ROBERT COLLEGE School of Engineering DEPARTMENT OF MECHANICAL ENGINEERING	Course: M.S. Thesis	Class: Grad. ME	Page:
	Lab. Design Thesis: "A study on the optimum design characteristics of compact heat-exchangers used in aircraft air-conditioning systems"		

R O B E R T   C O L L E G E

S C H O O L   O F   E N G İ N E E R I N G

Department
of
Mechanical Engineering

Thesis for M.S. degree

" A STUDY ON THE OPTİMUM DESIGN CHARACTERISTICS OF COMPACT HEAT-EXCHANGERS USED IN AIRCRAFT AIR-CONDITIONING SYSTEMS "

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T A B L E O F CONTENTS

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NOMENCLATURE

A_i - Inner surface area, ft^2

A_o - Outer " " , ft^2

A_f - Minimum free flow area, ft^2

C_2, C_4 - Dimensionless numbers depending on l_l/D_o, l_t/D_o

C - Capacity rate, Btu / hr F

C_p - Specific heat at constant pressure, Btu / lb F

D_i - Interior tube diameter, ft

D_o - Outer " " , ft

$F = \frac{(NTU)_o}{(NTU)_i}$

f - fanning friction factor, dimensionless

g - Acceleration of gravity = 32.2×3600^2 ft / hr^2

G - mass velocity, based on minimum free flow area, lb/hr ft^2

h - film coefficient of heat transfer, Btu / hr ft^2 F

J - $N_{St}\, N_{Pr}^{2/3}$, dimensionless

k - Conductivity, Btu / hr F ft

K_c - Entrance loss coefficient, dimensionless

K_e - Exit " " , "

K_p - Pass " " , "

L - Tube length, ft

l_t - Transverce pitch, ft

l_l - Longitudinal " , ft

L'' - Non-flow length, ft

L' - The flow length of outside gas, ft

M - Mach number , dimensionless

N - Total number of tubes

NTU - Total number of transfer units = $\frac{UA}{(WC_p)_{min}}$ dimensionless

NTU_i - $\frac{h_i\ A_i}{W_i\ C_{pi}}$, interior number of transfer units , dimensionless

NTU_o - $\frac{h_o\ A_o}{W_o\ Cp_o}$, exterior " " " " "

N_t - The number of tubes per row normal to the outside gas flow

N_1 - The number of tubes per pass

$(NTU)_p$ - The number of transfer units per pass

n - Number of passes

p_l - $\frac{l_l}{D_o}$, dimensionless

p_t - $\frac{l_t}{D_o}$, dimensionless

P - Total friction power loss , ft #/ hr

p - pressure of gases , psfa

$\bar{R}$ - Gas constant , Btu / lb F

r_w - The ratio of mass flow rate of outside gas to that of inside gas

R - Hydraulic radius for flow normal to tubes = $\frac{A_{fo}}{A_o}$. L'

$s = r_w \frac{C_{po}}{C_{pi}}$, dimensionless

t - Tube wall thickness , ft

T - Temperature , °R

U - Over-all coefficient of heat transfer , Btu / ft^2 hr

V - Volume , cu ft

v_i - Specific volume of inside gas , cu ft / lb

v_o - Specific volume " outside " , "

W_i - Mass flow rate of the interior gas , lb / hr

W_o - " " " " " exterior " , "

w - Total weight of tubes , lb

Dimensionless Groups :

N_{St} - Stanton number $= \frac{h}{G\, C_p}$

N_{Pr} - Prandtl " $= \frac{\mu Cp}{k}$

N_{Re} - Reynolds " $= \frac{D\, G}{\mu}$

Greek Letters :

δ_i - v_{im} / v_{i1} , dimensionless

δ_o - v_{om} / v_{o1} , "

ε - Effectiveness of the heat-exchanger , dimensionless

ε_p - " per pass of the heat-exchanger , "

λ - t / D_i , dimensionless

μ - Absolute viscosity , lb / hr ft

σ - Ratio of free flow to frontal area , dimensionless

ρ - Density of the tube material , lb / cu ft

τ - Stress , psf

φ_1 - Dimensionless factor , used in calculating $(\Delta p)_o$

φ_2 - " " , " " " $(\Delta p)_i$

Subscripts :

o - refers to the outer gas

i - refers to the outer gas

I - " " inlet conditions

2 - " " outlet "

m - " " mean "

INTRODUCTION

In this work we have tried to indicate the optimum design of a cross-flow bare tube heat-exchanger to be used in aircraft air-conditioning system , with volume, and weight being the principle quantities to be minimized, given the maximum allowable power loss .

In particular we have attempted to solve completely the following problem :

To design an optimum heat-exchanger , given the mass flow rate inlet temperature and pressure and leaving temperature of the hot gas, and the entering temperature and pressure of the cold gas .

The formulas developed or collected from references can be used in the solution of many similar problems .

We have reached the conclusion that the minimum weight and volume cannot be achieved without increasing the non-flow dimension to very large values . But since this leads to awkward ducting ,one must restrict the non-flow dimension .

We have also noticed that the usage of tubes with smaller diameters tends to decrease the weight and also the volume .

As for the optimum tube spacings no definite answer is given . Although it may be stated that they should be chosen as small as possible , provided the non-flow dimension is not excessive .

The reason we used bare tubes instead of finned surfaces is that the latter type surfaces though compact will make the heat-exchanger heavier for a particular design than the tubular type. This is due to the fact that the fins are not so effective in transmitting the heat than the bare tube surface. (See pp. 619, Reference 2)

All the quantities used are assumed to be given in ft, #, lb, hr, ^{o}R, Btu system.

The Derivation Of various Design Equations

The effectiveness- NTU relations :

The effectiveness of the heat-exchanger is given by

$$\varepsilon = \frac{Cp_i \ W_i (T_{i1} - T_{i2})}{(Cp)W_{min} \ (T_{i1} - T_{o1})} \qquad \text{pp. 185 , ref. 5}$$

since $\frac{T_{i1} - T_{i2}}{T_{i1} - T_{o1}}$ is usally close to I we must make

$(Cp\ W)_{min} = (Cp\ W)_i$, otherwise to ensure a large effectiveness large NTU is needed . The effect of large NTU on weight will be discussed later .

$$\varepsilon = \frac{Cp_i \ W_i \ (T_{i1} - T_{i2})}{(Cp\ W)_{min} \ (T_{i1} - T_{o1})} = \frac{T_{i1} - T_{i2}}{T_{i1} - T_{o1}} \qquad (I)$$

From page 12 ref. I

for $C_{max} = C_{mixed} = W_o \ Cp_o$ $\qquad C_{min} = C_{unmixed} = W_i \ Cp_i$

effectiveness per pass is given by

$$\varepsilon_p = r_W \frac{Cp_o}{Cp_i} \left[I - e^{-\frac{\Gamma \ Cp_i}{r_W Cp_o}} \right] \qquad (2) \qquad \text{where}$$

$$\Gamma = I - e^{-NTU_p} \qquad (3)$$

The assumption being that the fluid flowing normal to tube banks is perfectly cross-mixed .

The over-all effectiveness is given by formula I7 (b) pp.I3 ref. I

$$\varepsilon = \frac{\left(\dfrac{I - \dfrac{\varepsilon_p Cp_i}{r_W\, Cp_o}}{I - \varepsilon_p}\right)^n - I}{\left(\dfrac{I - \dfrac{\varepsilon_p Cp_i}{r_W\, Cp_o}}{I - \varepsilon_p}\right)^n - \dfrac{Cp_i}{r_W\, Cp_o}} \qquad (4)$$

the assumption being that the hot air completely mixes between passes . This is conservative since the " degree of mixing is not known , and since as the degree of mixing increases the effectiveness decreases .

$$NTU = n\ NTU_p \qquad (5)$$

Equating the heat transfer between the hot and cold gases of the exchanger

$$W_i\ Cp_i\ \Delta T_i = W_o\ Cp_o\ \Delta T_o$$

Let $r_W = \dfrac{W_o}{W_i}$ (6)

hence $$r_W = \frac{Cp_i}{Cp_o} \cdot \frac{\Delta T_i}{\Delta T_o} \qquad (7)$$

From fig. I it is apparent that

$$r_W > \frac{Cp_i}{Cp_o} \cdot \frac{\Delta T_i}{T_{i1} - T_{o1}} = \frac{Cp_i}{Cp_o} \times \varepsilon$$

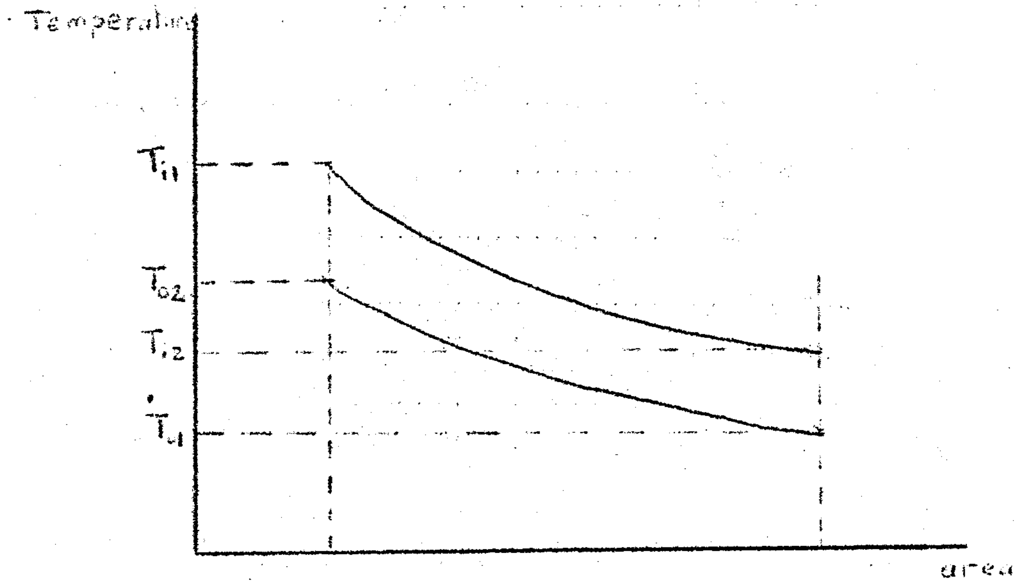

Fig. 1

Determination Of Pressure losses :

As is well-known the sum of the pressure drop ratios in the heat- exchangers is directly proportional to the decrease in the cyclo thermal efficiency. Therefore we must keep it below a certain value. In our problem it is a priori given.

If we design the heat-exchanger for lower pressure drops than the allowable value than the exchanger will be unnecessarly large. Hence we must make the total pressure drop equal to the specified value.

The pressure drop for flow of gas normal to tubes is given by the formula

$$\Delta p_o = \rho_1 \frac{4 f_o n N_1 C_o^2 v_o}{2 g} \qquad (8)$$

(formula 53 , pp.974 ; ref.3)

where $\varphi_1 > 1$ takes care of the losses at entrance and exit.

and $$f_G = f_{Grimison} = \frac{C_4}{(N_{Re})^{0.13}} \quad \text{(pp.975, ref.3)}$$

where C_4 depends on the geometry of tube arrangements, and is presented in figures 6 and 7 of reference 3 . The assumption being that the flow is turbulent and that the Reynolds number is based on outside diameter of tubes.

As for inner gas flow the pressure drop will be

$$\Delta p_i = \frac{G_i^2}{2\,g}\, v_{i1}\Big[(K_c + 1 - \sigma^2) + 2\Big(\frac{v_{i2}}{v_{i1}} - 1\Big) + f_i \frac{A_i}{A_{if}} \frac{v_{im}}{v_{i1}} - (1 - \sigma^2 - K_e)\frac{v_{i2}}{v_{i1}} + (n - 1)\, K_p\Big] \qquad (9)$$

It appears from figure 7 (ref.) that the increase of the number of passes beyond 3 or 4 does not increase the effectiveness appreciably, whereas according to the formula (8) above the pressure loss increases as the number of passes increases. Hence there is probably no gain in volume beyond n = 3 . Also there is another argument against increasing the number of passes.

If we increase the number of passes the non-flow dimension increases.

Since all the losses in equation (9) are small in compa -

rison with fanning friction losses we can write

$$\Delta p_i = \frac{\varphi_2 f_i G_i^2 A_i v_i}{2g\, A_{if}} \qquad (10)$$

where $\varphi_2 > 1$ is introduced to take care of varios losses except the fanning friction losses.

For turbulent flow f_i can be approximated by

$$f_i = \frac{0.046}{(R_{Re})_i^{0.2}} = 0.046 \left(\frac{\mu_i}{D_i}\right)^{0.2} G_i^{-0.2} \qquad (11)$$

(pp. 969, ref. 3)

$$v_i \cong \frac{T_{i1} + T_{i2}}{2\, T_{i1}}\, v_{i1} = \delta_i v_{i1} \qquad (12)$$

$$v_o \cong \left(1 + \frac{1}{2 x_v} \cdot \frac{Cp_i}{Cp_o} \cdot \frac{\Delta T_i}{T_{o1}} \right) = \delta_o v_{o1} \qquad (13)$$

The friction power loss is given by

$$P = W_i\, v_i \Delta p_i + W_o\, v_o \Delta p_o = (G_i A_{if})(v_i)\left(\frac{\varphi_2 f_i G_i^2 A_i v_i}{2g\, A_{if}}\right) +$$

$$(G_o\, A_{of})(v_o) \cdot \frac{\varphi_1 4\, C_4\, nH_1\, G_o^2 v_o}{2g \left(\frac{G_o D_o}{\mu_o}\right)^{0.13}}$$

Sustituting f_i from (11)

$$P = 0.023 \frac{\varphi_2 A_i\ v_i^2}{g} \left(\frac{\mu_i}{D_i}\right)^{0.2} G_i^{2.8} + \frac{2nH_1\ v_o^2\ \varphi_1 C_4}{g} \left(\frac{\mu_o}{D_o}\right)^{0.13} A_{of} G_o^{2.87}$$

Letting
$$\Lambda_i = 0.023 \frac{\varphi_2 A_i\ v_i^2}{g} \left(\frac{\mu_i}{D_i}\right)^{0.2} \qquad (14)$$

$$\Lambda_o = \frac{2nH_1\ v_o^2\ \varphi_1 C_4}{g} \left(\frac{\mu_o}{D_o}\right)^{0.13} A_{of} \qquad (15)$$

$$P = \Lambda_i G_i^{2.8} + \Lambda_o G_o^{2.87} \qquad (16)$$

Determination of Heat Transfer Characteristics :

For the flow inside the pipes the following dimensionless group versus N_{Re} defines the heat transfer characteristics of the surface :

$$J = N_{St}\ N_{Pr}^{2/3} = \left(\frac{h}{G\ Cp_i}\right) \left(\frac{Cp}{k}\right)_i^{2/3} \qquad (17)$$

For flow through round tubes at N_{Re} from 5000 to 200 000 the value of J is given by

$$J = 0.023 / (N_{Re})_i^{0.2} \qquad (18) \qquad \text{(see ref.3 , pp.969)}$$

where $(N_{Re})_i = \dfrac{D_i\ G_i}{\mu_i}$

From (17) using (18)

$$h_i = \frac{0.023\ Cp_i\ G_i}{(N_{Re})_i^{0.2}\ (N_{Pr})_i^{2/3}} = \frac{0.023\ Cp_i}{(N_{Pr})_i^{2/3}} \left(\frac{\mu}{D_i}\right)^{0.2} G_i^{0.8} \qquad (19)$$

For the cold air side

$$h_o = 1.10\ C_2\ Cp_o\ G_o\ (N_{Pr})_o^{-\frac{2}{3}}\ (N_{Re})_o^{-0.4} \qquad (20)$$

(formula 52a , pp.974 , ref.3)

where C_2 is a number depending on the geometry of tube arrangements . It is given in figures 4 and 5 of ref. 3 .

Now the over-all coefficient of heat transfer, U, is given by

$$\frac{1}{U\ A} = \frac{1}{h_i\ A_i} + \frac{1}{h_o\ A_o} \qquad (21)$$

where we neglected the tube wall resistance to heat flow.

$$h_i\ A_i = \frac{0.023\ Cp_i}{(N_{Pr})_i^{2/3}} \left(\frac{\mu}{D_i}\right)^{0.2} A_i\ G_i^{0.8} = X_i\ G_i^{0.8} \qquad (22)$$

where
$$X_i = \frac{0.023\ Cp_i}{(N_{Pr})_i^{2/3}} \left(\frac{\mu}{D_i}\right)^{0.2} A_i \qquad (23)$$

$$h_o\ A_o = 1.10\ C_2\ Cp_o\ (N_{Pr})_o^{-\frac{2}{3}} \left(\frac{\mu_o}{D_o}\right)^{0.4} A_o\ G_o^{0.6} = X_o\ G_o^{0.6} \qquad (24)$$

where $$\chi_o = 1.10\ C_2\ Cp_o (N_{Pr})_o^{-2/3} \left(\frac{\mu_o}{D_o}\right)^{0.4} Ao \qquad (25)$$

substituting $h_i\ A_i$, $h_o\ A_o$ from (22) and (24) into (21) and multiplying both sides by $W_i\ Cp_i$ we get

$$\frac{W_i\ Cp_i}{U\ A} = \frac{1}{NTU} = W_i\ Cp_i \left(\frac{1}{\chi_i G_i^{0.8}} + \frac{1}{\chi_o G_o^{0.6}}\right) \qquad \text{or}$$

$$\frac{1}{\chi_i G_i^{0.8}} + \frac{1}{\chi_o G_o^{0.6}} = \frac{1}{W_i\ Cp_i\ NTU} \qquad (26)$$

We shall minimize P assuming that G_i and G_o are main variables and such that G_o , G_i satisfy (26) .

It will be found that (see the appendix for derivation) P will be minimum if G_i / G_o has the value

$$Gi / G_o = \gamma = \frac{(5700\ C_2\ C_4)^{0.278}}{(N_{Re})_o^{0.0361}} \left(\frac{Cp_o}{Cp_i} \frac{N_{Pri}}{N_{Pr_o}} \frac{\varphi_1}{\varphi_2} \frac{R}{I_1}\right)^{0.278} \left(\frac{v_o}{v_i}\right)^{0.555} \left(\frac{D_o}{D_i}\right)^{0.444} \left(\frac{\mu}{\mu}\right) \qquad (27)$$

In this equation $(N_{Re})_o$ comes to a low power, therefore an assumed value of $(N_{Re})_o$ does not affect Gi / G_o ratio very much .

Expressions for weight and volume of the core of the heat-exchanger :

The total number of tubes is given by

$$N = n\, N_l\, N_t \qquad (28)$$

The weight of the core is given by

$$w = N \frac{\pi}{4} (D_o^2 - D_i^2)\, L\, \rho \qquad (29)$$

where ρ is the density of the tube material

$$w = \frac{\pi \rho}{2} t\, N\, 2\, D_i \left(1 + \frac{t}{D_i} \right) L = (\pi\, D_i\, L\, N)\rho t\, (1 + \lambda) = A_h\, t\, (1 + \lambda) \qquad (30)$$

$L'' =$ non- flow length $= N_t\, l_t$

$L' = n\, N_l\, l_l$

$$V = L\, L'\, L'' = n\, N_l\, N_t\, L\, l_l\, l_t = N\, L\, l_l\, l_t \qquad (31)$$

From (30) and (31)

$$V = \frac{w\, l_l\, l_t}{\frac{\pi}{4}(D_o^2 - D_i^2)\rho} \qquad (32)$$

We shall assume that $\lambda = \frac{t}{D_i}$ is independent of the tube size .

$$D_i = \frac{t}{\lambda} \qquad (33) \qquad\qquad D_o = \frac{1+2\lambda}{\lambda}\, t \qquad (34)$$

The relationship between weight, Power loss, NTU, thickness of tubes and spacing of tubes :

By means of (27) and (16) one obtains (see appendix)

$$G_o = \frac{M\, P^{0.357}\, t^{0.428}}{v^{0.357}} \qquad (35)$$

$$G_i = \frac{\gamma_I\, P^{0.357}\, t^{0.428}}{v^{0.357}} \qquad (36)$$

where

$$M = \left[\frac{0.023\, \varphi_2 v_i^2}{\rho g(I+\lambda)} (\lambda\mu_i)^{0.2}\, \gamma^{2.8} + \frac{2\, v_o^2\, C_4(I+2\lambda)}{g(I+\lambda)} \cdot \frac{n}{l_1}\left(\frac{\lambda\mu_o}{1+2\lambda}\right)^{0.2} (N_{Re})^{0.07}\right] \quad (37)$$

Hence M also does not vary much with $(N_{Re})_o$.

Substituting G_i , G_o from (35) , (36) into (26)

$$\frac{I}{\dfrac{S_1 P^{0.285}}{t^{0.857}}\, v^{0.715}} + \frac{I}{\dfrac{S_2\, P^{0.214}}{t^{0.143}}\, v^{0.786}} = \frac{I}{Cp_i\, W_i\, NTU} \qquad (38)$$

where

$$S_1 = 0.023\, Cp_i\, (N_{Pr})_i^{-2/3}\, \frac{(\lambda\mu_i)^{0.2}}{\rho(I+\lambda)}\, (\gamma_I)^{0.8} \qquad (39)$$

$$S_2 = 1.10\ G_2\ Cp_o\ (N_{Pr})_o^{-2/3} \left(\frac{\lambda \mu_2}{1+2\lambda}\right)^{0.4} \frac{1+2\lambda}{\rho(1+\lambda)}\ M^{0.6} \qquad (40)$$

From equation (38) it is apparent that w decreases if NTU is decreased. But from the graph given in the illustative example it is clear that NTU decreases if r_v increases. Hence w decreases if r_v increases. However as we shall see later that the non-flow dimension increases and the other 2 dimensions decrease if r_v increases.

Therefore we must restrict r_v to obtain good proportions. From the same equation it is apparent that as t is decreased w decreases. From equation (32) volume also decreases with decreasing t .

From the above discussion it is clear that we must choose a tube with smallest possible diameter. However for technological impossibilities and stregth considerations restrics t .

If a gas with high velocity flows with high velocity flows normal to tube banks the maximum estimated bending stress is given by

$$\tau = \frac{8\ \delta P_o (L/D_o)^2}{\pi \left[1 - \left(\frac{D_i}{D_o}\right)^4\right]} \qquad (41)$$

where P_o is the rise of pressure of the outside gas due to the impact with tubes

$$\delta P_o = P_o \left[1 - \left(1 + \frac{k-1}{2} M^2 \right) \right]^{\frac{k}{k-1}} \quad (42)$$ see ref. 6

Eq. (38) also shows the variation of W with P. As the allowable friction loss is decreased weight (and also volume) increases, and vice versa .

Determination of $(NTU)_o$ and $(NTU)_i$.-

If we let

$$F = \frac{(NTU)_o}{(NTU)_i} = \frac{h_o A_o}{W_o \; Cp_o} \Big/ \frac{h_i A_i}{W_i \; Cp_i} = \frac{h_o \; D_o \; Cp_i}{h_i \; D_i \; r_w \; Cp_o} \quad (43)$$

Consider

$$\frac{1}{U A} = \frac{1}{h_i \; A_i} + \frac{1}{h_o \; A_o}$$

multiplying both sides by $W_i \; Cp_i = (W \; Cp)_{min}$

$$\frac{1}{NTU} = \frac{1}{(NTU)_i} + \frac{Cp_i / Cp_o}{r_w \; (NTU)_o} \quad (44)$$

From (43) and (44)

$$NTU_i = (NTU) \frac{Cp_i / Cp_o + r_w \; F}{r_w \; F} \quad (45)$$

$$NTU_o = (NTU) \frac{Cp_i / Cp_o + r_w \; F}{r_w} \quad (46)$$

But $$F = \frac{1}{r_W}\,\beta \qquad (47)$$ where

$$\beta = 7\,\frac{C_2^{0.778}}{C_4^{0.222}}\cdot\frac{1}{(N_{Re})_o^{0.171}}\cdot\left(\frac{D_o}{D_i}\right)^{0.444}\cdot\left(\frac{\mu_o}{\mu_i}\right)^{0.111}\cdot\left(\frac{\varphi_2}{\varphi_1}\right)^{0.222}\cdot\left(\frac{l_1}{R}\right)^{0.222}\cdot$$

$$\cdot\left(\frac{v_i}{v_o}\right)^{0.444}\left(\frac{Cp_i}{Cp_o}\right)^{0.222}\left(\frac{(N_{Pr})_i}{(N_{Pr})_o}\right)^{0.444} \qquad (48)$$ (see appendix)

Hence

$$(NTU)_i = (NTU)\cdot\frac{\left(Cp_i/Cp_o + \beta\right)}{\beta} \qquad (49)$$

$$(NTU)_o = (NTU)\cdot\frac{\left(Cp_i/Cp_o + \beta\right)}{r_w} \qquad (50)$$

Determination of tube lengths .-

From equation (I7) and (I8)

$$h_i = \frac{0.023\,Cp_i\,G_i}{(N_{Re})_i^{0.2}\,(N_{Pr})_i^{2/3}}$$

$$NTU_i = \frac{h_i\,A_i}{W_i\,Cp_i} = \frac{0.023\,G_i\,A_i}{(N_{Re})_i^{0.2}\,(N_{Pr})_i^{2/3}} = \frac{0.023\,(4L/D_i)\,n}{(N_{Re})_i^{0.2}\,(N_{Pr})_i^{2/3}} \qquad (51)$$

$$\left(\text{Since}\quad G_iA_i / W_i = \frac{A_i}{A_{if}} = \frac{(\pi D_i\,L)\,N}{\left(\frac{\pi D_i^2}{4}\right)\frac{N}{n}} = \frac{4L}{D_i}\,n\right)$$

From equation (49) and (51)

$$L = 10.88 \frac{(NTU)}{n} \cdot \left(1 + \frac{Cp_i / Cp_o}{\beta} \right) (N_{Re})_o^{0.2} (N_{Pr})_i^{2/3} D_i \quad (52)$$

Determination of L' .-

$$(NTU)_o = \frac{h_o A_o}{W_o Cp_o} = \frac{1.10 \ C_2 \ Cp_o \ G_o \ (N_{Pr})_o^{-2/3} \ (N_{Re})_o^{-0.4} \ A_o}{W_o \ Cp_o} \ldots (53)$$

(Since $W_o = G_o A_{of} = G_o A_o \frac{R}{L'}$

substituting into (53) and solving for L' we get

$$L' = 0.909 \frac{R}{C_2 r_v} (NTU) (\beta + Cp_i / Cp_o) (N_{Pr})_o^{2/3} (N_{Re})_o^{0.4} \ldots (54)$$

Determination of the number of tubes .-

The number of tubes will be found from

$$W_i / G_i = A_{fi} = \frac{N}{n} \frac{\pi D_i^2}{4}$$

$$N = \frac{4 n W_i}{\pi D_i^2 G_i} \quad (55)$$

The effect of increasing r_w on weight and volume .-

Let $\quad l_1 = D_o\, p_1 \qquad l_t = D_o\, p_t$

for fixed $t,\ p_1,\ p_t$

$w \propto (NTU)^{4/3}$ (approx.) $\qquad V \propto (NTU)^{4/3}$ (approx.)

$L \propto NTU \qquad L' \propto \frac{NTU}{r_w} \qquad L'' \propto r_w \,/\, (NTU)^{2/3}$ (approx.)

Hence the non-flow dimension increases whereas w, V decrease if r_w .

The restriction of r_w amounts to restricting the non-flow dimension. The decrease of non-flow dimension is a desirable property, because with large non-flow dimension ducting becomes awkward .

The best one can do is therefore to assume a value of $(NTU)_p$ and r_w from the graph of $r_w - (NTU)_p$. Probably best way is to assume that $\quad NTU_p = K \ln \frac{1}{\sqrt[n]{1-\varepsilon}} \quad$ where is the asymptotic value of NTU_p and K is a factor such that

$\left| \frac{d(NTU)_p}{d r_w} \right|$ is not too small.

The effect of decreasing p_l, p_t on weight and volume.-

$R = D_o f(p_l, p_t, \text{arrangement})$

For in-line tubes $R \cong \frac{\pi}{8} D_o (p_t - 1)$

This formula agrees very well with the given data in table 12, ref. 1

S_1 and S_2 do not vary much with p_l, p_t. Hence w is almost constant. (See the illustrative problems)

But from formula (32) volume decreases if p_l, p_t is decreased. However the non-flow dimension increases if p_l, p_t decreases.

ILLUSTRATIVE EXAMPLE

To design an optimum heat-exchanger of cross-flow bare tube type with both fluids being air , the hot air flowing inside the tubes.

The design data is given below :

Maximum allowable power loss . . . . 5 HP

Volume flow rate of hot air . . . . IO ft^3/s

Entering pressure of hot air . . . . 22 psia

Entering temperature of hot air . . . . IIO° F

Leaving temperature of hot air . . . . 68° F

Entering pressure of cold air . . . . I4.7 Psia

Entering temperature of cold air . . . . 50° F

The number of passes on the tube side 2

Thetubes are arranged in line

PROCEDURE

1) Calculate ε from formula 1
2) Draw $\varepsilon_p - r_w$
3) Assume $(NTU)_p$ from graph of $\varepsilon_p - r_w$
4) Assume the inner and outer diameters of tubes
5) Assume the type of tube arrangements (in-line , staggered)
6) Assume l_l , l_t .(they should be chosen as small as possible)
7) Assume a reasonable value for $(N_{Re})_o$
8) Calculate γ
9) Calculate M
10) Calculate S_1, S_2
11) From formula 38 calculate w
12) " " 35 " G_o
13) Check $(N_{Re})_o$ by repeating 8-12
14) Calculate N from formula 55
15) Calculate β
16) Calculate L from formula 52
17) Calculate L' from formula 54
18) From $nN_l = \frac{L'}{l_l}$ obtain nN_l
19) From $L'' = \frac{N}{nN_l} l_t$ obtain L"
20) Check the non-flow dimension to see if it is within permissable limit
21) From $N = nN_l N_t$ obtain N_t
22) From $V = LL'L''$ obtain volume. Check this against formula 32

ROBERT COLLEGE School of Engineering DEPARTMENT OF MECHANICAL ENGINEERING	Course : M . S . Thesis	Class : Grad. M.E.	Page : 25
	Lab. Design Thesis	"A study on the optimum design characteristics of compact heat-exchangers used in aircraft air-conditioning systems"	

SOLUTION   SEQUENCE OF THE DESIGN PROBLEM

I) $v_{i1} = RT_{i1} / p_i = 53.3 \times 570 / 22 \times I44 = 9.59$ cu ft / lb

$v_{o1} = R\,T_{o1} / p_o = 53.3 \times 5I0 / I4.7 \times I44 = I2.83$ cu ft / lb

$W_i = \frac{dV}{dt} / v_{i1} = I0 \times 3600 / 9.59 = 3750$ lb / hr

$$\varepsilon = \frac{T_{i1} - T_{i2}}{T_{i1} - T_{o1}} = \frac{II0 - 68}{II0 - 50} = 0.7$$

2) Since $C_{pi} = C_{po} = 0.240$ Btu /lb F $\varepsilon = 0.7$, $n = 2$

$$c = r_V \quad \text{and} \quad e^{-I} + e^{-(NTU)_p} = \frac{(r_V - I)^{r_V}\left(\frac{I - \frac{0.7}{r_V}}{0.3}\right)^{r_V/2}}{\left[r_V\left(\frac{I - \frac{0.7}{r_V}}{0.3}\right)^{I/2} - I\right]^{r_V}}$$

$(Ntu)_p - r_V$ graph is next drawn

3) Take K = I.5 and therfore

$(NTU)_p = 0.9$ $r_V = I.58$

4) Assume that tubes are made of Aliminum for lightness, and that $D_i = 0.250''$ $D_o = 0.300''$ $t = 0.025''$ $D_o / D_i = I.2$ $\lambda = 0.I$

5) Assume that the tubes are arranged in-line .

6) Assume $l_l = I.75\,D_o = 0.525''$ $l_t = 2\,D_o = 0.6''$

7) Assume that $(N_{Re})_o = 5000$

8) From fig. 4, Reference 2

$C_2 = 0.292$

from fig. 6 , ref.2

$$C_4 = 0.150$$

$$(N_{Pr})_i = (N_{Pr})_o = 0.712$$

$$R = \frac{\pi}{8} D_o \left(\frac{l_t}{D_o} - 1 \right) = \frac{3.14}{8} \times 0.3 \ (2 - 1) = 0.118''$$

$$\delta_o = 1 + \frac{1}{2 \, r_w} \cdot \frac{(C_p)_i}{(C_p)_o} \cdot \frac{\Delta T_i}{T_{o1}} = 1 + \frac{1}{2 \times 1.58} \times \frac{42}{510} = 1.026$$

$$v_o = 1.026 \times 12.83 = 13.16 \ \text{cu ft / lb}$$

$$\delta_i = \frac{T_{i1} + T_{i2}}{2T_{i1}} = 0.5 + \frac{528}{2 \times 570} = 0.964$$

$$v_i = 9.59 \times 0.964 = 9.24 \ \text{cu ft / lb}$$

Equating the heat transfer between hot and cold fluids

$$W_i \, C_{pi} \, \Delta T_i = r_w \, W_i \, C_{po} \, \Delta T_o$$

$$\Delta T_i = r_w \Delta T_o$$

$$T_{o2} = T_{o1} + \frac{\Delta T_i}{r_w} = 50 + \frac{42}{1.58} = 76.5 \ ^oF$$

$$T_{om} = \frac{50 + 76.5}{2} = 63.2 \ ^oF$$

$$T_{im} = \frac{110 + 68}{2} = 89 \ ^oF$$

$$\mu_o = 0.0434 \ \text{lb / hr ft}$$

$$\mu_i = 0.0452 \ \text{lb / hr ft}$$

$$\gamma = \frac{\left(5700 \times 0.292 \times 0.150 \times \frac{1.1}{1.15}^{0.278} \times \frac{0.118}{0.525}\right)\left[\left(\frac{13.16^5}{9.24}\right) \times (1.2)^4 \times \frac{0.0434}{0.0452}\right]^{0.1}}{5000^{0.0361}}$$

$= 2.93$

9) For Aluminum $= 2.7 \times 62.4 = 168.3$ lb / cu ft

$$M = \left[\frac{0.023 \times 1.15 \times 9.24^2}{168.3 \times 32.2 \times 3600^2 \times 1.1}(0.1 \times 0.0452)^{0.2}(2.93)^{2.8} + \frac{2 \times 1.1 \times 13.16^2 \times 0.15 \times 1.2 \times 0.118}{168.3 \times 32.2 \times 3600^2 \times 1.1 \times 0.525}\left(\frac{0.1 \times 0.0434}{1.2}\right)^{0.2} (5000)^{0.97}\right]^{\frac{1}{2.8}}$$

$= 2400$

$$S_1 = 0.023 \times (0.712)^{-\frac{2}{3}} \times 0.24\,(0.00452)^{0.2}\frac{1}{168.3 \times 1.1}(2.93 \times 2400)$$

$= 0.01523$

$$S_2 = 1.10 \times 0.292 \times (0.712)^{-\frac{2}{3}} \times 0.24 \times \left(\frac{0.00434}{1.2}\right)^{0.4}\frac{1.2\,(2400)^{0.6}}{168.3 \times 1.1}$$

$= 0.0071$

11) $P = 5 \times 550 \times 3600 = 9.9 \times 10^6$ ft / hr

$P^{0.285} = 98.8$; $P^{0.214} = 31.5$

$$t^{0.857} = \left(\frac{1}{40 \times 12}\right)^{0.857} = 0.00513$$

$$t^{1.143} = \frac{1}{480^{1.143}} = 0.000851$$

$$C_{pi}\ W_1\ 2\ (NTU)_p = 0.24 \times 3750 \times 2 \times 0.9 = 1620$$

$$\frac{S_1\ P^{0.285}}{t^{0.857}} = \frac{0.01523 \times 98.8}{0.00513} = 293$$

$$\frac{S_2\ P^{0.214}}{t^{1.143}} = \frac{0.0071 \times 31.5}{0.000851} = 263$$

substituting into equation 38

$$\frac{1}{293\ w^{0.715}} + \frac{1}{263\ w^{0.786}} = \frac{1}{1620} \qquad \text{or}$$

$$f(w) = \frac{5.53}{w^{0.715}} + \frac{6.15}{w^{0.786}} = 1$$

We shall solve this equation by trial and error

Assume $w = 25$ lb

$25^{0.715} = 10 \qquad\qquad 25^{0.786} = 12.6$

$f(25) = 1.041 > 1$

Hence $w = 25$ lb is too small

$f(26) = 1.015 > 1$

Next assume $w = 27$ lb

$f(27) = 0.987 < 1$

By linear interpolation between $w=26$ and $w=27$ we get $w=26.5$

$$G_o = \frac{N\ P^{0.357} t^{0.428}}{v^{0.357}} = \frac{2400}{(480)^{0.428}} \left(\frac{9.9 \times 10^{6}}{26.5} \right)^{0.357} = 16560 \text{ lb/hr ft}^2$$

$$(N_{Re})_o = \frac{D_o G_o}{\mu_o} = \frac{0.300 \times 16560}{12 \times 0.0434} = 11300$$

13) Next assume $(N_{Re})_o = 11300$

$$\gamma = \frac{3.03 \times 1.313}{11300^{0.0361}} = 2.84$$

$$N = \left[9.93 \times 10^{-12} (2.84)^{2.8} + 64.9 \times 10^{-12} (11300)^{0.07} \right]^{-0.357}$$

$N = 2450$

$$S_1 = 1.27 \times 10^{-5} (2.84 \times 2450)^{0.8} = 0.01493$$

$$S_2 = 6.66 \times 10^{-5} (2450)^{0.6} = 0.00721$$

$$\frac{S_1\ P^{0.285}}{t^{0.857}} = 287$$

$$\frac{S_2\ P^{0.214}}{t^{1.143}} = 267$$

$$f(v) = \frac{5.65}{v^{0.715}} + \frac{6.06}{v^{0.786}} = 1$$

The approximate root of this equation is $v = 26.7$ lb

$$G_o = \frac{2450}{14} \times \frac{9.9 \times 10^{6}}{26.7}^{0.357} = 16800 \text{ lb / hr ft}^2$$

Which is very close to the previos value

Hence take $G_o = 16800$ lb / hr ft^2; $v = 26.7$ lb, $\gamma = 2.84$

$$G_1 = \gamma G_o = 2.84 \times 16800 = 47800 \text{ lb / hr ft}^2$$

14) From equation 55

$$N = \frac{4 \times 2 \times 3750 \times 144}{3.14 \times \frac{1}{16} \times 47800} = 461 \text{ tubes}$$

15) $(N_{Re})_o^{0.171} = 4.92$

$$\beta = 7 \times \frac{0.292}{4.92}^{0.778} \left[\frac{1}{(0.15)^2} \times (1.2)^4 \times \frac{0.0434}{0.0452} \times \left(\frac{1.15}{1.1}\right)^2 \times \left(\frac{0.525}{0.118}\right)^2 \times \left(\frac{9.24}{13.16}\right)^4 \right]^{0.1}$$

$= 1.08$

16) $L = 10.88 \times 0.9 \left(1 + \frac{1}{1.08} \right) \left(\frac{47800 \times 0.25}{0.0452 \times 12} \right)^{0.2} \times \frac{0.25}{12} \times (0.712)^{2/3}$

$= 2.25$ ft

17) $L' = 0.909 \times \frac{0.118}{0.292 \times 12 \times 1.58} \times 1.8 \, (1.08 + 1) \, (0.797) \left(\frac{16800 \times 0.3}{0.0434 \times 12} \right)^{0.4}$

2.28 ft

18) $nN_1 = \frac{2.28 \times 12}{0.525} = 52.2$ take 52 tubes

19) $L'' = \frac{N}{n N_1} . l_t = \frac{461}{52} \times \frac{0.6}{12} = 0.444$ ft

20) We shall assume that the non-flow dimension found above is within permissable limits .

21) $N_t = \frac{N}{nN_1} = \frac{461}{52} = 8.85$ Take 9

$$V = L\,L'\,L'' = 2.25 \times 2.28 \times 0.444 = 2.28 \text{ cu ft}$$

AS A CHECK we note that

$$V = \frac{w\,l_l\,l_t}{\frac{\Pi}{4}(D_o^2 - D_i^2)} = \frac{26.7 \times 0.525 \times 0.6}{\frac{\Pi}{4} \times 168.3\,(0.09 - 0.0625)} = 2.31 \text{ ft}^3$$

If we take $nN_l = 52$ $N_t = 9$ so $N = 52 \times 9 = 468$ 461

however the error introduced will be negligable in comparison with all the error due to simplifying assumptions .

Weight will increase to the value

$$26.7 \times \frac{468}{461} = 27.1 \text{ lb}$$

Let us check the tubes for strength.

Since the pressure difference between the inner and outer gases is not large we need check only for bending due to the outside pressure.

$$V_o = G_o\,v_o = \frac{16200 \times 13.16}{3600} = 61.4 \quad \text{ft/ sec}$$

$$M_o = \frac{61.4}{49.1\sqrt{523}} = 0.0546$$

Since M is small

$$\frac{p_{oT}}{p_o} \cong \left(1 + \frac{k}{2} M^2 \right) = 1 + 0.7 \times 0.0546^2 = 1.00209$$

$$p_o = 0.00209 \times 14.7 = 0.0307 \quad \text{psi}$$

$$= \frac{8 \times 0.0307 \left(\frac{2.25 \times 12}{0.3}\right)^2}{3.14\left[1 - \frac{1}{(1.2)^4}\right]} = 1225 \text{ psi}$$

Since the yield strength of even the weakest Aluminum is 30 ksi

$$\text{Design factor} \quad \frac{30}{1.225} = 24.5$$

Hence it is safe .

PROBLEM 2 .-

Solve the same problem under the assumption that

$p_t = 1.5 \qquad p_l = 1.5$,and the tubes are arranged in-line

6) $l_l = 1.5\ D_o = 0.45''$; $\qquad l_t = 1.5\ D_o = 0.45''$

7) Assume that $(N_{Re})_o = 10\ 000$

8) From fig. 4 , ref. 2

$C_2 = 0.294$

From fig. 6 , ref.2

$C_4 = 0.250$

$$R = \frac{\pi}{8} D_o\ (p_t - 1) = 0.0588''$$

$$\gamma = \frac{\left(5700 \times 0.294 \times 0.250 \times \frac{1.1}{1.15} \times \frac{0.0588}{0.45}\right)^{0.278} \left[\left(\frac{13.16}{9.24}\right)^5 (1.2)^4 \cdot \frac{0.0434}{0.0452}\right]^{0.111}}{10000^{0.0361}}$$

$= 2.84$

9) $$M = \left[\frac{0.023 \times 1.15 \times 9.24^2}{168.3 \times 32.2 \times 3600^2 \times 1.1} \cdot (0.1 \times 0.0452)^{0.2} \cdot (2.84)^{2.8} + \right.$$

$$+ \frac{2 \times 1.1 \times 13.16^2 \times 0.250 \times 1.2 \times 0.0588}{168.3 \times 32.2 \times 3600^2 \times 1.1 \times 0.45} \left(\frac{0.1 \times 0.0434}{1.2} \right)^{0.2} \left(10\,000 \right)^{0.07} \Bigg]^{-\frac{1}{2.8}}$$

$$= 2570$$

$$S_1 = 0.023 \left(0.712 \right)^{-2/3} \times 0.24 \left(0.00452 \right)^{0.2} \frac{1}{168.3 \times 1.1} \left(2.84 \times 2570 \right)^{0.8}$$

$$= 0.0156$$

$$S_2 = 1.10 \times 0.294 \left(0.712 \right)^{-2/3} \times 0.24 \left(\frac{0.00434}{1.2} \right)^{0.4} \frac{1.2}{168.3 \times 1.1} \left(2540 \right)^{0.6}$$

$$= 0.0074$$

$$\frac{S_1 P^{0.285}}{t^{0.857}} = \frac{0.0156 \times 98.8}{0.00513} = 300$$

$$\frac{S_2 P^{0.214}}{t^{1.143}} = \frac{0.0074 \times 31.5}{0.000851} = 274$$

$$f(w) = \frac{5.4}{w^{0.715}} + \frac{5.92}{w^{0.786}} = 1$$

whose root is $w = 25.4$ lb

$$G_o = \frac{2570}{(480)^{0.428}} \left(\frac{9.9 \times 10^6}{25.4} \right)^{0.357} = \frac{2570 \times 100}{14.02} = 18350 \ \text{lb/ hr ft}^2$$

$$(N_{Re})_o = \frac{D_o \ G_o}{\mu_o} = \frac{0.300 \times 18350}{12 \times 0.0434} = 10550$$

It is very close to the assumed value , hence we need not 8-12

$$G_i = \gamma G_o = 2.84 \times 18350 = 52100 \quad lb / hr\ ft^2$$

14) $$N = \frac{4 \times 2 \times 3750 \times 144}{\pi \times \frac{1}{16} \times 52100} = 424 \quad \text{tubes}$$

15) $$\beta = 7 \; \frac{0.294^{0.778}}{4.86} \left[\frac{1}{(0.25)^2} (1.2)^4 \left(\frac{0.0434}{0.0452}\right) \left(\frac{1.15}{1.1}\right)^2 \left(\frac{0.45}{0.0588}\right)^2 \left(\frac{9.24}{13.16}\right)^4 \right]^{0.181}$$

$$= 1.129$$

16) $$L = 10.88 \times 0.9 \left(1 + \frac{1}{1.129}\right) (10550)^{0.2} (0.712)^{2/3} \; \frac{0.25}{12} = 1.963'$$

17) $$L' = 0.903 \times \frac{0.0588}{12 \times 0.294 \times 1.58} \times 1.8 (1.129 + 1) 0.797 \times 40.7 = 1.192'$$

18) $$nN_1 = \frac{1.192 \times 12}{0.45} = 31.8 \qquad \text{take} \quad 32$$

19) $$L'' = \frac{424}{32} \times \frac{0.45}{12} = 0.497''$$

20) We shall assume that the non-flow dimension found above is within permissable limits .

21) $$N_t = \frac{424}{32} = 13.2 \qquad \text{take} \quad N_t = 14$$

22) $$V = L\, L'\, L'' = 1.963 \times 1.192 \times 0.514 = 1.21 \quad ft^3$$

As a check we note that

$$V = \frac{25.4 \times 0.45 \times 0.45}{\frac{\pi}{4} \times 168.3 (0.09 - 0.0625)} = 1.415 \quad ft^3$$

The discrepancy between the two values is . However with more accurate data the discrepancy may be smaller .

Comparison of two heat-exchangers shows that the weight and especially the volume of the second heat-exchanger is smaller only its non-flow dimension is somewhat larger .

CONCLUSION

The heat-exchangers to be used in aircraft air-conditioning systems must be leight, and also have small volume. Since leight weight is preferable to compactness tubular type of heat-exchangers are used in these applications.

We have reached the conclusion that the diameter of tubes should be as small as possible. As this tends to decrease weight and volume. The only limitation of diameter being strength .

It is best to decrease the spacingsof tubes as much as possible. However it requires excessive time and effort to make little error in the spacings if they are narrow: very accurate spacings being necessary since the performance of the heat-exchanger may be very much different with inaccurate spacings. Also good shape requirements restricts p_t , p_l .

As for the flow rate of the outside gas it may be stated that it should be as large as possible, provided that the non-flow dimension will not be excessive.

The number of passes should not be too large . Because a very little gain in volume and weight is made at the cost of increasing the non-flow dimension.

Recommendations for future work.-

Whether in-line tubes or staggered tubes should be prefered must be checked by solving the same problem with both kinds of arrangements.

Itx is desirable to derive analogous results for other type surfaces (plate fin, pin-fin, matrix surfaces, etc.) and under different conditions. Once the performance characteristics is known for each type of surface one can select the best type of surface according to performance characteristics curves, and then design according to the formulas already developed. No doubt the preparation of all these curves requires great deal of time. Probably years !

However, we believe that, since the performances of various extended surface type heat-exchangers are similar, the solution of even a single problem with a particular extended surface will indicate the things to do to obtain "The best heat-exchanger" satisfying the given conditions.

Now

$$\lim_{s \to \infty} \left(1 - \frac{1}{s}\right)^{s} = e^{-1}$$

$$\lim_{s \to \infty} \left(1 - \frac{\varepsilon}{s}\right)^{\frac{s}{n}} = e^{-\frac{\varepsilon}{n}}$$

$$\lim_{s \to \infty} \left[\left(1 - \frac{\varepsilon}{s}\right)^{\frac{1}{n}} - \frac{\sqrt[n]{1-\varepsilon}}{s}\right]^{s} = e^{-\frac{\varepsilon}{n} - \sqrt[n]{1-\varepsilon}}$$

Hence for $s \to \infty$

$$e^{-1 + e^{-(NTU)_p}} = \frac{e^{-1}\, e^{-\frac{\varepsilon}{n}}}{e^{-\frac{\varepsilon}{n} - \sqrt[n]{1-\varepsilon}}} \qquad \text{or}$$

$$\lim_{s \to \infty} (NTU)_p = \ln \frac{1}{\sqrt[n]{1-\varepsilon}}$$

APPENDIX

Calculation of mean specific volumes.-

Assuming that the temperature varies linearly with area

From formula (25) pp. 22, ref. (I)

$$\frac{v_{om}}{v_{o1}} \cong \frac{p_{o1}}{p_{avg}} \cdot \frac{T_{avg}}{T_{o1}} \cong \frac{T_{o1} + T_{o2}}{2} \Big/ T_{o1} = 1 + \frac{\varepsilon_o}{2} \frac{(\Delta T_o)}{T_{o1}} = 1 + \frac{1}{2 r_w} \frac{C_{p_i}}{C_{p_o}} \frac{\Delta T_i}{T_{o1}} =$$

$$\frac{v_{im}}{v_{i1}} \cong \frac{T_{avg}}{T_{i1}} \cong \frac{T_{i1} + T_{i2}}{2\, T_{i1}} = \delta_i$$

Calculation of optimum G_i / G_o .-

$$\frac{1}{X_i\, G_i^{0.8}} + \frac{1}{X_o\, G_o^{0.6}} = \frac{1}{W_i\, C_{p_i}\, NTU} = \text{const.}$$

Hence we shall minimize $P = \Lambda_i G_i^{2.8} + \Lambda_o G_o^{2.87}$

under the condition that G_i and G_o satisfy the above equation

Let ξ be the Lagrange multiplier. Consider

$$\psi = \Lambda_i G_i^{2.8} + \Lambda_o G_o^{2.87} + \xi \left(\frac{1}{X_i\, G_i^{0.8}} + \frac{1}{X_o\, G_o^{0.6}} \right)$$

For minimum P we set $\frac{\partial \psi}{\partial G_i} = 0$ and $\frac{\partial \psi}{\partial G_o} = 0$

If one eliminates ξ

$$\frac{G_i^{3.6}}{G_o^{3.47}} = 1.37 \frac{\Lambda_o X_o}{\Lambda_i X_i}$$

This formula is similar to formula (79) of ref. 3, except that the constants $\Lambda_o, X_o, \Lambda_i, X_i$ are now different .

Substituting the values of $\Lambda_o, X_o, \Lambda_i, X_i$ in the above formula and simplifying we get

$$\frac{G_i^{3.6}}{G_o^{3.47}} = 1.37 \, \frac{2nN_1 v_o^2 \varphi_1 C_4}{g} \left(\frac{\mu_o}{D_o}\right)^{0.13} \Lambda_g \times 1.10 \, C_2 C_{p_o} (N_{Pr})_o^{-2/3} \left(\frac{\mu_o}{D_o}\right)^{0.4} A_o$$

$$\times \frac{2g}{\varphi_2 A_i v_i^2 \, 0.046} \left(\frac{D_i}{\mu_i}\right)^{0.2} \frac{(N_{Pr})_i^{2/3}}{0.023 \, C_{p_i}} \left(\frac{D_i}{\mu_i}\right)^{0.2} \frac{1}{A_i}$$

Using also

$$\frac{A_{of}}{A_o} = \frac{R}{L'}$$ formula (1), pp.33, ref. (I) .

where R is the hydraulic radius for the flow normal to tube banks. It is given in table I2, Ref. I .

$$\frac{G_i}{G_o} = \gamma = \frac{(5700 \, C_2 C_4)^{0.278}}{(N_{Re})_o^{0.0361}} \left(\frac{v_o}{v_i}\right)^{0.555} \left(\frac{\varphi_1}{\varphi_2}\right)^{0.278} \left(\frac{R}{l_c}\right)^{0.278} \left(\frac{D_o}{D_i}\right)^{0.444} \left(\frac{\mu_o}{\mu_i}\right)^{0.111} \left(\frac{C_{p_o}}{C_{p_i}}\right)^{0.278} \left[\frac{(N_{Pr})_i^{1/3}}{(N_{Pr})_o^{1/3}}\right]$$

which is similar to eq. (8I) ref.3

<u>Determination of G_i, G_o in terms of power loss.-</u>

Since

$$P = \frac{0.023 \, \varphi_2 v_i^2}{\rho g (1+\lambda)} \frac{w}{t} \left(\frac{\mu_i}{D_i}\right)^{0.2} \gamma^{2.8} G_o^{2.8} + \frac{2 \varphi_1 v_o^2 C_4 R (1+2\lambda)}{\rho g (1+\lambda) \, l_c} \frac{w}{t} \left(\frac{\mu_o}{D_o}\right)^{0.13} G_o^{2.[illegible]}$$

Substituting $G_i = \gamma G_o$

$$P = \frac{w}{t} G_o^{2.8} \left[\frac{0.023 \varphi_2 v_i^2}{\rho g (1+\lambda)} \frac{(\lambda \gamma_i)^{0.2}}{t^{0.2}} \gamma^{2.8} + \frac{2\varphi_4 v_o^2 C_4 (1+2\lambda) R}{\rho g (1+\lambda) l_e} \left(\frac{\lambda \gamma_o}{1+2\lambda} \right)^{0.2} \frac{1}{t^{0.2}} (N_{Re})_o^{0.07} \right]$$

solving for G_o we get

$$G_o = \left(\frac{P t^{1.2}}{w} \right)^{\frac{1}{2.8}} \left[\frac{0.023 \varphi_2 v_i^2}{\rho g (1+\lambda)} (\lambda \gamma_i)^{0.2} \gamma^{2.8} + \frac{2\varphi_4 v_o^2 C_4 (1+2\lambda) R}{\rho g (1+\lambda) l_e} \left(\frac{\lambda \gamma_o}{1+2\lambda} \right)^{0.2} (N_{Re})_o^{0.07} \right]$$

$$G_o = M \frac{P^{0.357} t^{0.428}}{w^{0.357}}$$

$$G_i = \gamma G_o = \gamma M \frac{P^{0.357} t^{0.428}}{w^{0.357}}$$

Determination of asymptotic value of $(NTU)_p$.-

Letting

$$s = r_w \frac{C_{p_o}}{C_{p_i}} \quad , \quad \varepsilon_p = s \left[1 - e^{-\frac{\Gamma}{s}} \right] \; , \; \Gamma = 1 - e^{-(NTU)_p}$$

where

$$\varepsilon = \frac{\left(\frac{1 - \frac{\varepsilon_p}{s}}{1 - \varepsilon_p} \right)^n - 1}{\left(\frac{1 - \frac{\varepsilon_p}{s}}{1 - \varepsilon_p} \right)^n - \frac{1}{s}}$$

Solving for $e^{-1 + e^{-(NTU)_p}}$ we get

$$e^{-1 + e^{-(NTU)_p}} = \frac{\left(1 - \frac{1}{s}\right)^s \left(1 - \frac{\varepsilon}{s}\right)^{\frac{s}{n}}}{\left[\left(1 - \frac{\varepsilon}{s}\right)^{\frac{1}{n}} - \frac{\sqrt[n]{1-\varepsilon}}{s} \right]^s}$$

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