"STUDY OF THE TURKISH, AMERICAN and FRENCH REINFORCED CONCRETE BUILDING CODES, WITH RESPECT TO SLAB DESIGN and THE YIELD-LINE THEORY"

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K. D.

INTRODUCTION

The purpose of this work is to compare the Turkish, American and French Reinforced Concrete Building Codes with respect to slab design. This comparison will involve the load and safety factors introduced by the codes, the approach of the Codes to the problem. Since all of the three Codes to be compared are guided by the "Elastic analysis of plates" a modern method "the yield-line theory" based on "ultimate strength" will also be presented as a criterion.

The work is divided into 5 Sections.

Section I includes the two theories, i.e. the Elastic analysis of Plates and the Yield-line theory.

In Section 2 the methods adopted by the three Codes are explained.

Section 3 includes numerical computations to serve for the proposed comparisons. The steps followed in Section 3 are: (a)The moment coefficients are obtained from tables accepted by Code (b)Moment coefficients are calculated through the yield-line method by the procedure explained at the beginning of Section3, (c)The slabs are designed by the working stresses of the three Codes, (d)Ultimate carrying capacities of the obtained sections are computed, (e)The load factors applied by the Codes are calculated by the procedure explained in Part 3.2.2.d.

The results obtained in Section 3 are discussed and compared in Section 4.

Section 5 includes the conclusions.

SECTION I

THEORIES ON SLABS

1.1 ELASTIC ANALYSIS OF PLATES

1.1.1 - The general equation of laterally loaded plates takes the form of a fourth degree partial differential equation[1]:

$$\nabla^4 w = \frac{\partial^4 w}{\partial x^4} + 2\frac{\partial^4 w}{\partial x^2 \partial y^2} + \frac{\partial^4 w}{\partial y^4} = \frac{q}{Eh^3/12(1-\nu^2)} = \frac{q}{D} \qquad (1)$$

where w = statical deflection of the plate,

q = distributed load on the plate,

h = depth of the plate,

E = modulus of elasticity of the material,

ν = Poisson's ratio,

D is referred to as the "flexural rigidity" of the plate.

The basic assumptions that are made for the derivation of Eq.(1) are the following:

a-The plates are of constant medium thickness,

b-Deflections are small compared to thickness,

c-Stretching of the middle plane is of negligible importance,

d-Shear deformation can be neglected.

e-The material of the plate is homogeneous, elastic, isotropic.

f-Flexural strains vary linearly through the depth of the slab.

The problem of bending of laterally loaded plates is reduced to the integration of Eq.(1), since the moments M_x, M_y, and M_{xy} are given by the expressions:

$$M_x = -D\left(\frac{\partial^2 w}{\partial x^2} + \nu\frac{\partial^2 w}{\partial y^2}\right) \qquad M_y = -D\left(\frac{\partial^2 w}{\partial y^2} + \nu\frac{\partial^2 w}{\partial x^2}\right)$$

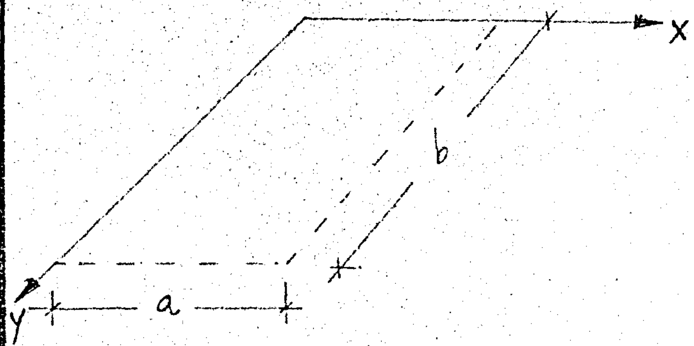
1 . For the derivation see S.Timoshenko and S.Woinowsky-Krieger "Theory of Plates and Shells" Pp. 79-82
Mc Graw Hill 1959

$$M_{xy} = D(1-\nu)\frac{\partial^2 w}{\partial x \partial y} \quad \text{(twisting moment)}$$

and the shearing forces Q_x and Q_y by:

$$Q_x = -D\frac{\partial}{\partial x}\left(\frac{\partial^2 w}{\partial x^2} + \frac{\partial^2 w}{\partial y^2}\right) \; ; \; Q_y = -D\frac{\partial}{\partial y}\left(\frac{\partial^2 w}{\partial x^2} + \frac{\partial^2 w}{\partial y^2}\right)$$

1.1.2- Boundary conditions



The term "w" in Eq.(1) has to satisfy the different boundary conditions of different support types. We find it useful to give here the most general support types for rectangular plates, and the boundary conditions to which they give rise.

a-Built-in edge: At a built-in edge the deflection of the plate is zero, and the slope of the tangent plane to the deflected middle surface along this edge is also zero.

Supposing the plate shown above is simply supported along the edge x=b we can write:

$$(w)_{x=b} = 0 \quad , \quad \left(\frac{\partial w}{\partial x}\right)_{x=b} = 0$$

b-Simply-Supported edge: The deflection of a plate at a simply-supported edge is zero. Similarly since at such an edge the plate is free to rotate the bending moments are zero. If the edge x=b is simply-supported we can write:

$$(w)_{x=b} = 0 \quad , \quad \left(\frac{\partial^2 w}{\partial x^2} + \nu\frac{\partial^2 w}{\partial y^2}\right)_{x=b} = 0$$

As $\frac{\partial^2 w}{\partial y^2}$ vanishes with "w" along the edge we should have $\frac{\partial^2 w}{\partial x^2} = 0$ or

$$\nabla w = \left(\frac{\partial^2 w}{\partial x^2} + \frac{\partial^2 w}{\partial y^2}\right)_{x=b} = 0$$

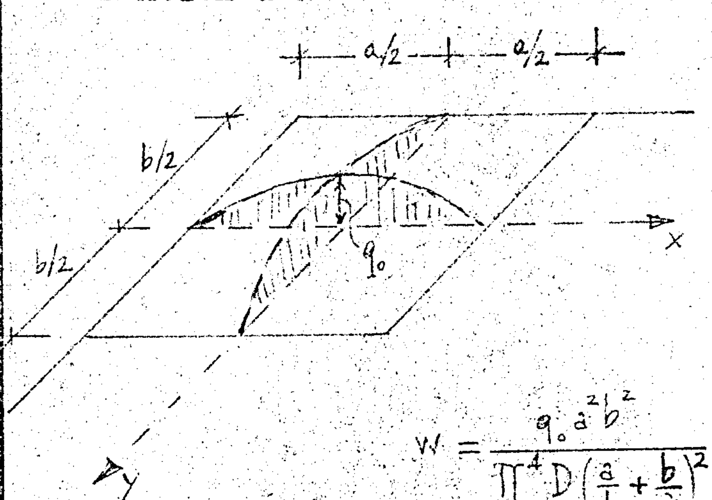
c-Free edge: If the edge x=b is free than the boundary conditions are shown to be[1]:

$$\left[\frac{\partial^3 w}{\partial x^3} + (2-\nu)\frac{\partial^3 w}{\partial x \partial y^2}\right]_{x=b} = 0$$

$$\left(\frac{\partial^2 w}{\partial x^2} + \nu\frac{\partial^2 w}{\partial y^2}\right)_{x=b} = 0$$

1.1.3.- Solutions of the differential equation of plates

For design purposes it is necessary to solve Eq.(1) and determine the deflection surface w of the plate. The determination of w enables one to calculate M_x, M_y etc. We will present here the results of three solutions of which the first represents a particular loading and the other two are universally applicable solutions. The results presented here are derived for rectangular plates simply supported on four sides, one can find the solutions for other support conditions by applying the boundary limitations mentioned above.



a. Let $q = q_0 \cos\frac{\pi x}{a} \sin\frac{\pi y}{b}$ representing the particular loading shown in the figure. With this assumption and a simply supported plate we get:

$$w = \frac{q_0 a^2 b^2}{\pi^4 D\left(\frac{a}{b}+\frac{b}{a}\right)^2} \cos\frac{\pi x}{a} \cos\frac{\pi y}{b}$$

$$M_x = \frac{q_0 ab}{\pi^2} \frac{\left(\frac{b}{a}+\nu\frac{a}{b}\right)}{\left(\frac{a}{b}+\frac{b}{a}\right)^2} \cos\frac{\pi x}{a} \cos\frac{\pi y}{b}$$

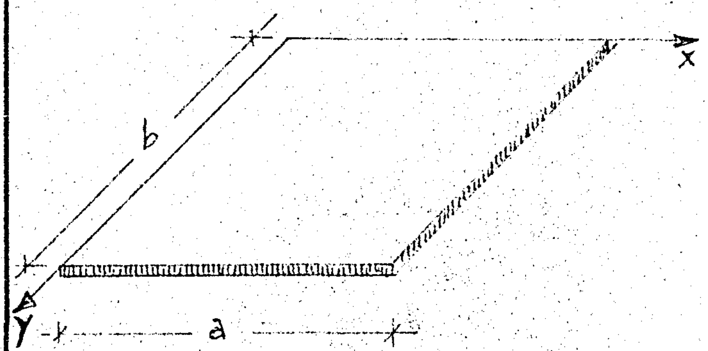
1 . Ibid pp.83-84

$$M_y = \frac{q_o ab}{\pi^2} \frac{\left(\frac{a}{b} + \nu \frac{b}{a}\right)}{\left(\frac{a}{b} + \frac{b}{a}\right)^2} \cos \frac{\pi x}{a} \cos \frac{\pi y}{b}$$

$$M_{xy} = \frac{-q_o ab}{\pi^2} \frac{(1-\nu)}{\left(\frac{a}{b} + \frac{b}{a}\right)^2} \sin \frac{\pi x}{a} \sin \frac{\pi y}{b}$$

b. Now let

$$q_{xy} = \sum_{n=1}^{\infty} \sum_{m=1}^{\infty} q_{mn} \sin \frac{n\pi x}{a} \sin \frac{m\pi y}{b}$$



In this form q_{xy} -a function of x and y - is generalized in such a way that it includes any sort of loading applied to a rectangular plate supported on all four sides. The coordinate axes are located as shown in the figure.

For this general case of loading, and a rectangular plate simply supported on all four sides the integration of Eq.(1) gives the following result[1]:

$$w = \frac{a^2 b^2}{\pi^4 D} \sum_{n=1}^{\infty} \sum_{m=1}^{\infty} \frac{q_{mn}}{\left(n^2 \frac{b}{a} + m^2 \frac{a}{b}\right)} \sin \frac{n\pi x}{a} \sin \frac{m\pi y}{b}$$

This solution is known as "Navier solution".

From the expression for w given above one can easily derive :

$$M_x = \frac{ab}{\pi^2} \sum_{n=1}^{\infty} \sum_{m=1}^{\infty} q_{mn} \frac{\left(n^2 \frac{b}{a} + \nu m^2 \frac{a}{b}\right)}{\left(n^2 \frac{b}{a} + m^2 \frac{a}{b}\right)^2} \sin \frac{n\pi x}{a} \sin \frac{m\pi y}{b}$$

$$M_y = \frac{ab}{\pi^2} \sum_{n=1}^{\infty} \sum_{m=1}^{\infty} q_{mn} \frac{\left(m^2 \frac{a}{b} + \nu n^2 \frac{b}{a}\right)}{\left(n^2 \frac{b}{a} + m^2 \frac{a}{b}\right)^2} \sin \frac{n\pi x}{a} \sin \frac{m\pi y}{b}$$

1. Ibid pp. 108-109

$$M_{xy} = \frac{-ab}{\pi^2} \sum_{n=1}^{\infty} \sum_{m=1}^{\infty} \frac{q_{mn}(1-\nu)mn}{\left(n^2\frac{b}{a}+m^2\frac{a}{b}\right)^2} \cos\frac{n\pi x}{a} \cos\frac{m\pi y}{b}$$

The coefficient q_{mn} which appears in all the moment expressions will be determined by performing the following integration:

$$q_{mn} = \frac{4}{ab} \int_0^b \int_0^a q_{xy} \sin\frac{n\pi x}{a} \sin\frac{m\pi y}{b} \, dx dy$$

The loads q_{xy} that we mostly encounter in practice are uniformly distributed i.e. $q_{xy} = p$ kg/m2 or p lb/sq.ft..For this particular case the integration above yields :

$$q_{mn} = \frac{16p}{\pi^2 mn}$$ where m and n are odd integers

Substituting q_{mn} in the deflection and moment expressions we get the following results:

$$w = \frac{16pa^2b^2}{\pi^6 D} \sum_{n=1,3,5...}^{\infty} \sum_{m=1,3,5...}^{\infty} \frac{\sin\frac{n\pi x}{a} \sin\frac{m\pi y}{b}}{mn\left(n^2\frac{b}{a}+m^2\frac{a}{b}\right)^2}$$

$$M_x = \frac{16pab}{\pi^4} \sum_{n=1,3,5...}^{\infty} \sum_{m=1,3,5...}^{\infty} \frac{\left(n^2\frac{b}{a}+\nu m^2\frac{a}{b}\right)}{mn\left(n^2\frac{b}{a}+m^2\frac{a}{b}\right)^2} \sin\frac{n\pi x}{a} \sin\frac{m\pi y}{b}$$

$$M_y = \frac{16pab}{\pi^4} \sum_{n=1,3,5..}^{\infty} \sum_{m=1,3,5..}^{\infty} \frac{\left(m^2\frac{b}{a}+\nu n^2\frac{a}{b}\right)}{mn\left(n^2\frac{b}{a}+m^2\frac{a}{b}\right)^2} \sin\frac{n\pi x}{a} \sin\frac{m\pi y}{b}$$

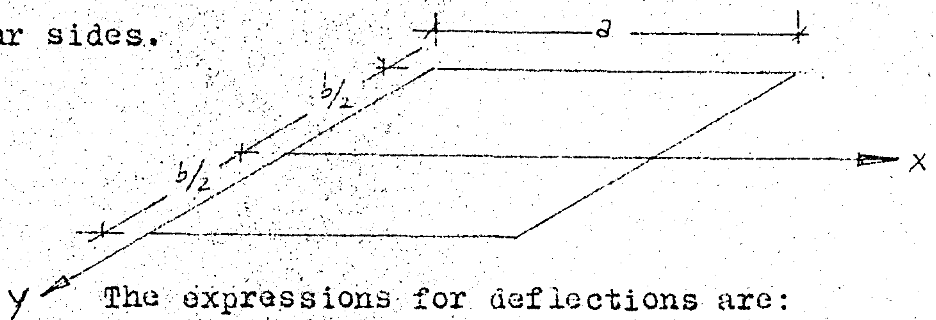
c.- The second general method of solution to be presented here is the one suggested by M.Lévy [1].He proposed to show the deflection w as:

$$w = w_1 + w_2$$

where w_1 is the deflection of a one way plate,and w_2 is the

1 . Comp. rend. Vol.129 pp.535-539,1899 and,
Timoshenko and Woinowsky-Krieger op.cit. pp.113-115

factor correcting for the fact that the plate is supported on all four sides.



The expressions for deflections are:

$$w_1 = \frac{qa^4}{24D}\left[\left(\frac{x}{a}\right)^4 - 2\left(\frac{x}{a}\right)^3 + \left(\frac{x}{a}\right)\right]$$

$$w_2 = \sum Y_n \sin\frac{n\pi x}{a}$$ where Y_n is a function of y only

w_1 satisfies $\nabla^4 w_1 = \frac{q}{D}$ and also the boundary conditions at the edges x=0 and x=a. The expression w_2 should evidently satisfy $\nabla^4 w_2 = 0$. Applying this condition and all boundary limitations we get:

$$w = \frac{qa^4}{24D}\left[\left(\frac{x^4}{a^4}\right) - 2\left(\frac{x}{a}\right)^3 + \frac{x}{a}\right] - \frac{4qa^4}{D}\sum_{n=1,3,5..}^{\infty}\frac{\sin\frac{n\pi x}{a}}{n^5\pi^5}\left[\frac{2+\frac{n\pi b}{2a}\tanh\frac{n\pi b}{2a}}{2\cosh\frac{n\pi b}{2a}}\cosh\frac{n\pi y}{a} - \frac{\frac{n\pi y}{a}\sinh\frac{n\pi y}{a}}{2\cosh\frac{n\pi b}{2a}}\right]$$

To get M_x and M_y we use :

$$M_x = -D\,\frac{\partial^2 w^{(0)}}{\partial x^2} \quad \text{and} \quad M_y = -D\,\frac{\partial^2 w^{(0)}}{\partial y^2}$$ i.e. we let $\nu = 0$

Then:

$$M_x^{(0)} = -\frac{qa^2}{2}\left[\left(\frac{x}{a}\right)^2 - \frac{x}{a}\right] - \frac{4qa^2}{\pi^3}\sum_{n=1,3,5..}^{\infty}\frac{\sin\frac{n\pi x}{a}}{n^3}\left[\frac{2+\frac{n\pi b}{2a}\tanh\frac{n\pi b}{2a}}{2\cosh\frac{n\pi b}{2a}}\cosh\frac{n\pi y}{a} - \frac{\frac{n\pi y}{a}\sinh\frac{n\pi y}{a}}{2\cosh\frac{n\pi b}{2a}}\right]$$

$$M_y^{(0)} = \frac{4qa^2}{\pi^3}\sum_{n=1,3,5..}^{\infty}\frac{\sin\frac{n\pi x}{a}}{2n^3\cosh\frac{n\pi b}{2a}}\left[\frac{n\pi b}{2a}\tanh\frac{n\pi b}{2a}\cosh\frac{n\pi y}{a} - \frac{n\pi y}{a}\sinh\frac{n\pi y}{a}\right]$$

The series obtained by this method converge very rapidly and the consideration of n=1 only, seems satisfactory for practical purposes.

To get the moments M_x and M_y for other values of ν we make use of the equations:

$$M_x^{(\nu)} = M_x^{(o)} + \nu M_y^{(o)}$$
$$M_y^{(\nu)} = M_y^{(o)} + \nu M_x^{(o)}$$

where the superscripts indicate the value of ν considered.

1.1.4 - Coefficients of bending moments by use of the theory of elasticity

Westergaard gives the following values for square panels [1]

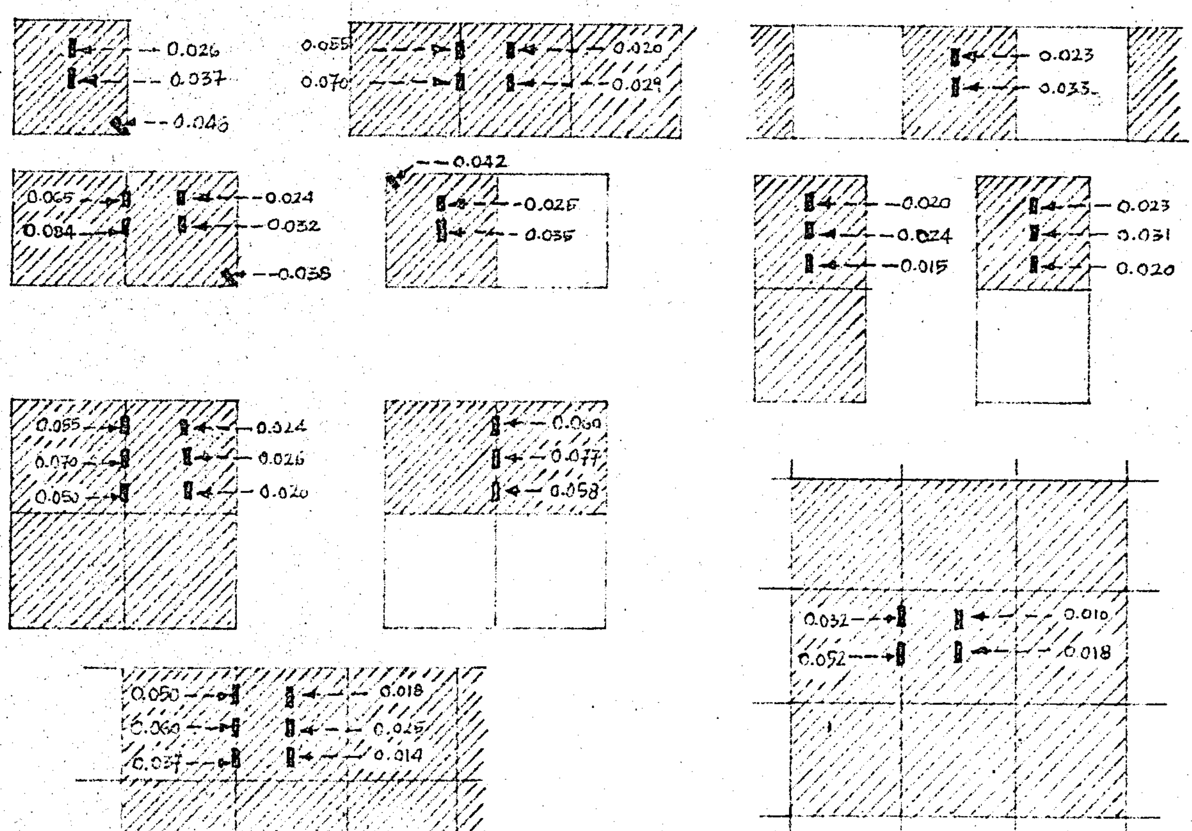


Fig 1 — $M = cqa^2$ (a = span length)

1. H.M. Westergaard - "Formulas for the design of rectangular floor slabs and the supporting girders" Proc. ACI Vol. XXII, 1926 pp. 27-28

The coefficients given in Fig. 1 were derived on the assumption that the slabs are of homogeneous, isotropic, elastic material with Poisson's ratio equal to zero. Each panel is simply supported on four sides but the slab is continuous over the interior beams. Each shaded panel carries a uniform load q.

1.2. THE YİELD-LİNE THEORY

~~1.2.1~~ - The second theory on slabs will be the "Yield line theory" developed mostly by K.W.Johansen.

The yield-line theory is concerned with the ultimate load carrying capacity of reinforced concrete slabs, and it will very probably be accepted as a design method by the codes of all countries, it is already largely practised in the Scandinavian countries.

The "yield-line" is the direction in which the curvature of a slab will be concentrated, it is also identified by the concentration of cracking. A "yield-line is analogous to the "yield hinge" or "plastic hinge" used in inelastic analyses of beams and frames. The terms positive and negative yield-lines are often used to distinguish between yield-lines giving tension in the bottom of a slab, and those giving tension in the top of the slab.

1.2.2 - One-way slabs

a- Simple span : As a first illustration the behaviour of a simply supported one-way slab with 0.5 percent reinforcement will be considered. Failure of such a structure will be initiated by yielding of the tension reinforcement at midspan, and large deflections will develop before the ultimate load is reached by

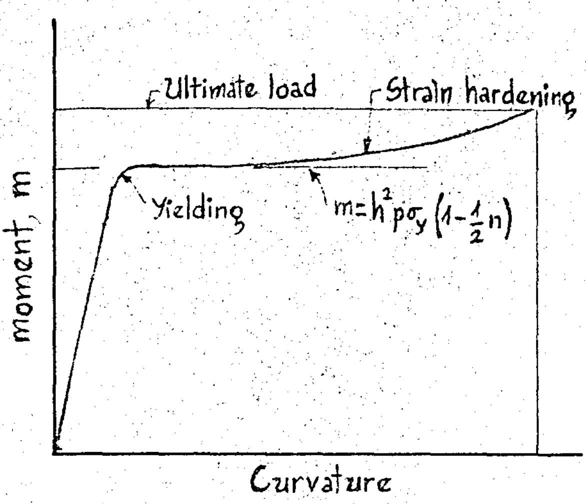


Fig.2 - Moment-Curvature relation.

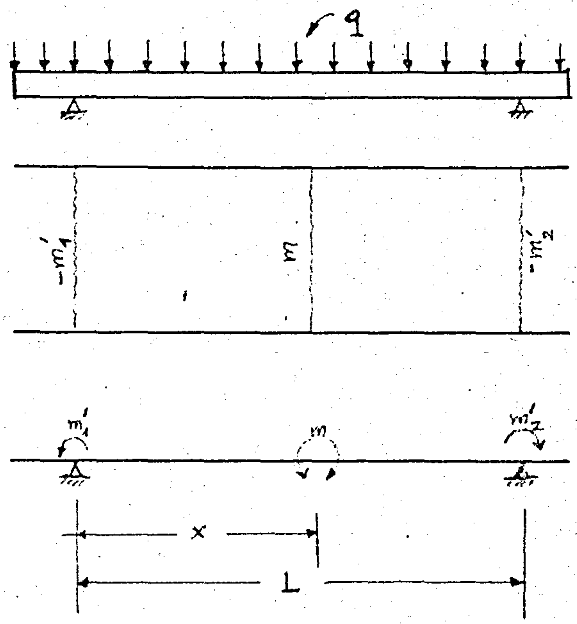


Fig.3-Restrained one-way slab

crushing of the concrete [1]. A yield-line will form across the slab at midspan.

The moment per unit width at the instant of failure by yielding of steel is given by the following expression [2]:

$$m = M/b = h^2 p\sigma_y(1 - 0.5n) \qquad (2)$$

where M = Moment over total width of slab

b = width of the slab

h = effective depth

p = percentage of reinforcement

σ_y = yield strength of steel

$n = p(\sigma_y/\sigma_b)$

The flexural strength of a uniformly loaded simple-

1. E.Hognestad "Yield line theory for the ultimate flexural strength of reinforced concrete slabs" Proc. ACI vol 49 1953 pp. 638
2. For the derivation see : Urquhart, O'Rourke, Winter "Design of concrete structures" Art.12-5 pp.445 McGraw Hill 1958

span slab should, therefore, be expressed for design purposes as:

$$m = qL^2/8 = h^2 p \sigma_y (1 - 0.5n)$$

where q = uniform load per unit area

L = span of slab

b-Restrained span: The one-way slab considered above is taken this time to be restrained at the supports (Fig.3). This boundary condition requires the formation of a mechanism for failure. This mechanism will be reached when a positive yield-line forms at the positive moment section and two negative yield-lines form at the supports. We can write by reference to Fig.3

$$m_1' + m = qx^2/2$$

$$m_2' + m = q(L - x)^2/2$$

where x gives the position of the positive yield-line.

The solution of this system gives

$$m = \frac{qL^2}{2\left(\sqrt{i_1+1} + \sqrt{i_2+1}\right)^2} \qquad x = \frac{L\sqrt{i_1+1}}{\sqrt{i_1+1} + \sqrt{i_2+1}}$$

where $i_1 = m_1'/m$ and $i_2 = m_2'/m$

For simple spans $m_1' = m_2' = 0$ therefore $i_1 = i_2 = 0$ and $m = qL^2/8$ at $x = L/2$. If $i_1 = i_2 = 1$ we obtain $m = m' = qL^2/16$, while the theory of elasticity for a fixed slab gives $m' = qL^2/12$ and $m = qL^2/24$. If a fixed slab is reinforced according to theory of elasticity $i_1 = i_2 = 2$, then the equation above also gives $m = qL^2/24$ and therefore $m' = qL^2/12$.

The equations derived above have been experimentally verified by von Kazinczy [1].

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1. See reference N° 17 in E.Hognestad loc.cit.pp655

1.2.3- Two-way slabs

We will first consider slabs reinforced in two orthogonal directions with such amounts of reinforcement as to give the same yield moment for any one of the two directions. This state of reinforcement is called "isotropic". If such a slab is overloaded, yielding will begin in regions of high moment, and with increased loads yield-lines will form and spread into a "yield-line pattern". the slab will reach a state of neutral equilibrium and will have used all of its load carrying capacity when the yield-lines spread to the slab edges.

"The yield-lines divide the slab into several parts and a heavy concentration of curvature takes place in the yield-lines since the plastic deformations are much larger then the elastic ones. Near the ultimate load it is reasonable to assume, therefore that the individual slab parts are plane, all deformations taking place in the yield-lines [1]. It then follows that the yield-lines

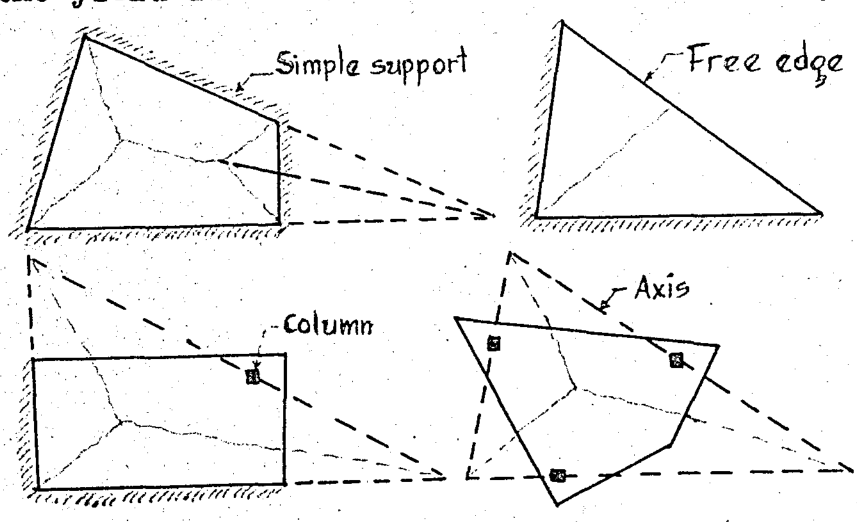

Fig. 4 - Typical yield-line patterns

1. Nylander, H. reports in "Korsarmerade betongplattor" (Bull.5 Royal Inst. of Tech. Div. of Building and Structural Engineering, Stockholm, 1950, pp.140) the results of tests on 7 uniformly loade slabs. These results show a concentration of curvature in the yield-lines.

must be straight, and deformations of the slab may be considered as rotations of the slab parts about axes in their supports. Furthermore a yield-line between two slab parts must pass through the intersection of axes of rotation of the two parts. Fig. 4 shows some typical yield-line patterns for various types of slabs. An axis of rotation must lie in a line of support and must pass through columns. In this manner the general nature of the possible yield-line pattern may be determined."[1]

To determine the yield-line pattern corresponding to the ultimate load of a slab Hognestad makes use of equilibrium conditions for the individual slab parts. Since the yield moments are principal moments, twisting moments are zero in the yield-lines, and the moment m per unit length of yield-line acts perpendicular to these lines. The total moment is equal to m times the length of the yield-line and is represented by a vector in the direction of the yield- line. The resulting moment for an individual slab part is then found by vector addition.

As an illustration to the method the case of a rectangular slab will be given. This problem is first solved by Ingerslev [2].

A rectangular slab subject to a uniform loading q and reinforced isotropically in two directions is considered fixed on all four edges. The moment at a fixed support depends primarily on the amount of negatine reinforcement as indicated by the equation:

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1. E.Hognestad loc.cit. pp.641
2. Ibid pp.641-642

$$m = h^2 p \sigma_y (1 - 0.5n)$$

Different amounts of reinforcement may be provided at the four edges so that $m'_1 = i_1 m$, $m'_2 = i_2 m$ etc. in which m is the positive yield moment in the field.

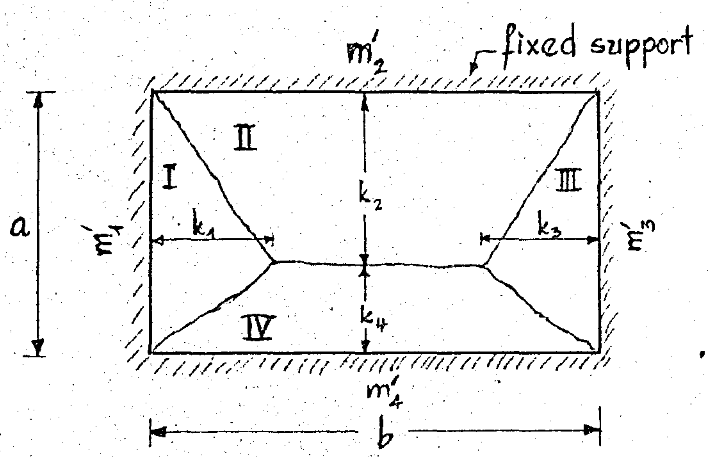


Fig.5 - Rectangular slab

Yield-lines form as shown in Fig.5 and the application of the equilibrium conditions to each of the four parts gives:

Part I - $ma\ (1 + i_1) = qak_1^2/2$

Part II- $mb\ (1 + i_2) = qk_1k_2^2/6 + qk_3k_2^2/6 + q(b-k_1-k_3)k_2^2/2$

Part III- $ma\ (1 + i_3) = qak_3^2/6$

Part IV - $mb\ (1 + i_4) = qk_1k_4^2/6 + qk_3k_4^2/6 + q(b-k_1-k_3)k_4^2/2$

From these four equations and ($k_2 + k_4 = a$),it is possible to determine m, k_1, k_2, k_3, and k_4 as functions of a , b , and the four i values.It is found that :

$$m = \frac{qa_r^2}{24}\left[\sqrt{3+\left(\frac{a_r}{b_r}\right)^2} - \frac{a_r}{b_r}\right]^2 \qquad (3)$$

in which the reduced side lengths "a_r" and "b_r" are given by:

$$a_r = \frac{2a}{\sqrt{1+i_2}+\sqrt{1+i_4}} \qquad b_r = \frac{2b}{\sqrt{1+i_1}+\sqrt{1+i_3}}$$

For a simply supported slab i = 0 and $a_r = a, b_r = b$. For simply supported square slab (a=b, i=0), $m = qa^2/24$. For a one-way slab i.e. $b_r = \infty$ we get $m = qa_r^2/8$.

Equation (3) is a general one, A.J.Ashdown has simplified it into[1]:

$$m(1+i_1) = \frac{qa^2}{24}\left[i_1+3-2\frac{a}{b}\sqrt{i_1}\right] \qquad (4)$$

1.2.4 - Design methods proposed by Hognestad

The yield-line theory eliminates the necessity of solving Eq.(1),and simplifies the analysis of even very complex slabs. E.Hognestad gives two methods to be used in practice when designing slabs by the yield-line method.

"The problem in design is generally to estimate the necessary yield moment m for a slab subject to given ultimate loads and with given dimensions and support conditions.The correct value of m is a maximum value resulting from the correct yield pattern and satisfying the equations of equilibrium" [2].

a-Method of successive approximations:

This method makes use of the wirtual work principle which states that for the slab as a whole,the work of the internal forces plus the work of the loads must equal zero,the work of reactions being zero as they act along axes of rotation.This principle is formulated as [3]:

$$\sum \vec{M}\vec{\theta} + \sum \iint q\delta \, dx\,dy = 0$$

in which summation is made over the entire slab,and integration is made over the individual slab parts.

In this formula $\vec{M}$ represents the resultants of the moments in the yield-lines, $\vec{\theta}$ the rotations,q the load and δ the

1. A.J.Ashdown "Design of slabs by the yield-line method"Concrete and constructional eng.g June 1960 pp.221-222
2. E.Hognestad loc.cit. pp.645
3. For the derivation see Ibid. pp.643-644.

displacements. The yield moment m is calculated by using the facts that $\vec{M}$ is proportional to m, and that the correct yield pattern will give the maximum m.

In the method of successive approximations a yield pattern is assumed which is in accord with the support conditions, and the necessary yield moment m is computed from the virtual work equation. Since, for the correct yield pattern, all m values should be equal, a check on the originally assumed yield pattern may be obtained by computing m for every individual slab part from equilibrium equations. If the m values thus computed differ considerably, the separate values will indicate how the pattern should be altered, and the first estimate of m from the virtual work equation will indicate how much the pattern should be changed.

The method of successive approximations will be illustrated by the example of a floor slab worked out by E. Hognestad.

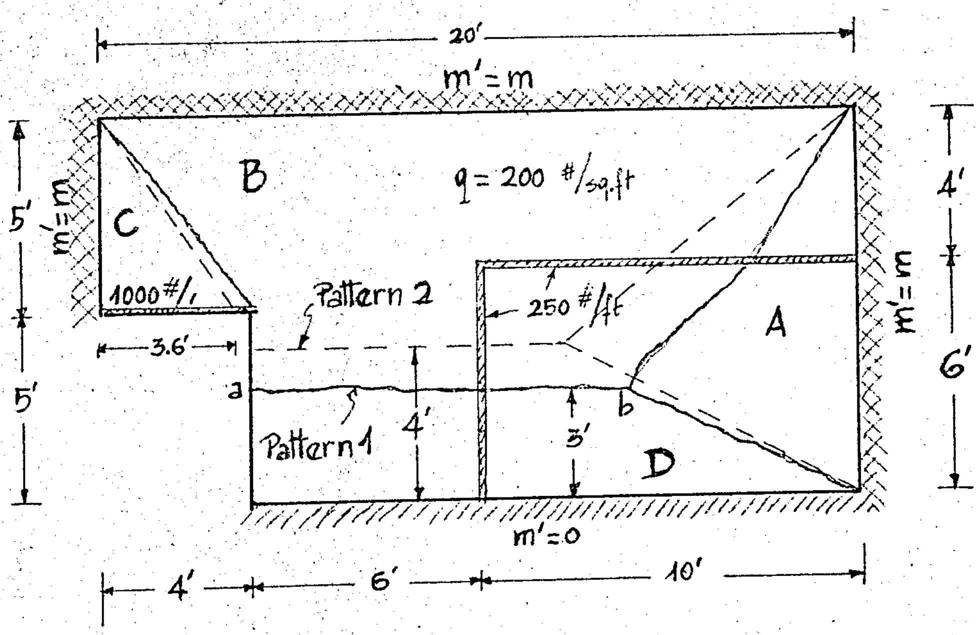


Fig. 6 - Analysis of floor slab

The slab shown in the figure is fixed or continuous on three edges, a negative reinforcement equal to positive reinforcement being chosen, which gives $m = m'$. The fourth edge is simply supported, $m' = 0$, and an opening with free edges is provided for a staircase. The indicated loads represent service loads multipled by proper load factors. Thus, a uniform load q=200 psf, a line load of 250 lb per ft from a partition wall, and a line load of 1000 lb per ft from the stair case are given.

A yield pattern is assumed, pattern N° 1; the assumption is guided by the fact that yield lines between two slab parts must pass through the intersection of the corresponding axes of rotation that is the supports. It is furthermore known that simply supported or free edges as well as openings "attract" yield-lines, while fixed edges "repel" lines [1].

The necessary yield moment m is first computed for each slab part separately by equilibrium of moments about the supports.

Part A- $10(m+m) = 200 \times 10 \times 6^2/6 + 250 \times 3.43^2/2$

$$20m = 13470 \quad \text{or } m_A = 674 \text{ lb.ft/ft}$$

Part B- $20(m+m) = 200(6\times7^2/6+10\times7^2/2+4\times5^2/6)+250(6.57\times4+3\times5.5) - m\times5\times4/5$

$$40m = 72790-4m \text{ or } m_B = 1653 \text{ lb.}$$

Part C- $5(m+m) = 200 \times 5 \times 4^2/6 + 1000 \times 4^2/2 + m \times 4 \times 4/5$

$$10m = 10670+3.2m \quad \text{or } m_C = 1568 \text{ lb}$$

Part D- $16(m+0) = 200(6 \times 3^2/6 + 10 \times 3^2/2) + 250 \times 3^2/2$

$$16m = 12030 \quad \text{or } m_D = 753 \text{ lb.}$$

It is seen that the m values vary from 674 to 1653 lb and the assumed yield pattern is therefore not the correct one. An estimate of the correct yield moment may nevertheless be obtained by applying the virtual work equation. A virtual deflection of unity

1. For the proof of this statement see Ibid. pp.642

along the yield line a-b gives the rotations: $\theta_A = 1/6$, $\theta_B = 1/7$, $\theta_C = (5/7)/4 = 1/5.6$, and, $\theta_D = 1/3$.

The equilibrium equations are written in such a way that the virtual work equations can be established easily.

$$\sum \vec{M}.\vec{\theta} = m(20/6 + 40/7 + 10/5.6 + 16/3) = 16.17m$$
$$= \sum \iint q\delta dxdy = 13470/6 + 72490/7 + 10670/5.6 + 12030/3$$
$$= 18560$$

which gives $m_1 = 1148$ lb for yield pattern N° 1.

It appears that the area of slab parts A and D must be increased, while those of B and C must be decreased. Such correction leads to pattern N° 2 which gives : $M_A = 1243$ lb, $m_B = 1182$ lb, $m_C = 1167$ lb, and $m_D = 1190$ lb. The corresponding virtual work equation gives $m_2 = 1192$ lb.

In this case the four m values are almost equal and m = 1192 lb is a satisfactory design value. It should be noticed that m_2 is only 3.8 % greater than m_1. Application of the virtual work equation to yield patterns reasonably close to the correct one gives yield moments only slightly less than the correct value.

b-Method of superposition

Although the yield-line theory is not a linear one and superposition is not theoretically applicable, Johansen[1] has shown that its use gives results on the safe side, as the sum of the yield moments corresponding to a number of individual loadings is greater than or equal to the yield moment corresponding to the sum of loadings.

1. Johansen, K.W. "Brudlinie teorier" Jul. Gjellerups Forlag, Copenhagen, Aug. 1943 pp. 189

1.2.5 -Determination of μ

The analyses and examples presented above were all concerned with slabs having an equal yield moment in all directions. In practical designs isotropic reinforcement is often not economical and it is necessary to determine the quantity of reinforcement which will satisfy both equilibrium equations and economical considerations.

To determine the value of the ratio of the yield moments in two orthogonal directions, that is $\mu = m_1/m$ A.J.Ashdown proposes the following way: the amount of reinforcement is proportional to m and m_1, therefore the cost of steel <u>s</u> is proportional to $m(1+\mu)ab$, substituting in eq.(4) we get:

$$s = qa^3b/24(\mu + 3 - 2a\sqrt{\mu}/b)$$

Differentiating s with respect to μ and equating to zero :

$$\frac{ds}{d\mu} = \frac{qa^3b}{24} - \frac{qa^4}{24\sqrt{\mu}}$$

or

$$\mu = \left(\frac{a}{b}\right)^2 = K^2$$

Substituting in Eq.(4):

$$m(1+K^2) = \frac{qa^2}{24}\left[K^2 + 3 - 2K^2\right]$$

or

$$m = \frac{qa^2}{24}\left[\frac{3-K^2}{1+K^2}\right] \qquad (5)$$

When designing a slab <u>m</u> is calculated from Eq.(5) then $m_1 = \mu m = K^2 m$.

1.2.6.- Orthogonally anisotropic reinforcement

The problem of orthogonally anisotropic reinforcement has also been considered by Johansen and it has been shown [1] that the analysis of slabs with yield moments m and $m' = im$ in one direction, and μm, $m' = i\mu m$ in an orthogonal direction may be reduced to the case of isotropic reinforcement ($\mu = 1$) by dividing the linear dimensions in the m direction by $\sqrt{\mu}$ and taking the same load q per unit area.

1- E.Hognestad loc.cit. pp.650-651

SECTİON 2

METHODS OF SLAB DESİGN

The coefficients of moments given at the end of the proceding section apply to plates of homogeneous, isotropic and elastic material, but they may be used as a guide in establishing formulas for the design of slabs of reinforced concrete, with the same pattern of panels. A reinforced concrete slab does not satisfy completely the usual assumptions of elasticity, isotropy and homogeneity but the elastic theory is widely used because nothing better has been available for a long time and comparisons with test results have indicated that computed moments and stresses are almost without exception on the safe side [1], provided that nominal working stresses are used in design.

In this section two methods of design will be presented. The first one -Westergaard's formulas- proposes a reduction in the moment coefficients obtained from elastic analysis since test results show a stress redistribution as the loads are increased. The second- Marcus' approximate solution- is a mathematical approximation method to obtain the maximum moments in a slab.

These two methods result from the elastic analysis of plates.

1. C.P.Siess and N.M.Newmark "Moments in two-way slabs" Univ.of Illinois Bull. Vol.47 N°43 February 1950, pp.53

2.1- WESTERGAARD'S COEFFICIENTS

Although quite old, the study of plates by H.M.Westergaard will be presented here because it constitutes the basis of the moment coefficients given in the "ACI Reinforced Concrete Building Code" as method 2, and on the other hand the values given therein are still serving authors as references in this subject.

The study by H.M.Westergaard is based on information of the following four kinds [1]:

1-Results obtained by the theory of elasticity,

2-Results of tests,

3-General information concerning the phenomenon of redistribution of stress resulting from redistribution of relative stiffness as the stresses increase,

4-Knowledge of the behaviour of flat slabs.

Six types of panels, represented in the following figure, are considered: (1)The single panel, (2)End panels of a row of panels, (3)Intermediate panels in a row, (4)Corner panels, (6)Interior panels.

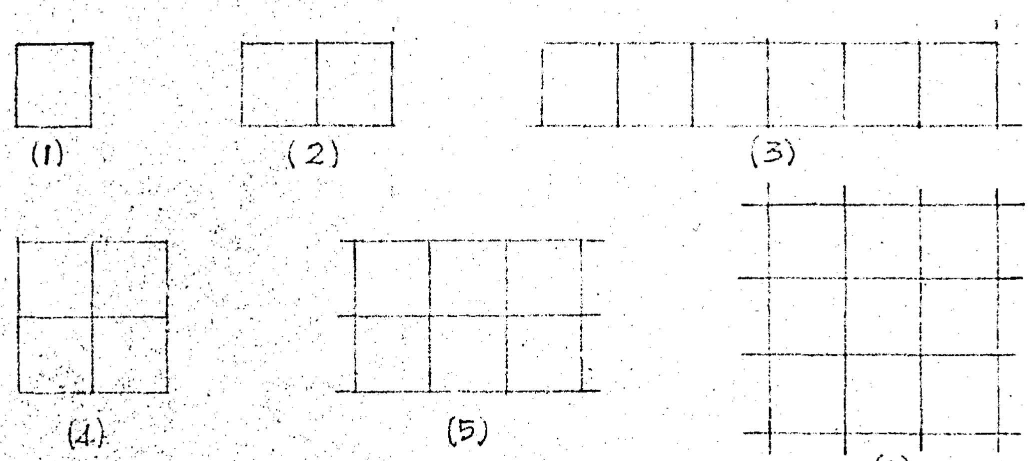


Fig-7 Types of panels.

1 . H.M.Westergaard "Formulas for the design of rectangular floor slabs and the supporting girders" Proc.ACI Vol.22(1926) pp.26

The tests conducted by H.M.Westergaard and W.A.Slater [1] have indicated the possibility of using smaller coefficients than those obtained for homogeneous elastic material by use of the theory of elasticity.

Prof.Westergaard summarizes the results of his tests in the following sentences:[2]

"The tests demonstrate quite clearly the phenomenon of redistribution of stress.This phenomenon may be explained in terms of what happens in the case of a rectangular interior panel.The load is assumed to be distributed uniformly over the whole floor and to be increased gradually from zero.With the smallest loads the panel will act most nearly like a homogeneous elastic slab. The greatest stresses will occur at the centers of the longer edges.The diagram of coefficients of bending,moments across the edge shows small values near the corners and large values at the middle.When the load is increased,the increase of bending moment corresponding to a given increase of deformation will become smaller at the middle of the edge where the stresses are large than near the corners where the stresses are small.That is,when the load increases,the stiffness of the material becomes relatively smaller at the middle of the edge than near the corners.The result is that the parts near the corners become relatively more active. One may say that stresses are thrown from the middle of the edge toward the corners,and the middle borrows strength from the sides.

1 . H.M.Westergaard and W.A.Slater "Moments and stresses in slabs" Proc.ACI Vol.17(1921) pp.415-538
2 . H.M.Westergaard Op.Cit. pp. 31-32

Accordingly, the diagram of coefficients of bending moments is flattened out, the ordinates becoming smaller at the middle and larger at the ends. At the same time, the fact that the deformations are smaller at the middle of the panel than at the longer edges, causes a second redistribution of the coefficients of bending moments, or borrowing of strength, with decreases of the negative moments at the edges and increases of the positive moments at the middle of the panel. Whenthe stresses near the middle increase, a third redistribution occurs, the diagram of positive moments across the longer line of symmetry is flattened out. Finally, there is a redistribution of action from the shorter span to the longer span. These redistributions result, generally speaking, in a lowering of the critical coefficients by which the slab would be designed."

The amount by which the moment coefficients will be lowered is determined by assuming that the flexure of a two-way slab supported on girders is no different from the flexure of a flat slab. Statics principles applied to a flat slab give 0.125 as the sum of the positive and negative moments. The corresponding coefficient is given to be 0.09 by the Joint Committee on Standard Specifications for Concrete and Reinforced concrete in the report of 1924 (Section 142). The difference of the two coefficients enable Westergaard to reduce the moment coefficients in the ratio of 0.09 to 0.125 , that is 28 percent, in the case of square panels supported on all four sides.

The following figure shows the set of coefficients proposed for the use in design of square panels supported on all four sides.

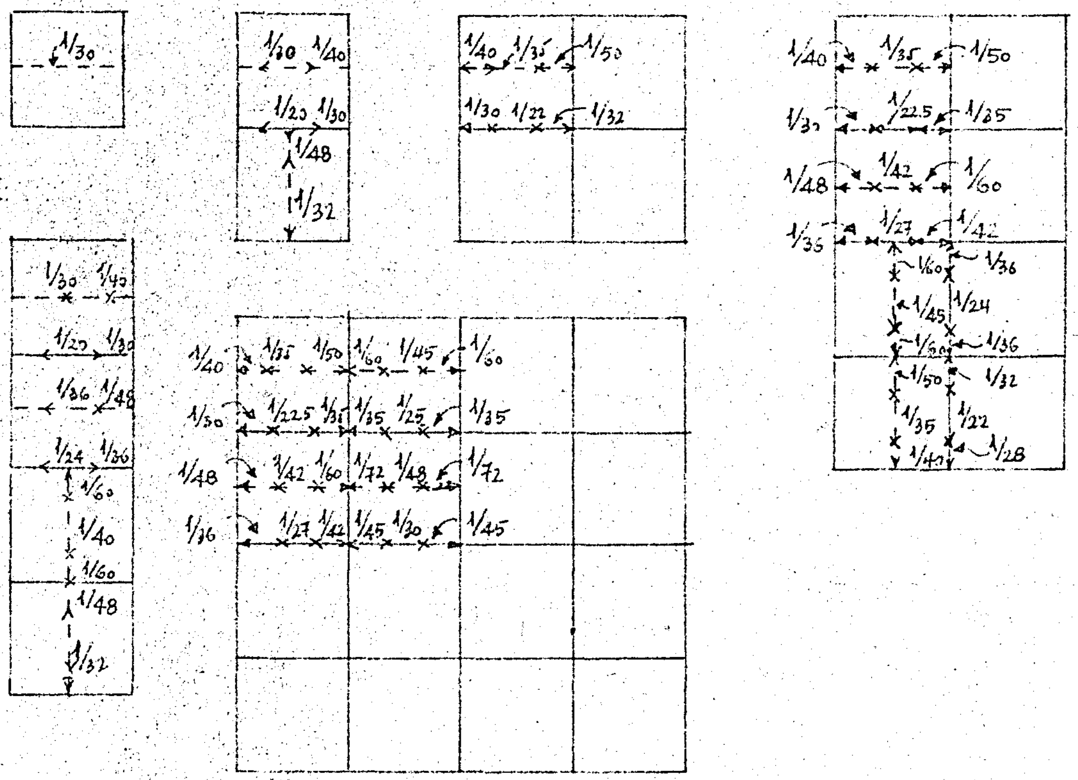


Fig. - 8 - Proposed coefficients of bending moments for the design of square panels of reinforced concrete.
$M=\alpha Qb$ = bending moment in the strip of width b; c = coefficient given by the numbers; Q = total uniform load on the panel.

These coefficients are obtained by using the coefficients of Fig.1 ,by taking into account the redistribution of stress, by permitting a reduction of coefficients to the extent of 28% and by assuming that the deflections of the girders are small compared to the deflections of the central portions of the loaded panel.

Fig. 9 shows sets of similar coefficients proposed for rectangular panels. The bending moments in the shorter span are expressed in terms of the ratio ε of the shorter span to the longer span by formulas of the type

$$M = \pm \frac{c_1}{1 + c_2 \varepsilon^3} q l^2 b \qquad (6)$$

where c_1 and c_2 are numbers. With $\varepsilon=1$ corresponding to square

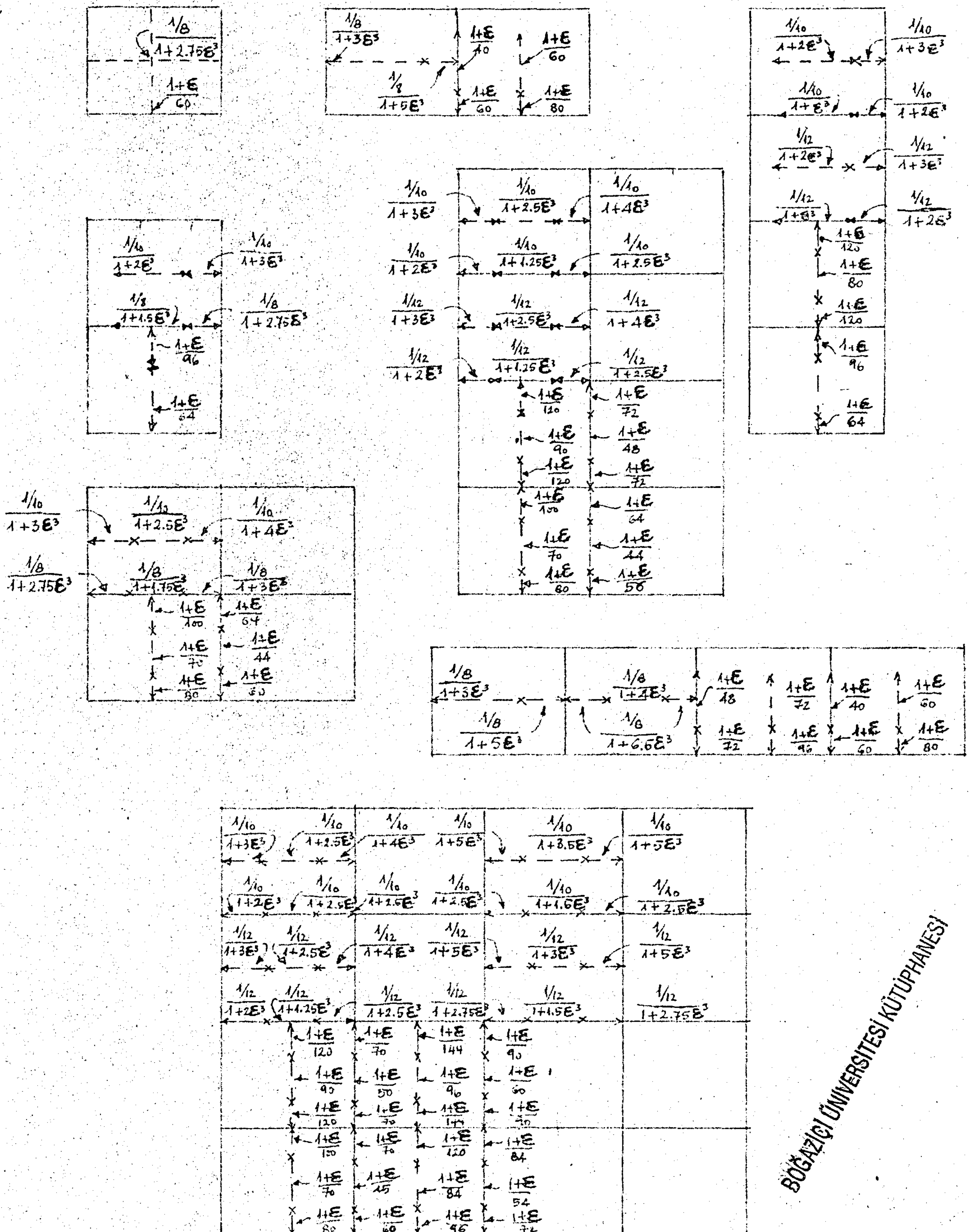


Fig.-9- Coefficients of bending moments as in Fig.3

slabs, the coefficients assume the values given in Fig.3 .One can find the moment coefficients for one-way slabs by taking m=0. The form of Eq.(2) agrees satisfactorily with the results found for rectangular panels by the theory of elasticity [1].

The moment coefficients given in the method 2 of the ACI Building Code are very nearly the ones proposed by Westergaard.

2.2 - MARCUS PROPOSITION

For the design of two-way reinforced concrete slabs Marcus proposes the use of the following equations:

$$\max M_x = M_x \left[1 - \frac{5}{6} \left(\frac{l_x}{l_y} \right)^2 \frac{M_x}{\mathfrak{M}_x} \right] \qquad (7)$$

$$\max M_y = M_y \left[1 - \frac{5}{6} \left(\frac{l_y}{l_x} \right)^2 \frac{M_y}{\mathfrak{M}_y} \right]$$

These equations are based on pure theoretical considerations. $\mathfrak{M}_x$ and $\mathfrak{M}_y$ are the moments in a simply supported one-way slab i.e. $\mathfrak{M}_x = q \frac{l_x^2}{8}$, $\mathfrak{M}_y = q \frac{l_y^2}{8}$. M_x and M_y are the moments in a two-way slab l_x wide and l_y long, computed by the elastic analysis of the slab.

In the maximum moment equations proposed by Marcus the reducing effect of torsional moment is accounted for.

Tables giving $\max M_x$ and $\max M_y$ for different values of l_x/l_y have been prepared. The Turkish practice is to use the tables given by PUCHER, LÖSER and BETON KALENDER.

The French Bulding Code also allows the use of the tables given By PUCHER and PİGEAUD.

1 . H.M.Westergaard and W.A.Slater Op.Cit. Fig.3-10 pp.431-438

SECTION 3

3.1 - In this section two types of slabs with different support conditions and side ratios will be designed. As "types" the four-edges-fixed and simply supported slabs are chosen, and 0.6, 0.8 and 1.0 are the side ratios considered.

The procedure outlined below is followed in this section

A- Slab designs

a-The moment coefficients corresponding to the following moments are taken from Pucher, ACI code and French code.

M_x = Moment at midspan in the short direction
M_y = Moment at midspan in the long direction
X_x = Moment at support in the short direction
X_y = Moment at support in the long direction

b-The slab is designed by the "Yield-line" theory, by taking $M_y/M_x = \mu$, $i = X_x/M_x$ and $i' = X_y/M_y$.

c-The slab is designed with the working stresses given by the Turkish, American and French codes.

d-The ultimate moment carrying capacities of the obtained sections are computed.

The load factors applied by different codes are calculated as explained in part 3.2.2. d .

B - Capacity Comparaisons

The capacity of a given slab with a given load q is computed by the yield-line theory, the Turkish, American and French Codes.

3.2. PROBLEM Design a slab with $l_x = 2.40$ m., $l_y = 4.00$ m ($l_x/l_y = 0.6$) fixed on all four edges and carrying a distributed load q of 500 kg/M².

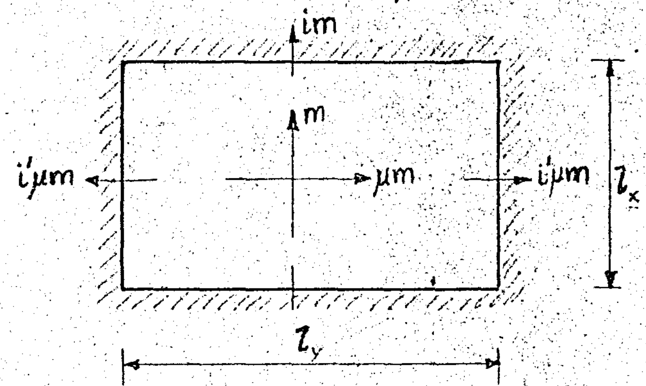
Available B-160 concrete $\overline{\sigma}_b = 160$ kg/cm²

ST-I steel $\sigma_y = 2400$ kg/cm²

In the following designs q is assumed to include the dead load of the slab.

3.2.1- Design by the yield-line method

Values for μ, i, and i' will be obtained from the coefficients given by Pucher, the ACI code and the French code.



a-Pucher coefficients used to compute μ, i and i'.

For a slab of the dimensions shown above Pucher gives:

$M_x = M = 0.03362\, ql_x^2$ $\quad M_y = \mu M = 0.00436\, ql_y^2$

$X_x = iM = -\frac{1}{12} 0.8853\, ql_x^2$ $\quad X_y = i'\mu M = -\frac{1}{12} 0.1147\, ql_y^2$

$$\mu = \frac{0.00436}{0.03362}\left(\frac{l_y}{l_x}\right)^2 = 0.1295\,(2.78) = 0.36$$

$$\sqrt{\mu} = 0.6$$

Dimensions for isotropic analysis:

$L_y = 4.00$ m $\quad l_x = 2.40/0.6 = 4.00$ m

$$i = \frac{0.8853}{12 \times 0.03362} = 2.02 \qquad i' = \frac{0.1147}{12 \times 0.00436} = 2.19$$

Now Eq.(3) will be used to compute the moment given by the yield line theory.

$$(l'_x)_r = \frac{2l'_x}{2\sqrt{1+2.02}} = 0.576\, l'_x \qquad (l_y)_r = \frac{2l_y}{2\sqrt{1+2.19}} = 0.560\, l_y$$

$$m = \frac{q(0.576)^2 l'^2_x}{24}\left[\sqrt{3+\left(\frac{0.576}{0.560}\right)^2\left(\frac{l'_x}{l_y}\right)^2} - \left(\frac{0.576}{0.560}\right)\left(\frac{l'_x}{l_y}\right)\right]^2 = 0.01375\, q\, l'^2_x$$

since $l'^2_x = \frac{1}{\mu} l^2_x$ $\qquad m = 0.0382\, q\, l^2_x = 0.110$ ton-m/m

The reinforcement will be designed by using Eq.(2)

$$m = h^2 p \sigma_y (1 - 0.5n)$$

We have $n = \frac{\sigma_y}{\overline{\sigma_b}} p = \frac{2400}{160} p = 15p$

Solving Eq.(2) for p we get:

$$p = \frac{1 \pm \sqrt{1 - 30m/h^2\sigma_y}}{15} \qquad (8)$$

To get a yield failure p should be below $p_b = 0.43\, \overline{\sigma_b}/\sigma_y = 0.0287$

In the present case m=110 kg m/m. Then $(h = 3.5\ cm.)$

$$30m/h^2\sigma_y = \frac{30 \times 110}{(3.5)^2 \times 2400} = 0.112$$

$$p = \frac{1 \pm \sqrt{0.888}}{15} = \begin{cases} 0.1295 > p_b & \text{(Gives compression failure)} \\ 0.0040 < p_b \end{cases}$$

$$F_e = pbh = 0.004 \times 100 \times 3.5 = 1.4\ cm^2/m$$

<u>b- ACI Code (Method 2) coefficients used to compute μ, i and i'</u>

For $l_x/l_y = 0.6$ the ACI code Method 2 gives

$M = 0.047\, ql^2_x$ $\qquad \mu M = 0.025\, ql^2_x$

$iM = 0.063\, ql^2_x$ $\qquad i'\mu M = 0.033\, ql^2_x$

$\mu = \frac{0.025}{0.047} = 0.532$ $\quad i = \frac{0.063}{0.047} = 1.34$ $\quad i' = \frac{0.033}{0.025} = 1.32$

Dimensions for isotropically reinforced slab:

$l_y = 4.00$ m $\quad l_x' = 2.40 / \sqrt{0.532} = 3.29$ m

$l_x' / l_y = 0.824$

$$(l_x')_r = \frac{2 l_x'}{2\sqrt{1+1.34}} = 0.654\, l_x' \qquad (l_y)_r = \frac{2 l_y}{2\sqrt{1+1.32}} = 0.656\, l_y$$

$$m = \frac{q (0.654)^2 l_x'^2}{24} \left[\sqrt{3 + \left(\frac{0.654}{0.656}\right)^2 (0.824)^2} - \left(\frac{0.654}{0.656}\right)(0.824) \right]^2$$

$$m = 0.0214\, q l_x'^2 = 0.0402\, q l_x^2 = 0.1158 \ \text{tm/m}$$

let $h = 3.5$ cm. then

$$\frac{30\, m}{h^2 \sigma_y} = 0.117 \qquad p = \frac{1 \pm \sqrt{0.883}}{16} = \begin{cases} 0.1295 > p_b \\ 0.0040 < p_b \end{cases}$$

$$F_e = 1.40 \ \text{cm}^2/\text{m}$$

c-French Code coefficients used to compute μ, i and i'

$M = 0.0632\, q l_x^2$ $\qquad \mu M = 0.326\, M$

$iM = 0.50\, M$ $\qquad i' \mu M = 0.50\, \mu M$

$\mu = 0.326 \qquad i = i' = 0.50$

Dimensions for isotropically reinforced slab:

$l_y = 4.00$ m $\qquad l_x = \frac{2.40}{\sqrt{0.326}} = 4.14$ m

$l_x'/l_y = 1.035 \qquad (l_x')_r = \frac{2 l_x'}{2\sqrt{1+0.50}} = 0.816\, l_x' \qquad (l_y)_r = 0.816\, l_y$

$$m = \frac{q (0.816)^2 l_x'^2}{24} \left[\sqrt{3 + (1)(1.035)^2} - (1)(1.035) \right]^2 = 0.0269\, q l_x'^2$$

$$m = 0.0825\, q l_x^2 = 0.2380 \ \text{tm/m}$$

$m = 238.0$ kgm/m $\qquad$ let $h = 3.5$ cm. Then

$$\frac{30\, m}{h^2 \sigma_y} = 0.242$$

$$p = \frac{1 \pm \sqrt{0.758}}{15} = \begin{matrix} \nearrow 0.125 > p_b \\ \searrow 0.00865 < p_b \end{matrix}$$

$$F_e = 0.0865 \times 3.5 \times 100 = 3.03 \ cm^2/m.$$

3.2.2- Design by Pucher coefficients and "Türkiye Köprü ve İnşaat cemiyeti" Betonarme Şartnamesi

a-The maximum positive moment will occur at the center of the slab in the short span direction. This moment will be [1]:

$$M_x = 0.03362\ ql_x^2 = 0.0968 \ \text{tm/m}$$

b-Design of reinforcement Allowable stresses are [2]:

$\sigma_b = 60 \ kg/cm^2$, $\sigma_e = 1400 \ kg/cm^2$

In order to have failure by yielding of the reinforcement the required useful depth is [3]:

$$h_r = 9.897\sqrt{\frac{M}{b_o}} = 9.897\sqrt{\frac{0.0968}{1.00}} \approx 3.1 \ cm$$

Assuming a slab thickness of 5cm. (Dead load $=0.05 \times 2400 = 120 kg/m^2$) we provide $h=5-1.5 = 3.5$ cm

The required steel area in the maximum moment direction is:

$$F_e = \frac{0.821 \times 0.0968}{0.035} = 2.28 \ cm^2/m$$

c-Calculation of the ultimate strength of the obtained secti

Eq.(2) will be used to compute the ultimate strength of the cross section with $F_e = 2.28 \ cm^2$, h=3.5 cm and width = 100 cm.

$$p = \frac{2.28}{3.5 \times 100} = 0.00652$$

p should be below $p_b \leq 0.43\ \sigma_b/\sigma_y$ in order to have yield failur

$$p_b = 0.43 \cdot \frac{160}{2400} = 0.0287 > 0.00652$$

1. A.Pucher "Lehrbuch des Stahlbetonbaues" Wien 1949 pp.159-162
2. From Türkiye Köprü ve İnşaat Cemiyeti Betonarme Şartnamesi
3. Prof.Benno Löser "Betonarme Hesap Metodları" Translated by Doç.Yük.Müh.Yusuf Berdan İstanbul 1955 pp.134-135

$$n = p\frac{\sigma_y}{\sigma_b} = 0.00652 \times \frac{2400}{160} = 15 \times 0.00652$$

$$m = (3.5)^2 (0.00652)(2400)\left(1 - \frac{15 \times 0.00652}{2}\right)$$

$$m = 183.2\ {}^{kg.m}/_{m} = 0.1832\ {}^{tm}/_{m} = 0.0637\, q l_x^2$$

d- We can now calculate the "load factor" resulting from the use of Pucher coefficients and Türkiye Köprü ve İnşaat Cemiyeti Betonarme Şartnamesi (TKİCBA Şart.)

The ultimate capacity computed in the previous part is equal to the yield-line moment with $q = q_{ult.}$

But in part 3.2.1.a the yield-line theory gave:

$$M = 0.0382\ ql_x^2$$

Equating the two moments:

$$0.0637\ ql_x^2 = 0.0382\ q_u l_x^2 \quad \text{where } q_u = \text{ultimate load}$$

or $$\text{Load Factor} = q_u/q = \frac{0.0637}{0.0382} = 1.67$$

3.2.3- Design by ACI coefficients and ACI working stresses

a- The maximum positive moment according to method 2 of the ACI Code will be:

$$M_x = 0.047\ ql_x^2 = 0.047 \times 0.500 \times (2.40)^2 = 0.135\ tm/m$$

b-Design of reinforcement Allowable stresses are:

$\sigma_b = 70\ kg/cm^2$ $\quad \sigma_e = 1400\ kg/cm^2$

For computation of the required depth we make use of B.Löser 's "Betonarme Hesap Metodları" Table 46.

$$h_r = 8.819\sqrt{\frac{0.135}{1.00}} \approx 2.8\ cm.$$

We again provide h=3.5 cm by taking the thickness of the slab 5cm, Then

$$f_e = \frac{0.833 \times 0.135}{0.035} = 3.22\ cm^2/m.$$

c-Calculation of the ultimate strength of the obtained section

$$p = \frac{3.22}{100 \times 3.5} = 0.00919 < p_b = 0.0287$$

$$n = 0.00919 \times 15$$

$$m = (3.5)^2 (0.00919)(2400)\left(1 - \frac{15 \times 0.00919}{2}\right) = 252.5 \ kgm/m$$

$$m = 0.253 \ tm/m = 0.0880 \ q l_x^2$$

d-

$$L.F. = \frac{0.0880}{0.0402} = 2.19$$

3.2.4 - Design by the coefficients and working stresses of the French Reinforced Concrete Code [1]

a- Maximum positive moment

$$M_x = 0.0632 \ q l_x^2 = 0.183 \ tm/m$$

b- Design of reinforcement: Allowable stresses are [2]:

$$\sigma_b = 96 \ kg/cm^2 \qquad \sigma_e = 1600 \ kg/cm^2$$

The moment that concrete should take is:

$$M = \frac{1}{2} \sigma_b \, b \, (0.403 \times 0.866) h_r^2 = 0.183 \ tm$$

wich gives $h_r = 3.3$ cm The slab thickness is again taken to be 5cm

$$F_e = \frac{M}{\sigma_e h} = \frac{0.183}{1.6 \times 0.035} = 3.26 \ cm^2$$

c- Calculation of the ultimate strength of the section

$$p = \frac{3.26}{3.5 \times 100} = 0.0093 < p_b \qquad n = 0.0093 \times 15$$

$$m = (3.5)^2 (0.0093)(2400)\left(1 - \frac{15 \times 0.0093}{2}\right) = 256 \ kgm/m$$

$$m = 0.256 \ tm/m = 0.0889 \ q l_x^2$$

d-

$$L.F. = \frac{0.0889}{0.0825} = 1.08$$

1. "Règles pour le calcul et l'exécution des constructions en béton armé" Mars 1961
2. Computed from Ibid. pp.55 and 67

3.3 - PROBLEM Design a slab $l_x = 4.00$ m by $l_y = 5.00$ m ($l_x / l_y = 0.8$) fixed on all the four edges.The loads and the available are the same as in problem 3 .2..q is again assumed to include the dead load of the slab.

3.3.1 -Design by the yield-line method

a-Pucher coefficents used to compute μ i and i'

For $l_x / l_y = 0.8$ Pucer gives

$M = 0.02583\, q l_x^2$ $\qquad \mu M = 0.01058\, q l_y^2$

$iM = -\frac{1}{12} 0.7094\, q l_x^2$ $\qquad i'\mu M = -\frac{1}{12} 0.2906\, q l_y^2$

$$\mu = \frac{0.01058}{0.02583} \cdot \left(\frac{l_y}{l_x}\right)^2 = (0.408)(1.56) = 0.638$$

$$\sqrt{\mu} = 0.798$$

$$i = \frac{0.7094}{12 \times 0.02583} = 2.28 \qquad i' = \frac{0.2906}{12 \times 0.01058} = 2.29$$

$$l_y = 5.00 \text{ m.} \qquad l'_x = \frac{4.00}{0.798} = 5.00 \text{ m.}$$

$$(l'_x)_r = \frac{2 l'_x}{2\sqrt{1 + 2.28}} = 0.553\, l'_x \qquad (l_y)_r = 0.553\, l_y$$

$$m = \frac{q (0.553)^2 l'^2_x}{24} \left[\sqrt{3 + (1)(1)^2} - 1\right]^2 = 0.0127\, q l'^2_x$$

$$m = 0.0199\, q l_x^2 = 0.159 \text{ tm/m}$$

let $h = 6.5$ cm. Then

$$\frac{30 m}{h^2 \sigma_y} = 0.0472 \qquad p = \frac{1 \pm \sqrt{0.9528}}{15} = \begin{cases} 0.00147 \\ 0.132 > p_b \end{cases}$$

$$F_e = 0.00147 \times 100 \times 6.5 = 0.955 \text{ cm}^2/\text{m}$$

b- ACI Code coefficients used to compute μ i and i'

$M = 0.036\ ql_x^2 \qquad \mu M = 0.025\ ql_x^2$

$iM = 0.048\ ql_x^2 \qquad i'\mu M = 0.033\ ql_x^2$

$\mu = \frac{0.025}{0.036} = 0.694 \qquad \sqrt{\mu} = 0.834 \qquad i = \frac{0.048}{0.036} = 1.33 \qquad i' = \frac{0.033}{0.025} = 1.32$

Dimensions for isotropic analysis

$l_y = 5.00\ m \qquad l'_x = \frac{4.00}{0.834} = 4.80\ m. \qquad l'_x/l_y = 0.96$

$(l'_x)_r = \frac{2l'_x}{2\sqrt{1+1.33}} = 0.655\ l'_x \quad ; \quad (l_y)_r = 0.655\ l_y$

$$m = \frac{q(0.655)^2 l'^2_x}{24}\left[\sqrt{3+(0.96)^2} - 0.96\right]^2 = 0.0186\ ql'^2_x$$

$$m = 0.0268\ ql_x^2 = 0.212\ tm/m$$

Let $h = 6.5$ cm. then $\frac{30m}{h^2\sigma_y} = 0.0630 \qquad p = \frac{1 \pm \sqrt{0.937}}{15} =$ ↗ 0.00227 ; ↘ $0.131 > p_b$

$$F_e = 0.00227 \times 100 \times 6.5 = 1.48\ cm^2/m$$

c -French Code coefficients used to compute μ i and i'

$M = 0.0462\ ql_x^2 \qquad \mu M = 0.512\ M \qquad iM = 0.50\ M \qquad i'\mu M = 0.50\ \mu M$

$\mu = 0.512 \qquad \sqrt{\mu} = 0.716 \qquad i = i' = 0.50$

$l_y = 5.00\ m. \qquad l'_x = \frac{4.00}{0.716} = 5.58 \qquad l'_x/l_y = 1.11$

$(l'_x)_r = 0.816\ l'_x \qquad (l_y)_r = 0.816\ l_y$

$$m = \frac{q(0.816)^2 l'^2_x}{24}\left[\sqrt{3+(1.11)^2} - (1.11)\right]^2 = 0.025\ ql'^2_x = 0.0488\ ql_x^2 = 0.390\ tm/m$$

Let $h = 6.5$ cm, then $\frac{30m}{h^2\sigma_y} = 0.116 \qquad p = \frac{1 \pm \sqrt{0.884}}{15} =$ ↗ 0.0040 ; ↘ $0.129 > p_b$

$$F_e = 0.0040 \times 6.5 \times 100 = 2.6\ cm^2/m$$

3.3.2 -Design by Pucher coefficients and TKİCBA Şart.

a - For the maximum positive moment with $l_x/l_y=0.8$ Pucher gives

$$M_x = 0.02583\ ql_x^2 = 0.206\ tm/m$$

b- Design of reinforcement $\sigma_b / \sigma_e = 60/1400$

$$h_r = 9.897\sqrt{\frac{0.206}{1.00}} = 4.5\ cm.$$

Assuming a slab thickness of 8 cm (Dead L.192 $kg/m^2 < 500$) we provide $h = 8.0 - 1.5 = 6.5$ cm.

The required steel area to take the maximum moment is:

$$F_e = \frac{0.821 \times 0.206}{0.065} = 2.60\ cm^2/m$$

c- Ultimate strength of the obtained section

$$p = \frac{2.60}{6.5 \times 100} = 0.0040 < p_b = 0.0287 \qquad n = 0.0040 \times 15$$

$$m = (6.5)^2 (0.0040)(2400)\left(1 - \frac{15 \times 0.004}{2}\right) = 394\ kgm/m = 0.394\ tm/m$$

$$m = 0.0493\ ql_x^2$$

d - Load factor $L.F = \frac{0.0493}{0.0199} = 2.46$

3.3.3 - Design by the ACI coefficients and working stresses

a- Maximum positive moment

$$M_x = 0.036\ ql_x^2 = 0.288\ tm/m$$

B- Design of reinforcement $\sigma_b / \sigma_e = 70/1400$

$$h_r = 8.819\sqrt{\frac{0.288}{1.00}} = 4.7\ cm.$$

The slab thickness is again taken to be 8 cm.

$$F_e = \frac{0.833 \times 0.288}{0.065} = 3.69\ cm^2/m$$

c- Ultimate strength of the obtained section

$$p = \frac{3.69}{6.5 \times 100} = 0.00567 < p_b$$

$$m = (6.5)^2 (0.00567)(2400)\left(1 - \frac{15 \times 0.00567}{2}\right) = 552 \ kgm/m = 0.552 \ tm/m$$

$$m = 0.0679 \ q l_x^2$$

d-Load factor $L.F. = \frac{0.0679}{0.0268} = 2.53$

3.3.4 Design by the French code coefficients and working stresses

a- Maximum positive moment

$M_x = 0.0462 \ q l_x^2 = 0.370$ tm/m

b- Design of reinforcement $\sigma_b / \sigma_e = 96/1600$

The moment that concrete should take is:

$$M = \frac{1}{2} \sigma_b \, b \, (0.403 \times 0.866) h_r^2 = 0.370$$

which gives h_r=4.7 cm. h=6.5 cm is provided again

$$F_e = \frac{0.370}{1.6 \times 0.065} = 3.55 \ cm^2/m$$

c- Ultimate strength of the obtained section

$$p = \frac{3.55}{6.5 \times 100} = 0.00546 < p_b$$

$$m = (6.5)^2 (0.00546)(2400)\left(1 - \frac{0.0546 \times 15}{2}\right) = 531 \ kgm/m = 0.531 \ tm/m$$

$$m = 0.0663 \ q l_x^2$$

d- Load factor

$$L.F. = \frac{0.0663}{0.0488} = 1.36$$

3.4 - PROBLEM Design a square slab 4.00 m by 4.00 m, fixed on all four edges. The loads and the available materials are the same as in the previous problems. q includes the dead load.

3.4.1 - Design by the yield-line method

a- Pucher coefficients used to compute μ i and i'

For a square slab μ = 1 and i = i'

$M = 0.0179\ ql_x^2 \qquad iM = -\frac{1}{12}\,0.5000 ql_x^2 \qquad i = 2.33$

$$(l_x)_r = \frac{2l_x}{2\sqrt{1+2.33}} = 0.548\, l_x = (l_y)_r$$

$$m = \frac{q(0.548)^2 l_x^2}{24}\left[\sqrt{3+1}-1\right]^2 = 0.0125\, q l_x^2 = 0.100 \text{ tm/m}$$

Let h = 6.5 cm. Then $\frac{30m}{h^2\sigma_y} = 0.0298 \qquad p = \frac{1 \pm \sqrt{0.9702}}{15} = \begin{cases} 0.00093 \\ 0.133 > p_b \end{cases}$

$$F_e = 0.00093 \times 100 \times 6.5 = 0.605 \text{ cm}^2/\text{m}.$$

b- ACI Code coefficients used to compute i

$M = 0.025\ ql_x^2 \qquad iM = -0.033\ ql_x^2 \qquad i = 1.32$

$$(l_x)_r = \frac{2l_x}{2\sqrt{1+1.32}} = 0.655\, l_x = (l_y)_r$$

$$m = \frac{q(0.655)^2 l_x^2}{24} = 0.0178\, q l_x^2 = 0.142 \text{ tm/m}$$

Let h = 6.5 cm. Then $\frac{30m}{h^2\sigma_y} = 0.0423$

$$p = \frac{1 \pm \sqrt{0.9577}}{15} = \begin{cases} 0.00147 \\ 0.132 > p_b \end{cases}$$

$$F_e = 0.00147 \times 100 \times 6.5 = 0.96 \text{ cm}^2/\text{m}.$$

c - French code coefficients used to compute i

$M = 0.0317\ ql_x^2 \qquad iM = 0.50\ M \qquad i = 0.50$

$$(l_x)_r = 0.816\ l_x = (l_y)_r$$

$$m = \frac{q(0.816)^2 l_x^2}{24} = 0.0277\ q l_x^2 = 0.220\ tm/m$$

let $h = 6.5$ cm. Then $\frac{30m}{h^2 \sigma_y} = 0.0654 \qquad p = \frac{1 \pm \sqrt{0.9346}}{15} = \nearrow 0.00226,\ \searrow 0.131 > p_b$

$$F_e = 0.00226 \times 6.5 \times 100 = 1.47\ cm^2/m.$$

3.4.2 - Design by Pucher coefficients and TKİCBA Şart.

a- Maximum positive moment:

$$M = 0.0179\ ql_x^2 = 0.143\ tm/m$$

b- Design of reinforcement $\qquad \sigma_b / \sigma_e = 60/1400$

$$h_r = 9.897 \sqrt{\frac{0.143}{1.00}} = 3.8\ cm.$$

6.5 cm is provided for h by taking the slab thickness 8 cm

$$F_e = \frac{0.821 \times 0.143}{0.065} = 1.81\ cm^2/m$$

c-Ultimate capacity of the obtained section

$$p = \frac{1.81}{6.5 \times 100} = 0.00278 < p_b$$

$$m = (6.5)^2 (0.00278)(2400)\left(1 - \frac{0.00278 \times 15}{2}\right) = 300\ kgm/m = 0.300\ tm/m$$

$$m = 0.0375\ ql_x^2$$

d-Load factor

$$L.F. = \frac{0.0375}{0.0125} = 3.00$$

3.4.3- Design by the ACI coefficients and working stresses

a- Maximum positive moment

$$M = 0.025\, q l_x^2 = 0.200 \quad tm/m$$

b- Design of reinforcement $\quad \sigma_b / \sigma_e = 70/1400$

$$h_r = 8.819 \sqrt{\frac{0.200}{1.00}} \simeq 4.0 \text{ cm.}$$

6.5 cm is provided for h by taking the slab thickness 8 cm

$$F_e = \frac{0.833 \times 0.200}{0.065} = 2.56 \text{ cm}^2/m$$

c-Ultimate capacity of the obtained section

$$p = \frac{2.56}{100 \times 6.5} = 0.00394 < p_b$$

$$m = (6.5)^2 (0.00394)(2400)\left(1 - \frac{0.00394 \times 15}{2}\right) = 396 \text{ kgm}/m = 0.396 \text{ tm}/m = 0.0495\, q l_x^2$$

d-Load factor $\quad L.F. = \frac{0.0495}{0.0178} = 2.78$

3.4.4 - Design by the French Code coeff. and working stresses

a- $M = 0.0317\, q l_x^2 = 0.254$ tm/m

b- Design of reinforcement $\quad \sigma_b / \sigma_e = 96/1600$

$$0.254 = \frac{1}{2} \sigma_b b (0.403 \times 0.866) h_r^2$$

which gives h_r=4.0 cm ,6.5 cm will be provided again

$$F_e = \frac{0.254}{1.6 \times 0.065} = 2.44 \text{ cm}^2/m$$

c- Ultimate strength of the obtained section

$$p = \frac{2.44}{6.5 \times 100} = 0.00375 < p_b$$

$$m = (6.5)^2 (0.00375)(2400)\left(1 - \frac{0.00375 \times 15}{2}\right) = 378 \text{ kgm}/m = 0.378 \text{ tm}/m = 0.0472\, q l_x^2$$

d-Load factor

$$L.F = \frac{0.0472}{0.0277} = 1.71.$$

3.5 - PROBLEM Design a slab with l_x=2.40 m, l_y=4.00 m (l_x/l_y=0.6), simply supported on all four edges and carrying a distributed load q of 500 kg/m²; Same materials as in the previous problems are available and q is assumed to include the dead load.

3.5.1 - Design by the yield-line method

a-PUCHER coefficients used to compute μ $(i=i'=0)$

$$M = 0.08127\ ql_x^2 \qquad \mu M = 0.01053\ ql_y^2$$

$$\mu = \frac{0.01053}{0.08127}\left(\frac{l_y}{l_x}\right)^2 = \frac{0.01053}{0.08127}(1.66)^2 = 0.357 \qquad \sqrt{\mu} \simeq 0.6$$

Dimensions of isotropically reinforced slab

$$l_y = 4.00\ m. \qquad l'_x = \frac{2.40}{0.6} = 4.00\ m.$$

Eq.(3) will be used again to compute the necessary moment capacity of a slab reinforced as above.

$$(l'_x)_r = l'_x \text{ since } i=0\ ; \quad (l_y)_r = l_y \text{ since } i'=0$$

Then

$$m = \frac{ql'^2_x}{24}\left[\sqrt{3+(1)^2} - 1\right]^2 = 0.0417\ ql'^2_x$$

$$m = 0.116\ ql_x^2 = 0.334\ tm/m$$

let $h = 6.5$ cm. Then $\frac{30m}{h^2\sigma_y} = 0.0994$

$$p = \frac{1 \pm \sqrt{0.900}}{15} = \begin{cases} 0.00333 \\ 0.130 > p_b \end{cases}$$

$$F_e = 0.00333 \times 100 \times 6.5 = 2.17\ cm^2.$$

b-ACI Code coefficients used to compute μ, i and i'

Although the slab is simply supported on all four sides the ACI Code requires that it be reinforced for negative moments near the supports, i.e. i and i' are not zero.

$M = 0.083\ ql_x^2 \qquad \mu M = 0.050\ ql_x^2$

$iM = 0.053\ ql_x^2 \qquad i'\mu M = 0.033\ ql_x^2$

$\mu = \frac{0.050}{0.083} = 0.602 \qquad \sqrt{\mu} = 0.777 \qquad i = i' = 0.66$

Dimensions of isotropically reinforced slab

$l_y = 4.00\ m. \qquad l'_x = \frac{2.40}{0.777} = 3.09\ m.$

$(l'_x)_r = \frac{2l'_x}{2\sqrt{1+0.66}} = 0.775\ l'_x \qquad (l_y)_r = 0.775\ l_y$

$$m = \frac{q(0.775)^2 l'^2_x}{24}\left[\sqrt{3+\left(\frac{0.775\ l'_x}{0.775\ l_y}\right)^2} - \left(\frac{0.775\ l'_x}{0.775\ l_y}\right)\right]^2 = 0.0251\ q l'^2_x$$

$$m = 0.0416\ q l_x^2 = 0.120\ tm/m$$

let $h = 6.5$ cm. Then $\frac{30m}{h^2\sigma_y} = 0.0356 \qquad p = \frac{1 \pm \sqrt{0.9644}}{15} = \begin{cases} 0.00120 \\ 0.1322 > p_L \end{cases}$

$$F_e = 0.00120 \times 100 \times 6.5 = 0.78\ cm^2/m.$$

c- French Code coefficients used to compute μ (i=i'=0)

$M = 0.0849\ ql_x^2 \qquad \mu = 0.435 \qquad \sqrt{\mu} = 0.66$

Dimensions of isotropically reinforced slab

$l_y = 4.00\ m. \qquad l'_x = \frac{2.40}{0.66} = 3.64\ m.$

$$m = \frac{q l'^2_x}{24}\left[\sqrt{3+\left(\frac{3.64}{4.00}\right)^2} - \left(\frac{3.64}{4.00}\right)\right]^2 = 0.0458\ q l'^2_x$$

$$m = 0.1054\ q l_x^2 = 0.304\ tm/m$$

let $h = 6.5$ cm. $\frac{30m}{h^2\sigma_y} = 0.0900$

$$p = \frac{1 \pm \sqrt{0.91}}{15} = \begin{cases} 0.0030 \\ 0.130 > p_L \end{cases}$$

$$F_e = 0.0030 \times 100 \times 6.5 = 1.95\ cm^2/m$$

3.5.2.-Design by Pucher coefficients and TKİCBA Şart.

a- Maximum positive moment

$$M = 0.08127\ ql_x^2 = 0.234\ \text{tm/m}$$

b- Design of reinforcement $\sigma_b / \sigma_e = 60/1400$

$$h_r = 9.897\sqrt{\frac{0.234}{1.00}} = 4.8\ \text{cm.}$$

Taking the thickness of the slab 8 cm, h=6.5 cm is provided.

$$F_e = \frac{0.821 \times 0.234}{0.065} = 2.96\ \text{cm}^2/\text{m}$$

c- Ultimate capacity of the obtainedsection

$$p = \frac{2.96}{100 \times 6.5} = 0.00455 < p_b$$

$$m = (6.5)^2(0.00455)(2400)\left(1 - \frac{0.00455 \times 15}{2}\right) = 0.445\ \text{tm/m}$$

$$m = 0.1545\ ql_x^2$$

d-Load factor $\quad L.F. = \frac{0.1545}{0.116} = 1.32.$

3.5.3. - Design by ACI Code

a- $M = 0.083\ ql_x^2 = 0.239$ tm/m

b- Design of reinforcement $\sigma_b / \sigma_e = 70/1400$

$h_r = 8.819\sqrt{\frac{0.239}{1.00}} = 4.3$ cm. h = 6.5 cm is provided again

$$F_e = \frac{0.833 \times 0.239}{0.065} = 3.06\ \text{cm}^2/\text{m}$$

c-Ultimate capacity of the obtained section

$$p = \frac{3.06}{6.5 \times 100} = 0.00471$$

$$m = (6.5)^2(0.00471)(2400)\left(1 - \frac{0.00471 \times 15}{2}\right) = 0.471\ \text{tm/m}$$

$$m = 0.1633\ ql_x^2$$

d-Load factor: $\quad L.F. = \frac{0.1633}{0.0416} = 3.93$

3.5.4 - Design by the French Code

a - $M = 0.0849\ ql_x^2 = 0.244$ tm/m

b - Design of reinforcement $\sigma_b / \sigma_e = 96/1600$

$$M = \frac{1}{2}\sigma_b\, b\,(0.403 \times 0.866)\, h_r^2 = 0.244 \text{ tm}$$

$h_r = 3.9$ cm $h = 6.5$ cm is provided

$$F_e = \frac{0.244}{1.6 \times 0.065} = 2.35 \text{ cm}^2/\text{m}$$

c - Ultimate capacity of the obtained section

$$p = \frac{2.35}{6.5 \times 100} = 0.00362$$

$$m = (6.5)^2(0.00362)(2400)\left(1 - \frac{0.00362 \times 15}{2}\right) = 0.356 \text{ tm/m}$$

$$m = 0.1235\ ql_x^2$$

d - Load factor $L.F. = \frac{0.1235}{0.1054} = 1.17.$

3.6 - PROBLEM Design a slab $l_x = 4.00$ m, $l_y = 5.00$ m, simply supported on all four sides and carrying a uniform load of 500kg/m². Same materials as in the previous problems are available, q is assumed to include the dead load.

3.6.1 - Design by the yield-line method

a- Pucher coefficients used to compute μ

$M = 0.05512\ ql_x^2$ $\mu M = 0.02258\ ql_y^2$

$$\mu = \frac{0.02258}{0.05512}\left(\frac{l_y}{l_x}\right)^2 = 0.408 \times (1.25)^2 = 0.64 \qquad \sqrt{\mu} = 0.8$$

Dimensions of isotropically reinforced slab

$$l_y = 5.00 \text{ m} \qquad l'_x = \frac{4.00}{0.8} = 5.00 \text{ m}.$$

$$(l'_x)_r = l'_x \qquad (l_y)_r = l_y$$

$$m = \frac{q l'^2_x}{24}\left[\sqrt{3 + (1)^2} - (1)\right]^2 = 0.0417\ q l'^2_x$$

$$m = 0.065\ ql_x^2 = 0.520 \text{ tm/m}$$

let $h = 6.5$ cm. Then $\frac{30m}{h^2\sigma_y} = 0.154$

$$p = \frac{1 \pm \sqrt{0.846}}{15} = \begin{cases} 0.00533 \\ 0.1280 > p_L \end{cases}$$

$$F_e = 0.00533 \times 100 \times 6.5 = 3.46 \ cm^2/m$$

b - ACI coefficients used to compute μ, i and i'

$M = 0.064 \ ql_x^2$ $\qquad$ $M = 0.050 \ ql_x^2$

$iM = 0.043 \ ql_x^2$ $\qquad$ $i'\mu M = 0.033 \ ql_x^2$

$$\mu = \frac{0.050}{0.064} = 0.782 \qquad \sqrt{\mu} = 0.885 \qquad i = 0.672 \qquad i' = 0.66$$

Dimensions of isotropically reinforced slab

$$l_y = 5.00 \ m. \qquad l'_x = \frac{4.00}{0.885} = 4.52 \ m \qquad l'_x/l_y = 0.9$$

$$(l'_x)_r = \frac{2l'_x}{2\sqrt{1+0.672}} = 0.775 \ l'_x \qquad (l_y)_r = \frac{2l_y}{2\sqrt{1+0.66}} = 0.775 \ l_y$$

$$m = \frac{q(0.775)^2 l'^2_x}{24}\left[\sqrt{3+(0.9)^2} - 0.9\right]^2 = 0.0258 \ ql_x^2$$

$$m = 0.0330 \ ql_x^2 = 0.266 \ tm/m$$

let $h = 6.5$ cm. Then $\frac{30m}{h^2\sigma_y} = 0.0788$

$$p = \frac{1 \pm \sqrt{0.9212}}{15} = \begin{cases} 0.00267 \\ 0.1305 > p_L \end{cases}$$

$$F_e = 0.00267 \times 100 \times 6.5 = 1.74 \ cm^2/m$$

c - French Code coefficients used to compute μ

$$M = 0.0615 \ ql_x^2 \qquad \mu = 0.684 \qquad \sqrt{\mu} = 0.828$$

Dimensions of isotropically reinforced slab

$$l_y = 5.00 \ m \qquad l'_x = \frac{4.00}{0.828} = 4.84 \qquad l'_x/l_y = 0.966$$

$$(l'_x)_r = l'_x \qquad (l_y)_r = l_y$$

$$m = \frac{q l_x'^2}{24}\left[\sqrt{3+(0.966)^2} - (0.966)\right]^2 = 0.0423\, q l_x'^2$$

$$m = 0.0618\, q l_x^2 = 0.495 \ tm/m$$

Let $h = 6.5$ cm. Then $\frac{30\, m}{h^2 \sigma_y} = 0.146$

$$p = \frac{1 \pm \sqrt{0.854}}{15} = \begin{cases} 0.0050 \\ 0.128 > p_b \end{cases}$$

$$F_e = 0.0050 \times 100 \times 6.5 = 3.25 \ cm^2/m.$$

3.6.2 - Design by Pucher coefficients and TKİCBA Şart.

a - $M = 0.05512\, q l_x^2 = 0.441$ tm/m

b - Design of reinforcement $\sigma_b / \sigma_e = 60/1400$

$h_r = 9.897\sqrt{\frac{0.441}{1.00}} = 6.5$ cm. h = 6.5 will br provided

$$F_e = \frac{0.821 \times 0.441}{0.065} = 5.57 \ cm^2/m$$

c - Ultimate capacity of the obtained section

$$p = \frac{5.57}{100 \times 6.5} = 0.00857$$

$$m = (6.5)^2(0.00857)(2400)\left(1 - \frac{0.00857 \times 15}{2}\right) = 0.814 \ tm/m$$

$$m = 0.102\, q l_x^2$$

d - Load factor: $L.F. = \frac{0.102}{0.065} = 1.57$

3.6.3 - Design by ACI code

a - $M = 0.064\, q l_x^2 = 0.512$ tm/m

b - Design of reinforcement $\sigma_b / \sigma_e = 70/1400$

$h_r = 8.819\sqrt{\frac{0.512}{1.00}} = 6.34$ cm. h = 6.5 cm will be provided

$$F_e = \frac{0.833 \times 0.512}{0.065} = 6.56 \ cm^2/m$$

c- Ultimate capacity of the section obtained

$$p = \frac{6.56}{6.5 \times 100} = 0.0101$$

$$m = (6.5)^2 (0.0101)(2400)\left(1 - \frac{0.0101 \times 15}{2}\right) = 0.941 \ tm/m$$

$$m = 0.118 \ q l_x^2$$

d - Load factor: $L.F. = \frac{0.118}{0.033} = 3.58$

3.2.4 - Design by the French Code

a - $M = 0.0615 \ q l_x^2 = 0.492$ tm/m.

b - Designof reinforcement $\sigma_b / \sigma_e = 96/1600$

$$M = \frac{1}{2} \sigma_b b (0.403 \times 0.866) h_r^2 = 0.492 \ tm.$$

$h_r = 5.42$ cm , h = 6.5 cm will be provided

$$F_e = \frac{0.492}{1.6 \times 0.065} = 4.74 \ cm^2/m$$

c - Ultimate capacity of the obtained section

$$p = \frac{4.74}{6.5 \times 100} = 0.00729 < p_L$$

$$m = (6.5)^2 (0.00729)(2400)\left(1 - \frac{0.00729 \times 15}{2}\right) = 0.700 \ tm/m$$

$$m = 0.0875 \ q l_x^2$$

d - Load factor: $L.F. = \frac{0.0875}{0.0618} = 1.41$.

3.7 - PROBLEM Design a square slab 4.00 m by 4.00 m, simply supported on all four sides and carrying a distributed load of 500 kg/m^2. Same materials as in the previous problems are available and q is assumed to include the dead load.

3.7.1 - Design by the yield-line method

a-Pucher coefficients used to compute i

Pucher gives $M_x = M_y = 0.03646 \ q l_x^2$ $\mu = 1$ $i = i' = 0$

For the case of square slabs, simply supported and with isotropic reinforcement the yield-line theory gives:

$$m = ql_x^2/24 = 0.0417\ ql_x^2 = 0.334\ tm/m$$

let $h = 6.5$ cm. Then $\frac{30m}{h^2\sigma_y} = 0.0994$

$$p = \frac{1 \pm \sqrt{0.900}}{15} = \begin{cases} 0.00333 \\ 0.130 > p_b \end{cases}$$

$$F_e = 0.00333 \times 100 \times 6.5 = 2.17\ cm^2/m$$

b- ACI Code coefficients used to compute i

$M = 0.050\ ql_x^2$ $\qquad iM = 0.033\ ql_x^2$ $\qquad i = 0.66$

$$(l_x)_r = \frac{2l_x}{2\sqrt{1+0.66}} = 0.775\ l_x \qquad (l_y)_r = 0.775\ l_y$$

$$m = \frac{q(0.775)^2 l_x^2}{24}\left[\sqrt{3+(1)^2} - (1)\right]^2 = 0.025\ ql_x^2 = 0.200\ tm/m$$

let $d = 6.5$ cm. Then $\frac{30m}{h^2\sigma_y} = 0.0596$

$$p = \frac{1 \pm \sqrt{0.94}}{15} = \begin{cases} 0.0020 \\ 0.131 > p_b \end{cases}$$

$$F_e = 0.0020 \times 100 \times 6.5 = 1.30\ cm^2/m$$

c - French Code coefficients used

The French Code gives $M = 0.0423\ ql_x^2$

$$M = 0.0417\ ql_x^2 \quad 0.334\ tm/m$$

From part (a) of this section

$$F_e = 2.17\ cm^2/m$$

3.7.2 - Design by Pucher coefficients and TKİCBA Şart.

a- $M\ 0.03646\ ql_x^2 = 0.292$ tm/m

b- Design of reinforcement $\sigma_b / \sigma_a = 60/1400$

$h_r = 9.897\sqrt{\frac{0.292}{1.00}} = 5.4$ cm. h = 6.5 cm will be provided

$$F_e = \frac{0.821 \times 0.292}{0.065} = 3.69\ cm^2/m$$

c- Ultimate capacity of the obtained section

$$p = \frac{3.69}{6.5 \times 100} = 0.00569$$

$$m = (6.5)^2 (0.00569)(2400)\left(1 - \frac{0.00569 \times 15}{2}\right) = 0.552\ tm/m$$

$$m = 0.0690\ ql_x^2$$

d-Load factor: $L.F. = \frac{0.0690}{0.0417} = 1.65$

3.7.3 -Design by the ACI code coeff.and working stresses

a- $M = 0.050\ ql_x^2 = 0.400$ tm/m

b- Design of reinforcement $\sigma_b / \sigma_a = 70/1400$

$h_r = 8.819\sqrt{\frac{0.400}{1.00}} = 5.6$ cm. H = 6.5 cm will be provided

$$F_e = \frac{0.833 \times 0.400}{0.065} = 5.12\ cm^2/m$$

c- Ultimate capacity of the obtained section

$$p = \frac{5.12}{6.5 \times 100} = 0.00789$$

$$m = (6.5)^2 (0.00789)(2400)\left(1 - \frac{0.00789 \times 15}{2}\right) = 0.753\ tm/m$$

$$m = 0.0942\ ql_x^2$$

d-Load factor: $L.F. = \frac{0.0942}{0.0250} = 3.76.$

3.7.4. - Design by the French Code

a - $M = 0.0423\, q l_x^2 = 0.338$ tm/m

b - Design of reinforcement $\sigma_b / \sigma_e = 96/1600$

$$M = \frac{1}{2} \sigma_b b (0.403 \times 0.866) h_r^2 = 0.338 \text{ tm}$$

$h_r = 4.5$ cm ; $h = 6.5$ cm will be provided

$$F_e = \frac{0.338}{1.6 \times 0.065} = 3.25 \text{ cm}^2/\text{m}$$

c - Ultimate capacity of the obtained section

$$p = \frac{3.25}{6.5 \times 100} = 0.0050$$

$$m = (6.5)^2 (0.0050)(2400)\left(1 - \frac{0.0050 \times 15}{2}\right) = 0.488 \text{ tm/m}$$

$$m = 0.061\, q l_x^2$$

d - Load factor : $L.F. = \dfrac{0.0610}{0.0417} = 1.46$

3.8 -CAPACITY COMPARISONS

Problem : Consider a reinforced concrete slab, fixed on all four edges (B-160 concrete, ST-I steel), 4.00 m by 4.00 m, 10.0 cm thick and isotropically reinforced with 4.00 cm^2 of steel per unit width. What is the maximum load that this slab can carry ?

(a) According the yield-line theory

(b) According the TKİCBA Şartnamesi

(c) According the ACI Reinforced Building Code

(d) According the French Reinforced con. Building Code

Percentage of steel $= p = \dfrac{4.00}{10 \times 100} = 0.0040$

Balance percentage $= p_b = 0.43 \dfrac{160}{2400} = 0.0287 > 0.0040$

Failure will occur by the yielding of the reinforcement.

3.8.1 - The ultimate moment that this section can take is:

$$m = (8.5)^2 (0.0040)(2400)\left(1 - \frac{0.0040 \times 15}{2}\right) = 674 \ kgm/m$$

The yield moment of a square plate with isotropic reinforcement is given by Eq.(3) as:

$$m = \frac{q l_x^2}{48}$$

Substituting $674 = \frac{(q)(4.00)^2}{48} \longrightarrow q = 2022 \ kg/m^2$

Introducing a load factor of 1.5

$$q_{allow} = 1350 \ kg/m^2$$

3.8.2 - TKİCBA Şart. $\sigma_b / \sigma_e = 60/1400$

$$M = \frac{f_e \times h}{k_3} = \frac{4.00 \times 8.5}{0.821} = 41.4 \ t.cm/cm = 414 \ kgm/m.$$

The moment coefficient given by Pucher for this case is:

$$\alpha_x = \alpha_y = 0.01794$$

$$M = 414 = 0.01794 \, q l_x^2 = 0.01794 \, (4.00)^2 q$$

$$q = 1440 \ kg/m^2$$

3.8.3 - ACI Code $\sigma_b / \sigma_e = 70/1400$

$$M = \frac{4.00 \times 8.5}{0.833} = 40.8 \ tcm/cm = 408 \ kgm/m$$

The moment coefficient given by ACI code is : $\alpha = 0.025$

$$M = 408 = 0.025 \, (4.0)^2 q$$

$$q = 1020 \ kg/m^2$$

3.8.4 - French Code $\sigma_b / \sigma_a = 96/1600$

$$M = (4.00)(1.6)(0.87 \times 8.5) = 47.4 \ tcm/cm = 474 \ kg.m/m$$

The moment coefficient given by the code is : $\alpha = 0.0318$

$$M = 474 = 0.0318 \, (4.0)^2 q$$

$$q = 933 \ kg/m^2.$$

SECTION 4

4.1 The results obtained from the previous section, summarized in Table I, will serve to derive some conclusions about the moment coefficients proposed by different codes and the load and safety factors introduced by the codes.

The information gathered in the preceding section enables one to draw the following curves :

(a) Moment coefficients given by codes vs. l_x / l_y,

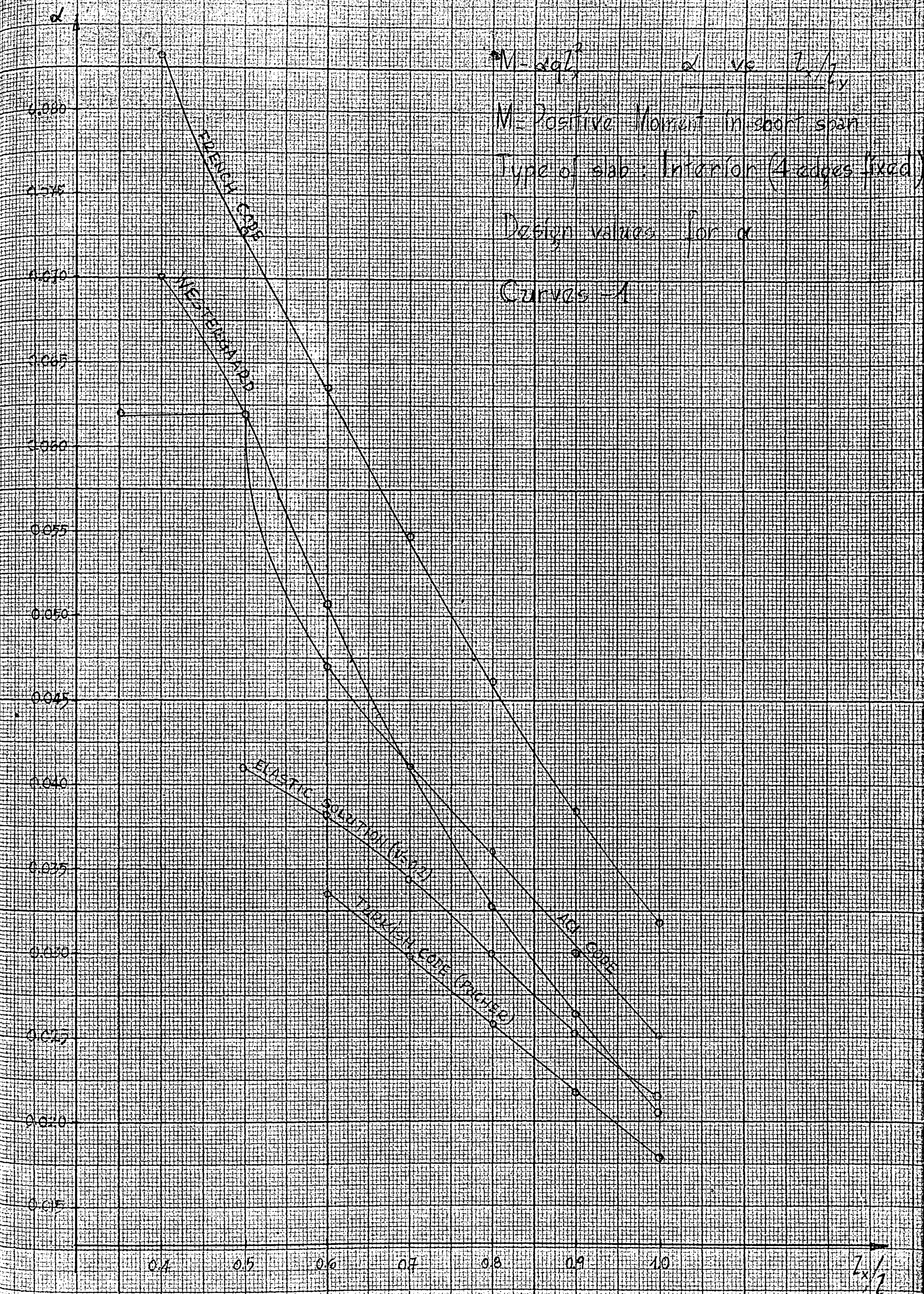
(b) Coefficients of the ultimate moments that can be taken by the designed sections vs. l_x / l_y,

(c) Coefficients given by the yield-line theory vs. l_x/l_y

It may be noticed that the curves showing moment coefficients given by codes vs l_x/l_y are all almost parallel to the curve obtained by elastic analysis. This fact demonstrates that the determination of the values of the moment coefficients is guided by the elastic analysis of plates. One thing to notice in this respect is that all curves, except one, lie above the curve representing the elastic solution in the case of interior, fixed slabs, whereas they all lie below the curve of elastic solution in the case of freely supported slabs. This fact may be explained as follows: the reduction of the moment coefficients is proposed by Westergaard, and is greately based on a similarity of flat and two-way slabs. In the case of interior slabs this similarity already doubtful, becomes still more doubtful, since the supporting beams, poured monolithically with the slab restrain flat-slab action; the behaviour of simply supported slabs rapproaches more the flat-slab action due to the absence of the torsional rigidity of the supporting members. This

TABLE - I

	l_x/l_y	α from Elastic analysis $\nu=0.2$	α from Westergaard	T.K.I.C.B.A Şart.			ACI CODE			FRENCH CODE		
				α PUCHER	α from Yield Line	α After design	α	α from Yield Line	α After design	α	α from Yield Line	α After design
SIMPLY SUPPORTED SLABS	0.6	0.0868	0.0782	0.08127	0.1160	0.1545	0.083	0.0416	0.1633	0.0849	0.1054	0.1235
	0.8	0.0627	0.0519	0.05512	0.0650	0.1020	0.064	0.0330	0.1180	0.0615	0.0618	0.0875
	1.0	0.0442	0.0334	0.03646	0.0417	0.0690	0.050	0.0250	0.0942	0.0423	0.0417	0.0610
INTERIOR SLABS	0.6	0.0381	0.0506	0.03362	0.0382	0.0637	0.047	0.0402	0.0880	0.0636	0.0825	0.0889
	0.8	0.0299	0.0329	0.02583	0.0199	0.0493	0.036	0.0268	0.0679	0.0461	0.0488	0.0663
	1.0	0.0213	0.0208	0.01794	0.0125	0.0375	0.025	0.0178	0.0495	0.0317	0.0277	0.0472



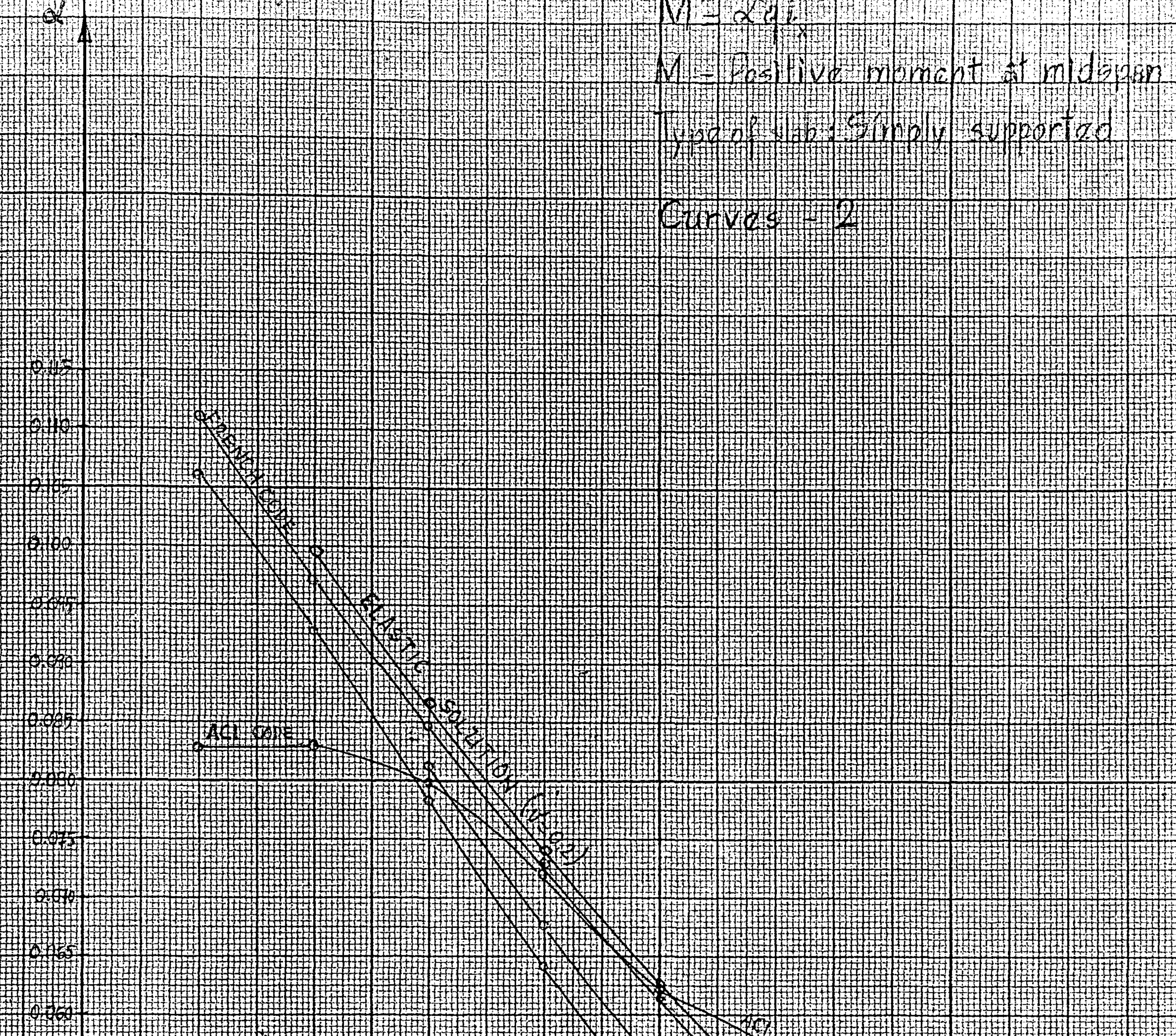
$M = \alpha q l_x^2$ α vs l_x/l_y
M = Positive Moment in short span
Type of slab : Interior (4 edges fixed)
Design values for α
Curves - 1
α
0.080
0.070
0.065
0.060
0.055
0.050
0.045
0.040
0.035
0.030
0.025
0.020
0.015
0.4
0.5
0.6
0.7
0.8
0.9
1.0
l_x/l_y
FRENCH CODE
WESTERGAARD
ELASTIC SOLUTION (ν=0.2)
TURKISH CODE (PUCHER)
ACI CODE

$M = \alpha q l_x^2$

M - Positive moment at midspan

Type of slab: Simply supported

Curves - 2

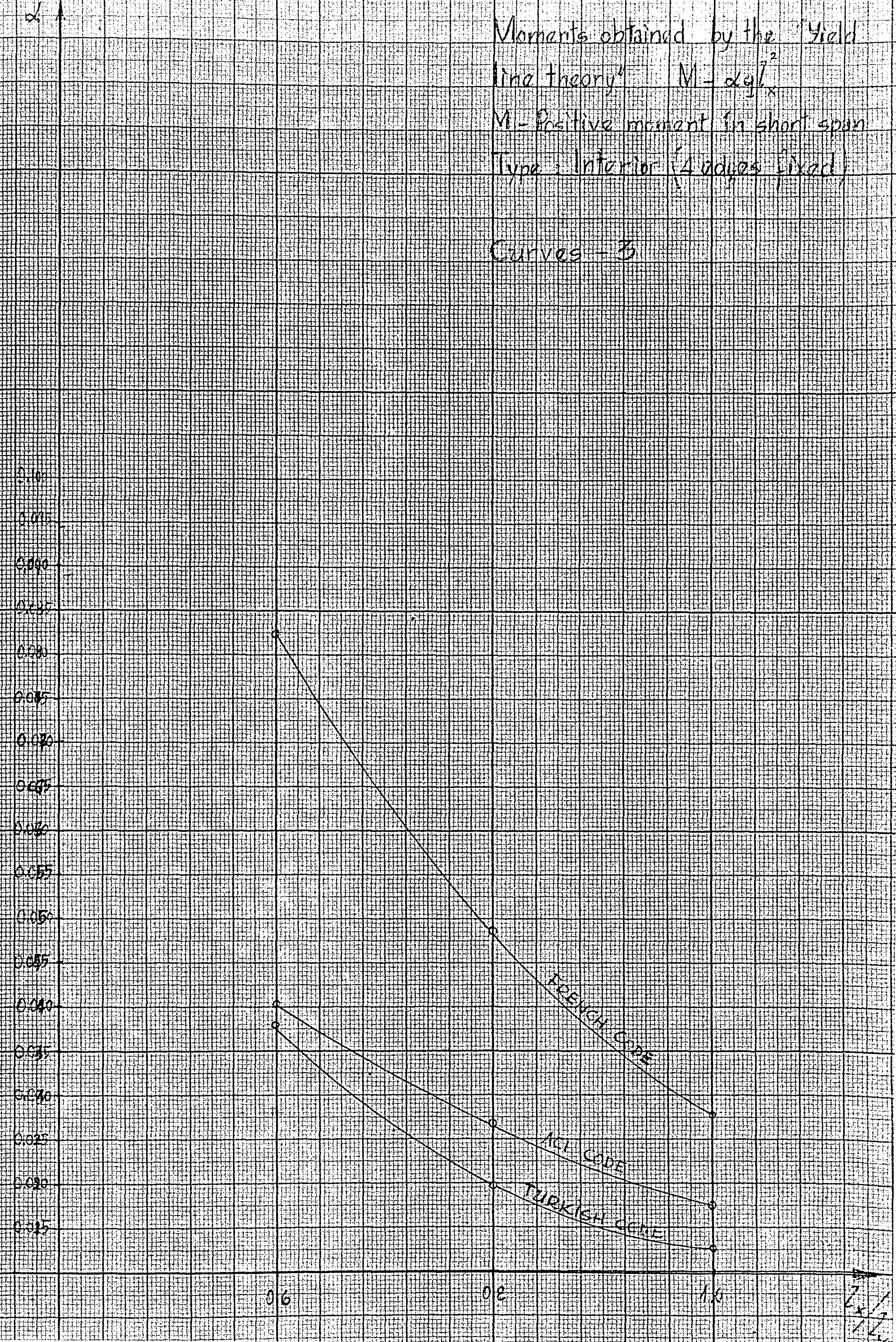


Moments obtained by the "Yield line theory" $M = \alpha q l_x^2$

M - Positive moment in short span

Type : Interior (4 edges fixed)

Curves - 3

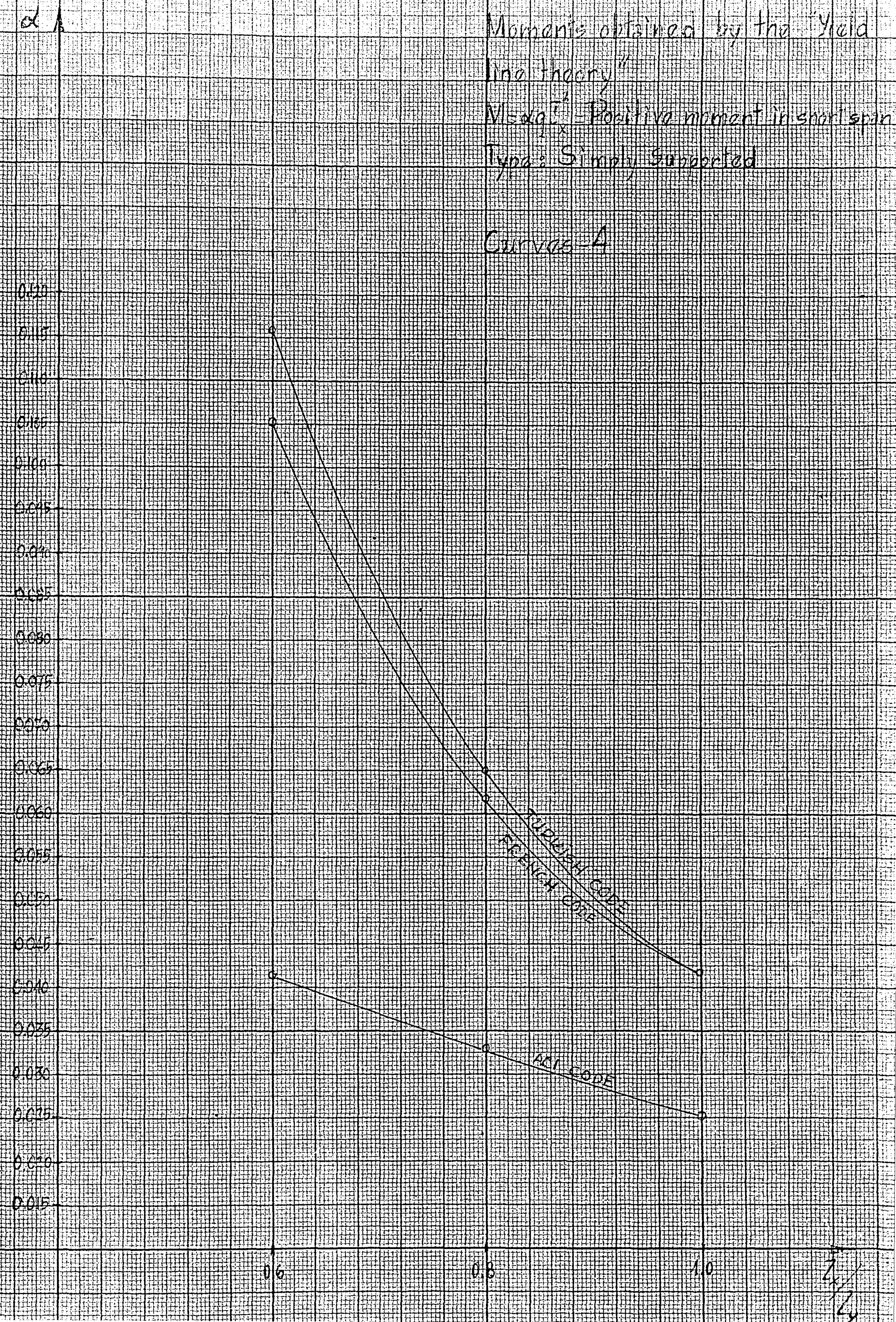


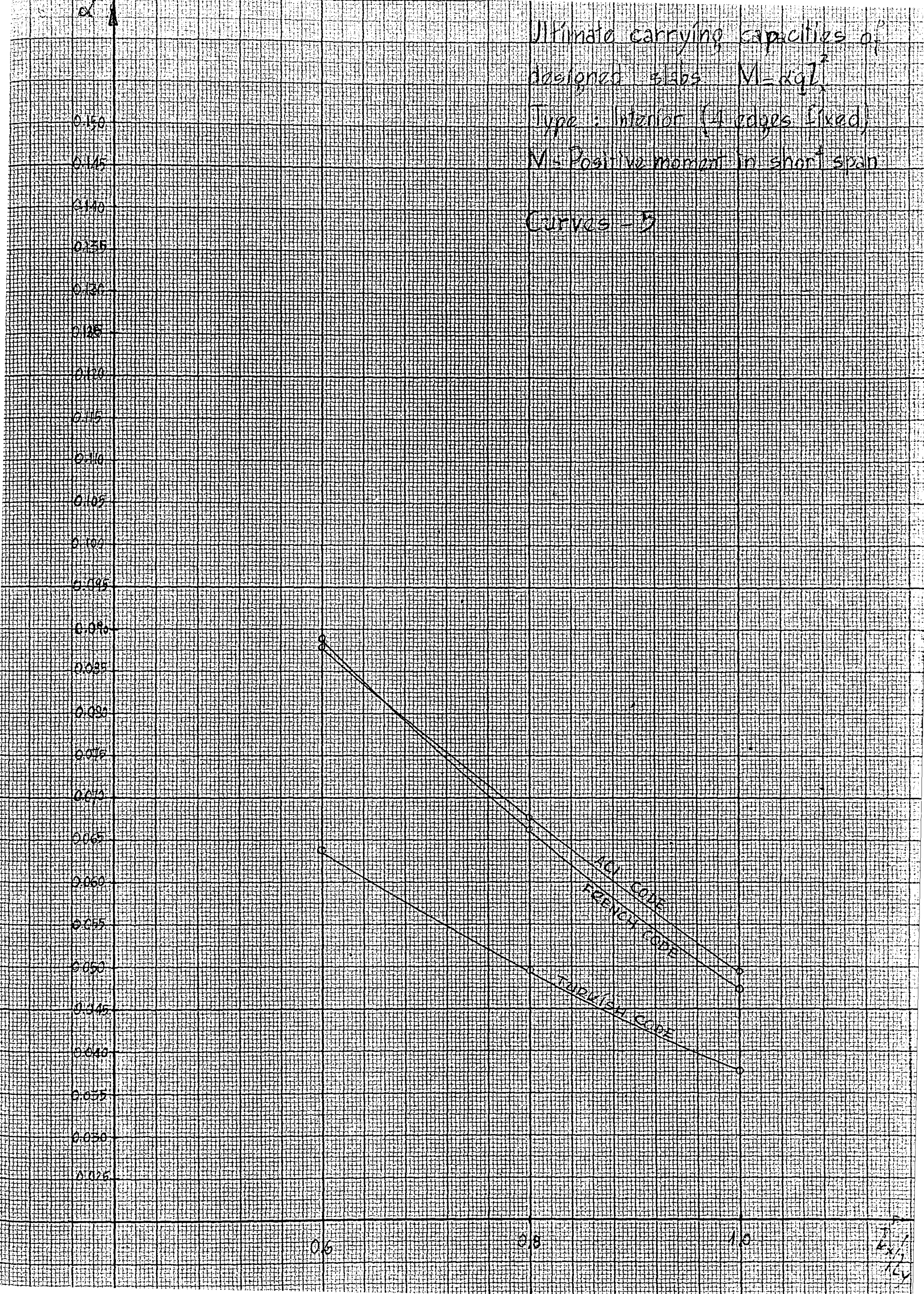
Moments obtained by the "Yield line theory"

$M = \alpha q l_x^2$ - Positive moment in short span

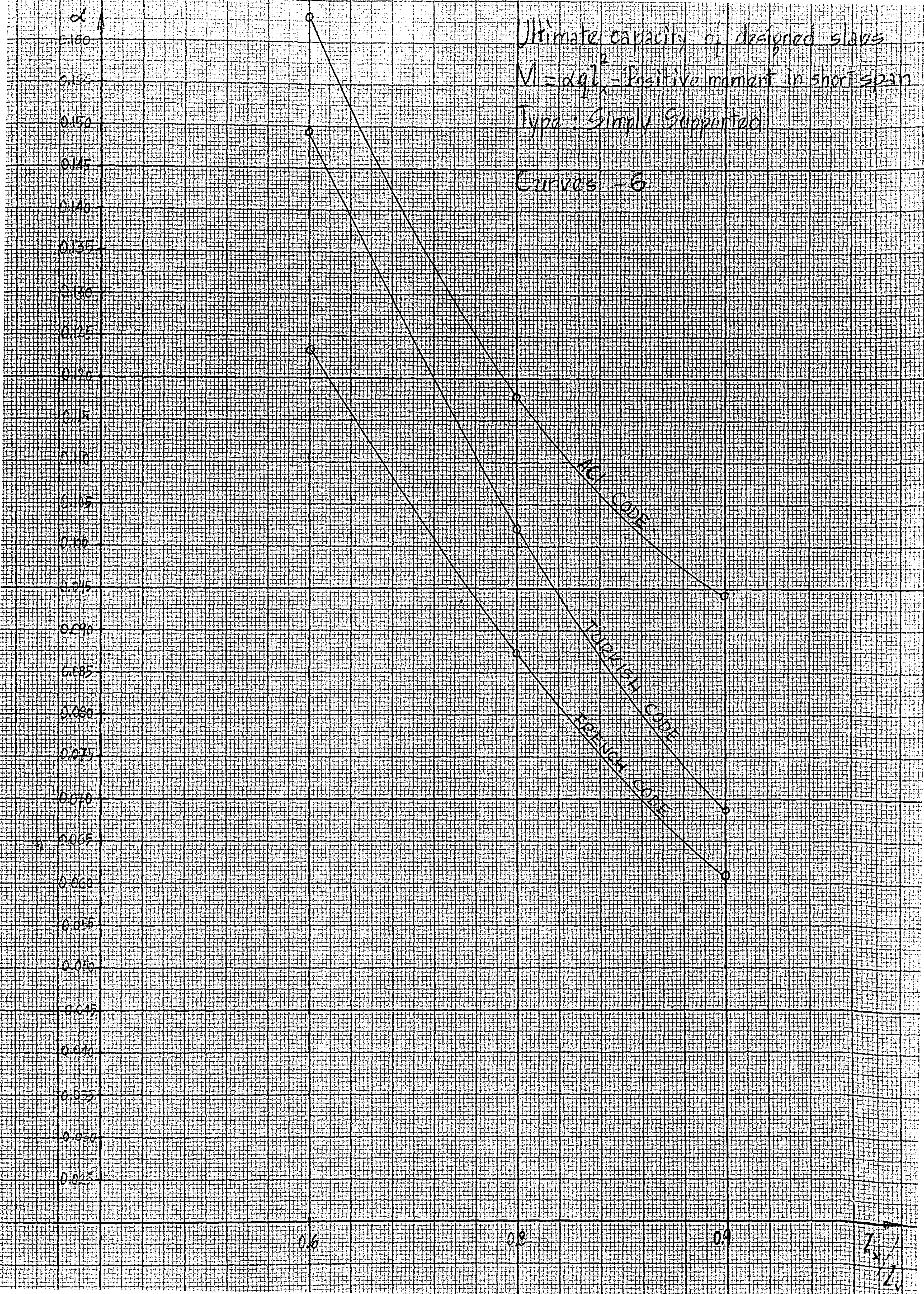
Type: Simply Supported

Curves-4





Ultimate carrying capacities of designed slabs $M = \alpha q l^2$
Type : Interior (4 edges fixed)
M : Positive moment in short span
Curves - 5
α
0.150
0.145
0.140
0.135
0.130
0.125
0.120
0.115
0.110
0.105
0.100
0.095
0.090
0.085
0.080
0.075
0.070
0.065
0.060
0.055
0.050
0.045
0.040
0.035
0.030
0.025
0.6
0.8
1.0
l_x / l_y
ACI CODE
FRENCH CODE
TURKISH CODE



Ultimate capacity of designed slabs
$M = \alpha q l_x^2$ - Positive moment in short span
Type : Simply Supported
Curves - 6
α
0.160
0.150
0.145
0.140
0.135
0.130
0.125
0.120
0.115
0.110
0.105
0.100
0.095
0.090
0.085
0.080
0.075
0.070
0.065
0.060
0.055
0.050
0.045
0.040
0.030
0.025
ACI CODE
TURKISH CODE
FRENCH CODE
0.6
0.8
0.9
l_x / l_y

difference in action of the two types of slabs explains why it has been possible to reduce the moment coefficients for simply supported slabs and not for the four-edges -fixed slabs.

In the following parts of the section the design coefficients given by the three codes will be studied.For this purpose it will be assumed that the moments computed by the yield-line theory are the real necessary moment-carrying capacities of the designed slabs.There is good reason in this assumption since it has been proved experimentally that the ultimate loads predicted by the yield line theory are on the average 80 to 90 percent of those observed in tests [1].

The load factors introduced by the codes were calculated in the previous section.The safety factors introduced by the use of allowable stresses of the codes will be calculated by dividing the ultimate moment carrying capacities of the desinged sections by the design moments specified by the codes.

4.2 Remarks about the codes

4.2.1- Türkiye Köprü ve İnşaat Cemiyeti B.A. Şartnāmesi

(a)Reference to Table I will show that the coefficients given by Pucher are always lower than the ones given by elastic analysis.This reduction is made possible by taking into account the torsional stiffness of the slab [2].The amount of this reduction is not as large,in reality,as is noticed in Table I and Curves 1 - 2,because the values given as the result of elastic analysis are computed for $\nu = 0.2$ which is quite higher than the real Poisson's ratio for concrete,which is about 0.12:Since a decrease

1 . E.Hognestad loc.cit. pp.652
2 . Beton Kalender pp.204 W.Ernst und Sohn - Berlin 1963

in ν means a decrease in the moment, the coefficients given by the elastic solution would be somewhat lower and thus the reduction introduced by Pucher would be smaller.

(b) Reference to curves 1 and 5, 2 and 6 indicates the differences between the ultimate capacities of the slabs designed by the Turkish Code and the corresponding design moments. These differences are due to the factor of safety introduced by the Code. The numerical value for the factor of safety will be obtained by dividing the ultimate capacity of the obtained section by the design moment. The load factors were already calculated in the preceding section. The following table gives a summary of the load and safety factors resulting from the use of the Pucher coefficients and TKİCBA Şart.

TABLE - II

	l_x/l_y	L.F.	F.S.
Simply supported slabs	0.6	1.32	0.1545/0.08127=1.90
	0.8	1.57	0.1020/0.05512=1.85
	1.0	1.65	0.0690/0.03646=1.89
Interior slabs	0.6	1.67	0.0637/0.03362=1.89
	0.8	2.46	0.0493/0.02583=1.91
	1.0	3.00	0.0375/0.01794=2.09

4.2.2. The ACI Code

(a) The moment coefficients given by the ACI Code lie between Pucher coefficients and the ones given by the French Code in the case of interior slabs. For simply supported slabs the curve of ACI coefficients is the only one that lies (partially) above the elastic solution curve. It is known that the ACI coefficients are inspired by Westergaard's coefficients, but as a look on curves 1 and 2 would show they are quite different from each other, the ACI values being higher in the most probable ranges of l_x/l_y. This difference is largely due to the wish of introducing a first factor of safety by increasing the value of the design moment

(b) The ACI Code allows the reduction of the steel area in the column strip in the ratio of 2/3 as compared to the steel area in the middle strip. This reduction would increase the necessary moment capacity determined from the yield-line theory. To account for this effect the load factor provided by the ACI Code will be reduced in the arbitrary ratio of 5/6. The factor of safety is calculated as in the previous case.

TABLE III

	l_x/l_y	L.F.	F.S.
Simply	0.6	3.93 x 5/6=3.28	0.1633/0.083=1.97
supported slabs	0.8	3.58 x 5/6=2.99	0.1180/0.064=1.85
	1.0	3.76 x 5/6=3.14	0.0942/0.050=1.88
Interior	0.6	2.19 x 5/6=1.83	0.0880/0.0470=1.87
slabs	0.8	2.53 x 5/6=2.11	0.0679/0.0360=1.88
	1.0	2.78 x 5/6=2.32	0.0495/0.0250=1.97

4.2.3 - The French Code

(a) The curves giving the moment coefficients of the French Code are clearly parallel to the curves representing the values obtained by elastic solution. In the case of interior slabs the coefficients of the code are very much above the values given by elastic solution, whereas in the case of simply supported slabs they are slightly below the elastic solution.This fact reflects the same way of thinking as Westergaard,that is a reduction is made on the coefficients of the panels which approximate flat slabs in their behaviour.A first factor of safety is again introduced ,as in the case of the ACI Code,for interior panels.

(b)The load and safety factors are calculated as it was done for TKİCBA Şart.

TABLE IV

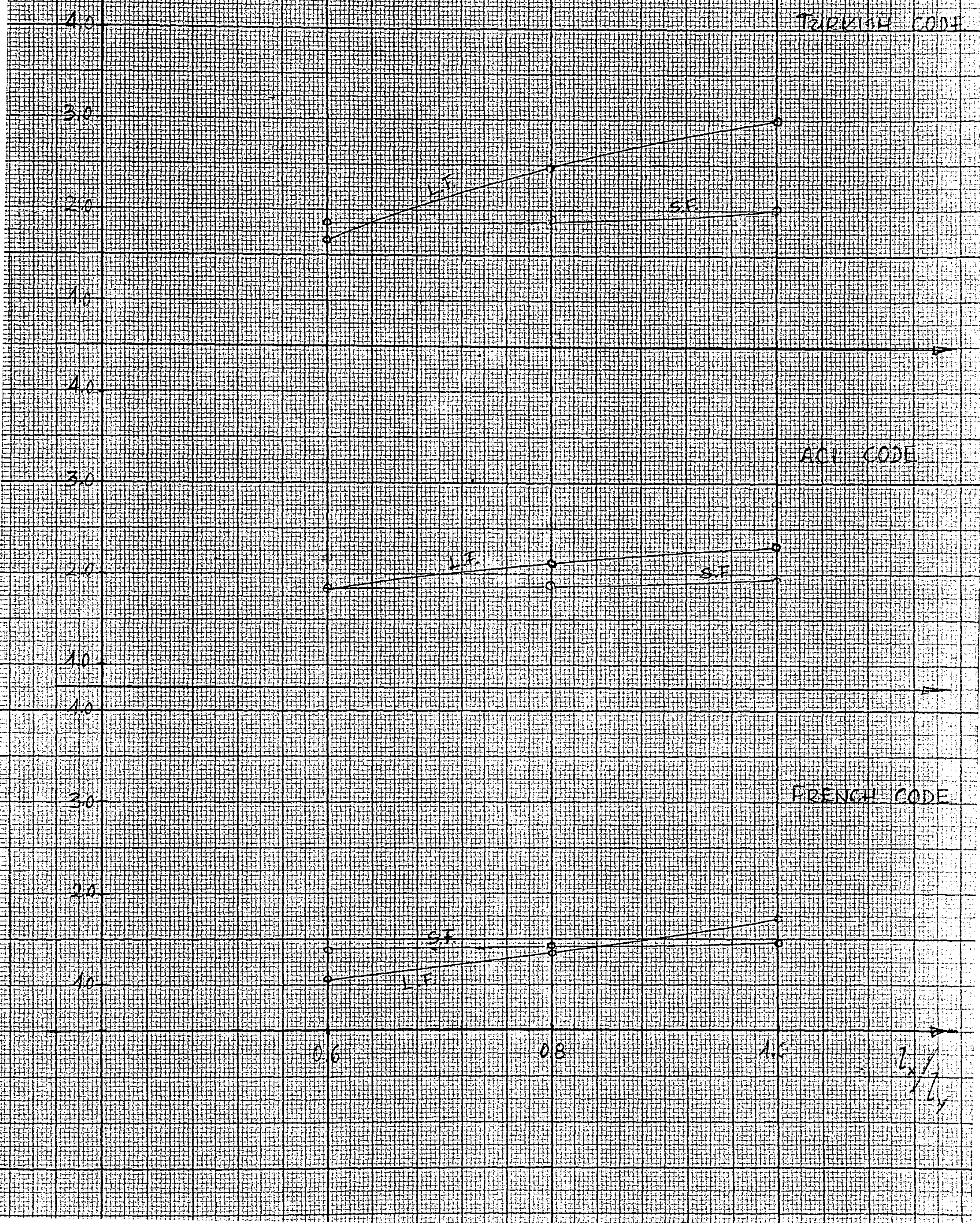
	l_x / l_y	L.F.	F.S.
Simply	0.6	1.17	0.1235/0.0849=1.46
supported	0.8	1.41	0.0875/0.0615=1.42
slabs	1.0	1.46	0.0610/0.0423=1.44
Interior	0.6	1.08	0.0889/0.0636=1.40
slabs	0.8	1.36	0.0663/0.0461=1.44
	1.0	1.71	0.0472/0.0317=1.49

Curves N° 7 and 8 represent load and safety factors vs. l_x/l_y .The curves 9 to 12 show the percentages of steel required by different methods of design,for different side ratios and support conditions.

Variation of Load and safety factors with l_x/l_y

Interior (4 edges fixed) slabs

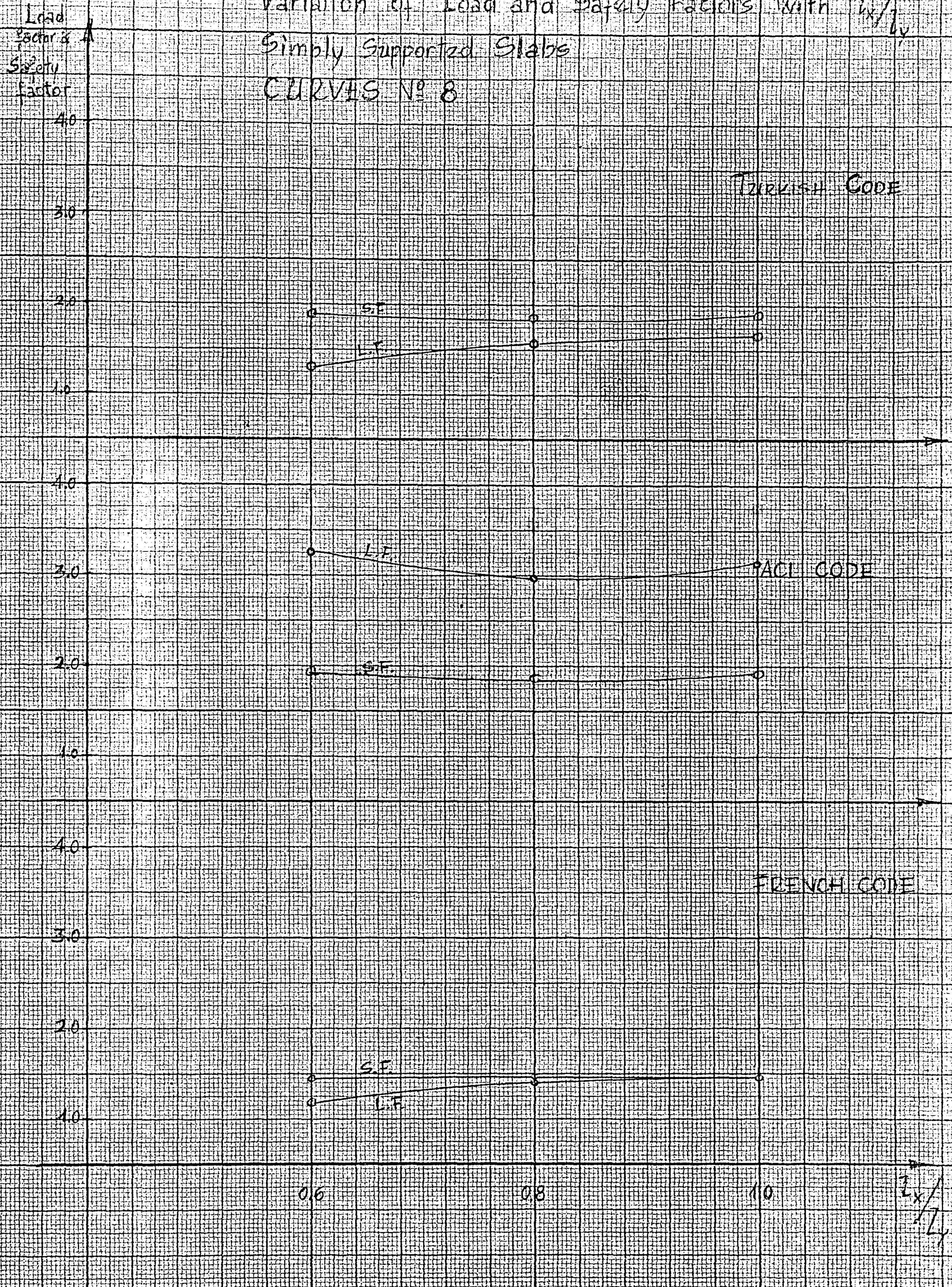
CURVES Nº 7



Variation of Load and Safety Factors with l_x/l_y

Simply Supported Slabs

CURVES № 8



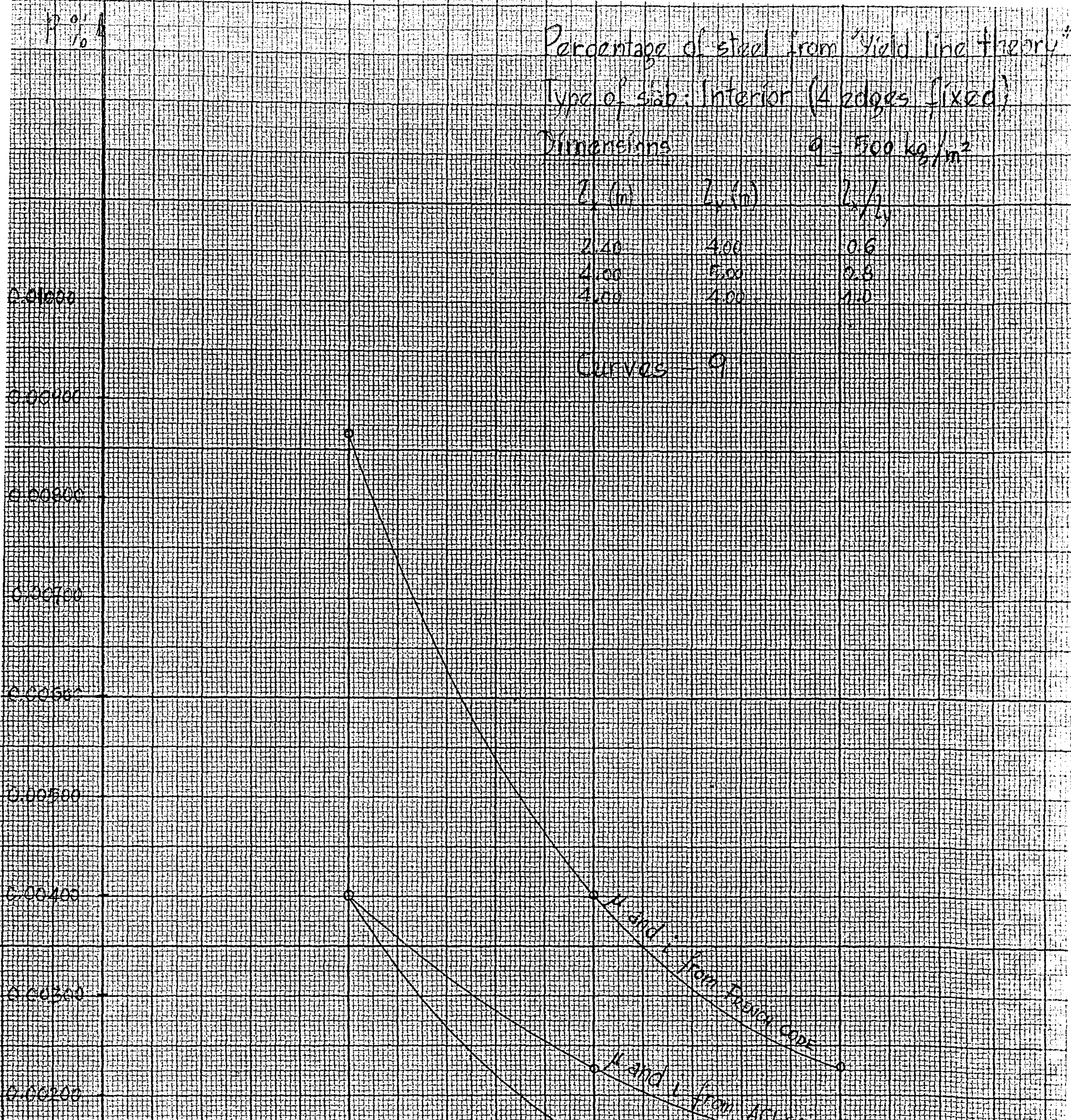
Percentage of steel from "Yield line theory"

Type of slab: Interior (4 edges fixed)

Dimensions $q = 500\ kg/m^2$

l_x (m)	l_y (m)	l_x/l_y
2.40	4.00	0.6
4.00	5.00	0.8
4.00	4.00	1.0

Curves - 9



The points are arbitrarily conected by straight lines

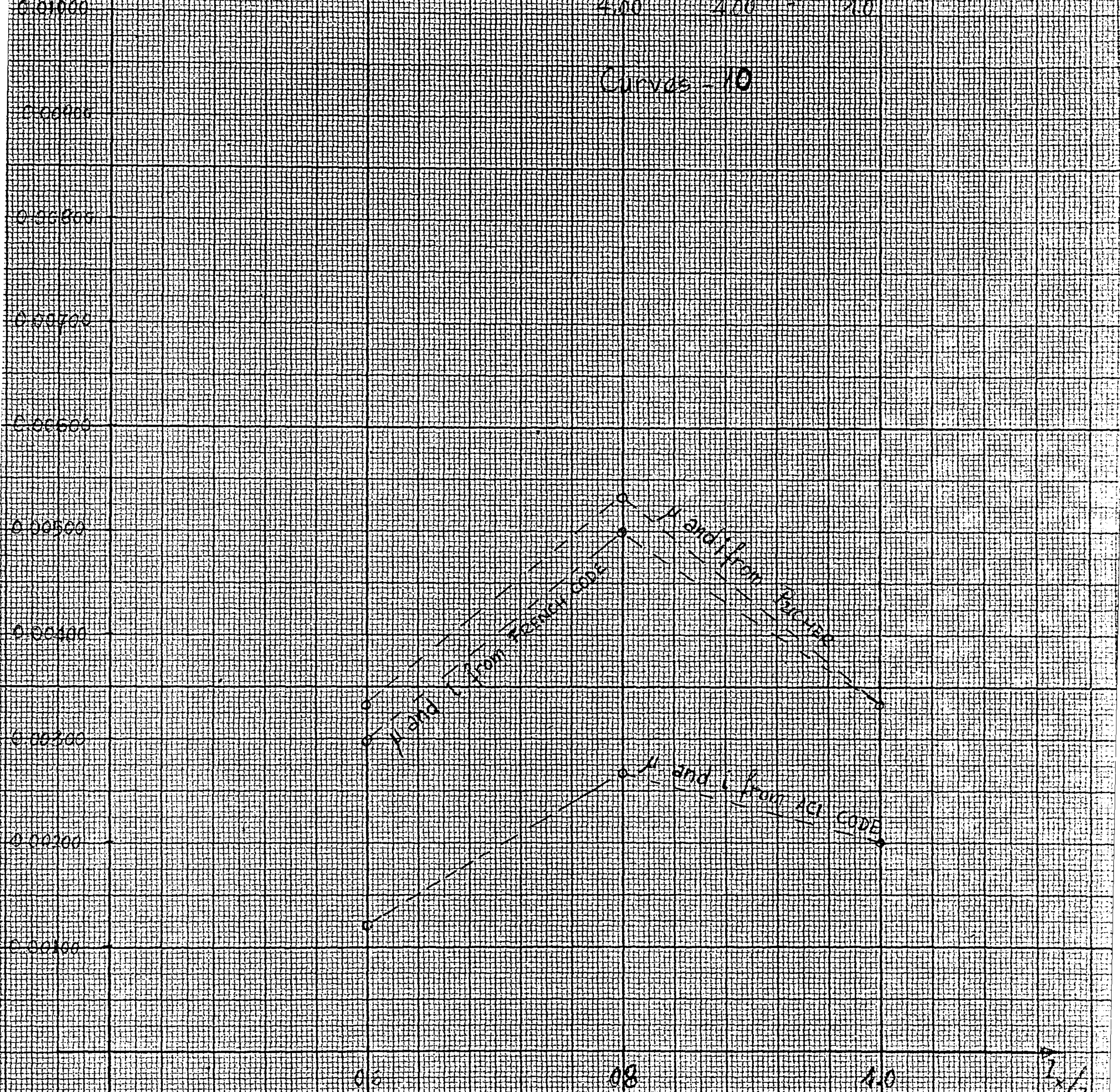
Percentage of steel from "Yield line theory"

Type of slab : Simply Supported

Dimensions : $q = 500\ kg/m^2$

L_x (m)	L_y (m)	L_x/L_y
2.40	4.00	0.6
4.00	5.00	0.8
4.00	4.00	1.0

Curves - 10



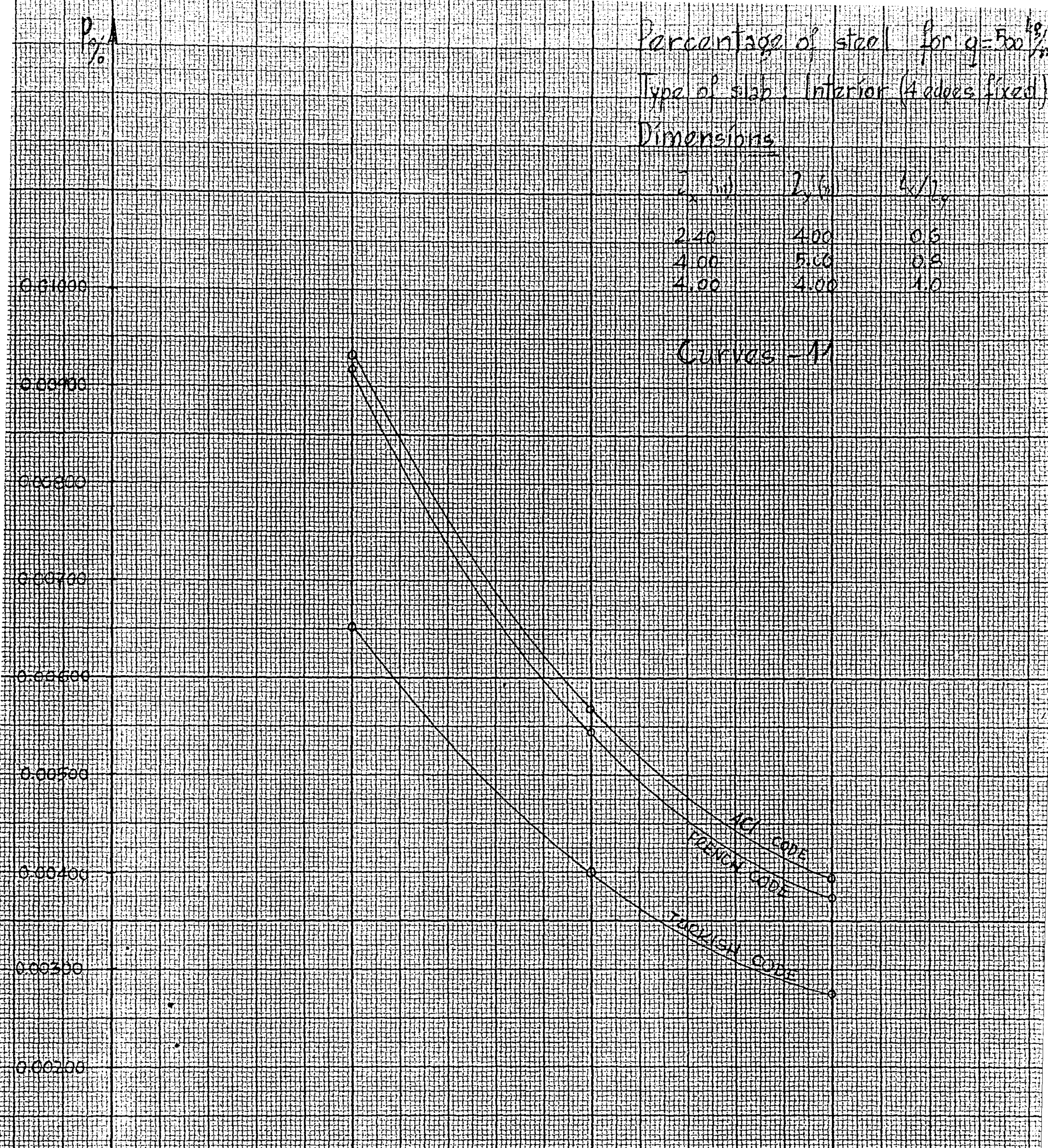
Percentage of steel for $q = 500\ kg/m^2$

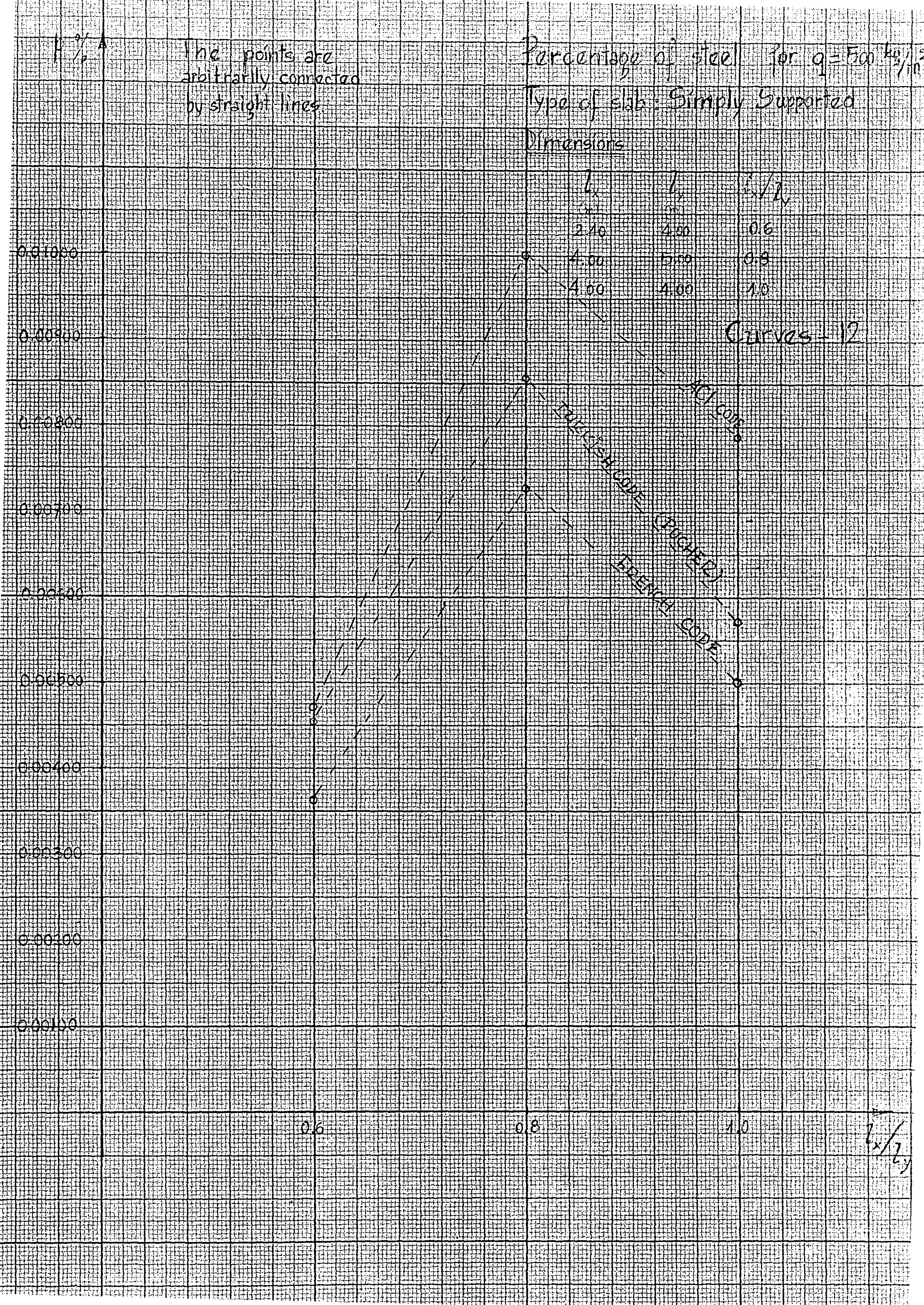
Type of slab: Interior (4 edges fixed)

Dimensions

l_x (m)	l_y (m)	l_x/l_y
2.40	4.00	0.6
4.00	5.00	0.8
4.00	4.00	1.0

Curves -11




μ %
The points are arbitrarily connected by straight lines
Percentage of steel for q = 500 kg/m²
Type of slab: Simply Supported
Dimensions
lx ly lx/ly
m m
2.40 4.00 0.6
4.00 5.00 0.8
4.00 4.00 1.0
Curves - 12
ACI CODE
TURKISH CODE (PUCHER)
FRENCH CODE
0.01000
0.00900
0.00800
0.00700
0.00600
0.00500
0.00400
0.00300
0.00200
0.00100
0.6
0.8
1.0
lx/ly

SECTION 5

CONCLUSIONS

The statements in this Section are based on the computations of Section 3, and it is well understood that they pertain only to the two particular support conditions that were considered.

The load factors and safety factors provided by the Codes will be discussed first.

Reference to Tables II, III and IV and Curves 7 and 8 will show that the factors of safety applied by the three Codes remain as constants. The average values being 1.92 for the Türkish Code, 1.90 for the ACI Code and 1.44 for the French Code. The large difference between the safety factor of the French Code and the two others arises from the fact that the allowable stresses of the French Code are based on careful consideration of the mechanical properties of the available materials the place where they are used. The average value 1.44 applies, therefore, only to the case of B-160 concrete and ST-I steel used for the construction of flexural members[1], this value would easily change for other constructional materials and types of members. The safety factors of the ACI and Turkish Codes are arbitrary numbers by which the ultimate strengths of the available materials are divided in order to obtain the allowable stresses, their numerical values will remain constant with different materials and for different types of structural elements.

The load factors were computed on the basis of the moments obtained by the application of the yield-line theory, these

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1. See Reference N° 2 on page 33

moments being accepted as the real necessary moment capacities of the section in hand. If the requirements of the Codes are strictly realised in a structure, then the resulting load factors will be those shown on Curves 7 and 8. But, as one may easily realise, the moments obtained by the use of the yield-line theory depend largely on the amount of reinforcement used at the supports, and a designer may provide large load factors - in cases of uncertainty about the live loads - by increasing the amount of reinforcement at the supports. (An increase of the steel area in the support means an increas of i, and consequently a decrease of m in the yield-line theory). The best example to this statement is given by the ACI Code in the case of simply supported slabs; this code provides a large load factor, compared to the ones of the Turkish and French Codes, by requiring the use of negative reinforcement near the simple supports.

Now it remains to answer the question "Which method to adopt for slab design?"

It has been proved experimentally that the ultimate loads predicted by the yield-line theory are on the average 80 to 90 percent of those observed in tests. There is good reason therefore in computing the necessary moment carrying capacities by this theory and designing accordingly by the application of a suitable load factor. The magnitude of the load factor will be mainly determined by the degree of with which the designer can predict the amounts of live loads. The only disadvantage of the yield-line theory would be the fact that it is the application of the "Upper bound theorem" of plasticity and all the possible mechanisms should be investigated for the determination of the ultimate load, since omissions could give serious results.

Although it seems as the best method of design, the yield-line theory is not yet accepted as a design method by the Codes we have considered. The best, "legal" method of design should then be determined, This determination is related to two main requirements: (a) The live loads being exactly known it is wanted that the slab be only as strong as to take these loads; (b) High ultimate strength is desired. In case (a) the ultimate strength of the slab is not important, therefore the required steel area should be the variable to be considered, the best solution in this case would be the one which is satisfied with the least amount of steel for a given load q. Curves 11 and 12 show that the Turkish and the French Codes give the best results for the cases of interior and simply supported slabs respectively. For case (b), where the ultimate capacity of the slab is the important variable, Curves 5 and 6 show that it would be advisable to apply the ACI Code.

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