THESIS
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LONGITUDINAL FINS OF RECTANGULAR CROSS SECTION IN HEAT TRANSFER

Thesis
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1965

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TABLE OF CONTENTS

Table of Symbols	i
Table of Figures	iii
Foreword	iv
PART I: Purpose of This Work	1
PART II: General Conclusions	5
PART III: Recommendations	10
PART IV: Analysis	12
Temperature Distribution	13
Heat Dissipation	17
Fin Effectiveness	19
Optimum Conditions	21
Temperature At the Tip	24
Surrounding Temperature Distribution	27
The Fin System	31
PART V: Experimentation	38
Object of the Experiment	39
Equipment	43
Procedure	47
Original Data Sheets	50
Developments on Collected Data	53
Discussion of Results	60
Conclusion	64

PART VI: Examples of Fin Design 69
Some Examples of Fin Design 70
PART VII: Appendices 77
A. Coefficients of Heat Transfer 78
B. Non-Continuous Longitudinal Fins 84
C. Comparison of Fin Metals 87
Bibliography 88

Drawing: Heat Exchanger Behind cover

TABLE OF SYMBOLS

A	Cross sectional flow area ft^2
a	Cross sectional area of the half fin ft^2
C	Wetted perimeter ft.
c_p	specific heat at constant pressure B/lb F
D	Diameter ft
D_h	Hydraulic diameter ft
e	2.72... (also written as exp.)
e_r	Fin effectiveness
F	Fin factor
F	Subscript denoting the fin system
f	A dimensionless factor used in finding the optimum fin conditions
f	Subscript denoting the fin
g	Mass flow rate per one-half fin lb_m/hr
G	Total mass flow rate lb_m/hr
h	Coefficient of heat transfer for convection B/hr ft^2F
H_F	Total rate of heat transfer from a number of fins B/hr
J_A	A dimensionless heat transfer factor
K	Constant of proportionality (dimensions as to use)
k_f	Thermal conductivity of the fin material B/hr ft F
L	Fin length ft
M	$= (h/k_f y_o)^{1/2}$
Nu	Dimensionless Nusselt Number
N	Number of fins
q	Rate of heat transfer from a pertion of the fin B/hr (usually per unit length of fin)

Q_F	Rate of heat transfer from one fin B/hr
Q	Rate of heat transfer from a fin system B/hr
Re	Dimensionless Reynold's Number
r	Ratio of the tip temperature difference to the base temperature difference
s	Fin spacing (= 2b) ft
s	Subscript denoting the surrounding fluid
T	Temperature F
T_o	Temperature of the fin base
T_s	Temperature of the surrounding fluid F
v	Velocity ft/hr
w	Fin width ft
W	Effective width of the fin ft
x	Direction perpendicular to y and z
y	Direction perpendicular to x and z
y_o	One-half thickness of the fin ft
z	Direction perpendicular to x and y
θ	Temperature difference F
θ_o	Temperature difference between the fin base and the surrounding fluid. F
θ_m	Mean temperature difference F
μ	Dynamic viscosity lb hr/ft^2

TABLE OF FIGURES

Fig.1	Temperature distribution along the fin	p.13
Fig.2	Infinitesimal element of the fin	p.15
Fig.3	Fin effectiveness	p.20
Fig.4	Plot of f vs m w	p.26
Fig.5	Single fin in the z-direction	p.27
Fig.6	Temperature difference along fin length	p.28
Fig.7	Fin in the z-direction	p.31
Fig.8	The fin system	p.34
Fig.9	Experimental set up and piping	p.42
Fig.10	Plot of h vs Re	p.56
Fig.11	Plot of e_r vs Re	p.57
Fig.12	Plot of F vs Re	p.58
Fig.13	Plot of Q vs Re	p.59
Fig.14	Plot of $(w/y_o)_{opt}$ vs (hy_o/k_f)	p.63
Photo 1	The experimental set up	p.41
Photo 2	The fin tubes	p.44
Photo 3	Inlet end of heat exchanger	p.45
Photo 4	Outlet end of heat exchanger	p.46
Fig.A-1	Plot of Nu vs Re	p.81

FOREWORD

Of the problems -whether they yet exist or not- Nature is the greatest of them all. It is a problem, very wide in general ways, but extremely particular in details, which introduces enchantment, extacy, wonder, curiosity, and involves conflicts. The human, in mind, body, and senses, has the courage (and he ought to be proud of it) to indulge his efforts in the solution of this great problem.

This brave human, on the other hand, is not an 'observer' who he pretends to be. He is, we must confess, a part of Nature, and in the purer sense he can never be an observer since he is a part of nature. However, this brave human claims that he has, apart from his courage, a constitution commonly called judgement which possibly enables him to have qualities of a perfect observer. Whether these qualities in turn can be traced down to his presumed courage is open to dispute.

Such ventures of observing Nature and trying to find explanations for the events thus observed have no doubt existed long before the word "science" came into use. As a phony observer of Nature I claim that such ventures were born with the very first sign of life, and they progressed as long as there was life. (Many people may tend to agree with this statement; I wish that all of us put together could prove it by undisputable evidence.) It appears that the initial ventures started due to the evil effects of nature, and that they were in the form to find a protective solution. The trend today has the tendency to change from protection to the satisfaction of wonder and curiousity, and thus a searching attitude rather than a fear

of evil and disaster. This probably is due to the fact that we have, by time, become used to Nature so as to enjoy most of it, and the fear of evil and disaster is rapidly diminishing.

The mind, particularly that of human, is in a process of development, and this development may be the starting point of his being the perfect obsever. That we recently have learned to inquire and ask for explanations, that we try to keep away from false influences, and even that we often have to and sometimes can disregard our own personal feelings is what we may call scientific attitude.

The most interesting process of scientific attitude is learning. As all other processes learning is carried out in many different ways and in many different steps. The optimum case, no doubt, is that the ways complement and the steps follow each other. Thus an efficient progress of mind is reached. It is also necessary to note that this progress while under way during learning proceeds a little further when something is learned; every knowledge is an introduction for further learnings.

What is strange but common is that learning never seems to end. Each new hill climbed allows us to see the ups and downs of more hills ahead. What is important, however, is the method by which we climb each hill, and this method developes further as we climb more hills. Thus the common link which connects all these hills is the progression of a method and the necessary mental development. So is a thesis.

1965 E. K.

PART I: PURPOSE OF THIS WORK

'The aim of our studies should be that of so guiding our mental powers that they are made capable of passing sound and true judgements on all that presents itself to us.'

Rule I; "Rules for the Guidance of Our Native Powers", Descartes, 1628.

This thesis has originated as the fulfilment of a requirement for a degree. Its purpose is to carry out a study on the subject of longitudinal fins in heat transfer. The explanation of this rather general statement can be done in two phases: The General purposes and the individual purposes or aims.

The general purposes involve mainly of carrying out a study, which includes the development of mind and the scientific attitude, devising of methods and their justification, and the transmission of thought to clear expression. As a student of science I believe that the particulars constituting the general purposes of any thesis can be enumerated as follows:

1. To gain knowledge of a phenomenon in which the student has interest.
2. To realise self-responsibility and receive minimum external help.
3. To develop one or more methods of progress, and to apply these methods under one's own responsibility.
4. To taste the urge of obtaining original results and learn how they can be possible.
5. To judge his own work.

The last of these items is probably one causing a great deal of misunderstandings between the authors and the readers. The case, however, is simple enough. The judgement of one's own work is his own satisfaction from it averaged for the times of starting, finishing, and long after it is finished.

As for the individual purposes or the aims of this thesis I wish to give a short account of how it started.

I had my first significant contact with the use of finned systems in my works involving air-conditioning equipment. I became aware that apart from manufacturers data very few information existed for the refrigerant evaporators and even less so for condensers which made use of fins.

My second contact with fins occured during a short study of nuclear reactor cooling systems. It appeared at the time that the design of fins were done without much care and once they fulfilled the requirements they were acceptable. Not much attention was paid to the comparative analysis of fins. My curiousity started with the economic as pect of fin systems and then wandered to the increases in outputs as applied to compactness of equipment. This initiated the aim of this thesis:

Is it possible to find fin systems which posses the properties of

a. maximum economy
b. maximum capacity

all at the same time. This question has been favourably answered in this work.

Initially I had planned that this analysis was to be general in character to include all types of fins and their behaviour under all possible conditions of operation. I soon found out that such general an analysis was too general to be practically possible. I had then two alternatives: To study specific fields of operation or to analize specific fin shapes. I chose to study a specific shape of fin: The fin of rectangular cross-section. The next point of decision came when the differential equations of the longitudinal and the spiral fins came out to be very different in form and solution. My topic was finally established when I preferred the longitudinal fins for analysis for their wider use and simpler mathematics.

The application of fins is unlimited if they are properly design to suit in the best way the job they are expected to do. How they can be properly designed is an interesting study, well worth the time and effort spent for it.

My proposed method consists of a theoretical analysis of the subject, a series of experiments, observations, and a discussion of the results thus reached. I believe that it is necessary to start from the very basic observations and to reach a point at which the results are useful and satisfactory.

PART II: GENERAL CONCLUSIONS

'Finally, we have to make use of all the aids afforded by understanding, imagination, sense and memory; first for the purpose of intuiting distincly the simples which come before the mind; secondly, in comparing the things into which we are inquiring with those we already know, in such fashion that they may thereby come to be likewise known; and thirdly, in finding the things which allow of thus being compared, in order that, so far as our human powers allow, nothing be omitted'.

Rule XII; "Rules for the Guidance of Our Native Powers", Descartes, 1628.

The use of fin tubes in heat exchange equipment increases the heat transfer rates up to a hundred per cent as compared to bare tubes, depending on their arrangement and the operating conditions.

Longitudinal fins which have short lengths are oten used for gases, usually for air. Systems of long longitudinal fins are preferred for liquids without limitations as long as they are in the phase to flow. What has been covered in this thesis is arrangements of long longitudinal fins, their analysis and design for heat transfer. Flow friction, pressure drops, and fouling rates and factors are not considered.

The analysis is carried out first for a single fin of finite dimensions under a set of operating conditions. In this portion the fin effectiveness is defined as the fraction of the fin surface which operates as an ordinary heat transfer surface. This leads to the determinatinn of the most economic fin. In other words, the dimensions thus derived give the fin of least material to accomplish the required job. Next was a study of the fin system defined as an arrangement of fins and the operatin conditions. This sudy indicated that the control of fin systems involves a very large number of dependent and independent variables. Throughout my thesis I have tried usually in vain, to arrive at a practical number of relationships which would account for all the possible variations of these variables. If this were achieved the amount of time and effort spent on fin design proble would be considerably shortened.

I cannot consider myself wholly succesful in this matter for

I do not feel that the study has ended or that it ever will.

My vision of all the variables related to the subject is that they are analogus to brothers and sisters of the same family and home. Their interactions and influences cannot easily be seen by a stranger unless he chose to live under the same roof with them to examine their conduct, behaviour, and effects under all possible circumstances.

In my hypotheses that all these different modes of behaviour could be explained, I was not wrong. It is possible to see almost all the individual circumstances of influence and predict the behaviour which resulted. However, to derive a few relationships which would explain all the possible interactions and all the resulting behaviours has remained as mysterious to me as it already was when I had started this work.

The solution of this sad phenomenon was simply simplification. By simplification in a manner to discard all conditions that were unfavourable, it was possible to express all individual cases of influence and the over-all inter-effects for cases which were favourable. By this method it was clear that the heat transfer through a fin was dependent on:

a. Geometric dimensions
b. Properties of the fin material
c. Properties of the surrounding fluid
d. The operating conditions

Once the case of the single fin was established it was necessary to examine the conditions under which this fin would be efficient and economical. The results in this case show that the heat transfer capacity of a system does not increase proportional to the extra surfaces added to the system by fins. The most efficient system is the one with no fins at all, and the addition of fins is a phenomenon of diminishing returns.

The idea to increase the capacity of a system in spite of a drop in the efficiency is not a new one, and it is inevitably applied inthis case. Since the analysis presumes the existance of fins, the efficiency of the system can never be 100 per cent. On the other hand, it is found that if there is to be a fin there can be a fin which requires the least amount of material for its purpose. This fin is defined as the most economic fin.

In the consideration of a system of fins it was found that each fin actually contributes only a fraction of its surface to the welfare of the system. This fraction is the effectiveness of the fin. Thus it is possible that the surface gained by the fin be less than the surface lost by locating the fin. This is a case most unfavourable in the fin systems.

Experiments confirmed that the addition of fins to a system resulted in diminishing returns, the over-all capacity of the system thus tending to reach a limiting value. After this limit the capacity follows a decreasing path.

At this point the results could be summerized in two statements:

a. There exists a fin of least material for a given set of operating conditions.

b. There is a set of conditions under which a fin system gives the maximum output.

The question then remained as to whether these two statements could be combined to define the fin system which was most favourable.

The final result is based on part of an analysis made by Avrami and Little, and presented in fig.14. The process of obtaining a fin system of both the least meterial and maximum output, although simple in words, involves a process of carefully satisfying three conditions given in Part V, the results of which can or should be checked by the relationships derived

to give the capacity of any fin system.

For laminar flow and fluids of high viscosity (Re low) the use of non-continuous fins can be made. (See Appendix B) These are specially preferable for fluids such as fuel oils, heavy oils, and other viscous fluids. At low Reynold's Numbers the non-continuous fins increase the heat transfer rates as high as twofold and also introduce advantages in fouling rates.

PART III: RECOMMENDATIONS

Although analytical studies of fin systems are being made few engineering design data on the subject exist. On the other hand the industrial applications of finned systems are growing every day.

On longitudinal fins this work does not cover friction and pressure drops, and the fouling rates of fin systems. This is a separate field of study and research which needs to be taken up parallel to the heat transfer theory.

I believe that the best way to develop design data for engineering applications of fins, is to set up a wide variety of standard finarrangements and to list in tables the geometric and the operational properties relative to the types (viscosity) of the fluids that may be used in the system. The development of such standards will be very helpful in simplifying the design procedure, and also the industry of fin manufacture.

Manufacture of fin tubes is another very interesting subject. It is also a field which is not fully developed in Turkey. If we consider the amount of heat exchange equipment used around us, the necessity of developments will be clearly seen.

The whole study of heat transfer is one which is very flexible and this has many immediate advantages. But in prectical applications the lack of standards, poor processes of manufacture all result in uneconomic and wasteful designs. The setting up ofworthy standards no doubt requires time and experience, but when done they will contribute a lot to the acceleration of progress.

Part IV: ANALYSIS

'In treating of the objects proposed for investigation what we have to examine is not what others have opined, nor what we ourselves may cojecture, but what we can clearly and evidently intuit, or can deduce with certainty: knowledge is not obtainable in any other way.'

Rule III;"Rules for the Guidance of Our Native Powers", Descartes, 1628.

TEMPERATURE DISTRIBUTION

We consider a fin of finite dimensions (fig.1). It is rectangular in cross section, and has a total thickness of 2a and width of w. The length of the fin is assumed to extend in the z-direction. The magnitude of this extension will be handled later in the analysis.

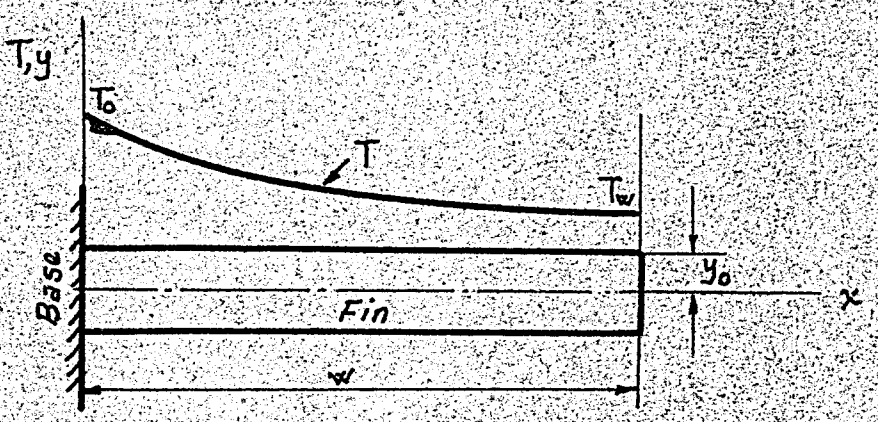

Fig.1

The case will be taken when a heat source exists at x = 0. The heat is dissipated in the fin by conduction and to the surroundings by convection. The effect of radiation is small especially in the case of forced convection thus it will not be considered.

The temperature at any point on the fin is denoted by T, which varies in all three dimensions. However, the thickness of the fin is very small and the variation of the temperature along the y-direction does not contribute to the problem.

We now take a small portion of the fin, a unit length in the z-direction, along which we assume that T_z remains constant. The temperature of the surroundings, denoted by T_s, is assumed to vary only in the z-direction. Although this is not actually true the surrounding fluid usually moves in the z-direction and the conditions along the x-direction are unchanged. In cases where the flow is not in the z-direction one would not be justified in making this assumption.

The portion of the fin under consideration is shown in fig.2. Before making the heat balance we must stop to observe that the tempearture gradient along the y-direction is zero at y=0. There is no heat transmission accross the two halves of the fin; this is a condition of symmetry and explains why we consider only one half of the fin.

From fig.2 we can see the following heat balance:

$$q_1 = q_2 + dq \tag{1}$$

and we know that

$$q_1 = -k_f a \frac{dT}{dx}_x \tag{2}$$

$$q_2 = -k_f a \frac{dT}{dx}_{x+dx} = -k_f a \left[\frac{dT}{dx} + \frac{d^2T}{dx^2} dx\right] \tag{3}$$

$$dq = h\, dx\, (T - T_s) \tag{4}$$

Substituting equations 2,3, and 4 into 1, we have:

$$-k_f a \frac{df}{dx} = -k_f a \left[\frac{dT}{dx} + \frac{d^2T}{dx^2} dx\right] + h\, dx\, (T - T_s) \tag{5}$$

which reduces into the following simple form

$$\frac{d^2T}{dx^2} = \frac{h}{k_f a} (T - T_s) \tag{6}$$

In order to siplify the calculations we now introduce the following terms:

$$m^2 = \frac{h}{k_f a} \tag{7}$$

$$\theta = (T - T_s) \tag{8}$$

With the substitution of (7) and (8) equation (6) becomes:

$$\frac{d^2\theta}{dx^2} - m^2 \theta = 0 \qquad (9)$$

Equation (9) has the solution

$$\theta = A \cosh mx + B \sinh mx \qquad (10)$$

In order to determine the two constants A and B, we have the following boundary conditions:

$$\text{when} \quad x = 0 \qquad \theta = \theta_o$$

$$\text{and when} \quad x = w \qquad \frac{d\theta}{dx} = 0$$

At this point we must consider the heat loss at the tip of the fin which was not included in the heat balance equations. This omission can be corrected approximately by using [1]:

$$w_{corr.} = w_{act} + a \qquad (11)$$

and the boundary conditions above would apply. The constants A and B are then evaluated as:

$$A = \theta_o \qquad (12)$$

$$B = - \theta_o \tanh mw \qquad (13)$$

and the solution to (9) becomes

$$\theta = \theta_o \left[\cosh mx - (\tanh mw) \sinh mx \right] \qquad (14)$$

[1] By Max Jacob.

HEAT DISSIPATION

Having the expression for the temperature distribution along w , the fin width, we can now derive the expression for the amount of heat which is dissipated from the fin to the surroundings. The heat transmission to the surroundings is by convection. From an infinitesimal element of the fin, the heat lost to the surroundings was indicated by eq.4 :

$$dq = h\,\theta\,dx \qquad (4)$$

By substituting eq.13 into 4, and integrating along the fin length we can express the total heat dissipated to the surroundings

$$q = h\,\theta_o \int_0^w \left[\cosh mx - (\tanh mw)\sinh mx\right] dx \qquad (15)$$

And, integrating between the limits:

$$q = \frac{h\,\theta_o}{m}(\tanh mw) \qquad (16)$$

On the other hand the same amount of heat must be conducted into the fin at the base (i.e. at x=0). In this case we have the following relationship:

$$q = -k_f y_o \left.\frac{dT}{dx}\right|_{x=0} \qquad (17)$$

From what we have already derived as the temperature distribution, we can differentiate eq.13 to get

$$\frac{dT}{dx} = \frac{d\theta}{dx} = \theta_o\, m \left[\sinh mx - (\tanh mw)\cosh mx\right] \qquad (18)$$

Thus at $x = 0$

$$\left.\frac{dT}{dx}\right|_{x=0} = -\theta_o\, m\, (\tanh mw) \qquad (19)$$

Substituting eq.19 into 17, we have:

$$q = k_f y_o\, m\, \theta_o\, (\tanh mw) \qquad (20)$$

which checks eq.16. Therefore we write the expression for the heat dissipated from the fin as:

$$q = \theta_o \sqrt{h k_f y_o}\, (\tanh \sqrt{h/k_f y_o}\; w) \qquad (21)$$

FIN EFFECTIVENESS

Effectiveness of a fin is a theoretical measure of the quality and efficiency of the fin. It is defined as the ratio of the amount of heat actually dissipated from the fin surface to the amount of heat which would be dissipated if the fin surface were held at the temperature of the base. In our case this ratio, denoted by e_r, can be expressed as:

$$e_r = \frac{q}{h\, w\, \theta_o} \qquad (22)$$

where q is obtained from eq.21, and

$$e_r = \frac{1}{w} \sqrt{k y_o / h}\, (\tanh \sqrt{h/k_f y_o}\, w) \qquad (23)$$

Recalling the definition of m from eq.8, the effectiveness of the fin which has a rectangular cross section and is longitudinal in shape, is simply

$$e_r = \frac{\tanh mw}{mw} \qquad (24)$$

Eq.24 is plotted in fig.3 to show the variation of e_r with respect to the changing mw. Note that the fin effectiveness reaches its maximum value as the mw tends towards zero, and decreases uniformly as mw attains large values.

OPTIMUM CONDITIONS

Observing the fin effectiveness curve, fig.3, we see that the most efficient case occurs when the fin dimensions are reduced to zero. But, since our analysis assumes the existance of finite fins of dimensions other than zero we are urged to disregard the point where (mw)=0. This also means that we sacrifice from the efficiency of the fin in order to increase the rate of heat transfer, and at thesame time introduce compactness to the heat exchange equipment.

Before going into details of finite fin dimensions other than zero let us first show that our statement above regarding the most efficient case which occurs when the fin is reduced to non-existance. We shall do this by differentiating eq.21 with respect to the variable y_o, and equating the differential to zero. We have:

$$\frac{dq}{dy_o} = 0 = \frac{\theta_o}{2}\sqrt{hk_f/y_o}\left[\tanh mw - mw\ \mathrm{sech}^2 mw\right] \qquad (25)$$

which reduces to:

$$\tanh mw = mw\ \mathrm{sech}^2 mw \qquad (26)$$

And simplifying we arrive at the condition where the highest heat transfer rate occurs relative to the variable y_o :

$$\frac{\sinh mw}{mw} = \frac{1}{\cosh mw} \qquad (27)$$

The value of (mw) which satisfies eq.27 is found to be equal to zero. This indicates a fin of infinite thickness and zero width or, in other words, no fin at all. This result is in agreement with the efficiency concept which gives the maximum effectiveness when (mw) is equal to zero.

However, in order to study actual fins we wish to try another method to reach an expression for the optimum coditions of a fin which has real thickness and width. This method searches for the conditions in which a fin of known dimensions gives the maximum output. The basic variable now is the half cross-sectional area (a) of the fin. It is defined as:

$$a = w\, y_o \tag{28}$$

If this paremeter is substituted into eq.21, the expression for heat dissipation becomes:

$$q = \theta_o \sqrt{h k_f y_o} \left[\tanh \sqrt{h/k_f}\, a y_o^{-3/2} \right] \tag{29}$$

We now differentiate eq.29 with respect to y_o for a given value of (a) and equate it to zero. We thus have:

$$\frac{\sinh mw}{3\, mw} = \frac{1}{\cosh mw} \tag{30}$$

The value of (mw) which satisfies eq.30 is found by trial and error to be[1]

$$mw = \sqrt{h/k_f}\, w y_o^{-\frac{3}{2}} = 1.4192 \tag{31}$$

---

(1) Max Jacob

As consequences of eq.31 the optimum length, thickness, and half cross-section area are calculated as below:

$$w = \frac{1.5957}{h} \left(\frac{q}{\theta_o} \right) \tag{32}$$

$$y_o = \frac{1.2642}{h\, k_f} \left(\frac{q}{\theta_o} \right)^2 \tag{33}$$

$$a = \frac{2.0192}{h^2\, k_f} \left(\frac{q}{\theta_o} \right)^3 \tag{34}$$

We see from this set of equations that for any set of given requirements, a fin can be found which has the least amount of material to fulfill those requirements. This is a characteristic of compactness.

TEMPERATURE AT THE TIP

The temperature along the width of the fin was expressed by eqs.13 and 14. Substituting in these equations the condition where $x = w$, the temperature at the tip of the fin is given by:

$$\theta_w = \theta_o \frac{1}{\cosh mw} \tag{35}$$

We had also found in the previous section that the optimum condition for a given half cross-section area occured when we had $mw = 1.4192$. Then if our fin is operating under the optimum condition we know that the temperature at its tip is

$$\theta_w = 0.45706\ \theta_o \tag{36}$$

or that the tip temperature for a fin of least material is a little below half of the temperature at the base. We must note, however, that the term temperature here actually implies the temperature differential defined as the difference between the base temperature and the temperature of the location in question.

For the general case, expressing the tip temperature as in eq.35, and substituting it into the expression for heat dissipation, eq.21, we get:

$$q = \theta o \sqrt{h\ k_f\ y_o\ (1 - r^2)} \tag{37}$$

where

$$r = \frac{\theta_w}{\theta_o} = \frac{1}{\cosh mw} \tag{38}$$

With eq.37, the set of equations for the main fin dimensions (eqs.32,33, and 34) become:

$$w = \frac{\cosh^{-1}(1/r)}{1 - r^2} \quad \frac{1}{h} \quad \left(\frac{q}{\theta_o}\right) \tag{39}$$

$$y_o = \frac{1}{1 - r^2} \quad \frac{1}{hk_f} \quad \left(\frac{q}{\theta_o}\right)^2 \tag{40}$$

$$a = \frac{\cosh^{-1}(1/r)}{(1 - r^2)^{3/2}} \quad \frac{1}{h^2 k_f} \quad \left(\frac{q}{\theta_o}\right)^3 \tag{41}$$

The last equation (eq.41), representing the cross-sectional area of the half fin, has introduced a new factor which is useful in design problems. We define

$$f = \frac{\cosh^{-1}(1/r)}{(1 - r^2)^{3/2}} \tag{42}$$

to which the fin cross-sectional area is directly proportional. Fig.4 shows the variation of (f) as a function of (mw), also related to (r). Note that (f) becomes a minimum, thus making the fin area (a) minimum, when (mw) = 0.457.

BOĞAZİÇİ ÜNİVERSİTESİ KÜTÜPHANESİ

SURROUNDING TEMPERATURE DISTRIBUTION

So far our analysis was only concerned with the temperature distribution within the fin. The equation for q expressed the amount of heat dissipated for a unit length of the fin in the z-direction. The aim in this section is to determine the temperature distribution of the surroundings along the fin length in the z-direction. For the surroundings the temperature variation in the x-direction is very small and thus the temperature can be considered as constant. It varies, however, in the z-direction as heat is gained along the fin length. The previous condition that the temperature of the fin base is constant prevails.

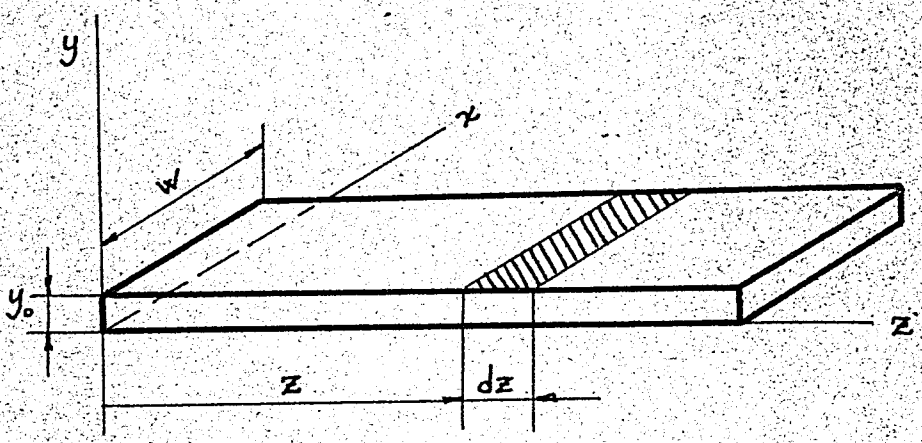

Fig.5

We now consider an infinitesimal portion of the fin as shown in fig.5. The temperature of the surroundings is a function of the variable (z) only, and the heat dissipated by the infinitesimal element is equal to the heat gained by the surrounding

fluid. This can be expressed as:

$$h\, w\, dz\, \theta_m = -g\, c_p\, d\theta_m \qquad (43)$$

where (θ_m) is the mean temperature difference along the x-direction, (g) is the flow of surrounding fluid per half-fin, and (c_p) is the specific heat of the surrounding fluid at constant pressure. The negative sign is due to the fact that while the left hand side of eq.43 is a loss of heat, the right hand side indicates that heat is gained. The integral form of eq.43 is:

$$-\int_0^L \frac{h\, w}{g\, c_p}\, dz = \int_{(\theta_m)_{in}}^{(\theta_m)_{out}} \frac{d\theta_m}{\theta_m} \qquad (44)$$

The significance of the limits of eq.44 as well as the shape of the surrounding temperature distribution are shown in fig.6.

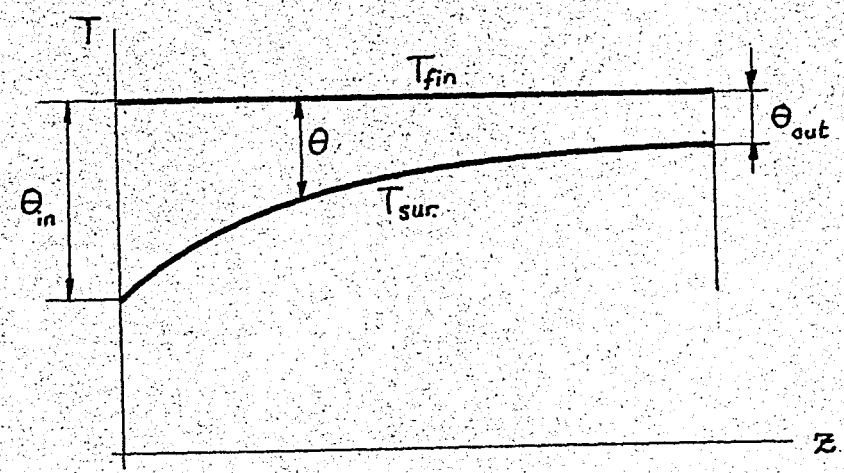

Fig.6

In tegrating both sides of eq.44 between the limits, we obtain:

$$-\frac{h\,w}{g\,c_p}\,L = \log\,\frac{(\theta_m)_{out}}{(\theta_m)_{in}} \tag{45}$$

or more explicitly

$$\frac{(\theta_m)_{out}}{(\theta_m)_{in}} = \exp\,(-hwl/gc_p) \tag{46}$$

The mean temperature difference is defined as:

$$\theta_m = \frac{1}{w}\int_0^w \theta\,dx \tag{47}$$

and the expression for (θ) was derived as the solution to the differential equation of heat balance (see eq.14). Substituting eq.14 into 47, and integrating between the limits, we have:

$$\theta_m = \frac{\theta_o}{m\,w}\,\tanh\,mw = \theta_o\,e_r \tag{48}$$

along the x-direction.

We see that, since (e_r) does not vary along the length of the fin, the ratio of temperature differences as required in in eq.46 is independent of the location in the x-direction. In other words, when used as a ratio, the fin effectiveness is cancelled and the actual temperature differences remain. Thus, eq.46 can now be expressed in the following convenient form:

$$\frac{(\theta_o)_{out}}{(\theta_o)_{in}} = \exp\,(-hwl/gc_p) \tag{49}$$

It must be noted that eq.49 is a very useful expression in the analysis of a fin. When as usual the inlet temperature difference is known, the fin and the flow characteristics are pictured then the outlet temperature is easily calculated. As can be seen, eq.49 constitutes a major part of the general equation for the capacity of a fin system, which will be derived in the following section.

We can put eq.49 into a general form to express the temperature at any point of the flow. The relationship then becomes a function of both x and z:

$$\theta_{x,z} = (\theta_x)_{in} \left[\exp\ (hw/gc_p)z\right] \qquad (50)$$

THE FIN SYSTEM

We shall conclude our analysis of the single fin by finding the total amount of heat which it dissipates to the surroundings.

With the results of the preceeding section we can express the mean temperature difference along the z-direction, as:

$$\theta_m(z) = (\theta_o)_{in}\, e_r \left[\exp\,(-hw/gc_p)z\right] \qquad (51)$$

Now consider the shaded portion of the fin shown in fig.7.

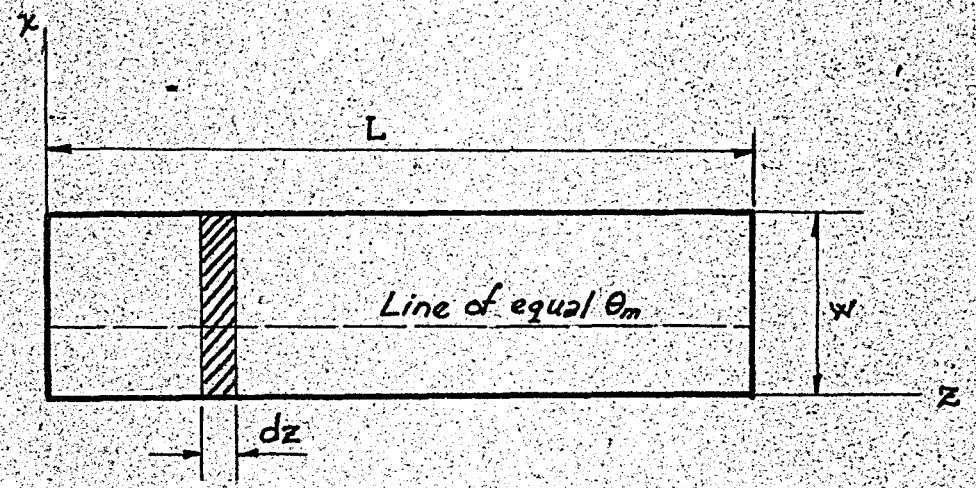

Fig.7

The heat loss from the area (w dz) can be written as:

$$\frac{1}{2}\, dQ_F = h\, w\, dz\, (\theta_o)_{in}\, e_r \left[\exp\,(-hw/gc_p)z\right] \qquad (52)$$

where the one-half indicates that the heat loss is from one side of the fin, and the subscript F stands for 'the fin only'.

Integrating eq.52 between the limits $0 \leq z \leq L$, we get:

$$Q_F = 2 g c_p (\theta_o)_{in} e_r \left[1 - \exp(-hwL/gc_p)\right] \qquad (53)$$

where Q_F is the total heat dissipation from one fin.

Let us now consider that we have N fins all of which undergo the same conditions of operation. The over-all heat dissipation which then takes place can be expressed by using eq. 53. If we denote by G, the total mass flow rate of the surrounding fluid, and by H_F the total (over-all) rate of heat dissipation, we have:

$$G = 2 g N \qquad (54)$$

$$H_F = Q_F N \qquad (55)$$

Substituting equations 54 and 55 into equation 53 we get:

$$H_F = G c_p (\theta_o)_{in} e_r \left[1 - \exp\left(-\frac{2NhwL}{Gc_p}\right)\right] \qquad (56)$$

as the over-all heat dissipation from a group of N fins all of which are subject to the same conditions of operation and have the same dimensions.

It is interesting to notice that in appearance eq.56 does not depend on the properties of the fin material. It should be remembered, however, that the fin effectiveness depends on the

conductivity (k_f) of the fin material, and it will be seen in Appendix A that the heat transfer coefficient for convection, (h), also depends on the conductivity of the fin material.

. . . . . .

So far in our analysis we have dealt with the single fin, and a group of N fins undergoing the same conditions and having the same dimensions. We have considered that all the heat transfer took place through the fins and none through the vacant portions of the base to which they are connected. However, since the fin <u>is</u> connected to its base, and usually more than one fins are connected to the same base, we shall widen our scope of vision to a 'fin system'.

We shall define the fin system as a geometric arrangement of fins and their base (of the same material in this case), and the conditions of their operation.

We know that the portion of the fin base between two fins is open and thus a heat transfer takes place through it. The amount of heat which is dissipated through the vacant portion of the base is sometimes very small and can be neglected. At other times when the fin spacing is large, this additional heat exchange becomes significant and cannot be neglected.

We had already called the fin thickness ($2y_o$), and now we shall denote the fin spacing, or in other words the portion of the base open to the surroundings, by (2b) or (s). Thus s = 2b. Such an arrangement is shown in fig.8.

It is certain that the total heat loss from the fin system is the sum of the heat loss through the fins and the heat loss through the spacing of fins. In the microscopic analysis of the single fin the heat dissipation was expressed as

$$q = \theta_o \sqrt{hk_f y_o}\ (\tanh \sqrt{h/k_f y_o}\ w) \qquad (21)$$

and the heat dissipation through one half of the fin spacing can be expressed as:

$$q_b = h\, b\, \theta_o \tag{57}$$

for a unit length of the fin.

The algebraic sum of equations 21 and 57 gives us the total heat dissipated to the surroundings by the fin system.

$$q_t = h\, \theta_o \left[b + \sqrt{k_f y_o / h}\, (\tanh \sqrt{h / k_f y_o}\, w) \right] \tag{58}$$

The term inside the brackets will be called the 'effective width' of the one-half fin system, and will be denoted by W. By definition we have

$$W = b + \frac{\tanh mw}{m} \tag{59}$$

or

$$W = b + we_r \tag{60}$$

The effective width is seen to be composed of the half fin spacing plus the amount of additional width that a fin actually contributes to the system. We also see that the contribution of a fin is the actual width of the fin reduced by its own effectiveness.

One immediate consequence of eq.60 is that a fin is not worth its existance unless

$$w\, e_r > y_o \tag{61}$$

otherwise it would be contributing lessthan the area which it occupies.

The application ofthe effective width to the method of the preceeding section can be done simply by substituting W for w in eq.43, and carrying out the integration along the fin length. Consequently, let us noww pass directly to the system of N fins and spacings. Then equating the heat lost by the fin arrangement to the heat gained by the surroundings, we have

$$2\,N\,h\,W\,dz\,\theta_o = -G\,c_p\,d\theta_o \qquad (62)$$

where G and c_p are mass flow rate and the specific heat at constant pressure of the surrounding fluid, respectively. Note that $\theta_o = T_o - T_s$,and $dT_s = -d\theta_o$. Integrating eq.62 between the limits from zero to L, we get

$$\frac{(\theta_o)_{out}}{(\theta_o)_{in}} = \exp\left(-\frac{2NhWL}{Gc_p}\right) \qquad (63)$$

which is similar to eq.49, but it covers the fin system more completely.

We had also expressed the mean temperature difference along the x-direction in eq.51. We can, again by substituting the conditions for the system of N fins and the effective width, W, alter eq.51 to one which will be valid for the complete system. Thus we have:

$$\theta_m(z) = (\theta_o)_{in}\, e_r\left[\exp\left(-\frac{2NhWz}{Gc_p}\right)\right] \qquad (64)$$

Then the heat lost by the system becomes

$$\int_o^L dQ = \int_o^L 2NhW(\theta_o)_{in}\, e_r\left[\exp\left(-\frac{2NhWz}{Gc_p}\right)\right] dz \qquad (65)$$

and integrating we have

$$Q = G c_p (\theta_o)_{in} e_r \left[1 - \exp\left(-\frac{2NhWL}{Gc_p}\right)\right] \qquad (66)$$

as the total quantity of heat dissipated to the surroundings by the complete fin system.

It is worthwhile to compare eq.66 with eq.56. We see that the extra surface now included in the consideration, the fin spacing, only changes the exponential term in these equations. Let us re-write the exponent in eq.66, and call it F, the fin factor:

$$F = \frac{2 N h W L}{G c_p} = \frac{2 N h L}{G c_p} (b + we_r) \qquad (67)$$

We wish to make the fin factor, F, as high in magnitude as possible with due care as to how the other terms of eq.66 behave.

Part V: EXPERIMENTATION

'If we have intuited several propositions, and propose todeduce something else from them, it is useful to run through them in a continuous and uninterrupted act of thought, to reflect on their relation to one another, and so far as is practicable, to apprehend distinctly several of them, all at once. For in this way our knowledge is rendered much more certain and the mind's capacity very greatly increased.'

Rule XI; "Rules for Guidance of Our Native Powers", Descartes, 1628.

OBJECT OF THE EXPERIMENT

A theory is built up in analytical terms on the basis of reason to describe the expectations of real happenings. It is developed by observation of these real happenings during their actual occurence. It is established when the singular inquiries are thoroughly explained making possible the understanding of all the related phenomenon in an over-all manner.

Certain relatinships concerning the use of fin systems have been established. They involve arrangements of longitudinal fins of rectangular cross section and their characteristics as applied to the transfer of heat from one media to another. They are the results of analytical methods in which a number of basic assumptions and simplifications have taken place, and threaten the validity of the results. These assumptions and simplifications are permissible as particular items, but it is difficult to judge whether they would, in multiple, cause important descrepancies in the theory.

It is necessary, at times, to pass our processes of thought through examinations of application both to check their validity, and also to reach a confirmed point from where on further developments can take place. This is the object of our experiment.

The wide use of fin systems seem to be largely flexible to an observer on the surface. We have established in the preceeding sections two important facts which are quite contradictory to the above statement. These can be summerized as:

1. There is a well defined set of dimensions of a fin which requires the least amount of material for a known rate of

heat exchange and operating conditions. This is the first important aspect of economy in the matter.

2. For the heat output of a fin system the relationship is exponential. The rate of output increase always decreases, tending to reach a limit. Thus the phenomenon is one of diminishing returns.

These two facts will be further developed in the following sections in referance to the experimental data.

EQUIPMENT

The experimental set, shown diagramatically in fig.9, was manufactured and installed in the labaratories of the Robert College Engineering School. The set consists of:

1. The heat exchanger (Dwg.01), manufactured of standard 3" steel pipe with flanged covers at each end. Water inlet and outlet by welded couplings at the two ends of the heat exchanger with a thermometer for each. Two 1/4" valves are provided across the heat exchanger for differential pressure measurements. (See photo 1)
2. One bare and three finned tube arrangements (photo 2). (One finned tube not shown since installed in the heat exchanger.) These tubes are slided into the 3" shell of the heat exchanger and locked by the flanges and rubber ring gaskets, one end being connected to the steam inlet (photo 3) with a thermometer and pressure gauge, the other connected to the condensate tank (photo 4). Saturated steam flows and condenses inside the fin tube while its heat is transferred to the water that flows outside the finned tube in the annular section of the heat exchanger. We have a tube with no fins
   a tube with four fins
   a tube with eight fins
   and a tube with sixteen fins.
3. Water and condensate collect valves and drains (photo 4). Through the collect valves water and steam condensate can be collected to measure the flow rates. The water

PROCEDURE

The heat exchanger in the experiment was used to heat water by using saturated steam in a fin system. Saturated steam was connected to the fin tube, and its temperature and pressure were indicated at the inlet side. The condensate which collected in a small tank was drained through a steam trap. The selection of saturated steam was to make available a constant temperature condensation, and thus a constant temperature at the fin base. This also meant that the pressure of the saturated steam had to be held constant throughout a series of readings.

Water to be heated entered the annular section of the heat exchanger on one side and flowed in between the fins until it reached the other end to exit. Both the inlet and the outlet temperatures of water flow were indicated on local thermometers.

Since there were four tubes (one bare and three with fins) it was necessary to perform the experimentation procedure four times, once for each tube. This required careful handling of the apparatus while the inner tube was being changed. When, after a change, all the piping were connected and tested for leaks, the steam supply readied at the required pressure, and the water supply checked, the apparatus was set for the experiment.

The readings were taken at constant steam temperature and pressure, at at various rates of water flow. The rate of water flow could easily be adjusted by the globe valve located at the water outlet.

The following is a step by step description of the procedure employed in taking the readings for one rate of water flow

including the initial start-up of the apparatus after the steam and water supplies were checked to be ready for use:

1. Fully open the drain valve for water.
2. Fully open water outlet globe valve.
3. Open fully but slowly the water inlet valve and see the full amount of water flow at the drain.
4. Slowly reduce the water flow rate by turning the globe valve at water outlet. Stop when the minimum flow rate for the experiment is observed.
5. Fully open the inlet side steam drain valve.
6. Slowly open the globe valve at steam inlet. The condensate collected in the supply pipe will drain through the drain pipe opened in (5). Wait until steam comes out which indicates that all the condensate has drained.
7. Fully open the two valves on the condensate side to allow free flow through the steam trap and drain. (When the apparatus has not been in use for a long time it is necessary to drain the condensate tank before proceeding with step 7.)
8. Close the steam drain valve at the inlet side. This will direct all the steam through the fin tube to the condensate side.
9. When condensate begins to come out through the steam trap, and the equipment has 'warmed up', slowly open the steam inlet valve to full.
10. Record steam inlet temperature and pressure, and observe that they remain constant throughout the following procedure.
11. Record water inlet temperature.
12. Collect water in a container at the outlet side and time the interval of collection. Record the time reading when the collection is over and the container is about full. The time should be kept by a stopwatch.

13. Weigh the water collected in the container and record the weight. (The weight of the container should be subtracted fro the reading on the scale.)
14. Record the water outlet temperature.
15. The readings are now finished. Increase the water flow rate by the necessary amount by using the globe valve at water outlet.
16. Repeat items 10 through 15 for the next readings. The test is finished when the maximum water flow rate is reached.

CAUTION: The steam inlet pressure and temperature has a tendency to vary as different water flow conditions are tested. There may be other causes for such effects. It is necessary to keep a frequent check of the steam conditions to keep them constant. The globe valve at the inlet side provides a relatively easy adjustment of pressure.

ORIGINAL DATA SHEETS

In this section exact duplicates of the original data sheets are presented. (The thermometers of the experimental set read degrees Centigrade. These are converted into degrees F.)

Bare Tube (No fins): April 1, 1965

Water Flow		Steam		Water	
lbs.	sec.	psig	F	inlet F	outlet F
18.0	80	53	302	53.6	109.4
21.5	68	53	302	53.6	93.2
6.6	15	54	302	55.4	88.7
20.5	35	53	302	53.6	77.0
9.8	10	54	302	55.4	71.6
21.0	26	52	298.2	53.6	70.7
22.5	18	51	298.2	54.5	67.1
20.0	14	53	302	55.4	65.3
22.5	12.6	51	298.2	53.6	62.6
22.0	8.5	51	298.2	53.6	60.8
20.5	6	50	298.2	52.7	59.0
20.5	5.6	50	298.2	52.7	57.2

The next test was on the 4-fin tube whose original data sheet is given on next page.

4-Fin Tube: April 3,1965

Run	Water Flow lbs.	sec.	Steam psig	F	Water inlet F	outlet F
1	18.5	112	46	294	59.0	129.2
2	21.0	73	46	294	57.2	109.4
3	22.0	60	46	294	57.2	99.5
4	21.0	44	45	292	57.2	87.8
5	20.0	30	44	292	55.4	80.6
6	23.0	25	42	290	53.6	75.2
7	21.0	19	40	287	53.6	69.8
8	22.0	16	39	287	51.8	68.0
9	22.5	13	39	287	51.8	64.4
10	21.5	10	40	287	51.8	62.6
11	20.0	8	38	284	51.8	60.8

8-Fin Tube: April 17,1965

1	17.5	125	55	302	59	147
2	20.0	60	55	302	56	118
3	21.0	35	55	302	55	90
4	21.0	28	54	302	55	82
5	22.5	22	54	302	55	79
6	26.0	19	53	302	57	75
7	25.0	12	52	300	57	70
8	21.0	9	51	300	55	68
9	23.5	9	51	300	54	64
10	12.0	3.5	51	298	54	63
11	11.0	3.5	50	298	54	63
12	13.5	3.5	50	297	54	61
13	13.0	3.0	50	297	54	61
14	12.0	2.5	50	297	54	61

16-Fin Tube: May 15, 1965

Run	Water Flow lbs.	sec	Steam psig.	F	Water inlet F	outlet F
1	22	88	48	295	61	133
2	21	46	48	295	61	109
3	23	35	48	295	61	97
4	23	23	48	295	61	84
5	25	18.5	48	295	61	79
6	25.5	16	48	295	61	77
7	26.5	13	48	295	61	75
8	21	10	48	295	61	73
9	16.5	6	48	295	61	72
10	15	5	47	295	61	71
11	18	5.3	47	295	61	69
12	16	4.6	47	295	61	68
13	16.5	4.7	47	295	61	68
14	15.5	3.7	47	295	61	67
15	15.5	3.3	47	295	61	67

DEVELOPMENTS ON COLLECTED DATA

The special point of interest in the experiments is that the observations are on pre-made arrangements of fins. Thus the geometric parameters such as the flow area, the hydraulic diameter, fin thickness, fin width, length, and spacing are constant for each arrangement. The advantage of this lies in the reduction of the variables in the phenomenon. The control of the operating conditions is made by changing the flow rate of water. It is possible to add another means of control by varying the steam pressure and temperature. This however, does not have an interesting effect since the base temperature difference has a direct proportionality with the output of the system.

The basic factor throughout all the experiments is the Reynold's Number. It is possible to express the Reynolds Number for each fin arrangement by the following simple relationship:

$$Re = K\,(G/\mu) \qquad (68)$$

where K is a geometric constant for each fin arrangement,

K = hydraulic diameter/flow area

G = flow rate of water

μ = dynamic viscosity of water

During the experiment G was manually varied, and the dynamic viscosity is a function of the water temperatures.

Many of the variables in the operation of fin systems have individual and at times complicated relationships with the Rey-

nold's Number. First to be considered among these relationships is that of the heat transfer coefficient for convection, (h). A detailed analysis of this item is given in Appendix A. The relationship is a well known one and empirically established by experimental data.

The heat transfer coefficients for our experiments are calculated by eq.A7 of Appendix A, and plotted against the Reynold's Number in fig.10.

Under the conditions of our experimental procedures, the heat transfer coefficient (h) has a direct influence on the fin effectiveness (e_r). Since the conductivity, the thickness, and the width of the fin are constant for each arrangement, the fin effectiveness varies as the hyperbolic tangent of the square root of the heat transfer coefficient. After expressing the heat transfer coefficient as a function of the Reynold's Number it is possible to see how (e_r) varies with the Reynold's Number. (e_r) is plotted against Re in fig.11. We thus resolve that the fin effectiveness can be predicted (with reservations of how the Prandtl Number behaves) for a known value of the Reynold's Number and a known geometry of the system.

The next of our building stones on the basis of Reynold's Number is the fin factor (F). The fin factor was defined in our analysis by eq.67:

$$F = \frac{2 N h L}{G c_p} (b + w e_r)$$

We see that for a pre-made arrangement of fins N, L, b, and w are constant (but L, the fin length can be varied as one wishes). G, h, c_p, and e_r are the variables which are also interrelated to each other.

The fin factor is plotted against the Reynold's Number in fig.12. It is easily seen that the value of F for one arrange-

ment can be equated to that of another system without altering the conditions of operation simply by adjusting the lengths (L) of each system.

Finally it is necessary, to see the overall output capacities of each system, to plot the heat outputs against the Reynold's Number. The heat outputs are calculated by using the experimental data according to the equation:

$$Q = G\, c_p\, \Delta T \qquad (69)$$

where $\Delta T = T_{out} - T_{in}$ for water.

Fig.13 shows the curves of Q vs Re for each fin arrangement. The discussion of these results are presented in the following section.

DISCUSSION OF RESULTS

Two primary observations are made on the foregoing data:

1. The rate of heat transfer for a known arrangement of fins does not increase as rapidly for high Reynold's Numbers as it does for lower. For very high values of the Reynold's Number the heat transfer rate tends to approach a limiting value.
2. The increase in the number of fins has a considerable effect on the over-all heat transfer capacity of the arrangement for a certain range of the number of fins. Further increase of the number of fins result in diminishing gains.

Both of these observations suggest the existance of an optimum fin arrangement for a required capacity of heat transfer. We shall first consider the conditions, and the capacity ranges that a known fin arrangement offers.

A known fin arrangement offers a very wide range of heat transfer capacities. These capacities are obtained by various combinations of:

a. Properties of the fluids employed.
b. Mass rate of flow (G).
c. The base temperature difference.
d. The heat transfer coefficient.

If a fluid is assigned to the system, the variables are then reduced to the last three of the above four items. Although the fluid properties still have an effect (as functions of temperature) on the capacity of a system this is not a dominating effect.

If we observe fig.13 carefully we see that the capacity of the system increases as the number of fins increases from zero to four. There is less an increase as the number of fins become eight. And finally a small decrease in the heat output occurs as the number of fins increase from eight to sixteen.

This decrease is due mainly to the fact that as the number of fins is increased the hydraulic diameter of the system has a considerable drop in magnitute. This drop in turn reduces the Reynold's Number for a given rate of mass flow. Smaller Reynold's Number and smaller hydraulic diameter give smaller heat transfer coefficients.

Let us now consider in what ways a decrease in the heat transfer coefficient (h) affects the overall capacity of the system. A decrease in (h) in the first place causes the fin effectiveness (e_r) to rise to a more favourable value. But the contribution of the fin effectiveness to the capacity of the system off occurs bothe as a direct proportionality and also as a member of the fin factor (defined by eq.67.) This double effect has a very important behaviour.

For a given Reynold's Number the following statements are true as the number of fins increase:

1. The fin effectiveness decreases until a certain number (N) is reached. After this value of N the fin effectiveness follows an increasing curve.
2. The heat transfer coefficient (h) increases tending to reach a limiting valne.
3. The mass rate of flow increases.
4. As in the fin factor, the product of N, h, and e_r, divided by G, the flow rate, has an increasing behaviour.
5. The term exp(-F) follows a decreasing curve tending to reach a limit.
6. Exp(-F) multiplied by the fin effectiveness has a decreasing and an increasing behaviour, possessing apoint of minima.

7. This point of minima indicates that there is a point where the heat output of a system reaches a maximum.

These seven statements represent the summary of a very complex relationship which can be seen through the experimental data but the analytical explanation requires very long and tedious work. An analysis of this kind has been carried out in detail by Ayrami and Little in 1952. They have explained that the ratio (w/y_o) has an optimum value for a given (hy_o/k_f), where the heat capacity of the system reaches its maximum. Their results are plotted in fig.14, $(w/y_o)_{opt}$ vs (hy_o/k_f). The ratio (hy_o/k_f) is a special Nusselt Number picked for this analysis. It is seen that (w/y_o) is zero for the special Nu = 0.8, and for larger values of Nu the fins have an insulating effect.

With these considerations the seven statements given above can be put into a general form where all possibilities of the fin dimensions and the ir conditions of operation are explained. For this, let us enumerate the results we have reached so far:

From the analysis:

1. Under certain conditions of operation there is a set of dimensions which constitute the fin which requires the least amount of material.

From experimental data:

2. There is a set of operating conditions which give the maximum heat output from an arrangement of fins.

CONCLUSION

Whether coincidentally or with due care, the two statements at the end of preceeding section, under very strict coditions, can be combined into one saying:

A fin system can be found which with its dimensions and the operating conditions requires the least amount of material and also gives the maximum heat output. This is possible if the fin system obeys the basic requirements of both the economic and the capacity aspects at the same time. The process is one of trial and error, and frequently very complicated.

We had shown by eq.31 that the fin of least material had the characteristic:

$$m\,w = 1.4192$$

Substituting this value into eq.24 we get:

$$e_r = \frac{\tanh(1.4192)}{1.4192} = 0.627 \qquad (70)$$

With these two values the heat transfer capacity of a fin arrangement whose fins are of least material can be expressed by using eq.56, as:

$$H_F = 0.627\ Gc_p(\theta_o)_{in}\left[1 - \exp\left(-\frac{4.07NL}{G\,c_p}\sqrt{hk_f v_o}\right)\right] \qquad (71$$

We must notice here that the terms $Gc_p(\theta_o)_{in}$ constitute the quantity of heat necessary to raise the surrounding fluid

temperature to the base temperature of the fin. In other words it is the capacity of a fin system which is infinitely long. We shall designate this quantity by H_∞, or

$$H_\infty = G\, c_p\, (\theta_o)_{in} \tag{72}$$

Eq. 71 then reduces to

$$H_F = 0.627\, H_\infty \left[1 - \exp\left(-4.03NL \frac{(\theta_o)_{in}}{H_\infty}\sqrt{hk_f y_o}\right)\right] \tag{73}$$

or a more convenient form of it can be expressed as:

$$H_F = 0.627\, H_\infty \left[1 - \exp\left(-4.03NL \frac{(\theta_o)_{in}\, k_f}{H_\infty}\sqrt{hy_o/k_f}\right)\right] \tag{73a}$$

where the last term under the radical we notice as the special Nusselt Number of Avrami and Little.

When the number of fins is small and the fin spacing is comparatively large we must make use of the effective width (W) for more precise calculations. Substitution of the conditions for the fin of least material into eq.60 gives:

$$W = b + 0.89\sqrt{k_f y_o/h} \tag{74}$$

We use this special case of the effective width in eq.66, which takes into account the heat transfer through the fin spacing. Eq.66 then becomes

$$Q = 0.627\, H_\infty \left[1 - \exp\left(-\frac{2.84NL}{H_\infty}\,(\theta_o)_{in}\,(b\sqrt{hk_f v_o} + 0.89\, k_f v_o)\right)\right] \qquad (75)$$

which is qdite a complicated expression for a process of trial and error. For practical purposes, a simpler procedure would be the use of eq.73a to approach the optimum conditions and then to make the final calculation of the exact capacity by eq.75.

Observing equations 73 and 73a, we see that although providing for the fin system economy and capacity it does not give a comfortable view of the combined phenomenon. It is necessary to prepare the conditions so that they satisfy two different requirements at the same time. To show that this is possible I find it necessary to demonstrate a hypothethical example:

Let us consider a system of fins, made of material whose conductivity is 223 B/hr.ft.F. The dimensions of each fin are as follows:

$$y_o = 0.0012 \text{ ft}$$

$$w = 0.073 \text{ ft}$$

In order to have the fin of least material this system needs to operate with a coefficient of heat transfer as

$$h = (1.4192)^2\, k_f\, y_o / w^2 = 109 \text{ B/hr.ft}^2\text{F}$$

Now to satisfy the conditions of Avrami and Little we have for the special Nusselt Number:

$$h\, y_o / k_f = \frac{109 \times 0.0012}{223} = 0.00059$$

Reading (w/y_o) in fig.14, we find that in order to have the maximum output from our system we must have:

$$w/y_o = 61$$

If we calculate the actual aspect ratio for our system we get

$$w/y_o = \frac{0.073}{0.00012} = 61$$

Thus the requirements for both the fin of least material and maximum capacity are satisfied. Our system is then the most economic and is working under conditions to give the maximum output.

To summerize the conclusion the following statements will be adequate:

I. For minimum material and maximum output a fin system should fulfil the necessary and sufficient conditions given below:

a) $$\sqrt{\frac{h}{k_f y_o}} = 1.4192 \qquad (31)$$

b) The intersaction of w/y_o and hy_o/k_f must lie on the curve of fig.14.

c) When conditions a and b are satisfied, the heat output of the system can be varied by the base temperature difference and the length of the same system according to the relation

$$H_f = 0.627\, H_\infty \left[1 - \exp\left(-4.03 NL \frac{(\theta_o)_{in} k_f}{H_\infty} \sqrt{\frac{hy_o}{k_f}}\right)\right] \qquad (73a)$$

where $H_\infty = G\, c_p\, (\theta_o)_{in}$

II. In the case when eq.73a becomes approximate due to the shape of the fin system (a condition where the fin spacing is large and the heat loss frm the base cannot be neglected) the results obtained by the above statement need to be checked and corrected by either or both of the following relationships:

a) $$Q = G\, c_p\, (\theta_o)_{in}\, e_r \left[1 - \exp\left(-\frac{2NhWL}{Gc_p}\right)\right] \qquad (66)$$

b)

$$Q = 0.627 H_\infty \left[1 - \exp\left(-2.84 \frac{NL(\theta_o)_{in}}{H_\infty} \left(b\sqrt{hk_r v_o} + 0.89 k_r v_o\right)\right)\right] \qquad (75)$$

PART VI: EXAMPLES OF FIN DESIGN

'For the perfect understanding of a question we must abstract it from all that is superfluous, rendering it as simple as possible, and resorting to enumeration, divide it into its minimal parts'

Rule XIII; "Rules for the Guidance of Our Native Powers", Descartes, 1628.

SOME EXAMPLES OF FIN DESIGN

The method employed in the design of fin systems depends to a large extent on the nature of the problem. Usually the best method is an initial assumption of a fin arrangement, and then the corrections and alterations necessary to fulfil the requirements can be made. This method enables the designer to possess a number of parameters on which he can base his calculations. In many cases, however, the final design turns out to be quite different from the initial assumption. Thus it is worthwhile to donate some effort to the selection of the initial arrangement, because a good selection will reduce the later work by a considerable amount.

Sometimes the design problem appears in a different manner. A good example is an already existing arrangement. The problem here could be the determination of the conditions under which the existing arrangement would be economical and give the maximum output.

Let us, as our first example consider the tube with eight fins which was used in the experimentation. By this selection we have a set of dimensions on which our calculations can depend. These dimensions are:

Flow area (A) = 0.0383 ft^2
Hydraulic diameter = 0.0662 ft
Length of fin-tube (L) = 2.834 ft
Fin width (w) = 0.073 ft
Fin thickness ($2y_o$) = 0.0026 ft y_o = 0.0013 ft
Fin spacing (2b) = 0.040 ft b = 0.020 ft

For this arrangement we shall try to find the conditions operation under which the fins would be of the least material and give the maximum output. W e know that the fins are made of copper which has a conductivity of

$$k_f = 222 \text{ B/hr.ft.F}$$

By equation 31 we can calculate the heat transfer coefficient which would make our fins of the least material.

$$h = (1.4192)^2 \frac{k_f y_o}{w^2}$$

$$= 2.015 \frac{222 \times 0.0013}{0.0053} = 110 \text{ B/hr.ft}^2\text{F}$$

We have as the aspect ratio of the fins

$$\frac{w}{y_o} = \frac{0.073}{0.0013} = 56$$

and the special nusselt number is:

$$\frac{hy_o}{k_f} = \frac{110 \times 0.0013}{222} = 0.00065$$

Checking by fig.14 we see that for this value of the Nu we must have the aspect ratio of 60. Al though the figures are close enough for practical purposes, let us see how we can have a closer agreement.

Let us for a second try increase the fin width to

$$w = 0.076 \text{ ft}$$

By eq.31 we must have the heat transfer coefficient

$$h = 2.015 \frac{222 \times 0.0013}{0.0058} = 101 \text{ B/hr ft}^2\text{F}$$

We now have as the aspect ratio

$$\frac{w}{y_o} = \frac{0.076}{0.0013} = 58.5$$

And the special Nusselt Number:

$$\frac{h\, y_o}{k_f} = \frac{101 \times 0.0013}{222} = 59.5 \times 10^{-5}$$

For this value of Nu, fig.14 gives that the aspect ratio should be 61. The difference has now dropped from 4 to 2.5, and this is an agreement close enough for our purposes.

Let us now suppose that our system has to heat water flowing at a rate of 2500 lb_m/hr, and the inlet base temperature difference of 250 F. We shall find the capacity of the system which we know to be maximum, and then see how we can double this capacity without disturbing the conditions of economy and maxima.

From eq.72 we have:

$$H_\infty = G\, c_p\, (\theta_o)_{in}$$

$$= 2500 \times 1 \times 250$$

$$= 625{,}000 \text{ B/hr}$$

as the capacity of the infinite system. We will use eq.73a to find the actual capacity.

$$H_F = 0.627\, H_\infty \left[1 - \exp\left(-4.03 NL \frac{(\theta_o)_{in} k_f}{H_\infty} \sqrt{h y_o / k_f}\right)\right]$$

Substituting we have

$$H_F = (.627)(625000)\left[1 - \exp\left(-\frac{4.03x8x2.834x250x222}{625000}\sqrt{.00060}\right)\right]$$

which gives:

$$H_F = 73,300 \text{ B/hr}$$

In order to double this value of heat capacity, let us find what length of the system is necessary. Since every term except (L) remains the same, we have:

$$H_F = 146,600 = (.627)(625000)\left[1 - \exp(-0.070\,L)\right]$$

We solve for (L), and find that

$$0.070\,L = 0.466$$

$$L = 6.67 \text{ ft}$$

The water flow rate of 2500 lb_m/hr gives us that the system operates at

$$Re = 1600$$

For any other values of Re it is necessary to repeat these calculations and see whether the system can be operating with the same optimum conditions. If not, the system is said to be inefficient.

Let us now try to find some other alteration which will increase the capacity of the system we have been considering without changing its length. The principles that the fins must be of least material, and the capacity must be maximum for the system, still hold.

We consider that the flow rate of water can be adjusted to give

$$h = 350 \text{ B/hr ft}^2\text{F}$$

and this time vary the fin width (w). From eq. 31 we have

$$w = 1.4192 \sqrt{\frac{k_f y_o}{h}}$$

$$= 1.4192 \sqrt{\frac{222 \times 0.0013}{350}}$$

$$= 0.0407 \text{ ft.}$$

We see that, as the heat transfer coefficient increases, the fin width should decrease to keep the effectiveness constant.

The aspect ratio of our new system is

$$\frac{w}{y_o} = \frac{0.0407}{0.0013} = 31.4$$

and the special Nusselt Number is:

$$\frac{h y_o}{k_f} = \frac{350 \times 0.0013}{222} = 0.00205$$

Fig.14 shows us that for Nu = 0.00205 the optimum aspect ratio is 33. That we actually have 31.4 can be considered acceptable for practical purposes.

We see from our experimental data that the heat transfer

coefficient h = 350 is obtained when the flow rate of water is about 12,300 lb_m/hr. Keeping our inlet base temperature difference unchanged, we have

$$H_\infty = 12{,}300 \times 1 \times 250 = 3{,}080{,}000 \text{ B/hr}$$

and applying eq.73a

$$H_F = 0.627\ (3{,}080{,}000)\left[1 - \exp\left(-\frac{4.03 \times 8 \times 2.834 \times 250 \times 222\sqrt{.00205}}{3{,}080{,}000}\right)\right]$$

we get:

$$H_F = 139{,}000 \text{ B/hr}$$

This capacity is a little lower than the capacity of the original system when its length was increased close to 2.5 times.

If, while increasing the flow rate, we had kept the width of the fin same as it was (this is the way the experiment was carried out) the heat output of the system would have been lower in spite of the fact that the fin surface was close to twice the value we have calculated above.

We shall see this difference now when we calculate the heat output of the original dimensions. We have

$$H_F = G c_p (e_o)_{in} e_r \left[1 - \exp\left(-\frac{2NLh_w}{Gc_p}\right)\right]$$

where the effectiveness now is

$$e_r = \frac{\tanh m_w}{m_w} = 0.378$$

Substituting this value to the above equation, we get:

$$H_F = 12300 \times 1 \times 250 \times .378 \left[1 - \exp\left(- \frac{2 \times 8 \times 2.834 \times 350 \times 0.073}{12300 \times 1}\right)\right]$$

$$H_F = 104{,}000 \text{ B/hr}$$

This is a good example which shows the meaning of output maximization. It shows that the capacity of the system is more dependent on the combination of all the conditions of operation and the dimensions of the arrangement than the absolute quantity of the heat transfer surface. The summary of the above example is that we have increased the capacity of a system by 35,000 B/hr (by about 30 %) simply cutting the fin surface in half, not considering the material that we have saved.

PART VII: APPENDICES

APPENDIX A: Coefficients of Heat transfer

The application of the finned heat transfer surfaces is usually with forced convection. In this section we shall consider the coefficient of heat transfer for forced convection.

The coefficient for heat heat transfer in forced convection is assumed to be in the form

$$h = K v^a D_h^b \mu^f k^i d^m c_p^n \qquad (A\text{-}1)$$

where K = constant of proportionality
v = mean velocity of the fluid
D_h = hydraulic diameter of the passage
μ = dynamic viscosity of the fluid
k = conductivity of the fin
d = density of the fluid
c_p = specific heat of the fluid at constant pressure

Applying dimensional analysis to eq.A-1 we have

$$(Bt^{-1}L^{-2}T^{-1}) = (Lt^{-1})^a (L)^b (Mt^{-1}L^{-1})^f (Bt^{-1}L^{-1}T^{-1})^i (ML^{-3})^m (Bt^{-1}M^{-1}T^{-1})^n \qquad (A\text{-}2)$$

Expanding and equating the exponents:

For B: $1 = i + n$
For t: $-1 = -a - f - i$
For L: $-2 = a + b - f - i - 3m$

For M: $0 = f + m - n$

For T: $-1 = -i - n$

The first and the last of these equations being identical we have six unknowns and four equations. We shall solve these equations in terms of the two unknowns m and n. We have:

$$a = m$$
$$b = m - 1$$
$$f = n - m$$
$$i = 1 - n$$

With these exponents eq.A-1 is put into the form

$$\frac{h D_h}{k} = C \left(\frac{v D_h d}{\mu} \right)^n \left(\frac{\mu c_p}{k} \right)^m \qquad (A-3)$$

where we recognize the dimensionless parameters:

$$Nu = \text{Nusselt Number} = \frac{h D_h}{k} \qquad (A-4)$$

$$Re = \text{Reynold's Number} = \frac{v D_h d}{\mu} = \frac{v D_h}{\nu} \qquad (A-5)$$

$$Pr = \text{Prandtl Number} = \frac{\mu c_p}{k} \qquad (A-6)$$

The constant of proportionality C, and the exponents are evaluated by experimental means. They are given as:

$$C = 0.023$$
$$n = 0.80$$
$$m = 0.4 \quad \text{for heating}$$
$$= 0.3 \quad \text{for cooling}$$

for turbulent flow. Eq.A-3, then, can be expressed in the following convenient form:

$$Nu = 0.023\ Re^{0.8}\ Pr^{m} \qquad (A-7)$$

where m depends on the type of application.

In the case of free convection, a relationship very similar to eq.A-7 is used.

$$Nu = C\ Gr^{n}\ Pr^{m} \qquad (A-8)$$

where Gr is the Grashof nuber, defined as:

$$Gr = \frac{g\,B\,D_h^3\,\theta}{\nu^2} \qquad (A-9)$$

For further information on eq.A-8 and A-9 the reader is referred to Chapter 23 of "Heat Transfer" by Max Jacob.

Eq.A-7 is useful for most practical purposes, but the fact that it assumes that the heat transfer coefficient is constant along the fin surface is not precisely true. The heat transfer coefficient actually varies throughout the fin surface. A short analysis of this variation is presented below.

Considering the local Nusselt number as

$$Nu_{x,z} = \frac{h_{x,z}\,Z}{k} \qquad (A-10)$$

and the Reynold's Number in a similar way

$$Re_{x,z} = \frac{v_{x,z}\,Z}{\nu_{x,z}} \qquad (A-11)$$

the variation of the heat transfer coefficient along the fin surface has been expressed by Jacob and Ghai (1950), as

$$Nu_{x,z} = C_o \, Re^n \left[1 - C_1 \left(\frac{w}{s}\right)^m \left(\frac{w-x}{w}\right)^p\right] \qquad (A\text{-}12)$$

where C_o and C_1 are constants, and w and s are the fin width and the fin spacing, respectively. The constants and the exponents m and p are experimentally determined.

If we define the mean coefficient of heat transfer as:

$$h_{x,m} = \frac{1}{Z} \int_{z_{oo}}^{Z} h_{x,z} \, dz \qquad (A\text{-}13)$$

then substituting $h_{x,z}$ from eq.A-12, and performing the integration, we get:

$$Nu_{x,m} = C_o \, n \, Re^n \left[1 - C_1 \left(\frac{w}{s}\right)^m \left(\frac{w-x}{w}\right)^p\right] \qquad (A\text{-}14)$$

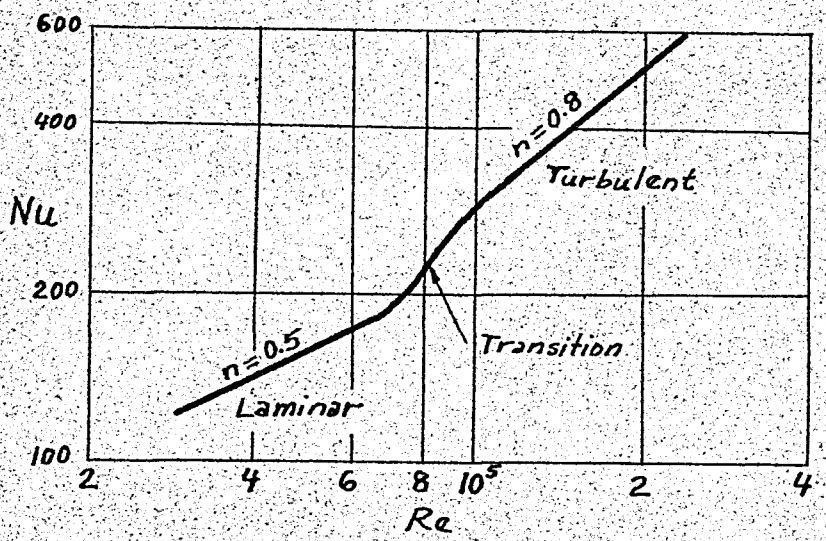

Fig. A-1

By using experimental data, eq.A-14 can be put into applicable forms. For the laminar region

$$Nu_{x,m} = 0.7\ Re^{0.5} \left[1 - 0.35 \left(\frac{w}{s}\right)^{1/3} \left(\frac{w-x}{w}\right)^{2}\right] \qquad (A-15)$$

and for the turbulent region

$$Nu_{x,m} = 0.03\ Re^{0.8} \left[1 - 0.23 \left(\frac{w}{s}\right)^{0.5} \left(\frac{w-x}{w}\right)^{3/2}\right] \qquad (A-16)$$

Fig.A-1 shows the behaviour of equations A-15 and A-16, as well as the conditions at the transition range.

As can be noticed eq.A-14 is variable in the x-direction. If we define

$$h_{m,m} = \frac{1}{w} \int_{0}^{w} h_{x,m}\, dx \qquad (A-17)$$

and substitute $h_{x,m}$ from equations A-15 and A-16 we obtain for the laminar region:

$$h_{m,m} = h_{w,m} \left[1 - 0.12 \left(\frac{w}{s}\right)^{1/3}\right] \qquad (A-18)$$

and for the turbulent region:

$$h_{m,m} = h_{w,m} \left[1 - 0.09 \left(\frac{w}{s}\right)^{1/2}\right] \qquad (A-19)$$

where $h_{w,m}$ is considered to be the heat transfer coefficient at the tips, and we should note that it can readily be calculated by equations A-3 and A-8.

Some numerical results of the Jacob and Ghani analysis are given in the following table:

w/s	Laminar region $100\ h_{m,m}/h_{w,m}$	Turbulent region $100\ h_{m,m}/h_{w,m}$
2	84.9	87.3
4	81.0	82.0
8	76.0	77.1
16	69.8	64.0

For errors due to over simplifications of this analysis, an empirical correction is given in the form:

$$(h_{m,m})_{corr.} = h_{m,m}\left[1 - 0.02\ w/s\right] \qquad (A-20)$$

APPENDIX B: Non-Continuous Longitudinal Fins

With experiments and development of fins it has been shown that the use of non-continuous fins over continuous fins for liquids give better results and advantages. Various studies of non-continuous fins were made by Gunter and Shaw (1940), and the following conclusions were reached:

1. The use of non-continuous fins instead of long continuous fins increases the heat transfer coefficient, h, in the ratio 2 to 1, where the length of the system is from 1.2 to 10 ft.

2. Correlation of the test data where L/D_h is the controlling factor, indicates that the curves given by Norris and Streid's[1] Fig.4 are conservative above Re = 200, because the results show the transition range beginning at this point.

3. The dimensionless function $(\mu/\mu_w)^{0.14}$ should be included in the J_A value[2] when correlating viscous fluids.

4. The pressure drop efficiency factor for non-continuous fins is comparable to that on long continuous fins where L is from 1 to 10 ft.

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(1) "Laminar Flow Heat Transfer Coefficients for Ducts" R.H.Norris and D.D.Streid Trans. A.S.M.E. Vol.62, 1940 pp.525-533

(2) $$J_A = \frac{h_f}{G\,c_p}\left(\frac{c_p\mu}{k}\right)^{2/3}\left(\frac{\mu}{\mu_w}\right)^{-0.14}$$

5. Correlation of test data on pressure drop gives fair agreement with Norris and Spofford[1] in the range of Re from 700 to 2000.

6. From a theoretical point of view, fouling has less effect on fin tubes than on bare tubes, for as fouling increases, the fin effectiveness increases, tending to counter balance the fouling effect.

7. A fouling factor of 200 for longitudinal fintubes in heavy fuel oil service, with steam temperatures not above 350 F, will normally assure at least one year of service without cleaning. Variations of operating conditions or intermittent operation may shorten this period.

In laminar flow where the Reynold's Number attains low values, the heat transfer rates are usually expected to be low in the case of liquids. In order to increase these heat transfer rates and have compact heat exchangers the alternate of employing noncontinuous fins is very useful. The individual lengths of these non-continuous fins rage from 2.45 ft to 1.2 ft. They are separated by very short (about 3/4") turbulence promoters. As the (L/D_h) ratio decreases it is observed that the dimensionless parameter J_A increases by 100 per cent. Test results also showed that the transition range for laminar and turbulent flow begins at Re = 100 to 200, and for laminar flow Re = 30 was taken as the reference point.

The following ranges of variables were covered in the tests of Gunter And Shaw:

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(1) "High Performence Fins for Heat Transfer"
R.H.Norris and W.A.Spofford
Presented at A.S.M.E. annual meeting, New York,
Dec.2, 1941

Fluids:	Gasolene, Kerosene	$Re > 2300$
	Light burning oils	$200 < Re < 8000$
	Medium fuel oils	$80 < Re < 300$
	Heavy fuel oils and asphalts	$Re < 100$

Steam temperatures	250 to 325 F
Average oil temperatures	115 to 235 F
Linear velocity	0.3 to 10 fps
$(\mu/\mu_w)^{0.14}$	0.85 to 1.9
$c_p \mu/k$	5 to 5000
L/D_h	37.6 to 313

APPENDIX C: Comparison of Fin Metals -

Metal	k B/hr ft F	density lb/cu.ft	V/V_{cu}	W/W_{cu}
Copper	222	556	1	1
Aluminum	121	169	1.83	0.556
Iron	30	487	7.33	6.43

V/V_{cu} is the volume of metal compared to the volume of copper for the fin of least material.

W/W_{cu} is the weight of metal compared to the weight of copper for the fin of least material.

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