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PAGE

THE EFFECT OF SHORT, RANDOMLY DISTRIBUTED
CLOSELY SPACED WIRES IN CONCRETE UPON ITS
ENGINEERING PROPERTIES

BY

ERFAN ŞANVER

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ERTAN ŞANVER

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CHAPTER I

INTRODUCTION

The relatively low tensile strength of concrete is normally considered an unavoidable deficiency of the material and ignoring this low value, either the tensile stress is carried only by the reinforcing steel or concrete is prestressed in order to cancel out tensile stresses due to dead load and live load by initial compression.

It is also known that reinforced concrete does not show the properties of a two-phase material. There are two phases present, namely, concrete and steel but the existence of one phase does not improve the basic properties of the other and behavior of the composite material is dictated by the individual behavior of each phase. In a two-phase material, however, the phase with low modulus of elasticity stretches and deforms under an applied load and stress is transferred to the other phase with high tensile strength. The total structure thereby absorbs a loading stress that would easily rupture the weaker component. Wood which combines a high-strength fiber, cellulose, with a plastic matrix, lignin and fiberglass in a plastic matrix show the above mentioned properties. (1)*

However, two-phase behavior can be attained and tensile strength can be improved in concrete considerably if the average distance between adjacent reinforcing elements can be reduced below a certain value. This fact can best be explained by crack arrest mechanism concept (2). According to this concept as a micro-crack forms in concrete, displacements develop in the material in the vicinity of the crack as a result of the stress field singularity at the crack edge. Steel reinforcement, however,

(*) Numbers in brackets refer to the books listed in Bibliography

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with its great rigidity opposes these displacements and resisting forces are developed due to the reinforcement.

With the help of the equations describing compatibility between adjacent points on the reinforcement and concrete, these opposing forces can be calculated and applying fracture mechanics concept, reduction in forces causing extension of cracks may be determined. It is theoretically shown that the stress required to extend a crack beyond the area enclosed by adjacent reinforcing rods is inversely proportional to the square root of the rod spacing. The effect is noticeable with spacings less than 0.5" and tensile strength increases rapidly with decrease of this value. This is achieved by reducing the spacing of reinforcement but, at the same time, decreasing reinforcement diameter in order to maintain steel percentage constant. As a result, reinforcing elements become fine wires at effective spacings.

An early suggestion of this idea came from Nervi who made experiments on the behavior of concrete slabs reinforced with closely spaced steel wire mesh. The spacing of the wires was in the order of 0.4 in. Nervi observed that the slabs exhibited great flexibility and that visible cracks did not occur until the steel was stressed nearly to its yield point.

Later on, beams with wires arranged parallel to one another along the direction of major principal stress gave consistent results. This was achieved by stringing individual wires through pierced plates at the ends of the forms. This had, of course, some limitations and difficulties in practical applications. (2)

Recently, concrete samples with short lengths of wire in random orientation but at nearly uniform spacing were tested in tension and gave consistent results with the theory. (3)

The object of the present thesis work is therefore to study the effect of average spacing of short randomly distributed closely

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spaced wires on various properties of concrete such as tensile, strength, compressive strength, modulus of elasticity and shrinkage; and to discuss the theoretical basis for the observed effects on the above-mentioned properties.

Results of several tests done on concrete indicate a well marked influence of the wires. Tensile and compressive strengths are increased about 1.5 times even with a small percentage of wire reinforcement of 2-3 %. Effect of wires on shrinkage also follows a definite pattern. Its effect being small at the beginning, as concrete hardens, towards the 21st day, the shrinkage of concrete samples with wires is in the order of 50 percent of those which are not reinforced. Dynamic modulus of elasticity tests done with the sonometer, however, give inconsistent results. This is mainly attributed to stress concentrations due to wires within the concrete mass.

CHAPTER II

THEORY

a) FRACTURE MECHANICS CONCEPT:

The fracture propagation phenomenon is best illustrated with a solid material, subjected to tensile stress, σ and containing a disk shaped crack. A. A. Griffith (4) explained it with the elastic energy release concept. Considering an internal crack having a radius a , the condition that assures crack extension was given by

$$\frac{\partial}{\partial a} (U - V) = 0 \quad (1)$$

Where U is the reduction in stored elastic energy due to the presence of crack and V is the crack surface energy. In plane stress, the reduction in stored elastic energy of a thin plate subjected to tensile stress, is

$$U = \frac{\pi \sigma^2 a^2}{E} \quad (2)$$

Where E is the modulus of elasticity. The crack surface energy of a brittle material, on the other hand, is given by

$$V = 4 T a \quad (3)$$

Where T is the surface tension. Substituting eq. (2) and (3) into eq. (1), we obtained

$$2T = \frac{\pi \sigma^2 a}{E} \quad (4)$$

If the rate of release of elastic energy is denoted by G , which is a function of loading and geometry; and the work rate required to extend a crack is represented as G_c , (a material property), then it was postulated by Irwin (5) that the stress perpendicular to the plane of the crack, at a point, at a distance of r apart

from the end of the crack, can be expressed as:

$$\sigma_y = K \frac{\cos \theta/2}{\sqrt{2r}} \left[1 + \sin \frac{\theta}{2} \sin \frac{\theta}{3} \right] \quad (5)$$

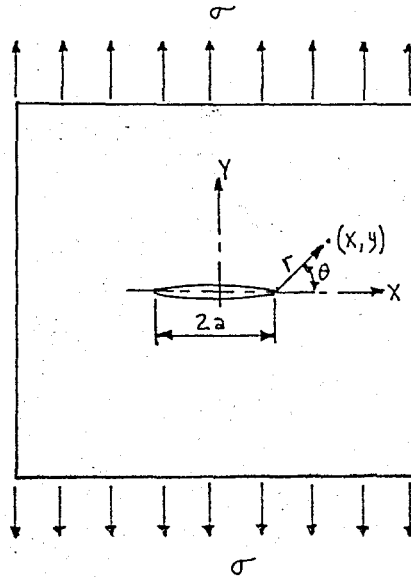


Figure 1

Where θ is the angle between radius r and the x -axis, and K is the stress concentration factor. K refers to the summation of the stress intensities each due to a different type of loading. A direct relationship between K and G was given for plane stresses as follows:

$$G = \frac{\pi K^2}{E} \quad (6)$$

Applying this analysis to concrete, the critical crack size can be estimated for a given tensile stress. The decrease in potential energy of a solid medium, acquiring a crack of radius a perpendicular to the axis of tensile stress is given by I.N. Sneddon (6) as

$$U = \frac{8(1-\mu^2)\sigma^2 a^3}{3E} \quad (7)$$

From definition of G :

$$G = \frac{\partial U}{\partial A} = \frac{\partial U}{\partial a} \frac{\partial a}{\partial A} \quad (8)$$

where A is the area of the crack surface. For a circular crack of radius a ,

$$A = \pi a^2$$

$$\frac{da}{dA} = 1/2\pi a$$

therefore:

$$G = \frac{4(1-\mu^2)\sigma^2 a}{\pi E} \quad (9)$$

As mentioned earlier, a crack starts to extend when the value of G becomes equal to G_c . Kaplan (7) observed that G_c for concrete varied between 0.1 in.-lbs./in.² and 0.02 in.-lbs./in.². Taking Poisson's Ratio, μ as 0.3 and E as 3×10^6 psi, critical crack size for a given tensile stress can be determined from eq. (9), equating G to G_c . For G_c 0.02 in.-lbs./in.², substituting into equation (9), critical crack radius is found to be 0.208 in. at a tensile stress of 500 psi and 0.052 in. at 1000 psi. It is seen that the critical crack radius is quite small and it does not differ much from holes and flaws normally present in concrete. This could be the real cause of failure of concrete at low tensile stresses. Now the question arises how to increase the tensile strength of concrete. In order to achieve this, concrete should be as compact as possible, eliminating internal holes and flaws. However, possibility of having a single crack, and propagation of it throughout the tension zone is a factor that should not be underestimated. Then, there remains the alternate way of increasing tensile strength. Assuming an internal crack has formed in concrete, propagation of it over the tensioned area could be prevented by a method called Fracture Arrest Mechanism (2), (8).

b) FRACTURE ARREST MECHANISM:

The mechanism is described by a concrete mass subjected to a tensile stress, σ (2,8). The reinforcement consisting of a rectangular array of steel wires at a spacing s and located parallel to the direction of the tensile stress.

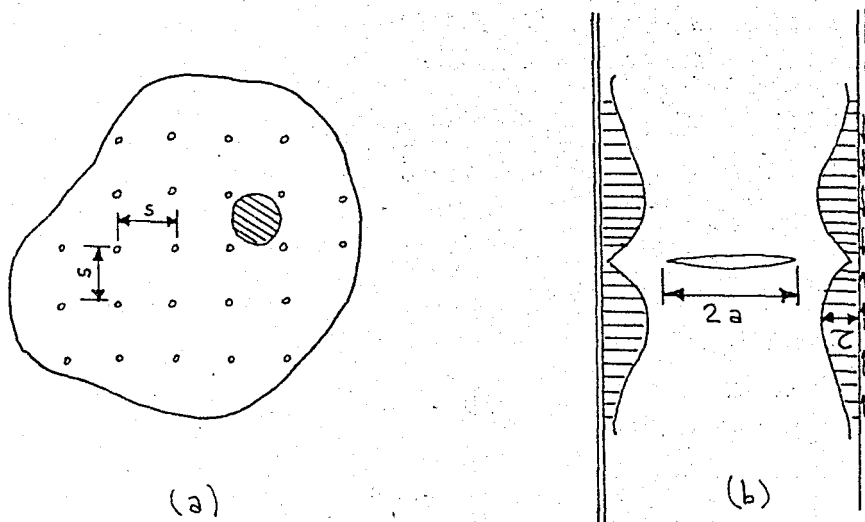


Figure 2

An internal flaw in the form of a circular crack is assumed in the concrete as shown in Fig. 2. The crack is centrally located between four adjacent wires and the diameter of the crack is taken equal to wire spacing, s . In the absence of a crack, the steel rods and the concrete will be stretched equally and there will be no tendency for one to move relative to the other. In the presence of a crack, however, the local concrete displacement caused by the extensional strains will be resisted by stiffer steel wire. The bond stress distribution will be as shown in Fig 2-b. In the plane of the crack the bond stress will be zero, it will increase to a maximum, and then decrease with increasing distance from the crack edge as the local displacements diminish. The developed bond stresses will act on the concrete as pinching forces, will produce a stress intensity factor K_f , which will have an opposite sense to the stress intensity factor due to different types of loading, defined earlier. Thus, the total stress intensity factor will be given by

$$K_T = K_\sigma - K_F \quad (10)$$

The spacing and cross-section of the reinforcing wires for effective crack control must be so chosen that the magnitude of K_F will be sufficient enough to cause reduction in K_t and permit larger over-all tensile stress. An additional provision for effective crack arrest, will be the fact that distributed forces should not exceed the bond strength between concrete and steel wires.

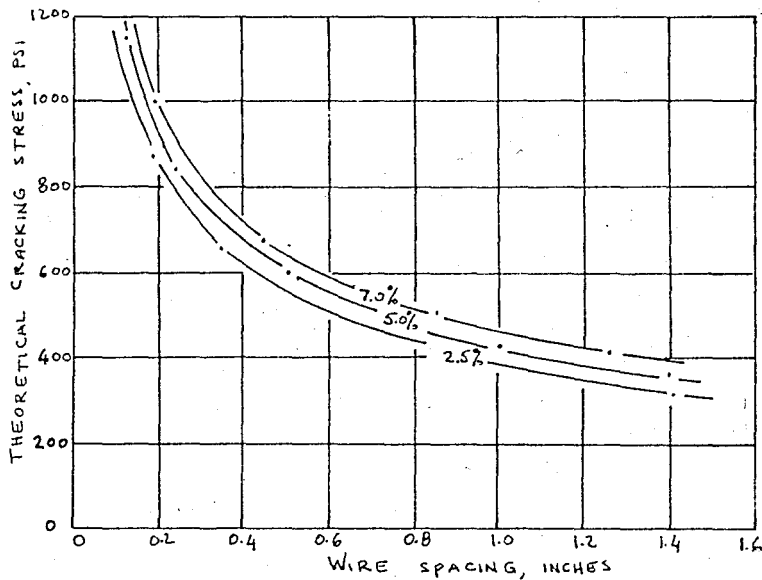


Figure 3 - Cracking stress as a function of wire spacing ($G_c = 0.02$ in.-lb per sq.in.) (6)

Theoretical solutions for a variety of reinforcement spacings and percentages of steel have been determined by Sneddon(6) for an experimentally determined G_c value and summarized in Fig.3. It is a plot of theoretical tensile cracking strength as a function of reinforcement spacing for several percentages of steel.

Figure 3 is significant in that it indicates true two-phase behavior at smaller spacings. For any percentage of steel, the

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smaller the spacing, the higher is the cracking stress. The knee is at a spacing of about 0.5 in., and a substantial increase in tensile strength occurs with smaller values of s . It should be further noted that this increase in strength is in proportion to the inverse square root of spacing s .

c) AVERAGE SPACING BETWEEN RANDOMLY DISTRIBUTED SHORT LENGTHS OF WIRE

The analysis will be given by assuming a uniform distribution of steel wires throughout the concrete mass. The validity of this assumption may be questionable but, since we make crude assumptions in treating concrete, such as homogeneity and isotropy, the above assumption is acceptable.

The assumption of uniform distribution of wires gives rise to another question, the efficiency of wires in crack control. Since wires are oriented randomly, only those wires which are parallel to, and those which make an angle less than 90° with the direction of tensile stress are stressed. This can be formulated in several ways. As an approximate solution, it can be assumed that only one-third of the wires are effective because there are three principal axes, tensile stress being parallel to one of these; or those wires having a component along the direction of the tensile stress are effective in crack control. The latter assumption employed by Romualdi and Mandel (3) forms the basis of the following analysis.

Taking a wire of length L , oriented in space making an angle θ with the x-y plane and ϕ with the x-axis, the projection of it in the x-direction is $L \cos \theta \cos \phi$. If there are N wires uniformly distributed, the average length of the wires effective in the x-direction will be calculated as

$$\frac{N \int_0^{\pi/2} \int_0^{\pi/2} L \cos \theta \cos \phi \, d\theta \, d\phi}{N \int_0^{\pi/2} \int_0^{\pi/2} d\theta \, d\phi} = 0.41 L \quad (11)$$

It follows that only 41 percent of the total amount of reinforcement is effective along each principal axis and with tensile stress parallel to one of these, 0.41 is the proper correction factor that will be required. The average distance between the

centroids of the wires will be given by

$$S = \sqrt[3]{\frac{V}{N}} \quad (12)$$

Where N is the number of wires in a volume V . Since 41 percent of the wires are effective only, the spacing between the centroids of effective wires will be

$$S_{ef} = \sqrt[3]{\frac{V}{0.41N}} \quad (13)$$

If there are n centroids per unit area, then

$$n = \left(\frac{1}{S_{ef}}\right)^2 \quad (14)$$

Assuming that the length of wires, L is greater than the effective wire spacing S_{ef} , there will be overlapping of wires which will increase the number of wires through a cross section. The number of wires per unit area will become

$$n_w = \left(\frac{1}{S_{ef}}\right)^2 \left(\frac{L}{S_{ef}}\right) = \frac{L}{S_{ef}^3} \quad (15)$$

Substituting eq.(13) into eq. (15), the average spacing of the wires will now be:

$$S = \frac{1}{\sqrt{n_w}} = \sqrt{\frac{S_{ef}^3}{L}} = \sqrt{\frac{V}{0.41NL}} \quad (16)$$

Designating the percentage of steel as p , volume of steel V_s and volume of reinforced concrete V_c ,

$$\frac{V_s}{V} \times 100 = p \quad (17)$$

Expressing the volume of each steel wire of length L and diameter d as:

$$v_s = \frac{\pi d^2 L}{4} \quad (18)$$

The number of wires, N will be:

$$N = \frac{V_s}{v_s} = \frac{4V_s}{\pi d^2 L} = \frac{pV}{25\pi d^2 L} \quad (19)$$

and substituting eq. (19) into eq. (16) the spacing will become:

$$S = 13.8 d \sqrt{\frac{1}{p}} \quad (20)$$

which expresses the spacing in terms of wire diameter and percentage by volume.

CHAPTER III

EXPERIMENTAL ANALYSIS

a) MATERIALS

1) Aggregates:

The aggregate used for concrete for determination of shrinkage and dynamic modulus of elasticity, was local sand passing a No. 4 sieve, and that, used for determination of tensile and compressive strengths was composed of sand and gravel with a maximum size of 1.0 inch. The gradation of each is represented in Fig. 4 and tables below.

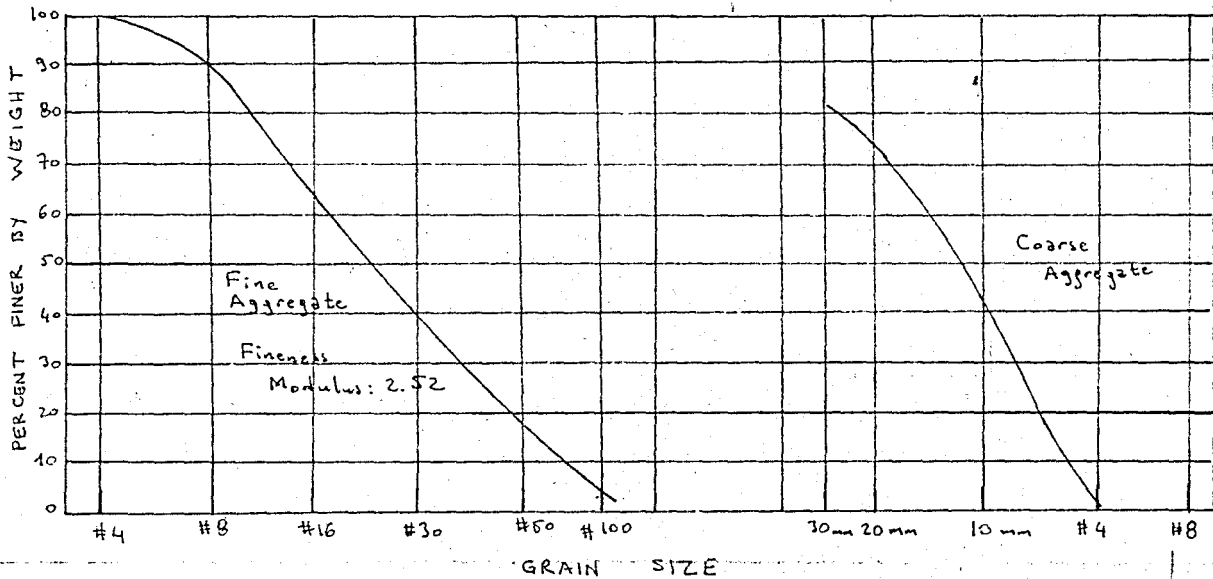


Figure 4 - Gradation curve of fine and coarse aggregates.

Fine Aggregate:

| | | | | | | |
|----------|-----|----|----|----|----|-----|
| Sieve No | 4 | 8 | 16 | 30 | 50 | 100 |
| % Finer | 100 | 94 | 80 | 52 | 16 | 3 |

Coarse Aggregate:

| | | | | |
|----------|----|------|------|-----|
| Sieve No | 1" | 3/4" | 3/8" | # 4 |
| % Finer | 89 | 80 | 50 | 0 |

Sand and gravel was mixed with a 1 to 1 ratio by weight and gave the following gradation:

Mixed Aggregate:

| | | | | | | | | | |
|-----------|----|------|------|-----|-----|------|------|------|-------|
| Sieve No. | 1" | 3/4" | 3/8" | # 4 | # 8 | # 16 | # 30 | # 50 | # 100 |
| % Finer | 95 | 90 | 75 | 50 | 47 | 40 | 26 | 8 | 1.5 |

2) Cement

Concrete was prepared with a normal portland cement produced by Ankara Çimento Sanayi T.A.Ş.

3) Steel Wire

The wire used, was cold drawn, bright finish (but untreated surface) high strength steel wire of 0.50 mm diameter. Several tensile tests were run and a tensile strength of 284000 psi was recorded on samples of 50 mm length. The average length of wires mixed with concrete was 1.0 inch. (See Fig.5)

b) MIX PROPORTIONS, MIXING, CURING AND PREPARATION FOR TESTING

The proportioning of concrete was done according to ACI specification (ACI 613-54) (9, 10). Before the actual mix was prepared, several trial batches with varying water-cement ratios were prepared in order to determine the suitable mix; since amount of water had a great effect on successful distribution of wires. If the mixture^{was} too dry, the wires tended to knit into balls; the same thing happened if there was too much water. Upon trials, it was decided to use a water-cement ratio of 5.5 gal per sack.

Procedure followed in proportioning is given below:

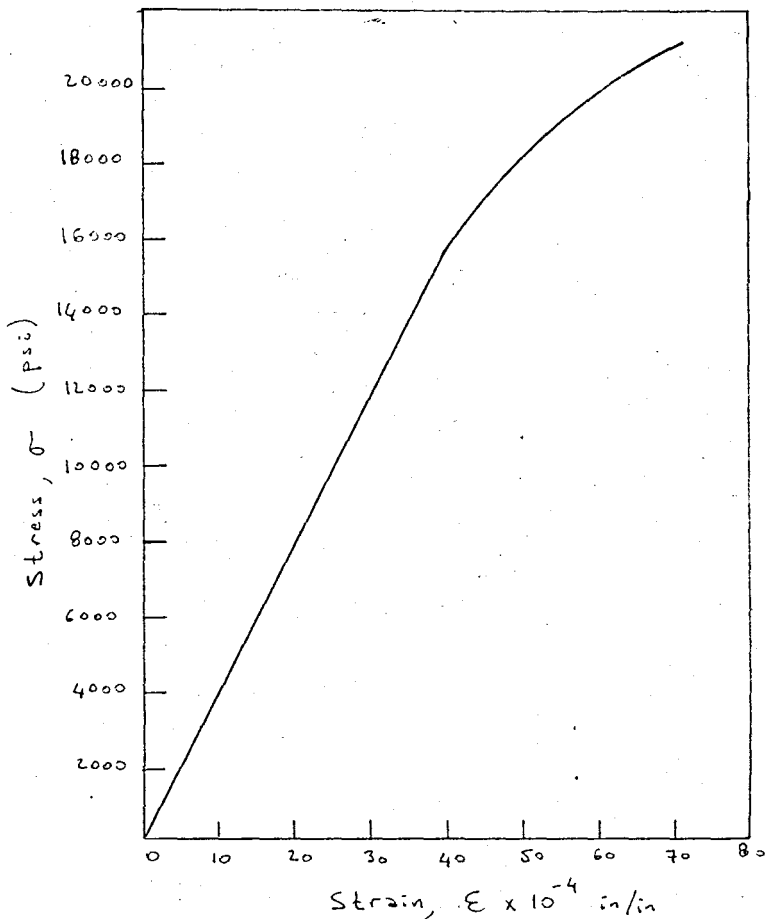


Figure 5 - Stress-Strain Curve of Steel Wire

- 1) W/C: 5.5 gal. water per sack of cement (0.49 by weight)
- 2) Fineness modulus of sand: 2.52
- 3) Max. Size of coarse aggregate: 1"
- 4) Bulk volume of dry-rodded coarse aggregate per unit volume of concrete 0.69
Dry unit weight: 1.96 gm/cm^3
- 5) Entrapped air: 1.5
- 6) Specific gravity of cement: 3.15
- 7) Sand dry unit weight: 1.65 gm/cm^3
- 8) Specific gravity of aggregates: 2.65
- 9) Water per dm^3 of concrete: 0.198 liter

For a 1 dm³ batch of concrete, the amount of each component used was:

Cement: 0.404 kg.
Water : 0.198 liter
Gravel: 0.875 kg.
Sand : 0.875 kg

Mixing was done with a trowel. The sand, gravel, cement and water were first mixed and then wires were subsequently added in small increments. No difficulty was encountered in mixing, up to a steel percentage of 3.0 by volume, but at higher percentages, concrete lost its workability and addition of wires was not possible anymore.

Specimens to be used for split-cylinder and compression tests were prepared in standard 6" x 12" cylinder molds. In order to see the effect of placement method on the distribution of wires in the concrete mass, some specimens were prepared by placing concrete in three layers, each layer being rodded 25 times with a tamping rod. Others were prepared by filling the molds with concrete and applying electrical vibrator afterwards. The latter method gave better wire distribution, wires orienting themselves in a horizontal direction in the former method.

Specimens for determination of shrinkage and dynamic elastic modulus, were poured in standard 2" x 2" x 11" prismatic molds (ASTM. C157-64 T) and were vibrated for two minutes on an electrical vibrator.

Twenty-four hours after pouring, molds were removed and specimens were immersed in water. Cylinders were kept in water for twenty-eight days and tested for compressive and split-cylinder strengths. Prisms were cured in water for seven days and then kept in air at a place free of air movements, with an average temperature of 71 F and a relative humidity of 52 %.

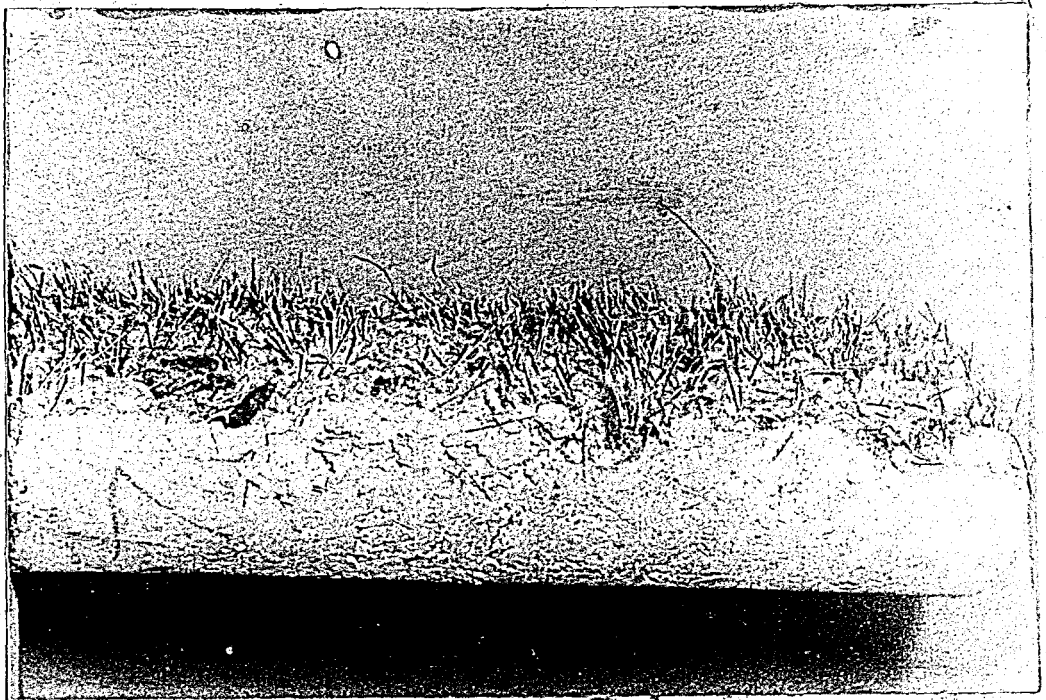


Figure 6-Distribution of Wires in a
Sample with 3 percent Reinforcement

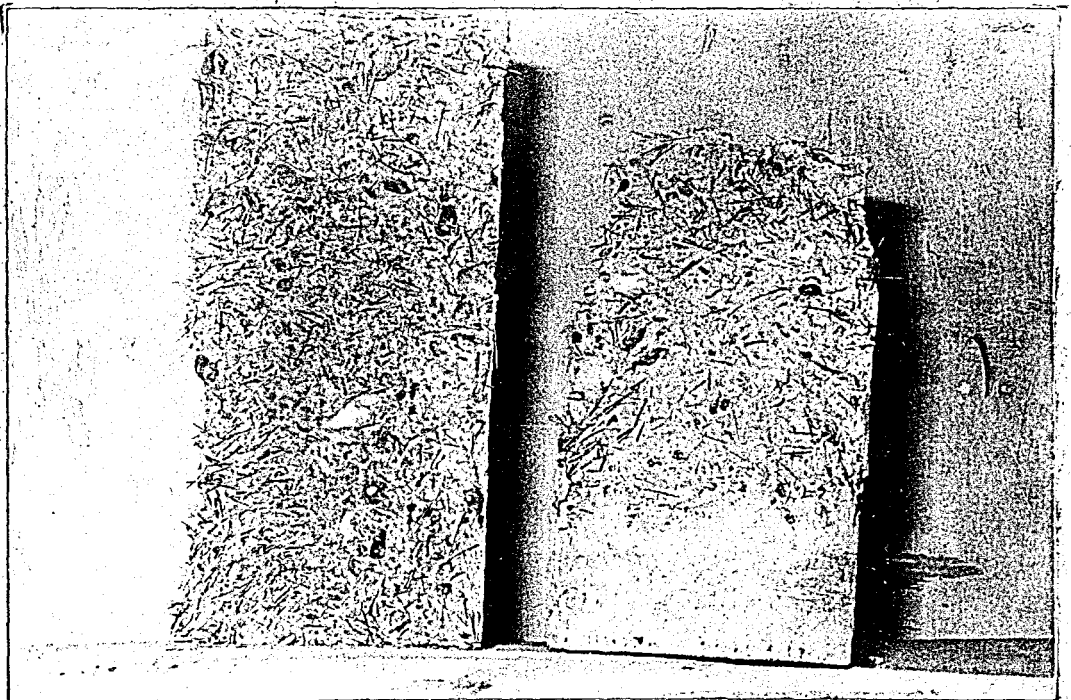


Figure 7-Split Cylinder and Compression
Test Samples

c) APPARATUS

The following instruments were employed for the preparation and testing of samples:

- 1) Universal Testing Machine:
 Model: Riehle, Screw type, mechanical, maximum loading capacity of 70,000 lbs.
- 2) Cylinder Testing Machine:
 Makers: Soiltest Incorporated, hydraulic, manually working, maximum loading capacity of 250,000 lbs., accuracy of 500 lbs.
- 3) Sonometer, Exciter, Pick-up Set:
 Makers: Soiltest Incorporated, Electro Products Laboratories, Model 4100, maximum frequency control of 20,000 cycles.
- 4) Dial Micrometer for shrinkage measurements:
 Sensitivity: 0.0001 inches
- 5) Dial Gage Set-up:
 Sensitivity: 0.005 inches
- 6) Electrical Hand Vibrator
 Model DU
 Stow Manufacturing Co.
- 7) Electrical Plane Vibrator
 Model VP 60
 Makers: Soiltest Incorporated
- 8) 6" x 12" cylindrical steel molds
- 9) 2" x 2" x 11" prismatic steel molds
- 10) Balance of 0.1 gm sensitivity

d) TESTING PROCEDURE

1. Compression Test:

Twenty-eight days old water cured specimens were taken out of water, top and bottom surfaces wiped dry and capped with a sulphur capping compound. They were kept under a wet burlap for two hours before they were tested. There were eight 6" x 12" cylinders prepared, with steel percentages of 0, 1, 2 and 3%, two samples being cast for each mix.

Cylinders were first tested in a Universal Testing Machine, applying the load at a rate of 400 lbs/sec. according to ASTM C39-64 (11). In order to obtain the stress-strain curve a dial page set-up was attached to the samples. Since the capacity of the testing machine was not enough to crush these specimens, load was slowly removed at 60,000 lbs and crushing stress and strain were determined by the hydraulic testing machine.

2) Split-Cylinder Test:

Four pairs of cylinders with the above mentioned steel percentages were water cured for 28 days. They were placed horizontally under the bearing plate of the Universal Testing Machine. Strips of Plywood about 1/8" thick, 1" wide and 12" long were placed on the upper and lower bearing elements to ensure uniform bearing (11). Cylinders were so positioned that the center of their upper bearing element coincided with the center of the upper bearing block of the machine. The load was applied at a rate of 400 lbs/sec.

The splitting tensile strength, T was calculated from the equation (11):

$$T = \frac{2P}{\pi DL}$$

where: P: maximum load at failure
L: length of the cylinder
D: diameter of the cylinder

3) Shrinkage Measurements:

Mortar prisms, 2 inch square and 11 inch long were cast with the same steel percentages as used above. Mortar was obtained by preparing the same mix that was used for the cylinders and sieving it through No. 4 sieve. The specimens contained a stainless steel gage plug at the centers of each of the ends projecting about 1/8" beyond these ends. Specimens were kept in the mold for 24 hours, removed from the molds and water cured for seven days. At the end of the seventh day, initial lengths of each were recorded with the dial micrometer, and their weights were determined. Samples were then kept in air at a place free of air movements. Shrinkage and weight measurements were repeated at weekly intervals.

4) Determination of Dynamic Modulus of Elasticity:

Similar specimens as for shrinkage measurements, were used for the determination of dynamic moduli of elasticity.

The samples to be tested were suspended on two wires placed at a distance of $1/4 L$ from each end. The vibrations were transmitted directly through the driver which was bearing on the end part of the sample. These vibrations were detected by the pick-up which was placed against the other end of the specimen. The vibrational frequency was read from the sonometer. Damping frequencies were not considered. Modulus of elasticity was calculated using equation (31) given in Appendix.

CHAPTER IV

RESULTS

a) TENSION TEST:

Details of the eight cylinders tested in indirect tension are presented in table 1 and relation of strength to steel percentage is illustrated in fig 8.

| Test No | Wire Percent | Splitting Load (lbs.) | Tensile Strength (psi) | Strength Ratio |
|---------|--------------|-----------------------|------------------------|----------------|
| 1 | Plain | 38 500 | 340 | } 1 |
| 2 | Plain | 40 500 | 358 | |
| 3 | 1.0 | 56 500 | 500 | 1.43 |
| 4 | 1.0 | 53 000 | 468 | 1.34 |
| 5 | 2.0 | 60 000 | 530 | 1.52 |
| 6 | 2.0 | 59 500 | 525 | 1.51 |
| 7 | 3.0 | 65 500 | 580 | 1.66 |
| 8 | 3.0 | 64 500 | 570 | 1.63 |

Table 1-Summary of Split Cylinder tests

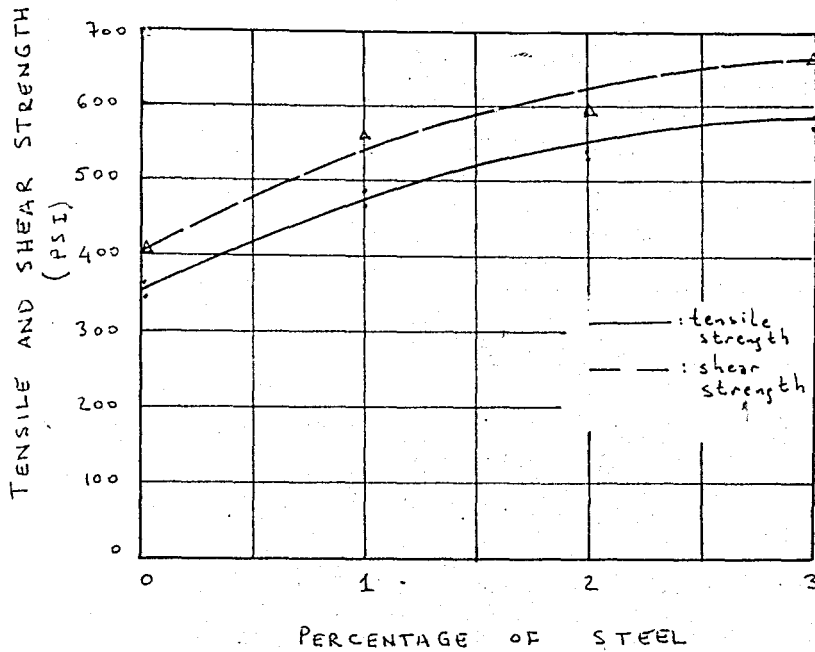


Figure 8-Tensile Strength and Shear Strength as a Function of Steel Percentage

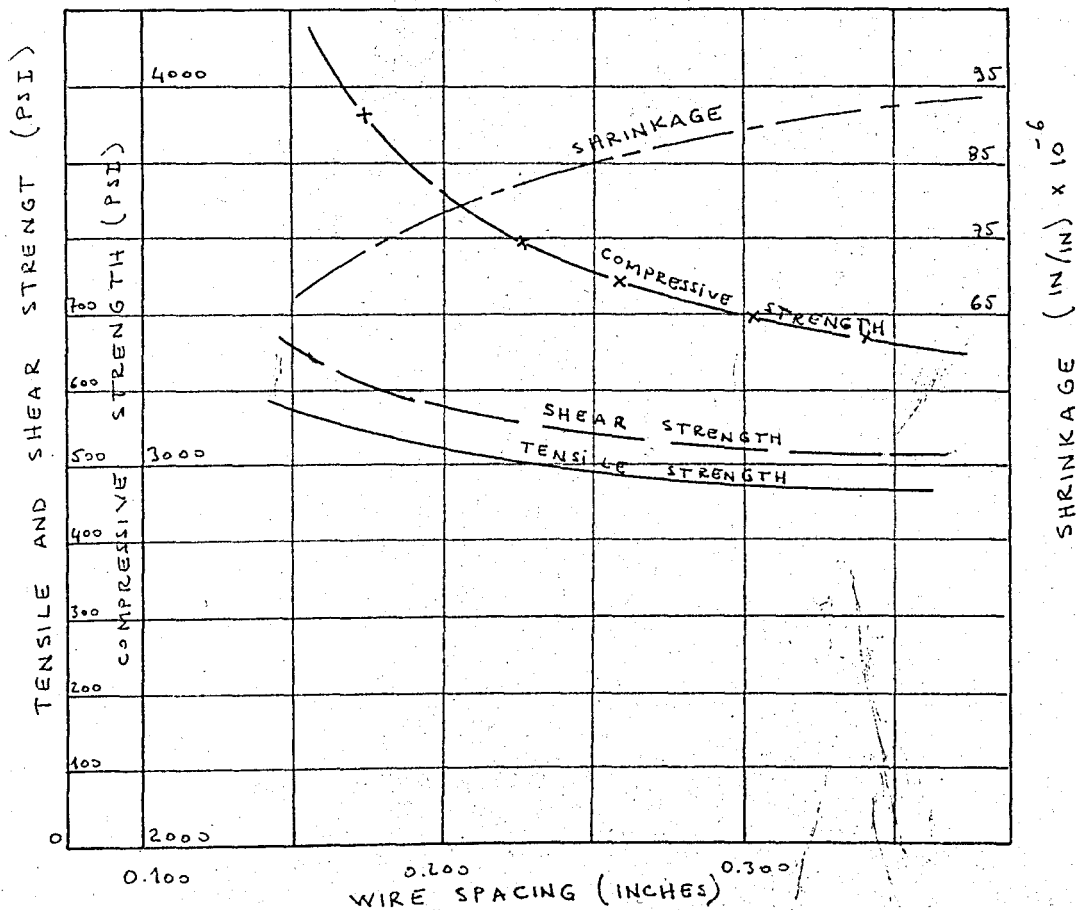


Figure 9-Variation in Properties of Concrete with Wire

b) COMPRESSION TEST:

The data obtained from compression tests are tabulated in table 2 and corresponding stress-strain curves are given in figure 10.

| Percentage of Steel | | | | | | | |
|---------------------|-----------------|----------------|-----------------|----------------|-----------------|----------------|-----------------|
| Plain | | 1 % | | 2 % | | 3 % | |
| Strain (x10 | Stress (psi) | Strain (x10 | Stress (psi) | Strain (x10 | Stress (psi) | Strain (x10 | Stress (psi) |
| 2.50 | 288 | 2.50 | 398 | 2.25 | 300 | 3.00 | 454 |
| 3.75 | 471 | 4.75 | 582 | 5.00 | 625 | 3.75 | 588 |
| 5.00 | 630 | 6.25 | 760 | 7.75 | 1000 | 5.50 | 845 |
| 6.25 | 805 | 7.55 | 920 | 9.75 | 1320 | 7.50 | 1130 |
| 7.50 | 935 | 9.75 | 1220 | 12.50 | 1650 | 8.50 | 1320 |
| 8.25 | 1030 | 11.50 | 1480 | 16.00 | 2300 | 9.50 | 1480 |
| 10.00 | 1290 | 13.50 | 1720 | 19.75 | 2650 | 10.00 | 1650 |
| 11.25 | 1460 | 14.75 | 1890 | 22.50 | 3050 | 11.50 | 1770 |
| 14.00 | 1830 | 16.25 | 2090 | 27.25 | 3550 | 12.50 | 1990 |
| 17.50 | 2170 | 19.00 | 2360 | 32.00 | 3740 | 13.50 | 2120 |
| 24.25 | 2420 | 35.00 | 3520 | 35.00 | 3600 | 29.00 | 4120 |
| 37.50 | 2910 | 47.50 | 3200 | | | 37.50 | 3850 |
| 47.50 | 2875 | | | | | | |

Table 2-Compression Test Data

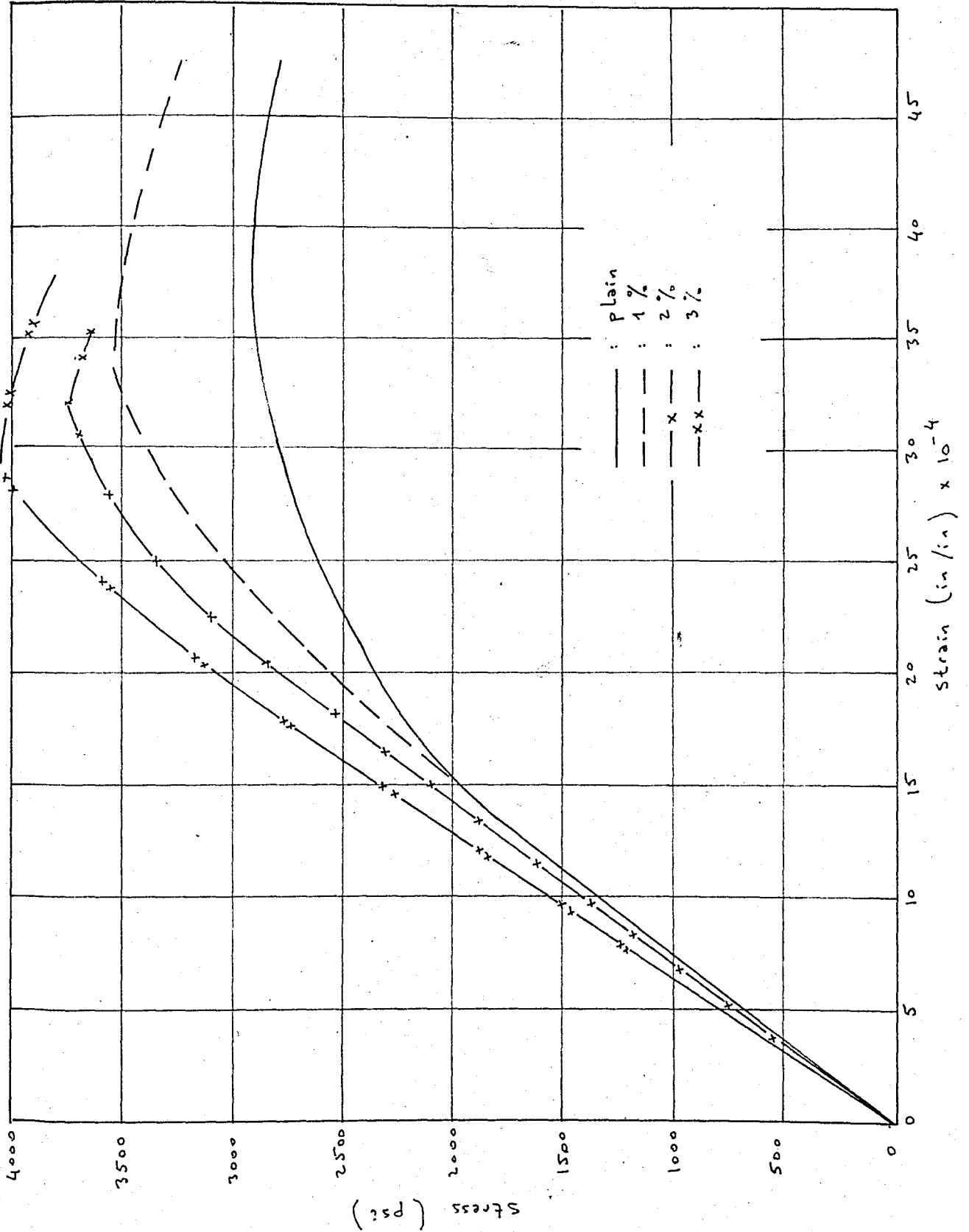


Fig 10- Stress-Strain Diagram

| Wire Percentage | Average Spacing (in) | Compress. Strength (psi) | Modulus of Elasticity | Strength Ratio ²² |
|-----------------|----------------------|--------------------------|-----------------------|------------------------------|
| Plain | = | 2910 | 1.30×10^6 | 1.00 |
| 1.0 | 0.272 | 3520 | 1.29×10^6 | 1.21 |
| 2.0 | 0.193 | 3740 | 1.43×10^6 | 1.28 |
| 3.0 | 0.157 | 4120 | 1.68×10^6 | 1.41 |

Table 3-Comparison of Compressive Strength for Different Wire Spacings

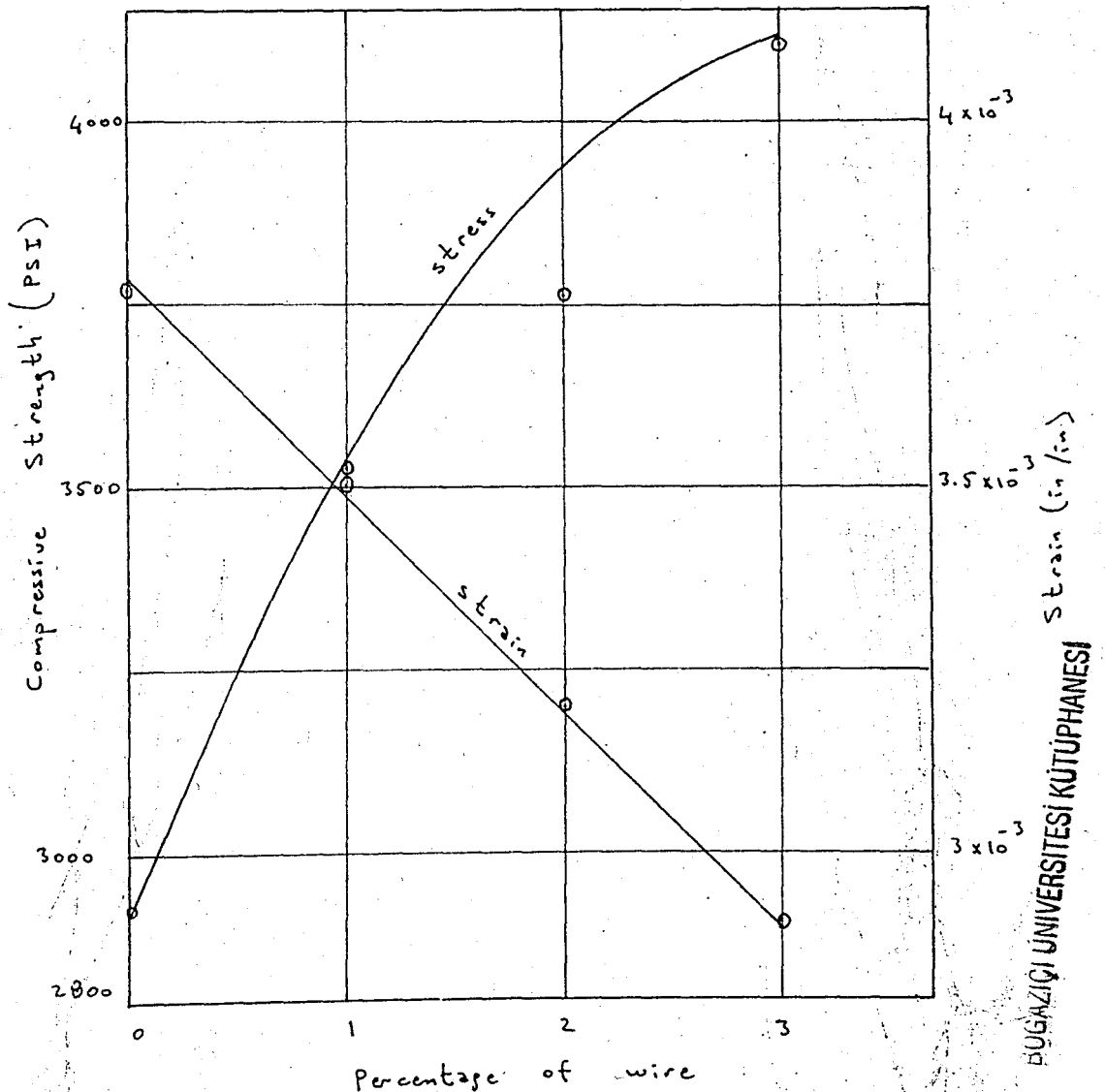


Figure 11-Compressive Strength and Strain at Max. Load

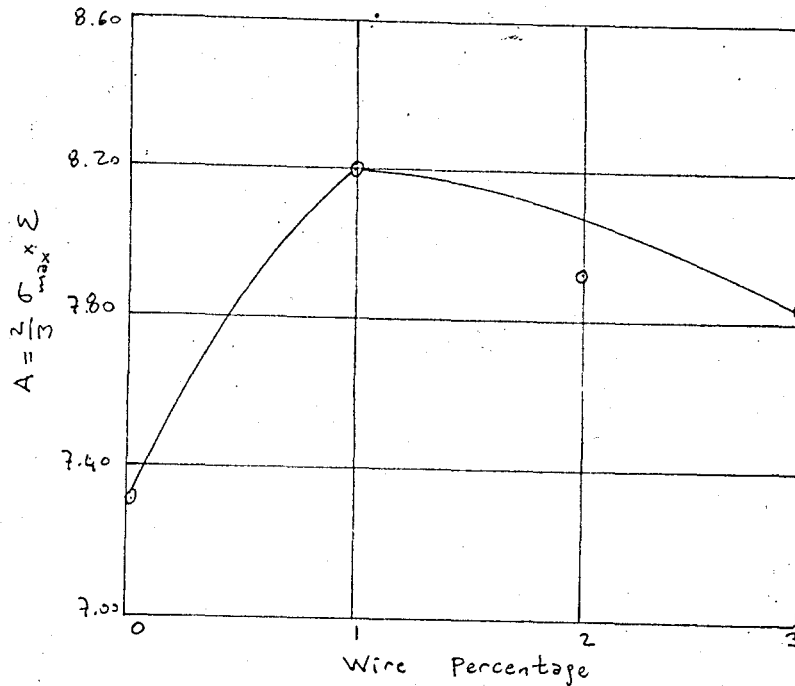


Figure 12-Strain Energy at Maximum Load
Versus Steel Percentage

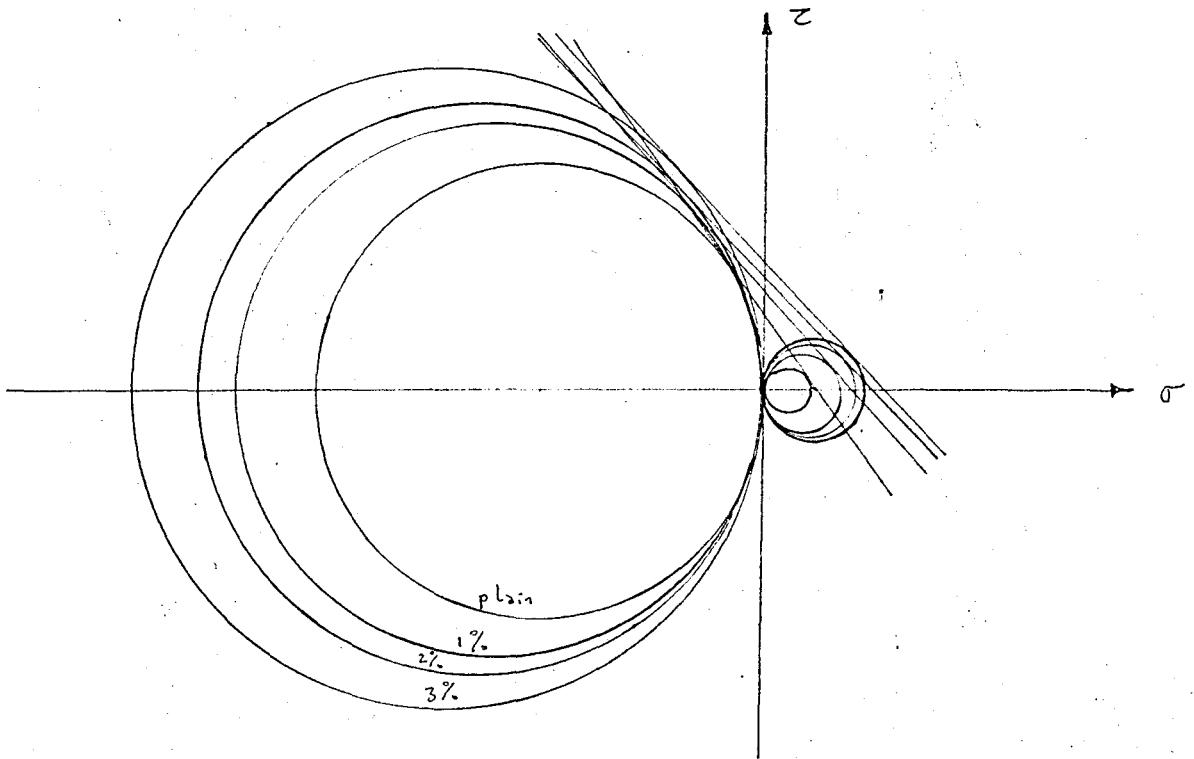
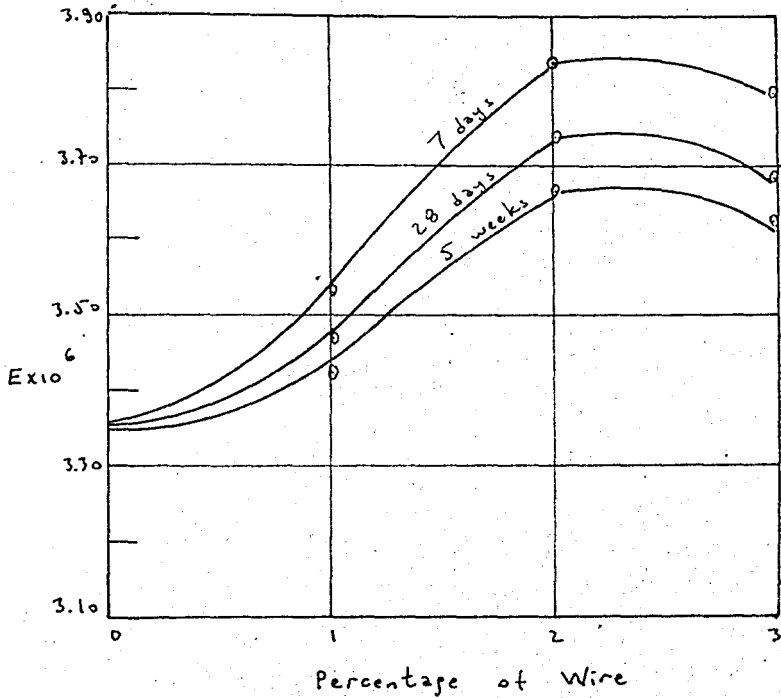


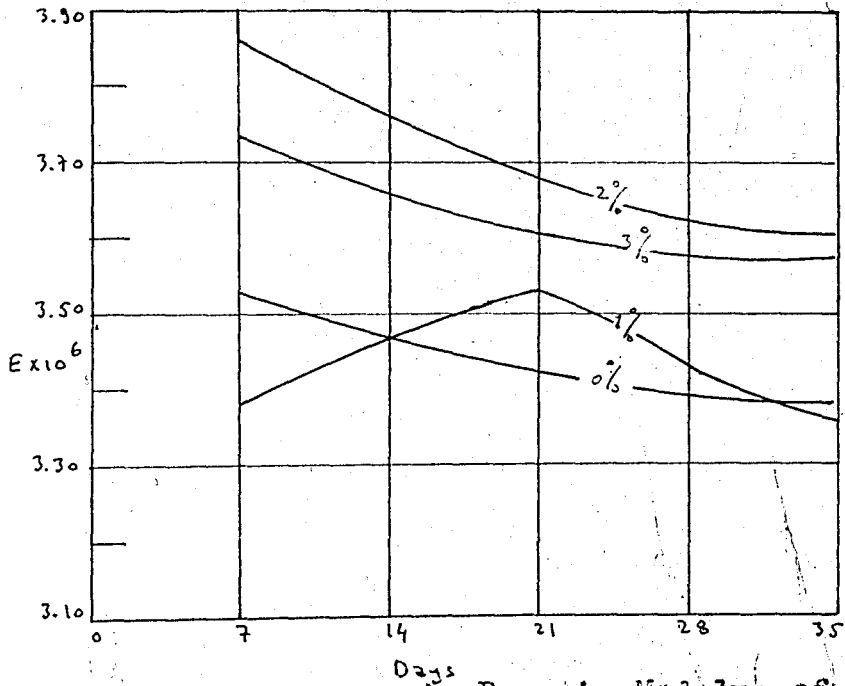
Figure 13-Shear Strength Determined
from Mohr's Circle

| Test No | Wire Percentage | 7 Days | | | 14 Days | | | 21 Days | | | 28 Days | | |
|---------|-----------------|-------------|--------|-----------------------|-------------|--------|-----------------------|-------------|--------|-----------------------|-------------|--------|-----------------------|
| | | Weight (gm) | Cycles | $E \times 10^6$ (psi) | Weight (gm) | Cycles | $E \times 10^6$ (psi) | Weight (gm) | Cycles | $E \times 10^6$ (psi) | Weight (gm) | Cycles | $E \times 10^6$ (psi) |
| 1 | Plain | 1647 | 1890 | 3.63 | 1579 | 1920 | 3.54 | 1569 | 1940 | 3.45 | 1565 | 1940 | 3.37 |
| 2 | Plain | 1625 | 1900 | 3.36 | 1557 | 1970 | 3.46 | 1545 | 1950 | 3.36 | 1544 | 1950 | 3.36 |
| 3 | 1 | 1667 | 1920 | 3.24 | 1596 | 1940 | 3.20 | 1583 | 1930 | 3.14 | 1582 | 1920 | 3.11 |
| 4 | 1 | 1674 | 1920 | 3.29 | 1604 | 1950 | 3.25 | 1594 | 1950 | 3.23 | 1592 | 1950 | 3.22 |
| 5 | 2 | 1728 | 1980 | 3.22 | 1660 | 1970 | 3.22 | 1648 | 1950 | 3.13 | 1646 | 1930 | 3.06 |
| 6 | 2 | 1711 | 1980 | 3.35 | 1648 | 2010 | 3.32 | 1638 | 2000 | 3.27 | 1635 | 2000 | 3.27 |
| 7 | 3 | 1730 | 1940 | 3.00 | 1661 | 1960 | 3.00 | 1652 | 1940 | 2.93 | 1648 | 1930 | 2.89 |
| 8 | 3 | 1725 | 1960 | 3.12 | 1661 | 1960 | 3.00 | 1650 | 1980 | 3.04 | 1649 | 1980 | 3.04 |

TABLE 4 - Summary of Dynamic Elastic Modulus Experiments



Percentage of Wire
Figure 14-Dynamic Modulus of Elasticity
as a Function of Wire Reinforcement



Days
Figure 15-Change in Dynamic Modulus of
Elasticity with Time

| Test No | Wire Percentage | 7 Days | | 14 Days | | 21 Days | | 28 Days | |
|---------|-----------------|--|--------------------|--|--------------------|--|--------------------|--|--------------------|
| | | $\frac{\Delta L}{L} \times 10^{-6}$ (in/in) | Loss of Wt (gm) | $\frac{\Delta L}{L} \times 10^{-6}$ (in/in) | Loss of Wt (gm) | $\frac{\Delta L}{L} \times 10^{-6}$ (in/in) | Loss of Wt (gm) | $\frac{\Delta L}{L} \times 10^{-6}$ (in/in) | Loss of Wt (gm) |
| 1 | Plain | 49 | 68 | 71 | 78 | 83 | 82 | 98 | 86 |
| 2 | Plain | 43 | 68 | 69 | 80 | 80 | 81 | 95 | 87 |
| 3 | 1 | 44 | 70 | 65 | 83 | 75 | 84 | 86 | 89 |
| 4 | 1 | 41 | 70 | 65 | 80 | 76 | 82 | 88 | 89 |
| 5 | 2 | 39 | 68 | 58 | 80 | 66 | 82 | 76 | 88 |
| 6 | 2 | 39 | 63 | 59 | 73 | 67 | 76 | 77 | 80 |
| 7 | 3 | 34 | 69 | 51 | 78 | 58 | 82 | 66 | 86 |
| 8 | 3 | 38 | 64 | 54 | 75 | 61 | 76 | 70 | 82 |

TABLE 5 - Shrinkage Data

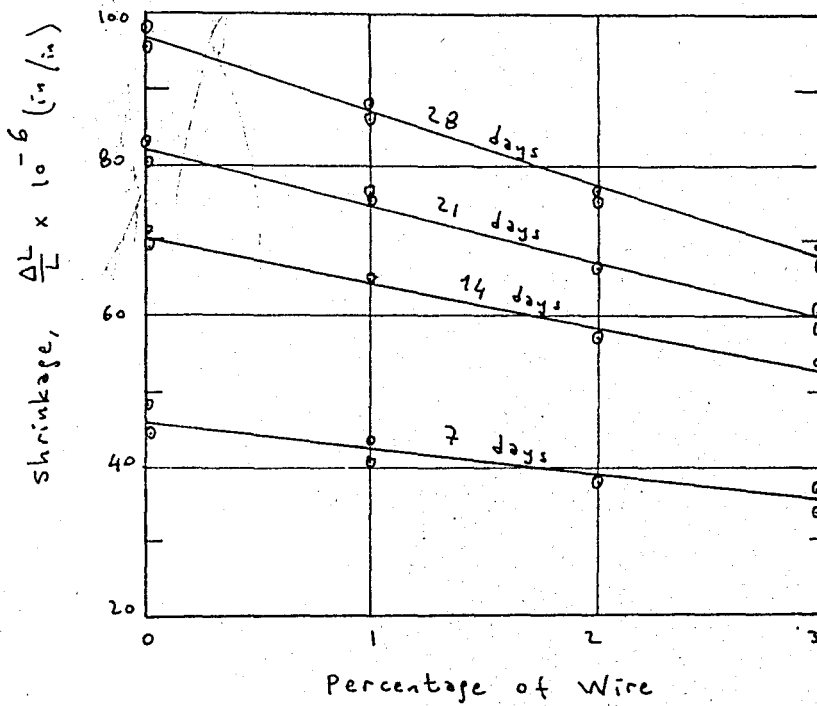


Figure 16-Variation of Shrinkage with Wire Percentage

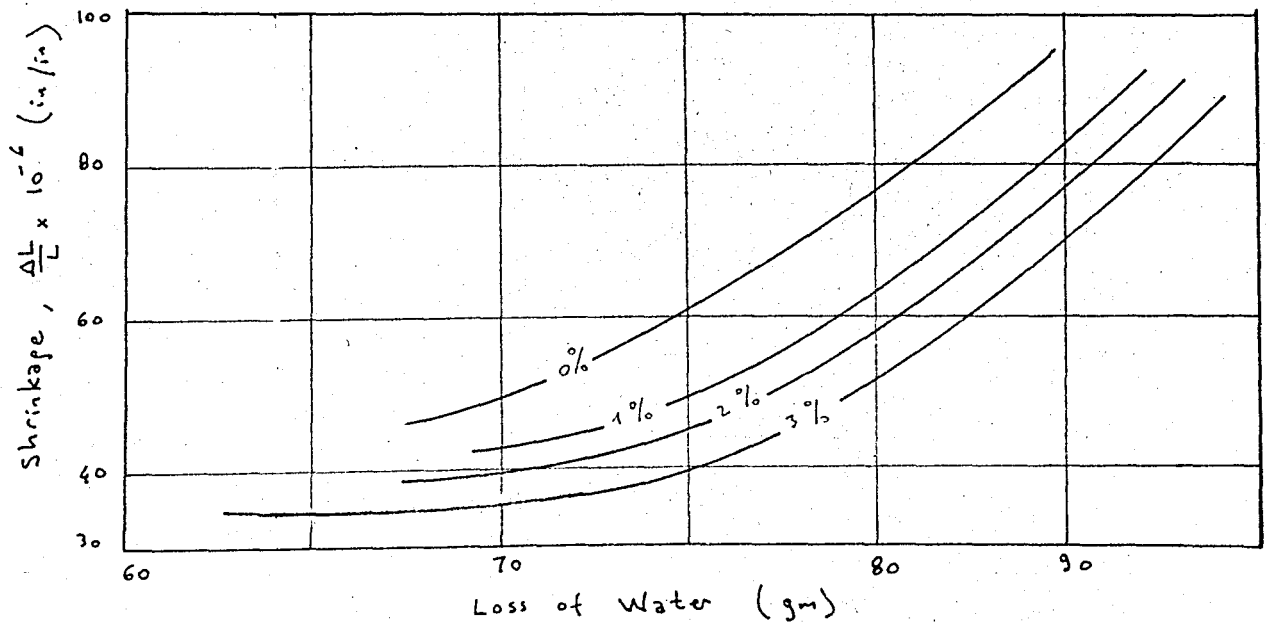


Figure 17-Shrinkage as a Function of Loss of Water

| | INDEX ^(x) | | | |
|----------------------|----------------------|------|------|------|
| Percentage of Wire | 0 | 1 | 2 | 3 |
| Compressive Strength | 1 | 2.48 | 3.86 | 4.95 |
| Tensile Strength | 1 | 2.16 | 3.26 | 4.25 |
| Amount of Shrinkage | 1 | 2.68 | 3.90 | 4.90 |

Table 6- Index Values for Various Properties of Wire Reinforced Concrete

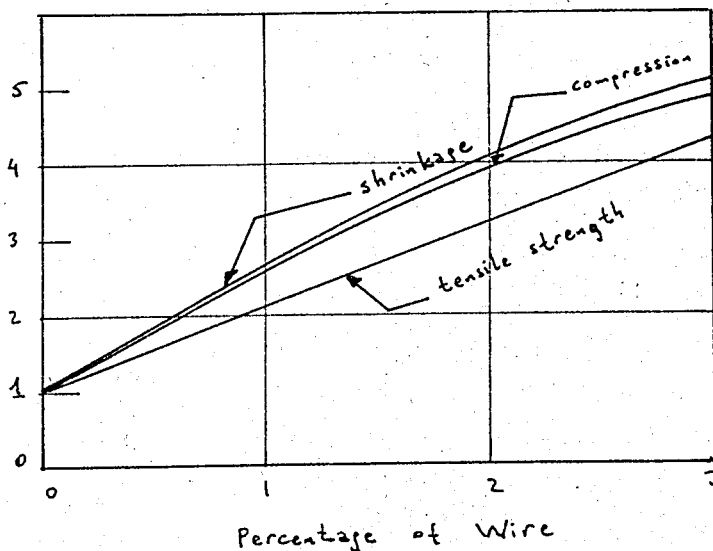


Figure 18-Index Values Plotted Against Percentage of Wire Reinforcement

(x) Cost-Strength Ratio of Plain Concrete is taken as the base.

CHAPTER V

DISCUSSION OF RESULTS

In the present work, the effect of including short, randomly distributed, closely spaced wire reinforcement on properties of concrete have been investigated. They are :

- a) Tensile strength
- b) Compressive strength
- c) Shear strength
- d) Strain at maximum load
- e) Strain energy at maximum load
- f) Static and dynamic moduli of elasticity
- g) Workability and density
- h) Shrinkage
- i) Economy.

The results have been presented in previous chapters.

In this chapter, we will discuss these results one by one and try to draw conclusions concerning the applicability of such reinforcement.

a) TENSILE STRENGTH

Split-cylinder test results summarized in table 1 show a definite increase in tensile strength of concrete with the addition of randomly distributed short wires and this follows a certain pattern which is a function of wire spacing (Figure 9), in agreement with the theory stated at the beginning. A question may arise whether the test method employed, that is, split-cylinder method, represents the true failure criteria. It is true that tensile strengths obtained from splitting tests are higher than the real values but it still serves our purpose, as it gives a comparative scale.

Figure 8 indicates the rate of change of tensile strength with the addition of wires. It is seen that rate of increase in strength being high at the beginning, it flattens out after 2 percent and reaches, more or less, to an optimum at 3 percent wire reinforcement. Tensile strength at this percentage is increased to more than 1.5 times as compared to plain concrete.

b) COMPRESSIVE STRENGTH

Table 3 shows the effect of wires on compressive strength of concrete. Increase in compressive strength is not as high as increase in tensile strength but it goes up to 141 percent with a wire of 3 percent. Looking at figure 11, we can conclude that increase in strength is high, up to 2 percent, after which, it slows down. From the pattern that the curve follows, we can conclude that an optimum is reached after 3 percent.

c) SHEAR STRENGTH

Shear strength is determined by employing Mohr's envelope (figure 13). Figure 8 indicates that shear strength shows a similar behaviour like tensile strength. That is, 50 percent increase in strength is attained with 2 percent reinforcement, after which, gain is negligible compared to steel used.

d) STRAIN AT MAXIMUM LOAD

Observation of figure 12 shows that there is a straight-line relationship between amount of wire reinforcement and strain at maximum load. Strain decreases with increase in steel percentage. This indicates that ductility of concrete decreases with addition of wires.

e) STRAIN ENERGY AT MAXIMUM LOAD

Strain energy is calculated by measuring the area under the stress-strain curve. The behavior of strain energy at maximum load with change in steel percentage is given in Fig. 12.

There is a rapid increase in strain energy with the addition of 1 percent wire after which, it does not change much. This shows that toughness of concrete is maximum with 1 percent of wire reinforcement, that is, more energy is absorbed before failure starts.

f) STATIC AND DYNAMIC MODULI OF ELASTICITY

Static modulus of elasticity is also affected by wires. Effect being negligible for low percentage of steel, static elastic modulus of concrete with 3 percent reinforcement is 1.3 times as much as that of plain concrete. This explains the reason why ductility and toughness are reduced with higher percentages, as stated in previous paragraphs.

Steel wires also increase the dynamic elastic modulus of concrete. The change is small at low steel percentages, increases rapidly after 1 percent and reaching to an optimum between 2 and 3 percent, begins to decline (figure 14). The decrease in dynamic modulus of elasticity for high steel percentages can be explained by decrease in workability and density of concrete which will be discussed in the following section.

However, behaviour of dynamic elastic modulus with time is rather erratic for various percentages of steel (figure 15). In plain concrete, dynamic modulus increases for the first two weeks after which, it decreases gradually, reaching to its initial seventh day value. For the samples with 1 and 2 percent reinforcements, there is a gradual decrease in dynamic modulus with time. The case is more complicated for samples of 3 percent reinforcement since there is no any definite pattern other than fluctuations. These may be due to stress concentrations present in the concrete mass caused by resistance of wires against shrinkage.

g) WORKABILITY AND DENSITY

Although the plain concrete mix was quite workable, it was observed during mixing that, addition of wires reduced workability to a great extent and even with 3 percent of wires, mixing was

quite difficult. Density measurements of fresh concrete also reflect this. Unit weight of concrete with 3 percent reinforcement was 138 lbs/cu.ft. whereas, it was 144 lbs/cu.ft. for low percentages of steel (weight and volume of steel are excluded for the above calculations).

h) SHRINKAGE

Measurements done on length changes reveal the effect of wires on the shrinkage of concrete. As stated before, first readings were taken after a water curing period of 7 days. The effect of wires is small during the first week of drying but it becomes effective afterwards. Figure 16 shows that magnitude of shrinkage is decreased considerably with the addition of wires and this is more defined for longer periods of drying.

i) ECONOMY

Feasibility of the method is investigated by making a cost analysis and finding out an index for each physical property of the concrete taking plain concrete as the base. Table 6 summarizes the results of calculations given in Appendix, part c. As seen from figure 18, it is not possible to find a steel percentage which will give a feasible solution with the current prices of steel wire. However, this can be attained by using some suitable industrial waste as reinforcement, in which case the cost will be decreased.

CHAPTER VI

CONCLUSION

Test results indicate that the two-phase property of wire reinforced concrete can be utilized in a large engineering field. Crack controlling characteristics suggest applications in flexible pavement design, construction of structures which are subjected to chemical attack from the surroundings, such as marine constructions and those exposed to sudden, high temperature changes. Increase in shear is advantageous for beams reducing shear reinforcement necessary. Due to its low shrinkage, this material can be used in highway and airfield constructions, where width of shrinkage joints is an important problem.

CHAPTER VII

SUGGESTIONS

The results of this thesis work have revealed that the material at hand has improved properties as compared to normal concrete, which make it convenient for a wide range of structural applications. There are still some questions to be answered. The resistance of wire reinforced concrete to impact loading, repeated loading, freezing and thawing should be investigated. Theoretical analysis discussed at the beginning suggests that these characteristics can also be improved by wire reinforcement.

THESIS

ROBERT COLLEGE GRADUATE SCHOOL
BEBEK, ISTANBUL

PAGE

A P P E N D I X

a) STATIC MODULUS OF ELASTICITY:

Plain:

| σ (psi) | $\epsilon \times 10^{-4}$ (in/in) | $\sigma_i \cdot \epsilon_i \times 10^{-4}$ | $\epsilon_i^2 \times 10^{-8}$ |
|-------------------|--------------------------------------|--|-------------------------------|
| 288 | 2.50 | 720 | 6.25 |
| 471 | 3.75 | 1770 | 14.10 |
| 630 | 5.00 | 3150 | 25.00 |
| 805 | 6.25 | 5040 | 39.00 |
| 955 | 7.50 | 7010 | 56.50 |
| 1050 | 8.25 | 8500 | 68.00 |
| 1290 | 10.00 | 12900 | 100.00 |
| 1460 | 11.25 | 16400 | 126.00 |
| Σ 6909 | Σ 54.50 | Σ 55490 | Σ 434.85 |

$$E = \frac{\sum_{i=1}^n (\sigma_i \cdot \epsilon_i) - n \cdot \bar{\sigma} \cdot \bar{\epsilon}}{\sum_{i=1}^n (\epsilon_i)^2 - n (\bar{\epsilon})^2} \quad (14)$$

$$\bar{\sigma} = \frac{\sum \sigma_i}{n} \quad \bar{\epsilon} = \frac{\sum \epsilon_i}{n}$$

$$\bar{\sigma} = \frac{6909}{8} = 865$$

$$\bar{\epsilon} = \frac{54.50 \times 10^{-4}}{8} = 6.80 \times 10^{-6}$$

$$E = \frac{5.55 - 8(865)(6.80) \times 10^{-4}}{434.85 \times 10^{-8} - 8(6.80)^2 \times 10^{-8}}$$

$$E = 1.3 \times 10^6 \text{ psi}$$

1 % Wire

| σ (psi) | $\epsilon \times 10^{-4}$ (in/in) | $\sigma_i \cdot \epsilon_i \times 10^{-4}$ | $\epsilon_i^2 \times 10^{-8}$ |
|-------------------|--------------------------------------|--|-------------------------------|
| 398 | 2.50 | 995 | 6.25 |
| 582 | 4.75 | 2760 | 22.60 |
| 760 | 6.25 | 4750 | 39.00 |
| 910 | 7.55 | 6860 | 57.00 |
| 1220 | 9.75 | 11900 | 95.00 |
| 1480 | 11.50 | 17000 | 132.00 |
| 1720 | 13.50 | 23200 | 182.00 |
| 1890 | 14.75 | 27900 | 218.00 |
| Σ 3950 | Σ 70.55 | Σ 95365 | Σ 751.85 |

$$\bar{\sigma} = \frac{8960}{8} = 1120$$

$$\bar{\epsilon} = \frac{70.55 \times 10^{-4}}{8} = 8.81 \times 10^{-6}$$

$$E = \frac{9.54 - 8(1120)(8.81) \times 10^{-4}}{751.85 - 8(8.81)^2 \times 10^{-4}}$$

$$E = 1.29 \times 10^6 \text{ psi}$$

2 % Wire

| σ (psi) | $\epsilon \times 10^{-4}$ (in/in) | $\sigma_i \cdot \epsilon_i \times 10^{-4}$ | $\epsilon_i^2 \times 10^{-8}$ |
|-------------------|--------------------------------------|--|-------------------------------|
| 300 | 2.25 | 675 | 5.10 |
| 625 | 5.00 | 3110 | 25.00 |
| 1000 | 7.75 | 7750 | 60.00 |
| 1320 | 9.75 | 12900 | 95.00 |
| 1650 | 12.50 | 20600 | 156.00 |
| 2300 | 16.00 | 36800 | 256.00 |
| 2650 | 19.75 | 52400 | 390.00 |
| 3050 | 22.50 | 68500 | 493.00 |
| Σ 12895 | Σ 95.50 | Σ 202835 | Σ 1480.10 |

$$\bar{\sigma} = \frac{12895}{8} = 1610$$

$$\bar{\epsilon} = \frac{95.50 \times 10^{-4}}{8} = 11.9 \times 10^{-4}$$

$$E = \frac{20.28 - 8(1610)(11.9) \times 10^{-4}}{1480. \times 10^{-8} - 8(11.9)^2 \times 10^{-8}}$$

$$E = 1.43 \times 10^6 \text{ psi}$$

3 % Wire

| σ (psi) | $\epsilon \times 10^{-4}$ (in/in) | $\sigma_i \cdot \epsilon_i \times 10^{-4}$ | $\epsilon_i^2 \times 10^{-8}$ |
|-------------------|--------------------------------------|--|-------------------------------|
| 454 | 3.00 | 1360 | 9.00 |
| 588 | 3.75 | 2210 | 14.10 |
| 845 | 5.50 | 4650 | 30.40 |
| 1130 | 7.50 | 8500 | 56.10 |
| 1320 | 8.50 | 11200 | 72.10 |
| 1480 | 9.50 | 14100 | 90.20 |
| 1600 | 10.00 | 16000 | 100.00 |
| 1770 | 11.00 | 19500 | 121.00 |
| Σ 9187 | Σ 58.75 | Σ 77520 | Σ 492.90 |

$$\bar{\sigma} = \frac{9187}{8} = 1146$$

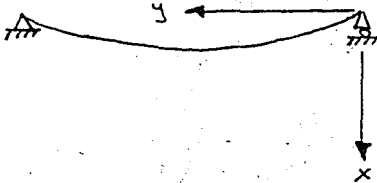
$$\bar{\epsilon} = \frac{58.75 \times 10^{-4}}{8} = 7.35 \times 10^{-4}$$

$$E = \frac{7.75 - 8(1146)(7.35) \times 10^{-4}}{492.9 - 8(7.35)^2 \times 10^{-8}}$$

$$E = 1.68 \times 10^6 \text{ psi}$$

b) DYNAMIC MODULUS OF ELASTICITY

From strenght of materials, deflection of a beam subjected to a moment is given by the differential equation



$$M = EI \frac{d^2 x}{dy^2} \quad (22)$$

$$V = \frac{dM}{dy} \quad (23)$$

where M : bending moment

V : shearing force

I : moment of inertia

x : deflection at the cross-section

y : distance from origin to the cross-section

If we assume a sinusoidal deflection along the beam

$$x = x_0 \cos \omega t \quad (24)$$

$$a = \frac{d^2 x}{dt^2} = -\omega^2 x_0 \quad (25)$$

Assuming mass is uniformly distributed, being m per unit length, the inertia force dF of element dy is given by

$$dF = m dy \omega^2 x_0 \quad (26)$$

$$\frac{dF}{dy} = m \omega^2 x_0 \quad (27)$$

$\frac{dF}{dy}$ is equal to the variation of shearing stress along the beam. Therefore :

$$\frac{dF}{dy} = \frac{dV}{dy} = \frac{d^2 M}{dy^2} = EI \frac{d^4 x_0}{dy^4} \quad (28)$$

from which it follows that

$$\frac{d^4 x_0}{dy^4} - \frac{m \omega^2}{EI} x_0 = 0$$

The general solution is given by G.W. Santen (12) as

$$x_0 = A_1 \sinh py + A_2 \cosh py + A_3 \sin py + A_4 \cos py \quad (29)$$

where

$$P = \sqrt[4]{\frac{mw^2}{EI}}$$

Constants can be determined from the boundary conditions.

For a simply supported beam of length l :

$$\begin{aligned} x_0 = 0, \quad M = 0 \quad \text{at} \quad y = 0 \\ x_0 = 0, \quad M = 0 \quad \text{at} \quad y = l \end{aligned}$$

and natural frequencies are

$$v_0 = \frac{n^2 \pi}{2} \sqrt{\frac{EI}{ml^4}} \quad (30)$$

where $n = 1, 2, 3, \dots$

However, this equation does not give reliable answers and some corrections should be made for torsional vibrations, etc.

The following formula takes care of the necessary correction

(13)

$$E = CW/v_0^2 \quad (31)$$

where W : weight

v_0 : natural frequency

$$C = \frac{4\pi^2 L^3}{g I m^4}$$

where L : length

g : gravitational constant

m : a constant which is equal to 4.730

C is corrected as $C = C' T$ where

$$T = 1 + 81.79 \left(\frac{r}{l}\right)^2 - \frac{1314 \left(\frac{r}{l}\right)^4}{1 + 81.09 \left(\frac{r}{l}\right)^2} - 125 \left(\frac{r}{l}\right)^4$$

for $\mu = 1/6$ and

$$C' = \frac{4\pi^2 L^3}{g I m^4}$$

c) COST ANALYSIS OF CONCRETE

(The prices are taken from the Bill of Quantities, published by the Ministry of Public Works.)

| | | |
|----|--|----------------------------|
| 1) | Cost of plain concrete : | 117.81 T.L./m ³ |
| | (including cost of materials, use of machinery and workmanship) | 117.81 T.L. |
| 2) | Concrete with 1 % reinforcement : | |
| | Cost of plain concrete : | 117.81 T.L. |
| | Cost of wires 10 x 7.88 x 3.00 | 236.00 T.L. |
| | | 353.81 T.L. |
| 3) | Concrete with 2 % reinforcement : | |
| | Cost of plain concrete : | 117.81 T.L. |
| | Cost of wires 20 x 7.88 x 3.00 | 472.00 T.L. |
| | | 589.81 T.L. |
| 4) | Concrete with 3 % reinforcement : | |
| | Cost of plain concrete : | 117.81 T.L. |
| | Cost of wires 30 x 7.88 x 3.00 | 708.00 T.L. |
| | | 825.81 T.L. |

$$I_n = \frac{C_n / C_0}{S_n / S_0} = \frac{S_0 / C_0}{S_n / C_n}$$

where I_n : index of concrete with a steel percentage of n.
 S_n : strength of concrete
 C_n : cost of concrete
 S_0 and C_0 : strength and cost of plain concrete.

Calculation of Index Values:

1) Compressive Strength:

$$\begin{aligned}
 \text{1\% Wire : } & \frac{S_0/C_0}{S_1/C_1} = \frac{2910/117.81}{3520/353.81} = 2.48 \\
 \text{2\% Wire : } & \frac{S_0/C_0}{S_2/C_2} = \frac{2910/117.81}{3740/589.91} = 3.86 \\
 \text{3\% Wire : } & \frac{S_0/C_0}{S_3/C_3} = \frac{2910/117.81}{4120/825.81} = 4.95
 \end{aligned}$$

2) Tensile Strength:

$$\begin{aligned}
 \text{1\% Wire : } & \frac{S_0/C_0}{S_1/C_1} = \frac{358/117.81}{500/353.81} = 2.16 \\
 \text{2\% Wire : } & \frac{S_0/C_0}{S_2/C_2} = \frac{358/117.81}{530/589.91} = 3.38 \\
 \text{3\% Wire : } & \frac{S_0/C_0}{S_3/C_3} = \frac{358/117.81}{580/825.81} = 4.25
 \end{aligned}$$

3) Shrinkage:

$$\begin{aligned}
 \text{1\% Wire : } & \frac{S_1 C_1}{S_0 C_0} = \frac{87 \times 353.81}{96.5 \times 117.81} = 2.68 \\
 \text{2\% Wire : } & \frac{S_2 C_2}{S_0 C_0} = \frac{76.5 \times 589.91}{96.5 \times 117.81} = 3.90 \\
 \text{3\% Wire : } & \frac{S_3 C_3}{S_0 C_0} = \frac{68 \times 825.81}{96.5 \times 117.81} = 4.90
 \end{aligned}$$

d) DETERMINATION OF WIRE SPACING

The average spacing of wires in the concrete mass for different wire percentages was calculated according to the equation(20) given in chapter III.

1% Wire :

$$S = \frac{13.8 \times 0.050}{2.54} \sqrt{\frac{1}{1}} = 0.272 \text{ inches}$$

2% Wire :

$$S = \frac{13.8 \times 0.050}{2.54} \sqrt{\frac{1}{2}} = 0.194 \text{ inches}$$

3% Wire :

$$S = \frac{13.8 \times 0.050}{2.54} \sqrt{\frac{1}{3}} = 0.157 \text{ inches}$$

BIBLIOGRAPHY :

1. Slayter, G., Two Phase Materials, Scientific American, V. 206, No. 1, Jan. 1962.
2. Romualdi, J.P. and Batson, G.B., The Mechanics of Crack Arrest in Concrete, ASCE Proceedings, Journal of the Engineering Mechanics Division, V. 89, EM3, June 1963, pp. 147-168
3. Romualdi, J.P. and Mandel, J.A., Tensile Strength of Concrete Affected by Uniformly Distributed and Closely Spaced Short Lengths of Wire Reinforcement, ACI Journal, Proceedings, V. 61, June 1964, pp. 657-670.
4. Griffith, A.A., The Phenomena of Rupture and Flow in Solids, Phil. Trans., Royal Soc. of London, V. 221, pp. 163-198.
5. Irwin, G.R., Analysis of Stress and Strains Near End of a Crack Traversing a Plate, Trans. ASME, V. 79, 1957, pp. 361-65.
6. Sneddon, I.N., The Distribution of Stress in the Neighborhood of a Crack in an Elastic Solid, Proceedings, Royal Society of London, Series A, V. 187, 1946, p. 229.
7. Kaplan, M.F., Crack Propagation and the Fracture of Concrete, ACI Journal, November 1961.
8. Romualdi, J.P. and Batson, G.B., The Behaviour of Reinforced Concrete Beams with Closely Spaced Reinforcement, ACI Journal, Proceedings, V.60, No. 6, June 1963, pp. 775-789.
9. ACI Committee 613, Recommended Practice for Selecting Proportions for Concrete, ACI Journal, Proceedings, V. 51, No. 1, Sept. 1954, pp. 49-64.
10. Troxell, C.E. and Davis, H.E., Composition and Properties of Concrete, c. 1956.
11. 1965 Book of ASTM Standards, Part 10.
12. Santen, G.W., Introduction to a Study of Mechanical Vibration, Philips Technical Library, 1958.
13. Electro Products Laboratories, Incorporated, Electro Sonometer, Chicago 40, Illinois.

14. Richmond, S.B., Statistical Analysis, The Ronald Press Company, New York, 1964, pp. 430-432.