REPLENISHMENT AND LATERAL TRANSSHIPMENT DECISIONS IN MULTI-PERIOD AND MULTI-LOCATION INVENTORY SYSTEMS

by

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ABSTRACT

REPLENISHMENT AND LATERAL TRANSSHIPMENT DECISIONS IN MULTI-PERIOD AND MULTI-LOCATION INVENTORY SYSTEMS

Inventory management has been researched for many years in the literature. The problem of determining the optimal inventory level directly affects the profitability of the systems. This thesis proposes an optimization framework for multi-item, multi-location inventory management systems to obtain optimal inventory levels for multiple periods. The distribution of demand is unknown, it is estimated based on the probabilities generated by a random forest. Backlog is not allowed, demand is lost in case of stock-out. The first analyses conclude that as the scenario number considered in the mathematical model increases, the system stabilizes and performs better due to the consideration of variation in demand. To evaluate the robustness of the model, the suggested linear programming model is tested on different inventory level cases and it is concluded that inventory cases have important effect on objective values of the system. The analyses are constructed on two systems with different management strategies: firstly, the system is considered to allow lateral transshipment along with the replenishment, then the system is assumed to allow only replenishment decisions. It is concluded that allowing transshipment decisions improve the lost sales of the system. After the case analyses, objective value distribution for scenarios are analyzed to evaluate the robustness of the proposed method. Finally, the periodic review strategy is tested and it is concluded that there is a significant difference between the systems with periodic review and single review strategy. In addition to the prediction methods used in the case analyses, smart MA is introduced and the analyses show that it is a powerful prediction method for the simulated system.

ÖZET

ÇOK DÖNEMLİ VE ÇOK KONUMLU ENVANTER SİSTEMLERİNDE DEPODAN İKMAL VE YANAL AKTARIM KARARLARI

En uygun envanter seviyesini belirleme sorunu, sistem karlılığını doğrudan etkiler ve uzun süredir araştırılmaktadır. Bu çalışmada, en uygun envanter kararlarını elde etmek için; çok ürünlü, çok konumlu envanter sistemi birden çok dönem için optimize edilmiştir. Talep dağılımının bilinmediği kabul edilerek oluşturulan matematiksel modelde, talep dağılımı random forest metodu sayesinde oluşturulmuştur. Stok adedinin sıfıra düşmesi durumunda karşılanamayan talep kaybedilir. İlk analizler, matematiksel modelde kullanılan senaryo sayısı artışını test etmiştir, artan senaryo sayısı talepteki değişkenliği daha iyi yansıttığı için sistemin daha istikrarlı ve daha iyi sonuç vermesini sağlamıştır. Ek olarak, önerilen model, farklı envanter seviyesi senaryolarında test edilmiştir. Yapılan testler sonucunda envanter seviyesinin sistem performansı üzerinde etkili olduğu görülmüştür. Analizler, iki aşamalı olarak sürdürülmüştür. İlk olarak, sistemin ikmal kararları ile birlikte yanal aktarmaya izin verdiği düşünülmektedir. Daha sonra sistemin yalnızca ikmal kararlarına izin verdiği varsayılır. Sistemin ikmal kararlarına ek olarak yanal aktarım kararı vermesi sistemde toplam kayıp satış tutarını azaltırken, envanter taşıma seviyesini artırmıştır. Farklı envanter seviyesi analizlerinden sonra sistemin sipariş kararı alma sıklığı için kullanılan strateji test edilmiştir. Yapılan testler sonucunda dinamik ve statik izleme stratejilerinin arasında önemli bir fark olduğu görülmüştür. Tahmin modellerinde random forest methoduna ek olarak, kayan ortalama ve örneklem ortalama yaklaşımı yöntemleri kullanılmıştır. Sistemler kullandıkları tahmin modellerine göre karşılaştırıldığında, rassal orman metodunun daha yüksek performans sağladığı görülmüştür.

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LIST OF SYMBOLS

c_{ijs}	Cost of shipping item i from location j to s
$cogs_i$	Purchase cost for item i
$demand_{ij}^t$	Demand for item i in location j at period t
$demand_{ijk}^t$	Demand for item i in location j at period t for scenario k
$holding$	Inventory holding cost rate
i	index for items
I	Set of items
inv_{ij}^t	Beginning inventory for item i at location j period t
inv_{ijk}^t	Beginning inventory for item i at location j period t for scenario k
j	index for locations
J	Set of origin locations
k	index for scenarios
K	Set of scenarios
$leadtime_{js}$	Lead time for shipment from j to s
$OUTL_{ij}^t$	Order up to level for item i at location j period t
$price_i$	Resale price for item i
s	index for destination locations
S	Set of destination locations
$sales_{ij}^t$	Sales for item i at location j period t
$sales_{ijk}^t$	Sales for item i at location j period t for scenario k
$shipment_{ijs}^t$	Shipment decision for item i from location j to location s at period t
$stock_{ij}$	Initial inventory for item i in location j
$stock_{ijk}$	Initial inventory for item i in location j for scenario k
t	index for periods
T	Set of periods

LIST OF ACRONYMS/ABBREVIATIONS

ANOVA	Analysis of variance
ARIMA	Auto-regressive integrated moving average method
Avg	Average
KPI	Key performance indicator
LS	Lost sales
MA	Moving Average
OUTL	Order up to level
Promo	Promotion
RF	Random Forest
SAA	Sample Average Approximation
WH	Warehouse
WOS	Weeks of Supply

1. INTRODUCTION

Inventory management has been researched for many years in the literature. Different policies are studied to determine the optimal inventory level for the locations with unknown demand. The problem of determining the optimal inventory level directly affects the profitability of the systems. Optimizing the inventory level enables the system to balance the lost revenue due to out of stock and inventory holding cost. In order to obtain optimal inventory levels, the studies in the literature focus on the same problem with different assumptions on the number of items, number of locations, time horizon, or cost structure in the system. While using only replenishment decisions is a common approach, there are also some approaches considering replenishment and transshipment decisions together as an inventory management strategy. This thesis aims to answer the questions of how the optimal decisions both for the replenishment and transshipment are generated under unknown demand distribution assumption and whether the decisions are changing based on the systems inventory level cases.

The solution to the stochastic replenishment problem starts with the classical newsvendor problem in the literature. It considers a system with single-item, single-period assumptions. However, due to resource capacity constraints or interdependent demand assumption of items, multi-item consideration is needed. Gallego (1996) and Smith (2000) both consider a multi-item inventory management problem. Gallego uses it to consider the resource capacity constraints as Smith prefers multi-item problem to consider the substitution effects. Erlebacher (2009) also considers a multi-item problem with capacity constraint but in order to consider the resource capacity, multi-location consideration is proposed in his study. As the number of locations increases, the ability to redistribute inventory among locations becomes more valuable. Therefore, as a shipment decision strategy, systems allowing lateral transshipment are studied in literature. Banerjee and Burton (2003) stated lateral shipment approaches are superior to a policy of no such shipments. Paterson et al. (2011) provide a literature review that categorizes the research on lateral transshipment, two different types are identified; proactive and reactive lateral transshipments. As reactive transshipment decisions are given after demand realizations, proactive transshipment decisions are given before demand realizations not to have stock-outs. In order to decide to move inventory from one location to another, demand for future periods should be considered. This creates a need for multi-period consideration. Karmarkar (1980) and Matsuyama (2006) provide multi-period extensions to the classical newsvendor problem. As multi-period inventory management problems are considered, the review frequency becomes important. Agrawal et. al. (2004) study the timing of the redistribution decisions and new inventory levels with dynamic programming.

The most critical part of the studies is the demand prediction, and the demand is assumed to be deterministic or stochastic in the studies. Stochastic demand assumption is more applicable for real life cases, this is why in the literature the demand is assumed to be stochastic and the probability distribution of demand is known. However, Bertsimas and Kallus (2020) offer a new optimization approach without making any assumptions on the distribution of demand [1]. They develop a new framework combining the capabilities of machine learning (ML) and operations research (OR). Additionally, they suggest using auxiliary data in addition to the sales observations in prediction methods.

In this thesis, a mathematical model with linear programming method is proposed to solve the optimization problem by better estimation of demand distribution based on conditional probabilities estimated by random forests. The framework suggested by Bertsimas and Kallus (2020) is used [1]. Multi-item, multi-location inventory management system is optimized for multiple periods. Backlog or back-order are not allowed and demand is lost in case of stock-out. Firstly, the mathematical model without making any assumptions on demand distribution is tested for different number of scenarios. It is seen that as the number of demand scenarios considered in the mathematical model increases, the system stabilizes and performs better due to the consideration of variation in demand. Then, the model with the highest scenario number is tested on different inventory level scenarios. Statistical tests show that inventory level cases have significant effects on objective values of the system. Therefore, the optimal strategies change according to the inventory level cases of the management system. The analyses consider two systems with different management strategies. Firstly, the system allow lateral transshipment along with the replenishment decisions. Then, the system is assumed to allow only replenishment decisions. As a result of the comparison, it is concluded that allowing lateral transshipment decisions improve the lost sales values of the system for the systems with high inventory levels. After analyzing the inventory level cases, the proposed optimization framework is analyzed to see if there are huge differences among the objectives for each demand scenario, so that robustness of the system is evaluated. Finally, the strategy used for the review period of the system is tested. Periodic review strategy is compared to the first-period review strategy for a system with three periods. It is shown that there is a significant difference between the systems with periodic and single review strategy and periodic strategy obtains lower lost sales values. In addition to the random forest, moving average and sample average approximation methods, smart moving average is also tested and it reveals that although the smart moving average is a powerful prediction method for the basic simulated system, random forest method performs better than the other systems for the systems with high inventory due to its wide range of variation consideration.

The thesis is organized as follows. Chapter 2 presents a brief literature review on the newsvendor problem and its extensions. Quick background information for prediction methods and the key performance indicators used are explained in details in Chapter 3. As a solution approach, constructed mathematical model and detailed system summary is covered in Chapter 4. It also focuses on how the stochastic nature of demand is handled in the mathematical model. Then computational results are summarized in Chapter 5. In conclusion, the approach is summarized and future research suggestions are presented in Chapter 6.

2. LITERATURE REVIEW

In the literature, inventory management has been researched for many years and it differs in many dimensions, such as the number of items or locations, time horizon, assumptions on demand characteristics, or cost structure. The main distinguishing feature is the assumption of demand characteristics, the literature can be categorized into two; studies with the assumption of deterministic demand and studies with the assumption of stochastic demand.

In the literature, inventory management studies with the stochastic demand started with the classical newsvendor problem. It considers a system with single-item, single-period assumptions. However, due to resource capacity constraints or interdependent demand assumption of items, multi-item consideration is needed. For the items with short selling season, such as newspaper, the single period assumption is reasonable, but if the remaining inventory is usable in the next periods then multi-period assumptions should be considered. As the number of periods in the inventory problem increases, redistribution of allocated inventory can be considered to improve the system performance. In a few words, it started with the classical newsvendor problem but its extensions are studied for years. To understand the categorization of the studies in the literature, the dimensions of inventory management problems are reviewed by Silver (2008) [2]. The summary of categorization that Silver (2008) studied can be seen from Figure 1.

The first dimension in Figure 1 is the single-item versus multiple-item consideration in inventory management. The single-item assumption is mostly considered to divide the problem into sub-problems because having multiple items increases the complexity of the models. As Silver (2008) mentioned, this dimension affects the scope of the calculations. Therefore, if the inventory problem can be divided into independent sub-problems, then the single item assumption is reasonable. However, in order to solve inventory management problems with resource capacity constraints, multi-item problem consideration is needed.

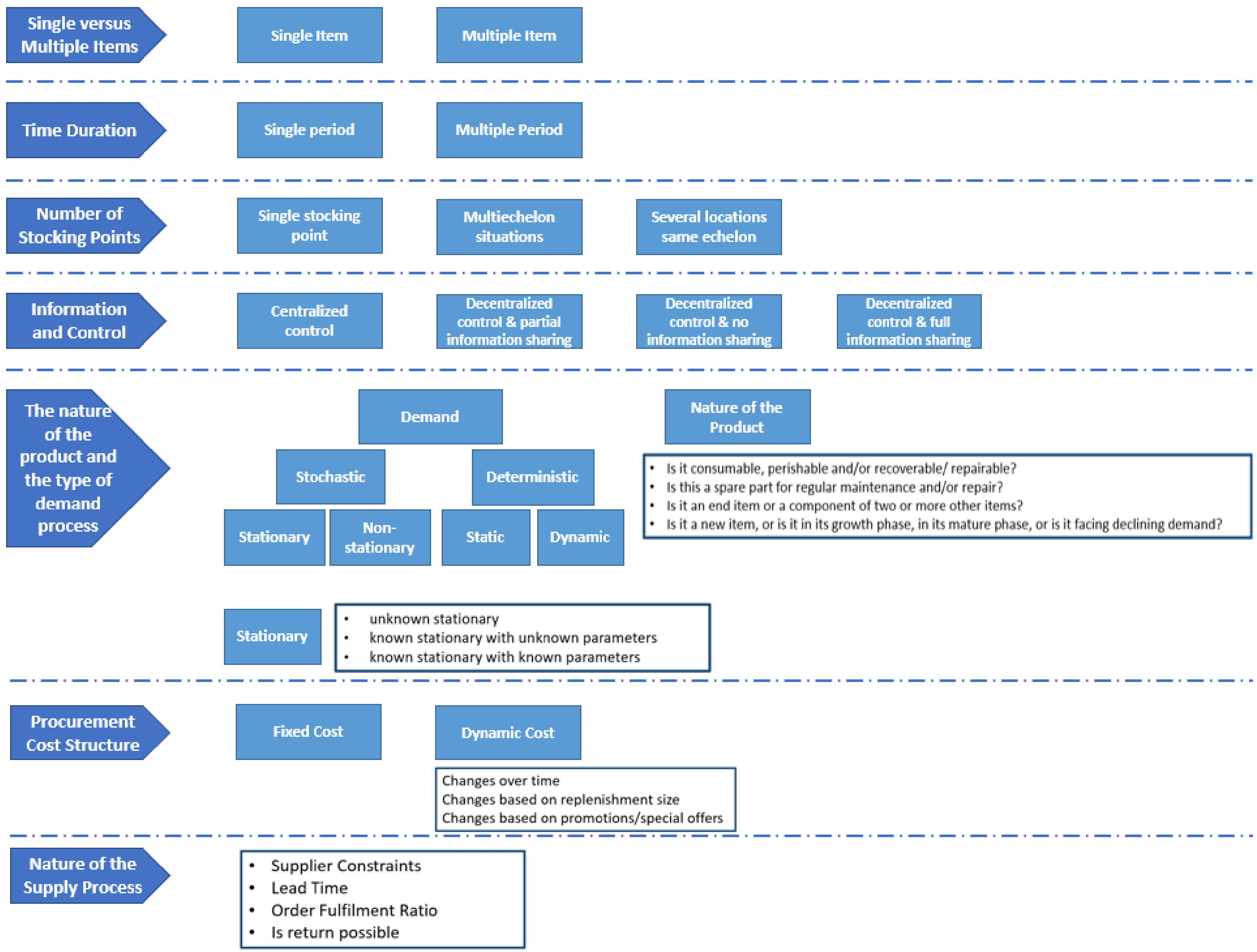


Figure 1: Dynamics of Inventory Management

Gallego (1996) mentioned, in case of the absence of resource constraints, single-item inventory problems can be solved by the economic order quantity formula [3]. If the resource capacity is considered then the problem should contain more than one item and having more than one item increases the complexity of the system. In addition to the systems with resource capacity, systems with the assumption of demand substitution also require multiple item consideration. Smith (2000) considers the substitution effect across items in the optimal inventory level calculations [4]. The substitution effect is calculated for a demand increase of a certain product when other items are out of stock. Planning promotion on certain items also requires considering cross effects among items. Therefore, items with interdependent demand patterns should be optimized in the same model. In this thesis, interdependent characteristics of the items are ignored but multi-item consideration is preferred in the optimization model because the problems with resource capacity can be solved easily by adding one or two constraints to the suggested model.

Erlebacher (2009) also considers a multi-item newsvendor problem with a single capacity constraint [5]. The capacity of the resources also requires considering the number of stocking locations which is another important dimension mentioned by Silver (2008). This dimension is categorized as, systems with a single stocking point, several locations in multi-echelon systems, or several locations in single-echelon systems [2]. In this thesis, a multi-location inventory problem with one distribution center is studied. For the systems with several locations, as there is a finite inventory in the system, transshipment decisions among the locations become valuable so the scope of the problem extends. Paterson et al. (2011) provide a literature review that categorizes the research on lateral transshipment [6]. Two different types of transshipment are identified: proactive and reactive transshipment. In reactive transshipment models, lateral transshipment is used to meet the demand in case of a location faces a stock-out while other locations have enough inventory on hand. In proactive transshipment models, the inventory is redistributed before demand realizations.

In the literature, reactive transshipment is considered as suitable for the supply chains having low transshipment costs such as the spare part environment and instead of holding a large amount of inventory, transshipment after the demand realizations is preferred. Proactive transshipment is much more suitable for the retail sector as handling cost is relatively dominant [6]. Therefore, in the systems with several stocking points, in addition to the replenishment decisions, transshipment decisions need to be considered. In this thesis, the optimal decisions are analyzed both for the system with only replenishment decisions, and the replenishment with lateral proactive transshipment decisions. This provides an output indicating the importance of the lateral transshipment in the supply chain. Banerjee and Burton (2003) stated lateral shipment approaches are superior to a policy of no such shipments, despite the increasing expense of transportation activity. However, the transshipment decisions and replenishment decisions are not integrated in the stated policy. Gross studied the static point lateral transshipment and ordering policies jointly [7]. Two locations are considered, and the lead time is assumed to be zero. For more than two locations a numerical iterative procedure is provided. However, in real life, most of the time lead time is positive. Diks and De Kok (1996) compare several heuristics for multiple locations with lead times [8]. A rationing policy is used for lateral transshipment. The ratios for the locations are externally given and the objective of the transshipment is to keep each location's inventory constant (a fraction of total projected inventory) [9]. In addition to the ordering policies in the literature, optimization solvers are used to obtain the ordering decisions. An optimization model generating optimal decisions for both replenishment and lateral transshipment is developed by Vicente(2015). It considers a multi-item, multi-echelon inventory system with normally distributed demand assumption [10]. Positive lead time and transportation capacities are considered in the Vicente's paper. In this thesis, positive lead time assumption is used but transportation capacity is not handled, multiple locations are studied and replenishment and transshipment decisions are generated jointly.

Inventory management problems generally focus on determining how much to ship, but another important question is when to ship if multi-period consideration is used. Silver (2008) categorized the inventory management systems for the time duration dimension as single vs multiple periods. The classical newsvendor problem focuses on single period systems but if the remaining inventory is usable in the next periods then multi-period assumptions should be considered. Karmarkar (1980) provides a multi-period extension with negligible lead times. It is shown that the characteristics of the optimal solutions are similar to the single-period problems [11]. However, this is due to the negligible lead time assumption. For the items with long selling seasons, Matsuyama (2006) extends the classical newsvendor problem to a multi-period problem in order to be able to take the unsatisfied demand and unsold quantity into account [12]. The aim was not only to determine ordering decisions but to determine optimal inventory level. Additionally, as upcoming promotions and special days are important factors on demand, multi-period consideration might decrease the stock-out possibilities for the systems with promotion activities.

Multi-period extensions needs to consider the timing of the shipments and determining new inventory levels periodically. Agrawal et. al. (2004) study the timing of the balancing shipments and optimal inventory levels with a dynamic programming. They also consider the effect of starting inventories to the proactive transshipment decisions [13]. Study shows that the value of transshipment decisions increases with the number of locations and dynamic transshipment policies are superior to the static ones especially in systems in which starting inventories at the retailers are balanced. Therefore, starting inventory level is an important factor and affects the optimal decisions. Initial supply level feature can be categorized under the nature of the supply process in Figure 1. For the nature of the supply process, supplier constraints, lead time, service levels are also important features. For the classical newsvendor problem, orders are fulfilled from the supplier without any constraints. However, supply in the system in real life may not be sufficient to fulfill shipment decisions. In this thesis, supplier has a finite inventory, lead times are deterministic and fixed. Cost structure of the inventory management systems might be analyzed under the supply process dimension, but Silver (2008) categorized the cost structure as a separate one. The unit value of a product may be fixed or can be determined based on special offers and promotions defined by suppliers. In this thesis, cost value and price of the items are assumed to be fixed.

Among all the categories discussed in the inventory management problem, demand characteristics are the most important and distinguishing one. The classical newsvendor problem and most of its extensions in the literature assume the complete knowledge of probability distribution of demand. Silver (2008) mentioned the dimensions of demand under the nature of demand process category. Demand is assumed to be either stochastic or deterministic in the literature [2]. Chiang (2006) considers a periodic review system for replenishment decisions and assumes the demand has a certain probability distribution [14]. Dynamic programming is used to propose two different solutions both for exceeding demand is lost or back-ordered. Bertsimas and Thiele (2004) stated that since the dynamic programming has a disadvantage as curse of dimensionality; a new optimization method is needed and the new approach should be designed without making any assumptions on the demand distribution. They propose a deterministic, numerically tractable methodology to solve the problem of optimally controlling supply chains subject to uncertain demand [15]. The solution does not include any demand distribution assumption. The neccesity of the knowledge of distribution is tested by Benzion et. al. (2010) and it reveals that the knowledge of the demand distribution does not necessarily improve the system performance. Therefore, without knowing the demand distribution, solving the inventory management problem with a high performance is possible. Ban and Rudin (2019) proposes a single-step solution for the newsvendor problem with the machine-learning algorithms. They mentioned the "big-data" concept in the paper and use the demand related features as well as historical data. Demand related features are also used as auxiliary variables by Bertsimas and Kallus (2020) and they develop a new framework combining the capabilities of machine learning (ML) and operations research (OR) [1].

In this thesis, the framework suggested by Bertsimas and Kallus (2020) is considered. The decision is how much to order from warehouses or to ship from other locations for multiple items, multiple locations. Therefore, the system with multi-item, multi-location problem is considered for multiple periods. Although the multi-item consideration is used, interdependent characteristics of the items are ignored in the prediction methods. Cost value and price of the items are assumed to be fixed and known. As obtaining the optimal shipment decisions the lead time is assumed to be positive and deterministic. In case of stock-outs, unfulfilled demand is lost. Demand distribution is unknown and a mathematical model is proposed to solve the optimization problem by better estimation of demand distribution based on conditional probabilities estimated by random forests.

3. BACKGROUND

In this section, details of prediction methods and performance metrics used in the following sections are discussed. According to the estimated demand with different prediction methods, replenishment and transshipment decisions are made. The system representation for a location can be seen from Figure 2. As these methods and metrics are explained, the following representations used in Figure 2 are used; i stands for items, as j represents the locations in the supply chain system and t is used for the decision periods in the system. inv_{ij}^t is the beginning inventory for item i, location j at period t. Demand for sales for item i at location j is represented as $demand_{ij}^t$ for period t. As $demand_{ij}^t$ is fulfilled, $sales_{ij}^t$ occurs. It indicates the sales amount at period t for item i and location j. In order to fulfill the demand, inventory is needed so the movement of inventory in the system is represented as $shipment_{ijs}^t$. It is the shipment amount for item i from location j to location s at period t.

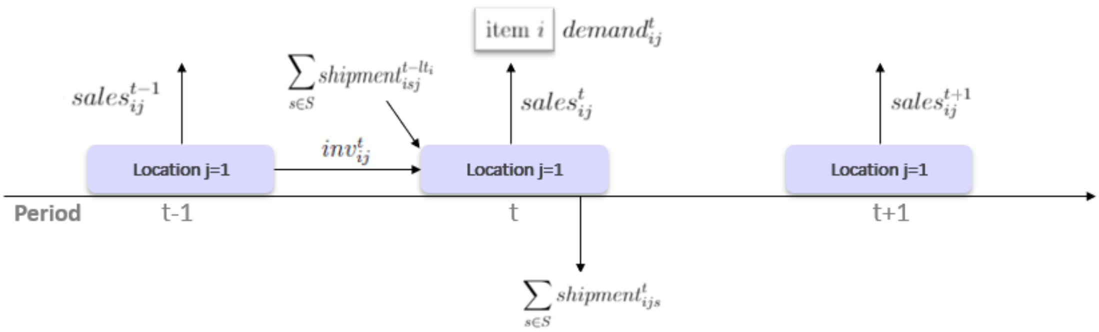


Figure 2: System flow for location j over periods

From Figure 2, it can be seen that location j has sales for items i at different periods. As these sales occur, in/out shipments have been realized. The details for the system flow are discussed in the system description section. In this section, prediction methods and different performance metrics are discussed to compare the accuracy of the shipment decisions.

3.1. Prediction

The predictive distribution of the demand is estimated by the following three different methods: moving average, sample average approximation, and random forest.

3.1.1. Simple Moving Average. Simple moving average is the most common method used in forecasting. The forecast values are calculated based on the average of the past m sales data. Predictions are deterministic and dependent on the previous sales values. This method provides point estimates, instead of a predictive distribution.

For $t > m$,

$$demand_{ij}^{t} = 1/m \sum_{z=t-m}^{t-1} sales_{ij}^{z} \tag{1}$$

For Figure 2, in order to predict $demand_{ij}^{t+1}$, by assuming m=2, the calculation is the average of $sales_{ij}^{t-1}$ and $sales_{ij}^{t}$

3.1.2. Sample Average Approximation. Sample Average Approximation is also a common method used in stochastic optimization problems. The demand can be approximated by a sample average estimate derived from the past sales data. After generating a random sample, of size N, the average of the members of the sample is used as demand. This method is used with $N = 1$ in order to keep the variance information in sales. Each sample taken from the population is used as demand prediction. This method provides a predictive distribution obtained from the realized sales observations.

3.1.3. Tree-based Ensembles. Tree-based ensembles are commonly used for supervised learning methods. It is one of the most effective machine learning methods for predictive analysis especially when auxiliary data is available. In the retail sector, past sales data, related previous price information, and web click information if an online channel is available, can be stored easily these days. These types of data can be considered as auxiliary data in the problem [1]. Any information that affects the sales can be used to train the tree-based models to obtain more accurate prediction results. For both categorical and continuous input and output variables, tree-based ensembles can be used. In order to predict a continuous variable, regression trees are used, whereas for categorical variables classification trees are used. In this supply chain problem, the aim is to predict demand, which means regression trees are needed. Regression trees follow a set of if-else conditions to calculate the predictions. Each branch in the trees split into a node, and at the end, the nodes that have no additional nodes coming off them are known as leaf nodes. Each individual leaf node has a single prediction.

Random forest is an ensemble learning method consisting of many regression trees. With the help of more than one regression tree, it may explain complex relations among variables. One of the prediction methods used in the following sections is the random forest. In order to see the effect of the number of trees in the random forest, 10 different scenarios are generated. The scenarios are determined at 10 intervals from 10 to 100 and each leaf value in the random forest is considered as a scenario. The details for the application of random forest are discussed in the analysis section.

3.2. Key Performance Indicators

In order to evaluate the proposed approaches, two key performance indicators are considered. The first indicator is selected as the lost sales ratio of the system:

$$LS = \frac{\sum_t lostsales^t}{\sum_t lostsales^t + \sum_t profit^t} \tag{2}$$

where

$$lostsales^t = \sum_{i \in I} \sum_{j \in J} (demand_{ij}^t - sales_{ij}^t) profit_i \tag{3}$$

In Equation (3) sum of lost profit at period t for all items and locations is calculated. In Equation (2), overall lost profit metric is calculated by dividing the total lost profit to total expected profit. Lost sales is an important metric to measure the system's inventory performance, having stock-outs does not only mean lost revenue but also impacts customer satisfaction and loyalty. However, it is a common mistake in the retail sector to hold high inventory to avoid lost sales. Excess inventory implies holding costs including the cost of goods, storage space, labor, and insurance. There is also an opportunity cost of the money invested in the inventory. In order to measure the excess inventory, weeks of supply metric is used as a second indicator of the performance of the system:

$$WOS = [\sum_{t} wos_t]/|T| \quad t \in T \tag{4}$$

$$s.t$$

$$wos_t = \frac{\sum_{j}\sum_{i} inv_{ij}^{t} cogs_i}{\sum_{y=t}^{t+n-1} \sum_{j}\sum_{i} demand_{ij}^{y} cogs_i/n} \tag{5}$$

where n is a parameter indicating the number of periods considered in the calculation of average demand. In Equation (5), weeks of supply values for each period is calculated. This metric indicates the number of periods that inventory value fulfills the average cost of future demand. In Equation (4), the overall weeks of supply value is calculated by averaging the weeks of supply values of each period. In the following sections, the system is evaluated based on lost sales and weeks of supply metrics in addition to the objective values of the optimization model.

4. APPROACH

4.1. Problem description

For the inventory management problem, a multi-item, multi-location system with one distribution center is considered. The aim is to determine shipment decisions among the locations by obtaining the optimal inventory values. All locations are reviewed periodically for both replenishment and transshipment decisions. The time that elapses between the shipment decision and the delivery of the goods is nonnegligible, in other words, replenishment and transshipment orders are delivered with a positive lead time. Initially, the problem is formulated with known demand, but in the following sections, stochastic demand with unknown distribution is considered. In the mathematical model, the following notations are introduced.

Indices

$i \in I = \{1, \ldots, \lvert I \rvert\}$	index for items
$j \in J = \{1, \ldots, \lvert J \rvert\}$	index for origin locations
$s \in S = \{1, \ldots, \lvert S \rvert\}$	index for destination locations
$t \in T = \{1, \ldots, \lvert T \rvert\}$	index for periods

Parameters

$stock_{ij}$	initial inventory for item i in location j
$demand^t_{ij}$	demand for item i in location j at period t
$cogs_i$	purchase cost for item i
$price_i$	resale price for item i
$leadtime_{js}$	leadtime for shipment from j to s
c_{ijs}	cost of shipping item i from location j to s
$holding$	inventory holding cost rate
wh	warehouse code in the system

<u>Variables</u>

inv_{ij}^{t}	variable indicating beginning inventory for item i at location j period t
$sales_{ij}^{t}$	variable indicating sales for item i at location j period t
$shipment_{ijs}^{t}$	variable indicating transfer decision for item i from location j to location s at period t

In the mathematical model, it is assumed that inventory, demand, cost, and price values for items, the lead time between the locations, the shipment cost, the holding cost parameters are known. The goal is to determine the inventory level and shipment quantities. Therefore, sales, inventory levels, and shipment quantities are defined as variables. $stock_{ij}$ is a parameter used to assign the initial inventory of items for each location, in other words, it is the inventory at $t = 0$. $demand_{ij}^{t}$ represents the demand for each item, location, and period. Demand for the distribution center is assumed to be zero for each item and period, as it is in the retail business, and it can only take non-negative values for locations.

Purchase cost as $cogs_i$ and resale price as $price_i$ are defined for each item, it is assumed that items at different locations have the same price and cost values. This assumption might be extended easily. However, it would not create additional value for our problem. Shipping items from one location to another causes not only operational costs but also opportunity costs and some possible damage costs. For some items, it is highly risky to ship from location to location due to its fragility, items may be damaged during the shipment. Opportunity costs of not keeping the items at the origin should be considered. This is why the cost of shipping an item c_{ijs} is designed to be based on item, origin, and destination locations. Additionally, the inventory holding cost at locations is considered. In order to calculate the inventory holding cost *holding* is used as a rate of return defined in the economy. With the help of the parameters defined above, replenishment and transshipment decisions are optimized to maximize the expected average total profit. Total profit is the sum of profit gained from sales, minus lost profit due to unfulfilled demand and cost of replenishment, transshipment, holding inventory.

Since a multi-period inventory system is considered, shipment decisions previously made should be taken into consideration as in-transit inventory and it is known that previous shipment decisions will arrive exactly after the shipment duration. So $leadtime_{js}$ is used to represent shipment duration, location to location, shipment duration may differ based on the distance between the locations or the method used in shipping (own trucks, third party transshipment etc.) The assumptions made for the inventory system will be explained in detail in the following section.

4.2. System Description

In order to construct the analyses on the suggested model, a system with multiple items and multiple locations is generated. It is assumed that transshipment decisions are made before demand realizations (proactive) and the aim is to satisfy possible demand. Events occur in the following order for period t:

- Arrival of inventory shipped in $t - leadtime$
- Shipment of transshipment and replenishment orders
- Realization of demand (Backlog is not an option, unfulfilled demand is lost)
- Remaining inventory is carried to the next period, $t + 1$.
- Inventory holding cost is incurred

The system flow for period t is summarized with the help of the visual representation in Figure **??**. Each flow in the figure is numbered to clarify the processes in detail. Holding cost for the period $t-1$ is calculated and the ending inventory is carried to the current period (0). Ending inventory for period $t-1$, in other words, beginning inventory for period t is represented as inv_{ij}^{t} in the model. The system for period t starts with the arrival of the goods that have been shipped in the previous periods (1). The shipment decision that arrives to location j at period t is represented as $\sum\limits_{s \in S} shipment_{isj}^{t-leadtime_{sj}}$.

The shipment decisions from location j at period t is calculated as $\sum_{s \in S} shipment^t_{ijs}$, after the shipment of the transshipment decision from the location (2), demand realization occur (3). In other words, demand can be fulfilled from the arrived inventory and can not use the inventory that is planned to be shipped. Fulfillment of demand leads to $sales^t_{ij}$. As a final step, the remaining inventory after the sales is carried to the next period (4) and it is represented as inv^{t+1}_{ij}.

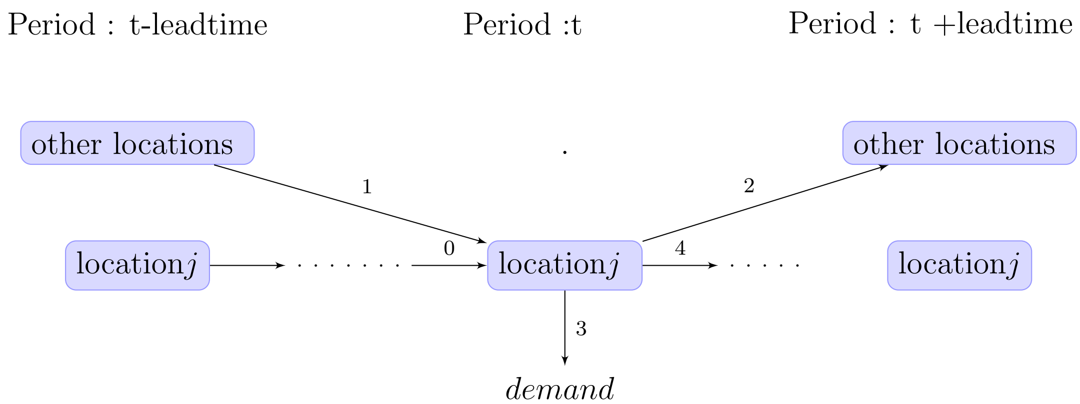


Figure 3: System Flow

In order to clarify the system, assumptions made on the parameters should be discussed. Lead time and cost parameters play an important role in supply chain decisions, also demand characteristics are important. The planning time horizon is discrete and periodic shipment decisions are considered, it is represented as t. Price and cost parameters for items are the same over locations. Initial inventory level (for time period $t = 0$) for each item - location is known. Although lateral transshipment lead time is mostly ignored in the lateral transshipment literature, replenishment and transshipment lead times are assumed to be positive and deterministic. It can be defined from location to location as it is in the real distribution systems. The inventory shipped in period t, arrives at $t+leadtime$ and the transportation lead times between locations are assumed to be given. Additionally, ordering and shipment costs are known. Transshipment cost is assumed to be higher than replenishment cost. Therefore, it would be preferred to ship from the warehouse instead of stores if inventory is available in the warehouse. Lost sales cost is equal to the sales price minus the cost of the item and it is assumed to be known. The backlog is not allowed. Unfulfilled demand is lost. In the following section, details of the optimization model for inventory management are studied.

4.3. Model Formulation

With the help of the variables defined in the previous sections, a mathematical model with linear programming method is constructed to obtain the optimal shipment decisions periodically.

$$\max z = \sum_{t \in T} \sum_{i \in I} \sum_{j \in J} (price_i - cogs_i) sales_{ij}^t$$
$$- \sum_{t \in T} \sum_{i \in I} \sum_{j \in J} inv_{ij}^t (cogs_i) holding - \sum_{i \in I} \sum_{t \in T} \sum_{j \in J} \sum_{s \in S} shipment_{ijs}^t c_{ijs}$$

s.t.

$$inv_{ij}^t = stock_{ij} \qquad \forall i \in I, \forall j \in J, \text{for t=1} \tag{4.6}$$

$$inv_{ij}^t - \sum_{s \in S} shipment_{ijs}^t \geq sales_{ij}^t \qquad \forall i \in I, \forall j \in J, \forall t \in T \tag{4.7}$$

$$inv_{ij}^t = inv_{ij}^{t-1} - sales_{ij}^{t-1} + \sum_{s \in S} (shipment_{isj}^{t-lt_i} - shipment_{ijs}^{t-1}) \qquad \forall i \in I, \forall j \in J, \forall t \in T \tag{4.8}$$

$$sales_{ij}^t \leq demand_{ij}^t \qquad \forall i \in I, \forall j \in J, \forall t \in T \tag{4.9}$$

$$\sum_{j \in J} shipment_{ijs}^t = 0 \qquad \forall i \in I, \forall t \in T, \forall s \in S, \text{ for s=wh} \tag{4.10}$$

$$shipment_{ijs}^t \geq 0 \qquad \forall i \in I, \forall j \in J, \forall t \in T, \forall s \in S \tag{4.11}$$

$$sales_{ij}^t \geq 0 \qquad \forall i \in I, \forall j \in J, \forall t \in T \tag{4.12}$$

$$inv_{ij}^t \geq 0 \qquad \forall i \in I, \forall j \in J, \forall t \in T \tag{4.13}$$

The objective function maximizes the profit which is equal to the total profit obtained by sales less the inventory holding cost, and transshipment costs. The first term represents the profit for items by multiplying the achieved sales with profit, while the second and third term stands for the the inventory holding cost and the transshipment cost due to the decision of shipping items from location j to location s respectively.

Constraints (4.6) assign the first-period beginning inventory to the given initial inventory. Constraints (4.7) make sure that sales realized in period t and store j is less than the beginning inventory of period t, store j less the sum of transshipment decisions given from store j to any other store for each item. In other words, transshipment decisions can not be higher than the inventory after realization of sales at store j. Constraints (4.8) stands for the inventory balance constraint. Beginning inventory for period t equals to the $t-1$ beginning inventory less the realized sales at period $t-1$ less the shipped amount at period $t-1$, plus the arrived inventory shipped at period $t - leadtime$. Constraint (4.9) guarantee that sales realized is less than demand. Constraints (4.10) ensures that transshipment to the warehouse is impossible. Constraints (4.11), (4.12),and (4.13) are the cardinality constraints indicating the positive rules of shipment decisions, inventory and sales.

This model determines the optimal decisions for a system with lateral transshipment and replenishment orders from warehouses. If lateral shipment use is not an option, then the following equation should be added to the model.

$$\sum_{s \in S} shipment^{t}_{ijs} = 0 \qquad \forall i \in I, \forall t \in T, \forall j \in J \text{ for } j \neq wh \tag{4.14}$$

Although the demand value represented as $demand^{t}_{ij}$ is assumed to be deterministic in the above model, it is stochastic and its distribution is unknown. Therefore, the assumption of deterministic demand is extended in the following section.

4.4. Model with Unknown Demand Distribution

The solution of the mathematical model with the full knowledge of demand provides the optimal decisions. However, demand has a stochastic nature in real life. Therefore it is predicted with different forecasting methods in the literature. Moving average, regression, and machine learning methods are examples of widely used prediction methods in inventory management problems. If the assumption for demand characteristic is stochastic, then the objective function should be maximizing *the ex-*

pected profit for inventory management decisions [1].

$$max_{decision \in Z}\, \mathbb{E}(G(decision; demand)) \tag{4.15}$$

Equation (4.15) indicates that the optimal decision should be the one that maximizes the expected gain. The equation can be rewritten with its four major components as follows:

$$G(decision; demand) : max_{decision \in Z}(Revenue - ShipmentCosts - HoldingCosts)$$

After realization of total revenue, lost sales, shipment, and holding costs are subtracted to calculate the gain. Before constructing the mathematical model, the prediction for demand should be generated. However, obtaining good predictions does not always provide good decisions in the inventory management systems, because it is not clear how to go from a good prediction to a good decision [1]. As suggested in [1], the problem stated in Equation (4.15) is transformed to:

$$max_{decision \in Z} \sum_{k \in K} w_k G(decision; demand^k) \tag{4.16}$$

Equation (4.16) indicates that the optimal decision should be the one that maximizes the weighted expected gain for different scenarios. Scenarios are represented by k in Equation (4.16). Instead of optimizing the shipment decisions by maximizing the gain function generated by a single prediction $demand^t_{ij}$, weighted average gain function for k different scenarios can be maximized. As a new index k for demand parameter $demand^t_{ijk}$ is introduced, the optimization model is modified accordingly. The new model is constructed as follows.

Indices

$i \in I = \{1, \ldots, \|I\|\}$	index for products
$j \in J = \{1, \ldots, \|J\|\}$	index for origin stores
$s \in S = \{1, \ldots, \|S\|\}$	index for destination stores
$t \in T = \{1, \ldots, \|T\|\}$	index for weeks
$k \in K = \{1, \ldots, \|K\|\}$	index for scenarios

Parameters

$stock_{ij}$	initial inventory for pairs
$demand^t_{ijk}$	demand for product i in store j at week t scenario k
$cogs_i$	purchase cost for product i
$price_i$	resale price for product i
$leadtime_{js}$	lead time from location j to location s
c_{ijs}	cost of shipping product i from store j to s
$holding$	inventory holding cost rate
wh	warehouse code in the system

Variables

inv^t_{ijk}	continuous variable indicating beginning inventory for product i at store j week t
$sales^t_{ijk}$	continuous variable indicating sales for product i at store j week t
$shipment^t_{ijs}$	continuous variable indicating shipment decision for product i from store j to store s at week t

$$\max z = 1/|K| \sum_{k \in K} \sum_{t \in T} \sum_{i \in I} \sum_{j \in J} (price_i - cogs_i) sales^t_{ijk}$$
$$- \sum_{k \in K} \sum_{t \in T} \sum_{i \in I} \sum_{j \in J} inv^t_{ijk}(cogs_i) holding - \sum_{i \in I} \sum_{t \in T} \sum_{j \in J} \sum_{s \in S} shipment^t_{ijs} c_{ijs}$$

s.t.

$$inv_{ijk}^{t} = stock_{ij} \qquad \forall i \in I, \forall j \in J, \forall k \in K \text{ for } t = 1 \quad (4.17)$$

$$inv_{ijk}^{t} - \sum_{s \in S} shipment_{ijs}^{t} \geq sales_{ijk}^{t} \qquad \forall i \in I, \forall j \in J, \forall t \in T, \forall k \in K \quad (4.18)$$

$$inv_{ijk}^{t} = inv_{ijk}^{t-1} - sales_{ijk}^{t-1} + \quad (4.19)$$

$$\sum_{s \in S} (shipment_{isj}^{t-leadtime_{sj}} - shipment_{ijs}^{t-1}) \qquad \forall i \in I, \forall j \in J, \forall k \in K, \forall t \in T$$

$$sales_{ijk}^{t} \leq demand_{ijk}^{t} \qquad \forall i \in I, \forall j \in J, \forall k \in K, \forall t \in T \quad (4.20)$$

$$\sum_{j \in J} shipment_{ijs}^{t} = 0 \qquad \forall i \in I, \forall t \in T, \forall t \in T \text{ for } s = wh \quad (4.21)$$

$$shipment_{ijs}^{t} \geq 0 \qquad \forall i \in I, \forall j \in J, \forall t \in T, \forall s \in S \quad (4.22)$$

$$sales_{ij}^{t} \geq 0 \qquad \forall i \in I, \forall j \in J, \forall t \in T \quad (4.23)$$

$$inv_{ij}^{t} \geq 0 \qquad \forall i \in I, \forall j \in J, \forall t \in T \quad (4.24)$$

The objective becomes the average gain of the scenarios. Constraints (4.17) assign the first-period beginning inventory to the initial inventory parameter for all scenarios. In other words, all scenarios start with the same inventory. Constraints (4.18) and (4.19) are the inventory balance constraints. For each scenario k, different sales and inventory values may occur, but the transshipment decisions are the same for all scenarios. This is why the transshipment variable does not depend on k as the sales variable is defined based on k. Constraint (4.20) guarantee that sales realized is less than demand. Constraints (4.21), (4.22), (4.23), and are (4.24) the same as the previous model.

In order to construct $demand_{ijk}^{t}$, three prediction methods are analyzed; sample average approximation, moving average, and random forest. For the moving average prediction, all scenarios are the same since the moving average method provides point estimates instead of predictive distributions. The details of the prediction methods and the related performance metrics are analyzed in the following section.

5. COMPUTATIONAL EXPERIMENTS AND RESULTS

Each supply chain system has its own characteristics, such as number of items, number of locations, and has different management strategies such as fulfillment ratio, supply management strategy or so on. To construct the analyses, a system with the following characteristics is simulated:

- Number of items: 5
- Number of selling locations: 5
- Number of distribution centers: 1
- Time period: 52

This system is assumed to use the order up to level method as the inventory management policy and order up to level is determined as the sum of the past four period sales:

$$OUTL_{ij}^{t} = \sum_{i=t-4}^{t-1} sales_t \tag{5.1}$$

The shipment quantities for each period are calculated by subtracting the inventory on-hand value from the order up to level in the system.

$$order_{ij}^{t} = max_{(0, OUTL_{ij}^{t} - inv_{ij}^{t})} \tag{5.2}$$

In order to apply the suggested methods to the system, it is assumed that the system works with the order up to level strategy for 40 weeks. Then the suggested methods are applied to the system for different cases. In the following sections, different cases with different management strategies for this system are constructed to see the robustness of the suggested methods. Firstly, demand data generation methods are explained in details, then the inventory level scenarios are generated to test the system with different total inventory cases. The mathematical model constructed in the previous sections is solved with Gurobi for each inventory level case. It is assumed that the system's shipment policy is to use both replenishment and lateral transshipment. Later, this assumption is changed to the system with only replenishment, and the effect of lateral transshipment decisions on the system performance is analyzed. Finally, the effect of shipment review frequency is tested. The shipment decisions for both replenishment and lateral transshipment are constructed for the future four periods in this system. The effect of the shipment review frequency is analyzed by comparing the KPI values of the system allowed to update the shipment decisions periodically and the system that freeze the decisions for future four periods.

5.1. Data Generation

In order to generate the demand data for the inventory management system, the auto-regressive integrated moving average method (ARIMA) is used. It is one of the most common methods used for creating time series data. Data generation is repeated for 5 times to test the robustness of the suggested methods on different environments. 52 data points are generated in each iteration for five items, four selling locations, and demand is assumed to be zero for one distribution center. ARIMA simulation has 3 different parameters, p as the number of time lags of the auto-regressive model, d as the number of times the data have had past values subtracted, and q as the order of the moving-average model. From the parameter set, p is set to 4, d and q are set to 0. So the order for the ARIMA model is (4,0,0) which is equivalent to AR(4). The AR coefficients are set to (0.35,0.25,0.20,0.05).

After the creation of initial data, random promotion points defined on weeks, products, stores, or product-stores are generated. Promotion points are constructed with random number generators for each week, product, and store. Even in real life, it is not allowed to apply more than one promotion on a store-product pair. This is why prioritization of promotion types is needed. The relative priority order is as follows: week promotion, product promotion, store promotion, product-store promotion.

To clarify, if a week is labeled as in-promotion, then additional product or store promotions are not allowed. Generated promotions are assumed to have a multiplicative effect on demand and the effect of promotions differs based on promotion type. Promotion effects for each type can be seen from Table 1. In case of having no promotion, generated demand is not modified. The promotion having the highest effect is the product-store type promotion and it is 1.6. The lowest effect is 1.3 and it is used for a store promotion and if a store is in promotion then the demand value of all products in that store is multiplied by 1.3. The product promotion effect is higher than the store promotion effect and it is 1.4 and having a product promotion label at a period means the demand value of that product in each store is multiplied by 1.4. Lastly, if a period is labeled as week promotion then demand values generated at that week are multiplied by 1.5 for all products and stores.

Table 1: Promotion Effects

Product Promo	**Store Promo**	**Week Promo**	**Multiplier**
False	False	False	1
False	False	True	1.5
False	True	True	1.5
False	True	False	1.3
True	False	False	1.4
True	False	True	1.5
True	True	True	1.5
True	True	False	1.6

Figure 4 represents the generated demand data aggregated by items for each period. The same data is also visualized based on location aggregated level at Figure 5. As it is stated before, the data generation step is replicated for 5 times, to provide a visual representation of the generated data, one of the replications is selected and used in the figures. The dashed line indicates the raw demand generated with ARIMA whereas the solid line indicates the final demand after promotion multipliers are applied to the raw demand. The test period for the analyzes starts at period 40 and it is separated by a vertical line in the graph.

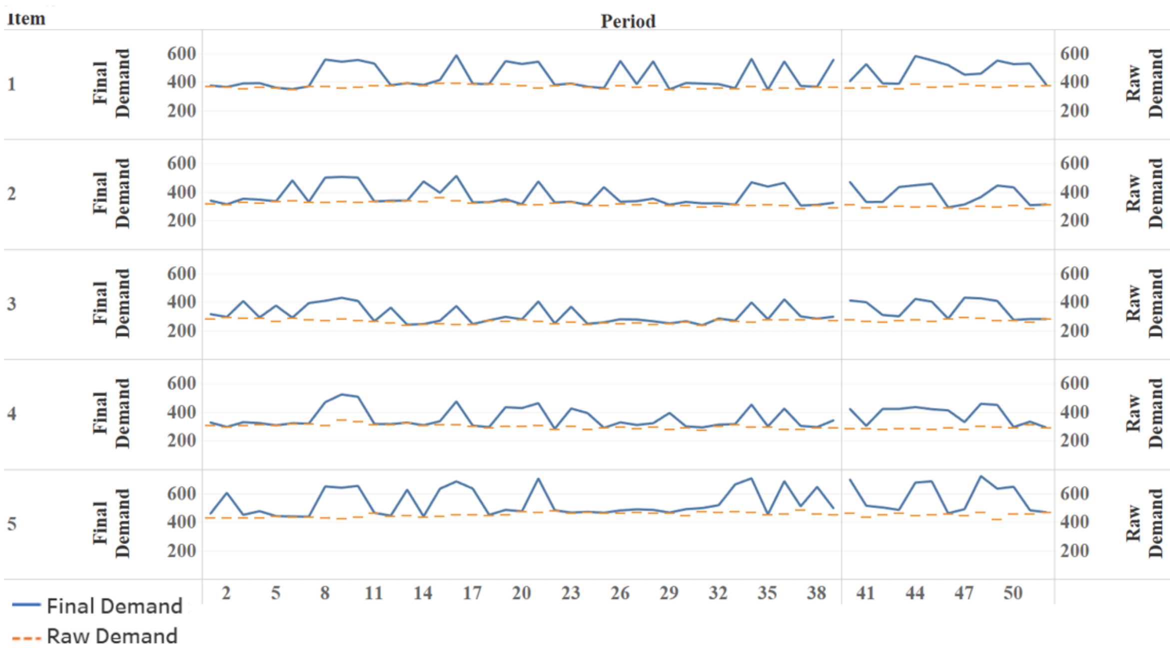


Figure 4: Generated raw and final demand data aggregated by item level

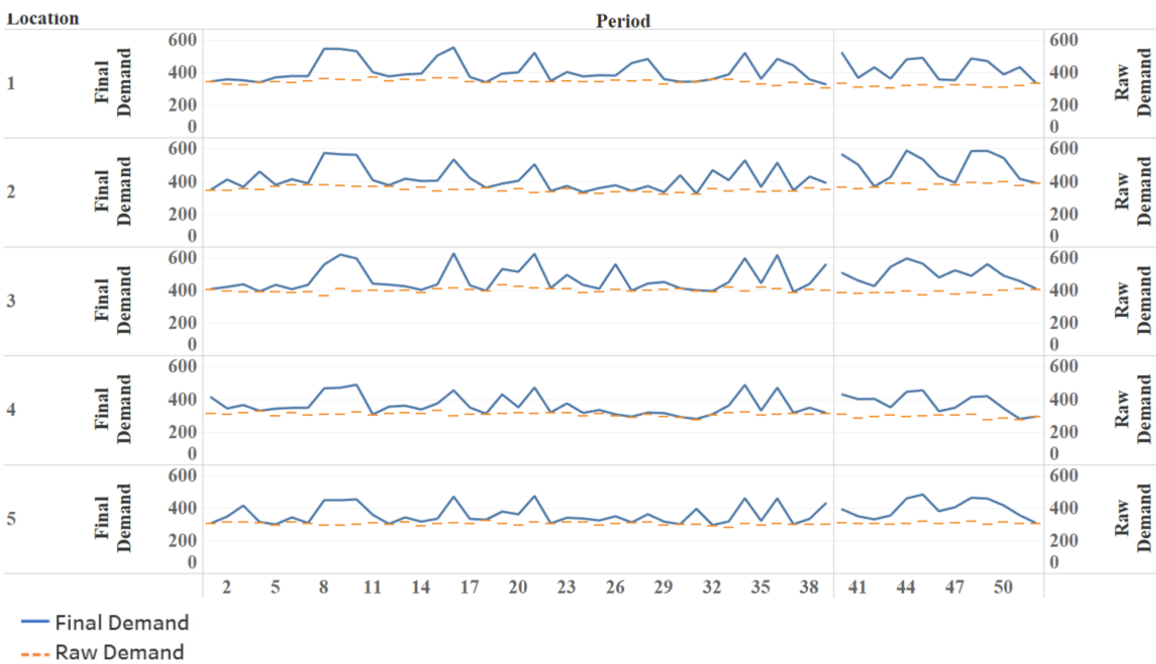


Figure 5: Generated raw and final demand data aggregated by location level

Inventory level case generations are studied after the demand generation step is completed. In order to evaluate the robustness of the approaches, nine inventory scenarios are generated. The inventory level scenarios are generated in two steps. Firstly, the ratio of the total system inventory to the total demand of the system is considered. Then cases for how much to allocate stores at the beginning of the system are considered. As a first step, for the beginning inventory level of the system, three different cases are considered:

- shortage in the system : supply ratio in the system is 0.5
- supply and demand balance : supply ratio in the system is 1.0
- excess in the system : supply ratio in the system is 1.5

The first case indicates that the total inventory level is not enough to fulfill the total demand in the system and lost sales is inevitable. In the second case, the inventory level is equal to the total demand of the system, in the case of knowing the exact demand of each period for item-location, zero lost sales value is attainable. In the third case, the total inventory level is much higher than the total demand of the system. As a second step, for each inventory level, the initial allocation ratios, which are calculated as the beginning total inventory of the stores over the total system inventory, are tested for three levels:

- warehouse-oriented allocation : the initial allocation ratio is 0.25
- balanced allocation : the initial allocation ratio is 0.50
- store-oriented allocation : the initial allocation ratio is 0.75

After determining the total system inventory level, beginning inventory levels for locations are calculated with the assumption of each allocation strategy. Warehouse oriented allocation indicates that the majority of inventory is reserved for later allocation, as store oriented allocation indicates the majority is allocated to stores at the beginning of the system. The ratios used in the calculation of beginning inventory levels are summarized in Table 2.

Table 2: Inventory Level Cases

Case Number	**Total Inventory / Total Demand**	**Initial Store Allocation Ratio**
1	0.5	0.25
2	0.5	0.50
3	0.5	0.75
4	1.0	0.25
5	1.0	0.50
6	1.0	0.75
7	1.5	0.25
8	1.5	0.50
9	1.5	0.75

After the calculations, the total allocated inventory to the locations is obtained, but allocation among locations is not handled yet. In order allocate the inventory to the locations, relative performances of the locations are considered. For each item-location, a performance ratio is calculated, the details for the calculation can be seen from Equation (5.3).

$$ratio_{ij} = \frac{\sum_{t=1}^{t=40} sales_{ij}^{t}}{\sum_{t=1}^{t=40} \sum_{j} sales_{ij}^{t}} \tag{5.3}$$

For each location-item, the ratio indicates the relative sales performance and it is calculated as the sum of the past sales data for item-location divided by the sum of the past sales data for the item.

Finally, in order to use in the calculations of the objective value, price and cost values for each item are generated. The margin values for items are set to 45% on average. The cost vector used for $cogs_i$ is (4.13, 12.93, 11.00, 9.25, 3.85), whereas the resale price vector for $price_i$ is (7.50, 23.50, 20,18. 5,70). The holding cost rate, *holding* is set to 10%. However, the cost of having inventory at warehouse is much lower than keeping the inventory at sales locations, so holding cost rate for the warehouse is set to 10% of *holding*. For the calculations of shipment costs, c_{ijs} is set to 11% of $cogs_i$. The details of the usage of the price and cost parameters can be seen from Section 4.3.

To summarize, demand data for this system is constructed with a time series data generation method and random promotions are applied with multiplicative effect assumption. Inventory data is generated for different cases so that in the following sections the system can be tested on different environments.

5.2. Effect of Number of Scenarios

In the previous sections, it is stated that demand is stochastic and its distribution is unknown and to handle the unknown distribution of demand, the optimization model is constructed over different demand scenarios. The framework suggested by Bertsimas and Kallus(2020) is used and for each prediction method, k number of scenarios are constructed [1]. In order to analyze the effect of the number of demand scenarios in the system, k is tested from 10 to 100. The objective value of the system and the KPI values indicated in the previous sections are analyzed for each scenario number.

After observing the system for 40 weeks, for each item, location, and period, four weeks of predictions are constructed for each prediction method for nine periods. To clarify the scenario generation process, details of constructing $demand_k$ for three prediction methods, moving average, random forest, and sample average approximation are discussed in the following sections.

5.2.1. Moving Average. For the simple moving average method, the forecast values are calculated based on the average of the past 4 sales data. For $t > 40$,

$$demand^t = 1/m \sum_{z=t-m}^{t-1} sales^z \tag{5.4}$$

As it is a deterministic prediction method; all demand scenarios (k) are the same and equal to the average of past 4 weeks sales because the variance of demand is not considered in the moving average method. Therefore, all of the scenarios indicates the same system, and there is not a difference between the model with 10 demand scenario and 100 demand scenarios.

5.2.2. Sample Average Approximation. In addition to the moving average predictions, sample average approximation is used to obtain a stochastic prediction. Demand scenario generation starts with picking random past weeks. After the random selection, future four weeks' sales data of the selected week is used as the future prediction. To clarify, in order to construct a scenario for an item-location with the sales data in Table 3; if week 27 is randomly selected, then the prediction starting from week 43 is as follows 8-12-18-18.

Table 3: Sample Sales Data

week	**sales**	**week**	**sales**
25	10	34	15
26	15	35	11
27	8	36	18
28	12	37	9
29	18	38	11
30	18	39	14
31	7	40	17
32	11	41	15
33	14	42	10

This procedure is repeated for k times in order to construct k scenarios for an item-location. This method provides variance for demand due to the random sampling and assumes that there is not a seasonality trend.

5.2.3. Random Forest. Simple moving average and the sample average approximation methods depend only on the previous sales values, but the random forest is a prediction method taking advantage of external attributes. Past sales data, item attributes indicating items/location performance, special day, or promotion flags can be used in the model. Predictions for each scenario(k) are obtained by generating k number of trees in the forest so that each tree's leaf value is used as a scenario. To clarify, for a random forest with 3 trees illustrated in Figure 6, each response value in each tree is used as a prediction set for a scenario.

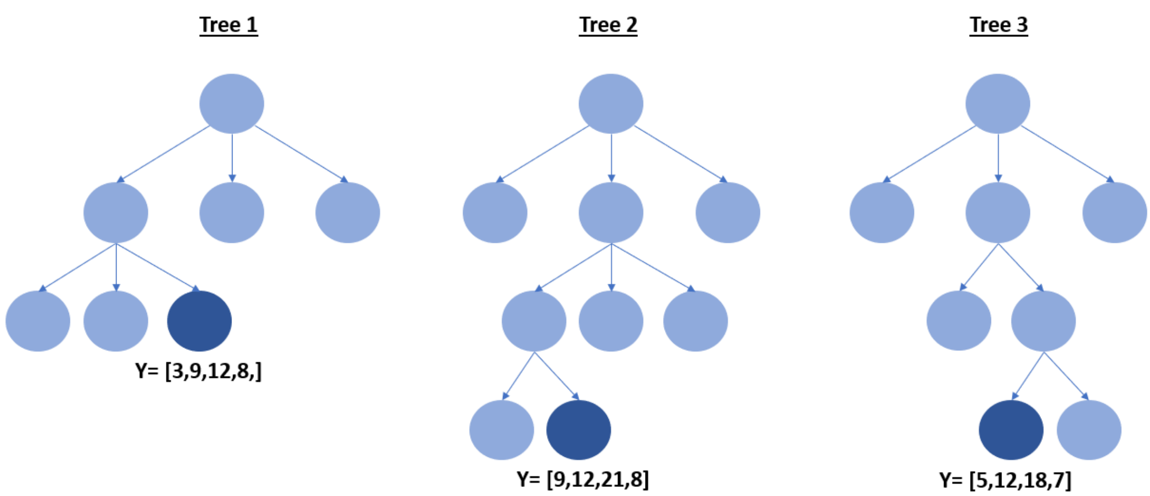


Figure 6: Illustrative random forest example

From Figure 6, it can be seen that the target variable is a list but not a single number. In order to obtain predictions for the future four periods at once, multi-objective random forest that generates multi-targets is used [16]. Kocev et. al. (2007) conclude that the results of the multi-objective random forests perform equally good or significantly better than the single-objective ones. The minimum number of samples required to split a node is assigned to 2, so that at the leaf node one observation is obtained. The number of features to consider when looking for the best split is set to 0.1, and bootstrap is allowed. The response contains four periods, and for an item-location with the solution in Figure 6; prediction results for scenarios at period t are constructed as in Table 4.

Table 4: Scenario predictions for Figure 6

$demand_{ij}$	k=1	k=2	k=3
t=1	3	9	5
t=2	9	12	12
t=3	12	21	18
t=4	8	8	7

For the three different prediction methods, the optimization model is run with different scenario numbers (from 10 to 100). The significance of scenario number in the optimization model is analyzed with the assumption of the system allowing lateral transshipment. ANOVA is used to test the significance of the number of scenarios. From Table 5, it is seen that number of scenarios and prediction methods and the interaction have significant effects on objective values.

Table 5: ANOVA results for number of scenarios and prediction methods

	Source	SS	DF	MS	F	p-unc	np2
0	NumberofScenarios	6.827291e+05	9	75858.784	24.943	0.0	0.652
1	PredictionMethod	1.789886e+07	2	8949428.855	2942.650	0.0	0.980
2	NumberofScenarios * PredictionMethod	4.489261e+05	18	24940.341	8.201	0.0	0.552
3	Residual	3.649538e+05	120	3041.282	NaN	NaN	NaN

The average objective values obtained in this analysis can be seen from Table 6. As it is stated before, the objective value of the optimization model is the profit in the system for nine weeks of periodic review after inventory and cost realizations. The objective calculation can be seen from Equation 4.16 and the mathematical model is stated in the Section 4.4. From Table 6, it can be stated that as scenario number increases, the objective values increase for SAA and RF methods while for the moving average method, objective values are the same in each scenario number, because de-

Table 6: Average objective values for each prediction method over different scenario numbers

	Prediction Method		
Scenario Number	Moving Average	Sample Average Approximation	Random Forest
10	80,479	80,267	80,836
20	80,479	80,395	81,114
30	80,479	80,344	81,142
40	80,479	80,432	81,230
50	80,479	80,485	81,244
60	80,479	80,500	81,262
70	80,479	80,457	81,247
80	80,479	80,499	81,284
90	80,479	80,475	81,237
100	80,479	80,510	81,301

mand variation is not considered in MA. Therefore, it is possible to conclude that as scenario number increases, the model gives better decisions. Since the change in the objective values are small and the cost structure has a huge effect on the objective values, lost sales and weeks of supply metrics are analyzed additionally. In order not to be dependent on the cost structure, KPI metrics practiced in the inventory management problems are used. Lost sales values for each scenario number can be seen from Figure 7. The lost sales ratio for the system is calculated as the profit lost due to unfulfilled demand divided by the expected profit for total demand.

In Figure 7, the ratio stands for the average of the lost sales ratios of each generated case (nine inventory scenarios, five replication in prediction methods, five different data) in the system. From the figure, it can be stated that as the scenario number increases, the lost sales ratio converges to its lowest value because increasing the variance in the model results in improvement and stabilization in the KPI metrics. For the moving average method, indicated by the solid line in Figure 7, increasing scenario number does not change the lost sales ratio, but for the RF and SAA methods, as scenario number increases, the optimization model gives better decisions in terms of lost sales. The second KPI value of the supply chain system is summarized in Fig-

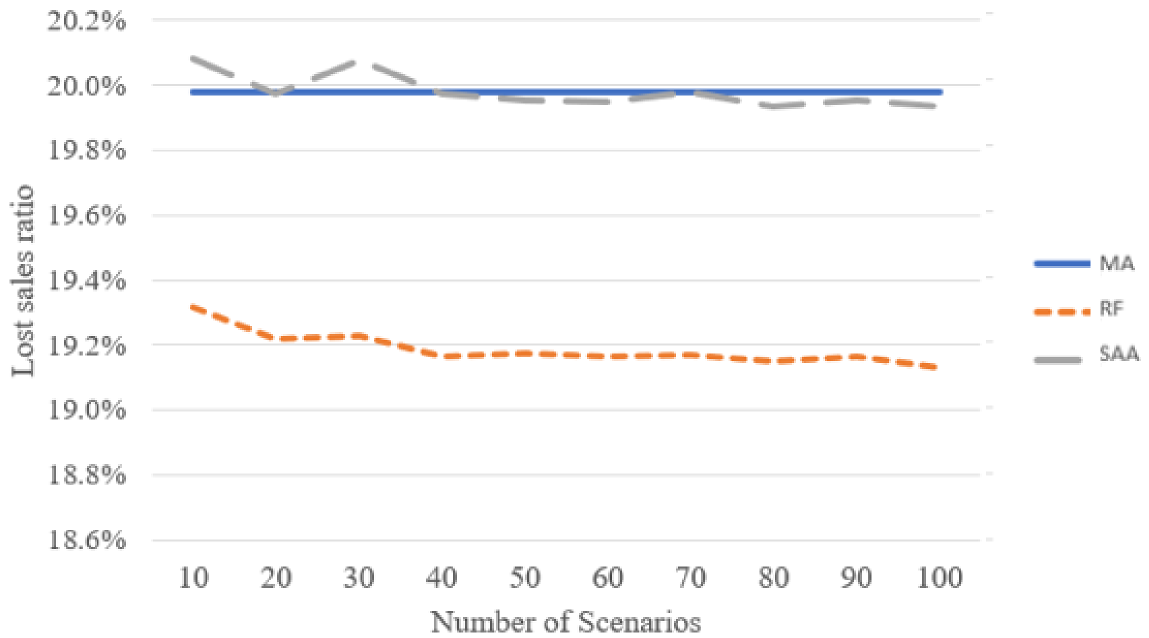


Figure 7: Average lost sales ratios for different scenario numbers

ure 8, the weeks of supply value for each period indicates the number of periods that inventory on-hand can fulfill the average demand and it is calculated with inventory holding cost and expected average demand value. The weeks of supply value on figures are the average of the weeks of supply of each period in each case (ten period-ends, nine inventory scenarios, five replication in prediction methods, five different data) in the system. From the figure, it can be stated that as scenario number increases, the weeks of supply values stabilize for all prediction methods and decrease dramatically for the random forest method.

As a result, the number of scenarios affects both the quality and the robustness of the system. As the number of scenarios used in the system increases, objective value obtained by the model increases. Moreover, lost sales value decreases and it converges to its lowest value. Having high scenario numbers the optimization model solves the problem by better estimation of demand distribution based on conditional probabilities estimated by prediction methods. For the next sections, the optimization model with 100 scenario number is used in the analyses.

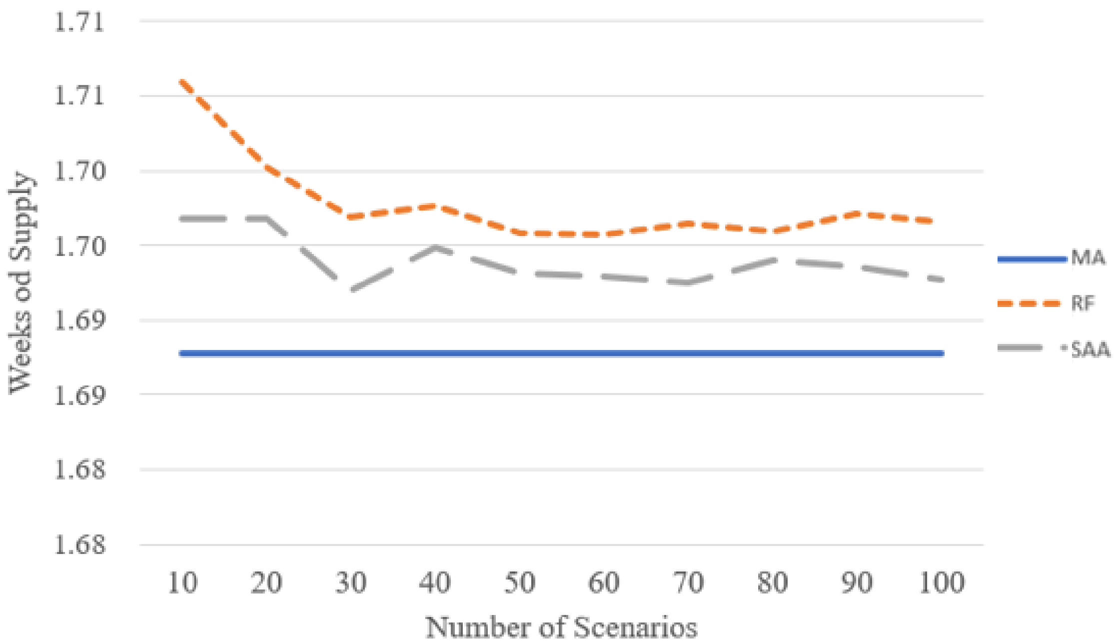


Figure 8: Average weeks of supply ratios for different scenario numbers

5.3. Case Analyses

This section analyzes the results for different inventory situations, nine inventory level cases. Firstly, it is analyzed with the assumption of the system allowing lateral transshipments, then the system is changed to the system with only replenishment decisions. In the previous section, it is stated that using the optimization model with higher scenario numbers performs better. Therefore, the optimization model with 100 scenarios is used in the case analyses. After choosing which optimization model to be used in the analyses, the model is tested on different environments to obtain the true performance and robustness of the suggested method. As constructing the environments, in order to test the robustness, five different data sets are analyzed for five different replications in each prediction method, so the analyses are constructed on 25 different systems.

5.3.1. Case Analyses with Lateral Transshipment.

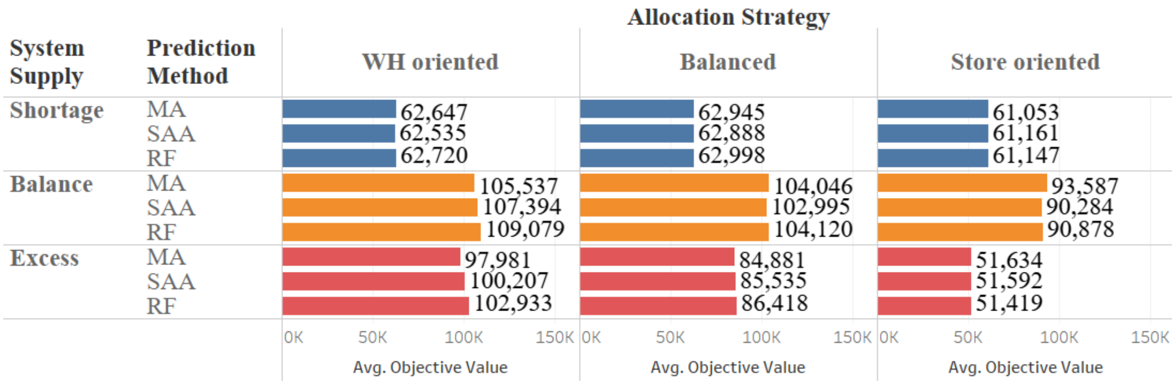
The system is analyzed with the assumption of allowing lateral transshipments, which means in addition to the shipment decision from the distribution center, shipment decision among locations is also possible. Firstly, in Table 7, it is tested if the inventory level cases and prediction methods have statistically significant effect on the objective values.

Table 7: ANOVA results for inventory level cases

	Source	SS	DF	MS	F	p-unc	np2
0	SystemSupply	1.688802e+11	2.0	8.444012e+10	2914.8	0.0	0.9
1	AllocationStrategy	5.834197e+10	2.0	2.917098e+10	1006.9	0.0	0.8
2	PredictionMethod	9.771693e+07	2.0	4.885846e+07	1.7	0.2	0.0
3	SystemSupply * AllocationStrategy	4.606897e+10	4.0	1.151724e+10	397.6	0.0	0.7
4	SystemSupply * PredictionMethod	1.189443e+08	4.0	2.973608e+07	1.0	0.4	0.0
5	AllocationStrategy * PredictionMethod	2.811207e+08	4.0	7.028017e+07	2.4	0.0	0.0
6	SystemSupply * AllocationStrategy * Prediction...	1.726517e+08	8.0	2.158146e+07	0.7	0.7	0.0
7	Residual	1.877251e+10	648.0	2.896992e+07	NaN	NaN	NaN

From Table 7, it is seen that system supply and allocation strategy have significant effects on objective values and their interaction effect is significant. Prediction method does not have a significant effect individually, while the interaction of the prediction method and the allocation strategy is significant. Therefore, it can be stated that inventory level cases have significant effects on the total profit of the system. Then, the objective values can be analyzed in details for each case. The average objective values of 25 different data set for each inventory level case is summarized in Table 8 for nine weeks of periodic review strategy. From Table 8, it can be stated that systems with

Table 8: Inventory level case objective analyses for systems with lateral transshipment



a shortage in supply have the lowest objective values for each allocation strategy. The objective values are changing around 60,000. Because of the shortage in supply, high lost sales values are occurred and the objective values become significantly less than the other systems. For the systems with shortage in supply, the store oriented allocation strategy has the lowest objective values as the system with store oriented allocation has higher inventory holding costs, because allocating the majority of inventory to stores at the beginning of the system results in keeping the inventory on hand for longer periods. However, in the WH oriented system, inventory is waiting at the warehouse to be allocated more accurately in the coming periods. Systems with a balance in supply have the highest objective values on average for each allocation strategy because the systems with excess in supply has higher inventory holding costs. For the same reasons stated above store oriented systems has lower objective values. Although the random forest prediction method has the highest objective value for most of the cases, the difference in objective values among the prediction methods used in each system is not notable.

Table 9: Inventory level case KPI analyses for systems with lateral transshipment

		Allocation Strategy					
		WH oriented		Balanced		Store oriented	
System Supply	Prediction Method	Avg. Wos	Avg. Ls Ratio	Avg. Wos	Avg. Ls Ratio	Avg. Wos	Avg. Ls Ratio
Shortage	MA	0.15	49.9%	0.27	49.9%	0.52	49.9%
	SAA	0.16	49.9%	0.28	49.9%	0.52	49.9%
	RF	0.15	49.9%	0.27	49.9%	0.52	49.9%
Balance	MA	0.69	9.2%	1.31	6.2%	2.43	3.7%
	SAA	0.70	8.0%	1.31	7.1%	2.43	6.1%
	RF	0.70	6.7%	1.32	6.0%	2.43	5.5%
Excess	MA	1.39	7.5%	2.89	3.4%	5.54	0.0%
	SAA	1.41	5.4%	2.90	2.8%	5.54	0.1%
	RF	1.43	2.6%	2.92	1.4%	5.54	0.0%

In addition to the objective value analysis, lost sales and weeks of supply values are analyzed. In Table 9, the average weeks of supply and lost sales values for inventory level cases are listed. In the case of a system with a shortage in supply, the prediction method or the allocation strategy does not matter for lost sales metric, because the lost sales ratios are the same. However, weeks of supply values increase from warehouse oriented to store oriented allocation strategy. This is because the system with store oriented strategy starts with higher inventory levels. For the systems with balance and excess in supply, allocation strategy and prediction methods become important. The improvement obtained with the random forest is significant for the systems with a balance or excess in supply for warehouse oriented systems. For the systems with excess supply, lost sales improvement among prediction methods can not be seen if store oriented allocation strategy is used. This is due to the inventory level is very high in the system, and this can be checked from weeks of supply values as well. In the case of having warehouse oriented allocation strategy for the systems with excess supply, the system operates with less than two weeks of inventory on average, but if the store oriented allocation is preferred then the system would have almost six weeks of supply on average which is very high as compared to other systems. Additionally, it can be seen that the lost sales ratio is also decreasing from warehouse oriented systems to store oriented systems. This is because starting with higher inventory in stores fulfill the unexpected demand. However, it also increases the weeks of supply values significantly.

As a result of the inventory case analyzes, it can be stated that; if the system with lateral transshipments has a shortage in supply, the importance of the prediction method decreases, but changing the allocation strategy provides improvement for weeks of supply values. If the system with lateral transshipments has a balance in supply, then the prediction method becomes important if the allocation strategy is warehouse oriented, and changing the allocation strategy mainly affects the weeks of supply values. Allocation strategy should be determined by the decision-makers with the consideration of the trade off between lost sales and weeks of supply values. If the system with lateral transshipments has an excess in supply, the importance of the allocation strategy increases because there is a significant change in the weeks of supply values among the allocation strategies.

<u>5.3.2. Case Analyses without Lateral Transshipment.</u> The importance of the allocation strategy changes with the shipment capabilities of the system. When the systems with the same data are operated without lateral transshipment, the results reveal the importance of lateral transshipment, especially for systems with excess in supply. The comparison between the systems with transshipment and without transshipment has a huge dependency on the holding and shipment cost structures.

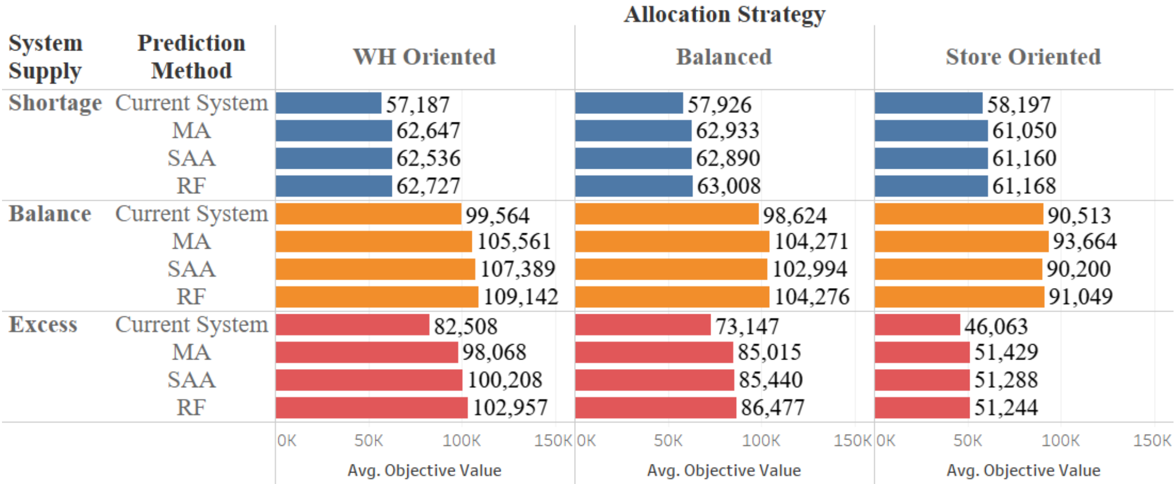
Table 10: ANOVA results for inventory level cases without lateral transshipment

	Source	SS	DF	MS	F	p-unc	np2
0	SystemSupply	1.695262e+11	2.0	8.476309e+10	2849.1	0.0	0.9
1	AllocationStrategy	5.883581e+10	2.0	2.941791e+10	988.8	0.0	0.8
2	PredictionMethod	1.095252e+08	2.0	5.476261e+07	1.8	0.2	0.0
3	SystemSupply * AllocationStrategy	4.667649e+10	4.0	1.166912e+10	392.2	0.0	0.7
4	SystemSupply * PredictionMethod	1.198360e+08	4.0	2.995899e+07	1.0	0.4	0.0
5	AllocationStrategy * PredictionMethod	2.770617e+08	4.0	6.926542e+07	2.3	0.1	0.0
6	SystemSupply * AllocationStrategy * Prediction...	1.739064e+08	8.0	2.173830e+07	0.7	0.7	0.0
7	Residual	1.927845e+10	648.0	2.975069e+07	NaN	NaN	NaN

Before analyzing the objective values for the system without lateral transshipment in details, the significance of the different cases are tested for the systems without lateral transshipment. From Table 10, it is seen that system supply and allocation strategy and the interaction have significant effects on objective values. The average objective values for the system without lateral transshipment can be seen from Table 11 in details.

In Table 11, the proposed methods are also compared with the current system strategy. As stated before, current system is managed with the OUTL strategy with 4 weeks of sales order up to level and the it does not allow lateral transshipment. It is seen that current system has the lowest objective values for each case, it means the total system profit is not as high as for the proposed methods. For the proposed methods without lateral transshipment, the objective values are very close to the objective values of the systems with lateral transshipment. In order to analyze the performance changes between the systems with and without lateral transshipment, KPI values for

Table 11: Inventory level case objective analyses for systems without lateral transshipment



the systems without lateral transshipment can be seen from Table 12.

The current system has the lowest lost sales ratios in Table 12. However, weeks of supply values are significantly higher because the current system's main purpose is to keep inventory at 4 weeks of supply. Yet, the proposed methods solve the inventory problem with the knowledge of 1 period of lead time and 1 period of review time, and do not aim to keep more than 2 weeks of supply due to nature of optimization. Therefore, comparing the system with 4 periods cover to the proposed methods is not compatible. In order to obtain compatible systems the current system should be run with 2 weeks of order up to level.

In case of simulating the current system with 2 weeks order up to level, weeks of supply values for the systems become compatible and the results can be seen from Table 13. It can be stated that for each case, the current system results in higher lost sales values and higher weeks of supply values.

After comparing the current system with the proposed methods, the effect of lateral transshipment can be analyzed from the same table, Table 13. For the systems with shortage, lost sales values are exactly the same with the systems using lateral

Table 12: Inventory level case KPI analyses for systems without lateral transshipment (Current System uses four weeks of sales as OUTL)

		Allocation Strategy					
		WH Oriented		Balanced		Store Oriented	
System Supply	Prediction Method	Avg. Wos	Avg. Ls Ratio	Avg. Wos	Avg. Ls Ratio	Avg. Wos	Avg. Ls Ratio
Shortage	Current System	0.49	49.9%	0.58	49.9%	0.70	49.9%
	MA	0.15	49.9%	0.27	49.9%	0.52	49.9%
	SAA	0.16	49.9%	0.28	49.9%	0.52	49.9%
	RF	0.15	49.9%	0.27	49.9%	0.52	49.9%
Balance	Current System	1.77	1.1%	2.09	1.1%	2.86	1.1%
	MA	0.69	9.2%	1.32	6.1%	2.44	3.6%
	SAA	0.70	8.1%	1.31	7.2%	2.43	6.1%
	RF	0.70	6.7%	1.32	6.0%	2.44	5.5%
Excess	Current System	2.65	0.0%	3.63	0.0%	5.67	0.0%
	MA	1.39	7.4%	2.90	3.3%	5.55	0.2%
	SAA	1.41	5.4%	2.91	2.8%	5.54	0.3%
	RF	1.43	2.7%	2.93	1.5%	5.55	0.2%

Table 13: Inventory level case KPI analyses for systems without lateral transshipment (Current System uses two weeks of sales as OUTL)

		Allocation Strategy					
		WH Oriented		Balanced		Store Oriented	
System Supply	Prediction Method	Avg. Wos	Avg. Ls Ratio	Avg. Wos	Avg. Ls Ratio	Avg. Wos	Avg. Ls Ratio
Shortage	Current System	0.18	49.9%	0.30	49.9%	0.53	49.9%
	MA	0.15	49.9%	0.27	49.9%	0.52	49.9%
	SAA	0.16	49.9%	0.28	49.9%	0.52	49.9%
	RF	0.15	49.9%	0.27	49.9%	0.52	49.9%
Balance	Current System	0.71	14.2%	1.34	9.3%	2.47	4.0%
	MA	0.69	9.2%	1.32	6.1%	2.44	3.6%
	SAA	0.70	8.1%	1.31	7.2%	2.43	6.1%
	RF	0.70	6.7%	1.32	6.0%	2.44	5.5%
Excess	Current System	1.41	12.0%	2.93	4.0%	5.55	0.2%
	MA	1.39	7.4%	2.90	3.3%	5.55	0.2%
	SAA	1.41	5.4%	2.91	2.8%	5.54	0.3%
	RF	1.43	2.7%	2.93	1.5%	5.55	0.2%

transshipment, because the inventory is limited. For the systems with warehouse oriented strategy, the system prefers to fulfill the demand from warehouse. Therefore, there is not a huge difference between the systems with or without transshipment decisions if allocation strategy is warehouse oriented. This can be checked from Table 14, for the warehouse oriented allocation strategy, the systems with lateral transshipment does not use the transshipment capability. The values in Table 14 indicates the total transshipment quantity ratio to the total shipment decisions. For the systems with shortage in supply and for the systems with warehouse oriented systems, lateral transshipment capability is not used and shipments from the warehouse is preferred. As the inventory in the system increases (excess in supply), if the allocation strategy is store oriented then it is seen that systems with lateral transshipment uses mainly lateral transshipment decisions and performs better especially for lost sales values. This is because the store oriented strategy for the systems without lateral transshipment blocks the inventory and the inventory can not be moved among locations, so this results in higher lost sales. For the systems with balance and store oriented strategy, weeks of supply values also decreases if the system allow lateral transshipment in case of balance and excess in supply.

Table 14: Lateral transshipment decisions percentage over total shipment decisions

		Allocation Strategy		
System Supply	Prediction Method	WH oriented	Balanced	Store oriented
Shortage	MA	0.0%	0.1%	3.0%
	SAA	0.0%	0.0%	0.2%
	RF	0.1%	0.1%	1.1%
Balance	MA	0.0%	1.1%	3.7%
	SAA	0.0%	0.3%	1.2%
	RF	0.1%	1.6%	5.5%
Excess	MA	0.6%	2.8%	62.3%
	SAA	0.0%	0.7%	38.3%
	RF	0.1%	3.5%	43.8%

For the systems with excess in supply, comparison of Table 11 and Table 8 shows that without lateral transshipment decisions, systems result in higher objective values. The decrease in the objective value if lateral transshipment is used is due to the additional transshipment costs. Although the objective values indicate that in some cases having only replenishment decisions in the system provide higher objectives, KPI values in Table 13 show that the systems without lateral transshipment have higher lost sales, and weeks of supply values are not changing too much. In other words, having lateral transshipment in the system provide better results in terms of KPIs of the system. In the following section, the distribution of the objective values for scenarios is analyzed.

5.4. Objective Distribution for Demand Scenarios

The case analyses are constructed with the optimization model with 100 scenarios. As stated in Equation (4.16), the objective value of the optimization model is the average gain function for all scenarios. However, in order to evaluate the robustness of the suggested method, the distribution of the gain values for each scenario should be analyzed. Although the case analyses are constructed over five different data set for five replications, in order to obtain compatible results in objective distribution analyses, one data set is used for one replication for the each prediction method. The robustness of the proposed model is tested for the system with the balanced allocation strategy for each system supply case. The objective results in this analysis contains four future weeks. The distribution of the objective values for the sample average approximation and random forest are visualized for each system supply case with histograms in Figure 9. As stated before, moving average method does not create predictive distribution but point estimates. Therefore, For the moving average method each scenario has the same objective and it is not reported in this figure.

From Figure 9, it can be stated that random forest prediction method results in wider distribution among demand scenarios than the sample average approximation. For each system supply case random forest has higher standard deviation values than SAA method. For the systems with shortage in supply, there are outlier scenarios for random forest method whereas the sample average approximation has a narrow distribution. For the balance and excess in supply, although the distribution is wider

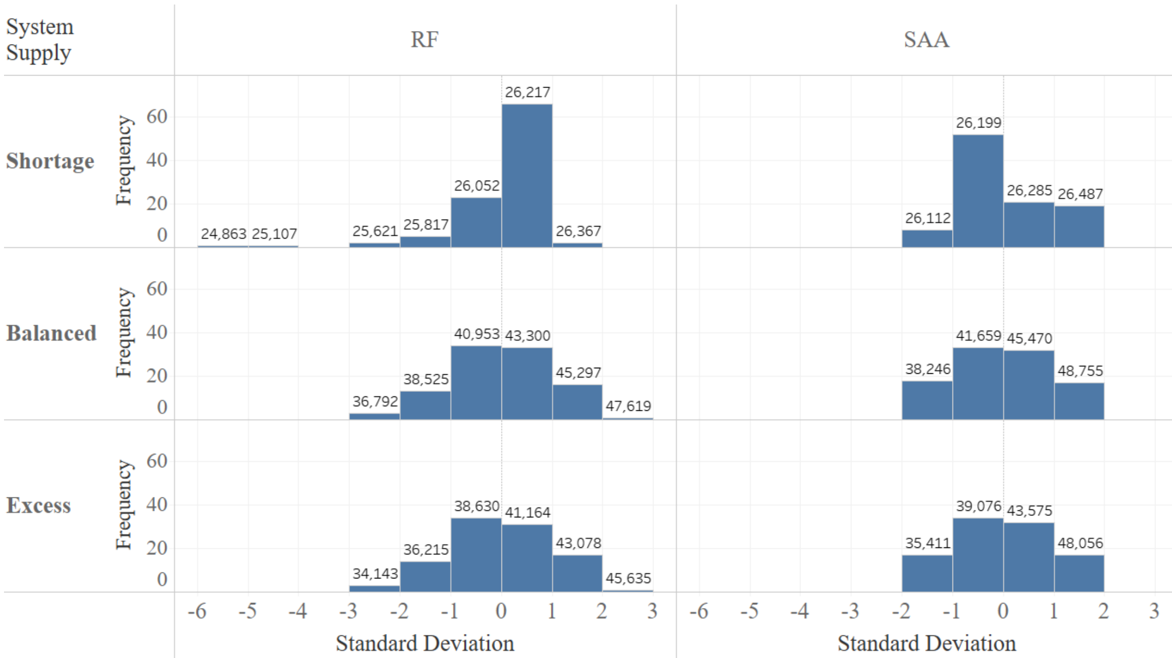


Figure 9: Objective values distribution for demand scenarios

for random forest method, outlier scenarios are not noted. As a result, it can be stated that the suggested method does not create high differences among scenarios if the system supply is not in shortage. In case of the systems with shortage in supply, random forest method does not provide a robust solution.

In addition to the distribution of the total system profit, store based analyses can be seen from Figure 10. This figure indicates the results of the system with balanced supply. It can be stated that random forest has again wider distribution compared to the SAA method. Moreover, among the stores there is not a notable difference in terms of distribution. Each store's distribution has nearly the same deviation from the mean.

In the following section, the effect of review period is analyzed for the systems with lateral transshipments for 100 scenarios.

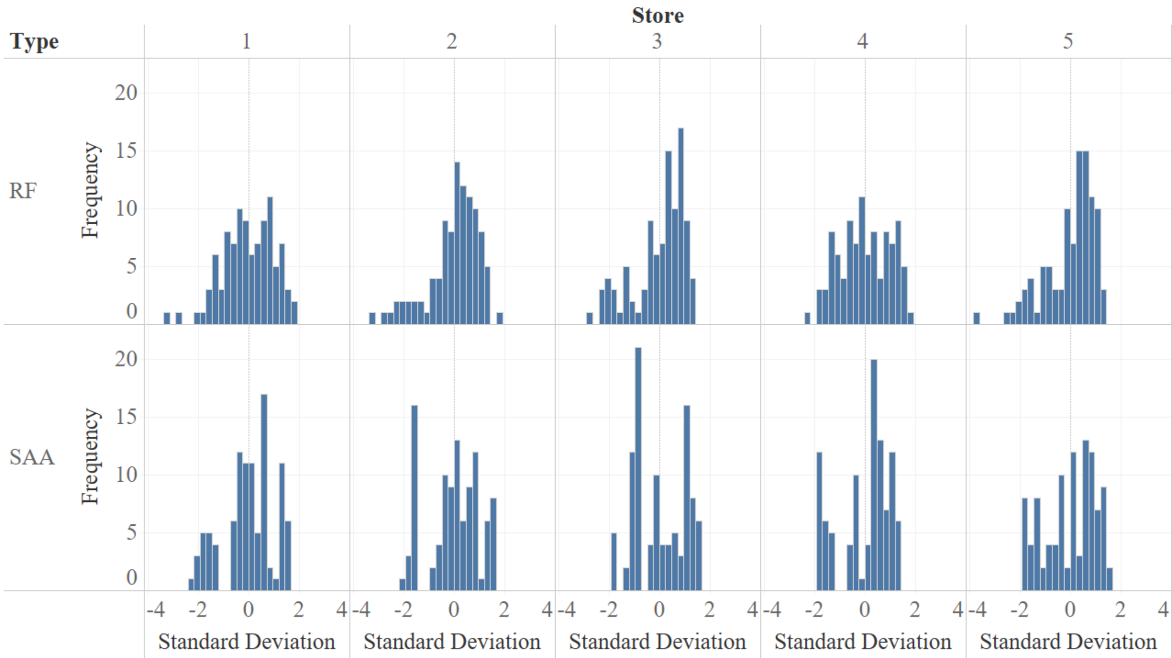


Figure 10: Store based objective values distribution for demand scenarios

5.5. Effect of Review Frequency

In the system, predictions are constructed for four future periods and shipment decisions obtained from the model is a decision set for four periods. Therefore, reviewing the system only once in four periods is possible. After obtaining the decisions for four periods, it may be preferred to freeze the decisions and apply the decisions for four periods without reviewing the system periodically. However, it can also be reviewed periodically, and making changes in the decision is possible. In this section, the simulated system is tested for both strategies: a system with periodic review for four periods and a system with one-time decision including four periods at $t = 40$. In order to obtain compatible results only the first three periods performance will be analyzed. The analyses are constructed for the system allowing lateral transshipment with the model using 100 demand scenarios. As in the previous section inventory level analyses were summarized, it is assumed that the system supply is on balance with demand, and the initial allocation is done based on the warehouse oriented strategy.

The objective values for the review frequency analysis can be seen from Table 15. If the review frequency of the system is to give decisions at $t = 40$ and not to change it in the future periods, then this strategy is called as single period review. If the decisions are reviewed at each period and changed periodically then the strategy is stated as periodic in the table. For the three prediction methods, periodic review strategy have higher objective values. However, the change in the objective values is not notable enough to conclude a superiority among the strategies. Therefore, it is tested statistically with a paired t-test. The p-value obtained from the paired t-test for objective values comparison is nearly 6.51e-10, this low p value indicates that there is a significant difference between these approaches.

Table 15: Objective values for review frequency analyses

	Review Frequency	
Prediction Method	Single Review	Periodic Review
MA	34,045	34,082
SAA	36,625	37,196
RF	36,821	37,729

In addition to the objective value comparison, KPI values are also analyzed. For the lost sales and weeks of supply KPIs, paired t-test provides low p-values and indicates that there is a significant difference between the review strategies. In order to see the difference, the KPI results of the review frequency analysis are summarized in Table 16.

Table 16: KPI values for review frequency analyses

	Review Frequency			
	Single Review		Periodic Review	
Prediction Method	Avg. Wos	Avg. Ls Ratio	Avg. Wos	Avg. Ls Ratio
MA	0.34	14.0%	0.36	13.8%
SAA	0.38	7.2%	0.42	6.2%
RF	0.37	6.2%	0.39	5.5%

For the three prediction methods, the systems operate better with a periodic review strategy. Although the weeks of supply values increases, the lost sales ratio change is important. Reviewing the system periodically enables to make decisions with live inventory, this is why it performs better than the single review strategy.

However for some systems single review strategy might have some other advantages, like less shipment/operational workforce planning. In such cases, if it is needed to use a single review strategy, selecting the prediction method becomes very important, because Table 16 shows that if the prediction method is random forest then the system performs better.

5.6. Performance Comparison of the Prediction Methods

Three different prediction methods are analyzed in the previous sections: Moving Average, Sample Average Approximation, and Random Forest. SAA and RF are the stochastic prediction methods whereas the MA is used as a deterministic benchmark. MA is widely used in the retail sector, both the basic moving average method and the smarter versions of it are studied. As stated before, the current system is a promotional environment, and future promotion flags are known during the shipment decision processes. Promotion features are already used by the random forest method, but the performance of the version of the moving average that considers promotions is unknown in the previous sections. In this section, the moving average method is extended to a version of averaging the last 4 sales with promotion if the future prediction is in promotion, and the last 4 non-promotion sales if the prediction period is not in promotion. The KPI values for the comparison can be seen from Table 17. For each cases, it can be stated that MA prediction method has a week performance. Although the SAA prediction method performs better than MA, there are two different prediction methods that performs better than SAA.

For the systems with shortage in supply, the lost sales ratios are the same and do not have a significant difference among the weeks of supply values. However, for the systems with balance in supply, Smart MA has the lowest lost sales and weeks of supply values for each allocation strategy. This is because the simulated system is a basic sales environment with promotion uplifts. Smart MA covers all the features of the system, but in case of a system with cannibalization and substitution effects moving average methods would not be sufficient. Random forest is a more powerful method as features

Table 17: KPI analyses of the prediction methods for different inventory cases

		Allocation Strategy					
		WH Oriented		Balanced		Store Oriented	
System Supply	Type	Avg. Wos	Avg. Ls Ratio	Avg. Wos	Avg. Ls Ratio	Avg. Wos	Avg. Ls Ratio
Shortage	MA	0.15	49.9%	0.26	49.9%	0.50	49.9%
	Smart MA	0.13	49.9%	0.25	49.9%	0.49	49.9%
	SAA	0.16	49.9%	0.26	49.9%	0.49	49.9%
	RF	0.14	49.9%	0.26	49.9%	0.49	49.9%
Balance	MA	0.66	9.2%	1.25	6.2%	2.32	3.7%
	Smart MA	0.59	4.3%	1.20	3.2%	2.29	1.8%
	SAA	0.67	8.1%	1.25	7.2%	2.32	6.1%
	RF	0.67	6.7%	1.26	6.1%	2.32	5.5%
Excess	MA	1.33	7.5%	2.76	3.4%	5.28	0.0%
	Smart MA	1.27	3.7%	2.73	1.8%	5.29	0.1%
	SAA	1.34	5.4%	2.77	2.7%	5.29	0.1%
	RF	1.36	2.7%	2.79	1.4%	5.28	0.0%

of the system increases because it considers all demand related features. In addition to the substitution and cannibalization effects, creating stock outs in the system would also mislead the moving average methods, and random forest is supposed to identify such links between the stock-out feature and sales information.

For the systems with excess in supply, it can be stated that the random forest is still the best prediction method in terms of lost sales performance. Lost sales values of smart moving average are higher than the lost sales values of the random forest. However, the smart moving average method keeps lower weeks of supply values. This can be explained by the variation considered in the optimization framework. As stated before for the optimization model, demand scenarios are generated and random forest provides a predictive distribution as the moving average provides point estimates. In case of having excess inventory on hand, thanks to the variation covered in the optimization model with the random forest prediction method, the system keeps inventory with a buffer and performs better in terms of lost sales value for each allocation strategy.

6. CONCLUSION

Shipment decisions for multi-period and multi-location inventory systems are studied in this thesis. After conducting a literature review study, it is seen that the complete knowledge of probability distribution of demand is a common assumption in the literature. However, Bertsimas and Kallus (2020) suggest a new optimization approach without making any assumptions on the distribution of demand. With the help of the suggested framework, a mathematical model as linear programming is used to solve the optimization problem by better estimation of demand distribution based on conditional probabilities estimated by random forests. The optimization problems are solved via Gurobi packages. In order to handle the stochasticity of demand nature, instead of using point estimates in the mathematical model, demand scenarios are generated by better estimation of demand distribution obtained by random forest and sample average approximation prediction methods. Moreover, to create a benchmark, the moving average method is used as an example for point estimate prediction methods. Different prediction points retrieved from the demand distribution are used as scenarios. The main goal of the model is to maximize the profit, by obtaining the optimal inventory levels with the consideration of each demand scenario.

To evaluate the robustness of the suggested method, a simulated system is used for the analyses. Firstly, the effect of the number of demand scenarios in the mathematical model is analyzed. It is seen that as the number of demand scenarios increases, the system stabilizes and performs better due to the consideration of variation in demand. Then the inventory level cases are tested because, during the literature review, it is seen that the starting inventory levels for locations and the review frequency are important for the system performance. Therefore, nine different inventory level cases are generated. Statistical tests show that inventory level cases have significant effects on the objective values of the system. Therefore, the optimal strategies might change according to the inventory level cases of the management system. After the comparison of the systems with and without lateral transshipments, it is concluded that allowing lateral transshipment decisions improves the lost sales values of the system escpipecially if the system is in excess in supply. The strategies should be selected with consideration of the trade-off between lost sales and weeks of supply. After the inventory level case analyses, in order to evaluate the robustness of the proposed methods for each inventory case, the distribution of the objective results of the mathematical model is analyzed. The results reveal that, the method using random forest prediction has a wider distribution for objective results over demand scenarios. Moreover, for the systems with shortage in supply, there are scenarios with outlier objective results, indicating that optimization model with random forest demand scenarios is not a robust solution. After analyzing the distribution of the system profit over scenarios, the strategy used for the review period of the system, periodic or single-period review, with lateral transshipment strategy is tested. The test concludes that there is a significant difference between the systems with periodic and single-period review strategy and periodic strategy obtains lower lost sales values. Finally, although the analyses reveal that the systems with the random forest method perform better than the other systems, smart moving average method is introduced and tested. For the systems with balance in supply, the smart moving average is a better method than the random forest. This is because the simulated system is a basic sales environment with promotion uplifts. For the systems with excess in supply, random forest method performs better than the smart moving average. This reveals that the variation in the random forest method provides the system to keep higher inventory compared to the others, and in case of keeping the inventory considering different demand scenarios the system performs better.

As future work, the same analyses can be conducted for different cost structures to evaluate the effect of the cost structure on the suggested method. The results might change for each inventory level cases if a different cost structure is considered. Moreover, as multi-period consideration is used, substitution effects among items can be taken into account in the prediction methods. For the complex systems, it can be tested if random forest provides better results compared to the smart moving average method. Creating stock-outs might also change the prediction method's power, it can be tested with additional simulated data sets. Multi-period consideration also enables to include resource capacity constraints in the suggested model. As the model allows lateral transshipment decisions, including distance information among locations can be considered and the effect of distance among locations on the optimal decisions can be analyzed. In addition to the shipment decisions, pricing decisions can be added as variables to the optimization model so the model can generate pricing and shipment decisions jointly. The suggested method can be tested with real data instead of a simulated data set. Therefore, auxiliary variables can be extended to obtain better predictions for the random forest prediction method. Additionally, demand scenario weights, w_k stated in Equation (4.16), can be studied with various methods as suggested in Bertsimas' framework. Finally, the suggested mathematical model uses linear programming assumptions and does not use integer constraints. However, integer programming can be used to solve the same problem but additional solutions should be suggested to prevent the possible run time issues for large data sets.

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