MANAGING IRRIGATION-INDUCED SALINITY: THE CASE OF HARRAN PLAIN

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MANAGING IRRIGATION-INDUCED SALINITY: THE CASE OF HARRAN PLAIN

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Thesis Abstract

“Managing Irrigation-induced Salinity: The Case of Harran Plain”

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In this study, irrigation-induced salinity is analysed through a dynamic model. In the model, all the farmers in two neighboring Water User Associations (WUAs), which are managed independently, contribute to the accumulation of a common groundwater. However, by assumption, they are asymmetrically affected by the rising watertable due to the slope in the region. Since each farmer’s benefit function depends on not only his own irrigation parcatices, but also the groundwater accumulation, there are strategic interactions among all the farmers. We use differential games as the methodology and the open-loop Nash equilibrium as the solution concept. We analyse the model under four different scenarios. We show that even when each farmer moves in cooperation with other member farmers in his/her own WUA, socially optimal level of groundwater accumulation cannot be reached. Under these conditions, two different input taxes based on the level of groundwater accumulation are offered to each type of farmers to induce them to take sustainable irrigation decisions.

Tez Özeti

Managing Irrigation-induced Salinity: The Case of Harran Plain

Hakan Karaca

Bu çalışmada, sulama-kaynaklı tuzlanma sorunu dinamik bir modelden yararlanılarak analiz edilmeye çalışılmıştır. Modelde, birbirinden bağımsız yönetilen 2 komşu sulama birliğinde yaşayan bütün çiftçiler ortak bir yeraltı suyunu paylaşmaktadır. Fakat, bölgedeki eğim yüzünden çiftçilerin yükselen taban suyundan asimetrik olarak etkilendikleri varsayılmıştır. Herbir çiftçinin fayda fonksiyonu kendi kullandıkları sulama suyu yanında yeraltı suyunun seviyesine de bağlı olduğu için, bütün çiftçiler arasında stratejik bir etkileşim söz konusudur. Bu çalışmada, metodoloji olarak diferansiyel oyunlar, çözüm konsepti olarak da Açık-Döngülü Nash Dengesi kullanılmıştır. Model 4 farklı senaryo üstünden işlenmiştir. Bu bağlamda, herbir çiftçi kendi sulama birliğindeki diğer çiftçilerle ortak hareket etse dahi, toplumsal olarak optimal olan yer altı suyu seviyesine ulaşılamayacağı gösterilmiştir. Bu koşullar altında, tipine göre herbir çiftçiye önerilmek üzere yeraltı suyuna bağlı olarak degişen 2 farklı girdi vergisi hesaplanmıştır.

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CHAPTER 1

1. Introduction

Fresh water as a renewable natural resource captures very important roles in our daily life. It can be used for private purposes as well as for commercial purposes. Among many sectors, which use water as an input, one of major consumers of fresh water is the agricultural sector, which uses it for irrigation purposes. On 18% of the total arable land in the world (approximately 230 million hectares), irrigation is used in agricultural practices. Moreover, more than 33% of the total agricultural output is produced in this irrigated land (Johansson, 2000). However, irrigated agriculture, which is much more productive in food production than non-irrigated agriculture, has also extensive negative effects on environment through irrigation-induced problems such as salinization and waterlogging.

Salinization is the accumulation of soluble salts at the soil surface, or at some point below the soil surface, to levels that have negative effects on plant growth and/or on soils (Ritzema et al., 1996). As mentioned in Wilchens (1999), leakage from poorly lined canals and reservoirs, excessive water application, and inadequate drainage of agricultural land are the main causes of irrigation-induced salinity. In the presence of salinization, plants cannot take up water and required nutrients from the soil. This inhibits plant growth or leads to possibly plant death (Janmaat, 2000). Waterlogging is the accumulation of excess water in the root zone of the soil. (Ritzema et al., 1996) As seepage from irrigation facilities and deep percolation from irrigation practices contribute to the accumulation of groundwater, water table begins to rise. If the water table is too high, in the sense that it is less than 2 m below the soil surface, then waterlogging occurs in the soil (Wilchens, 1999). In this case, the roots of most plants cannot get the oxygen they need sufficiently such that their growth are inhibited. Under waterlogging, the groundwater can evaporate due to capillary action or can be transpired by plants. This induces an overaccumulation of salts in the rootzones of the plants. Under these circumstances, the effects of waterlogging worsens (Janmaat, 2000; Saysel and Barlas, 2001).

Irrigation water management has vital importance in the presence of salinization and waterlogging. Deep percolation from irrigation practices is one of the major source of groundwater accumulation which can cause a rise in the level of watertable, which, in turn, may result in waterlogging problem. Two main tools to cope with irrigation-induced problems are to construct appropriate drainage facilities and pricing irrigation water effectively. In order to lower deep percolation at the regional scale, large-scale drainage projects have been constructed (Legras and Lifran, 2006). By the help of an appropriately constructed drainage facility, irrigation water is sent away before it enters to groundwater (Ritzema et al., 1996).

An important problem related with the efficient use of irrigation water is how to price it. There is an extensive literature on pricing of irrigation water. The price of the irrigation water should reflect appropriate incentives to the consumers of it such that irrigation water should be allocated in an economically efficient manner (Johansson, 2000). This corresponds to the case in which the price of water equals the marginal cost of supplying an additional unit of water plus the scarcity value of the resource (Johansson, 2000). Inefficient use of irrigation water can result in irrigation induced problems, salinity and waterlogging. One type of inefficiency in irrigation water management can be due to the gap between the scarcity value of irrigation water and its current management value (Legras and Lifran, 2006). Another type of inefficiency can originate from the gap between individual and social valuations of an unconfined aquifer as a stock of water.(Legras and Lifran, 2006) This gap is firstly stated by Wilchens (1999). Wilchens (1999) emphasized the importance of the assimilative capacity of unconfined aquifers. He states that if the opportunity costs of assimilative capacity of an unconfined aquifer are not communicated to users of irrigation water through the irrigation water prices or allocation procedures, the users will not take into account long-term consequences of their irrigation and leaching activities.

In another study, Legras and Lifran (2006) modelled the strategic interactions in irrigation water applications between the farmers in an irrigation district, who are located above a common aquifer and constrained by its limited assimilative capacity. The authors modelled the assimilative capacity as dynamics of groundwater accumulation in which there is a recharge into groundwater by deep percolation from irrigation water practices of the farmers and a discharge from groundwater in the form of a discharge to the surface water system and intrusion into rootzone.[1] They assume a homogenous environment in the sense that groundwater accumulation affects all the farmers

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[1] Modelling of groundwater accumulation in Legras and Lifran (2006) depends on the model developed by Saysel and Barlas (2001). Saysel and Barlas (2001) modelled irrigation, drainage, groundwater discharge and groundwater intrusion to analyze rootzone salinization.

in the same degree. In additon, they also assume that all the farmers are members of only one irrigation district. By excluding third party externalities, they analyze cooperative and non-cooperative behaviours of all the farmers within the borders of the irrigation district. They showed that when all farmers behave non-cooperatively, an overaccumulation of groundwater occurs when compared to socially optimal level of groundwater accumulation, which is derived under the cooperative behaviour of all the farmers. Lastly, they offer a dynamic taxation scheme applied to make non-cooperative farmers follow their socially optimal irrigation water paths. In other words, they offer such an incentive scheme that each farmer realizes the opportunity costs of assimilative capacity of the underlying aquifer such that socially optimal groundwater accumulation is achieved.

Many large-scale irrigation projects are initiated in 20th century to introduce irrigated agriculture in arid and semi-arid areas in many developing countries such as Egypt, Turkmenistan, India, Pakistan, China and Turkey. Due to extensive increases in agricultural productivity, large scale irrigation projects are seen as the key strategy for development in rural areas in these countries. GAP (Turkish acronym of Southeastern Anatolia Project) region irrigation is such a large-scale irrigation project the target of which is to irrigate 1,7 million ha upon its completion (Kendirli et al., 2005; Kibaroglu, 2002). By the end of 2003, the total land which was began to be irrigated by DSI (Genaral Directorate of State Hydraulic Works) is 215 080 ha. Şanlıurfa-Harran Irrigations[2] (120 000 ha) constitutes the largest part of the GAP region irrigations (Kendirli, Cakmak and Ucar, 2005).

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[2] The green rectangle in figure 1 shows the Harran Plain Irrigations.

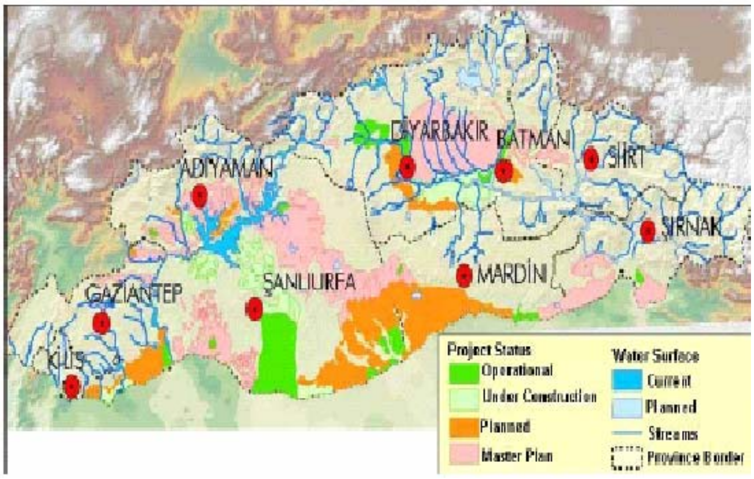


figure 1: The different stages of irrigation projects in GAP region (Ayguney, 2002)

1.A. A Brief Description of Harran Plain

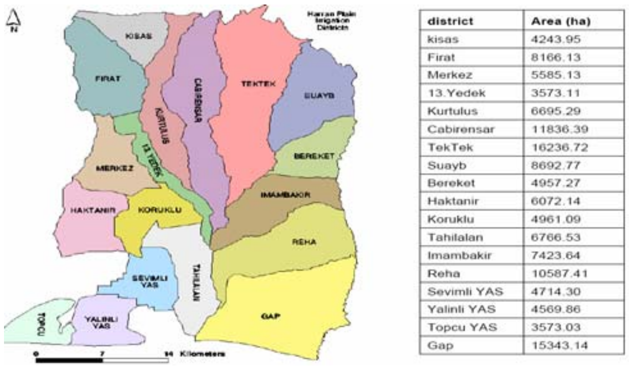
Atatürk dam is the main source for irrigation water used in Harran Plain. In addition, in lower parts of the plain, groundwater is used for irrigation purposes. The water from the Atatürk dam is conveyed via the Şanlıurfa Tunnels (two tunnels, each 26.4 km long by 7.6 m diameter, capable of carrying up to 328 m3 second−1 that can irrigate an area of more than 400,000 ha) (Tsur et al., 2003). The Atatürk dam on the Euphrates is the largest of the 22 dams owned by the GAP and is among the ten largest worldwide (completed in 1992, with a reservoir capacity of $48.7\,\mathrm{km}^3$) (Tsur et al., 2003). The huge Atatürk reservoir implies no water scaricty and the conveyance facilities impose no capacity constrains for Harran Plain.

When GAP was at planning stage, the area allocated to cotton production was 1,43%. After the completion of GAP, it was projected that cotton would be cultivated on 35% of all the irrigated land. However, on lands where irrigated agriculture begins, the lands that are allocated to the cultivation of cotton are much more larger when compared to the projected lands. In 1994, 21% of total lands have been used in cotton production largely depending on groundwater. By 1995, Şanlıurfa-Harran Plain has been introduced to irrigated agriculture. Following this, the share of cotton cultivated lands in the region rose to 60% by 1995, 70% by 1996, 85% by 1997, 88% by 1998, 90% by 1999, 94% by 2000 and 91% by 2001 (Kun et al., 2005). Monoculture cropping of cotton is widespread in Harran Plain because of convenience of accessing the tools and machinery for cotton farming, a stable price and a secure market demand for the crop (Adaman and Ozertan, 2007).

Irrigation-induced problems, salinization and waterlogging, have been becoming more severe in Harran Plain since the introduction of irrigation in agricultural production in 1995 (Adaman and Ozertan, 2007; Kendirli, Cakmak and Ucar, 2005). Before the introduction of irrigation, the total land area with critical highest water table levels between 0 and 2.00 was 2,747 ha. In 2001, this area reached to 40,780 ha (Kendirli, Cakmak and Ucar, 2005). Main causes of salinization and waterlogging in Harran Plain are improper functioning and insufficient drainage systems, lack of crop diversification, excessive irrigation practices, and the slope from north to south, which will be explained in the coming section (Adaman and Ozertan, 2007; Kendirli, Cakmak and Ucar, 2005;Tekinel, 2002).

After 1993, there has been significant institutional changes in irrigation water management in Turkey, as well as in the Harran Plain. DSI transferred its water rights to manage irrigation water to water user associations (WUAs). The main reasons for this transfer is the huge budget deficits of DSI (State

Hydraulic Works) and to induce farmers who are the users of water to participate in irrigation water management (Cakmak, Beyribey and Kodal, 2004; Unver and Gupta, 2003). The main tasks of a WUA are to allocate irrigation water among member farmers, to price water, to do organization and maintenance services, to collect water fees from farmers, and to achieve a balanced budget (Cakmak, Beyribey and Kodal, 2004; Unver and Gupta, 2003). There are 22 WUAs which are managed independently[3] from each other within the borders of Şanlıurfa. The WUAs in Harran Plain are shown in Figure 2.



district	Area (ha)
kisas	4243.95
Firat	8166.13
Merkez	5585.13
13.Yedek	3573.11
Kurtulus	6695.29
Cabirensar	11836.39
TekTek	16236.72
Suayb	8692.77
Bereket	4957.27
Haktanir	6072.14
Koruklu	4961.09
Tahilalan	6766.53
Imambakir	7423.64
Reha	10587.41
Sevimli YAS	4714.30
Yalinli YAS	4569.86
Topcu YAS	3573.03
Gap	15343.14

figure 2: Harran Plain Irrigation Districts (Ozdogan, 2003)

1.B. In This Thesis:

In this thesis, we analyze a group of farmers who are located above a common aquifer. We assume that there is no scarcity in water resources and no capacity constraints in conveyance facilities. Monocultural cropping pattern is the prevailing crop pattern. All the farmers contribute to the accumulation of a

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[3] The independence of WUAs from each other means each WUA allocate and price the irrigation water within its borders by using different methods depending on its specific needs and demands of its member farmers (Cakmak, Beyribey and Kodal, 2004).

common groundwater through their irrigation practices. However, there is a slope from North to South in the sense that the farmers in North are less severely affected by the groundwater accumulation than the farmers in South. We also assume that the farmers in the North and the farmers in the South participate in different water user associations (WUAs)[4]. So, two water WUAs are hydrologically connected since all the farmers in two WUAs contribute to the accumulation of the same groundwater, but the WUAs are managed independently. Theoretically, one of many aims of a WUA should be to achieve sustainable agriculture[5] by taking into account all of its members.

Under these settings, the model is introduced in section 2. We analyze four different scenarios depending on this model in section 3 and the results are shown for a numerical example. In case 1, all the farmers move in cooperation. In case 2, all the farmers move noncooperatively. In case 3, the farmers in WUA_N moves in cooperation within WUA_N and the farmers in WUA_S moves in cooperation within WUA_S, while the farmers in different WUAs compete against each other. In case 4, the farmers in WUA_N moves noncooperatively within WUA_N and the farmers in WUA_S moves in cooperation within WUA_S, while the farmers in different WUAs compete against each other. In this paper, we analyze the time path for groundwater accumulation which each farmer contributes to through his own irrigation practices. However, as groundwater accumulates, there occurs a negative externality by each farmer on other

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[4] This is a strong assumption, but not unrealistic for developed countries. In Johansson (2000), it is mentioned that there are two models of WUAs: The American model depends on hydraulic boundaries, but The Asian model depends on social boundaries in order to provide direct participation of all member farmers.

[5] Salinization and waterlogging are two irrigation induced problems that threaten agricultural practices. In waterlogging context, sustainable agriculture can be achieved with the least level of water table or groundwater accumulation.

farmers through rising water table which creates irrigation-induced problems such as waterlogging and salinization. By assumption, each WUA is managed independently in the sense that they price or allocate irrigation water in such a way that each WUA takes into account only its member farmers' interest at best. Within a WUA, there are two extreme policies in the allocation of irrigation water among member farmers. The first policy is to supply irrigation water to the extent which each farmer demands irrigation water only by taking into account his own benefit. The second one is the allocation of irrigation water among member farmers by taking into account their joint benefit of member farmers. In this respect, case 1 is a benchmark case to analyze the socially optimal outcome, because in this case, each farmer determines the amount of irrigation water by taking into account not only his own benefit but also the benefits of all other farmers. We compare case 2, 3, and 4 with case 1. In case 2, no WUA takes action against the negative externality caused by groundwater accumulation. In case 4, only the WUA_S takes action against groundwater accumulation such that the farmers in WUA_S moves in cooperation, but due to the independency of WUAs, farmers in different WUAs compete against each other. In case 3, both WUA_N and WUA_S take action against groundwater accumulation such that the farmers in WUA_N moves in cooperation, the farmers in WUA_S moves in cooperation, but due to the independency of WUAs, farmers in different WUAs compete. With a numerical example, we show the suboptimality of the outcomes in case 2, case 3 and case 4 when compared to the socially optimal case 1. In section 4, an optimal dynamic taxation scheme is derived to achieve the socially optimal outcome. Then, we conclude.

CHAPTER 2

2. The Model:

We assume that there are two water user associations (WUAs) within the region: water user association in the North is denoted by WUA_N and the one in the South is denoted by WUA_S. Each association consists of n member farmers. The farmers in WUA_N are indexed by i=1,2,…,n and the farmers in WUA_S are indexed by j=n+1,n+2,…,2n. Each farmer i and j produces a homogenous good from a unique input, w_i & w_j, where w_i is the irrigation water usage of a farmer in WUA_N and w_j is the irrigation water usage of a farmer in WUA_S. All the farmers are identical in the sense that they all use the same technology in their production process, their discount rates are equal, and each farmers owns a farm that is identical in size to farms of other farmers.

When water is applied for irrigation purposes, some part of it will not be absorbed by the crop such that the irrigation water which is not absorbed goes down through the soil. This percolation water accumulates in groundwater. Similar to Legras and Lifran (2006), we assume that all the water used in irrigation percolates into groundwater. Percolation water can have beneficial effects such as removing salts from the rootzone (Wilchens, 1999). This can be done both during the irrigation season and the dormant season. As done by Legras and Lifran (2006), we assume that leaching takes place only during the irrigation season.

Groundwater accumulation is treated as a stock of pollution. This is a dynamic process in the sense that there is a recharge into and discharge from the groundwater. There are two kinds of recharge: percolation water and natural recharge (precipitations minus evaporation). In this study, recharge into groundwater consists of only percolation water. We assume that all the farmers in WUA_N and WUA_S are located above the same aquifer such that percolation water of all of them have an impact on the same groundwater. Discharge to surface water system and intrusion into the rootzone make up the discharge from groundwater. Both types of discharge depend on the level of watertable level. As watertable approaches the surface of the soil, both of them increase in extent. The dynamics of groundwater accumulation is described by the following equation:

$$(1) \qquad \dot{G}(t) = \sum_{i=1}^{n} w_i(t) + \sum_{j=n+1}^{2n} w_j(t) - \gamma \cdot G(t) \text{, with}$$

$$(2) \qquad G(0) = G_0 \geq 0 \ ,$$

where $G(t)$ is the accumulation of groundwater at time t, G_0 is the initial level of groundwater accumulation and γ is the groundwater discharge fraction. This fraction represents the assimilative capacity[6] of underlying aquifer. As γ increases, assimilative capacity increases. We also assume that $\gamma > 0$.

Groundwater accumulation has negative effects on agricultural production. The major effect is the rise of watertable, which in turn activates

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[6] This term is taken from Wilchens (1999). It defines the capacity by how much an aquifer can dicharge water.

two mechanisms: capillary action and waterlogging. Through capillary action, salty water reaches the rootzone of the crops. As stated above, this can inhibit the growth of the crop and possibly lead to the death of the crop. Like Legras and Lifran (2006), we do not account for these type of damages, and restrict our analysis to the waterlogging problem. In sum, we assume that all farmers are negatively affected by groundwater accumulation.

Importantly, we assume that there is a slope from north to south in the region such that the farmers in WUA_N are less severely affected by groundwater accumulation than those in WUA_S although all the farmers contribute to the accumulation of the groundwaterr. Hence, excess water use of farmers within the WUA_N possibly impose additional negative externalities on those within the WUA_S. This asymmetry will be incorporated into the benefit functions of farmers explained below.

The time horizon in the problem is infinite with $t \in [0,\infty)$, where t denotes the point in time analyzed. The benefit of any farmer at each instant of time depends on the water used in irrigation and the level of groundwater accumulation. Since the farmers in different WUAs are affected by groundwater accumulation differently, we have two types of benefit functions. For each farmer i in WUA_N, the benefit function is defined as:

(3) $$F(w_i(t),G(t),a) = \alpha_1 + \alpha_2 \cdot w_i(t) - \alpha_3 \cdot \left[w_i(t)\right]^2 - \frac{a}{2} \cdot \left[G(t)\right]^2 .$$

For each farmer j in WUA_S, the benefit function is defined as:

(4) $$F(w_j(t),G(t),b) = \alpha_1 + \alpha_2 \cdot w_j(t) - \alpha_3 \cdot \left[w_j(t)\right]^2 - \frac{b}{2} \cdot \left[G(t)\right]^2 .$$

We assume that $\alpha_1 > 0$, $\alpha_2 > 0$, $\alpha_3 > 0$ and $b > a > 0$ such that the signs of the partial derivatives are given as $\frac{\partial F(w_i,G,a)}{\partial w_i} > 0$, $\frac{\partial^2 F(w_i,G,a)}{\partial w_i^2} < 0$, $\frac{\partial F(w_i,G,a)}{\partial G} < 0$, $\frac{\partial^2 F(w_i,G,a)}{\partial G^2} < 0$, $\frac{\partial F(w_j,G,b)}{\partial w_j} > 0$, $\frac{\partial^2 F(w_j,G,b)}{\partial w_j^2} < 0$, $\frac{\partial F(w_j,G,b)}{\partial G} < 0$, and $\frac{\partial^2 F(w_j,G,b)}{\partial G^2} < 0$. The benefit functions are composed of two parts: the production part in which irrigation water is the only input and the damage part which represents negative effect of groundwater accumulation. Concave production functions are widely used in literature (Smith and Tsur, 1997; Cetin and Bilgel, 2002). The damage part of the benefit function is convex in groundwater accumulation. This means that as the groundwater accumulates, the water table rises. And this has negative impacts on the soil at an increasing rate as the water table comes closer to the surface of the soil. The assumption on the parameters a and b says that the benefit of any farmer decreases at an increasing rate as the groundwater accumulation increases, and also since b is greater than a, groundwater accumulation affects the farmers in WUA_S more severely than the farmers in WUA_N due to the slope that runs from north to south in the region.

CHAPTER 3

3. Groundwater Accumulation Under 4 Different Cases:

Now, we analyze the time path of farmers' selection of irrigation water amounts and the groundwater accumulation under 4 different cases, given the environment characterized in the previous section. We follow a continuous time approach with infinite time horizon. We use differential games as the appropriate tool for the analyses. The reasons are twofold. The 2n farmers included in the model have first an impact on the common aquifer, and second, the life-long benefit of each farmer from cropping his/her field depends both on his/her own choice of amount of irrigation water applied and the remaining 2n-1 farmers' choices of amounts of irrigation water applied through groundwater accumulation. In this respect, each farmer's irrigation practice depends on the level of the groundwater accumulation and affects the level of groundwater accumulation in future such that the state equation shows how the groundwater accumulation evolves over time.

3.A. Case 1- All the farmers move in cooperation with all other farmers:

In the first case, we would like to derive the socially optimal outcome as a solution to cooperative game involving all the farmers in the region. The socially optimal outcome is defined as the level of groundwater accumulation for which the total of life-long benefits of all the farmers is maximized. We assume that only those farmers who have an impact on groundwater accumulation are negatively affected by this accumulation such that other externalities like downstream salinity are not taken into account. We assume that there exists a central water authority (CWA), which can derive and provide the solution of the cooperative game for all the farmers in both regions. In the cooperative game, all the farmers in WUA_N and WUA_S should jointly[7] maximize the sum of all farmers' life-long benefits subject to groundwater accumulation. This accumulation actually hurts them and it is the consequence of their own decisions on quantity of water applied. So , CWA's problem is

$$\max_{w_1,\ldots,w_{2n}} \sum_{i=1}^{n}\int_{0}^{\infty} F(w_i,G,a)\cdot e^{-r.t}\cdot dt+\sum_{j=n+1}^{2n}\int_{0}^{\infty} F(w_j,G,b)\cdot e^{-r.t}dt$$

$$\text{subject to } \dot{G}=\sum_{i=1}^{n} w_i+\sum_{j=n+1}^{2n} w_j-\gamma\cdot G \quad \text{and} \quad G(0)=G_0\geq 0$$

where r denotes the market's discount rate.

PROPOSITION 1. Assuming interior solutions, the following equations denote the time path of quantity of irrigation applied and resulting groundwater accumulation under socially optimal outcome. When all farmers move in full cooperation, groundwater accumulation is given by:

$$(3) \qquad G^c(t)=(G_0-G_\infty^c)\cdot e^{s_1\cdot t}+G_\infty^c,$$

and the adjoint variable for each farmer is:

$$(4) \qquad \lambda(t)=n\cdot(a+b)\cdot\left[\frac{1}{(s_1-r-\gamma)}\cdot G^c(t)-\frac{s_1}{(r+\gamma)\cdot(s_1-r-\gamma)}\cdot G_\infty^c\right].$$

The quantity of irrigation applied by each farmer is:

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[7] We assume that the joint benefit of the farmers who cooperate is simply sum of their individual benefits such that the joint benfit function is purely utilitarian (Mas-colell, Whinston and Green, 1995).

(5)

$$w^c(t)=\frac{n\cdot(a+b)\cdot\left[\frac{1}{(s_1-r-\gamma)}\cdot G^c(t)-\frac{s_1}{(r+\gamma)\cdot(s_1-r-\gamma)}\cdot G^c_\infty\right]+\alpha_2}{\alpha_3},$$

where the steady-state level of groundwater accumulation is:

(6) $$G^c_\infty=\frac{2\cdot n\cdot\alpha_2\cdot(r+\gamma)}{\gamma\cdot\alpha_3\cdot(r+\gamma)+2\cdot n^2\cdot(a+b)},$$

and s_1 gives the negative root of the equation:

$$s^2-r\cdot s-\gamma^2-\gamma\cdot s-\frac{2\cdot n^2\cdot(a+b)}{\alpha_3}=0.$$

Proof. The current value Hamiltonian is:

$$H(w_i,w_j,G,\lambda)=\sum_{i=1}^{n}(\alpha_1+\alpha_2\cdot w_i-\frac{\alpha_3}{2}\cdot w_i^2-\frac{a}{2}\cdot G^2)+\sum_{j=n+1}^{2n}(\alpha_1+\alpha_2\cdot w_j-\frac{\alpha_3}{2}\cdot w_j^2-\frac{b}{2}\cdot G^2)$$
$$+\lambda^*\cdot(\sum_{i=1}^{n}w_i+\sum_{j=n+1}^{2n}w_j-\gamma\cdot G).$$

λ^* is interpreted as the dynamic shadow cost of groundwater accumulation. For all the farmers, the necessary conditions for optimality are as follows:

(7) $$\lambda^*=\alpha_3\cdot w-\alpha_2,$$

(8) $$\dot{\lambda}^*=(r+\gamma)\cdot\lambda^*+n\cdot(a+b)\cdot G^c,$$

using (1) and (2), and the transversality condition is,

(9) $$\lim_{t\to\infty}e^{-r\cdot t}\cdot\lambda^*(t)\cdot G^c(t)=0.$$

As the current value Hamiltonian is concave in all irrigation water choices, these conditions are also sufficient.□ [8]

Equation (7) states that when all the farmers agree to full cooperation, optimal irrigation water choice of each farmer in WUA_N and WUA_S at each

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[8] Full derivation of the proof of proposition 1 is given in Appendix A.

point of time should be such that the shadow cost of groundwater accumulation equals the net marginal benefit of irrigation water usage. According to equation (8), for each farmer in WUA_N and WUA_S, the shadow cost of groundwater accumulation evolves over time at the rate $(r+\gamma)$ with an additive term of total marginal damage to all farmers.

The steady-state level of groundwater accumulation has the following relation:

$$(10) \qquad \alpha_2 - \alpha_3 \cdot w_\infty^c = -\lambda_\infty^* = \frac{n \cdot (a+b)}{r+\gamma} \cdot G_\infty^c ,$$[9]

where w_∞^c and λ_∞^c are steady-state levels.

Equation (10) states that at the steady-state, for each farmer in WUA_N and WUA_S, the valuation of the individual marginal benefit from amount of irrigation water applied equals the present value of total marginal damage of the steady-state level of groundwater accumulation. In other words, when all the farmers move in full cooperation, each farmer in WUA_N and WUA_S internalizes the externality generated by the groundwater accumulation by taking into account the impact of his/her own amount of irrigation water applied on all the farmers in his/her WUA and those in the other WUA. The solutions derived in this case are consistent with the ones derived in Legras and Lifran (2006).

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[9] Relation is taken from Appendix A.

3.B. Case 2- All the farmers move noncooperatively:

In this second case, we analyze a differential game in which all the farmers in WUA_N and WUA_S move non-cooperatively within their own WUA. Since farmers in different WUAs compete in exploiting the assimilative capacity of the unconfined aquifer by using their irrigation water, this case corresponds to a situation where all the farmers (2n farmers) play non-cooperatively. We use an open-loop information structure, which is also used in Legras and Lifran (2006). In this information structure, each farmer's irrigation water choice at time t depends on time t and the initial level of groundwater accumulation, G_0 .[10](Legras and Lifran, 2006) The solution concept we derive under this information structure is the open-loop Nash equilibrium (OLNE). In analyzing the waterlogging problem, OLNE can be beneficial since farmers generally do not have relevant information about the depth of watertable.

So, each farmer i in WUA_N will maximize his/her own life-long benefit subject to the groundwater accumulation which is damaging the production process, but all the other farmers in WUA_N and WUA_S also influence this accumulation. Farmer i's problem is defined as: (for i=1,..,n)

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[10] As mentioned in Legras and Lifran [2], in open-loop Nash equilibrium (OLNE), each farmer commits himself /herself to his/her chosen irrigation water time path and does not respond to observed variations in groundwater accumulation. Benchekroun and van Long [26] stated that in the literature on OLNE, it has been proved that no player can gain by deviating from his/her equilibrium strategy at any time t along the equilibrium path. In other words, it is time consistent.

$$\max_{w_i} \int_0^{\infty} F(w_i,G,a)\cdot e^{-r.t}\cdot dt$$

$$\text{subject to } \dot{\mathrm{G}} = \sum_{i=1}^{n} w_i + \sum_{j=n+1}^{2n} w_j - \delta\cdot G \quad \text{and} \quad G(0) = G_0 \geq 0.$$

Each farmer j in WUA_S will maximize his/her own life-long benefit subject to groundwater accumulation which is damaging, but on which all other farmers in WUA_N and WUA_S have impact. Farmer j’s problem is given by: (for j=n+1,…,2n)

$$\max_{w_j} \int_0^{\infty} F(w_j,G,b)\cdot e^{-r.t}\cdot dt$$

$$\text{subject to } \dot{\mathrm{G}} = \sum_{i=1}^{n} w_i + \sum_{j=n+1}^{2n} w_j - \delta\cdot G \quad \text{and} \quad G(0) = G_0 \geq 0.$$

PROPOSITION 2. Assuming interior solutions, the following equations denote the time path of amounts of irrigation applied and respective groundwater accumulation. When all farmers move non-cooperatively, the groundwater accumulation becomes:

$$(11) \qquad G^{nc}(t) = (G_0 - G_{\infty}^{nc})\cdot e^{s_2\cdot t} + G_{\infty}^{nc},$$

with the adjoint variable for farmer i in WUA_N is:

$$(12) \qquad \lambda_i(t) = a\cdot\left[\frac{1}{(s_2-r-\gamma)}\cdot G^{nc}(t) - \frac{s_2}{(r+\gamma)\cdot(s_2-r-\gamma)}\cdot G_{\infty}^{nc}\right]$$

CWA’s problem is he irrigation water choice for farmer i in WUA_N is:

$$(13) \qquad w_i^{nc}(t) = \frac{a\cdot\left[\frac{1}{(s_2-r-\gamma)}\cdot G^{nc}(t) - \frac{s_2}{(r+\gamma)\cdot(s_2-r-\gamma)}\cdot G_{\infty}^{nc}\right] + \alpha_2}{\alpha_3},$$

the adjoint variable for farmer j in WUA_S is:

$$\lambda_j(t) = b \cdot \left[\frac{1}{(s_2 - r - \gamma)} \cdot G^{nc}(t) - \frac{s_2}{(r+\gamma)\cdot(s_2 - r - \gamma)} \cdot G^{nc}_{\infty} \right] \tag{14}$$

the irrigation water choice for farmer j in WUA_S is:

$$w_j^{nc}(t) = \frac{b \cdot \left[\frac{1}{(s_2 - r - \gamma)} \cdot G^{nc}(t) - \frac{s_2}{(r+\gamma)\cdot(s_2 - r - \gamma)} \cdot G^{nc}_{\infty} \right] + \alpha_2}{\alpha_3}, \tag{15}$$

the steady-state level of groundwater accumulation is:

$$G^{nc}_{\infty} = \frac{2 \cdot n \cdot \alpha_2 \cdot (r+\gamma)}{\gamma \cdot \alpha_3 \cdot (r+\gamma) + n \cdot (a+b)}, \tag{16}$$

and s_2 is the negative root of the equation: $s^2 - r \cdot s - \gamma^2 - \gamma \cdot s - \frac{n \cdot (a+b)}{\alpha_3} = 0$.

Proof. The current value Hamiltonian for each farmer i in WUA_N is:

$$H_i(w_i, G, \lambda_i) = (\alpha_1 + \alpha_2 \cdot w_i - \frac{\alpha_3}{2} \cdot w_i^2 - \frac{a}{2} \cdot G^2) + \lambda_i^* \cdot (\sum_{i=1}^{n} w_i + \sum_{j=n+1}^{2n} w_j - \gamma \cdot G).$$

λ_i^* is interpreted as the dynamic shadow cost of groundwater accumulation for each farmer i in WUA_N. For each farmer i in WUA_N, the necessary conditions for optimality are as follows:

$$\lambda_i^* = \alpha_3 \cdot w_i - \alpha_2, \tag{17}$$

$$\dot{\lambda}_i^* = (r+\gamma) \cdot \lambda_i^* + a \cdot G^{nc}, \tag{18}$$

and using (1) and (2), and the transversality condition is ,

$$\lim_{t \to \infty} e^{-r \cdot t} \cdot \lambda_i^*(t) \cdot G^{nc}(t) = 0. \tag{19}$$

The current value Hamiltonian for each farmer j in WUA_S is:

$$H_j(w_j, G, \lambda_j) = (\alpha_1 + \alpha_2 \cdot w_i - \frac{\alpha_3}{2} \cdot w_i^2 - \frac{b}{2} \cdot G^2) + \lambda_j^* \cdot (\sum_{i=1}^{n} w_i + \sum_{j=n+1}^{2n} w_j - \gamma \cdot G).$$

λ_j^* is interpreted as the dynamic shadow cost of groundwater accumulation for each farmer j in WUA_S. For each farmer j in WUA_S, the necessary conditions for optimality are as follows:

(20) $$\lambda_j^* = \alpha_3 \cdot w_j - \alpha_2 ,$$

(21) $$\dot{\lambda}_j^* = (r+\gamma)\cdot \lambda_j^* + b\cdot G^{nc} ,$$

and also (1) and (2), and the transversality condition is,

(22) $$\lim_{t\to\infty} e^{-r\cdot t}\cdot \lambda_j^*(t)\cdot G^{nc}(t) = 0 .$$

As all current value Hamiltonians are concave in quantity of irrigation water applied, these conditions are also sufficient. □ [11]

Equation (17) and equation (20) state that when all the farmers move non-cooperatively, optimal irrigation water usage of each farmer in WUA_N and WUA_S at each point of time should be such that the shadow cost of groundwater accumulation equals the net marginal benefit of irrigation water choice. According to equation (18) and equation (21), for each farmer in WUA_N and WUA_S, shadow cost of groundwater accumulation evolves over time at the rate $(r+\gamma)$ with an additive term of marginal damage to only himself/herself.

The steady-state level of groundwater accumulation is given by:

(23) $$\alpha_2 - \alpha_3 \cdot w_{i\infty}^{nc} = -\lambda_{i\infty}^* = \frac{a}{r+\gamma}\cdot G_\infty^{nc} ,^{12}$$

(24) $$\alpha_2 - \alpha_3 \cdot w_{j\infty}^{nc} = -\lambda_{j\infty}^* = \frac{b}{r+\gamma}\cdot G_\infty^{nc} ,^{13}$$

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[11] Full derivation of the proof of proposition 2 is given in Appendix B.

[12] Relation is taken from Appendix B.

where $w_{i\infty}^{c}$, $w_{j\infty}^{c}$, $\lambda_{i\infty}^{c}$ and $\lambda_{j\infty}^{c}$ are steady-state levels.

Equation (23) states that at the steady-state, for each farmer in WUA_N, the valuation of the individual marginal benefit from irrigation water applied equals the present value of marginal damage of the steady-state level of groundwater accumulation only to himself/herself. In other words, when all the farmers move non-cooperatively, a farmer in WUA_N does not internalize the externality generated by the groundwater accumulation by taking into account the impact of his/her own irrigation water applied on other member farmers in WUA_N and the farmers in WUA_S. Equation (24) states that at the steady-state, for each farmer in WUA_S, the valuation of the individual marginal benefit from quantity of irrigation water applied equals the present value of marginal damage of the steady-state level of groundwater accumulation to only himself/herself. In other words, when all the farmers move non-cooperatively, a farmer in WUA_S does not internalize the externality generated by the groundwater accumulation by taking into account the impact of his own irrigation water usage on other member farmers in WUA_S and the farmers in WUA_N. The solutions derived in this case are consistent with the ones derived in Legras and Lifran (2006).

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[13] Relation is taken from Appendix B.

3.C. Case 3- The farmers in WUA_N moves in cooperation within WUA_N and the farmers in WUA_S moves in cooperation within WUA_S , while the farmers in different WUAs compete:

In case 3, we analyze a situation in which the farmers in WUA_N decide their amounts of irrigation water applied in cooperation within the borders of WUA_N, and the farmers in WUA_S decide their amounts of irrigation water applied in cooperation within the borders of WUA_S. Importantly, farmers in different WUAs compete in their irrigation water usages. We use the open-loop case as the informational structure and the OLNE as the solution concept.

Since each farmer i in WUA_N moves in cooperation with the other farmers in WUA_N, he/she should maximize total life-long benefits of all the farmers in WUA_N. This task can be fulfilled by the administration board of WUA_N. So, the problem of the board of WUA_N is:

$$\max_{w_1,\ldots,w_n} \sum_{i=1}^{n} \int_0^{\infty} F(w_i,G,a)\cdot e^{-r.t}\cdot dt$$

$$\text{subject to } \dot{G} = \sum_{i=1}^{n} w_i + \sum_{j=n+1}^{2n} w_j - \gamma\cdot G \quad \text{and} \quad G(0) = G_0 \geq 0.$$

Since each farmer j in WUA_S moves in cooperation with the other farmers in WUA_S, he/she should maximize total life-long benefits of all the farmers in WUA_S. This task can be fulfilled by the administration board of WUA_S. So, the problem of the board of WUA_S is:

$$\max_{w_{n+1},\dots,w_{2n}} \sum_{j=n+1}^{2n} \int_0^{\infty} F(w_i, G, b) \cdot e^{-r.t} \cdot dt$$

$$\text{subject to } \dot{G} = \sum_{i=1}^{n} w_i + \sum_{j=n+1}^{2n} w_j - \gamma \cdot G \quad \text{and} \quad G(0) = G_0 \geq 0.$$

PROPOSITION 3. Assuming interior solutions, the following equations denote the time path of quantities of irrigation water applied and groundwater accumulation. When all the farmers in WUA_N move in cooperation within the borders of WUA_N, all the farmers in WUA_S move in cooperation within the borders of WUA_S, but farmers in different WUA compete in their quantities of irrigation water applied irrigation water choice, groundwater accumulation becomes:

(25) $$G_3(t) = (G_0 - G_{3\infty}) \cdot e^{s_3 \cdot t} + G_{3\infty},$$

the adjoint variable for farmer i in WUA_N is:

(26) $$\lambda_{3i}(t) = n \cdot a \cdot \left[\frac{1}{(s_3 - r - \gamma)} \cdot G_3(t) - \frac{s_3}{(r + \gamma) \cdot (s_3 - r - \gamma)} \cdot G_{3\infty} \right]$$

the quantity of irrigation water applied by farmer i in WUA_N is:

(27) $$w_{3i}(t) = \frac{n \cdot a \cdot \left[\frac{1}{(s_3 - r - \gamma)} \cdot G_3(t) - \frac{s_3}{(r + \gamma) \cdot (s_3 - r - \gamma)} \cdot G_{3\infty} \right] + \alpha_2}{\alpha_3},$$

the adjoint variable for farmer j in WUA_S is:

(28) $$\lambda_{3j}(t) = n \cdot b \cdot \left[\frac{1}{(s_3 - r - \gamma)} \cdot G_3(t) - \frac{s_3}{(r + \gamma) \cdot (s_3 - r - \gamma)} \cdot G_{3\infty} \right]$$

the quantity of irrigation water applied by farmer j in WUA_S is:

(29) $$w_{3j}(t) = \frac{n \cdot b \cdot \left[\frac{1}{(s_3 - r - \gamma)} \cdot G_3(t) - \frac{s_3}{(r + \gamma) \cdot (s_3 - r - \gamma)} \cdot G_{3\infty} \right] + \alpha_2}{\alpha_3},$$

the steady-state level of groundwater accumulation is:

(30) $$G_{3\infty}=\frac{2\cdot n\cdot\alpha_2\cdot(r+\gamma)}{\gamma\cdot\alpha_3\cdot(r+\gamma)+n\cdot(a+b)},$$

and s_3 is the negative root of the equation: $$s^2-r\cdot s-\gamma^2-\gamma\cdot s-\frac{n^2\cdot(a+b)}{\alpha_3}=0.$$

Proof. The current value Hamiltonian for each farmer i in WUA_N is:

$$H_i(w_i,G,\lambda_{3i})=\sum_{i=1}^{n}(\alpha_1+\alpha_2\cdot w_i-\frac{\alpha_3}{2}\cdot w_i^2-\frac{a}{2}\cdot G^2)+\lambda_{3i}^*\cdot(\sum_{i=1}^{n}w_i+\sum_{j=n+1}^{2n}w_j-\gamma\cdot G).$$

λ_{3i}^* is interpreted as the dynamic shadow cost of groundwater accumulation for each farmer i in WUA_N. For each farmer i in WUA_N, the necessary conditions for optimality are as follows:

(31) $$\lambda_{3i}^*=\alpha_3\cdot w_{3i}-\alpha_2,$$

(32) $$\dot{\lambda}_{3i}^*=(r+\gamma)\cdot\lambda_{3i}^*+n\cdot a\cdot G_3,$$

and using (1) and (2), and the transversality condition is,

(33) $$\lim_{t\to\infty}e^{-r\cdot t}\cdot\lambda_{3i}^*(t)\cdot G_3(t)=0.$$

The current value Hamiltonian for each farmer j in WUA_S is:

$$H_j(w_j,G,\lambda_{3j})=\sum_{i=n+1}^{2n}(\alpha_1+\alpha_2\cdot w_j-\frac{\alpha_3}{2}\cdot w_j^2-\frac{b}{2}\cdot G^2)+\lambda_{3j}^*\cdot(\sum_{i=1}^{n}w_i+\sum_{j=n+1}^{2n}w_j-\gamma\cdot G).$$

λ_{3j}^* is interpreted as the dynamic shadow cost of groundwater accumulation for each farmer j in WUA_S. For each farmer j in WUA_S, the necessary conditions for optimality are as follows:

(34) $$\lambda_{3j}^*=\alpha_3\cdot w_{3j}-\alpha_2,$$

(35) $$\overset{\square}{\lambda}{}^{*}_{3j} = (r+\gamma)\cdot\lambda^{*}_{3j} + n\cdot b\cdot G_3,$$

and using (1) and (2), and the transversality condition is,

(36) $$\lim_{t\to\infty} e^{-r\cdot t}\cdot\lambda^{*}_{3j}(t)\cdot G_3(t) = 0\,.$$

As all current value Hamiltonians are concave in irrigation water usages, these conditions are also sufficient. □ [14]

Equations (31) and (34) state that when each farmer moves in cooperation with other farmers in his/her own WUA, but competes with farmers who are not in his/her own WUA, optimal quantity of irrigation water applied by each farmer in WUA_N and WUA_S at each point of time should be set such that the shadow cost of groundwater accumulation equals the net marginal benefit of irrigation water choice. According to equations (32) and (35), for each farmer in WUA_N and WUA_S, the shadow cost of groundwater accumulation evolves over time at the rate $(r+\gamma)$ with an additive term of marginal damage to all the farmers in his/her own WUA.

The steady-state level of groundwater accumulation has the following relations:

(37) $$\alpha_2 - \alpha_3\cdot w_{3i\infty} = -\lambda^{*}_{3i\infty} = \frac{n\cdot a}{r+\gamma}\cdot G_{3\infty},$$ [15]

(38) $$\alpha_2 - \alpha_3\cdot w_{3j\infty} = -\lambda^{*}_{3j\infty} = \frac{n\cdot b}{r+\gamma}\cdot G_{3\infty},$$ [16]

where $w_{3i\infty}$, $w_{3j\infty}$, $\lambda_{3i\infty}$ and $\lambda_{3j\infty}$ are steady-state levels.

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[14] Full derivation of the proof of proposition 3 is given in Appendix C.

[15] Relation is taken from Appendix C.

[16] Relation is taken from Appendix C.

Equation (37) states that at the steady-state, for each farmer in WUA_N, the valuation of the individual marginal benefit from irrigation water choice equals the present value of marginal damage of the steady-state level of groundwater accumulation to all farmers in WUA_N. In other words, when each farmer moves in cooperation with other farmers in his/her own WUA, but competes with farmers who are not in his/her own WUA, a farmer in WUA_N internalizes the externality generated by the groundwater accumulation by taking into account the impact of his/her own quantity of irrigation water applied on other member farmers in WUA_N. Equation (38) states that at the steady-state, for each farmer in WUA_S, the valuation of the individual marginal benefit from irrigation water choice equals the present value of marginal damage of the steady-state level of groundwater accumulation to all farmers in WUA_S. In other words, when each farmer moves in cooperation with other farmers in his/her own WUA, but competes with farmers who are not in his/her own WUA, a farmer in WUA_S internalizes the externality generated by the groundwater accumulation by taking into account the impact of his own quantity of irrigation water applied on other member farmers in WUA_S.

3.D. Case 4- The farmers in WUA_N moves noncooperatively within WUA_N and the farmers in WUA_S moves in cooperation within WUA_S , while the farmers in different WUAs compete:

Lastly, in case 4, we analyze a situation in which the farmers in WUA_S decide on their amount of irrigation water applied in cooperation within the borders of WUA_S, the farmers in WUA_N decide their amount of irrigation water applied non-cooperatively within the borders of WUA_N, and also the farmers in different WUAs compete in their amounts of irrigation water applied. We use the open loop case as informational structure and the OLNE as the solution concept.

Since each farmer i in WUA_N behaves non-cooperatively with all the other farmers in WUA_N and WUA_S, he/she should maximize only his/her own life-long benefit subject to groundwater accumulation which hurts him/her, but on which all the farmers in WUA_N and WUA_S are affected. Farmer i's problem is: (for i=1,..,n)

$$\max_{w_i} \int_0^{\infty} F(w_i,G,a)\cdot e^{-r.t}\cdot dt$$

$$\text{subject to } \dot{G}=\sum_{i=1}^{n} w_i + \sum_{j=n+1}^{2n} w_j - \delta\cdot G \quad \text{and} \quad G(0)=G_0 \geq 0.$$

Since each farmer j in WUA_S moves in cooperation with the other farmers in WUA_S, he/she should maximize the total life-long benefits of all the farmers in WUA_S. This task can be fulfilled by the administration board of WUA_S. So, the problem of the board of WUA_S can be stated as:

$$\max_{w_{n+1},\ldots,w_{2n}} \sum_{j=n+1}^{2n} \int_0^{\infty} F(w_i, G, b) \cdot e^{-r.t} \cdot dt$$

$$\text{subject to } \dot{G} = \sum_{i=1}^{n} w_i + \sum_{j=n+1}^{2n} w_j - \gamma \cdot G \qquad \text{and} \qquad G(0) = G_0 \geq 0.$$

PROPOSITION 4. Assuming interior solutions, the following equations denote the time path of amount of irrigation water applied and groundwater accumulation. When all the farmers in WUA_S move in cooperation within the borders of WUA_S, all the farmers in WUA_N move non-cooperatively within the borders of WUA_N, and also farmers in different WUA compete in their amounts of irrigation water applied, the groundwater accumulation becomes:

(39) $$G_4(t) = (G_0 - G_{4\infty}) \cdot e^{s_4 \cdot t} + G_{4\infty},$$

the adjoint variable for farmer i in WUA_N is:

(40) $$\lambda_{4i}(t) = a \cdot \left[\frac{1}{(s_4 - r - \gamma)} \cdot G_4(t) - \frac{s_4}{(r+\gamma) \cdot (s_4 - r - \gamma)} \cdot G_{4\infty} \right]$$

the irrigation water choice for farmer i in WUA_N is:

(41) $$w_{4i}(t) = \frac{a \cdot \left[\frac{1}{(s_4 - r - \gamma)} \cdot G_4(t) - \frac{s_4}{(r+\gamma) \cdot (s_4 - r - \gamma)} \cdot G_{4\infty} \right] + \alpha_2}{\alpha_3},$$

the adjoint variable for farmer j in WUA_S is:

(42) $$\lambda_{4j}(t) = n \cdot b \cdot \left[\frac{1}{(s_4 - r - \gamma)} \cdot G_4(t) - \frac{s_4}{(r+\gamma) \cdot (s_4 - r - \gamma)} \cdot G_{4\infty} \right]$$

the irrigation water choice for farmer j in WUA_S is:

(43) $$w_{4j}(t) = \frac{n \cdot b \cdot \left[\frac{1}{(s_4 - r - \gamma)} \cdot G_4(t) - \frac{s_4}{(r+\gamma) \cdot (s_4 - r - \gamma)} \cdot G_{4\infty} \right] + \alpha_2}{\alpha_3},$$

the steady-state level of groundwater accumulation is:

(44) $$G_{4\infty} = \frac{2 \cdot n \cdot \alpha_2 \cdot (r+\gamma)}{\gamma \cdot \alpha_3 \cdot (r+\gamma) + n \cdot a + n^2 \cdot b)},$$

and s_4 is the negative root of the equation: $s^2 - r \cdot s - \gamma^2 - \gamma \cdot s - \frac{(n^2 \cdot b + n \cdot a)}{\alpha_3} = 0$.

Proof. The current value Hamiltonian for each farmer i in WUA_N is:

$$H_i(w_i, G, \lambda_{4i}) = \alpha_1 + \alpha_2 \cdot w_i - \frac{\alpha_3}{2} \cdot w_i^2 - \frac{a}{2} \cdot G^2 + \lambda_{4i}^* \cdot (\sum_{i=1}^{n} w_i + \sum_{j=n+1}^{2n} w_j - \gamma \cdot G).$$

λ_{4i}^* is interpreted as the dynamic shadow cost of groundwater accumulation for each farmer i in WUA_N. For each farmer i in WUA_N, the necessary conditions for optimality are as follows:

(45) $$\lambda_{4i}^* = \alpha_3 \cdot w_{4i} - \alpha_2,$$

(46) $$\overset{\square}{\lambda}{}_{4i}^* = (r+\gamma) \cdot \lambda_{4i}^* + a \cdot G_4,$$

and using (1) and (2), and the transversality condition is,

(47) $$\lim_{t \to \infty} e^{-r \cdot t} \cdot \lambda_{4i}^*(t) \cdot G_4(t) = 0.$$

The current value Hamiltonian for each farmer j in WUA_S is:

$$H_j(w_j, G, \lambda_{4j}) = \sum_{i=n+1}^{2n} (\alpha_1 + \alpha_2 \cdot w_j - \frac{\alpha_3}{2} \cdot w_j^2 - \frac{b}{2} \cdot G^2) + \lambda_{4j}^* \cdot (\sum_{i=1}^{n} w_i + \sum_{j=n+1}^{2n} w_j - \gamma \cdot G).$$

λ_{4j}^* is interpreted as the dynamic shadow cost of groundwater accumulation for each farmer j in WUA_S. For each farmer j in WUA_S, the necessary conditions for optimality are as follows:

(48) $$\lambda_{4j}^* = \alpha_3 \cdot w_{4j} - \alpha_2,$$

(49) $$\overset{\square}{\lambda}{}_{4j}^* = (r+\gamma) \cdot \lambda_{4j}^* + n \cdot b \cdot G_4,$$

and using (1) and (2), and the transversality condition is,

$$(50) \qquad \lim_{t\to\infty} e^{-r\cdot t}\cdot\lambda^{*}_{4j}(t)\cdot G_4(t)=0\,.$$

As all current value Hamiltonians are concave in irrigation water choices, these conditions are also sufficient. □ [17]

Equations (45) and (48) state that when all the farmers in WUA_S move in cooperation within the borders of WUA_S, all the farmers in WUA_N move non-cooperatively within the borders of WUA_N, and also farmers in different WUA compete in their amounts of irrigation applied, optimal irrigation water choice of each farmer in WUA_N and WUA_S at each point of time should be such that the shadow cost of groundwater accumulation equals the net marginal benefit of irrigation water choice. According to equation (49), for each farmer in WUA_S, the shadow cost of groundwater accumulation evolves over time at the rate $(r+\gamma)$ with an additive term of marginal damage to all the farmers in his/her own WUA. According to equation (46), for each farmer in WUA_N, the shadow cost of groundwater accumulation evolves over time at the rate $(r+\gamma)$ with an additive term of marginal damage only to himself/herself.

The steady-state level of groundwater accumulation has the following relations:

$$(51) \qquad \alpha_2-\alpha_3\cdot w_{4i\infty}=-\lambda^{*}_{4i\infty}=\frac{a}{r+\gamma}\cdot G_{4\infty}\,,^{18}$$

$$(52) \qquad \alpha_2-\alpha_3\cdot w_{4j\infty}=-\lambda^{*}_{4j\infty}=\frac{n\cdot b}{r+\gamma}\cdot G_{4\infty}\,,^{19}$$

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[17] Full derivation of the proof of proposition 4 is given in Appendix D.

[18] Relation is taken from Appendix .D

[19] Relation is taken from Appendix D.

where $w_{4i\infty}$, $w_{4j\infty}$, $\lambda_{4i\infty}$ and $\lambda_{4j\infty}$ are the steady-state levels.

Equation (51) states that at the steady-state, for each farmer in WUA_N, the valuation of the individual marginal benefit from irrigation water choice equals the present value of marginal damage of the steady-state level of groundwater accumulation only to himself/herself. In other words, when all the farmers in WUA_S move in cooperation within the borders of WUA_S, all the farmers in WUA_N move non-cooperatively within the borders of WUA_N, and also farmers in different WUA compete in their amounts of irrigation water applied, a farmer in WUA_N does not internalize the externality generated by the groundwater accumulation by taking into account the impact of his/her own irrigation water choice on other member farmers in WUA_N and the farmers in WUA_S. Equation (52) states that at the steady-state, for each farmer in WUA_S, the valuation of the individual marginal benefit from irrigation water choice equals the present value of marginal damage of the steady-state level of groundwater accumulation to all the farmers in WUA_S. In other words, when all the farmers in WUA_S move in cooperation within the borders of WUA_S, all the farmers in WUA_N move non-cooperatively within the borders of WUA_N, and also farmers in different WUA compete in their amounts of irrigation water applied, a farmer in WUA_S internalizes the externality generated by the groundwater accumulation by taking into account the impact of his/her own irrigation water choice on other member farmers in WUA_S.

3.E. Numerical Illustrations:

In this section, we present a numerical example of our model under the 4 different cases described above. The values of the parameters we use in illustrations are: $\alpha_1 = 0.1$, $\alpha_2 = 3.2$, $\alpha_3 = 0.4$, a=0.000001, b=0.0000031[20], r=0.02, γ= 0.005[21] and n=4. The time path for groundwater accumulation for case 1, case 2, case 3 and case 4 are shown in figure 3, figure 5, figure 7 and figure 9, respectively. The time path for amounts of irrigation water choices of each farmer i in WUA_N and each farmer j in WUA_S for case 1, case 2, case 3 and case 4 are shown in figure 4, figure 6, figure 8 and figure 10, respectively.

Case 1:

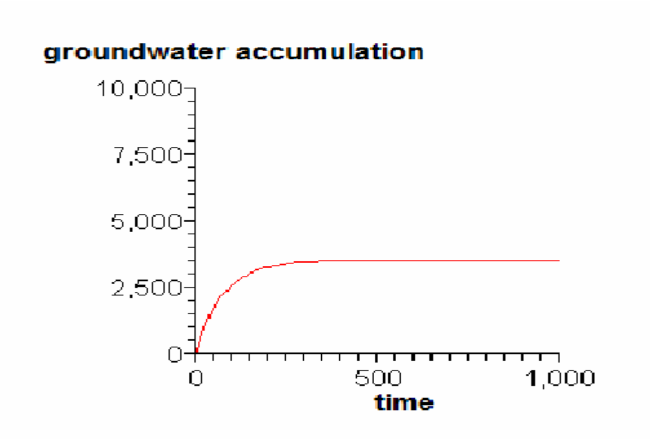

figure 3: groundwater accumulation in case 1

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[20] By holding other parameters constant, for b>0.0000031, the irrigation water applied by farmer j in WUA_S becomes less than 0 in case 4.

[21] By holding other parameters constant, for $\gamma < 0.004$, the irrigation water applied by farmer j in WUA_S becomes less than 0 in case 3 and case 4.

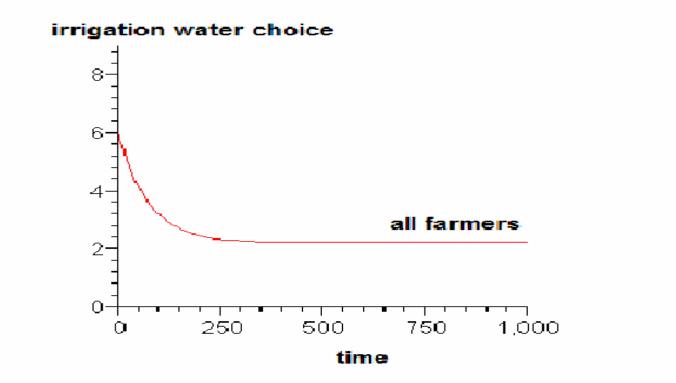

figure 4: irrigation water choices in case 1

for case 2:

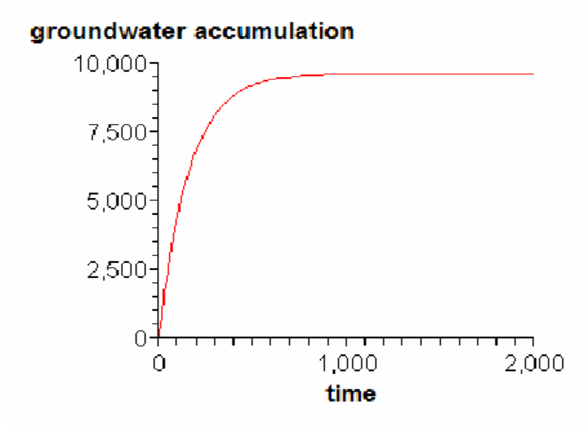

figure 5: groundwater accumulation in case 2

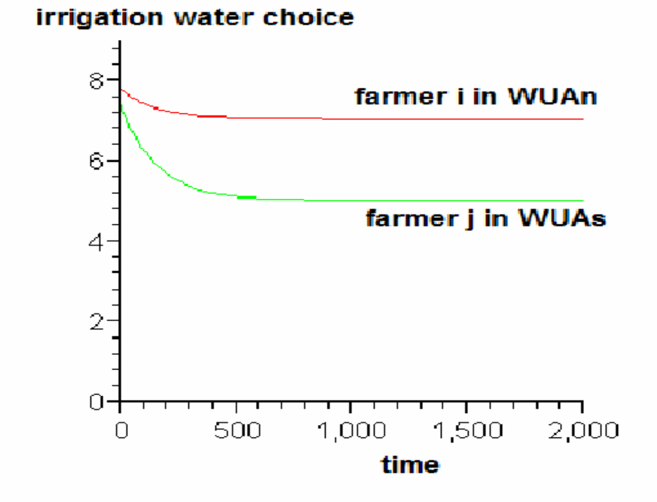

figure 6: irrigation water choices in case 2

In figure 6, the amounts of irrigation water applied are shown for case 2, in which all farmers move non-cooperatively. The difference between quantities of irrigation water applied by the farmers in different WUAs originates from the slope from north to south such that as groundwater accumulates, the farmers in WUA_N are less severely affected than those in WUA_S. So, the shadow price of groundwater accumulation is different for farmers in different WUAs. In other words, the farmers in WUA_S value the groundwater accumulation more than the farmers in WUA_N such that each farmer j in WUA_S uses less irrigation water than each farmer i in WUA_N.

for case 3:

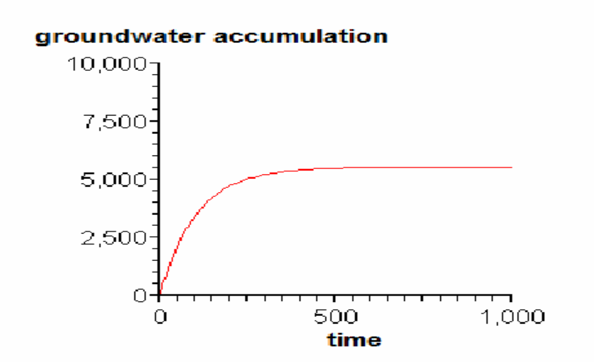

figure 7: groundwater accumulation in case 3

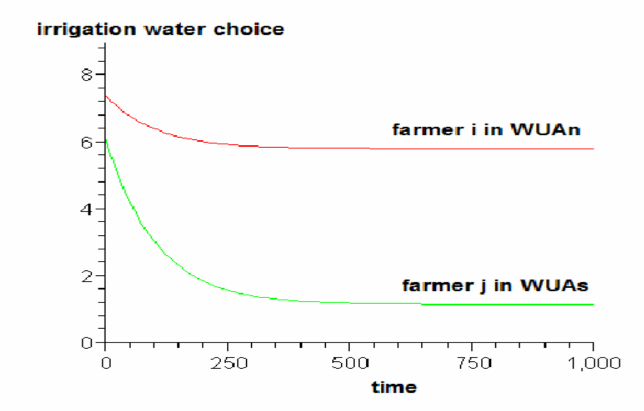

figure 8: irrigation water choices in case 3

for case 4:

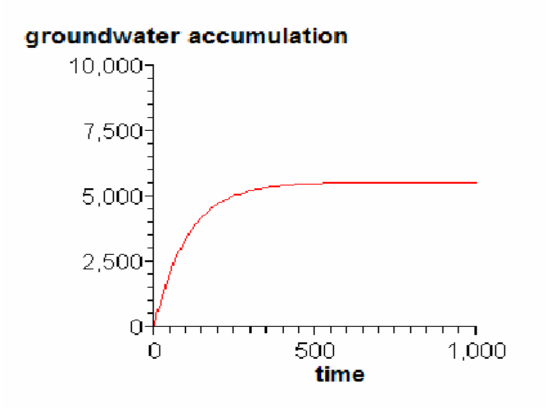

figure 9: groundwater accumulation in case 4

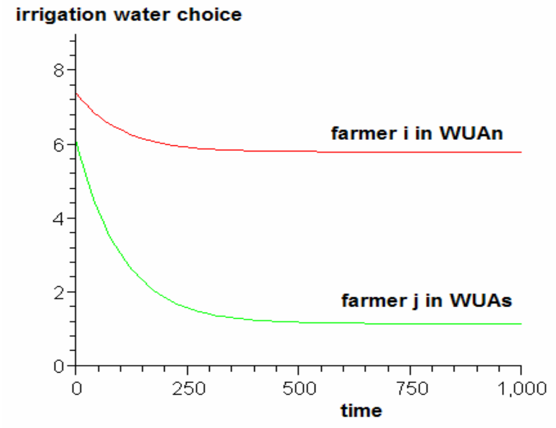

figure 10: irrigation water choice in case 4

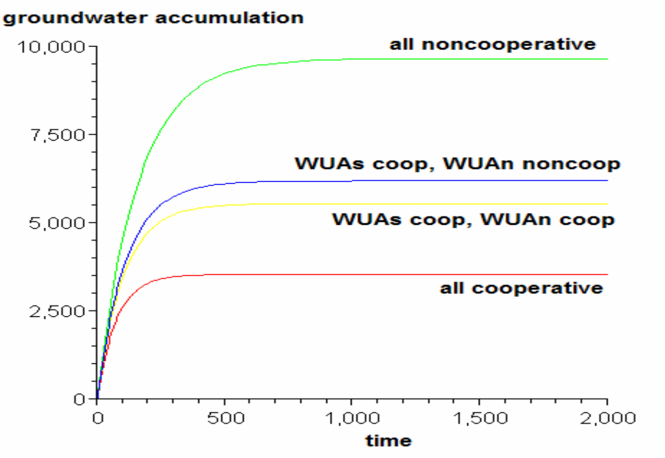

figure 11: groundwater accumulation in all cases

In figure 11, time paths for groundwater accumulation under 4 cases are shown. In section 3, the steady-state levels of groundwater accumulation under 4 cases were derived. These levels were derived as; in case 1, $G_{\infty}^{c}=\dfrac{2\cdot n\cdot\alpha_2\cdot(r+\gamma)}{\gamma\cdot\alpha_3\cdot(r+\gamma)+2\cdot n^2\cdot(a+b)}$ by (6);

In case 2, $G_{\infty}^{nc}=\dfrac{2\cdot n\cdot\alpha_2\cdot(r+\gamma)}{\gamma\cdot\alpha_3\cdot(r+\gamma)+n\cdot(a+b)}$ by (16);

in case 3, $G_{3\infty}=\dfrac{2\cdot n\cdot\alpha_2\cdot(r+\gamma)}{\gamma\cdot\alpha_3\cdot(r+\gamma)+n\cdot(a+b)}$ by (30);

in case 4, $G_{4\infty}=\dfrac{2\cdot n\cdot\alpha_2\cdot(r+\gamma)}{\gamma\cdot\alpha_3\cdot(r+\gamma)+n\cdot a+n^2\cdot b)}$ by (44).

PROPOSITION 5. Under the 4 cases analysed in section 3, for any a and b with $b>a>0$, the following relationship holds: $G_{\infty}^{nc}>G_{4\infty}>G_{3\infty}>G_{\infty}^{c}$.

Legras and Lifran (2006), have found that when there is only one irrigation district and all farmers move non-cooperatively within the borders of irrigation district, the socially optimal level of groundwater accumulation cannot be achieved, where socially optimal level is the level of groundwater accumulation achieved when all farmers in the irrigation district move in cooperation within the borders of the irrigation district. So, our result of $G_{\infty}^{nc}>G_{\infty}^{c}$, is consistent with the one found in Legras and Lifran (2006). However, when there are more than one WUA, where they are hydrologically connected within the same region, even if each farmer moves in cooperation with other member farmers in his/her own WUA, the socially optimal level of groundwater accumulation still cannot be achieved, since $G_{\infty}^{nc}>G_{3\infty}>G_{\infty}^{c}$.

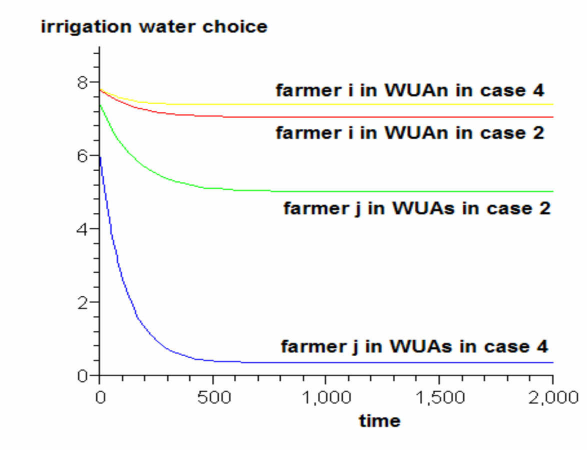

figure 12: irrigation water choices under case 2 and case 4

In figure 12, we compare the amounts of irrigation water applied by farmers under case 2 and case 4. Whereas in case 2 all the farmers move non-cooperatively, in case 4 each farmer in WUA_S moves in cooperation with other member farmers in WUA_S, the farmers in WUA_N move non-cooperatively and the farmers in different WUAs compete among each others. So, the difference between two cases is how the farmers in WUA_S decide on their amounts of irrigation water applied. As mentioned in section 3.B, when all the farmers move non-cooperatively, no farmer in WUA_N or WUA_S takes into account the negative impact of his/her irrigation water application on other farmers in his/her own or the other WUA through the groundwater accumulation. But, in case 4, farmers in WUA_S move in cooperation such that each farmer in WUA_S takes into account the negative impact of his/her irrigation water application on other farmers in WUA_S through groundwater accumulation. In figure 10, it is shown that irrigation water choice for farmer j in WUA_S in case 4 is less than the irrigation water choice for farmer j in

WUA_S in case 2 at any time and the amount of irrigation water applied by the farmer i in WUA_N in case 4 is larger than that by farmer i in WUA_N in case 2 at any time. This is due to the fact that each farmer j in WUA_S is aware of how he/she affects other farmers in WUA_S through groundwater accumulation and internalizes the damage to himself/herself and other farmers in WUA_S, so, each farmer j in WUA_S decreases his quantity of irrigation water applied in case 4 compared to his/her quantity selected in case 2. Since we have a complete information model, each farmer i in WUA_N is aware of how irrigation water choice of farmer j in WUA_S will change when farmers in WUA_S move in cooperation with each other (case 4). So, each farmer in WUA_N will respond to the change by increasing his/her amount of irrigation water applied in case 4 when compared to the one in case 2.

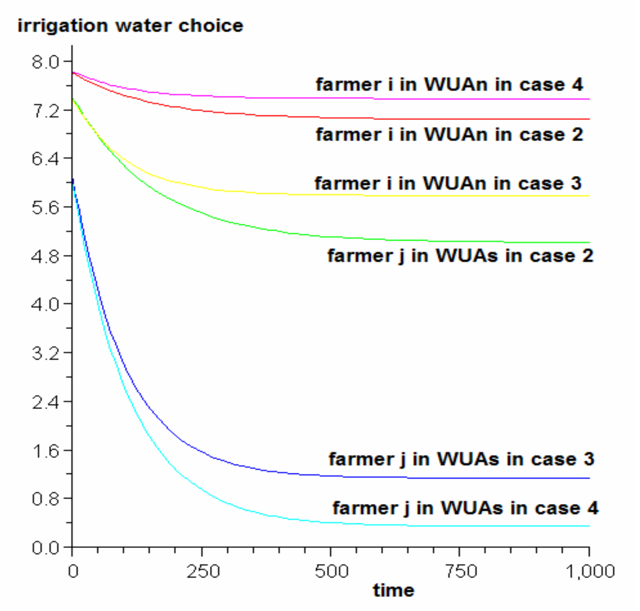

figure 13: irrigation water choices under case 3 and case 4

Figure 13 is an extended version of figure 10, to which irrigation water choices under case 3 are added. In case 3, each farmer moves in cooperation with other farmers in his/her own WUA and the farmers in different WUAs do not cooperate while deciding how much irrigation water to apply. So, in case 3, each farmer internalizes the damage only to all farmers in his/her own WUA. When irrigation water choices in case 2 and case 3 considered, each farmer in both WUAs uses less irrigation water in case 3 than in case 2. The reason is that each farmer in WUA_N or in WUA_S takes into account not only the damage to himself/herself (case 2) through the groundwater accumulation, on which all farmers have impact, but also the damage to all the farmers in his/her own WUA (case 3). When we compare the cases 3 and 4, each farmer in WUA_N uses less irrigation water in case 3 than in case 4, and each farmer in WUA_S uses more irrigation water in case 3 than in case 4. In case 3, each farmer in WUA_N takes into account not only the damage to himself/herself through the groundwater accumulation, on which all farmers have impact, but also the damage to all the farmers in WUA_N. So, when compared to case 4, each farmer i in WUA_N decreases his/her amount of irrigation water applied. Since we are in the complete information case, each farmer j in WUA_S is aware of how irrigation water choices change in case 3 such that each farmer j in WUA_S will respond to this change by increasing his/her own amount of irrigation water applied.

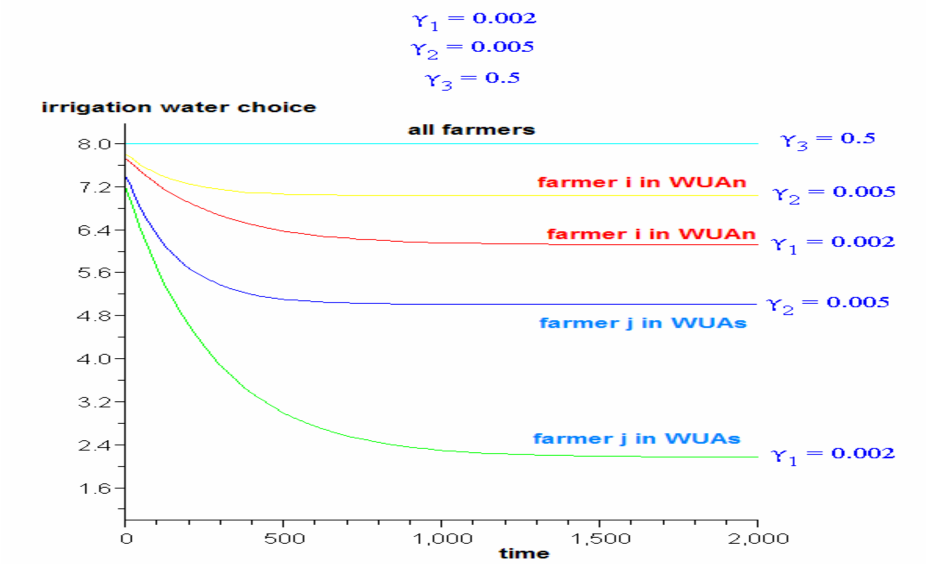

figure 14: irrigation water choices under case 2 for different discharge fractions

In figure 14, the amounts of irrigation water choices are shown for different discharge fractions in case 2[22]. As farmers apply irrigation water, the groundwater accumulates such that through damage part in benefit functions of the farmers, this puts a constraint on how much irrigation water to use in production. The groundwater discharge fraction, γ, determines how much water will be drained from the aquifer for a given level of groundwater accumulation. As it increases, more water is discharged for any given level of groundwater accumulation such that the constraint imposed by groundwater accumulation is relaxed. For example, for $\gamma_1 < \gamma_2$, each farmer in WUA_N and WUA_S use irrigation water more aggressively since for any given level of irrigation water used by them the groundwater accumulates less or, in other words, although they make the same contribution to the groundwater, less irrigation water accumulates in groundwater. In figure 13, it is also shown that,

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[22] The values of the parameters we use in illustrations are: $\alpha_1 = 0.1$, $\alpha_2 = 3.2$, $\alpha_3 = 0.4$, a=0.000001, b=0.0000031[22], r=0.02, and n=4.

if the groundwater discharge fraction is sufficiently high, groundwater accumulation puts no constraint on irrigation water choices of the farmers both in WUA_N and WUA_S such that all farmers use irrigation water to the extent as if there were no damage part in their benefit functions.

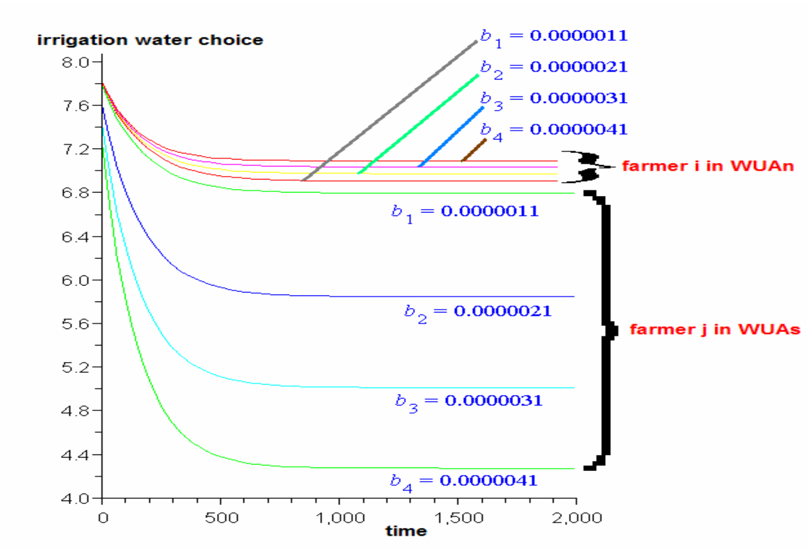

figure 15: irrigation water choices for different slope parameters

In figure 15, irrigation water choices of each farmer in WUA_N and WUA_S in case 2 are shown for different values on slope parameters[23], b, of benefit functions for each farmer in WUA_S. As mentioned in section 2, the parameters, a and b, represent the slope from north to south such that a<b implies that as groundwater accumulates, a farmer in WUA_S will be affected more negatively than in the one in WUA_N. Holding a constant, $b_1 < b_2$ implies that the degree of slope increases such that the farmers in WUA_S will be

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[23] The values of the parameters we use in illustrations are: $\alpha_1 = 0.1$, $\alpha_2 = 3.2$, $\alpha_3 = 0.4$, a=0.000001, r=0.02, $\gamma = 0.005$ and n=4

affected more negatively by groundwater accumulation. This will make each farmer in WUA_S apply less irrigation water. However, each farmer in WUA_N will respond to this change by increasing his/her amount of irrigation water applied.

CHAPTER 4

4. An Optimal Dynamic Taxation Scheme:

In section 3, we derived the time paths of groundwater accumulation for 4 different scenarios. We had the following assumptions in our model: The region is hydrologically connected in the sense that all the farmers are located above a common aquifer, there is a slope from north to south, the farmers in the north form the association WUA_N, the farmers in the south form WUA_S and each WUA is managed independently from each other to the contrary, Legras and Lifran (2006) assumed that even if each farmer moves in cooperation with other member farmers in his/her own WUA, the socially optimal level of groundwater accumulation cannot be reached. Similar to Legras and Lifran (2006), we will derive an optimal dynamic stock-based input tax[24] in order to induce non-cooperative farmers to follow the socially optimal time paths[25] of irrigation water choices and groundwater accumulation.[26] The general formulation of an input-based tax is: for farmer i in WUA_N, $T(w_i,G)=(m_1 \cdot G+k_1)\cdot w_i$ and for farmer j in WUA_S, $T(w_j,G)=(m_2 \cdot G+k_2)\cdot w_j$, where $m_1 \geq 0$, $m_2 \geq 0$, $k_1 \geq 0$ and $k_2 \geq 0$. We

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[24] This is a taxation scheme which is put on amount of irrigation water applied and depends on level of groundwater accumulation by a stock-based input tax.

[25] By socially optimal time paths we mean the time paths of irrigation water choices and groundwater accumulation realized when all farmers move in cooperation (case 1).

[26] Genaral properties of input taxes are discussed in Legras and Lifran, [2]. Through an input tax which is not dependent on groundwater accumulation, although the socially optimal level of groundwater accumulation can be reached, the farmers do not follow socially optimal time paths of irrigation water choices.

will deal only with the tax rates which depend on the groundwater accumulation. The information structure is the open-loop information structure and the solution concept is the OLNE.

As we apply the tax to non-cooperative agents, the problem of farmer i in WUA_N is: (for i=1,..,n),

$$\max_{w_i} \int_0^{\infty} \{F(w_i,G,a) - T_i(w_i,G)\} \cdot e^{-r.t} \cdot dt$$

$$\text{subject to } \dot{G} = \sum_{i=1}^{n} w_i + \sum_{j=n+1}^{2n} w_j - \delta \cdot G \quad \text{and} \quad G(0) = G_0 \geq 0.$$

The problem of farmer j in WUA_S is: (for j=n+1,..,2n),

$$\max_{w_j} \int_0^{\infty} \{F(w_j,G,b) - Tj(w_j,G)\} \cdot e^{-r.t} \cdot dt$$

$$\text{subject to } \dot{G} = \sum_{i=1}^{n} w_i + \sum_{j=n+1}^{2n} w_j - \delta \cdot G \quad \text{and} \quad G(0) = G_0 \geq 0.$$

PROPOSITION 6. There exist two farmer-specific taxation schemes of the form, $T(w_i,G) = (m_1 \cdot G + k_1) \cdot w_i$ for each farmer i in WUA_N, and $T(w_j,G) = (m_2 \cdot G + k_2) \cdot w_j$ for each farmer j in WUA_S with $m_1 \geq 0$, $m_2 \geq 0$, $k_1 \geq 0$ and $k_2 \geq 0$, which induce farmers who move non-cooperatively to follow the socially optimal amounts of irrigation water application and groundwater accumulation. The optimal parameters are the following:

$$m_1 = \frac{\alpha_3 \cdot (\gamma + s_1) \cdot (s_1 - \gamma - r) - 2 \cdot n \cdot a}{2 \cdot n \cdot r + (2 \cdot n + 1) \cdot \gamma + s_1 \cdot (1 - 2 \cdot n)} > 0,$$

$$k_1 = m_1 \cdot \frac{s_1 \cdot (1 - 2 \cdot n)}{2 \cdot n \cdot (r + \gamma)} \cdot G_{\infty}^{c} + \frac{s_1 \cdot \alpha_3 \cdot (r - s_1)}{2 \cdot n \cdot (r + \gamma)} \cdot G_{\infty}^{c} + \alpha_2 > 0,$$

$$m_2 = \frac{\alpha_3 \cdot (\gamma + s_1) \cdot (s_1 - \gamma - r) - 2 \cdot n \cdot b}{2 \cdot n \cdot r + (2 \cdot n + 1) \cdot \gamma + s_1 \cdot (1 - 2 \cdot n)} > 0,$$

$$k_2 = m_2 \cdot \frac{s_1 \cdot (1 - 2 \cdot n)}{2 \cdot n \cdot (r + \gamma)} \cdot G_\infty^c + \frac{s_1 \cdot \alpha_3 \cdot (r - s_1)}{2 \cdot n \cdot (r + \gamma)} \cdot G_\infty^c + \alpha_2 > 0 .$$

Proof. See Appendix E.

4.A. Numerical Illustrations:

In this section, we present a numerical example of taxation schemes for each farmer in WUA_N and WUA_S. The values of the parameters we use in illustrations are: $\alpha_1 = 0.1$, $\alpha_2 = 3.2$, $\alpha_3 = 0.4$, a=0.000001, b=0.0000031[27], r=0.02, γ= 0.005[28] and n=4 .

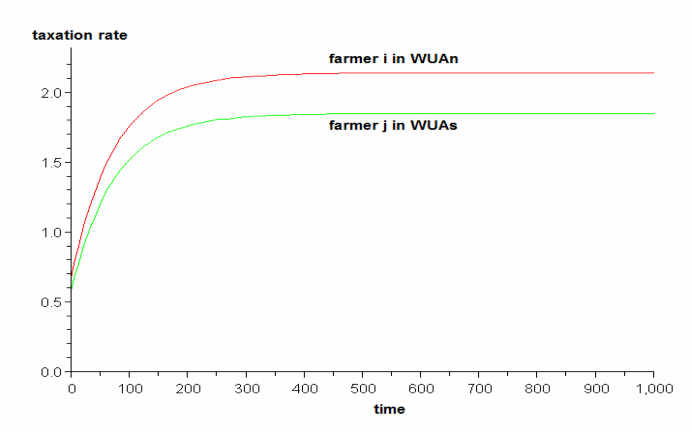

figure 16: taxation rates

By proposition 6, it can be seen that analytically $m_1 > m_2$ and $k_1 > k_2$ such that at each instant of time, the tax imposed on each farmer in WUA_N is

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[27] By holding other parameters constant, for b>0.0000031, irrigation water choice of farmer j in WUA_S becomes less than 0 in case 4.

[28] By holding other parameters constant, for $\gamma < 0.004$, irrigation water choice of farmer j in WUA_S becomes less than 0 in case 3 and case 4.

greater than the tax imposed on each farmer in WUA_S. In figure 16, how taxation rates for each farmer in WUA_N and WUA_S change over time is displayed. For all the farmers, taxation rates increase as time passes depending on the accumulation of groundwater. The taxation rates signal that as groundwater accumulation increases, all the farmers must be punished more severely in order to give them the incentive to use less irrigation water. As can be seen from figure 14, the taxation rate for each farmer i in WUA_N is greater than the taxation rate for each farmer j in WUA_S. The reason is, since each farmer i in WUA_N is less negatively affected by groundwater accumulation than each farmer j in WUA_S, each farmer i in WUA_N uses irrigation water more aggressively than each farmer j in WUA_S. So, each farmer i in WUA_N must be punished more aggressively than each farmer j in WUA_S in order to induce all the farmers in the region to follow their socially optimal time path for irrigation water choice. The taxation schemes must be applied to all the farmers in the region simultaneously when they move non-cooperatively. This is due to the necessity of a central water agency to interfere with the management of irrigation water in order to implement such dynamic taxation schemes to induce all the farmers to follow their socially optimal time paths for irrigation water choices.

CHAPTER 5

5. Conclusion:

In this thesis, we analyzed the irrigation induced problems in a region with a slope running from North to South. We also assume that the farmers in the South and North participate in different WUAs, which are managed independently. Although all farmers contribute to the accumulation of the common groundwater, farmers in the South and in the North are negatively affected by the groundwater accumulation by differing degrees due to the slope. We showed that even if each farmer moves in cooperation with other member farmers in his/her own WUA in deciding how much irrigation water to use, the level of the groundwater accumulation exceeds the socially optimal level of the groundwater accumulation. As a solution we proposed dynamic taxation schemes to those farmers who move non-cooperatively in order to make them follow their socially optimal irrigation water paths. With this setting, the more the groundwater accumulates, the bigger the taxation rates become. This means that we should penalize these farmers in a greater extent as the groundwater accumulates more with time. Also, at each point in time, the tax rate imposed on each farmer in the WUA_N is greater than that imposed on each farmer in the WUA_S. By assumption, farmers who are members of the WUA_N are less severely affected by the groundwater accumulation (to which all the farmers in the region contribute) through their irrigation practices than farmers who are members of the WUA_S. We showed that this results in excess/more aggressive use of irrigation water by farmers in the WUA_N compared with those in the WUA_S at each instant of time. This is the reason why farmers in the WUA_N must be penalized more severely than those in the WUA_S through time.

Based on our results we can deduce the following policy implications for the case of the Harran Plain. Within the Harran Plain salinization increased significantly after the introduction of irrigation in agricultural production in 1995. The parts of Harran Plain which are under severe salinization are the areas that are also waterlogged. These are mostly the depression sections of the Harran Plain [15]. A technical solution to the problem will be constructing appropriate drainage systems in problematic areas. This has a vital importance in the sense that it will increase the assimilative capacity of the underlying unconfined aquifer since leaching requirements in dormant season increase due to salt accumulation in the soil. But the main reasons of waterlogging especially in lower parts of the Harran Plain are not only restricted with irrigation water practices in these parts, but also the irrigation water practices in upper parts of the region. So, the salinization and the waterlogging problems cannot be solved by only the efforts of WUAs that are located in the Southern parts of the Harran Plain. A level of coordination must be provided between the WUAs in the upper parts and the lower parts. This task can be fulfilled by a central water authority such as the State Hydraulic Works in Turkey.

From the model, it can also be inferred that steady-state level of the groundwater accumulation can be lowered by growing less water intensive crops. A lower steady-state level of the groundwater accumulation corresponds to a lower probability of waterlogging. So, by broadening the crop patterns and giving incentives to farmers to grow less water intensive crops can be helpful in reducing the intensity of the waterlogging problem in the region. A similar argument can be supported for investing in better irrigation technologies such as drip irrigation which uses less water more effectively or subsidizing the construction of such systems in order to reduce the intensity of waterlogging problem in the region.

APPENDICES

Appendix A

Proof of proposition 1:

CWA's problem is

$$\max_{w_1,\dots,w_{2n}} \sum_{i=1}^{n}\int_0^{\infty} F_i(w_i,G,a)\cdot e^{-r.t}\cdot dt+\sum_{j=n+1}^{2n}\int_0^{\infty} F_j(w_j,G,b)\cdot e^{-r.t}\cdot dt$$

subject to $\dot{G}=\sum_{i=1}^{n} w_i+\sum_{j=n+1}^{2n} w_j-\gamma\cdot G$ and $G(0)=G_0\geq 0$.

Since we use open-loop Nash equilibrium, we solve the model through Hamiltonians. We apply the same method to other cases.

The current value Hamiltonian is:

$$\begin{aligned}H(w_i,w_j,G,\lambda)=&\sum_{i=1}^{n}(\alpha_1+\alpha_2\cdot w_i-\frac{\alpha_3}{2}\cdot w_i^2-\frac{a}{2}\cdot G^2)+\sum_{j=n+1}^{2n}(\alpha_1+\alpha_2\cdot w_j-\frac{\alpha_3}{2}\cdot w_j{}^2-\frac{b}{2}\cdot G^2)\\&+\lambda\cdot(\sum_{i=1}^{n} w_i+\sum_{j=n+1}^{2n} w_j-\gamma\cdot G).\end{aligned}$$

Assuming interior solutions, first order necessary conditons (FONCs) are:

for each i=1,2,..,n:

$$\frac{\partial H}{\partial w_i}=0 \;\Leftrightarrow\; \alpha_2-\alpha_3\cdot w_i^c+\lambda=0 \;\Rightarrow\; w_i^c=\frac{\lambda+\alpha_2}{\alpha_3}.$$

(a1) $w_i^c=\dfrac{\lambda+\alpha_2}{\alpha_3}$.

for each j=n+1,n+2,..,2n:

$$\frac{\partial H}{\partial w_j}=0 \;\Leftrightarrow\; \alpha_2-\alpha_3\cdot w_j^c+\lambda=0 \;\Rightarrow\; w_j^c=\frac{\lambda+\alpha_2}{\alpha_3}.$$

(a3) $w_j^c = \dfrac{\lambda + \alpha_2}{\alpha_3}$.

We need to derive adjoint equation: $\dot{\lambda} = r \cdot \lambda - \dfrac{\partial H^*}{\partial G}$, where H^* is maximized Hamiltonian. Maximized Hamiltonian is obtained by putting optimal control variables (here, they corresponds to optimal irrigation water choices, w_i^c and w_j^c) into current value Hamiltonian. So, $H^* = H(w_i^c, w_j^c, G, \lambda)$. Then,

(a4) $\dfrac{\partial H^*}{\partial G} = -n \cdot a \cdot G^c - n \cdot b \cdot G^c - \gamma \cdot \lambda$.

Put (a4) into adjoint equation such that we obtain:

$\dot{\lambda} = r \cdot \lambda - (-n \cdot a \cdot G^c - n \cdot b \cdot G^c - \gamma \cdot \lambda)$,

(a5) $\dot{\lambda} = (r + \gamma) \cdot \lambda + n \cdot (a + b) \cdot G^c$.

Here, (a1), (a3) and (a5) are FONCs. Since $\dfrac{\partial^2 H}{\partial w_i \partial w_i} = -\alpha_3 < 0$ and $\dfrac{\partial^2 H}{\partial w_j \partial w_j} = -\alpha_3 < 0$, Hamiltonian is concave in w_i and w_j such that the FONCs are also sufficient.

We put our optimal controls, w_i^c and w_j^c, into state equation:

$$\dot{G} = \sum_{i=1}^{n} w_i^c + \sum_{j=n+1}^{2n} w_j^c - \gamma \cdot G^c \Rightarrow \dot{G} = \sum_{i=1}^{n} (\frac{\lambda + \alpha_2}{\alpha_3}) + \sum_{j=n+1}^{2n} (\frac{\lambda + \alpha_2}{\alpha_3}) - \gamma \cdot G^c \Rightarrow$$

(a6) $\dot{G} = \dfrac{2 \cdot n \cdot \lambda}{\alpha_3} + \dfrac{2 \cdot n \cdot \alpha_2}{\alpha_3} - \gamma \cdot G^c$.

Differential equations (a5) and (a6) form the modified Hamiltonian dynamical system (MHDS). To solve the model, we need to solve MHDS. First, we find steady-state levels of λ and G. Set $\dot{\lambda} = 0$ and $\dot{G} = 0$.

$$\dot{\lambda}=0 \;\Rightarrow\; 0=(r+\gamma)\cdot\lambda_\infty+n\cdot(a+b)\cdot G_\infty^c \;\Rightarrow\; \lambda_\infty=-\frac{n\cdot(a+b)\cdot G_\infty^c}{(r+\gamma)}$$

(a7) $\lambda_\infty=-\dfrac{n\cdot(a+b)\cdot G_\infty^c}{(r+\gamma)}$.

$$\dot{G}=0 \;\Rightarrow\; 0=\frac{2\cdot n\cdot\lambda_\infty}{\alpha_3}+\frac{2\cdot n\cdot\alpha_2}{\alpha_3}-\gamma\cdot G_\infty^c \;\Rightarrow\; G_\infty^c=\frac{2\cdot n\cdot\lambda_\infty}{\alpha_3\cdot\gamma}+\frac{2\cdot n\cdot\alpha_2}{\alpha_3\cdot\gamma}$$

(a8) $G_\infty^c=\dfrac{2\cdot n\cdot\lambda_\infty}{\alpha_3\cdot\gamma}+\dfrac{2\cdot n\cdot\alpha_2}{\alpha_3\cdot\gamma}$. Put (a7) into (a8):

$$G_\infty^c=\frac{2\cdot n}{\alpha_3\cdot\gamma}\cdot\left(-\frac{n\cdot(a+b)\cdot G_\infty^c}{(r+\gamma)}\right)+\frac{2\cdot n\cdot\alpha_2}{\alpha_3\cdot\gamma} \;\Rightarrow$$

$$G_\infty^c\left(\frac{\alpha_3\cdot\gamma\cdot(r+\gamma)+2\cdot n^2\cdot(a+b)}{\alpha_3\cdot\gamma\cdot(r+\gamma)}\right)=\frac{2\cdot n\cdot\alpha_2}{\alpha_3\cdot\gamma} \;\Rightarrow$$

(a9) $G_\infty^c=\dfrac{2\cdot n\cdot\alpha_2\cdot(r+\gamma)}{\alpha_3\cdot\gamma\cdot(r+\gamma)+2\cdot n^2\cdot(a+b)}$.

We need to solve MHDS, where we know G_0, G_∞^c and λ_∞. So, from (a5) and (a6),

$$\begin{aligned}\dot{\lambda}&=(r+\gamma)\cdot\lambda+n\cdot(a+b)\cdot G^c\\ \dot{G}&=\frac{2\cdot n\cdot\lambda}{\alpha_3}-\gamma\cdot G^c+\frac{2\cdot n\cdot\alpha_2}{\alpha_3}\end{aligned} \;\Rightarrow\; \dot{y}=A\cdot y\text{, where } y=\begin{pmatrix}\lambda\\ G\end{pmatrix}\text{ and}$$

$A=\begin{pmatrix} r+\gamma & n\cdot(a+b)\\ \dfrac{2\cdot n}{\alpha_3} & -\gamma\end{pmatrix}$. To solve MHDS, we need to find the eigenvalues of the matrix A. The characteristic polynomial is $s^2-r\cdot s-\gamma^2-\gamma\cdot s-\dfrac{2\cdot n^2\cdot(a+b)}{\alpha_3}=0$. To ensure stability, we choose the negative root of the characteristic polynomial. Let s_1 be the negative root. We know that optimal groundwater accumulation has the following form: $G^c(t)=D\cdot e^{s_1\cdot t}+E$. We need to find the parameters D and E.

We know G_0 and G_∞^c. Then,

$G^c(0) = D \cdot e^{s_1 \cdot 0} + E = G_0 \Rightarrow D + E = G_0$

$G^c(\infty) = D \cdot e^{s_1 \cdot \infty} + E = G_\infty^c \Rightarrow D \cdot 0 + E = G_\infty^c \Rightarrow E = G_\infty^c$.

So, $D + E = G_0 \Rightarrow D = G_0 - E = G_0 - G_\infty^c \Rightarrow D = G_0 - G_\infty^c$.

So, $G^c(t) = \left(G_0 - G_\infty^c\right) \cdot e^{s_1 \cdot t} + G_\infty^c$.

We know that the adjoint variable has the following form: $\lambda(t) = D \cdot e^{s_1 \cdot t} + E$.

We know that $\lambda_\infty = -\dfrac{n \cdot (a+b) \cdot G_\infty^c}{(r+\gamma)}$. Then,

$\lambda(\infty) = D \cdot e^{s_1 \cdot \infty} + E = \lambda_\infty \Rightarrow D \cdot 0 + E = \lambda_\infty \Rightarrow E = \lambda_\infty$.

Then, $\lambda(t) = D \cdot e^{s_1 \cdot t} + \lambda_\infty$. $\Rightarrow \lambda(t) = D \cdot e^{s_1 \cdot t} - \dfrac{n \cdot (a+b) \cdot G_\infty^c}{(r+\gamma)}$. We need to find the parameter D. $\lambda(t) = D \cdot e^{s_1 \cdot t} - \dfrac{n \cdot (a+b) \cdot G_\infty^c}{(r+\gamma)}$ has to satisfy the adjoint equation (a5). So,

$\dot{\lambda} = D \cdot s_1 \cdot e^{s_1 \cdot t} = (r+\gamma) \cdot \left(D \cdot e^{s_1 \cdot t} - \dfrac{n \cdot (a+b) \cdot G_\infty^c}{(r+\gamma)} \right) + n \cdot (a+b) \cdot G^c \quad \Rightarrow$ By putting $G^c(t) = \left(G_0 - G_\infty^c\right) \cdot e^{s_1 \cdot t} + G_\infty^c$ into equation,

$$D \cdot \left(s_1 - r - \gamma\right) \cdot e^{s_1 \cdot t} = -n \cdot (a+b) \cdot G_\infty^c + n \cdot (a+b) \cdot \left[\left(G_0 - G_\infty^c\right) \cdot e^{s_1 \cdot t} + G_\infty^c \right] \Rightarrow$$

$$D \cdot \left(s_1 - r - \gamma\right) \cdot e^{s_1 \cdot t} = n \cdot (a+b) \cdot \left(G_0 - G_\infty^c\right) \cdot e^{s_1 \cdot t} \Rightarrow D = \frac{n \cdot (a+b) \cdot \left(G_0 - G_\infty^c\right)}{\left(s_1 - r - \gamma\right)}.$$

So,

$\lambda(t) = \dfrac{n \cdot (a+b) \cdot \left(G_0 - G_\infty^c\right)}{\left(s_1 - r - \gamma\right)} \cdot e^{s_1 \cdot t} - \dfrac{n \cdot (a+b) \cdot G_\infty^c}{(r+\gamma)} \Rightarrow$ So, adjoint variable is:

(a10) $\lambda(t)=n\cdot(a+b)\cdot\left[\frac{1}{(s_1-r-\gamma)}\cdot G^c(t)-\frac{s_1}{(r+\gamma)\cdot(s_1-r-\gamma)}\cdot G^c_\infty\right].$

for each i=1,2,..,n and each j=n+1,n+2,..,2n: (from FONCs)

$$w_i^c=\frac{\lambda+\alpha_2}{\alpha_3}=w_j^c \Rightarrow$$

$$w_i^c=w_j^c=w^c=\frac{n\cdot(a+b)\cdot\left[\frac{1}{(s_1-r-\gamma)}\cdot G^c(t)-\frac{s_1}{(r+\gamma)\cdot(s_1-r-\gamma)}\cdot G^c_\infty\right]+\alpha_2}{\alpha_3}$$

.

The transversality condition is also satisfied: $\lim_{t\to\infty} e^{-r\cdot t}\cdot\lambda(t)\cdot G^c(t)=0$.

Another method to derive $\lambda(t)$ is:

We know that $G^c(t)=\left(G_0-G^c_\infty\right)\cdot e^{s_1\cdot t}+G^c_\infty$.

We also know that $\lambda(t)$ satisfies (a6).

$$\dot{G}=s_1\cdot\left(G_0-G^c_\infty\right)\cdot e^{s_1\cdot t}=\frac{2\cdot n\cdot\lambda}{\alpha_3}+\frac{2\cdot n\cdot\alpha_2}{\alpha_3}-\gamma\cdot\left(G_0-G^c_\infty\right)\cdot e^{s_1\cdot t}+G^c_\infty \Rightarrow$$

$$\dot{G}=s_1\cdot\left(G_0-G^c_\infty\right)\cdot e^{s_1\cdot t}=\frac{2\cdot n\cdot\lambda}{\alpha_3}+\frac{2\cdot n\cdot\alpha_2}{\alpha_3}-\gamma\cdot G^c \Rightarrow$$

$$s_1\cdot\left(G^c-G^c_\infty\right)=\frac{2\cdot n\cdot\lambda}{\alpha_3}+\frac{2\cdot n\cdot\alpha_2}{\alpha_3}-\gamma\cdot G^c \Rightarrow$$

$$s_1\cdot G^c-s_1\cdot G^c_\infty=\frac{2\cdot n\cdot\lambda}{\alpha_3}+\frac{2\cdot n\cdot\alpha_2}{\alpha_3}-\gamma\cdot G^c \Rightarrow$$

$$\left(s_1+\gamma\right)\cdot G^c-s_1\cdot G^c_\infty=\frac{2\cdot n\cdot\lambda}{\alpha_3}+\frac{2\cdot n\cdot\alpha_2}{\alpha_3} \Rightarrow$$

$$\lambda=\frac{\left(\left(s_1+\gamma\right)\cdot G^c-s_1\cdot G^c_\infty\right)\cdot\alpha_3}{2\cdot n}-\alpha_2 \Rightarrow$$

Put $\lambda(t)$ into (a3):

$$w_i^c = w_j^c = w^c = \frac{\left(\dfrac{\left(\left(s_1+\gamma\right)\cdot G^c - s_1\cdot G_\infty^c\right)\cdot\alpha_3}{2\cdot n} - \alpha_2\right) + \alpha_2}{\alpha_3} \Rightarrow$$

$$\text{(a11)} \quad w_i^c = w_j^c = w^c = \frac{\left(\left(s_1+\gamma\right)\cdot G^c - s_1\cdot G_\infty^c\right)}{2\cdot n} \quad \square$$

Appendix B

Proof of proposition 2:

Firstly, we will solve the problem of each farmer in WUA_N. Secondly, we will solve the problem of each farmer in WUA_S. Farmer i's problem is: (for i=1,..,n)

$$\max_{w_i} \int_0^\infty F_i(w_i,G,a)\cdot e^{-r.t}\cdot dt$$

subject to $\overset{\square}{\mathrm{G}} = \sum_{i=1}^{n} w_i + \sum_{j=n+1}^{2n} w_j - \delta\cdot G$ and $G(0) = G_0 \geq 0$

The current value Hamiltonian is:

$$H_i(w_i,G,\lambda_i) = \alpha_1 + \alpha_2\cdot w_i - \frac{\alpha_3}{2}\cdot w_i^2 - \frac{a}{2}\cdot G^2 + \lambda_i\cdot\left(\sum_{i=1}^{n} w_i + \sum_{j=n+1}^{2n} w_j - \gamma\cdot G\right).$$

Assuming interior solutions, first order necessary conditons (FONCs) are:

$$\frac{\partial H_i}{\partial w_i} = 0 \Leftrightarrow \alpha_2 - \alpha_3\cdot w_i^{nc} + \lambda_i = 0 \Rightarrow w_i^{nc} = \frac{\lambda_i + \alpha_2}{\alpha_3}.$$

$$\text{(a12)} \quad w_i^{nc} = \frac{\lambda_i + \alpha_2}{\alpha_3}.$$

We need to derive adjoint equation: $\dot{\lambda}_i = r \cdot \lambda_i - \frac{\partial H_i^*}{\partial G}$, where H_i^* is maximized Hamiltonian. Maximized Hamiltonian is obtained by putting optimal control variable, w_i^{nc}, into current value Hamiltonian. So, $H_i^* = H_i(w_i^{nc}, G, \lambda_i)$. Then,

(a13) $\frac{\partial H_i^*}{\partial G} = -a \cdot G^{nc} - \gamma \cdot \lambda_i$.

Put (a12) into adjoint equation such that we obtain:

$\dot{\lambda}_i = r \cdot \lambda_i - (-a \cdot G^{nc} - \gamma \cdot \lambda_i)$,

(a14) $\dot{\lambda}_i = (r + \gamma) \cdot \lambda_i + a \cdot G^{nc}$.

Here, (a12) and (a14) are FONCs. Since $\frac{\partial^2 H_i}{\partial w_i \partial w_i} = -\alpha_3 < 0$, H_i is concave in w_i such that the FONCs are also sufficient.

The problem of farmer j in WUA_S is: (for j=n+1,…,2n)

$$\max_{w_j} \int_0^\infty F_j(w_j, G, b) \cdot e^{-r.t} \cdot dt$$

subject to $\dot{G} = \sum_{i=1}^{n} w_i + \sum_{j=n+1}^{2n} w_j - \delta \cdot G$ and $G(0) = G_0 \geq 0$

The current value Hamiltonian is:

$$H_j(w_j, G, \lambda_j) = \alpha_1 + \alpha_2 \cdot w_j - \frac{\alpha_3}{2} \cdot w_j^2 - \frac{a}{2} \cdot G^2 + \lambda_j^* \cdot \left(\sum_{i=1}^{n} w_i + \sum_{j=n+1}^{2n} w_j - \gamma \cdot G\right).$$

Assuming interior solutions, first order necessary conditons (FONCs) are:

$$\frac{\partial H_j}{\partial w_j} = 0 \iff \alpha_2 - \alpha_3 \cdot w_j^{nc} + \lambda_j = 0 \Rightarrow w_j^{nc} = \frac{\lambda_j + \alpha_2}{\alpha_3}.$$

(a15) $w_j^{nc} = \frac{\lambda_j + \alpha_2}{\alpha_3}$.

Since $\frac{\partial^2 H_j}{\partial w_j \partial w_j} = -\alpha_3 < 0$, H_j is concave in w_j such that the FONCs are also sufficient.

We need to derive adjoint equation: $\dot{\lambda}_j = r \cdot \lambda_j - \frac{\partial H_j^*}{\partial G}$, where H_j^* is maximized Hamiltonian. Maximized Hamiltonian is obtained by putting optimal control variable, w_j^{nc}, into current value Hamiltonian. So, $H_j^* = H_j(w_j^{nc}, G, \lambda_j)$. Then,

(a16) $\frac{\partial H_j^*}{\partial G} = -b \cdot G^{nc} - \gamma \cdot \lambda_j$.

Put (a16) into adjoint equation such that we obtain:

$\dot{\lambda}_j = r \cdot \lambda_j - (-b \cdot G^{nc} - \gamma \cdot \lambda_j)$,

(a17) $\dot{\lambda}_j = (r + \gamma) \cdot \lambda_j + b \cdot G^{nc}$.

Here, (a15) and (a16) are FONCs. Since $\frac{\partial^2 H_j}{\partial w_j \partial w_j} = -\alpha_3 < 0$, H_j is concave in w_j such that the FONCs are also sufficient.

We put our optimal controls, w_i^{nc} and w_j^{nc}, into state equation:

$$\dot{G} = \sum_{i=1}^{n} w_i^{nc} + \sum_{j=n+1}^{2n} w_j^{nc} - \gamma \cdot G^{nc} \quad \Rightarrow \quad \dot{G} = \sum_{i=1}^{n} (\frac{\lambda_i + \alpha_2}{\alpha_3}) + \sum_{j=n+1}^{2n} (\frac{\lambda_j + \alpha_2}{\alpha_3}) - \gamma \cdot G^{nc}$$

$\Rightarrow$

(a18) $$\dot{G} = \frac{\sum_{i=1}^{n} \lambda_i}{\alpha_3} + \frac{2 \cdot n \cdot \alpha_2}{\alpha_3} + \frac{\sum_{j=n+1}^{2n} \lambda_j}{\alpha_3} - \gamma \cdot G^{nc}.$$

Differential equations (a14), (a17) and (a18) form the modified Hamiltonian dynamical system (MHDS). To solve the model, we need to solve MHDS.

First, we find steady-state levels of λ_i for i=1,..,n, λ_j for j=n+1,…,2n and G.

Set $\dot{\lambda}_i = 0$, $\dot{\lambda}_j = 0$ and $\dot{G} = 0$.

$$\dot{\lambda}_i = 0 \Rightarrow 0 = (r+\gamma)\cdot\lambda_{i\infty} + a\cdot G_\infty^{nc} \Rightarrow \lambda_{i\infty} = -\frac{a\cdot G_\infty^{nc}}{(r+\gamma)}$$

(a19) $\lambda_{i\infty} = -\frac{a\cdot G_\infty^{nc}}{(r+\gamma)}$,

$$\dot{\lambda}_j = 0 \Rightarrow 0 = (r+\gamma)\cdot\lambda_{j\infty} + b\cdot G_\infty^{nc} \Rightarrow \lambda_{j\infty} = -\frac{b\cdot G_\infty^{nc}}{(r+\gamma)}$$

(a20) $\lambda_{j\infty} = -\frac{b\cdot G_\infty^{nc}}{(r+\gamma)}$,

$$\dot{G} = 0 \quad \Rightarrow \quad 0 = \frac{n\cdot\lambda_{i\infty}}{\alpha_3} + \frac{n\cdot\lambda_{j\infty}}{\alpha_3} + \frac{2\cdot n\cdot\alpha_2}{\alpha_3} - \gamma\cdot G_\infty^{nc} \quad \Rightarrow$$

$$G_\infty^{nc} = \frac{n\cdot\lambda_{i\infty}}{\alpha_3\cdot\gamma} + \frac{n\cdot\lambda_{j\infty}}{\alpha_3\cdot\gamma} + \frac{2\cdot n\cdot\alpha_2}{\alpha_3\cdot\gamma}$$

(a21) $G_\infty^{nc} = \frac{n\cdot\lambda_{i\infty}}{\alpha_3\cdot\gamma} + \frac{n\cdot\lambda_{j\infty}}{\alpha_3\cdot\gamma} + \frac{2\cdot n\cdot\alpha_2}{\alpha_3\cdot\gamma}$. Put (a19) and (a20) into (a21):

$$G_\infty^{nc} = \frac{n}{\alpha_3\cdot\gamma}\cdot\left(-\frac{a\cdot G_\infty^{nc}}{(r+\gamma)}\right) + \frac{n}{\alpha_3\cdot\gamma}\cdot\left(-\frac{b\cdot G_\infty^{nc}}{(r+\gamma)}\right) + \frac{2\cdot n\cdot\alpha_2}{\alpha_3\cdot\gamma} \Rightarrow$$

$$G_\infty^{nc}\left(\frac{\alpha_3\cdot\gamma\cdot(r+\gamma) + n\cdot(a+b)}{\alpha_3\cdot\gamma\cdot(r+\gamma)}\right) = \frac{2\cdot n\cdot\alpha_2}{\alpha_3\cdot\gamma} \Rightarrow$$

(a22) $G_\infty^{nc} = \frac{2\cdot n\cdot\alpha_2\cdot(r+\gamma)}{\alpha_3\cdot\gamma\cdot(r+\gamma) + n\cdot(a+b)}$.

We need to solve MHDS, where we know G_0, G_∞^{nc}, $\lambda_{i\infty}$ and $\lambda_{j\infty}$. So, from (a14), (a17) and (a18),

$$\begin{aligned} &\dot{\lambda}_1 = (r+\gamma)\cdot\lambda_1 + a\cdot G^{nc} \\ &. \\ &. \\ &\dot{\lambda}_{2n} = (r+\gamma)\cdot\lambda_{2n} + a\cdot G^{nc} \\ &\dot{G} = \frac{\sum_{i=1}^{n}\lambda_i}{\alpha_3} + \frac{\sum_{j=n+1}^{2n}\lambda_j}{\alpha_3} - \gamma\cdot G^{nc} + \frac{2\cdot n\cdot\alpha_2}{\alpha_3} \end{aligned} \quad \Rightarrow \quad \dot{y} = A\cdot y\text{, where } y = \begin{pmatrix} \lambda_1 \\ \lambda_2 \\ . \\ . \\ . \\ \lambda_{2n} \\ G^{nc} \end{pmatrix} \text{ and}$$

$$A = \begin{pmatrix} r+\gamma & 0 & . & . & . & 0 & a \\ 0 & r+\gamma & 0 & . & . & . & . \\ . & 0 & r+\gamma & 0 & . & . & a \\ . & . & 0 & r+\gamma & 0 & . & b \\ . & . & . & 0 & r+\gamma & 0 & . \\ 0 & . & . & . & 0 & r+\gamma & b \\ \frac{1}{\alpha_3} & \frac{1}{\alpha_3} & \frac{1}{\alpha_3} & \frac{1}{\alpha_3} & \frac{1}{\alpha_3} & \frac{1}{\alpha_3} & -\gamma \end{pmatrix}$$

such that the dimension of y is $(2\cdot n+1)\times 1$ and the dimension of A is $(2\cdot n+1)\times(2\cdot n+1)$

To solve MHDS, we need to find the eigenvalues of the matrix A. The characteristic polynomial is $s^2 - r\cdot s - \gamma^2 - \gamma\cdot s - \frac{n\cdot(a+b)}{\alpha_3} = 0$. To ensure stability, we choose the negative root of the characteristic polynomial. Let s_2 be the negative root. We know that optimal groundwater accumulation has the following form: $G^{nc}(t) = D\cdot e^{s_2\cdot t} + E$. We need to find the parameters D and E.

We know G_0 and G_∞^{nc}. Then,

$G^{nc}(0) = D\cdot e^{s_2\cdot 0} + E = G_0 \;\Rightarrow\; D + E = G_0$

$G^{nc}(\infty) = D\cdot e^{s_2\cdot\infty} + E = G_\infty^{nc} \;\Rightarrow\; D\cdot 0 + E = G_\infty^{nc} \;\Rightarrow\; E = G_\infty^{nc}$.

So, $D + E = G_0 \;\Rightarrow\; D = G_0 - E = G_0 - G_\infty^{nc} \;\Rightarrow\; D = G_0 - G_\infty^{nc}$.

So, $G^{nc}(t) = \left(G_0 - G_\infty^{nc}\right) \cdot e^{s_2 \cdot t} + G_\infty^{nc}$.

We know that the adjoint variables has the following forms: $\lambda_i(t) = D \cdot e^{s_2 \cdot t} + E$ for farmers in WUA_N and $\lambda_j(t) = K \cdot e^{s_2 \cdot t} + L$ for farmers in WUA_S. We know that $\lambda_{i\infty} = -\dfrac{a \cdot G_\infty^{nc}}{(r+\gamma)}$. Then,

$\lambda_i(\infty) = D \cdot e^{s_2 \cdot \infty} + E = \lambda_{i\infty} \;\Rightarrow\; D \cdot 0 + E = \lambda_{i\infty} \;\Rightarrow\; E = \lambda_{i\infty}$.

Then, $\lambda_i(t) = D \cdot e^{s_2 \cdot t} + \lambda_{i\infty}$. $\Rightarrow$ $\lambda_i(t) = D \cdot e^{s_2 \cdot t} - \dfrac{a \cdot G_\infty^{nc}}{(r+\gamma)}$. We need to find the parameter D. $\lambda_i(t) = D \cdot e^{s_2 \cdot t} - \dfrac{a \cdot G_\infty^{nc}}{(r+\gamma)}$ has to satisfy the adjoint equation (a13).

So, $\dot{\lambda}_j = D \cdot s_2 \cdot e^{s_2 \cdot t} = (r+\gamma) \cdot \left(D \cdot e^{s_2 \cdot t} - \dfrac{a \cdot G_\infty^{nc}}{(r+\gamma)} \right) + a \cdot G^{nc}$ $\Rightarrow$ By putting $G^{nc}(t) = \left(G_0 - G_\infty^{nc}\right) \cdot e^{s_2 \cdot t} + G_\infty^{nc}$ into equation,

$$D \cdot \left(s_2 - r - \gamma\right) \cdot e^{s_2 \cdot t} = -a \cdot G_\infty^{nc} + a \cdot \left[\left(G_0 - G_\infty^{nc}\right) \cdot e^{s_2 \cdot t} + G_\infty^{nc} \right] \Rightarrow$$

$D \cdot \left(s_2 - r - \gamma\right) \cdot e^{s_2 \cdot t} = a \cdot \left(G_0 - G_\infty^{nc}\right) \cdot e^{s_2 \cdot t} \;\Rightarrow\; D = \dfrac{a \cdot \left(G_0 - G_\infty^{nc}\right)}{\left(s_2 - r - \gamma\right)}$. So,

$\lambda_i(t) = \dfrac{a \cdot \left(G_0 - G_\infty^{nc}\right)}{\left(s_2 - r - \gamma\right)} \cdot e^{s_2 \cdot t} - \dfrac{a \cdot G_\infty^{nc}}{(r+\gamma)}$ $\Rightarrow$ So, adjoint variable is:

(a23) $$\lambda_i(t) = a \cdot \left[\frac{1}{(s_2 - r - \gamma)} \cdot G^{nc}(t) - \frac{s_2}{(r+\gamma) \cdot (s_2 - r - \gamma)} \cdot G_\infty^{nc} \right].$$

for each i=1,2,..,n: (from FONCs)

$$w_i^{nc} = \frac{\lambda_i + \alpha_2}{\alpha_3} \qquad \Rightarrow$$

$$w_i^{nc} = \frac{a \cdot \left[\frac{1}{(s_2 - r - \gamma)} \cdot G^{nc}(t) - \frac{s_2}{(r+\gamma)\cdot(s_2 - r - \gamma)} \cdot G_\infty^{nc} \right] + \alpha_2}{\alpha_3}.$$

We know $\lambda_{j\infty} = -\frac{b \cdot G_\infty^{nc}}{(r+\gamma)}$. Then,

$\lambda_j(\infty) = K \cdot e^{s_2 \cdot \infty} + L = \lambda_{j\infty} \Rightarrow K \cdot 0 + L = \lambda_{j\infty} \Rightarrow L = \lambda_{j\infty}$.

Then, $\lambda_j(t) = K \cdot e^{s_2 \cdot t} + \lambda_{j\infty}$. $\Rightarrow \lambda_j(t) = K \cdot e^{s_2 \cdot t} - \frac{b \cdot G_\infty^{nc}}{(r+\gamma)}$. We need to find the parameter K. $\lambda_j(t) = K \cdot e^{s_2 \cdot t} - \frac{b \cdot G_\infty^{nc}}{(r+\gamma)}$ has to satisfy the adjoint equation (a13).

So, $\dot{\lambda}_j = K \cdot s_2 \cdot e^{s_2 \cdot t} = (r+\gamma) \cdot \left(K \cdot e^{s_2 \cdot t} - \frac{b \cdot G_\infty^{nc}}{(r+\gamma)} \right) + b \cdot G^{nc} \Rightarrow$ By putting

$G^{nc}(t) = \left(G_0 - G_\infty^{nc}\right) \cdot e^{s_2 \cdot t} + G_\infty^{nc}$ into equation,

$$K \cdot (s_2 - r - \gamma) \cdot e^{s_2 \cdot t} = -b \cdot G_\infty^{nc} + b \cdot \left[\left(G_0 - G_\infty^{nc}\right) \cdot e^{s_2 \cdot t} + G_\infty^{nc} \right] \Rightarrow$$

$K \cdot (s_2 - r - \gamma) \cdot e^{s_2 \cdot t} = b \cdot \left(G_0 - G_\infty^{nc}\right) \cdot e^{s_2 \cdot t} \Rightarrow K = \frac{b \cdot \left(G_0 - G_\infty^{nc}\right)}{\left(s_2 - r - \gamma\right)}$. So,

$\lambda_j(t) = \frac{b \cdot \left(G_0 - G_\infty^{nc}\right)}{\left(s_2 - r - \gamma\right)} \cdot e^{s_2 \cdot t} - \frac{b \cdot G_\infty^{nc}}{(r+\gamma)} \Rightarrow$ So, adjoint variable is:

(a24) $\lambda_j(t) = b \cdot \left[\frac{1}{(s_2 - r - \gamma)} \cdot G^{nc}(t) - \frac{s_2}{(r+\gamma)\cdot(s_2 - r - \gamma)} \cdot G_\infty^{nc} \right]$.

for each j=n+1,n+2,..,2n: (from FONCs)

$$w_j^{nc}=\frac{\lambda_j+\alpha_2}{\alpha_3} \qquad \Rightarrow$$

$$w_j^{nc}=\frac{b\cdot\left[\frac{1}{(s_2-r-\gamma)}\cdot G^{nc}(t)-\frac{s_2}{(r+\gamma)\cdot(s_2-r-\gamma)}\cdot G_\infty^{nc}\right]+\alpha_2}{\alpha_3}.$$

The transversality condition is also satisfied: $\lim_{t\to\infty} e^{-r\cdot t}\cdot\lambda_i(t)\cdot G^{nc}(t)=0$ and

$\lim_{t\to\infty} e^{-r\cdot t}\cdot\lambda_j(t)\cdot G^{nc}(t)=0$. $\square$

Appendix C

Proof of proposition 3:

Firstly, we will solve the problem of the board of WUA_N. Secondly, we will solve the problem of the board of WUA_S. The problem of the board of WUA_N is:

$$\max_{w_1,\dots,w_n} \sum_{i=1}^{n}\int_0^\infty F_i(w_i,G,a)\cdot e^{-r.t}\cdot dt$$

$$\text{subject to } \dot{G}=\sum_{i=1}^{n}w_i+\sum_{j=n+1}^{2n}w_j-\gamma\cdot G \quad \text{and} \quad G(0)=G_0\geq 0$$

The current value Hamiltonian is:

$$H_i(w_i,G,\lambda_{3i})=\sum_{i=1}^{n}(\alpha_1+\alpha_2\cdot w_i-\frac{\alpha_3}{2}\cdot w_i^2-\frac{a}{2}\cdot G^2)+\lambda_{3i}^*\cdot(\sum_{i=1}^{n}w_i+\sum_{j=n+1}^{2n}w_j-\gamma\cdot G).$$

Assuming interior solutions, first order necessary conditons (FONCs) are:

$$\frac{\partial H_i}{\partial w_i}=0 \Leftrightarrow \alpha_2-\alpha_3\cdot w_{3i}+\lambda_{3i}=0 \Rightarrow w_{3i}=\frac{\lambda_{3i}+\alpha_2}{\alpha_3}.$$

(a25) $w_{3i}=\dfrac{\lambda_{3i}+\alpha_2}{\alpha_3}$.

We need to derive adjoint equation: $\dot{\lambda}_{3i} = r\cdot\lambda_{3i} - \frac{\partial H_i^*}{\partial G}$, where H_i^* is maximized Hamiltonian. Maximized Hamiltonian is obtained by putting optimal control variable, w_{3i}, into current value Hamiltonian. So, $H_i^* = H_i(w_{3i}, G, \lambda_{3i})$. Then,

(a26) $\frac{\partial H_i^*}{\partial G} = -n\cdot a\cdot G_3 - \gamma\cdot\lambda_{3i}$.

Put (a26) into adjoint equation such that we obtain:

$\dot{\lambda}_{3i} = r\cdot\lambda_{3i} - (-n\cdot a\cdot G_3 - \gamma\cdot\lambda_{3i})$,

(a27) $\dot{\lambda}_{3i} = (r+\gamma)\cdot\lambda_{3i} + n\cdot a\cdot G_3$.

Here, (a25) and (a27) are FONCs. Since $\frac{\partial^2 H_i}{\partial w_i \partial w_i} = -\alpha_3 < 0$, H_i is concave in w_i such that the FONCs are also sufficient.

The problem of the board of WUA_S is:

$$\max_{w_{n+1},\dots,w_{2n}} \sum_{j=n+1}^{2n}\int_0^{\infty} F_j(w_i,G,b)\cdot e^{-r.t}\cdot dt$$

$$\text{subject to } \overset{\square}{\mathrm{G}} = \sum_{i=1}^{n} w_i + \sum_{j=n+1}^{2n} w_j - \gamma\cdot G \quad \text{and} \quad G(0) = G_0 \geq 0$$

The current value Hamiltonian is:

$$H_j(w_j,G,\lambda_{3j}) = \sum_{i=n+1}^{2n}(\alpha_1 + \alpha_2\cdot w_j - \frac{\alpha_3}{2}\cdot w_j^{\,2} - \frac{b}{2}\cdot G^2) + \lambda_{3j}^*\cdot(\sum_{i=1}^{n} w_i + \sum_{j=n+1}^{2n} w_j - \gamma\cdot G).$$

Assuming interior solutions, first order necessary conditons (FONCs) are:

$$\frac{\partial H_j}{\partial w_j} = 0 \iff \alpha_2 - \alpha_3\cdot w_{3j} + \lambda_{3j} = 0 \Rightarrow w_{3j} = \frac{\lambda_{3j} + \alpha_2}{\alpha_3}.$$

(a28) $w_{3j} = \frac{\lambda_{3j} + \alpha_2}{\alpha_3}$.

We need to derive adjoint equation: $\dot{\lambda}_{3j} = r \cdot \lambda_{3j} - \frac{\partial H_j^*}{\partial G}$, where H_j^* is maximized Hamiltonian. Maximized Hamiltonian is obtained by putting optimal control variable, w_{3j}, into current value Hamiltonian. So, $H_j^* = H_j(w_{3j}, G, \lambda_{3j})$. Then,

(a29) $\frac{\partial H_j^*}{\partial G} = -n \cdot b \cdot G_3 - \gamma \cdot \lambda_{3j}$.

Put (a29) into adjoint equation such that we obtain:

$\dot{\lambda}_{3j} = r \cdot \lambda_{3j} - (-n \cdot b \cdot G_3 - \gamma \cdot \lambda_{3j})$,

(a30) $\dot{\lambda}_{3j} = (r + \gamma) \cdot \lambda_{3j} + n \cdot b \cdot G_3$.

Here, (a28) and (a30) are FONCs. Since $\frac{\partial^2 H_j}{\partial w_j \partial w_j} = -\alpha_3 < 0$, H_j is concave in w_j such that the FONCs are also sufficient.

We put our optimal controls, w_{3i} and w_{3j}, into state equation:

$$\dot{G} = \sum_{i=1}^{n} w_{3i} + \sum_{j=n+1}^{2n} w_{3j} - \gamma \cdot G_3 \Rightarrow \dot{G} = \sum_{i=1}^{n} (\frac{\lambda_{3i} + \alpha_2}{\alpha_3}) + \sum_{j=n+1}^{2n} (\frac{\lambda_{3j} + \alpha_2}{\alpha_3}) - \gamma \cdot G_3 \Rightarrow$$

$$\dot{G} = \frac{\sum_{i=1}^{n} \lambda_{3i}}{\alpha_3} + \frac{2 \cdot n \cdot \alpha_2}{\alpha_3} + \frac{\sum_{j=n+1}^{2n} \lambda_{3j}}{\alpha_3} - \gamma \cdot G_3 \Rightarrow$$

$$\dot{G} = \frac{n \cdot \lambda_{3i}}{\alpha_3} + \frac{2 \cdot n \cdot \alpha_2}{\alpha_3} + \frac{n \cdot \lambda_{3j}}{\alpha_3} - \gamma \cdot G_3 \Rightarrow$$

(a31) $\dot{G} = \frac{n \cdot \lambda_{3i}}{\alpha_3} + \frac{2 \cdot n \cdot \alpha_2}{\alpha_3} + \frac{n \cdot \lambda_{3j}}{\alpha_3} - \gamma \cdot G_3$.

Differential equations (a27), (a30) and (a31) form the modified Hamiltonian dynamical system (MHDS). To solve the model, we need to solve MHDS.

First, we find steady-state levels of λ_{3i} , λ_{3j} and G. Set $\dot{\lambda}_{3i}=0$, $\dot{\lambda}_{3j}=0$ and $\dot{G}=0$.

$$\dot{\lambda}_{3i}=0 \;\Rightarrow\; 0=(r+\gamma)\cdot\lambda_{3i\infty}+n\cdot a\cdot G_{3\infty} \;\Rightarrow\; \lambda_{3i\infty}=-\frac{n\cdot a\cdot G_{3\infty}}{(r+\gamma)}$$

(a32) $\lambda_{3i\infty}=-\dfrac{n\cdot a\cdot G_{3\infty}}{(r+\gamma)}$,

$$\dot{\lambda}_{3j}=0 \;\Rightarrow\; 0=(r+\gamma)\cdot\lambda_{3j\infty}+n\cdot b\cdot G_{3\infty} \;\Rightarrow\; \lambda_{3j\infty}=-\frac{n\cdot b\cdot G_{3\infty}}{(r+\gamma)}$$

(a33) $\lambda_{3j\infty}=-\dfrac{n\cdot b\cdot G_{3\infty}}{(r+\gamma)}$,

$$\dot{G}=0 \;\Rightarrow\; 0=\frac{n\cdot\lambda_{3i\infty}}{\alpha_3}+\frac{n\cdot\lambda_{3j\infty}}{\alpha_3}+\frac{2\cdot n\cdot\alpha_2}{\alpha_3}-\gamma\cdot G_{3\infty} \;\Rightarrow$$

$$G_{3\infty}=\frac{n\cdot\lambda_{3i\infty}}{\alpha_3\cdot\gamma}+\frac{n\cdot\lambda_{3j\infty}}{\alpha_3\cdot\gamma}+\frac{2\cdot n\cdot\alpha_2}{\alpha_3\cdot\gamma}$$

(a34) $G_{3\infty}=\dfrac{n\cdot\lambda_{3i\infty}}{\alpha_3\cdot\gamma}+\dfrac{n\cdot\lambda_{3j\infty}}{\alpha_3\cdot\gamma}+\dfrac{2\cdot n\cdot\alpha_2}{\alpha_3\cdot\gamma}$. Put (a31) and (a32) into (a33):

$$G_{3\infty}=\frac{n}{\alpha_3\cdot\gamma}\cdot\left(-\frac{n\cdot a\cdot G_{3\infty}}{(r+\gamma)}\right)+\frac{n}{\alpha_3\cdot\gamma}\cdot\left(-\frac{n\cdot b\cdot G_{3\infty}}{(r+\gamma)}\right)+\frac{2\cdot n\cdot\alpha_2}{\alpha_3\cdot\gamma} \;\Rightarrow$$

$$G_{3\infty}\left(\frac{\alpha_3\cdot\gamma\cdot(r+\gamma)+n^2\cdot(a+b)}{\alpha_3\cdot\gamma\cdot(r+\gamma)}\right)=\frac{2\cdot n\cdot\alpha_2}{\alpha_3\cdot\gamma} \;\Rightarrow$$

(a35) $G_{3\infty}=\dfrac{2\cdot n\cdot\alpha_2\cdot(r+\gamma)}{\alpha_3\cdot\gamma\cdot(r+\gamma)+n^2\cdot(a+b)}$.

We need to solve MHDS, where we know G_0, $G_{3\infty}$, $\lambda_{3i\infty}$ and $\lambda_{3j\infty}$. So, from (a27), (a30) and (a31),

$$\dot{\lambda}_{3i} = (r+\gamma)\cdot\lambda_{3i} + n\cdot a\cdot G_3$$
$$\dot{\lambda}_{3j} = (r+\gamma)\cdot\lambda_{3j} + n\cdot b\cdot G_3 \qquad \Rightarrow \dot{y} = A\cdot y \text{, where } y = \begin{pmatrix} \lambda_{3i} \\ \lambda_{3j} \\ G_3 \end{pmatrix} \text{ and}$$
$$\dot{G} = \frac{n\cdot\lambda_{3i}}{\alpha_3} + \frac{n\cdot\lambda_{3j}}{\alpha_3} - \gamma\cdot G_3 + \frac{2\cdot n\cdot\alpha_2}{\alpha_3}$$

$A = \begin{pmatrix} r+\gamma & 0 & n\cdot a \\ 0 & r+\gamma & n\cdot b \\ \frac{n}{\alpha_3} & \frac{n}{\alpha_3} & -\gamma \end{pmatrix}$ such that the dimension of y is 3×1 and the dimension of A is 3×3.

To solve MHDS, we need to find the eigenvalues of the matrix A. The characteristic polynomial is $s^2 - r\cdot s - \gamma^2 - \gamma\cdot s - \frac{n^2\cdot(a+b)}{\alpha_3} = 0$. To ensure stability, we choose the negative root of the characteristic polynomial. Let s_3 be the negative root. We know that optimal groundwater accumulation has the following form: $G_3(t) = D\cdot e^{s_3\cdot t} + E$. We need to find the parameters D and E.

We know G_0 and $G_{3\infty}$. Then,

$$G_3(0) = D\cdot e^{s_3\cdot 0} + E = G_0 \;\Rightarrow\; D + E = G_0$$

$$G_3(\infty) = D\cdot e^{s_3\cdot\infty} + E = G_{3\infty} \;\Rightarrow\; D\cdot 0 + E = G_{3\infty} \;\Rightarrow\; E = G_{3\infty}.$$

So, $D + E = G_0 \;\Rightarrow\; D = G_0 - E = G_0 - G_{3\infty} \;\Rightarrow\; D = G_0 - G_{3\infty}$.

So, $G_3(t) = (G_0 - G_{3\infty})\cdot e^{s_3\cdot t} + G_{3\infty}$.

We know that the adjoint variables has the following forms: $\lambda_{3i}(t) = D\cdot e^{s_3\cdot t} + E$ for WUA_N and $\lambda_{3j}(t) = K\cdot e^{s_3\cdot t} + L$ for WUA_S. We know that $\lambda_{3i\infty} = -\frac{n\cdot a\cdot G_{3\infty}}{(r+\gamma)}$. Then,

$\lambda_{3i}(\infty) = D \cdot e^{s_3 \cdot \infty} + E = \lambda_{3i\infty} \Rightarrow D \cdot 0 + E = \lambda_{3i\infty} \Rightarrow E = \lambda_{3i\infty}$.

Then, $\lambda_{3i}(t) = D \cdot e^{s_3 \cdot t} + \lambda_{3i\infty}$. $\Rightarrow \lambda_{3i}(t) = D \cdot e^{s_3 \cdot t} - \dfrac{n \cdot a \cdot G_{3\infty}}{(r+\gamma)}$. We need to find the parameter D. $\lambda_{3i}(t) = D \cdot e^{s_3 \cdot t} - \dfrac{n \cdot a \cdot G_{3\infty}}{(r+\gamma)}$ has to satisfy the adjoint equation (a27).

So, $\dot{\lambda}_{3i} = D \cdot s_3 \cdot e^{s_3 \cdot t} = (r+\gamma) \cdot \left(D \cdot e^{s_3 \cdot t} - \dfrac{n \cdot a \cdot G_{3\infty}}{(r+\gamma)} \right) + n \cdot a \cdot G_3 \Rightarrow$ By putting

$G_3(t) = (G_0 - G_{3\infty}) \cdot e^{s_3 \cdot t} + G_{3\infty}$ into equation,

$$D \cdot (s_3 - r - \gamma) \cdot e^{s_3 \cdot t} = -n \cdot a \cdot G_{3\infty} + n \cdot a \cdot \left[(G_0 - G_{3\infty}) \cdot e^{s_3 \cdot t} + G_{3\infty} \right] \Rightarrow$$

$$D \cdot (s_3 - r - \gamma) \cdot e^{s_3 \cdot t} = n \cdot a \cdot (G_0 - G_{3\infty}) \cdot e^{s_3 \cdot t} \Rightarrow D = \frac{n \cdot a \cdot (G_0 - G_{3\infty})}{(s_3 - r - \gamma)}.$$ So,

$\lambda_{3i}(t) = \dfrac{n \cdot a \cdot (G_0 - G_{3\infty})}{(s_3 - r - \gamma)} \cdot e^{s_3 \cdot t} - \dfrac{n \cdot a \cdot G_{3\infty}}{(r+\gamma)} \Rightarrow$ So, adjoint variable is:

(a36) $$\lambda_{3i}(t) = n \cdot a \cdot \left[\frac{1}{(s_3 - r - \gamma)} \cdot G_3(t) - \frac{s_3}{(r+\gamma) \cdot (s_3 - r - \gamma)} \cdot G_{3\infty} \right].$$

For each farmer in WUA_N: (from FONCs)

$$w_{3i} = \frac{\lambda_{3i} + \alpha_2}{\alpha_3} \Rightarrow$$

$$w_{3i} = \frac{n \cdot a \cdot \left[\frac{1}{(s_3 - r - \gamma)} \cdot G_3(t) - \frac{s_3}{(r+\gamma) \cdot (s_3 - r - \gamma)} \cdot G_{3\infty} \right] + \alpha_2}{\alpha_3}.$$

We know that $\lambda_{3j\infty} = -\dfrac{n \cdot b \cdot G_{3\infty}}{(r+\gamma)}$. Then,

$\lambda_{3j}(\infty) = K \cdot e^{s_3 \cdot \infty} + L = \lambda_{3j\infty} \Rightarrow K \cdot 0 + L = \lambda_{3j\infty} \Rightarrow L = \lambda_{3j\infty}$.

Then, $\lambda_{3j}(t) = K \cdot e^{s_3 \cdot t} + \lambda_{3j\infty}$. $\Rightarrow$ $\lambda_{3j}(t) = K \cdot e^{s_3 \cdot t} - \dfrac{n \cdot b \cdot G_{3\infty}}{(r+\gamma)}$. We need to find the parameter K. $\lambda_{3j}(t) = K \cdot e^{s_3 \cdot t} - \dfrac{n \cdot b \cdot G_{3\infty}}{(r+\gamma)}$ has to satisfy the adjoint equation (a30).

So, $\dot{\lambda}_{3i} = K \cdot s_3 \cdot e^{s_3 \cdot t} = (r+\gamma) \cdot \left(K \cdot e^{s_3 \cdot t} - \dfrac{n \cdot b \cdot G_{3\infty}}{(r+\gamma)} \right) + n \cdot b \cdot G_3$ $\Rightarrow$ By putting

$G_3(t) = (G_0 - G_{3\infty}) \cdot e^{s_3 \cdot t} + G_{3\infty}$ into equation,

$$K \cdot (s_3 - r - \gamma) \cdot e^{s_3 \cdot t} = -n \cdot b \cdot G_{3\infty} + n \cdot b \cdot \left[(G_0 - G_{3\infty}) \cdot e^{s_3 \cdot t} + G_{3\infty} \right] \Rightarrow$$

$K \cdot (s_3 - r - \gamma) \cdot e^{s_3 \cdot t} = n \cdot b \cdot (G_0 - G_{3\infty}) \cdot e^{s_3 \cdot t}$ $\Rightarrow$ $K = \dfrac{n \cdot b \cdot (G_0 - G_{3\infty})}{(s_3 - r - \gamma)}$. So,

$\lambda_{3j}(t) = \dfrac{n \cdot b \cdot (G_0 - G_{3\infty})}{(s_3 - r - \gamma)} \cdot e^{s_3 \cdot t} - \dfrac{n \cdot b \cdot G_{3\infty}}{(r+\gamma)}$ $\Rightarrow$ So, adjoint variable is:

(a37) $$\lambda_{3j}(t) = n \cdot b \cdot \left[\frac{1}{(s_3 - r - \gamma)} \cdot G_3(t) - \frac{s_3}{(r+\gamma) \cdot (s_3 - r - \gamma)} \cdot G_{3\infty} \right].$$

For each farmer in WUA_S : (from FONCs)

$$w_{3j} = \frac{\lambda_{3j} + \alpha_2}{\alpha_3} \Rightarrow$$

$$w_{3j} = \frac{n \cdot b \cdot \left[\dfrac{1}{(s_3 - r - \gamma)} \cdot G_3(t) - \dfrac{s_3}{(r+\gamma) \cdot (s_3 - r - \gamma)} \cdot G_{3\infty} \right] + \alpha_2}{\alpha_3}.$$

The transversality condition is also satisfied: $\lim\limits_{t \to \infty} e^{-r \cdot t} \cdot \lambda_{3i}(t) \cdot G_3(t) = 0$ and $\lim\limits_{t \to \infty} e^{-r \cdot t} \cdot \lambda_{3j}(t) \cdot G_3(t) = 0$. □

Appendix D

Proof of proposition 4:

Firstly, we will solve the problem of the farmers in WUA_N. Secondly, we will solve the problem of the board of WUA_S. The problem of each farmer i in WUA_N is:

$$\max_{w_i} \int_0^{\infty} F_i(w_i,G,a)\cdot e^{-r.t}\cdot dt$$

$$\text{subject to } \dot{G}=\sum_{i=1}^{n} w_i+\sum_{j=n+1}^{2n} w_j-\delta\cdot G \quad \text{and} \quad G(0)=G_0\geq 0$$

The current value Hamiltonian is:

$$H_i(w_i,G,\lambda_{4i})=\alpha_1+\alpha_2\cdot w_i-\frac{\alpha_3}{2}\cdot w_i^2-\frac{a}{2}\cdot G^2+\lambda_{4i}^{*}\cdot(\sum_{i=1}^{n} w_i+\sum_{j=n+1}^{2n} w_j-\gamma\cdot G).$$

Assuming interior solutions, first order necessary conditons (FONCs) are:

$$\frac{\partial H_i}{\partial w_i}=0 \Leftrightarrow \alpha_2-\alpha_3\cdot w_{4i}+\lambda_{4i}=0 \Rightarrow w_{4i}=\frac{\lambda_{4i}+\alpha_2}{\alpha_3}.$$

(a38) $w_{4i}=\dfrac{\lambda_{4i}+\alpha_2}{\alpha_3}.$

We need to derive adjoint equation: $\dot{\lambda}_{4i}=r\cdot\lambda_{4i}-\dfrac{\partial H_i^{*}}{\partial G}$, where H_i^{*} is maximized Hamiltonian. Maximized Hamiltonian is obtained by putting optimal control variable, w_{4i}, into current value Hamiltonian. So, $H_i^{*}=H_i(w_{4i},G,\lambda_{4i})$. Then,

(a39) $\dfrac{\partial H_i^{*}}{\partial G}=-a\cdot G_4-\gamma\cdot\lambda_{4i}.$

Put (a38) into adjoint equation such that we obtain:

$\dot{\lambda}_{4i} = r\cdot\lambda_{4i} - (-a\cdot G_4 - \gamma\cdot\lambda_{4i})$,

(a40) $\dot{\lambda}_{4i} = (r+\gamma)\cdot\lambda_{4i} + a\cdot G_4$.

Here, (a38) and (a40) are FONCs. Since $\frac{\partial^2 H_i}{\partial w_i \partial w_i} = -\alpha_3 < 0$, H_i is concave in w_i such that the FONCs are also sufficient.

The problem of the board of WUA_S is:

$$\max_{w_{n+1},\ldots,w_{2n}} \sum_{j=n+1}^{2n}\int_0^{\infty} F_j(w_i,G,b)\cdot e^{-r.t}\cdot dt$$

subject to $\dot{G} = \sum_{i=1}^{n} w_i + \sum_{j=n+1}^{2n} w_j - \gamma\cdot G$ and $G(0) = G_0 \geq 0$

The current value Hamiltonian is:

$$H_j(w_j,G,\lambda_{4j}) = \sum_{j=n+1}^{2n}(\alpha_1 + \alpha_2\cdot w_j - \frac{\alpha_3}{2}\cdot w_j^{\,2} - \frac{b}{2}\cdot G^2) + \lambda_{4j}^{*}\cdot(\sum_{i=1}^{n} w_i + \sum_{j=n+1}^{2n} w_j - \gamma\cdot G).$$

Assuming interior solutions, first order necessary conditons (FONCs) are:

$$\frac{\partial H_j}{\partial w_j} = 0 \iff \alpha_2 - \alpha_3\cdot w_{4j} + \lambda_{4j} = 0 \Rightarrow w_{4j} = \frac{\lambda_{4j} + \alpha_2}{\alpha_3}.$$

(a41) $w_{4j} = \frac{\lambda_{4j} + \alpha_2}{\alpha_3}$.

We need to derive adjoint equation: $\dot{\lambda}_{4j} = r\cdot\lambda_{4j} - \frac{\partial H_j^*}{\partial G}$, where H_j^* is maximized Hamiltonian. Maximized Hamiltonian is obtained by putting optimal control variable, w_{4j}, into current value Hamiltonian. So, $H_j^* = H_j(w_{4j},G,\lambda_{4j})$. Then,

(a42) $\frac{\partial H_j^*}{\partial G} = -n\cdot b\cdot G_4 - \gamma\cdot\lambda_{4j}$.

Put (a42) into adjoint equation such that we obtain:

$$\dot{\lambda}_{4j} = r \cdot \lambda_{4j} - (-n \cdot b \cdot G_4 - \gamma \cdot \lambda_{4j}),$$

(a43) $\dot{\lambda}_{4j} = (r+\gamma) \cdot \lambda_{4j} + n \cdot b \cdot G_4$.

Here, (a41) and (a43) are FONCs. Since $\dfrac{\partial^2 H_j}{\partial w_j \partial w_j} = -\alpha_3 < 0$, H_j is concave in w_j such that the FONCs are also sufficient.

We put our optimal controls, w_{4i} and w_{4j}, into state equation:

$$\dot{G} = \sum_{i=1}^{n} w_{4i} + \sum_{j=n+1}^{2n} w_{4j} - \gamma \cdot G_4 \quad \Rightarrow \quad \dot{G} = \sum_{i=1}^{n} \left(\frac{\lambda_{4i} + \alpha_2}{\alpha_3}\right) + \sum_{j=n+1}^{2n} \left(\frac{\lambda_{4j} + \alpha_2}{\alpha_3}\right) - \gamma \cdot G_4$$

$\Rightarrow$

$$\dot{G} = \frac{\sum_{i=1}^{n} \lambda_{4i}}{\alpha_3} + \frac{2 \cdot n \cdot \alpha_2}{\alpha_3} + \frac{\sum_{j=n+1}^{2n} \lambda_{4j}}{\alpha_3} - \gamma \cdot G_4 \quad \Rightarrow$$

$$\dot{G} = \frac{\sum_{i=1}^{n} \lambda_{4i}}{\alpha_3} + \frac{2 \cdot n \cdot \alpha_2}{\alpha_3} + \frac{n \cdot \lambda_{4j}}{\alpha_3} - \gamma \cdot G_4 \Rightarrow$$

(a44) $\dot{G} = \dfrac{\sum_{i=1}^{n} \lambda_{4i}}{\alpha_3} + \dfrac{n \cdot \lambda_{4j}}{\alpha_3} - \gamma \cdot G_4 + \dfrac{2 \cdot n \cdot \alpha_2}{\alpha_3}$.

Differential equations (a40), (a43) and (a44) form the modified Hamiltonian dynamical system (MHDS). To solve the model, we need to solve MHDS. First, we find steady-state levels of λ_{4i} for i=1,..,n, λ_{4j} and G. Set $\dot{\lambda}_{4i} = 0$, $\dot{\lambda}_{4j} = 0$ and $\dot{G} = 0$.

$$\dot{\lambda}_{4i} = 0 \Rightarrow 0 = (r+\gamma) \cdot \lambda_{4i\infty} + a \cdot G_{4\infty} \Rightarrow \lambda_{4i\infty} = -\frac{a \cdot G_{4\infty}}{(r+\gamma)}$$

(a45) $\lambda_{4i\infty} = -\dfrac{a \cdot G_{4\infty}}{(r+\gamma)}$,

$$\dot{\lambda}_{4j} = 0 \;\Rightarrow\; 0 = (r+\gamma)\cdot\lambda_{4j\infty} + n\cdot b\cdot G_{4\infty} \;\Rightarrow\; \lambda_{4j\infty} = -\frac{n\cdot b\cdot G_{4\infty}}{(r+\gamma)}$$

(a46) $\lambda_{4j\infty} = -\dfrac{n\cdot b\cdot G_{4\infty}}{(r+\gamma)}$,

$$\dot{G} = 0 \;\Rightarrow\; 0 = \frac{n\cdot\lambda_{4i\infty}}{\alpha_3} + \frac{n\cdot\lambda_{4j\infty}}{\alpha_3} + \frac{2\cdot n\cdot\alpha_2}{\alpha_3} - \gamma\cdot G_{4\infty} \;\Rightarrow$$

$$G_{4\infty} = \frac{n\cdot\lambda_{4i\infty}}{\alpha_3\cdot\gamma} + \frac{n\cdot\lambda_{4j\infty}}{\alpha_3\cdot\gamma} + \frac{2\cdot n\cdot\alpha_2}{\alpha_3\cdot\gamma}$$

(a47) $G_{4\infty} = \dfrac{n\cdot\lambda_{4i\infty}}{\alpha_3\cdot\gamma} + \dfrac{n\cdot\lambda_{4j\infty}}{\alpha_3\cdot\gamma} + \dfrac{2\cdot n\cdot\alpha_2}{\alpha_3\cdot\gamma}$. Put (a45) and (a46) into (a47):

$$G_{4\infty} = \frac{n}{\alpha_3\cdot\gamma}\cdot\left(-\frac{a\cdot G_{4\infty}}{(r+\gamma)}\right) + \frac{n}{\alpha_3\cdot\gamma}\cdot\left(-\frac{n\cdot b\cdot G_{4\infty}}{(r+\gamma)}\right) + \frac{2\cdot n\cdot\alpha_2}{\alpha_3\cdot\gamma} \;\Rightarrow$$

$$G_{4\infty}\left(\frac{\alpha_3\cdot\gamma\cdot(r+\gamma) + n\cdot a + n^2\cdot b}{\alpha_3\cdot\gamma\cdot(r+\gamma)}\right) = \frac{2\cdot n\cdot\alpha_2}{\alpha_3\cdot\gamma} \;\Rightarrow$$

(a48) $G_{4\infty} = \dfrac{2\cdot n\cdot\alpha_2\cdot(r+\gamma)}{\alpha_3\cdot\gamma\cdot(r+\gamma) + n\cdot a + n^2\cdot b}$.

We need to solve MHDS, where we know G_0, $G_{4\infty}$, $\lambda_{4i\infty}$ and $\lambda_{4j\infty}$. So, from (a40), (a43) and (a44),

$$\begin{array}{l} \dot{\lambda}_{41} = (r+\gamma)\cdot\lambda_{41} + a\cdot G_4 \\ . \\ . \\ \dot{\lambda}_{4n} = (r+\gamma)\cdot\lambda_{4n} + a\cdot G_4 \\ \dot{\lambda}_{4j} = (r+\gamma)\cdot\lambda_{4j} + n\cdot b\cdot G_4 \\ \dot{G} = \dfrac{\sum_{i=1}^{n}\lambda_{4i}}{\alpha_3} + \dfrac{n\cdot\lambda_{4j}}{\alpha_3} - \gamma\cdot G_4 + \dfrac{2\cdot n\cdot\alpha_2}{\alpha_3} \end{array} \;\Rightarrow\; \dot{y} = A\cdot y\,,$$

where

$$y = \begin{pmatrix} \lambda_{41} \\ . \\ . \\ \lambda_{4n} \\ \lambda_{4j} \\ G \end{pmatrix} \; y = \begin{pmatrix} \lambda_{41} \\ . \\ . \\ \lambda_{4n} \\ \lambda_{4j} \\ G \end{pmatrix} \text{and} \quad A = \begin{pmatrix} r+\gamma & 0 & . & . & . & 0 & a \\ 0 & r+\gamma & 0 & . & . & . & a \\ . & 0 & r+\gamma & 0 & . & . & a \\ . & . & 0 & r+\gamma & 0 & . & a \\ . & . & . & 0 & r+\gamma & 0 & a \\ 0 & . & . & . & 0 & r+\gamma & n \cdot b \\ \frac{1}{\alpha_3} & \frac{1}{\alpha_3} & \frac{1}{\alpha_3} & \frac{1}{\alpha_3} & \frac{1}{\alpha_3} & \frac{n}{\alpha_3} & -\gamma \end{pmatrix}$$

such that the dimension of y is $(n+2)\times 1$ and the dimension of A is $(n+2)\times(n+2)$.

To solve MHDS, we need to find the eigenvalues of the matrix A. The characteristic polynomial is $s^2 - r \cdot s - \gamma^2 - \gamma \cdot s - \frac{n \cdot a + n^2 \cdot b}{\alpha_3} = 0$. To ensure stability, we choose the negative root of the characteristic polynomial. Let s_4 be the negative root. We know that optimal groundwater accumulation has the following form: $G_4(t) = D \cdot e^{s_4 \cdot t} + E$. We need to find the parameters D and E.

We know G_0 and $G_{4\infty}$. Then,

$$G_4(0) = D \cdot e^{s_4 \cdot 0} + E = G_0 \;\Rightarrow\; D + E = G_0$$

$$G_4(\infty) = D \cdot e^{s_4 \cdot \infty} + E = G_{4\infty} \;\Rightarrow\; D \cdot 0 + E = G_{4\infty} \;\Rightarrow\; E = G_{4\infty}.$$

So, $D + E = G_0 \;\Rightarrow\; D = G_0 - E = G_0 - G_{4\infty} \;\Rightarrow\; D = G_0 - G_{4\infty}$.

So, $G_4(t) = (G_0 - G_{4\infty}) \cdot e^{s_4 \cdot t} + G_{4\infty}$.

We know that the adjoint variables has the following forms: $\lambda_{4i}(t) = D \cdot e^{s_4 \cdot t} + E$ for each farmer i in WUA_N and $\lambda_{4j}(t) = K \cdot e^{s_4 \cdot t} + L$ for WUA_S.

We know that $\lambda_{4i\infty} = -\dfrac{a \cdot G_{4\infty}}{(r+\gamma)}$. Then,

$\lambda_{4i}(\infty) = D \cdot e^{s_4 \cdot \infty} + E = \lambda_{4i\infty} \Rightarrow D \cdot 0 + E = \lambda_{4i\infty} \Rightarrow E = \lambda_{4i\infty}$.

Then, $\lambda_{4i}(t) = D \cdot e^{s_4 \cdot t} + \lambda_{4i\infty}$. $\Rightarrow \lambda_{4i}(t) = D \cdot e^{s_4 \cdot t} - \dfrac{a \cdot G_{4\infty}}{(r+\gamma)}$. We need to find the

parameter D. $\lambda_{4i}(t) = D \cdot e^{s_4 \cdot t} - \dfrac{a \cdot G_{4\infty}}{(r+\gamma)}$ has to satisfy the adjoint equation (a40).

So, $\dot{\lambda}_{4i} = D \cdot s_4 \cdot e^{s_4 \cdot t} = (r+\gamma) \cdot \left(D \cdot e^{s_4 \cdot t} - \dfrac{a \cdot G_{4\infty}}{(r+\gamma)} \right) + a \cdot G_4 \Rightarrow$ By putting

$G_4(t) = (G_0 - G_{4\infty}) \cdot e^{s_4 \cdot t} + G_{4\infty}$ into equation,

$D \cdot (s_4 - r - \gamma) \cdot e^{s_4 \cdot t} = -a \cdot G_{4\infty} + a \cdot \left[(G_0 - G_{4\infty}) \cdot e^{s_4 \cdot t} + G_{4\infty} \right] \Rightarrow$

$D \cdot (s_4 - r - \gamma) \cdot e^{s_4 \cdot t} = a \cdot (G_0 - G_{4\infty}) \cdot e^{s_4 \cdot t} \Rightarrow D = \dfrac{a \cdot (G_0 - G_{4\infty})}{(s_4 - r - \gamma)}$. So,

$\lambda_{4i}(t) = \dfrac{a \cdot (G_0 - G_{4\infty})}{(s_4 - r - \gamma)} \cdot e^{s_4 \cdot t} - \dfrac{a \cdot G_{4\infty}}{(r+\gamma)} \Rightarrow$ So, adjoint variable is:

(a49) $\lambda_{4i}(t) = a \cdot \left[\dfrac{1}{(s_4 - r - \gamma)} \cdot G_4(t) - \dfrac{s_4}{(r+\gamma) \cdot (s_4 - r - \gamma)} \cdot G_{4\infty} \right]$.

For each farmer in WUA_N: (from FONCs)

$$w_{4i} = \frac{\lambda_{4i} + \alpha_2}{\alpha_3} \Rightarrow$$

$$w_{4i} = \frac{a \cdot \left[\dfrac{1}{(s_4 - r - \gamma)} \cdot G_4(t) - \dfrac{s_4}{(r+\gamma) \cdot (s_4 - r - \gamma)} \cdot G_{4\infty} \right] + \alpha_2}{\alpha_3}.$$

We know that $\lambda_{4j\infty} = -\dfrac{n \cdot b \cdot G_{4\infty}}{(r+\gamma)}$. Then,

$\lambda_{4j}\left(\infty\right) = K \cdot e^{s_4 \cdot \infty} + L = \lambda_{4j\infty} \;\Rightarrow\; K \cdot 0 + L = \lambda_{4j\infty} \;\Rightarrow\; L = \lambda_{4j\infty}$.

Then, $\lambda_{4j}\left(t\right) = K \cdot e^{s_4 \cdot t} + \lambda_{4j\infty}$. $\Rightarrow$ $\lambda_{4j}\left(t\right) = K \cdot e^{s_4 \cdot t} - \frac{n \cdot b \cdot G_{4\infty}}{(r+\gamma)}$. We need to find the

parameter K. $\lambda_{4j}\left(t\right) = K \cdot e^{s_4 \cdot t} - \frac{n \cdot b \cdot G_{4\infty}}{(r+\gamma)}$ has to satisfy the adjoint equation (a43).

So, $\dot{\lambda}_{4i} = K \cdot s_4 \cdot e^{s_4 \cdot t} = (r+\gamma) \cdot \left(K \cdot e^{s_4 \cdot t} - \frac{n \cdot b \cdot G_{4\infty}}{(r+\gamma)} \right) + n \cdot b \cdot G_4$ $\Rightarrow$ By putting

$G_4\left(t\right) = \left(G_0 - G_{4\infty}\right) \cdot e^{s_4 \cdot t} + G_{4\infty}$ into equation,

$$K \cdot \left(s_4 - r - \gamma\right) \cdot e^{s_4 \cdot t} = -n \cdot b \cdot G_{4\infty} + n \cdot b \cdot \left[\left(G_0 - G_{4\infty}\right) \cdot e^{s_4 \cdot t} + G_{4\infty}\right] \Rightarrow$$

$$K \cdot \left(s_4 - r - \gamma\right) \cdot e^{s_4 \cdot t} = n \cdot b \cdot \left(G_0 - G_{4\infty}\right) \cdot e^{s_4 \cdot t} \;\Rightarrow\; K = \frac{n \cdot b \cdot \left(G_0 - G_{4\infty}\right)}{\left(s_4 - r - \gamma\right)}.$$

So,

$$\lambda_{4j}\left(t\right) = \frac{n \cdot b \cdot \left(G_0 - G_{4\infty}\right)}{\left(s_4 - r - \gamma\right)} \cdot e^{s_4 \cdot t} - \frac{n \cdot b \cdot G_{4\infty}}{(r+\gamma)}$$

$\Rightarrow$ So, adjoint variable is:

(a50) $$\lambda_{4j}(t) = n \cdot b \cdot \left[\frac{1}{(s_4 - r - \gamma)} \cdot G_4(t) - \frac{s_4}{(r+\gamma) \cdot (s_4 - r - \gamma)} \cdot G_{4\infty} \right].$$

For each farmer in WUA_S: (from FONCs)

$$w_{4j} = \frac{\lambda_{4j} + \alpha_2}{\alpha_3} \Rightarrow$$

$$w_{4j} = \frac{n \cdot b \cdot \left[\frac{1}{(s_4 - r - \gamma)} \cdot G_4(t) - \frac{s_4}{(r+\gamma) \cdot (s_4 - r - \gamma)} \cdot G_{4\infty} \right] + \alpha_2}{\alpha_3}.$$

The transversality condition is also satisfied: $\lim_{t \to \infty} e^{-r \cdot t} \cdot \lambda_{4i}(t) \cdot G_4(t) = 0$ and $\lim_{t \to \infty} e^{-r \cdot t} \cdot \lambda_{4j}(t) \cdot G_4(t) = 0$. □

Appendix E

Proof of proposition 6:

We try to find two taxation schemes, $T_i(w_i,G)=(m_1\cdot G+k_1)\cdot w_i$ for the farmers in WUA_N, and $T_j(w_j,G)=(m_2\cdot G+k_2)\cdot w_j$ for the farmers in WUA_S, in order to induce all the farmers to follow the time paths for their socially optimal irrigation water choices when all the farmers move non-cooperatively.

The problem of farmer i in WUA_N is: (for i=1,..,n),

$$\max_{w_i} \int_0^{\infty} \{F_i(w_i,G,a)-T_i(w_i,G)\}\cdot e^{-r.t}\cdot dt$$

$$\text{subject to } \dot{G}=\sum_{i=1}^{n} w_i+\sum_{j=n+1}^{2n} w_j-\delta\cdot G \qquad \text{and} \qquad G(0)=G_0\geq 0$$

The current value Hamiltonian is:

$$H_i(w_i,G,\lambda_i)=\alpha_1+\alpha_2\cdot w_i-\frac{\alpha_3}{2}\cdot w_i^2-\frac{a}{2}\cdot G^2-(m_1\cdot G+k_1)\cdot w_i+\lambda_i\cdot(\sum_{i=1}^{n} w_i+\sum_{j=n+1}^{2n} w_j-\gamma\cdot G).$$

Assuming interior solutions, first order necessary conditons (FONCs) are:

$$\frac{\partial H_i}{\partial w_i}=0 \;\Leftrightarrow\; \alpha_2-\alpha_3\cdot w_i^*-m_1\cdot G-k_1+\lambda_i=0 \;\Rightarrow\; w_i^*=\frac{\lambda_i+\alpha_2-m_1\cdot G-k_1}{\alpha_3}$$

(a51) $$w_i^*(G)=\frac{\lambda_i+\alpha_2-m_1\cdot G-k_1}{\alpha_3}.$$

We need to derive adjoint equation: $\dot{\lambda}_i=r\cdot\lambda_i-\dfrac{\partial H_i^*}{\partial G}$, where H_i^* is maximized Hamiltonian. Maximized Hamiltonian is obtained by putting optimal control variable, $w_i^*\left(G\right)$, into current value Hamiltonian. So, $H_i^* = H_i(w_i^*\left(G\right),G,\lambda_i)$. Then,

$$H_i^*(w_i^*\left(G\right),G,\lambda_i) = \alpha_1 + \alpha_2 \cdot w_i^*\left(G\right) - \frac{\alpha_3}{2} \cdot \left[w_i^*\left(G\right)\right]^2 - \frac{a}{2} \cdot G^2$$
$$-(m_1 \cdot G + k_1) \cdot w_i^*\left(G\right) + \lambda_i \cdot (w_i^*\left(G\right) + \sum_{\substack{l=1 \\ l \neq i}}^{n} w_l + \sum_{j=n+1}^{2n} w_j - \gamma \cdot G). \Rightarrow$$

(a52) $$\begin{aligned}\frac{\partial H_i^*}{\partial G} &= \alpha_2 \cdot \frac{\partial w_i^*}{\partial G} - \alpha_3 \cdot w_i^* \cdot \frac{\partial w_i^*}{\partial G} - a \cdot G - m_1 \cdot w_i^* \\ &- \left(m_1 \cdot G + k_1\right) \cdot \frac{\partial w_i^*}{\partial G} + \lambda_i \cdot \frac{\partial w_i^*}{\partial G} + \lambda_i \cdot \left(-\gamma\right)\end{aligned}.$$

(a53) $\frac{\partial w_i^*}{\partial G} = \frac{-m_1}{\alpha_3}$. Put (a53) into (a52) :

$$\frac{\partial H_i^*}{\partial G} = \alpha_2 \cdot \left(\frac{-m_1}{\alpha_3}\right) - \alpha_3 \cdot \left(\frac{\lambda_i + \alpha_2 - m_1 \cdot G - k_1}{\alpha_3}\right) \cdot \left(\frac{-m_1}{\alpha_3}\right) - a \cdot G$$
$$- m_1 \cdot \left(\frac{\lambda_i + \alpha_2 - m_1 \cdot G - k_1}{\alpha_3}\right) - \left(m_1 \cdot G + k_1\right) \cdot \left(\frac{-m_1}{\alpha_3}\right) + \lambda_i \cdot \left(\frac{-m_1}{\alpha_3}\right) + \lambda_i \cdot \left(-\gamma\right)$$

$\Rightarrow$

$$\frac{\partial H_i^*}{\partial G} = \alpha_2 \cdot \left(\frac{-m_1}{\alpha_3}\right) - a \cdot G - \left(m_1 \cdot G + k_1\right) \cdot \left(\frac{-m_1}{\alpha_3}\right) + \lambda_i \cdot \left(\frac{-m_1}{\alpha_3}\right) + \lambda_i \cdot \left(-\gamma\right) \Rightarrow$$

$$\frac{\partial H_i^*}{\partial G} = -m_1 \cdot \left(\frac{\alpha_2 + \lambda_i - \left(m_1 \cdot G + k_1\right)}{\alpha_3}\right) - a \cdot G - \gamma \cdot \lambda_i \Rightarrow$$

(a54) $\frac{\partial H_i^*}{\partial G} = -\gamma \cdot \lambda_i - a \cdot G - m_1 \cdot w_i^*$.

Put (a54) into adjoint equation such that we obtain:

$\dot{\lambda}_i = r \cdot \lambda_i - \left(-\gamma \cdot \lambda_i - a \cdot G - m_1 \cdot w_i^*\right)$,

(a55) $\dot{\lambda}_i = \left(r + \gamma\right) \cdot \lambda_i + a \cdot G + m_1 \cdot w_i^*)$.

Here, (a51) and (a55) are FONCs. We look for a taxation scheme which will induce the farmer follow his socially optimal time path. So, for farmers in

WUA_N, we try to find $\{w^c, T_i\}$ such that this pair satisfies FONCs, (a51) and (a55). Firstly, we need to put (a51) into (a55).

We know from (a11), $w_i^c = w^c\left(G^c\right) = \dfrac{\left(\left(s_1+\gamma\right)\cdot G^c - s_1\cdot G_\infty^c\right)\cdot\alpha_3}{2\cdot n}$.

Put (a11) into (a51) such that (a51) becomes:

$$w^c\left(G^c\right) = \frac{\lambda_i + \alpha_2 - m_1\cdot G^c - k_1}{\alpha_3} \Rightarrow$$

(a56) $\lambda_i = \alpha_3\cdot w^c\left(G^c\right) - \alpha_2 + m_1\cdot G^c + k_1$

Take derivative of both sides of expression in (a56) with respect to time:

(a57) $\dot{\lambda}_i = \alpha_3\cdot\dfrac{\partial w^c\left(G^c\right)}{\partial G^c}\cdot\dfrac{\partial G^c(t)}{\partial t} + m_1\cdot\dfrac{\partial G^c(t)}{\partial t}$

Put (a56) and (a57) into (a55):

$$\begin{aligned}\alpha_3\cdot\frac{\partial w^c\left(G^c\right)}{\partial G^c}\cdot\frac{\partial G^c(t)}{\partial t} + m_1\cdot\frac{\partial G^c(t)}{\partial t} &= \left(r+\gamma\right)\cdot\left(\alpha_3\cdot w^c\left(G^c\right) - \alpha_2 + m_1\cdot G^c + k_1\right)\\ &\quad + a\cdot G^c + m_1\cdot w^c\left(G^c\right)\end{aligned}$$

$\Rightarrow$

(a58)
$$\begin{aligned}\alpha_3\cdot\frac{\partial w^c\left(G^c\right)}{\partial G^c}\cdot\frac{\partial G^c(t)}{\partial t} &= \left(r+\gamma\right)\cdot\left(\alpha_3\cdot w^c\left(G^c\right) - \alpha_2 + m_1\cdot G^c + k_1\right)\\ &\quad + a\cdot G^c + m_1\cdot\left(w^c\left(G^c\right) - \frac{\partial G^c(t)}{\partial t}\right)\end{aligned}.$$

We need to find $\dfrac{\partial w^c\left(G^c\right)}{\partial G^c}$ and $\dfrac{\partial G^c(t)}{\partial t}$. By (a11), $\dfrac{\partial w^c\left(G^c\right)}{\partial G^c} = \dfrac{\left(s_1+\gamma\right)}{2\cdot n}$.

Since $G^c(t) = \left(G_0 - G_\infty^c\right)\cdot e^{s_1\cdot t} + G_\infty^c$,

$$\frac{\partial G^c(t)}{\partial t} = s_1\cdot\left(G_0 - G_\infty^c\right)\cdot e^{s_1\cdot t} = s_1\cdot\left(G^c(t) - G_\infty^c\right)$$

Put $w^c\left(G^c\right)$, $\dfrac{\partial w^c\left(G^c\right)}{\partial G^c}$ and $\dfrac{\partial G^c(t)}{\partial t}$ into (a58):

$$\alpha_3 \cdot \left(\frac{\left(s_1 + \gamma\right)}{2 \cdot n} \right) \cdot \left(s_1 \cdot \left(G^c\left(t\right) - G^c_\infty \right) \right) = \left(r + \gamma\right) \cdot \left(\alpha_3 \cdot \left(\frac{\left(\left(s_1 + \gamma\right) \cdot G^c\left(t\right) - s_1 \cdot G^c_\infty \right) \cdot \alpha_3}{2 \cdot n} \right) - \alpha_2 + m_1 \cdot G^c\left(t\right) + k_1 \right.$$

$$\left. + a \cdot G^c\left(t\right) + m_1 \cdot \left(\left(\frac{\left(\left(s_1 + \gamma\right) \cdot G^c\left(t\right) - s_1 \cdot G^c_\infty \right) \cdot \alpha_3}{2 \cdot n} \right) - \left(s_1 \cdot \left(G^c\left(t\right) - G^c_\infty \right) \right) \right) \right)$$

For the above equation to hold, the following two equations must hold.

$$G^c\left(t\right) \left[\alpha_3 \cdot \left(\frac{\left(s_1 + \gamma\right)}{2 \cdot n} \right) \cdot s_1 - a + m_1 \cdot s_1 - m_1 \cdot \left(r + \gamma\right) - \frac{m_1 \cdot \left(s_1 + \gamma\right)}{2 \cdot n} - \frac{\alpha_3 \cdot \left(r + \gamma\right) \cdot \left(s_1 + \gamma\right)}{2 \cdot n} \right] = 0$$

$$G^c_\infty \left[-\alpha_3 \cdot \left(\frac{\left(s_1 + \gamma\right)}{2 \cdot n} \right) \cdot s_1 - m_1 \cdot s_1 + \frac{m_1 \cdot s_1}{2 \cdot n} + \frac{\alpha_3 \cdot \left(r + \gamma\right) \cdot s_1}{2 \cdot n} \right] + \left(r + \gamma\right) \cdot \left(\alpha_2 - k_1\right) = 0$$

m_1 and k_1 are deduced from the above two equations.

$$m_1 = \frac{\alpha_3 \cdot (\gamma + s_1) \cdot (s_1 - \gamma - r) - 2 \cdot n \cdot a}{2 \cdot n \cdot r + (2 \cdot n + 1) \cdot \gamma + s_1 \cdot (1 - 2 \cdot n)} > 0$$

and $k_1 = m_1 \cdot \frac{s_1 \cdot (1 - 2 \cdot n)}{2 \cdot n \cdot (r + \gamma)} \cdot G^c_\infty + \frac{s_1 \cdot \alpha_3 \cdot (r - s_1)}{2 \cdot n \cdot (r + \gamma)} \cdot G^c_\infty + \alpha_2 > 0$.

To derive the taxation that must be applied to farmers in WUA_S, the same procedures should be applied. Only subscriptions will change. The results for $T_j(w_j, G) = (m_2 \cdot G + k_2) \cdot w_j$ are:

$$m_2 = \frac{\alpha_3 \cdot (\gamma + s_1) \cdot (s_1 - \gamma - r) - 2 \cdot n \cdot b}{2 \cdot n \cdot r + (2 \cdot n + 1) \cdot \gamma + s_1 \cdot (1 - 2 \cdot n)} > 0,$$

$$k_2 = m_2 \cdot \frac{s_1 \cdot (1 - 2 \cdot n)}{2 \cdot n \cdot (r + \gamma)} \cdot G^c_\infty + \frac{s_1 \cdot \alpha_3 \cdot (r - s_1)}{2 \cdot n \cdot (r + \gamma)} \cdot G^c_\infty + \alpha_2 > 0.$$

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