REVENUE MANAGEMENT WITH DYNAMIC PRICING IN COMPETITIVE MARKET

by

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ABSTRACT

REVENUE MANAGEMENT WITH DYNAMIC PRICING IN COMPETITIVE MARKET

In this study, we formulate the seasonal product-pricing problem as a dynamic programming model analytically and discuss some structural properties of the optimal policy and the optimal value function. We consider discrete time dynamic pricing model where a seller needs to sell multiple items over a finite time horizon when the firms adjust their prices in accordance with the competition, on hand inventory and the time remaining in both monopolistic and oligopolistic market environment. Demands are Bernoulli process with probability lambda; for cases in which customer reservation prices follow normal distribution, we derive the optimal policy in a closed form. Upon arrival, a customer either purchases one unit of item if the posted price is lower than his/her reservation price, or leaves empty-handed and maybe wait for an appropriate posted price. After purchasing the item, some of the customers, who have bought an item, will return one unit of the item to the seller with binomial distribution for a full refund. We assume that a returned item can be resold to the future customers. The product's price needs to be adjusted dynamically to incorporate new demand information, to balance supply with demand over the sale horizon in order to maximize the expected total revenue when the sale horizon ends. We conduct numerical studies to develop insights on the sensitivity of the optimal policies to the various probability parameters and to evaluate the performance of expected revenue function under different arrival probability, price coefficient, cancelation probability and initial inventory levels in both monopolistic and oligopolistic market with full refund. The majority of research published in the literature, assumes that the market is monopolistic. Our numerical results show that, competition and cancelation are important issues to consider that affects the expected revenue.

ÖZET

REKABET ORTAMINDA DİNAMİK FİYATLANDIRMA İLE GETİRİ YÖNETİMİ

Bu çalışmada, sezonluk bir ürünün dinamik programlama kullanarak analitik olarak fiyatlandırılması, en uygun politika ve hedef fonksiyonun bazı yapısal özellikleri incelenmiştir. Satıcının ürünlerinin fiyatlarını, kesikli ve sonlu bir zamanda rekabet dikkate alındığında, elindeki stokuna ve satış için kalan zamanına bağlı olarak dinamik fiyatlandırması üzerinde çalışılmıştır. Müşterilerin rezervasyon fiyatlarının normal dağılım, taleplerin Bernoulli dağılımı ile lamda olasılıkla olduğu varsayıldığında değer fonksiyonu yazılmıştır. Müşteri geldiği taktirde, eğer sunulan ürün fiyatı kendi rezervasyon fiyatından düşükse ya bir birim ürünü alır yada daha uygun bir fiyatı beklemek üzere ürünü almaz. Bazı müşteriler almış oldukları bir birim ürünü Binom dağılıma göre iade edip tam para iadesi isteyebilirler. İade olan ürün sezon bitinceye kadar gelecek olan müşterilere tekrar satılabilir. Ürünün fiyatı satış süresi bitinceye kadar beklenen toplam geliri maksimize etmek, yeni talepleri karşılamak ve dengelemek için dinamik olarak ayarlanmalıdır. Hedef fonksiyonun çeşitli parametrelere duyarlılığını incelemek, rekabet ortamında ve tekel pazarda tam para iadesi olduğu varsayıldığında farklı talep, fiyat duyarlılığı, iptal olasılığı ve başlangıç stok miktarlarında beklenen getiriyi maksimize etmek için bazı nümerik çözümler yapılmıştır. Litaretürde basılı olan çalışmaların çoğunluğu pazarın tekel olduğunu varsayar. Matematiksel çözümlerimiz rekabet ve yapılan iadenin dikkate alınması gereken konular olduğunu ve beklenen getiriyi ne şekilde etkilediğini gösterir.

TABLE OF CONTENTS

LIST OF FIGURES

LIST OF TABLES

LIST OF SYMBOLS/ABBREVIATIONS

$\mathbf{k} = (k_1, k_2)$	On hand inventory vector
k_i	On hand inventory for firm i
$\mathbf{p} = (p_1, p_2)$	Price vector
$\mathbf{p}^* = (p_1^*, p_2^*)$	Optimal price vector
p_i	Posted price by firm i
p_i^*	Optimal posted price by firm i
q	Cancelation probability
$q_i(\mathbf{p})$	Choice probability for firm i
s_i	Initial inventory level for firm i
t	Remaining time
T	Sale horizon
$U_i(\mathbf{k}, t)$	Expected revenue for firm i
X_i	Utility of a customer from purchasing one unit of product
Z_i	Gumbel random variable for i=0, ..., n
α_i	Quality, brand image and the popularity of firm i's product
β	Price response coefficient
γ	Euler's constant
λ	Arrival probability
GDP	Gross Domestic Product
i.i.d.	independently and identically distributed
MNL	Multinomial Logit Model
PLC	Product Life Cycle
TUIK	State Statistics Institute

1. INTRODUCTION

Revenue management or yield management is concerned with dynamic pricing of perishable products. The "perishability" of the products means that the product has short selling periods (seasonality of the product), during which inventory management and pricing strategies are central while the replenishment of inventory is prevented. This is a realistic trend since most products such as medicine; dairy products and chemicals start to deteriorate once they are produced. The inventory of seats on a particular flight, the inventory of rooms at a hotel at a particular night, the tables at the restaurant all perish at certain times. Retailers and service providers have the opportunity to enhance their revenues through optimal pricing of their perishable products that must be sold within a fixed period of time. Therefore, the seller has a fixed inventory on hand and must decide on how to price the product over the remaining selling horizon.

Firms aim to do their best in terms of their performance criteria. They may have different objectives related with their profits, their market shares, service levels and their operating costs. In order to achieve these objectives, firms can follow different strategies such as, more efficient transportation, marketing strategies and more profitable manufacturing methods.

Better demand management via dynamic pricing policies becomes an important goal to control revenue. There should be some strategy that will dynamically adjust prices to sell the right products to each right customer at the right price that the customer is willing to pay (perfect price discrimination) at the right time. The main idea behind revenue management is to divide the market into multiple customer classes and to provide different types of products with different prices to each class (market segmentation). There is a trade-off when setting prices. If the retailers set the prices too low, they will loose customers' surplus; if they set the prices too high, they will lose some customers and will have the risk of having product surplus at the end of the horizon. However, it is almost impossible for a firm to know each individual's valuation for each product. Even if it is known, it will not be fair to charge each customer differently.

Customers' knowledge about a product can affect their decision to purchase and their willingness to pay. In general, customers are less informed about a product at the beginning of its life cycle and a retailer can influence future demand via word-of-mouth (from previous sales) and advertisement.

Purchasing behavior of the customers affects the retailer's pricing decisions over time. Customers are grouped as myopic and strategic in accordance with their purchasing behavior. Myopic customers purchase the product immediately if the price is below their reservation price, without considering future prices. Strategic (rational) customers consider possible future prices of the product when making decision. Therefore, the seller should consider carefully the effects of his price over customers' current and future decisions. Dynamic pricing decisions of a seller facing strategic customers is more complex since the seller has to consider the effects of future as well as current prices on customers' purchasing decisions.

Reservation price is the most important factor that defines the purchasing behavior. It is the maximum amount that the customer is willing to pay for a particular product. Posted price is the price that is offered by the firm to maximize their revenues over the planning horizon, while taking the heterogeneity of the population into consideration. If the posted price is lower than the reservation price of the customer, the customer buys the product otherwise she does not. The difference between the maximum amount that the customers are willing to pay for the product and the amount they actually pay is called customers' surplus. This difference represents the customers' gain. It is usually assumed that the reservation price is a random variable with a continuous distribution over a population of customers and this distribution may change over time. The reasons for the variance in the reservation prices can be stated as the heterogeneity in the market (difference in income, age etc.) and a lack of information about the customer's tastes and needs.

Almost all of the studies conclude that the initial level of the inventory affects the revenue. If the retailer has excess initial inventory, the prices should be set low compared with threshold price of the customer, to sell out all of the on hand inventory. On the other hand, if the initial inventory is low and demand is known to be higher than the on hand inventory, the prices should be set high compared with threshold price of the customer. In that case, the products will be sold to customers with higher reservation prices. Low prices with low initial inventory lead to the loss of the customer surplus. The retailer will then experience lost sales, loss of goodwill and decreases in market share since the inventory will be depleted before the horizon ends.

The arrival process of customers is another factor that affects the retailers' pricing decisions. Campaigns and promotions can affect the arrival pattern of the customers. Due to the higher arrival rate, the probability of having customers with higher reservation prices increases and high-price products are sold. However, when the arrival probability is small, it is more convenient to set the low prices. Otherwise, the small number of arriving customers will not purchase the product and this will result in excess on-hand inventory also loss of the customer.

Another factor that affects the pricing decision of the seller is the length of the planning horizon. If the planning horizon is short, then the initial price should be set lower. On the other hand, if we have a long planning horizon, we can set the initial price higher. So, we can get the customer surplus.

Pricing of a product in a competitive market is more difficult than in a monopolistic one. In the absence of direct competition, it can be estimated how a price change will affect sales simply by analyzing buyers' price response. However, if there is competition, the firms can change buyers' alternatives by changing their prices.

Most firms sell multiple products. The effect of one product's sales on another's can be either adverse or favorable. If adverse, then the products are "substitutes", currently offered by a number of industries including air cargo, tour operators and Internet stores. Most substitutes are different brands in the same product class. If one product's sales favorably affect the sales of another, then the products are “complements”.

Incorporating business rules is disconnection between the most of academic literature and the practice in pricing decisions. While e-commerce makes it technically possible for sellers to change prices as often as desired, historical pricing behavior and market norms usually imply that there are a limited number of prices that are used in the marketplace. If the seller faces strategic customers, committing to a rule, such as infrequent and small price drops might encourage purchases earlier in the time horizon and hence increase the seller’s profits.

Business rules have restriction such as:

- The allowed number and frequency of markdowns.
- Min-max discount levels or maximum lifetime discount.
- The minimum number of weeks before an initial markdown can occur.
- The types of markdowns allowed.

In addition to the main characteristics discussed above, there are other factors that can influence a dynamic pricing policy, such as seasonality, salvage value and external shocks to demand.

The rest of the thesis is organized as follows. In Chapter 2, an extensive literature review about revenue management, and in Chapter 3, revenue management and also dynamic pricing concepts are presented. In Chapter 4, the problem under consideration is defined and its formulated model is introduced. Depending on model variables and restrictions, the objective function of expected revenue is obtained. In Chapter 5, numerical results are given. Finally conclusions, general results and extensions of this study are stated in Chapter 6.

In this study, products are assumed to be perishable, so cannot be replenished and need to be sold out in a finite time. Therefore, the retailer has a fixed inventory of products at the starting period. The prices are set at the beginning of the period and are adjusted until the end of the sale horizon in accordance with the intensity of customer arrivals, cost of price changing, on hand inventory, cancelation probability, price response of the customers, customer reservation price, firm's reputation, the length of the planning horizon, behaviors of the competitors and prices of the substitutable products. Cost of price update is closely related with advances in information technologies, so it directly affects revenue management success. There is no published paper that concerns multiple items in competitive market with cancelation. We have extended existing models to see the performance of optimal policy function.

2. LITERATURE REVIEW

Dynamic pricing has been studied and developed extensively in the literature of the revenue management. Kincaid and Darling introduced dynamic pricing in 1963 [1]. The revenue management has been developed from the activity of airlines selling discounted price tickets, which started in the 1970's. Many of the earlier studies in the literature concentrated on the problem of knowing when to reject the request for discounted seats from lower fare class customers to reserve some seats for higher fare class customers in order to maximize profits.

There are numerous of research papers in the literature dealing separately with dynamic pricing or fixed number of price changes of perishable products, timing and optimal duration of price changes also exclusively dealing with the problem of overbooking, cancelation, no-shows while discritizing the continuous time interval to solve the model more efficiently. Monopolistic dynamic pricing models have been well studied in the literature by Gallego and van Ryzin [5] [8] and Lin [13]. In the rest of this section, we will review the most frequently referred studies related with pricing of perishable products in the context of revenue management.

Bitran and Leong [2], Badinelli and Olsen [3] and Bitran and Mondschein [4] studied the actual hotel sales and reservations planning problem in which stays are not limited to single days and with multiple room types. They introduced the concept of customer segmentation. Previous studies considered the capacity allocation and the yield management problems independently. In this paper, they showed how they could be coordinated. They also provided rules to assist in accepting reservations. The rules can be applied to sales management and control of discount offers. The models demonstrated that the use of customer segmentation can increase profits.

Gallego and van Ryzin [5] proposed a model where customers arrive in accordance with a Poisson process, whose reservation prices are i.i.d. (independently and identically distributed). The retailer was allowed to change the price in real-time in order to maximize the expected total revenue. In this model, the optimal policy was shown in closed form only where the customer arrival rate was defined as a function of a posted price and the reservation price is exponentially distributed, but not in general. The researchers studied the structural properties and proposed the fixed price heuristic policy, which is asymptotically optimal as the expected sales volume tends to infinity. However, this heuristic cannot address adequately the situation when inventory is relatively small compared to the number of customers. They report that their policy provides revenue that is only five per cent to 12 per cent below the optimal revenue when the number of items is fewer than 10 and it is nearly optimal for more than 20 items.

Bitran and Wadhwa [6] defined the seasonal product pricing problem as the problem of determining the dynamic optimal pricing policy for a retail product under a limited sale period and a fixed stock of product. In this paper, they proposed and investigated some conjectures about the structure of the problem. They considered both deterministic and stochastic demand versions of the problem. The first three conjectures state that the optimal price is non-increasing in the level of inventory and non-decreasing in the level of demand, and the marginal revenue function is non-increasing in the level of inventory. The fourth conjecture states that the retailer does not benefit from reserving some stock for future periods if the demand for the product is known to decrease over time. They showed through a series of counter-intuitive examples that these conjectures are not true in general. They derived a set of sufficient conditions under which the conjectures can be guaranteed to hold and present examples of some demand functions and distributions, which satisfy these conditions.

Badinelli [7] focused on the decision problem given accurate estimates of demand rates for the Markov demand processes from each market segment, leaving the forecasting issue as a separate research topic. The treatment of unknown demand rates necessarily involves adaptive policies. The model developed in the paper was intended as a base upon which to build an adaptive model.

Gallego and van Ryzin [8] studied a multi-product problem where demand for each product was a stochastic point process with an intensity that was a function of the vector of prices for products and the time at which these prices were offered. An upper bound on the optimal expected revenue was established by analyzing a deterministic version of the problem.

Mcgill and van Ryzin [9] reviewed research on transportation revenue management. In their research, there were developments in forecasting, overbooking, seat inventory control and pricing. They supplied a brief overview about revenue management and extensions. Their survey included a glossary of revenue management terminology and a bibliography of over 190 references.

Elmaghraby and Keskinocak [10] provided a review of the literature and current practices in dynamic pricing. Their focus was on dynamic pricing in the presence of inventory considerations. This paper constituted a good summary for dynamic pricing policies.

An [11] considered continuous time dynamic pricing model, where a seller needed to sell a single item, over a finite time horizon. The objective of the seller was to dynamically adjust the price in order to maximize the expected total revenue when the sale horizon ends and cancelation was taken into account. By discretizing both time and the allowable price set, they try to approximate the continuous model. Then an algorithm was presented for this discrete time model.

Lin [12] studied a sequential dynamic pricing model where a seller sold a given stock to a random number of customers and the seller knew the distribution of the total number of future customers. They formulated the seller's problem as a stochastic dynamic programming model and developed an algorithm to compute the optimal policy when the total number of future customers was distributed such as deterministic, geometric, and bounded. They then applied the results from this sequential dynamic pricing model to the case where customers arrived according to a continuous-time point process, developed an optimal heuristic policy and showed that the numerical results were nearly optimal.

Lin and Li [13] then studied continuous time point process. They developed a dynamic pricing policy that applied to arbitrary customer choice models and arbitrary customer arrival process. Then some numerical examples were demonstrated to show the efficiency of the policy.

Anjos et al. [14] presented a family of continuous pricing functions for which the optimal pricing strategy could be explicitly characterized and easily implemented. These pricing functions were the basis for a general pricing methodology, which was particularly well suited for application in the context of an increasing role for the Internet as a means to market goods and services.

Lin and Sibdari [15] developed a game-theoretic model to describe real-time dynamic price competitions between firms that sell substitutable products, and use the model to predict the market equilibrium. Based on findings from the model, they developed a heuristic dynamic-pricing strategy with which a firm did not need to keep track of its competitors' inventory levels. They used numerical examples to demonstrate the benefits of incorporating this game-theoretic model to set prices and the efficiency of the heuristic strategy.

In this section, we reviewed a limited number of papers related with pricing of perishable products. Solution techniques for revenue management problems that are used are Markov decision processes, mathematical programming approaches, multiple vs. single optimization and hybrid formulation.

3. WHAT IS REVENUE MANAGEMENT?

What is known as revenue management (or yield management, or perishable asset revenue management) is as old as business itself. The main objective is to respond to whom which price to ask, which offers to accept, when to offer a lower price, whether to try selling at a later point in time or in a different market. Seller can issue full price revenue by protecting some of the products for full fare customers or can sell the products for lower fare customers by discounted price as shown in Figure 3.1. The seller experiences the trade-off between the revenue loss and spoilage loss; the revenue loss associated with selling an item at a low price and losing the opportunity to sell it at a higher price and waiting for higher price customers then losing the opportunity to sell the item to lower price customers. It is literally the tactical approach for pricing items dynamically differently during the sale horizon to capture customer demand (customer willingness to pay) and maximize profits. It has gained an increasing popularity in also retail settings and academic research recently.

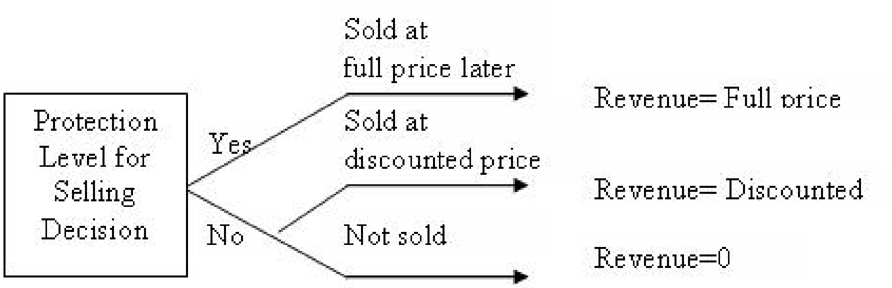

Figure 3.1. Protection level for selling decision

The main focus in revenue management research has been on the allocation of limited capacity to different demand classes while maximizing profits, when the fundamental decision is whether to accept or reject the demand. Perishable products have largely fixed short-term costs and small variable costs per customer; thus, in most situations, it is sufficient to seek policies that maximize revenues. There is lower risk in accepting a current booking request than in waiting for later possible bookings. In this domain, most of the literature on pricing decisions focuses on industry-level trends and implications, rather than operational level decisions [10].

American Airlines had revenue increase of $500 million per year and Delta Airlines uses similar systems to generate additional revenues of $300 million per year by using revenue management techniques [16]. Airlines are the oldest users of revenue management. However, these practices also appear in other service industries and used more commonly in reservation-based industries with perishable inventories by an increasing number of hotels, travel, car rental companies, cruise ship lines even by restaurants.

Also manufacturers are using revenue management techniques to increase profits. Manufacturing capacity is perishable, too. Retailers have recently begun to adopt revenue management, especially in the fashion apparel, consumer electronics and toy sectors. Retail demand is highly volatile and uncertain, customers' valuations change rapidly over time and with short selling seasons, long production and distribution lead times, and also supply is quite inflexible. The typical revenue management techniques are not sufficient for handling retail fashion inventory because of the unit of inventory that vary and must be replaced by complex non-linear techniques to appropriately model, forecast and optimize the price. The markdown optimization solution process begins with a bottom-up analysis of historical sales and inventory behavior also consumer response to markdowns [17]. The energy sector has been a recent adopter of revenue management methods as well, principally in the area of managing the sale of pipeline capacity for gas transportation. Again, energy demands are volatile, uncertain and the technology for generating and transmitting electricity and gas can be quite inflexible.

The common characteristics of these industries are the perishability of the products, demand variability, uncertainty, customer heterogeneity and small marginal cost differences. Most are subject to some sort of supply or production inflexibility. Firms are now utilizing increasingly sophisticated revenue management techniques such as; market segmentation, price discrimination, overbooking and inventory control. In the airline industry, economy class customers are early buyers who are not willing to pay as much as the business class customers, who usually purchase later. On the other hand, in the fashion industry, early buyers are willing to pay higher prices than the bargain hunters, who usually shop towards the end of the season. Although dynamic pricing may be either infeasible or costly to apply at an extreme degree, that firms change their prices number of times during the selling period.

Revenue management determines and updates the availabilities of different products to different customer segments, variable costs, opportunity costs, price responsiveness by segment and strategic goals subject to business and physical constraints in order to maximizing operating profit as shown in Figure 3.2 and 3.3.

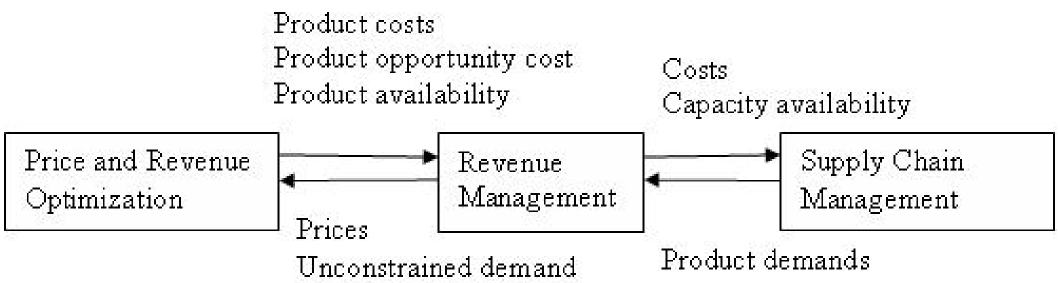

Figure 3.2. Price and revenue optimization

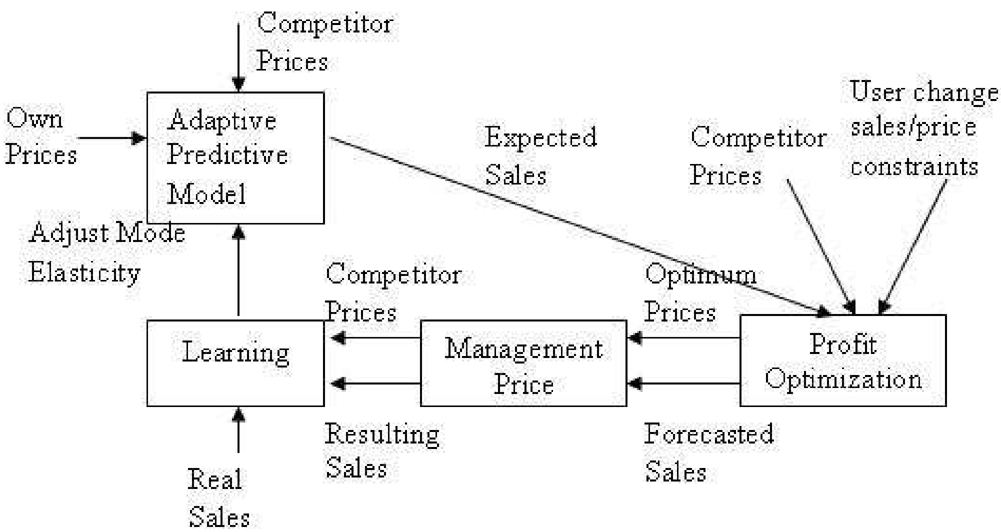

Figure 3.3. Revenue management concept

Revenue management addresses three basic categories of demand management decisions such as; what information how frequently should be exchanged and what the reliability and stability of the system is.

- Structural Decisions (strategic decisions): What products to create; which selling format to use; which segmentation to use; which terms of trade to offer (discounts, cancellation/refund options); brand image,
- Pricing Decisions (controllable and affected by uncertainty): How to set posted prices; how to price across product categories; how to price over time; how to markdown (discount) and promotion decisions over the PLC (Product Life Cycle);
- Quantity Decisions (uncertainty): Whether to accept or reject an offer, how to allocate capacity to different segments, how to bundle products.

Demand in three dimensions, a single cell in the Figure 3.4,indicates a particular customer's valuation for a particular product at a particular point in time. Revenue management addresses the pricing, timing, structural and quantity decisions a firm makes in trying to exploit the potential of this multidimensional "demand landscape" [18].

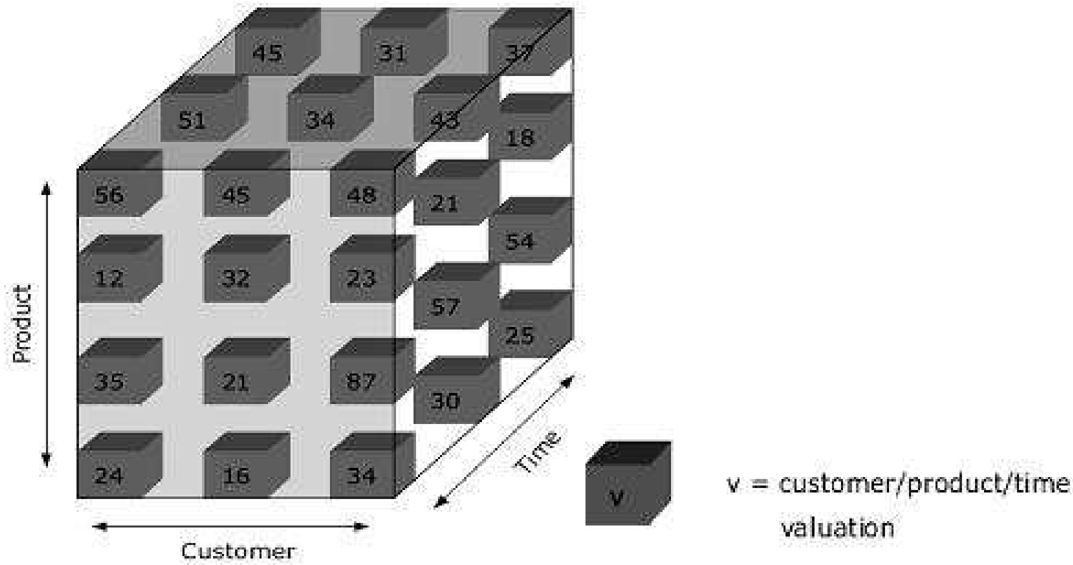

Figure 3.4. Conceptual view of a firm's demand landscape

3.1. What Does a Seasonal Product Mean and What are Its Features?

Once the initial order is placed, they often have no flexibility in placing reorders. The seasonal product-pricing problem can be defined to find dynamic optimal pricing policy for retail products that include a fixed stock of product and a limited sale period.

3.1.1. Perishability and Production Inflexibility

It is expensive or impossible to store excess resource. For short life cycle products, replenishing inventory during the selling season might not be possible (which is often the case with seasonal products), due to long procurement lead times.

In this case, the retailer needs to make pricing decisions given a fixed amount of inventory. The production inflexibility characteristics are shared by service industries, such as; hotels, cruise ship lines, car rental companies, theaters, airlines and sporting venues, radio / TV broadcasters. For other products, inventory can be replenished over time to satisfy the demand.

3.1.2. Limited Sale Period

Seasonal products have limited sale horizon so limited life. The seller should set prices to sell out the products until the end of sale horizon to maximize their revenues.

3.1.3. Salvage Value

Products remaining unsold at the end of the sale horizon are disposed with little or no salvage value. The salvage value is also a decision variable, depending on the choice of the seller among multiple liquidation channels.

3.1.4. Customer Heterogeneity

Customers are heterogeneous in their valuations. Customers for air travel and hotels have widely varying patterns of usage and behavior in terms of when they purchase, how flexible their plans are, and how they place valuations on the need of consumption. The firm can differentiate among customer segments, and each segment has a different demand curve. Purchase restrictions and refundability requirements help to segment the market between leisure and business customers.

3.1.5. The Threshold Price

The threshold price plays a central role in revenue management. The threshold price shows each customer's financial situation, the strength of his/her desire and the value that the customer places on the service.

3.1.6. Demand Variability and Uncertainty

Demand might follow a different pattern over time due to factors such as holidays, seasons and weekends. It is often not feasible to predict demand accurately at the beginning of the season and due to seasonality or shocks, more uncertainty occurs about future demand. Commitments need to be made when future demand is uncertain. Protection from low fare customers should be done before knowing how many high fare customers will arrive.

3.1.7. External Shocks to Demand

External events, which are out of control, might influence the demand for certain products. For example, an unexpected weather condition, a "global" or an economic event might decrease the demand for many goods, including travel services. So demand forecasting can be so hard to predict.

3.1.8. Cross-Elasticity

The demand of a product might depend on the other complement and substitute products. The demand for the private brand would depend not only on the price of the private brand, but also on the price of the generic brand.

3.1.9. Price as a Signal of Quality

Revenue management is more suitable for the products where price is not a status symbol and not a significant signal of value. The same unit of capacity can be used to deliver many different products or services. For instance, hotel rooms are essentially the same, no matter whether they are used by business or leisure travelers.

3.1.10. Information Systems Infrastructure and Business Rules

Depending on the sales channel, changing prices might be costly. Attempting to apply revenue management in industries that do not have databases or transactions systems can be time consuming, expensive and risky. In contrast, changing prices on the Internet can be done virtually at no cost and just with a mouse click.

3.2. A Brief Overview of Dynamic Pricing

3.2.1. What is Dynamic Pricing?

Dynamic pricing is a business strategy that adjusts the price of a product in real time in order to allocate the right service to the right customer at the right time order to maximize profit as shown in Figure 3.5. Clearly, the ability of the seller is to maximize revenues. Ideally, if the seller can update the sales price continuously, with no cost incurred for price changes, then it would be optimal to set the price exactly to the known customer's valuation of the product at each particular moment. The strategy we propose in this thesis is particularly suitable for companies who sell products that cannot be replenished and unsold products have little or no salvage value, such as; airline seats, hotel rooms, restaurants and even tickets.

Top management with market share and profit targets, marketing department with market knowledge and finance department with cost and performance monitoring, decides pricing strategies and optimize the expected revenue over time. By analyzing demand elasticity in terms of products, geographies, sales channels and prime customer segments, firms usually discover an "indifference zone" around the average market price for a product. Within this zone, customers are indifferent to small pricing changes, whereas outside the zone, demand rises (falls) dramatically in response to a price decrease (increase). Decreasing prices below a certain point, however, does not have a significant impact on purchases. According to a report by McKinsey and Company, prices for well-known brand-name consumer health and beauty products can be raised as much as 17 per cent without a change in consumer reaction. However, the range in which price variations are acceptable for certain financial services is less than one per cent [18]. A thorough understanding of demand elasticity helps sellers avoid needlessly reducing prices to attract customers (thereby lowering profitability) in situations where customers are price-inelastic (unlikely to respond to lower prices).

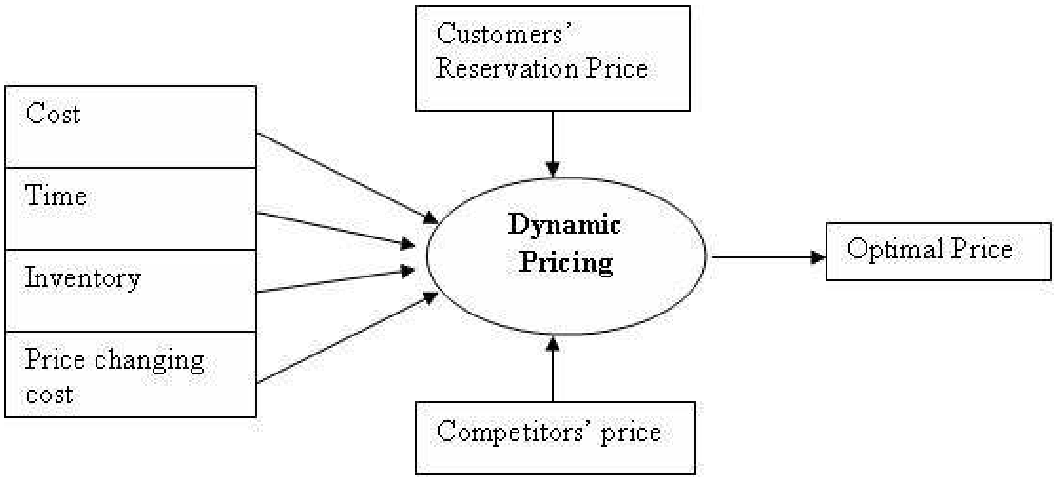

Figure 3.5. Dynamic pricing

Frequent price changes may be time consuming and may even lead to "nervousness" of demand. Therefore, the firm sets limit to price changes or move prices slowly over time.

3.2.2. Why Dynamic Pricing?

Market environment (demand, reservation price, cost, competitors' price) changes over time. For many years, dynamic pricing has proven to be an effective tool to increase revenue in the airline and other service industries. Most studies, however, focused on monopolistic models and ignored the fact of competition. A dynamic pricing strategy would not benefit a firm much if the firm assumes it enjoys monopolistic in the market. An important assumption of such a monopolistic model is that the distribution of the random demand depends solely on the price set by the monopolistic firm. Nowadays, consumers can easily compare prices on the Internet and a customer can easily track the real-time price of an item.

The demand for a product depends not only on its price, but also on the price of a similar product from a different supplier. A firm that enjoys monopolistic power uses its price as a tool to induce demand, with the objective to maximize the total expected revenue when the sale ends. The firm increases the price if the sale goes well to reserve products for potential later high fare customers. If the sale does not go well, the firm may lower the price to induce sales to sell out the inventory. The optimal policy is often difficult to find analytically, so in many cases the researchers first establish structural properties of the optimal policy, and then develop an algorithm to compute or develop efficient heuristic policies. There is limited research in revenue management of perishable products that concerns competitions between firms.

3.3. Pricing of Perishable Products

Through detailed analysis of all items in a category, PLC' s begin to emerge. The PLC is a mathematical representation of the sales behavior of an item or group of items at "regular (full)" price. Demand estimation takes into account the existence of competitors, their prices, seasonality and macroeconomic factors and where an item is in its life cycle. The resulting demand model is then validated and its parameters updated frequently throughout the selling horizon as actual sales unfold.

In the literature pricing is divided into four categories depending on PLC such as; new product pricing, competitive pricing, product line pricing and cost-based pricing situations. Depending on the product type and on the market it is sold, a firm may need to use two or more of these strategies at the same time. However, this complicates the pricing decisions.

There are two kinds of pricing depending on customer behavior such as, hidden price case and the revealed price case [7]. In the hidden price case, the reservation caller does not reveal his/her willingness to pay, making the resulting transaction between the caller and the reservation system a probabilistic event. If the quoted price is not larger than the price that the caller is willing to pay, then a reservation is made. In the revealed price case, the caller identifies himself/ herself as one of a particular group entitled to a certain fare. In this case, the reservation system can either accept or reject the reservation attempt with prior knowledge of the resulting lost or gained product such as for the convention groups, military or government travelers, buyers of special vacation packages.

4. MODEL AND PRELIMINARIES

4.1. Why Tourism Product?

The tourism industry is one of the top three industries worldwide in terms of volume and economic significance. According to the TUIK (State Statistics Institute), the industry has generated revenues of $13,203 million and accounted for approximately six per cent of the Turkey's GDP (Gross Domestic Product) in 2003 [19]. Tourism products are

- Confidence based good: The tourist cannot assess the properties and qualities of the tourism product a priori before consumption.
- Experience gained good: The tourist's satisfaction is mainly based on the experience derived from the consumption of the product and less on the properties of the product itself. This may lead to completely different evaluations of the same product depending on customer heterogeneity.
- Complex: It usually consists of a large number of independent suppliers. These suppliers may show great variation in their behavior and characteristics.
- Perishable: Tourism products are time critical, must be consumed in a specific time frame so cannot be stored. Customers may be willing to give up their cost savings and accept higher prices as time passes, thus transferring part of their surplus to the seller.

The seasonality of the tourism product and the elasticity of the industry show fluctuations in demand and may be mismatching with the supply. They typically have to face problems such as overstocking, non-vacancies, or empty seats even during the high season where demand is higher than supply. So the most important issue is to balance supply and demand.

4.2. Electronic Market

Dynamic pricing application to the electronic market is especially valuable because the sellers on the Internet use a centralized database that involves minimal cost for price update. For that reason, dynamic pricing will become more important and popular in the emerging electronic market. Posted price is commonly used for a good that is sold at a take-it-or-leave-it price determined by the seller in various markets.

In the past, the posted prices were usually static. This was mainly due to the absence of accurate demand information, the high transaction costs associated with changing prices and the huge investments required for software and hardware necessary for implementing a dynamic pricing strategy. Advances in information technology and e-commerce have altered this situation, by increasing the amount of information that the sellers can gather about customers, providing sellers with a medium for changing prices at low (or no) cost and increasing their connectivity with customers.

E-commerce opens the door for dynamic pricing policies. They can easily analyze what should be sold, where and when and to understand not only what is sold, but also what is not sold and why. Web-stores can only learn what the most profitable prices are by price experimentations.

4.3. Model and Assumptions

Every seller of a product or service faces a number of fundamental decisions. Seasonal product sellers have a certain time to sell their products when market conditions are most favorable. It is difficult to forecast the demand accurately because of the structure of the seasonal product. Therefore, sellers have an incentive to use the price, to control the demand in order to maximize their total revenue. This is the principal reason why the sellers price items differently for different customers. It is literally the tactical approach for dynamically pricing items differently during the sale horizon to capture the customer demand (customer willingness to pay) and maximize profits.

4.3.1. Demand and Demand Forecasting

The profitability of the firm depends largely on how well it uses its capacity. However, demands have associated uncertainties. They must determine how to allocate capacities to the customers who are willing to pay different rates and at the same time, to manage the operation with these uncertainties. A decision about price today may affect the information about future pricing decisions as shown in Figure 4.1.

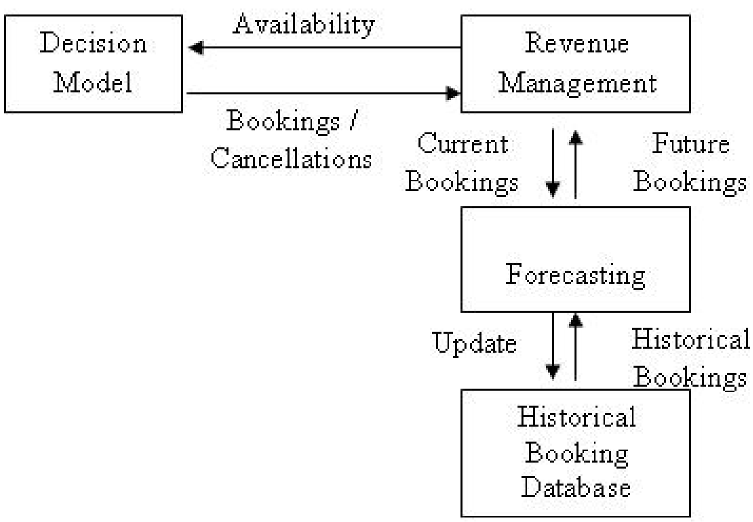

Figure 4.1. Decision model

Historical demand is used to predict future demand. In the case of a new firm or a rapidly changing market and competitive environment, historical demand data may not produce good estimates of future demand rates. In some industries, greater weight is given to the most recent demand patterns since customer preferences change rapidly.

The demand processes from any two segments must be independent. The demand associated to these customers is stochastic because some of them do not show up, even though they have reservations and also to avoid the assumption that customers have to wait in the store until they purchase the good.

Even though the major sources of demand are random, some types of demand can be controlled. Limiting the number of reservations to accept and reject some demands in the case of insufficient capacity, controls the demand.

The level of demand means the size of the market, and motivates the availability of competitive products and the attractiveness of the product. The Web-store is confronted with a population of customers that is large enough to be considered as infinite. This assumption means that on the market, the population of customers is large enough for none of them to have a significant influence on the Web-store's pricing strategy.

We take into account scarcity of goods (finite inventory levels) as well as finite number of periods. They may leave and return at a later date. In this thesis we study optimal pricing strategies when a producer faces a stochastic arrival of customers, considering a fixed capacity and a finite planning horizon while not considering demand forecasting. We assume that the demand rates for each segment is independent and each rate at each time period is known.

4.3.2. Arrival

The uncertainty about the arrival times of customers to the store adds significant complexity to the customers' purchasing decisions and the seller's pricing strategy. A seller faced with declining customer valuations, uncertain arrival times and a finite number of products, can maximize the expected revenues from sales by continuously changing the price over the season.

Accepting early bookings increases the certainty of getting enough business. Examples of early booking sources are package tour operators and convention organizers. These early bookings tend to fetch lower rates and therefore, hotels may refuse some of them in the hope of getting more business customers later.

As the Web-store does not know how a particular customer reacts to the offer he makes, he cannot take his strategies into account in his own pricing policy. The Web-store must assume that each customer he faces behaves like an average and representative individual of the whole population (homogeneity assumption). Moreover, the Web-store must also suppose that the probability distribution representing the behaviors of the customers does not change as time goes on (stationarity assumption).

The customers sequentially arrive with a constant and known probability. At most one arrival per unit time is allowed and the customers can only buy one unit of the good. This assumption defines the dynamics of the customers' arrival. Most stochastic economic models assume that the customers sequentially arrive at a constant rate and that each of them can only buy same given quantity of the good. We formulate our optimal policy assuming that the customer arrivals follow a Bernoulli process with a known probability.

4.3.3. Cancelation

It is well known that each customer who makes a reservation has a propensity to cancel that reservation before the booking date. After a customer makes a reservation he/she either consumes the package or cancels the package on or before the booking date. The probability that a customer will cancel is likely to be based on the customer's personal budget, changes in travel plans, alternative packages offered by competitors and the desirability to the customer of the package that was offered. These considerations, in general, would make the occurrence of a cancelation and the timing of a cancelation dependent on the package reserved, the time of the reservation and the market segment from which the customer came.

A no-show is considered to be a cancelation that takes place in the last time period. The number of cancelations and no-shows can be highly variable. Though expected no-show rate is around 15 per cent, indicating the problem's magnitude and difficulty. We say a reservation survived if it has not been canceled or failed to show.

We assume that the customers behave independently of one another with a known probability. In this thesis we do not consider no-shows.

4.3.4. Overbooking

Another important component in the revenue management is the use of overbooking when there is a chance that the risks of cancelations and no-shows appear. When a prospective guest with reservation, arriving in good time, finds no available room in the hotel, an overbooking is said to have taken place. Overbooking occurs because hotels sometimes overbook reservations to keep occupancy levels high. If a customer that had booked a ticket on an airline flight and not show up for the departure, the airline may end up flying an empty seat resulting in lost revenue for the company. In order to account for such no-shows, airlines routinely overbook their flights: based on the historical rate of no-shows. If the demand can't be served at the requested time, then the customer is lost. When overbooking of a particular room-type occurs, hotel operators can choose between turning away the prospective guest and giving her, at no additional cost, a better room and freebies. In addition to loss of revenue and extra costs, the fear of goodwill loss makes hotel management desire to see this happen as rarely as possible.

Downgrading a room, on the other hand, adds a contribution to profit though smaller than what it is potentially capable of. Nevertheless, the downgraded room may have remained vacant and contributed nothing.

The hotel may be motivated to stop offering rooms before making a certain number of reservations because the increase in the expected cost of walking customers due to another reservation outweighs the expected gain in yield. The highest rate for each room-type is often referred to as the rack rate for that room-type. Cost of over sale for each guest class is its room rate. In this thesis we do not consider overbooking.

4.4. Observability of Inventory and Web Based Pricing

In several retailing environments, customers cannot observe the prevailing quantity of items in stock. Therefore, customers' purchasing decisions, would depend on their beliefs about the existing level of inventory, as well as their belief about other customers' beliefs. If customers decide to pay less, they should wait for appropriate lower prices. On the other hand, postponement of purchases can be risky to the customers themselves, when inventory is limited.

Capacity that is not sold has a zero opportunity cost. At the tactical level, the manager must decide whether to accept particular demand for the product for a particular target date and considering future stochastic requests and cancelations. In this way, no revenues are collected. It is at the operational level that the capacity is actually sold and revenues are collected. At the operational level, the manager knows the total number of reservations within each market segment. However, the resulting number of reservations that turn into sales is a random variable because of the no shows. Therefore, due to the cancelations at the tactical level and no shows at the operational one, managers usually overbook in order to maximize the total expected profit. When doing this, they must take into account the trade-offs of having idle capacity due to no shows and cancelations and turning down customers with the corresponding rejection costs.

Web stores display their prices in electronic catalogues so bargaining is not possible. The Web-store does not know the customers' response function. In most market situations, the seller does not know the customers' response function. It is of course the same for a store on the Internet. In our study, we suppose that the Web-store starts its activity without any prior information on this probability distribution.

The pricing problem of a Web-store is an inter-temporal decision problem under uncertainty in which the time of the decision process is defined by the customers' arrival probability.

With the goal of balancing demand and supply, early applications of dynamic pricing methods have been mainly in industries where the short term capacity (supply) is difficult to change, such as airlines, cruise ships, hotels, electric utilities, sporting events, and health care.

From an implementation perspective, in most of these industries it was possible to control prices in a centralized fashion and prices could be changed at little or no cost. Hence, dynamic pricing practices have been especially popular in service industries with sufficiently large markets, where companies have long used computer systems to help them increase their revenues and better utilize their existing inventory/capacity. However, despite significant improvements in reducing supply chain costs via improved inventory management, a large portion of retailers still lose millions of dollars annually due to lost-sales and excess inventory.

Even with limited ability to change prices today, early users of the new pricing decision-support software have reported improved financial performance, quick return on investment, and no negative impact on price image.

4.5. The Model Description

Table 4.1. Model Parameters

T	Sale horizon
t	Time remaining
i	Index of firm
s_i	Initial inventory level
k_i	Current inventory level
λ	Arrival probability
q	Cancelation probability
$q_0(p)$	Probability of customer rejection
$q_i(p)$	Probability of customer acceptance
$U_i(k,t)$	Expected revenue
$\mathbf{p}$	Price vector

Dynamic price adjustment in both monopolistic and oligopolistic firms who offer substitutable products is examined in this study. We consider two firms that own a fixed capacity of identical items for sale to a random number of customers and which must be consumed over a finite time horizon on the event of cancelation. We assume that there are T time periods, where we count the time period in the reverse chronological order so that period one represents the last period. We also know that each firm has limited initial inventory, which is s_i for i=1,2 at the beginning of the sale horizon and the on hand inventory as k_i for i=1,2 at any time. The state of the system is defined as the remaining item and the remaining time. Due to the complexity of solving continuous time formulation, we divide the planning horizon into time intervals small enough so that we can assume the number of arrivals or cancelation in each interval is either zero or one. Reducing the length of the time intervals, we can get as close as we want to the continuous time formulation. Each firm offers one product in each time period and starts with a given number of items at the beginning of the sale horizon. Inventory cannot be replenished during the sale and unsold items at the end of period have no salvage value. As the sale goes on, each firm sets its product price and dynamically changes its product price at any time. After each period, the retailer checks the current inventory level and determine the price to be charged in the next period. The customers show up sequentially, while the seller can post a different price for each customer. Customers arrive from period T to period one according to a Bernoulli process such that in each period the probability of a customer arrives is λ and the probability that no customer arrival is (1- λ). When a customer arrives, he first compares these substitutable products and their prices and then either buys one unit of item of his most preferable product if the posted price is lower than his reservation price, or leaves empty-handed. We use the MNL (Multinomial Logit Model) that is commonly used in marketing science to describe a customer's discrete choice. A customer purchases a product not only depending on its price, but also on prices of other substitutable products with probability $q_i(\mathbf{p})$ ($\mathbf{p}$ is the posted price vector that is $\mathbf{p} = (p_1, p_2)$) for firm i for i=1,2 or leaves empty handed with probability $q_0(\mathbf{p})$ (there is no revenue from the customer at time t, which is equivalent to no customer arrives at time t). Customer choice in each period is a random variable whose distribution depends on price (the probabilistic demand case). We assume that demand is a smooth and decreasing function of price. $U_i(\mathbf{k}, t)$ denote the expected revenue from time t and onward if the seller has k_i items on hand, t time remaining and uses the optimal policy. $U_i(\mathbf{k}, t)$ is determined by conditioning on whether the customer arrives or not and whether the arriving customer buys the item or not and also whether cancelation occurs or not. After each customer arrival, there is a probability that the customer can make cancelation with probability q due to various causes. The probability that no cancelation occurs is given as $(1-q)^{s_i-k_i}$. At each period, instantaneous expected revenue is calculated. The optimal pricing policy at time t is derived as the maximizer to the function $U_i(\mathbf{k}, t)$. Upon cancelation, the seller receives a returned item and issues a refund and sells the item later to another customer. The model structure is shown in detail in Figure 4.2. The seller is not allowed to sell the returned item in the same period. Overbooking and no-shows are not permitted. We do not consider inventory holding costs or discount rates in this problem.

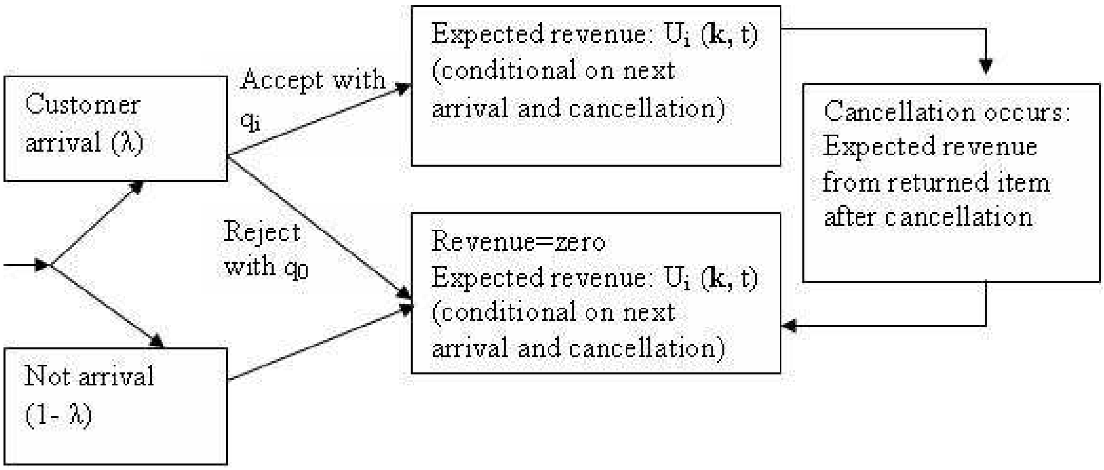

Figure 4.2. Model structure

We formulate the dynamic pricing problem as a dynamic programming model and derive the structural properties of the optimal policy and the optimal value function. The seller's objective is to post a potentially different price for each customer in order to maximize the expected total revenue.

Most studies focus on deriving structural properties of the optimal policy and developing heuristics that are easy to implement. Because there are multiple firms in the game, each firm's optimal policy depends on what the other firms do. By assuming each firm knows the real-time inventory levels of all the other firms, we characterize the price and the expected revenue. Keeping track of the inventory levels of the other firms can be expensive or sometimes impossible. We propose a practical policy that requires the stocks of the other firms as public information.

4.6. Formulation of Customer Choice

We use the MNL to describe customer discrete choice that each customer acts independently to maximize its own utility. Customer choice behavior is an important phenomenon within revenue management systems. MNL models are also known "conditional logit", "discrete choice" and "universal logit" models. All of them can be viewed as special cases of a general model of utility maximization.

The MNL model is an individual-level response model that helps to analyze and explain the choices of individual customers. In the MNL, customers are utility maximizers and the utility of each choice is a random variable. Modeling utility as random can reflect either heterogeneity in preference among individual customers or the presence of unobservable explanatory variables in the utility. In either case, as a result of uncertainty in the utilities, the choice outcome of any given customer is uncertain. At the beginning of each time period, each firm sets its product price. A customer compares the prices of all products and then decides whether to buy one unit of his most preferable product or not to buy at all. We assume only one product is sold at any point of time. Thus, customers face a binary choice to buy or not to buy. It is the most widely used discrete choice model in practical applications.

Specifically, for each customer, the utility for purchasing one product from firm i at price p_i is equal to

$$X_i = \alpha_i - \beta p_i + Z_i, \quad i = 1, 2.$$

where the parameter α_i models the quality, brand image and the popularity of firm i' s product, and β represents the price response coefficient. The utility of no purchase is

$$X_0 = Z_0.$$

The random variables Z_i, for $i = 0$, 1, 2, models the preference of each customer and are i.i.d. Gumbel random variables. Its distribution function is

$$P(Z_i \leq z) = \exp(-(e^{-\frac{z}{\mu}+\gamma})), \quad z \geq 0$$

$$\gamma = \lim_{k \to \infty} \frac{1}{i} - \ln(k) \cong 0,5772$$

where μ is a scale parameter and $\gamma \approx$ 0,5572 is Euler' s constant. The variance of the distribution is $\pi^2/6\mu^2$.

When facing a price vector $\mathbf{p} = (p_1, p_2)$, where p_i denotes the posted price by firm i, a customer will maximize its utility. In other words, a customer will buy one unit of item from firm i with the probability that a given firm i chooses alternative as

$$q_i(p_i) = P(X_i = \max_{j=1,2} X_j) = \frac{e^{\alpha_i - \beta p_i}}{1 + \sum_{j=1}^{2} e^{\alpha_j - \beta p_j}}, \quad i = 1, 2 \tag{4.1}$$

and will leave empty-handed with probability

$$q_0(p_i) = \frac{1}{1 + \sum_{j=1}^{2} e^{\alpha_j - \beta p_j}}, \quad i = 1, 2. \tag{4.2}$$

Where we have defined α_i -β p_i= 0 and interpreted as the no-purchase decision. When calculating $q_i(\mathbf{p})$, for $i = 0$, 1, 2, we have let $\mu = 1$ without loss of generality, because we can absorb the scale parameter μ into the constants α_i for $i = 1$, 2 and β. Without loss of generality, we assume the salvage value of any unsold item is zero.

We have assumed that there are two firms in the market. The first firm behaves as monopolistic and the second enters to the market behaves as oligopolistic.

4.6.1. Case One: Firm One is Monopolistic

When there is only one firm in the market, the arriving customer accepts the posted price set by the monopolistic firm one and buys the product with probability q_1, that is calculated using Equation 4.1 and it is

$$q_1(p_1) = \frac{e^{\alpha_1 - \beta \mathrm{p}_1}}{1 + e^{\alpha_1 - \beta \mathrm{p}_1}}$$

and rejects the posted price and decides not to buy the product with probability q_0, that is calculated using Equation 4.2 and it is

$$q_0(p_1) = \frac{1}{1 + e^{\alpha_1 - \beta \mathrm{p}_1}}.$$

4.6.2. Case Two: Firm One is Monopolistic and Firm Two is Oligopolistic

When there are two firms in the market, the customer accepts the posted price set by the firm one with probability q_1, that is calculated using Equation 4.1 and it is

$$q_1(p_1, p_2) = \frac{e^{\alpha_1 - \beta \mathrm{p}_1}}{1 + e^{\alpha_1 - \beta \mathrm{p}_1} + e^{\alpha_2 - \beta \mathrm{p}_2}}$$

and the customer accepts the posted price set by the firm two with probability q_2, that is calculated using Equation 4.1 and it is

$$q_2(p_1, p_2) = \frac{e^{\alpha_2 - \beta \mathrm{p}_2}}{1 + e^{\alpha_1 - \beta \mathrm{p}_1} + e^{\alpha_2 - \beta \mathrm{p}_2}}$$

and rejects both of the prices with probability q_0, that is calculated using Equation 4.2 and it is

$$q_0(p_1, p_2) = \frac{1}{1 + e^{\alpha_1 - \beta \mathrm{p}_1} + e^{\alpha_2 - \beta \mathrm{p}_2}}.$$

4.7. Model Description of Dynamic Pricing in Competitive Market with Cancelation

4.7.1. Case One: Firm One is Monopolistic

There is only one firm (firm one) in the market. The maximum posted price p_1^* is calculated to maximize its expected revenue $U_1(k_1, t)$ from the last period t=1 to T for each level of k_1 and t, level of the remaining inventory and the remaining time respectively. The boundary condition is

$$U_1(k_1, 0) \text{ or } U_1(0, t) = 0.$$

This means that the sale period is closed so the firm ends up offering a price when the remaining inventory or remaining time is equal to zero.

We assume that a cancelation occurs in accordance with the binomial distribution. If we assume that A_i is the total number of items that is sold by firm i, then it is equal to s_i - k_i. If we assume that cancelation occurs in each period, then the cancelation probability is

$$\sum_{n=1}^{A_i} \begin{pmatrix} A_i \\ n \end{pmatrix} q^1(1-q)^{A_i-1}$$

$$\sum_{n=1}^{s_i-k_i} \begin{pmatrix} s_i - k_i \\ n \end{pmatrix} q^1(1-q)^{s_i-k_i-1} = (s_i - k_i)\, q(1-q)^{s_i-k_i-1}$$

So if no cancelation occurs, this probability reduces to

$$\begin{pmatrix} s_i - k_i \\ 0 \end{pmatrix} q^0(1-q)^{s_i-k_i} = (1-q)^{s_i-k_i}$$

and if cancelation occurs it reduces to

$$1-(1-q)^{s_i-k_i}.$$

So the optimal value function can be defined as

$$U_1\left(k_1,t\right)=\max_{p_1}\left[\begin{array}{l}(1-q)^{s_1-k_1}\\ \left\{\begin{array}{l}\lambda\left[\begin{array}{l}q_1(p_1)(p_1+U_1(k_1-1,t-1))\\ +q_0(p_1)(U_1(k_1,t-1))\end{array}\right]\\ +(1-\lambda)(U_1(k_1,t-1))\end{array}\right\}\\ +(1-(1-q)^{s_1-k_1})\\ \left\{\begin{array}{l}\lambda\left[\begin{array}{l}q_1(p_1)(p_1+U_1(k_1,t-1)-p_1)\\ +q_0(p_1)(U_1(k_1+1,t-1)-p_1)\end{array}\right]\\ +(1-\lambda)(U_1(k_1+1,t-1)-p_1)\end{array}\right\}\end{array}\right] \quad (4.3)$$

The optimal value function shows that customer arrives with probability λ. When a customer arrives, he/she either accepts or rejects the price p_1. If the customer accepts to buy, then instantaneous revenue p_1 is gained, if not then there is no revenue from the arriving customer. If there is no customer arrival, then also no revenue gained as well. This situation is valid for both cancelation and no cancelation case. If there is cancelation, then full refund is given for each returned item.

4.7.2. Optimal Policy for the Last Period for Case One

For each state maximum posted price p_1^*, optimum expected revenue $U_1(k_1,t)$, customer acceptance probability of p_1 that is $q_1(p_1)$ and customer rejection probability of p_1 that is, $q_0(p_1)$ are calculated.

First consider a monopolistic case where firm one is the only firm in the market. Because there is only one firm in the market, each customer will purchase one unit of firm one's product at price p_1 with probability $q_1(p_1)$ or leaves empty-handed with probability $1 - q_1(p_1)$, according to our MNL choice model as previously introduced. To maximize the expected total revenue for firm one, let $U_1(k_1, t)$ denote the expected additional revenue if firm one still has k_1 items in inventory with t time periods remaining. Firm one can find its optimal dynamic pricing strategy by solving the dynamic program with boundary conditions $U_1\ (k_1,\ 0) = U_1\ (0,\ t) = 0$. Let p_1^* denote the maximizer of the preceding dynamic program; in other words, p_1^* is firm one's optimal price when it has k_1 units of product with t time periods remaining. In the rest of the study, we will refer to this optimal strategy as the monopolistic strategy. From firm one's standpoint, the monopolistic strategy is easy to compute, as well as its revenue function $U_1(k_1, t)$. For example, if we let $\alpha_1 = 4$, $\beta = 0.1$, $\lambda = 0.1$, then we can compute $U_1(20, 600) = 895.59$, iteratively.

At the last period because of the boundary condition $U_1\ (k_1,\ 0) = 0$, the firm tries to maximize its expected revenue from the arriving customer, that is calculated using Equation 4.3.

$$U_1\,(k_1, 1) = \max_{p_1} \left[(1-q)^{s_1 - k_1} \left\{ \lambda \left[q_1(p_1)(p_1) \right] \right\} + (1 - (1-q)^{s_1 - k_1}) \left\{ \begin{array}{c} \lambda \left[q_0(p_1)(-p_1) \right] \\ (1-\lambda)(-p_1) \end{array} \right\} \right]$$

4.7.3. Case Two: Firm One is Monopolistic and Firm Two is Oligopolistic

There are two firms (firm one and firm two) in the market. The maximum posted price p_2^* is calculated to maximize its expected revenue $U_2(k_1, k_2, t)$ from the period $t = 1$ to T for each level of remaining level of inventory. The boundary condition is

$$U_2(k_1, k_2, 0) \text{ or } U_2(k_1, 0, t) = 0.$$

It means the firm ends up offering a price when the remaining inventory or remaining time is equal to zero and the sale period is closed. We do not need a boundary condition for $k_1 = 0$, because after firm one sells out its stock, its product price-from firm two's and the customers' standpoints-is equal to 1, which allows firm two to recursively solve the dynamic program.

We can find the optimal policy for firm two by solving the following dynamic program $U_2(k_1, k_2, t)$ as

$$U_2(k_1,k_2,t) = \max_{p_2} \left[\begin{array}{l} (1-q)^{s_1-k_1}(1-q)^{s_2-k_2} \\ \left\{ \begin{array}{l} \lambda \left[\begin{array}{l} q_0(p_1,p_2)(U_2(k_1,k_2,t-1)) \\ +q_1(p_1,p_2)(U_2(k_1-1,k_2,t-1)) \\ +q_2(p_1,p_2)(U_2(k_1,k_2-1,t-1)+p_2) \end{array} \right] \\ +(1-\lambda)(U_2(k_1,k_2,t-1)) \end{array} \right\} \\ +(1-(1-q)^{s_1-k_1})(1-q)^{s_2-k_2} \\ \left\{ \begin{array}{l} \lambda \left[\begin{array}{l} q_0(p_1,p_2)(U_2(k_1+1,k_2,t-1)) \\ +q_1(p_1,p_2)(U_2(k_1,k_2,t-1)) \\ +q_2(p_1,p_2)(U_2(k_1+1,k_2-1,t-1)+p_2) \end{array} \right] \\ +(1-\lambda)(U_2(k_1+1,k_2,t-1)) \end{array} \right\} \\ +(1-q)^{s_1-k_1}(1-(1-q)^{s_2-k_2}) \\ \left\{ \begin{array}{l} \lambda \left[\begin{array}{l} q_0(p_1,p_2)(U_2(k_1,k_2+1,t-1)-p_2) \\ +q_1(p_1,p_2)(U_2(k_1-1,k_2+1,t-1)-p_2) \\ +q_2(p_1,p_2)(U_2(k_1,k_2,t-1)+p_2-p_2) \end{array} \right] \\ +(1-\lambda)(U_2(k_1,k_2+1,t-1)-p_2) \end{array} \right\} \\ +(1-(1-q)^{s_1-k_1})(1-(1-q)^{s_2-k_2}) \\ \left\{ \begin{array}{l} \lambda \left[\begin{array}{l} q_0(p_1,p_2)(U_2(k_1+1,k_2+1,t-1)-p_2) \\ +q_1(p_1,p_2)(U_2(k_1,k_2+1,t-1)-p_2) \\ +q_2(p_1,p_2)(U_2(k_1+1,k_2,t-1)+p_2-p_2) \end{array} \right] \\ +(1-\lambda)(U_2(k_1+1,k_2+1,t-1)-p_2) \end{array} \right\} \end{array} \right] \tag{4.4}$$

The participation of firm two can potentially bring significant effect on firm one's revenue if firm one ignores the competition. Consider the same example with $\alpha_1 = 4$, $\beta = 0.1$, λ= 0.1, and $U_1(20, 600) = 895.59$.

4.7.4. Optimal Policy for the Last Period for Case Two

At the last period because of the boundary condition, the firm tries to maximize its expected revenue from the arriving customer, that is calculated using Equation 4.4.

$$U_2\left(k_1,k_2,1\right)=\max_{p_2}\left[\begin{array}{l}(1-q)^{s_1-k_1}(1-q)^{s_2-k_2}\\ \{\lambda\left[q_2(p_1,p_2)(p_2)\right]\}\\ +(1-(1-q)^{s_1-k_1})(1-q)^{s_2-k_2}\\ \{\lambda\left[q_2(p_1,p_2)(p_2)\right]\}\\ +(1-q)^{s_1-k_1}(1-(1-q)^{s_2-k_2})\\ \left\{\begin{array}{l}\lambda\left[\begin{array}{l}q_0(p_1,p_2)(-p_2)\\ +q_1(p_1,p_2)(-p_2)\end{array}\right]\\ +(1-\lambda)(-p_2)\end{array}\right\}\\ +(1-(1-q)^{s_1-k_1})(1-(1-q)^{s_2-k_2})\\ \left\{\begin{array}{l}\lambda\left[\begin{array}{l}q_0(p_1,p_2)(-p_2)\\ +q_1(p_1,p_2)(-p_2)\end{array}\right]\\ ++(1-\lambda)(-p_2)\end{array}\right\}\end{array}\right]$$

4.8. Structural Properties

We assume that customer reservation price strictly increases in a posted price $\mathbf{p}$. A customer buys an item if his/her reservation price exceeds a posted price and the seller receives the instantaneous revenue $\mathbf{p}$ from a customer with the probability $q_i(\mathbf{p})$ for i = 1,2. In other words, if the posted price is p_i, the probability a customer will purchase one item is $q_i(\mathbf{p})$ for i= 1, 2 when $\mathbf{p} \geq 0$. We assume that $q_i(\mathbf{p})$ is strictly decreasing in $\mathbf{p}$ as shown in Figure 4.3. With this assumption, we can interpret the seller's decision as choosing the probability of successfully selling one item instead of choosing the price $\mathbf{p}$.

The boundary condition of the optimal policy $\mathbf{p}^*$ for both monopolistic and oligopolistic case is derived as the maximizer of

$$U_1(k_1,1) = \max_{p_1}\left[(1-q)^{s_1-k_1}\{\lambda[q_1(p_1)(p_1)]\} + (1-(1-q)^{s_1-k_1})\left\{\begin{array}{l}\lambda[q_0(p_1)(-p_1)]\\ +(1-\lambda)(-p_1)\end{array}\right\}\right]$$

can be defined as

$$U_1(k_1,1) = \max_{p_1}\left[p_1(\lambda q_1(p_1) - 1 + (1-q)^{s_1-k_1})\right]$$

and

$$U_2(k_1,k_2,1) = \max_{p_2}\left[\begin{array}{l}(1-q)^{s_1-k_1} * (1-q)^{s_2-k_2}\\ \{\lambda[q_2(p_1,p_2)(p_2)]\}\\ +(1-(1-q)^{s_1-k_1})(1-q)^{s_2-k_2}\\ \{\lambda[q_2(p_1,p_2)(p_2)]\}\\ +(1-q)^{s_1-k_1}(1-(1-q)^{s_2-k_2})\\ \left\{\begin{array}{l}\lambda\left[\begin{array}{l}q_0(p_1,p_2)(-p_2)\\ +q_1(p_1,p_2)(-p_2)\end{array}\right]\\ +(1-\lambda)(-p_2)\end{array}\right\}\\ +(1-(1-q)^{s_1-k_1})(1-(1-q)^{s_2-k_2})\\ \left\{\begin{array}{l}\lambda\left[\begin{array}{l}q_0(p_1,p_2)(-p_2)\\ +q_1(p_1,p_2)(-p_2)\end{array}\right]\\ +(1-\lambda)(-p_2)\end{array}\right\}\end{array}\right]$$

can be defined as

$$U_2(k_1,k_2,1) = \max_{p_2}\left[p_2(\lambda q_2(p_1,p_2)((1-q)^{s_1-k_1}+1) + (1-q)^{s_2-k_2} - (1-q)^{s_1-k_1} - 1)\right]$$

Since there is no customer after time zero a seller only needs to maximize the instantaneous revenue from a customer at time one.

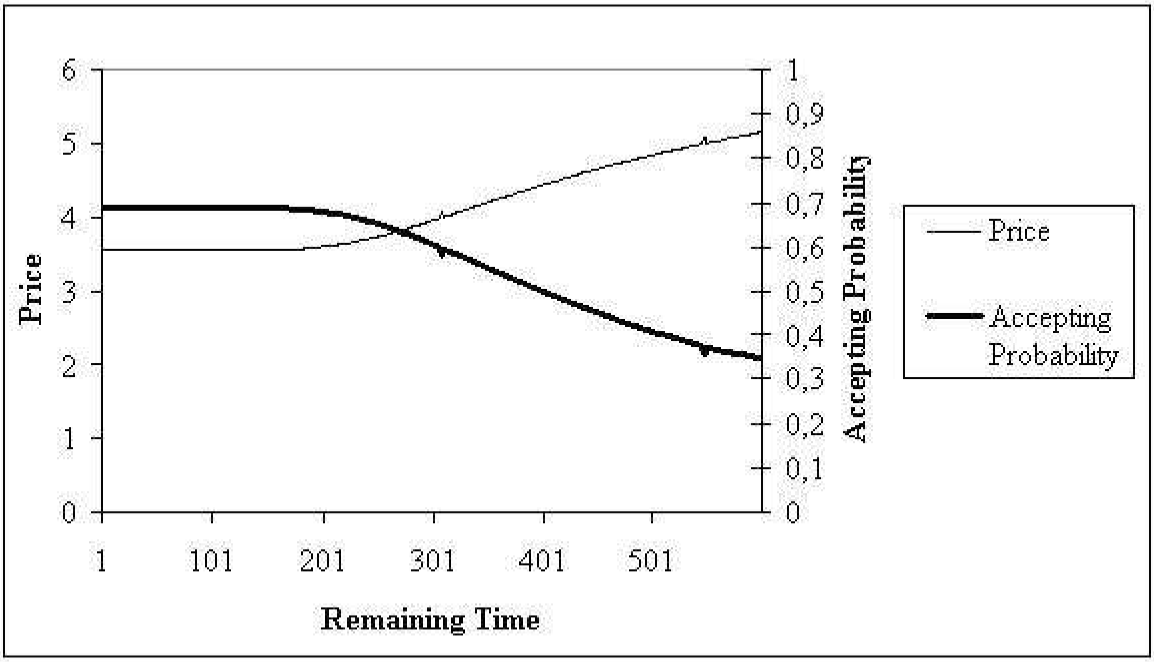

Figure 4.3. Price versus accepting probability when $q = 0$, $\alpha_1 = 4$, $\beta = 0.9$

4.9. Formulation of the Optimal Policy

Consider a seller who has k_i items for sale and knows the distribution of the total number of customers - that is $\lambda\, T$. In the beginning of the sale, the seller simply waits for the first customer to show up. With probability (1-λ) no customer shows up, so for that period, no purchase is done and the additional revenue is equal to zero. On the other hand, with probability λ, the first customer shows up, so the seller sets a product price and lets the customer decide if she wants to purchase an item. After the first customer leaves, the seller waits for the second customer. By repeating this process, it is shown that $U_i(\mathbf{k}, t)$ is bounded and continuous. It is, therefore, finite which shows that the expected revenue $U_i(\mathbf{k}, t)$ is compact. With these recursive equations, we can compute $U_i(\mathbf{k}, t)$ for any level of inventory, starting with the following boundary conditions

$$U_i(.,.,0) = U_i(.,0,.) = 0, \quad \text{i} = 1,2$$

Theorem 4.1 *$U_i(\boldsymbol{k}, t)$ is non-decreasing, concave in* $\boldsymbol{p}$ *as reflected in Figure 4.4.*

The first derivative of $U_i(\mathbf{k}, t) \geq 0$. Also, the second derivative of $U_i(\mathbf{k}, t) \leq 0$. Therefore, $U_i(\mathbf{k}, t)$ is non-decreasing and concave in p.

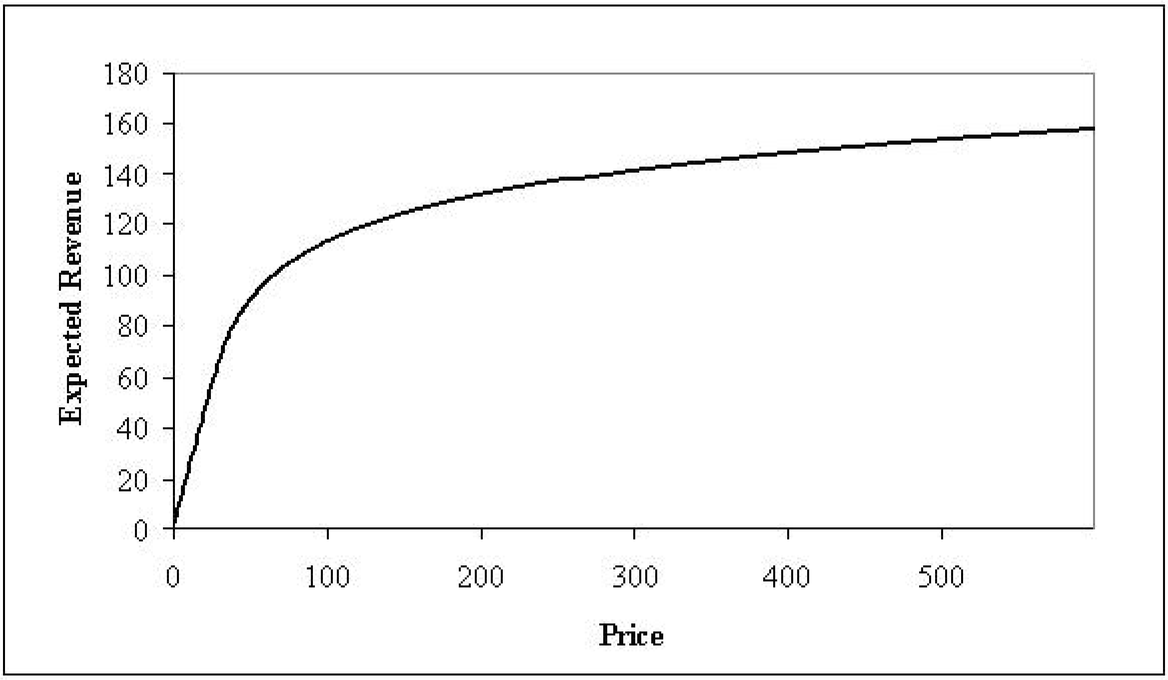

Figure 4.4. Expected revenue versus price

When $p_i \rightarrow \infty$, *then* $\lambda = 0$ (e.g. customers have bounded wealth) and the revenue rate is

$$\mathbf{p}\lambda = \sum_{i=1}^{2} \mathrm{p_i}\lambda$$

It is bounded for all $p_i \in P$ and has a finite maximizer. This demand model assumes that customer decisions only depend on the current price vector and not on past and/or future pricing decisions.

A more general model would allow customers to learn the firm's pricing policy and adjust their actions accordingly and thus incorporate the strategic interaction between the firm and the customers' collective behavior. While this may seem appealing, it leads to a complicated game-theoretic analysis and is often avoided both in the revenue management literature and in practice.

$$\lim_{p_i \to \infty} q_i(\mathbf{p}) = \lim_{p_i \to \infty} p_i q_i(\mathbf{p}) = 0, \quad \text{i} = 1, 2$$

where $q_i(\mathbf{p})$ is the customer acceptance probability of the item, that is offered by the firm i. It is monotonically decreasing in its price p_i. Therefore, a firm can set the price to infinity if it does not want to offer its product to a particular customer, in which case the expected revenue is equal to zero. This implies that the revenue function is bounded. In addition, if a firm sells out its product before the sale ends, then its product price remains equal to infinity for the rest of the sales horizon. In other words, when the firm sells out type i products, mathematically it can set $p_i = \infty$ to effectively remove type i products from the shelf. Let $p_1^*(\mathbf{k}, t)$ denote the optimal solution. In other words, the optimal price for one type i product to the arriving customer is $p_1^*(\mathbf{k}, t)$ if the current inventory is k_i. In particular, we have $p_1^*(\mathbf{k}, t) = \infty$ if $k_i = 0$.

Suppose there are two firms and upon arrival of a new customer, firm i sets its product price to p_i, for i = 1, 2. Let $\mathbf{p} = (p_1, p_2)$ denote the joint price vector. The payoff function (the immediate expected revenue from the arriving customer) for firm i is

$$p_i q_i(\mathbf{p}) = \frac{p_i e^{\alpha_\mathrm{i} - \beta \mathrm{p_i}}}{1 + \sum_{j=1}^{2} e^{\alpha_\mathrm{j} - \beta \mathrm{p_j}}}, \quad \text{i} = 1, 2$$

By conditioning on whether a customer shows up in period one, we can conclude that the expected revenue for firm i in equilibrium at the beginning of period one is equal $\lambda\ p_i^*\ q_i(\mathbf{p}^*)$, where $\mathbf{p}^*$ is the myopic price vector and no cancelation occurs. The total number of customers follows a binomial distribution with expected value equal to $\lambda\ T$ = 0,1*600 = 60. To obtain the total number of sales, we multiply the number of potential customers by the proportion of customers that actually buy, given the current price.

4.10. Optimal Policy in General

We derive structural properties of the optimal policy and use those properties to develop efficient algorithms to compute the optimal policy. The first derivative of the payoff function

$$\frac{p_1 e^{\alpha_1 - \beta p_1}}{1 + e^{\alpha_1 - \beta p_1}}$$

is

$$-e^{\alpha_1 - \beta p_1}(\beta p_1 - 1 - e^{\alpha_1 - \beta p_1})/(1 + e^{\alpha_1 - \beta p_1})^2 = 0.$$

.

Since $e^{\alpha_1 - \beta p_1}$ is always greater than zero, the solution must satisfy

$$\beta p_1 - 1 = e^{\alpha_1 - \beta p_1}$$

We can see that β p_1 - 1 is strictly increasing function of p_1, while $e^{\alpha_1 - \beta p_1}$ is a strictly decreasing. Also when p_1=0, β p_1 - 1 = -1, while $e^{\alpha_1 - \beta p_1} = e^{\alpha_1} > 0$. Therefore, there is only one solution to $p_1 q_1(\mathbf{p})$=0, denoted as p_1^*.

When $p_1 < p_1^*$, β p_1 - 1 - $e^{\alpha_1 - \beta p_1} < 0$. So we have $p_1 q_1(\mathbf{p}) > 0$, which means $p_1 q_1(\mathbf{p})$ is strictly increasing. When $p_1 > p_1^*$, β p_1 - 1 - $e^{\alpha_1 - \beta p_1} > 0$. So we have $p_1 q_1(\mathbf{p}) < 0$, which means $p_1 q_1(\mathbf{p})$ is strictly decreasing. Overall, $p_1 q_1(\mathbf{p})$ has only one global maximum at p_1^*.

Theorem 4.2 *The best response function is uniformly bounded for all p_i [15].*

Proof: Because

$$q_i(\mathbf{p}) = \frac{e^{\alpha_i - \beta p_i}}{1 + \sum_{j=1}^{n} e^{\alpha_j - \beta p_j}} \leq \frac{e^{\alpha_i - \beta p_i}}{1 + e^{\alpha_i - \beta p_i}} \leq \frac{e^{\alpha_i}}{1 + e^{\alpha_i}}$$

It follows that

$$[1 - \beta p_i(1 - q_i(p))] \leq 1 - \beta p_i(\frac{1}{1 + e^{\alpha_i}})$$

Because the right hand side becomes negative if $p_i > (1 + e^{\alpha_i}) \ / \ \beta$,

$$\frac{\partial p_1 q_1(\mathbf{p})}{\partial p_i} < 0$$

In other words, firm i' s payoff function $p_1 q_1(\mathbf{p})$ decreases in p_i for $p_i > (1 + e^{\alpha_i})$ / β, regardless what the other firms do. Consequently firm i should never set a price greater than $(1 + e^{\alpha_i}) \ / \ \beta$.

5. NUMERICAL RESULTS

Pricing at discrete time points, which hinders a seller to post appropriate prices for the customers in continuous time, is more restrictive. Hence, every time a customer arrives, the seller experiences the opportunity loss in revenue. In order to reduce this opportunity loss, the seller should increase the frequency of posted price determination. The more prices the sellers identify, the better approximation they can achieve. However, this also leads into the more computational effort.

Optimal policy for $\mathbf{p}^*$ is derived once the distribution of the reservation price is known. In addition, it is clear that for any given time t, $\mathbf{p}^*$ increases in λ regardless of the cancelation probability q. It would be reasonable to post higher price for each arriving customer, if there is higher demand for an item. In other words, $\mathbf{p}$ increases as λ increases which is reflected in Figure 5.1, when the initial state of the system is defined as the remaining time $t = 600$ and the on hand inventory $k_1 = 20$.

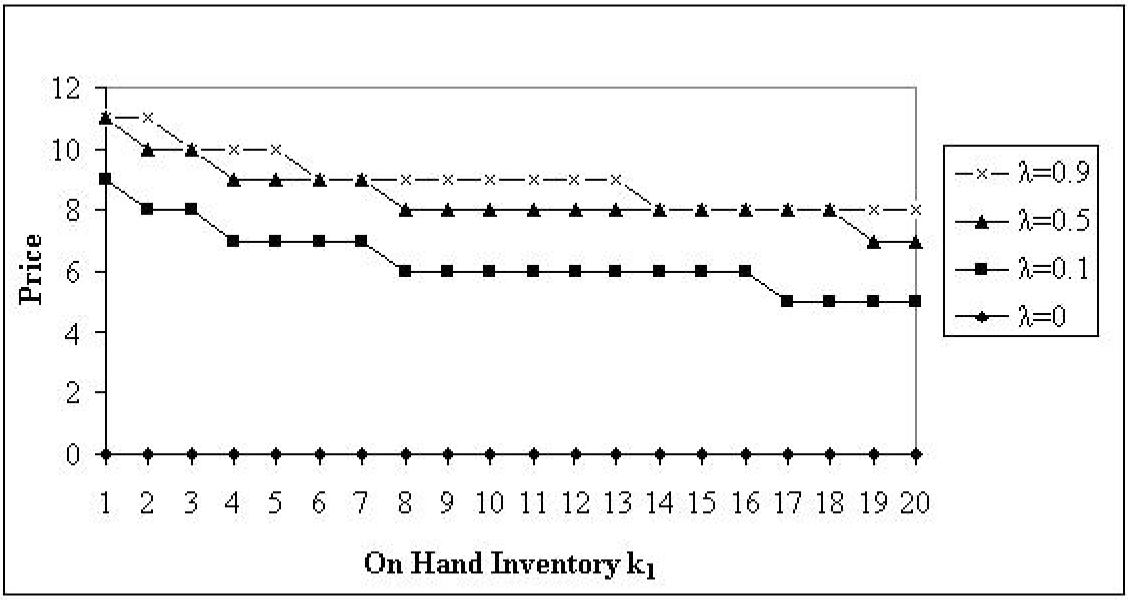

Figure 5.1. Pricing strategy of the firm one when $q = 0.9$, $\alpha_1 = 4$, $\beta = 0.9$ due to k_1

Due to the higher arrival probability, there would be higher customer demand for the item and hence there would be more variation in the reservation prices of the customers. Therefore, it is obvious that the more (variable) prices the sellers have, the more precisely they can respond to these different reservation prices. Then, the opportunity revenue loss can be less and better approximation can be achieved. Hence, the sellers should increase the number of prices in the arrival probability.

It is also important to be aware that it is never optimal to post any price less than $\mathbf{p}^*$ at any given time. The customer's reservation price is assumed i.i.d. and it is known by the seller. Posting such a price results in lower instantaneous expected revenue from a customer and higher probability that the item is sold out. For the sake of illustration, the expected revenue function is portrayed in Figure 5.2 for the case $\lambda = 0.1$, $\alpha_1 = 4$, $\beta = 0.1$ and q=0. Obviously, it is useless to post any price less than $\mathbf{p}^*$ at any time. Therefore the expected revenue is not less than the given function.

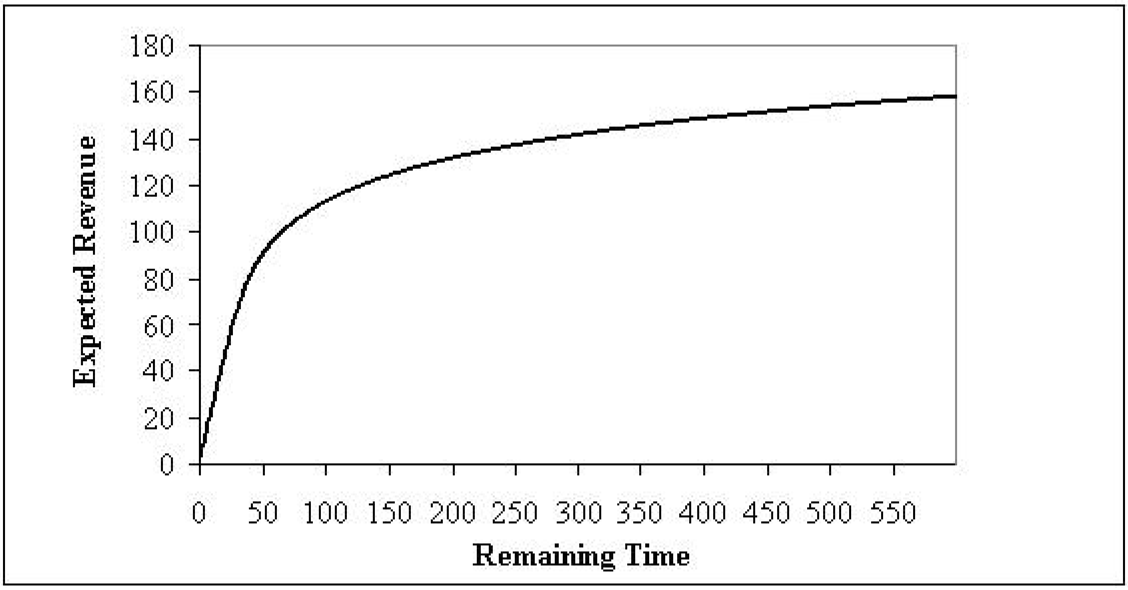

Figure 5.2. Expected revenue versus time remaining

Suppose that there are two firms and the firm one has more chance to sell out his inventory at the last period than firm two. Firm one will be able to yield at least as much revenue than the firm two, until the business is closed, whether or not cancelation is taken into account as shown in Table 5.1. In case of no cancelation, posted price is affected by arrival probability directly. If the arrival probability is high and then the posted price is set higher. If the price markdowns are rarely advertised, then the arrival probability is independent of price, but their purchase depends on the price, that is posted.

Table 5.1. Pricing strategy and expected revenue of the firm two with respect to k_2 when t=1, k_1=1, $q = 0.1$, $\lambda = 0.1$, $\alpha_1 = 4$, $\alpha_2 = 5$, $\beta = 0.1$

t	k_1	k_2	p_1	U_1	p_2	U_2
1	1	1	32	2,20	12	0,08
1	1	2	32	2,20	12	0,09
1	1	3	32	2,20	13	0,12
1	1	4	32	2,20	14	0,14
1	1	5	32	2,20	15	0,17
1	1	6	32	2,20	16	0,21
1	1	7	32	2,20	17	0,25
1	1	8	32	2,20	17	0,29
1	1	9	32	2,20	18	0,35
1	1	10	32	2,20	19	0,42
1	1	11	32	2,20	20	0,49
1	1	12	32	2,20	21	0,58
1	1	13	32	2,20	22	0,69
1	1	14	32	2,20	23	0,81
1	1	15	32	2,20	24	0,95
1	1	16	32	2,20	26	1,11
1	1	17	32	2,20	27	1,30
1	1	18	32	2,20	28	1,53
1	1	19	32	2,20	29	1,78
1	1	20	32	2,20	31	2,09

The expected revenue increases when the on hand inventory and the remaining time increase even if cancelation is taken into account. If the initial stock of firm two is smaller than firm one, then firm one dominates the market. Therefore, the monopolistic strategy performs well for firm one. On the other hand, if the initial stock of firm two is larger than firm one, then using the monopolistic strategy for firm one tends to set the price too high. This will cause to have undesired inventory at the end of the horizon. Consequently, this will make firm one's expected revenue with the monopolistic strategy much lower than it would expect.

The fact that with the monopolistic strategy, firm one will have unwanted inventory at the end, when firm two enters the market. Taking into account the competition, a rational firm one would want to change its strategy to maximize its own expected revenue. After firm one adopts a new strategy, firm two would also want to make adjustments, and then firm one, and so on, until each firm's strategy is the best response against the other firm.

We can approximate computing $U_i(\mathbf{k}, t)$ for an arbitrary distribution, if q_2 is stochastically larger than q_1, denoted by $q_2 >_{st} q_1$, that is, $P(q_2 > i) > P(q_1 > i)$ for i = 1, 2. Then $U_2(k_2, t) > U_1(k_1, t)$ as shown in Table 5.2. Suppose there are two firms: firm one is in state (k_1, t) and firm two is in state (k_2, t). Let firm one uses the optimal policy and let firm two mimics firm one's policy. Based on this assumption, firm two has more customers, then the firm two will generate no less revenue than firm one does with this mimicking policy.

Table 5.2. Pricing strategies and the expected revenues of the firm one and the firm two when $q = 0$, $\lambda = 0.1$, $\alpha_1 = 4$, $\alpha_2 = 5$, $\beta = 0.1$

t	k_1	p_1	q_1	U_1	k_2	p_2	q_2	U_2
400	20	42	0,5	751,19	20	45	0,50171	762,45
450	20	42	0,45016	796,72	20	45	0,47548	826,32
500	20	44	0,40131	834,77	20	48	0,42237	882,08
550	20	45	0,37754	867,26	20	49	0,40755	930,53
600	20	46	0,35434	895,50	20	51	0,36877	972,55

5.1. Monopolistic Case

In the monopolistic case, we have assumed that there is only one firm in the market and it follows monopolistic strategy. We tried to show how the expected revenue and the posted prices are affected by price response coefficient, on hand inventory, the remaining time, the arrival and the cancelation probability.

Prices are decreasing in the level of the initial inventory to increase the probability that all of the goods on hand will be sold. If the initial inventory is low, then the products will be sold to those customers with higher reservation prices. Low prices with low initial inventory levels lead to the loss of the customer surplus. The retailer will then experience lost sales and loss of goodwill. Then the market share decreases, since the inventory will be depleted before the horizon ends. Prices are increasing in the arrival probability as shown in Figure 5.3. Due to the higher arrival probability, the probability of having customers with higher reservation prices increases. However, when the arrival probability is small, it is more convenient to set the prices lower. Otherwise, the small number of arriving customers will not purchase the product. This will result in increased excess on-hand inventory and customer loss.

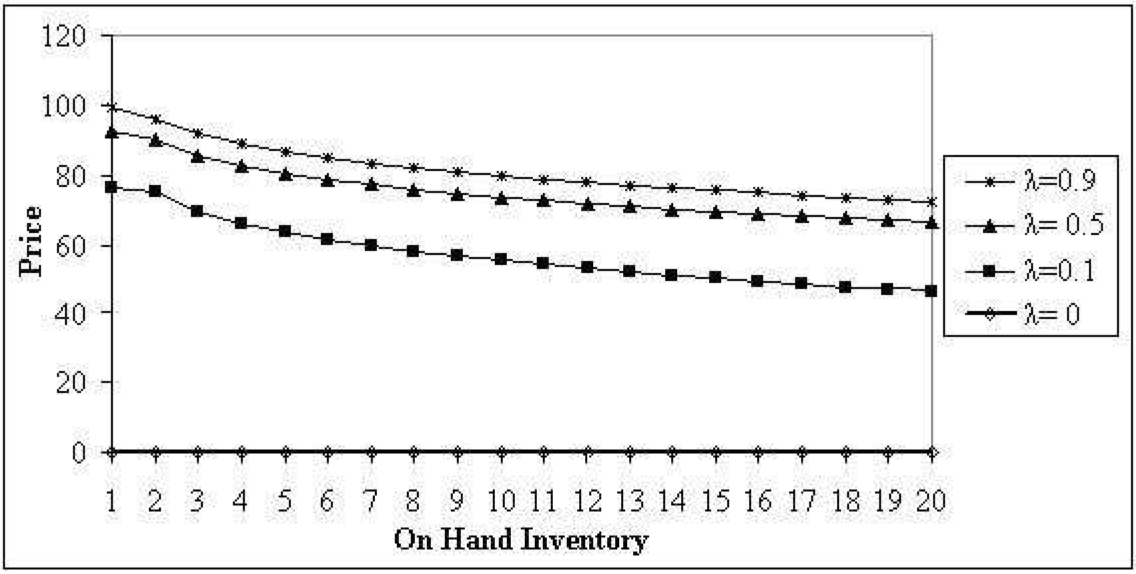

Figure 5.3. Pricing strategy of the firm one for different λ when $q = 0$, $\alpha_1 = 4$, $\beta = 0.1$ due to k_1

The expected revenue increases with the initial level of the inventory and with the arrival probability of the customers. When the arrival probability is zero, the expected revenue is zero, too. The maximum expected revenue function is non-decreasing and concave in the remaining inventory as shown in Figure 5.4.

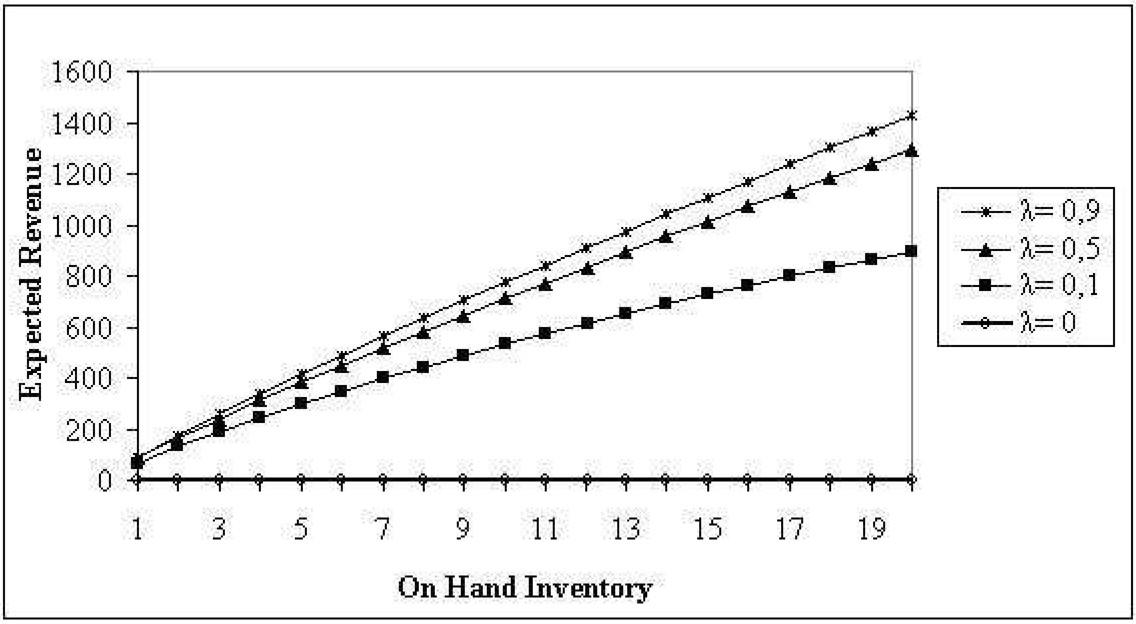

Figure 5.4. Expected revenue of the firm one for different λ when $q = 0$, $\alpha_1 = 4$, $\beta = 0.1$ due to k_1

If the planning horizon is short, then the initial price should be set lower. On the other hand, if we have long planning horizon, we can set the initial price higher. Because customers' valuations for seasonal goods tend to peak at the beginning of the selling period and decline with time t. Optimal price monotonically increases in the remaining time t (the time monotonic property) and goes to infinity as t goes to infinity, which is interpreted as the longer the available time to sell the item, the higher the prices are posted by a seller. Specifically, the more customers arrive with interest in the item, the more likely it is for the seller to yield higher revenue since customer arrival does not cost them anything at all as shown in Figure 5.5.

$\mathbf{p}^*$ increases in the remaining time t. $q_1(\mathbf{p})$ decreases in posted price $\mathbf{p}$ and also in the remaining time t. The higher the posted price for an item, the lower the customer acceptance probability. In particular, when the posted prices are sufficiently high, the customer acceptance probability would be almost equal to zero. For this reason, the instantaneous revenue from a customer would be converged to zero as the posted price goes to infinity. Another interpretation could be that customer arrivals do not mean anything when a seller has a sufficiently large amount of time for sale. The more time remaining, the higher the probability that a customer cancels the item.

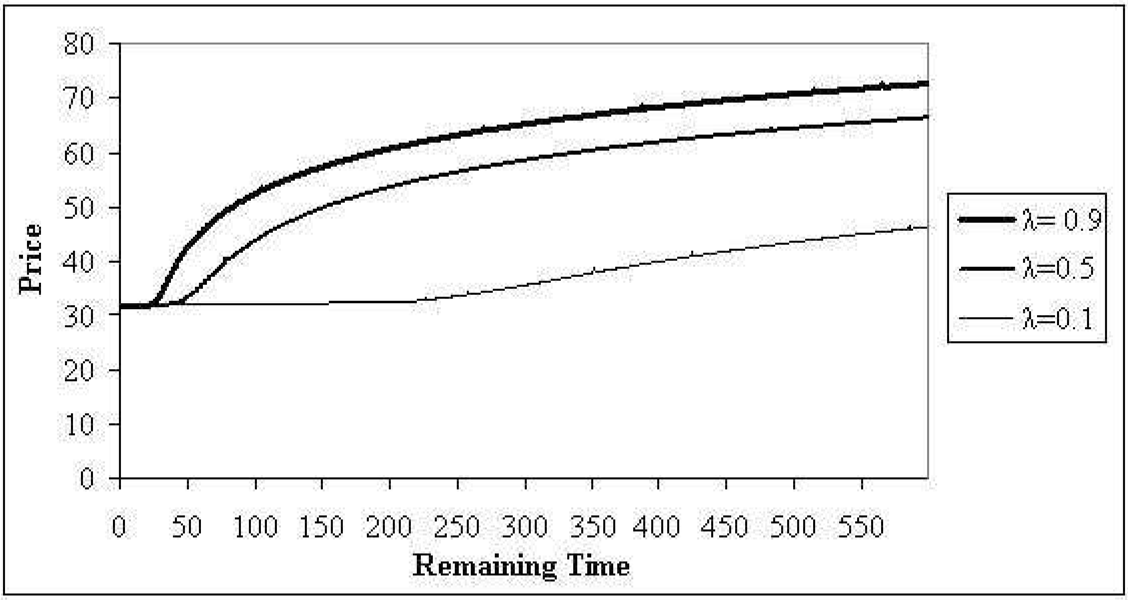

Figure 5.5. Pricing strategy of the firm one for different λ when $q = 0$, $\alpha_1 = 4$, $\beta = 0.1$ due to t

At the beginning of the sale horizon expected revenue is higher. It decreases with the length of the sale horizon and in the posted prices. The maximum expected revenue function is non-decreasing and concave in the remaining time as shown in Figure 5.6. $U_1(k_1, t)$ monotonically increases in the remaining time t. In this case, the more time available for sale, the more advantageous for the seller to sell the item since the seller has more customer arrivals and thus more opportunity to sell the item. However, this is not necessarily true. It actually changes its behavior depending on the ratio of the arrival probability to the cancelation probability as shown in Figure 5.7.

Expected revenue and the posted price are maximum at the beginning of the sale horizon (targeting for higher reservation price customers), decrease by the remaining time (targeting for lower reservation price customers) and especially with the decreasing ratio of λ / q. The expected revenue is maximum when the remaining time is highest and the on hand inventory is lowest. That is max $U_1(k_1, t)$ for $k_1 = 1$ and $t = 600$.

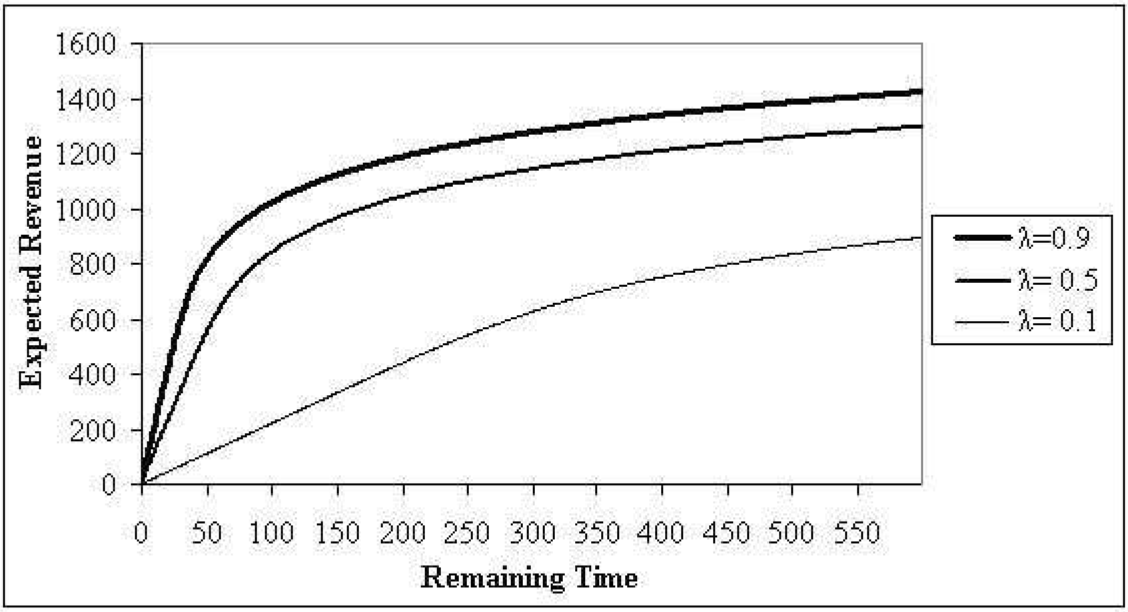

Figure 5.6. Expected revenue of the firm one for different λ when $q = 0$, $\alpha_1 = 4$, $\beta = 0.1$ due to t

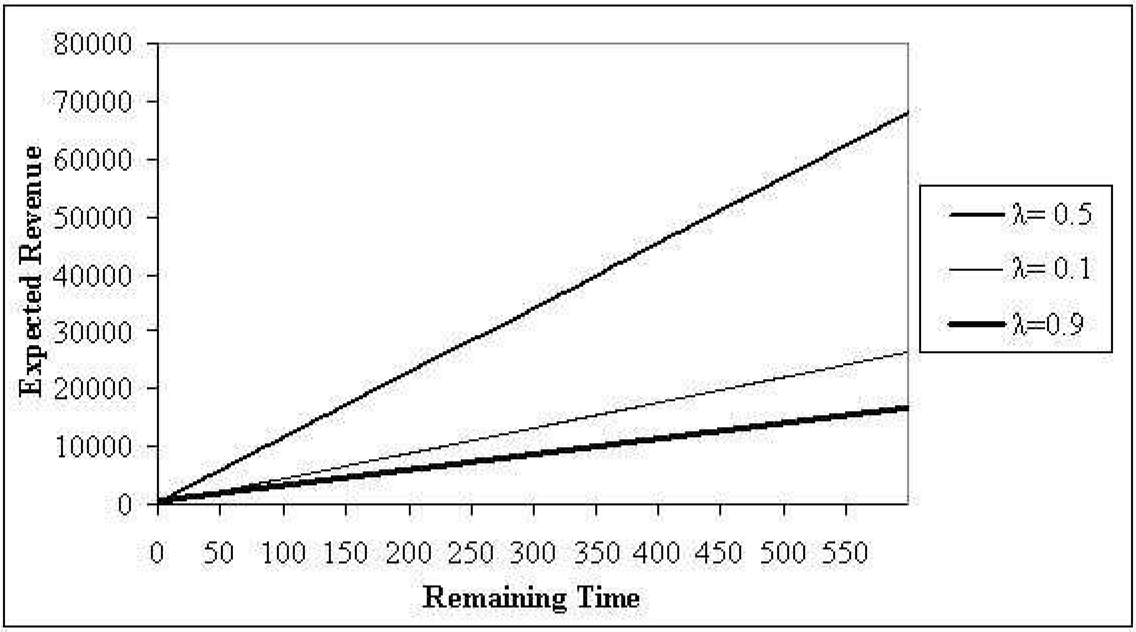

Figure 5.7. Expected revenue of the firm one for different λ when $q = 0.9$, $\alpha_1 = 4$, $\beta = 0$ due to t

In case of the price response coefficient is zero, the posted price is set as the maximum among the other variables. In this case the customer is not sensitive to the prices that is set by the seller. The expected revenue is increasing with the on hand inventory and with the ratio of λ / q, except q is equal to λ, when there is probability of cancelation as shown in Figure 5.8. If the price response is higher, prices are set lower and if it is small, the prices are set higher and the posted price increases as the on hand inventory decreases. Expected revenue for lower price response coefficient decreases with the on hand inventory gradually as shown in Figure 5.9.

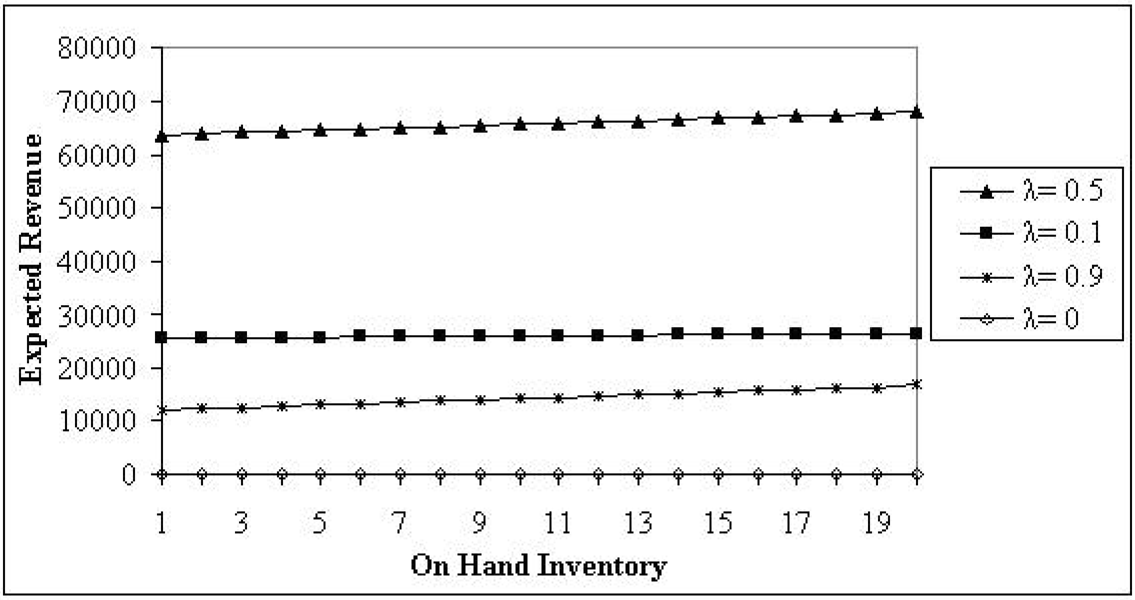

Figure 5.8. Expected revenue of the firm one for different λ when q = 0.9, α_1 = 4, β = 0 due to k_1

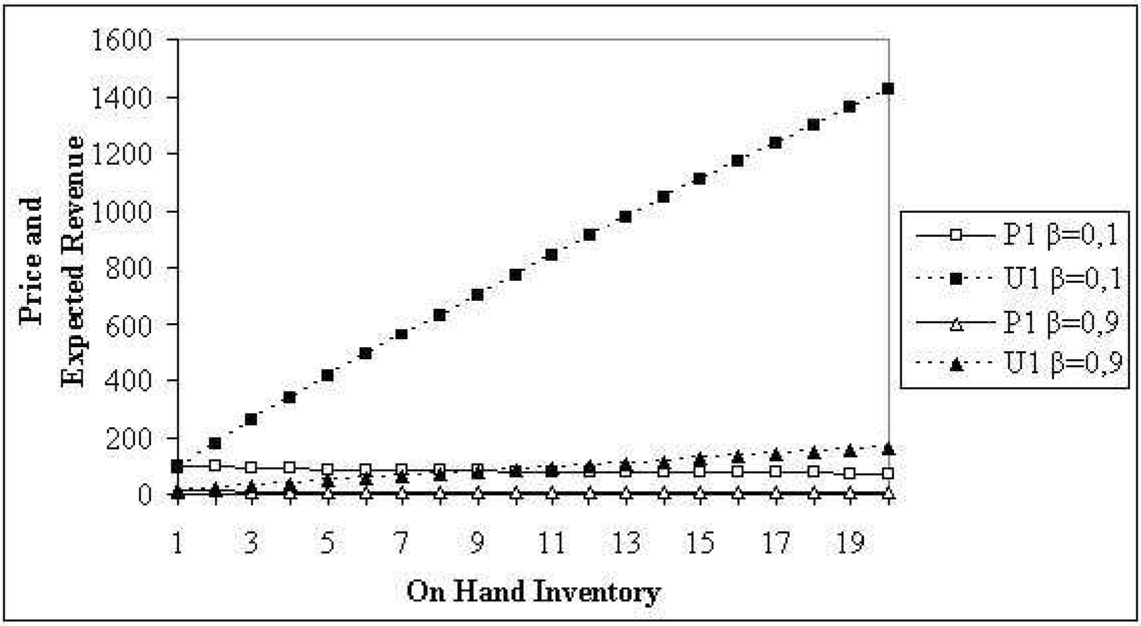

Figure 5.9. Expected revenue and pricing strategy of the firm one when λ = 0.9, q = 0, α_1 = 4, β = 0.1 or 0.9 due to k_1

The posted price starts to be more stable, if more cancelation occurs. Expected revenue is always higher in the no cancelation case. It shows that at each time cancelation occurs expected revenue becomes lower. Also the seller tries to post less variable prices not to have cancelations. And towards to end of sale horizon posted price for no cancelation occurring case starts to be less than more cancelation occurring case. So the sellers guarantee that more products are sold, when there is no cancelation as shown in Figure 5.10 and 5.11.

It is clear that the expected revenue is higher, when the ratio λ / q is in the higher range. In this case, a seller has more chance to sell the item and fewer chances to experience cancelation since there are more customer arrivals with interest in the item. In other words, the customer demand for the item is higher and hence the seller is more likely to sell out of the item until the end of the sale horizon. Therefore, the item is more valuable.

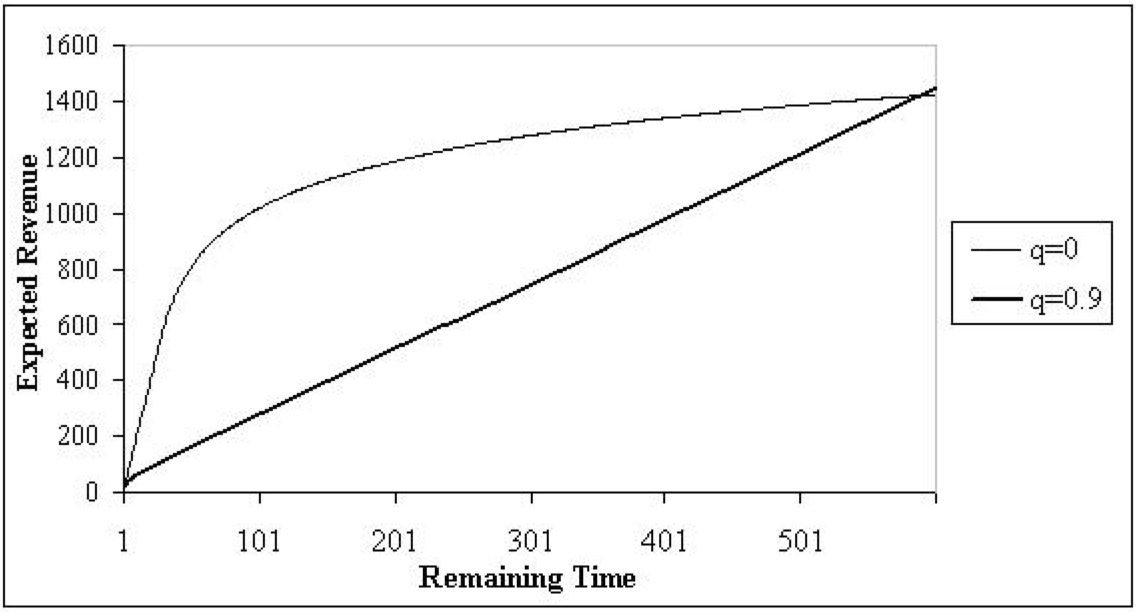

Figure 5.10. Expected revenue of the firm one when q = 0 or 0.9, α_1 = 4, β = 0.1, λ = 0.9 due to t

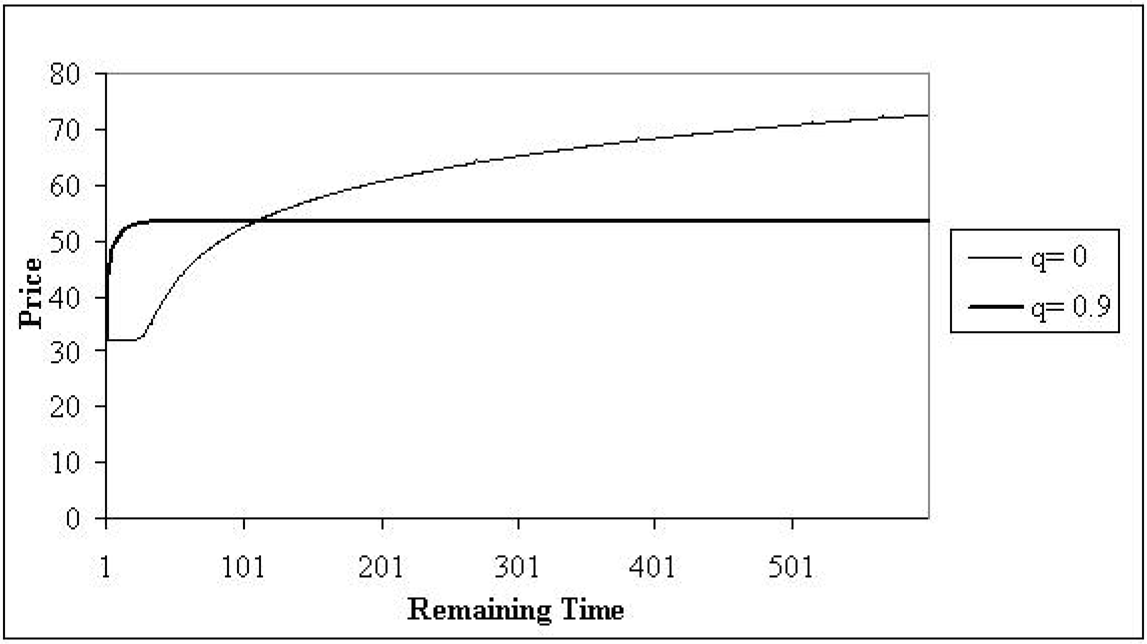

Figure 5.11. Pricing strategy of the firm one when $q = 0$ or 0.9, $\alpha_1 = 4$, $\beta = 0.1$, $\lambda = 0.9$ due to t

It can clearly be seen that, for any given time, the optimal posted price $\mathbf{p}^*$ and the expected revenue, increases as the ratio λ / q increases. Posted price is the highest, when price response coefficient zero and the λ / q is the highest. On the other hand, when the ratio λ / q is in the lower range, the expected revenue is much lower. In this case, a customer is more likely to cancel the item after purchasing it; the revenue that a seller yields from a customer is more likely to be refunded to the customer. Hence, even though the seller posts higher prices in response to the higher arrival probability and then yields some higher revenue, this revenue is more likely to be canceled out. As a result, the item is not expected to be sold out at the end of the sale horizon in this case. Therefore, the item is expected to be less valuable and the revenue is expected to be much lower, no matter how much the seller would post the price in response to the magnitude of the arrival probability.

As a matter of fact, the seller should put more focus on the lower reservation prices of customers to reduce the opportunity revenue loss. In this case the seller is now more likely to sell the item later in the time horizon than earlier since the less time remaining, the lower the probability that a customer cancels the item. The seller, thus, is more likely to experience the opportunity revenue loss later in the time horizon.

It is more advantageous if the seller has less available time, since there is less probability that a customer cancels the item and thus the seller has a slightly better chance to sell out of the item by the end of the sale horizon. Therefore, the expected revenue would be slightly higher later in the time horizon. In summary, how much revenue a seller can expect depends on the ratio of the arrival probability to the cancelation probability. If the ratio is in the higher range, the revenue is expected to be higher in response to the arrival probability. Otherwise, the effect of cancelations would be so significant that it would adversely affect the revenue of the seller. In a full refund system, cancelation decreases the revenue of the seller every time it occurs. The customer receives full refund and experiences the opportunity loss. This causes the seller to post the prices in the rest of the time horizon that are lower than the first posted price. This means that, once cancelation occurs, the total revenue afterward cannot be higher than the first yielded revenue. As a result, cancelation prevents the seller from yielding high revenue as long as a full refund policy is adopted.

5.2. Oligopolistic Case

In the oligopolistic case, we have assumed that there are two firms in the market; one follows monopolistic where the other follows oligopolistic strategy. We tried to show how the competition, expected revenue and the posted prices are affected by price response coefficient, on hand inventory, the remaining time, the arrival and the cancelation probability.

State of the system can be defined as $(k_1, k_2, 600)$ from $k_1 = 1$ to 20 and $k_2 =$ 1 to 20. (e.g. state one is $(1, 1, 600)$ where state 20 is $(1, 20, 600)$) As λ increases posted price increases in competitive market and the firm two posts higher price at the beginning of the sale horizon. Sudden price increase occurs when the firm two has one item. When λ increases, the firm two posts higher prices because of the competition when compared with the firm one and then it expects more revenue. The firm two sets more variable prices due to the competition as shown in Figure 5.12 and 5.13.

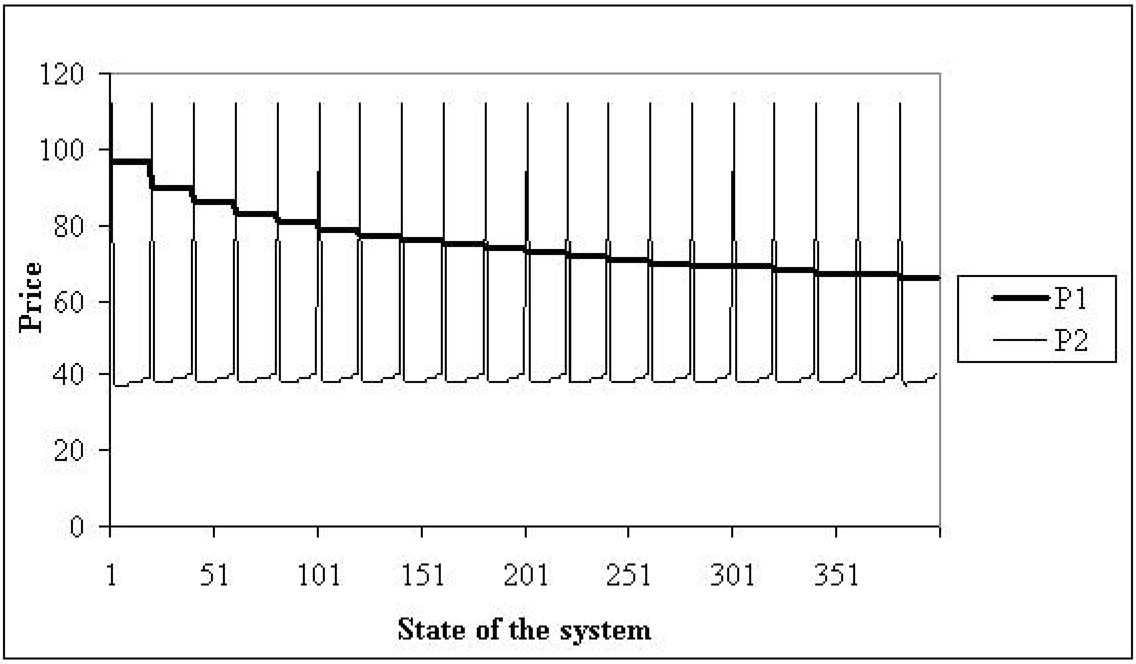

Figure 5.12. Pricing strategies of the firm one and the firm two when $q = 0.1$, $\alpha_1 = 4$, $\alpha_2 = 5$, $\beta = 0.1$, $\lambda = 0.5$ due to the state of the system when T=600

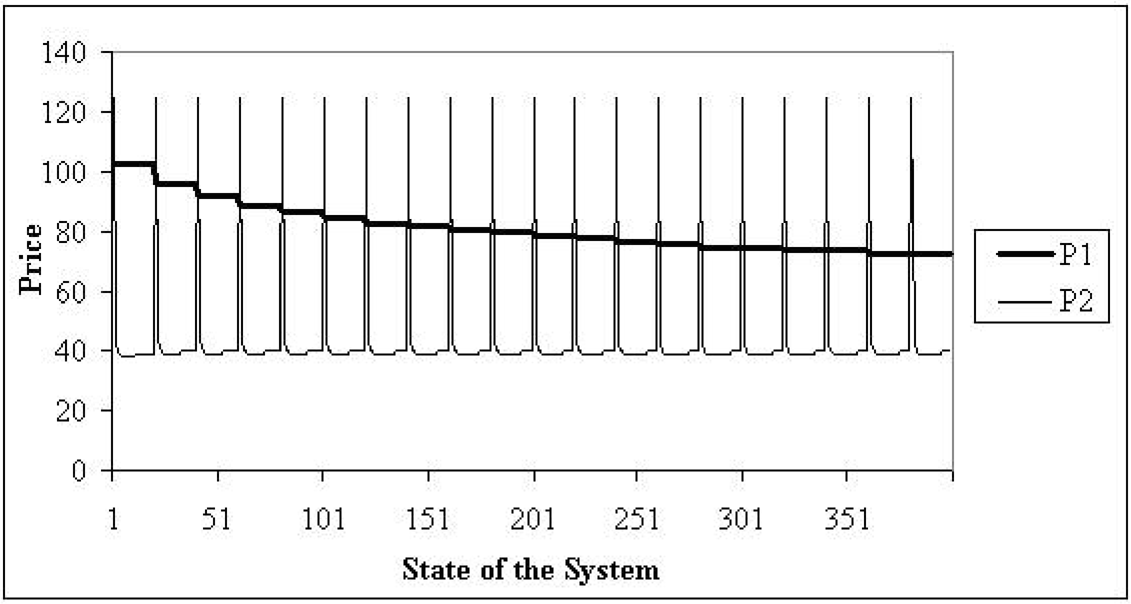

Figure 5.13. Pricing strategies of the firm one and the firm two when $q = 0.1$, $\alpha_1 = 4$, $\alpha_2 = 5$, $\beta = 0.1$, $\lambda = 0.9$ due to state of the system when $T = 600$

When the on hand inventory decreases, then the firm two posts lower price than the firm one. The firm two decreases its posted price and then starts to post higher price than the firm one, when it has one item on hand in accordance with the other on hand inventory levels for the same remaining time as shown in Figure 5.14. So q_2 increases and then decreases. It is independent from the remaining time and dependent on the on hand inventory.

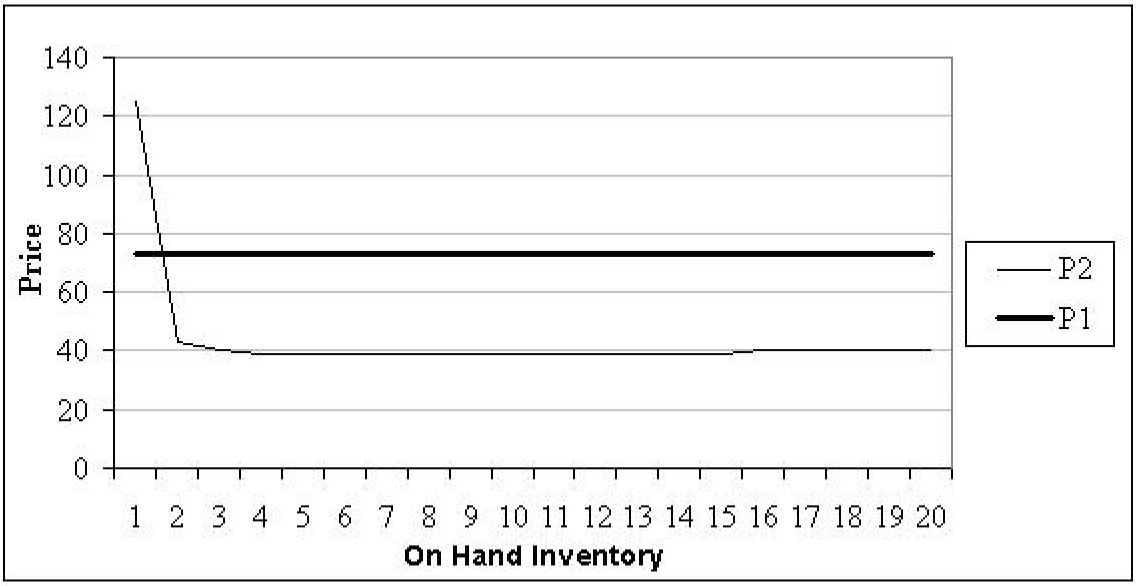

Figure 5.14. Pricing strategies of the firm one and the firm two when $q = 0.1$, $\alpha_1 = 4$, $\alpha_2 = 5$, $\beta = 0.1$, $\lambda = 0.9$ due to k_2 when $T = 600$

Higher β means that the customers are more sensitive to the posted price. The firm two starts to post much more lower prices to capture the demand, so expects to sell more, while the firm one posts less variable prices. If the customers are sensitive to the prices, then the firm two posts lower prices except when it has only one item because of taking into account competition. The firm two expects much more revenue as it decreases its prices. Also their expected revenues decrease by increasing price response of the customers as shown in Figure 5.15 and 5.16.

As cancelation probability increases, the posted price decreases to capture the demand. When cancelation probability is zero, then the firm two posts higher prices. When it has only one item on hand, then sudden increase occurs in the posted price. Due to the increasing posted price, the firm two starts to expect less revenue as shown in Figure 5.17 and 5.18.

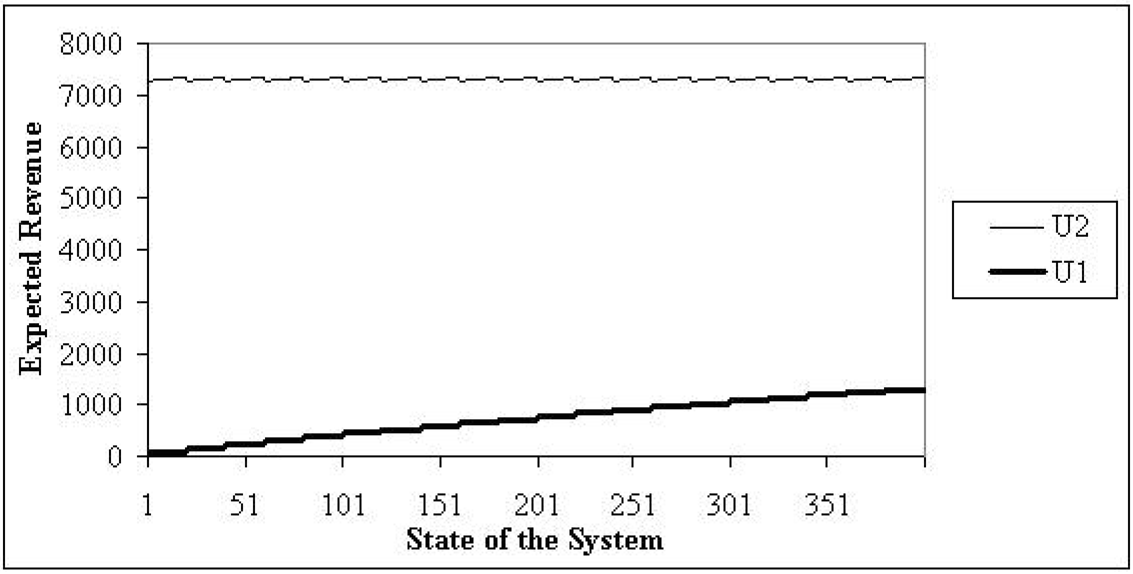

Figure 5.15. Expected revenues of the firm one and the firm two when $q = 0.1$, $\alpha_1 = 4$, $\alpha_2 = 5$, $\beta = 0.1$, $\lambda = 0.5$ due to the state of the system when $T = 600$

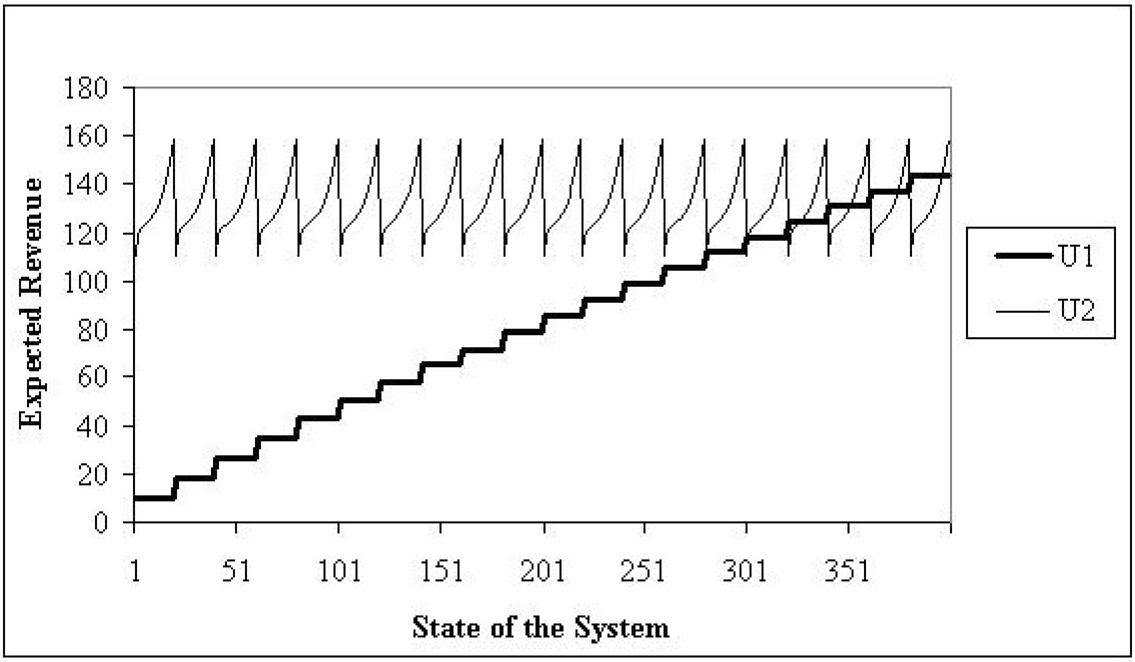

Figure 5.16. Expected revenues of the firm one and the firm two when $q = 0.1$, $\alpha_1 = 4$, $\alpha_2 = 5$, $\beta = 0.9$, $\lambda = 0.5$ due to the state of the system when $T = 600$

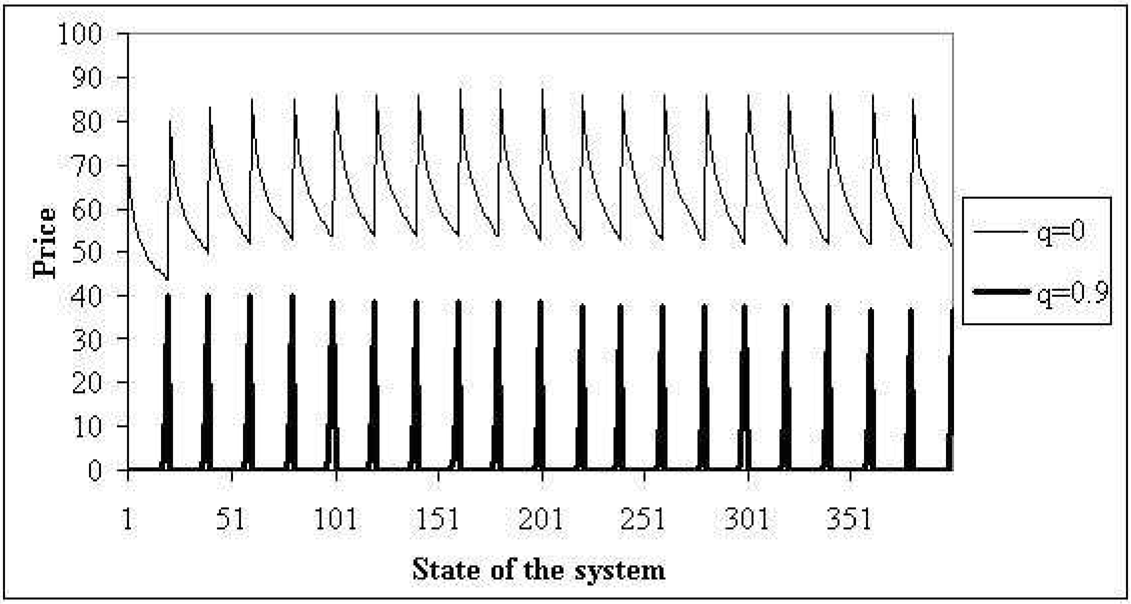

Figure 5.17. Pricing strategy of the firm two when $\alpha_1 = 4$, $\alpha_2 = 5$, $\beta = 0.1$, $\lambda = 0.1$ due to the state of the system for different q

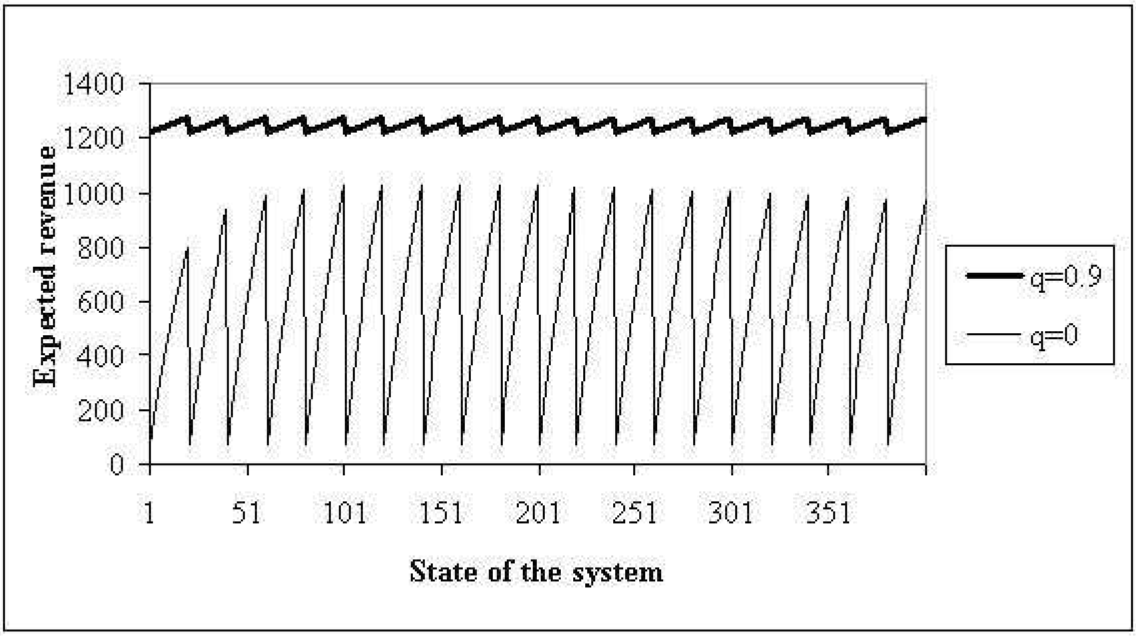

Figure 5.18. Expected revenue of the firm two when $\alpha_1 = 4$, $\alpha_2 = 5$, $\beta = 0.1$, $\lambda = 0.1$ due to the state of the system for different q

When cancelation probability is comparable smaller, then the expected revenue of the firm two increases as the ratio of arrival probability to cancelation probability as shown in Figure 5.19. As cancelation occurs, when the firm one behaves as monopolistic and the firm two as oligopolistic, p_1 stays same for different levels of k_2, because of not taking into consideration the competition. As k_2 decreases, the firm two starts to post lower prices because of competition, to maximize its expected revenue. Also when cancelation probability increases, p_2 decreases and U_2 increases because the firm two expects to sell out the on hand inventory with lower posted prices.

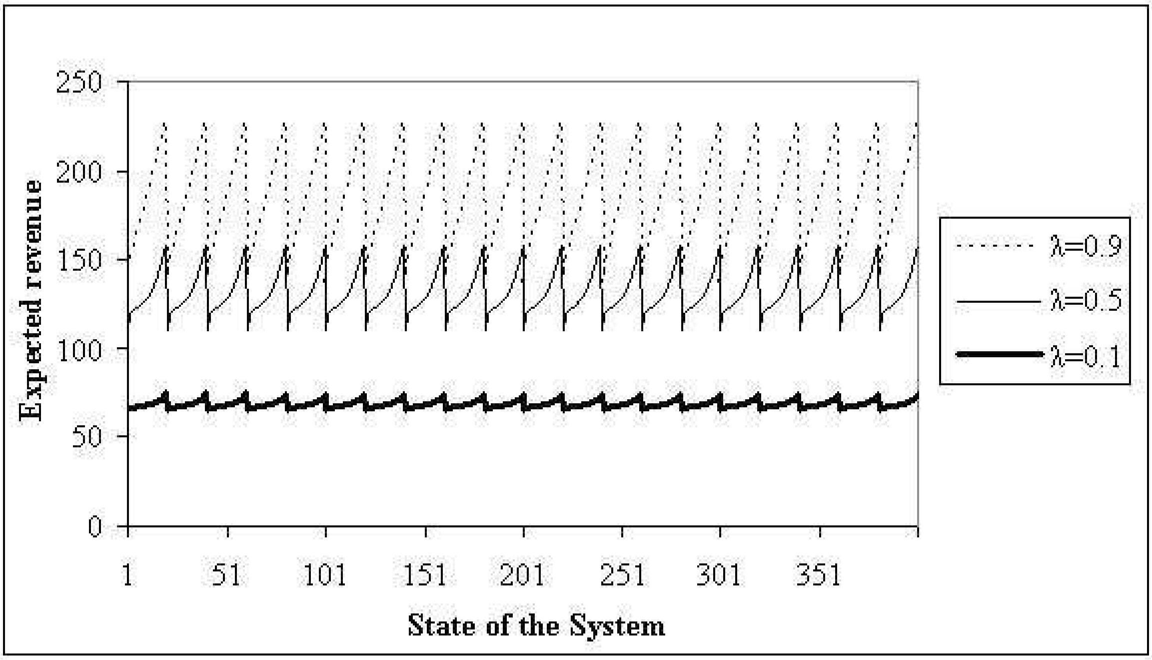

Figure 5.19. Expected revenue of the firm two when $q = 0.1$, $\alpha_1 = 4$, $\alpha_2 = 5$, $\beta = 0.9$, due to the state of the system for different λ

At the beginning of the sale horizon, if the firm two has comparable lower inventory (e.g., $k_2 = 1$) than the firm one, then the firm two posts higher prices because it tries to maximize its revenue. When they have same level of inventory, the firm two posts lower price than the firm one to capture the demand as shown in Figure 5.20. The firm one expects much more expected revenue at the beginning of the sale horizon because of having more on hand inventory as reflected in 5.21.

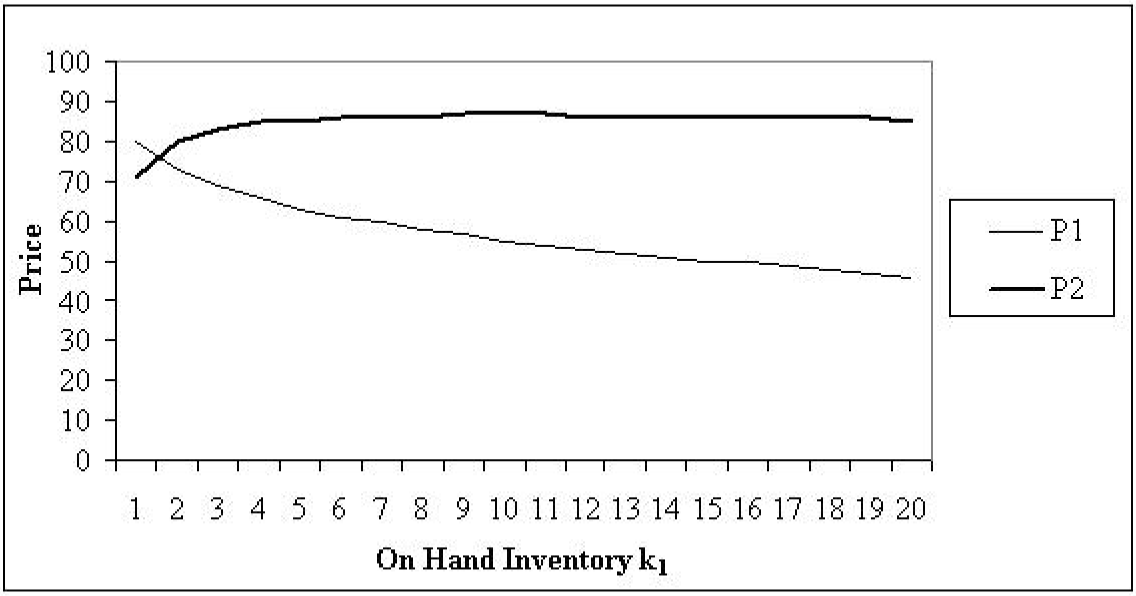

Figure 5.20. Pricing strategies of the firm one and the firm two when $\lambda = 0.1$, $q = 0$, $\alpha_1 = 4$, $\alpha_2 = 5$, $\beta = 0.1$ due to k_1 when $T = 600$

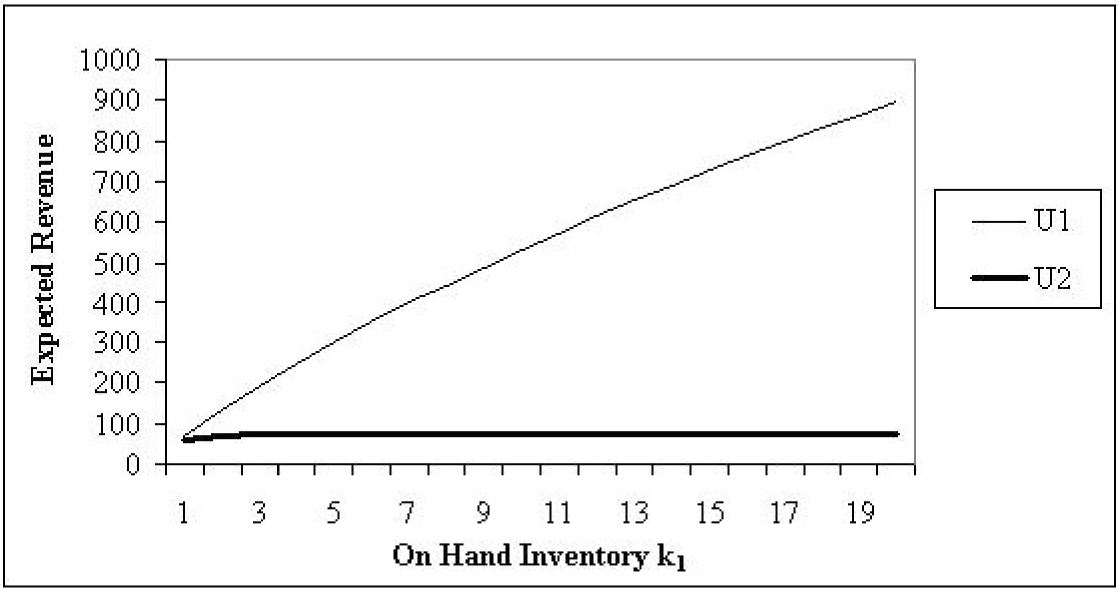

Figure 5.21. Expected revenues of the firm one and the firm two when $\lambda = 0.1$, $q = 0$, $\alpha_1 = 4$, $\alpha_2 = 5$, $\beta = 0.1$ due to k_1 when $T = 600$

In case of no cancelation, if the firm two has much lower on hand inventory, then the firm two posts higher prices because of competition. On the other hand, the firm two posts lower prices according to the firm one because of competition, when there is cancelation probability, except when the state is $(20, 1, 600)$. Because the firm two has more time available to sell out the inventory. If there is cancelation probability, then the firm two posts lower prices than no cancelation case but expects more revenue. As cancelation happens, the firm two starts to expect more revenue due to the lower prices as shown in Table 5.3.

Table 5.3. Pricing strategies and the expected revenues of the firm one and the firm two when $\lambda = 0.1$, $\alpha_1 = 4$, $\alpha_2 = 5$, $\beta = 0.1$ and $k_1 = 20$

T	k_1	k_2	p_1	U_1 **(q=0, q=0.1)**	p_2 **(q=0)**	U_2 **(q=0)**	p_2 **(q=0.1)**	U_2 **(q=0.1)**
600	20	1	46	895,50	85	75,07	69	1177,16
600	20	2	46	895,50	78	142,77	19	1187,78
600	20	3	46	895,50	74	205,96	19	1190,81
600	20	4	46	895,50	71	265,79	20	1193,72
600	20	5	46	895,50	68	322,90	21	1196,66
600	20	6	46	895,50	66	377,67	22	1199,62
600	20	7	46	895,50	65	430,37	23	1202,61
600	20	8	46	895,50	63	481,18	24	1205,63
600	20	9	46	895,50	62	530,24	24	1208,68
600	20	10	46	895,50	61	577,66	25	1211,77
600	20	11	46	895,50	59	623,50	26	1214,90
600	20	12	46	895,50	58	667,84	27	1218,06
600	20	13	46	895,50	57	710,73	28	1221,27
600	20	14	46	895,50	56	752,20	30	1224,52
600	20	15	46	895,50	55	792,28	31	1227,82
600	20	16	46	895,50	54	831,00	32	1231,17
600	20	17	46	895,50	54	868,37	33	1234,57
600	20	18	46	895,50	53	904,41	34	1238,03
600	20	19	46	895,50	52	939,14	36	1241,55
600	20	20	46	895,50	51	972,55	37	1245,15

As decreasing level of on hand inventory k_1, p_2 increases and q_2 decreases. Then at certain level of on hand inventory because of competition and due to cancelation, p_2 starts to decrease as shown in Table 5.4. In that case the firm two tries to sell out on hand inventory.

Table 5.4. Pricing strategies and the expected revenues of the firm one and the firm two when $\lambda = 0.1$, $\alpha_1 = 4$, $\alpha_2 = 5$, $\beta = 0.1$ and $k_2 = 20$

T	k_1	k_2	p_1	U_1 **(q=0, q=0.1)**	p_2 **(q=0)**	U_2 **(q=0)**	p_2 **(q=0.1)**	U_2 **(q=0.1)**
600	1	20	80	70,04	44	798,47	39	1249,81
600	2	20	73	132,80	50	939,45	40	1249,57
600	3	20	69	191,13	52	989,46	40	1249,23
600	4	20	66	246,19	53	1011,90	40	1248,90
600	5	20	63	298,64	54	1022,70	**40**	1248,59
600	6	20	61	348,86	54	1027,63	**39**	1248,28
600	7	20	60	397,12	54	1029,19	39	1247,99
600	8	20	58	443,62	54	1028,63	39	1247,70
600	9	20	57	488,50	**54**	1026,65	39	1247,43
600	10	20	55	531,88	**53**	1023,68	39	1247,17
600	11	20	54	573,84	53	1020,03	39	1246,93
600	12	20	53	614,45	53	1015,82	39	1246,71
600	13	20	52	653,76	53	1011,20	38	1246,50
600	14	20	51	691,83	53	1006,24	38	1246,30
600	15	20	50	728,68	52	1001,02	38	1246,12
600	16	20	50	764,34	52	995,59	38	1245,97
600	17	20	49	798,84	52	990,00	38	1245,80
600	18	20	48	832,19	52	984,28	38	1245,59
600	19	20	47	864,41	51	978,45	37	1245,37
600	20	20	46	895,50	51	972,55	37	1245,15

The participation of firm two can potentially bring significant effect on the firm one's revenue. If the firm one ignores the competition, when the initial stock of the firm two is much smaller than the firm one, then the firm one still dominates the market. Therefore the monopolistic strategy performs well for the firm one, when there is no cancelation probability. On the other hand, if the initial stock of the firm two is larger than the firm one, then using this monopolistic strategy tends to set the price too high, which will leave unwanted inventory on hand. Consequently, the firm one's expected revenue with the monopolistic strategy becomes much lower than it would expect.

In a market where there is price competition, managers should set a lower price and safety protection level for full fare customers than in a monopolistic market. In a market where inventory competition dominates, managers should set a higher price and safety protection levels than a monopolistic firm would. Interestingly, in a market where the two levels of competition are matched, managers should set a lower price and a higher safety protection levels than a monopolistic.

6. CONCLUSION

In this study, we have studied the discrete time dynamic pricing model, where multiple items are sold over a finite time horizon in a competitive market.

We have formulated the dynamic pricing problem as a dynamic programming model and have derived the structural properties of the optimal pricing policy and the optimal value function. In particular, we have derived the optimal policy in a closed form and have gained some insight into the optimal policy.

The optimal design of a pricing mechanism shows, where the seller who wants to sell fixed level of inventory, posts a decreasing price at each period. Customers' decision is how much (if any) to purchase. Acting strategically, a customer might choose not to purchase at a given period, even if that price is below his reservation price, hoping to purchase at a later time period with lower price to increase his surplus.

Competition affects revenue management decisions, so it is an important factor to consider. When prices are fixed, competition leads to increase in high-fare customers and decrease in low-fare customers. When prices are decision variables, competition leads to decrease in average prices, but low fares may go up. In both cases, firms focus more on high-fare products under competition. Dynamic pricing reduces sales variability significantly. The benefits of dynamic pricing are the greatest, when the demand starts high and decreases over time. The benefits of dynamic pricing are the smallest, when the demand starts low and increases over time. It can be explained by the presence of capacity constraints and the fact that more time is needed in order to replenish inventory.

Expected revenue increases depending on the initial level of the inventory. Even in the case when the demand is not known, so cannot be forecasted and the retailer has excess initial inventory. The prices should be set lower compared with valuation of customers to increase the probability that all of the goods on hand will be sold.

On the other hand, if the initial inventory is low and demand is known to be higher than the on hand inventory, the prices should be set higher compared with valuation of customers. In that case, the products will be sold to those customers with higher reservation prices. Low price with low initial inventory levels lead to the loss of the customer surplus. The retailer will then experience lost sales and the market share decreases since the inventory will be depleted before the horizon ends.

If the planning horizon is short, then the initial price should be set lower. On the other hand, if we have a long planning horizon, we can set the initial price higher. As t increases, the posted price should be increased to sell out the on hand inventory.

More initial inventory and/or longer remaining time to sell goods lead to the higher expected revenue. The expected revenue function is non-decreasing and concave in the remaining inventory level and the remaining time. The optimal price decreases as the inventory increases and the optimal price increases if there is more time to sell for a given level of inventory.

Due to the higher arrival probability, the probability of having customers with higher reservation prices increases and the products with higher prices are sold. However, when the arrival probability is lower, it is more convenient to set the prices lower. Otherwise, the small number of arriving customers will not purchase the product. It will lead to increased holding cost, excess on hand inventory and loss of the customers. The seller does not benefit from reserving some stock of the product for future periods (limiting sales in early periods), if the level of demand is decreasing over time.

The expected revenue in a full refund system is worsened every time when cancelation occurs. So, the seller cannot take advantage of the cancelation with resale as an opportunity to improve the total revenue in a full refund system. The ratio of the arrival probability to cancelation probability influences the expected revenue of a seller. Actually, it turns out that cancelation is so significant that its effect cannot be ignored. If the arrival probability is not much higher than the cancelation probability, the seller cannot yield higher revenue.

If the price markdowns are rarely advertised, then the customers have little information about prices before they go to a store. Hence, customers' arrival probability is independent of price, but their purchase depends on price. In fact, customer reservation price strictly increases in a posted price $\mathbf{p}$. Those arriving early in the season have higher valuations than those arriving later (strictly decreasing function of $t \in [0, T]$). Customers' valuations for seasonal goods tend to peak at the beginning of the selling period and decline with time.

There are few possible research extensions. Batch demand is an important extension because in many cases a customer needs to buy multiple units of the same product. Secondly, some sort of partial refund policy (time dependent and also demand dependent refund policy) should be considered to see if a seller could take advantage of the cancelation to increase expected revenue. In addition, in order to describe the process more dynamically, the time-dependent arrival and cancelation probability should be considered. Also, customer segmentation can be incorporated to the model. The solutions of these models will help a management to decide on more realistic pricing strategies by making better price discrimination.

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