

OPTIMAL COMMODITY PROCUREMENT WITH IMPERFECT DELIVERY

by

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ABSTRACT

OPTIMAL COMMODITY PROCUREMENT WITH IMPERFECT DELIVERY

For a number of commodity markets, including many agricultural and energy products, a typical problem faced by decision makers is to determine the optimal spot and forward contracting quantities when both the product demand and the spot price are random. While satisfying uncertain demand, the lack of storability possibilities for certain commodities –electricity for example– adds another layer of complexity to the procurement policy optimization as the non-storable feature is ruling out the classical buy-and-hold argument for them [1]. Under the circumstances, firms increasingly engage in forward contracts for future deliveries, as well as, to avoid price fluctuations and benefit from lower transaction costs. However, along with the increasingly volatile business environments, there is the risk of suppliers not delivering the agreed quantities. Delivery risks particularly hold a great importance in the profitability of firms as they have no perfect guarantee that the quality or the quantity of the delivered goods absolutely meets their expectations. Still, ‘uncertain deliveries’ is not discussed much in the literature and most of the time, other drivers mentioned above is held individually rather than a multivariate setting. Recognizing the imperfect delivery aspect of forward procurement, integrated with stochastic price and demand environment, this thesis presents a new trivariate approach for examining policy-optimization models regarding non-storable commodities. For each of the variables –price, demand and delivery of supplies– evolving in a geometric Brownian motion, an optimal forward contracting policy is proposed and algorithmically characterized by means of stochastic optimization. The most significant result is that profit improvements can be achieved by introducing uncertain delivery notion into procurement decision making. We also demonstrate that it can be valuable to model delivery interactions with price and demand into the procurement models, and we present several possible ways to model these interactions. We present structure, algorithms, and heuristics for our models.

ÖZET

HATALI TESLİMAT ALTINDA OPTİMAL TEDARİK POLİTİKALARI

Pek çok tarım ve enerji ürünü de dahil olmak üzere piyasaların birçoğunda, raslantısal ürün talebi ve spot fiyatlar altında, en uygun spot ve vadeli tedarik miktarlarını belirlemek karar vericilerin karşılaştığı tipik bir problemdir. Değişken taleplerin karşılanmasında, elektrik gibi birtakım ürünler için depolanabilirlik olanaklarının sınırlı oluşu bu tip ürünler için geleneksel ‘al ve tut’ argümanını devre dışı bırakırken [1], depolanamama özelliği tedarik politikası optimizasyonuna karmaşıklığın bir başka katmanını ekler. Dolayısıyla, vadeli teslim kaydıyla alım yapabilmenin yanı sıra, fiyat dalgalanmalarından kaçınmak ve düşük işlem maliyetlerinden faydalanmak için firmaların forward sözleşmelere yönelimi giderek artmaktadır. Ancak hızla değişebilen iş ortamları ile beraber, tedarikçilerin anlaşılan ürün miktarını teslim etmeme riski vardır. Ulaştırılan ürünlerin miktar ve kaliteleri ile firma beklentilerinin birebir karşılanması garantisi edilemediğinden, teslimat riskleri firmaların karlılığında özellikle önemlidirler. Bununla birlikte, literatürde, kusurlu teslimat konusu şimdiye dek çok tartışılmamış, söz edilen diğer değişkenler ise birlikte kapsanmaktan ziyade, ağırlıklı olarak ayrı ayrı ele alınmışlardır. Bu çalışmada, vadeli satın alma kontratlarının hatalı teslim boyutu tanınmış ve raslantısal fiyat ve talep çevresine dahil edilerek, depolanamayan ürünlere ilişkin tedarik politikalarının optimizasyonu için üç değişkenli yeni bir yaklaşım sunulmuştur. Her bir değişken —talep, fiyat ve teslimat boyutu— geometrik Brownian hareket süreci ile modellenmiş ve önerilen optimal forward kontrat politikaları stokastik optimizasyon yöntemleri ile saptanmıştır. Sonuçta, tedarik miktarlarının belirlenmesi sürecine teslimatlardaki belirsizlik kavramının dahil edilmesiyle firmaların karlarında iyileşmeler sağlanabileceği gösterilmiştir. Ayrıca, teslimat boyutu ile fiyat ve talep arasındaki ilişkilerin de modellenerek tedarik miktarlarının belirlenmesine dahil edilmesinin faydalı olabileceği gösterilmiş ve bu korelasyonların çeşitli olası modelleri önerilmiştir. Model yapıları ile birlikte algoritma ve sezgisel yöntemler sunulmuştur.

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LIST OF SYMBOLS

A	Spot transaction cost
B	Forward transaction cost
c	Correlation coefficient between log spot demand and nominal price
$D(t)$	Demand forecast process
dt	Infinitesimal time increment
dW_D	Increments to underlying Brownian Motion of demand forecast process
dW_L	Increments to underlying Brownian Motion of availability index process
dW_S	Increments to underlying Brownian Motion of price process
$\mathbb{E} [\]$	Expectation operator
$F(t)$	Forward nominal price process
$L(t)$	Availability index process
q	Quantity to procure forward at time 0
t	Time index
T	Specified period of time (maturity)
V	Value of optimally procuring in spot and forward markets
θ	Correlation coefficient between log spot demand and availability index
μ_D	Log spot demand mean
μ_F	Log spot nominal price mean
μ_L	Log availability index mean
ρ	Correlation coefficient between log spot nominal price and availability index
σ_D	Log spot demand standard deviation
σ_L	Log availability index standard deviation
σ_S	Log spot nominal price standard deviation
$\Phi()$	Standard normal function
$\Phi^{-1}()$	Inverse standard normal function

$\mathbb{1}_{\{\cdot\}}$ Indicator function of an event $\{\cdot\}$

LIST OF ACRONYMS/ABBREVIATIONS

GBM	Geometric Brownian Motion
MLN	Martingale Lognormal

1. INTRODUCTION

As the global markets of today offer more purchasing and selling opportunities and pose new challenges in supply chain operations to firms, procurement decisions grow into one of the most crucial problems faced by managers. To meet an uncertain demand with fluctuating prices, most of firms engage in commodity-trading activities over forward markets for future deliveries as well as spot markets with deliveries immediately after ordering. However, stochastic demand and prices, which are common occurrences in a wide variety of industries today, can be accompanied by uncertainties in the delivery of commodities, such as arrival of fewer products than contracted amounts. Recognizing this uncertainty, in this study, we examine policy-optimization models for a firm operating in spot and forward markets and faced by variable price and demand in addition to variable supplies. We refer to this problem as the “optimal commodity procurement with imperfect delivery” problem.

Profitability of a firm highly depends on controlling costs in managing the supply chain, and from the standpoint of procurement management, the use of forward market operations to manage procurement in advance of demand realization can serve to improve the firm’s profitability. Firms have increasing interest in forward procurement since the transaction costs are typically lower in forward markets than those in spot and forward markets are less risky as they reduce the quantities traded at the more volatile spot price [2]. Also, forward procurement results in lower logistics costs than spot procurement since they can be planned and arranged in advance [3]. However, demand is highly unpredictable in most cases and it is hard to procure entire demand from forward market. Then, the spot market comes into play. As delivery and payment take place immediately, one can buy commodities from spot in order to satisfy the missing part of demand on the date of usage. Since decision makers to satisfy uncertain demand apply spot markets, many supply chain researchers have focused on spot markets also, in order to understand how these markets can be used more effectively for different purposes including: (i) the procurement of products (raw materials, components and commodities), (ii) trading in the market under different circumstances

(such as low/high demand realizations, favourable price differentials, etc.) and (iii) design of supply chain contracts in which both buyers and sellers can conduct operations in the spot market [4]. That is to say, dealing with trading strategies both in spot and forward markets has become the issue of importance regarding operations management field.

The prevailing part of trading problems both in spot and forward markets is constituted by non-storable commodities, such as electricity, natural gas or commodities subject to perishability. Because non-storable commodities cannot be carried over to the current period from previous period, they challenge the concept of inventory. This may bring an increase in the variability while satisfying an uncertain demand. Eggs, for instance, are considered non-storable because they spoil quickly; a fresh egg is quite different from a month-old egg, and quantity or quality characteristics change rapidly for non-storables [5]. As most of energy sources have lack of storage or limited storage accessibility, non-storability is quite peculiar behaviour of energy commodities. For instance, it is difficult to move electricity forward in time. However, prices show significant changes through time. So, specifically focusing on energy markets in this thesis, we are interested in the optimization of forward procurement quantities in the case of no inventory.

While the non-storability feature of the underlying commodities is ruling out the classical buy-and-hold argument [1], most firms try to find an optimal balance of forward contracts and spot sourcing. Concerning that price and demand processes could be highly volatile in today's markets, firms buy for future demand in order to avoid commodity price fluctuations and generally, they make procurement decisions regarding certain deliveries at agreed times. However, this may lead to the fallacy of the optimal order quantity concept. We do not live in an ideal world and things can go wrong, forward contracted amounts may not be fully delivered at the delivery dates. Then, is it still optimal to order "that much" from forward markets? Consider for example, Ericsson. In March of 2000, the semiconductor plant of Philip's Electronics in New Mexico –as the key supplier of Ericsson– caught on fire. Since Ericsson failed to recover the shortage quickly, the company lost 400 million Euro by the year-end [6].

Apple sets another example upon losing many customer orders during a supply shortage of DRAM chips after an earthquake hit Taiwan in 1999 [6]. In the event of BP Oil Spill in 2010, apart from 184 million gallons (almost 4.4 million barrels) of oil spilled and ongoing harm on seafood supply and marine ecosystems, BP estimates it will deliver 75,000 barrels per day (b/d) less output until 2015 while the International Energy Agency (IEA) has said that 300,000–800,000 b/d could be lost industry-wide by mid-decade [7,8]. As the latest example, Gazprom, the Russian state-controlled gas company, threatened disruption of gas supplies to Ukraine on June 2014. It may be due to political tension between Russian and Ukrainian governments but as a supplier, Gazprom may reduce supplies to Ukraine –a key transit country for 15 % of Europe’s gas supplies [10]. Besides, it is reported that companies suffering from supply chain disruptions experienced 33-40% lower stock returns relative to their industry benchmarks [11]. For such detrimental cases but more familiar to our daily lives, electricity supply interruptions stand out. While electricity is typically non-storable, supply can be interrupted for a number of reasons, planned (i.e. maintenances) or unplanned (i.e. storm damages). For example, between 2003 and 2012, roughly 679 power outages – each affecting at least 50,000 customers– occurred in the United States due to weather events and costs the economy billions of dollars a year in lost output and wages, spoiled inventory, delayed production, inconvenience and damage to grid infrastructure. Over this period, weather-related outages are estimated to have cost the U.S. economy an inflation-adjusted annual average of 18 to 33 billion dollars [9]. Obviously, it is not necessarily optimal to trade under the assumption of exact and perfect deliveries.

Being exposed to various types of risks (natural or man-made, i.e. weather conditions, damages to commodities in transit, inaccurate shipment or poor quality of traded commodities), firms discuss supply contracts which may serve as important tools for reducing risk exposure in volatile commodity markets. Particularly, due to the complex patterns of consumption and the limited or no storability of energy products, some contracts in the energy markets are structured to incorporate the flexibility of delivery (with respect to both the timing and the amount of energy used) which are known as swing or take-or-pay options [12]. However, this may not be possible in the current setups of many supply chains. In another respect, this type of contracts are

devised and considered for pricing and hedging purposes in finance literature rather than procurement operations in supply chain management.

As discussed, the optimality of procurement policies governed by just stochastic demand and price processes is questionable without ensuring perfect deliveries of ordered amounts. So today, managers and decision makers are also interested in delivery uncertainties. (Hewlett-Packard, for example, introduced a new procurement risk management strategy that generated over 425 million \$ in cumulative cost savings between 2001 and 2007 [13]). Such uncertainties occur when there is a mismatch in the quality, consistency or the amount of the product delivered upon stated in the forward contract. To deal with this issue, incorporating ‘shortfall or oversupply risk’ into the procurement planning decisions will be meaningful and help decision makers in deciding how much to order from forward markets while considering delivery ‘surprises’.

For all of its cruciality for the firms faced with volatile prices and demand, delivery uncertainty in commodity markets, still, has not discussed much in the literature in such a setting. The novelty of this thesis is the extension of the optimal procurement decision problem in spot and forward markets into a modeling that explicitly considers the effect of delivery uncertainty. This extension enables us to obtain, with zero-inventory holding, the optimal forward contract quantities in the context of delivery risk, and to figure out the impact of ignoring delivery risk on the optimal profit. That is to say, to the best of our knowledge, this thesis considers random price, demand and deliveries simultaneously in procurement decision making over both spot and forward markets for the first time.

To do so, we consider the delivery percentage as a stochastic process from now on and apply stochastic models, particularly stochastic framework of geometric Brownian motion (GBM), to formulate stochastic optimization problems that capture the random fluctuations of price, demand and delivery amounts. In practice, it is usually found that level of supplies does not exceed the promised amount but falls short, so we begin with imposing a boundary upon the familiar stochastic framework of GBM for the delivery process. Solving the formulated problems by adapting well-known formulas

and an approximation approach, we end up with optimal and near-optimal contract quantities. Then, we relax the boundary constraint on the delivery percentage, so as to obtain closed-form expression for the optimal order quantities associated with all kinds of –more or less– deliveries. The models under approximation and non-constrained delivery process serve for functional benchmarking as well and help us to appreciate the exact consideration of imperfect deliveries in stochastic procurement optimizations.

Justifying the delivery uncertainty in an optimal procurement policy, we comment on the applications of our framework in turn. Energy products have always been commodities with volatile demand and prices. Lossy deliveries are also typical of energy sources. Variation of the delivered amounts of commodities may occur due to leakages or evaporation of gas and liquids, or theft or consumption during the transportation of commodities. Hence, we particularly consider energy markets in the thesis. On the other hand, the proposed framework in this thesis may arise in a number of application areas, e.g procurement of commodities ranging from perishable goods to livestock, risk management in supply chains, evaluation of supply contracts and supplier capabilities, and thereafter order allocation among suppliers. To illustrate, consider cattle procurement as a typical example of livestock market. Cattle production fluctuates unpredictably based on weather conditions (i.e. long-term droughts) and economic environment. Then, the fluctuation of cattle production leads to price fluctuations in addition to the seasonality effects on the availability of cattle [14]. Regarding the non-storable nature of livestock commodities, the suggested model in this thesis works very well for cattle procurement. Moreover, as the zero-inventory and just-in-time movement of goods become dominant, sensitivity increases for the supply chains which rely on timely delivery of products and services to meet customer demand [15]. Variability is detrimental to the performance of such storable-asset-supply chains since it may cause costs in the form of stock outs, poor capacity utilisation, and costly buffers [16]. The storability problem also arises in the travel and leisure industry. So, our model is eligible for such supply chains, aside from non-storable commodities.

To enhance the applicability of our framework, it is useful to distinguish between the time dimension as medium term, as well as, short and long terms. Regarding price

elasticity of demand on a time scale, we consider different correlation structures and develop our model variations. By doing so, we use a price dependent demand model, at first, with an independently stochastic delivery process for the medium term. That is because; demand side cannot tolerate price changes and is relatively price sensitive in the medium term, as adequate resources are available around for consumers to switch their preferences. This typical case arises in a variety of industries. As governments use taxes to reduce consumption of certain commodities, Espey [17] shows that 10% hike in the price of gasoline lowers quantity demanded by 2.6%, for example. Food prices are subject of great concern also, as rising commodity prices hit the poor very hard. The price elasticity of demand for food is very high in developing countries and a change in price leads to a relatively higher decrease in demand [18].

In contrast to medium term, consumer demand response is limited in most markets for the short term. This arises from the fact that consumers do not have the ability to change their supply sources or consumption levels in the short run, since there exist inadequate alternatives and time for switching. To illustrate, at residential level, the short-run demand for energy for heating is found to be price-inelastic in Spain for the period 2006-2008 [19]. However, supply levels may influence prices. In the event of California electricity crisis of 2000-2001, during the prolonged period of tight supply, demanders were exposed to sustained high prices [2].

When it is of interest to investigate the long run, it can be expected that fluctuations in demand are associated with price and availability as well. The long term market assures that adequate resources exist and establishes a transparent competitive process for identifying and pricing resources [2]. Hence, prices fluctuate considerably due to varying demand –in fact, they tend to increase as the costs of production and distribution increase as demand increases in the long term. Higher prices make suppliers more competitive and able to develop high-quality supplier capability [20] and demand can be modeled as an availability-sensitive stochastic process as customers make their decisions based on the supplier capability. For further research, there may be a lot to assume aside from these correlations. Focusing on energy markets, that is because energy demand and prices are not solely determined by the size of availability

but also by the economy, possible new laws and regulations governing energy choices (such as emissions of carbon dioxide and other gases), the policies and political stability of petroleum-rich nations, lifestyle choices and business decisions, climate change, and the pace of developments in science and engineering [21]. Deferring the discussion of each individual factor to a different research, these extensions relate more closely to the delivery structure of the required product, equivalently ‘availability of supplies’ in our work.

It is therefore meaningful that all models developed here will have their uses in different time dimensions. Essentially, they may serve to gain some insight into the multi-period problem.

After deriving the optimal contract quantities under different correlation structures, we demonstrate the value of such contracts. We further develop closed-form solutions and effective heuristics for various special cases of our models for comparison. The solutions provide insights on the effects of modelling delivery uncertainty and price & supply delivery correlation into the procurement decision making.

Our numerical results identify a notable improvement in profit, in some cases exceeding 2.5%, when the stochastic nature of deliveries –or supply availabilities, equivalently– is considered. Such improvements are measured in thousands or millions of dollars in some markets and this highlights the importance of imperfect deliveries to devise procurement management policies for traded commodities.

The rest of the thesis proceeds as follows: Chapter 2 is devoted to the review of the related literature. Chapter 3 describes the base model and assumptions, introduces stochastic processes of our variables as price and demand are jointly distributed and finally, represents how to construct expressions for optimal order quantities by adapting three different approaches. In Chapter 4, similar expressions are derived to study the joint behavior of price and availability, and the joint behavior of all three variables, respectively. Chapter 5 then concentrates on the numerical scheme and provides numerical examples, as well as plots, aimed to analyze the penalty weights for

the solutions of special cases given and the effects of parameters. Conclusions are summarized in Chapter 6, with some discussion on the limitations of our model and possible extensions. The appendix collects proofs and more details on the objective functions considered, as well as plots of model results to identify additional insights. All the numerical computations and plots are carried out using R-commands.

2. LITERATURE SURVEY

The literature on procurement models in stochastic environments has been particularly abundant in the last decades. Many aspects have been explored but mainly models subject to stochastic price and demand fluctuations (possibly correlated) have received significant attention. Frequently, profit maximization or cost minimization objectives have been employed in the literature –aside from different optimization objectives such as minimizing the risk of failing a target level, i.e. service level. As one of the papers studying optimal ordering decisions in the context of profit maximization, Gurnani and Tang [22] analyze a newsvendor problem with two ordering periods while the demand is uncertain and the purchasing cost in the second period is stochastic. Their model allows demand forecast updating by utilizing the market signals observed between the first and second instants but also, purchasing price could be higher (or lower) in the second instant. So, the risk-neutral retailer faces a trade-off between more accurate forecast and a potentially higher unit cost at the second instant. Gurnani and Tang characterize the conditions under which the retailer would defer the ordering decision to the second instant and the optimal policy to allocate the optimal order quantity to be placed at both instants in their paper.

Under uncertain market conditions, firms invest in inventory and optimal inventory policies have been studied in different settings by several researchers over the years [23–26]. Kalyon [23] studies a single-item multi-period inventory model where the purchase price follows a Markov process and the distribution of demand in each period depends on the current price. It is assumed that in the current period the price is known before making the ordering decision. The objective is minimizing the total cost consisting of inventory holding costs, shortage costs and purchasing costs. For both a finite and infinite planning horizon, the problem is formulated as a stochastic dynamic program, the form and bounds of optimal policies are derived and also, computational approaches exploiting structure are discussed in the paper. In a cost minimization framework, stochastic demand and prices are also investigated by Wang [24]. Dealing with inventory replenishment policies when purchase prices goes down over time in a

multi-period model, Wang proves the optimality of myopic inventory policy and characterize the conditions regarding cost parameters under which myopic stocking policies are optimal. With a myopic policy, the optimal stocking decision at each period is simply to bring the inventory to a specific order-up-to level, however, the problem of determining optimal procurement policies becomes more complex if the prices follow a continuous distribution. Berling and Rosling [25] consider a model with stochastic demand and stochastic prices allowing no-arbitrage opportunities and discuss how the financial risks and option valuation will affect the inventory policies. Purchase price follows a stationary logarithmic Wiener process in their paper while evaluating the effect of the systematic risk of the purchase price on the optimal policy and on the capital cost of holding inventory. Studying a periodic review inventory model with zero lead time and a fixed set-up cost, they conclude that systematic risk of stochastic demand is very small compared to risk of purchase price for the inventory management. More recently, Berling and Martinez-de-Albeniz [26] consider optimal inventory policies when both the product demand and purchase price are random. They show that for Poisson demand, a price-dependent base-stock policy is optimal and they characterize the optimal base-stock level, equivalently the threshold prices for which an order should or should not be placed. The procedure provided to calculate thresholds yields closed-form solutions when price follows a geometric Brownian motion and implicit solutions under the Ornstein-Uhlenbeck price model. The papers discuss uncertainty and inventories but not spot market use, mainly.

Then, it comes to permit risk-averse decision makers to procure products also from the spot market, hereby it could be possible to reduce inventory related costs and improve profit. One of the earliest studies on the dynamics of futures and spot market is presented by Pindyck [27]. Providing a detailed discussion on the relationship between spot prices, futures prices, and inventory behavior, Pindyck proposes that for storable commodities, inventories plays crucial role in the spot price of the commodity. As one of the several papers closely related to our topic, Seifert *et al.* [28] develop a model that determines explicitly the combination of the optimal order quantity to purchase via forward contracts and the optimal quantity to purchase via spot markets in a single period problem. In their model, demand and spot follow a bivariate normal

distribution. To characterize the optimal usage of spot markets in the procurement of commodities, they include the results for models where spot market is used for both buying and selling, as well as restricted to either buying or selling. They demonstrate that procuring a moderate fraction via the spot market and the rest via forward contract can lead to significant profit improvements and spot markets become increasingly relevant as the variance in demand increases.

Other references on this topic include Goel and Gutierrez [3, 29, 30]. Integrating spot markets in the optimal procurement policies of stochastic price and demand settings, Goel and Gutierrez [3] explore the value of incorporating information about spot and futures market prices in making procurement decisions. For a multi-period, periodic-review inventory setting, they propose optimal policies for buying and disposing of inventory in spot and forward markets in a way that optimal forward procurement policies are characterized by a fixed band and optimal spot procurement policies are described as a regulated band. The thresholds of the regulated band depend on the spread between spot and futures prices and other firm-specific parameters. While the thresholds of the regulated band are also a function of the forward procurement band, they provide computational algorithm to obtain the optimal band thresholds for both policies. According to the numerical results from this paper, it is possible to reduce inventory related costs significantly by incorporating spot and futures price information in decision making, and as well, price-risk reduction and the potential savings in transportation costs associated with forward procurement could incline firms to procure a significant fraction of goods from forward markets, using spot procurement only to fine tune stocking levels and recover from emergencies. Goel and Gutierrez [29] also address the procurement and distribution policies in the context of a multi-echelon distribution system consisting of a central warehouse and multiple non-identical retailers to satisfy random demand under fluctuating prices. Within the framework of a periodic review inventory system for a two echelon distributive supply chain setting, they formulate a model to compute approximate, but near optimal, procurement and distribution policies, and they quantify the value of the flexibility provided by the spot market in the procurement process and the value of incorporating the market generated price information in decision making. Goel and Gutierrez model commodity prices using the

two factor price model of Schwartz and Smith [31] –in which natural logarithm of the spot price at time t evolves according to a two-dimensional Brownian motion– in their papers [3, 29].

In the literature, one other model to solve the procurement problem of a firm operating in forward and spot markets is suggested by Secomandi and Kekre [32]. Under both volatile demand and prices, they develop a model of commodity procurement for a single future date but the firm can trade multiple times in the forward market, in advance of this date. Formulating this problem as a Markov decision process, they point out that the whole dynamic optimization can provide a better solution overall when not only the update of the forward price, but also the update the demand forecast is taken into account in procurement decision making.

As using spot markets in procurement of commodities is an active area of research, comprehensive review on supply chain operations in the presence of a spot market is provided in a paper by Haksoz and Seshadri [4]. In particular, they classify the literature into studies addressing to optimal procurement policies and those related to the valuation of supply contracts, and then, provide new results on supply contract valuation by permitting abandonment. Reviewing on impact of spot markets on supply chains, we put more emphasis on optimal procurement policy approaches than contract valuation, and in this respect, Haksoz and Seshadri [4] is still valuable through critically surveying the literature for models related to buying and selling excess inventory in the spot market, procurement in the spot market and buyer–seller coordination, multi-period models of procurement incorporating the spot market and so on.

Different from the literature above, Secomandi [33] considers capacity constraints on storage facilities, so on the inventories. The limits on inventories give rise to optimal trading policies which, at each decision time, depends both on the realized spot price and the inventory availability, rather than only the realized spot price as traditionally. Then, the optimal policy structure is characterized by two stage and spot price dependent basestock targets. The spot price evolves according to a one factor model, in particular a seasonal mean reverting model but the author also finds a re-

optimized deterministic model that can perform very well in this context. But the point is that approach developed in this paper is to assess the value of storage under variable demand and prices. Alternatively, Devalkar *et al.* [34] modeled the sourcing as an integrated optimization problem of procurement, processing, and trade of commodities for a firm subject to capacity constraints on procurement and processing. In a multi-period setting, they show that the procurement and processing decisions in any period are governed by price-dependent inventory thresholds. In essence, as the issues of processing and storage limitations are taken into account, we learn more about the typical procurement problem from these papers and how it can relate to the procurement policies of non-produced (but procured for usage or further production) and non-storable commodities.

Production and storage are two supply drivers mitigating the risk of mismatch in supply and demand, but may nonexistent in the settings of certain markets [35] –energy and financial markets, for example. Accordingly, there is now a substantial literature agreeing that storage limitation is a major volatility driver while non-storable commodities are characterized by the immutable fact that the quantity or quality changes frequently [36]. Concerning the volatile environment of non-storables, a number of papers analyze the variant of forward and spot trading in this respect [1, 35–42].

While Naik and Leuthold [37] conduct a study to identify the cash and futures price relationships for non-storable commodities attributing to cattle and hog markets, Botterud *et al.* [38] analyze the relationship between spot and futures prices in the Nord Pool power exchange, which is a very large multinational market place for electricity –highly liquid and also trades a wide range of electricity derivatives. By addressing the relationship between financial and physical, spot and forward markets, Skantze and Ilic [39] attempt to explore the arbitrage pricing theory approach to modeling forward prices, and evaluate its relevance for non-storable commodities. From joint dynamics of electricity spot and forward markets, they arrive at stochastic models which quantify future uncertainty in the marketplace. Dong and Liu [40] examine the bilateral supply contracts of non-storable commodities in the presence of spot markets. The authors derive the unique equilibrium forward contract in closed form and

reveal the value of forward contracts, from risk-hedging perspective, that justify their existence in spite of spot market presence. They also show the relationship between the spot and forward price as the equilibrium forward price can be non-monotonic in the spot price volatility and can decrease as the initial spot price increases but the forward price is also dependent on the manufacturer's market power. However, the common theme in this line of research is managing the financial risks, that is, the focus is on the hedging role of forward markets or pricing purposes, i.e. futures, forwards and options trading as well as designating market power. The consideration of physical deliveries in supply chains is rarely investigated.

A related optimal problem is analyzed by Secomandi and Kekre [41]. Given the lower transaction costs for forward contracts compared to spot purchases, they study a model of procurement by using both the forward and spot markets for the commodities that cannot be stocked. With joint evolution of stochastic future demand and forward price –demand and price are jointly lognormally distributed and satisfy a martingale condition– they derive the structure of the optimal commodity procurement policy in the closed form expression. After quantifying the value of the forward procurement option on realistic natural gas instances, they further suggest that procuring the demand forecast in the forward market is nearly optimal. Finally, they discuss an extended model in which the aggregate time period is subdivided into equally spaced dates such that there is still a single purchase time but a fraction of the quantity purchased is delivered on each spaced date. Understanding the structure of their decision making through stochastic modeling, we extend their model to the case in which it is not certain that the all forward contracted amount is fully delivered at the delivery date. By integrating uncertainty to the delivery percentage, we end up with three stochastic processes-variant of Secomandi and Kekre's model. Hence, another category of literature relevant to our topic is the stream of research dealing with delivery uncertainty.

Some models developed in the literature rely on utilizing swing contracts which incorporate flexibility-of-delivery into the options. Swing options are common and important form of options found in energy sector and in this context, studying the electricity and natural gas markets in particular, Jaillet *et al.* [12] propose a pricing

method of swing options by dynamic programming approach.

Supply uncertainties appear in environmental literature around water availability, including the works of Fisher and Rubio [43] and Bhaduri and Manna [44]. Fisher and Rubio [43] consider the impact of changing climate –which may bring an increase in the variability of precipitation in many regions, as well as increased evaporation– and the flow into the water reserves, which is uncertain, that a measure of the uncertainty. They examine the implications of the hypothesized increase in the variability of water flows for investment in water reserves, i.e. the determination of long-run water storage capacity. Assuming that water resources evolve through time according to a geometric Brownian motion, they derive the steady state conditions of the corresponding stochastic control problem. Similarly, Bhaduri and Manna [44] investigate the impact of water supply uncertainty stemming from climate change, but also water storage capacity at a farm level on the decision of farmers to invest in efficient irrigation technology under a flexible water price regime. Following Fisher and Rubio, they also assume that water supply follows a geometric Brownian motion and in the framework of a stochastic dynamic programming model, they identify a complementary relationship between storage capacity at a farm level and investments in efficient irrigation. The numerical results in this paper suggest that a risk averse farmer may invest more in efficient irrigation only if the variance in water supply is very high.

Two other papers that discuss supply uncertainty are Popescu and Seshadri [45] and Benth and Meyer-Brandis [46]. In the paper of Popescu and Seshadri [45], noting the greatness of financial impact of supply shortages, availability risk is recognized –in addition to prices risk, inventory risk and demand risk—but from the modeling point of view, the paper assumes that forward contracts are firm quantity commitments between buyers and producers, such that order cancellations and refunds are not permitted. However, the authors make a mention of that such a frictionless setting is unrealistic for many industries where it is costly to firm quantity commitments between buyers and suppliers. In the paper by Benth and Meyer-Brandis [46], the notion of the information yield is introduced for the future supply uncertainty and pricing of forward contracts in the electricity market is discussed based on the enlargement of the

information filtration. Considering the non-storability feature of the commodity, the paper is based on the idea of including future demand and supply information in the forward price formation. The future supply information is sometimes the future temperature predictions that affect the production and the availability of the commodity, or sometimes the maintenance of a major electricity power plant so that the market knows that the power plant will be taken off the next the whole next month and a significant part of the supply side in the market falls out. According to the paper, the information about the delivery of electricity for the maintenance period should be incorporated in pricing electricity forwards.

Motivated by the flour milling industry, Tan [47] contributes to optimal procurement and hedging research in the integrated operational and financial risks management literature by his thesis. He explores the optimal procurement policies of a firm-value-maximizing flour miller –taking demand uncertainty, volatile wheat purchase price, uncertain wheat quality and not entirely fixed wheat supply amount into account– in the presence of forward contracts and spot market. He further derives the value of downward substitution in commodity markets. Results imply that myopic base-stock policies are optimal and the % benefit of downward substitution exponentially increases with quality requirement uncertainty.

Existence of both volume and price uncertainties at the supply end is distinguished by also Fu *et al.* [48]. Differently, the optimal portfolio strategy is investigated for procurement, in a single period setting when both the demand and the spot price are random (either correlated or independent). Then, it turns out to be that two carefully selected suppliers are enough to produce near-optimal performance in most cases, instead of sticking to one contract or maintaining an extensive list of contracts with suppliers.

As is seen, models in the literature are stylized in various ways. Readers may refer to Tang [6] for a review and discussions of the quantitative models to manage supply chain risks –operational or disruption. Addressing the use of supply/ demand/ product/ information management strategies, the author also relates the supply chain

risk management strategies examined in the literature with actual practices and then, concludes the paper with some suggestions for future research.

Although supply uncertainty is an important topic in the field of procurement optimization, the literature emphasizes more on demand and price uncertainties. Mainly, this might be due to the rare availability of (or lack of easy access to) delivery data of the commodities procured, which is necessary for the determination of a delivery pattern. But recently, Pineda and Conejo [49] provide an availability scenario of a power generating unit referring mean time to failure and mean time to repair of the unit. Concerning the supply side of markets, the authors aim not only to maximize the expected profit but also to reduce the profit variability caused by price and availability risks (due to unexpected unit failures). They describe a multi-stage stochastic model in their tutorial paper to determine the optimal forward and option contracting decisions for a risk-averse power producer. They note that when trading in electricity markets, forward contracts and insurances are derivatives that reduce either the price or the availability risk, respectively, but then, electricity options are financial derivatives that can be used by power producers to hedge against both price and availability risk. They treat price and availability uncertainty rather carefully but neglect demand uncertainty.

To our knowledge, although there are papers studying similar problems, none includes demand, price and delivery uncertainties simultaneously, when trading over forward and spot markets for non-storable assets. So, they may be limited in depicting the real world. Although existing models in the literature only deal with one or two aspects of the problem at a time, Pineda and Conejo's availability risk consideration [49] together with Secomandi and Kekre's model [41] comprise valuable first steps in the proposed thesis. Our thesis contributes to the literature on procurement policy modeling from spot and forward markets for a non-storable commodity under stochastic demand and price by incorporating stochasticity in delivery structure of the quantities procured forward, and by considering its possible correlations with price and demand.

For accommodating different correlation structures for the three variables at hand, we inspire from the approach of Ausubel and Cramton [2] to the usage of forward markets in order to improve electricity market design. They complement the spot market with two forward markets, one medium term and one long term, based on the issue of consumer demand inelasticity. In the medium term, consumer demand response is limited in most markets, particularly for energy, and a forward market lets suppliers and demanders lock in energy prices and quantities. In the long term, a forward market assures adequate resources are available when they are needed most and consumers have ability to express their preferences. We address this issue in more detail as we introduce different correlations for the evolution of the price, demand and availability.

Thereupon, two or more relevant variables involved in this thesis may correlate with each other in specified manners and so, some model and formulas developed rely on characterizing the product moments of bivariate lognormals when a multiplicative constraint is imposed. The explicit formulas derived by Lien and Balakrishnan enable [50] us to calculate these moments.

Summarizing key results from the literature on optimization of procurement policies, we first turn our attention to the case of correlated price and demand process subject to delivery uncertainty, after describing the optimization model.

3. DESCRIPTION OF THE MODEL

In this chapter, we consider the role of forward procurement option when trading with stochastic demand and prices. We motivate and introduce our proposed model framework for energy markets when supply is unreliable and the underlying commodity is known to be a non-storable. Our main focus is to argue for the relevance of delivery risk in procurement optimization.

In order to set the notation and make this model setup, we apply the framework studied by Secomandi and Kekre in their working paper [41] but our model configures a stochastic factor $L(t)$ to incorporate uncertainty into the delivery of forward contracted amount.

Here, we take $L(t)_{0 \leq t \leq T}$ to be an indicator which gives a signal on the availability of the commodity in the market at time t . That is, we relate ‘the success of the supplier or the market for the delivery of the required product’ to a theoretical value, i.e. availability index. To illustrate, if the commodity availability is sensitive to weather conditions, then $L(t)$ represents a forecast for weather conditions at time t , or else if the underlying commodity is perishable, $L(t)$ may imply a stochastic perishability factor for this item.

In the remainder of this thesis, ‘availability index’ and ‘delivery percentage’ terms (or delivery uncertainty) are deployed interchangeably to referring delivery risk, but mainly, ‘availability index’ is used to denominate the stochastic process of delivery in our modelling structure.

3.1. Base Model

In the base model, we consider the case of single time period with length T such that the firm can trade once in the forward market, at time 0, for the purpose of satisfying an uncertain demand encountered at a some specified future date T , for

a single item. Demand over this time period is a random variable whose value only becomes known at time T , and at this time, demand must be fully satisfied. Since the considered commodities under such a problem setting are not storable or have limited storability, no inventory is held over time; but then, it is possible for the firm to satisfy the remaining demand by spot procurement as well as selling the residual supply in spot market. Also, it is noted that buying nothing from forward market but satisfying all demand by spot purchase is feasible but may be inferior to any other positive amount of commodity procured from the forward market.

At time 0, the buyer has to decide the forward procurement quantity q regarding its unbiased time-zero forecast for the demand at time T , that is $D(0)$. At time T , demand $D(T)$ occurs and the amount arriving differs from the contracted quantity q by a factor of quality-availability index $\tilde{L}(T)$.

Here, $\tilde{L}(T)$ is a function of the availability index $L(T)$, $\tilde{L}(T) = f(L(T)) \in [0, 1]$. For example, consider a case where $L(t)$ takes on values $\{0, 1, \dots, m\}$, each indicating a possible availability level for period T . Then, a possible form of the function $f(x)$ could be:

$$\tilde{L}(T) = f(L(T)) = \mathbb{1}_{\{L(T) \in R_i\}} * \alpha_i \quad (3.1)$$

where $\mathbb{1}_{\{\cdot\}}$ is the indicator function and $\alpha_i \in [0, 1]$. If $L(T)$ is a nonnegative random variable, then

$$\tilde{L}(T) = f(L(T)) = e^{-L(T)} \quad (3.2)$$

is another example for such a function.

In the succeeding sections, we will derive our results for various forms of $f(L(T))$. In particular, in Section 3.3, we will consider the function $f(x) = \min_{x \geq 0} \{1, x\}$. As this

model does not yield a closed form expression for the optimal procurement amount, in Section 3.4, we propose an approximation, and eventually in Section 3.5, we let $f(x) = x$ assuming that $f(L(T)) = L(T)$ in $[0, \infty)$.

$L(T)$ is a random variable, hence $\tilde{L}(T)$ is a random variable, too. $\tilde{L}(T)$ can be viewed as a key dimension of our model since it is assumed that quantity ordered q will arrive as quantity received $q.\tilde{L}(T)$ at the promised date. The difference between these two volumes prompts decision makers to engage in spot trading.

At time T , if realized demand $D(T)$ is less than the quantity delivered $q.\tilde{L}(T)$, the residual supply amount $q.\tilde{L}(T) - D(T)$ is sold in the spot market. Else, in the case of excess demand, occurring when demand $D(T)$ is greater than the quantity $q.\tilde{L}(T)$, the additional $D(T) - q.\tilde{L}(T)$ amount is purchased from spot market.

Figure 3.1 depicts the major framework of the procurement decision at hand and delivery uncertainty consideration incorporated into it.

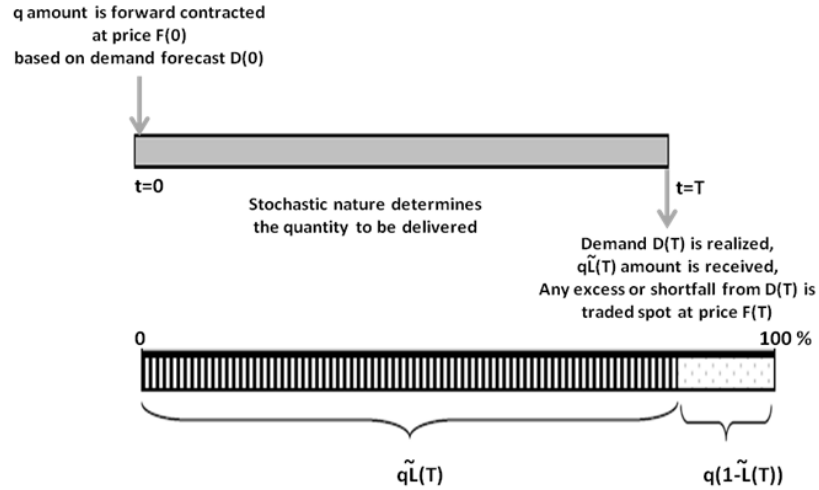


Figure 3.1. Timeline and framework of the problem.

Carrying out trade transactions in spot and forward markets, the firm incurs transaction costs. In our model, the transaction costs in the spot and forward markets

are referred as A and B , respectively. Then, under the assumption that $(A, B) \in (0, 1)$, we further assume that transaction costs in the forward market are lower than those in spot market, that is $(B < A)$. This assumption is justified because spot transactions are subject to time constraints and physical limits, inducing higher costs, whereas the lead time associated with forward market transactions allow for more cost efficient scheduling of production and transportation of the traded commodity [3].

It must be noted that in order to apply the valuation framework under the presence of transaction costs of Jouini and Kallal [51], referred as equivalent measure, transaction costs must be assumed to be a time-dependent function which does not decrease in the time interval $[0, T]$, that is: $B \equiv B(0) \leq B(t) \leq B(T) \equiv A$, $t \in [0, T]$. Consistently, here we simply allow transaction costs to be time-independent parameters with the limiting condition that $(B < A)$.

The actions allowed for a firm are spot purchase, forward purchase and spot sell. Forward sell of commodities previously purchased is disallowed since this is suboptimal under no-arbitrage valuation framework. Also, under no inventory model, it is not possible to buy commodities at one point in time, store them and sell at a later point in time –in order to exploit price variabilities, for example. The firm pays $(1 + A)F(T)$ for one unit spot purchase and receives $(1 - A)F(T)$ for one unit spot sell. Making one unit forward purchase has the price of $(1 + B)F(0)$.

Evoking marginal analysis, $(1 + A)F(T)$ can also be considered as the cost of not satisfying the demand at time T , so the marginal underage cost comes out as $(1 + A)F(T) - (1 + B)F(0)$. Similarly, $(1 - A)F(T)$ is the revenue generated from selling leftover units spot and the marginal overage cost turns out to be $(1 + B)F(0) - (1 - A)F(T)$. The optimal solution is supposed to balance the expected marginal underage and overage costs. Introduction of uncertainty component to the quantity received makes the interpretation of marginal costs particularly important. That is because, unavailability of the supply may affect the acquisition of the underlying commodity even at a higher price.

The decision variable is the quantity q to procure forward at time 0, that is, the quantity of forward contract to buy at the price of $F(0)$ knowing that the demand and quantity arrived at the time of delivery, i.e. time T , are stochastic. Money is exchanged only at time T . Hence, expected profit function to be maximized is expressed in terms of time- T money. Adapted from the formulation of Secomandi and Kekre's model, the expected profit at the termination date of forward contract is written as:

$$V := \max_{0 \leq q} \mathbb{E} \left[(1 - A)F(T)\{q\tilde{L}(T) - D(T)\}^+ - (1 + A)F(T)\{q\tilde{L}(T) - D(T)\}^- \right. \\ \left. - (1 + B)F(0)q\tilde{L}(T) \right] \quad (3.3)$$

where $(.)^+ = \max\{0, .\}$, $(.)^- = -\min\{0, .\}$ and the expression “ $:=$ ” stands for “defined as”.

The first term inside the expectation is revenue generated by the spot sell of excess contracted supply while the second term stands for the cost of spot purchase in the case of supply shortfall, and the last term inside expectation is the amount paid for forward purchase.

Using the conventions $(x - y)^- = \max\{0, y - x\}$ and $\max\{0, y - x\} = y - x + \max\{0, x - y\}$ and rearranging the terms, the objective function is rewritten as:

$$V := \max_{0 \leq q} \mathbb{E} \left[(1 + A)F(T)\{q\tilde{L}(T) - D(T)\} - 2AF(T)\{q\tilde{L}(T) - D(T)\}^+ \right. \\ \left. - (1 + B)F(0)q\tilde{L}(T) \right] \quad (3.4)$$

$$V := \max_{0 \leq q} (1 + A)\mathbb{E} \left[F(T)q\tilde{L}(T) \right] - (1 + A)\mathbb{E} [F(T)D(T)] \\ - 2A\mathbb{E} \left[F(T)\{q\tilde{L}(T) - D(T)\}^+ \right] - (1 + B)F(0)q\mathbb{E} [\tilde{L}(T)] \quad (3.5)$$

It is straightforward to verify that the objective function is proper concave in q and goes toward $-\infty$ as $q \rightarrow \infty$, one can easily check this concave nature by taking first and second order derivatives of the above profit function with respect to q . (See Appendix A.1 for more details on the proof of concavity proposition.)

Thus, differentiating Equation (3.5) with respect to q and equating to 0, the first order optimality condition:

$$(1 + A)\mathbb{E} \left[F(T)\tilde{L}(T) \right] - 2A\mathbb{E} \left[F(T)\tilde{L}(T)\mathbb{1}_{\{D(T) \leq q\tilde{L}(T)\}} \right] - (1 + B)F(0)\mathbb{E} \left[\tilde{L}(T) \right] = 0 \quad (3.6)$$

Rearranging this equation yields:

$$\mathbb{E} \left[F(T)\tilde{L}(T)\mathbb{1}_{\{D(T) \leq q\tilde{L}(T)\}} \right] = \frac{(1 + A)\mathbb{E} \left[F(T)\tilde{L}(T) \right] - (1 + B)F(0)\mathbb{E} \left[\tilde{L}(T) \right]}{2A} \quad (3.7)$$

Regardless of a specific joint probability distributions for spot demand, nominal price and availability index, (3.7) represents the optimality condition for our procurement decision.

3.2. Processes

In order to solve the formulated stochastic optimization problem, we require specific information about the probability distributions of spot demand, nominal price and availability index. So, our purpose here is to assume some specific distributions for these variables.

Following the original paper by Secomandi and Kekre, we assume Martingale log-normal (MLN) model for our three processes, such that demand, price and availabil-

ity index processes follow lognormal distribution and satisfy the martingale property. That is to say; spot nominal price, demand and availability index evolve during time as known stochastic processes and under equivalent measure, their expectations are just equal to forward nominal price, current demand forecast and availability index.

3.2.1. The Demand Forecast Process

Demand to be realized by the firm for the underlying commodity evolves continuously, as a stochastic process. We may expect that the demand for commodities traded obeys the lognormal distribution. The empirical evidence for demands which are well-represented by lognormal distribution was provided by Brown in 1959 [52]. In fact, Wiener process demand appears in many papers including the works of Puterman, Sulem and Dohi *et al.* who propose inventory management policies for the demands following ordinary Wiener processes with drift [53–55] while a continuous time inventory model when the cumulative demand following a Wiener process was discussed at first by Bather [56].

Hence, demand forecast is assumed to be a stochastic process $D(t)$ which solves the following stochastic differential equation:

$$dD(t) = \sigma_D D(t) dW_D(t) \quad (3.8)$$

where $W_D(t)$ is a standard Brownian motion and σ_D is volatility term. Specifying geometric Brownian motion (GBM) with zero drift to describe the progression of demand forecasts over time, we have:

$$D(t) = D(0) \exp\left\{-\frac{1}{2}\sigma_D^2 t + \sigma_D W_D(t)\right\} \quad (3.9)$$

with $D(0)$ as the initial demand forecast at time 0.

Owing to the MLN nature of the demand forecast process, the expected value of spot demand reduces to the demand forecast at the forward trading time, that is:

$$\mathbb{E}[D(T)] = D(0) \quad (3.10)$$

3.2.2. The Forward Price Process

Regarding the evolution of nominal prices, prior research shows that geometric Brownian motion is highly relevant [12, 41]. To explain further, referring $S(t)$ as the spot nominal price at time t of the underlying forward contract, it follows a diffusion process specified as:

$$dS(t) = \sigma_S S(t) dW_S(t) \quad (3.11)$$

where σ_S is the volatility in log-prices and $W_S(t)$ is a standard Brownian motion. Under the risk-neutral probability and arbitrage free assumption, this model captures the relationship between spot and forward prices, provided a constant interest rate. Forward price at time t for delivery of the commodity at time T , that is $F(t)$, must be given by:

$$F(t) = S(t) \exp\{r(T - t)\} \quad (3.12)$$

where r is the risk-free interest rate.

Then, for $t = 0$ and $t = T$, we have the following:

$$F(0) = S(0)\exp\{rT\} \quad (3.13)$$

$$F(T) = S(T) \quad \Rightarrow \quad \mathbb{E}[F(T)] = \mathbb{E}[S(T)] \quad (3.14)$$

Also, the spot price of the underlying commodity at time zero is equal to the discounted value of the expected spot price at time T :

$$S(0) = \exp\{-rT\}\mathbb{E}[S(T)] \quad (3.15)$$

$$\exp\{rT\}S(0) = \mathbb{E}[S(T)] \quad \Rightarrow \quad \mathbb{E}[F(T)] = \exp\{rT\}S(0) \quad (3.16)$$

Hence, the expected time- T forward nominal price is simply the time-zero forward nominal price:

$$\mathbb{E}[F(T)] = F(0) \quad (3.17)$$

which shows that forward nominal price is a martingale. Running from spot to forward prices, the forward nominal price of commodity at time t follows GBM with volatility σ_S :

$$dF(t) = \sigma_S F(t) dW_S(t) \quad (3.18)$$

Therefore,

$$F(t) = F(0)\exp\left\{-\frac{1}{2}\sigma_S^2 t + \sigma_S W_S(t)\right\} \quad (3.19)$$

To incorporate the dependency within nominal price and demand forecast processes, we have:

$$dW_S(t)dW_D(t) = c dt \quad (3.20)$$

where $dW_S(t)$ and $dW_D(t)$ are correlated increments to standard Brownian motions with correlation parameter c .

We assume the spot demand and nominal prices to follow GBM, and accordingly, their underlying distributions are lognormal. Hence, assuming bivariate lognormal distribution of spot nominal price and spot demand, it is implied that the natural logarithms of $F(t)$ and $D(t)$ variables are jointly normally distributed with standard deviations σ_S and σ_D , and means $\ln F(0) - \frac{\sigma_S^2}{2}$ and $\ln D(0) - \frac{\sigma_D^2}{2}$.

3.2.3. The Availability Index Process

Up to this point, we are amenable to the common assumption of the most literature as GBM is extensively used for modeling price processes in financial studies as a natural choice [57], and as demand models are often described by GBM [58]. GBM is also applied to different models, such as evolution of water levels in reservoirs, as it is originated to describe particle movement in physics essentially [43]. It may be remarked that since a GBM is defined nonnegative, it provides a more realistic structure for all of our variables.

We should note, also, that log-normal approximation is particularly appropriate when no discrete scale of measurement is known but estimated indirectly. Examples are estimates of phytoplankton population by chlorophyll content or rate of photosynthesis [59]. This can be very fair for energy commodities while we have no relevant data associated with delivery amounts but we can use failure rates (e.g. power failures) or pipeline leak statistics (e.g. flow rate) or such rates to measure. Hence, for modeling the dynamic behavior of the quantity arrived, i.e. availability index, it will be plausible that the availability process is assumed to be a Wiener process. The basic idea is that the index created keeps the track of randomly varying quantities of traded commodity which is very analogous to water levels in reservoirs and cannot take negative values.

Accordingly, to describe availability index process, we assume a continuous-time stochastic process with zero drift. Our availability index stochastically fluctuates according to a geometric Brownian motion and satisfies the following stochastic differential equation:

$$dL(t) = \sigma_L L(t) dW_L(t) \quad (3.21)$$

where σ_L is the log-availability index volatility and $W_L(t)$ stands for standard Brownian motion. For the initial availability index value $L(0)$, we obtain:

$$L(t) = L(0) \exp\left\{-\frac{1}{2}\sigma_L^2 t + \sigma_L W_L(t)\right\} \quad (3.22)$$

Note that, by assumption, $L(t)$ is a martingale process satisfying:

$$\mathbb{E}[L(T)] = L(0) \quad (3.23)$$

Stating lognormal distribution for the underlying distribution of availability index, random variable $\ln L(t)$ is normally distributed with standard deviation σ_L and mean $\ln L(0) - \frac{\sigma_L^2}{2}$.

Implications of these distributions are used in the remainder of this thesis to characterize expressions relevant to our problem.

3.3. A Special Case: Availability Index Process is Truncated

Assuming lognormal distribution for the availability index, its range now extends from zero to infinity. In such setting, availability index could have values greater than 1 with a positive probability implying that percentage delivered to the firm can be in the excess of contracted amount. Although several models - such as financial transactions or some physical conversions - allow augmenting amount of flow during transfers, this rarely occurs for our kind of settings, particularly within energy procurement. Generally, lossy deliveries are expected in our model due to, for instance, leakage, evaporation, etc. It is now desirable to make some refinement in our concept of the availability index. We therefore restrict the support of the distribution of $L(t)$ to be $[0, 1]$ (i.e. there is no physical delivery larger than the contracted quantity).

In this section, we will examine the solution procedure with the availability index $L(t)$ to be a lognormal random variable truncated at the critical value 1 by using exact expressions in calculation. We consider the special case the function f :

$$f(x) = \begin{cases} x & \text{if } x \leq 1 \\ 1 & \text{if } x > 1 \end{cases} \quad (3.24)$$

and we define $\tilde{L}(T) = \min\{L(T), 1\}$ such that:

$$\tilde{L}(T) = \begin{cases} L(T) & \text{if } L(T) \leq 1 \\ 1 & \text{if } L(T) > 1 \end{cases} \quad (3.25)$$

The first order condition for profit maximization is given by Equation (3.7) and since spot nominal price and availability index processes are uncorrelated, the expectation of their product is equal to $F(0)\mathbb{E}[\tilde{L}(T)]$, and the right hand side of the equation can be reduced as:

$$\mathbb{E} \left[F(T)\tilde{L}(T)\mathbb{1}_{\{D(T) \leq q\tilde{L}(T)\}} \right] = \frac{(A - B)F(0)\mathbb{E}[\tilde{L}(T)]}{2A} \quad (3.26)$$

Note that inside the expectation on the left hand side, we have a multiplicative constraint such that:

$$\mathbb{1}_{\{D(T) \leq q\tilde{L}(T)\}} = \begin{cases} 1 & \text{if } D(T)\tilde{L}^{-1}(T) \leq q \\ 0 & \text{if } otherwise \end{cases} \quad (3.27)$$

In order to establish further results, we need to evaluate expectations of multiplicatively constrained form as well as partial expectations. For this reason, the explicit formulas derived by Lien and Balakrishnan for the moments of bivariate lognormal statistics under such multiplicative constraints [50] and the corollary of lognormal partial expectation [60] will be given first, before moving on.

Theorem 3.1. For bivariate lognormal random variables denoted by X and Y such that $\ln(X)$ and $\ln(Y)$ are bivariate normal $\sim N(\mu_X, \mu_Y, \sigma_X, \sigma_Y, \rho)$, it holds that:

$$\mathbb{E}[X^m Y^n \mathbb{1}_{\{X^a Y^b \leq K\}}] = e^Q \Phi\left(\frac{\ln(K) - \mu - \{am\sigma_X^2 + (bm + an)\sigma_{XY} + bn\sigma_Y^2\}}{\sqrt{a^2\sigma_X^2 + 2ab\rho\sigma_X\sigma_Y + b^2\sigma_Y^2}}\right)$$

with mean μ equal to the sum of $a\mu_X + b\mu_Y$ and covariance term $\sigma_{XY} = \rho\sigma_X\sigma_Y$, where e^Q denotes the joint moment term $\mathbb{E}[X^m Y^n]$ as is given by:

$$e^Q = \mathbb{E}[X^m Y^n] = \exp\left\{m\mu_X + n\mu_Y + \frac{1}{2}m^2\sigma_X^2 + mn\rho\sigma_X\sigma_Y + \frac{1}{2}n^2\sigma_Y^2\right\}$$

Corollary 3.2. Suppose x is a lognormal random variable and $f(x)$ is defined to be its probability density function with parameters μ and σ as the mean and standard deviation of the variable's natural logarithm, respectively. Then, the partial expectation of this lognormal random variable from 0 to K :

$$\mathbb{E}[x \mathbb{1}_{\{x \leq K\}}] = \int_0^K x f(x) dx = e^{\mu + \frac{1}{2}\sigma^2} \Phi\left(\frac{\ln(K) - \mu - \sigma^2}{\sigma}\right)$$

where $\Phi()$ is the cumulative density function of standard normal.

Similarly, the partial moments for higher orders, i.e. $\gamma \geq 1$, can be constructed as:

$$\mathbb{E}[x^\gamma \mathbb{1}_{\{x \leq K\}}] = \int_0^K x^\gamma f(x) dx = e^{(\mu + \frac{1}{2}\sigma^2)\gamma} \Phi\left(\frac{\ln(K) - \mu - \sigma^2\gamma}{\sigma}\right)$$

Now, along with the above theorem and partial expectation corollary, the expectations involved in Equation (3.26), $\mathbb{E}[\tilde{L}(T)]$ and $\mathbb{E}[F(T)\tilde{L}(T)\mathbb{1}_{\{D(T) \leq q\tilde{L}(T)\}}]$, can be

decomposed by this means:

$$\mathbb{E} \left[\tilde{L}(T) \right] = \mathbb{E} \left[L(T) \mathbb{1}_{\{L(T) \leq 1\}} \right] + \mathbb{E} \left[\mathbb{1}_{\{L(T) > 1\}} \right] \quad (3.28)$$

$$= L(0) \Phi \left(-\frac{\mu_L}{\sigma_L} - \sigma_L \right) + \Phi \left(\frac{\mu_L}{\sigma_L} \right) \quad (3.29)$$

$$\begin{aligned} \mathbb{E} \left[F(T) \tilde{L}(T) \mathbb{1}_{\{D(T) \leq q \tilde{L}(T)\}} \right] &= \mathbb{E} \left[F(T) L(T) \mathbb{1}_{\{D(T) \leq q L(T)\}} \mathbb{1}_{\{L(T) \leq 1\}} \right] \\ &\quad + \mathbb{E} \left[F(T) \mathbb{1}_{\{D(T) \leq q\}} \mathbb{1}_{\{L(T) > 1\}} \right] \end{aligned} \quad (3.30)$$

It is easy to see that the second expectation on the right side of (3.30) can be written as:

$$\mathbb{E} \left[F(T) \mathbb{1}_{\{D(T) \leq q\}} \right] \mathbb{E} \left[\mathbb{1}_{\{L(T) > 1\}} \right] = \mathbb{E} \left[F(T) \mathbb{1}_{\{D(T) \leq q\}} \right] P(L(T) > 1) \quad (3.31)$$

while the first expectation term can be expressed as the integral of the instantaneous availability index as follows:

$$\mathbb{E} \left[F(T) L(T) \mathbb{1}_{\{D(T) \leq q L(T)\}} \mathbb{1}_{\{L(T) \leq 1\}} \right] = \int_0^1 \mathbb{E} \left[F(T) x \mathbb{1}_{\{D(T) \leq qx\}} \right] dF_{L(T)}(x) \quad (3.32)$$

where $F_{L(T)}(\cdot)$ represents the cumulative distribution function of availability index.

Evaluating the above expressions by the multiplicatively constrained moment formula of bivariate lognormal variables, we can deduce an expression for the left hand

side of Equation (3.30) as:

$$\begin{aligned}
LHS = & \int_0^1 \left(F(0)x \Phi \left(\frac{\ln(qx) - \mu_D}{\sigma_D} - c\sigma_S \right) \right) dF_{L(T)}(x) \\
& + F(0)\Phi \left(\frac{\ln(q) - \mu_D}{\sigma_D} - c\sigma_S \right) \Phi \left(\frac{\mu_L}{\sigma_L} \right)
\end{aligned} \tag{3.33}$$

To ease notation, set $P1 : \Phi \left(-\frac{\mu_L}{\sigma_L} - \sigma_L \right)$ and $P2 : \Phi \left(\frac{\mu_L}{\sigma_L} \right)$. Substituting (3.33) and the new notation into Equation (3.30) yields the following proposition.

Proposition 3.3. *Under the MLN model, the value of the optimal forward procurement quantity with the truncated availability index parameter, $L(T) \in [0, 1]$, is given by the solution of the following equation:*

$$\begin{aligned}
& \int_0^1 x \Phi \left(\frac{\ln(qx) - \mu_D}{\sigma_D} - c\sigma_S \right) dF_{L(T)}(x) + \Phi \left(\frac{\ln(q) - \mu_D}{\sigma_D} - c\sigma_S \right) P2 \\
& = \frac{A - B}{2A} \{L(0)P1 + P2\}
\end{aligned} \tag{3.34}$$

This expression is pre-conditioned by positive volatilities of $\sigma_D, \sigma_L > 0$.

We end up with the finding that our decision variable q appears in two different places on the left, and the left hand side is non-invertible with the implication of no closed form solution for this model. Although we do not have an analytical solution for the forward procurement quantity, we can still calculate it numerically but the numerical solution is available only when the right hand side of (3.34) is in the range $(0, 1)$. This is intuitive because it can be interpreted as a critical ratio and we want to choose the forward contracted quantity q such that all realized demand is satisfied from the forward contracted quantity with no need of spot purchase to satisfy the leftover demand with a probability equal to this critical ratio.

This proposition also shows that the optimal forward procurement quantity is not affected by the forward and spot nominal prices but the log spot nominal price standard deviation. As the correlation coefficient between log spot demand and nominal price appears as a multiplier for the standard deviation of the natural logarithm of the spot nominal price, its effect is scaled by this price-demand correlation.

The R-function that calculates optimal order quantity to procure forward is presented next:

```
Find_q_E<-function(D0,F0,L0,varD,varL,varS,c,A,B){
#D0...Time-zero demand forecast for time T
#F0...Time-zero nominal price of the forward contract with delivery time T
#L0...Time-zero availability index forecast for time T
#varD...Variance of demand forecast
#varL...Variance of availability index
#varS...Variance of spot nominal price standard deviation
#c...Correlation coefficient between the log spot demand and nominal price
#A...Transaction cost in spot market
#B...Transaction cost in forward market

#sigmaD...Log spot demand standard deviation
#sigmaL...Log availability index standard deviation
#sigmaS...Log spot nominal price standard deviation
sigmaD<-sqrt(log((varD/(D0^2))+1))
sigmaL<-sqrt(log((varL/(L0^2))+1))
sigmaS<-sqrt(log((varS/(F0^2))+1))

#muD...mean of log spot demand
#muL...mean of log availability index
muD<-log(D0)-((sigmaD^2)/2)
muL<-log(L0)-((sigmaL^2)/2)

P1<-pnorm(-(muL/sigmaL)-sigmaL)
P2<-pnorm(muL/sigmaL)

#r...right hand side of equation
r<-{(A-B)*{L0*P1+P2}}/(2*A)

f1 <- function(x){
f2 <- function(y){
dlnorm(y,meanlog=muL,sdlog=sigmaL,log=FALSE)*y*pnorm((log(x*y)-muD-c*sigmaS*sigmaD)/sigmaD)}
return(f2)}

firstTERM <-function(x) {integrate(f1(x), lower=0, upper=1)$value}
secondTERM<-function(x) {P2*pnorm((log(x)-muD-c*sigmaS*sigmaD)/sigmaD)}
lhs<-function(x){return(firstTERM(x)+secondTERM(x)-r)}
l<-0.000001
u<-D0*1000
q<- uniroot(function(x) lhs(x), lower=l, upper=u, tol = 1e-9)$root
q
}
```

Figure 3.2. R-code for Decision under jointly distributed Price & Demand: Availability Index Truncated.

3.4. Results with the Approximated Model

From the standpoint of computational efficiency, presenting the expectations by exact expressions with integral terms may be inferior. This motivates the use of approximated models and we discuss an approximation model for the optimal forward procurement policy with availability index process truncated at 1.

The first order optimality condition to be satisfied is again (3.26) and we readily obtained an expression for the right hand side of the equation. Then, consider the below expression previously stated in Section 3.3:

$$\begin{aligned} \mathbb{E} \left[F(T) \tilde{L}(T) \mathbb{1}_{\{D(T) \leq q \tilde{L}(T)\}} \right] &= \mathbb{E} \left[F(T) L(T) \mathbb{1}_{\{D(T) \leq q L(T)\}} \mathbb{1}_{\{L(T) \leq 1\}} \right] \\ &+ \mathbb{E} \left[F(T) \mathbb{1}_{\{D(T) \leq q\}} \mathbb{1}_{\{L(T) > 1\}} \right] \end{aligned} \quad (3.30)$$

The expansion of the second expectation value on the right-hand side of the equation is stated beforehand as:

$$\mathbb{E} \left[F(T) \mathbb{1}_{\{D(T) \leq q\}} \mathbb{1}_{\{L(T) > 1\}} \right] = F(0) \Phi \left(\frac{\ln(q) - \mu_D}{\sigma_D} - c\sigma_S \right) \Phi \left(\frac{\mu_L}{\sigma_L} \right) \quad (3.35)$$

and in this section, to avoid applying integral calculus, we approximated the first right-hand expectation such that:

$$\mathbb{E} \left[F(T) L(T) \mathbb{1}_{\{\frac{D(T)}{L(T)} \leq q\}} \mathbb{1}_{\{L(T) \leq 1\}} \right] = \mathbb{E} \left[F(T) \mathbb{1}_{\{\frac{D(T)}{L(T)} \leq q\}} \right] \mathbb{E} \left[L(T) \mathbb{1}_{\{L(T) \leq 1\}} \right] \quad (3.36)$$

Here in this expansion, we do not take into account the correlation among the GBM processes $\frac{D(T)}{L(T)}$ and $L(T)$, as a simplifying assumption.

Then, we specify a new process $Y(T) = D(T)L^{-1}(T)$ and bearing in mind that demand is price dependent, we mean to approximate the joint distribution of $F(T)$ and $Y(T)$, so that:

$$(\ln F(T), \ln Y(T)) \sim N(\mu_F, \mu_Y = \mu_D - \mu_L, \sigma_S^2, \sigma_Y^2 = \sigma_D^2 + \sigma_L^2)$$

and the covariance between $\ln F(T)$ and $\ln Y(T)$ is $c\sigma_S\sigma_D$.

By lognormal partial expectation and the moment expression of joint lognormal variables when a multiplicative constraint is present, it follows that under approximated model, the left hand side of Equation (3.26) is:

$$\begin{aligned} LHS = & F(0) \Phi \left(\frac{\ln(q) - \mu_D + \mu_L - c\sigma_S\sigma_D}{\sqrt{\sigma_D^2 + \sigma_L^2}} \right) L(0) \Phi \left(-\frac{\mu_L}{\sigma_L} - \sigma_L \right) \\ & + F(0) \Phi \left(\frac{\ln(q) - \mu_D}{\sigma_D} - c\sigma_S \right) \Phi \left(\frac{\mu_L}{\sigma_L} \right) \end{aligned} \quad (3.37)$$

Deploying the previously stated notation for convenience, Proposition 3.4 establishes an equation by approximated expression for the quantity to procure forward under the MLN model.

Proposition 3.4. *Suppose spot nominal price and demand forecast are correlated GBMs and all nominal price, demand and availability index processes satisfy the MLN property. Then, the optimal order quantity for forward procurement under the approx-*

imated model is calculated by the equation:

$$L(0)P1 \Phi \left(\frac{\ln(q) - \mu_D + \mu_L - c\sigma_S\sigma_D}{\sqrt{\sigma_D^2 + \sigma_L^2}} \right) + P2 \Phi \left(\frac{\ln(q) - \mu_D}{\sigma_D} - c\sigma_S \right)$$

$$= \frac{A - B}{2A} \{L(0)P1 + P2\} \quad (3.38)$$

if the parameters of the volatilities σ_D, σ_L are not zero.

Notice that we have our decision variable q inside two different standard normal functions on the left hand side of the approximated model, which implies that there is no simple form of analytical solution available for this model. The R-function to numerically obtain the optimal order quantity to procure from forward market is provided below:

```
Find_q_A<-function(D0,F0,L0,varD,varL,varS,c,A,B){

sigmaD<-sqrt(log((varD/(D0^2))+1))
sigmaL<-sqrt(log((varL/(L0^2))+1))
sigmaS<-sqrt(log((varS/(F0^2))+1))

muD<-log(D0)-((sigmaD^2)/2)
muL<-log(L0)-((sigmaL^2)/2)

P1<-pnorm(-(muL/sigmaL)-sigmaL)
P2<-pnorm(muL/sigmaL)

#r...right hand side of equation
r<-{(A-B)*{L0*P1+P2}}/(2*A)
l<-0
u<-D0*1000
lhs<-function(x){
firstTERM<-function(x) {L0*P1*pnorm((log(x)-muD+muL-c*sigmaS*sigmaD)/sqrt(sigmaD^2+sigmaL^2))}
secondTERM<-function(x) {P2*pnorm((log(x)-muD-c*sigmaS*sigmaD)/sigmaD)}
return(firstTERM(x)+secondTERM(x))
}
q<- uniroot(function(x) lhs(x) - r, lower=l, upper=u, tol = 1e-9)$root
q
}
```

Figure 3.3. R-code for Decision by Approximation Method under jointly distributed Price & Demand: Availability Index Truncated.

3.5. Results with the Non-constrained Model

Regarding computational aspect, approximated model, also, does not provide clear-closed form of solution. So, we now proceed to produce an alternative model which considers the availability index process as non-constrained, that is, availability index process as a true log-normal variable can occupy a continuous set of values from 0 to ∞ . Taking the process of $L(t)$ into account rather than $\tilde{L}(t)$, the profit function in (3.5) to be maximized takes the form of:

$$\begin{aligned} V &:= \max_{0 \leq q} (1 + A) \mathbb{E} [F(T)qL(T)] - (1 + A) \mathbb{E} [F(T)D(T)] \\ &\quad - 2A \mathbb{E} [F(T)\{qL(T) - D(T)\}^+] - (1 + B)F(0)q \mathbb{E} [L(T)] \end{aligned} \quad (3.39)$$

which yields the first order optimality condition as:

$$\mathbb{E} [F(T)L(T)\mathbb{1}_{\{D(T) \leq qL(T)\}}] = \frac{(1 + A) \mathbb{E} [F(T)L(T)] - (1 + B)F(0) \mathbb{E} [\tilde{L}(T)]}{2A} \quad (3.40)$$

Adding the assumption of non-constrained lognormality of availability index process through its whole range, permits us to switch from the implicit expressions for the forward procurement quantities to the explicit statement of optimal amount of supply to procure forward, and the following proposition specifies the closed form solution for the optimal procurement value.

Proposition 3.5. *Under the MLN model and the non-constrained availability index process, $L(T) \in [0, \infty)$, the amount of supply to purchase in the forward market that*

optimizes the procurement decision is simply given by:

$$q^* = \exp \left\{ \Phi^{-1} \left(\frac{A - B}{2A} \right) \left(\sqrt{\sigma_D^2 + \sigma_L^2} \right) + \mu_D - \mu_L + c\sigma_S\sigma_D - \sigma_L^2 \right\} \quad (3.41)$$

This result follows from Theorem 3.1 –that the left hand side of Equation (3.7) can be written as:

$$\mathbb{E} [F(T)L(T)\mathbb{1}_{\{D(T) \leq qL(T)\}}] = F(0)L(0)\Phi \left(\frac{\ln(q) - \mu_D + \mu_L - c\sigma_S\sigma_D + \sigma_L^2}{\sqrt{\sigma_D^2 + \sigma_L^2}} \right) \quad (3.42)$$

while under the MLN model and in the case of uncorrelated price and availability index processes, $\mathbb{E} [F(T)L(T)] = F(0)L(0)$ and $\mathbb{E} [L(T)] = L(0)$ hold.

R-code below shows the commands to obtain optimal forward procurement quantity under non-constrained availability index process:

```
Find_q_NC<-function(D0,F0,L0,varD,varL,varS,c,A,B){
  sigmaD<-sqrt(log((varD/(D0^2))+1))
  sigmaL<-sqrt(log((varL/(L0^2))+1))
  sigmaS<-sqrt(log((varS/(F0^2))+1))

  muD<-log(D0)-((sigmaD^2)/2)
  muL<-log(L0)-((sigmaL^2)/2)

  #r...right hand side of equation
  r<-(A-B)/(2*A)
  termT0add<-muD-muL+c*sigmaS*sigmaD-sigmaL^2
  termT0multiply<-sqrt(sigmaD^2+sigmaL^2)
  x<-qnorm(r)*termT0multiply+termT0add
  q<-exp(x)
  q
}
```

Figure 3.4. R-code for Decision under jointly distributed Price & Demand: Availability Index Non-constrained.

4. EXTENSIONS TO THE BASE MODEL

We next examine several extensions of our base model. The first extension allows demand and price processes to be independent but price varies jointly with availability index. Next we extend our model to a procurement scenario in which all of our variables –price, demand and availability– are correlated. For the first scenario, we find an analytic formulation and approximation under some restrictions and for both scenarios, closed-form optimal policies for the non-constrained availability case.

4.1. Bivariate Price-Availability Index Scenario

Initially, we consider the case of correlated demand and price processes. In this section, we relax this assumption and derive optimal policies for the case in which demand is independent of price but price and availability index are treated as correlated random variables. It seems reasonable to relax correlated demand and price, since in the short term, energy sector is not so flexible. For example, demand for electricity is often unresponsive to spot price fluctuations [2]. People place demand on energy regardless of price information. While the demand for commodities in the energy sector is inelastic in the short-run, the available supply for such commodities is supposedly rising and falling for economic reasons [20]. For instance, higher prices make suppliers more competitive and may develop high-quality supplier capability as well as that they encourage the extraction of energy products from more difficult locations with the promise of covering the additional costs [20]. Hence, bivariate distribution to model spot price and availability index is considered satisfactory.

This correlation is also valid for other types of commodities rather than energy products. Concerning the modeling of productivity and quality effects from relative prices, Nguyen [61] explains price variability by productivity shocks and dynamics besides productivity shocks as quality innovation. While pointing out a new and growing literature on inferring quality from price information, the author allows the coexistence of productivity shock and quality innovation in his study and provides the detailed

modeling of prices. For our case, quality is some function of available quantity and so price and availability could have a comovement pattern, either positive or negative.

For jointly modeling the spot nominal price and availability, we impose

$$dW_S(t)dW_L(t) = \rho dt \quad (4.1)$$

where $dW_L(t)$ and $dW_S(t)$ are correlated increments to standard Brownian motion with the correlation parameter ρ .

Introducing correlation between spot nominal price and availability index, it is no longer possible to combine $\mathbb{E} [F(T)\tilde{L}(T)]$ and $F(0)\mathbb{E} [\tilde{L}(T)]$ as like terms. Also, the availability index variable should lie in the interval $[0, 1]$ to be consistent with our base model specification. Hence, deploying the truncated variable $\tilde{L}(T)$ again, the first order optimality condition is given by Equation (3.7), in that case. Then, it necessary to evaluate the expectations of the following form $\mathbb{E} [F(T)\tilde{L}(T)\mathbb{1}_{\{D(T) \leq q\tilde{L}(T)\}}]$, $\mathbb{E} [F(T)\tilde{L}(T)]$ and $\mathbb{E} [\tilde{L}(T)]$.

We readily obtain an expression for the expectation $\mathbb{E} [\tilde{L}(T)]$ (in Section 3.3), and by Theorem 3.1 applied to $\mathbb{E} [F(T)\tilde{L}(T)]$, we have:

$$\mathbb{E} [F(T)\tilde{L}(T)] = \mathbb{E} [F(T)L(T)\mathbb{1}_{\{L(T) \leq 1\}}] + \mathbb{E} [F(T)\mathbb{1}_{\{L(T) > 1\}}] \quad (4.2)$$

$$\begin{aligned} &= F(0)L(0) \exp\{\rho\sigma_S\sigma_L\} \Phi\left(-\frac{\mu_L}{\sigma_L} - \rho\sigma_S - \sigma_L\right) \\ &+ F(0) \Phi\left(\frac{\mu_L}{\sigma_L} + \rho\sigma_S\right) \end{aligned} \quad (4.3)$$

Here, e^Q , as the joint moment of $F(T)$ and $L(T)$, is not equal to $F(0)L(0)$ but with the correlation between spot nominal price and availability index variables, it is now $F(0)L(0) \exp\{\rho\sigma_S\sigma_L\}$.

Likewise, for the left-hand expectation of (3.7), we have:

$$\begin{aligned} \mathbb{E} \left[F(T) \tilde{L}(T) \mathbb{1}_{\{D(T) \leq q \tilde{L}(T)\}} \right] &= \mathbb{E} \left[F(T) L(T) \mathbb{1}_{\{D(T) \leq q L(T)\}} \mathbb{1}_{\{L(T) \leq 1\}} \right] \\ &+ \mathbb{E} \left[F(T) \mathbb{1}_{\{D(T) \leq q\}} \mathbb{1}_{\{L(T) > 1\}} \right] \end{aligned} \quad (4.4)$$

It is straightforward to rewrite the latter expectation on the right as:

$$\mathbb{E} \left[F(T) \mathbb{1}_{\{D(T) \leq q\}} \mathbb{1}_{\{L(T) > 1\}} \right] = \mathbb{E} \left[F(T) \mathbb{1}_{\{L(T) > 1\}} \right] P(D(T) \leq q) \quad (4.5)$$

and evaluating this expression by the multiplicatively constrained moment of bivariate lognormal variables, we obtain:

$$\mathbb{E} \left[F(T) \mathbb{1}_{\{L(T) > 1\}} \right] P(D(T) \leq q) = F(0) \Phi \left(\rho\sigma_S + \frac{\mu_L}{\sigma_L} \right) \Phi \left(\frac{\ln(q) - \mu_D}{\sigma_D} \right) \quad (4.6)$$

To find an expansion for the first expectation on the right hand side, note that it can be expressed as the integral of the instantaneous demand forecast as follows:

$$\mathbb{E} \left[F(T) L(T) \mathbb{1}_{\{D(T) \leq q L(T)\}} \mathbb{1}_{\{L(T) \leq 1\}} \right] = \int_0^q \mathbb{E} \left[F(T) L(T) \mathbb{1}_{\{\frac{x}{q} \leq L(T) \leq 1\}} \right] dF_{D(T)}(x) \quad (4.7)$$

where $F_{D(T)}(\cdot)$ represents the cumulative distribution function of demand.

By applying the formula provided by Lien and Balakrishnan for the moments of bivariate lognormal random variables which are multiplicatively constrained in both sides and introducing $H : F(0)L(0) \exp\{\rho\sigma_S\sigma_L\}$ for ease of notation, it is possible to rewrite the integral as:

$$\int_0^q H \left(\Phi \left(-\frac{\mu_L}{\sigma_L} - \rho\sigma_S - \sigma_L \right) - \Phi \left(\frac{\ln\left(\frac{x}{q}\right) - \mu_L}{\sigma_L} - \rho\sigma_S - \sigma_L \right) \right) dF_{D(T)}(x) \quad (4.8)$$

Additionally, $P3 : \Phi \left(-\frac{\mu_L}{\sigma_L} - \rho\sigma_S - \sigma_L \right)$ and $P4 : \Phi \left(\frac{\mu_L}{\sigma_L} + \rho\sigma_S \right)$ are defined and then, Proposition 4.1 establishes the integral property of the optimal policy.

Proposition 4.1. *Under the MLN model and bivariate spot nominal price and availability index distribution when $L(t)$ is restricted to $[0, 1]$, optimal value of the forward procurement quantity decision satisfies the following equation:*

$$\begin{aligned} & \int_0^q H \left(P3 - \Phi \left(\frac{\ln\left(\frac{x}{q}\right) - \mu_L}{\sigma_L} - \rho\sigma_S - \sigma_L \right) \right) dF_{D(T)}(x) + F(0)P4 \Phi \left(\frac{\ln(q) - \mu_D}{\sigma_D} \right) \\ &= \frac{(1 + A)\{H P3 + F(0)P4\} - (1 + B)F(0)\{L(0)P1 + P2\}}{2A} \end{aligned} \quad (4.9)$$

which requires nonzero spot demand and availability index volatilities.

It must be pointed out that in order to obtain a value for our decision variable q , it is necessary to evaluate integrals of the above-stated form which has q inside the integrand term and as the boundary value also. Such an equation is non-invertible and does not explicitly solve q .

R-commands to numerically calculate the optimal quantity to contract forward is presented next.

```
Find_q_E<-function(D0,F0,L0,varD,varL,varS,rho,A,B){
#rho...Correlation coefficient between the log spot nominal price and availability index
sigmaD<-sqrt(log((varD/(D0^2))+1))
sigmaL<-sqrt(log((varL/(L0^2))+1))
sigmaS<-sqrt(log((varS/(F0^2))+1))

muD<-log(D0)-((sigmaD^2)/2)
muL<-log(L0)-((sigmaL^2)/2)

H<-F0*L0*exp(rho*sigmaS*sigmaL)
P1<-pnorm(-(muL/sigmaL)-sigmaL)
P2<-pnorm(muL/sigmaL)
P3<-pnorm(-(muL/sigmaL)-rho*sigmaS-sigmaL)
P4<-pnorm((muL/sigmaL)+rho*sigmaS)

#r...right hand side of equation
r<-{(1+A)*{H*P3+F0*P4}-(1+B)*F0*{L0*P1+P2}}/(2*A)
f1 <- function(x){
f2 <- function(y){
H*dlnorm(y,muD,sigmaD,log=FALSE)*(P3-pnorm(((log(y/x)-muL)/sigmaL)-rho*sigmaS-sigmaL))}
return(f2)}

firstTERM <-function(x) {integrate(f1(x), lower=0, upper=x)$value}
secondTERM<-function(x) {F0*P4*pnorm((log(x)-muD)/sigmaD)}
lhs<-function(x){return(firstTERM(x)+secondTERM(x)-r)}
l<-0.000001
u<-D0*1000
q<- uniroot(function(x) lhs(x), lower=l, upper=u, tol = 1e-9)$root
q
}
```

Figure 4.1. R-code for Decision under jointly distributed Price & Availability Index Process: Availability Index Truncated.

4.1.1. Results with the Approximated Model

We now proceed with the same type of reasoning in Section 3.4. Since it is not easy to find the antiderivative of resulted integral term which come along with some computational inconvenience, the integral term is replaced with an approximation.

Working only on the left hand side of the first order optimality condition given by Equation (3.7), it consists of two parts as the availability index variable having values

less than 1 and greater than 1:

$$\mathbb{E} \left[F(T) L(T) \mathbb{1}_{\{D(T) \leq qL(T)\}} \mathbb{1}_{\{L(T) \leq 1\}} \right] + \mathbb{E} \left[F(T) \mathbb{1}_{\{D(T) \leq q\}} \mathbb{1}_{\{L(T) > 1\}} \right] \quad (4.10)$$

For the second expectation term, the following statement has been derived (refer to the preceding section):

$$\mathbb{E} \left[F(T) \mathbb{1}_{\{D(T) \leq q\}} \mathbb{1}_{\{L(T) > 1\}} \right] = F(0) \Phi \left(\frac{\mu_L}{\sigma_L} + \rho \sigma_S \right) \Phi \left(\frac{\ln(q) - \mu_D}{\sigma_D} \right) \quad (4.11)$$

However, it is the difficult part to obtain a clear-closed form expression for the first one. Then, we approximated this expectation such that:

$$\mathbb{E} \left[F(T) L(T) \mathbb{1}_{\{D(T) \leq qL(T)\}} \mathbb{1}_{\{L(T) \leq 1\}} \right] = \mathbb{E} \left[F(T) L(T) \mathbb{1}_{\{D(T) \leq qL(T)\}} \right] P\{L(T) \leq 1\} \quad (4.12)$$

Again, the principle remains the same and the expectation term, on the right, is expanded according to the moment expression of joint lognormal variables when such a multiplicative constraint is present. For this purpose, we introduce two new bivariate lognormal variables. Defining $Y_1(T)$ and $Y_2(T)$ as $F(T)L(T)$ and $D(T)L^{-1}(T)$, respectively, the expectation takes the form of $\mathbb{E} \left[Y_1(T) \mathbb{1}_{\{Y_2(T) \leq q\}} \right]$. Since the product and quotients of lognormally distributed random variables are again lognormal,

$$\begin{aligned} (\ln Y_1(T), \ln Y_2(T)) &\sim N(\mu_{Y_1}, \mu_{Y_2}, \sigma_{Y_1}^2, \sigma_{Y_2}^2) \\ &\sim N(\mu_F + \mu_L, \mu_D - \mu_L, \sigma_S^2 + 2\rho\sigma_S\sigma_L + \sigma_L^2, \sigma_D^2 + \sigma_L^2) \end{aligned}$$

with the covariance between $\ln Y_1(T)$ and $\ln Y_2(T)$ as $(-\rho\sigma_S\sigma_L - \sigma_L^2)$.

The point to be taken into consideration here is that $e^Q = \mathbb{E}[Y_1^1(T)Y_2^0(T)]$ is equivalent to $\mathbb{E}[F(T)L(T) \mid L(T) \leq 1]$ rather than $\mathbb{E}[F(T)L(T)]$.

Thus, the first part of the left hand side expectation of (3.7), namely (4.12) is obtained as:

$$F(0)L(0)\exp\{\rho\sigma_S\sigma_L\} \Phi\left(-\frac{\mu_L}{\sigma_L} - \rho\sigma_S - \sigma_L\right) \Phi\left(\frac{\ln(q) - \mu_D + \mu_L + \rho\sigma_S\sigma_L + \sigma_L^2}{\sqrt{\sigma_D^2 + \sigma_L^2}}\right) \quad (4.13)$$

Since the terms $\mathbb{E}[\tilde{L}(T)]$ and $\mathbb{E}[F(T)\tilde{L}(T)]$ are readily at hand from previous sections, it is not necessary to reproduce them. Substituting Equations (3.29), (4.3), (4.11) and (4.13) into (3.7), with the associated notation, yields the approximate expression presented in the next proposition.

Proposition 4.2. *Suppose that spot nominal price and availability index are correlated under the MLN model as availability index process is truncated at 1. Then, the optimal forward procurement quantity under the approximated model holds:*

$$\begin{aligned} & H P3 \Phi\left(\frac{\ln(q) - \mu_D + \mu_L + \rho\sigma_S\sigma_L + \sigma_L^2}{\sqrt{\sigma_D^2 + \sigma_L^2}}\right) + F(0) P4 \Phi\left(\frac{\ln(q) - \mu_D}{\sigma_D}\right) \\ &= \frac{(1+A)\{H P3 + F(0) P4\} - (1+B)F(0)\{L(0)P1 + P2\}}{2A} \end{aligned} \quad (4.14)$$

when $\sigma_D \neq 0$ and $\sigma_L \neq 0$.

Once again, the clear-closed form expression for the optimal forward procurement value is not characterized but it is possible to solve q numerically.

R-code routine that generates the value of optimal forward procurement quantity under this approximated model is given below:

```
Find_q_A<-function(D0,F0,L0,varD,varL,varS,rho,A,B){

sigmaD<-sqrt(log((varD/(D0^2))+1))
sigmaL<-sqrt(log((varL/(L0^2))+1))
sigmaS<-sqrt(log((varS/(F0^2))+1))

muD<-log(D0)-((sigmaD^2)/2)
muL<-log(L0)-((sigmaL^2)/2)

H<-F0*L0*exp(rho*sigmaS*sigmaL)
P1<-pnorm(-(muL/sigmaL)-sigmaL)
P2<-pnorm(muL/sigmaL)
P3<-pnorm(-(muL/sigmaL)-rho*sigmaS-sigmaL)
P4<-pnorm((muL/sigmaL)+rho*sigmaS)

#r...right hand side of equation
r<-{(1+A)*H*P3+(1+A)*F0*P4-(1+B)*F0*{L0*P1+P2}}/(2*A)
termT0subtract<-(muD-muL-rho*sigmaS*sigmaL-(sigmaL^2))
termT0divide<-sqrt(sigmaD^2+sigmaL^2)
l<-0
u<-D0*1000
lhs<-function(x){
firstTERM<-function(x) {H*P3*pnorm((log(x)-termT0subtract)/termT0divide)}
secondTERM<-function(x) {F0*P4*pnorm((log(x)-muD)/sigmaD)}
return(firstTERM(x)+secondTERM(x))
}
q<- uniroot(function(x) lhs(x) - r, lower=l, upper=u, tol = 1e-9)$root
q
}
```

Figure 4.2. R-code for Decision by Approximation Method under jointly distributed Price & Availability Index Process: Availability Index Truncated.

4.1.2. Results with the Non-constrained Model

As already discussed in Section 3.5, models with non-constrained availability index processes provide a relatively simple and generally analytical formulation for the optimal forward procurement quantity. Therefore, it comes to develop a model with non-constrained availability index variable for bivariate distribution of spot nominal price and availability index processes.

Letting $L(t)$ to cover its whole domain, the first order optimality condition is written in the way:

$$\mathbb{E} [F(T)L(T)\mathbb{1}_{\{D(T)\leq qL(T)\}}] = \frac{(1+A)\mathbb{E}[F(T)L(T)] - (1+B)F(0)\mathbb{E}[L(T)]}{2A} \quad (3.40)$$

and by the martingale hypothesis, the unbiased estimate of the availability index at the delivery date is just the currently observed value of it, that is, $\mathbb{E}[L(T)] = L(0)$.

Then, the following statements are the immediate consequence of Theorem 3.1 and the joint moment rule for the bivariate lognormal variables, respectively:

$$\mathbb{E} \left[F(T)L(T)\mathbb{1}_{\{\frac{D(T)}{L(T)} \leq q\}} \right] = F(0)L(0)\exp\{\rho\sigma_S\sigma_L\}\Phi \left(\frac{\ln(q) - \mu_D + \mu_L + \rho\sigma_S\sigma_L + \sigma_L^2}{\sqrt{\sigma_D^2 + \sigma_L^2}} \right) \quad (4.15)$$

$$\mathbb{E} [F(T)L(T)] = F(0)L(0)\exp\{\rho\sigma_S\sigma_L\} \quad (4.16)$$

Taking all together yields the next proposition.

Proposition 4.3. *Under the MLN model and bivariate price and availability index processes in the case of the non-constrained availability index, $L(T) \in [0, \infty)$, the optimal forward procurement quantity is explicitly characterized by:*

$$\begin{aligned} q^* &= \exp \left\{ \Phi^{-1} \left(\frac{(1+A)-(1+B)\exp\{-\rho\sigma_S\sigma_L\}}{2A} \right) \left(\sqrt{\sigma_D^2 + \sigma_L^2} \right) \right. \\ &\quad \left. + \mu_D - \mu_L - \rho\sigma_S\sigma_L - \sigma_L^2 \right\} \end{aligned} \quad (4.17)$$

It is worth pointing out that, if the mean and volatility parameters of the underlying availability index process are eligible such that $L(T)$ has the values below 1 at the most of the time, this case becomes more realistic and Proposition 4.3 provides very good approximation for the models with truncated availability index processes.

R-function that returns the optimal order quantity under non-constrained distribution of availability index $L(t)$ is provided next:

```
Find_q_NC<-function(D0,F0,L0,varD,varL,varS,rho,A,B){
sigmaD<-sqrt(log((varD/(D0^2))+1))
sigmaL<-sqrt(log((varL/(L0^2))+1))
sigmaS<-sqrt(log((varS/(F0^2))+1))

muD<-log(D0)-((sigmaD^2)/2)
muL<-log(L0)-((sigmaL^2)/2)

#r...right hand side of equation
r<-((1+A)-(1+B)*exp(-rho*sigmaS*sigmaL))/(2*A)
termT0add<-muD-muL-rho*sigmaS*sigmaL-sigmaL^2
termT0multiply<-sqrt(sigmaD^2+sigmaL^2)
x<-qnorm(r)*termT0multiply+termT0add
q<-exp(x)
q
}
```

Figure 4.3. R-code for Decision under jointly distributed Price & Availability Index Process: Availability Index Non-constrained.

4.2. Trivariate Procurement Scenario

After considering the problems with correlation and insignificance among the demand and price variables in the first and second set of models, as an extension, we model all three random variates -the spot demand, nominal price and availability index- as correlated random variables in this section. Thus, we again incorporate the standard assumption of correlated price-demand relationship to our model, in addition to the correlation between demand and availability index.

Configuring price and demand as correlated random variables does not conflict with the independent price-demand assumption in Section 4.1, because demand on en-

ergy is less likely to react to price fluctuations in the short term, but in the long run, high prices stimulate firms to look for available substitutes or to economize on their energy consumption. Depending on availability and price of substitute commodities, buyers can replace their inputs with other types of energy. According to Pindyck [27], price fluctuations themselves (whether caused by fluctuations in demand or something else, such as speculative buying and selling) can cause consumption and production to fluctuate. Thus, volatility in price can be accompanied by volatilities of demand and availability. It may be interesting here to note that availability of the commodity indicates pretty much the supplier capability to deliver what it promises, so our availability index. Furthermore, there are examples of energy crises in which sustained changes in availability of energy products lead to changes in demand for those, [62]. Hence, it is reasonable to assume that the changes in availability levels are accompanied by changes in demand. Saying the other way around, this section works through the typical case of that the price of the traded commodity is established based on the supply (i.e. availability) and demand in the market.

Introducing θ as the correlation coefficient between the natural logarithms of spot demand and availability index, we have the following:

$$dW_D(t)dW_L(t) = \theta dt \quad (4.18)$$

in addition to previously imposed correlations:

$$dW_S(t)dW_D(t) = c dt \quad (4.19)$$

$$dW_S(t)dW_L(t) = \rho dt \quad (4.20)$$

where $dW_D(t)$, $dW_L(t)$ and $dW_S(t)$ are correlated increments to standard Brownian

motions with stated correlation parameters. Such extension, however, adds complexity to the calculation of the optimal procurement quantities under truncated availability index process, so it is only considered the procurement policy under non-constrained availability index process in trivariate case.

The purpose here is, again, to find an optimal procurement policy so as to maximize the total expected profit at time T which is given by Equation (3.39) and so, the first order optimality condition is identical to Equation (3.40) derived in Chapter 3.

Considering correlations between variables, we exploit the property that the product of two lognormal random variables remains lognormal and thus, it is possible to reduce our trivariate model to bivariate lognormals. So, proceeding in a similar fashion with Section 4.1, we again specify $Y_1(T) = F(T)L(T)$ and $Y_2(T) = D(T)L^{-1}(T)$.

Notice that the variance and covariance terms are now different, that is:

$$\begin{aligned} (\ln Y_1(T), \ln Y_2(T)) &\sim N(\mu_{Y_1}, \mu_{Y_2}, \sigma_{Y_1}^2, \sigma_{Y_2}^2) \\ &\sim N(\mu_F + \mu_L, \mu_D - \mu_L, \sigma_S^2 + 2\rho\sigma_S\sigma_L + \sigma_L^2, \sigma_D^2 - 2\theta\sigma_D\sigma_L + \sigma_L^2) \end{aligned}$$

and the covariance between $\ln Y_1(T)$ and $\ln Y_2(T)$ is $(c\sigma_S\sigma_D - \rho\sigma_S\sigma_L + \theta\sigma_D\sigma_L - \sigma_L^2)$ in that case.

Then, it is straightforward to deploy the explicit joint moment formulas for bivariate lognormal variables under the presence of a multiplicative constraint and the main result of this section is the explicit calculation of the forward procurement quantity as stated in the following proposition.

Proposition 4.4. *The optimal forward procurement quantity for the trivariate case with the non-constrained availability index process, $L(T) \in [0, \infty)$, under the MLN*

model is given by:

$$q^* = \exp \left\{ \Phi^{-1} \left(\frac{(1+A)-(1+B) \exp\{-\rho\sigma_S\sigma_L\}}{2A} \right) \left(\sqrt{\sigma_D^2 - 2\theta\sigma_D\sigma_L + \sigma_L^2} \right) \right. \\ \left. + \mu_D - \mu_L + c\sigma_S\sigma_D - \rho\sigma_S\sigma_L + \theta\sigma_D\sigma_L - \sigma_L^2 \right\} \quad (4.21)$$

The implication is again that, to define the optimal forward procurement value uniquely, the model parameters must be eligible so that correlation structure and volatility characteristics of the underlying processes satisfy:

$$\left(\frac{(1+A)-(1+B) \exp\{-\rho\sigma_S\sigma_L\}}{2A} \right) \in (0, 1) \quad (4.22)$$

R-function to calculate optimal quantity to purchase forward under trivariate – joint spot demand, nominal price and availability index – process while availability index is non-constrained is performed as in the following box.

```
Find_q_NC_allCORRELATED<-function(D0,F0,L0,varD,varL,varS,rho,c,theta,A,B){
#theta...Correlation coefficient between the log spot demand and availability index

sigmaD<-sqrt(log((varD/(D0^2))+1))
sigmaL<-sqrt(log((varL/(L0^2))+1))
sigmaS<-sqrt(log((varS/(F0^2))+1))

muD<-log(D0)-((sigmaD^2)/2)
muL<-log(L0)-((sigmaL^2)/2)

#r...right hand side of equation
r<-((1+A)-(1+B)*exp(-rho*sigmaS*sigmaL))/(2*A)
termT0add<-muD-muL+c*sigmaS*sigmaD-rho*sigmaS*sigmaL+theta*sigmaD*sigmaL-(sigmaL^2)
termT0multiply<-sqrt(sigmaD^2-2*theta*sigmaD*sigmaL+sigmaL^2)
x<-qnorm(r)*termT0multiply+termT0add
q<-exp(x)
q
}
```

Figure 4.4. R-code for Decision: Trivariate Procurement Process.

5. COMPUTATIONAL RESULTS

The details of the implementation and the numerical results for the routines are provided in this chapter.

5.1. Data

This study does not apply the models on real data due to the nonexistence of relevant data of the spot and forward markets, but makes use of possible data intervals based on the studied paper by Secomandi and Kekre [41] and gives the essential framework of how to proceed when the necessary data are available.

To do so, the study makes use of three types of data set, namely those that characterize the underlying asset with bivariate price and demand accompanied by a univariate availability index, with bivariate price and availability index accompanied by an insensitive but stochastic demand, and finally with a trivariate -price, demand and availability index- profit function.

Table 5.1 next summarizes the baseline parameter values that are intended to be representative of actual values. Following Secomandi and Kekre [41], as they study a hypothetical natural gas distributor operating in U.S., demand units are given in MMBtu (one million British thermal unit) and prices are \$ per MMBtu. To understand the behaviour of the optimal purchase policy with changing parameters, we change one parameter –a pair of parameters for transaction costs– at a time through implementation while keeping the other parameters fixed at the base values. Thus, all combinations of values are considered in computations as each combination is feasible.

Table 5.1. Parameter values employed in the numerical implementation.

	Parameter Values
$D(0)$	10
$F(0)$	5
$L(0)$	0.7, 0.9, 0.95
(A, B)	(0.0375, 0.025), (0.05, 0.025), (0.05, 0.0125)
$Var(D(T))$	1, 3
$Var(L(T))$	0.05, 0.1, 0.2
$Var(S(T))$	0.5, 1, 2
for Set 1: c^*	-0.1, 0.4, 0.6
for Set 2: ρ^*	-0.05, 0.2, 0.35
for Set 3: θ^*	-0.1, 0.1, 0.3

*Feasible combinations of c , ρ and θ are considered in Set 3, for trivariate scenario

As table implies, 216 combinations of different parameter values formed for each transaction costs pair in Set 1 and Set 2, corresponding bivariate price-demand and bivariate price-availability, respectively. In addition to common parameters, for trivariate scenario, there are three parameters varying on the specified values on the table. As they stand for correlation values between variables, computation routine is performed for 756 combinations (for each pair of transaction costs) under which correlation structures are feasible.

Since we specify the length of the time horizon as single period, we evaluate the parameters and objective function at time $t = T = 1$ –the maturity of the forward contract– by employing MLN model properties.

To obtain the insights into the effect of parameters on the value of forward procurement policy, we set parameter variances, i.e. $Var(L(T))$, to certain values. Associated standard deviations are computed inside the R-routines as they corresponds to different values for each initial value and variance combination –for example, by using

$\sigma_L = \sqrt{\ln((\text{Var}(L(T))/(L(0)^2)) + 1)/T}$, we end up with 9 different σ_L values ranging from 0.23 to 0.59, and similarly for other variables.

5.2. Numerical Implementation

We compute optimal purchase quantity and optimal profit for various cases. To evaluate the effectiveness of the approximation and the model considering the availability index process as non-constrained on the continuous set of 0 to ∞ , we include π_A and π_{NC} such that:

$$\pi_A = \frac{V(q^*) - V(q_A^*)}{V(q^*)} \quad (5.1)$$

$$\pi_{NC} = \frac{V(q^*) - V(q_{NC}^*)}{V(q^*)} \quad (5.2)$$

where $V(q_A^*)$ and $V(q_{NC}^*)$ are profits achieved by optimal quantities under approximated and non-constrained models, respectively. In order to quantify likely contribution of incorporating delivery risk, we report the effect of ignoring supply risk on the optimal profit:

$$\pi_S = \frac{V(q^*) - V(q_{L=1}^*)}{V(q^*)} \quad (5.3)$$

Lastly, to measure the value of using the forward purchase option, we compute:

$$\pi_F = \frac{V(q^*) - V(q=0)}{V(q^*)} \quad (5.4)$$

which compares the acting optimally in the forward market with procuring all demand from spot.

5.2.1. Procurement Scenario under Bivariate Price & Demand

In this section, we analyze the profit improvement over the bivariate price-demand model that can be achieved by taking delivery percentage risk into account. We base our comparison on the expected profit obtained by the optimal contract quantity when availability index is not included to the model. Further, we deal with model comparison in order to identify an implied alternative use of other models. Table 5.2 summarizes the results.

Table 5.2. Summary of numerical results for bivariate price-demand scenario.

Quantity	Value Range
q^*	8.56 - 12.59
$V(q^*)$	(-53.31) - (-50.78)
π_A	0.000% - 0.048%
π_{NC}	0.001% - 0.431%
π_S	0.011% - 0.278%
π_F	0.064% - 3.077%

It may be essential to note that the profit function value is negative since it is mainly the cost of forward and spot procurement. The function includes the revenue obtained by spot selling activities but purchasing costs dominates the function. Stating the other way around, our objective is maximizing the minus cost.

As seen, profit is improved when uncertainty of delivery percentage is modelled into the decision making. The resulting optimal profit is higher as 0.011% in the worst case. It sounds as a small improvement but roughly, the value of incorporating delivery uncertainty into procurement policies is to be in 26–147 thousand dollars range on the optimal profit. Clearly, there is also a benefit of forward procurement as the

penalty of not using the forward purchase option exceeds 3% in some cases. Besides, non-constrained model performs slightly behind the exact model, while approximation performs very well, even achieves the same objective function value as the exact model, in numerous instances.

Average behaviour of model for different values of parameters are provided in Table 5.3.

Table 5.3. Performance summary for bivariate price-demand scenario.

Increase in	q^*	$V(q^*)$	π_A	π_{NC}	π_S	π_F
$L(0)$	$\searrow$	$\nearrow$	$\nearrow$	$\nearrow$	$\searrow$	$\nearrow$
$Var(D(T))$	$\searrow$	$c \leq 0 \nearrow$ $c > 0 \searrow$	$\searrow$	$\searrow$	$\nearrow$	$\searrow$
$Var(L(T))$	$\searrow$	$\searrow$	$\nearrow$	$\nearrow$	$\searrow$	$\searrow$
$Var(S(T))$	$c \leq 0 \searrow$ $c > 0 \nearrow$	$c \leq 0 \nearrow$ $c > 0 \searrow$	NC	NC	NC	NC
c	$\nearrow$	$\searrow$	NC	NC	NC	NC

NC stands for inconclusiveness -the influence of the parameter is very small or statistically insignificant

Across all our instances for the base model, the volume procured in the forward market decreases when availability of commodities and variability in the delivery of them are expected to increase. Also the applicability of approximated and non-constrained models weakens and the penalty of ignoring delivery risk decreases. Namely, when availability is high in the market, decision makers do not need to smooth imperfect deliveries by forward procurement and do not prefer to buy more from forward as increasing variability of deliveries is a measure of the supplier incapability. When delivery percentages get close to the contracted quantities, our problem converges to the previously studied cases where just stochastic price and demand exists for the product (refer [41] for such a problem) and the value of modelling supply availability decreases.

With respect to the stochastic price-demand relationship, the effect of demand variance, as well as price variance, differ on the behaviour of contract arrangements. To further illustrate, the next figure plots the impact of demand variance on the optimal profit value relative to price-demand correlation. As seen, profit increases with demand variability when the correlation is negative, however, the decrease in the profit value with increasing demand variance becomes stronger as the magnitude of price-demand correlation evolves in positive direction. Intuitively, it follows from that as demand varies greatly, it is more likely that the contracted amount departs more from the realized demand at the delivery date. So, the firm trades in spot with higher transaction costs and volatile prices, resulting a decrease in profit. (Increasing demand volatility decreases forward procurement quantities and the option value in the instances of Secomandi and Kekre [41] also, as demand correlation with price is positive.) However, when price and demand are negatively correlated, it may be the case demand declines than expected levels and the procured surplus value is sold spot with higher prices (or vice versa), leading higher profits for firm.

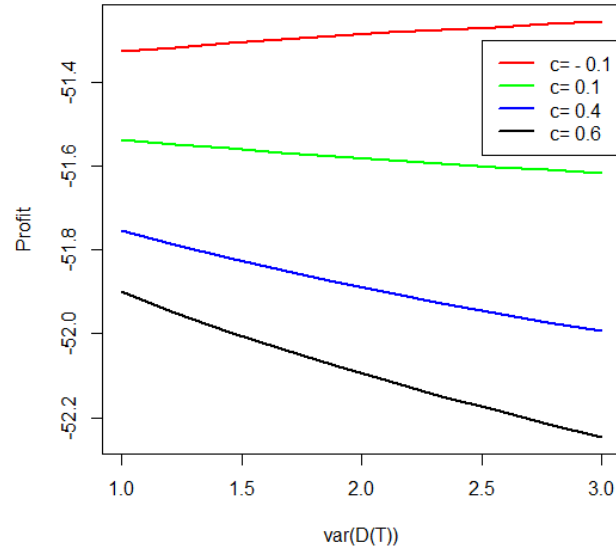


Figure 5.1. Optimal profit as a function of variability in demand, $Var(D(T))$, with respect to the correlation between log spot demand and nominal price, c .

$$D(0) = 10, F(0) = 5, L(0) = 0.7, Var(L(T)) = 0.05, Var(S(T)) = 0.5, A = 0.0375, \\ B = 0.025$$

Similar results are obtained for increased price variance and the associated plots of the optimal policy in terms of contract quantity and profit value over the varying price variability parameter for the different values of price-demand correlation are provided in Appendix B.1.

Another observation is that, transaction costs in the spot and forward markets, which are referred as A and B , respectively, matter on the behavior of purchase policy with respect to other parameters. We find that, usually, optimal forward quantities and profit values decreases or increases faster as the difference between transaction costs gets larger. To illustrate, optimal profit as a function of variability in demand, $Var(D(T))$ is re-plotted with respect to different price-demand correlation values and also the transaction costs this time (see Figure 5.2). Graphically speaking, more sloping curves of $A = 0.05$ and $B = 0.0125$ values imply that it is the difference of spot and forward transaction costs leading to lower profits for a constant change in demand variability and a given value of price-demand correlation.

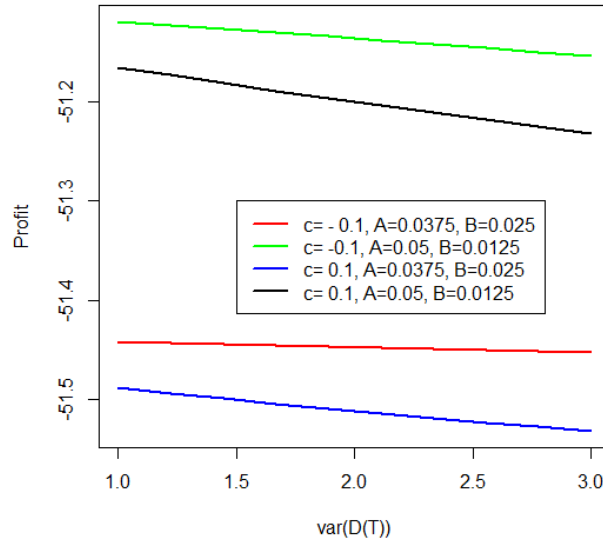


Figure 5.2. Optimal profit as a function of variability in demand, $Var(D(T))$, with respect to the correlation between log spot demand and nominal price, c , and transaction costs, A and B .

$$D(0) = 10, F(0) = 5, L(0) = 0.7, Var(L(T)) = 0.05, Var(S(T)) = 0.5$$

Again, as the difference between transaction costs gets larger, an upward trend in the effect of ignoring supply risk on the optimal profit, that is π_S , is observed.

5.2.2. Procurement Scenario under Bivariate Price & Availability Index

In this section we present the numerical example of optimal procurement policy using Proposition 4.1 and illustrate its value function. To compare the numerical efficiency of the exact model, numerical results are given for both approximated and non-constrained models described in Sections 4.1.1 and 4.1.2, respectively.

It should be mentioned that in the practical implementation of the exact quantity expression under bivariate price and availability index scenario, the choice of parameters is crucial. From Proposition 4.1, the quantity q takes place as a boundary of an integral as well as a denominator of a fraction inside the integrand term, so it is hard to solve for q .

The R-code provided (see Figure 4.1) can solve for it numerically, however, it can be still problematic. By the choice of parameters, if q to be yield becomes too large, the method might diverge, while a too small result for it, might stuck in division by zero error. That is to say, the code provided in the Section 4.1 is limited in scope, but the ultimate goal is to implement the basic computational approach in the routine. So, as a practical matter, optimization model built around q_{NC} (the optimal quantity in the case of availability index is non-constrained over its horizon $[0, \infty)$) could be a workable alternative.

We facilitated the R-code computing optimal order quantity in Proposition 4.1 by searching a reasonable interval around its non-constrained reciprocity.

```

Find<-function(D0,F0,L0,varD,varL,varS,rho,A,B){

sigmaD<-sqrt(log((varD/(D0^2))+1))
sigmaL<-sqrt(log((varL/(L0^2))+1))
sigmaS<-sqrt(log((varS/(F0^2))+1))

muL<-log(L0)-((sigmaL^2)/2)
muD<-log(D0)-((sigmaD^2)/2)
H<-F0*L0*exp(rho*sigmaS*sigmaL)
P1<-pnorm(-(muL/sigmaL)-sigmaL)
P2<-pnorm(muL/sigmaL)
P3<-pnorm(-(muL/sigmaL)-rho*sigmaS-sigmaL)
P4<-pnorm((muL/sigmaL)+rho*sigmaS)
#r...right hand side of equation
r<-{(1+A)*{H*P3+F0*P4}-(1+B)*F0*{L0*P1+P2}}/(2*A)

f1 <- function(x){
f2 <- function(y){
H*dlnorm(y,muD,sigmaD,log=FALSE)*(P3-pnorm(((log(y/x)-muL)/sigmaL)-rho*sigmaS-sigmaL))}
return(f2)}

firstTERM <-function(x) {integrate(f1(x), lower=0, upper=x)$value}
secondTERM<-function(x) {F0*P4*pnorm((log(x)-muD)/sigmaD)}

q<-Find_q_NC(D0,F0,L0,varD,varL,varS,rho,A,B)

quan<-seq(q*0.5,q*5,length=100000)

le<-length(quan)
left<-matrix(0,ncol=le,nrow=1)

for(i in 1:le){
left[1,i]<-firstTERM(quan[i])+secondTERM(quan[i])

if(left[1,i]-r>=0){
return(quan[i])
break
}
}
}

```

Figure 5.3. R-code for Decision under jointly distributed Price & Demand by Interval Searching Method: Availability Index Truncated.

As a correlation between price and supply delivery is introduced in the model, that is ρ , we also investigate the penalty of ignoring delivery risk-spot price correlation here, in addition to the penalty of ignoring supply risk. For this purpose, we include the below notation with the corresponding equation as:

$$\pi_c = \frac{V(q^*) - V(q_{\rho \equiv 0}^*)}{V(q^*)} \quad (5.5)$$

Results of the numerical implementation performed in this section are reported in Table 5.4 below.

Table 5.4. Summary of numerical results for bivariate price-availability index scenario.

Quantity	Value Range
q^*	8.62 - 18.49
$V(q^*)$	(-51.99) - (-49.55)
π_A	0.018% - 0.243%
π_{NC}	0.000% - 0.654%
π_S	0.030% - 1.349%
π_c	0.002% - 0.668%
π_F	0.313% - 5.954%

With our proposed bivariate price-availability index scheme, the effect of ignoring supply risk on the optimal profit seems to be growing as it strikes more away from zero, than the previously reported for the bivariate price-demand scenario in Section 5.2.1. The value of incorporating delivery uncertainty into optimization amounts to improving the profit by about 18–669 thousand dollars. It is also found that forward procurement option plays a significant role in inducing higher profits. The magnitude of optimization benefit over forward markets can be up to the range 0.9–2.95 million dollars while pure-spot-procurement penalty is within about 0.3 to 6 percent of the optimal profit. Further, the results from the non-constrained model agree very well with the exact solution but the model tends to charge larger penalties than the approximated model, on average. Again, they are both applicable since they perform reasonably well as alternatives to the exact modelling.

To foster our insights, Table 5.5 reports the behavior of the optimal quantity, profit and the associated penalty terms as we let the parameters increase over their ranges. It is seen that decomposing total effects into two parts as the parameter $\rho \leq 0$ and $\rho > 0$ for some cases is fairly essential.

Table 5.5. Performance summary for bivariate price-availability index scenario.

Increase in	q^*	$V(q^*)$	π_A	π_{NC}	π_S	π_c	π_F
$L(0)$	$\searrow$	$\rho \leq 0 \nearrow$ $\rho > 0 \searrow$	$\nearrow$	$\nearrow$	$\searrow$	$\searrow$	$\rho \leq 0 \nearrow$ $\rho > 0 \searrow$
$Var(D(T))$	$\searrow$	$\searrow$	$\searrow$	$\searrow$	$\nearrow$	$\nearrow$	$\searrow$
$Var(L(T))$	$\rho \leq 0 \searrow$ $\rho > 0 \nearrow$	$\rho \leq 0 \searrow$ $\rho > 0 \nearrow$	$\nearrow$	$\nearrow$	$\rho \leq 0 \searrow$ $\rho > 0 \nearrow$	$\nearrow$	$\rho \leq 0 \searrow$ $\rho > 0 \nearrow$
$Var(S(T))$	$\rho \leq 0 \searrow$ $\rho > 0 \nearrow$	$\rho \leq 0 \searrow$ $\rho > 0 \nearrow$	NC	NC	$\nearrow$	$\nearrow$	$\rho \leq 0 \searrow$ $\rho > 0 \nearrow$
ρ	$\nearrow$	$\nearrow$	NC	NC	$\nearrow$	$\nearrow$	$\nearrow$

NC stands for inconclusiveness -the influence of the parameter is very small or statistically insignificant

In practice, we expect that there would be a decrease in the forward trading quantities as the commodity availability in the market increases because the buyer incurs a smaller risk per delivery. The downward trends in the penalty terms of ignoring supply risk and supply-price correlation are fairly predictable with the same reasoning. In the event that price and availability of supplies are positively correlated and prices are to go up, availability also increases and the firms can sell the surplus in spot with higher prices and the overall profit, as well as the value of forward procurement increases. Table 5.5 reports these effects as expected.

Another effect that is identified clearly is that an increase in $VarD(T)$ decreases the optimal quantity to procure forward, overall profit and the relative advantage forward procurement (See π_F). As the magnitude of demand variance is larger than others, it accounts mainly for the variability of our model performance indicators.

Results also demonstrate that the value of forward contracts to the buyer increases with increased volatility of the spot price, with increased volatility of supply deliveries, as well as increased correlation between spot price and supply delivery (when positive).

It is meaningful as managing supply and price uncertainties is key for our model.

For illustration purposes, plots are provided in Appendix B.2.

As stated earlier, transaction costs associated with spot markets are assumed to be higher than those in forward markets in our model, in line with real world occurrences. Recall that the difference between them matters on the analysis of results. This result is caused by the fact that in the case of bivariate price and demand with delivery structure is simply independent of them, the $(A - B)$ difference appear on the right side of propositions. The model is governed by the difference of transaction costs. On the other hand, for bivariate price-availability index case, $(1 + A)$ term –associated with spot purchase– starts to dominate the right side. It appears to be difficult to identify the effect of $(1 + B)$ term, if any. Hence, we make use of this information to plot the following figure and also we observe that with an increasing spot transaction cost A , penalties of ignoring delivery uncertainty and its correlation with price, π_S and π_c respectively, increases. (For π_S values regarding the spot and forward market transaction costs, see Table B.1 in Appendix B.3).

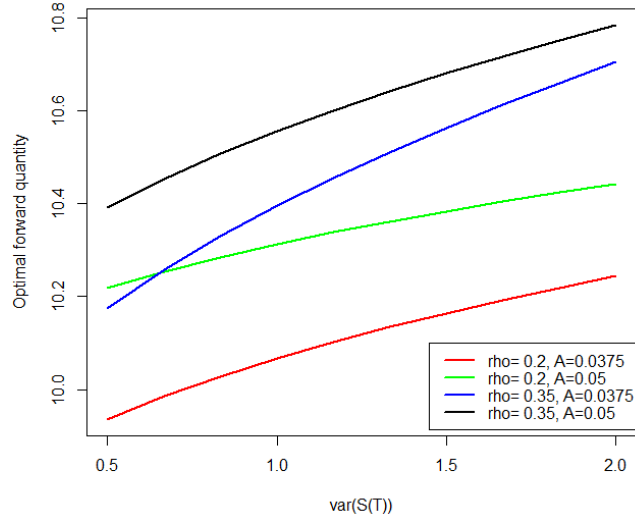


Figure 5.4. Optimal profit as a function of variability in demand, $Var(S(T))$, with respect to the correlation coefficient between log spot nominal price and availability index, ρ , and spot transaction cost A .

$$D(0) = 10, F(0) = 5, L(0) = 0.9, Var(D(T)) = 1, Var(L(T)) = 0.1, B = 0.025$$

5.2.3. Trivariate Procurement Scenario

To gain further insights into the behaviour of the models, numerical studies to investigate the effects of parameters under trivariate correlation structure are conducted next.

Table 5.6. Summary of numerical results for trivariate scenario.

Quantity	Value Range
q^*	4.44 - 28.48
$V(q^*)$	(-53.74) - (-48.21)
π_S	0.000% - 2.609%
π_c	0.000% - 2.845%
π_F	0.095% - 8.569%

The reported values in the above table imply that the penalty terms for ignoring delivery uncertainty and its correlation with price in the procurement may result zero as they add no value to our optimal profit or may increase beyond the 2.5% which can result in significant reductions in our possible profit. Such findings do not invalidate the basic finding of our model, that is the inclusion of uncertain aspect of deliveries to decision making improves our overall profit. It is the long-run commodity market dynamics appear here. As discussed earlier, in the long term, our three variable are all correlated and the correlation values between them sometimes scales up their relative effect and sometimes fluctuations in one variable are smoothed by fluctuations of another variable. For instance, the demand for future is likely to be higher when the expected delivery levels or availability are low for a commodity. Since this increases the likelihood of price increments in the market, suppliers look for further sources to make more products available to the market. When they are capable to deliver more commodities, the overall supply availability does not matter much for firms over the period. It seems that the promises of higher delivery percentage outweigh the uncertain delivery risk. On the other hand, in the markets with high concentration, for example crude oil, this is not case. Any danger of invasion or war for the provider countries

results sustained increases in crude oil prices, which directly increases demand for oil and this will again causes the availability or delivered amounts to fall and prices to rise further. Therefore, to account for the variability in the uncertain deliveries is valuable for the profitability of firms.

Table 5.7 evaluates the effects of various input parameters on the model behavior.

Table 5.7. Performance summary for trivariate scenario.

Increase in	q^*	$V(q^*)$	π_S	π_c	π_F
$L(0)$	$\searrow$	$\searrow$	$\searrow$	$\searrow$	$\rho \leq 0 \nearrow$ $\rho > 0 \searrow$
$Var(D(T))$	$\searrow$	$\searrow$	$\rho \leq 0 \searrow$ $\rho > 0 \nearrow$	NC	$\searrow$
$Var(L(T))$	$\rho \leq 0 \searrow$ $\rho > 0 \nearrow$	$\rho \leq 0 \searrow$ $\rho > 0 \nearrow$	$\nearrow$	$\nearrow$	$\rho \leq 0 \searrow$ $\rho > 0 \nearrow$
$Var(S(T))$	$\rho \leq 0 \searrow$ $\rho > 0 \nearrow$	$\rho \leq 0 \searrow$ $\rho > 0 \nearrow$	$\nearrow$	$\nearrow$	$\rho \leq 0 \searrow$ $\rho > 0 \nearrow$
c	$\nearrow$	$\searrow$	$\nearrow$	$\nearrow$	NC
ρ	$\nearrow$	$\nearrow$	$\nearrow$	$\nearrow$	$\nearrow$
θ	$\nearrow$	$\nearrow$	$\nearrow$	$\nearrow$	$\nearrow$

NC stands for inconclusiveness -the influence of the parameter is very small or statistically insignificant

The first row is intuitive. If deliveries are not expected to be much missing, policies are to procure less forward, and ignoring supply uncertainty in modeling is biased less. It must be noted that negative price-availability index correlation implies lower prices expected in future under increasing availability structure. So, the penalty of procuring entirely from spot is large for the instances concerned.

We find that, usually, firms tend to place larger orders forward as variances of the variables increase and they are highly and positively correlated with each other.

Under these circumstances, it can be concluded that it is more valuable to model imperfect delivery and its price correlation into the forward procurement decision as the corresponding penalty terms increases.

For related figures –The optimal procurement policy with respect to $Var(L(T))$ and $Var(S(T))$, respectively– refer Appendix B.3.

5.2.4. Comparison of Scenarios

In the last part of computational analysis, the optimal forward contracted quantities of the different models are compared. It is observed that for all sets of transaction costs, bivariate price-availability index relationship lead to the higher forward orders than trivariate scenario and the model with bivariate price-demand relationship performs in between.

Table 5.8. Forward contract quantities under different scenarios.

q^*	A=0.0375, B=0.025		A=0.05, B=0.025		A=0.05, B=0.0125	
	Average	Range	Average	Range	Average	Range
Case 1	9.60	8.86-10.64	10.16	9.39-11.38	11.00	10.10-12.59
Case 2	10.37	8.62-16.33	10.71	9.33-15.19	11.68	10.06-18.49
Case 3	9.75	4.44-26.95	10.04	6.14-19.87	11.51	7.49-28.48

However, the reported values are very close to each other and to identify insightful results, it is important to distinguish negative and positive correlations between our variables.

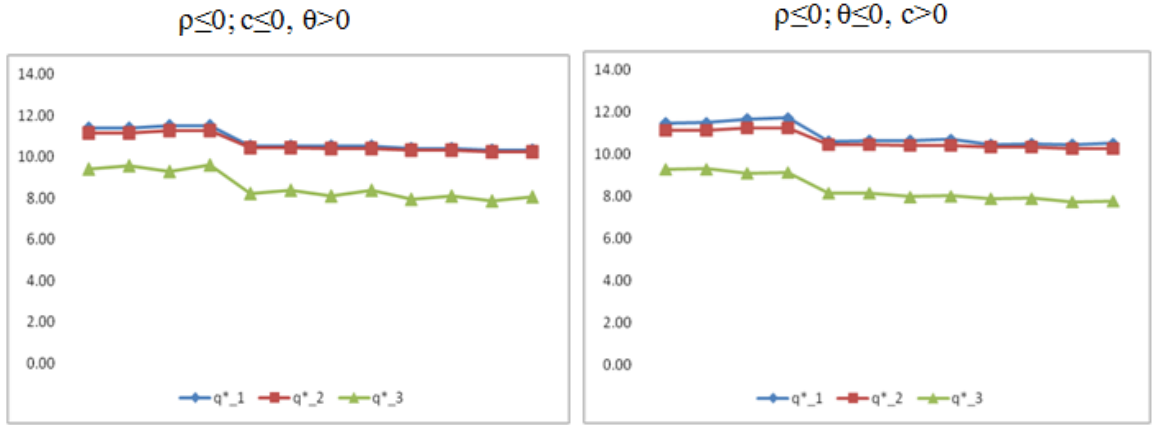


Figure 5.5. Behavior of contract quantities under different scenarios with respect to the correlations, $\rho \leq 0$.

$D(0) = 10$, $F(0) = 5$, $L(0) = 0.7$, $Var(D(T)) = 1$, $Var(L(T)) = 0.1$, $Var(S(T)) = 1$,
 $A = 0.05$, $B = 0.0125$

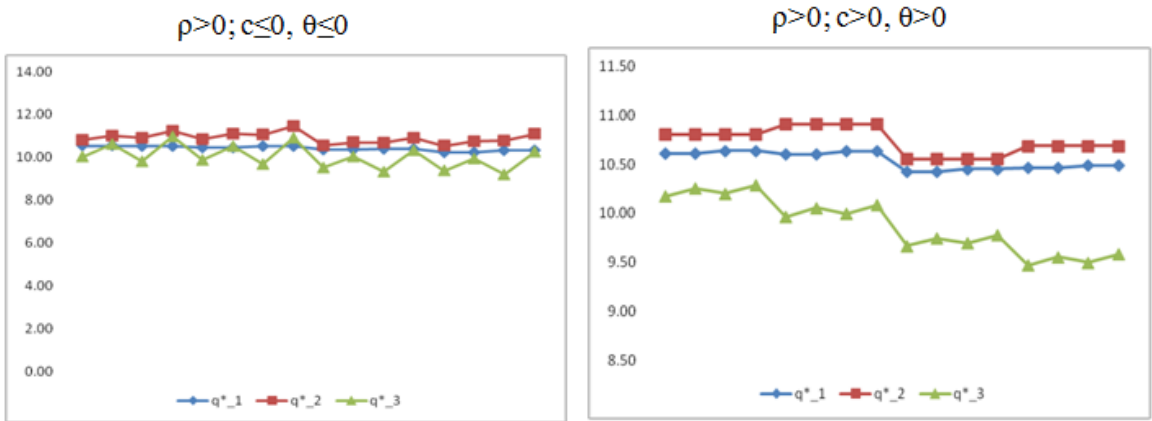


Figure 5.6. Behavior of contract quantities under different scenarios with respect to the correlations, $\rho > 0$.

$D(0) = 10$, $F(0) = 5$, $L(0) = 0.7$, $Var(D(T)) = 1$, $Var(L(T)) = 0.1$, $Var(S(T)) = 1$,
 $A = 0.05$, $B = 0.0125$

It appears that the model corresponding to the trivariate case usually results in lower forward contracted quantities. As volatility increases along with the correlations between price, demand and delivery, the relative advantage of forward market is reduced. Especially, when they are all positively correlated, it is better to order less from forward market.

For Cases 1 and 2, forward procurement quantities under bivariate price and demand scenario do not differ from those under bivariate price-availability conditions at all, when supply availability is negatively correlated with price. Regardless of demand-price correlation is negative or positive, Case 1 solutions are very close to Case 2 solutions, may be slightly higher in some instances.

However, when supply availability and price are positively correlated, Case 2 model results above Case 1 model. The delivery risk, independent from demand and prices in Case 1, may not affect the optimal order quantities much but when it is positively correlated with prices, it is better to order higher volumes from forward, the plots imply.

Table 5.9 and Table 5.10 summarize the results across all our instances.

Table 5.9. Forward contract quantities under different scenarios, $\rho \leq 0$.

q^*	A=0.0375, B=0.025		A=0.05, B=0.025		A=0.05, B=0.0125	
	Average	Range	Average	Range	Average	Range
Case 1	9.56	8.86-10.64	10.12	9.39-11.38	10.96	10.10-12.59
Case 2	9.25	8.62-10.25	9.90	9.33-11.03	10.72	10.06-12.21
Case 3	6.87	4.44-9.88	8.05	6.14-10.95	9.35	7.49-12.22

Table 5.10. Forward contract quantities under different scenarios, $\rho > 0$.

q^*	A=0.0375, B=0.025		A=0.05, B=0.025		A=0.05, B=0.0125	
	Average	Range	Average	Range	Average	Range
Case 1	9.61	8.86-10.64	10.17	9.39-11.38	11.02	10.10-12.59
Case 2	10.82	8.62-16.33	11.03	9.64-15.19	12.06	10.34-18.49
Case 3	9.60	7.46-17.03	9.70	7.87-19.87	10.87	9.19-25.48

Further, during numerical study, we observe that the approximated and non-constrained models order slightly fewer quantities than the optimal model, in general.

As they still yield very close to the exact optimal solutions, they are applicable when making the procurement decision if applied in this context.

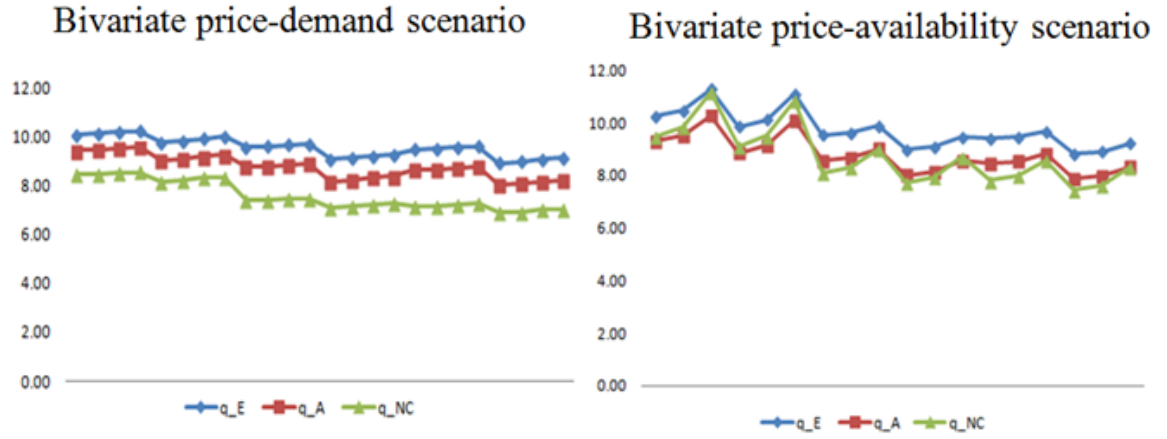


Figure 5.7. Forward contract quantities under 3 approaches.

6. CONCLUSION

In this thesis, we show the existence of an optimal policy for the commodity-trading activities over both forward and spot markets under a three-variable model of the profit function –assuming that the purchase prices, demand and delivery percentage of the contracted amounts are stochastic.

Traditionally, deciding on procurement quantities depends exclusively on the demand forecast and spot nominal prices (either correlated or independent) as demand and prices are expected to fluctuate considerably over short periods of time but the contract volumes are to be ideally delivered at the agreed dates. Distinguishing the existence of volume uncertainties at the supply end, we extend the traditional models through stochastic deliveries included in modeling the procurement policies, in this thesis.

We describe a stochastic model in a single period setting to formulate a procurement strategy and determine the optimal procurement quantity in order to maximize profit through forward contracting. The structure of the optimal contract amounts is derived and the routine that quickly computes the optimal quantities with associated profits is provided. We then propose an approximation procedure to compute heuristic policies and also we allow the arrival of more quantities than the contracted –in addition to deficient deliveries– to compare our base model against them. In the case of non-constrained model, the solution algorithms simplify considerably and the optimal closed-form contract quantities are obtainable. For other cases, very efficient numerical procedures are developed. In general, it is found that the approximated and non-constrained models order slightly fewer quantities than the optimal model and they can perform very well when making the procurement decision if applied in this context.

Further, as the market becomes more volatile, in addition to shifts in demand, price and supply structure, we think that the procurement managers should be able

to anticipate possible interactions between them and plan their respective sourcing strategies, accordingly. By modeling the dynamics of commodities under different correlation structures, we present a number of different characterizations of our models and we discuss applications of these characterizations. Considering the market driven characteristics –stochastic price, demand and availability– in addition to non-storable nature of the commodities, it is deduced that our models are good for procurement-policy optimization in a number of commodity markets, from energy to livestock or products under just-in-time management.

Overall, modeling imperfect deliveries and its possible correlations with price into the procurement decisions is proved to be valuable and the beneficial role of forward procurement, rather than entirely procuring in the spot, is identified. We believe that this paper enhances our understanding about the impact of delivery risks and unreliable suppliers in a supply chain. The other insightful observations from this study are: (i) an increased demand volatility typically decreases the quantity to procure forward, (ii) as transaction costs in spot market increases, it becomes more valuable to include supply uncertainty into the procurement policies.

As stated, this thesis studies the unexplored part of the delivery of forward procurements, particularly in the energy markets. It aims to make a start on the analysis and for simplicity, execution horizon is assumed to be one period. Although time dimension value is relatively considered for short, medium and long terms through different correlation structures, in general, the concept of the time value in multiperiod setting should be introduced in the models. By doing such, it can be possible to account for the variability in the uncertain parameters through time and an extended model may let the update of decisions by considering several procurement dates as more accurate demand, price and supply availability information becomes available.

Incorporating delivery risk into the optimization of forward procurement contracts, we believe, this area of research has a promise to advance the theory and managerial practice of commodity procurement and distribution. Even though the purpose of analyzing the imperfect delivery issue and to proposing an optimal policy

to evaluate the benefit of its inclusion is met, there still exists some room for extension. One limitation lies in the modeling of delivery risk. It can be bothersome to evaluate the delivery process over certain ranges, in our model. Relaxing our assumption of lognormality, more realistic modelling of uncertain deliveries –i.e. a distribution whose boundaries are restricted to certain values– can be built upon this model.

In addition, an interesting and related question can be how to pool the delivery risk for the forward contracts. Benefit of dual or multiple sourcing, so as to reduce the impact of delivery risks, can be considered. Instead of sticking to one supplier, building up a supplier network and the size and conditions of any possible supplier portfolio (in a manner similar to reviewed in [48]) can be matters of further research. Splitting orders between a reliable and an unreliable supplier that offers a purchase price discount also deserves further investigation.

One other possible extension is to relax our assumption of deterministic transaction costs. Combining the framework in our model with transaction costs evolving through time, one can study the supply chains of commodities whose spot and forward market transaction costs are sensitive to product demand and price.

APPENDIX A: OBJECTIVE FUNCTION ANALYSIS

This thesis has started with modeling the procurement policies in spot and forward markets and a decision making problem has been constructed and solved through stochastic modeling. In this section, we begin with showing the concavity of our objective function in q and then, we introduce the expansions of our baseline objective functions that may be of use to evaluate the value of forward procurement, the value of spot procurement and the value of optimally procuring both in the forward and spot markets.

Profit function to be maximized in Sections A.2, A.3, A.5 and A.6 comes directly from (3.5), while it defines the availability in terms of the original (non-constrained) lognormal distribution and corresponds to (3.39) in Sections A.4, A.7 and A.8.

A.1. Proof of Concavity Proposition

With availability index truncation at 1, the profit function is restated below:

$$\begin{aligned} V(q) &= (1 + A)\mathbb{E}\left[F(T)\left(q\tilde{L}(T) - D(T)\right)\right] \\ &\quad - 2A\mathbb{E}\left[F(T)\{q\tilde{L}(T) - D(T)\}^+\right] - (1 + B)F(0)q\mathbb{E}\left[\tilde{L}(T)\right] \end{aligned} \quad (\text{A.1})$$

where $V()$ stands for the value function of the total profit.

Straightforward application of the differential calculus rules gives the following

as the first derivative of our objective function $V(q)$ with respect to q :

$$\begin{aligned} \frac{dV(q)}{dq} &= (1 + A)\mathbb{E} \left[F(T)\tilde{L}(T) \right] - 2A\mathbb{E} \left[F(T)\tilde{L}(T)\mathbb{1}_{\{D(T) \leq q\tilde{L}(T)\}} \right] \\ &\quad - (1 + B)F(0)\mathbb{E} \left[\tilde{L}(T) \right] \end{aligned} \quad (\text{A.2})$$

Note that our variable term q appears only between the curly brackets of indicator function which is found inside the second expectation and it is the same as writing:

$$\begin{aligned} -2A\mathbb{E} \left[F(T)\tilde{L}(T)\mathbb{1}_{\{D(T) \leq q\tilde{L}(T)\}} \right] &= -2A \int_{x=0}^{\infty} \int_{y=0}^1 \int_{z=0}^{qy} x y f(x, y, z) dz dy dx \\ &= -2A \int_{x=0}^{\infty} x \int_{y=0}^1 y \left(\int_{z=0}^{qy} f(x, y, z) dz \right) dy dx \end{aligned} \quad (\text{A.3})$$

when x , y and z stand for $F(t)$, $L(t)$ and $D(t)$, respectively. Then, taking the second derivative with respect to q , we get:

$$\frac{d^2V(q)}{dq^2} = -2A \int_{x=0}^{\infty} \int_{y=0}^1 x y^2 f(x, y, qy) dy dx < 0 \quad (\text{A.4})$$

It follows from (A.4) that $V(q)$ is concave. Since $\frac{dV(0)}{dq} > 0$ and $\frac{dV(\infty)}{dq} < 0$ for $t = T$ (whether $F(t)$ and $L(t)$ are correlated or not), there exists a unique and finite $q^* \in (0, \infty)$ that maximizes $V(q)$ and satisfies the first order condition given by Equation (3.7).

A.2. Expansion of Objective Function under Jointly Distributed Price & Demand

We begin with the case in which spot nominal price and demand forecast are correlated and both are independent of availability index. The expanded version of the profit –stated in (3.5)– is simply:

$$\begin{aligned}
 V &= (1 + A)q \mathbb{E} \left[F(T) \tilde{L}(T) \right] - (1 + A) \mathbb{E} [F(T)D(T)] \\
 &\quad - 2Aq \mathbb{E} \left[F(T) \tilde{L}(T) \mathbb{1}_{\{D(T) \leq q \tilde{L}(T)\}} \right] + 2A \mathbb{E} \left[F(T)D(T) \mathbb{1}_{\{D(T) \leq q \tilde{L}(T)\}} \right] \\
 &\quad - (1 + B)F(0)q \mathbb{E} \left[\tilde{L}(T) \right]
 \end{aligned} \tag{A.5}$$

Owing to the independent nature of price and availability index, it is possible to combine like terms as such:

$$\begin{aligned}
 V &= (A - B)F(0)q \mathbb{E} \left[\tilde{L}(T) \right] - (1 + A) \mathbb{E} [F(T)D(T)] \\
 &\quad - 2Aq \mathbb{E} \left[F(T) \tilde{L}(T) \mathbb{1}_{\{D(T) \leq q \tilde{L}(T)\}} \right] + 2A \mathbb{E} \left[F(T)D(T) \mathbb{1}_{\{D(T) \leq q \tilde{L}(T)\}} \right]
 \end{aligned} \tag{A.6}$$

Setting the coefficients of the expectation terms apart for a while, we can decompose the total profit into four parts:

I. $\mathbb{E} \left[\tilde{L}(T) \right]$

II. $\mathbb{E} [F(T)D(T)]$

$$\text{III. } \mathbb{E} \left[F(T) \tilde{L}(T) \mathbb{1}_{\{D(T) \leq q \tilde{L}(T)\}} \right]$$

$$\text{IV. } \mathbb{E} \left[F(T) D(T) \mathbb{1}_{\{D(T) \leq q \tilde{L}(T)\}} \right]$$

For part I and part III, below statements has been attained in Section 3.3 regarding the right-truncated availability index:

$$\text{I} = L(0) P1 + P2 \quad (\text{A.7})$$

$$\text{III} = \int_0^1 F(0)x \Phi \left(\frac{\ln(qx) - \mu_D}{\sigma_D} - c\sigma_S \right) dF_{L(T)}(x) + F(0) \Phi \left(\frac{\ln(q) - \mu_D}{\sigma_D} - c\sigma_S \right) P2 \quad (\text{A.8})$$

By definition, part II is the joint moment of lognormally distributed variables:

$$\text{II} = F(0) D(0) \exp\{c\sigma_S \sigma_D\} \quad (\text{A.9})$$

and part IV can be constructed as follows:

$$\begin{aligned} & \mathbb{E} \left[F(T) D(T) \mathbb{1}_{\{D(T) \leq q \tilde{L}(T)\}} \right] \\ &= \mathbb{E} \left[F(T) D(T) \mathbb{1}_{\{D(T) \leq q L(T)\}} \mathbb{1}_{\{L(T) \leq 1\}} \right] + \mathbb{E} \left[F(T) D(T) \mathbb{1}_{\{D(T) \leq q\}} \mathbb{1}_{\{L(T) > 1\}} \right] \end{aligned} \quad (\text{A.10})$$

$$\begin{aligned} &= \int_0^1 \mathbb{E} \left[F(T) D(T) \mathbb{1}_{\{D(T) \leq qx\}} \right] dF_{L(T)}(x) + \mathbb{E} \left[F(T) D(T) \mathbb{1}_{\{D(T) \leq q\}} \right] P(L(T) > 1) \end{aligned} \quad (\text{A.11})$$

By Theorem 3.1 applied to the integrand and the offside expectation,

$$\begin{aligned}
\text{IV} &= F(0)D(0)\exp\{c\sigma_S\sigma_D\} \int_0^1 \Phi\left(\frac{\ln(qx) - \mu_D}{\sigma_D} - c\sigma_S - \sigma_D\right) dF_{L(T)}(x) \\
&+ F(0)D(0)\exp\{c\sigma_S\sigma_D\} \Phi\left(\frac{\ln(q) - \mu_D}{\sigma_D} - c\sigma_S - \sigma_D\right) \Phi\left(\frac{\mu_L}{\sigma_L}\right) \quad (\text{A.12})
\end{aligned}$$

Let us introduce some additional notation:

$$G : F(0)D(0)\exp\{c\sigma_S\sigma_D\}$$

Taken together, the overall profit function is expressed as:

$$\begin{aligned}
V &= (A - B)qF(0)\left\{L(0)P1 + P2\right\} - (1 + A)G \\
&- 2AqF(0)\left\{\int_0^1 x \Phi\left(\frac{\ln(qx) - \mu_D}{\sigma_D} - c\sigma_S\right) dF_{L(T)}(x) \right. \\
&\quad \left. + \Phi\left(\frac{\ln(q) - \mu_D}{\sigma_D} - c\sigma_S\right) P2\right\} \\
&+ 2AG\left\{\int_0^1 \Phi\left(\frac{\ln(qx) - \mu_D}{\sigma_D} - c\sigma_S - \sigma_D\right) dF_{L(T)}(x) \right. \\
&\quad \left. + \Phi\left(\frac{\ln(q) - \mu_D}{\sigma_D} - c\sigma_S - \sigma_D\right) P2\right\} \quad (\text{A.13})
\end{aligned}$$

A.3. Expansion of Approximated Objective Function under Jointly Distributed Price & Demand

The terms involving integrals, III and IV, are now simplified by following the same procedure used in approximating the integral term in Section 3.4. Approximation for part III is readily available as:

$$\text{III} = F(0) \left\{ L(0) P1 \Phi \left(\frac{\ln(q) - \mu_D + \mu_L - c\sigma_S\sigma_D}{\sqrt{\sigma_D^2 + \sigma_L^2}} \right) + P2 \Phi \left(\frac{\ln(q) - \mu_D}{\sigma_D} - c\sigma_S \right) \right\} \quad (\text{A.14})$$

Working on the IV. part:

$$\begin{aligned} & \mathbb{E} \left[F(T) D(T) \mathbb{1}_{\{D(T) \leq q\tilde{L}(T)\}} \right] \\ &= \mathbb{E} \left[F(T) D(T) \mathbb{1}_{\{D(T) \leq qL(T)\}} \mathbb{1}_{\{L(T) \leq 1\}} \right] + \mathbb{E} \left[F(T) D(T) \mathbb{1}_{\{D(T) \leq q\}} \mathbb{1}_{\{L(T) > 1\}} \right] \end{aligned} \quad (\text{A.15})$$

Approximating the first expectation and enjoying the independency of availability index from price and demand for the second expectation:

$$= \mathbb{E} \left[F(T) D(T) \mathbb{1}_{\{D(T) \leq qL(T)\}} \right] P(L(T) \leq 1) + \mathbb{E} \left[F(T) D(T) \mathbb{1}_{\{D(T) \leq q\}} \right] P(L(T) > 1) \quad (\text{A.16})$$

Then, Theorem 3.1 yields the following:

$$= G(1 - P2) \Phi \left(\frac{\ln(q) - \mu_D + \mu_L - c\sigma_S\sigma_D - \sigma_D^2}{\sqrt{\sigma_D^2 + \sigma_L^2}} \right) + G P2 \Phi \left(\frac{\ln(q) - \mu_D}{\sigma_D} - c\sigma_S - \sigma_D \right) \quad (\text{A.17})$$

Thus, the original objective function under the approximation assumption takes the form of:

$$\begin{aligned}
V &= (A - B) q F(0) \left\{ L(0) P1 + P2 \right\} - (1 + A) G \\
&- 2A q F(0) \left\{ L(0) P1 \Phi \left(\frac{\ln(q) - \mu_D + \mu_L - c\sigma_S \sigma_D}{\sqrt{\sigma_D^2 + \sigma_L^2}} \right) \right. \\
&\quad \left. + P2 \Phi \left(\frac{\ln(q) - \mu_D}{\sigma_D} - c\sigma_S \right) \right\} \\
&+ 2A G \left\{ (1 - P2) \Phi \left(\frac{\ln(q) - \mu_D + \mu_L - c\sigma_S \sigma_D - \sigma_D^2}{\sqrt{\sigma_D^2 + \sigma_L^2}} \right) \right. \\
&\quad \left. + P2 \Phi \left(\frac{\ln(q) - \mu_D}{\sigma_D} - c\sigma_S - \sigma_D \right) \right\} \tag{A.18}
\end{aligned}$$

A.4. Expansion of Objective Function under Jointly Distributed Price & Demand with Non-constrained Availability Index

When availability index is not constrained to be in the set $[0, 1]$, the profit function is stated as:

$$\begin{aligned}
V &= (A - B) F(0) q \mathbb{E} [L(T)] - (1 + A) \mathbb{E} [F(T) D(T)] \\
&- 2A q \mathbb{E} [F(T) L(T) \mathbb{1}_{\{D(T) \leq qL(T)\}}] + 2A \mathbb{E} [F(T) D(T) \mathbb{1}_{\{D(T) \leq qL(T)\}}] \tag{A.19}
\end{aligned}$$

which involves four main parts to evaluate:

I. $\mathbb{E} [L(T)]$

$$\text{II. } \mathbb{E} [F(T)D(T)]$$

$$\text{III. } \mathbb{E} [F(T)L(T)\mathbb{1}_{\{D(T) \leq qL(T)\}}]$$

$$\text{IV. } \mathbb{E} [F(T)D(T)\mathbb{1}_{\{D(T) \leq qL(T)\}}]$$

We have I and III from Section 3.5. Also, notice that part II does not depend on availability index, whether it is truncated or not, and it has been given in A.2. Eventually, it is straightforward to evaluate part IV by the product moment formula of multiplicatively constrained bivariate lognormal distribution and we end up with:

$$\begin{aligned} V &= (A - B) q F(0)L(0) - (1 + A)G \\ &- 2A q F(0)L(0) \Phi \left(\frac{\ln(q) - \mu_D + \mu_L - c\sigma_S\sigma_D + \sigma_L^2}{\sqrt{\sigma_D^2 + \sigma_L^2}} \right) \\ &+ 2A G \Phi \left(\frac{\ln(q) - \mu_D + \mu_L - c\sigma_S\sigma_D - \sigma_D^2}{\sqrt{\sigma_D^2 + \sigma_L^2}} \right) \end{aligned} \quad (\text{A.20})$$

A.5. Expansion of Objective Function under Jointly Distributed Price & Availability Index

Relaxing the correlation between spot demand forecast and nominal price when adopting a correlation between availability index and nominal price instead, the profit function given by (A.5) is decomposed into five main parts now:

$$\text{I. } \mathbb{E} [F(T)\tilde{L}(T)]$$

$$\text{II. } \mathbb{E} [F(T)D(T)]$$

$$\text{III. } \mathbb{E} [F(T)\tilde{L}(T)\mathbb{1}_{\{D(T) \leq q\tilde{L}(T)\}}]$$

$$\text{IV. } \mathbb{E} \left[F(T) D(T) \mathbb{1}_{\{D(T) \leq q \tilde{L}(T)\}} \right]$$

$$\text{V. } \mathbb{E} \left[\tilde{L}(T) \right]$$

We have from Section 4.1:

$$\text{I. } \mathbb{E} \left[F(T) \tilde{L}(T) \right] = H P3 + F(0) P4 \quad (\text{A.21})$$

$$\text{III. } \mathbb{E} \left[F(T) \tilde{L}(T) \mathbb{1}_{\{D(T) \leq q \tilde{L}(T)\}} \right]$$

$$= \int_0^q H \left(P3 - \Phi \left(\frac{\ln(\frac{x}{q}) - \mu_L}{\sigma_L} - \rho \sigma_S - \sigma_L \right) \right) dF_{D(T)}(x) + F(0) P4 \Phi \left(\frac{\ln(q) - \mu_D}{\sigma_D} \right) \quad (\text{A.22})$$

and from Section 3.3:

$$\text{V. } \mathbb{E} \left[\tilde{L}(T) \right] = L(0) P1 + P2 \quad (\text{A.23})$$

In the case of independent spot demand and nominal price, it holds that:

$$\mathbb{E} [F(T) D(T)] = F(0) D(0) \quad (\text{A.24})$$

for part II.

Deploying integral expression to evaluate expectation, part IV is given by:

$$\begin{aligned}
\text{IV. } & \mathbb{E} \left[F(T) D(T) \mathbb{1}_{\{D(T) \leq q \tilde{L}(T)\}} \right] \\
&= \mathbb{E} \left[F(T) D(T) \mathbb{1}_{\{\frac{D(T)}{q} \leq L(T)\}} \mathbb{1}_{\{L(T) \leq 1\}} \right] + \mathbb{E} \left[F(T) D(T) \mathbb{1}_{\{D(T) \leq q\}} \mathbb{1}_{\{L(T) > 1\}} \right] \\
&= \int_0^q \mathbb{E} \left[F(T) x \mathbb{1}_{\{\frac{x}{q} \leq L(T) \leq 1\}} \right] dF_{D(T)}(x) + \mathbb{E} \left[F(T) \mathbb{1}_{\{L(T) > 1\}} \right] \mathbb{E} \left[D(T) \mathbb{1}_{\{D(T) \leq q\}} \right]
\end{aligned} \tag{A.25}$$

When applied to the above integrand expression and offside expectation, Theorem 3.1 yields:

$$\begin{aligned}
&= F(0) \int_0^q x \left(\Phi \left(-\frac{\mu_L}{\sigma_L} - \rho \sigma_S \right) - \Phi \left(\frac{\ln(\frac{x}{q}) - \mu_L}{\sigma_L} - \rho \sigma_S \right) \right) dF_{D(T)}(x) \\
&+ F(0) \Phi \left(\frac{\mu_L}{\sigma_L} + \rho \sigma_S \right) D(0) \Phi \left(\frac{\ln(q) - \mu_D}{\sigma_D} - \sigma_D \right)
\end{aligned} \tag{A.26}$$

Substituting the obtained expressions for parts I–V and bringing their coefficients back, the expansion of profit function comes out as:

$$\begin{aligned}
V &= (1 + A) q \{ H P3 + F(0) P4 \} - (1 + A) F(0) D(0) \\
&- 2A q \left\{ \int_0^q H \left(P3 - \Phi \left(\frac{\ln(\frac{x}{q}) - \mu_L}{\sigma_L} - \rho \sigma_S - \sigma_L \right) \right) dF_{D(T)}(x) \right. \\
&\quad \left. + F(0) P4 \Phi \left(\frac{\ln(q) - \mu_D}{\sigma_D} \right) \right\}
\end{aligned}$$

$$\begin{aligned}
& + 2A F(0) \left\{ \int_0^q x \left((1 - P4) - \Phi \left(\frac{\ln(\frac{x}{q}) - \mu_L}{\sigma_L} - \rho\sigma_S \right) \right) dF_{D(T)}(x) \right. \\
& \quad \left. + D(0)P4 \Phi \left(\frac{\ln(q) - \mu_D}{\sigma_D} - \sigma_D \right) \right\} \\
& - (1 + B) q F(0) \left\{ L(0)P1 + P2 \right\}
\end{aligned} \tag{A.27}$$

A.6. Expansion of Approximated Objective Function under Jointly Distributed Price & Availability Index

The preceding profit function sets up two integral terms while evaluating the expectations, for the parts III and IV. Our proposed method in Section 4.1.1 approximates the value of integral function in III by treating $D^{-1}(T)L(T)$ and $L(T)$ as they are unrelated. Following the similar manner, part IV results in:

$$\begin{aligned}
\text{IV} &= \mathbb{E} \left[F(T)D(T) \mathbb{1}_{\{D(T) \leq qL(T)\}} \mathbb{1}_{\{L(T) \leq 1\}} \right] + \mathbb{E} \left[F(T)D(T) \mathbb{1}_{\{D(T) \leq q\}} \mathbb{1}_{\{L(T) > 1\}} \right] \\
&= \mathbb{E} \left[F(T)D(T) \mathbb{1}_{\{D(T) \leq qL(T)\}} \right] \mathbb{E} \left[\mathbb{1}_{\{L(T) \leq 1\}} \right] + \mathbb{E} \left[F(T) \mathbb{1}_{\{L(T) > 1\}} \right] \mathbb{E} \left[D(T) \mathbb{1}_{\{D(T) \leq q\}} \right]
\end{aligned} \tag{A.28}$$

By Theorem 3.1:

$$\begin{aligned}
&= F(0)D(0) \Phi \left(-\frac{\mu_L}{\sigma_L} \right) \Phi \left(\frac{\ln(q) - \mu_D + \mu_L + \rho\sigma_S\sigma_L - \sigma_D^2}{\sqrt{\sigma_D^2 + \sigma_L^2}} \right) \\
&+ F(0) \Phi \left(\frac{\mu_L}{\sigma_L} + \rho\sigma_S \right) D(0) \Phi \left(\frac{\ln(q) - \mu_D}{\sigma_D} - \sigma_D \right)
\end{aligned} \tag{A.29}$$

Hence, the approximations lead to the following profit function:

$$\begin{aligned}
V &= (1 + A) q \{ H P 3 + F(0) P 4 \} - (1 + A) F(0) D(0) \\
&- 2 A q \left\{ H P 3 \Phi \left(\frac{\ln(q) - \mu_D + \mu_L + \rho \sigma_S \sigma_L + \sigma_L^2}{\sqrt{\sigma_D^2 + \sigma_L^2}} \right) \right. \\
&\quad \left. + F(0) P 4 \Phi \left(\frac{\ln(q) - \mu_D}{\sigma_D} \right) \right\} \\
&+ 2 A F(0) D(0) \left\{ (1 - P 2) \Phi \left(\frac{\ln(q) - \mu_D + \mu_L + \rho \sigma_S \sigma_L - \sigma_D^2}{\sqrt{\sigma_D^2 + \sigma_L^2}} \right) \right. \\
&\quad \left. + P 4 \Phi \left(\frac{\ln(q) - \mu_D}{\sigma_D} - \sigma_D \right) \right\} \\
&- (1 + B) q F(0) \{ L(0) P 1 + P 2 \}
\end{aligned} \tag{A.30}$$

A.7. Expansion of Objective Function under Jointly Distributed Price & Availability Index with Non-constrained Availability Index

The profit function (A.5) can be reduced to the non-constrained availability index case of this model when we assume $L(T)$ – change of variable for $\tilde{L}(T)$. In this case, the profit function to be maximized is:

$$\begin{aligned}
V &= (1 + A) q \mathbb{E} [F(T) L(T)] - (1 + A) \mathbb{E} [F(T) D(T)] \\
&- 2 A q \mathbb{E} [F(T) L(T) \mathbb{1}_{\{D(T) \leq q L(T)\}}] + 2 A \mathbb{E} [F(T) D(T) \mathbb{1}_{\{D(T) \leq q L(T)\}}] \\
&- (1 + B) F(0) q \mathbb{E} [L(T)]
\end{aligned} \tag{A.31}$$

If expand the contained expectations,

$$\text{I. } \mathbb{E} [F(T)L(T)] = F(0)L(0) \exp\{\rho\sigma_S\sigma_L\} = H \quad (\text{A.32})$$

$$\text{II. } \mathbb{E} [F(T)D(T)] = F(0)D(0) \quad (\text{A.33})$$

$$\text{III. } \mathbb{E} [F(T)L(T)\mathbb{1}_{\{D(T)\leq qL(T)\}}] = H \Phi \left(\frac{\ln(q) - \mu_D + \mu_L + \rho\sigma_S\sigma_L + \sigma_L^2}{\sqrt{\sigma_D^2 + \sigma_L^2}} \right) \quad (\text{A.34})$$

$$\text{IV. } \mathbb{E} [F(T)D(T)\mathbb{1}_{\{D(T)\leq qL(T)\}}] = F(0)D(0)\Phi \left(\frac{\ln(q) - \mu_D + \mu_L + \rho\sigma_S\sigma_L - \sigma_D^2}{\sqrt{\sigma_D^2 + \sigma_L^2}} \right) \quad (\text{A.35})$$

$$\text{V. } \mathbb{E} [L(T)] = L(0) \quad (\text{A.36})$$

Then, adding I–V with the corresponding coefficients lead to:

$$\begin{aligned} V &= (1+A)qH - (1+A)F(0)D(0) \\ &\quad - 2AqH\Phi \left(\frac{\ln(q) - \mu_D + \mu_L + \rho\sigma_S\sigma_L + \sigma_L^2}{\sqrt{\sigma_D^2 + \sigma_L^2}} \right) \\ &\quad + 2AF(0)D(0)\Phi \left(\frac{\ln(q) - \mu_D + \mu_L + \rho\sigma_S\sigma_L - \sigma_D^2}{\sqrt{\sigma_D^2 + \sigma_L^2}} \right) \\ &\quad - (1+B)qF(0)L(0) \end{aligned} \quad (\text{A.37})$$

A.8. Expansion of Objective Function under Trivariate Procurement Process with Non-constrained Availability Index

From the same type of reasoning that gave Equations (A.20) and (A.37), the original expression for the profit –when all three processes are correlated and availability is not restricted to be in the range $[0, 1]$ – expands fully as:

$$\begin{aligned}
 V &= (1 + A) q H - (1 + A) G \\
 &- 2A q H \Phi \left(\frac{\ln(q) - \mu_D + \mu_L - c \sigma_S \sigma_D + \rho \sigma_S \sigma_L - \theta \sigma_D \sigma_L + \sigma_L^2}{\sqrt{\sigma_D^2 - 2\theta \sigma_D \sigma_L + \sigma_L^2}} \right) \\
 &+ 2A G \Phi \left(\frac{\ln(q) - \mu_D + \mu_L - c \sigma_S \sigma_D + \rho \sigma_S \sigma_L - \sigma_D^2 + \theta \sigma_D \sigma_L}{\sqrt{\sigma_D^2 - 2\theta \sigma_D \sigma_L + \sigma_L^2}} \right) \\
 &- (1 + B) q F(0) L(0)
 \end{aligned} \tag{A.38}$$

APPENDIX B: PLOTS – BEHAVIOR OF THE MODELS WITH RESPECT TO PARAMETERS

B.1. Plots for the Procurement Scenario under Bivariate Price & Demand

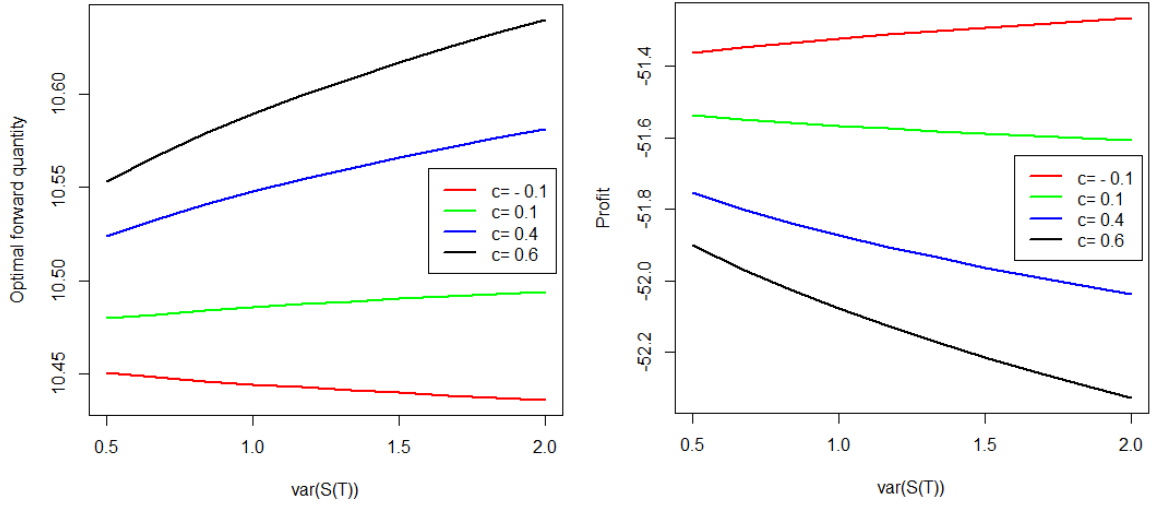


Figure B.1. Optimal policy as a function of variability in price, $Var(S(T))$, and the correlation between log spot demand and nominal price, c .

$D(0) = 10$, $F(0) = 5$, $L(0) = 0.7$, $Var(D(T)) = 1$, $Var(L(T)) = 0.05$, $A = 0.0375$, $B = 0.025$

From Figure B.1, it appears that a risk-averse decision maker may invest more in forward procurement as the variance in price increases when the correlation between log spot demand and nominal price is positive and on the rise. (See the last two rows of Table 5.3, also.) On the other hand, variance in the commodity prices decreases the firm's share of forward procurement for the negative values of demand-price correlation.

B.2. Plots for the Procurement Scenario under Bivariate Price & Availability Index

Given the spot price and price variance, Figure B.2 compares the forward procurement quantities and associated profit valuations under different price and availability index correlation values.

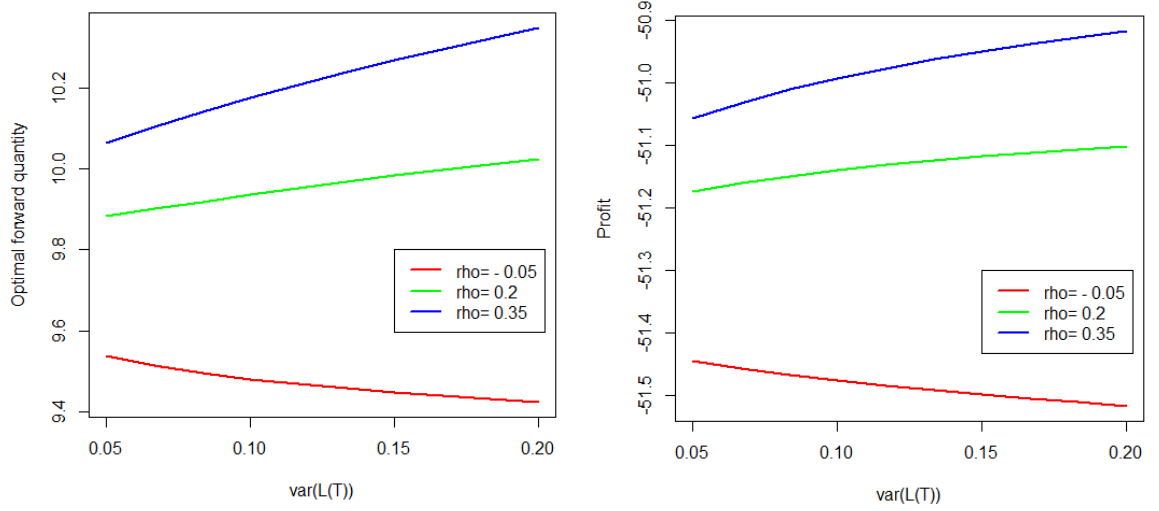


Figure B.2. Optimal policy as a function of variability in supply availability, $Var(L(T))$, with respect to the correlation coefficient between log spot nominal price and availability index, ρ .

$$D(0) = 10, F(0) = 5, L(0) = 0.9, Var(D(T)) = 1, Var(S(T)) = 0.5, A = 0.0375, B = 0.025$$

When assuming spot nominal price and delivery percentage as positively correlated stochastic processes, an increase in $Var(L(T))$ leads to a relatively higher amount of forward orders and also profits in the case of large correlation values. The impact of $Var(L(T))$ increase is on the contrary when correlation coefficient ρ is negative, however. That is also the case implied over $Var(S(T))$ change, as can be seen from Figure B.3.

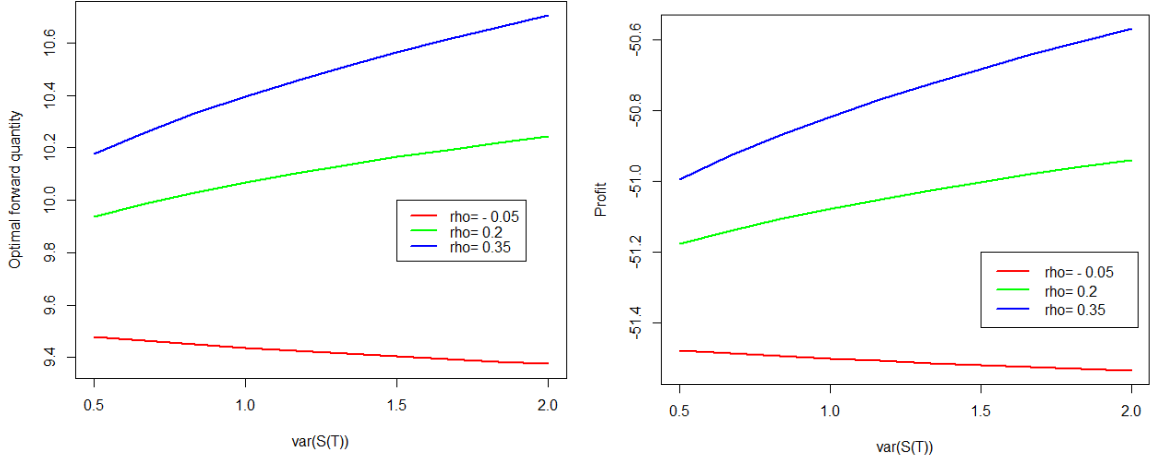


Figure B.3. Optimal policy as a function of variability in price, $Var(S(T))$, with respect to the correlation coefficient between log spot nominal price and availability index, ρ .

$$D(0) = 10, F(0) = 5, L(0) = 0.9, Var(D(T)) = 1, Var(S(T)) = 0.5, A = 0.0375, B = 0.025$$

B.3. Plots for the Trivariate Procurement Scenario

A higher volatile environment for the underlying product –with higher volatilities of variables and their interactions– its more difficult to identify the effects of parameters clearly. Capturing the delivery uncertainty in non-constrained form in trivariate case, the followings are observed.

The predominant effect of the demand forecast volatility on the value of optimal forward contract quantities relative to price-availability index correlation parameter is evident in the following figure.

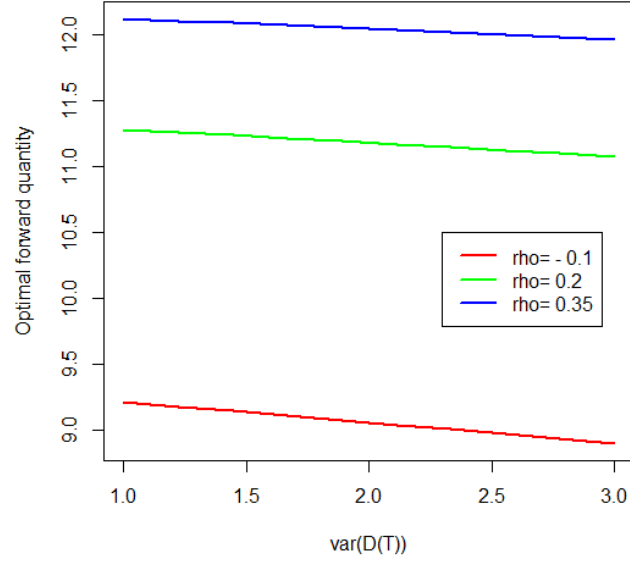


Figure B.4. Optimal profit as a function of variability in demand, $Var(D(T))$, and the correlation coefficient between log spot nominal price and availability index, ρ .

$D(0) = 10$, $F(0) = 5$, $L(0) = 0.7$, $Var(L(T)) = 0.05$, $Var(S(T)) = 0.5$, $c = 0.2$, $\theta = 0.1$,
 $A = 0.0375$, $B = 0.025$

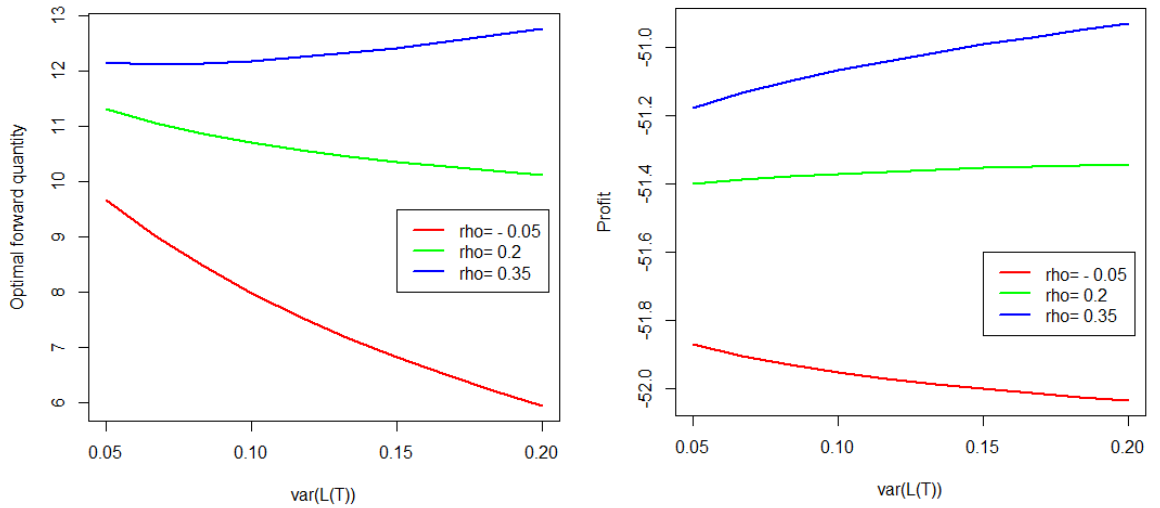


Figure B.5. Optimal policy as a function of variability in supply availability, $Var(L(T))$, with respect to correlation coefficient between log spot nominal price and availability index, ρ .

$D(0) = 10$, $F(0) = 5$, $L(0) = 0.7$, $Var(D(T)) = 1$, $Var(S(T)) = 0.5$, $c = 0.4$, $\theta = 0.1$,
 $A = 0.0375$, $B = 0.025$

As can be observed from Figure B.5, for low levels of availability uncertainty, the impact of price-availability index correlation ρ is low. For higher levels of uncertainty, the effect of the correlation between price and availability emerges much more evident. Accordingly, under the optimal contract, ρ has a limited impact on the profit when the variance in availability of supplies is low. However, for higher levels of uncertainty, the more the price-availability correlation in effect on the overall profit.

Also, the effect of price variance $Var(S(T))$ on the optimum order quantity and profit is similar to the effect of $Var(L(T))$ on those models.

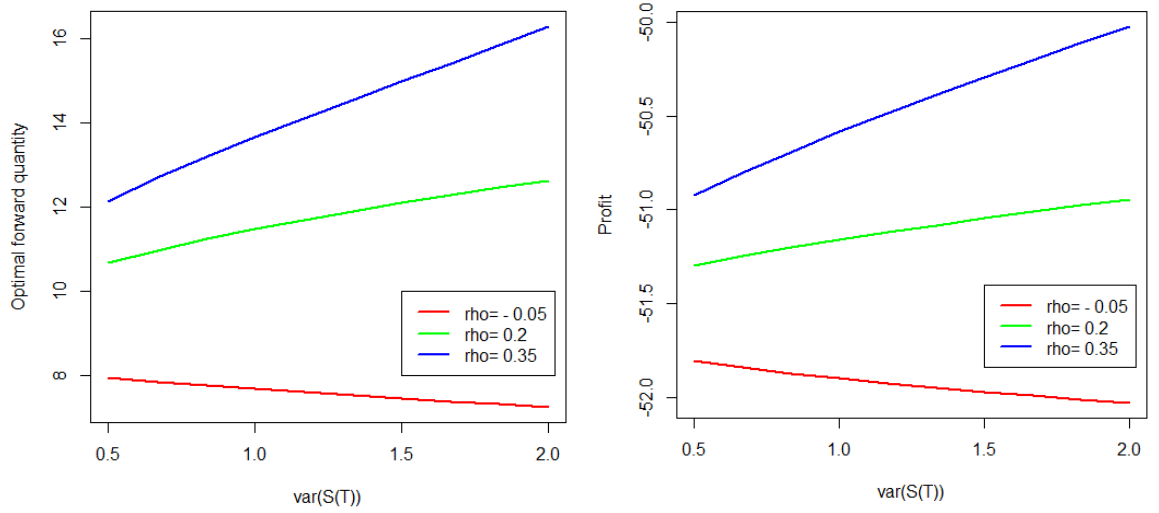


Figure B.6. Optimal policy as a function of variability in price, $Var(S(T))$, with respect to correlation coefficient between log spot nominal price and availability index, ρ .

$$D(0) = 10, F(0) = 5, L(0) = 0.7, Var(D(T)) = 1, Var(L(T)) = 0.1, c = 0.2, \theta = 0.1, \\ A = 0.0375, B = 0.025$$

Finally, in order to observe the relevance of supply risk with respect to transaction costs, Table B.1 is provided, covering for all three scenarios.

Table B.1. Penalty of ignoring supply uncertainty with respect to transaction costs.

q^*	A=0.0375, B=0.025		A=0.05, B=0.025		A=0.05, B=0.0125	
	Average	Range	Average	Range	Average	Range
Case 1	0.024	0.011-0.063	0.57	0.027-0.149	0.111	0.052-0.278
Case 2	0.104	0.002-1.000	0.135	0.015-0.944	0.212	0.035-1.349
Case 3	0.154	0.0-1.611	0.173	0.0-2.154	0.246	0.0-2.609

It reveals that as transaction costs in spot market increases, it becomes more valuable to include supply uncertainty into the procurement policies. Indeed, it is the spot transaction cost A to govern the values in Case 2 and the difference between spot and transaction costs $A - B$ for Case 1 and 3. This can be inferred from the relevant propositions as the terms including transaction costs appear on the right hand side of their equations and dominate the proposition results in the way they appear (as a difference $A - B$ or as a multiplier $1 + A$).

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