, we cannot assign R_k 's equally among systematic factors. Finally, we calculate the loading of each firm-specific or idiosyncratic risk factor by Equation (2.11) so that the standardized asset returns have a variance of one.

$$b_k^2 = 1 - \sum_j \sum_l a_{kj} a_{kl} \rho_{jl}$$

ρ_{jl} is the correlation between the systematic factors j and l . Here, we use correlations instead of covariances while factor loadings a_{kj} 's include standard deviations of systematic factors.

$$b_k^2 = 1 - \sum_j \sum_l R_k w_{kj} \frac{\sigma_j}{\sigma_k} R_k w_{kl} \frac{\sigma_l}{\sigma_k} \rho_{jl}$$

$$b_k^2 = 1 - \frac{R_k^2}{\sigma_k^2} \sum_j \sum_l w_{kj} \sigma_j w_{kl} \sigma_l \rho_{jl}$$

$$b_k^2 = 1 - \frac{R_k^2}{\sigma_k^2} \sum_j \sum_l w_{kj} w_{kl} \text{Cov}(z_j, z_l)$$

$$b_k = \sqrt{1 - R_k^2} \quad (2.11)$$

Modeling of the multi-factor default structure requires daily or weekly stock (equity) data of the obligors within a credit portfolio and variance-covariance matrix of the systematic factors. If the equity data are not available, for instance if the shares of the

corresponding obligors are not publicly traded in the stock exchange market, then time series data of the value of those obligors' assets and the knowledge of debt figures over the same period that the asset price data cover are necessary. However, these data are harder to obtain than the equity data.

2.1.4. Drawbacks from Literature

One important drawback of this framework is the usage of average transition probabilities, which are calculated as historical average of migration and default data (Crouhy *et al.*, 2000). As a result, every obligor within the same rating group has the same transition and default probabilities. Furthermore, if and only if an obligor migrates to another rating, default probability of that firm is adjusted accordingly. This is a rather discrete modeling of default. In fact, as Crouhy *et al.* (2000) also argue, default rates are continuous over time whereas ratings are not. Moody's KMV strongly opposes this conjecture of J. P. Morgan. Similarly, Schmid (2004) also counters the fact that CreditMetrics disregard default rate volatilities. He then emphasizes that default intensities are also correlated with business cycles and industries to which the obligors belong. Yet, Schmid (2004) asserts, there is not enough information in the market to estimate transition matrices with respect to business cycles but the estimation of different matrices is possible for industries.

Another disadvantage of this framework is that it requires a wide data set since it is not a default only model but a Mark-to-Market model. Even when the required data are achievable, it is most likely that some of the estimates will have statistically low significance levels (Schmid, 2004). In addition, Schmid (2004) affirms that because of the capability of rating agencies in catching the rating changes, both the probability of maintaining the last rating and the default probability overrate the true probabilities, forcing some other transition probability estimates underestimate their true figures.

Altman (2006) sees the recovery rate process adopted by CreditMetrics as another drawback of the framework while it handles recovery only at the time of default and generally uses a beta distribution to assess recovery rates. He provides significant empirical evidence that recovery rates and default probabilities are correlated events, and

recovery rate process should be included as a factor in systematic risk of an obligor. In fact, none of the portfolio VaR models apply a similar methodology.

As a final disadvantage, it should be added that this methodology is only applicable to the firms with a known rating (Schmid, 2004). Considering that most of the firms in Turkey do not have a rating, this is very essential if implemented in Turkey. In such situations, Schmid (2004) suggests the use of observable financial data of those firms to calculate fundamental financial ratios so that by matching them to the ones of the firms with known ratings, it can be possible to determine the unknown ratings.

2.2. KMV Portfolio Manager

This section briefly explains the Portfolio Manager Framework built for credit risk quantification, gives a general idea of the calibration of this model, and then states the drawbacks of the framework mentioned in literature.

2.2.1. The Framework

KMV Portfolio Manager is another mark-to-market portfolio VaR model while it also considers credit quality changes due to rating migrations. Also, as explained in the previous section the framework does not use transition matrices and instead it models the default rates in a continuous manner. This approach can be seen as an enhancement of J. P. Morgan's CreditMetrics. Crouhy *et al.* (2000) shows and explains the results of Moody's KMV's simulation test that reveals significant deviations of actual default and transition probabilities from average probabilities. Moreover, Portfolio Manager covers the calculation of each obligor's actual default probability, which is named as Expected Default Frequency (EDF) by Moody's KMV. Another substantial improvement of Portfolio Manager is that these EDFs are firm specific, so any rating or scoring system can be used to match these probabilities (Crouhy *et al.*, 2000). On the other hand, this framework is still based on Merton's (1974) firm valuation approach (Schmid, 2004). Simply, if the market value of a firm's assets falls below the total debt value, then that firm is said to default. Therefore, the price of a put option written on the asset value of a firm with a strike price equal to that firm's total debt gives us the risk.

Contrary to Creditmetrics, KMV Portfolio Manager uses asset values of firms as risk drivers since correlated asset values straightforwardly act as a trigger to correlated default events (Bessis, 2002). Kealhofer and Bohn (2001) explain the model used by Portfolio Manager as follows. Portfolio Manager first derives current asset value and asset volatility (percent change) of a firm (obligor) from time series data of that firm's equity value and its fixed liabilities. As seen in Figure 2.2, equity is nothing but the difference

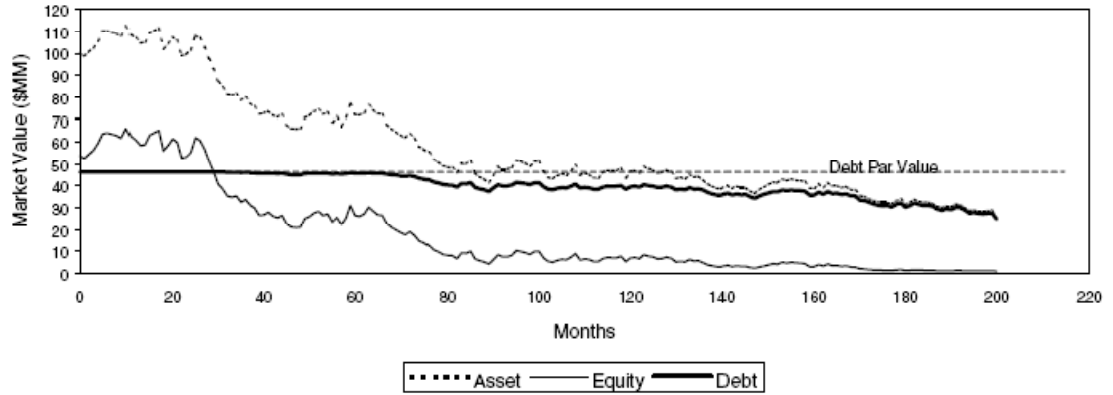


Figure 2.2. Time series of asset, equity and debt values (Kealhofer and Bohn, 2001)

between the value of a firm's assets and its liabilities; moreover, current equity value can be directly calculated by the following famous Black-Scholes-Merton option pricing formula under the assumption that the percentage change in a firm's underlying assets follows the stochastic process shown in Equation (2.13).

$$E = A\Phi(d_1) - e^{-rT}D\Phi(d_2) \quad (2.12)$$

$$d_1 = \frac{\ln(A/D) + \left(r_f + \frac{\sigma_A^2}{2}\right)T}{\sigma_A\sqrt{T}}$$

$$d_2 = d_1 - \sigma_A\sqrt{T}$$

In Equation (2.12), A , E , σ_A , r_f , Φ , D , and T are equity value, market value of assets, asset volatility, risk-free interest rate, cumulative standard normal distribution function, total

debt of the firm, and time horizon, respectively. Furthermore, W in Equation (4.2) is a Standard Brownian Motion (or Wiener Process), and μ_A is the drift.

$$dA = \mu_A A dt + \sigma_A A dW \quad (2.13)$$

To explain it further, Kealhofer and Bohn (2001) assert that shareholders of a firm can be seen as holders of a call option on the firm's asset value with a strike price equal to its liabilities. So, the shareholders can choose to exercise the option and pay the debt value or choose to default and pay a rate of the debt value to the lender considering whether the option is in the money or out of the money. However, based on the studies of Oldrich Vasicek and Stephen Kealhofer (VK), Moody's KMV uses an extended version of Black-Scholes-Merton (BSM) formulation. The differences between these two models are summarized in Table 2.1.

Table 2.1. Comparison of BSM and VK EDF models (Bohn, 2006)

Black-Scholes-Merton	Vasicek-Kealhofer EDF Model
Two classes of Liabilities: Short Term Liabilities and Common Stock	Five Classes of Liabilities: Short Term and Long Term Liabilities, Common Stock, Preferred Stock, and Convertible Stock
No Cash Payouts	Cash Payouts: Coupons and Dividends (Common and Preferred)
Default occurs only at Horizon.	Default can occur at or before Horizon.
Default barrier is total debt.	Default barrier is empirically determined.
Equity is a call option on Assets, expiring at the Maturity of the debt.	Equity is a perpetual call option on Assets
Gaussian relationship between probability of default (PD) and distance to default (DD).	DD-to-EDF mapping empirically determined from calibration to historical data.

To find the asset volatility, Crosbie and Bohn (2003) use the formulation in Equation (2.14), where A , E , σ_A , and σ_E are asset value, equity price, and their volatilities, respectively.

$$\sigma_E = \frac{A}{E} \Delta^E \sigma_A \quad (2.14)$$

Δ^E is the equity delta (the change in the equity value with respect to the change in the asset value, $\frac{\partial E}{\partial A}$), which is equal to $\Phi(d_1)$. Lu (2008) gives the details of the estimation of asset volatility and drift as follows. First, similar to the asset value changes, equity itself follows a Geometric Brownian Motion and thus equity return process is a stochastic process shown in Equation (2.15).

$$dE = \mu_E E dt + \sigma_E E dW \quad (2.15)$$

From Equation (2.12) we know that equity price is a function of time and asset price, which itself is an *Ito Process*. So, the change in equity price can be obtained by *Ito-Doebelin Formula* in the following way:

$$\begin{aligned} dE &= \frac{\partial E}{\partial A} dA + \frac{\partial E}{\partial t} dt + \frac{1}{2} \frac{\partial^2 E}{\partial A^2} (dA)^2 + \text{higher order terms} \\ &= \frac{\partial E}{\partial A} (\mu_A A dt + \sigma_A A dW) + \frac{\partial E}{\partial t} dt + \frac{1}{2} \frac{\partial^2 E}{\partial A^2} (\mu_A A dt + \sigma_A A dW)(\mu_A A dt + \sigma_A A dW) \end{aligned}$$

and

$$dE = \frac{\partial E}{\partial A} (\mu_A A dt + \sigma_A A dW) + \frac{\partial E}{\partial t} dt + \frac{1}{2} \frac{\partial^2 E}{\partial A^2} A^2 \sigma_A^2 dW dW$$

while $dt dt = 0$ and $dW dt = 0$. Next, since the term $dW dW = dt$ (variance of a Standard Brownian Motion), we can rearrange the equation as:

$$dE = \left(\frac{\partial E}{\partial A} \mu_A A + \frac{\partial E}{\partial t} + \frac{1}{2} \frac{\partial^2 E}{\partial A^2} A^2 \sigma_A^2 \right) dt + \frac{\partial E}{\partial A} \sigma_A A dW \quad (2.16)$$

Then, by comparing Equation (2.15) and (2.16);

$$\sigma_E E = \frac{\partial E}{\partial A} \sigma_A A$$

$$\mu_E E = \frac{\partial E}{\partial A} \mu_A A + \frac{\partial E}{\partial t} + \frac{1}{2} \frac{\partial^2 E}{\partial A^2} A^2 \sigma_A^2$$

Finally,

$$\sigma_A = \frac{\sigma_E E/A}{\partial E / \partial A}$$

$$\mu_A = \frac{\mu_E E - \frac{\partial E}{\partial t} - \frac{1}{2} \frac{\partial^2 E}{\partial A^2} A^2 \sigma_A^2}{\frac{\partial E}{\partial A} A}$$

$\frac{\partial^2 E}{\partial A^2}$ is the *Equity Gamma*, and $\frac{\partial E}{\partial t}$ is the *Equity Theta*. Moreover, they are calculated by the following formulas.

$$\Gamma^E = \frac{\partial^2 E}{\partial A^2} = \frac{\Phi(d_1)}{A \sigma_A \sqrt{T-t}} > 0$$

$$\theta^E = \frac{\partial E}{\partial t} = -\frac{A \Phi(d_1) \sigma_A}{2\sqrt{T-t}} - r D e^{-r(T-t)} \Phi(d_2)$$

Hence, since we already know the equity delta, the drift parameter of asset values becomes:

$$\mu_A = \frac{\mu_E E - \theta^E - \frac{1}{2} \Gamma^E A^2 \sigma_A^2}{\Delta^E A}$$

Then, if we know the equity price, equity drift and volatility, and debt of the corresponding firm at any point in time, we can determine the market value of the assets and asset volatility by solving Equation (2.12) and (2.14) simultaneously. Finally, given asset volatility and drift, default probability (*DP*) at any time, t , is calculated by Equation (2.17) where A_t is the current value of assets, and T is the time horizon. Here, drift parameter is used instead of a risk-free rate. This is merely because in real markets, expected return on

assets of a firm is usually greater than the risk-free rate. Therefore, by the integration of a drift parameter, we disregard the assumption of risk-neutral world.

$$\begin{aligned}
DP &= Pr[A_T < D_T] \\
&= Pr \left[A_t \exp \left\{ \left(\mu_A - \frac{\sigma_A^2}{2} \right) (T - t) + \sigma_A W_{T-t} \right\} < D_T \right] \\
&= Pr \left[W_{T-t} < \frac{\ln \left(\frac{D_T}{A_t} \right) - \left(\mu_A - \frac{\sigma_A^2}{2} \right) (T - t)}{\sigma_A} \right] \\
&= Pr \left[\varepsilon < \frac{\ln \left(\frac{D_T}{A_t} \right) - \left(\mu_A - \frac{\sigma_A^2}{2} \right) (T - t)}{\sigma_A \sqrt{T - t}} \right] \\
&= Pr[\varepsilon < -DD] \\
&= \Phi(-DD)
\end{aligned} \tag{2.17}$$

Now, we know that DP is nothing but $\Phi(-DD)$, and this also complies with the fact that if standardized return of a firm's assets fall below a certain level, this is an indication of a default. Moreover, DD in Equation (2.17) is called implied distance-to-default and calculated by Equation (2.18). In other words, it is the measure of how many units of standard deviation (units of asset return volatility) a firm is away from default. DP is the shaded area in Figure 2.3.

$$DD = \frac{\ln \frac{A_t}{D_T} + \left(\mu_A - \frac{1}{2} \sigma_A^2 \right) (T - t)}{\sigma_A \sqrt{T - t}} \tag{2.18}$$

Up until now, we explained the KMV's BSM model. However, actual Portfolio Manager of Moody's KMV uses VK EDF model. One noticeable deficiency of BSM

model is that a firm can only default at maturity and that the model only considers short-term liabilities.

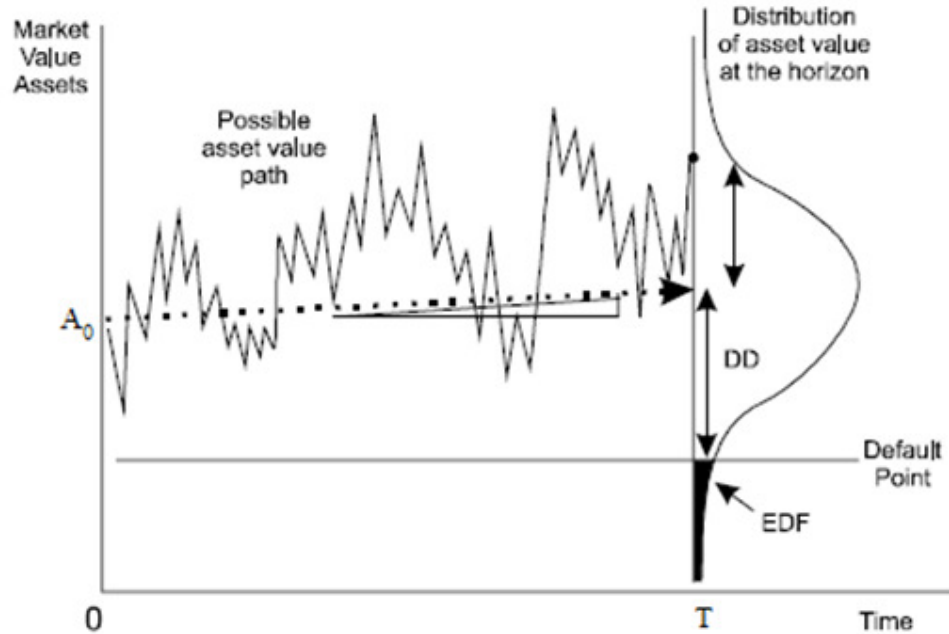


Figure 2.3. Future asset value distribution (Crosbie and Bohn, 2003)

In VK's EDF model, after estimating asset values and asset value volatilities, Portfolio Manager calculates each obligor's distance-to-default parameter by the following equation.

$$DD = \frac{[\text{Market Value of Assets}] - [\text{Default Point}]}{[\text{Market Value of Assets}][\text{Asset Volatility}]} = \frac{E(A_1) - D}{\sigma}$$

Here, $E(A_1)$ refers to the expected market value of assets in one year. Moreover, as explained by Crouhy *et al* (2001), Portfolio Manager finds it more efficient to choose the default point (D) of a firm as a summation of the short-term liabilities to be paid within the time horizon and half of the long-term liabilities.

Then, it empirically maps these distance-to-default values to default probabilities by examining the time series of default data. Basically, the framework determines the default probabilities for a five year horizon by finding how many of the firms that used to have the

same DD during any time in the past defaulted after within one to five years. As a result, this method handles obligors with the same DD as equal and attains DPs to each obligor, accordingly.

Furthermore, to estimate future asset values, Moody's KMV also introduces asset return correlations through a multifactor model. The framework adopts a three-level multifactor model. Crouhy *et al.* (2001) explain this multi-factor model in the following manner. The first level consists of a single composite factor (CF). This factor is composed of industry and country risk factors at the second level of this model with weights determined based on the obligor's allocations in different countries and industries. Next, at the third level, country and industry factor returns accommodate global economic risk, regional risk, industrial sector risk, and industry and country specific risks. Thus, at the first level, asset return of an obligor looks like;

$$r_k = \beta_k CF_k + \epsilon_k$$

At the second level:

$$CF_k = \sum_m \alpha_{km} C_m + \sum_n \alpha_{kn} I_n$$

which is subject to

$$\sum_m \alpha_{km} = 1$$

$$\sum_n \alpha_{kn} = 1$$

while α_{km} 's and α_{kn} 's are allocations of the k^{th} obligor's business among countries and industries, correspondingly. I_n is the industry return, whereas C_m is the country return. If we combined these three levels and write the asset return in terms of global economic, regional, sector, and industry-specific and country-specific risk factors, this multifactor

model would look like the one of CreditMetrics. However, Portfolio Manager calculates factor loadings in a different way. Following this step, Portfolio Manager also derives the pair-wise correlations from this multifactor model and then utilizes them in portfolio optimization. According to Saunders and Allen (2002), the framework uses its multifactor model to generate the standard normal variate, ε , in Equation (2.19), which is the solution of the stochastic process in Equation (2.13).

$$\ln(A_T) = \ln(A_0) + \left(\mu_A - \frac{\sigma_A^2}{2} \right) T + \varepsilon \sigma_A \sqrt{T} \quad (2.19)$$

In brief, the framework generates a bunch of possible future asset returns via normal distribution and then uses these simulated values together with asset volatilities to calculate the distance-to-default of each obligor (Jacobs, 2004). If an obligor defaults, the framework incurs a recovery rate. Else, by mapping these distance-to-default values to empirical default rates, Portfolio Manager finds the relative default probability and the new credit spread, which are obtained through corporate bond data (usually over LIBOR curve – see Crouhy *et al.*, 2001), for each obligor. Then, the framework uses these default probabilities in finding the present value of the future cash flows. This step is carried out by risk-neutral bond pricing. Crouhy *et al.* (2001) give the risk-neutral pricing of a bond or a loan subject to default risk through their future cash flows as follows.

$$PV = (1 - LGD) \sum_{i=1}^n \frac{C_i}{(1 + r_f^i)^{t_i}} + LGD \sum_{i=1}^n \frac{(1 - Q_i)C_i}{(1 + r_f^i)^{t_i}} \quad (2.20)$$

r_f^i is the risk-free rate (or discount rate) at the horizon, t_i , C_i is the cash flow in the i^{th} period, and Q_i is the cumulative risk-neutral default probability. In practice, the empirically determined default probabilities, in other words EDFs, are actual measures and to be able to use Equation (2.20), we need to convert these actual probabilities to risk-neutral probabilities. Crouhy *et. al* (2001) explain the derivation of risk-neutral probabilities in detail. The DD in Equation (2.18) is equal to d_2^* in Black-Scholes-Merton option pricing formula when we disregard the risk-neutral assumption. Hence, $\Phi(-d_2^*)$ gives us the actual default probability. However, in a risk-neutral world, d_2 is a function of r_f , not μ_A . By

using the relation between the risk-neutral d_2 and actual d_2^* (Equation (2.21)) we can calculate the cumulative risk-neutral default probability, Q_T , from EDF by Equation (2.22).

$$d_2^* = d_2 + \frac{(\mu_A - r_f)}{\sigma_A} \sqrt{T} \quad (2.21)$$

$$Q_T = \Phi \left(\Phi^{-1}(EDF) + \frac{(\mu_A - r_f)}{\sigma_A} \sqrt{T} \right) \quad (2.22)$$

Next, after determining these risk-neutral probability measures, the framework revalues obligors' remaining cash flows via Equation (2.20) and generates portfolio value distributions. Besides these value distributions, Portfolio Manager provides unexpected losses of all individual obligors. The terminology, "unexpected loss" (UL) stands for the standard deviation of loss resulting from individual obligors. When the obligors are examined as individuals but not within a portfolio, and when LGD assessments and default probabilities are known, then the calculation of UL due to an obligor is straightforward.

Default process of an individual obligor within a year is nothing but a Bernoulli trial. Thus, the standard deviation of losses due to an individual obligor is:

$$UL_i = LGD_i \sqrt{DP_i(1 - DP_i)}$$

Nevertheless, portfolio managers desire to determine the unexpected loss of the whole portfolio. That is simply the sum of individual ULs in case of independent obligors which is never the case in practice. When obligors are correlated, we need to define all pair-wise correlations. When the correlations are defined, UL of a portfolio can be calculated by the following equation:

$$UL_P = \sqrt{\sum_{i=1}^n \sum_{j=1}^n w_i w_j UL_i UL_j \rho_{ij}}$$

with a constraint;

$$\sum_{i=1}^n w_i = 1$$

where w_i is the weight (or proportion) of the i^{th} exposure in the portfolio, UL_i is the unexpected loss due to the i^{th} obligor, and ρ_{ij} is the correlation between the i^{th} and the j^{th} obligor. Yet, in practice it is not efficient and computationally cheap to calculate all pair-wise correlations among the obligors of a large credit portfolio. That is why Portfolio Manager introduces a multifactor model to describe the correlation structure. After building the previously explained multifactor model, Portfolio Manager derives also the pair-wise correlations from this model and utilizes them in portfolio optimization by examining unexpected losses.

In comparison to CreditMetrics, Portfolio Manager uses beta distribution for recovery rate. Yet, its mean parameter is user-defined. Besides, KMV Portfolio Manager provides absolute risk contributions whereas CreditMetrics provides marginal risk contributions of obligors of a portfolio. Here, it is worthy to note that KMV Portfolio Manager is the only framework that offers portfolio optimization (Bessis, 2002). Finally, we give the implementation steps of KMV Portfolio Manager as follows:

- (i) Calculate the current asset value of each obligor
- (ii) Derive asset value volatility and drift for each obligor
- (iii) Define a multifactor model for asset log-returns and find the corresponding factor loadings
- (iv) Simulate asset values by using the multifactor model
- (v) Determine DD by using the pre-determined drift and volatility parameters
- (vi) Calculate EDF from DD regarding the log-normality assumption of asset movements
- (vii) Carry out the risk-neutral valuation of the remaining payments
- (viii) Obtain the portfolio value distribution for each simulated horizon.

2.2.2. Calibration

Calibration of the Portfolio Manager framework consists of assessing the factor loadings in the multifactor model and deriving actual and risk-neutral default probabilities

through the calibration of Sharpe ratio (SR) and a time parameter, θ . In this section, we explain the calibration methodology as explained by Crouhy *et al.* (2001).

In the multifactor model, factor loadings of industry returns within the composite factor are simply the average of asset allocation percent of that industry and sales within that industry as a percent of total sales. Factor loadings of country returns are calculated in a similar fashion. For instance, if sales within an industry are 45 per cent of the total sales, and asset allocated to that industry is 35 per cent of the total assets, then the weight for that industry (x) is:

$$\alpha_{kx} = \frac{0.45 + 0.35}{2} = .4$$

Next, for the calibration of SR and θ to the risk-neutral default probabilities, Portfolio Manager makes use of Capital Asset Pricing Model (CAPM). First, it extracts beta coefficient (β) from the single factor CAPM shown below. Here, π is the market risk premium and expressed by Equation (2.23), where μ_M is the mean return of the market portfolio (for instance a country index, such as DOWJONES, S&P 500, or DAX).

$$\mu_A - r_f = \beta\pi$$

$$\pi = \mu_M - r_f \quad (2.23)$$

Normally, β is calculated by the following formula, where r_A , r_M , $\rho_{A,M}$, and σ_M are asset value return, market return, correlation between asset value and the market portfolio, and volatility of the market portfolio, respectively.

$$\beta = \frac{cov(r_A, r_M)}{var(r_M)} = \rho_{A,M} \frac{\sigma_A}{\sigma_M}$$

If we divide both sides in the single factor CAPM by σ_A , we obtain Equation (2.24).

$$\frac{\mu_A - r_f}{\sigma_A} = \frac{\beta\pi}{\sigma_A} \quad (2.24)$$

When we replace β in Equation (2.24);

$$\frac{\mu_A - r_f}{\sigma_A} = \rho_{A,M} \frac{\pi}{\sigma_M} \quad (2.25)$$

The ratio, $\frac{\pi}{\sigma_M}$, in Equation (2.25) is called the market Sharpe Ratio (*SR*), which is the excess return per unit of market volatility for the market portfolio. KMV replaces $\frac{\mu_A - r_f}{\sigma_A}$ in Equation (2.22) by Equation (2.25). Moreover, because in practice,

- (i) it is not very easy to statistically determine the market premium,
- (ii) *DP* is not exactly equal to the shaded area under DPT in Figure 2.3, and
- (iii) asset returns do not precisely have a normal distribution,

KMV chooses to estimate Q_T in Equation (2.22) by calibrating *SR* and θ using bond data via the following equation.

$$Q_T = \Phi(\Phi^{-1}(EDF) + \rho_{A,M} SR T^\theta) \quad (2.26)$$

For the calibration, we need the market data. From Equation (2.20), we can write the equation seen below:

$$e^{-s_i t_i} = [(1 - LGD) + (1 - Q_{t_i})LGD]e^{-r_f^i t_i}$$

where i refers to the valuation of the i^{th} cashflow, and s_i is the continuously compounded spot rate (we can convert the discrete return rates into continuous by $\ln(1 + r_f^i)$). Thus, obligor's corporate spread can be expressed by Equation (2.27). Finally, if we replace Q_{t_i} in Equation (2.27) with Equation (2.26), we can use the resulting equation to calibrate *SR* and θ via the least-square sense.

$$s_i - r_f^i = -\frac{1}{t_i} \ln(1 - Q_{t_i} LGD) \quad (2.27)$$

2.2.3. Drawbacks from Literature

Although Moody's KMV maps distance-to-default values to appropriate default probabilities, in practice it may not be possible to implement the same methodology while the default data in hand can be limited or incomplete or may not be enough to map these measures. That is why Moody's KMV uses a 30 year default data to map DDs to suitable EDFs. Although the log-normality assumption, which is also adopted by BSM EDF model, of asset values can still be employed to find the EDFs of the obligors as explained in the previous sections, today's KMV finds this methodology inefficient because of the deficiencies stated in Section 2.2.2.

2.3. CreditRisk+

In contrast to the previously presented models, CreditRisk+ is a pure actuarial model (Crouhy *et al.*, 2001). In other words, the framework tries to model the default rate distributions, and via probability density function of number of total defaults it finds the analytical loss distribution of a bond or a loan portfolio. This chapter gives a general look to the model, states a number of calibration tips, and finally points out the drawbacks that we came across in the literature.

2.3.1. The Framework

CreditRisk+ of Credit Suisse First Boston (CSFB) is a default-mode type model – it does not involve rating migrations- but analogous to KMV Portfolio Manager, it is a framework that models default rates in a continuous manner. In addition, CreditRisk+ underlines the fact that it is not possible to know the precise time of a default or the exact number of total defaults in a credit portfolio within a certain time horizon, since default events are consequences of several different successive events (CSFB, 1997). Thus, CreditRisk+ tries to determine the distribution of number of defaults over a period by defining default rates and their volatilities but does not deal with the time of the defaults.

CreditRisk+ requires four types of input; obligors' credit exposures, default rates and their volatilities, and recovery rates. Furthermore, to determine default probabilities of the obligors and their volatilities over time, CSFB (1997) suggests the use of credit spreads in the market or use of obligors' ratings as a proxy (by deriving a common default rate and volatility for each credit rating from historical data of rating changes). Therefore, CreditRisk+ tries to combine the discrete nature of rating transitions with the continuous nature of default rates while CSFB (1997) emphasizes that one-year default rates change simultaneously with the state of economy and several other factors resulting in a deviation from average default rates.

The aim of the framework is to find the total loss distribution. In 2002, Bessis refers to that CreditRisk+ easily generates a portfolio's analytical loss distribution under several assumptions as regards the loss distributions of portfolio segments and their dependency structure. As a start, let's assume default rates are constant over time. This causes the distribution of total number of default events in a credit portfolio, in case of independent sub-portfolios, to be Poisson (recall that sum of independent Poisson processes is again Poisson). This can easily be proved by using probability generating functions. First, we need to write the generating function of default of a single obligor.

$$F_A(z) = (1 - p_A)z^0 + p_A z^1 = 1 + p_A(z - 1)$$

p_A is nothing but a success probability of a Bernoulli trial that is for this case the default process. Hence, the generating function of number of defaults in a portfolio or a sub-portfolio is the product of $F_A(z)$'s:

$$F(z) = \sum_{n=0}^{\infty} P(\text{number of defaults} = n)z^n$$

$$F(z) = \prod_A F_A(z) \tag{2.28}$$

By substituting for $F_A(z)$ and upon rearrangement, Equation (2.28) becomes:

$$F(z) = e^{\mu(z-1)} \tag{2.29}$$

where μ is the expected number of total defaults and calculated as:

$$\mu = \sum_A p_A$$

By using the Taylor Series expansion of $e^{\mu z}$ around zero in Equation (2.29), we obtain the generating function below, which is the generating function of a Poisson distribution.

$$F(z) = \sum_{n=0}^{\infty} \frac{\mu^n z^n}{n!} e^{-\mu}$$

Moreover, in order to introduce variability in default rates, CSFB (1997) assumes that annual default rates are distributed due to a Gamma distribution with a dependency assumption between sectors. The probability density function (pdf) of Gamma distribution is given in Equation (2.30).

$$g(x; \alpha, \beta) = x^{\alpha-1} \frac{e^{-\frac{x}{\beta}}}{\beta^\alpha \Gamma(\alpha)} \text{ for } x > 0 \quad (2.30)$$

CreditRisk+ primarily provides a methodology for the analytical derivation of a portfolio loss distribution in the simplest case, which is the case of independent portfolio segments, by means of generating function of the portfolio loss (generating function is, as a result, the product of sub-portfolios' generating functions). First of all, we should define the generating function of number of defaults in a portfolio segment, k , in terms of a pdf, for instance $f(x)$, of that segment's aggregated default rate.

$$F_k(z) = \sum_{n=0}^{\infty} P(\text{number of defaults} = n) z^n = \sum_{n=0}^{\infty} z^n \int_0^{\infty} P(n|x) f(x) dx$$

When Equation (2.29) is used, the generating function becomes the following:

$$F_k(z) = \int_0^{\infty} e^{x(z-1)} f(x) dx$$

If $f(x)$ is replaced with the pdf of a gamma distribution with a shape parameter, α , and a scale parameter, β :

$$\begin{aligned} F_k(z) &= \int_0^{\infty} e^{x(z-1)} x^{\alpha-1} \frac{e^{-\frac{x}{\beta}}}{\beta^{\alpha} \Gamma(\alpha)} dx \\ &= \frac{1}{\beta^{\alpha} \Gamma(\alpha)} \int_0^{\infty} e^{x(z-1)} x^{\alpha-1} e^{-\frac{x}{\beta}} dx \\ &= \frac{1}{\beta^{\alpha} \Gamma(\alpha)} \int_0^{\infty} e^{-y} \left(\frac{y}{-z+1+\beta^{-1}} \right)^{\alpha-1} \frac{dy}{(-z+1+\beta^{-1})} \end{aligned}$$

Since $\Gamma(\alpha) = \int_0^{\infty} e^{-y} y^{\alpha-1} dy$ is the definition of the gamma function;

$$F_k(z) = \frac{1}{\beta^{\alpha} (-z+1+\beta^{-1})^{\alpha}}$$

Yet, we will use the following notation for $F_k(z)$:

$$F_k(z) = \left(\frac{1 - \hat{p}_k}{1 - \hat{p}_k z} \right)^{\alpha_k}$$

where $\hat{p}_k = \frac{\beta_k}{1+\beta_k}$. Then, substituting the Taylor series expansion for $\left(\frac{1}{1-\hat{p}_k z} \right)^{\alpha_k}$;

$$F_k(z) = (1 - \hat{p}_k)^{\alpha_k} \sum_{n=1}^{\infty} \binom{n + \alpha_k - 1}{n} \hat{p}_k^n z^n$$

As a result, the probability of n defaults within a portfolio segment is:

$$P(\text{number of defaults} = n) = (1 - \hat{p}_k)^{\alpha_k} \binom{n + \alpha_k - 1}{n} \hat{p}_k^n \quad (2.31)$$

which is the probability mass function (pmf) of a Negative Binomial distribution.

Negative Binomial distribution is the distribution of number of failures but with enough number of trials that will end up with certain number of successes. Besides, the derivation of Equation (2.31) could be easily derived without the use of generating functions, as well.

Next, we need to find the generating function of the portfolio loss. For the fixed default rate case, CSFB (1997) first defines the generating function of a portfolio segment with a common exposure band (all obligors in that sub-portfolio are assumed to have the same exposure, net of recoveries) as below where n refers to number of defaults within that portfolio segment and then proves that the generating function of the total portfolio loss is as shown in Equation (2.32). Moreover, v_j and m in the following equations are common exposure in segment j in units of L , and number of segments in the portfolio, respectively. All losses in CreditRisk+ framework are written in terms of units of some predefined L . Thus, exposure bands are multiple of this L .

$$G_j(z) = \sum_{n=0}^{\infty} P(\text{number of defaults} = n) z^{n v_j} = \sum_{n=0}^{\infty} \frac{e^{-\mu_j} \mu_j^n}{n!} z^{n v_j} = e^{-\mu_j + \mu_j z^{v_j}}$$

$$G(z) = \prod_{j=1}^m G_j(z) = \exp\left(-\sum_{j=1}^m \mu_j + \sum_{j=1}^m \mu_j z^{v_j}\right) \quad (2.32)$$

Then, CSFB (1997) tries to find the analytical distribution of the portfolio loss by utilizing the property of generating functions seen in Equation (2.33) where A_n is the probability that portfolio loss will be equal to n units of L .

$$P(\text{portfolio loss} = nL) = \frac{1}{n!} \left. \frac{d^n G(z)}{dz^n} \right|_{z=0} = A_n \quad (2.33)$$

The recurrence relation obtained by CSFB (1997) for A_n is the following equation.

$$A_n = \sum_{j: v_j \leq n} \frac{l_j}{n} A_{n-v_j}$$

l_j is the expected loss of the j^{th} portfolio segment and thus calculated by:

$$l_j = v_j \times \mu_j$$

So, for instance, the probability of zero loss resulting from a whole portfolio is:

$$A_0 = G(0) = e^{-\mu} = e^{-\sum_{j=1}^m \mu_j} = e^{-\sum_{j=1}^m \frac{l_j}{v_j}}$$

Furthermore, with gamma distributed default rates, the generating function of portfolio loss becomes the following equation where indices are a bit different than in Equation (2.32); j 's refer to individual obligors whereas k refers to portfolio segment that includes those obligors. Also, CSFB (1997) verifies that this generating function, when the variability of

$$G(z) = \prod_{k=1}^n G_k(z) = \prod_{k=1}^n \left(\frac{1 - \hat{p}_k}{1 - \frac{\hat{p}_k}{\mu_k} \sum_{j=1}^{m_k} \frac{l_j^{(k)}}{v_j^{(k)}} z^{v_j^{(k)}}} \right)^{\alpha_k} \quad (2.34)$$

default rates tends to zero and number of segments to infinity, converges to the one in Equation (2.32). Moreover, CSFB (1997) introduces two polynomials, $A(z)$ and $B(z)$, as follows in order to achieve a recurrence relation for the distribution of portfolio loss from Equation (2.34).

$$\frac{d}{dz} (\log G(z)) = \frac{1}{G(z)} \frac{dG(z)}{dz} = \frac{A(z)}{B(z)}$$

$$A(z) = a_0 + a_1 z^1 + \dots + a_r z^r$$

$$B(z) = b_0 + b_1 z^1 + \dots + a_s z^s$$

The resulting recurrence relation is:

$$A_{n+1} = \frac{1}{b_0(n+1)} \left(\sum_{i=0}^{\min(r,n)} a_i A_{n-i} - \sum_{j=0}^{\min(s-1,n-1)} b_{j+1}(n-j) A_{n-j} \right)$$

In addition to all the features explained previously, CreditRisk+ provides a methodology to establish correlated loss distributions of portfolio segments. In comparison to the previous models, the framework again uses a multifactor model to introduce correlation between portfolio segments with regard to loss distributions and default event. CreditRisk+ first groups the obligors within a portfolio with respect to their exposures, net of recoveries, risk levels and several other factors so that it can convert number of defaults into loss distributions. Next, it models the default process of each obligor group or say, portfolio segment as a mixed Poisson process with a mixed Poisson parameter, $q_i \times \mu_i$, where q_i is the mixing variable for portfolio segment i , and μ_i is the average default intensity of that segment (Bessis, 2002). CreditRisk+ develops a multifactor model on the mixing variable, q_i , which is a random variable with an expectation of one. This way, CSFB group supports their idea of variability in default intensities over time and several other external factors. Additionally, even though these mixed Poisson processes are correlated, because they are correlated through mixing variables, and because a linear multifactor model is used, sum of these processes is also a mixed Poisson process (Bessis, 2002). This eases the analytical calculation of the portfolio loss distribution also because the mixed Poisson variable of the whole portfolio is a linear combination of individual Poisson parameters and mixing variables. It is simply the weighted average of the mixing variables, with weights equal to Poisson parameters as shown in Equation (2.35).

$$q = \frac{\sum_i \mu_i \times q_i}{\sum_i \mu_i} \quad (2.35)$$

However, the framework does not provide a method for defining the sensitivities of q_i 's to those external factors. So, they are all user-defined together with the factors. Nevertheless, this in practice will definitely allow us to capture the effect of economic cycles and be helpful in stress testing. The factor model looks like:

$$q_i = \beta_{i1}x_1 + \beta_{i2}x_2 + \beta_{i3}x_3 + \dots$$

where x_j 's are external factors. In order to include the correlation, CreditRisk+ groups the obligors with respect to these factors. For example, let's consider a firm, A, which is active in both Germany and England. So, the firm A will be affected by both country indices and should be present as an obligor in the portfolio group of German firms, and English firms as well. As a consequence of this grouping, it is essential to allocate the weights of the default rate and exposure of A to related segments or factors (these groups are more like factors than segments). The following two equations are used to calculate the mean default rate (μ_k) and default rate volatility (σ_k) of factor k by using the allocation vector θ where $\hat{\mu}_A$ is the mean default rate and $\hat{\sigma}_A$ is the standard deviation the default rate of the obligor, A.

$$\mu_k = \sum_A \theta_{Ak} \hat{\mu}_A$$

$$\sigma_k = \sum_A \theta_{Ak} \hat{\sigma}_A$$

These equations are subject to Equation (5.9). In other words, θ_{Ak} is a percent allocation and therefore, it should add up to one with a summation over factors (we are dividing the attributes of an obligor into several different groups).

$$\sum_{k=1}^n \theta_{Ak} = 1 \quad (2.36)$$

In fact, the external factors here are more like systematic factors. Hence, as also Bessis (2002) points out, the linear relation between the factors and the obligors becomes deterministic leaving no room for idiosyncratic risk. To cover this deficiency, CreditRisk+ framework offers an additional (artificial) portfolio segment (and thus a factor) which is independent of the remaining portfolio segments.

Another feature of the framework is that it provides also the individual risk contributions of obligors. The CreditRisk+ framework defines the risk contributions in two different ways shown in Equation (2.37) and (2.38) where E_A is the exposure of obligor A,

and σ is the deviation of the total portfolio loss. Yet, it uses the second definition while that way risk contributions add up to standard deviation of the portfolio loss.

$$RC_A = E_A \frac{\partial \sigma}{\partial E_A} \quad (2.37)$$

$$RC_A = \frac{E_A \partial \sigma^2}{2\sigma \partial E_A} \quad (2.38)$$

Then, CreditRisk+ uses the first and second moments of the generating function of the portfolio loss to extract the mean and the variance of both the portfolio itself and the sub-portfolios. For instance, in a multifactor structure, CSFB (1997) determines the variance of the total loss by Equation (2.39).

$$\sigma^2 = \sum_{k=1}^n l_k^2 \left(\frac{\sigma_k}{\mu_k} \right)^2 + \sum_A l_A v_A \quad (2.39)$$

By substituting for σ^2 in Equation (2.38), RC_A simplifies to

$$RC_A = \frac{E_A \dot{\mu}_A}{\sigma} \left(E_A + \sum_k \left(\frac{\sigma_k}{\mu_k} \right)^2 l_k \theta_{Ak} \right)$$

Then, CreditRisk+ uses these risk contributions in concentration analysis. Due to the variance-covariance formulation of σ^2 in Equation (2.40) where $\hat{\sigma}_A$ is the standard deviation of obligor A's default event indicator, it is straightforward to check whether these risk contributions add up to σ (CSFB, 1997).

$$\sum_A RC_A = \frac{1}{2\sigma} \sum_A E_A \frac{\partial \sigma^2}{\partial E_A} = \frac{2\sigma^2}{2\sigma} = \sigma$$

$$\sigma^2 = \sum_A \sum_B \rho_{AB} E_A E_B \hat{\sigma}_A \hat{\sigma}_B \quad (2.40)$$

The final feature of CreditRisk+ is that it gives the default event correlations by the following formula. This formula indicates that if two obligors of a credit portfolio do not have a common factor, then their default event correlation is zero since not a single systematic factor influences them both (CSFB, 1997).

$$\rho_{AB} = (\dot{\mu}_A \dot{\mu}_B)^{1/2} \sum_{k=1}^n \theta_{Ak} \theta_{Bk} \left(\frac{\sigma_k}{\mu_k} \right)^2$$

Finally, we give the steps for implementing CreditRisk+:

- (i) Determine the exposures of the obligors, net of recoveries
- (ii) Find a base unit of loss, L , and divide the portfolio into exposure bands
- (iii) Extract average default rates and default rate volatilities for each rating group from bond market
- (iv) Define a multifactor model as explained in this section and determine portfolio segments, accordingly
- (v) Calculate θ_{Ak} 's for each obligor
- (vi) Find the generating function of portfolio loss.
- (vii) Try to obtain a recurrence relation for the discrete probabilities of portfolio by using the properties of generating functions

2.3.2. Calibration

Calibration of CreditRisk+ is not well-defined. Most of the parameters and sensitivities are user-defined. Nevertheless, there are two calibration tips that we would like to explain. First one is about the parameterization of the gamma distribution. The mean and variance of a gamma distribution is calculated via the following equations in terms of its shape and scale parameters.

$$\mu_k = \alpha_k \beta_k$$

$$\sigma_k^2 = \alpha_k \beta_k^2$$

Hence, when we know the means and standard deviations of default rates, we need to choose the shape and scale parameters of each sectoral default rate distribution as below.

$$\alpha_k = \frac{\mu_k^2}{\sigma_k^2}$$

$$\beta_k = \frac{\sigma_k^2}{\mu_k}$$

The second calibration tip is the choice of the mixing variables in case a multifactor model is used. CreditRisk+ proposes the use of time series data of default to calibrate q_i 's so that they can mimic the actual default volatilities (Bessis, 2002). Moreover, the framework models this mixing variable in such a way that its expectation stays as one so that the number of defaults within a time interval does not always tend to ascend or descend in time. Bessis (2002) states that the framework further uses another property of mixed Poisson distribution. We know that if the mixing variable has a Gamma distribution, then the corresponding default event distribution is again Negative Binomial. On the other hand, we still need a mixed Poisson variable whose expectation is one. Thus, it is necessary to use a standardised Gamma distribution. Since $E[x] = \frac{\alpha}{\beta}$, we should choose $\alpha = \beta = h$. Then, the analytical density function of the portfolio mixing variable, q , becomes:

$$h(q) = \left[\frac{1}{\Gamma(h)} \right] \int_0^{hq} e^{-z} z^{h-1} dz$$

whose expectation is one and variance, $1/h$. As a result, expected number of defaults in a unit interval (one year for our case) stays as μ_k but variance turns out to be:

$$\sigma^2(n) = \mu_k + \mu_k^2/h$$

2.3.3. Drawbacks from Literature

There are several weaknesses of the framework. First, factors in multifactor model and the sensitivities are user-defined, and indeed, they are not very clear how to be defined. When a Gamma distribution is used for mixing variables, CreditRisk+ again does not clarify how to determine parameters of the Gamma distribution. Additionally, CreditRisk+ uses fixed exposure-at-default values. Since the framework does not estimate the time of defaults, it cannot value losses with respect to remaining cash flows as KMV Portfolio Manager does. However, there are also statistics for recovery rates of an obligor's total exposure.

Another disadvantage might be the use of exposure bands because as also affirmed by CSFB (1997), introducing exposure bands into the model correspond to an approximation of the risk capital. On the other hand, CSFB (1997) states that this approximation cannot be crucial in assessing the overall risk if the exposure bandwidth is chosen very small compared to the average exposure of a portfolio and if the number of obligors is large enough.

Crouhy *et al.* (2001) states an additional drawback of the framework as; since interest rates are deterministic in CreditRisk+, similar to the previously discussed models it does not involve market risk, and in contrast to the previous two models, it does not include rating-transition risk. A final drawback of CreditRisk+ is that in comparison to KMV Portfolio Manager and CreditMetrics, it cannot handle derivative products such as options and swaps, adequately (Crouhy *et al.*, 2001).

2.4. McKinsey's CreditPortfolioView

This section explains another credit-risk model called CreditPortfolioView (CPV) introduced by McKinsey & Company. Following sections give the basics of the framework, several calibration tips, and the drawbacks mentioned for this model.

2.4.1. The Framework

CPV is a framework that is based on the econometric modelling of default and transition rates (Bessis, 2002). In fact, CPV develops an econometric model on default rates, embeds a correlation structure inside this model, and uses an operator to translate the information generated from this econometric model into the changes in transition rates. The remaining step of the framework consists of using these new transition rates to obtain new obligor ratings and revaluing credit payments and exposures, accordingly. Moreover, Bessis (2002) states that the framework tries to connect DPs to economic cycles through cyclical dynamics of industries and countries while CreditMetrics, Portfolio Manager, and CreditRisk+ do not deal with this relation directly. Crouhy *et al.* (2001) also point out the presence of the link between DPs and economic cycles by emphasizing how close credit cycles and business cycles progress. In addition, researchers such as Crouhy *et al.* (2001), Bessis (2002), and Smithson (2003) agree that CPV is a model better suitable for speculative grade obligors, for instance, BB and down-rated obligors since they are more sensitive to macroeconomic factors.

Like most of the other credit risk models, CPV also requires the time series of default data. Next, combining this information with the time series of economic variables, it can derive the credit-worthiness of an industry group. CPV requires the examination of a portfolio as several sub-portfolios partitioned due to several criteria, such as credit quality. Furthermore, the framework requires a model for economic variables in order to be able to predict their future values. Then, by using these predicted values, it determines the default probabilities through a logit function. Therefore, default rates change with the factors, and this way firms become correlated because of the common factors and correlated noise terms affecting their default rates.

As a primary step, CPV divides the credit portfolio into N portfolio segments, for instance, with respect to country or industries, and then assigns an index, Y_i , to each segment (Bessis, 2002). Next, it matches these indices to several explanatory variables, X_k 's, which are country-industry specific macroeconomic factors such as the long-term interest rates, government expenses, foreign exchange rates, growth in gross domestic product, and the unemployment rate. Moreover, CPV uses a multifactor model such as the

one in Equation (2.41) in determining the country-industry indices, where the error term is independent of the factors.

$$Y_{i,t+1}(X_1, X_2, \dots, X_k) = \beta_{i,0} + \beta_{i,1}X_{1,t+1} + \beta_{i,2}X_{2,t+1} + \dots + \beta_{i,k}X_{k,t+1} + \epsilon_{i,t+1} \quad (2.41)$$

Then, it models the default rates as a logit function of country-industry indices and therefore the economic variables so that the rates stay in the range of 0 and 1. This type of non-linear models is called logit regression models. Moreover, as Rachev *et al.* (2007) state, these are models used to predict an event probability conditioned on several numerical or categorical factors via cumulative probability function of logistic distribution shown in Equation (2.42) where $y_{i,t+1}$ in our case is the default indicator. In other words, CPV uses such a function to assign a probability to default for a certain country and industry.

$$DP_{i,t+1}(Y_{i,t+1}) = P(y_{i,t+1} = 1 | Y_{i,t+1}) = \frac{1}{1 + e^{-Y_{i,t+1}}} \quad (2.42)$$

This model of CPV is a conditional model while the country-industry index is conditioned on macroeconomic factors, and each macroeconomic factor is conditioned on its observed past values, past error terms, and an error term specific to that factor. Bessis (2002) defines this model as an Auto Regressive Integrated Moving Average (ARIMA) but shows the ARIMA model of a macroeconomic factor by the following equation.

$$X_{k,t+1} = \alpha_1 X_{k,t} + \alpha_2 X_{k,t-1} + \dots + \gamma_1 e_{k,t} + \gamma_2 e_{k,t-1} + e_{k,t+1} \quad (2.43)$$

Equation (2.43), in fact, exhibits an Auto Regressive Moving Average (ARMA) model. Rachev *et al.* (2007) explains the use of ARIMA models as follows. Most of the time, the time series of an economic factor is not stationary, in other words, it has a varying mean and variance over time. Yet, in these cases, time series of the changes in the same factor may have a mean and variance that do not vary with time. Therefore, it might be more reliable to model the series $\Delta X_{k,t}$ instead of $X_{k,t}$. These difference models are called ARIMA models.

Smithson (2003) also states that CPV uses an ARMA model with moving average part ($e_{k,t}$ and $e_{k,t-1}$ in Equation (2.43)) consisting of time series different than $X_{k,t}$'s. Nonetheless, Rachev *et al.* (2007) affirm that these different time series are merely previously observed disturbances (noise, residual or error) for an ARMA model. Besides the approaches of Smithson (2003) and Bessis (2002), for the implementation of CPV, Crouhy *et al.* (2001) suggests the use of a simple Auto Regressive (AR) model, which models the factor to be predicted only with respect to its previous observations (without the addition of previously observed noise terms). Yet, it is accepted that there is no reason for the error terms, $e_{k,t+1}$'s to be independent, and that the error term vector e_{t+1} is identically normally distributed with mean zero and a variance-covariance vector, Σ_e . There are also cross-correlations between country-industry specific error terms ϵ_{t+1} and e_{t+1} . Additionally, country-industry specific error terms are also correlated. Thus, while generating DPs, all error terms are generated simultaneously with regard to their covariances. Furthermore, as Crouhy *et al.* (2001) chooses to do for simplicity, it is useful to define a variance-covariance matrix that involves all correlations as shown below.

$$\Sigma = \begin{bmatrix} \Sigma_{\epsilon} & \Sigma_{\epsilon,e} \\ \Sigma_{e,\epsilon} & \Sigma_e \end{bmatrix}$$

$\Sigma_{\epsilon,e}$ and $\Sigma_{e,\epsilon}$ are cross-correlation matrices. As a result, we can now generate error vector, ξ_{t+1} , seen below by Cholesky decomposition, which is explained in detail in Chapter 4.

$$\xi_{t+1} = \begin{bmatrix} \epsilon_{t+1} \\ e_{t+1} \end{bmatrix}$$

After the determination of common DP for speculative grade obligors within each country and industry, CPV compares these probabilities to their long-term historical averages (which can be obtained, i.e. from Moody's or Standard&Poor) in order to determine the state of the economic cycle. If $DP_{i,t+1}$ is greater than its long-term average ϕDP for a speculative grade obligor, then it means the economy is in recession (Crouhy *et al.*, 2001). Otherwise, it means the economy is in expansion. Crouhy *et al.* (2001) state that with $DP_{i,t+1}/\phi DP$ ratios, CPV adjust the rating-transition matrix, ϕM , which is based on a

wide historical data, such as one of Standard&Poor or Moody's. In recession, the framework uses a shift operator to shift the probabilities in the matrix ϕM through right and to left, otherwise (Smithson, 2003). The idea behind is that in case of a recession, downgrades in ratings and default frequencies increase but when in expansion, upgrades increase. In this sense, KMV and McKinsey adopt the same idea that both default rates and the rating-transition rates vary over time (Crouhy *et al.*, 2001). Yet, Portfolio Manager uses a microeconomic approach as explained in Chapter 2.2.

So, CPV basically generates rating-transition matrices conditional on the economical state. The framework simulates macroeconomic factors and country-industry indices by generating random error terms, and then generates the distributions of default and transition rates for each rating group within each industry of each country (Crouhy *et al.*, 2001). Furthermore, every time the framework simulates a default probability for each country-industry group in that credit portfolio, it calculates a loss rate for the whole portfolio by the following equation;

$$LR = \sum_i DP_i \times w_i$$

w_i is the exposure weight of that portfolio segment in that credit portfolio. As Bessis (2002) explains, from the cumulated frequencies of the generated portfolio loss rates, CPV determines the loss percentiles, as well.

Another feature of CPV is that it can generate multi-period transition matrices by generating transition matrices for consecutive time periods. Let, for instance, M_t be a rating-transition matrix CPV generates for period t . Then, Crouhy *et al.* (2001) show the calculation of multi-period transition matrix by the following equation.

$$M_T = \prod_{t=1}^T M_t$$

Hence, by simulating the matrix M_T , one can also discover the distribution of cumulative conditional default and rating-transition rates for any rating class over any time horizon.

Finally, in the valuation step, Smithson (2003) asserts that CPV can use either a mark-to-market approach by utilizing credit spreads just like Portfolio Manager does or a default-only approach and can focus only on the loss resulting from default. As also stated by Smithson (2003), we here give the steps of the framework as follows.

- (i) Simulate the future values of macroeconomic factors
- (ii) Use these to determine the country-industry indices and the corresponding default rates
- (iii) With these simulated default rates and a shift operator defined on ϕM , adjust the historical averages of transition rates to obtain M_t
- (iv) Simulate new ratings of the obligors via M_t and revalue each portfolio segment
- (v) Sum across all portfolio segments in order to achieve the total portfolio value for time period t
- (vi) Replicate steps (i) to (v) for the whole planning horizon
- (vii) Replicate steps (i) to (vi), a large number of times for the precision of the estimates.

2.4.2. Calibration

There are three calibration issues for CPV: calibration of the logit regression model, fitting the data to ARMA model, and the definition of the shift operator on ϕM .

Rachev *et al.* (2007) assert that because the logit regression model is not a linear model, it is not possible to fit data to this kind of models by least squares methods. Thus, Rachev *et al.* (2007) suggest the use of Maximum Likelihood methods. Simply, we need to find such parameter values that maximize the likelihood function seen in Equation (2.44), where k and i refer to the k^{th} country-industry segment and i^{th} observation, respectively. Moreover, F is a cumulative probability function for default, which in logit models is the function in Equation (2.42), and y_i is a default indicator.

$$\ell = \prod_i F(\beta_{k,0} + \beta_{k,1}X_{1,i} + \dots + \beta_{k,n}X_{n,i})^{y_i} \times \left(1 - F(\beta_{k,0} + \beta_{k,1}X_{1,i} + \dots + \beta_{k,n}X_{n,i})\right)^{1-y_i} \quad (2.44)$$

Next calibration issue is the parameter estimation of ARMA models. To estimate the parameters of an ARMA model, Rachev *et al.* (2007) propose three approaches with three different estimators, namely, Yule-Walker Estimator, Least Squares Estimator, and Maximum Likelihood Estimator. Rachev *et al.* (2007) give details of these approaches and also explain the validation procedures of ARMA models.

The last issue is about how to define a shift parameter conditional on the state of the economy. Smithson (2003) suggests the use of past data of upgrades and downgrades. In other words, he uses historical rating-transition matrices to find an adequate shift operator based on the economical state.

2.4.3. Drawbacks from Literature

There are a few limitations of the CreditPortfolioView framework mentioned in literature. First, the framework requires a wide data set for the estimation of the model parameters. Also, Crouhy *et al.* (2001) highlight that it is not easy to find reliable default data for each industry sector within each country.

Another drawback of the framework is that although for this thesis, we tried to gather the necessary information to implement the model from different references, Bessis (2002) points out that CPV gives only the model of default rates and the correlation structure while leaving the calibration and the necessary statistical fits to the end-users.

Final limitation we came across in literature is the rate shifting procedure. Crouhy *et al.* (2001) question whether such a shifting or adjusting procedure on transition rates is superior to a simple Bayesian model, where the procedure is carried out by the expertise of a credit department of the bank regarding the current state of the economy.

3. BASEL II CAPITAL ACCORD

Basel II developed by a group worldwide highly reputable in financial and banking industry is essentially a framework that consists of standards for measuring, managing and supervising capital requirements of a bank. Basel Committee on Banking Supervision (BCBS), a committee that is a part of BIS, announced this new capital adequacy framework thought to cope both with financial innovations, which 1988 Basel I Accord no longer could and with uprising risk complexity of the financial world, in June 1999 (Ong, 2005). In other words, the new accord is an enhanced and sounder version of Basel I Accord. Moreover, BCBS is a board, which was initially established in 1975 by the heads of G-10 countries' central banks and is now formed by senior representatives of central banks of 13 countries, namely, Belgium, Canada, France, Germany, Italy, Japan, Luxembourg, Netherlands, Spain, Sweden, Switzerland, the United Kingdom, and the United States, and senior representatives of executive authorities in banking (BCBS, 2006). The committee meetings take place in Basel, Switzerland, and that is why BCBS announces every standard with the name, Basel.

Basel I Capital Accord was the first standard introduced for internationally active banks of G-10 countries to manage their risk and capital adequacy. Yet, the more complex risk structure that had evolved since 1988 and also the financial innovations that had taken place till 1999 gave rise to a need for a new framework for the regulatory capital requirements (BCBS, 1999). Contrary to Basel I, Basel II adds capital requirements for credit risk. Capital requirements of Basel II are recommendations for how much capital a bank should hold in order to be able to cover its unexpected loss. Furthermore, Basel II also assesses capital for market and operational risk.

BCBS (1999) emphasizes that Basel II should be adapted to the developments and changes in the financial world and further defines the supervisory objectives of the new accord as follows. The new capital accord should

- always support and advance the coherency and robustness of the banking and financial industry,

- never be an important cause of competitive inequality in the market, in fact, it should try to maintain and also improve the competitive equality,
- form a framework that handles the risks an internationally active bank bears more thoroughly,
- concentrate on internationally active banks while its fundamentals should also be capable of addressing the risks of banks with less complex structure.

In addition to *Minimum Capital Requirements* first introduced by Basel I, in Basel II BCBS introduces two new concepts, namely, *Supervisory Review Process* and *Market Discipline*, to explain the supervisory process, widens the scope of *Minimum Capital Requirements*, and calls these three key concepts as three pillars of Basel II. Following subsections explain the three pillars and give the links between Basel II and the models that are used commonly in practice.

3.1. Minimum Capital Requirements

For the calculation of minimum capital requirements of a bank, Basel II considers three risk sources, which are credit, market, and operational risk. Market risk is the risk that total value of a bank's investments or assets will be subject to a level of decrease because of the change in market factors (BCBS, 2006). Furthermore, BCBS (2006) affirms that Market risk involves foreign exchange risk, risk due to "interest rate instruments and equities in the trading book", and "commodities risk throughout the bank". Market risk also covers the counterparty risk due to swaps, forwards, futures, forward rate agreements, options, and credit derivatives, such as credit default swaps. Basel II suggests two approaches for measuring the market risk; the Standardized Measurement Method and the Internal Models Approach.

Operational risk, on the other hand, is the risk of losses that incapability or failures of a bank's employees, internal systems or processes, or certain external factors cause (BCBS, 2006). Moreover, the Basic Indicator Approach, the Standardized Approach, and the Advanced Measurement Approaches are the methodologies the Basel II framework offers for the operational risk measurement.

BCBS (2006) recommends the ratio between the regulatory capital, which is the minimum capital, recommended by Basel II, that a bank should hold in order to cover its risk (unexpected portion of loss), and total risk weighted assets to be not lower than eight per cent. Basel II uses this minimum capital ratio of eight per cent in the calculation of minimum capital requirements. Finally, to find the total capital requirement, Basel II simply sums the capital requirements for all three risk sources; operational, market, and credit. Furthermore, there are two approaches to calculate the minimum capital requirement for credit risk. The following two sections explain the basics of these approaches, which are standardized and internal ratings-based approaches.

3.1.1. Standardized Approach

In the standardized approach, Basel II divides the assets into risk classes, and for the assets within the same risk class it assigns a fixed risk-weight (these weights are given by the framework) in order to define the riskiness of each asset from that class. For instance, risk weight of an asset changes whether the claim is on a sovereign, a multilateral development bank, a bank, a public sector entity, a securities firm, or a corporate. It also changes when the loan is secured by residential property, or by commercial real estate. Credit exposures due to, for example, credit card debts, leases such as installment loans, auto loans and educational loans, and personal loans are considered in a retail portfolio and weighted accordingly. In addition, Basel II charge higher risk weights to low rated assets while they contribute higher risks. As also BCBS (2006) explains, Basel II supports the credit risk mitigation techniques, such as collateralization, guarantees, and credit derivatives but also assigns risk weights to these products while foretelling that they may still contribute different risks. Moreover, risk weights vary with respect to credit worthiness of an asset, as well. For that manner, standardized approach of Basel II suggests the use of an external credit assessment institution's (i.e. Standard&Poor) mapping of risk characteristics of assets to different credit ratings. However, these external credit assessment institutions must be approved by the regulatory supervisors. After the assignments of risk weights, the regulatory capital is calculated by the following formula where w_i is the corresponding risk weight for the i^{th} asset, and EAD is the exposure-at-default, which is defined as the remaining exposure due to that asset in the event of a default in Basel II.

$$\text{Regulatory Capital} = \sum_i w_i \times EAD_i \times 0.08$$

3.1.2. Internal Ratings-Based Approach

Basel II internal ratings-based (IRB) approach allows banks to use their own estimates of default probability (DP), loss given default (LGD), EAD, and effective maturity (M) in calculating the capital requirement of a particular credit exposure if the data, models, and the procedures used by those banks to estimate these risk measures are approved by regulatory supervisors (BCBS, 2006). Yet, supervisors may see it necessary for a bank to use supervisory value for some risk components instead of internal estimates (BCBS, 2006).

First, BCBS (2006) states that exposures of a bank must be classified under five different asset classes; corporate, sovereign, bank, retail, and equity. Then, for the calculation of risk parameters specified in the previous paragraph, the risk-weighted assets and capital requirements, Basel II offers two approaches, which are Foundation IRB (F-IRB) approach and Advanced IRB (A-IRB) approach. For corporate, sovereign, and bank exposures, a bank, under F-IRB approach, uses its own estimates of only DP and certain values defined by Basel II for the other risk parameters but under A-IRB approach it uses its own data and models to also estimate LGD, EAD, and M. In any case, the regulatory capital is calculated as capital requirement, K , times EAD for that exposure. Moreover, under IRB approach, the asset correlation, ρ , capital requirement, K , and risk-weighted assets, RWA are calculated by the following formulas where b is a maturity adjustment, CEL is the conditional expected loss, and Φ and Φ^{-1} are cumulative density function (cdf) and inverse cdf of standard normal distribution, respectively. For sovereign, corporate, and bank exposures:

$$\rho = 0.12 (1 - e^{-50 \times DP}) / (1 - e^{-50}) + 0.24 [1 - (1 - e^{-50 \times DP}) / (1 - e^{-50})]$$

$$b = (0.11852 - 0.05478 \times \ln(DP))^2$$

$$CEL = LGD \times \Phi \left[\frac{\Phi^{-1}(DP)}{\sqrt{1-\rho}} + \sqrt{\frac{\rho}{1-\rho}} \Phi^{-1}(0.999) \right] \quad (3.1)$$

$$K = [CEL - DP \times LGD] \times \frac{1 + (M - 2.5) \times b}{1 - 1.5 \times b}$$

$$RWA = K \times EAD \times 12.5$$

For retail exposures:

$$K = CEL - DP \times LGD$$

$$RWA = K \times EAD \times 12.5$$

In RWA calculations, 12.5 is the reciprocal of minimum capital ratio, eight per cent. In addition, effective maturity, M, under A-IRB approach is generally calculated as:

$$M = \frac{\sum_i t_i C_i}{\sum_i C_i}$$

where C_i is the cash flow in the i^{th} period. Furthermore, within Basel II framework, the asset correlation for retail exposures is defined as constant in case of mortgage and revolving retail exposures. For other types of retail exposures, it is calculated by the following equation.

$$\rho = 0.03 (1 - e^{-35 \times DP}) / (1 - e^{-35}) + 0.16 [1 - (1 - e^{-35 \times DP}) / (1 - e^{-35})]$$

Consequently, the asset correlation function of Basel II is such that the correlation always stays between 12 per cent and 24 per cent for sovereign, corporate, and bank exposures, and between three per cent and 16 per cent for retail exposures except mortgage and revolving retail exposures.

Besides the previously referred exposure types, BCBS (2006) suggests two different approaches for the calculation of risk-weighted assets for equity exposures and calls them as market-based approach and PD/LGD approach.

3.1.2.1. The Statistical Model behind the IRB Approach. BCBS neither gives the mathematical background of Basel II framework in detail nor explains the model behind this framework. Yet, Gürtler et. al (2008) reveal several key concepts of the capital accord as follows.

First, *CEL* in Equation (3.1) is nothing but a Value-at-Risk (VaR) statistics calculated via an asymptotic single-factor model, which Basel II defines on normalized asset returns as shown in Equation (3.2) where X_k is the normalized asset return of the k^{th} asset, Z is the common systematic risk factor affecting all assets or exposures, ϵ_k is the nonsystematic portion of the risk which is asset-specific, and ρ_k is the correlation for that asset. Thus, the correlation of two assets is simply $\sqrt{\rho_i}\sqrt{\rho_j}$. In addition, systematic factors are factors that constitute the non-diversifiable part of the risk. Unlike nonsystematic risk factor, they can be deterministic. Systematic factors can be chosen such that they give the state of the economy. For instance, a country index can be chosen as a systematic factor.

$$X_k = \sqrt{\rho_k}Z + \sqrt{1 - \rho_k}\epsilon_k \quad (3.2)$$

The factor model of Basel II is asymptotic because it assumes that the portfolio is asymptotically fine-grained with infinitely many small exposures resulting in the perfect diversification of nonsystematic risks. Moreover, the model assumes that both risk factors are standard (since the return is normalized) normally distributed, and so does the asset return. Next, if we define the *DP* as the probability that the normalized asset return falls below some value, x_k , then x_k should be equal to $\Phi^{-1}(DP)$. Hence, the *DP* given the value of the global systematic factor, Z , can be expressed as:

$$P(X_k < \Phi^{-1}(DP) | Z) = P(\sqrt{\rho_k}Z + \sqrt{1 - \rho_k}\epsilon_k < \Phi^{-1}(DP) | Z)$$

$$\begin{aligned}
&= P\left(\epsilon_k < \frac{\Phi^{-1}(DP) - \sqrt{\rho_k}Z}{\sqrt{1 - \rho_k}} \mid Z\right) \\
&= \Phi\left[\frac{\Phi^{-1}(DP) - \sqrt{\rho_k}Z}{\sqrt{1 - \rho_k}}\right]
\end{aligned} \tag{3.3}$$

Equation (3.3) is the second product term of *CEL* calculation in Equation (3.1) given the 0.999-quantile of Z . In other words, if we stress the systematic factor to obtain a downturn DP (corresponding for instance to a downturn in economy), we can calculate a *CEL* as defined in Basel II, and then determine the VaR and the regulatory capital (without a maturity adjustment) of a credit portfolio by the following equations since we adopt a single factor model. The second term in Equation (3.4) is the expected loss of the credit portfolio.

$$\begin{aligned}
VaR_\alpha &= \sum_k EAD_k \times LGD_k \times \Phi\left[\frac{\Phi^{-1}(DP) - \sqrt{\rho_k}\Phi^{-1}(1 - \alpha)}{\sqrt{1 - \rho_k}}\right] \\
Regulatory\ Capital &= VaR_\alpha - \sum_k DP_k \times LGD_k \times EAD_k
\end{aligned} \tag{3.4}$$

3.2. Supervisory Review Process and Market Discipline

This section briefly explains the two new but complementary pillars of Basel II. The pillar two of Basel II is the supervisory review process. The objective of this framework is to make sure that the capital a bank holds complies with the bank's total risk profile and its general risk policy (BCBS, 1999). Thus, by monitoring the bank's activities, models, strategies, and capital position, supervisors can detect whether the capital position of a bank does not compensate its risk, at an early stage as pointed out by BCBS (1999). Supervisors should foresee when a bank's capital adequacy will fall below minimum requirements or not be sufficient anymore due to certain changes in its risk profile and act early to intervene the process. Essentially, supervisors, should check whether a bank's capital ratios are consistent with minimum ratios required by Basel II, act early in case of any inconsistency and insufficiency in the risk management process, validate the internal

models, procedures, and the data used for risk estimates. In other words, supervisors should validate a bank's methodologies and data used for measuring operational, market, credit, interest rate, liquidity, and other types of risk and verify its capital assessments to these risk sources. Moreover, supervisors can and should require a bank to process over the minimum requirements when necessary and ask the bank to use regulatory values instead of one or more internal estimates. This way banks can have more efficient and robust risk valuation and management systems.

The third and the final pillar is market discipline, which strengthens the other two pillars in banking. In fact, it can increase the reliability of any financial system by contributing to capital adequacies and any supervisory process. Generally, it gives banks incentives to operate in a safer and more robust way (BCBS, 1999). Fundamentally, it is helpful to foresee future losses due to a bank's risky assets and future market conditions, and to assess the risk management system of a bank. Also, supervisors see this pillar as a strong and crucial complementing tool of risk assessment and management processes. To conclude, BCBS (2006) presumes that presenting disclosures, which will enable the other market players to assess capital structure and risk measurement processes of a bank, "is an effective means of informing the market about a bank's exposure to those risks and provides a consistent and understandable disclosure framework that enhances comparability".

3.3. Link between Basel II and the Models Used in Practice

There are studies, such as Gürtler *et al.* (2008) and Jacobs (2004) that try to link the widely used credit risk models to the Basel II framework. For instance, Jacobs (2004) underlines the presence of a connection between the Basel II framework and the CreditMetrics model when combined with CreditRisk+. Moreover, as explained previously Basel II uses a single-factor asset value model to introduce correlation. The model in Equation (3.2) is one factor version of the multi-factor model of CreditMetrics seen in Equation (2.1) while again systematic factor is assumed to have a normal distribution. However, Basel II adopts a pure default model but uses the idea of a default threshold as in Equation (2.6).

Gürtler *et al.* (2008) state that the asymptotic single factor model of Basel II is a good approximation of a multifactor model in case of perfectly diversified portfolios. Yet, we can adjust the asset value based multifactor models suggest to get a similar model to the one of Basel II. Furthermore, BCBS (2006) also allows the use of multifactor models to calculate capital requirements for the exposures that Basel II calls equity exposures. Under IRB approach of Basel II, we believe that the frameworks, Portfolio Manager, CreditRisk+, and CreditPortfolioView can be applied to a credit portfolio to estimate the default probabilities of obligors, and then, these estimates can be directly used in minimum capital requirement calculations. Also, it may be reasonable to implement a single-factor CreditMetrics model or modify any multifactor asset model, as Gürtler *et al.* (2008) suggest, to obtain a single-factor model in order to calculate conditional default probabilities like in Equation (3.3) and use these risk measures consistent with Basel II to calculate the regulatory capital under IRB approach. Yet, all four models explained in this thesis can be used to find an economic capital, and then this capital can be compared to the regulatory capital of Basel II without a maturity adjustment.

4. OUR CALIBRATION OF CREDITMETRICS MULTIFACTOR MODEL

During the implementation of CreditMetrics, because we came across several restrictions, due to lack of data or the types of the systematic factors we chose, namely country indices, and country-industry indices, we decided to alter a few steps of the CreditMetrics' multi-factor modeling and developed a regression-based approach. In the following chapter, we try to calibrate a small credit portfolio to real data with this approach to determine the factor loadings of the CreditMetrics multifactor model. For the method we used, daily or weekly data of the stocks and of the systematic factors are necessary. This section explains the methods and estimation of the parameters necessary for this modeling.

In determining the factor loadings of a multi-factor model, although CreditMetrics obtains these loadings as explained in Section 2.1.3, we choose to use directly the regression coefficients as factor loadings and next, we standardize the equity returns (our latent variables) resulting from these loadings. After this normalization process, we achieve the final loadings. In addition, in contrast to CreditMetrics, we prefer to use log-returns instead of percent returns. Since we will fit a regression, we first need to transform the daily or weekly data of the obligors' equities, country indices, and country-industry indices into log-returns. Furthermore, by using historical data of these systematic factors and obligors' equities, we calculate the log-returns by Equation (4.1), where \ln indicates the natural logarithm function.

$$r_t = \ln\left(\frac{E_t}{E_0}\right) \quad (4.1)$$

E_t is the value of the equity or index after t units of time with an initial value of E_0 . If we think E_t in yearly basis, then it matches the definition of our latent variable, X_k in Equation (4.2). Next, we simply regress the daily or weekly log-return data of the systematic factors that an obligor is associated (e.g., according to in which countries and industries a firm operates) over the log-returns of the related obligor's equity and obtain a regression

function like the one shown in Equation (4.2) where Z_i is a systematic factor, for each obligor in the portfolio. In our case, we have two systematic factors for each obligor. This method requires us to fit number of obligors many regressions to define a multi-factor model. The motive behind our implication is that CreditMetrics uses the asset allocations among industries in order to determine the systematic factor loadings. However, we used country and country-industry indices as systematic factors during this study. Hence, it is not explicit how one can agree on such allocations among such factors of different kinds. In case we decide to assign one for the allocation (w_{kj} in Equation (2.9)) of both factors, indicating a full allocation to a single country and a single industry within that country, then regarding the CreditMetrics' multi-factor model, for the obligors within the same country-industry group, the only parameter that makes the distinction between factor loadings is R^2 statistics. In other words, factor loadings become linearly dependent. So, this way the model cannot be reliable since it becomes a single factor model, and the correlation structure might highly deviate from the true structure.

$$X_k = a_{k1}Z_1 + a_{k2}Z_2 + \dots + a_{kd}Z_d + \epsilon_k \quad (4.2)$$

Moreover, since we assume that asset value changes follow a Geometric Brownian Motion, r_t in Equation (4.1) is normally distributed with mean $(\mu_E - (\sigma_E^2/2))t$ and variance $\sigma_E^2 t$. Hence, in order to standardize (or normalize) these log-returns we can use Equation (2.8). Yet, because we determine the equity log-returns through a multi-factor model, in order to normalize these returns, we need to estimate variance-covariance matrix of the systematic factors so that afterwards we can utilize the Cholesky decomposition of this matrix and achieve the variance of each obligor's equity log-return. In order to do so, first step is to estimate the volatility parameters of the systematic factors by applying Equation (4.3) and (4.4). We could also estimate the drift parameters but it will not be necessary for our calculations.

$$v = (\mu_E - (\sigma_E^2/2))t = \frac{\sum_{i=1}^n r_{t_i}}{n} \quad (4.3)$$

$$\sigma_E^2 = \frac{\sum_{i=1}^n (r_{t_i} - v)^2}{n - 1} \quad (4.4)$$

By using these equations together with the covariances of the systematic factors, we can achieve the variance of each obligor's equity log-return. Nevertheless, if, for instance, daily returns are used, the volatility (σ_E) and also the drift (μ_E) we estimate with these equations will be in daily terms. Yet, we are interested in yearly log-returns. To achieve annual drift and volatility, we need to multiply these estimates by 258, number of trading days a year.

After the volatility estimation, next step is to estimate the covariances from the time series data of systematic factors and thus finally obtain the variance-covariance matrix of these factors. However, if we work with daily returns, we again need to multiply these covariances by 258. The following step is the normalization of the latent variable, X_k seen in Equation (4.2). In fact, Equation (4.2) and (4.1) are very similar and we will make use of this fact in factor loading calculations. First of all, when we compare these two equations, b_k in Equation (4.2) appears to be equal to one. Moreover, we know that Z_j has a multivariate normal distribution with a variance-covariance matrix, Σ , and that each normal variable has a mean and a standard deviation, σ_j . Since we would like to standardize X_k , we can directly generate Z_j 's with a zero mean instead of subtracting the mean from the generated latent variable. This is why we do not need to estimate the drift parameters. Then, in order to find the variances, we utilize Cholesky decomposition and convert the dependent factor loadings into independent factor loadings. First, by using Cholesky decomposition we write X_k 's as follows;

$$X_k = [a_{k1} \quad a_{k2} \quad a_{k3} \quad \dots][\mathbb{L}] \begin{bmatrix} \bar{Z}_1 \\ \bar{Z}_2 \\ \bar{Z}_3 \\ \vdots \end{bmatrix} + \epsilon_k \quad (4.5)$$

where \bar{Z}_j 's are independent standard normal variables, and \mathbb{L} is the Cholesky decomposition of Σ . So, what we call as independent factor loadings are,

$$[\bar{a}_{k1} \quad \bar{a}_{k2} \quad \bar{a}_{k3} \quad \dots] = [a_{k1} \quad a_{k2} \quad a_{k3} \quad \dots][\mathbb{L}] \quad (4.6)$$

Then, the variance of the latent variable is nothing but,

$$\sigma_k^2 = \sum_{j=1}^d \bar{a}_{kj}^2 + 1 \quad (4.7)$$

To standardize the log-returns, we need to divide them by the total volatility (σ_k) of the latent variable as in Equation (4.8) and (4.9). The addition of one in Equation (4.7) is due to the IID normally distributed error term, ϵ_k . Now for each obligor we can easily determine systematic and non-systematic factor loadings by Equation (4.8) and (4.9), respectively and generate log-returns via correlation matrix of the systematic factors instead of Σ while we already include the factor volatilities (σ_j 's) inside the factor loadings. Therefore, in the end we have a multi-factor model for correlation that looks analogous to the one of CreditMetrics but is based on a regression analysis rather than an intuitive allocation of factors.

$$\hat{a}_{kj} = a_{kj} \frac{\sigma_j}{\sigma_k} \quad (4.8)$$

$$\hat{b}_k = 1/\sigma_k \quad (4.9)$$

5. A REAL-WORLD CREDIT PORTFOLIO

Following the aims stated in Chapter 1, in this chapter, we create a real-world portfolio that consists of 25 real firms, of which we could find sensible data. We chose these 25 firms with respect to whether there is available data on the country-industry indices they belong to. By using the historical data of these 25 firms' equity returns, we try to implement our calibration method explained in Chapter 4 to define a multifactor model for this credit portfolio. Later in the following chapters, through a CreditMetrics approach explained in Chapter 2.1 in detail, we simulate this credit portfolio with random exposures assigning fixed values to the other simulation parameters and investigate the resulting default loss distributions together with several risk measures and the portfolio value distributions. Then, we utilize the factor loadings of this portfolio with obligors from the real world to artificially generate a multifactor model for a much larger credit portfolio in order to achieve results that are more realistic and present our simulation results over that artificial large credit portfolio. In the meantime, we also give and explain our *R* codes for every step of our implementations.

5.1. Portfolio

Our small credit portfolio consists of 25 publicly traded firms from both the United States (USA) and United Kingdom (UK) from four different industries, namely, Telecommunication, Technology, Aerospace, and Pharmacy. All the data we used were obtained from *Bloomberg Data Services*. Below in Table 5.1, we give the information of 25 obligors we selected for our credit portfolio. We chose to use Standard & Poor's and Fitch's long-term issuer credit ratings. However, Sage Group appeared to be a non-rated cooperation. During this study, we assumed a rating of AA for Sage Group.

Table 5.1. Selected 25 obligors

Obligor	Stock Symbol	Rating	Country-Industry Indication Matrix					
			USA	UK	TEL	TECH	AERO	PHARM
BT Group	BTA	BBB+	0	1	1	0	0	0
Cable & Wireless	CW	BB-	0	1	1	0	0	0
Vodafone Group	VOD	A-	0	1	1	0	0	0
Sage Group	SGE	AA-	0	1	0	1	0	0
Invensys	ISYS	BB+	0	1	0	1	0	0
The Boeing Company	BA	A+	1	0	0	0	1	0
General Dynamics Corporation	GD	A	1	0	0	0	1	0
Goodrich Corporation	GR	BBB+	1	0	0	0	1	0
Honeywell International Inc.	HON	A	1	0	0	0	1	0
Lockheed Martin Corporation	LMT	A-	1	0	0	0	1	0
Northrop Grumman Corporation	NOC	BBB+	1	0	0	0	1	0
Precision Castparts Corporation	PCP	BBB+	1	0	0	0	1	0
Raytheon Company	RTN	A-	1	0	0	0	1	0
United Technologies Corporation	UTX	A	1	0	0	0	1	0
Abbott Laboratories	ABT	AA	1	0	0	0	0	1
Allergan Inc.	AGN	A	1	0	0	0	0	1
Bristol-Myers Squibb Company	BMY	A+	1	0	0	0	0	1
Eli Lilly and Company	LLY	AA	1	0	0	0	0	1
Johnson & Johnson	JNJ	AAA	1	0	0	0	0	1
Merck & Co. Inc.	MRK	AA-	1	0	0	0	0	1
Mylan Inc.	MYL	BB-	1	0	0	0	0	1
Pfizer Inc.	PFE	AAA-	1	0	0	0	0	1
Schering-Plough Corporation	SGP	A-	1	0	0	0	0	1

Table 5.1. continues

Watson Pharmaceuticals Inc.	WPI	BBB-	1	0	0	0	0	1
Wyeth	WYE	A+	1	0	0	0	0	1

5.2. Step by Step Multifactor Modeling in R

This section explains step-by-step how one can obtain a multifactor model via our calibration method in *R*. With respect to the information given in Table 5.1, we decided to choose country (FTSE for UK and S&P 500 for USA) and country-industry indices the firms belong to as regressors. Our data, observed between the years 1998 and 2009, are composed of 2773, 2521, and 2760 daily returns of, respectively, the FTSE-Telecommunication stocks, FTSE-Technology stocks, and S&P 500 Aerospace and Pharmacy stocks.

```
ftseTel<-read.table(file=file.choose(),header=TRUE)
ftseTech<-read.table(file=file.choose(),header=TRUE)
spAero<-read.table(file=file.choose(),header=TRUE)
spPharm<-read.table(file=file.choose(),header=TRUE)
ftseTel.logrt<-getLogreturns(ftseTel)
ftseTech.logrt<-getLogreturns(ftseTech)
spAero.logrt<-getLogreturns(spAero)
spPharm.logrt<-getLogreturns(spPharm)
ftseTel.reg<-fitRegression(ftseTel.logrt)$coefficients
ftseTech.reg<-fitRegression(ftseTech.logrt)$coefficients
spAero.reg<-fitRegression(spAero.logrt)$coefficients
spPharm.reg<-fitRegression(spPharm.logrt)$coefficients
```

Figure 5.1. Regression for the multifactor model

In Figure 5.1, the *R* code for achieving the regression coefficients of Equation (4.2) is shown. For this coding, data should be grouped with respect to country and industry, with country and industry indices appearing in the first columns. Our *R* function *getLogreturns* uses Equation (4.1). Also, *fitRegression* function uses the bundled function, *lm* of *R* for

linear regression but it also returns R^2 statistics together with the regression coefficients as seen in Figure 5.2.

```

fitRegression<-function(returns,n=2,omitIntercept=TRUE){
Rsquare<-0
betas<-matrix(0,(dim(returns)[2]-n),(n+1))
for(i in 1:(dim(returns)[2]-n)){
  if(omitIntercept)
    regres<-lm(returns[,i+n]~0+returns[,1:n])
  else
    regres<-lm(returns[,i+n]~returns[,1:n])
  betas[i,]<-regres$coefficients
  anovaTable<-anova(regres)
  Rsquare[i]<-anovaTable[1,2]/sum(anovaTable[,2])
}
list(coefficients=betas,Rsquare=Rsquare)
}

```

Figure 5.2. *R* function to obtain regression coefficients and R^2 statistics

Next step is to obtain the variance-covariance matrix of country and industry indices but because we work with daily log-returns, we need to multiply this matrix by 258 in order to attain the yearly variance and covariances. We determined this variance-covariance matrix by the built-in *R* function, *var*, by using a matrix that includes daily log-returns of country and country-industry indices as input of this *R* function. It is possible to generate multi-variate normal variables with zero mean instead of subtracting the total mean from the obtained latent variable, which in our case is the asset log-return. To standardize the asset returns, we need to divide them by the total variance of the latent variable. In *R*, the standardization is performed as seen in Figure 5.3, where *chol* is the *R* function for Cholesky decomposition and following command lines are direct implementation of Equation (4.7), (4.8), and (4.9), successively but *W* is the factor loadings matrix including all obligors.

```

W<-matrix(0,25,6)
W[1:3,2:3]<-ftseTel.reg
W[4:5,c(2,4)]<-ftseTech.reg
W[6:14,c(1,5)]<-spAero.reg
W[15:25,c(1,6)]<-spPharm.reg
L<-t(chol(varcov))
variances<-rowSums((W%*%L)^2)+1
W<-t(t(W)*sqrt(diag(varcov)))/sqrt(variances)
b<-1/sqrt(variances)

```

Figure 5.3. Normalization of the asset returns

In Table 5.3, we reveal the factor loadings of this 25 obligor credit portfolio achieved after normalization, and in Table 5.4, the independent factor loadings obtained by Equation (4.6) (via Cholesky decomposition of the systematic factors' correlation matrix in Table 5.2). Moreover, in these tables, Tel, Tech, Aero, and Pharm are used as aliases for Telecommunication, Technology, Aerospace, and Pharmacy, respectively. In addition, the R^2 statistics of these regression functions change between 18 per cent and 70 per cent. Due to our observations, in general, the higher the rating is, the higher is the R^2 . Probably, the reason behind is that higher rated firms have a higher weight in the corresponding country and industry index calculations and thus are influenced by (or influence) the index more severely.

Table 5.2. Correlation matrix of the systematic factors

	S&P 500	FTSE	FTSE-Tel	FTSE-Tech	S&P-Aero	S&P-Pharm
S&P 500	1	0.1666873	0.120928	0.053435	0.71532	0.6404117
FTSE	0.1666873	1	-0.06029	-0.0276	0.115844	0.123425
FTSE-Tel	0.1209284	-0.060289	1	-0.03524	0.093363	0.1111678
FTSE-Tech	0.0534352	-0.0276	-0.03524	1	0.006575	0.0071613
S&P-Aero	0.7153195	0.1158437	0.093363	0.006575	1	0.4864952
S&P-Pharm	0.6404117	0.123425	0.111168	0.007161	0.486495	1

In Table 5.3, it is observed that in the USA column there are several values that are close to and not significantly different than zero.

Table 5.3. Factor loadings for 25 obligors

Obligors	Systematic Factor Loadings						Idiosyncratic Factor Loadings
	USA	UK	TEL	TECH	AERO	PHARM	
BTA	0	0.1425806	0.128799	0	0	0	0.9824947
CW	0	0.1793713	0.1029657	0	0	0	0.9795157
VOD	0	0.1199214	0.2344714	0	0	0	0.9664535
SGE	0	0.0382374	0	0.3190158	0	0	0.9473332
ISYS	0	0.2069573	0	0.1097896	0	0	0.972815
BA	-0.0310626	0	0	0	0.295543	0	0.9616773
GD	-0.0005587	0	0	0	0.182309	0	0.9833153
GR	0.07751834	0	0	0	0.187744	0	0.9684638
HON	0.04447985	0	0	0	0.244825	0	0.9604702
LMT	-0.0406334	0	0	0	0.214347	0	0.9822752
NOC	-0.018415	0	0	0	0.174636	0	0.9867948
PCP	0.08242359	0	0	0	0.175595	0	0.9703952
RTN	-0.0408795	0	0	0	0.22437	0	0.9803617
UTX	0.06159623	0	0	0	0.19686	0	0.9695897
ABT	0.00706124	0	0	0	0	0.1797816	0.9828544
AGN	0.07361375	0	0	0	0	0.1199923	0.9843116
BMY	0.01129407	0	0	0	0	0.2199564	0.973812
LLY	-0.000011	0	0	0	0	0.2303796	0.9731025
JNJ	-0.0000414	0	0	0	0	0.1735257	0.984834
MRK	-0.0064829	0	0	0	0	0.2439325	0.9708143
MYL	0.1008972	0	0	0	0	0.0839385	0.9858634
PFE	0.00253175	0	0	0	0	0.2460604	0.9688395
SGP	0.00452081	0	0	0	0	0.2485178	0.9678737
WPI	0.08024763	0	0	0	0	0.085059	0.9887278
WYE	-0.0094438	0	0	0	0	0.2472472	0.9704485

Table 5.4. Independent factor loadings of 25 obligors

Obligors	Systematic Factor Loadings						Idiosyncratic Factor Loadings
	USA	UK	IT	TECH	AERO	PHARM	
BTA	0.03934	0.13008	0.12742	0	0	0	0.9824947
CW	0.04235	0.16846	0.10186	0	0	0	0.9795157
VOD	0.04834	0.09911	0.23196	0	0	0	0.9664535
SGE	0.02342	0.02589	-0.0144	0.31801	0	0	0.9473332
ISYS	0.04036	0.2	-0.005	0.10944	0	0	0.972815
BA	0.18035	-0.001	0.00197	-0.0093	0.2063	0	0.9616773
GD	0.12985	-0.0006	0.00121	-0.0058	0.12726	0	0.9833153
GR	0.21182	-0.0006	0.00125	-0.0059	0.13105	0	0.9684638
HON	0.21961	-0.0008	0.00163	-0.0077	0.1709	0	0.9604702
LMT	0.11269	-0.0007	0.00143	-0.0068	0.14962	0	0.9822752
NOC	0.10651	-0.0006	0.00116	-0.0055	0.1219	0	0.9867948
PCP	0.20803	-0.0006	0.00117	-0.0055	0.12257	0	0.9703952
RTN	0.11962	-0.0008	0.00149	-0.0071	0.15662	0	0.9803617
UTX	0.20241	-0.0007	0.00131	-0.0062	0.13742	0	0.9695897
ABT	0.1222	0.00304	0.00638	-0.0045	0.00707	0.13764	0.9828544
AGN	0.15046	0.00203	0.00426	-0.003	0.00472	0.09187	0.9843116
BMY	0.15216	0.00372	0.0078	-0.0055	0.00864	0.1684	0.973812
LLY	0.14753	0.0039	0.00817	-0.0057	0.00905	0.17638	0.9731025
JNJ	0.11109	0.00293	0.00616	-0.0043	0.00682	0.13285	0.984834
MRK	0.14973	0.00413	0.00866	-0.0061	0.00959	0.18676	0.9708143
MYL	0.15465	0.00142	0.00298	-0.0021	0.0033	0.06426	0.9858634
PFE	0.16011	0.00416	0.00873	-0.0061	0.00967	0.18839	0.9688395
SGP	0.16367	0.0042	0.00882	-0.0062	0.00977	0.19027	0.9678737
WPI	0.13472	0.00144	0.00302	-0.0021	0.00334	0.06512	0.9887278
WYE	0.1489	0.00418	0.00877	-0.0062	0.00972	0.1893	0.9704485

6. SIMULATION WITH CREDITMETRICS

Simulation can be used as a tool for examining the credit risk. With the help of simulation, it may be possible to assess the credit portfolio policies of a bank and to more or less foresee to what degree of exposure the risk is bearable. Here in this chapter, we mainly focus on two kinds of simulation; default loss (DL) simulation and mark-to-market (MtM) simulation. Even though CreditMetrics is a mark-to-market model as explained in the previous chapters, DL is a very popular topic among regulators and studies that are based on statistical modeling of credit risk. Furthermore, we give the necessary inputs for these simulations, the *R* coding, and results of the simulations we carried over our 25-obligor credit portfolio explained in the previous chapter.

6.1. Default Loss Simulation

Default loss distribution alone, or in other words loss resulting merely from default is one of the most favorite topics under Credit Risk among academicians. Basel II also focuses essentially on default loss as explained by both the Basel Committee on Banking Supervision (BCBS) (2006) and Engelmann and Rauhmeier (2006). Default loss is nothing but the loss incurred in the event of a default, and in simulation studies this loss is examined yearly. No other loss other than default is handled or handled within these simulations. Moreover, there are also several academicians working on the simulation of default loss, such as Kalkbrener *et al.* (2007), Glasserman and Li (2005), Sak and Hörmann (2008), and Grundke (2008). In conclusion, default loss distribution is one of the main interests of several previous and on-going studies among both practitioners and academicians.

In this section, we give the inputs and parameters we used for the default loss simulation of our 25-obligor credit portfolio, then explain our *R* function, *simDefaultLoss_CM* and finally present the simulation results.

6.1.1. Simulation Inputs

Before the simulation we first need to determine and attain the required inputs, and rearrange these according to the model. There are six inputs of a default simulation:

- (i) Ratings of obligors (or bonds) and regrouping of these ratings as in Table 6.1.
- (ii) Obligors' exposures (or bond par values)
- (iii) Factor loadings of the multi-factor model
- (iv) Rating-transition matrix
- (v) Correlation between systematic factors
- (vi) Parameters of the recovery rate distribution

Table 6.1. Long-term senior debt rating symbols (Schmid, 2004)

Investment-grade ratings		
<i>S&P</i>	<i>Moody's</i>	<i>Interpretation</i>
AAA	Aaa	Highest quality, extremely strong High quality
AA+	Aa1	
AA	Aa2	
AA-	Aa3	
A+	A1	Strong payment capacity
A	A2	
A-	A3	
BBB+	Baa1	Adequate payment capacity
BBB	Baa2	
BBB-	Baa3	
Speculative-grade ratings		
<i>S&P</i>	<i>Moody's</i>	<i>Interpretation</i>
BB+	Ba1	Likely to fulfill obligations ongoing uncertainty
BB	Ba2	
BB-	Ba3	
B+	B1	High risk obligations
B	B2	
B-	B3	
CCC+	Caa1	Current vulnerability to default
CCC	Caa2	
CCC-	Caa3	
CC		
C	Ca	In bankruptcy or default, or other marked shortcoming
D		

For our simulation study, we uniformly generated credit exposures between $1000(\times 10^3)$ and $10000(\times 10^3)$ assuming that these are in dollar terms. Moreover, we used

the factor loadings in Table 5.3. Table 6.2 shows the ratings and credit exposures of our 25 obligors, and their rating groups with respect to Table 6.1. For our simulation study, we

Table 6.2. Rating categories and credit exposures of 25 obligors

Obligor	Rating	Rating Group	Exposure ($\times 10^3$)
BTA	BBB+	4	4027
CW	BB-	5	4427
VOD	A-	3	3184
SGE	AA-	2	9999
ISYS	BB+	5	5573
BA	A+	3	7154
GD	A	3	3853
GR	BBB+	4	9925
HON	A	3	2924
LMT	A-	3	8462
NOC	BBB+	4	9746
PCP	BBB+	4	7999
RTN	A-	3	6602
UTX	A	3	5249
ABT	AA	2	1668
AGN	A	3	6866
BMY	A+	3	8606
LLY	AA	2	8564
JNJ	AAA	1	3883
MRK	AA-	2	7556
MYL	BB-	5	7549
PFE	AAA-	1	2876
SGP	A-	3	9931
WPI	BBB-	4	3181
WYE	A+	3	6221

uniformly generated credit exposures between $1000(\times 10^3)$ and $10000(\times 10^3)$ assuming that these are in dollar terms. Moreover, we used the factor loadings in Table 5.3.

Next, it is necessary to choose or estimate a long-term one-year rating transition matrix. If it is to be estimated, then the historical data to be used should at least cover one whole economic cycle. The transition matrices in Table 6.3 and Table 6.4 are two example matrices provided by two rating agencies. Yet, we chose to use the rating-transition matrix of Standard & Poor (Table 6.3) during our study. Rating transition matrix in Table 6.4 is an implied transition matrix achieved by Gupton *et al.*(1997) via a least-square fit using a cumulative default table of *Moody's*. In addition to these global matrices, there are also regionally estimated transition matrices. For instance, Vazza *et al.* (2008) give the transition matrices of Europe and USA together with a global transition matrix estimated with data until 2007. Schmid (2004) also discusses the need for different transition matrices for different states of the economic cycle; however, concludes that there is not enough data to estimate all the elements of such matrices.

Table 6.3. Rating transition matrix of S&P (Schmid, 2004)

	AAA	AA	A	BBB	BB	B	CCC	Default
AAA	93.06	6.29	0.45	0.14	0.06	0	0	0
AA	0.59	90.99	7.59	0.61	0.06	0.11	0.02	0.01
A	0.05	2.11	91.43	5.63	0.47	0.19	0.04	0.05
BBB	0.03	0.23	4.44	88.98	4.7	0.95	0.28	0.39
BB	0.04	0.09	0.44	6.07	82.73	7.89	1.22	1.53
B	0	0.08	0.29	0.41	5.32	82.06	4.9	6.95
CCC	0.1	0	0.31	0.63	1.57	9.97	55.82	31.58

Another essential input is the correlation matrix of the systematic factors. This matrix can be determined by studying the time-series data of the systematic factors together. The correlation matrix we obtained in R is shown in Table 5.2.

The last simulation input is the required parameters of the recovery rate distribution. Since we adopt CreditMetrics framework, these are the shape parameters of beta distribution. Beta distribution has two shape parameters. In the literature, recovery rates are

determined with respect to asset types and/or credit ratings and/or whether an asset is securitized or not. Yet, for this thesis, we intuitively chose the average recovery rate as 0.4 (40 per cent expected recovery of the exposure in an instant of default) and so the shape parameters of the beta distribution, accordingly (*see* Figure 6.1 for the distribution we assumed for recovery rates).

Table 6.4. An implied transition matrix of Gupton *et al.* (1997)

Initial	Rating at the Year End (%)							
	AAA	AA	A	BBB	BB	B	CCC	Default
AAA	87.74	10.93	0.45	0.63	0.12	0.1	0.02	0.02
AA	0.84	88.23	7.47	2.16	1.11	0.13	0.05	0.02
A	0.27	1.59	89.05	7.4	1.48	0.13	0.06	0.03
BBB	1.84	1.89	5	84.21	6.51	0.32	0.16	0.07
BB	0.08	2.91	3.29	5.53	74.68	8.05	4.14	1.32
B	0.21	0.36	9.25	8.29	2.31	63.89	10.13	5.58
CCC	0.06	0.25	1.85	2.06	12.34	24.86	39.97	18.6

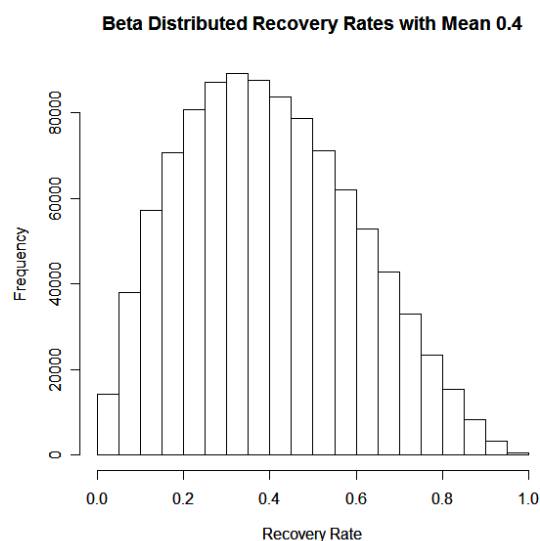


Figure 6.1. Recovery rate distribution

When this distribution (Figure 6.1. Recovery rate distribution) is compared to Figure 6.2, it appears as if it matches the recovery distributions suggested by Gupton *et al.* (1997) for a subordinated bond or a senior subordinated bond.

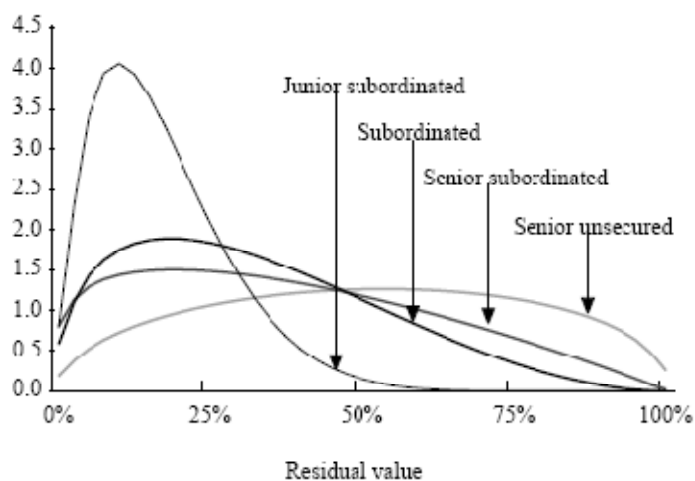


Figure 6.2. Recovery Rate Distribution regarding Seniority (Gupton *et al.*, 1997)

6.1.2. Default Loss Simulation in R

In this section, we give the details of our R function, `simDefaultLoss_CM`. Figure 6.3 shows the input parameters and initial steps of `simDefaultLoss_CM`. Although `varcov` parameter is referred to the variance-covariance matrix of systematic factors, in our examples, instead, we used the correlation matrix of these factors while we utilized the standardized asset log-returns and included the variances into factor loadings as shown in Equation (4.8).

```
simDefaultLoss_CM<-function(
rep=10000,  # number of replications
  horizon=1,  # planning horizon
  rt_prob,    # ratings transition matrix (percent)
  exposures, # exposures or par value of debts/bonds
  ratings,   # recent ratings of the obligors/bonds
```

```

loadings.a, # loadings of systematic risk factors
loadings.b, # loadings of idiosyncratic risk factors
varcov,     # variance covariance matrix of the factors
w_shape1=2, # shape parameter 1 of beta distributed recovery rates
w_shape2=3  # shape parameter 2 of beta distributed recovery rates
){
i_default<-dim(rt_prob)[2]
numberOfFactors<-dim(loadings.a)[2]
numberOfObligors<-length(ratings)
require(MASS)
initial_ratings<-ratings
rt_prob<-rt_prob/100
nRating<-dim(rt_prob)[1]

```

Figure 6.3. Parameter entry and initial steps of *simDefaultLoss_CM*

First, *simDefaultLoss_CM* generates all the systematic factors (Figure 6.4) that will be used throughout the whole simulation at one shot. This way is computationally much more efficient in *R* than generating these factors inside the inner replication one by one or batch by batch in case of a sufficient availability of memory. Next step is to determine the rating thresholds with respect to Equation (2.8) as in Figure 6.4, below. Here we use the *MASS* package of *R* which contains the built-in function *mvrnorm* for the generation of multinormal variates. In fact, we could also use Cholesky decomposition as illustrated in Figure 6.4 right after the generation of systematic factors.

```

systematicFactors<-array(t(mvrnorm(horizon*rep,rep(0,numberOfFactors),varcov)),
dim=c(numberOfFactors,horizon,rep))
# or by Cholesky decomposition;
# cholesky<-t(chol(varcov))
#
#           systematicFactors<-array(rnorm(horizon*rep*numberOfFactors),
c(numberOfFactors,horizon,rep))
temp.thresholds<-matrix(0,nRating,nRating)
for(i in 1:nRating){
  if(i==1)

```

```

        temp.thresholds[i,]<-                                -qnorm(
cumsum(rt_prob[1,(nRating+1):2])[nRating:1],lower.tail=FALSE)
        else{
            temp.thresholds[i,1:(i-1)]<-qnorm(cumsum(rt_prob[i,          1:(i-
1)]),lower.tail=FALSE)
            temp.thresholds[i,i:nRating]<--qnorm(cumsum(rt_prob[i,
(nRating+1):(i+1)][(nRating-i+1):1],lower.tail=FALSE)
        }
        temp.thresholds[i,]<-sort(temp.thresholds[i,])
    }
}

```

Figure 6.4. Generating the systematic factors and achieving the transition thresholds in *R*

Right after starting the replications, one substantial task that *simDefaultLoss_CM* carries out is, in order to speed up the simulation, to assign the corresponding thresholds to each one of the obligors considering the fact that array arithmetic is more efficient in *R* than the use of loops (Figure 6.5).

```

for(year in 1:horizon){
    for(i in 1:nRating)
        thresholds[ratings==i,]<-
matrix(rep(temp.thresholds[i,],length(ratings[ratings==i])),length(ratings[ratings==i]),nRating,byrow=TRUE)

```

Figure 6.5. Threshold assignment to the obligors due to ratings

Following the threshold assignment, the function generates asset returns for the non-defaulted obligors (they may have been defaulted in the previous year, so we don't need to consider their returns anymore) and then, as regards these returns, determine the new ratings as in Figure 6.6.

Finally, the function generates a recovery rate from a beta distribution and sums the EADs of the defaulted obligors net of recoveries for the simulated year (Figure 6.7). Then, the function returns the replicated default losses for each year of the simulation horizon.

```

numberOfNotDefaulted<-sum(ratings!=(i_default+1))
x[ratings!=(i_default+1)]<-
weightedFactors[,year][ratings!=(i_default+1)]+loadings.b[ratings!=(i_default+1)]*rnorm(
numberOfNotDefaulted)
ratingsAndIndices<-getRatings_CMetrics(x,ratings,i_default,thresholds)
ratings<-ratingsAndIndices$ratings

```

Figure 6.6. Obtaining the asset returns and new ratings

```

defaultLoss[year]<-sum(exposures[ratings==i_default]*(1-rbeta(sum(ratings==i_default),
w_shape1,w_shape2))) #EAD = PAR (1- LGD)
ratings[ratings==i_default]<-i_default+1
}
ratings<-initial_ratings
res[j,]<-defaultLoss

```

Figure 6.7. Calculation of default loss in R

6.1.3. Simulation Results

In R-Software Environment, we carried out Monte Carlo Simulations with 100 000 repetitions (the loss resulting from each year is replicated). By a PC with a two GHz Dual Core processor and two GB memory, DL simulation of a 25-obligor credit portfolio takes 6-6.5 min. Moreover, we calculated the Value-at-Risk (VaR), Expected Shortfall (ES), and Economic Capital for each year of the simulation (Table 6.5). For a loss distribution, ES is the average loss given that the loss is greater than VaR that has a significance level of α . Furthermore, Economic Capital is the excessive loss over the expected portfolio loss, in other words, the difference between VaR and the expected loss. Generally, Economic Capital is stated as the capital at risk. BCBS (2006) calls economic capital as unexpected loss and names it as risk itself while stating that all banks must cover their expected loss anyway. In our calculation of VaR, we used a significance level of 0.01 for which Basel II Capital requirements suggest a level of 0.001.

Table 6.5. Figures of DL simulation

$VaR_{0.01}$	$ES_{0.01}$	<i>Economic Capital</i>
5940.141	7253.655	5672.77
6664.357	8083.472	6265.646
7149.191	8737.95	6640.163
7590.513	9326.99	6983.563
7909.894	9725.42	7235.442

As seen in Table 6.5, the figures gets higher through the end of the horizon, meaning our risk is getting higher every year (recall that these figures are in million dollars). This is not unexpected because the total intensity of default tends to increase over time. Also, because of the characteristics of the transition process, which is a Markov chain, the default state is an absorbing state. In other words, if an obligor defaults, it stays in that state. Besides, due to the rating transition matrix we use, number of expected years (can be calculated by utilizing Markov chain properties) before default vary between 102 and 11 years, for ratings from AAA to CCC, respectively. Furthermore, the increase in the economic capital in Table 6.5 proves that a bank should update its economic capital every year.

6.2. Mark-To-Market Simulation

Mark-to-market simulation is a key tool to assess rates or spreads for obligors of different rating groups. In contrast to default loss simulation, mark-to-market simulation enables us to reflect the loss due to the change in credit worthiness of obligors to the annual portfolio value figures, and decide whether the spreads applied really cover the risk. This section explains how one can carry out a mark-to-market simulation by CreditMetrics. We again point out the necessary inputs of such a simulation, give the simulation inputs we used for our example, explain our R coding in detail, and then give the results of our simulation.

6.2.1. Simulation Inputs

There are 10 main inputs of CreditMetrics, namely;

- (i) Ratings of obligors (or of bonds)
- (ii) Obligators' exposures (or bond par values)
- (iii) Factor loadings of the multi-factor model
- (iv) Rating-transition matrix
- (v) Correlation between systematic factors
- (vi) Parameters of the recovery rate distribution
- (vii) Credit spreads (risky-bond spreads)
- (viii) Risk-free rate
- (ix) Debt maturities (or bond maturities)
- (x) Cash flows

For the mark-to-market simulation of our 25-obligor credit portfolio, we chose the first six simulation inputs the same as in our default loss simulation. Yet, the last three inputs are new and not used in a default loss simulation.

The first new input is credit spreads with respect to ratings. Credit spread is the difference between the rate of a debt or a risky bond and the rate of a riskless asset or bond of the same nature, for instance, of the same rating. In other words, it is the premium thought to cover the risk. We used different (but constant over time) spreads for different rating groups. There are different ways to calculate the credit spread. For our simulation studies, we utilized the following intuitive equation where cs is our credit spread.

$$DP \times (1 + r_f - LGD) EAD + (1 - DP)(1 + r_f + cs) EAD = (1 + r_f) EAD \quad (6.1)$$

This equation implies that in one year, if an obligor defaults, we will earn the risk-free interest rate, r_f , minus a loss rate (LGD in Equation (6.1)), and if not, we suppose we will earn the risk-free rate plus our credit spread but that after one year we will on the average earn the risk-free rate over the exposure (EAD) we give away as a loan.

$$cs = \frac{LGD \times DP}{1 - DP} \quad (6.2)$$

Kealhofer and Bohn (2001) adopt the same idea and utilizes the following equation to calculate the credit spread:

$$E[r] = DP \times (r_f - LGD) + (1 - DP)(1 + r_f + cs)$$

$E[r]$ is the expected return in one year. Moreover, Kealhofer and Bohn (2001) call Equation (6.2) as expected loss premium, which can only compensate the actuarial risk of default. However, we need an additional risk premium to cover our loss resulting from our systematic risk, which cannot be diversified, and to gain a return more than the risk-free rate over this investment. So, our spread is calculated by the following equation.

$$cs = \text{expected loss premium} + \text{risk premium}$$

We assumed a one per cent risk premium and calculated a credit spread for each rating class by using the DPs in Table 6.3. Table 6.6 shows the spreads we used with the risk premium added. Yet, our credit spread calculation does not consider our losses due to the changes in credit-worthiness of obligors and can compensate our default risk for a one year horizon. However, by including transition probabilities, this calculation can be extended in such a way that it seizes a multi-year perspective. Then, iteratively each year's credit spread for a certain rating group can be solved. Nevertheless, this thesis does not cover the estimation of exact spreads. Besides, Monte Carlo simulation is also a tool to assess the spreads a bank uses. Therefore, we can still assess our spreads by interpreting our simulation results.

Another way to calculate credit spread is the cash flow approach Crouhy *et al.* (2001) explains. For a single cash flow, the credit spread can be derived from the following equation where the terms are the same as in Equation (2.20).

$$\frac{(1 - LGD) \times C_i + LGD \times (1 - Q_i) \times C_i}{(1 + r_f^i)^{t_i}} = \frac{C_i}{(1 + r_f^i + cs_i)^{t_i}} \quad (6.3)$$

Table 6.6. Credit spreads

Rating	Spread (%)
AAA	1.000
AA	1.006
A	1.030
BBB	1.234
BB	1.932
B	5.481
CCC/C	28.693

The numerator on the left-hand side of Equation (6.3) can be also written in the following way.

$$(1 - Q_i) \times C_i + Q_i \times (1 - LGD) \times C_i$$

Therefore, when $t_i = 1$, the credit spread is:

$$cs = \frac{LGD \times (1 + r) \times Q}{1 - LGD \times Q}$$

Yet, there are also studies that focus on the estimation of risky-bond spreads, such as the one of Hull *et al.* (2005).

After the credit spreads, the next simulation input is the risk-free rate for each year of the horizon. Risk-free rates are often chosen among the treasury-bond rates or in addition, a stochastic process is used for the randomness of the risk-free rate, or a forecasting method is applied; nonetheless, we used a constant risk-free rate of 4.25 per cent within our simulation studies. Next, without loss of generality, we assumed a fix maturity of six years for each obligor, and simulated a shorter horizon than these maturities. Finally, while determining the cash flows (future payments of the obligors), we handled the credit exposures of obligors as bonds with coupon payments. In other words, obligors pay every year only the interest of their exposures. Therefore, we determined these cash flows or

coupon payments with respect to the spreads supposing again these spreads will cover the risk we bear. For instance, a BBB rated obligor with an exposure of 10000\$ has annual payments of 564\$ ($10000 \times (4.25+1.39)/100$).

Furthermore, it is important to recall that, for our example, spot rates are equal to the forward rates while we use a constant risk-free rate throughout the whole horizon. In the next section, we explain our R function, *sim_CreditMetrics* step by step and later give the results of our Monte Carlo Simulation carried out in R .

6.2.2. Mark-to-Market Simulation in R

This section explains the structure and methodology of our R function, *sim_CreditMetrics*. In Figure 6.8, the first part which includes the inputs of *sim_CreditMetrics* and the brief explanations of the abbreviations is shown. In order to use *sim_CreditMetrics*, it is required to construct the future cash flows in matrix form. For the construction of this matrix, the R function, *cashFlow* can be used by giving the exposures, maturities, ratings, risk-free rate, and credit spreads as inputs to the function. Moreover, *sim_CreditMetrics* also requires the spot rates of each year with regard to ratings. Therefore, this argument needs to be a matrix, too.

```
sim_CreditMetrics<-function(
  rep=10000, # number of replications
  horizon=1, # planing horizon to be examined
  rt_prob, # ratings transition matrix (percent)
  exposures, # exposures or par value of debts/bonds
  ratings, # recent ratings of the obligors/bonds
  cashFlows, #cashflows regarding each obligor/bond (as a matrix with dim,
  numberofObligors x max(bond maturities), with the exposure/par value included in the
  cashflow of the last year and with zero values in case of no flow)
  loadings.a, # loadings of systematic risk factors
  loadings.b, # loadings of idiosyncratic risk factors
  varcov, # variance-covariance matrix of the standardized factors
  (=correlation matrix)
```

```

    r=4.25,      # risk free rates of the planing horizon in percent with dim 1 x
max(bond maturities)
    forwardZeros, # forward zero-coupon (or spot) rates (percent) with respect to rating
groups and with dim nRating x max(bond maturities)-1
    w_shape1=2, # shape parameter 1 of beta distributed recovery rates
    w_shape2=3, # shape parameter 2 of beta distributed recovery rates
    updateFlow=FALSE # If TRUE then when an obligor's rating changes, so does its
interest rate and annual payments
  ){

```

Figure 6.8. Simulation inputs of *sim_CreditMetrics*

After achieving the dimensions of the input variables and completing the necessary adjustments or modifications of the input variables such as the conversion of rates from percent to decimals, *sim_CreditMetrics* generates all the systematic factors and determines the rating thresholds the same way *simDefaultLoss_CM* does (Figure 6.4). Again, it also assigns the corresponding thresholds to each one of the obligors as in Figure 6.5. Then, *sim_CreditMetrics* generates the latent variables and for each year throughout the horizon, it determines the rating changes via the function, *getRatings_CMetrics* (see APPENDIX A.1) as regards that year's asset returns as shown in Figure 6.6. Also, for *sim_CreditMetrics*, the threshold assignment in Figure 6.5 is very crucial for each replication and each year of the simulation because since the ratings change every year, so do the corresponding thresholds.

Following the realization of new ratings, if *sim_CreditMetrics* is asked for an update of future cash flows (in other words spread applied to an obligor with a rating change) in case of a rating change (for instance, if an obligor is paying a five per cent of its exposure every year, when its rating downgrades, it will, from that time on, pay, for example, seven per cent due to that change in its credit worthiness.), it looks for a difference between the ratings of this year and the ratings of the previous year as in Figure 6.9. Subsequently, *sim_CreditMetrics* updates the corresponding cash flows by the function, *getNewFlows* (see APPENDIX A.1). Next, by the R code in Figure 6.10, *sim_CreditMetrics* finds the total value of the payments received from non-defaulted firms and adds this to the recoveries, the present value of the previous years' receivables invested over a risk-free

instrument, and the present value of the future payments of those non-defaulted firms by using Equation (2.3) in order to find the credit portfolio value of that year.

```

if(updateFlow&&(year!=bondMaturity)){
  boolean<-as((ratings!=preRatings)*(ratings<i_default),"logical")
  if(sum(boolean)!=0)
    cashFlows[boolean,(year+1):bondMaturity]<-
getNewFlows(as.matrix(cashFlows[boolean,(year+1):bondMaturity]),exposures[boolean],r
atings[boolean],nRating,forwardZeros[,year:(bondMaturity-1)]) }

```

Figure 6.9. Updating cash flows regarding rating changes

```

receivedPayments[year]<-sum(cashFlows[ratings<i_default,year])
if(year!=1) receivedPayments[year]<-receivedPayments[year]+receivedPayments[year-
1]*(1+r[year-1])
# receivables from nondefaulted plus previous receivables invested over risk free rate
PVofNondefaulted<-0 # discounted future cashflows
if(year!=bondMaturity)
# because we only discount the future cashflows (of nondefaulted obligors). At the end of
the horizon, there is no future cashflows.
for(k in 1:nRating)
  if((bondMaturity-year)!=1)
    PVofNondefaulted<-
PVofNondefaulted+sum(cashFlows[ratings==k,-(1:year)]%*%(1+forwardZeros[k,
year:(bondMaturity-1)])^(1:(bondMaturity-year)))
  else
    PVofNondefaulted<-
PVofNondefaulted+sum(cashFlows[ratings==k,-(1:year)]*(1+forwardZeros[k,
year:(bondMaturity-1)])^(1:(bondMaturity-year)))
PVofDefaulted<-
sum(exposures[ratings==i_default]*rbeta(sum(ratings==i_default), w_shape1,w_shape2))
#EAD = PAR (1- LGD)
ratings[ratings==i_default]<-i_default+1

```

```

portfolioValue[year]<-
PVofNondefaulted+PVofDefaulted+receivedPayments[year]
```

Figure 6.10. Calculation of portfolio value in *R*

In Figure 6.10, the code line “ratings[ratings==i_default]<i_default+1” is very crucial for the simulation of upcoming year in distinguishing the recently defaulted firms from previously defaulted firms. In the absence of this line, the *sim_CreditMetrics* would continue generating recovery rates for pre-defaulted obligors.

Final Step is the initialization of the parameters. Also, to improve the precision, again it replicates the simulation of the whole path (from year one to the end of horizon).

6.2.3. Simulation Results

For MtM simulation, we carried out 100 000 replications, as well. Also, we ran the same simulations by allowing an update in the cash flow matrix in the case of rating transitions. In other words, if, for instance, an obligor is downgraded from BBB to B, from that time on, it pays a higher interest with respect to the credit spread of its new rating, which is B for this example. In this section, we give and discuss the results of these two simulations.

Simulation in *R*, again with the same PC specifications as before, takes approximately eight min. However, when we perform a simulation with yearly updated cash flows, it increases to 18-19 min. This level of computational cost is yet to be expected. Next, we converted the simulated portfolio values into yearly returns calculated over the total exposures at the beginning of the horizon by the following equation.

$$r_i = \left[\sqrt[i]{\left(\frac{\text{Portfolio Value of the } i^{\text{th}} \text{ year}}{\text{Total Exposure}} \right)} - 1 \right] \times 100$$

Figure 6.11 presents the yearly returns where blue lines signify the average yearly return after five years. All histograms in Figure 6.11 prove that our credit portfolio is not

exposed to a significant level of risk. Pr column of Table 6.7 also verifies this fact. Pr , here, is the probability that the portfolio value is greater than the risk-free investment for that year. Yet, it is notable that the worst credit worthiness within our 25-obligor credit portfolio is due to two BB rated obligors as can be seen from Table 5.1. Moreover, the remaining obligors are charged with very similar credit spreads.

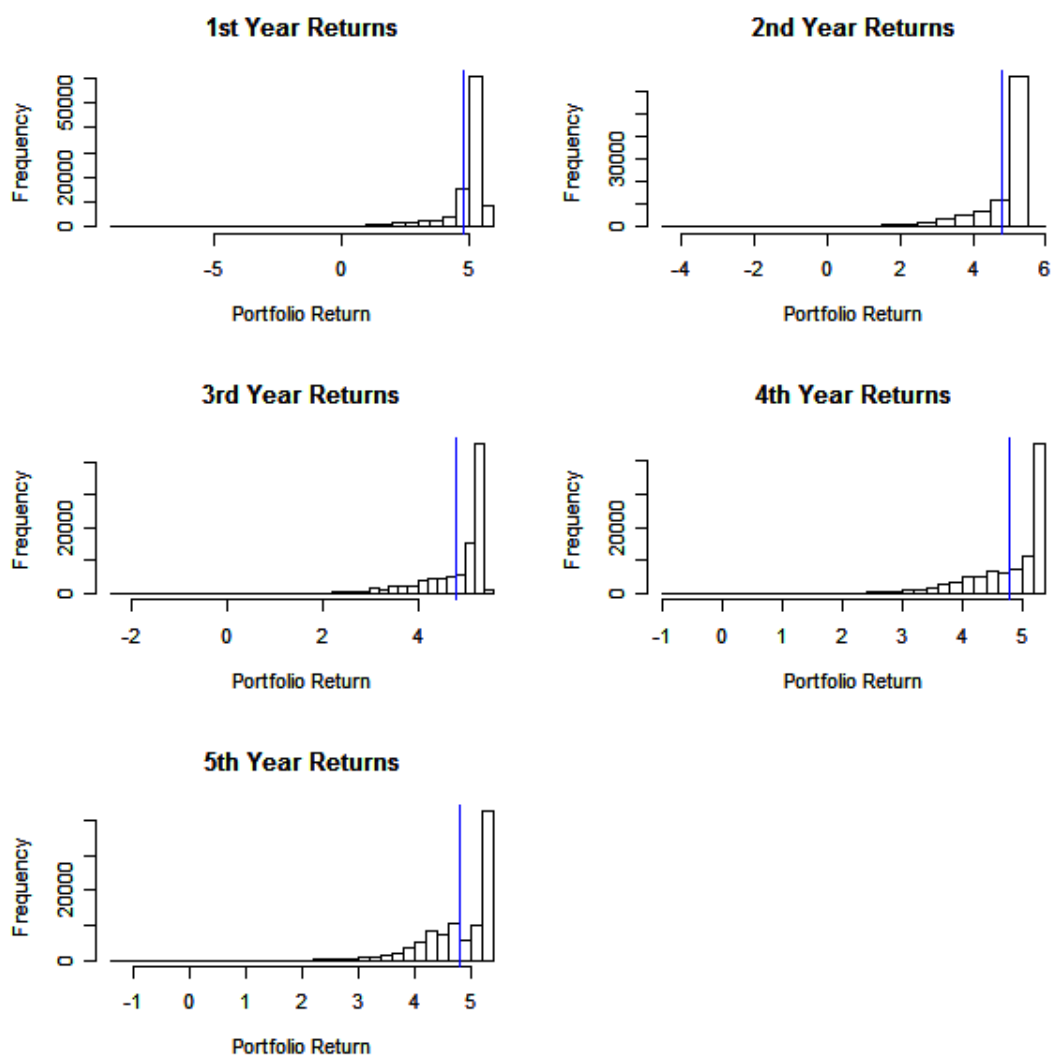


Figure 6.11. Annual returns of the 25-obligor credit portfolio

Figure 6.11 also implies that in the average we gain an interest rate close to the applied spread plus risk-free rate. Furthermore, the effect of default is not very clear from the histograms but still the tendency of the histograms to widen through both directions with

time (away from the average portfolio value) is easily observable from the last two columns of Table 6.7.

Table 6.7. Figures of the 25-obligor credit portfolio

Year	$VaR_{0.01}$	$ES_{0.01}$	Pr	<i>Difference between risk-free investment and $VaR_{0.01}$</i>
1	157097	155404	0.8691	5559.15
2	161501	158955	0.82403	8067.76
3	166844	163672	0.82032	9931.27
4	172588	169272	0.81476	11700.9
5	179453	175545	0.82624	12668.2

In addition, Figure 6.12 displays the portfolio return distributions resulting from a simulation with yearly updated cash flows. It is obvious that for the first year our portfolio is not exposed to any kind of risk. Besides, for the first year, this investment can be referred as risk-free with a rate of spread plus risk-free rate we assumed. Moreover, although Table 6.8 shows deterioration in VaR and ES figures when compared to Table 6.7, Pr and *Difference between risk-free investment and VaR* figures verify that when cash

Table 6.8. Figures of 25-obligor credit portfolio with yearly updated cash flow

Year	$VaR_{0.01}$	$ES_{0.01}$	Pr	<i>Difference between risk-free investment and $VaR_{0.01}$</i>
1	158172	156787	0.93946	4484.61
2	162659	160801	0.89805	6909.79
3	168719	165750	0.88681	8056.31
4	175077	171733	0.88978	9211.98
5	182142	178395	0.90543	9978.64

flows are updated, portfolio value distributions improve with reference to a risk-free investment.

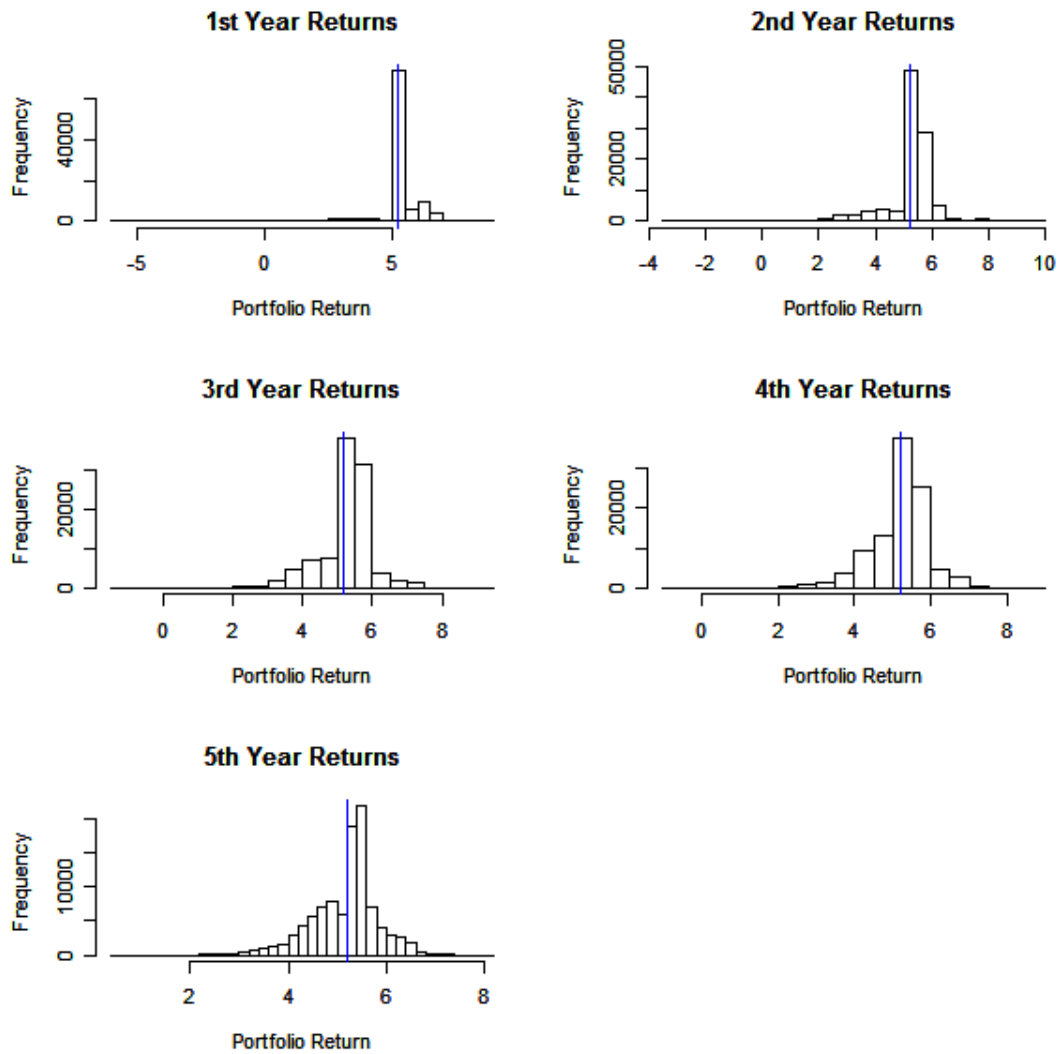


Figure 6.12. Annual returns of the 25-obligor credit portfolio with yearly updated cash flow

7. LARGER CREDIT PORTFOLIOS

A real credit portfolio of a bank probably consists of thousands or at least hundreds of obligors, and a simulation study of a 25-obligor credit portfolio will most likely not reveal the true distributions, figures, and thus, of course, the real risk. Moreover, the existence of several other studies, such as the one of Glasserman and Li (2005), proves the importance of the distributions resulting from large portfolios. Hence, in order our simulation results to converge to real figures and distributions, we generated an artificial credit portfolio taking the estimated parameters of our previous credit portfolio example as a reference. In short, we thought that if we had had 1000 obligors in our credit portfolio, again from UK and USA, and from four different sectors, which are Telecommunication, Technology, Aerospace, and Pharmacy, then we would probably have ended up with factor loadings not too different from the ones shown in Table 5.3. This section first explains how we obtained such a portfolio in *R*, and then gives the results of mark-to-market simulations with CreditMetrics for randomly selected ratings.

7.1. Generating an Artificial Credit Portfolio in *R*

In order to create a credit portfolio of 1000 obligors, we first determined the distributions of systematic factor loadings with respect to the countries and industries. In other words, we determined the mean and variance of each column (disregarding the zeros within that column) that belongs to the systematic factors in Table 5.3. Then, we calculated the covariances between these columns. Finally, we generated 250 obligors from each group via normal distribution, and since there are four groups, that made 1000 obligors.

We generated the artificial portfolio in *R* by our function, *generateArtificial*. The function requires two inputs; *n*, number of obligors to be generated from each group, and *W*, systematic factor loadings that will be used as a reference. Furthermore, Figure 7.1 and Figure 7.2 show two essential successive steps of our artificial factor loading generation. The first and most important step is to determine the group that each line (or each obligor) of the reference factor loading matrix belongs to. This step is performed by the function,

giveGroups, regarding that a systematic factor loading matrix will consist of several zero and non-zero elements (this is one of the typical characteristics of a systematic factor loading matrix while an obligor cannot be included in all the countries, industries, and

```
group<-giveGroups(W)
numberOfGroups<-max(group)
loadings.statistics<-giveStatistics(W)
loadings.variances<-loadings.statistics$variances
loadings.means<-loadings.statistics$means
```

Figure 7.1. Variance and mean calculation of the columns of a factor loading matrix

other factors). Then, as in Figure 7.1, variances and means of each column of the reference matrix (see APPENDIX A.3 for R codes of *giveStatistics*) are determined. The next and also the last step is to find the correlation of the columns for each group (*group.corr* in Figure 7.2), form their variance-covariance matrices by the function *giveVarCov* and generate normally distributed factor loadings for each group with these covariances and column means via *genArtificialGroup*.

```
for(id in 1:numberOfGroups){
  group.corr<-giveCorr(W,group,id)
  group.varcov<-
giveVarCov(loadings.variances[(W!=0)[group==id,][1,],group.corr)
  W_new[((id-1)*n+1):(n*id),]<-
genArtificialGroup(n,(W!=0)[group==id,][1,],loadings.means[(W!=0)[group==id,][1,],gro
up.varcov)
}
```

Figure 7.2. Finding the correlation of columns of a factor loading matrix and generating artificial loadings

After the generation of systematic factor loadings, determining the idiosyncratic factor loadings is trivial by applying a Cholesky decomposition to the systematic factor loadings matrix. Figure 7.3 shows how to perform this idiosyncratic factor loading calculation in *R*.

```
W_Artificial<-generateArtificial(250,W)
b_Artificial<-sqrt(1-rowSums((W_Artificial%*%t(chol(corMatrix)))^2))
```

Figure 7.3. R-commands to generate artificial factor loadings

In Figure 7.3, *W_Artificial*, *corMatrix*, and *b_Artificial* are the matrix of the generated systematic factor loadings, correlation matrix of the systematic factors, and idiosyncratic factor loadings, respectively. *chol* is the built-in function of *R* for Cholesky decomposition.

However, during the artificial data generation process, we couldn't generate factor loadings for the second group, which involves the firms belonging to the Technology sector of UK. This was because there were only two observations for that group. The resulting variance-covariance matrix was not a positive definite matrix, and since the function *genArtificialGroup* uses Cholesky decomposition, and because the Cholesky decomposition (a unique lower triangular matrix with strictly positive entries on the diagonal) requires the matrix of interest to be positive definite, we encountered an error in *R*. Later, we found another firm from that group over *Bloomberg Data Services*. Moreover, the systematic factor loadings we calculated for that firm is given below in Table 7.1. Then the data for that group was sufficient for the variance-covariance matrix to be positive definite.

Table 7.1. Systematic factors of the additional obligor

Obligor	Systematic Factors					
	USA	UK	IT	TECH	AERO	PHARM
Autonomy Corp PLC	0	0.032248	0	0.358044	0	0

7.2. Simulation Setting

For the simulation of our 1000-obligor credit portfolio, we used the spreads in Table 6.6, the assumption of fixed annual payments of interest, a constant risk-free rate of 4.25 per cent, and a fixed maturity of six. We generated the exposures uniformly as in the previous portfolio example, and generated randomly three different rating sets. For the first

set, we generated 1000 ratings uniformly only among top ratings – from AAA to A. The second set consists of A and lower ratings, and the final set of 1000 ratings are generated due to the distribution, shown in Figure 7.4, of non-financial issuers across Standard & Poor’s Credit Class. For further inspections, we also carried out these simulations with the spreads calculated with the formula below and again add a one per cent risk premium to those spreads. Indeed, the resulting spreads are slightly less than the ones in Table 6.6.

$$cs = LGD \times DP \quad (7.1)$$

7.3. Simulation Results

In this section, we will first give the resulting figures of a DL simulation of our 1000-obligor credit portfolios of different rating sets, then examine the return histograms we achieved by MtM simulations over these three portfolios, both with fixed cash flows and yearly updated cash flows, and finally, we will compare the simulations with two different spreads for the case of updated cash flows. Furthermore, in *R*, DL simulation of such a credit portfolio takes 22-24 min whereas MtM simulation takes 26-28 and 42-45 min with yearly updated cash flows.

First, the figures of DL simulations seen in Table 7.2, Table 7.3, and Table 7.4 illustrate significant discrepancies between the losses resulting from the three rating sets we used. It is seen that in the case of highly rated obligors (AAA-A), the loss is clearly lower than in the other two cases and so is the economic capital required to cover the risk.

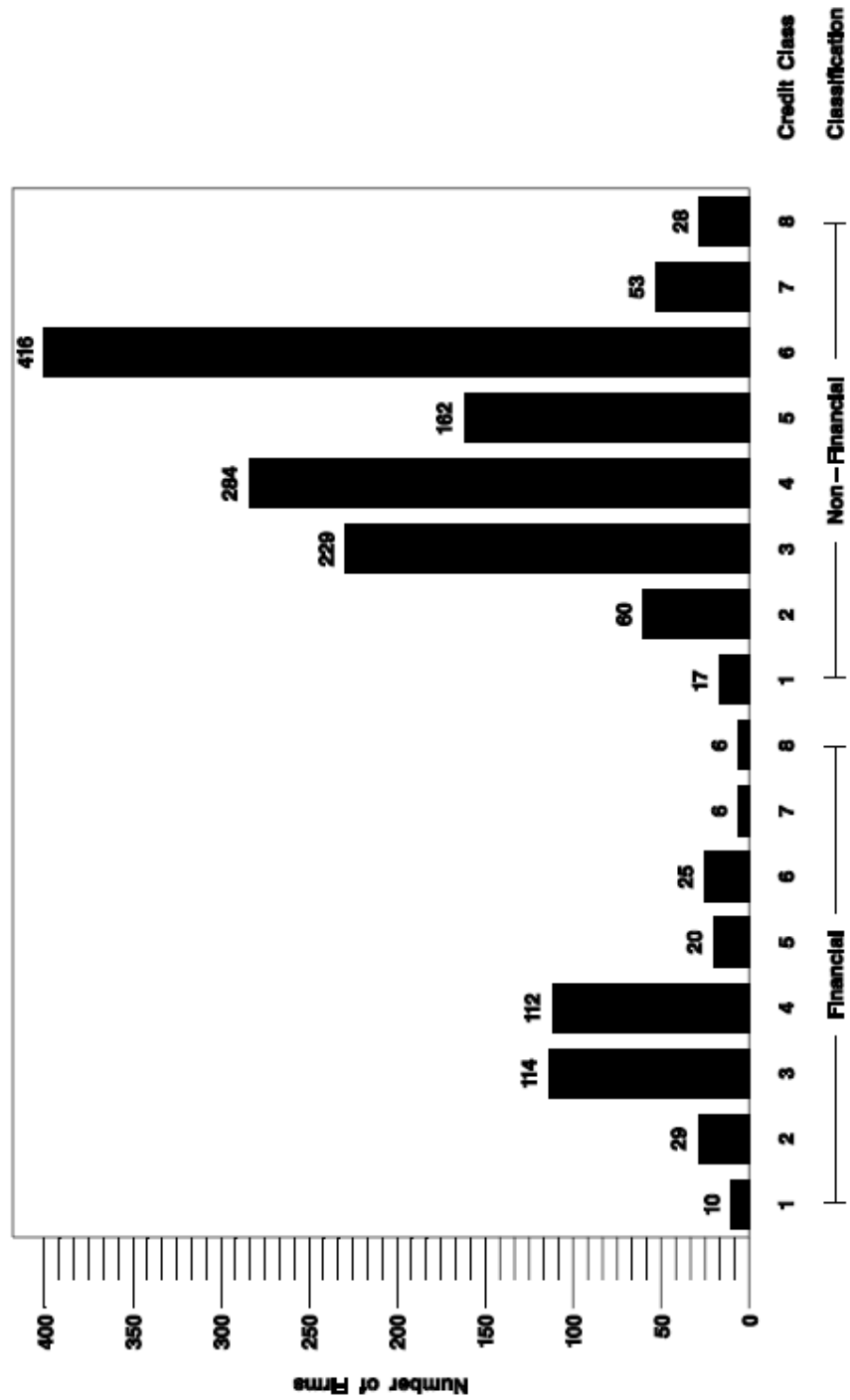


Figure 7.4. Distribution of financial and non-financial issuer groups across S&P credit class (Bohn, 1999)

Table 7.2. Figures of DL simulation (1000 AAA-A rated obligors)

Year	$VaR_{0.01}$	$ES_{0.01}$	<i>Economic Capital</i>
1	7675.94	9372.853	7014.561
2	10582.17	12703.48	9105.819
3	13270.69	15730.5	10894.19
4	15850.83	18516.32	12469.29
5	18332.47	21269.83	13902.7

There is a slight improvement in the figures presented in Table 7.4 when compared to the ones in Table 7.3. This result is also reasonable while the total default intensity of the portfolio that consists of A to CCC rated firms is higher than the one of the mixed portfolio.

Table 7.3. Figures of DL simulation (1000 A-CCC rated obligors)

Year	$VaR_{0.01}$	$ES_{0.01}$	<i>Economic Capital</i>
1	330549.3	341263.8	73007.96
2	250818.3	260641.3	66303.63
3	202229.2	211584.3	60556.12
4	170696.5	179711	55542.82
5	150199.4	158991.8	51987.82

Moreover, when Table 7.3 and Table 7.4 are compared, there is a severe difference between the figures of the first year. The reason behind this difference can be realized by examining Table 7.5, which shows the number of obligors each portfolio has from each rating. Because of the high default intensity of CCC rated obligors (*see* Table 6.3), several of CCC rated obligors of the second portfolio shown in Table 7.5 probably default in the first year (also *see* APPENDIX B.2 for default loss distributions). That is probably why there is that much difference between the first year's figures.

Table 7.4. Figures of DL simulation (1000 originally rated obligors)

Year	$VaR_{0.01}$	$ES_{0.01}$	<i>Economic Capital</i>
1	196042.7	205289.6	57869.7
2	187669.3	196420.3	57703.1
3	176672.1	185671.2	56934.65
4	164083.7	172987	54286.66
5	152044.6	160328.3	51788.23

Table 7.5. Rating distribution within each 1000-obligor portfolio

	AAA	AA	A	BBB	BB	B	CCC
Portfolio with AAA-A ratings	333	343	324	0	0	0	0
Portfolio with A-CCC ratings	0	0	206	203	198	206	187
Portfolio with mixed ratings	14	56	181	223	110	376	40

The following figures again illustrate the annual return distributions of our three portfolios. Figure 7.5, Figure 7.6, and Figure 7.7 are the results of the simulations with fixed cash flows, and Figure 7.8, Figure 7.9, and Figure 7.10 are with yearly updated cash flows.

As seen in Figure 7.5, although the returns are above the return of a risk-free investment, on the average we cannot get our risk premium. This may not be due to non-diversifiable risk portion because our portfolio consists of only AAA, AA, and A rated firms, and because of rating-transition matrix we used firms seem to start to default after the third or fourth year. Moreover, this is also probably because credit quality of our portfolio tends to decrease barely after the third or fourth year. This is because our spreads are very close to each other except for B and lower ratings.

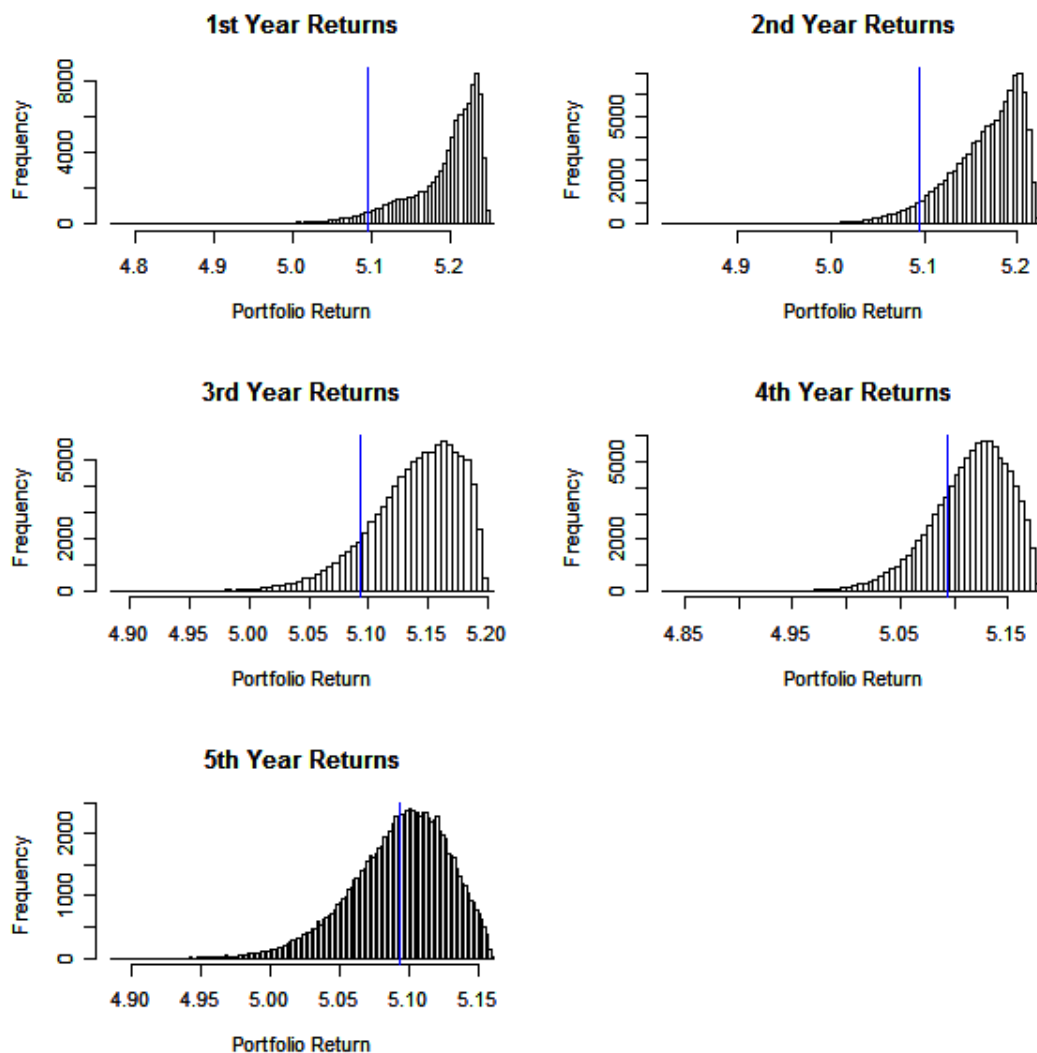


Figure 7.5. Annual returns of 1000-obligor credit portfolio (AAA-A)

In Figure 7.6, on the other hand, there are returns less than the risk free investment. As expected, through the end of the horizon, the average return of that year is getting close to the average return obtained after five years. Besides, the return distribution is more symmetric than it is in Figure 7.5 while the spreads and different default intensities of ratings from much more different credit qualities compensate each other. This is easier to observe in Figure 7.7. Furthermore, even though the worst case scenario points out very low returns in both Figure 7.6 and Figure 7.7, the tails of the distributions get heavier through both directions resulting in much higher returns than in Figure 7.5 for only the first year. Yet, in Figure 7.5, too, we start seeing more like a two-tailed distribution in the fifth

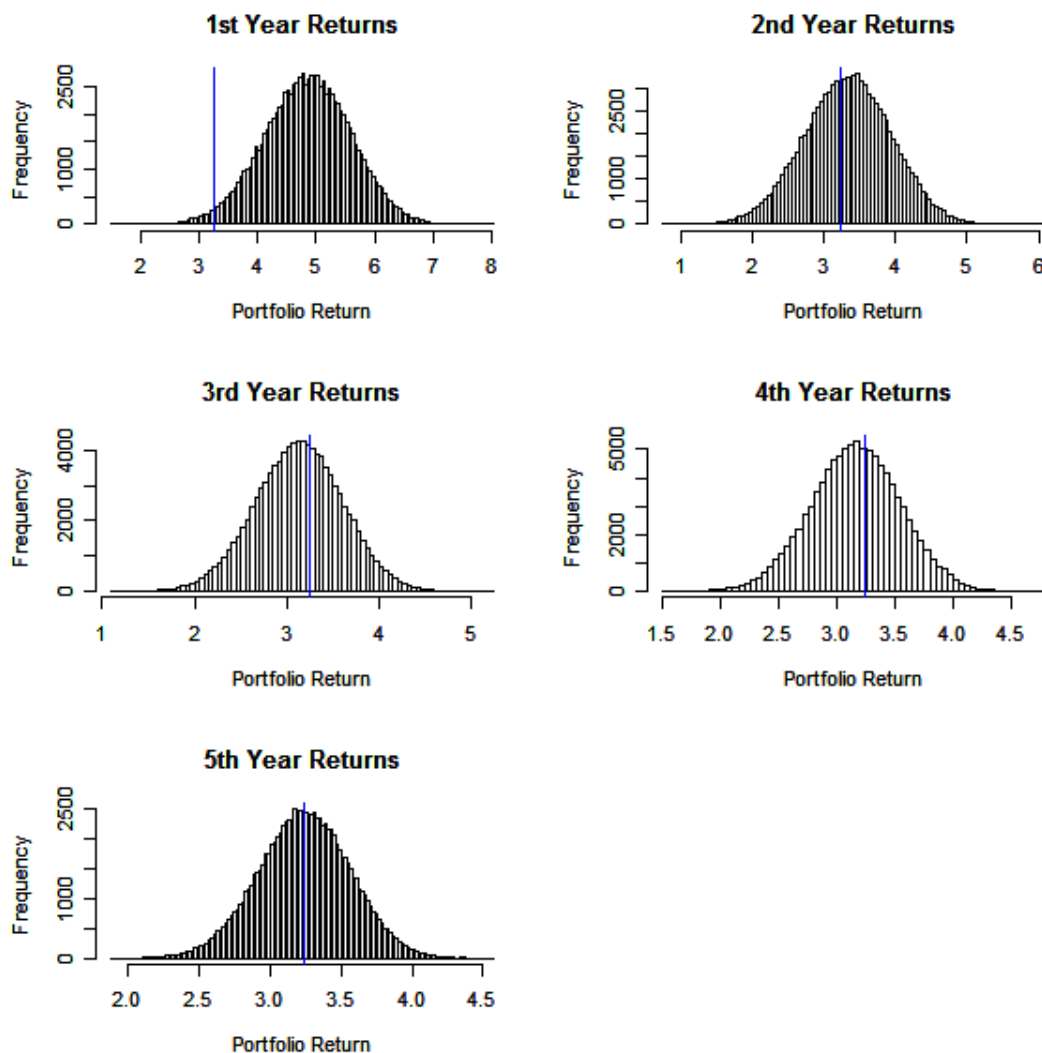


Figure 7.6. Annual returns of 1000-obligor credit portfolio (A-CCC)

year's returns. This means, at the end of the fifth year, even the obligors that have high ratings at the beginning of the horizon start to default after several transitions to other states (or ratings).

It is easily observable that our spread is sufficient for only the first year's credit risk. Hence, probably different spread policies should be applied for different years of the planning horizon since each year occupies different levels of risk. After the first year, our expected loss and risk premiums cannot compensate the transition and the default risk.

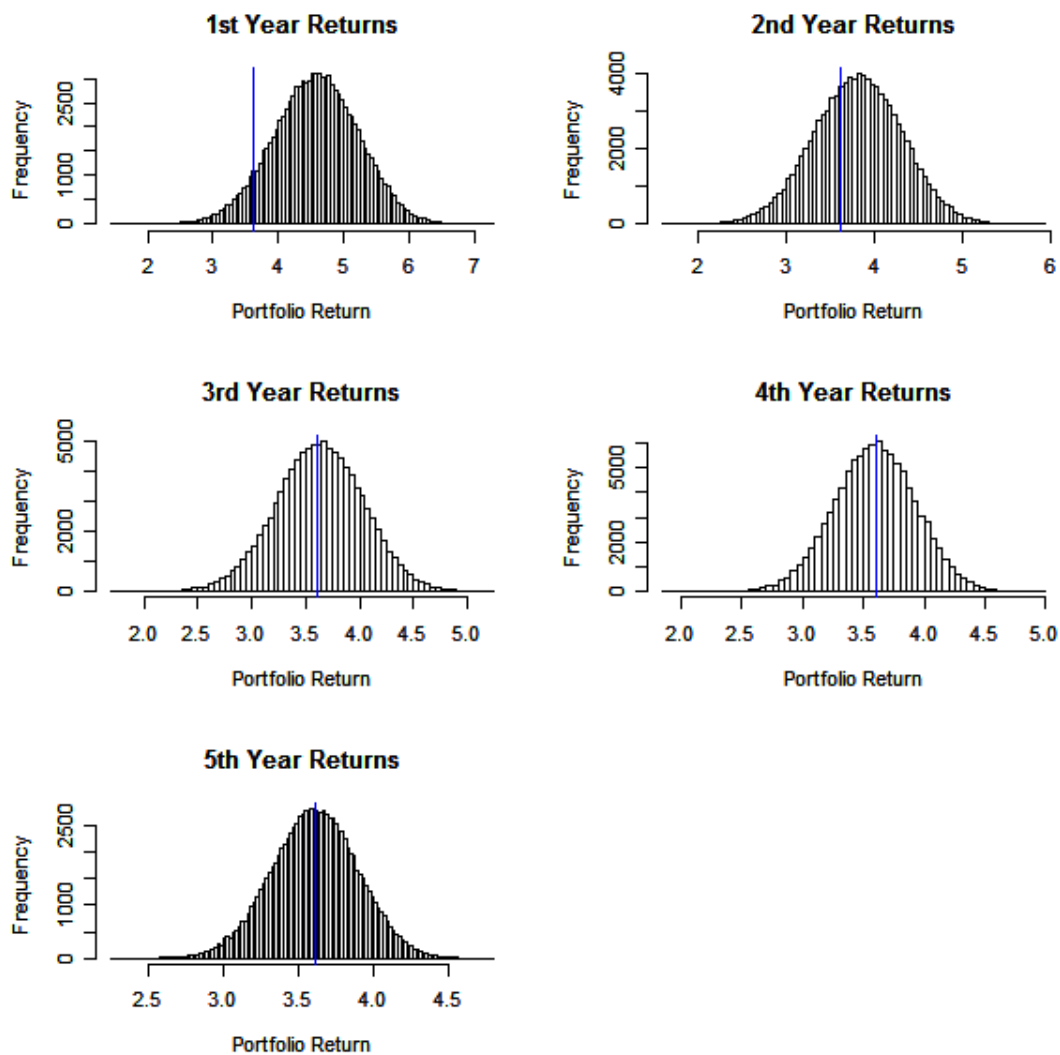


Figure 7.7. Annual returns of 1000-obligor credit portfolio (mixed ratings)

Next, when we look at Figure 7.8, we see that for the portfolio of AAA-A rated obligors, first year do not embrace any risk. Then after the second year, it seems like defaults start to appear with a very low frequency. Besides, this time the returns are generally higher than in the case of fixed cash flows (Figure 7.5).

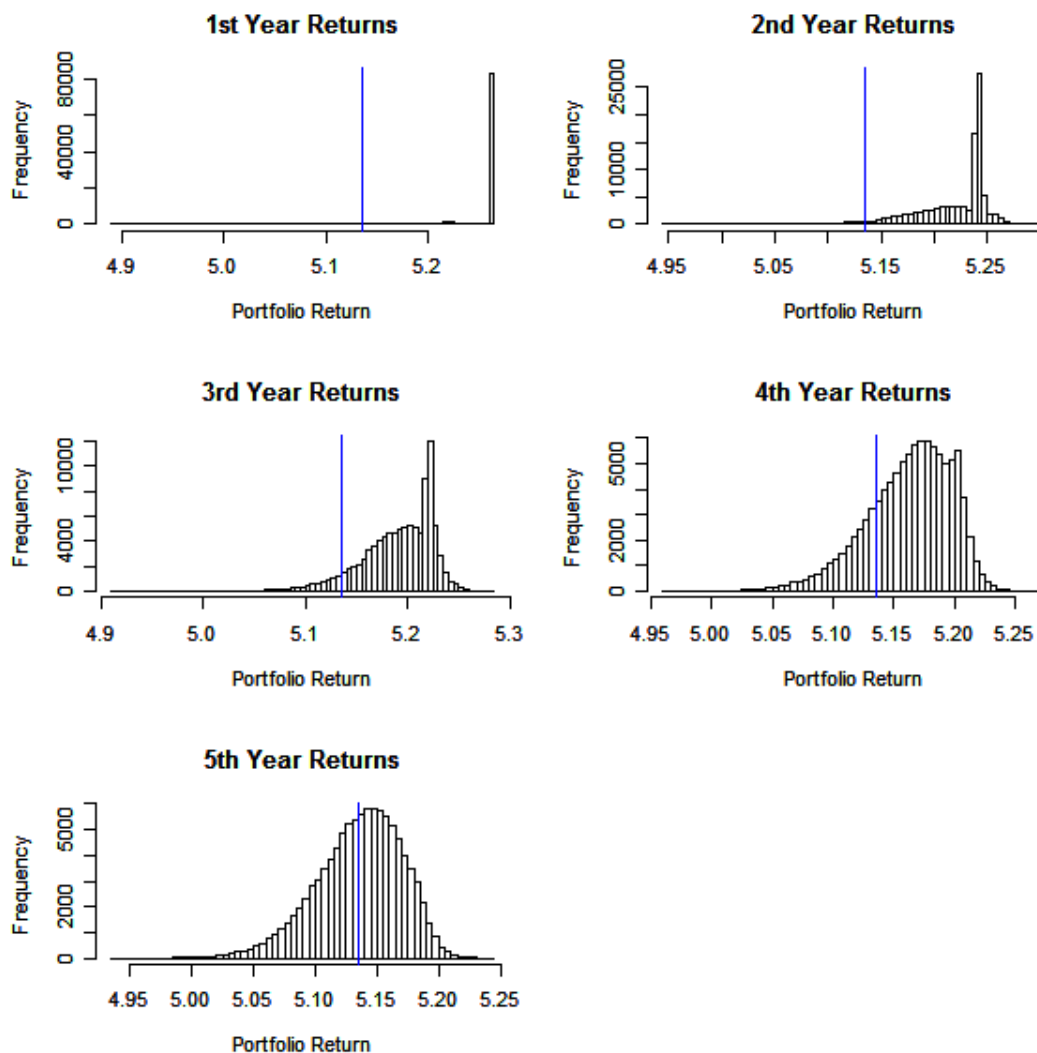


Figure 7.8. Annual returns of 1000-obligor credit portfolio (AAA-A) with updated flow

Figure 7.9 also presents improvements in case cash flows are updated with respect to rating transitions. However, after the second year, we still get less than the risk-free rate. So, our spread is still insufficient in covering our default risk after the second year even though we get rid of the transition risk. Moreover, the variance and the distribution (heavy-tailed through higher values) of our recovery rate may have a slight effect on the variance of our return distributions. Beta distribution has a variance of:

$$\sigma^2 = \frac{\alpha\beta}{(\alpha + \beta)^2(\alpha + \beta + 1)}$$

where α and β are shape parameters of the beta distribution. In our simulation setting, recovery rates have a variance of 0.04.

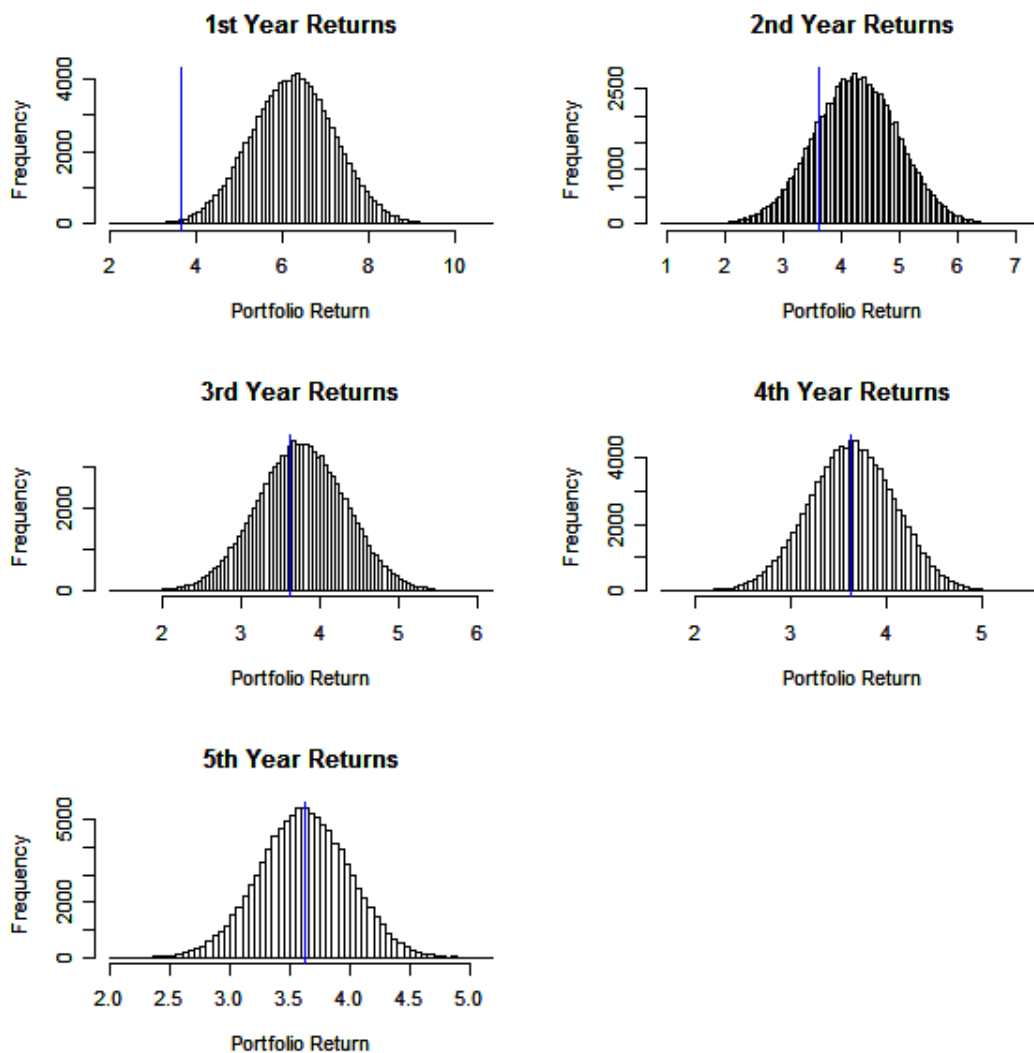


Figure 7.9. Annual returns of 1000-obligor credit portfolio (A-CCC) with updated flow

Additionally, for the first year of our simulations, the probability of higher returns looks higher in portfolios with mixed ratings and with A-CCC rated obligors than it is in the portfolio with AAA-A rated obligors. Nonetheless, the risk is still higher in these two because of their risk profiles and the spreads we used. Furthermore, simulation of the portfolio with A-CCC rated obligors reveals higher probabilities for higher returns but on the average it has a lower return than the portfolio with mixed ratings has. This is probably

because of the nature (number of obligors from speculative grades is notably higher - see Table 7.5) of our A-CCC credit portfolio. In our A-CCC credit portfolio, for the first year several CCC rated obligors are likely to default, and during the first couple of years, credit worthiness of A and BBB rated obligors (while their numbers in the portfolio are relatively high, and spreads are close to each other for AA, A, and BBB ratings) do most likely not change drastically.

Then, when Figure 7.7 and Figure 7.10 are examined, a minor improvement is seen for the first two years when cash flows are updated but again with the default frequencies increasing on a yearly basis, our yearly return falls below the risk-free rate after two years. Besides we can barely obtain the risk-free rate for a two year horizon.

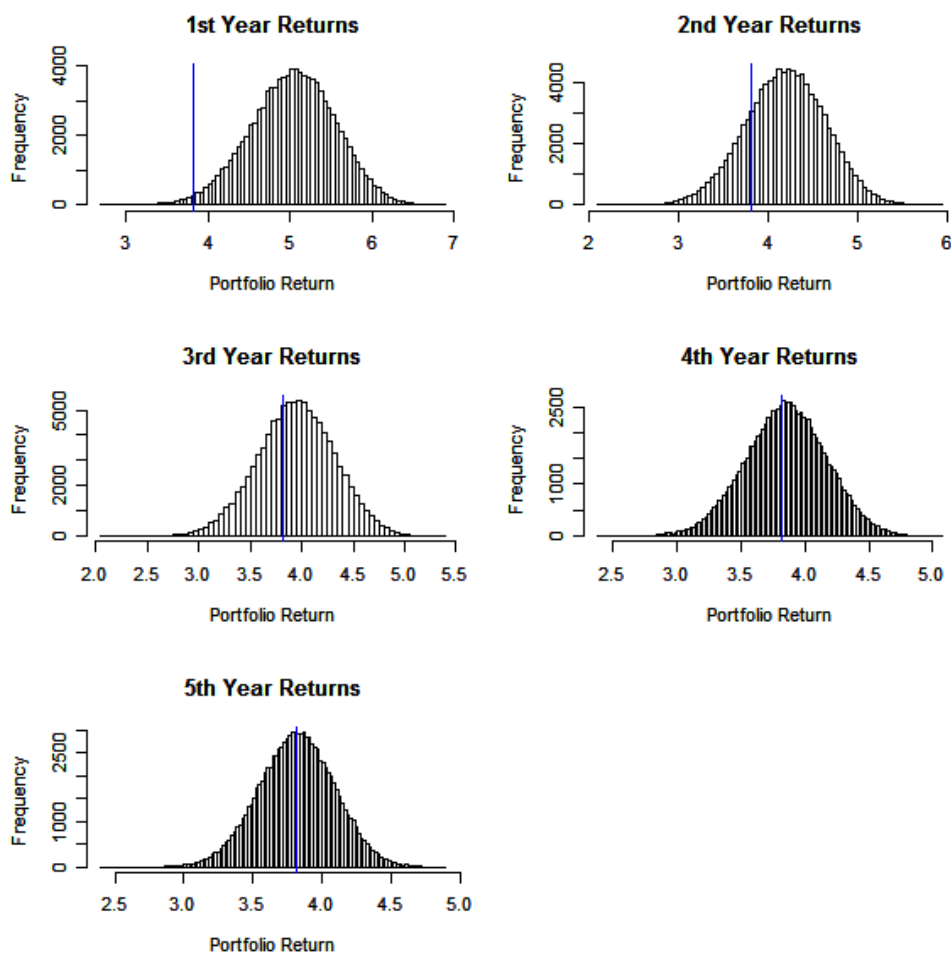


Figure 7.10. Annual returns of 1000-obligor credit portfolio (mixed ratings) with updated flow

Finally, when we used the spreads calculated by Equation (7.1), we obtained the portfolio value distributions seen in Figure 7.11, Figure 7.12, and Figure 7.13 with cash flows updated with respect to rating changes (*see* APPENDIX B.1 for the distributions in case of no update in cash flows). We utilized credit spreads for the compensation of default losses and a risk premium of one per cent. Moreover, when compared to Figure 7.8, Figure 7.9, and Figure 7.10, these graphs exhibit the exact difference between the two different spreads we used since the cash flows are updated within the simulations and thus our portfolios are exposed only to default risk, not transition risk (change in the credit quality).

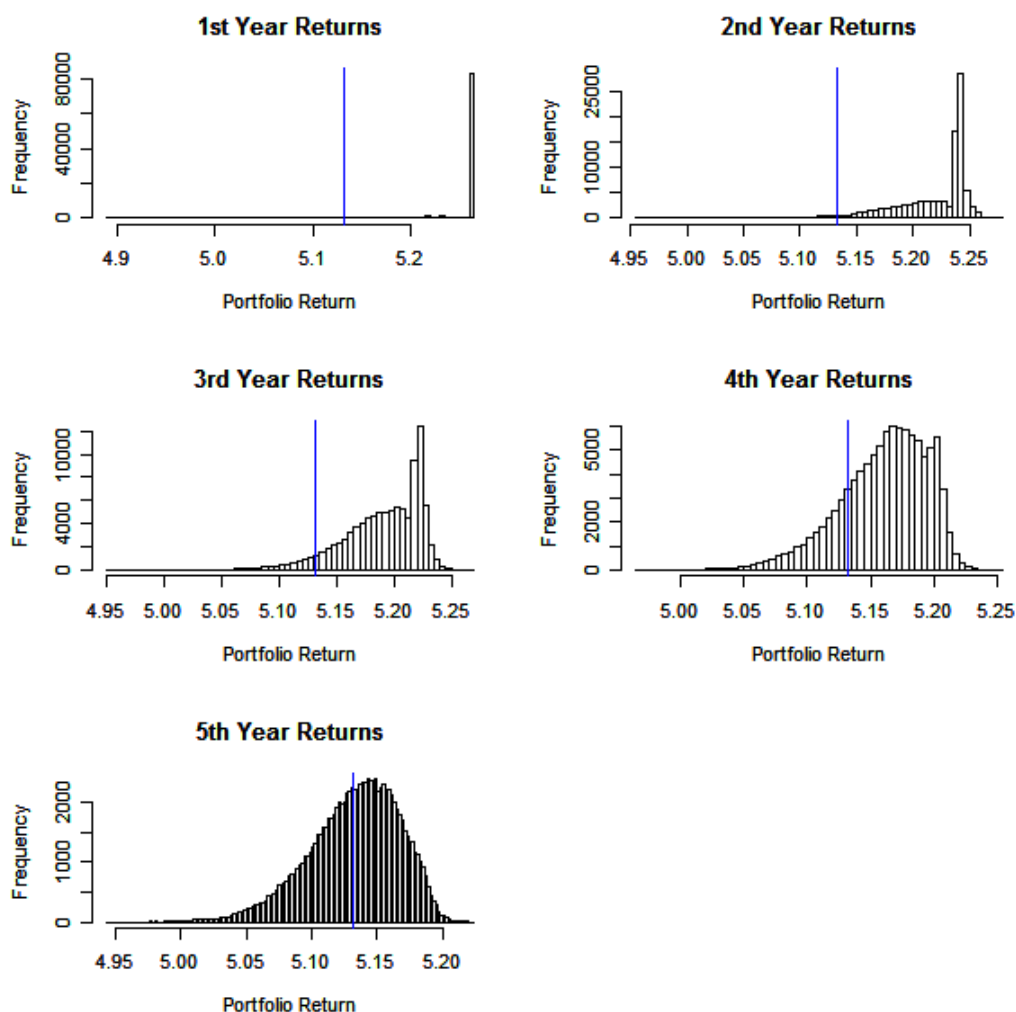


Figure 7.11. Annual returns of 1000-obligor credit portfolio (AAA-A) with updated flow and slightly lower spreads

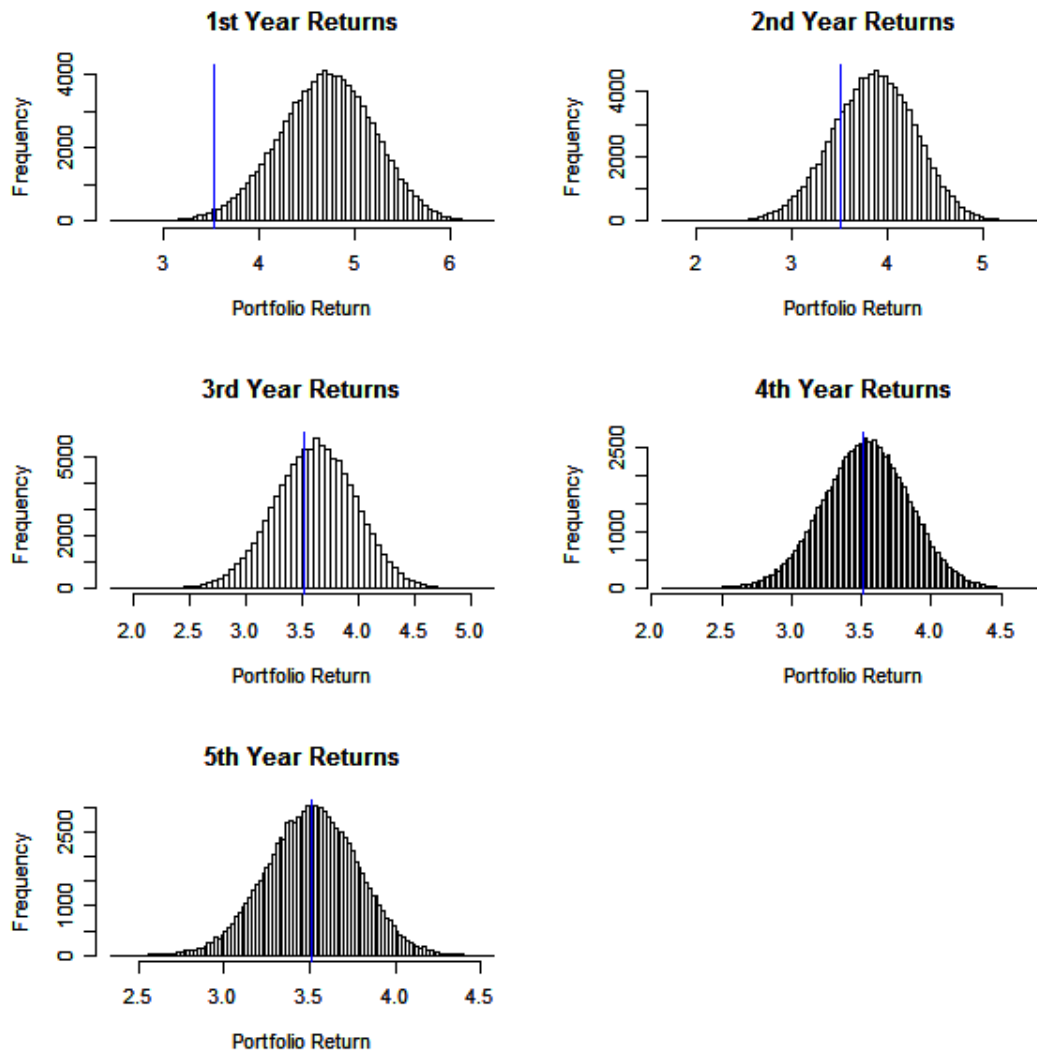


Figure 7.12. Annual returns of 1000-obligor credit portfolio (A-CCC) with updated flow and slightly lower spreads

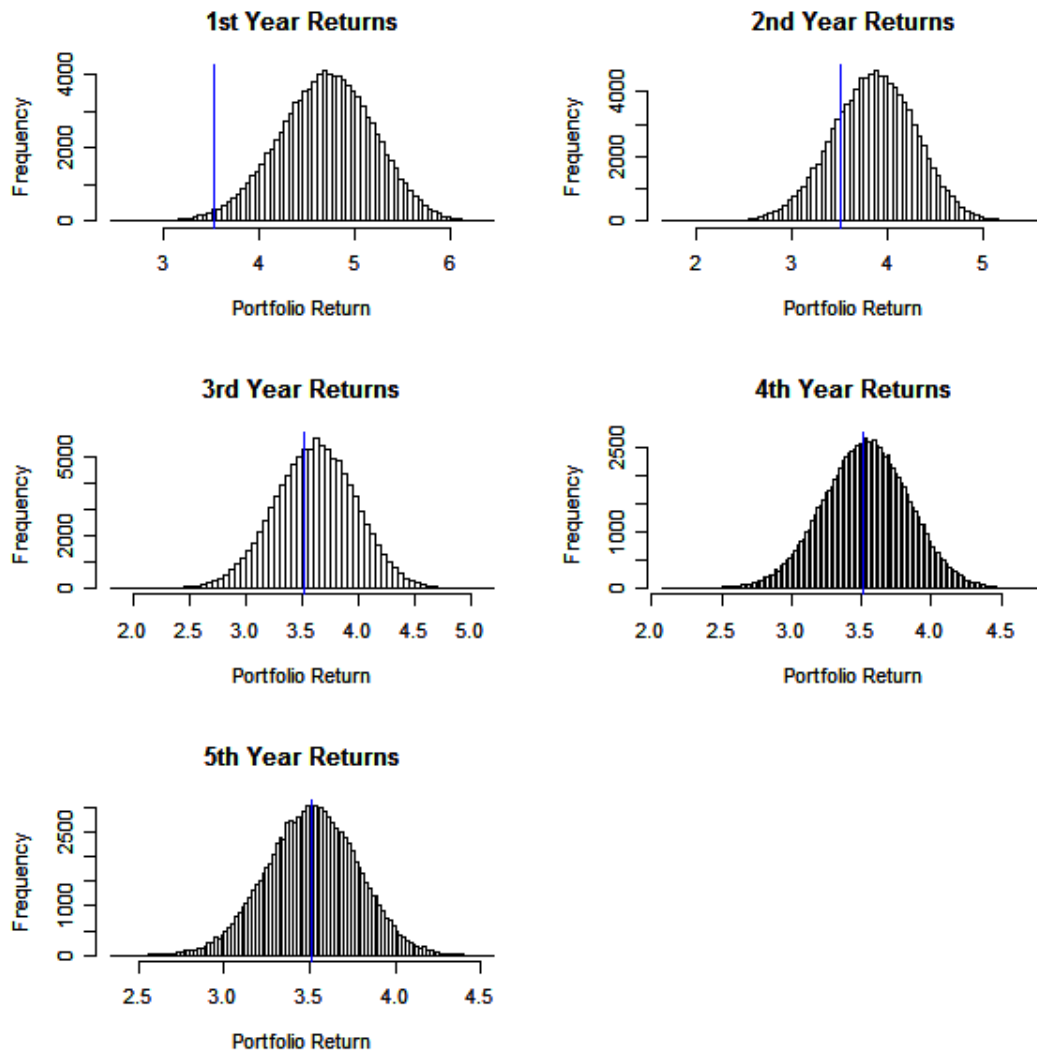


Figure 7.13. Annual returns of 1000-obligor credit portfolio (AAA-A) with updated flow and slightly lower spreads

8. CONCLUSIONS

The aim of this thesis was to understand the state-of-art of credit risk modelling in practice, the mathematical or statistical models behind the methodologies used widely in banking and financial industries, and their applicability; find relevant calibration methods for these models; apply one of those methodologies to a real credit portfolio; observe the true nature of a real credit portfolio by utilizing Monte Carlo simulations and thus reveal the effects of credit portfolio concentration and parameters such as credit spreads on risk-return profile of a credit portfolio. Another aim was to make the links between the regulatory approaches, statistical modelling, and the models accepted among practitioners. Pursuing this aim, we have reached the following conclusions.

In this thesis, we saw that the most widely used models among practice are CreditMetrics, Portfolio Manager, CreditRisk+, and CreditPortfolioView. We have given the details and the mathematical backgrounds of these four models. Apparently, there are several implications of these models. First, CreditMetrics is a model that requires the ratings of the obligors and the long-term average rating-transition and default rates assuming these rates are constant over time. Hence, CreditMetrics cannot illustrate the effect of business cycles and thus can underestimate the credit risk in case of downturns in the economy and overestimate in case the economy is in expansion. Moreover, the framework adopts a discrete time Markov chain model for the transitions. Portfolio Manager, on the other hand, models default probabilities in a more continuous manner. We think that the most useful and important characteristic of Portfolio Manager does not necessarily require the ratings of obligors. Yet, if the data in hand is sufficient, Portfolio Manager can still map its estimates of distance-to-default parameter to any rating system in order to calculate the corresponding spreads from the bond market. On the other hand, one vital deficiency of both CreditMetrics and Portfolio Manager is that they both require the historical equity data for every asset in the portfolio as input, and thus, as also stated by BCBS (2006), these models cannot be used to model the assets that are not publicly traded in the market. Indeed, that is one of the strengths of Basel II; it involves the pricing of assets even if they are not publicly traded and therefore there is no equity data available for those assets. CreditRisk+ is another well-known model, which models the variation in

default rates and mathematically tries to derive a closed-form expression for the total default loss distribution of a credit portfolio. CreditRisk+ does not cover transition risk and cannot model the effects of business cycles. Hence, although the framework introduces the default variation attribute, it is possible that CreditRisk+ cannot reflect the extreme losses resulting from downturns in the economic conditions because of the factor choice of CreditRisk+ and its assumptions to simplify the mathematical derivations. For example, the framework assumes a common exposure for obligors from the same group and in that sense, CreditRisk+ is an approximation model. Moreover, most of the parameters, such as LGD and sensitivities of default rates to the factors are not provided by the framework and are user-defined. The least favorable model is CreditPortfolioView, which is an econometric model conditioned on the levels of several macroeconomic factors. It is a very effective framework while it can reflect the changes in economic conditions to rating transition and default rates. Furthermore, it can be combined with a model that uses a rating-transition matrix, such as CreditMetrics. This way, for instance, CreditMetrics can attain a continuous nature. Basically, CreditPortfolioView predicts the future states of several macroeconomic factors with econometric models and utilizes these predictions to update a long-term average rating-transition matrix. However, it requires very wide default data on every industry sector in a credit portfolio.

Another conclusion of this thesis is on the basics of the Basel II framework and the link between Basel II, the models used in practice, and the statistical models used, for instance, by Glasserman and Li (2005) and Kalkbrener *et al.* (2007). A first obvious conclusion is that Basel II has several assumptions and drawbacks connected with these assumptions. For instance, it uses a single-factor model assuming that the portfolio of interest is perfectly diversified, which may not be the case in real-world credit portfolios. Moreover, it defines constant risk weights to assets but does not give the motivation behind these assignments. Another conclusion is that the models used in practice are applicable to Basel II under IRB approach. For instance, these models can be applied so that a bank can assess its own DP estimates. Next, most of the recent and popular studies are based on asset value models even though it is possible to model the default intensities with reduced-form models explained by Duffie and Singleton (2003) or by actuarial models like CreditRisk+ or by econometric models similar to CreditPortfolioView. Asset value models adopt the idea of thresholds for rating transitions and default. The idea of a default

threshold, for instance, the total debts of an obligor triggering a default, on the asset value distribution is very common among studies based on statistical models, the frameworks used in practice, and the regulatory approaches despite the fact that some adopt a MtM approach while the others adopt a pure default approach. Depending on how strong the correlations are and the level of diversification in the credit portfolio, the risk measures quantified by these models can converge to each other or diverge from each other.

We also point out the difficulties of developing and calibrating multifactor asset models such as CreditMetrics. The available data are not easy to obtain and are often very limited. It is more sensible to implement multifactor asset models if it is possible to access very wide financial data and/or if, regarding the timetable of the model implementation, it is feasible to develop own systematic factors, for instance, if there is time for creating own indices for the industries. It is not likely to find usable data on all the country-industries.

Next, our simulation results reveal that when using constant spreads a bank has to charge high spreads and gets most probably high returns for the first years but returns not much higher than the risk-free rate through the end of a contract with, for instance, a five or six year maturity. Also, a risk premium is necessary to cover the non-diversifiable risk and in case the spreads are not calculated regarding transition risks, the risk premium takes a higher importance. Although we carried out simulations from an updated cash flow perspective, our interviews reveal that at least in Turkey interest rates cannot be updated due to the downgrades or upgrades in the ratings - loan contracts do not cover such cases. Nevertheless, that way a bank can obviously, as expected, compensate the losses resulting from the changes in credit worthiness of an obligor. Yet, if it is not possible for a bank to update its interest rate policy due to rating changes, its portfolio is not perfectly diversified, and the spreads are not accurate, then the transition risk is a major risk component, which is not covered by Basel II or several other studies such as Glasserman and Li (2006). However, there are also studies in the literature such as Grundke (2008) that include transition risk in credit risk quantification. Besides, if a risk manager desires to adjust his capital position periodically and if the credit portfolio is liquid, in other words, if it is possible to dispose the loans (call the exposures back) or update the credit spread policy due to rating migrations, we believe that MtM approach is a more sensible and useful

method since one can see the annual value distributions of a credit portfolio throughout the whole planning horizon. Otherwise, we affirm that DL simulations will be of higher value.

Furthermore, regarding the simulations of portfolios of different risk profiles - rating groups resulting in different credit qualities – we understand that although there is significantly less risk when a portfolio is formed only with highly rated firms from investment grade, that way there is no chance to achieve high returns. Also, when obligors are chosen only among low ratings (speculative grade), even though the returns of lower ratings are higher than the returns of higher ratings, the risk increases drastically. Therefore, it appears that for a large credit portfolio, investment grade firms should be used to compensate the risk together with appropriate credit spreads, and speculative grade obligors should be chosen carefully to increase the return of the portfolio. In other words, concentration of a credit portfolio is very crucial in establishing the risk-return profile of a credit portfolio. Another result of our simulation studies is that a small credit portfolio of, for example, 25 obligors cannot reveal the true nature of a real credit portfolio.

For future studies, we can choose to apply and calibrate the remaining three models, namely, Portfolio Manager, CreditRisk+, and CreditPortfolioView, compare the simulation results of these models, and investigate the further integration of these models to the Basel II framework. Also, risk contributions of individual obligors can be inspected for portfolio optimization. Finally, more factors that will explain the asset movements can be sought to be added to our multi factor model to increase the R^2 - statistics and therefore the soundness of the regressions.

A final comment is that simulation is without a doubt a very effective tool in assessing the credit spreads and risk profile of a credit portfolio, in inspecting the loss and value distributions resulting from such portfolios and the concentration of those portfolios, and in validating a credit risk model.

APPENDIX A: R CODES

A.1. R Codes for Mark-to-Market Simulation with CreditMetrics

```

sim_CreditMetrics<-function(
  rep=10000, # number of replications
  horizon=1, # planing horizon to be examined
  rt_prob, # ratings transition matrix (percent)
  exposures, # exposures or par value of debts/bonds
  ratings, # recent ratings of the obligors/bonds
  cashFlows, # cashflows regarding each obligor/bond (as a matrix with dim,
numberOfObligors x max(bond maturities),
# with the exposure/par value included in the cash flow of the last
year and with zero values in case of no flow)
  loadings.a, # loadings of systematic risk factors
  loadings.b, # loadings of idiosyncratic risk factors
  varcov, # variance covariance matrix of the systematic factors
  r=4.25, # risk free rates of the planning horizon in percent with dim 1 x
max(bond maturities)
  forwardZeros, # forward zero-coupon (spot) rates (percent) with respect to rating
groups and with dim nRating x max(bond maturities)-1
  w_shape1=2, # shape parameter 1 of beta distributed recovery rates
  w_shape2=3, # shape parameter 2 of beta distributed recovery rates
  updateFlow=FALSE # If TRUE then when an obligor's rating changes, so does
its interest rate and annual payments
){
i_default<-dim(rt_prob)[2]
numberOfFactors<-dim(loadings.a)[2]
numberOfObligors<-length(ratings)
require(MASS)
initial_ratings<-ratings
if(updateFlow)

```

```

initial_cashFlows<-cashFlows
rt_prob<-rt_prob/100
forwardZeros<-forwardZeros/100
r<-r/100
nRating<-dim(rt_prob)[1]
bondMaturity<-dim(cashFlows)[2]
horizon<-min(bondMaturity,horizon)
if(length(r)==1) r<-rep(r,(horizon-1))
systematicFactors<-
array(t(mvnorm(horizon*rep,rep(0,numberOfFactors),varcov)),dim=c(numberOfFactors,h
orizon,rep))
# generating all the required random variates at one shot makes the simulation a lot faster
in R
temp.thresholds<-matrix(0,nRating,nRating)
for(i in 1:nRating){
  if(i==1)
    temp.thresholds[i,<--qnorm(cumsum(rt_prob[1,
(nRating+1):2])[nRating:1],lower.tail=FALSE)
  else{
    temp.thresholds[i,1:(i-1)]<-qnorm(cumsum(rt_prob[i,
1:(i-1)]),lower.tail=FALSE)
    temp.thresholds[i,i:nRating]<--qnorm(cumsum(rt_prob[i,
(nRating+1):(i+1)][(nRating-i+1):1],lower.tail=FALSE)
  }
  temp.thresholds[i,<-sort(temp.thresholds[i,])
}
thresholds<-matrix(0,numberOfObligors,nRating)
res<-array(1,c(rep,horizon))
for(j in 1:rep){
  weightedFactors<-as.matrix(loadings.a)%*%systematicFactors[,j]
# with dimensions, numberOfObligors x horizon
# or by Cholesky decomposition;
# cholesky<-t(chol(varcov))

```

```

#zz<-
array(mnorm(horizon*rep*numberOfFactors),c(numberOfFactors,horizon,rep))
# for(j in 1:rep) weightedFactors<- loadings.a%%cholesky%%zz[:,j]
x<-rep(0,numberOfObligors)
portfolioValue<-0
receivedPayments<-0
for(year in 1:horizon){
  for(i in 1:nRating) thresholds[ratings==i,<-
matrix(rep(temp.thresholds[i,],length(ratings[ratings==i])),length(ratings[ratings==i]),nRating,byrow=TRUE)
    numberOfNotDefaulted<-sum(ratings!=(i_default+1))
    x[ratings!=(i_default+1)]<-
weightedFactors[:,year][ratings!=(i_default+1)]+loadings.b[ratings!=(i_default+1)]*mnorm(
numberOfNotDefaulted)
  if(updateFlow&&(year!=bondMaturity)) preRatings<-ratings
    ratingsAndIndices<-getRatings_CMetrics(x,ratings,i_default,thresholds)
    ratings<-ratingsAndIndices$ratings
    x<-ratingsAndIndices$x
  if(updateFlow&&(year!=bondMaturity)){
    boolean<-as((ratings!=preRatings)*(ratings<i_default),"logical")
    if(sum(boolean)!=0)
      cashFlows[boolean,(year+1):bondMaturity]<-
getNewFlows(as.matrix(cashFlows[boolean,(year+1):bondMaturity]),exposures[boolean],ratings[boolean],nRating,forwardZeros[,year:(bondMaturity-1)])
  }
  receivedPayments[year]<-sum(cashFlows[ratings<i_default,year])
  if(year!=1)
    receivedPayments[year]<-
receivedPayments[year]+receivedPayments[year-1]*(1+r[year-1])
# receivables from nondefaulted plus previous receivables invested over risk free rate
  PVofNondefaulted<-0 # discounted future cashflows
  if(year!=bondMaturity)

```

```

# because we only discount the future cashflows (of nondefaulted obligors). At the end of
the horizon, there is no future cashflows.
      for(k in 1:nRating)
        if((bondMaturity-year)!=1)
          PVofNondefaulted<-
PVofNondefaulted+sum(cashFlows[ratings==k,-
(1:year)]%*%(1+forwardZeros[k,year:(bondMaturity-1)])^(1:(bondMaturity-year)))
        else
          PVofNondefaulted<-
PVofNondefaulted+sum(cashFlows[ratings==k,-(1:year)]*(1+forwardZeros[k,
year:(bondMaturity-1)])^(1:(bondMaturity-year)))

          PVofDefaulted<-
sum(exposures[ratings==i_default]*rbeta(sum(ratings==i_default), w_shape1,w_shape2))
          #EAD = PAR (1- LGD)
          ratings[ratings==i_default]<-i_default+1
          portfolioValue[year]<-
PVofNondefaulted+PVofDefaulted+receivedPayments[year]      # Portfolio value /
wealth / total assets
        }
      ratings<-initial_ratings
      if(updateFlow)
        cashFlows<-initial_cashFlows
      res[j,]<-portfolioValue
    }
  res}

```

Figure A.1. R code of sim_CreditMetrics

```

getNewFlows<-function(cashFlows,exposures,ratings,nRating,forwardZeros){
  n<-length(exposures)
  for(i in 1:nRating)
    if(length(ratings[ratings==i])!=0)

```

```

        cashFlows[ratings==i,]<-
(as.matrix(exposures[ratings==i])%%as.matrix(forwardZeros)[i,])*(cashFlows[ratings==i
,]!=0)
    if(is.matrix(cashFlows))
        whichColumns<-rowSums(cashFlows!=0)
    else
        whichColumns<-sum(cashFlows!=0)
    cashFlows[cbind(1:n,whichColumns)]<- cashFlows[cbind(1:n,whichColumns)] +
exposures
    cashFlows }

```

Figure A.2. R code of getNewFlows

```

getRatings_CMetrics<-function(x, ratings, i_default, thresholds){
    newRatings<-0
    whichInterval<-(x[ratings!=(i_default+1)]>thresholds[ratings!=(i_default+1),])
    newRatings[ratings!=(i_default+1)]<-i_default-rowSums(whichInterval)
    newRatings[ratings==(i_default+1)]<-(i_default+1)
    x[newRatings!=ratings]<-0
    x[newRatings==i_default]<--100
    list(ratings=newRatings, x=x) }

```

Figure A.3. R code of getRatings_CMetrics

```

cashFlow<-function(exposures,maturities,ratings,r,creditSpreads){
    n<-length(exposures)
    cashFlows<-matrix(0,n,max(maturities))
    for(i in 1:n){
        cashFlows[i,1:maturities[i]]<-
exposures[i]*(r/100+creditSpreads[ratings[i]]/100)
        cashFlows[i, maturities[i]]<- cashFlows[i, maturities[i]] + exposures[i]
    }
    cashFlows }

```

Figure A.4. R code of cashFlow

```

econCapital <- function(res,exposures,r,alpha=.01, type="V"){
# to obtain the risk figures such as VaR and ES
VaR<-ES<-Pr<-opportunity<-capital<-0
rep<-dim(res)[1]
if(type=="V"){      # for portfolio value distribution
  for(i in 1:dim(res)[2]){
    VaR[i]<-sort(res[,i])[ rep*alpha]
    ES[i]<-mean(res[res[,i]<VaR[i],i])
    Pr[i]<-mean(res[,i]>(sum(exposures)*(1+r)^i))
    opportunity[i]<- (sum(exposures)*(1+r)^i)- VaR[i]
    capital[i]<- mean(res[,i])- VaR[i]
  }
data<-data.frame(VaR=VaR,ES=ES, Pr=Pr, OppCost=opportunity, EconCapital=capital)
}else{
  # for loss distribution (loss resulting only from default)
for(i in 1:dim(res)[2]){
VaR[i]<-sort(res[,i])[ rep*(1-alpha)]
ES[i]<-mean(res[res[,i]>VaR[i],i])
capital[i]<- VaR[i]-mean(res[,i])
}
data<- data.frame(VaR=VaR,ES=ES, EconCapital=capital)
}
data}

```

Figure A.5. R code of econCapital

```

value2return<-function(# converts the portfolio values into yearly returns
  portfolioValue, # Simulation Results as a data frame
  exposures      # Exposures of the obligors
){
endOfHorizon<-dim(portfolioValue)[2]
totalExposure<-sum(exposures)
returns<-matrix(0, dim(portfolioValue)[1],endOfHorizon)
for(i in 1:endOfHorizon){
  returns[,i]<-(portfolioValue[,i]/totalExposure)^(1/i)
}
}

```

```

        returns[,i]<- (returns[,i]-1)*100      # in order to obtain percent
returns
}
averageReturn<-(mean(portfolioValue[,endOfHorizon])/totalExposure)^(1/endOfHorizon)
averageReturn<-(averageReturn-1)*100
list(returns=returns, average=averageReturn)}

```

Figure A.6. R code of value2return

```

gen_ratings<-function(numberOfObligors, numberOfRatings, weights=c(1,2,3,4,3,2,1)){
# Generates random ratings
  ratings<-runif(numberOfObligors)
  cumPr<-cumsum(weights/sum(weights))
  for(i in 1:numberOfRatings)
    ratings[ratings<cumPr[i]]<-i          # (i/numberOfRatings)<-i
  ratings}

```

Figure A.7. R code of gen_ratings

```

giveSpreads<-function(rt_prob,LGD=.6,rp=.01){
# Calculates the necessary one year spreads
rt_prob<-rt_prob/100
EDF<-rt_prob[,dim(rt_prob)[2]]
ELpre<-(LGD*EDF)/(1-EDF)
ELpre+rp}

```

Figure A.8. R code of giveSpreads

A.2. R Codes for Default Loss Simulation with CreditMetrics

```

simDefaultLoss_CM<-function(
  rep=10000, # number of replications
  horizon=1, # planing horizon to be examined
  rt_prob,   # ratings transition matrix (percent)
  exposures, # exposures or par value of debts/bonds
  ratings,   # recent ratings of the obligors/bonds

```

```

    loadings.a, # loadings of systematic risk factors
    loadings.b, # loadings of idiosyncratic risk factors
    varcov,     # variance covariance matrix of the factors
    w_shape1=2, # shape parameter 1 of beta distributed recovery rates
    w_shape2=3 # shape parameter 2 of beta distributed recovery rates
){
i_default<-dim(rt_prob)[2]
numberOfFactors<-dim(loadings.a)[2]
numberOfObligors<-length(ratings)
require(MASS)
initial_ratings<-ratings
rt_prob<-rt_prob/100
nRating<-dim(rt_prob)[1]
systematicFactors<-
array(t(mvnorm(horizon*rep,rep(0,numberOfFactors),varcov)),dim=c(numberOfFactors,h
orizon,rep))
# generating all the required random variates at one shot makes the simulation a lot faster
in R
temp.thresholds<-matrix(0,nRating,nRating)
for(i in 1:nRating){
  if(i==1)
    temp.thresholds[i,<--qnorm(cumsum(rt_prob[1,
(nRating+1):2])[nRating:1],lower.tail=FALSE)
  else{
    temp.thresholds[i,1:(i-1)]<-qnorm(cumsum(rt_prob[i,
1:(i-1)]),lower.tail=FALSE)
    temp.thresholds[i,i:nRating]<--qnorm(cumsum(rt_prob[i,
(nRating+1):(i+1)])[(nRating-i+1):1],lower.tail=FALSE)
  }
  temp.thresholds[i,<-sort(temp.thresholds[i,])
}
thresholds<-matrix(0,numberOfObligors,nRating)
res<-array(1,c(rep,horizon))

```

```

for(j in 1:rep){
  if(numberOfFactors==1)
    weightedFactors<-loadings.a*systematicFactors[:,j]
  else
    weightedFactors<-loadings.a%%systematicFactors[:,j]
    # with dimensions, numberOfObligors x horizon
  x<-rep(0,numberOfObligors)
  defaultLoss<-0
  for(year in 1:horizon){
    for(i in 1:nRating)
      thresholds[ratings==i,<-
matrix(rep(temp.thresholds[i,],length(ratings[ratings==i])),length(ratings[ratings==i]),nRating,byrow=TRUE)
      numberOfNotDefaulted<-sum(ratings!=(i_default+1))
      x[ratings!=(i_default+1)]<-
weightedFactors[:,year][ratings!=(i_default+1)]+loadings.b[ratings!=(i_default+1)]*rnorm(
numberOfNotDefaulted)
      ratingsAndIndices<-getRatings_CMetrics(x,ratings,i_default,thresholds)
      ratings<-ratingsAndIndices$ratings
      x<-ratingsAndIndices$x
      defaultLoss[year]<-sum(exposures[ratings==i_default]*(1-
rbeta(sum(ratings==i_default), w_shape1,w_shape2))) #EAD = PAR (1- LGD)
      ratings[ratings==i_default]<-i_default+1
    }
    ratings<-initial_ratings
    res[j,]<-defaultLoss
  }
}
res}

```

Figure A.9. R code of simDefaultLoss_CM

A.3. R Codes for Artificial Portfolio Generation

```

giveGroups<-function(W){
# groups a matrix that includes zeros with respect to the common columns of the rows

```

```

ind<-(W!=0)
group<-rep(0,dim(W)[1])
numberOfColumn<-dim(ind)[2]
groupID<-0
while(length(group[group==0])!=0){
  groupID<-groupID+1
  toBeClustered<-ind[group==0,][1,]
  clustered<-rowSums(t(t(ind[group==0,])==toBeClustered))
  group[group==0][clustered==numberOfColumn]<-groupID }
group}

```

Figure A.10. R code of giveGroups

```

giveStatistics<-function(W){ # gives variance and mean of each column of a matrix
  variances<-0
  means<-0
  for(k in 1:dim(W)[2]){
    Wk<-W[,k]
    Wk<-Wk[Wk!=0]
    variances[k]<-var(Wk)
    means[k]<-mean(Wk) }
  data.frame(variances=variances, means=means)}

```

Figure A.11. R code of giveStatistics

```

giveCorr<-function(W,group, groupID){
# gives the correlation matrix of the columns of a matrix that belongs to the groupID
  ind<-(W!=0)
  cor(W[group==groupID,ind[group==groupID,][1,]])}

```

Figure A.12. R code of giveCorr

```

giveVarCov<-function(variances, corr){
# forms the variance-covariance matrix of columns of a matrix

```

```

# should be done group by group. Here the vector, variances, is the variances of the
columns, which belong to the targetted group
  L<-length(variances)
  deviations<-sqrt(variances)
  varcov<-matrix(deviations,L,L,byrow=TRUE) # or simply varcov<-
deviations%*%t(deviations)*corr
  varcov<-varcov*deviations
  varcov<-varcov*corr
  varcov}

```

Figure A.13. R code of giveVarCov

```

genArtificialGroup<-function(n,ind, # instead of ind<-(subW!=0)[1,]
  mean,varcov){
# randomly generates factors for a group of obligors
  batch<-dim(varcov)[1]
  artificial<-matrix(0,n,length(ind))
  artificial[,ind]<-t( t(chol(varcov))%*%matrix(rnorm(batch*n,mean),batch) )
  artificial}

```

Figure A.14. R code of genArtificialGroup

```

generateArtificial<-function(n,W,cutoff.max=NULL,cutoff.min=NULL){
# generates an artificial systematic factor loading matrix with n many obligors for each
group in the reference matrix
  group<-giveGroups(W)
  numberOfGroups<-max(group)
  loadings.statistics<-giveStatistics(W)
  loadings.variances<-loadings.statistics$variances
  loadings.means<-loadings.statistics$means
  W_new<-matrix(0,n*numberOfGroups,dim(W)[2])
  for(id in 1:numberOfGroups){
    group.corr<-giveCorr(W,group,id)
    group.varcov<-
giveVarCov(loadings.variances[(W!=0)[group==id,][1,],group.corr)

```

```

        print(group.varcov)
        W_new[((id-1)*n+1):(n*id),]<-
genArtificialGroup(n,(W!=0)[group==id,][1,],loadings.means[(W!=0)[group==id,][1,],gro
up.varcov)
    }
    W_new}

```

Figure A.15. R code of generateArtificial

A.4. R Codes for Regression of Log>Returns

```

getLogreturns<-function(data){
# Requires a data frame as an input, e.g. daily or weekly prices
    numberOfStocks<-length(data)
    data<-log(data)
    N<-dim(data[1])[1]
    logrt<-array(0,dim=c(N-1,numberOfStocks))
    for(i in 1:numberOfStocks)
        logrt[,i]<-data[-1,i]-data[-N,i]
    print(colnames(data))
    logrt}

```

Figure A.16. R code of getLogreturns

```

fitRegression<-function(returns,n=2,omitIntercept=TRUE){
Rsquare<-0
betas<-matrix(0,(dim(returns)[2]-n),(n+1))
for(i in 1:(dim(returns)[2]-n)){
    if(omitIntercept)
        regres<-lm(returns[,i+n]~0+returns[,1:n])
    else
        regres<-lm(returns[,i+n]~returns[,1:n])
    betas[i,]<-regres$coefficients
    anovaTable<-anova(regres)
    Rsquare[i]<-anovaTable[1,2]/sum(anovaTable[,2])
}
}

```

```

}
list(coefficients=betas, Rsquare=Rsquare)}

```

Figure A.17. R code of fitRegression

```

getDriftAndVol<-function(data){
# Calculates the drift and volatility of equity
# Requires a data frame as input
  driftInd<-1;volInd<-2;meanPercInd<-3;varPercInd<-4;
  numberOfStocks<-length(data)
  res<-array(0,dim=c(numberOfStocks,4))
  for(i in 1:numberOfStocks){
    N<-dim(data[i])[1]
    logrt<-log(data[2:N,i]/data[1:(N-1),i])
    v<-sum(logrt)/N
    sigma2<-sum((logrt-v)^2)/(N-1)
    muLog<-v+sigma2/2
    res[i,driftInd]<-muLog*258; res[i,volInd]<-sqrt(sigma2*258);
# These are annualized drift and volatility regarding log normality assumption.
    rt<-(data[2:N,i]-data[1:(N-1),i])/data[1:(N-1),i]
    muPerc<-mean(rt); varPerc<-var(rt);
    # These are mean and variance of daily percent changes.
    res[i,meanPercInd]<-muPerc; res[i,varPercInd]<-varPerc;
  }
  print(c("drift", "volatility", "meanPercentReturn", "varPercentReturn"));
  res}

```

Figure A.18. R code of getDriftAndVol

APPENDIX B: ADDITIONAL HISTOGRAMS

B.1. Additional Value Distributions

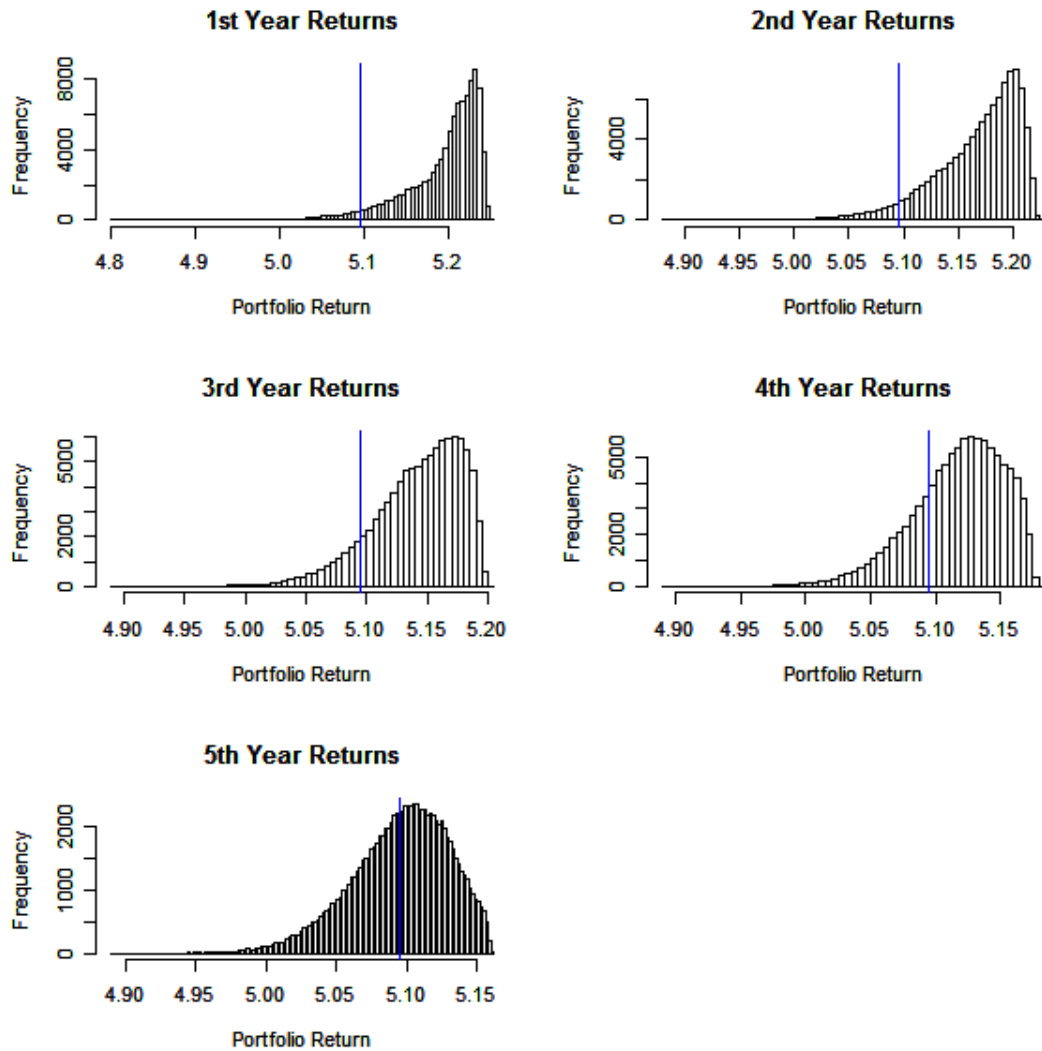


Figure B.1. Annual returns of 1000-obligor credit portfolio (AAA-A) with slightly smaller spreads

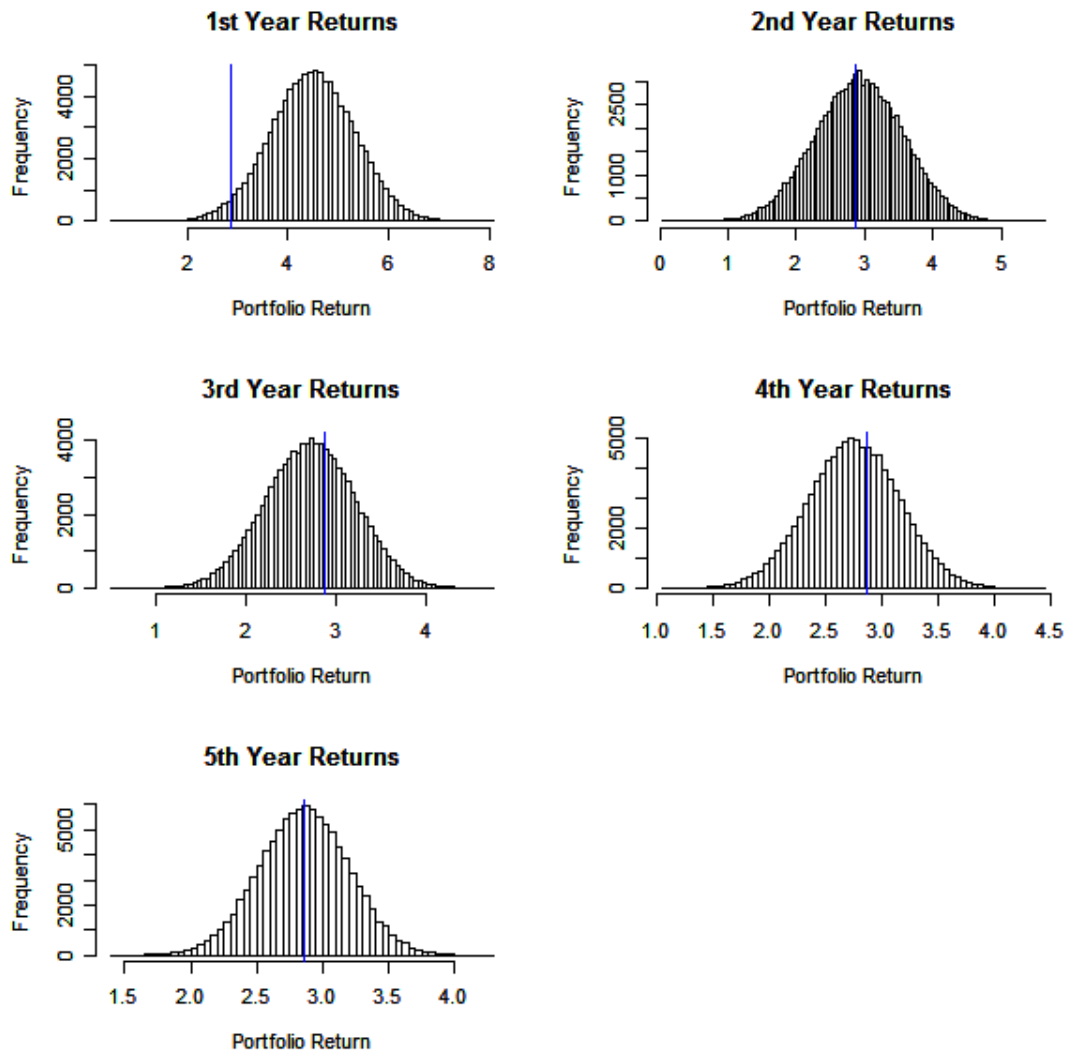


Figure B.2. Annual returns of 1000-obligor credit portfolio (A-CCC) with slightly smaller spreads

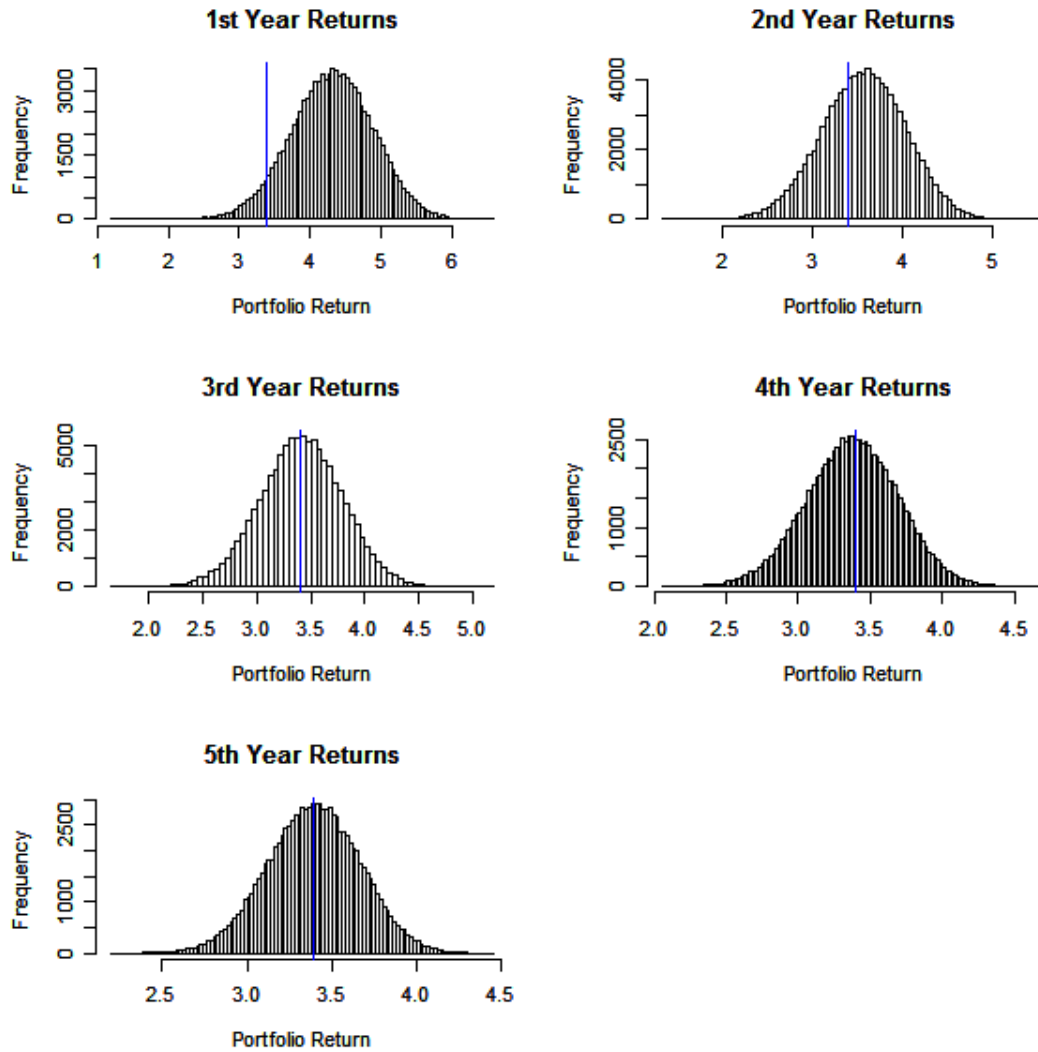


Figure B.3. Annual returns of 1000-obligor credit portfolio (mixed ratings) with slightly smaller spreads

B.2. Loss Distributions

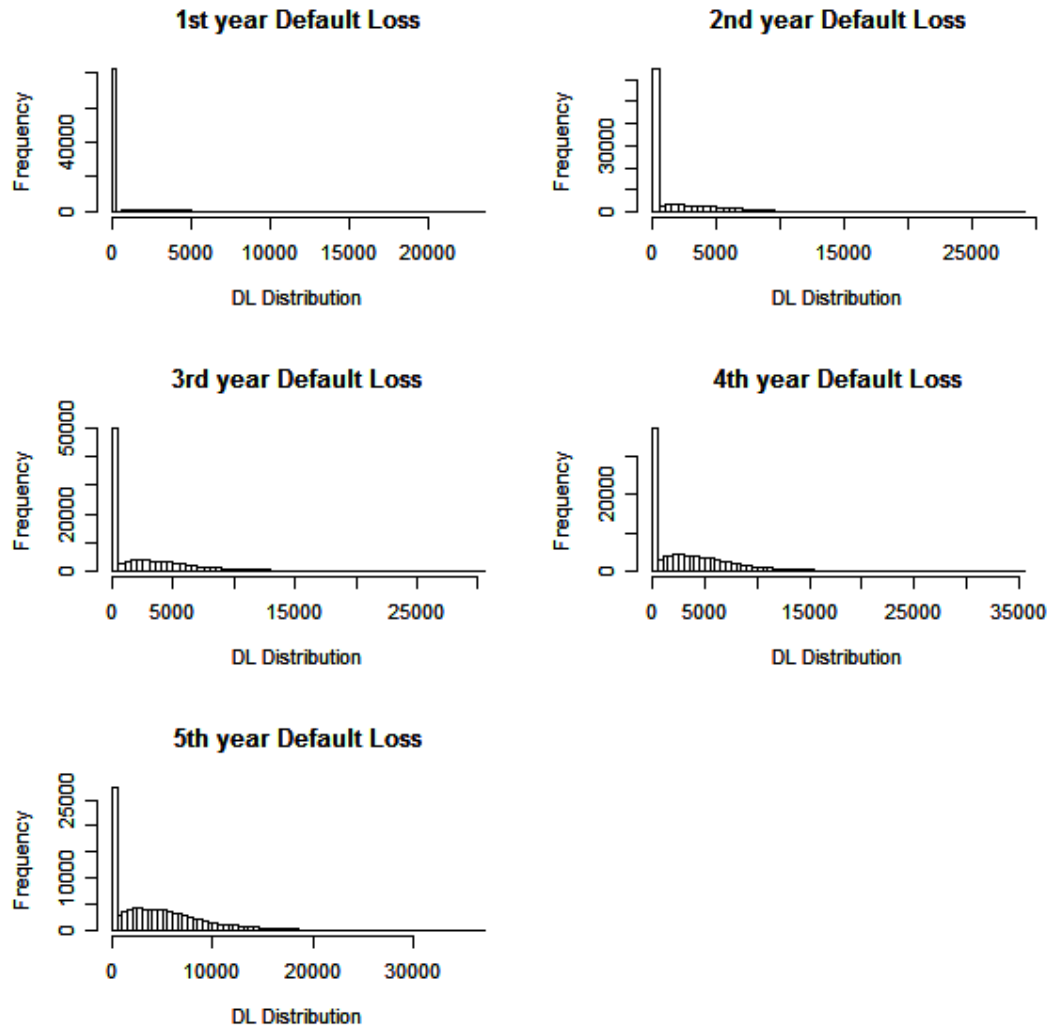


Figure B.4. Yearly default loss distributions of 1000-obligor credit portfolio (AAA-A)

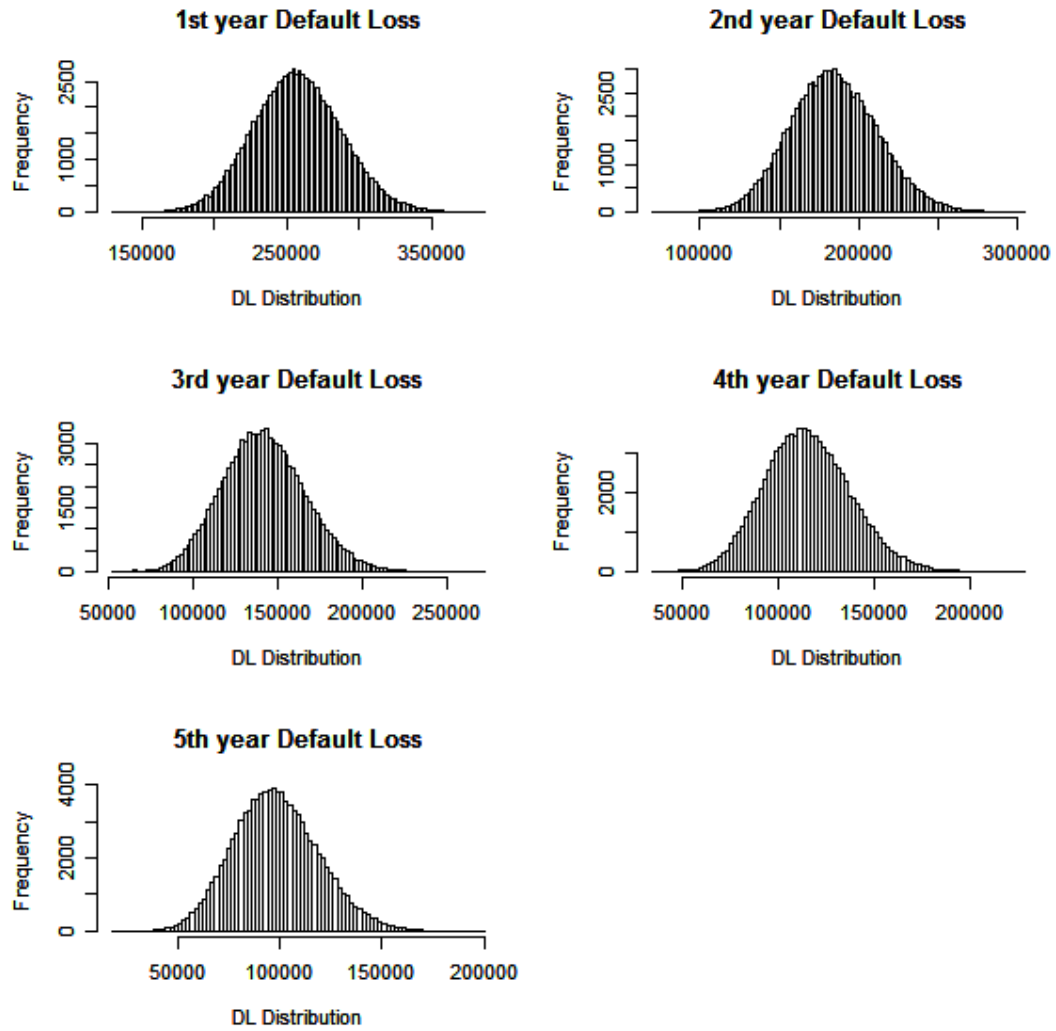


Figure B.5. Yearly default loss distributions of 1000-obligor credit portfolio (A-CCC)

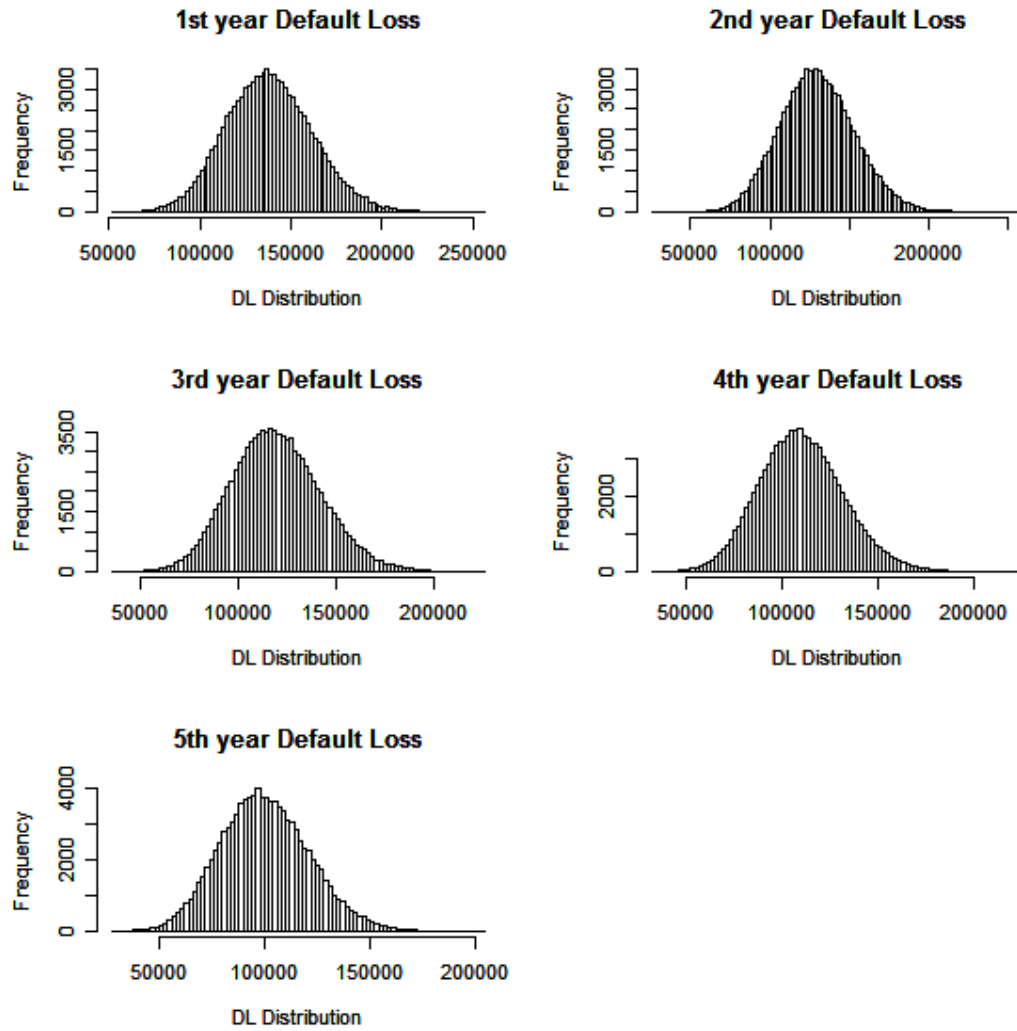


Figure B.6. Yearly default loss distributions of 1000-obligor credit portfolio (mixed ratings)

9. REFERENCES

- Anbar, A., 2006, "Credit Risk Management in Turkish Banking Sector: A Survey Study", *Electronic Journal of Social Sciences*, Vol. 5, No. 17, pp. 10-24.
- Altman, E. I., 2006, "Default Recovery Rates and LGD in Credit Risk Modeling and Practice: An Updated Review of the Literature and Empirical Evidence", *Working Paper*, New York University Salomon Center, USA.
- Bessis, J., 1999, *Risk Management in Banking*, Second Edition, John Wiley & Sons, 2002.
- Basel Committee on Banking Supervision (BCBS), 1999, "A New capital Adequacy Framework", Bank for International Settlements.
- Basel Committee on Banking Supervision (BCBS), 2006, "International convergence of capital measurement and capital standards", Bank for International Settlements.
- Bohn R. J., 1999, "Characterizing Credit Spreads", *Working Paper*, University of California, Berkeley.
- Bohn, J. R., 2006, "Structural Modeling in Practice", Moody's KMV Company.
- Choudhry, M., 2003, *The Bond & Money Markets: Strategy, Trading, Analysis*, Butterworth Heinemann, Burlington AM, Great Britain.
- Crosbie P. J., and J. R. Bohn, 2003, "Modelling Default Risk", Moody's KMV Company.
- Crouhy, M., D.Galai, and R.Mark, 2000, "A Comparative Analysis of Current Credit Risk Models", *Journal of Banking and Finance*, Vol. 24, pp. 59-117.
- Crouhy, M., D.Galai, and R.Mark, 2001, *Risk Management*, McGraw-Hill, New York.

- CSFB (Credit Suisse First Boston), CreditRisk+™ - A Credit Risk Management Framework, 1997, London.
- Duffie, D., and K. J. Singleton, 2003, *Credit Risk: Pricing, Measurement, and Management*, Princeton University Press.
- Engelmann, B., and R. Rauhmeier, 2006, *The Basel II Risk parameters: Estimation, Validation, and Stress Testing*, Springer, Berlin Heidelberg, Germany.
- European Central Bank (ECB), 2007, “The Use of Portfolio Credit Risk Models in Central Banks”, *Occasional Paper Series*, No. 64.
- Fatemi, A., and I. Fooladi, 2006, “Credit Risk Management: A Survey of Practices”, *Managerial Finance*, Vol. 32, No. 3, pp. 227-233.
- Glasserman P., and L. Li, 2005, “Importance Sampling for Portfolio Credit Risk”, *Management Science*, Vol. 51, No. 11, pp. 1643-1656.
- Grundke P., 2008, “Importance sampling for integrated market and credit portfolio models”, *European Journal of Operational Research*, doi:10.1016/j.ejor.2007.12.028.
- Gupton G. M., C. C. Finger, and M. Bhatia, 1997 (updated in 2007), “CreditMetrics – Technical Document”, J. P. Morgan, New York.
- Gürtler, M., M. Hibbeln and C. Vöhringer, 2008, “Adjusting Multi-Factor Models for Basel II-Consistent Economic Capital”, *Working Paper Series*, Finanz und Wirtschaft.
- Hull, J., M. Predescu, and A. White, 2005, “Bond Prices, Default Probabilities, and Risk Premiums”, *Journal of Credit Risk*, Vol. 1, No. 2, pp. 53-60.

- Jacobs, T. A., 2004, "Risk Models and Basel II: A Review of the Literature", *Seminar in Macro Finance and Financial Institutions*, University of Illinois.
- Kalkbrener, M., A. Kennedy, and M. Popp, 2007, "Sensible and Efficient Capital Allocation for Credit Portfolios", *The Journal of Computational Finance*, Vol. 11 No. 2, pp. 45-78.
- Kealhofer S., and J. R. Bohn, 2001, "Portfolio management of default risk", KMV Corporation.
- Löffler, G., 2004, "Implied Asset Value Distributions", *Applied Financial Economics*, Vol. 14, No. 12, pp. 875 – 883.
- Lu, Y. S., 2008, *Default Forecasting in KMV*, Master of Science Thesis, University of Oxford.
- Merton R. C., 1974, "On the Pricing of the Corporate Debt: The Risk Structure of Interest Rates", *Journal of Finance*, Vol. 29, No. 2, pp. 449-470.
- Oktaý and Temel, 2007, "A Field Study of Credit Risk Management In Commercial Banks in the Framework of Basel II Criteria", *Zonguldak Karaelmas University Journal of Social Sciences*, Vol. 3, No. 6, pp. 163-185.
- Ong, M. K., 2005, *The Basel Handbook: A Guide for Financial Practitioners*, Risk Books, London, England.
- Rachev, S. T., S. Mittnik, F. J. Fabozzi, S. M. Focardi, and T. Jašić, 2007, *Financial Econometrics: From Basics to Advanced Modeling Techniques*, John Wiley & Sons Inc., New Jersey, Canada.
- Sak H., and W. Hörmann, 2008, "Fast Simulation in Credit Risk", *Quantitative Finance*,
Cached: URL: //http.manuscriptcentral.com/tantf/rquf.

Saunders, A., and L. Allen, 2002, *Credit Risk Management: New Approaches to Value at Risk and Other Paradigms*, John Wiley & Sons, Inc., New York (Simultaneously Canada).

Schmid, B., 2004, *Credit Risk Pricing Models – Theory and Practice*, Springer, Berlin Heidelberg, Germany.

Smithson, C., 2003, *Credit Portfolio Management*, John Wiley & Sons Inc., New Jersey, Hoboken, Canada.

Smithson, C., S. Brannan, D. Mengle, and M. Zmiewski, 2002, *Results from the 2002 Survey of Credit Portfolio Management Practices*, www.rutterassociates.com.

Vazza, D., D. Aurora, and N. Kraemer, 2008, “Default, Transition and Recovery: 2007 Annual Global Corporate Default Study and Rating Transitions”, Standard & Poor’s.