RISK QUANTIFICATION OF MIXED PORTFOLIOS CONTAINING BONDS AND STOCKS

by

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ABSTRACT

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In this study, we design a method to quantify the expected return and risk of mixed portfolios containing bonds and stocks using the historical data and expert opinion about the future changes in the market. Vasicek model was the model we used as the short-rate model. Short-rate models are single factor models that give the possibility to price bonds with different time-to-maturities with a single random variable, the short-rate. We implemented the expert's expectation of future short-rate into Vasicek model to simulate future short-rates and developed a parameter calibration to obtain a yield curve closer to the current market yield curve. We used the lognormal multinormal model to simulate the future log-returns of the stocks. The design of a model for bonds and stock in a single portfolio requires information on the dependence between short-rate changes and bond returns. We calculated the empirical correlation between short-rate and stock returns for a large number of American stocks. The results showed that the empirical correlations were all close to zero. So we assumed that the changes of the short-rate and of the stock prices are independent. It is therefore not difficult to simulate many possible future portfolio values and to represent them in a histogram. It is also easy to use those simulated values to calculate the mean return and the risk measures VaR and CVaR of the portfolio.

ÖZET

BONO VE HİSSE SENEDİ İÇEREN PORTFÖYLERİN RİSK ÖLÇÜMÜ

Bu çalışmada, tarihi verileri ve piyasadaki değişikliklere dair uzman tahminlerini kullanarak bono ve hisse senedi içeren portföylerin beklenen getiri ve risk ölçümünü yapan bir metot geliştirdik. Vasicek model, bizim kullandığımız kısa vade faiz oranı modelidir. Kısa vade faiz oranı modelleri, farklı vadelere sahip bonoların fiyatlamasını tek bir rassal değişken,kısa vade faiz oranı, kullanarak mümkün kıla tek faktörlü modellerdir. Kısa vade faiz oranı simulasyonu için gelecekteki kısa vade faiz oranı hakkındaki uzman beklentilerini Vasicek modeline eklemledik ve piyasadaki getiri eğrisine daha yakın bir getiri eğrisi elde etmek için bir parametre kalibrasyonu geliştirdik. Gelecekteki hisse senedi fiyatlarının getirilerini ve simule etmek için Logreturn multinormal modeli kullandık. Tek bir portföyde bono ve hisse senedi içeren bir model, kısa vade faiz oranı ile hisse senedi getirileri arasındaki bağımlılık bilgisi gerektirir. Çok sayıda Amerikan hisse senedi getirisi ile kısa vade faiz oranının deneysel korelasyonunu hesapladık. Sonuçlar gösterdi ki deneysel korelasyonların tamamı sıfıra yakın. Bu sebeple kısa vade faiz oranındaki değişiklikler ile hisse senedi fiyatlarındaki değişikliklerin birbirinden bağımsız olduğunu varsaydık. İşte bu yüzden çok sayıdaki gelecek portföy değerlerini simule etmek ve bunları bir histogramda göstermek zor değildir. Simulasyonla elde edilen bu değerleri, portföyün ortalama getiri ve risk ölçütleri Riske Maruz Değer ve Koşullu Riske Maruz Değer'i hesaplamak için kullanmak da kolaydır.

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1. INTRODUCTION

The economic and intelligent growth of the world in the last century created huge opportunities for financial products and entire financial markets. Today, financial markets have an unbelievable size. As of 2010, the stock market had a size of 55 trillion USD around the world. (The 51 stock exchange that are members of World Federation of Exchanges-WFE) [1]. The bond market is in a similar situation. As of 2009, the global bond market had an estimated size of 82.2 trillion USD which is dominated by 31.2 trillion USD (%37.9 of the global market) according to the Bank for International Settlements data [2]. These sizes of the financial instruments, stocks and bonds, shows that the investors and traders all around the world need expertise of risk quantification, portfolio management, portfolio optimizing etc.

A bond is a contract between the issuer (borrower) and the bondholder (lender). The issuer promises to pay the bondholder interest. In other words, the issuer of the bond will repay to the lender/investor the amount borrowed plus interest over a specified period of time (maturity) [3]. Stock is a representation of capital paid or invested into a business entity. A stock exchange is a form of exchange which provides services for stock brokers and traders to trade stocks, bonds, and other securities. There is a saying in finance markets: “Do not put all the eggs into one basket” This is the idea of portfolios. The term portfolio refers to any collection of financial assets such as stocks, bonds, and cash. Portfolios may be held by individual investors and/or managed by financial professionals, hedge funds, banks and other financial institutions. It is a generally accepted principle that a portfolio is designed according to the investor’s risk tolerance, time frame and investment objectives.

The aim of this thesis is to design a method that uses simple stochastic models and R-codes to quantify the expected return and risk of mixed portfolios containing bonds and stocks. This method needs to be simple, easy to understand and to apply. Thus it should offer a practical solution for risk quantification for individual or corporate investors.

The method we suggest for this aim uses the market data such as historical stock prices and bond rates and the features of the investment like the investment period, the assets included, the weights of these assets, initial market data and the expert opinion on future average short-rate and future stock returns. It uses simulation to calculate many possible final values of the investment. It then returns to the user the results of the simulations and calculations that visualize the distribution of final wealth of the investor at the end of a certain investment period. To give this information simply and clearly, we prefer to give the histogram of the final investment values. This method also calculates the risk of the investment using the Value-at-Risk (VaR) and Conditional Value-at-Risk (CVaR). Therefore, our approach can serve as a decision support for the investor by calculating all these chracteristics. It also gives information about the expected return of the investment.

The thesis is organized as follows: The investment possibilities and portfolios are defined in Chapter 2. Short-rate and Vasicek model, short-rate simulation and bond pricing algorithms are explained in Chapter 3. Chapter 4 describes the assessing of bond portfolio investments using the Vasicek model. In Chapter 5, for bond portfolios risk and mean return calculations are discussed. Moreover Chapter 6 includes the calculation of mean return and risk for mixed portfolios containing stocks and bonds. Chapter 7 explains all the steps of the method that is developed to quantify the risk of mixed portfolios and gives the results and comparison of some investment examples.

2. SINGLE PERIOD INVESTMENT PROBLEM

Investment is the purchase of an asset or item with the hope that it will generate income or appreciate in the future and be sold at the higher price [7]. Companies or individuals desire to invest their savings or the cash surplus into instruments which are numerously available in the market. These investments contain both the hope or expectation of gaining more than invested wealth and also fear (possibility) of loosing some of the invested wealth. These two feature of investment are also called expected return and risk. Any investor would like find investments that promise higher return and carry small risk. So method that calculates expected return and risk for investments are important for investors to decide which investments to select. In our study, we try to guide the finance department of a company or an individual –we will call her *investor* later- who desires to carefully select her investment. Classically for any investment there is a trade-off between the risk taken and the return expected. We assume the investor does not like to take much risk for some reason, she needs the amount of money that provides the cash flow at the end of the investment period but she naturally likes to have a higher return. Speaking of the investment period, we assume that the investment period is short means 1 month to 1 year at most and the investor makes only one investment decision at the start of the period. This is practically rational as the transaction costs are high.

2.1. Investment Possibilities

Since we assume that the risk appetite of the investor is quite low, she desires mainly to invest into conservative assets such as bonds and perhaps small amount of stocks and not into any derivatives. The main asset of the investment is bonds just because of the lower risk. The investor has no chance to gamble with her money by investing it into risky assets. But still to increase the income without a dramatic increase in the risk, she can also invest into stocks. We assume that there are 1 months to 10 years to maturity government bonds and stocks available in the market. The investment is a single period investment, we assume that there will not be a chance to close a position and open a new one (reinvesting) during this period. For the sake of simplicity, the investment horizon fix to 6 months. The risk-free investment is defined as investing the entire fund into the 6 months bond.

2.2. Our Investment Problem

In this thesis, we make the experiments and calculations for only the US Government bonds which has the time to maturity 6 months, 1,2,3,5,7 and 10 years and the stocks quoted in S& P500 and the indices. The methods used and developed in this study can be also applied to any country's market that the historical data available. Please note that the bonds and stocks should be from the same country in order to both assets can be valued in the same currency without the foreign exchange rates.

We have chosen some investment examples and make the calculations on them, these are levelled from risk-free to moderate risk. Suppose that the investor has \$10000 and she wants to make a 6 month investment for a reason like she has a cash outflow 6 month later. The amount is chosen \$10000 to simplify the results and comments, that does not mean that the target investors are such small ones, the target is the middle class investor that has the investment opportunity \$500K to \$5M. The standard investment examples we consider are the following

Standard Investment a: 10000 into 6 month bonds (Risk-Free Investment).

Standard Investment b: \$5000 into 6 month and \$5000 into 2 year bond).

Standard Investment c: \$5000 and \$5000 into 6 month and 5 year bonds).

Standard Investment d: $5000 into 6 month and $3000 into 2 year bonds and $2000 into S& P500 index.

Standard Investment e: $4000 into 6 month and $3000 into 2 year bonds, $2000 into S& P500 index and $1000 into PBI.

2.3. Uncertainty of Investment

Risk is defined in most references as *"the potential that an action or activity (including the choice of inaction) will lead to a loss"*. For financial risk, there can be a definition as "the quantifiable likelihood of loss or less-than-expected returns" [4].

The best known type of risk in financial risk management is probably market risk, the risk of a change in the value of a financial position due to changes in the value of the underlying components on which that position depends, such as stock prices, exchange rates, commodity prices, etc. The next important risk category is interest rate risk that an investment's value will change due to a change in the absolute level of interest rates, in the spread between two rates, in the shape of the yield curve or in any other interest rate relationship. An other category is credit risk, the risk of not receiving promised repayments on outstanding investments such as loans and bonds, because of the "default" of the borrower. A further risk category that has received a lot of recent attention is operational risk, the risk of losses resulting from inadequate or failed internal processes, people and systems, or from external events [11]. In this study, only the market risk (for stocks) and the interest rate risk (for bonds) are considered.

Government bonds are commonly considered investments as without default risk and the investors will get their principle investment and interest at maturity. This is not the only way to convert a bond into cash. The investor can sell her bonds before maturity with a discounted market price in the secondary bond market. If the interest rates increase, the price of the bonds decrease and vice versa. That is why the investor has the "interest rate risk" even if there is no default risk.

More literally, if $P(t,T)$ denotes the price of a zero-coupon bond at time t with maturity T, the initial price of that bond at time $t=0$ is $P(0,T)$ and the loss a time $t>0$ will be,

$$Loss(t) = P(0.T - P(t,T) \tag{2.1}$$

This formula (2.1) explains that the investor invests $P(0,T)$ amount of money to buy the bond and by selling the bond convert it in to cash for $P(t,T)$ amount of money at time t. Consequently, if the interest rate level in the market is higher at time t than at time 0, this leads to some loss of the investor. To avoid misunderstanding note that, the investor will be definitely paid of the par-value of the bond independent from the interest rates changes if she waits till the maturity.

These sources of risk can be modelled and these models can be used to calculate the possible final values and risks of the investments. In later parts of this thesis, the Vasicek Short-Rate model will be used to quantify the interest rate risk and the multi-normal logreturn model will be used for quantifying the market risk.

2.4. Assessing Possible Outcomes of the Investment Expert Opinion

As we mentioned in the previous section, the investment includes uncertainty and there is no way to know what will exactly happen in the future. Still we can model the changes and calculate the possibilities of events (price levels or investment values) using stochastic methods. If we have no tool to do such calculations or no interest lose time such activities, there is still a way to make some prediction of an investment's value in the future. This can be done asking out their opinion on the future changes of the bond rates or the expected return of bonds. Although this is a primitive approach, it can help us to understand how the possible changes in the market affect the value of the investment. Suppose that the standard investment b (\$5000 into 6 month and

$5000 into 2 year bond)) is chosen to invest. We assume that today is 31.12.2009. Therefore the initial 6 month bond rate is 0.0094,the 1 year bond rate is 0.0134 and the 2 year bond rate is 0.0156. Expert opinions; one says the rates will remain the same, one says the rates will increase by a 0.004 in 6 months and become 0.0134 for 6 month bond, 0.0174 for 1 year bond and 0.0196 for 2 year bonds, and one expert says the rates will decrease by 0.002 and become 0.0074 for 6 month, 0.0114 for 1 year and 0.0136 for 2 year bonds. Using linear interpolation, the 1.5 year bond rate will be 0.0145, 0.0185 and 0.0125 for the expert opinions respectively. We also need these rates because the 2 year maturity bond will be 1.5 year maturity bond after 6 months from start of the investment.

The initial prices of the bonds are 0.995311 and 0.969287 respectively. Note that the par values of the bonds are 1. With the initial invested capital, the quantities that can be bought of bonds have a par value of 1 are 5023.555 and 5158.459. Let us see the final value of the investment for a 6 month period for these 3 different expert opinion. Since the 6 month bond price will be equal to its par value, the final wealth will be equal to $10071.03 = (5023.555*1+5158.459*0.9784848) if we assume that the first guess is correct. The guess that suppose the rates will increase results in a final value of $10040.84= (5023.555*1+5158.459*0.9726315). And finally according to the last guess that the rates will decrease results in a final value of $10086.19= (5023.555*1+5158.459*0.9814247). After these calculations, if we assign some probabilities to each opinion, the expected return can also be calculated. Assume that the opinions have the probabilities %30, %55 and %15 respectively, the expected return will be $10056.70. Clearly this "ad hoc" method is not enough for a modern investor. This kind of approach can not qualified the risk even can not give a hint about risk. So stochastic models are required to attach probabilities to future changes then we may qualify the risk reasonably. The next chapters include these models and demonstrate their application.

3. SHORT RATE MODELS

The interest rate risk of bond investment which depends much on the time to maturity, can be quantified using bond formulas. The only unifying approach where we have a realistic model of the different influences the change of interest rate has on bonds with different maturity is, a stochastic model for the development of the interest rate. The easiest possible model is a model that considers the only short-rate. Therefore, in this study, the short-rate model is highly important to achieve the main goal of the study. The advantage of having such a model is that this model allows to quantify the risk of bonds with different maturity. One can calculate possible different prices with this model.

The short rate is not a rate that can be simply observed from the market because it is defined as the limit of the bond interest rate for maturity tending to zero [15]. The yield of the shortest maturity government bonds(with 1 or 3 months maturity) or the overnight interest rates of the central banks can be considered as an observable short rate. The short rate changes over time, therefore the short rate is denoted by r_t.

3.1. Bond Basics

A bond is a form of borrowing by using governments, municipalities or companies [11]. A bond is a financial claim by which the issuer, or the borrower, is committed to paying back to the bondholder, or the lender, the cash amount borrowed, called "principal", plus interest calculated on this amount during a given period of time (maturity) [12]. Although private companies mostly banks can issue bonds, in this study we will consider only government bonds. It's because of majority of the bond market all around the world is government bonds. Government bond contracts can be supposed as loans given to a government. Basically, governments borrow fund to finance their treasury and promise to repay this money plus an interest rate at a fixed time called maturity. Although every loan has a default risk, stable governments' bond are generally considered to have no default risk. There are some definition about bonds below [9]:

The Par-Value (Par amount, face value, Principle Value or Maturity Value) is the amount paid at maturity by the issuer to the bondholder. This amount contains both the borrowed money and interest.

Coupon Bonds are bonds which have coupon payments. These payment is the interest rate paid by the issuer to the bond holder. These payments are generally paid periodicly before maturity. In this study, only zero-coupon bonds are considered.

Zero-Coupon Bonds pay no coupons in other word no regular interest. There is only a single payment at maturity. This means that the bondholder receives the money invested and the interest at once at maturity. This is simply like a bank account that the investor invest some amount of money into a bank for a certain time and withdraws the invested money plus interest.

Yield (Yield to Maturity) is the interest rate earned by investing a bond. This is a general measure of return of the bond which includes time-to-maturity, market price of the bond and the par-value of the bond. The yield is also called internal rate of return.

Yield ($R(t,T)$) can be expressed as simply compounded constant interest rate from time t to maturity T. This is the rate that produces interest income in the time interval t-T and as the capital is bond price $P(t,T)$ and the total amount at the maturity is one unit. In other word, the rate makes the interest payments plus bond price equal to 1. The yield formula is the following:

$$L(t,T) = \frac{1 - P(t,T)}{(T-t)P(t,T)} \,.$$

Therefore the bond price is

$$P(t,T) = \frac{1}{1 + L(t,T)(T-t)}\,.$$

This also called ”yield in simple-compounding and some market including Borsa İstanbul (formerly Istanbul Stock Exchange) use this method for bond pricing.

Remember the definition; the yield is the rate equating the bond price plus interest income to one so the formula for this definition is [10]:

$$P(t,T)\, e^{R(t,T)(T-t)} = 1\,.$$

Elementary manipulations of that equation lead to the formula for the bond price

$$P(t,T) = 1\, e^{-R(t,T)(T-t)}\,,$$

and to the yield formula:

$$R(t,T) = \frac{\log 1 - \log P(t,T)}{(T-t)} = \frac{-\log P(t,T)}{(T-t)}\,.$$

The aim of giving these definitions and formulas is to clarify the relationship between yield and the bond price. The conclusion is this part, the bond price is the discounted par value by the yield.

3.2. Vasicek Model

The Vasicek Model that was introduced by Oldrich Vasicek in 1977 is a “one factor model” describing the changes of the interest rate. This model is based upon the idea of a mean reverting short-rate process [13].

Vasicek assumed that the instantaneous short rate, r, acquires under risk-neutral measure the following dynamics:

$$dr_t = \beta(\mu - r_t)dt + \sigma dW_t, r(0) = r_0 \tag{3.1}$$

where r_0, β, μ and σ are positive constants.

The parameters of the model are β (mean-reversion), μ (long-term mean) and σ (volatility of the short rate). dW_t denotes the standard Brownian motion. The idea of mean-reversion implies that in the long run all trajectories of r will evolve around a mean level μ. This can also be inferred from (3.1). Whenever the short rate is below μ the drift of the process, $\mu - r_t$, is positive; whenever the short rate is above μ the drift of the process, $\mu - r_t$, is negative. Thus in every case r_t is pushed towards the level μ.

3.3. Parameter Estimation

Simulation of the short rate and bond pricing in Vasicek Model requires the model parameters β (mean-reversion), μ (long-term mean) and σ (volatility of the short rate) which were introduced before. By the Maximum Likelihood Estimation method (MLE) historical data should be used to estimate the parameters. Closed-form maximum likelihood estimates for Vasicek model parameters are [10]:

$$\hat{\alpha} = \frac{n\sum_{i=1}^{n} r_i r_{i-1} - \sum_{i=1}^{n} r_i \sum_{i=1}^{n} r_{i-1}}{n\sum_{i=1}^{n} r_{i-1}^2 - (\sum_{i=1}^{n} r_{i-1})^2}$$

$$\hat{\mu} = \frac{\sum_{i}^{n}[r_i - \hat{\alpha} r_{i-1}]}{n(1-\hat{\alpha})}$$

$$\hat{V^2} = \frac{1}{n}\sum_{i=1}^{n}\left[r_i - \hat{\alpha} r_{i-1} - \hat{\mu}(1-\hat{\alpha})\right]^2$$

where dt is the time step between observed proxies $r_0, r_1, ..., r_n$ of r and,

$$\hat{\beta} = \frac{-\log(\hat{\alpha})}{dt}$$

$$\hat{\sigma^2} = \frac{2\beta\hat{V^2}}{(1 - e^{-2\hat{\beta}dt})}$$

The 3 months maturity US monthly yield data from 1982 to 2009 [14] are used to estimate the parameters of the Vasicek Model for the US Bond Market. The R-function that calculates the MLE of the Vasicek model parameters is given in Appendix A.1 . The calculated MLE's for the US data as in Figure 3.1:

```
Vasicek_MLE = Vasicek_MLE(data=dataBond$m03,dt = 1/258)
Vasicek_MLE
#          mu        beta      sigma
#[1] 0.03036066 0.19337252 0.01371011
```

Figure 3.1. R-Code for calculating the MLE of the Vasicek model for US data.

As seen, for the US data with 3 months time to maturity the parameter estimates are:

$$\hat{\mu} = 1.809735577, \hat{\beta} = 0.006711272, \hat{\sigma} = 0.0.270466131$$

It is possible that the Vasicek model parameters change in the time. The volatility of any financial data may increase in the time of uncertainty and decrease in the time of stability. This is true also for short rate data. If only last 5 year of data are considered, the estimated parameters change to Figure 3.2:

```
> Vasicek_MLE (data=dataBond$m03[5507:6710],dt = 1/258)
#         mu        beta       sigma
#[1] 0.02871737 0.29108787 0.01328808
```

Figure 3.2. R-Code for calculating the MLE of the Vasicek model for US data(2004-2009).

It can be seen in Figure 3.2, the parameters are not the same. The standard deviation is higher for the last five year which means the last five years was a more volatile.

3.4. Bond Pricing with Short-Rate Following the Vasicek Model

Bond pricing with short-rate models is based on the yield definition because this kind of pricing equals the bond price to the expected value of its maturity value discounted by the short-rate process. So the price of a zero coupon bond which has the unit maturity value is:

$$P(t,T) = E_t\left(\exp(-\int_t^T r_s\, ds)\right) . \tag{3.2}$$

Continuous-compounding is used and the expectation is taken over r_t; since, r is *random*. This formula is similar to the bond price formula with continuously-compounded yield. But as the short rate is random we have to take the expectation of the integral.

The Vasicek Model is a "Equilibrium Term Structure Model" and it does not lead to arbitrage free bond prices. In this kind of models the model bond prices differ from the market bond prices which means that such models can not be perfectly calibrated to the yield curve. Still it is possible to calculate the prices, and the model is useful in practise. The expectation of the integral formula given in 3.2 for the Vasicek model defined in 3.1 can be calculated in closed form. A zero-coupon bond price using the Vasicek short-rate model is:

$$P(t,T) = A(t,T)e^{-B(t,T)r_t}\,, \tag{3.3}$$

where

$$\begin{aligned} A(t,T) &= \exp\left\{\left(\mu - \frac{\sigma^2}{2\beta^2}\right)[B(t,T) - T + t] - \frac{\sigma^2}{4\beta}B(t,T)^2\right\} \\ B(t,T) &= \frac{1}{\beta}\left[1 - e^{-\beta(T-t)}\right]\,. \end{aligned}$$

With the formula above, it is clear that the only required information to calculate a bond price are the time to maturity, the short-rate and the parameters μ, β, σ. Using the formula above it is easy to calculate the yield $R(t,T)$ (continuously compounded yield) of the Vasicek model.

$$P(t,T) = e^{-R(t,T)(T-t)}$$

Pluging the bond price of the Vasicek model into the equation above gives;

$$A(t,T)e^{-B(t,T)r(t)} = e^{-R(t,T)(T-t)}$$

Simple algebra then leads to the yield function

$$R(t,T) = \frac{(rB - \log(A))}{(T-t)} \tag{3.4}$$

This is very important because the practical use of a short-rate model can be measured by comparing the yield curve that the model implies and the yield curve of the market. The calibration of Vasicek model to the market yield curve will be discussed in Section 3.6

The R-function that calculates the bond price is given in the Appendix A.2. By using that R-function, the bond prices can be calculated as shown in Figure 3.3:

```
para<- Vasicek_MLE (data=dataBond$m03[5507:6710],dt = 1/258)


VasicekPriceYield(r=0.0046,tau=1,para,priceyn=1)
#[1] 0.9931134 price of a 1 year maturity bond
VasicekPriceYield(r=0.0046,tau=2,para,priceyn=1)
#[1] 0.9823534 price of a 2 year maturity bond
VasicekPriceYield(r=0.0046,tau=5,para,priceyn=1)
#[1] 0.9349813 price of a 5 year maturity bond
VasicekPriceYield(r=0.0046,tau=10,para,priceyn=1)
#[1] 0.8349424  price of a 10 year maturity bond
```

Figure 3.3. R-Code for calculating the MLE of the Vasicek model for US data(2004-2009).

3.5. Short-Rate Simulation

3.5.1. Short-Rate Simulation Using MLE Parameters

For the Vasicek model the exact distribution of r_t is known for a given value of r_s. It is a normal distribution with the following parameters [10];

$$r_t \sim N\left(r_s\, e^{-\beta(t-s)} + \mu\left(1 - e^{-\beta(t-s)}\right), \frac{\sigma^2}{2\beta}\left[1 - e^{-2\beta(t-s)}\right]\right) \tag{3.5}$$

To simulate r_t at times $0 = t_0 < t_1 < \cdots < t_n$ exact, we can thus easily use the recursion:

$$r_{t_{i+1}} = e^{-\beta(t_{i+1}-t_i)} r_{t_i} + \mu\left(1 - e^{-\beta(t_{i+1}-t_i)}\right) + \sigma\sqrt{\frac{1}{2\beta}\left(1 - e^{-2\beta(t_{i+1}-t_i)}\right)}Z \tag{3.6}$$

where Z is a vector of iid. standard normal variates. Note that this simple simulation is the exact simulation of the process. The R-code that shows the simulation of the Vasicek process by using exact simulation is given in Appendix A.3.

3.5.2. Short-Rate Simulation Using Assumption on the Expectation of the Future Short-Rate

The short-rate simulation using the MLE parameters introduced in 3.3 uses only historical short-rate data as the MLE only depend on the historical short-rate data. The Vasicek model assumes that the short-rate will achieve the long-term mean (μ) in the long-run. This implies the assumption that the expectation of a future short-rate will be always higher than the current short-rate. For the standard case that the current yield curve is increasing. This expectation of the short-rate is often not realistic for the bond market. So in cases where the market believes the rates will decrease in the future the calibration to the current yield curve implies that expected short-rate is higher. For this reason, the idea of using the assumption on the expectation of the future short-rate, which is firstly introduced in this study, may help to over come this drawback of the Vasicek model. The expectation of future short-rate can be assumed by some expert on market or the investor herself.

This method changes the short-rate process a bit. The short rate process follows a normal distribution as below:

$$r_t \sim N\left(r_s\, e^{-\beta(t-s)} + \mu\left(1 - e^{-\beta(t-s)}\right), \frac{\sigma^2}{2\beta}\left[1 - e^{-2\beta(t-s)}\right]\right) \tag{3.7}$$

The mean of this distribution is

$$r_s\, e^{-\beta(t-s)} + \mu\left(1 - e^{-\beta(t-s)}\right) \tag{3.8}$$

this method suggests replacing this mean with the expert's prediction of short-rate. The predicted short-rate is denoted by r_p so the distribution of short-rate at time t is becomes below:

$$r_t \sim N\left(r_p, \frac{\sigma^2}{2\beta}\left[1 - e^{-2\beta(t-s)}\right]\right) \tag{3.9}$$

As the distribution of the short-rate is altered this way, the simulation algorithm becomes as the following:

r_t at times $0 = t_0 < t_1 < \cdots < t_n$

$$r_{t_{i+1}} = r_p + \sigma\sqrt{\frac{1}{2\beta}\,(1 - e^{-2\beta(t_{i+1}-t_i)})}Z_{i+1} \tag{3.10}$$

It is expected from this development, if the investor has an opinion of the future behaviour or she wants to make a what-if analysis, this algorithm will give more precise results when quantifying the risk and expected return of bond portfolios. If the inestor has no special knowledge it is also sensible to assume that the expectation of the short-rate is equal to the current short-rate. The R-code functioning the simulation of the short-rate using the assumption on the expectation of the future short-rate is given in Appendix A.4.

3.6. Parameter Calibration to the Market Yield Curve

The Vasicek Model has three parameter long-term mean (μ), mean-reversion (β) and standard deviation (σ). In this study the parameters were estimated by Maximum Likelihood Estimation using an R-function and these parameters effect the simulation of the short-rate, the price of the bond (value of the portfolio) and of course the yield curve. Therefore, choosing these parameters correctly has a great importance. The long-term mean (μ) is supposed to be a level that the short-rate tends to return to. In other words, the short-rate process is supposed to return to the long-term mean. The MLE is using only the historical short-rate data to estimate the parameters. Its quality can be measured by calculating the yield curve implied by the estimated parameters and compare it to the market yield curve. For the US Government Bond Market, data from 04.01.1982 to 31.12.2009 and for 3 and 6 months, 1,2,3,5,7 and 10 year yields is used as historical data. The 3 month bond rate is accepted as the short-rate. On the date 31.12.2009, the yields are 0.0046 0.0094, 0.0134, 0.0156, 0.0180, 0.0280, 0.0329, 0.0401 respectively and the short-rate is 0.0046. The MLE parameters are in Figure 3.4.

```
Vasicek_MLE = Vasicek_MLE(data=dataBond$m03,dt = 1/258)
Vasicek_MLE
#          mu       beta      sigma
#[1] 0.03036066 0.19337252 0.01371011
```

Figure 3.4. R-Code for calculating the MLE of the Vasicek model for US data.

As we calculate the yields for maturities 3 and 6 months and 1,2,3,5,7 and 10 years, the results of the R-Code are in Figure 3.5.

```
yield<-VasicekPriceYield(r=r,tau=tau, para=param,priceyn=0)
yield
#[1] 0.005210873 0.005798878 0.006910460 0.008902094 0.010627983
#[6]0.013445753 0.015619922 0.018039251
```

Figure 3.5. R-Code for calculating the yields for US data from 04.01.1982 to 31.12.2009.

It is useful to compare the market yield (real yield) and the calculated yield curve implied by the MLE to see the difference (Figure 3.6).

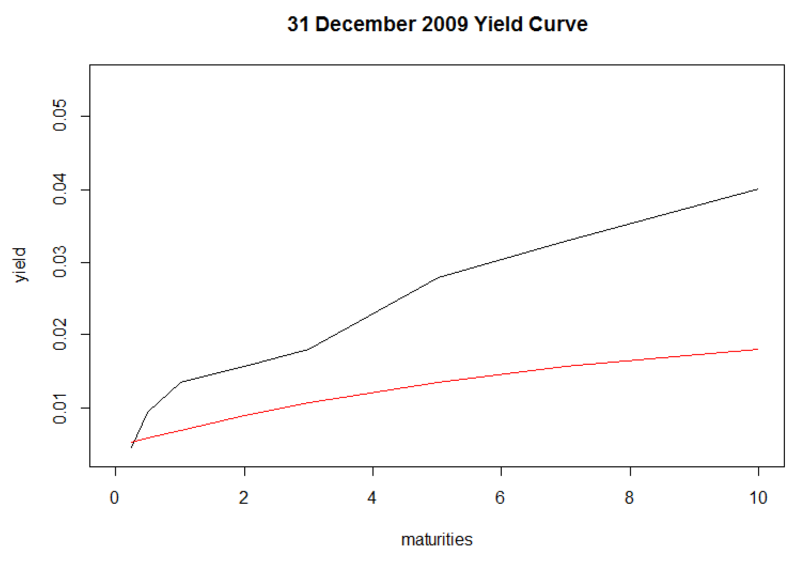

Figure 3.6. The plot of Market Yield Curve on 31.12.2009 and the Calculated Yield Curve by using Vasicek MLE parameters.

The calculated yield with the MLE parameters is quite different from the real one. The black line is the yield curve observed in the market and the red line is the calculated yield curve. This is a result of Vasicek Model's being an equilibrium short-rate model. This difference of the yield curves also indicates a big difference of the bond prices. There is a possibility to overcome this problem: Calibration of the Vasicek model parameters to the current yield curve.

The calibration can be simply done by changing only the long-term mean (μ)

and the mean reversion parameter (β). After a small number of trials, a much better fitting yield curve can be found. Note that the standard deviation is not changed to keep the volatility to the value observed from the short-rate data.

```
cal_param
[1] 0.05184900 0.32000000 0.01371011
cal_yield<-VasicekPriceYield(r=r,tau=tau, para=cal_param,priceyn=0)
cal_yield
[1] 0.0064387 0.0081791 0.0113892 0.0168709 0.0213301 0.0280032
[7] 0.0326131 0.0371748
```

Figure 3.7. R-Code for calculating the yields using the calibrated Vasicek model parameters.

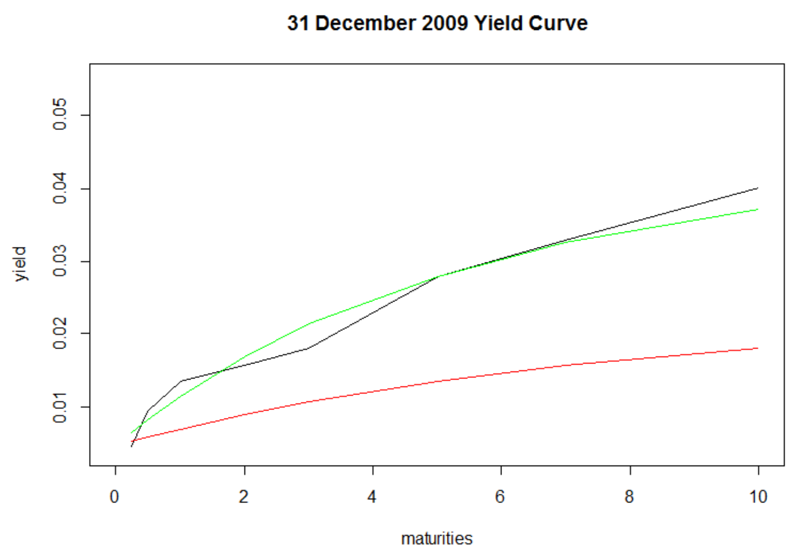

Figure 3.8. The plot of Market Yield Curve on 31.12.2009 and the Calculated Yield Curve by using calibrated and MLE Vasicek parameters.

In the Figure 3.8 the black line is the yield curve observed in the market, the red line is the yield curve calculated using the MLE parameters and the green line is the yield curve calculated by using the calibrated parameters. The calibration is clearly successful in obtaining a better fit of the calculated yield curve to the observed one. When trying this simple calibration method for different dates in the past, the results are similar.

The experiments were done for the 29.12.2006, 31.12.2003 and 31.12.1999. The plots of the observed yield curve of the date (black line) ,the calculated yield using the MLE parameters and the calculated yield using the calibrated parameters are shown.

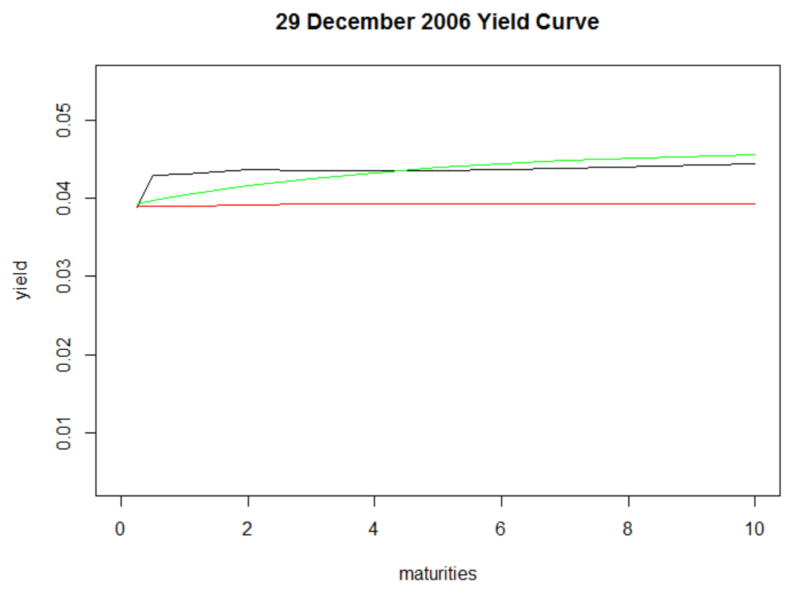

Figure 3.9. The plot of Market Yield Curve on 29.12.2006 and the Calculated Yield Curve by using calibrated and MLE Vasicek parameters.

As can be seen in Figure 3.9, 3.10 and 3.11 all experiments shows that this approach helps to find a better fitting yield curve. Thus the calculated bond prices are closer to the market price of the bonds than when using the MLE parameters.

The calibration of the Vasicek model parameters is used only for the bond price calculations. For the simulation of the short-rate, the MLE parameters σ and β are neccessary together with the assumed expected short-rate r_p. Note that the parameter μ is not necessary when simulating the future short-rate ,f we use the expected short-rate r_p.

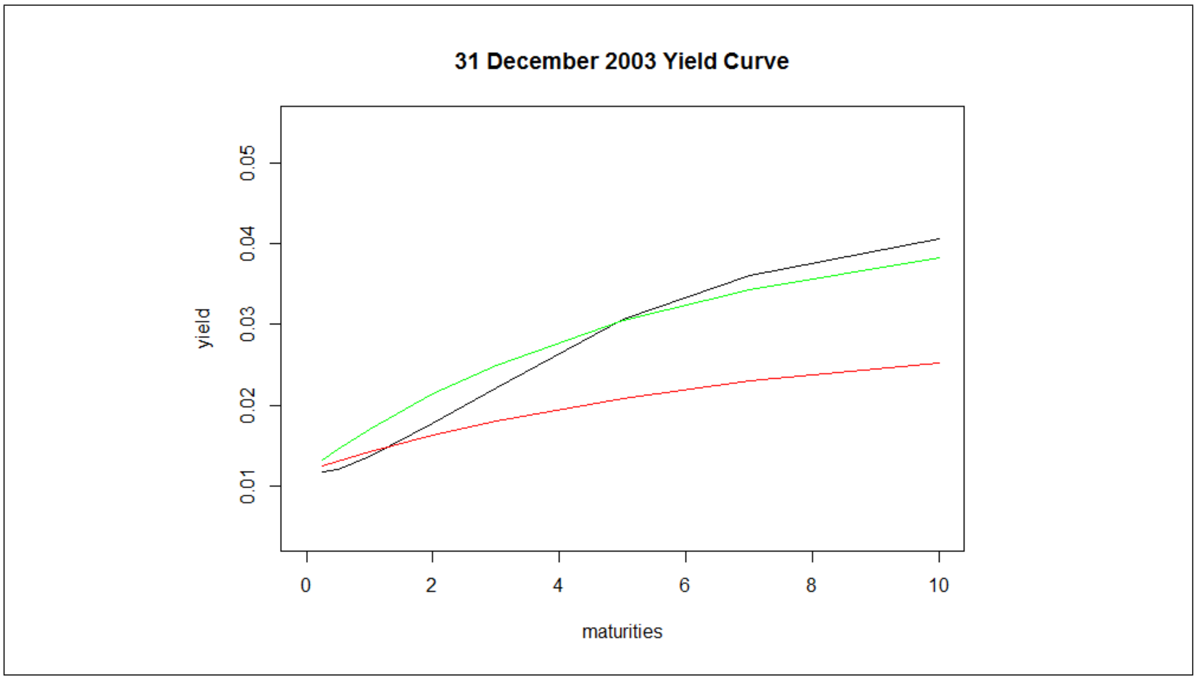

Figure 3.10. The plot of Market Yield Curve on 31.12.2003 and the Calculated Yield Curve by using calibrated and MLE Vasicek parameters.

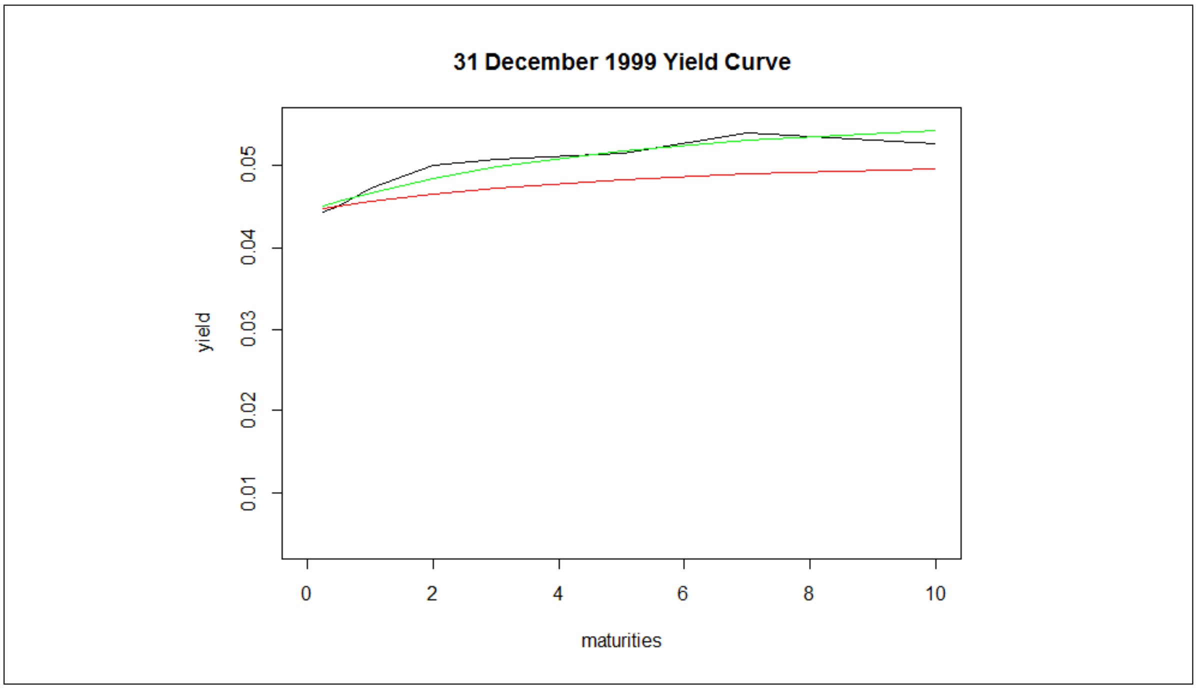

Figure 3.11. The plot of Market Yield Curve on 31.12.1999 and the Calculated Yield Curve by using calibrated and MLE Vasicek parameters.

4. ASSESSING BOND PORTFOLIO INVESTMENT USING THE VASICEK MODEL

The term portfolio refers to any collection of financial assets such as stocks, bonds, commodity and cash. Generally accepted principles about portfolios is that portfolios are designed according to the owner's risk tolerance, time frame and investment goals [6].

In the previous chapter, the Vasicek Model was introduced as short-rate model. The pricing formula of the previous chapter is for a single bond. In this chapter, the formula to simulate the final value of a bond portfolio using Vasicek Model is explained.

4.1. Short-Rate Simulation

The short-rate is the key element of the bond pricing algorithm. In the investment environment, the initial short-rate is known so the initial price of the bonds and the initial value of the bond portfolio can be calculated if the parameters of the Vasicek model are known. In this environment the short-rate at the end of the investment period can not be known. But simulation of possible future values may help the investor to understand the behaviour of the portfolio. As there is no way to know the future, a well designed simulation will be helpful to quantify the future possible returns and risks of a portfolio.

There are two possible ways to simulate the short-rate, First it is possible to use the MLE of Vasicek Model, second, as a new idea we have suggested to use some predicted short-rate values as the mean of the simulation process and the mean reversion parameter (β) and the standard deviation (σ) parameters are taken from the MLE.

4.1.1. Short-Rate Simulation Using MLE Parameters

This is the basic version of the short-rate simulation. Remember the short-rate process of the Vasicek Model:

$$dr_t = \beta(\mu - r_t)dt + \sigma dW_t, r(0) = r_0 \tag{4.1}$$

where $r_0, \mu, \beta and \sigma$ are constant.

The distribution of the short-rate and the simulation formula of the short-rate and the R-code functions belongs to these were given in the previous section. This approach can be used to simulate n different short-rate values. So with these short-rate values, n different bond prices can be calculated.

4.1.2. Short-Rate Simulation Using Assumption on the Expectation of the Future Short-Rate

As we mentioned before, there is no way to know the future but some expert predictions or the belief in the market or of the investor may be helpful to improve the simulation.

The idea is that, for a short period of time such as 6 months or at most one year, the expert on the bond market may have a guess and as the time period is quite short, this guess is assumed to have a high possibility to occur. This is sensible because instead of pure historical data, the algorithm can include some belief about the future. This method can also be used to generate different scenarios. For different predictions, the expected returns and risks can be qualified. This method changes the short-rate process a bit. The short rate process follows a normal distribution as below:

$$r_t \sim N\left(r_s\, e^{-\beta(t-s)} + \mu\left(1 - e^{-\beta(t-s)}\right), \frac{\sigma^2}{2\beta}\left[1 - e^{-2\beta(t-s)}\right]\right) \tag{4.2}$$

The mean of this distribution is

$$r_s\, e^{-\beta(t-s)} + \mu\left(1 - e^{-\beta(t-s)}\right) \tag{4.3}$$

this method suggests replacing this mean with the expert's prediction of short-rate. The predicted short-rate is denoted by r_p so the distribution of the short-rate at time t is done in the following way:

$$r_t \sim N\left(r_p, \frac{\sigma^2}{2\beta}\left[1 - e^{-2\beta(t-s)}\right]\right) \tag{4.4}$$

As the distribution of the short-rate is altered this way, the simulation algorithm is based on the following recursion:

r_t at times $0 = t_0 < t_1 < \cdots < t_n$

$$r_{t_{i+1}} = r_p + \sigma\sqrt{\frac{1}{2\beta}\left(1 - e^{-2\beta(t_{i+1}-t_i)}\right)}Z_{i+1} \tag{4.5}$$

4.1.3. Comparetive Examples of Short-Rate Simulation

Two simulation approaches for the Vasicek Model were suggested. To clarify the similarities and the differences of these two algorithms, we continue with some examples.

For the US Government Bond market, the 3-month bond rate is accepted as the short-rate. Daily bond data from 04.01.1982 to 31.12.2009 are used as the historical data and the risk horizon is 6 month. The 3 month bond rate on 31.12.2009 is the current short-rate $r_0 = 0.0046$. The MLE parameters are used as the Vasicek model parameters for this simulation.

```
r<-Vasicek_simul(n=10^6,T=1/2,para=c(0.03036066,0.19337252,0.01371011),
 r0=0.0046)
mean(r)
#[1] 0.006969472
sd(r)
#[1] 0.009248242
quantile(r,0.99)
       99%
0.02846169
```

Figure 4.1. R-Code simulating the short-rate for a 6 months risk horizon.

The mean of the 10^6 simulated short-rates is 0.00697 which is higher than the initial short-rate.

Figure 4.2 shows the histogram of the simulated short-rate using the MLE parameters. The green line is the initial short-rate and the blue line is mean of the simulated short-rate. The red line is the %99 quantile of the simulated short-rate.

At the same time and under the same conditions, if the investor desires to use some expert opinion or belief of herself about the future mean of the short-rate, the results are shown in Figure 4.3 and Figure 4.4.

As expected, the mean of the simulated short-rate using the predicted short-rate is very close to the predicted short-rate. The standard deviation of two simulations is the same. This is because, we do not change any parameters that effects the standard deviation. We can see the difference of these two process is only a shift of the histogram. No randomness or volatility is changed for these two algorithms.

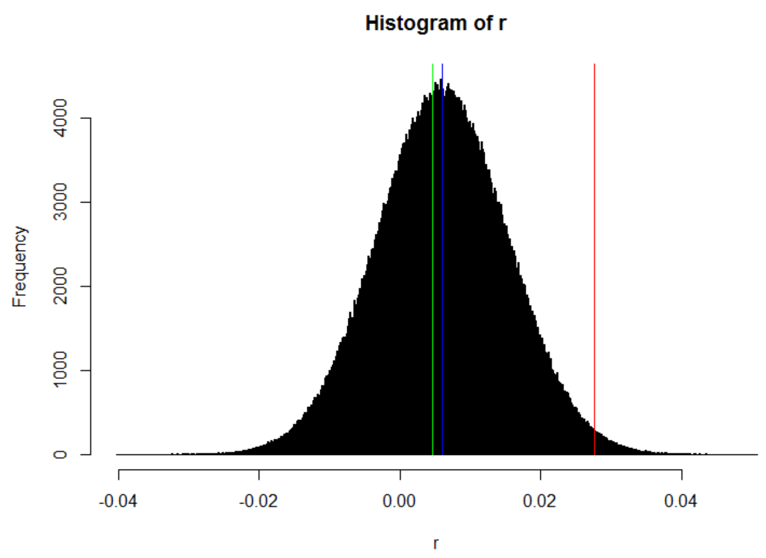

Figure 4.2. The histogram of simulated short-rate using MLE parameters for a 6 months risk horizon.

```
r<-Vasicek_simul(n=10^6,T=1/2,para=c(0.03036066,0.19337252,0.01371011),
 r0=0.0046,r_guessed=0.01)
 mean(r)
#[1] 0.009999208
sd(r)
#[1] 0.009238681
quantile(r,0.99)
       99%
0.031526353
```

Figure 4.3. R-Code simulating the short-rate for a 6 months risk horizon using predicted short-rate.

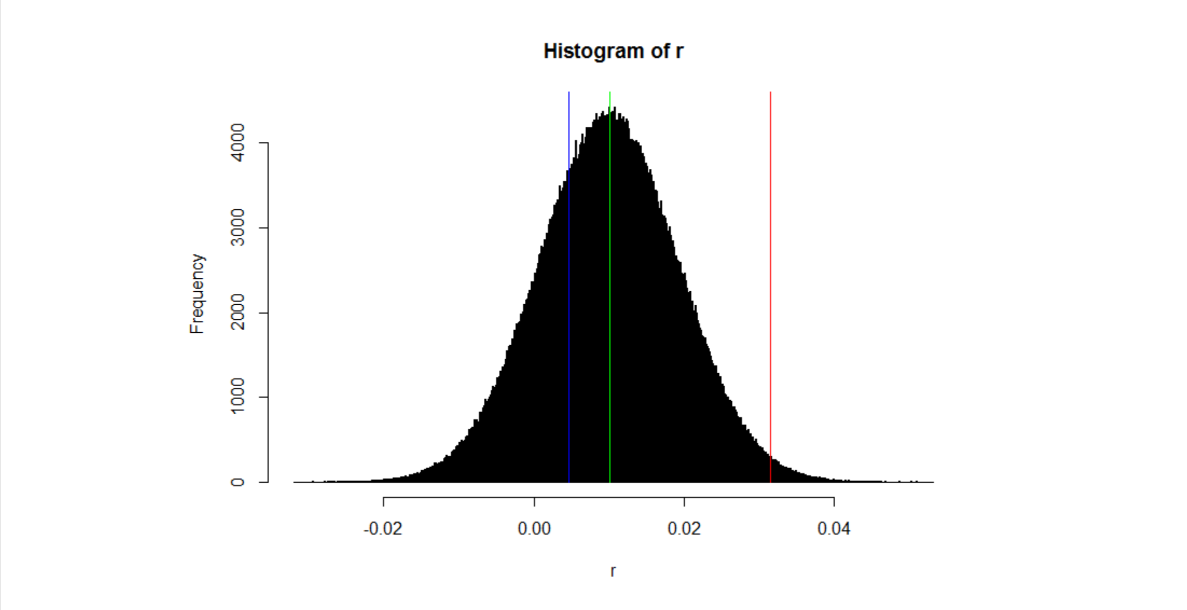

Figure 4.4. The histogram of simulated short-rate using predicted short-rate for a 6 months risk horizon.

4.2. Bond Pricing Using Calibrated Vasicek Model Parameters and the Simulated Short-Rate

The previous steps for bond pricing such as parameter estimation and parameter and short-rate simulation are discussed. Simulation of future bond prices using short-rate simulation with prediction or the calibrated Vasicek model parameters is the topic of this section. The bond pricing algorithm requires only the short-rate (r), the time to maturity $(T-t)$ and the Vasicek model parameters (μ, β, σ). The short-rate is taken from the short-rate simulation process, the time to maturity is constant and given and the calibrated Vasicek model parameters are constant. Here we assume that the shape of the yield curve will not change dramatically in a 6 month investment period. So the Vasicek model parameters that we calibrate will hopefully also be closely fitting at the end of the investment period and the bond pricing will be closer to the market. For a longer investment period such as more than a year this calibration is less sensible because in such a long term the shape of the yield curve may change and thus the calibrated parameters can not model future bond prices well.

The Vasicek model parameters are different for short-rate simulation and for bond pricing. In the short-rate simulation process, the unchanged MLE Vasicek model parameters are used but for the bond pricing the calibrated Vasicek model parameters are used. This difference should be clarified for the the reader.

Remember the bond pricing for the Vasicek model:

$$P(t,T) = A(t,T)e^{-B(t,T)r_t}\,, \tag{4.6}$$

where

$$\begin{aligned} A(t,T) &= \exp\left\{\left(\mu - \frac{\sigma^2}{2\beta^2}\right)[B(t,T) - T + t] - \frac{\sigma^2}{4\beta}B(t,T)^2\right\} \\ B(t,T) &= \frac{1}{\beta}\left[1 - e^{-\beta(T-t)}\right]\,. \end{aligned}$$

As the simulation comes in, that means that future comes in too. This algorithm is designed not only for initial bond prices or for a single constant short-rate but also for calculating future bond prices for many simulated short-rates.

The R-function that calculates the bond price is given in Appendix A.5. An experiment of that bond pricing is shown in Figure 4.5 is useful.

Figure 4.6 shows the histogram of the simulated prices of a 1 year bond by using a predicted short-rate mean and the calibrated Vasicek model parameters.

4.3. Simulated Portfolio Final Values and Histograms

The bond pricing algorithm defined before is only for a single bond. But the investment problem this study works on requires a portfolio with more than one bond. "Simulated portfolio final values" are calculated from the simulated short-rates. The algorithm is a simulation because the different short-rate values used for calculating bond prices are random. Remember the bond pricing of a single bond for Vasicek

```
Vasicek_MLE(data=dataBond$m03,dt = 1/258)
#        mu       beta      sigma
# 0.03036066 0.19337252 0.01371011
para<-Vasicek_MLE(data=dataBond$m03,dt = 1/258)
#The MLE Vasicek model parameters that will be used for short-rate
#simulation process.
cal_para<-c(0.0518490,0.32,0.01371011)
#The calibrated Vasicek model parameters that will be used for
#bond pricing algorithm.

r<-Vasicek_simul(para=param,r0=r0,r_guessed=0.01, n,T=1/2)
#r is the vector holding simulated short-rate for a 6 month investment
#period.
#r_guessed is the predicted mean short-rate
BondPrice<-VasicekPriceYield(r,T=1/2,TBond=1,tau=TBond-T,para=cal_para,
priceyn=1)
mean(BondPrice)
#0.9957398is mean of the simulated bond price of a 1 year maturity bond.
```

Figure 4.5. R-Code calculating the bond price using prediction and calibrated Vasicek model parameters.

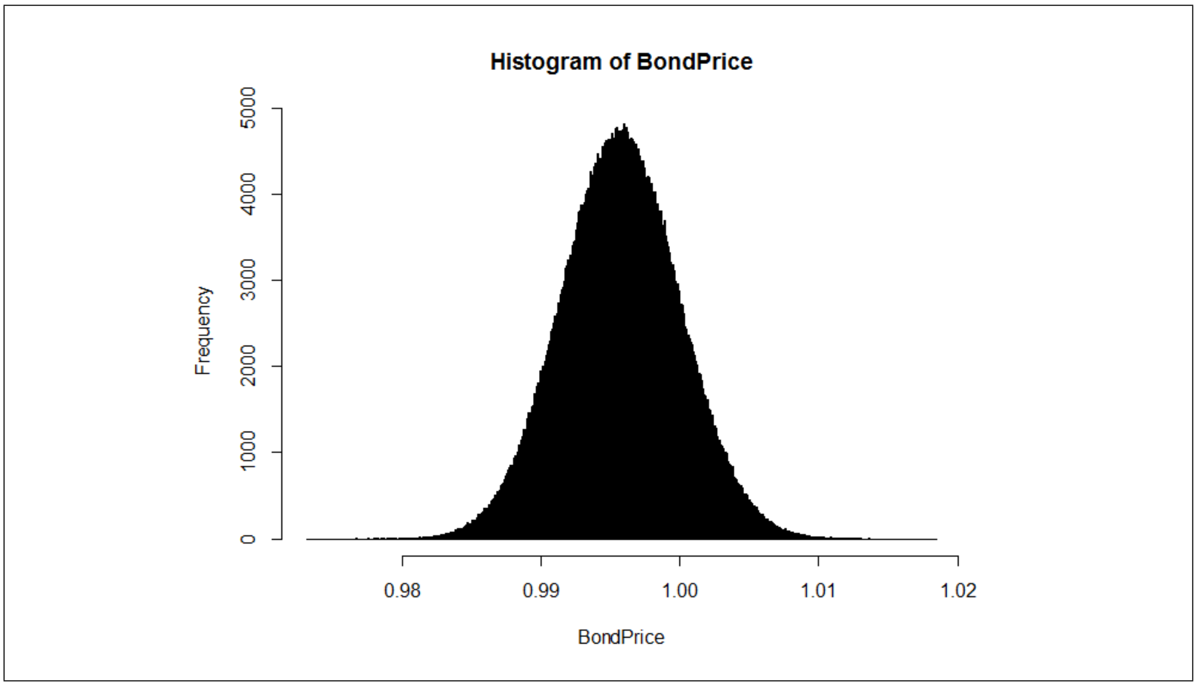

Figure 4.6. The histogram of simulated 1 year bond price using prediction and calibrated Vasicek model parameters.

model in (4.6). If the investment is distributed to several bonds, the only differences between bonds are their time to maturities and their weights. Suppose that a portfolio includes bonds with time to maturities $(T_1 - t)$ and $(T_2 - t)$ and with the weights w_1 and w_2. The value of the portfolio at time t is below:

$$V(t) = w_1 * A(t, T_1)e^{B(t,T_1)r_t} + w_2 * A(t, T_2)e^{B(t,T_2)r_t} \tag{4.7}$$

So the value of a bond portfolio for Vasicek model is the following:

$$V(t) = \sum_{i=1}^{n} w_i * A(t, T_i)e^{B(t,T_i)r_t} \tag{4.8}$$

The R-function that calculates the value of the portfolio is simply a sum of weighted prices of the different bonds. The user should input the times to maturity and the weights. This R-code is given in Figure 4.7 and Figure 4.8.

```
FinalPortValue<-function(n,T,r0,r_guessed,MLE_para,cal_para,TBond,CiB){

#The function calculating final value of bond portfolio containing
#stocks and bonds
#r0... is the initial short-rate
#r_guessed... is the predicted short-rate for the end of the investment
#period,
#cal_para...calibrated Vasicek Model Parameters
#TBond..is the vector holding time-to-maturities of the bonds in years
#T...is the investment period in years
#CiB...is the vector holding the amount of money invested into Bonds.
#The order has to be the same with the TBond. TBond=c(1/2,1) and
#CiB=c(200,400) means that $200 invested into 6month bond and $400
#invested into 1 year bond.


QiB<-CiB/VasicekPriceYield(r=r0,tau=TBond,para=cal_para,priceyn=1)
#QiB... is the vector holding quantity of each bond in the portfolio

Value0<-sum(CiB)
#The invested amount of money at the begining of invested period.

r<-Vasicek_simul_P(MLE_para,r0=r0,r_guessed=r_guessed, n,T=1/2,TBond)
#simulated short-rates for the end of the investment period

ValueT<-colSums(QiB*(VasicekPriceYield(r,tau=TBond-T,para=cal_para)))
#Total simulated value of the portfolio at the end of the time horizon
```

Figure 4.7. R-Code calculating the final value of the bond portfolio.

```
RiskFreeValue<-Value0/VasicekPriceYield(r0,tau=0.5,para=cal_para)
#The total wealth if all the money invested into 6 month bond

sorted_ValueT<-sort(ValueT)
CVaR<-mean(sorted_ValueT[1:length(ValueT)*0.01])



windows()
hist(ValueT,1000)
lines(rep(Value0,2),c(0,10^4), type="l",col="brown")
lines(rep(quantile(ValueT,0.01),2),c(0,10^4),type="l",col="red")
lines(rep(RiskFreeValue,2),c(-100,10^4),type="l",col="yellow")

res<-c(mean(ValueT),mean(ValueT)/Value0 ,
Value0-quantile(ValueT,0.01),Value0-CVaR)
names(res)<-c("MeanFinalWealth","MeanReturn","%1VaR","%1CVaR")

res
}
```

Figure 4.8. R-Code calculating the final value of the bond portfolio.

As an experiment, suppose that a portfolio is consist of 1 year and 5 year bonds. The weights are defined to be the capital invested into the bonds. In a portfolio, \$5000 and \$5000 were invested into 1 year and 5 year bonds respectively. The investment period is 6 months and the prediction of the average short-rate is 0.01. The R-function that calculates the value of this bond portfolio at the end of the investment period is given in Figure 4.9 and the histogram of the value of the portfolio is given in Figure 4.10

```
FinalPortValue<-Bond_Port(n=10^6,T=1/2,r0=0.0046,r_guessed=0.01,
MLE_para=c(0.19337252,0.01371011),cal_para=c(0.0518490,0.32,0.01371011),
TBond=c(0.5),CiB=10000)

mean(FinalPortValue)
#[1] 10063.365
```

Figure 4.9. R-Code calculating the final value of the bond portfolio.

The brown line in the histogram in Figure 4.10 indicates the invested capital and the green line indicates the mean of the calculated final portfolio values, the yellow line is final value of the risk-free investment (the investment that all the capital invested into 6 month bonds), and the red line is the 0.01 quantile of the calculated final portfolio values. The reason that we include the risk-free investment is to supply possibility to compare the quality of the investment. The relation of the 0.01 quantile of the calculated final portfolio values with a risk measure will be discussed in the next chapter.

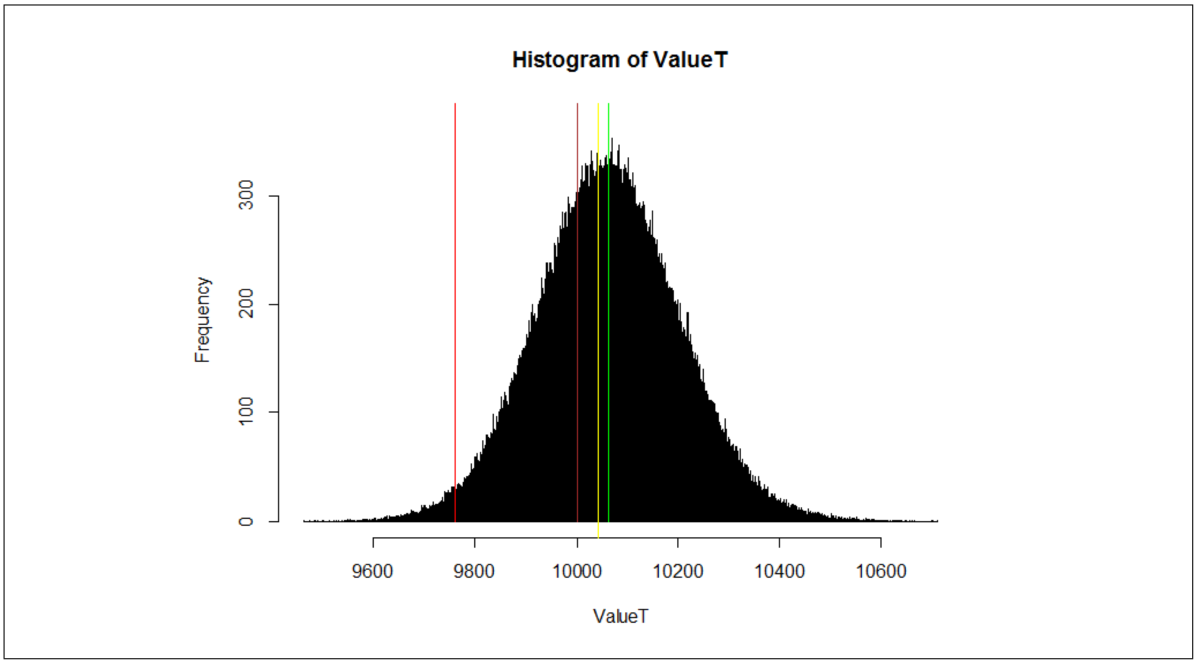

Figure 4.10. The histogram of simulated bond portfolio final value using prediction and calibrated Vasicek model parameters.

5. RISK AND MEAN RETURN CALCULATIONS FOR BOND PORTFOLIOS

Any investor desires to make the best investment but what makes an investment good or qualified? Any investment includes the hope of gaining more than the invested capital but this is never for free. Almost all investment also include the probability of loosing money which means they include risk. So an investment that gives the hope of higher return and includes low risk can be called a good investment. Thus expected return and risk are the most important quality measures of investments. An investor who knows about the expected return and the probability of loosing a certain amount can decide about investing into a portfolio. Also for a comparison, the return of the risk-free investment may be helpful. For this reason, we suggest a method to calculate both risk and return to help the investor to choose her investment.

5.1. Risk Measures

The definition of financial risk as "the quantifiable likelihood of loss or less-than-expected returns" [4] was given before. A main issue for risk management is to define a risk measure. Several different approaches to measure the risk are in use. In this thesis, Value-at-Risk (VaR) and Conditional Value-at-Risk (CVaR) which is also called Expected Shortfall are used.

5.1.1. Value at Risk (VaR)

VaR is the answer to the question *"How much money might I lose?"* based on probabilities and within parameters set by the risk management [5]. It has become popular in the last two decades.

Given some confidence level $\alpha \in (0,1)$ the VaR of a portfolio at the confidence level α is given by the smallest number l such that the probability that the loss L

exceeds l is not larger than $(1-\alpha)$ [4].

$$VaR_\alpha = inf\left\{l \in \mathbb{R} : P(L > l) \leq 1-\alpha\right\} \tag{5.1}$$

We build the algorithm to calculate the final investment value distribution instead of the loss distribution. Therefore, the 0.01 quantile (in the case $\alpha = 0.01$) of the final value distribution is the value that corresponding the %1VaR value. To calculate the %1VaR of the investment, we simply subtract the 0.01 quantile of the final values from the starting invested capital. VaR is popular due to its conceptual simplicity, ease of computation and ready applicability for the risk management field.

5.1.2. Conditional Value at Risk (CVaR)

Conditional Value at Risk (CVaR), in other word Expected shortfall (ES) is a second risk measure approach which is closely related to VaR. It is preferred by risk managers because of being sub- additive which assures its coherence as a risk measure. Nevertheless VaR is still popular in the financial world mainly because of having an intuitive interpretation. CVaR is the expected amount of loss of a portfolio given that the loss exceeds the VaR in some investment horizon under a given confidence level [16].

$$CVaR_\alpha = E(L|L > VaR_\alpha) \tag{5.2}$$

Clearly $CVaR_\alpha$ depends only on the distribution of V and clearly $CVar_\alpha > VaR_\alpha$. For continuous risk distributions, expected shortfall can be interpreted as the expected loss that is incurred in the event that the VaR is exceeded [4].

5.2. Mean Return and Risk Calculations

5.2.1. Mean Return and Risk Calculation With Simulation

How to simulated the final value of a portfolio was explained previously. The mean percentage return and risk calculation of the bond portfolios is based on the same simulation. The mean return is the difference of the mean value of the final portfolio and the starting invested capital over starting invested capital. It should give a ratio that is positive. The VaR of the portfolio is calculated simply by observing the quantile of the portfolio final values than subtract this quantile from the starting invested capital. For instance, the %1VaR of the portfolio is the difference of the starting invesment value and the %1 quantile of the simulated final portfolio values. The CVar of the portfolio is the mean value of the difference of the starting investment value and all the final portfolio values less than the %1 quantile. In other words, mean of all the losses that greater than the %1VaR.

```
FinalValueMixedPort(n=10^5,T=1/2,r0=0.0046,r_guessed=0.01,
MLE_para=c(0.19337252,0.01371011),cal_para=c(0.0518490,0.32,0.01371011),
TBond=c(0.5,1,2),CiB=c(4000,3000,3000))
MeanFinalWealth   MeanReturn   %1VaR   %1CVar
10048.7769        1.004878     57.516  72.9598
```

Figure 5.1. R-Codes for mean final value and VaR of the bond portfolio.

Figure 5.1 shows R-code that calculates the final values, mean return and risk of the investment consisting of %40 6 month bond, %30 1 year and %30 2 year bond for a 6 month investment period. Figure 5.2 shows the histogram of this investment's simulated final values and the $(1-\alpha)$ quantile of the final value of the investment which is $V_0 - VaR$ (the red line), mean final value of the investment (the green line), the risk-free investment final value (the yellow line) and the starting invested capital (the brown line). We aim to take advantage of visualizing by reporting these histograms. As we will apply this code to a number of different investments with different assets and weights,

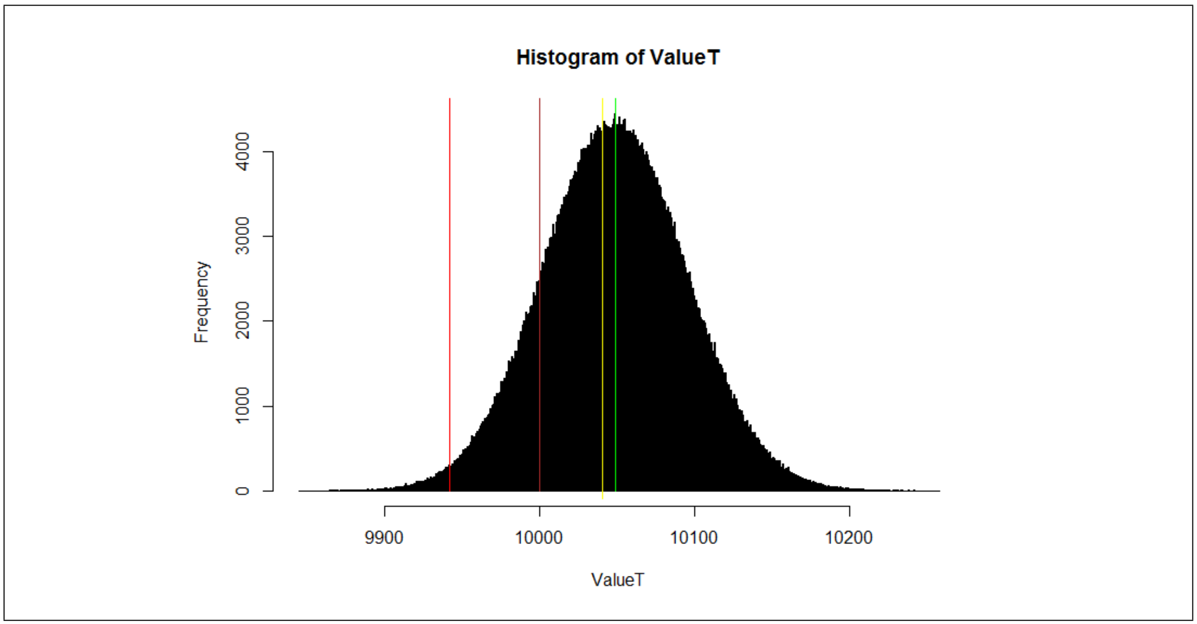

Figure 5.2. The histogram of simulated bond portfolio final value with VaR calculation.

seeing the histogram is valuable for understanding the characteristic of the investment. We prefer to give the final value distribution instead of the loss distribution. Our purpose of doing that is to show the investor the wealth that she possible will have at the end of the investment period.

5.2.2. Mean Return and Risk Calculation In Closed Form

As mentioned before, the short-rate process is normally distributed and this causes the value of the portfolio ($V(t)$) to be normal. The short-rate simulation using predicted mean value is the following:

$$r_{t_{i+1}} = r_p + \sigma\sqrt{\frac{1}{2\beta}\left(1 - e^{-2\beta(t_{i+1}-t_i)}\right)}Z \tag{5.3}$$

In the formula Z denotes the set of standard normal random numbers this is the core of the simulation. r_{t_i+1} is simply a linear transformation of Z. To obtain a closed form of the quantile value of the short-rate process, Z must be replaced the $1-\alpha$-quantile of the standard normal distribution.

In this formula, using the quantiles of this set instead of the set of random numbers makes possible to calculate the quantile of the short-rate and so the quantile of the portfolio final value. The closed form of the short-rate is the following:

$$r_{t_\alpha} = r_p + \sigma\sqrt{\frac{1}{2\beta}\left(1 - e^{-2\beta(t_)}\right)}qnorm(\alpha) \tag{5.4}$$

where $\alpha \in (0, 1)$

With this closed form equation, without any simulation, the "critical" short-rate can be calculated for a given quantile. How can this information be used for bond portfolio risk quantification? The answer is simple: For increasing the short-rate lowers the bond prices and vice versa. So the higher quantile of the short-rate corresponds the lower quantile of the portfolio value. For instance, the %99 quantile of the short-rate gives the %1 quantile of the portfolio value.

The R-code for the closed form is given in Appendix A.7. A small example is given as in the following R-code;

As seen in Figure 5.3 the short-rate quantile is the same for both methods. The prices calculated with these same short-rate are naturally the same. The plot in Figure 5.4 indicates the final portfolio values for given quantile of the short-rate. The quantiles are given from 0.01 to 0.99 with 0.01 step size. Note that the higher quantile of the short-rate causes a lower portfolio value.

```
Vasicek_closed_P(T=1/2,para=param,r0=0.0046,r_guessed=0.01, alpha=0.99,
TBond=1)
#[1] 0.03150521
r<-Vasicek_simul_P(T=1/2,para=param,r0=0.0046,r_guessed=0.005, n=10^6,
TBond=1)
quantile(r,0.99)
#       99%
#0.03150776
```

Figure 5.3. R-Codes for calculating and simulating the %99 quantile of short-rate.

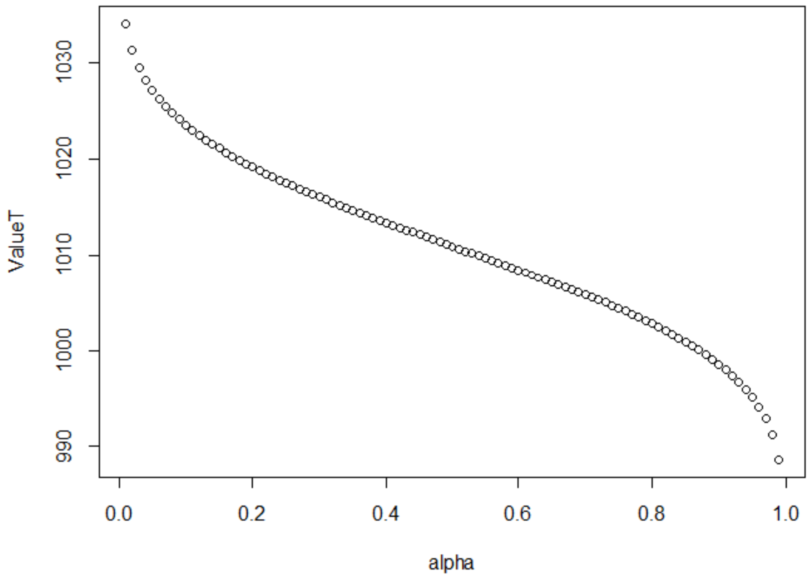

Figure 5.4. The plot of the portfolio final value for given quantile of short-rate.

6. RISK AND MEAN RETURN CALCULATIONS FOR MIXED PORTFOLIOS INCLUDING BONDS AND STOCKS

The investment problem that is studied in this thesis considers a conservative investment that includes only government bonds and possibly a small amount of stocks. These two instruments are from the same country with same currency so there is no currency risk. The main sources of the risk of this investment are the interest rate risk for bonds and the market risk for the stocks. For the calculation of the return and risk of the portfolio a simulation algorithm is developed. Before building this algorithm, the correlation between the sources of the risk for this investment have to be inspected.

6.1. Empirical Correlations Between Short-Rate and Stock Returns

The elements of risk of this investment are the interest rate risk and market risk. The interest rate risk of the bond can be modelled and quantified by the Vasicek model. The Vasicek model uses the short-rate as input. The market risk can be quantified by using simulation. This simulation uses Geometric Brownian Motion for the log-return of the stocks. So for a simulation algorithm that quantifies this mixed portfolio's risk, the correlation between the change in the short-rate and the logreturn of the stocks must be measured. This is even more important as there is a general believe that an increase in the interest rates has a negative effect on stock prices.

Daily data are used for this correlation test. The daily 3 and 6 months and 1,2,3,5,7,10 years yield data for US Government bonds and the daily closing prices of S&P500 index and 38 stocks from S&P500 were gathered. The daily increments of the yields and the daily logreturns of the index and the stocks were calculated together with the correlation between them. The results show that, the correlation between short-rate and stock prices is very small and it can be assumed that they are independent. The correlation matrix of the daily increments of the yields and the daily logreturns of the stocks are given in Appendix B.1. The correlation matrix of the weekly and monthly increments of the yields and weekly and monthly logreturns of the stocks are given in Appendix B.2 and B.3 respectively.

The maximum and minimum correlation between the daily logreturns of the 39 stock prices and the daily increments of the short-rate are %3.43 and %-8.55. They are %5.03 and %-5.58 for weekly and %10.06 and -%11.04 for monthly observations respectively.

For the daily data, the strongest correlation (%3.43) is between increment of 3 month bond yield (short-rate) and logreturn of MRK's (Merck and Co. Inc.)closing stock price, the strongest negative correlation (%-8.55) was observed between GT (The Goodyear Tire and Rubber Company) closing stock price and month bond yield (short-rate).

To investigate the influence of the size of the time step, the correlation for weekly and monthly data were calculated as well. The R-function that calculates the correlations is given in Appendix A.8 and the correlation matrix for weekly and monthly calculations you can find in Appendix A.9 and Appendix A.10. The value of the correlation is measures the linear dependency or independency of the two processes.

As these correlation are all closed to 0, it can be easily concluded that the correlation between stock price and short-rate proceses can be ignored and the model can be developed with the assumption that the stock prices and interest rates are independent.

6.2. Mean Return and Risk Simulations for Mixed Portfolios

As the previous section showed the small correlation between short-rate and stock price and lead to the assumption that these two process are independent, the simulation algorithm becomes quite easy. For the bond portfolio valuation, a simulation algorithm was already introduced previously. It is then necessary to simulate the possible future stock prices. After simulating the future stock prices, with the given weights, it is only left to sum up the weighted bond and stock prices to calculate the final investment values and than the mean return and the risk. We use the same risk measures and the difference between the mean of the simulated final values and the starting investment as the mean return. The R-code for this simulation is presented in Figure 6.1, 6.2, 6.3 and 6.4.

As an experiment, suppose an investment that consists of 1 year, 5 years bonds and S& P500 index and EXC stocks. The investing capital into each assets are \$4000,\$4000,\$1000,\$1000 respectively. R-code and results are in the Figure 6.5 and the histogram is in Figure 6.6.

This investment will probably have a final value histogram values shown in Figure 6.6. The red line shows the %1-quantile of the investment final value, the brown line shows the initial investment, the yellow line is the risk-free investment and the green line is the mean of the final values. As a result, the mean final value is \$10146.02 for this investment and the %1-quantile of the investment final value is \$9496.86. This investment results in an expected return of \$146.02, with a %1VaR of \$503.14. The mean of the loss if the loss exceeds %1VaR is \$580.91 (%1CVaR).

```
require(quantmod)
require(matrixStats)
dataBond<-read.table(file="Bond.txt",header=TRUE)
#Historic Bond data contains 3month-10years bond rates until 29.11.2013
dataBond<-dataBond[1:6710,]
#Historic Bond data contains 3month-10years bond rates until 31.12.2009

symbols<-c("^GSPC","ED","EXC","CVX","PBI","DIS")
getSymbols(symbols,from="1982-01-01",to="2009-12-31",src="yahoo")
#the S&P500 index historical data

dataStock<-data.frame(GSPC[,4],ED[,4],EXC[,4],CVX[,4],PBI[,4],DIS[,4])
#gathering the daily closing prices of the stocks

logret<-matrix(0,length(dataStock[,1])-1,length(dataStock[1,]))
for(j in 1:length(dataStock[1,])){
    for(i in 2: length(dataStock[,1])){
    logret[i-1,j]<-log(dataStock[i,j]/dataStock[i-1,j])
    }
    }
#Calculating daily logreturns of the stocks
logret<-data.frame(logret)
#saving the logreturns as a data frame

muS<-colMeans(logret)
#Mean values of daily logreturns of each stock
```

Figure 6.1. R-Code functioning the simulation of mixed investment.

```
sS<-colSds(logret)
#Standard deviation values of daily logreturns of each stock

R<-cor(logret)
#R... is the correlation matrix of stock logreturns

s0<-c(1115.1,45.43,48.87,76.99,22.76,32.25)
#the initial close values of the stocks on 29.11.2009

FinalValueMixedPort<-function(n,T,r0,r_guessed,MLE_para,cal_para,
TBond,CiB,s0,muS,sS,R,CiS){

#The function calculating final value of mixed portfolio containing
#stocks and bonds
#n...number of simulations
#T...is the investment period in years
#r0... is the initial short-rate
#r_guessed... is the predicted short-rate for the end of the period,
#MLE_para...Estimated (MLE) Vasicek Model Parameters
#beta and sigma
#cal_para...calibrated Vasicek Model Parameters
#TBond..is the vector holding time-to-maturities of the bonds in years
#CiB...is the vector holding the amount of money invested into Bonds.
s0...the initial prices of stocks
muS...vector holding the drift of logreturns of stocks
sS...vector holding the standard deviation of the logreturns of stocks
R...is the correlation matrix of logreturns of stocks
#CiS... is the vector holding the amount of money invested into stocks.
```

Figure 6.2. R-Code functioning the simulation of mixed investment.

```
Zmatrix<-t(chol(R))%*% matrix(rnorm(n*length(muS)),ncol=n,
nrow=length(muS))
ndays<-T*258
sT<-t(s0*exp(muS*ndays+sS*sqrt(ndays)*Zmatrix))
sT
#The simulated stock prices for the end of the investment period



QiS<-CiS/s0
#QiS... is the vector holding quantity of each stock in the portfolio
QiB<-CiB/VasicekPriceYield(r=r0,tau=TBond,para=cal_para,priceyn=1)
#QiB... is the vector holding quantity of each bond in the portfolio

Value0<-sum(CiS)+sum(CiB)
#The invested amount of money at the begining of invested period.

r<-Vasicek_simul_P(n,T=1/2,MLE_para,r0=r0,r_guessed=r_guessed, TBond)
#simulated short-rates for the end of the investment period

ValueT<-colSums(QiB*(VasicekPriceYield(r,tau=TBond-T,para=cal_para,
priceyn=1)))+ colSums(QiS*t(sT))
#Total simulated value of the portfolio at the end of the time horizon
RiskFreeValue<-(sum(CiB)+sum(CiS))/VasicekPriceYield(r0,tau=0.5,
para=cal_para,priceyn=1)
#The total wealth if all the money invested into 6 month bond

sorted_ValueT<-sort(ValueT)
CVaR<-mean(sorted_ValueT[1:length(ValueT)*0.01])
```

Figure 6.3. R-Code functioning the simulation of mixed investment.

```
windows()
hist(ValueT,1000)
lines(rep(Value0,2),c(0,10^4), type="l",col="brown")
lines(rep(quantile(ValueT,0.01),2),c(0,10^4),type="l",col="red")
lines(rep(RiskFreeValue,2),c(-100,10^4),type="l",col="yellow")

res<-c(mean(ValueT),mean(ValueT)-Value0,
Value0-quantile(ValueT,0.01),Value0-CVaR)
names(res)<-c("MeanFinalWealth","MeanReturn","%1VaR","%1CVaR")

res
}
```

Figure 6.4. R-Code functioning the simulation of mixed investment.

```
FinalValueMixedPort(n=10^6,T=1/2,r0=0.0046,r_guessed=0.01,
MLE_para=c(0.19337252,0.01371011),cal_para=c(0.0518490,0.32,0.01371011),
TBond=c(1,5),CiB=c(4000,4000),s0=c(1115.1,22.76),
muS=c(0.000312377,0.0001795194),sS=c(0.0116165,0.01790273),
R=matrix(c(1,0.382196,0.382196,1),2,2),CiS=c(1000,1000))

MeanFinalWealth   MeanReturn   %1VaR    %1CVaR
10146.024583      1.014602    503.139   580.911
```

Figure 6.5. R-Codes for calculating the standard investment b.

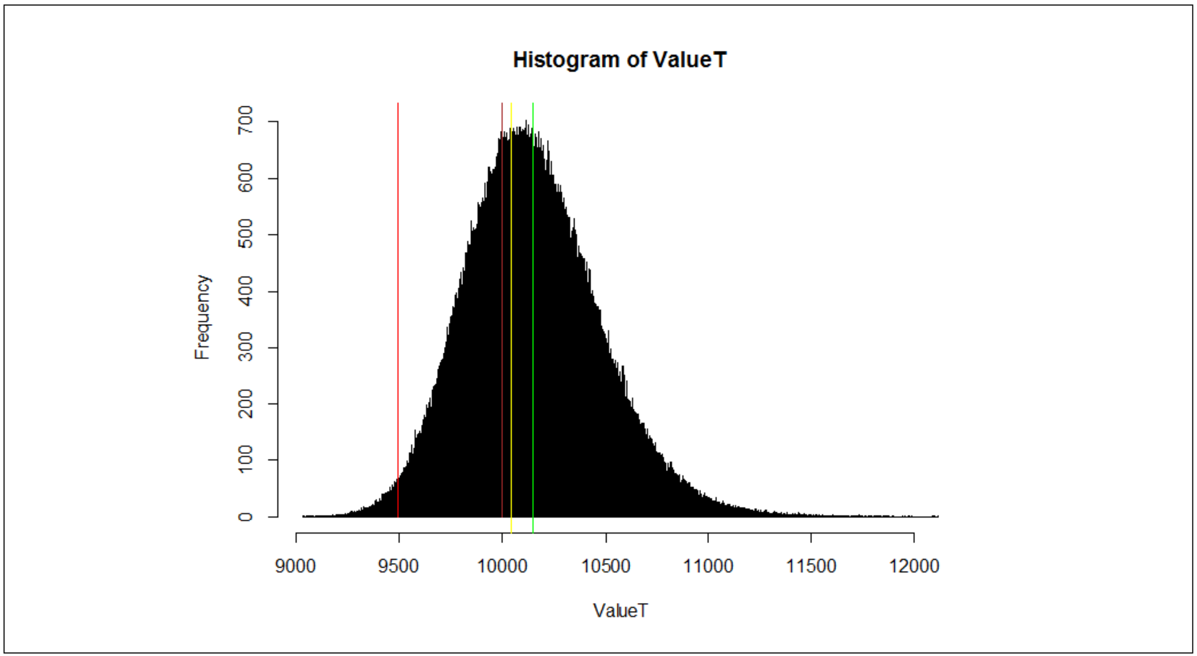

Figure 6.6. The histogram of final values of mixed portfolio.

7. ASSESSING INVESTMENTS IN PRACTICE

In this thesis, we developed a method that can be used to simulate the histogram of the final wealth, to quantify the risk and calculate the expected return for bond portfolios, stock portfolios and mixtures of these two. We presented the components of this method in previous chapters. In this chapter, we clarify the steps of the methods, the necessity of these steps, the assumptions that are made and we apply the method to the standard investments that we presented in Chapter 2.

7.1. The Algorithm Step by Step

We first list all steps necessary for the analysis.

Step I: Selecting the data set. First it is necessary to decide about the data set and to collec these data. Our method can be used only for investments that include bonds and stocks from the same country. For bonds, the shortest government bond rate data are used as short-rate data. For our examples we prefer to use US government bonds and take the 3 month bond rate as the short-rate. We downloaded the bond data from the US Federal Reserve [14]. Then the stocks and indices that may be considered must be chosen and their data collected. We choose 38 stocks from the NewYork Stock Exchange that are all part of S& P500 index.

Step II: Maximum Likelihood Estimation of Vasicek model parameters. Using the MLE method for the 3 month bond yield data from 04.01.1982 to 31.12.2009, we estimate the parameters of the Vasicek model.

Step III: Calibrating the Vasicek Model Parameters. We calibrate the Vasicek model parameters μ and β in order to obtain a yield curve better fitting to the market yield curve. Please note that we do not alter the σ parameter obtained by MLE. Doing this calibration we assume that the shape of the yield curve in the market will not change very much during the investment period. So the calibrated parameters that fit the yield curve of the Vasicek model to the initial market yield curve also fit the market yield curve at the end of the investment period. These calibrated parameters are used only for the bond price calculations, not for the simulation of the short-rate value.

Step IV: Calculating the logreturns and the parameters. We calculate the logreturns of the stocks from the historic closing prices. Then we estimate the correlation matrix and the standard deviations of the logreturns (volatilities of the stocks). The mean of the logreturns could be also estimated using the historical data. As this estimation implies high uncertainty we use instead the investor's assumption of the expected return of the considered stock. If she has no assumption on these a simple default assumption is the assumption that the return of the stock is equal to the return of longest maturity (10 years) government bond. These parameters are then used to simulate future stock prices using the multinormal logreturn model.

Step V: Observing the expert opinion. The expert opinion is the belief of the investor or some expert on the bond market about the expected short-rate value r_p at the and of the investment period. The idea of this step is to give a chance to the investor to make a scenario analysis and to avoid the always increasing short-rate mean of the Vasicek model. The value observed from the expert opinion is subjective and has an influence on the final values that will be simulated.

Step VI: Short-rate simulation by using the estimated parameters. In this step, we simulate the short-rate values for the end of the investment period. The MLE of the parameters β and σ and the expert opinion and the time horizon are the input for this step. The simulated short-rates in this step are the input of the bond pricing step.

We usually simulate 10^6 short-rate values.

Step VII:Calculation of the bond prices by using the calibrated parameters. This step is using the simulated short-rates and the calibrated Vasicek model parameters to calculate the total value of the bond portfolio at the end of the investment period. The number of bond portfolio value is the same as the number of simulated short-rates.

Step VIII: Simulation of the future stock prices. By using the multinormal model, we simulate the logreturns and using them we calculate the future stock prices. For this step, the inputs are the output of Step IV. The number of simulated stock prices is the same as the number of simulated short-rates.

Step IX: Calculation of the final values of the investment. This is simply the weighted sum of the simulated stock prices and bond prices. This is possible due to the assumption of independence of short-rate and stock prices. The output of this step is a vector holding the simulated final values of the investment.

Step X: Calculation of mean return and risk of the investment. After we obtain the simulated final values of the investment in the previous step, we calculate the average of these values which is a good estimate for the "Mean Final Wealth"; dividing this value by the starting invested capital, we obtain the "Mean Return". The α quantile of this distribution gives us the quantile of the final wealth which corresponds to the VaR. We calculate the VaR simple subtracting this quantile from the starting invested capital. We also do the same to every single final value smaller than this quantile value to calculate their mean and to obtain CVaR.

7.2. The R-Functions

Before using the R-function we developed, it is helpful to give a short description of these functions.

```
FinalPortValue(n=10^4, T=0.5, r0, r_guessed=r0,
               MLE_para, cal_para, TBond, CiB)
FinalValueMixedPort(n=10^4, T=0.5, r0, r_guessed=r0,
               MLE_para, cal_para, TBond, CiB,
               muS, sS, R, CiS)
The functions return n final wealth values and estimates the mean return
and the risk measures of the bond portfolio.

Parameters:
n .. number of simulated final values
T .. investment period in years
r0 .. initial short-rate
rguessed .. guessed short-rate for the end of the investment period
            its default value is r0, expected short rate is equal to r0
MLEpara .. MLE of Vasicek Model Parameters beta and sigma
cal_para .. calibrated Vasicek Model Parameters mu, beta and sigma
TBond .. vector holding time-to-maturity in years of each bond
CiB .. vector holding the amount of money invested into each bond
muS .. yearly mean vector of the log-returns of the stocks
sS .. yearly volatility vector of the log-returns of the stocks
R .. correlation matrix of the log-returns
CiS .. vector holding the amount of money invested into each stock
```

7.3. Mean Return and Risk Calculations for the Standard Investments

In this section of the study, we will calculate the risk and mean return for the standard investments that was mentioned in Chapter 2 with the methods we have developed here. We use the data for both bonds and stock from 04.01.1982 to 31.12.2009 and assume that we are on 04.01.2010 and our investment horizon is 6 months from today. Remember the standard investments:

Standard Investment a; 10000 into 6 month bonds (Risk-Free Investment)

Standard Investment b; $5000 into 6 month and $5000 into 2 year bond

Standard Investment c; $5000 and $5000 into 6 month and 5 year bonds respectively.

Standard Investment d; $5000 and $3000 into 6 month and 2 year bonds and $2000 into S& P500 index

Standard Investment e; $4000 and $3000 into 6 month and 2 year bonds, $2000 into S& P500 index and $1000 into PBI

Before presenting the calculations and results, it is useful to interpret the assumptions. We assume that the shape of the yield curve in the bond market will not change much till the end of the investment period. This allows us to make the calibration of the Vasicek parameters to have better fitting yield curve to the market and use them for bond price calculations. We also assume that the investor has an idea (herself or of some experts) about the average of the short-rate at the end of the investment period. This is kind of a guess coming from experience not a mathematical or statistical prediction. The user-investor also may not use this option so the algorithm takes the mean of the simulated short-rate same in the Vasicek model. In order to have unity, we take this guessed mean short-rate as 0.01 while the initial short-rate is 0.0046, this is also realistic just because the increase tendency in the bond market due to the American FED's recent politics. An other important assumption is the independence of the short-rate and the stock prices. The data we investigated showed the correlation is close to 0. So we assume them to be independent. This assumption allows us to simulate the bond and stock prices separately and then to sum them up.

The first example is called the risk-free investment because bonds with maturity equal to the investment horizon are assumed to be risk free. Thus the final investment value at the end of the investment period will be the par value of the bond. This risk-free investment value is important for comparison. We will also show the investor that her investment will probably gain "these" values and the risk-free investment will gain "that" value. So she can decide if the return she expect from the investment is enough to accept the risk of the investment or if she prefers the risk-free investment. The R-code and results are in the Figure 7.1.

```
FinalPortValue(n=10^6,T=0.5,r0=0.0046,r_guessed=0.01,
MLE_para=c(0.19337252,0.01371011),cal_para=c(0.0518490,0.32,0.01371011),
TBond=c(0.5)CiB=10000)


MeanFinalWealth    MeanReturn
 10040.97963        1.0041
```

Figure 7.1. R-Codes for calculating the standard investment a the risk-free investment.

This risk-free investment assures that the investor who invests \$10000 gains \$10040.98.

Standard investment b which is investing \$5000 into 6 month and \$5000 into 2 year bond. R-code and results are in the Figure 7.2 and the histogram is in Figure 7.3

This investment will probably have a final value histogram values shown in Figure

```
FinalPortValue(n=10^6,T=0.5,r0=0.0046,r_guessed=0.01,
MLE_para=c(0.19337252,0.01371011),cal_para=c(0.0518490,0.32,0.01371011),
TBond=c(0.5,2),CiB=c(5000,5000))

MeanFinalWealth   MeanReturn   %1VaR    %1CVaR
10050.33873         1.0050     77.03     95.304
```

Figure 7.2. R-Codes for calculating the standard investment b.

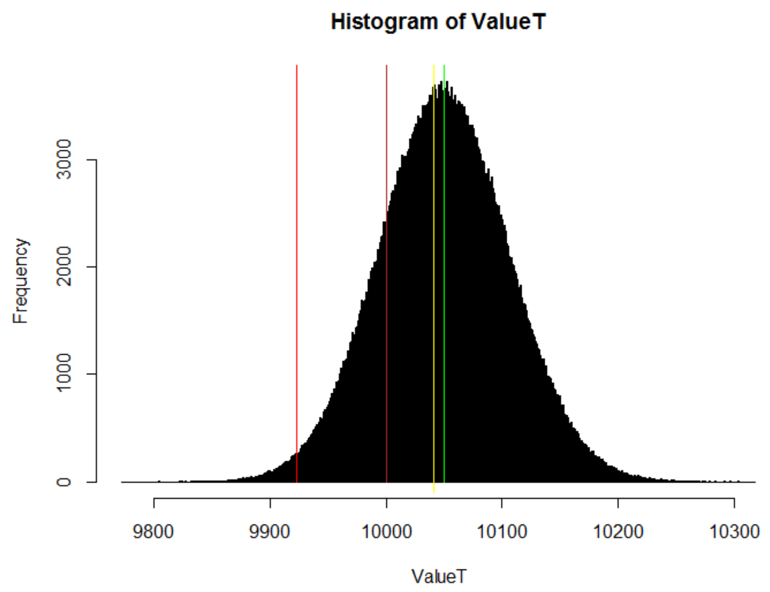

Figure 7.3. The histogram of final values of standard investment b.

7.3. The red line shows the %1-quantile of the investment final value, the brown line shows the initial investment, the yellow line is the risk-free investment and the green line is the mean of the final values. As a result, the mean final value is $10050.34 for this investment and the %1-quantile of the investment final value is $9922.27. This investment results in an expected return of $50.34, with a %1VaR of $77.03. The mean of the loss if the loss exceeds %1VaR is $95.30 (%1CVaR).

Standard investment c which is investing $5000 into 6 month and $5000 into 5 year bond can be named conservative investment. R-code and results are in the Figure 7.4 and the histogram is in Figure 7.5.

```
FinalPortValue(n=10^6,T=0.5,r0=0.0046,r_guessed=0.01,
MLE_para=c(0.19337252,0.01371011),cal_para=c(0.0518490,0.32,0.01371011),
TBond=c(0.5,5),CiB=c(5000,5000))


MeanFinalWealth   MeanReturn  %1VaR    %1CVaR
10059.65         1.005965     193.078   228.52
```

Figure 7.4. R-Codes for calculating the standard investment c.

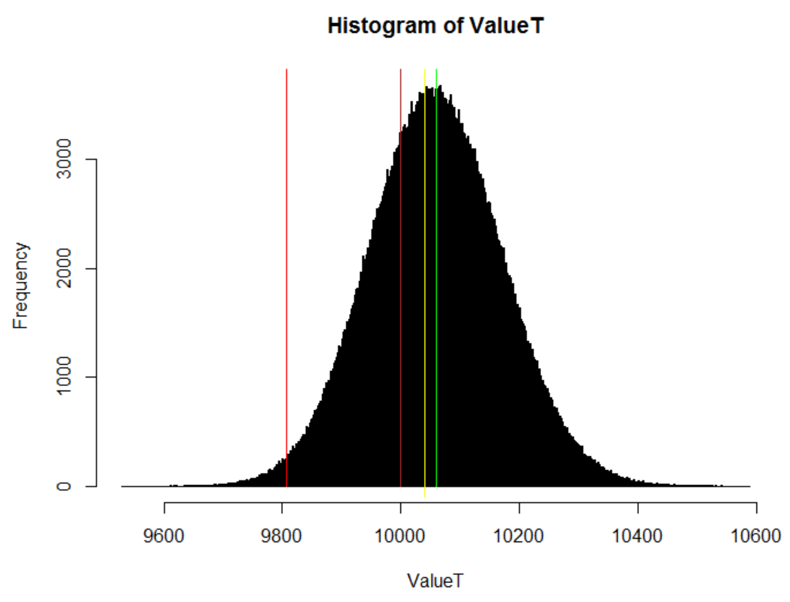

Figure 7.5. The histogram of final values of standard investment c.

This investment will probably have some final values that in the values in the histogram that is shown in Figure 7.5. The red line shows the %1-quantile of the investment final value, the brown line shows the initial investment, the yellow line is the risk-free investment and the green line is the mean of the final values. As a result, the mean final value is $10059.65 for this investment and the %1-quantile of the investment final value is $9806.92. This investment results in an expected return of $ 59.65, with a %1VaR of $193.08. The mean of the loss if the loss exceeds %1VaR is $228.52 (%1CVaR).

The standard investment d defined in Chapter 2 was investing $5000 and $3000 into 6 month and 2 year bonds and $2000 into S& P500 index. The results and the histogram of the final values are shown in Figure 7.6 and Figure 7.7.

```
FinalValueMixedPort(n=10^6,T=1/2,r0=0.0046,r_guessed=0.01,
MLE_para=c(0.19337252,0.01371011),cal_para=c(0.0518490,0.32,0.01371011),
TBond=c(0.5,2),CiB=c(5000,3000),s0=1115.1,muS=0.000312377,sS=0.0116165,
R=1,CiS=2000)

MeanFinalWealth   MeanReturn  %1VaR   %1CVaR
10138.4327        138.4327   436.31   504.09
```

Figure 7.6. R-Codes for calculating the standard investment d.

This investment will probably have some final values range of the histogram that is shown in Figure 7.7. The red line shows the corresponding final value to %1VaR, the brown line shows the initial investment, the yellow line is the risk-free investment and the green line is the mean of the final values. As a result, the mean final value is $10138.43 for this investment and the corresponding final value to %1VaR is $9563.69. Most probably this investment results in $138.43 return, with a %1 probability of a loss of more than $436.31 (%1VaR Loss) or more loss. The mean of the loss if the loss exceeds the %1VaR is $504.09 (%1CVaR).

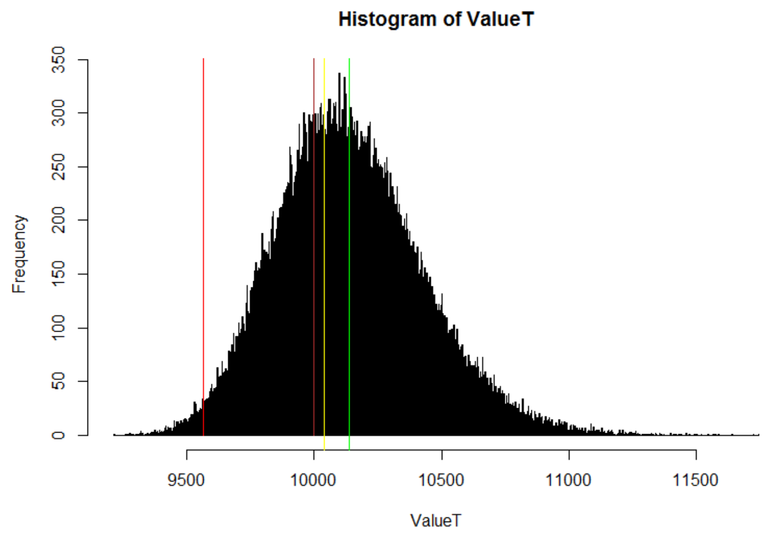

Figure 7.7. The histogram of final values of standard investment d.

The standard investment *e* defined in Chapter 2 was the investing \$4000 and \$3000 into 6 month and 2 year bonds, \$2000 into S& P500 index and \$1000 into PBI respectively. The results and the histogram of final values are shown in Figure 7.8 and 7.9.

```
FinalValueMixedPort(n=10^6,T=1/2,r0=0.0046,r_guessed=0.01,
MLE_para=c(0.19337252,0.01371011),cal_para=c(0.0518490,0.32,0.01371011),
TBond=c(0.5,2),CiB=c(4000,3000),s0=c(1115.1,22.76),
muS=c(0.000312377,0.0001275194),sS=c(0.0116165,0.024497),
R=matrix(c(1,0.3847338,0.3847338,1),2,2),CiS=c(2000,1000))


MeanFinalWealth   MeanReturn  %1VaR   %1CVaR
10191.15         1.019115     752.33    856.83
```

Figure 7.8. R-Codes for calculating the standard investment d.

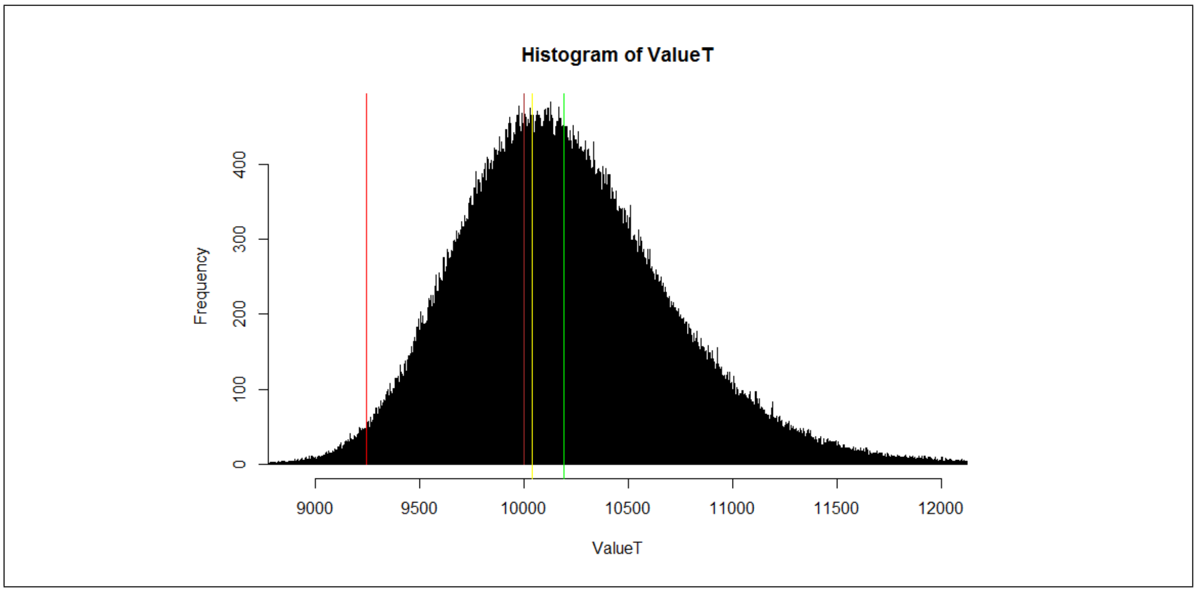

Figure 7.9. The histogram of final values of standard investment e.

7.4. The Comparison of the Standard Investments

In the previous sections, five different standard investments were defined. The first standard investment was constructed with 6 month bonds only and defined as the "Risk-Free Investment". The second and third standard investment was consisting only bonds with different times to maturity. Standard investment b has 6 months and 2 years bonds and standard investment c has 6 months and 5 years bonds with equal weights. Standard investment d and e are adding stocks to the bonds. Standard investment d has 6 months and 3 years bonds and S& P500 and standard investment e has the same assets with d and additionally PBI stocks. The characteristics and the histograms of the final values of these two standard investments were given above. We believe that seeing the whole results together is helpful to understand the effects of the assets chosen and their influence on the mean return and risk of the investment. Figure 7.10 presents the histogram of the final values of standard investment b, c, d and e.

As can be seen, the standard deviation of the investments increases from b to e. This is a sign of the increase of the risk. Yet the mean of the final values are moving to the right which shows increase of the mean return. Table 6.1 shows the mean final value, mean return, VaR and CVaR of the standard investments.

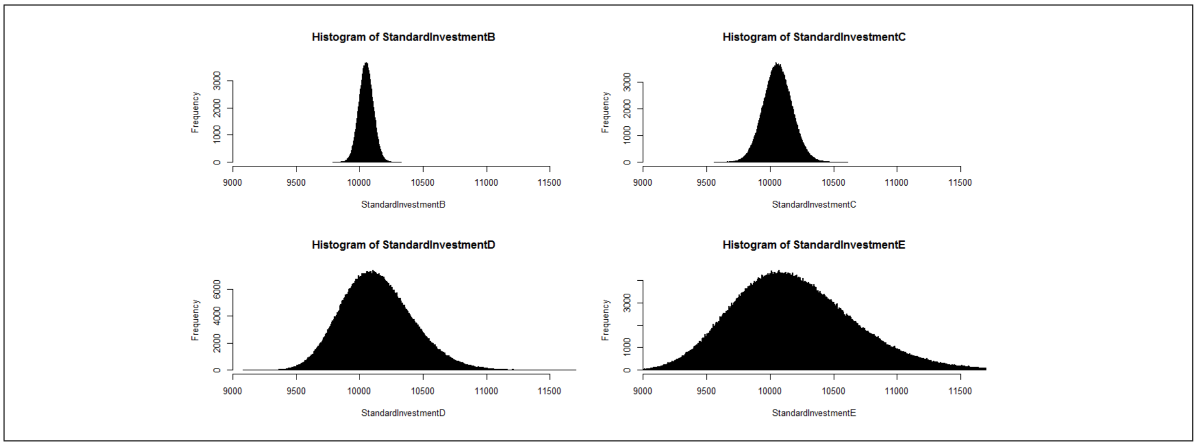

Figure 7.10. The histograms of final values of standard investments b,c,d,e.

Investment	Mean Final Value	Mean Return	%1VaR Loss	%1CVaR Loss
a	10048.98	1.004898	0	0
b	10050.35	1.005035	76.823	95.148
c	10059.72	1.005972	193.954	229.489
d	10139.05	1.013905	435.539	503.393
e	10191.15	1.019115	752.334	856.814

Table 7.1. The mean return and risk results of the standard investments a,b,c,d,e.

The risk (VaR) of the investments is increasing much faster than the mean return. For instance the standard investment d has \$139.05 as mean return but its VaR is \$435.54. Standard investment e has \$191.15 as its mean return but its VaR is \$752.33. The increase in the return is %38.19 and the increase in the risk is %72.74. The investor will decide to which investment to invest according to her risk appetite. It is important to realize that the above simulation results are strongly influenced by the assumption that we made. The assumption that the guessed short-rate mean (r_p) is equal to 0.01 (which is an increase) directly implies that long termed stock will have a low return. For instance, if we change the guessed short-rate mean (r_p) from 0.01 to 0.003 which means the expert believes the interest rates will go down, the mean return will positively change. It is similar for other assumptions.

8. CONCLUSIONS

In this study, we aimed to design a method that quantifies the risk of mixed portfolios containing bonds and stocks. The tool we developed is able to do this task. It calculates the risk (VaR and CVaR) and the mean return for bond, stock and mixed portfolios by using simple stochastic models, MLE, historical data and expert opinion about future changes.

It was demonstrated that risk can be sensibly quantified using stochastic models such as Vasicek model and multi-normal model and Maximum Likelihood estimation. We used the Vasicek model and MLE to model the short-rate and simulate future bond prices. The multi-normal model is used to simulate future stock prices. By the help of these models, we can simulate a histogram of the final wealth for the investments. This histogram indicates the possible final wealth at the end of the investment period. This is visualized by use of histograms. So the investor can clearly see the possible return and its probability (high-modarate-low) looking at the histogram.

We introduced the possibility to include the believes of the market and expert opinions into quantitative methods. We believe that we contributed to the literature and supply a useful tool for practitioners by developing this additive. The return estimates are mainly influenced by the believes in our method. Therefore making good guesses on future expectations of short-rate and stock returns are important. Yet even if the guesses may not be considered correct the method gives a chance to inspect the results of different believes or guesses.

APPENDIX A: R-CODES

A.1. R Codes for the MLE of the Parameters of the Vasicek Model

```
Vasicek_MLE <- function(data=dataBond$m03,dt = 1/258){
# returns a vector with the estimates for the
# parameters mu, beta and sigma of the Vasicek model
# data ... vector of short rate data
# dt ... time interval (in years) between the data points given in data
N = length(data)
rate <- data[2:N]
lagrate<- data[1:(N-1)]
alpha<-(N*sum(rate*lagrate) - sum(rate)*sum(lagrate))/
(N*sum(lagrate*lagrate)
- sum(lagrate)*sum(lagrate))
beta_hat = -log(alpha)/dt
mu_hat = sum(rate-alpha*lagrate ) / (N*(1-alpha))
v2hat<-sum((rate-lagrate*alpha-mu_hat*(1-alpha))^2)/N
sigma_hat<-sqrt(2*beta_hat*v2hat/(1-alpha^2))
res<-c(mu_hat,beta_hat,sigma_hat)
names(res) <- c("mu","beta","sigma")
return(res)
}
```

A.2. R Codes for the Bond Price and the Yield Calculations for the Vasicek Model

```
VasicekPriceYield<-function(r,tau=TBond-T,para,priceyn=1){
# r ... r(t) current value of short rate
# tau ... vector holding the time to maturity
#para ... vector holding the parameters of the Vasicek model
 mu =para[1]
 beta =para[2]
 sigma =para[3]
 B=(1-exp(-beta*tau))/beta
 A=((B-tau)*(beta^2*mu-0.5*sigma^2)/beta^2 - sigma^2*B^2/(4*beta))
 if(priceyn){
  return(exp(A-B*r))
 }else return((r*B-A)/tau)
}
```

A.3. R Codes for Exact Simulation of Short-Rate.

```
Vasicek_simul<- function(para,r0,n,T=1/2,TBond){
 # exact simulation of n many short rate
 # para ... parameters of the Vasicek model
 # r0 ... starting value of the interst rate process
 # n ... number of paths that are generated
 # T ... Time horizon
 mu=para[1]
 beta=para[2]
 sigma= para[3]
 r=rep(0,n)
 r[1]=r0
 r= r0*exp(-beta*T) + mu*(1-exp(-beta*T))+(
 sigma*sqrt((1-exp(-2*beta*T))/(2*beta))*rnorm(n))
 r
 }
```

A.4. R Codes for Simulation of Short-Rate Using Predictions.

```
Vasicek_simul <- function(para=param,r0,r_guessed,n,T=1/2,TBond){
 # exact simulation of n many short rate
 # para ... parameters of the Vasicek model
 # r0 ... starting value of the interst rate process
 # n ... number of paths that are generated
 # T ... Time horizon
 mu=para[1]
 beta=para[2]
 sigma= para[3]
 r=rep(0,n)
 r[1]=r0
 r= r_guessed+(sigma*sqrt((1-exp(-2*beta*T))/(2*beta))*rnorm(n))


r
 }
```

A.5. R-Code for Calculation of the Bond Price Using Prediction and Calibrated Vasicek Model Parameters

```
Vasicek_MLE(data=dataBond$m03,dt = 1/258)
#        mu       beta      sigma
# 0.03036066 0.19337252 0.01371011
para<-Vasicek_MLE(data=dataBond$m03,dt = 1/258)
#The MLE Vasicek model parameters that will be used for short-rate
#simulation process.
cal_para<-c(0.0518490,0.32,0.01371011)
#The calibrated Vasicek model parameters that will be used for
#bond pricing algorithm.

r<-Vasicek_simul(para=param,r0=r0,r_guessed=0.01, n,T=1/2)
#r is the vector holding simulated short-rate for a 6 month investment
#period.
#r_guessed is the predicted mean short-rate
BondPrice<-VasicekPriceYield(r,T=1/2,TBond=1,tau=TBond-T,para=cal_para,
priceyn=1)
mean(BondPrice)
#0.9957398is mean of the simulated bond price of a 1 year maturity bond.
```

A.6. R Codes for Risk and Mean Return Calculations of a Bond Portfolio

```
require(matrixStats)
dataBond<-read.table(file="Bond.txt",header=TRUE)
#Historic Bond data contains 3 month-10 years bond rates until
29.11.2013
dataBond<-dataBond[1:6710,]
#Historic Bond data contains 3 month-10 years bond rates until
31.12.2009

FinalPortValue<-function(r0,r_guessed,cal_para,TBond,T,n,priceyn=F,CiB,
CiS=rep(0,6)){
#The function calculating final value of bond portfolio containing
#stocks and bonds
#r0... is the initial short-rate
#r_guessed... is the predicted short-rate for the end of the investment
#period,
#cal_para...calibrated Vasicek Model Parameters
#TBond..is the vector holding time-to-maturities of the bonds in years
#T...is the investment period in years
#CiB...is the vector holding the amount of money invested into Bonds.
#The order has to be the same with the TBond. TBond=c(1/2,1) and
#CiB=c(200,400) means that $200 invested into 6month bond and $400
#invested into 1 year bond.
```

```
Vasicek_MLE <- function(data=dataBond$m03,dt = 1/258){
# returns a vector with the estimates for the
# parameters mu, beta and sigma of the Vasicek model
# data ... vector of short rate data
# dt ... time interval (in years) between the data points given in data
N = length(data)
rate <- data[2:N]
lagrate<- data[1:(N-1)]
alpha<-(N*sum(rate*lagrate) - sum(rate)*sum(lagrate))/
(N*sum(lagrate*lagrate)
- sum(lagrate)*sum(lagrate))
beta_hat = -log(alpha)/dt
mu_hat = sum(rate-alpha*lagrate ) / (N*(1-alpha))
v2hat<-sum((rate-lagrate*alpha-mu_hat*(1-alpha))^2)/N
sigma_hat<-sqrt(2*beta_hat*v2hat/(1-alpha^2))
res<-c(mu_hat,beta_hat,sigma_hat)
names(res) <- c("mu","beta","sigma")
return(res)
}

param<- Vasicek_MLE(dataBond$m03,dt=1/258)

Vasicek_simul_P <- function(para=param,r0,r_guessed,n,T=1/2,TBond){
 # exact simulation of n many of the short rate process and returns
 them as a matrix which have same elements on column
 # para ... parameters of the Vasicek model
 # r0 ... starting value of the interst rate process
 # n ... number of paths that are generated
 # T ... Time horizon
```

```
mu=para[1]
 beta=para[2]
 sigma= para[3]
 r=rep(0,n)
 r[1]=r0
r= r_guessed+(sigma*sqrt((1-exp(-2*beta*T))/(2*beta))*rnorm(n))
 op.mat<-matrix(1,nrow=n,ncol=length(TBond))
 r<-(t(r*op.mat))
 r }

VasicekPriceYield<-function(r,tau=tau,para=cal_para,priceyn=F){
# r ... r(t) current value of short rate
# tau ... vector holding the time to maturity
#para ... vector holding the parameters of the Vasicek model
 mu =para[1]
 beta =para[2]
 sigma =para[3]
 B=(1-exp(-beta*tau))/beta
 A=((B-tau)*(beta^2*mu-0.5*sigma^2)/beta^2 - sigma^2*B^2/(4*beta))
 if(priceyn){
  return(exp(A-B*r))
 }else return((r*B-A)/tau)
}

QiB<-CiB/VasicekPriceYield(r=r0,tau=TBond,para=cal_para,priceyn=1)
#QiB... is the vector holding quantity of each bond in the portfolio

Value0<-sum(CiB)
#The invested amount of money at the begining of invested period.
```

```
r<-Vasicek_simul_P(para=param,r0=r0,r_guessed=r_guessed, n,T=1/2,TBond)
#simulated short-rates for the end of the investment period

ValueT<-colSums(QiB*(VasicekPriceYield(r,tau=TBond-T,para=cal_para)))
#Total simulated value of the portfolio at the end of the time horizon

RiskFreeValue<-Value0/VasicekPriceYield(r0,tau=0.5,para=cal_para)
#The total wealth if all the money invested into 6 month bond

sorted_ValueT<-sort(ValueT)
CVaR<-mean(sorted_ValueT[1:length(ValueT)*0.01])



windows()
hist(ValueT,1000)
lines(rep(Value0,2),c(0,10^4), type="l",col="brown")
lines(rep(quantile(ValueT,0.01),2),c(0,10^4),type="l",col="red")
lines(rep(RiskFreeValue,2),c(-100,10^4),type="l",col="yellow")

res<-c(mean(ValueT),mean(ValueT)/Value0,
Value0-quantile(ValueT,0.01),Value0-CVaR)
names(res)<-c("MeanFinalWealth","MeanReturn","%1VaR","%1CVaR")

res
}
```

A.7. R Codes for Closed Form Calculation of Risk of a Bond Portfolio

```
require(matrixStats)
dataBond<-read.table(file="Bond.txt",header=TRUE)
#Historic Bond data contains 3 month-10 years bond rates until
29.11.2013
dataBond<-dataBond[1:6710,]
#Historic Bond data contains 3 month-10 years bond rates until
31.12.2009

param<-c(0.03036066, 0.19337252, 0.01371011)
CalBondPort<-function(r0,r_guessed,cal_para,TBond,T,alpha,CiB,priceyn=F)
{

Vasicek_closed_P <- function(para=param,r0,r_guessed,alpha,T=1/2,TBond)
{
 # exact simulation of n many of the short rate process and returns
 them as a matrix which have same elements on column
 # para ... parameters of the Vasicek model
 # r0 ... starting value of the interst rate process
 # n ... number of paths that are generated
 # T ... Time horizon
 mu=para[1]
 beta=para[2]
 sigma= para[3]
```

```
 r=rep(0,length(alpha))
 r[1]=r0
 r= r_guessed+(sigma*sqrt((1-exp(-2*beta*T))/(2*beta))*qnorm(alpha))
 op.mat<-matrix(1,nrow=length(alpha),ncol=length(TBond))
 r<-(t(r*op.mat))
 r }

r<-Vasicek_closed_P(para=param,r0=0.0046,r_guessed=0.005, alpha,T=1/2,
TBond)

VasicekPriceYield<-function(r,tau=tau,para=cal_para,priceyn=F){
# r ... r(t) current value of short rate
# tau ... vector holding the time to maturity
#para ... vector holding the parameters of the Vasicek model
 mu =para[1]
 beta =para[2]
 sigma =para[3]
 B=(1-exp(-beta*tau))/beta
 A=((B-tau)*(beta^2*mu-0.5*sigma^2)/beta^2 - sigma^2*B^2/(4*beta))
 if(priceyn){
  return(exp(A-B*r))
 }else return((r*B-A)/tau)
}

QiB<-CiB/VasicekPriceYield(r=r0,tau=TBond,para=cal_para,priceyn=1)
#QiB... is the vector holding quantity of each bond in the portfolio

r<-Vasicek_closed_P(para=param,r0=0.0046,r_guessed=0.01, alpha,T=1/2,
TBond)
```

```
Value0<-sum(CiB)

ValueT<-colSums(QiB*(VasicekPriceYield(r,tau=TBond-T,para=cal_para,
priceyn=1)))

RiskFreeValue<-(Value0)/VasicekPriceYield(r0,tau=0.5,para=cal_para,
priceyn=1)
#The total wealth if all the money invested into 6 month bond

plot(alpha,ValueT)
lines(rep(Value0,2),c(0,10^4), type="l",col="brown")
lines(rep(quantile(ValueT,0.01),2),c(0,10^4),type="l",col="red")
lines(rep(RiskFreeValue,2),c(-100,10^4),type="l",col="yellow")
VaR5<-Value0-ValueT[95]
VaR1<-Value0-ValueT[99]
res<-c(VaR5,VaR1)
names(res)<-c("VaR%5","VaR%1")
res
}
```

A.8. R Codes for Calculation of Daily Correlation Between Short-Rates and Stock Prices

```
require(quantmod)
dataBondD<-read.table(file="BondDailyRaw.txt",header=TRUE)
symbols<-c("^GSPC","AA","AET","APA","AXP","BA","BAX","BMY","CAT","CVX",
"DD","DIS","ED",
"EIX","EXC","F","GE","GT","HON","HPQ","HUM","IBM","KO","KR","LLY","LMT",
"LUV","MCD","MMM","MO","MRK","MSI","PBI","PG","SLB","UNP","UTX","WMT",
"XOM")
getSymbols(symbols,from="2000-01-03",to="2009-12-31",src="yahoo")
inc.r_m03<-dataBondD$m03[-1]-dataBondD$m03[-2396]
inc.r_m06<-dataBondD$m06[-1]-dataBondD$m06[-2396]
inc.r_y01<-dataBondD$y01[-1]-dataBondD$y01[-2396]
inc.r_y02<-dataBondD$y02[-1]-dataBondD$y02[-2396]
inc.r_y03<-dataBondD$y03[-1]-dataBondD$y03[-2396]
inc.r_y05<-dataBondD$y05[-1]-dataBondD$y05[-2396]
inc.r_y07<-dataBondD$y07[-1]-dataBondD$y07[-2396]
inc.r_y10<-dataBondD$y10[-1]-dataBondD$y10[-2396]

dGSPC<-as.vector(GSPC[days,4])
dAA<-as.vector(AA[days,4])
dAET<-as.vector(AET[days,4])
dAPA<-as.vector(APA[days,4])
dAXP<-as.vector(AXP[days,4])
dBA<-as.vector(BA[days,4])
```

```
dBAX<-as.vector(BAX[days,4])
dBMY<-as.vector(BMY[days,4])
dCAT<-as.vector(CAT[days,4])
dCVX<-as.vector(CVX[days,4])
dDD<-as.vector(DD[days,4])
dDIS<-as.vector(DIS[days,4])
dED<-as.vector(ED[days,4])
dEIX<-as.vector(EIX[days,4])
dEXC<-as.vector(EXC[days,4])
dF<-as.vector(F[days,4])
dGE<-as.vector(GE[days,4])
dGT<-as.vector(GT[days,4])
dHON<-as.vector(HON[days,4])
dHPQ<-as.vector(HPQ[days,4])
dHUM<-as.vector(HUM[days,4])
dIBM<-as.vector(IBM[days,4])
dKO<-as.vector(KO[days,4])
dKR<-as.vector(KR[days,4])
dLLY<-as.vector(LLY[days,4])
dLMT<-as.vector(LMT[days,4])
dLUV<-as.vector(LUV[days,4])
dMCD<-as.vector(MCD[days,4])
dMMM<-as.vector(MMM[days,4])
dMO<-as.vector(MO[days,4])
dMRK<-as.vector(MRK[days,4])
dMSI<-as.vector(MSI[days,4])
dPBI<-as.vector(PBI[days,4])
dPG<-as.vector(PG[days,4])
dSLB<-as.vector(SLB[days,4])
dUNP<-as.vector(UNP[days,4])
```

```
dUTX<-as.vector(UTX[days,4])
dWMT<-as.vector(WMT[days,4])
dXOM<-as.vector(XOM[days,4])

logreturn<- function(prices){
# it calculates and returns the log-returns of the input vector
# prices ... input vector of (daily) prices
n<-length(prices);
log(prices[-1]/prices[-n]) #returned value
}


gspc<-logreturn(dGSPC)
aa<-logreturn(dAA)
aet<-logreturn(dAET)
apa<-logreturn(dAPA)
axp<-logreturn(dAXP)
ba<-logreturn(dBA)
bax<-logreturn(dBAX)
bmy<-logreturn(dBMY)
cat<-logreturn(dCAT)
cvx<-logreturn(dCVX)
dd<-logreturn(dDD)
dıs<-logreturn(dDIS)
ed<-logreturn(dED)
eıx<-logreturn(dEIX)
```

```
exc<-logreturn(dEXC)
f<-logreturn(dF)
ge<-logreturn(dGE)
gt<-logreturn(dGT)
hon<-logreturn(dHON)
hpq<-logreturn(dHPQ)
hum<-logreturn(dHUM)
ıbm<-logreturn(dIBM)
ko<-logreturn(dKO)
kr<-logreturn(dKR)
lly<-logreturn(dLLY)
lmt<-logreturn(dLMT)
luv<-logreturn(dLUV)
mcd<-logreturn(dMCD)
mmm<-logreturn(dMMM)
mo<-logreturn(dMO)
mrk<-logreturn(dMRK)
msı<-logreturn(dMSI)
pbı<-logreturn(dPBI)
pg<-logreturn(dPG)
slb<-logreturn(dSLB)
unp<-logreturn(dUNP)
utx<-logreturn(dUTX)
wmt<-logreturn(dWMT)
xom<-logreturn(dXOM)
```

```
corDataD<-data.frame(gspc,aa,aet,apa,axp,ba,bax,
bmy,cat,cvx,dd,dis,ed,eix,exc,f,ge,gt,
hon,hpq,hum,ibm,ko,kr,lly,lmt,luv,mcd,
mmm,mo,mrk,msi,pbi,pg,slb,unp,utx,wmt,
xom,inc.r_m03,inc.r_m06,inc.r_y01,
inc.r_y02,inc.r_y03,inc.r_y05,
inc.r_y07,inc.r_y10)
cor(corDataD)



write.csv(cor(corDataD),file="DailyCorrelations.csv")
```

A.9. R Codes for Calculation of Weekly Correlation Between Short-Rates and Stock Prices

```
require(quantmod)
dataBondw<-read.table(file="BondWeeklyRaw.txt",header=TRUE)
symbols<-c("^GSPC","AA","AET","APA","AXP","BA","BAX","BMY","CAT","CVX",
"DD","DIS","ED",
"EIX","EXC","F","GE","GT","HON","HPQ","HUM","IBM","KO","KR","LLY","LMT",
"LUV","MCD","MMM","MO","MRK","MSI","PBI","PG","SLB","UNP","UTX","WMT",
"XOM")
getSymbols(symbols,from="1982-01-02",to="2009-12-31",src='yahoo')
getSymbols("^GSPC",from="1982-01-02",to="2009-12-31",src="yahoo")
inc.r_m03<-dataBondw$m03[-1]-dataBondw$m03[-1402]
inc.r_m06<-dataBondw$m06[-1]-dataBondw$m06[-1402]
inc.r_y01<-dataBondw$y01[-1]-dataBondw$y01[-1402]
inc.r_y02<-dataBondw$y02[-1]-dataBondw$y02[-1402]
inc.r_y03<-dataBondw$y03[-1]-dataBondw$y03[-1402]
inc.r_y05<-dataBondw$y05[-1]-dataBondw$y05[-1402]
inc.r_y07<-dataBondw$y07[-1]-dataBondw$y07[-1402]
inc.r_y10<-dataBondw$y10[-1]-dataBondw$y10[-1402]
wGSPC<-as.vector(GSPC[fridays,4])
wAA<-as.vector(AA[fridays,4])
wAET<-as.vector(AET[fridays,4])
wAPA<-as.vector(APA[fridays,4])
wAXP<-as.vector(AXP[fridays,4])
wBA<-as.vector(BA[fridays,4])
```

```
wBAX<-as.vector(BAX[fridays,4])
wBMY<-as.vector(BMY[fridays,4])
wCAT<-as.vector(CAT[fridays,4])
wCVX<-as.vector(CVX[fridays,4])
wDD<-as.vector(DD[fridays,4])
wDIS<-as.vector(DIS[fridays,4])
wED<-as.vector(ED[fridays,4])
wEIX<-as.vector(EIX[fridays,4])
wEXC<-as.vector(EXC[fridays,4])
wF<-as.vector(F[fridays,4])
wGE<-as.vector(GE[fridays,4])
wGT<-as.vector(GT[fridays,4])
wHON<-as.vector(HON[fridays,4])
wHPQ<-as.vector(HPQ[fridays,4])
wHUM<-as.vector(HUM[fridays,4])
wIBM<-as.vector(IBM[fridays,4])
wKO<-as.vector(KO[fridays,4])
wKR<-as.vector(KR[fridays,4])
wLLY<-as.vector(LLY[fridays,4])
wLMT<-as.vector(LMT[fridays,4])
wLUV<-as.vector(LUV[fridays,4])
wMCD<-as.vector(MCD[fridays,4])
wMMM<-as.vector(MMM[fridays,4])
wMO<-as.vector(MO[fridays,4])
wMRK<-as.vector(MRK[fridays,4])
wMSI<-as.vector(MSI[fridays,4])
wPBI<-as.vector(PBI[fridays,4])
wPG<-as.vector(PG[fridays,4])
wSLB<-as.vector(SLB[fridays,4])
wUNP<-as.vector(UNP[fridays,4])
```

```
wUTX<-as.vector(UTX[fridays,4])
wWMT<-as.vector(WMT[fridays,4])
wXOM<-as.vector(XOM[fridays,4])



logreturn<- function(prices){
# it calculates and returns the log-returns of the input vector
# prices ... input vector of (daily) prices
n<-length(prices);
log(prices[-1]/prices[-n]) #returned value
}

gspc<-logreturn(wGSPC)
aa<-logreturn(wAA)
aet<-logreturn(wAET)
apa<-logreturn(wAPA)
axp<-logreturn(wAXP)
ba<-logreturn(wBA)
bax<-logreturn(wBAX)
bmy<-logreturn(wBMY)
cat<-logreturn(wCAT)
cvx<-logreturn(wCVX)
dd<-logreturn(wDD)
dıs<-logreturn(wDIS)
ed<-logreturn(wED)
eıx<-logreturn(wEIX)
```

```
exc<-logreturn(wEXC)
f<-logreturn(wF)
ge<-logreturn(wGE)
gt<-logreturn(wGT)
hon<-logreturn(wHON)
hpq<-logreturn(wHPQ)
hum<-logreturn(wHUM)
ıbm<-logreturn(wIBM)
ko<-logreturn(wKO)
kr<-logreturn(wKR)
lly<-logreturn(wLLY)
lmt<-logreturn(wLMT)
luv<-logreturn(wLUV)
mcd<-logreturn(wMCD)
mmm<-logreturn(wMMM)
mo<-logreturn(wMO)
mrk<-logreturn(wMRK)
msı<-logreturn(wMSI)
pbı<-logreturn(wPBI)
pg<-logreturn(wPG)
slb<-logreturn(wSLB)
unp<-logreturn(wUNP)
utx<-logreturn(wUTX)
wmt<-logreturn(wWMT)
xom<-logreturn(wXOM)
```

```
corDataw<-data.frame(gspc,aa,aet,apa,axp,ba,bax,
bmy,cat,cvx,dd,dis,ed,eix,exc,f,ge,gt,
hon,hpq,hum,ibm,ko,kr,lly,lmt,luv,mcd,
mmm,mo,mrk,msi,pbi,pg,slb,unp,utx,wmt,
xom,inc.r_m03,inc.r_m06,inc.r_y01,
inc.r_y02,inc.r_y03,inc.r_y05,
inc.r_y07,inc.r_y10)
cor(corDataw)



write.csv(cor(corDataw),file="WeeklyCorrelations.csv")
```

A.10. R Codes for Calculation of Monthly Correlation Between Short-Rates and Stock Prices

```
require(quantmod)
dataBondw<-read.table(file="BondWeeklyRaw.txt",header=TRUE)
dataBondM<-dataBondw[(1:350)*4,]
symbols<-c("^GSPC","AA","AET","APA","AXP","BA","BAX","BMY","CAT",
"CVX","DD","DIS","ED",
"EIX","EXC","F","GE","GT","HON","HPQ","HUM","IBM","KO","KR","LLY","LMT",
"LUV","MCD","MMM","MO","MRK","MSI","PBI","PG","SLB","UNP","UTX","WMT",
"XOM")
getSymbols(symbols,from="1982-01-02",to="2009-12-31",src="yahoo")
inc.r_m03<-dataBondM$m03[-1]-dataBondM$m03[-350]
inc.r_m06<-dataBondM$m06[-1]-dataBondM$m06[-350]
inc.r_y01<-dataBondM$y01[-1]-dataBondM$y01[-350]
inc.r_y02<-dataBondM$y02[-1]-dataBondM$y02[-350]
inc.r_y03<-dataBondM$y03[-1]-dataBondM$y03[-350]
inc.r_y05<-dataBondM$y05[-1]-dataBondM$y05[-350]
inc.r_y07<-dataBondM$y07[-1]-dataBondM$y07[-350]
inc.r_y10<-dataBondM$y10[-1]-dataBondM$y10[-350]
end.month<-fridays[(1:350)*4]
mGSPC<-as.vector(GSPC[end.month,4])
mAA<-as.vector(AA[end.month,4])
mAET<-as.vector(AET[end.month,4])
mAPA<-as.vector(APA[end.month,4])
mAXP<-as.vector(AXP[end.month,4])
```

```
mBA<-as.vector(BA[end.month,4])
mBAX<-as.vector(BAX[end.month,4])
mBMY<-as.vector(BMY[end.month,4])
mCAT<-as.vector(CAT[end.month,4])
mCVX<-as.vector(CVX[end.month,4])
mDD<-as.vector(DD[end.month,4])
mDIS<-as.vector(DIS[end.month,4])
mED<-as.vector(ED[end.month,4])
mEIX<-as.vector(EIX[end.month,4])
mEXC<-as.vector(EXC[end.month,4])
mF<-as.vector(F[end.month,4])
mGE<-as.vector(GE[end.month,4])
mGT<-as.vector(GT[end.month,4])
mHON<-as.vector(HON[end.month,4])
mHPQ<-as.vector(HPQ[end.month,4])
mHUM<-as.vector(HUM[end.month,4])
mIBM<-as.vector(IBM[end.month,4])
mKO<-as.vector(KO[end.month,4])
mKR<-as.vector(KR[end.month,4])
mLLY<-as.vector(LLY[end.month,4])
mLMT<-as.vector(LMT[end.month,4])
mLUV<-as.vector(LUV[end.month,4])
mMCD<-as.vector(MCD[end.month,4])
mMMM<-as.vector(MMM[end.month,4])
mMO<-as.vector(MO[end.month,4])
mMRK<-as.vector(MRK[end.month,4])
mMSI<-as.vector(MSI[end.month,4])
mPBI<-as.vector(PBI[end.month,4])
mPG<-as.vector(PG[end.month,4])
mSLB<-as.vector(SLB[end.month,4])
```

```
mUNP<-as.vector(UNP[end.month,4])
mUTX<-as.vector(UTX[end.month,4])
mWMT<-as.vector(WMT[end.month,4])
mXOM<-as.vector(XOM[end.month,4])

logreturn<- function(prices){
# it calculates and returns the log-returns of the input vector
# prices ... input vector of (daily) prices
n<-length(prices);
log(prices[-1]/prices[-n]) #returned value
}

gspc<-logreturn(mGSPC)
aa<-logreturn(mAA)
aet<-logreturn(mAET)
apa<-logreturn(mAPA)
axp<-logreturn(mAXP)
ba<-logreturn(mBA)
bax<-logreturn(mBAX)
bmy<-logreturn(mBMY)
cat<-logreturn(mCAT)
cvx<-logreturn(mCVX)
dd<-logreturn(mDD)
dıs<-logreturn(mDIS)
ed<-logreturn(mED)
eıx<-logreturn(mEIX)
```

```
exc<-logreturn(mEXC)
f<-logreturn(mF)
ge<-logreturn(mGE)
gt<-logreturn(mGT)
hon<-logreturn(mHON)
hpq<-logreturn(mHPQ)
hum<-logreturn(mHUM)
ıbm<-logreturn(mIBM)
ko<-logreturn(mKO)
kr<-logreturn(mKR)
lly<-logreturn(mLLY)
lmt<-logreturn(mLMT)
luv<-logreturn(mLUV)
mcd<-logreturn(mMCD)
mmm<-logreturn(mMMM)
mo<-logreturn(mMO)
mrk<-logreturn(mMRK)
msı<-logreturn(mMSI)
pbı<-logreturn(mPBI)
pg<-logreturn(mPG)
slb<-logreturn(mSLB)
unp<-logreturn(mUNP)
utx<-logreturn(mUTX)
wmt<-logreturn(mWMT)
xom<-logreturn(mXOM)
```

```
corDataM<-data.frame(gspc,aa,aet,apa,axp,ba,bax,

bmy,cat,cvx,dd,dıs,ed,eıx,exc,f,ge,gt,

hon,hpq,hum,ıbm,ko,kr,lly,lmt,luv,mcd,

mmm,mo,mrk,msı,pbı,pg,slb,unp,utx,wmt,

xom,inc.r_m03,inc.r_m06,inc.r_y01,

inc.r_y02,inc.r_y03,inc.r_y05,

inc.r_y07,inc.r_y10)

cor(corDataM)



write.csv(cor(corDataM),file="MonthlyCorrelations.csv")
```

A.11. R Codes for Risk and Mean Return Calculations of Mixed Portfolios

```
require(quantmod)
require(matrixStats)
dataBond<-read.table(file="Bond.txt",header=TRUE)
#Historic Bond data contains 3month-10years bond rates until 29.11.2013
dataBond<-dataBond[1:6710,]
#Historic Bond data contains 3month-10years bond rates until 31.12.2009

symbols<-c("^GSPC","ED","EXC","CVX","PBI","DIS")
getSymbols(symbols,from="1982-01-01",to="2009-12-31",src="yahoo")
#the S&P500 index historical data

dataStock<-data.frame(GSPC[,4],ED[,4],EXC[,4],CVX[,4],PBI[,4],DIS[,4])
#gathering the daily closing prices of the stocks

logret<-matrix(0,length(dataStock[,1])-1,length(dataStock[1,]))
for(j in 1:length(dataStock[1,])){
    for(i in 2: length(dataStock[,1])){
    logret[i-1,j]<-log(dataStock[i,j]/dataStock[i-1,j])
    }}
#Calculating daily logreturns of the stocks

logret<-data.frame(logret)
#saving the logreturns as a data frame
```

```
muS<-colMeans(logret)
#Mean values of daily logreturns of each stock

sS<-colSds(logret)
#Standard deviation values of daily logreturns of each stock

R<-cor(logret)
#R... is the correlation matrix of stock logreturns

s0<-c(1115.1,45.43,48.87,76.99,22.76,32.25)
#the initial close values of the stocks on 29.11.2009

FinalValueMixedPort<-function(n,T,r0,r_guessed,MLE_para,cal_para,
TBond,CiB,s0,muS,sS,R,CiS){

#The function calculating final value of mixed portfolio containing
#stocks and bonds
#n...number of simulations
#T...is the investment period in years
#r0... is the initial short-rate
#r_guessed... is the predicted short-rate for the end of the period,
#MLE_para...Estimated (MLE) Vasicek Model Parameters
#beta and sigma
#cal_para...calibrated Vasicek Model Parameters
#TBond..is the vector holding time-to-maturities of the bonds in years
#CiB...is the vector holding the amount of money invested into Bonds.
s0...the initial prices of stocks
muS...vector holding the drift of logreturns of stocks
sS...vector holding the standard deviation of the logreturns of stocks
```

```
R...is the correlation matrix of logreturns of stocks
#CiS... is the vector holding the amount of money invested into stocks.

Zmatrix<-t(chol(R))%*% matrix(rnorm(n*length(muS)),ncol=n,
nrow=length(muS))
ndays<-T*258
sT<-t(s0*exp(muS*ndays+sS*sqrt(ndays)*Zmatrix))
sT
#The simulated stock prices for the end of the investment period

QiS<-CiS/s0
#QiS... is the vector holding quantity of each stock in the portfolio
QiB<-CiB/VasicekPriceYield(r=r0,tau=TBond,para=cal_para,priceyn=1)
#QiB... is the vector holding quantity of each bond in the portfolio

Value0<-sum(CiS)+sum(CiB)
#The invested amount of money at the begining of invested period.

r<-Vasicek_simul_P(n,T=1/2,MLE_para,r0=r0,r_guessed=r_guessed, TBond)
#simulated short-rates for the end of the investment period

ValueT<-colSums(QiB*(VasicekPriceYield(r,tau=TBond-T,para=cal_para,
priceyn=1)))+ colSums(QiS*t(sT))
#Total simulated value of the portfolio at the end of the time horizon
RiskFreeValue<-(sum(CiB)+sum(CiS))/VasicekPriceYield(r0,tau=0.5,
para=cal_para,priceyn=1)
#The total wealth if all the money invested into 6 month bond
```

```
sorted_ValueT<-sort(ValueT)
CVaR<-mean(sorted_ValueT[1:length(ValueT)*0.01])

windows()
hist(ValueT,1000)
lines(rep(Value0,2),c(0,10^4), type="l",col="brown")
lines(rep(quantile(ValueT,0.01),2),c(0,10^4),type="l",col="red")
lines(rep(RiskFreeValue,2),c(-100,10^4),type="l",col="yellow")

res<-c(mean(ValueT),mean(ValueT)-Value0,
Value0-quantile(ValueT,0.01),Value0-CVaR)
names(res)<-c("MeanFinalWealth","MeanReturn","%1VaR","%1CVaR")

res
}
```

APPENDIX B: MATRICES

B.1. Correlation Matrix of Daily Stock Prices and Bond Rates

	3 month	6 month	1 year	2 years	3 years	5 years	7 years	10 years
GSPC	-0.012	-0.003	0.001	-0.014	-0.013	-0.011	-0.007	-0.001
AA	0.012	0.003	-0.007	-0.016	-0.016	-0.018	-0.022	-0.018
AET	-0.007	0.002	0.003	-0.024	-0.024	-0.029	-0.029	-0.030
APA	0.009	0.025	0.030	0.025	0.029	0.033	0.024	0.028
AXP	-0.048	-0.017	-0.012	-0.014	-0.010	-0.008	-0.005	0.000
BA	-0.040	-0.034	-0.043	-0.037	-0.029	-0.017	-0.009	-0.004
BAX	0.012	0.032	0.008	0.011	0.010	0.022	0.027	0.032
BMY	0.028	0.037	0.034	0.002	-0.003	-0.005	-0.007	-0.007
CAT	-0.018	-0.002	0.001	-0.024	-0.019	-0.019	-0.015	-0.009
CVX	0.019	0.047	0.039	0.022	0.022	0.020	0.018	0.016
DD	-0.010	-0.007	-0.013	-0.026	-0.022	-0.019	-0.011	-0.003
DIS	-0.035	-0.027	-0.024	-0.032	-0.035	-0.024	-0.019	-0.011
ED	-0.033	-0.032	-0.038	-0.029	-0.021	-0.001	0.008	0.012
EIX	-0.003	0.012	0.017	0.003	0.008	0.020	0.024	0.022
EXC	0.008	0.018	0.012	-0.004	-0.002	0.001	-0.001	0.002
F	-0.059	-0.047	-0.041	-0.043	-0.048	-0.039	-0.027	-0.023
GE	-0.025	-0.032	-0.022	-0.023	-0.018	-0.026	-0.024	-0.022
GT	-0.086	-0.065	-0.033	-0.025	-0.022	-0.018	-0.013	-0.005
HON	-0.061	-0.049	-0.040	-0.047	-0.047	-0.040	-0.034	-0.029
HPQ	0.000	-0.007	-0.003	-0.008	-0.011	-0.016	-0.019	-0.019
HUM	0.008	0.006	0.008	-0.017	-0.022	-0.018	-0.016	-0.021
IBM	0.009	-0.003	-0.010	-0.016	-0.021	-0.011	-0.003	0.000

	3 month	6 month	1 year	2 years	3 years	5 years	7 years	10 years
KO	-0.033	-0.018	-0.028	-0.015	-0.015	-0.006	-0.004	-0.005
KR	-0.011	0.002	-0.025	-0.044	-0.042	-0.043	-0.042	-0.045
LLY	-0.028	-0.010	-0.008	-0.017	-0.024	-0.023	-0.022	-0.017
LMT	-0.032	-0.033	-0.030	-0.024	-0.022	-0.009	-0.008	-0.010
LUV	0.003	0.002	0.012	0.008	-0.006	-0.009	0.000	-0.002
MCD	0.000	0.010	-0.012	-0.015	-0.013	-0.006	-0.007	-0.004
MMM	-0.015	-0.021	-0.012	-0.023	-0.026	-0.026	-0.012	-0.012
MO	0.014	0.009	0.009	0.005	0.007	0.011	0.017	0.022
MRK	0.034	0.033	0.018	0.003	-0.003	0.002	0.009	0.009
MSI	-0.020	-0.014	-0.025	-0.044	-0.046	-0.040	-0.043	-0.044
PBI	0.007	-0.001	-0.014	-0.015	-0.019	-0.017	-0.019	-0.016
PG	-0.004	-0.026	-0.020	-0.011	-0.007	0.001	0.005	0.002
SLB	0.018	0.030	0.040	0.013	0.016	0.018	0.014	0.024
UNP	-0.025	-0.004	0.002	-0.005	-0.001	0.000	0.006	0.014
UTX	-0.033	-0.021	-0.027	-0.021	-0.022	-0.014	-0.013	-0.010
WMT	-0.030	-0.043	-0.040	-0.038	-0.039	-0.042	-0.037	-0.035
XOM	0.004	0.013	0.019	0.015	0.018	0.009	0.005	0.005

B.2. Correlation Matrix of Weekly Stock Prices and Bond Rates

	3 month	6 month	1 year	2 years	3 years	5 years	7 years	10 years
GSPC	0.010	-0.010	-0.014	0.013	0.006	0.000	-0.030	-0.040
AA	0.015	0.028	0.042	0.088	0.094	0.107	0.092	0.080
AET	0.023	0.001	-0.032	-0.052	-0.060	-0.063	-0.080	-0.077
APA	0.023	0.016	0.020	0.039	0.046	0.060	0.056	0.053
AXP	-0.015	-0.025	-0.027	-0.015	-0.021	-0.026	-0.052	-0.056
BA	0.041	0.034	0.033	0.045	0.034	0.029	0.010	0.001
BAX	0.011	0.018	0.018	0.010	0.007	-0.003	-0.018	-0.017
BMY	-0.012	-0.029	-0.047	-0.040	-0.046	-0.058	-0.073	-0.076
CAT	0.013	0.005	0.017	0.043	0.048	0.058	0.050	0.041
CVX	-0.004	-0.014	-0.018	0.001	0.006	0.008	-0.004	-0.010
DD	0.007	-0.005	-0.008	0.014	0.012	0.015	0.003	-0.002
DIS	0.050	0.061	0.064	0.080	0.083	0.079	0.051	0.044
ED	-0.037	-0.037	-0.048	-0.065	-0.084	-0.098	-0.121	-0.120
EIX	0.009	-0.002	0.000	0.001	-0.007	-0.015	-0.029	-0.028
EXC	-0.058	-0.075	-0.091	-0.093	-0.098	-0.097	-0.103	-0.103
F	-0.033	-0.037	0.003	0.039	0.044	0.045	0.031	0.012
GE	-0.008	-0.017	0.003	0.025	0.022	0.024	0.003	-0.002
GT	0.047	0.037	0.048	0.091	0.087	0.088	0.065	0.058
HON	0.011	0.005	0.016	0.038	0.044	0.039	0.023	0.017
HPQ	0.033	0.035	0.042	0.069	0.060	0.057	0.040	0.031
HUM	0.011	0.001	-0.009	-0.028	-0.025	-0.031	-0.049	-0.049
IBM	0.018	0.004	0.009	0.035	0.034	0.036	0.020	0.022

	3 month	6 month	1 year	2 years	3 years	5 years	7 years	10 years
KO	0.009	0.011	0.006	-0.004	-0.007	-0.016	-0.025	-0.027
KR	0.025	0.011	0.004	0.014	0.008	0.000	-0.007	-0.016
LLY	0.049	0.027	0.006	0.000	-0.013	-0.023	-0.041	-0.043
LMT	0.013	0.005	-0.012	-0.015	-0.028	-0.033	-0.045	-0.031
LUV	0.026	-0.004	-0.008	-0.002	-0.011	-0.010	-0.027	-0.031
MCD	-0.035	-0.045	-0.019	-0.002	-0.004	-0.008	-0.016	-0.016
MMM	0.019	0.015	0.021	0.034	0.036	0.038	0.028	0.021
MO	0.021	0.025	0.027	0.015	0.001	-0.009	-0.016	-0.024
MRK	-0.021	-0.057	-0.066	-0.064	-0.063	-0.054	-0.071	-0.067
MSI	0.040	0.044	0.044	0.071	0.072	0.078	0.072	0.069
PBI	-0.024	-0.048	-0.058	-0.056	-0.059	-0.064	-0.072	-0.081
PG	0.020	0.015	0.002	-0.003	-0.009	-0.014	-0.016	-0.026
SLB	0.007	-0.006	-0.001	0.026	0.032	0.039	0.033	0.027
UNP	-0.003	-0.009	-0.013	-0.011	-0.013	-0.005	-0.019	-0.020
UTX	0.014	0.000	-0.005	0.005	0.001	-0.001	-0.025	-0.026
WMT	0.009	-0.008	-0.016	-0.021	-0.027	-0.034	-0.036	-0.047
XOM	-0.014	-0.036	-0.019	0.000	0.000	0.002	-0.003	-0.004

B.3. Correlation Matrix of Monthly Stock Prices and Bond Rates

	3 month	6 month	1 year	2 years	3 years	5 years	7 years	10 years
GSPC	0.014	-0.055	-0.065	-0.053	-0.066	-0.090	-0.126	-0.143
AA	0.003	-0.004	0.025	0.076	0.085	0.075	0.058	0.043
AET	0.041	-0.020	-0.060	-0.103	-0.129	-0.156	-0.186	-0.184
APA	0.106	0.059	0.041	0.043	0.050	0.037	0.024	0.012
AXP	-0.006	-0.062	-0.079	-0.110	-0.129	-0.142	-0.161	-0.174
BA	0.103	0.075	0.083	0.092	0.077	0.063	0.027	0.016
BAX	-0.028	-0.044	-0.040	-0.053	-0.053	-0.061	-0.084	-0.079
BMY	-0.065	-0.089	-0.090	-0.092	-0.101	-0.118	-0.137	-0.144
CAT	-0.016	-0.019	0.013	0.047	0.050	0.048	0.036	0.023
CVX	0.010	-0.019	-0.027	-0.004	0.005	-0.004	-0.014	-0.021
DD	0.009	-0.023	-0.022	-0.012	-0.008	-0.018	-0.026	-0.031
DIS	0.047	0.048	0.046	0.079	0.090	0.090	0.079	0.073
ED	0.091	0.044	-0.002	-0.059	-0.097	-0.126	-0.134	-0.131
EIX	-0.013	-0.023	-0.021	-0.029	-0.046	-0.059	-0.078	-0.079
EXC	-0.106	-0.154	-0.198	-0.240	-0.259	-0.277	-0.294	-0.291
F	-0.065	-0.080	-0.045	-0.003	-0.004	-0.024	-0.041	-0.071
GE	-0.086	-0.110	-0.093	-0.097	-0.101	-0.101	-0.120	-0.131
GT	0.007	-0.032	-0.025	-0.008	-0.010	-0.027	-0.040	-0.059
HON	0.001	-0.033	-0.020	0.011	0.018	0.009	-0.011	-0.015
HPQ	0.024	0.007	0.010	0.034	0.022	0.012	-0.005	-0.022
HUM	0.047	-0.022	-0.038	-0.060	-0.059	-0.074	-0.095	-0.094
IBM	-0.025	-0.048	-0.041	-0.028	-0.025	-0.027	-0.040	-0.034

	3 month	6 month	1 year	2 years	3 years	5 years	7 years	10 years
KO	0.020	0.001	0.006	0.004	0.004	-0.001	-0.006	0.002
KR	-0.060	-0.081	-0.093	-0.084	-0.084	-0.083	-0.082	-0.086
LLY	0.035	0.007	-0.002	-0.002	-0.014	-0.036	-0.060	-0.068
LMT	0.023	0.009	-0.012	-0.038	-0.043	-0.062	-0.076	-0.065
LUV	0.027	-0.009	-0.013	0.002	-0.004	-0.015	-0.029	-0.043
MCD	0.044	0.046	0.057	0.052	0.029	0.017	-0.003	-0.012
MMM	-0.033	-0.050	-0.060	-0.067	-0.065	-0.060	-0.069	-0.064
MO	0.082	0.052	0.042	0.010	0.003	-0.009	0.001	0.001
MRK	-0.058	-0.101	-0.116	-0.138	-0.149	-0.153	-0.164	-0.156
MSI	0.094	0.043	0.032	0.052	0.048	0.051	0.039	0.033
PBI	-0.110	-0.136	-0.120	-0.089	-0.091	-0.107	-0.124	-0.138
PG	-0.072	-0.071	-0.061	-0.047	-0.035	-0.036	-0.035	-0.040
SLB	0.066	0.048	0.044	0.071	0.087	0.078	0.068	0.062
UNP	-0.027	-0.078	-0.079	-0.064	-0.059	-0.048	-0.046	-0.036
UTX	0.011	-0.038	-0.042	-0.035	-0.037	-0.034	-0.053	-0.043
WMT	0.045	0.012	0.005	0.000	-0.020	-0.039	-0.045	-0.070
XOM	-0.047	-0.070	-0.081	-0.067	-0.070	-0.076	-0.083	-0.083

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