A WELFARE COMPARISON OF LIFE-CYCLE INVESTMENT STRATEGIES FOR TURKEY

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A WELFARE COMPARISON OF LIFE-CYCLE INVESTMENT STRATEGIES FOR TURKEY

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2019

DECLARATION OF ORIGINALITY

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ABSTRACT

A Welfare Comparison of Life-Cycle Investment Strategies for Turkey

We perform an in-depth welfare comparison of the most common life-cycle investment strategies provided by retirement funds or suggested by classical portfolio theory in the case of households in Turkey. To perform our benchmarking, we construct heterogeneous agents who work and invest throughout their lifetime, using parameters calibrated from the historical data. We find that to households with upper-to-middle income, individually customized portfolios result in considerable welfare gains, while "off-the-shelf" life-cycle portfolio allocations perform better for households with lower income. We also show that life-cycle investment options outperform "fixed over the lifetime" options. Finally, we find that risk-averse individuals with volatile wages, can maximize their welfare by investing in housing as suggested by Munk (2016).

ÖZET

Yaşamboyu Yatırım Stratejilerin Türkiye için Refah Kıyaslaması

Bu çalışmamızda, klasik portföy teorisinin ve endüstri uzmanların sunduğu yaşamboyu yatırım stratejilerin refah seviyelerini kıyaslayacağız. Kıyaslamayı gerçekleştirmek için, tarihi verilerden belirlediğimiz parametrelere göre, iş hayatı boyunca yatıran heterojen ajanları kuracağız. Düşük maaşlı ajanlar için klasik modellerin daha karlı olduğunu ve orta-ve-üst maaşlı ajanlar için kişiselleştirilmiş modellerin daha karlı olduğunu öğreneceğiz. Munk (2016)'un sunduğu modeli kullanarak, riskten kaçınan ajanların, ev yatırımını yaparak, refah seviyelerini maksimize edebileceğini göreceğiz.

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CHAPTER 1

INTRODUCTION

1.1 Theory and heuristics of life-cycle investments

One of the most important investment decisions, individuals face in their lives, is investment in a retirement portfolio. Historically, retirement funds paid out fixed annuities to all of their clients. However, in 1952, with the rise of post-war instability, Teachers Insurance and Annuity Association (TIAA) allowed households to diversify their retirement portfolios for the first time. Academic economists, who were among the clients of TIAA, have reacted by allocating 50% of their funds to riskless bonds and 50% to risky stocks. Performing a deeper analysis, Markowitz (1952) has created a Modern Portfolio Theory and founded a field of financial economics.

With the development of the field, life-cycle investment strategies, which proposed adjusting portfolios at every age, gained recognition. Many funds offered "investment menus" — predetermined portfolio allocations for every age, to help their clients make optimal decisions without dealing with the complex theory. However, Turkish consulting firms have not included easy-to-comprehend lifecycle strategies (investment menus) in their bulletins, and didn't try to spread transparent information. We will fill this important gap in our paper, but firstly we will review the state of Turkish pension system.

1.2 Turkish pension system

The main pension funds in Turkey have been public for a long time: three main options existed: SSK for public and private sector workers, ES for civil servants, and Bag-Kur for self-employed workers and farmers. In 2006 they all merged into SGK. Private pensions have gained pace recently. As of January, 2017 a new clause of Turkish Labor Law came into action, that automatically enrolled every wage earner younger than 45 years into "Individual Retirement Scheme" — a private pension fund. To further incentivize people not to opt out, the government promised to subsidize 25% of their monthly contributions (as long as this wouldn't exceed 25% of minimal wage). According to PwC research, this has tremendously increased fund sizes. Under the current system individuals may retire after contributing to a pension fund for at least 10 years and at least reaching the age of 56.

The largest retirement funds in Turkey are listed in the Table 1. All of them offer 3-4 default investment options with varying degrees of riskiness but they are not lifecycle investment strategies mentioned above. They also provide flexible investment options with ability to change portfolio allocation up to six times a year, but they are not very popular as they assume active involvement in the portfolio and require a certain level of financial literacy. Not much academic research has been done on Turkish pension systems, and the existing research doesn't provide easy solutions. A recent example of this is Iscanoglu-Cekic's (2016) paper which doesn't consider life cycles and uses dynamic programming in the solution, which is also not accessible to wide audience.

1.3 Focus of this thesis

In this thesis we will consider the general framework of lifecycle investments and its historical evolution within the field of financial economics. We will take a look at standard heuristics suggested by financial consultancies and compare their welfare outcomes with those of optimal solutions given by theory. We will use the latest theoretical findings by Munk (2016) and show that optimal solutions can be both efficient and easy to comprehend without use of complex dynamic optimization results.

Next chapter will review all the relevant literature in this field and show the theoretical developments. Chapter 3 will summarize the theoretical framework and model, we will use in our simulation. Chapter 4 will explain the data sources and the

Table 1. Largest Turkish Pension Funds

Fund name	Fund size
AvivaSA Emeklilik ve Hayat	14.8 bln
Anadolu Hayat Emeklilik	14.1 bln
Garanti Emeklilik ve Hayat	11.1 bln
Allianz Yasam ve Emeklilik	10.4 bln
Vakif Emeklilik	6.1 bln

Source: Pension Monitoring Center (2018)

structure of our simulation. Chapter 5 will present the results of the simulation and Chapter 6 will conclude our findings. The used sources will be listed in Bibliography. All the relevant proofs will be available in Appendices.

CHAPTER 2

LITERATURE REVIEW

The field of financial economics has gone through big changes since its foundation by Markowitz (1952) and Tobin (1958). They pioneered the mean-variance analysis, which, given some assumptions, suggested that if investors cared about maximizing returns (mean) and minimizing risks (variance), then the optimal ratio of stocks to bonds in a single-period portfolio would be fixed for everyone, the share of former being equal to:

$$\alpha = \frac{\mu - R_f}{\gamma \sigma^2} \tag{2.1}$$

Merton (1971) generalized the problem to multiple periods using dynamic programming and found that it is optimal for all households to repeat the same fixed mean-variance solution every period.

These results were inconsistent with the popular financial advice suggesting that younger investors should have higher share of stocks in portfolio, and older investors — higher share of bonds. This advice was summarized by the famous rule of thumb:

$$\alpha_t = (100 - t)\% \tag{2.2}$$

Although Samuelson (1969) denied that risk-aversion changes by age, dismissing this advice would question the rationality of investors and "constitute *prima facie* evidence that people do not optimize" (Canner, Mankiw, and Weil (1997)).

Bodie, Merton, and Samuelson (1992) solved this problem by adding human capital into the Merton (1971)'s dynamic model and found that for complete markets and constant risk-free labor income, the optimal share of stocks in a portfolio is:

$$\alpha_t = \frac{\mu - R_f}{\gamma\sigma^2}\left(\frac{F_t + L_t}{F_t}\right) \tag{2.3}$$

Steady depletion of human capital L_t relative to the financial wealth F_t throughout life, explained the higher share of stocks in younger people. Cocco, Gomes, and J. (2005) extended this idea to the case of variable labor income, to find a recursive solution which could be approximated by the following rule of thumb:

$$\alpha_t = \begin{cases} 100\% & t < 40 \\ (200 - 2.5t)\% & t \in [40, 60] \\ 50\% & t > 60 \end{cases} \tag{2.4}$$

However, the hump-shaped lifetime stock share graph, observed by Chang, Hong, and Karabarbounis (2014) of Federal Reserve, instead of expected downward sloping one, suggested the presence of an opposing force.

Cocco (2005) found that this force was housing investment, which, due to its large size, crowded out all stocks from younger investors' portfolios. Flavin and Yamashita (2002) supported this view by showing that younger people, who already own the house, tend to invest more aggressively, as was expected by Bodie et al. (1992).

Munk (2016) found the same patterns using a series of one-period mean-variance optimizations without any dynamic stochastic modeling tools.

Finally, Ascheberg, Kraft, Munk, and Weiss (2013) illustrated the existence of long-term cointegration among house prices, stock prices, and labor income, the fact often omitted by previous portfolio researchers for simplicity. In our analysis we will not neglect the correlations as being equal to zero.

CHAPTER 3

MODEL

3.1 House prices, stock prices and labor income series

In accordance with Campbell, Cocco, Gomes, and Maenhout (1999) and Olear (2016) we model the labor income process as a function of individiual characteristics $f(t, Z_{it})$ plus idiosyncratic shocks v_{it}. Upon reaching the retirement age R, an individual receives a certain percentage λ of his/her last wage:

$$Y_{i,t+1} = \begin{cases} Y_{it}(1 + f(t+1, Z_{i,t+1}) + v_{it}), & t < R \\ \lambda(1 + f(R, Z_{iR}) + v_{iR}), & t \geq R \end{cases} \tag{3.1}$$

We model labor income, house prices, and stock prices as Geometric Brownian Motions with drifts μ_L, μ_H, μ_S and volatilities σ_L, σ_H, σ_S, satisfying the discrete version of Ascheberg et al. (2013)'s correlation structure, that is having nonzero correlations $\rho_{HS}, \rho_{HL}, \rho_{SL}$. See Appendix A for details.

3.2 Optimal portfolio

Along with the investment strategies, described in Equations 2.1 - 2.4, we consider in our benchmarking, the strategy proposed by Munk (2016), who stated that in the presence of housing, the optimal stock (π) and housing (π_h) shares can be solved analytically as follows:

$$\pi_{t+1} = \frac{1}{\gamma(1-\rho_{SH}^2)\sigma_S} \cdot \frac{F_t + L_t}{F_t} \left(\frac{\mu_S - r_f}{\sigma_S} - \rho_{SH} \frac{\mu_H - r_f}{\sigma_H} \right) - \frac{L_t}{F_t} \cdot \frac{\sigma_L}{\sigma_S} \frac{\rho_{SL} - \rho_{SH}\rho_{HL}}{1 - \rho_{SH}^2} \tag{3.2a}$$

$$\pi_{h,t+1} = \frac{1}{\gamma(1-\rho_{SH}^2)\sigma_H} \cdot \frac{F_t + L_t}{F_t} \left(\frac{\mu_H - r_f}{\sigma_H} - \rho_{SH} \frac{\mu_S - r_f}{\sigma_S} \right) - \frac{L_t}{F_t} \cdot \frac{\sigma_L}{\sigma_H} \frac{\rho_{HL} - \rho_{SH}\rho_{SL}}{1-\rho_{SH}^2} \tag{3.2b}$$

Setting $\rho_{SH} = 0$ and $\rho_{HL} = 0$, gives the optimal stock share by Munk (2016) in the absence of housing:

$$\pi_{t+1} = \frac{1}{\gamma\sigma_S} \cdot \frac{F_t + L_t}{F_t} \left(\frac{\mu_S - r_f}{\sigma_S} \right) - \frac{L_t}{F_t} \cdot \frac{\sigma_L}{\sigma_S} \rho_{SL} \tag{3.3}$$

3.3 Welfare measurement

We use stochastic constant relative risk-aversion utility function to compare welfare resulting from different income patterns:

$$E_1[U(c)] = \sum_{t=1}^{T} \delta^{t-1} \prod_{j=0}^{t-1} p_j \cdot \frac{c_{it}^{1-\gamma}}{1-\gamma} \tag{3.4}$$

where p_k is the probability of survival between ages $k-1$ and k. Unlike Cocco et al. (2005), we neglect the bequest motives, assuming that the retired person consumes all of his/her income at any given time.

3.4 Retirement income

The funds invested in retirement are modeled to be paid back in annuities, not withdrawn immediately. Further, to include housing investment in welfare calculation, we use “reverse mortgages” — annuities, paid to retired individuals in return for inheriting their house after their death. This is a plausible analysis tool, because it allows to liquidify the housing possessions, although such financial instrument is not yet available in Turkey.

Thus, at the age of retirement $R = 65$, the price of owned house is calculated and is added to the matured pension amount (MP) to obtain total wealth:

$$W_{65} = H_{65} + MP \tag{3.5}$$

All of the W_{65} is used to buy an annuity which will annually repay an individual:

$$A_t = W_{65} \cdot \left(1 + \sum_{t=66}^{100} \frac{\prod_{j=66}^{t} p_j}{(1+r_f)^{t-65}}\right)^{-1} \tag{3.6}$$

CHAPTER 4

DATA AND SIMULATION

In this section, we go over data sources, perform parameter calibrations and derive all investment strategies for our simulation.

4.1 Data sources

We use historical monthly BIST 30[1] and REIDIN[2] data from 2004 to 2014, to construct stock and house price series respectively. Figures 1 and 2 illustrate the general upward trend in both series, with a collapse during 2008 crisis.

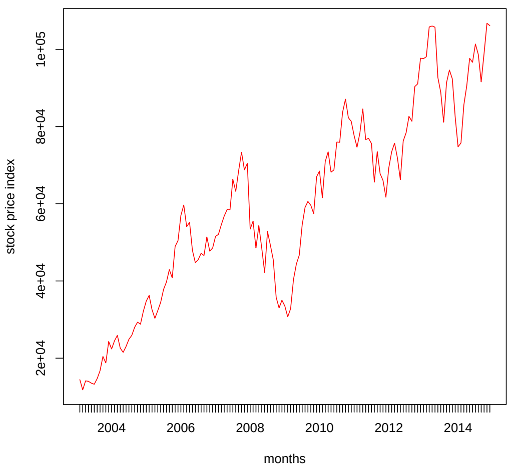


Figure 1. BIST30 Turkish stock market prices

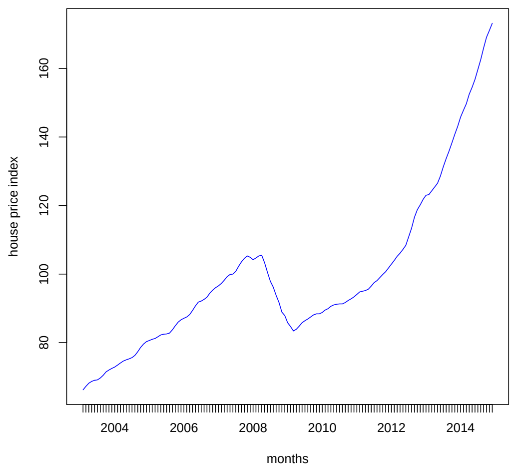


Figure 2. Reidin Turkish house price index

We construct labor income series from Turkish Statistical Institute (2014)'s repeated cross-sectional study, and, in line with Aktuğ, Kuzubaş, and Torul (2017), we aggregate the data to obtain a pseudo-panel with 55 thousand data points for 170 households from 2001 to 2014. Figure 3 displays the hump-shaped lifetime income

[1]BIST 30 is an index measuring the stock performance of 30 largest companies in Turkey

[2]REIDIN provides residential sales price index for Turkey, using data, covering 200,000 house listings in 62 cities and 221 counties, per month, weighted by population, and calculated using Laspeyres' formula.

distribution, consistent with the results of Aktuğ et al. (2017), who analyzed labor income profiles in Turkey, and Ben-Porath (1967), who predicted a decline in productivity, as workers get older.

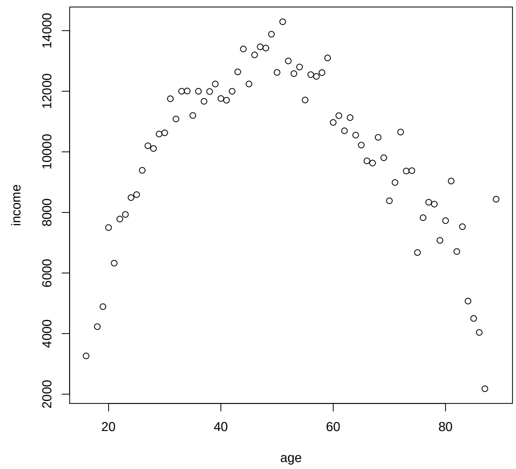


Figure 3. Median Turkish salaries by age

4.2 Default parameters

Similarly to Munk (2016), we start our simulation with a 25-year-old individual who invests in his/her retirement for 40 years until he/she reaches retirement at 65. Similarly to Torul and Öztunalı (2018), we set the default relative risk aversion coefficient for Turkish households at 1.5, and the subjective discount rate — at 0.89.

We deflate the nominal wages, stock and house prices by CPI and work with real variables.

We obtain annual rate of return on stocks 6.69% with volatility 38.44%, by annualizing long-term ARMA(2,2) forecasts of monthly return and volatility, based on historical BIST 30 data, mentioned above (see Appendix B.1).

Similarly, we use long-term ARMA(1,1) forecast of monthly return and volatilities on housing, and find annual real rate of return on housing 0.67% with 5.42% volatility (see Appendix B.2).

Risk-free rate 12% is given by OECD (2018) forecast[3], and, upon subtracting the medium-term inflation rate forecast $\pi = 9\%$ by TCMB (2018), is equal to 3% per annum with zero volatility.

We consider real wage growth rates separately for different types of agents, but before introducing heterogeneity, the ARMA(5,2) forecast gives the volatility 4% (see Appendix B.3).

In our simulation, the house-stock and house-wage contemporaneous correlations are given by 0.27 and 0.35 respectively.

Survival probabilities for all ages are provided by Turkish Statistical Institute (2018) and illustrated in Figure 4.

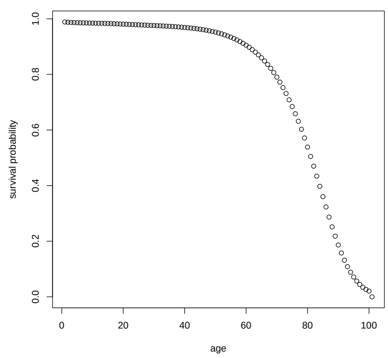


Figure 4. Survival probabilities by age

All of the above findings are summarized in Table 2.

4.3 Heterogeneity parameters

Combining the approaches of Olear (2016) and Munk (2016), we use wage growth rate, stock-income correlation and relative risk aversion level, to model heterogeneities among agents.

[3]Data was obtained before the Turkish currency and debt crisis of 2018

Table 2. Benchmark Parameters

Parameter	Description	Value
Y	Beginning age	25
R	Retirement age	65
T	Lifespan (years)	100
γ	Risk aversion	1.5
β	Discount rate	0.89
r_f	Risk-free rate	0.03
μ_s	Expected stock returns	0.0669
μ_h	Expected housing returns	0.0067
σ_s	Stock returns volatility	0.3844
σ_h	Housing returns volatility	0.0542
σ_w	Wage growth volatility	0.036
ρ_{hs}	House-stock correlation	0.24
ρ_{hw}	House-wage correlation	0.37
p_{25}	Survival probability at age 25	0.978
p_{65}	Survival probability at age 65	0.86
p_{100}	Survival probability at age 100	0

4.3.1 Heterogeneity in education

We model the heterogeneity in education using differences in wage growth rates. Figure 5 shows the lifetime labor income series for different levels of education. Notice that the curves are almost flat for the lower education levels and hump-shaped for the higher. We use undergraduate education, high school education, and no schooling, to model "steep", "moderate", and "flat" wage dynamics respectively. Performing regressions of wages on age, with kinks at $t = 40$ and $t = 55$:

$$\Delta \log(wage_{it}) = \alpha_0 + \alpha_1 \cdot d_{40} + \alpha_2 \cdot d_{55} \tag{4.1}$$

we estimate growth rates for different education levels, as summarized in Table 3 (see Appendix B.4 for regression results and line fits).

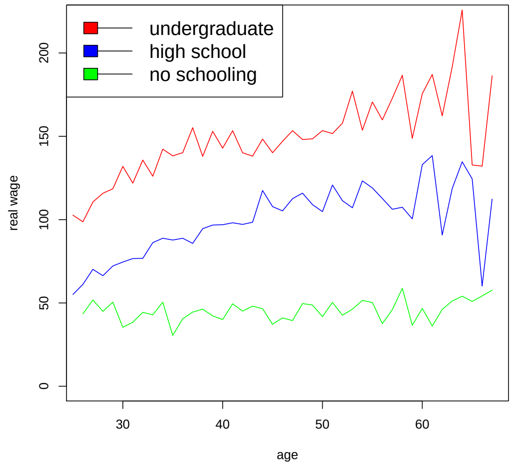


Figure 5. Lifetime real wage dynamics by education level.

Table 3. Estimated Benchmark Wage Growth Rates μ_w

Age	Flat	Moderate	Steep
25-40	0%	3.8%	2.2%
41-55	0%	1.4%	1.2%
56-65	0%	0%	1.5%

We assume steep wage earners (college graduates) to have a starting real salary of 100, and moderate and flat wage earners to have a starting real salary of 50. This is consistent with the historical data (see Figure 5) and the relevant discussion by Olear (2016). This difference in starting values also explains why "steep" wage growth rates are less than "moderate" ones.

4.3.2 Heterogeneity in sectors of work

We model the heterogeneity work sectors using corresponding stock-wage correlations (ρ_{ws}). Figure 6 illustrates how, during 2008 crisis, income in financial sectors ($\rho_{ws} = 0.44$) dropped drastically, while it was unaffected in education and agriculture ($\rho_{ws} = 0.08$). We use three measures of ρ_{sw} for our benchmark: 0, 0.2 and 0.4 (see Table 4).

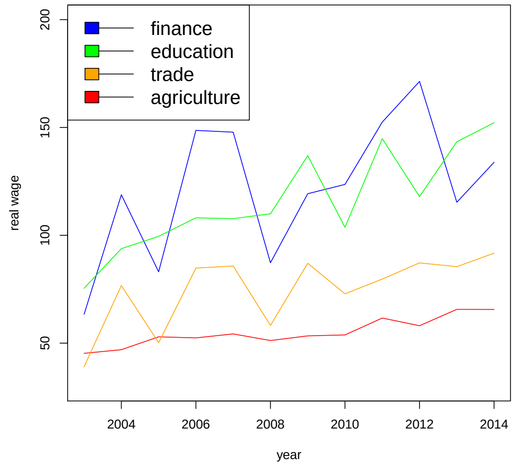


Figure 6. Historical real wage dynamics by sector.

Table 4. Benchmark Wage to Stock Correlations

	Low	Moderate	High
ρ_{sw}	0	0.2	0.4

4.3.3 Individual heterogeneity

We model individual heterogeneity using different risk aversion levels of investors, as summarized in Table 5.

Table 5. Coefficients of Risk Aversion

Values	default	low	moderate	high
γ	1.5	3	5	10

4.4 Capital series

4.4.1 Human capital

Human capital at any period is the discounted sum of all future wages until retirement with the discount factor r_f. To construct the individualized capital we used steep, moderate and flat wage series mentioned in the previous section. Figure 7 illustrates the current human capital for flat, moderate, and steep wages for every age.

4.4.2 Financial capital

Financial capital evolves according to dynamic investment diagram in Figure 9. Every period, a certain percentage c (3% for Turkey) of the wage w_t is invested in a retirement portfolio, while the previously invested amount accrues interest at portfolio rate of return. Figure 8 demonstrates the evolution of financial capital and its confidence interval for a naive fixed investment strategy (50% in stocks and 50% in bonds) by an individual with “steep” wage curve.

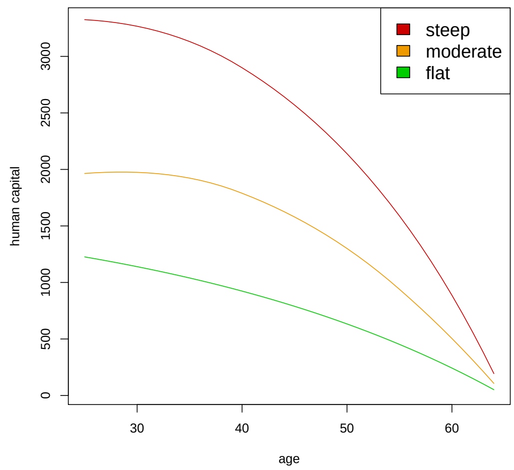


Figure 7. Human capital at every age for steep, moderate and flat wages.

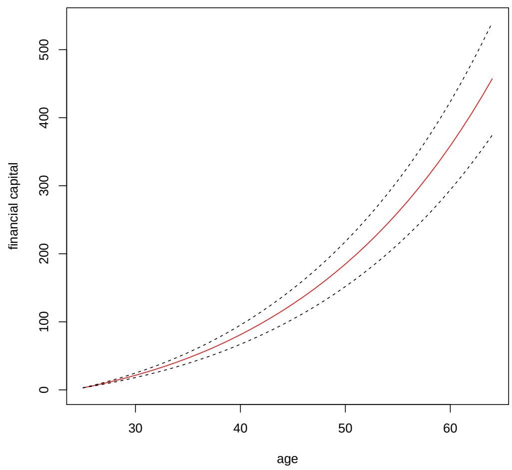


Figure 8. Financial capital at every age.

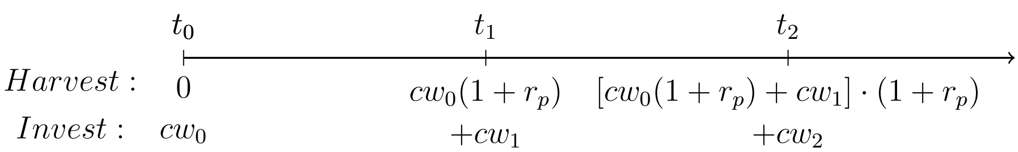


Figure 9. Law of motion of financial capital.

4.5 Investment strategies

Below, we present the life-cycle investment strategies to be benchmarked.

4.5.1 Homogeneous life-cycles

Homogeneous life-cycles are strategies, common to all individuals, regardless of their idiosyncratic characteristics. We benchmark strategies given by equations 2.1, 2.2, 2.4, and an aggressive portfolio allocation, offered by Turkish banks (60% in stocks, 40% in bonds). Figure 10 illustrates the stock shares in these investment strategies.

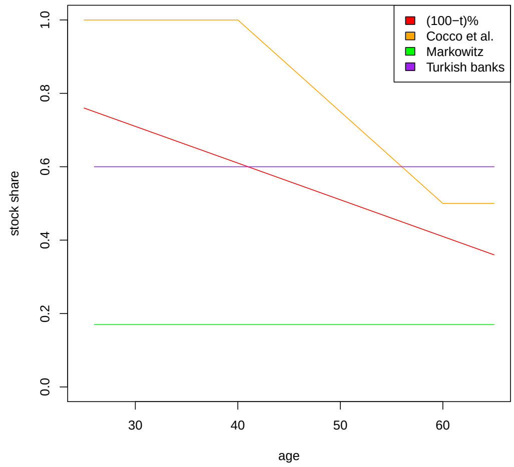
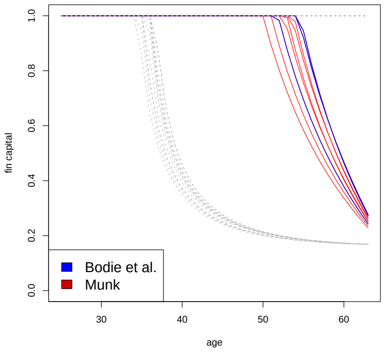


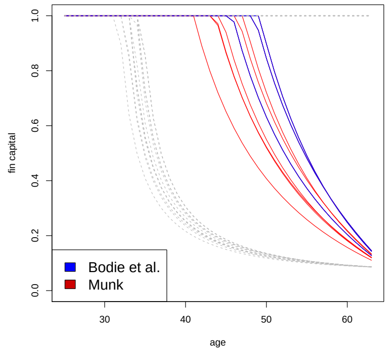
Figure 10. Share of stocks in a portfolio at every age, proposed by homogeneous life-cycle strategies

4.5.2 Heterogeneous life-cycles

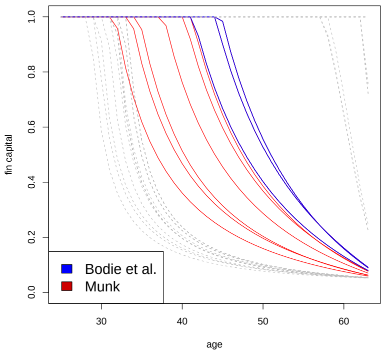
Heterogeneous life-cycles take idiosyncratic characteristics into consideration. We benchmark strategies by Bodie et al. (1992) (see Equation 2.3), and Munk (2016) (see Equation 3.3), which are illustrated in Figure 11 by blue and red curves respectively. The dashed lines illustrate the 68%-confidence interval ($\pm 1\sigma$). The figure proposes younger investors to allocate all of their funds in stocks, and gradually, through life, decrease their share in the portfolio — the more risk-averse they are or the flatter their wage curve is, the sooner. It also shows that, other things being equal, Munk (2016)'s strategy without housing, is less aggressive than Bodie et al. (1992)'s, which is consistent with equations 2.3 and 3.3.



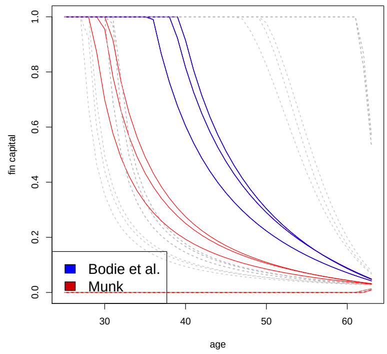
(a) $\gamma = 1.5$



(b) $\gamma = 3$



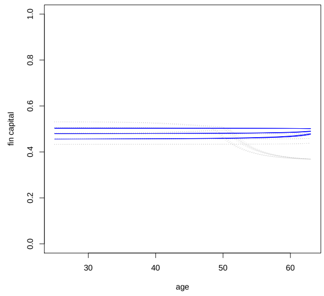
(c) $\gamma = 5$



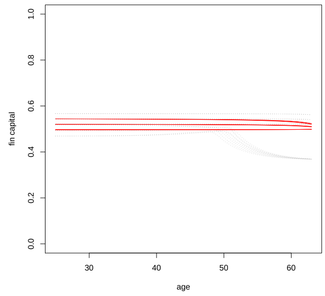
(d) $\gamma = 10$

Figure 11. Shares of stocks in a portfolio, suggested by Bodie, Merton, and Samuelson (1992) and Munk (2016).

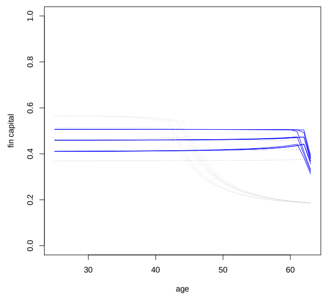
Figure 12 shows the stock and house shares, suggested by Munk (2016) for flat, moderate and steep labor income curves, low, moderate and high stock-wage correlations, and different levels of risk aversion. It confirms that steeper labor income curves results in a larger share of stocks in portfolio, and the more risk averse individuals are, the sooner they decrease both housing and stock investment, and buy more bonds.



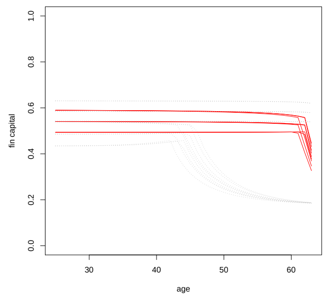
(a) Stocks for $\gamma = 1.5$



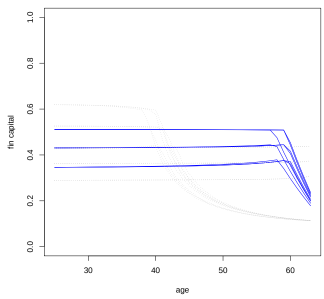
(b) Housing for $\gamma = 1.5$



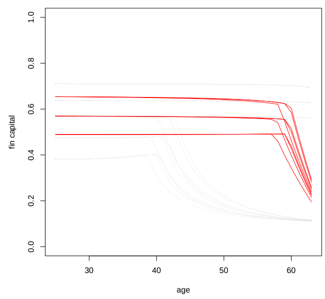
(c) Stocks for $\gamma = 3$



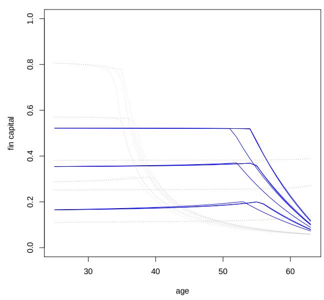
(d) Housing for $\gamma = 3$



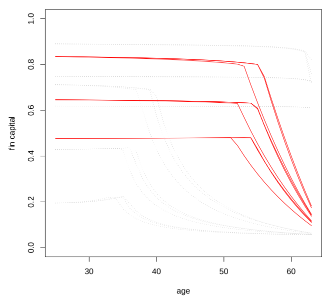
(e) Stocks for $\gamma = 5$



(f) Housing for $\gamma = 5$



(g) Stocks for $\gamma = 10$



(h) Housing for $\gamma = 10$

Figure 12. Munk's stock and housing shares for different wage growth, stock-wage correlation and risk aversion levels

CHAPTER 5

RESULTS

In this section, we calculate the accumulated wealth for every investment option above and benchmark the resulting expected utilities.

5.1 Accumulated wealth

After the lifetime of investing, households accumulate different levels of wealth. Tables 6 and 7 show the mean of 10,000 stock and house price realizations from a Monte Carlo simulation.

Table 6. Monte Carlo Results of Accumulated Wealth for Homogeneous Investment Strategies

wages	steep	moderate	flat
Markowitz	345	203	131
$100 - age$	458	266	179
Cocco et al.	567	327	226
Turkish banks	494	287	193

5.2 Annuities

Retired individuals will sell all of their accumulated wealth to buy annuities and consume them fully until their death, enjoying the utility of this consumption. Recall that we ignore bequest motives and savings after the retirement. We define annuities by dividing the total wealth before retirement by the discount factor $1 + \sum_{t=66}^{100} \frac{p_t}{(1+r_f)^{t-65}}$. Using survival probabilities, obtained from TUIK, and risk-free rate of return, described in the previous chapter, we calculate the discount factor as 13.73. The annuities are obtained by dividing all the values in the Tables 6 and 7 by 13.73.

Table 7. Monte Carlo Results of Accumulated Wealth for Heterogeneous Investment Strategies

wages	steep			moderate			flat		
ρ_{ws}	high	moderate	low	high	moderate	low	high	moderate	low
$\gamma = 1.5$									
Bodie et al.	563	563	563	327	327	327	214	214	214
Munk (no housing)	539	547	554	313	317	321	204	208	210
Munk (housing)	413	427	441	241	249	257	158	163	169
$\gamma = 3$									
Bodie et al.	498	498	498	290	290	290	187	187	187
Munk (no housing)	457	478	494	267	279	287	171	179	185
Munk (housing)	532	553	566	309	321	328	200	209	214
$\gamma = 5$									
Bodie et al.	455	455	455	266	266	266	170	170	170
Munk (no housing)	380	426	454	223	249	265	141	159	169
Munk (housing)	509	523	535	296	304	311	191	197	201
$\gamma = 10$									
Bodie et al.	408	408	408	239	239	239	151	151	151
Munk (no housing)	380	426	454	223	249	265	141	159	169
Munk (housing)	509	523	535	296	304	311	191	197	201

5.3 Expected utilities

Recall that we have defined the wages in real terms, dividing the nominal wages by CPI. Therefore we can plug the values, obtained above, to the CRRA utility function as consumption level. Tables 8-11 contains utilities of mean values of accumulated wealth from different scenarios of a Monte Carlo experiment, mentioned above.

Tables 8-11 show the following results:

- Cocco et al.'s $(200 - 2.5 \cdot age)\%$ performs better, on average, than any other portfolio.
- Even a naive life-cycle investment portfolio $(100 - age)\%$ overperforms fixed-over-lifetime Markowitz.

- All models perform better for higher risk aversion and worse for lower risk aversion.
- Munk's solution performs worse for flat wages than for steep wages.
- Munk's solution with housing is better than without housing when $\gamma > 1.5$.
- When $\rho_{ws} = 0$, Bodie's solution is almost equal to Munk's solution without housing, with the former performing slightly better than the latter.
- Munk's solution with housing performs better for sectors with low stock-wage correlation, being a low-risk investment.

Table 8. Summary of Expected Utilities from Simulation for $\gamma = 1.5$

wages	steep			moderate			flat		
ρ_{ws}	high	moderate	low	high	moderate	low	high	moderate	low
Markowitz	-2.456	-2.456	-2.456	-3.202	-3.202	-3.202	-3.986	-3.986	-3.986
$100 - age$	-2.132	-2.132	-2.132	-2.798	-2.798	-2.798	-3.410	-3.410	-3.410
Cocco et al.	-1.916	-1.916	-1.916	-2.523	-2.523	-2.523	-3.035	-3.035	-3.035
Turkish banks	-2.053	-2.053	-2.053	-2.693	-2.693	-2.693	-3.284	-3.284	-3.284
Bodie et al.	-1.923	-1.923	-1.923	-2.523	-2.523	-2.523	-3.119	-3.119	-3.119
Munk (no housing)	-1.965	-1.951	-1.938	-2.579	-2.563	-2.547	-3.194	-3.164	-3.149
Munk (housing)	-2.245	-2.208	-2.173	-2.939	-2.891	-2.846	-3.630	-3.574	-3.510

Table 9. Summary of Expected Utilities from Simulation for $\gamma = 3$

wages	steep			moderate			flat		
ρ_{ws}	high	moderate	low	high	moderate	low	high	moderate	low
Markowitz	-0.0049	-0.0049	-0.0049	-0.0141	-0.0141	-0.0141	-0.0338	-0.0338	-0.0338
$100 - age$	-0.0028	-0.0028	-0.0028	-0.0082	-0.0082	-0.0082	-0.0181	-0.0181	-0.0181
Cocco et al.	-0.0018	-0.0018	-0.0018	-0.0054	-0.0054	-0.0054	-0.0114	-0.0114	-0.0114
Turkish banks	-0.0024	-0.0024	-0.0024	-0.0070	-0.0070	-0.0070	-0.0156	-0.0156	-0.0156
Bodie et al.	-0.0023	-0.0023	-0.0023	-0.0069	-0.0069	-0.0069	-0.0166	-0.0166	-0.0166
Munk (no h.)	-0.0028	-0.0025	-0.0024	-0.0081	-0.0075	-0.0070	-0.0198	-0.0181	-0.0170
Munk (h.)	-0.0021	-0.0019	-0.0018	-0.0061	-0.0056	-0.0054	-0.0145	-0.0133	-0.0127

Table 10. Summary of Expected Utilities from Simulation for $\gamma = 5$

wages	steep			moderate			flat		
ρ_{ws}	high	moderate	low	high	moderate	low	high	moderate	low
Markowitz	-4e-06	-4e-06	-4e-06	-0.00003	-0.00003	-0.00003	-0.00019	-0.00019	-0.00019
$100 - age$	-1e-06	-1e-06	-1e-06	-0.00001	-0.00001	-0.00001	-0.00005	-0.00005	-0.00005
Cocco et al.	-1e-06	-1e-06	-1e-06	-0.00001	-0.00001	-0.00001	-0.00002	-0.00002	-0.00002
Turkish banks	-1e-06	-1e-06	-1e-06	-0.00001	-0.00001	-0.00001	-0.00004	-0.00004	-0.00004
Bodie et al.	-1e-06	-1e-06	-1e-06	-0.00001	-0.00001	-0.00001	-0.00007	-0.00007	-0.00007
Munk (no h.)	-3e-06	-2e-06	-1e-06	-0.00002	-0.00001	-0.00001	-0.00014	-0.00004	-0.00004
Munk (h.)	-1e-06	-1e-06	-1e-06	-0.00001	-0.00001	-0.00001	-0.00004	-0.00004	-0.00004

Table 11. Summary of Expected Utilities from Simulation for $\gamma = 10$

wages	steep			moderate			flat		
ρ_{ws}	high	moderate	low	high	moderate	low	high	moderate	low
Markowitz	-1.7e-13	-1.7e-13	-1.7e-13	-2e-11	-2e-11	-2e-11	-1.0e-09	-1.0e-09	-1.0e-09
$100 - age$	-1.0e-14	-1.0e-14	-1.0e-14	-1.8e-12	-1.8e-12	-1.8e-12	-6.3e-11	-6.3e-11	-6.3e-11
Cocco et al.	0	0	0	-2.8e-13	-2.8e-13	-2.8e-13	-7.7e-12	-7.7e-12	-7.7e-12
Turkish banks	-1.0e-14	-1.0e-14	-1.0e-14	-9e-13	-9e-13	-9e-13	-3.2e-11	-3.2e-11	-3.2e-11
Bodie et al.	-4.0e-14	-4.0e-14	-4.0e-14	-4.7e-12	-4.7e-12	-4.7e-12	-2.9e-10	-2.9e-10	-2.9e-10
Munk (no h.)	-5.7e-13	-1.7e-13	-4.0e-14	-6.6e-11	-1.9e-11	-4.8e-12	-4e-09	-1.2e-09	-2.9e-10
Munk (h.)	-3.0e-14	-1.0e-14	-1.0e-14	-3.3e-12	-1.8e-12	-1.2e-12	-1.9e-10	-1e-10	-6.6e-11

CHAPTER 6

CONCLUSION

In this thesis, we have reviewed the concept of "life-cycle investment" and summarized the common models and heuristics. We have presented our model as an application of Munk's (2016) recent findings and Olear's (2016) simulation techniques into Turkish retirement market.

We have collected historical data to calibrate and estimate the best parameters to be used in our simulation. Using these parameters, we have constructed heterogeneous agents, who worked and invested throughout their lifetime. We considered different investment models that our hypothetical agents would use and calculated the resulting investment capitals. Finally, we have calculated and compared the welfare effects of all popular models to the individualized Munk's solutions.

We have concluded that heuristic, provided by Cocco (2005) based on their underlying dynamic programming model, provides a higher utility on average than any other investment option.

We also found that even naive life-cycle investment heuristics perform better than fixed Markowitz solution.

Our simulation showed that for risk-averse individuals with riskless wages, it is best to invest in housing in a proportion proposed by Munk.

We propose these models to Turkish pension providers, as these options will increase their retirement welfare considerably.

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APPENDIX A

ASCHEBERG'S CORRELATION STRUCTURE

The structure that returns desired correlation coefficients ρ_{SL}, ρ_{SH} and ρ_{HL} is as follows:

$$\frac{\Delta S_{t+1}}{S_t} = \mu_S + \sigma_S \cdot \epsilon_{St} \tag{A.1a}$$

$$\frac{\Delta H_{t+1}}{H_t} = \mu_H + \sigma_H \cdot \left(\rho_{SH}\epsilon_{St} + (\sqrt{1-\rho_{SL}^2})\epsilon_{Ht} \right) \tag{A.1b}$$

$$\frac{\Delta Y_{t+1}}{Y_t} = \mu_L + \sigma_L \cdot \left(\rho_{SL}\epsilon_{St} + \left(\frac{\rho_{HL} - \rho_{SH}\rho_{SL}}{\sqrt{1-\rho_{SH}^2}} \right) \epsilon_{Ht} + \left(\sqrt{1-\rho_{SL}^2 - (\frac{\rho_{HL} - \rho_{SH}\rho_{SL}}{\sqrt{1-\rho_{SH}^2}})^2} \right) \epsilon_{Lt} \right) \tag{A.1c}$$

To derive this, let Σ be a correlation matrix of a vector $X = (x_1, x_2, ..., x_K)$. Also, let $\Sigma = LL'$ be a Cholesky decomposition of this matrix.
Notice that the variance-covariance matrix of an i.i.d. random vector $\Omega = (\epsilon_1, \epsilon_2, ..., \epsilon_K)$ with variances equal to 1, is an identity matrix. Thus, the product $L\Omega$ has the same correlation structure as X:

$$cov(L\Omega) = E[(L\Omega)(L\Omega)'] = E[L\Omega\Omega' L'],$$
$$cov(L\Omega) = L \cdot E[\Omega\Omega'] \cdot L' = L \cdot var(\Omega) \cdot L',$$
$$cov(L\Omega) = L \cdot I \cdot L' = LL' = \Sigma$$

The conclusion comes from the fact that the Cholesky decomposition of a correlation matrix R:

$$R = \begin{bmatrix} 1 & \rho_{SH} & \rho_{SL} \\ \rho_{SH} & 1 & \rho_{HL} \\ \rho_{SL} & \rho_{HL} & 1 \end{bmatrix}$$

can be easily calculated to be equal to Q:

$$Q = \begin{bmatrix} 1 & 0 & 0 \\ \rho_{SH} & \sqrt{1-\rho_{SH}^2} & 0 \\ \rho_{SL} & \frac{\rho_{HL}-\rho_{SH}\rho_{SL}}{\sqrt{1-\rho_{SH}^2}} & \sqrt{1-\rho_{SL}^2 - (\frac{\rho_{HL}-\rho_{SH}\rho_{SL}}{\sqrt{1-\rho_{SH}^2}})^2} \end{bmatrix}$$

APPENDIX B

PARAMETER CALIBRATIONS

B.1 Stock returns

We use log differences on our monthly stock price data to obtain historical monthly rates of return. The Augmented Dickey-Fuller test strongly shows that the modified series is stationary at 1% significance level.

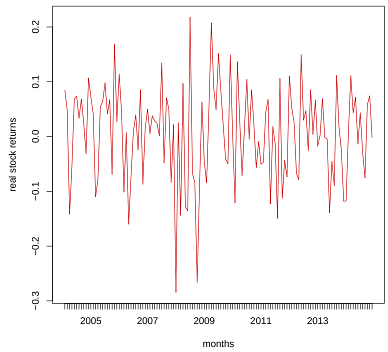

Figure B1. Historical monthly rates of return on stocks

We use Akaike Information Criterion to find the optimal lag order for ARIMA estimation — $p = q = 2$.

Performing ARIMA(2,0,2) estimation, we obtain the long-term forecasted monthly rate of return $\mu^S_{mon} = 0.541\%$ and volatility $\sigma^S_{mon} = 11.1\%$.
Finally, we annualize these values as follows:

$$\mu_S = (1 + \mu^S_{mon})^{12} - 1 = 6.69\% \tag{B.1}$$

$$\sigma_S = \sigma^S_{mon} \cdot \sqrt{12} = 38.44\% \tag{B.2}$$

B.2 Housing returns

Similarly, we log-differentiate monthly house prices to obtain growth rates. The house market collapse of 2008 brings large external shock, causing the Augmented Dickey-Fuller test to only find stationarity at 10% significance level for 3 lags at most. Again, using Akaike Information Criterion, we find $p = q = 1$.

ARIMA(1,0,1) estimation provides long-term forecasts for monthly rate of return and volatility as $\mu^H_{mon} = 0.06\%$ and $\sigma^H_{mon} = 1.57\%$.
Annualization gives:

$$\mu_H = (1 + \mu^H_{mon})^{12} - 1 = 0.67\% \tag{B.3}$$

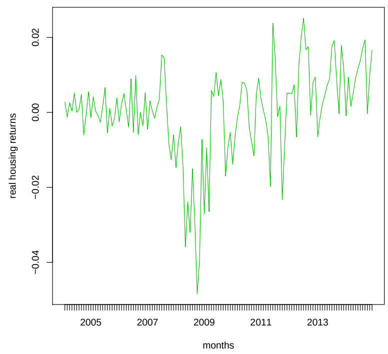

Figure B2. Historical monthly rates of return on housing

$$\sigma_H = \sigma^H_{mon} \cdot \sqrt{12} = 5.42\% \tag{B.4}$$

B.3 Labor income volatility

The Augmented Dickey-Fuller test returns stationarity for up to 9 lags at 5% significance level.

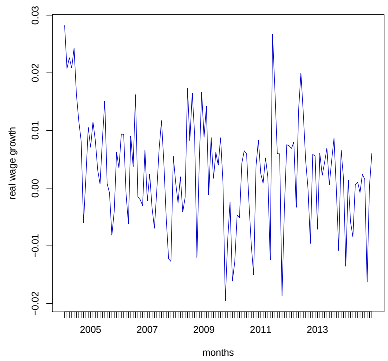

Figure B3. Historical monthly wage growth rates

Akaike Information Criterion suggests $p = 5$ and $q = 2$.

ARIMA(5,0,2) estimation gives monthly volatility $\sigma^L_{mon} = 1.04\%$, which annualizes as follows:

$$\sigma_L = \sigma^L_{mon} \cdot \sqrt{12} = 3.59\% \tag{B.5}$$

B.4 Wage regression results

The regressions of wage growth rates by age with kinks at 40 and 55, return the following coefficients.

The growth rates can be calculated using these coefficients. Note that we have rounded the growth rates for the flat wages to 0. Figure B4 illustrates how the estimated rates fit the data:

Table B1. Wage regression results by age

	flat	moderate	steep
(intercept)	0.1%	0.4%	1.5%
d40	-0.7%	3.3%	0.7%
d55	1.4%	0.9%	-0.4%

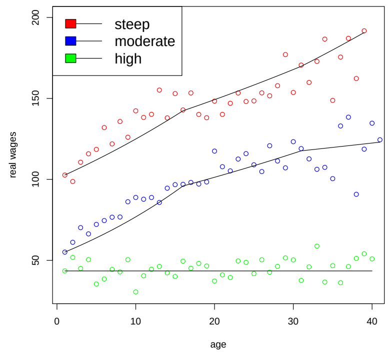

Figure B4. Fitted values from wage regressions