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METHODS OF ANALYSIS OF EARTH PRESSURES ON RETAINING WALLS

A THESIS SUBMITTED IN PARTIAL FULFILLMENT
OF THE REQUIREMENTS FOR THE DEGREE OF
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PREFACE

The purpose of this work is to give a thorough, but relatively short review, analysis and discussion of the most popular methods of earth pressure calculation behind retaining walls at present, thus making it possible to be a useful reference on the specific subject mentioned. At the same time, it aims to investigate the validity of Rankine's method starting with the assumption that Coulomb's method gives correct results and the soil is cohesionless, which is done in the form of comparison of the two methods.

In Part I, the subject has been introduced under the headings: definitions, uses, types, acting forces, and historical review of retaining walls and classes of earth pressures. In Part II, popular methods of earth pressure calculation has been considered. In Part III, a comparison between Coulomb's and Rankine's methods has been made, in table form and corresponding graphs, for different possible cases with two numerical examples at the end.

It is possible to extend Part III, if cohesion of the soil is also included, which will be a subject for further study.

TABLE OF CONTENTS

PART I - INTRODUCTION 1
Definitions 2
Uses 5
Types 6
Forces acting on retaining walls 9
Classes of earth pressures 13
Historical review 15

PART II - KNOWN METHODS OF EARTH PRESSURE CALCULATION
I) COULOMB'S METHOD 24
Assumptions 24
The method 28
a) Active Pressures 29
b) Passive Pressures 39
II) RANKINE'S METHOD 42
a) Active and Passive Rankine State in Semi-infinite Cohesionless Masses 46
b) Active and Passive Rankine State in Semi-infinite Cohesive Masses 50
III) TRIAL WEDGE METHOD 54
A) Cohesionless Soils 54
a) Active Pressures 54
b) Passive Pressures 58
B) Cohesive Soils 61
a) Active Pressures 61
b) Passive Pressures 67

IV) FRICTION CIRCLE METHOD 70
A) Cohesionless Soils 70
a) Active Pressures 70
B) Cohesive Soils 74
a) Active Pressures 74
V) LOGARITHMIC SPIRAL METHOD 79
VI) J. BRINCH HANSEN'S METHOD 82
a) Assumptions 82
b) Main Principles 85
c) Stresses in Rupture-Circle 88
d) Factors of Safety 98
e) Main Steps in the Theory 101
f) Solution of Retaining wall
Problems by Hansen's Method 104
1) General 104
2) Sloping Surface and Inclined
Back 107
An Example 109
3) Horizontal Surface and
Vertical Back 111
g) Summary of Hansen's Method 112
VII) CANTILEVER AND COUNTERFORT WALLS 116
VIII) DISCUSSION 120
a) Summary of Rankine's and
Coulomb's Conditions 123
b) Trial Wedge Method 125
c) Friction Circle Method 127
d) J. Brinch Hansen's Method 128
e) Original Contributions by
Brinch Hansen 130

PART III - COMPARISON OF RANKINE'S AND COULOMB'S METHODS

A) Rankine's Method for Vertical Walls 135
B) Coulomb's Method for Vertical Walls 138
Comparison for Vertical Walls 143
C) Coulomb's Method for Inclined Walls 148
D) Rankine's Method for Inclined Walls 155
Comparison for Inclined Walls 159
Two Comparative Numerical Examples 168
Conclusion 175
Symbols 178
Bibliography 180

PART I

I N T R O D U C T I O N

DEFINITIONS

A wall designed to maintain a difference in the elevations of the ground surfaces on each side of the wall is called a retaining wall. The earth whose ground surface is at the higher elevation is commonly called the backfill, and the wall is said to retain this backfill.

A retaining wall which supports the end of a bridge in addition to the backfill is called an abutment. The side of the wall adjacent to the backfill is called the back, and the side which is exposed for most of its height is called the face. The bottom surface of the wall is called the base. Also, the bottom portion, commonly constructed separately, is called the base or the footing. The line of intersection of the back and base is called the heel, and the line of intersection of the front of the wall and the base is called the toe. The batter of the face or back of a retaining wall is its inclination with the vertical. The slope or inclination of a ground surface is usually expressed as the length of horizontal projection per unit of vertical projection.

Earth which lies above a horizontal plane at the elevation of the top of a wall is called a surcharge. If the ground surface slopes upward from the top of a wall the wall is said to have an inclined or sloping surcharge. If the ground surface slopes downward from the top of a wall, the wall is said to have a negative surcharge. The backfills of retaining walls may carry railroad

tracks, highways, buildings, bins, stored materials, etc. These are called surcharge loads. In making earth pressure computations, the effect of a surcharge load is often provided for by adding an imaginary earth fill above the ground to produce a vertical pressure per unit of surface area equal to the average pressure per unit of surface area exerted by the surcharge load. This imaginary surcharge is called an equivalent surcharge.

Earth pressure is the force acting between the earth and a structure such as, for example, a wall. If the wall does not move at all, we have the so-called earth pressure at rest. If the wall is forced against the earth, we have passive pressure, and, if the wall moves away from the earth, we have active pressure.

The pressure centre of a wall is the point, in which the corresponding earth pressure resultant intersects the wall. The rotation centre of a wall is the point, about which the wall rotates in the state of failure.

The condition of failure for the earth is: $\tau = c + \sigma \tan \phi$ known as Coulomb's Law, where c (the cohesion) and ϕ (the friction angle) are constants. If $c = 0$, the earth is said to be cohesionless, and if $\phi = 0$, frictionless.

When a sliding movement takes place between wall and earth we have: $f = a + e \tan \delta$, where e and f are the unit normal and tangential earth pressures respectively, whereas a (the adhesion) and δ (the wall friction angle) are constants.

If $a = 0$ and $\delta = 0$, the wall is said to be perfectly smooth, and, in the case of $a = c$ and $\delta = \phi$, perfectly rough.

A line of rupture is a curve, for which the normal and shear stresses in any small element satisfy the condition of failure. This condition is also satified by the stresses in a line forming angles $90^{\circ} \pm \phi$ with the line of rupture.

When the condition of failure is fulfilled for any point within a certain finite area, this area is called a rupture-zone (or plastic zone), and the corresponding rupture-figure is termed a zone-rupture.

When the failure condition is fulfilled for all points on a certain curve only, this curve is a rupture-line, and the corresponding rupture-figure is termed a line-rupture.

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USES

Retaining walls are extensively used in connection with railways, highways, bridges, canals, and many other engineering works. Some of the more common uses are:

a) In a railway or highway constructed along a slope.

b) In elevated highway or railway to achieve grade separation. Subways or underpasses for cross traffic may be provided through the fill.

c) In depressed highway or railway in an underpass or subway to achieve grade separation. Bridges are constructed over the cut to provide for cross traffic.

d) Retaining walls forming the sides of a canal. This type of construction is used when the width is restricted so that sloping banks cannot be used.

e) In flood walls, which confine flood water of a river within an established channel.

f) Retaining walls supporting the earth fill, forming the approach to a bridge. Such a retaining wall is called an abutment.

g) Retaining walls used to support the earth fill adjacent to a building.

h) Retaining walls which confine stored material such as ore, coal, sand or gravel.

i) Retaining walls used to retain the ends of an approach fill for a bridge. The walls are usually placed at an angle with the normal to the direction of the fill. They are called

wing walls.

j) Retaining wall constructed at the junction of earth fill and masonry section of a dam. Such an arrangement is frequently used when it is considered unsafe to construct a spillway over an earth fill.

k) In tunnels, mining, and military fortifications.

TYPES

Usually retaining walls are of four general types:

a) Solid gravity wall - The simplest profile is trapezoidal in shape. The concrete for all except very low walls is commonly poured in two or more operations. When possible, the concrete for the lower portion, called the base or footing, is poured in an excavated trench and allowed to set. The forms for the upper portion are erected with the base as a support. One or more keys, whose combined horizontal area occupies from one-third to one-fourth of the horizontal section, are provided to resist the tendency for the upper portion to slide on the base because of the lateral thrust of the retained earth. The contact surface may be roughened to accomplish this result. Solid gravity walls are so proportioned that stresses are avoided, except that low computed tensile stresses are permitted at vertical sections through toe projection. Small tensile stresses are also permitted along the back if no horizontal construction joints are present.

Because of its massive construction, this type of wall is more resistant to destructive agencies than walls made up of thin reinforced sections. Also, since the unit stresses in the body of the wall are nominal and since its stability depends only on its weight, partial disintegration of the concrete is not as serious as in the heavily reinforced types.

b) Semigravity Wall - By introducing a relatively small amount of reinforcing in the back of the wall to tie the vertical stem to the base, a more slender vertical stem can be used. This ordinarily results in a greater toe projection, in order to maintain the required base width, so it is also necessary to provide a relatively small amount of reinforcing in the toe, to carry tensile stresses if they are larger than permissible tensile stresses in concrete.

Because of the considerable saving in concrete accomplished by a relatively small amount of steel, the semigravity type of wall may prove more economical than the solid gravity wall. Also, it has the same advantages in durability as the latter type but not to the same degree.

c) Cantilever Wall - It is made up of three cantilever beams: the vertical stem, the toe projection, and the heel projection. It is also called a T-wall because it resembles an inverted T. For higher walls, it may prove economical to provide fillets in the corners. The sliding resistance may sometimes be increased by a downward projection below the base called a key. It is usually placed near the center of the base so as

to provide end embedment for the vertical reinforcing in the stem. It is more effective in resisting sliding if placed at the heel and it is sometimes placed at the toe. This type of wall is usually more economical for ordinary heights and is more widely used than any other type in spite of the durability advantages of the solid gravity and semigravity walls. It has been used for walls as high as 125 ft or more.

d) Counterfort wall - Greater economy may sometimes be secured, by means of the counterfort wall. This economy is gained by making the vertical cantilevers deep, but rather than making them continuous along the wall, as in the cantilever wall, they are made into counterforts spaced at intervals along the wall. The earth is kept from spilling through between the counterforts by a vertical face slab anchored to the face of the counterforts by horizontal steel ties. The heel slab carries the vertical component of the backfill load longitudinally to the counterforts, and the toe acts as a cantilever in the same manner as in the cantilever wall. A key may be used to increase sliding resistance.

The buttressed wall is used occasionally under special conditions. It resembles a cantilever wall, but the vertical cantilever members supporting the face slab are placed on the front of the wall, instead of the back, and are called buttresses rather than counterforts.

FORCES ACTING ON RETAINING WALLS

In considering walls in which the cross section remains constant for a considerable distance, the unit which is selected for computation purposes is that included between two parallel transverse vertical planes 1 ft apart and parallel to the paper. When length is not specifically mentioned, the length being considered is 1 ft. It is assumed that forces acting on the two transverse planes neutralize each other. If a counterfort or buttressed wall is being considered, a portion of the wall bounded by vertical transverse planes passing through the centers of the counterforts, or through the mid-points between counterforts, is selected. The forces are the following:

1) The weight of the wall - The resultant of this weight acts through the center of gravity of the portion above the horizontal section considered.

2) The earth pressure against the back of the wall - The pressures considered are those due to the lateral thrust of the retained earth. The resultant force, acts against the back of the wall at an angle with the normal to the back of the wall.

3) The Normal Components of the Foundation Pressures - These are considered as being distributed linearly across the base giving a trapezoidal pressure diagram. The resultant of these forces acts through the centroid of the pressure diagram. The base of a wall is usually horizontal to the normal pressures,

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called the foundation or bearing pressures, are usually vertical.

4) The Horizontal Compononts of the Foundation Pressures - The distribution of these pressures is not usually considered but, if considered, it is usually assumed as corresponding to the distribution of the normal forces, if the wall is founded on cohesionless soil unless a key is provided. If the wall is founded on a soil with cohesion and without internal friction, the distribution is assumed to be uniform.

5) The Earth Pressure Against the Face of the Wall - The bottom of a retaining wall must be placed below the frost line at a sufficient depth to avoid heaving, if there is a possibility that freezing can occur, and at a depth where soil with the required bearing capacity can be reached. The earth in front of the wall may exert resistance to forward movement against the buried portion of the wall. This is known as passive pressure. The resultant force adds to the stability of the wall but is sometimes omitted from stability computations because of uncertainties concerning its magnitude.

6) Bridge Reactions, Tractive Effect, and Centrifugal Force - The loads due to the weight of a bridge and to the loads on the bridge are transmitted to the bridge seat through pedestals. The most severe conditions exist at the fixed end because the horizontal tractive force due to acceleration or deceleration of the live load, as well as the vertical reaction

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due to weight and the loads on the bridge, act at this end. If a bridge is located on a curve, the centrifugal force due to a moving vehicle sometimes must be considered.

7) Surcharge Loads - A train on a track passing over a bridge, or a loaded track or series of tracks paralleling a retaining wall, places heavy loads on the ground surface under tracks. Similarly, highway loads cause the same effect. These loads, in turn, increase the lateral earth pressures.

8) Water and Seepage Pressures - Water which accumulates in back of a wall and comes in contact with the wall exerts full hydrostatic pressure regardless of the presence of the backfill, but the earth pressure is reduced because of the buoyant effect of the water on the weight of the submerged soil, where possible, such a condition is avoided by installing a drainage system.

9) Uplift - Occasionally water is retained behind a wall which has no drainage system. If the foundation is permeable, a relatively small quantity of water seeps under the wall and emerges at the ground surface above the toe. Uplift pressures exist along the bottom of the wall due to the pressure of the water in the pores of the foundation material.

10) Vibration - The vibration resulting from railway or highway traffic, moving machinery, and other causes may increase the lateral earth pressures. Thus it may be desirable in some instances to make allowances for vibration.

11) Impact - Suddenly applied loads, such as moving trains and trucks, subject the supporting backfill to impact forces. However, the cushioning due to the large mass of soil in the backfill reduces this effect to such an extent that impact is not usually considered in computing lateral earth pressures against retaining walls and abutments.

12) Earthquake Shocks - The lateral earth pressure against a retaining wall may be increased momentarily by an earthquake shock. Such an increase is not usually significant but it is sometimes given special consideration by an increase of approximately 10 % in the lateral earth pressure.

13) Ice Thrust and Frost Action - The upper portion of a poorly drained soil may become saturated and then freeze. Any thrust which may develop from the expansion which results when water crystallizes into ice is small, but as the temperature of the frozen layer increases because of a rise in atmospheric temperature and solar radiation, significant expansion may occur and cause the wall to tilt slightly. In general, if conditions are such that an ice thrust develops, the magnitude of this thrust may be so large that it is not possible to design a wall to resist it. To avoid such pressures suitable drains and backfill material must be installed.

14) Expansion due to Changes in Water Content - Clay soils expand when the moisture content increases and shrink when it decreases. Expansion causes the lateral earth pressure to increase but the amount of increase is limited by the yielding

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of the wall. This phenomenon continues through the years and may cause a wall to tilt to such an extent that it has to be replaced. The fluctuations in the moisture content are greater at the ground surface and decrease with the depth below the surface. The effect of the phenomenon therefore decreases with the depth below the surface and usually does not extend beyond a depth of 4 ft. There is no simple and reliable procedure for computing the magnitude of the increase in lateral earth pressure due to this cause.

CLASSES OF EARTH PRESSURES

The lateral earth pressures in a natural soil deposit which has remained undisturbed are referred to as natural earth pressures or earth pressures at rest. The magnitude of these pressures depends upon the manner in which the soil is deposited and the displacement of the wall as well as upon the physical properties of the soil. Minimum lateral pressures will exist in a loosely dumped fill, whereas the pressures will be relatively high in deposits compacted by tamping or rolling. Earth pressures at rest may exist in deposits retained laterally by structures which do not yield appreciably under the action of the earth pressure. It is not possible to compute the magnitudes of earth pressures at rest. Such values can only be determined experimentally. Earth pressure at rest is a special case of elastic equilibrium.

Active earth pressures exist in a soil deposit which has been extended laterally, such as the backfill of a retaining wall, founded on soil which has tilted or moved horizontally a small but sufficient amount due to the pressures exerted by the backfill and the yielding of the foundations. The magnitude of active earth pressure is larger for soft clays and loose sands than for stiff clays and dense sands.

Passive earth pressures exist in a soil deposit which has been compressed laterally by the movement of a structure which abuts against it, or in some other manner. The magnitude of the lateral compression of the soil required to develop the passive state of stress in a given soil is somewhat greater than that of the lateral extension required to produce the active state of stress in that soil. The magnitude of the lateral compression of the soil required to develop both active and passive state of stress, depends upon the kind of soil, the manner in which it is deposited, and the pattern of deformation. In other words, the effect of displacement of a retaining structure on lateral earth pressure can always be shown as:

Passive earth pressure > Earth pressure at rest > Active earth pressure.

HISTORICAL REVIEW

In ancient times, the retaining walls to support the Egyptian and Babylonian defenses and fortifications, hundreds of feet in height certainly required some knowledge of earth pressures even if the knowledge was empirical.

In the latter part of the 17th century French military engineers contributed some empirical and analytical data pertaining to earth pressure on retaining walls for the design of fortifications. The famous French military engineer Vauban (1633-1707) gave some empirical rules for gauging the thickness of retaining walls.

In 1691 Bullot in his treatise assumed that, upon the displacement of the retaining wall all the retained soil wedge ABC, in the figure below, behind the wall and above a 45° plane, AC, through the toe, A, of the retaining wall would tend to slide down as a solid mass upon this plane.

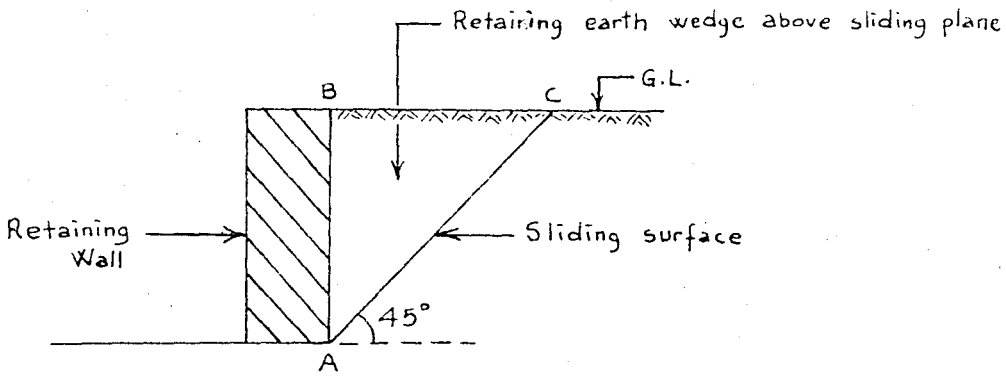


Fig. 1

Bullet resolved the weight of the sliding earth wedge, ABC, into two components, namely normally and tangentially to the 45° sliding plane. This tangential component of the weight Bullet considered as the thrust or earth pressure on the retaining wall.

Rondelet (1734-1829) dealt with earth pressure problems similarly to Bullet.

In 1726 Couplet tried to establish the earth-pressure theory scientifically, assuming the soil to be composed of small, spherical particles. In other words, he considered soil as an ideal material. This assumption, however, did not give useful and reliable results in practice.

Later some other authors assumed the position of the sliding plane to coincide with the slope of the angle of repose of the backfill material, and still others subtracted from the tangential weight-component a certain percent for friction.

Then came the period of the Classical Earth-Pressure Theories. Up to 1773 almost all theoretical considerations in calculating the lateral earth pressure of an ideal soil upon a retaining wall were based on the presupposition of a definite, arbitrarily assumed, sliding or failure plane.

Charles Augustin Coulomb (1736-1806) a famous French scientist and also a military engineer, considered it as his principal task to determine the real or true position of the

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sliding surface mathematically and thus gave a scientific basis to the theory. Coulomb is regarded as the founder of the so-called "Classical Earth-Pressure Theory". The period from Coulomb to second half of the 19th century is usually known as "the period of the classical Earth-Pressure Theories". In 1776, Coulomb introduced most of the basic concepts and assumptions still generally used in earth pressure calculations. He stated the so-called Coulomb's Law of shearing resistance: $\tau = c + \sigma \tan \phi$ and he introduced the principle of determining the active earth pressure as a maximum value. Further by assuming that the lines of rupture were straight he derived the well-known Coulomb formula for active earth pressure on a retaining wall. In his essay on the application of the rules of maximum and minimum, Coulomb presented an analysis of the so-called "sliding wedge theory", to bring about the calculations of the earth pressure against a retaining wall, as well as the critical height of retaining walls. He applies the laws of friction and cohesion for solid and granular bodies, and he determines the earth pressure upon a retaining wall from the "wedge of maximum pressure". He also treated the problem of surcharge on the ground surface.

The laws in Coulomb's earth pressure theory in short, are: 1) the frictional resistance on any sliding surface is equal to a fraction (constant) of the normal pressure on that surface,

and 2) the cohesive resistance is equal to a constant (cohesion) times the area of the sliding surface.

These laws represent the shear strength of non-cohesive and cohesive soil material, respectively, and that the combination of the two represents the shear strength of a cohesive-frictional soil material. The shear strength in Coulomb's assumption is uniformly distributed over the rupture plane.

His ideas on earth pressure still prevail in their principal points, with a few exceptions, and are used and recognized as valid even today, particularly when dealing with the calculations of stability of retaining walls.

As the time passed, Coulomb's theory underwent some alterations and new developments. However, these developments were based mostly on ideal, granular, non-cohesive soil such as dry sand or clean gravel possessing friction only between the particles.

Francais, Navier, and possibly others investigated special cases in which the ground surface behind the retaining wall is inclined and surcharged instead of being horizontal and unloaded.

Poncelet (178[illegible]-1867), a famous geometer and founder of projective geometry, extended Coulomb's theory in 1840, giving an elegant, graphical method of finding the magnitude of the earth pressure on the wall, vertical as well as for inclined wall sur-

faces on the backfill side, and for arbitrarily broken polygonal ground surfaces. Also, he was the first to apply the direction of the earth pressure inclined to the normal of the wall under an angle of friction between the soil and wall material.

In 1866, Karl Culmann (1821-1881) gave the Coulomb-Poncelet theory a geometrical formulation. Culmann's theory is the most general one permitting the graphical solution of the most complicated cases of earth pressure upon a retaining wall.

Alexandre Collin dealt with ruptures of slopes of canals and dams.

William J. Macgorn Rankine (1820-1872) published in 1857 a notable theory on earth pressure and equilibrium of earth masses, thus offering an analytical method of dimensioning retaining walls. The theory is based on an infinitesimal wedge within the mass of a uniform soil of laterally unlimited extent having a plane ground surface and subjected to its own weight only. The soil is assumed to be homogeneous, granular, cohesionless, and incompressible. By further assuming the lines of rupture to be straight, and with the aid of the boundary conditions at the ground surface, he developed the well-known Rankine formulae for active and passive states of failure in the ground. Rankine's theory is based on the principle of the internal stress condition in soil.

Coulomb-Poncelet earth pressure theories were elaborated by

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graphical analyses also by Rebhann, in 1871, Weyrauch, and others.

Engesser, in 1880, Jean Résal in 1910, A. L. Bell in 1915, William Cain in 1916, M. S. Ketchum in 1916 made significant contributions to earth pressure theory by their publications.

Coulomb and Rankine dealt only with straight rupture-lines. Kötter in 1903 derived the differential equation governing the stresses in a curved rupture-line, but because it was not simple it was rarely used.

To test the validity of the earth pressure theories, Müller-Breslau, in 1906 performed some relatively extensive and elaborate experiments with a large-scale model retaining wall. The results of his work showed that the earth pressure on the retaining wall may sometimes be larger than that calculated by means of Coulomb's theory. Also the manner of manipulation of the sand and the methodology of its loading and unloading may influence the magnitude of the earth pressure considerably.

Besides Müller-Breslau, contributions to the earth pressure theory were later likewise made by O. Franzius, H. Arey, in 1936, J. Feld, K. Terzaghi, in 1925 and 1941, the DEGEBO, G. P. Tschebotarioff and others with their experimental research with earth-retaining structure models on a large scale.

Different theories of plasticity were developed, chiefly, by Saint-Venant (1871), von Mises (1913), Prandtl (1920,1927),

Hencky (1920), Nadai (1928), Jurgenson (1934) and Odgvist (1934). Although most of these theories were developed formetals they could also be applied to earth. Based on Coulomb's failure condition Prendtl determined, in 1920, the rupture-figures corresponding to the pressure of weightless or frictionless earth on a perfectly rough wall. In 1926, von Karmann solved the corresponding problem for heavy earth with internal friction.

W. Feilenius, in 1927, made stability investigations and earth pressure calculations based on the maximum principle but assuming circular rupture-lines. The circular rupture-line enables a simple analysis to be made for frictionless earth, because the unknown normal stresses do not enter into the moment equation about the centre of the circle. Later investigations by Skempton, in 1948, Codling and Odenstad in 1950 have shown that this so-called $\phi = 0$ method usually gives very reliable results.

Stability and earth pressure calculations can be made for earth with internal friction, when suitable logarithmic spirals are used as rupture-lines. The first to make extensive use of this method was L. Rendulic (1935 and 1940), who determined the general relation between the magnitude of the earth pressure on a smooth vertical wall and the location of its pressure centre. He was also the first to calculate a composite rupture.

In 1936, Karl Terzaghi drew attention to the fact that the

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deformations in the earth must be compatible with the movements of the wall. As a result, the earth pressure must be a function of these movements. Brinch Hansen, in 1953, has made this very important fact one of the basic principles of his new method.

J. Ohde, in 1938, investigated in great detail the two special cases of a vertical wall rotating about its lower and upper edge respectively. In the latter case he assumed a circular rupture-line and made use of Kötter's equation. This, in combination with the 3 equilibrium conditions for the earth mass above the rupture-line, enabled him to determine the magnitude of the earth pressure and the location of its pressure centre.

Recently, theories of plasticity have been developed mainly by Sokolovski (1942), Freudenthal (1950), Nadai (1950), Drucker, Hodge and Prager (1950-51), Brinch Hansen (1952).

The most remarkable earth pressure tests have been made recently by Tschebotarioff (1948-1951) and Rowe (1952), who have both proposed empirical calculation methods for anchored sheet walls.

PART II

KNOWN METHODS OF EARTH PRESSURE CALCULATION

I) C O U L O M B'S M E T H O D

(For Cohesionless Soils)

ASSUMPTIONS

The basic assumptions underlying Coulomb's earth pressure theory are as follows:

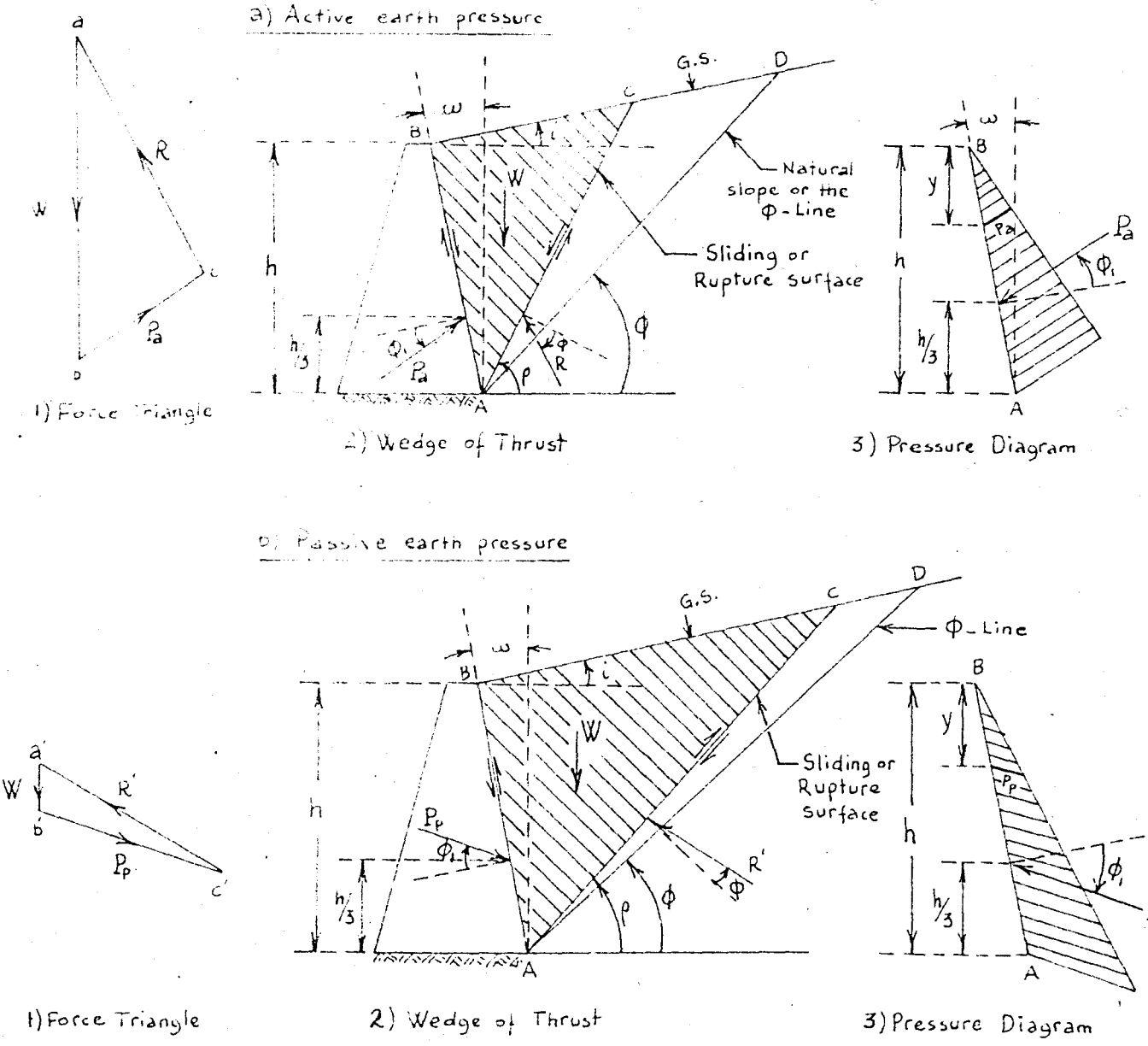


Fig. 2 Principal sketches of Coulomb's earth pressure-retaining wall system

1) The soil behind the retaining wall is considered to be dry; homogeneous; isotropic; elastically undeformable but breakable, granular material; endowed with internal friction but possessing no cohesion; capable of resisting compressive and shear stresses only. Such a soil is termod as an ideal soil.

2) The soil, depending upon its properties, forms a natural slope angle, ϕ , with the horizontal and the soil may be filled to this angle without rupturing and sliding down. Today this angle is understood to be the angle of internal friction of the soil, ϕ . The concept of friction (a friction of the normal load to the slidin surfaco) was clear to Coulomb.

3) Coulomb assumed that if the wall yields, then rupture of the backfill soil takes place and a soil wedge is torn off from the rest of the soil mass. In the case of an active earth pressure due to its own weight, the soil wedge slides sideways and downward over the rupture surface, thus exertin a lateral pressure upon the wall. In the case of passive earth resistance, the soil wedge, due to forcin the wall against the backfilled earth mass, slides sideways and upward on the rupture surface. The pressure at rest of the wall is not considered in this theory.

4) For the sake of convenience in analysis, the rupture surface is assumed to be a plane. The rupture surface passes through the heel of the wall under a certain angle, ρ . The rupture

BOĞAZİÇİ ÜNİVERSİTESİ KÜTÜPHANESİ

surface actually is not a plane, but is curved. This fact was known to Coulomb.

5) The sliding wedge itself is considered as a rigid body.

6) For a rupture plane within the soil mass, as well as between the inner face of the wall and the soil, the classical Newton's law of friction is valid:

$$f = \tan \phi = \frac{T}{N}$$

where f = coefficient of friction

ϕ = the proper angle of friction

T = the force tangential to the shearing surface

N = the force normal to the shearing surface

At equilibrium, the resulting stresses upon any surface within the ideal soil mass may be offset from the normal to that surface by an angle which does not exceed the angle of internal friction of the soil material - an angle which depends upon the physical properties of the given ideal soil.

7) The friction is distributed uniformly on the plane sliding or rupture surface.

8) The position and direction of the earth pressure, P , is assumed to be known. The pressure, P, acts on the inner face of the wall at a point one third of the height of the wall above the base of the wall, and makes an angle, ϕ_1 , with the normal of the inner (back) face of the wall. Angle ϕ_1 is

the angle of friction between wall and backfill materials. Coulomb assumed the wall friction to be zero.

9) The problem of determining the earth pressure, active or passive, is solved for a two-dimensional case (in plane). This is to say that the retaining wall is assumed to be of infinite length and linear, and that the profile of the retained earth mass, its weight, and other pressure and stress conditions in soil remain constant along the length of the wall. This assumption permits the performance of earth pressure calculations for a section of soil which is one linear unit of measure long perpendicular to the drawing plane.

10) When sliding of the soil wedge is just impending, the earth pressure theory gives two limiting pressure values: the least and the greatest values (active and passive) compatible with equilibrium.

11) The magnitudes of the active and passive earth pressures are calculated from the following considerations: Among the infinitely large number of rupture surfaces which can be passed through the heel-point, A , the most dangerous is that for which the active earth pressure, P_a , becomes a maximum (the wall must resist even the greatest pressure in the given system).

In the case of passive earth resistance, P_p , the most dangerous rupture surface is that for which P_p becomes

THESIS
ROBERT COLLEGE GRADUATE SCHOOL
BEBEK ISTANBUL

a minimum. If the soil mass ruptures at a minimum force, it will rupture at a force larger than minimum. Or, the question can be asked: what is the minimum force necessary to tear off a soil wedge, ABC, from a soil mass when the wall is forced against the backfilled soil ? Thus, the magnitude of the earth pressure is calculated by the application of the rules of maximum and minimum from the equilibrium condition, i.e., when rupture of the soil wedge is just impending.

Coulomb's theory treats the soil mass of the sliding soil wedge in its entirety.

The above assumptions permits one to treat the question of earth pressure as a statically determinate problem.

T H E M E T H O D

Coulomb assumed that the active earth pressure against the back of a wall, as shown in Fig. 2 a) is due to the thrust exerted by a wedge of soil between the back of the wall and a surface of rupture AC, which he assumed to be plane. This wedge tends to move downward along the back of the wall and along the surface of rupture as indicated by the arrows. The obliquity of the thrust against the back of the wall is equal to the angle of friction ϕ_1 , of the soil on the wall if any adhesion of the soil to the wall is neglected. The

THESIS
ROBERT COLLEGE GRADUATE SCHOOL
BEBEK, ISTANBUL

obliquity of the resultant pressure against the rupture surface is determined by the shearing resistance of the soil which is, in turn, determined by friction and cohesion. The form of Coulomb's formulas which has come into use neglects cohesion, so the obliquity of the resultant pressure along the plane of rupture becomes equal to ϕ , the angle of internal friction of the soil.

Coulomb did not consider the passive state of stress. However, a formula known by his name has been developed for passive pressure, following his basic assumptions. It recognizes that the thrust on the pressure surface in Fig. 2 b) tends to move a wedge of earth upward along a plane of rupture, as shown by the arrows, and that the wedge also tends to move upward along the pressure surface, as shown in the same manner.

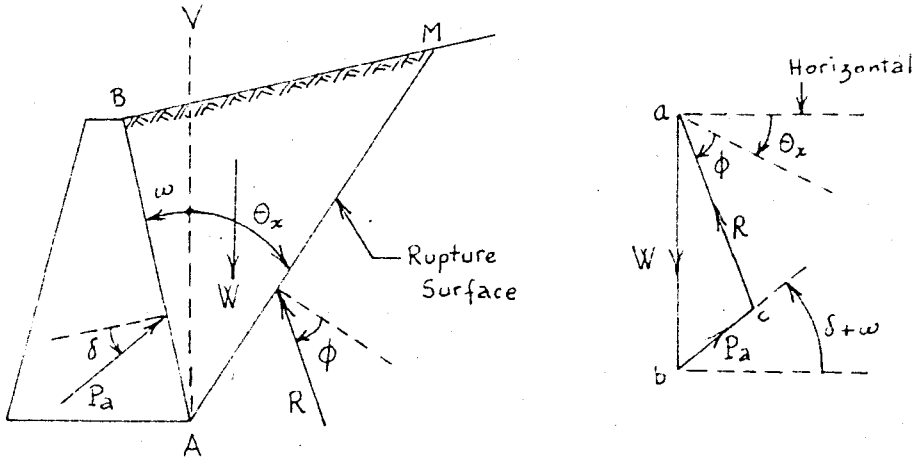
a) Active Pressures

Derivations of Formula - Coulomb's formula for active pressure applies to a plane ground surface. A graphical construction published by G. Rebhann in 1871 for determining resultant active earth pressures and planes of rupture serves as a convenient basis for deriving Coulomb's and Rankine's formulas for active pressure. When this construction is used directly, it gives results more quickly than they can be found by substitution in Coulomb's formula, and in addition locates

the plane of rupture for which no formula is available for Coulomb's conditions. Also, it can be used to determine the active pressures and planes of rupture for some irregular ground surfaces for which no formulas are available. This discussion is confined to the construction for active earth pressure because the construction for passive earth pressure is not particularly useful. Since Rebhann's method assumes a plane surface of rupture for conditions when the surface actually is curved, it includes the same theoretical error that is involved in other solutions such as Coulomb's method and the trial wedge method. However, it has been demonstrated that this error is not significant and can be disregarded. The use of Rebhann's construction has the same limitations as Coulomb's theory. It applies to solid gravity walls of usual proportions. When Coulomb's conditions prevail, Rebhann's construction gives the same results as Coulomb's formula, that we are going to derive, for active earth pressure when the ground surface is plane at least from the top of the wall to the point of intersection of the plane of rupture and the ground surface. The profile of the ground surface beyond this point does not affect the pressure on the wall.

The basic procedure involved in Rebhann's construction for active earth pressure may be outlined as follows with reference to Fig. 3. The wall in (a) yields a sufficient amount to develop the active state of stress in the backfill and causes

the plane of rupture AM to form. The wedge of earth between the back of the wall AB and the assumed plane of rupture AM, and whose weight is W, is held in equilibrium by the resultant active pressure P_a on AB and the resultant pressure



(a) Wall and Wedge of Thrust (b) Force Triangle

Fig. 3. Basic procedure for Rebhann's construction

R on AM. The active pressure P_a has an assumed obliquity of δ and, since rupture occurs along AM, the force R has an obliquity of ϕ below the normal. In order to determine the magnitude of P_a, per unit length of wall, proceed as follows:

1) Write an expression for W which is equal to the area ABM in (a), multiplied by γ, the unit weight of the soil. All the dimensions of the wedge are known constants except the angle θ_x.

2) Draw the force triangle abc in (b). The directions of W and P_a are known. The magnitude of W and the direction of R involve known values for the unit weight of the soil, the dimensions of the wall, the profile of the ground surface, and the known angle Θ_x . From the force triangle it is seen that

$$P_a = W \frac{\sin a}{\sin c}$$

3) To determine the value of Θ_x which will yield the maximum value of P_a, differentiate the expression for P_a with respect to Θ_x , equate the derivative to zero and solve for Θ_x . The value of Θ_x will be the angle of rupture α .

4) To determine the active pressure which is the maximum value of P_a, substitute the value of Θ_x , obtained in step 3 in the expression for P_a obtained in step 2.

The Rebhann's construction in Fig. 4 is carried out in the following steps:

1) Draw a line from A making an angle ϕ with the horizontal, and intersecting the surface at G.

2) From B, draw BH perpendicular to AG.

3) Draw BI at an angle ($\omega + \delta$) with BH.

4) Describe a semicircle with AG as a diameter. The center of the circle falls at J.

5) From I draw a perpendicular to AG until it intersects the semicircle at K.

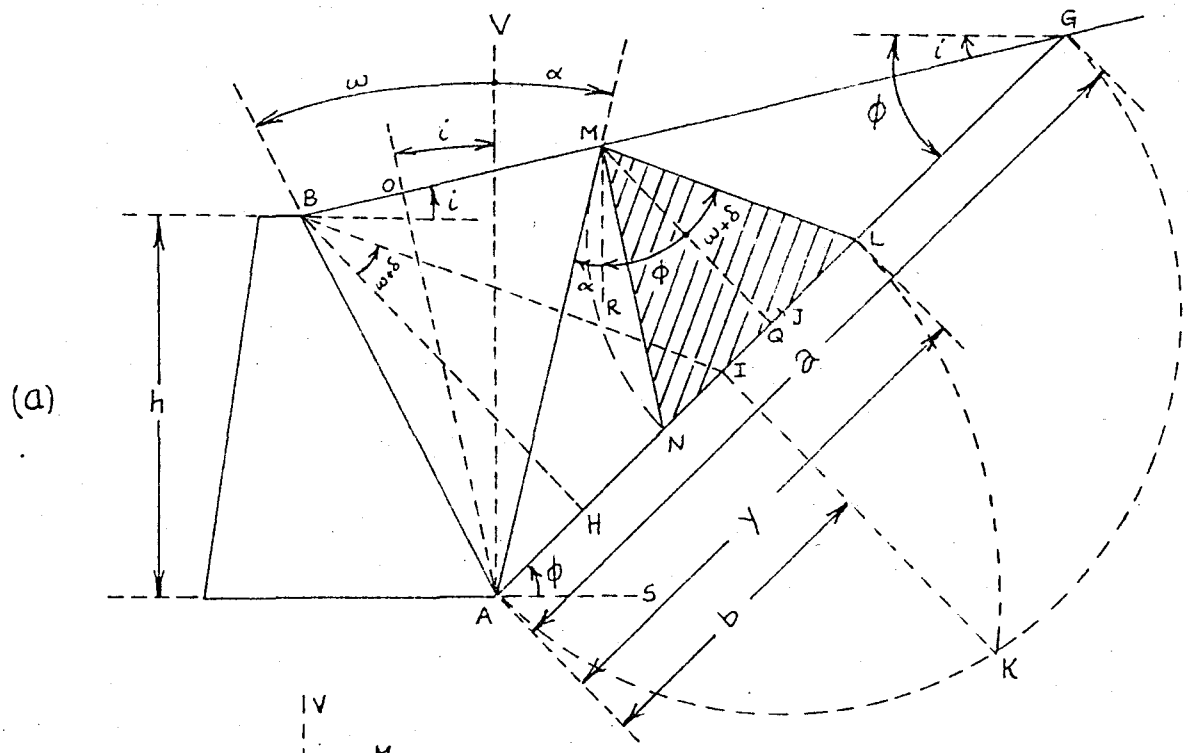
6) With A as a center and AK as a radius draw an arc which intersects AG at L.

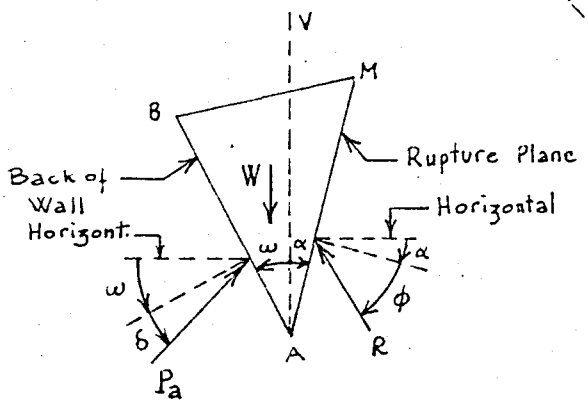
7) Draw ML parallel to BI.

8) With L as a center and ML as a radius draw an arc which intersects AG at N.

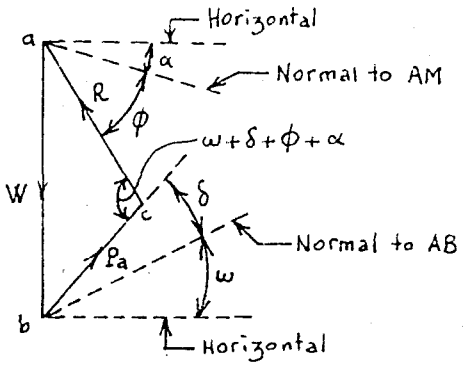
9) Draw MN.

10) Draw AM.





(b) Wedge of Thrust



(c) Force triangle for wedge of thrust

Fig. 4. Rebhann's Construction

The shaded area LMN multiplied by γ , the unit weight of the soil, is the resultant active pressure P_a which has the assumed obliquity of δ. The line AM is the trace of the plane of rupture and the angle VAM is the angle of rupture α.

The expression for W, the weight of the wedge of thrust is:

$$W = \frac{1}{2} \gamma \, AO \, . \, BM \qquad (a)$$

The expression for P_a, the resultant active pressure is determined as follows:

Draw MQ perpendicular to AG. Since ML is parallel to BI by step 7 of the construction and the angle IBH is equal to $(\omega + \delta)$ by step 3 of the construction, the angle LMQ = the angle I B H = $(\omega + \delta)$.

Draw the vertical line MR. Then the angle QMR = the angle SAG = ϕ by step 1 of the construction. By geometry angle RMA = angle VAM = α

Therefore

total angle LMA = angle LMQ + angle QMR + angle RMA

$= (\omega + \delta + \phi + \alpha)$

From the figure

angle LAM = $90 - \phi - \alpha$

In the triangle LMA

$$\frac{LM}{AL} = \frac{\sin LAM}{\sin LMA} = \frac{\sin(90^\circ - \phi - \alpha)}{\sin(\omega + \delta + \phi + \alpha)} \qquad (b)$$

The wedge of thrust ABM in Fig. 4b, whose weight is W, is held in equilibrium by the active pressure P_a and the force R on the plane AM acting in the directions shown. The force triangle abc for these forces is shown in Fig. 4c. From this force triangle

$$\frac{P_a}{W} = \frac{\sin a}{\sin c} = \frac{\sin(90^\circ - \phi - \alpha)}{\sin(\omega + \delta + \phi + \alpha)} \qquad (c)$$

From equations (b) and (c)

$$\frac{P_a}{W} = \frac{LM}{AL} \quad \text{or} \quad P_a = W \frac{LM}{AL} \qquad (d)$$

From (a) and (d)

$$P_a = \frac{1}{2} \gamma \cdot AO \cdot BM \cdot \frac{LM}{AL} \qquad (e)$$

In the similar triangles BIG and MLG in Fig. 4(a)

$$\frac{BM}{BG} = \frac{IL}{IG} \quad \text{or} \quad BM = BG \frac{IL}{IG} \qquad (f)$$

and

$$\frac{LM}{LG} = \frac{IB}{IG} \quad \text{or} \quad LM = LG \frac{IB}{IG} \qquad (g)$$

From (e), (f) and (g)

$$P_a = \frac{1}{2} \gamma \cdot AO \cdot BG \frac{IL}{IG} \cdot LG \frac{IB}{IG} \cdot \frac{1}{AL}$$

$$P_a = \frac{1}{2}\gamma\left(\frac{AO.BG.IB}{(IG)^2}\right)\frac{IL.LG}{AL} \qquad (h)$$

The location of the plane of rupture is determined as follows: All of the quantities in the above equation, except those which follow the parenthesis, are constants determined by the weight of the soil, the dimensions of the wall, and the profile of the ground surface. The remaining quantities vary with the assumed position of the plane of rupture AM.

To give the maximum value of P_a, which is the true value of the resultant active pressure, the quantity $\frac{IL.LG}{AL}$ in equation (h) must have its maximum value.

Let $AG = a$ $\quad AI = b$ and $AL = y$

then $IL = y - b$ and $LG = a - y$

or

$$\frac{IL.LG}{AL} = \frac{(y-b)(a-y)}{y} = a - \frac{ab}{y} - y + b \qquad (i)$$

to find the maximum, we differentiate equation (i) and equate the derivative to zero, and solve for y.

$$\frac{d}{dy}\left(a - \frac{ab}{y} - y + b\right) = 0 - \frac{ab}{y^2} - 1 + 0 = 0$$

from which: $y = \sqrt{ab}$ $\qquad (j)$

The length of each line in Fig. 4 would increase directly with h.

The derivation of Coulomb's formula consists of expressing the lengths of lines given in equation (h) in terms of h and the trigonometric functions of the various significant angles.

In the equation (h) all of the terms included in the parenthesis are constants depending upon h and the constant angles i, ω, δ and ϕ. The quantities which follow the parenthesis all depend upon the position of L which, in turn, depends upon the position of the ruture plane AM.

From Fig. 4

$$AO = AB \cos(\omega - i)$$

In the triangle ABG

$$BG = AB \frac{\sin(90^\circ - \phi + \omega)}{\sin(\phi - i)}$$

In the triangle ABI

$$IB = AB \frac{\sin(90^\circ - \phi + \omega)}{\sin(90^\circ - \delta - \omega)}$$

From Fig. 4

$$IL = y - b \qquad LG = a - y \qquad AL = y \qquad IG = a - b$$

$$\frac{IL \cdot LG}{AL} = \frac{(y-b)(a-y)}{y} = a - \frac{ab}{y} - y + b$$

but from equation (j) $\quad y = \sqrt{ab}$

Therefore $\quad \frac{IL \cdot LG}{AL} = a - \sqrt{ab} - \sqrt{ab} + b = \frac{(a - \sqrt{ab})^2}{a}$

Substituting in equation (h)

$$P_a = \frac{1}{2}\gamma \cdot AB\cos(\omega - i)\, AB\,\frac{\sin(90° - \phi + \omega)}{\sin(\phi - i)} \cdot AB\,\frac{\sin(90° - \phi + \omega)}{\sin(90° - \delta - \omega)} \cdot \frac{(a - \sqrt{ab})^2}{a(a-b)^2}$$

$$P_a = \frac{1}{2}\gamma \cdot (AB)^2 \frac{\cos(\omega - i)}{\sin(\phi - i)} \cdot \frac{\cos^2(\phi - \omega)}{\cos(\delta + \omega)} \cdot \frac{AB}{a}\left(\frac{a - \sqrt{ab}}{a - b}\right)^2 \qquad (k)$$

Various quantities in this equation may be evaluated as follows:

$$AB = \frac{h}{\cos\omega} \qquad \frac{AB}{a} = \frac{\sin(\phi - i)}{\sin(90° + i - \omega)} = \frac{\sin(\phi - i)}{\cos(\omega - i)}$$

$$\frac{a - \sqrt{ab}}{a - b} = \frac{1 - \sqrt{ab/a^2}}{1 - (b/a)} = \frac{1 - \sqrt{b/a}}{1 - (b/a)} = \frac{1}{1 + \sqrt{b/a}}$$

$$\frac{b}{AB} = \frac{\sin(\delta + \phi)}{\sin(90° - \delta - \omega)} \qquad \frac{a}{AB} = \frac{\sin(90° + i - \omega)}{\sin(\phi - i)} = \frac{\cos(\omega - i)}{\sin(\phi - i)}$$

$$\frac{b}{a} = \frac{\sin(\delta + \phi)}{\sin(90° - \delta - \omega)} \cdot \frac{\sin(\phi - i)}{\cos(\omega - i)} = \frac{\sin(\delta + \phi) \cdot \sin(\phi - i)}{\cos(\delta + \omega) \cdot \cos(\omega - i)}$$

Substituting these values in (k)

$$P_a = \frac{1}{2}\gamma h^2 \frac{1}{\cos^2\omega} \cdot \frac{\cos(\omega - i)}{\sin(\phi - i)} \cdot \frac{\cos^2(\phi - \omega)}{\cos(\delta + \omega)} \cdot \frac{\sin(\phi - i)}{\cos(\omega - i)} \cdot$$

$$\cdot \frac{1}{\left(1 + \sqrt{\frac{\sin(\delta + \phi) \cdot \sin(\phi - i)}{\cos(\delta + \omega) \cdot \cos(\omega - i)}}\right)^2}$$

which may be simplified to:

$$P_a = \frac{1}{2}\gamma h^2 \frac{\cos^2(\phi-\omega)}{\cos^2\omega \cdot \cos(\delta+\omega)\left(1+\sqrt{\frac{\sin(\delta+\phi)\sin(\phi-i)}{\cos(\delta+\omega)\cos(\omega-i)}}\right)^2} \qquad (1)$$

$$P_a = \frac{1}{2}\gamma h^2 K_a$$

This is Coulomb's formula for the resultant active earth pressure against the back of a retaining wall.

It should be noted that the resultant pressure does not equal the area of the pressure diagram in Fig. 2a3, but is proportional to that area because the unit pressures on AB are in terms of unit lengths measured along AB. The variation in unit pressure along the back of the wall is linear as shown in Fig. 2a3, so the resultant pressure acts at the lower third point of AB. The unit pressure per unit of vertical projection of the pressure surface and at a vertical depth y below the top of the wall is given by $p_a = \gamma y K$, and per unit of the pressure surface by $p_a = \gamma y \cos\omega K$.

b) Passive Pressures

As it is seen in Fig. 2a and Fig. 2b, the wedge of thrust and force triangle for passive pressure are the same as those for active pressure except that the obliquities δ or ϕ_1 and ϕ of P_p and R' for passive pressure are laid off above the normals whereas these obliquities for P_a and R for active pressure are laid off below the normals. If the signs of

δ and ϕ and the sign before the radical in the active pressure formula (1) are changed, formula for the resultant passive pressure will result. This formula is

$$P_p = \frac{1}{2}\gamma h^2 \frac{\cos^2(\phi+\omega)}{\cos^2\omega \cdot \cos(\omega-\delta)\left(1-\sqrt{\frac{\sin(\phi+\delta)\cdot\sin(\phi+i)}{\cos(\omega-\delta)\cdot\cos(\omega-i)}}\right)^2} \qquad (2)$$

$$P_p = \frac{1}{2}\gamma h^2 K_p$$

It should be noted that the resultant pressure does not equal to area of the pressure diagram in Fig. 2b3 but is represented by that area for the reason given in discussing the pressure diagram in Fig. 2a3. The variation of the unit pressure along the pressure surface is linear as shown, so the resultant pressure acts at the lower third point of AB. The unit pressure, at a depth y below the top of the pressure surface, is seen to be as follows:

In terms of a unit area measured along the vertical projection of the pressure surface

$$p_p = \gamma y K_p$$

In terms of a unit area measured along the pressure surface

$$p_p = \gamma y \cos\omega K_p$$

As explained before the rupture surface is not plane, as assumed in the derivation of the formula for passive pressure, but may be markedly curved. Except when δ and i are small,

The curvature is so pronounced as to introduce significant errors on the unsafe side, so the use of formula (2) is limited.

In most cases of passive pressure encountered in the design of retaining wall the conditions are those to which Coulomb's formula applies but, as stated above the errors involved in this formula are significant and on the unsafe side except when δ and i are small. If Rankine's formula for passive pressure is used, even though the conditions are those to which Coulomb's formula applies, the results will be on the safe side and the margin of safety will not be so excessive as to constitute poor design.

THESIS
ROBERT COLLEGE GRADUATE SCHOOL
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II) R A R K I N ' S M E T H O D

A semi-infinite mass is a homogeneous mass bounded by a horizontal plane and extending to infinity downward and in every horizontal direction. The state of stress in every point of the deposit can be represented by a circle of stress in Mohr's diagram. In Mohr's diagram every circle which does not touch or intersect the lines of rupture represents a state of elastic equilibrium. The term "elastic equilibrium" does not imply any definite relation between stress and strain. It merely imples that an infinitely small increase of the stress difference produces no more than an infinitely small increase of the strain. On the other hand, if the circlesof stress touch the lines of rupture, an infinitely small increase of the stress differences produces a steady incease of the corresponding strain. This is plastic flow. The flow is preceded by a state of plastic equilibrium. Through any point on the horizontal axis of Mohr's diagram for instance, through point Z in Fig. 5b, an infinite number of different circles can be traced which satisfy the condition for elastic equilibrium provided one of the principal stresses is equal to the abscissa OZ of the point. Yet there are only two circles through point Z which satisfy the conditions for plastic equilibrium. One of these circles is located on the right-hand side and the other on the left-hand side of point Z. Therefore, in contrast to the two states of plastic equilibrium,

represented by these two circles, the state of elastic equilibrium or of rest is statically indeterminate.

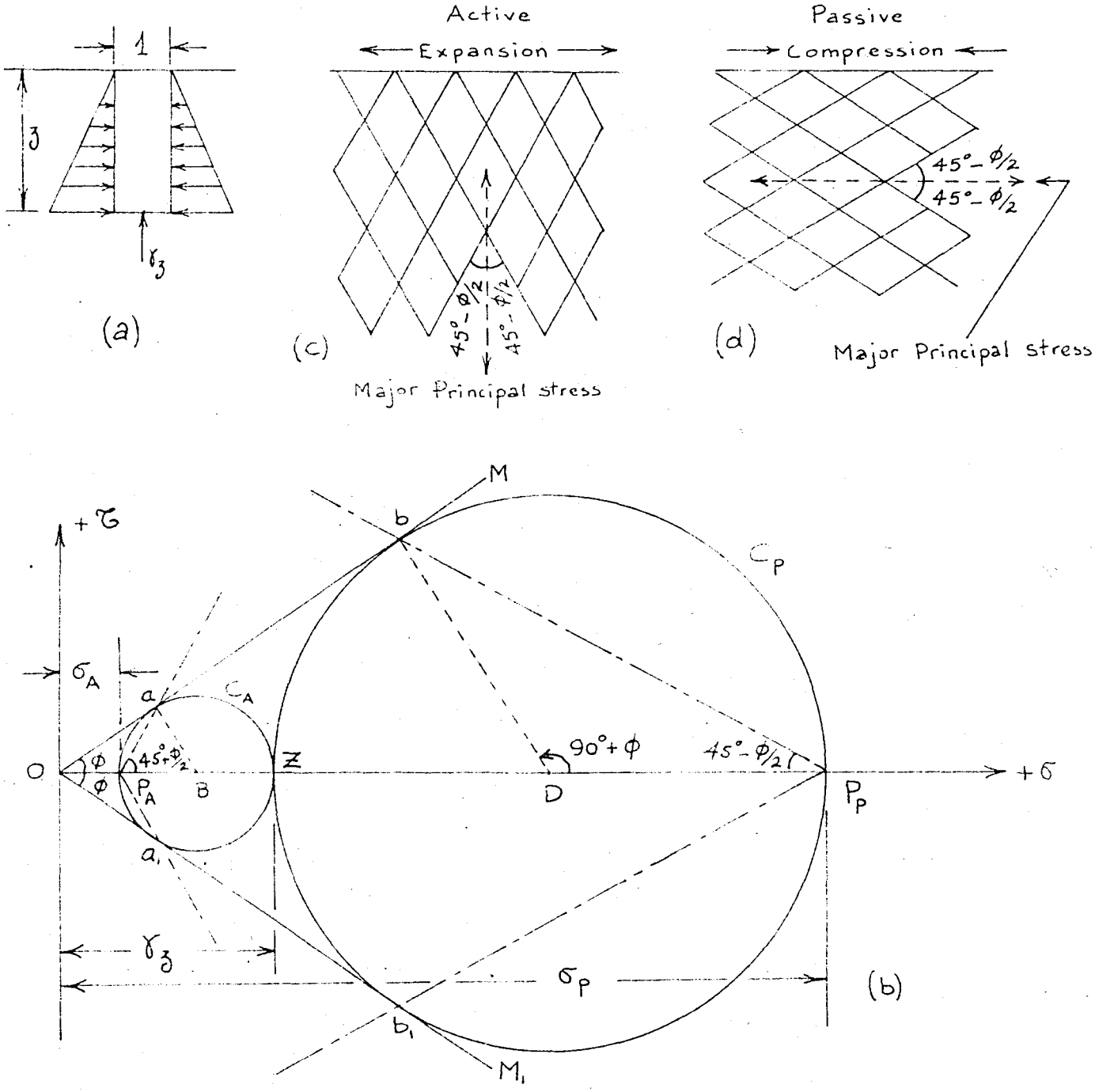


Fig. 5 - Semi-infinite cohesionless mass with horizontal surface. (a) Stresses at boundaries of prismatic elements; (b) graphic representation of state of stress at failure; (c) shear pattern for active state; (d) shear pattern for passive state.

THESIS
ROBERT COLLEGE GRADUATE SCHOOL
BEBEK, ISTANBUL

Let us assume a homogeneous mass with a horizontal surface. The unit weight of the mass is γ . Every vertical section through the mass represents a plane of symmetry for the entire mass. Therefore the shearing stresses on vertical and on horizontal sections are equal to zero. Fig. 5 a represents a prismatic block from this mass, with a width equal to unity. The deformation of the mass occurs parallel to the plane of the drawing. Since the shearing stresses on the vertical sides of the prism are equal to zero, the normal stress, $\sigma_v = \gamma z$, on the base of the prism is a principal stress.

The initial state of elastic equilibrium of the mass of which the prism is an element can be changed into a state of plastic equilibrium by either stretching the entire mass uniformly in a horizontal direction or compressing it uniformly in the same direction. If we stretch it, the pressure on the vertical sides of the prism decreases until the conditions for plastic equilibrium are satisfied while the pressure on the base remains unchanged. Any further stretching merely causes a plastic flow without changing the state of stress. The transition from the state of plastic equilibrium to that of plastic flow represents the failure of the mass. Since the weight of the soil helps in producing an expansion in a horizontal direction the subsequent failure is called active failure.

On the other hand, if we compress the soil in a horizontal direction the pressure on the vertical sides of the prism increases while the pressure on its base remains unchanged. Since the lateral compression of the soil is resisted by the weight of the soil the subsequent failure by plastic flow is called a passive failure.

Thus the transition of the soil from a state of elastic equilibrium to a state of plastic equilibrium can be accomplished by two different operations, lateral stretching or lateral compression. In either case the transition involves shear failure along two sets of surfaces of sliding (see Figs. 5 c and 5 d).

The problem consists in determining the stresses associated with the states of plastic equilibrium in the semi-infinite mass and the orientation of the surface of sliding. This problem was solved for the first time by Rankine (1857). Therefore, the plastic states which are produced by stretching or by compressing a semi-infinite mass of soil parallel to its surface will be called the active and the passive Rankine states, respectively.

If the active or the passive Rankine state exists only in one part of a semi-infinite mass with a horizontal surface, this part will be called a Rankine zone. With a Rankine zone the shear pattern is identical with one of the two patterns indicated in Figs. 5 c or 5 d.

a) Active and Passive Rankine State in a Semi-Infinite Cohesionless Mass

Fig. 5 a shows a prismatic element of a semi-infinite, cohesionless mass with a horizontal surface. The unit weight of the material is equal to γ and the stress conditions for failure are determined by the line of rupture OM in Fig. 5 b with the equation

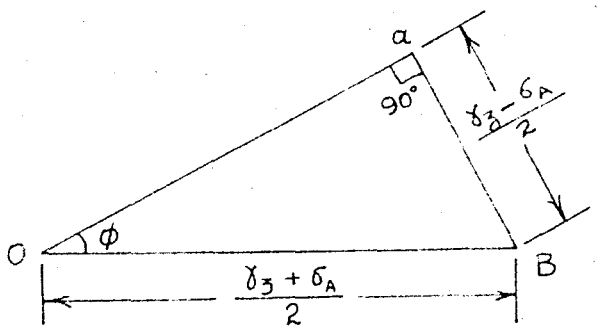
$$\tau = \sigma \tan \phi$$

The normal stress on the base of the elements is equal to the weight γz of the element. Since the shearing stress on horizontal sections is equal to zero, the normal stress γz on the base of the element is a principal stress. In Mohr's diagram (Fig. 5 b) this principal stress is represented by the distance OZ.

In order to produce the active Rankine state in the soil, we must stretch it in a horizontal direction until the stress conditions for plastic equilibrium are satisfied. Since the transition to the active Rankine state involves a decrease of the horizontal principal stress at a constant value of the vertical one, the circle of rupture which represents the active Rankine state at depth z is located on the left-hand side of point Z. It touches the line of rupture OM at point a. The corresponding pole will be called the active pole, P_A. The pole is located at the intersection of the circle and a

line drawn through Z parallel to the plane on which the stress γ_3 acts. Since this plane is horizontal and since, in addition, point Z is located on the horizontal axis, the active pole P_A is located at the point of intersection between the circle and the horizontal axis of Mohr's diagram. The planes of shear failure in Fig. 5 c are parallel to the lines $P_A a$ and $P_A a_1$ in Fig. 5b. Both sets of planes rise at an angle of $45 + \phi/2$ to the horizontal. The normal stress σ_A (active pressure) on a vertical section at depth z below the surface is obtained from Fig. 5 b as follows:

Take triangle OBa in Fig. 5b.



Utilizing law of tangents

$$\frac{\frac{\gamma_3 + \sigma_A}{2} + \frac{\gamma_3 - \sigma_A}{2}}{\frac{\gamma_3 + \sigma_A}{2} - \frac{\gamma_3 - \sigma_A}{2}} = \frac{\tan \tfrac{1}{2}(90° + \phi)}{\tan \tfrac{1}{2}(90° - \phi)}$$

Simplifying

$$\frac{\sigma_A}{\gamma_3} = \frac{\tan(45° - \phi/2)}{\tan(45° + \phi/2)} = \frac{\dfrac{1 - \tan \phi/2}{1 + \tan \phi/2}}{\dfrac{1 + \tan \phi/2}{1 - \tan \phi/2}}$$

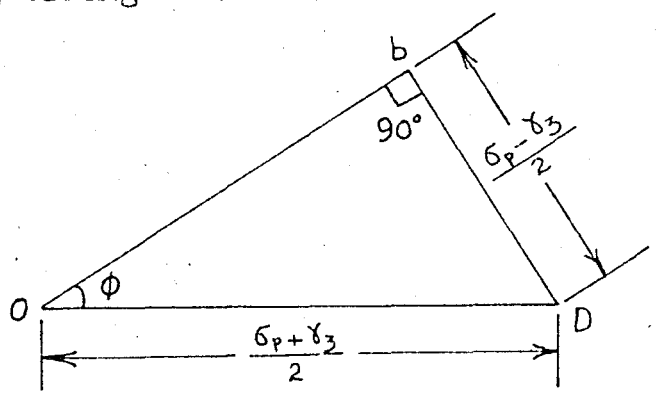
$$\frac{\sigma_A}{\gamma_3} = \left(\frac{1 - \tan \phi/2}{1 + \tan \phi/2}\right)^2 = \tan^2(45° - \phi/2)$$

Total active pressure on a retaining wall with vertical back and horizontal backfill, having a height, h can be found as:

$$P_A = 1/2\, \gamma h^2 \tan^2(45° - \phi/2)$$

If the failure of the soil is preceded by lateral compression, involving an increase of the horizontal principal stress, the state of stress at the instant of failure must be represented in Fig. 5 b by the circle through Z which is tangent to the rupture line OM at point b. The corresponding planes of shear (Fig. 5 d) are parallel to $P_p b$ and $P_p b_1$ (Fig. 5 b). They form an angle of $45° + \phi/2$ with the vertical . The stress σ_p (passive pressure) is obtained from Fig. 5 b as follows:

Taking triangle ODb



Using Law of Tangents

$$\frac{\frac{\sigma_p+\gamma_3}{2}+\frac{\sigma_p-\gamma_3}{2}}{\frac{\sigma_p+\gamma_3}{2}-\frac{\sigma_p-\gamma_3}{2}}=\frac{\tan\frac{1}{2}(90^\circ+\phi)}{\tan\frac{1}{2}(90^\circ-\phi)}$$

Simplifying

$$\frac{\sigma_p}{\gamma_3}=\frac{\tan(45^\circ+\phi/2)}{\tan(45^\circ-\phi/2)}=\frac{\frac{1+\tan\phi/2}{1-\tan\phi/2}}{\frac{1-\tan\phi/2}{1+\tan\phi/2}}$$

$$\frac{\sigma_p}{\gamma_3}=\left(\frac{1+\tan\phi/2}{1-\tan\phi/2}\right)^2=\tan^2(45^\circ+\phi/2)$$

Total passive pressure on a retaining wall with vertical back and horizontal backfill having a height h, can be found as:

$$P_p=\frac{1}{2}\gamma h^2\tan^2(45^\circ+\phi/2)$$

By means of Mohr's diagram, total active and passive pressures on retaining walls with vertical backs and inclined backfills can be found similarly. For a retaining wall with height, h, the formulas are as follows:

$$P_A=\frac{1}{2}\gamma h^2\cos i\,\frac{\cos i-\sqrt{\cos^2 i-\cos^2\phi}}{\cos i+\sqrt{\cos^2 i-\cos^2\phi}}$$

and

$$P_p = \tfrac{1}{2}\gamma h^2 \cos i \; \frac{\cos i + \sqrt{\cos^2 i - \cos^2 \phi}}{\cos i - \sqrt{\cos^2 i - \cos^2 \phi}}$$

b) Active and Passive Rankine State in Semi-Infinite Cohesive Masses

Fig. 6 a is a vertical section through a prismatic element of a semi-infinite mass of cohesive soil with a horizontal surface. The unit weight of the soil is γ and the stress conditions for failure are determined by the line of rupture $M_o M$ (Fig. 6 b) whose equation is:

$$\tau = c + \sigma \tan \phi$$

Since every vertical section through a homogeneous, semi-infinite mass with a horizontal surface represents a plane of symmetry, the shearing stresses on these sections are equal to zero and the normal stresses on both the vertical sides and on the base of the element are principal stresses. The normal stress σ_v on the base element is equal to the weight of the prism, γz.

In Mohr's diagram (Fig. 6 b) the stress σ_v is represented by the distance OZ. The lines of rupture $M_o M$ and $M_o M_1$ intersect the vertical axis of the diagram at a distance c from the origin O. The corresponding active Rankine state is represented by a circle C_A located on the left-hand side of point Z, tangent to the lines of rupture. If this circle intersect the

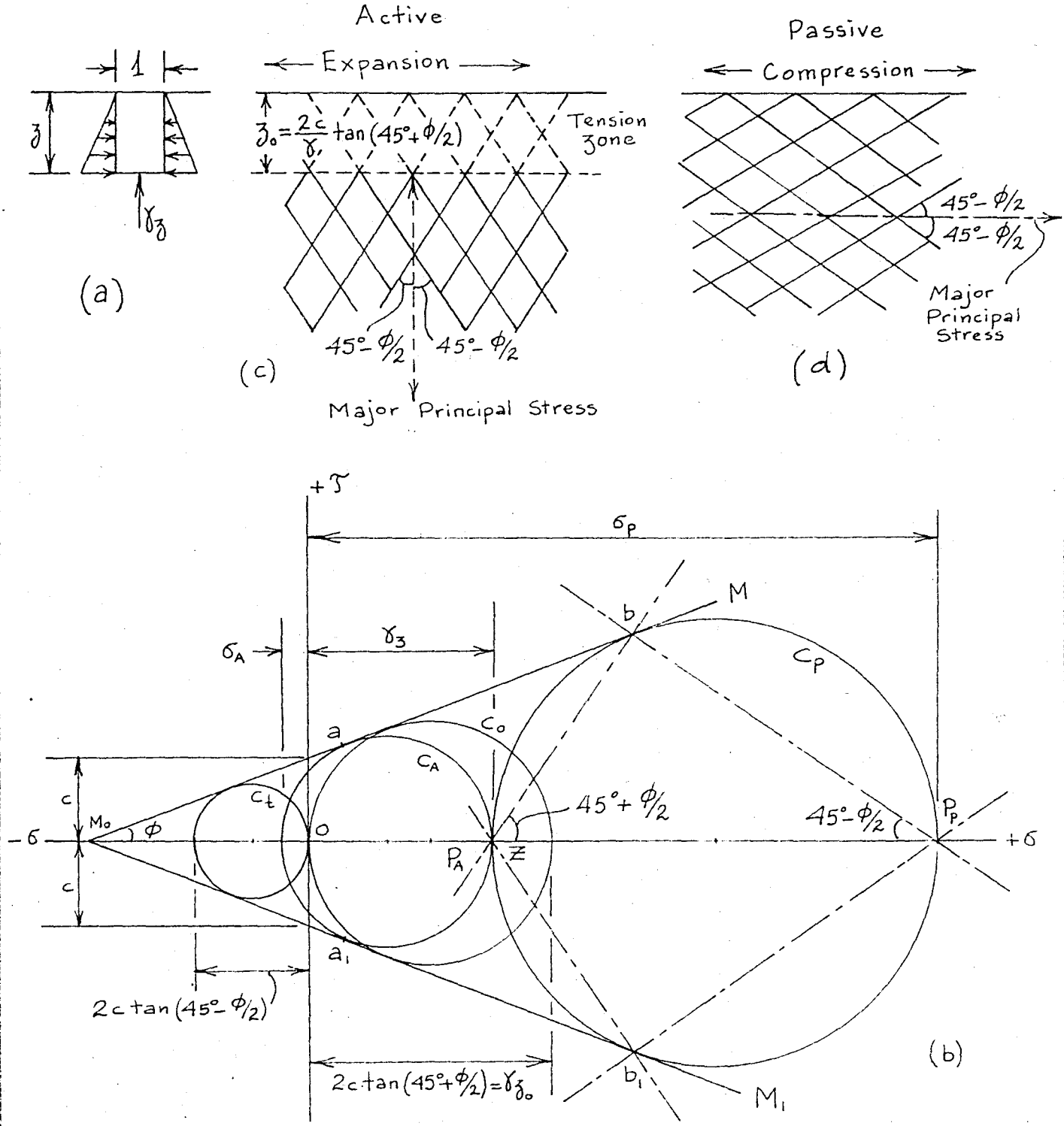


Fig. 6 – Semi-infinite cohesive mass with horizontal surface.

(a) Stresses at boundaries of prismatic element (b) graphic representation of state of stress at failure (c) shear pattern for active state (d) Shear pattern for passive state.

horizontal axis on the left-hand side of the origin O, as shown in Fig. 6 b, the horizontal principal stress at depth z is a tensile stress. For $Z = 0$ we obtain the circle C_t, whose diameter represents the simple tensile strength of the soil. As z increases, the corresponding tensile stress decreases, and at a certain depth Z_o the tensile stress becomes equal to zero. The state of stress at which the soil starts to flow at a depth z_o is represented by a circle C_o which passes through the origin and touches the lines of rupture. From the geometrical relations indicated in Fig. 6 b it may be shown by trigonometry that

$$z_o = \frac{2c}{\gamma} \tan\left(45^\circ + \phi/2\right)$$

Between the surface of ground and depth z_o the active Rankine state involves a state of tension in a horizontal direction.

If z is greater than z_o the entire corresponding circle of rupture is located on the right-hand side of the origin O. For any depth z the pole P_A coincides with the left-hand point of intersection between the circle and the horizontal axis. Fig. 6 c shows the two sets of planes of shear for $Z > Z_o$. They are parallel to the lines P_Ab and P_Ab_1 in Fig. 6 b, which connect the the pole with the points of contact between the circle of stress and the lines of rupture. They form an angle of $45^\circ - \phi/2$ with the vertical. The normal stress σ_A (active pressure) on a vertical section at depth z below the surface is obtained from Fig. 6 b as:

$$\sigma_A = \gamma(z - z_0) \tan^2(45° - \phi/2)$$

Total active pressure on a vertical plane whose height is $\underline{h}$ can be found as:

$$P_A = \tfrac{1}{2}\gamma(h - z_0)^2 \tan^2(45° - \phi/2)$$

For the passive Rankine state, produced by a lateral compression of the soil, all the circles of stress which represent a state of failure are located entirely on the right-hand side of point C, because in this state the gravity stress γz is the smallest principal stress. As a consequence the soil fails at every depth by shear. The circle C_p represents the state of stress at failure for an arbitrary depth z. The lines of rupture are tangent to it at b and b_1 and the two sets of shear planes are parallel to the lines $P_p b$ and $P_p b_1$ (Fig. 6 b).

The stress σ_p (passive pressure) is obtained from Fig. 6 b as:

$$\sigma_p = \gamma z \tan^2(45° + \phi/2) + 2c \tan(45° + \phi/2)$$

The total passive pressure on a vertical plane whose height is h is:

$$P_p = \frac{1}{2}\gamma h^2 \tan^2(45° + \phi/2) + 2ch \tan(45° + \phi/2)$$

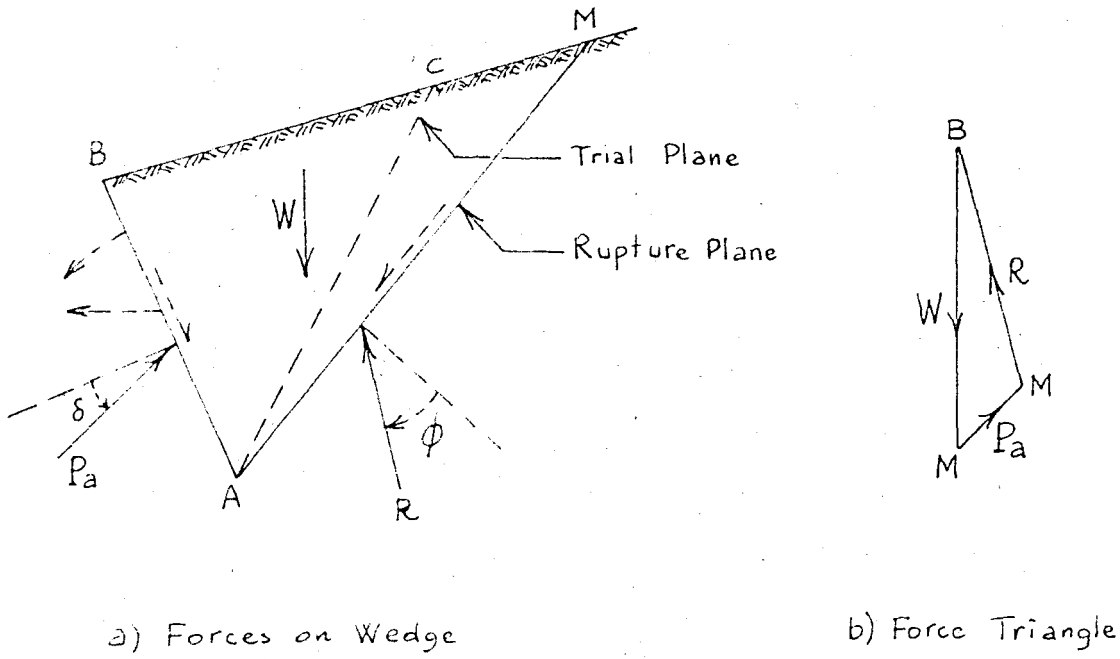
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III) TRIAL WEDGE METHOD

A) COHESIONLESS SOILS

a) Active Pressures

In Fig. 7 the pressure surface AB yields by rotating in a counterclockwise direction about A and may also yield to the left, as indicated by broken arrows, to create an active state of stress. This movement causes an inner surface of rupture to form.



a) Forces on Wedge

b) Force Triangle

Fig. 7 - Trial wedge and force triangle for active pressure.

It is assumed that this surface is a plane AM. The wedge of soil BAM moves downward a small amount along the plane of rupture and along the pressure surface, as indicated by the broken arrows. This wedge whose weight is W, is held in equilibrium by

the resultant active pressure acting on the surface AB and the resultant force R acting on the rupture plane AM. Since the wedge moves downward along AB, the force P_a acts with a known or assumed obliquity δ below the normal to oppose this movement. Similarly the force R acts with an obliquity ϕ below the normal because rupture is occuring along this plane. Here, our job is to determine the direction of AM.

To locate the plane of rupture for the active state of stress and determine the resultant active pressure, several trial planes, such as Ac in Fig. 7 a) and the resulting values of W, are used. The force triangles, corresponding to the one, shown in Fig. 7 b), for the plane AM, are constructed. In each trial we know both the magnitude and the direction of W, and the directions of P_a and R only. In this manner, several values of P_a are found. The largest value of P_a which can be obtained is the true value of the resultant active pressure, and the corresponding plane, which is thereby obtained, is the plane of rupture. The true value of P_a is designated by MM in Fig.7 b).

In constructing the force triangles corresponding to the various trial planes, the obliquities of R for all trial planes are assumed to equal ϕ when, actually, they are less than ϕ for all planes except the rupture plane. The assumed shearing resistances along all planes except the rupture plane have, therefore been too large. Since the shearing resistance along the trial plane reduces the value of P_a, the values of P_a for

for all planes except the rupture plane are too small, the plane which yields the maximum value of P_a is the rupture plane, and this maximum value of P_a is the active pressure.

By using several trial planes, constructing several force polygons, plotting a curve showing the values of P_a for various angles between the trial planes and the vertical, and selecting the maximum value of P_a and the corresponding trial plane, the active pressure, plane of rupture, and angle of rupture can be determined.

The procedure for finding the resultant active pressure P_a against the back of a retaining wall, and the angle of rupture is illustrated in Fig. 8. The same δ and ϕ values are used throughout the trials, in each force polygon.

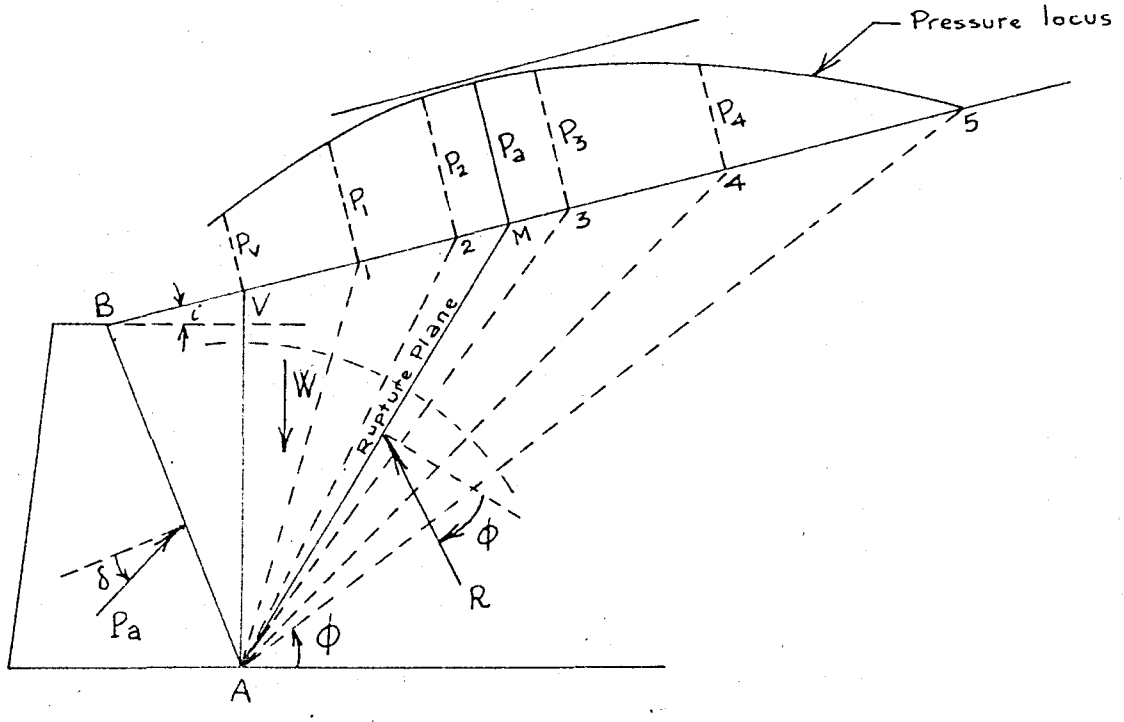


Fig. 8. Active pressure by trial wedge method.

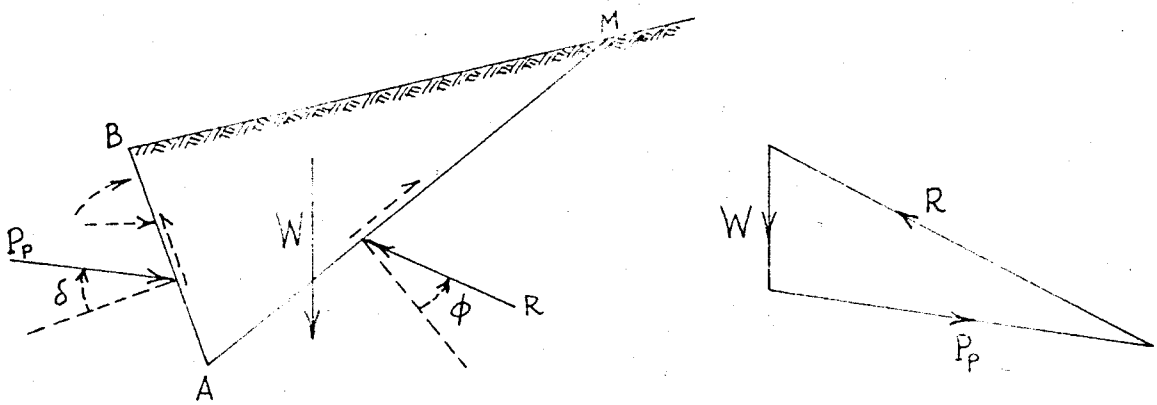
It is usually convenient to make one trial plane vertical, such as AV, another at an angle ϕ with the horizontal, such as A5 in the figure, and to subdivide the arc included between these planes into five equal segments. No plane should be located between the vertical plane and the back of the wall because the rupture plane cannot be in this region. In each trial drawing different force polygons for different trial planes we obtain the values of different active pressure, P's, we plot them according to a suitable scale, and then we draw the pressure locus, as shown in the figure. The peak point of the locus will give $P_{max} = P_a$ which is the true active pressure, thus providing the location of rupture plane, AM.

It can be seen from the figure that the plane of rupture cannot be located precisely by this method. However, the position of this plane can vary through several degrees, and still yield values of P_a which are quite close to the correct value. Also, where the surface of rupture is not plane, satisfactory values for the active earth pressure can be secured if it is assumed to be plane.

Trial wedge method is applicable to irregular ground surfaces too. The procedure is essentially the same as for the regular ground surface already explained.

b) Passive Pressures

In Fig. 9 the pressure surface AB moves toward the backfill by rotating in a clockwise direction about A, and it may also move to the right, as indicated by broken arrows, to create a passive state of stress. This movement causes a surface of rupture to form. It is assumed that this surface is a plane AM.



a) Forces on Wedge b) Force Triangle

Fig. 9 - Passive Wedge and force triangle.

The wedge BAM is forced upward a small amount along the plane of rupture and along the pressure surface, as indicated by broken arrows. The wedge whose weight is W, is in equilibrium because of the resultant passive pressure, P_p acting on the surface AB and the resultant force R acting on the plane of rupture AM. Since the movement of the wedge is upward along AB, the force P_p acts with an obliquity δ above the normal to oppose this movement. Similarly, the force R acts

with an obliquity ϕ above the normal to oppose the upward movement of the wedge. If the direction of the plane of rupture were known, the magnitude and direction of W and the direction of R would be known. It is first necessary to determine the direction of the rupture plane.

To locate the plane of rupture for the passive state of stress and determine the resultant passive pressure, several trial planes, corresponding to AM in Fig. 9 a) are used and several force triangles, corresponding to the one in Fig. 9 b) are constructed. In this manner several values of P_p are found. In each trial we know both the direction and magnitude of W, and the directions of P_p and R only. The smallest value of P_p found throughout the trials is the resultant passive pressure and the corresponding plane is the plane of rupture. The obliquities of the pressures R on all trial planes are assumed to equal ϕ when, actually, they are less than ϕ for all planes except the rupture plane. The assumed shearing resistances on all planes, except the rupture plane, have therefore been too large. Since these shearing resistances contribute to the value of P_p, the values of P_p for all planes except the rupture plane are too large, the plane which yields the minimum value of P_p is the rupture plane, and this minimum value of P_p is the passive pressure.

By using several trial planes, constructing several force polygons, plotting a curve showing the values of P_p for various angles between the trial planes and the vertical, and selecting the minimum value of P_p and the corresponding angle with the vertical, the passive pressure and the angle of rupture can be determined.

The procedure for finding the resultant passive pressure P_p against the back of a retaining wall, and the angle of rupture is illustrated in Fig. 10. The same δ and ϕ values are used throughout the trials in each force polygon.

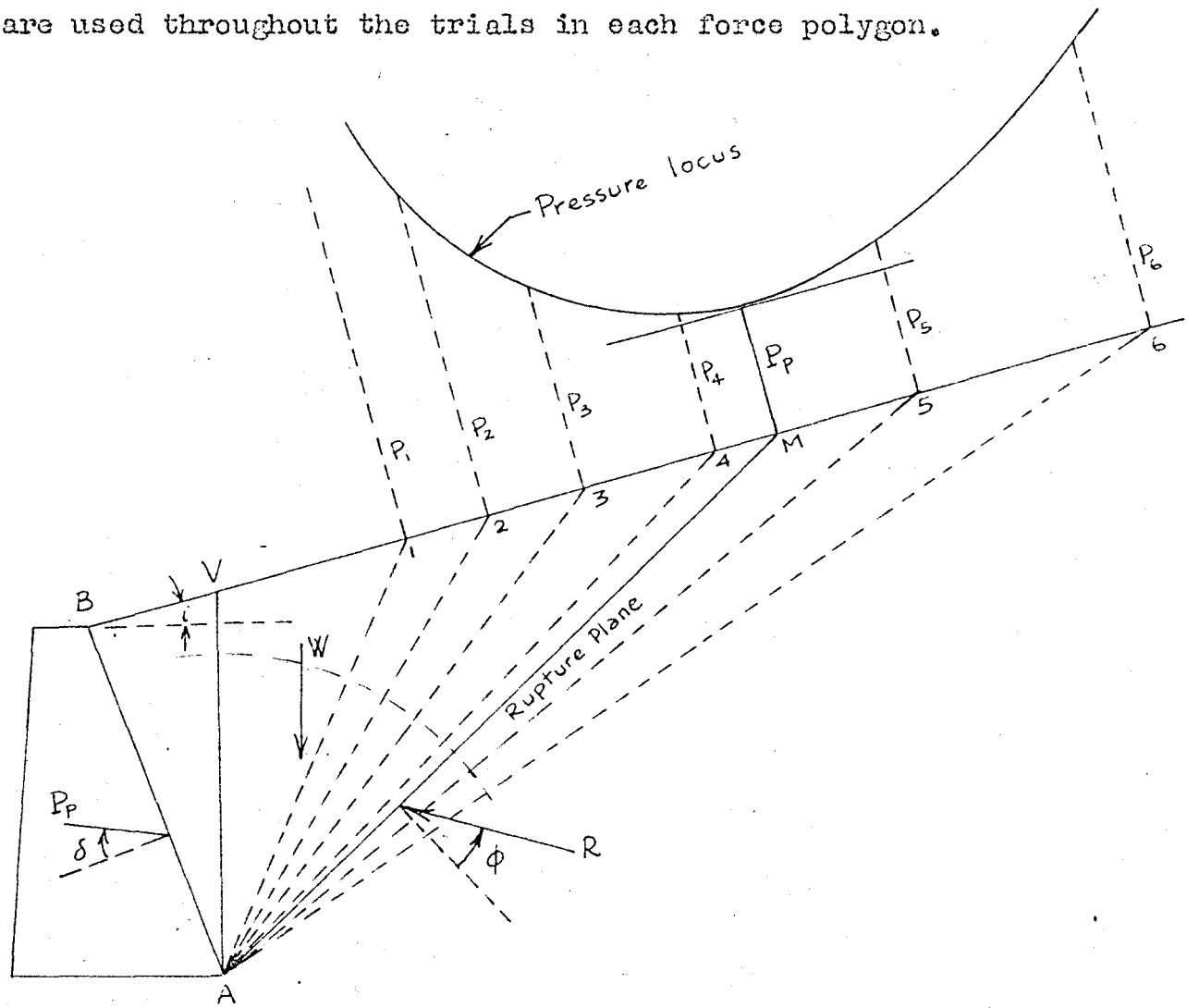


Fig. 10 Passive pressure by trial wedge method.

The pressure locus is drawn utilizing different force polygons for different trial planes. The minimum value of the locus gives $P_p = P_{min}$ which is the true passive pressure, drawn to same scale. This gives the correct location of the rupture plane, AN.

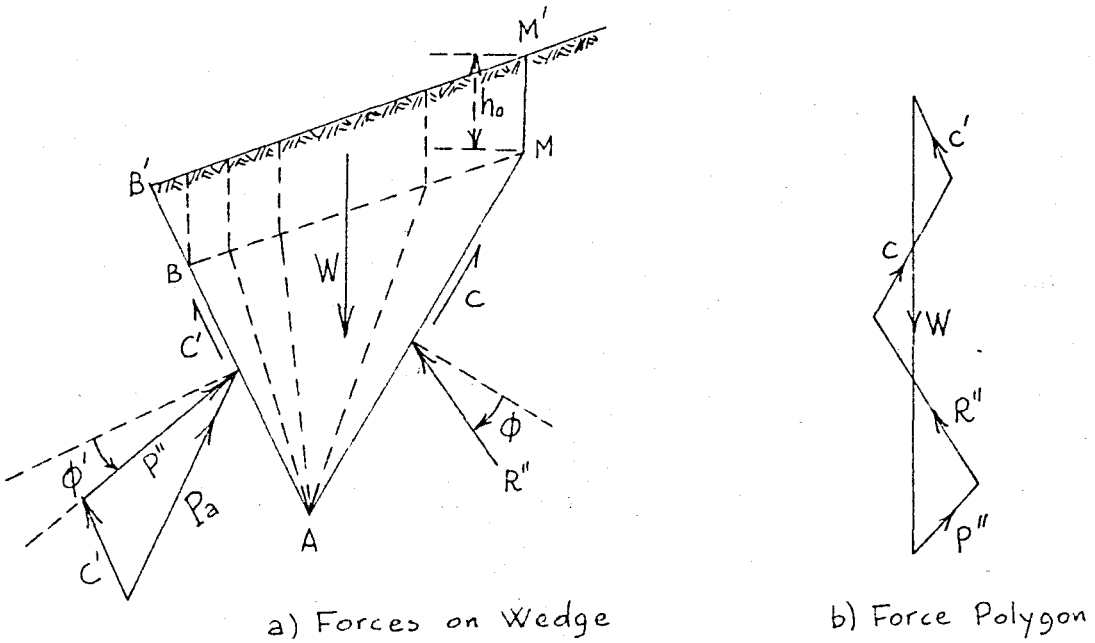
B) COHESIVE SOILS

The procedures are similar to those explained for cohesionless soils, but in addition to the friction along the trial plane, which was considered for cohesionless soils, the effect of cohesion is included in accordance with Coulomb's formula for shearing strength of soils, $S = c + \sigma \tan \phi$. If sliding occurs along a solid surface such as the back of a retaining wall, consideration is given to adhesion between the soil and that surface in addition to friction. This adhesion contributes to the stability of a wall in resisting active pressures and increases the resistance to movement offered by passive pressures.

a) Active Pressures

When a retaining wall with a cohesive backfill tilts or moves slightly outward due to the pressure of the retained earth, the active state of stress develops in the backfill. For walls with plane backs and of the usual proportions, sliding occurs along the back so no outer surface of rupture forms, but an inner surface of rupture forms. As shown in Fig. 11

this surface does not extend all the way from the heel of the wall to the ground surface as in cohesionless backfills, but vertical cracks, such as MM', develop.



a) Forces on Wedge

b) Force Polygon

Fig. 11 - Wedge of thrust for active state of stress for cohesive soils.

The depth of cracking, h_o , for a plane ground surface, regardless of its inclination is given by the formula:

$$h_o = \frac{2c}{\gamma} \tan (45° + \phi/2)$$

where c = unit cohesion

γ = unit weight of soil

ϕ = angle of internal friction

In Fig. 11 a) it is assumed that the location of the plane of rupture AM is known. Therefore, the magnitude and line of action of W, the weight of the wedge of thrust B'AMM', are known. This known force is held in equilibrium by the following forces:

1) The resultant P" of the normal reaction and the friction between the back of the wall and the backfill. This force has an obliquity of ϕ' to oppose the downward movement of the wedge, where ϕ' is the angle of friction between the soil and back of wall. The direction of P" is therefore known.

2) The adhesion C' between the back of the wall and the backfill. This force acts along the back of the wall and is equal in magnitude to the length AB multiplied by c', the unit adhesion between the wall and the backfill. This force acts upward to oppose the downward movement of the wedge. Its magnitude and direction are therefore known.

3) The resultant R" of the normal reaction and the friction along the inner surface of rupture AM. This force has an obliquity of ϕ in the direction to oppose the downward movement of the wedge. The direction of R" is therefore known.

4) The cohesion C along the inner surface of rupture AM. The magnitude of this force equals the length AM multiplied by c, the unit cohesive strength of the soil. It acts upward

to opporse the downward movement of the wedge. Its magnitude and direction are therefore known.

Since the directions of all the forces acting on the wedge of thrust and the magnitudes of all but P" and R" are known, the unknown magnitude of these forces can be determined by constructing a force polygon as shown in Fig. 11 b).

If the location of the inner plane of rupture were known, as has been assumed, the magnitude of P" would be known from the force polygon in b). By combining P" and C', as shown in Fig. 11 a) the magnitude and direction of active pressure P_a against the back of the wall could be found. However, since the location of this plane is not known, in the trial wedge method, its location is determined by trial. Force polygons corresponding to that shown in b) are constructed for several trial planes. The force polygon giving the maximum value of P" is the true value of P" to be used in finding the active pressure, P_a. Thus having found maximum P" we can locate the corresponding inner plane of rupture. The same ϕ' and ϕ values are used in every trial force polygon.

The following considerations assure the validity of the method:

1) The obliquity of R" is less than ϕ for all trial planes except the true rupture plane. This is true because the relative movement necessary to fully mobilize the

frictional resistance occurs only along surfaces where the soil is ruptured by shearing forces. Similarly, the full unit cohesive resistance of the soil is only utilized along the true rupture plane.

2) Since the obliquity of R" is assumed to be ϕ and the full cohesive strength of the soil is assumed for all trial planes, the assumed shearing resistances along the planes other than the true rupture plane therefore are larger than the actual but unknown shearing resistances along those planes.

3) Since the shearing resistance along trial planes reduces the value of P", the values obtained for P" for all trial planes other than the true plane of rupture are smaller than the true value.

4) Since the active pressure, P_a is the resultant of P" and C' and since C' is constant for all trial planes, the true value of P is determined by the same condition as the true value of P".

The procedure for determining the resultant active pressure for a wall with a plane back retaining cohesive soil with a plane ground surface is illustrated in Fig. 12.

The several trial planes in Fig. 12 are located by describing an arc with A as a center; and dividing this arc into equal intercepts, the trial planes with the corresponding vertical cracks are drawn. A pressure locus is drawn utilizing

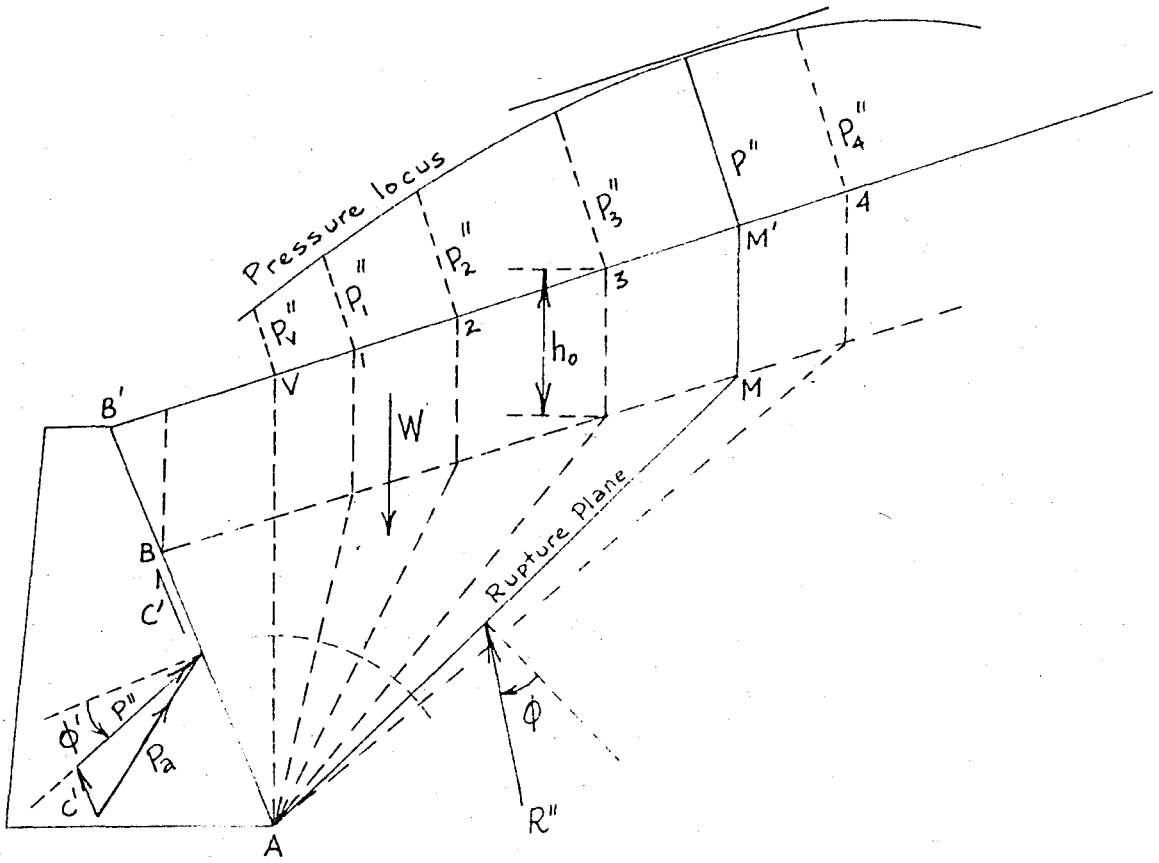


Fig. 12 - Active pressure for cohesive backfill with plane ground surface.

the values of P" taken from the force polygons in each trial. The maximum value of P" determines the rupture plane. Thus the resultant active pressure, P_a is found by combining C' and P" vectors. The procedure for computing the active pressure against a gravity section wall with plane back, supporting a backfill with irregular ground surface, is similar to the one already explained above.

The procedure which has been outlined satisfies the conditions that the sum of the vertical components and the sum of the horizontal components of all the forces acting on the wedge must each equal to zero, because the force polygon closes. However, it does not satisfy the condition that the sum of the moments, about any point, of all the forces acting on the wedge must equal zero. It is not possible to satisfy this requirement if the inner surface of rupture is assumed to be plane because actually it is curved. However, the resulting error is unimportant.

b) Passive Pressures

The basic procedure for computing passive pressures differs from that for computing active pressures as follows:

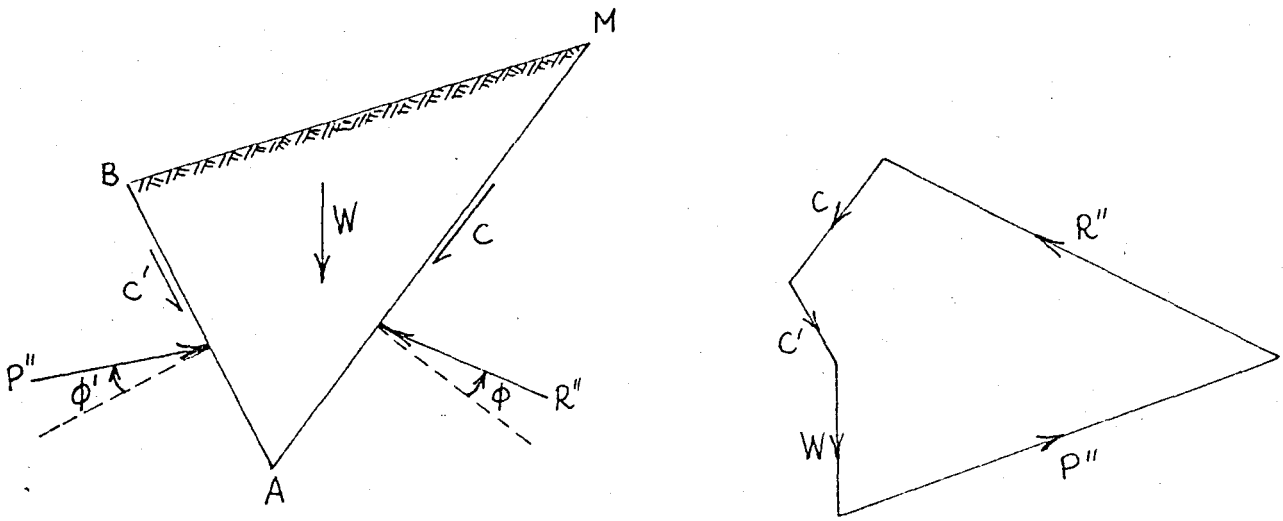
1) Cracks do not form in the backfill for the passive state of stress because the soil is in compression.

2) The wedge of thrust tends to move upward rather than downward.

3) The friction and cohesion components along the rupture surface act downward to oppose the upward motion.

4) The friction and adhesion components along the solid pressure surface act downward to oppose the upward motion of the wedge.

The general case for passive state is illustrated in Fig. 13 and Fig. 14.



a) Forces on Wedge

b) Force polygon

Fig. 13 - Wedge of thrust for passive state of stress for cohesive soils

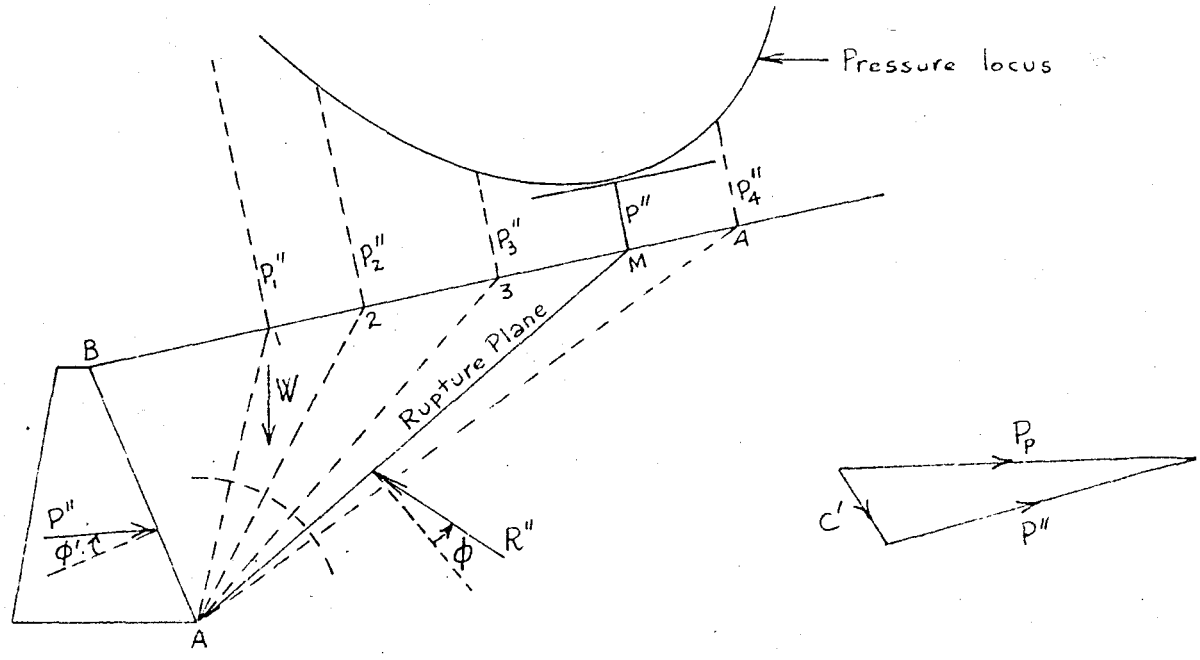


Fig. 14 - Passive pressure for cohesive backfill with plane ground surface

The procedure is similar to the one explained already for passive state in cohesionless soils. The final passive pressure, P_p is found by combining C' and P'' vectors as shown in Fig. 14.

IV) FRICTION CIRCLE METHOD

A) COHESIONLESS SOILS

a) Active Pressures

Rankine's active state of stress exists in the wedge BA'M in Fig. 15. The directions of the planes BA' and A'M can be computed quite easily, but the wedge cannot be drawn because the position of point A' is not known. Its position is determined by the condition that the surface of rupture is so located that the computed active pressure on the surface AB is the maximum value which can be obtained from any assumed surface of rupture. In the friction circle method the surface of rupture is located by trial, as in the trial wedge method.

The procedure for the friction circle method for computing active pressures is illustrated in Fig. 15 and is as follows:

1) Determine the directions of the surface of sliding BA' and the plane portion A'M of the surface of rupture by the following direction formulas:

$$\left.\begin{aligned} \alpha &= \tfrac{1}{2}(90^\circ - \phi) + \tfrac{1}{2}(\epsilon - i) \\ \beta &= \tfrac{1}{2}(90^\circ - \phi) - \tfrac{1}{2}(\epsilon - i) \end{aligned}\right\} \quad \text{where} \quad \sin\epsilon = \frac{\sin i}{\sin\phi}$$

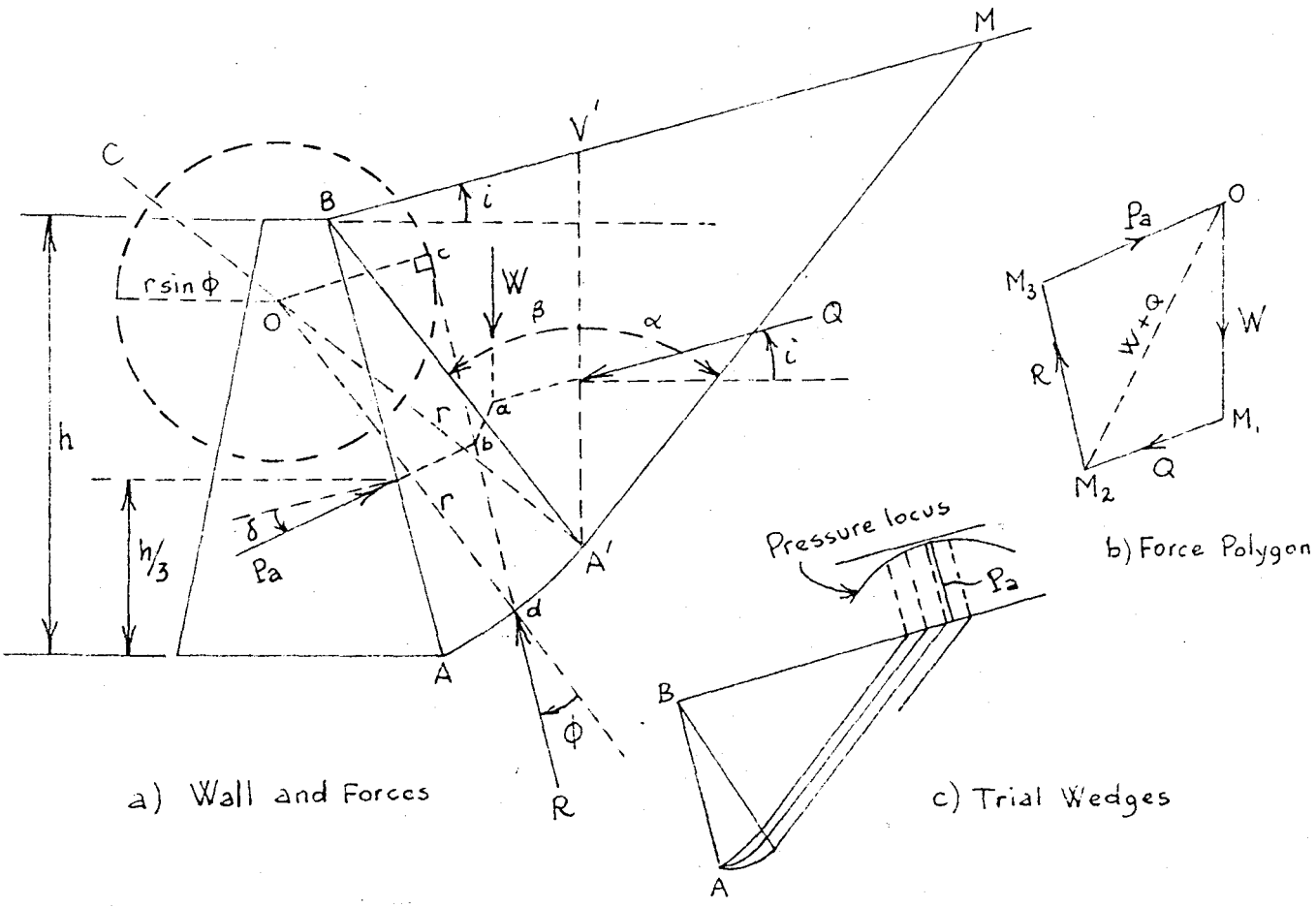


Fig. 15 - Active pressure for inclined ground surface by friction circle method.

2) Select A', the point of intersection of these planes, arbitrarily.

3) Draw the line of action of the resultant active pressure P_a acting through the lower third point of AB and with an obliquity δ in the direction to oppose the downward movement of the wedge.

4) Erect A'C perpendicular to A'M at A'. Draw the arc AA' with its center at a trial point O on this perpendicular.

The radius of this circle is designated by r. It is obvious that point A' must lie below a line through A and parallel to A'N.

5) Determine the weight of the block of soil AA'V'B and the line of action of its resultant weight W.

6) Compute the active pressure, Q on the vertical plane A'V' by Rankine's formula using the height A'V' = h_1 :

$$Q = \tfrac{1}{2}\gamma h_1^2 \left(\cos i \, \frac{\cos i - \sqrt{\cos^2 i - \cos^2 \phi}}{\cos i + \sqrt{\cos^2 i - \cos^2 \phi}} \right)$$

Q acts at the lower third point of A'V' and parallel to the ground surface. Extend Q until it intersects the line of action of W at point a in Fig. 15 a).

7) Determine the direction of OM_2, the resultant of W and Q, by drawing the force triangle OM_1M_2 in b). From point a in a), draw a line parallel to OM_2 until it intersects the line of action of P_a at point b.

8) Describe a circle in a) with its center at O on A'C and its radius equal to $r \sin \phi$. This is called the friction circle or the ϕ-circle, which gives the method its name.

9) Draw a line through point b in a) and tangent to the friction circle at point c to give a positive obliquity and thereby oppose the downward movement of the wedge. This will

intersect the rupture surface at point d. The line cd is the line of action of R, the resultant pressure on the surface A A'. It is seen that, by geometry, R makes an angle ϕ with the normal to the rupture surface at point d, which is the radius dO.

10) Draw a line through M_2 in b) parallel to R, and a line through O parallel to P_a. These lines intersect at M_3. The resultant active pressure P_a is equal to the vector M_3O for the assumed surface of rupture.

11) Repeat the procedure with different locations for A' until the maximum value of P_a is found. This is the true value for the resultant active pressure, and the corresponding surface of rupture is the true surface of rupture.

To assist in determining the maximum value of P_a, the pressure locus in Fig. 15 c) for various trial positions of A' is drawn. The maximum value of P_a as indicated by this curve is the true value for the active pressure.

The basis of the procedure is the same for the case where the entire rupture surface is arc of a circle.

B) COHESIVE SOILS

When sliding occurs along the pressure surface, even though the wall does not obstruct the formation of the outer surface of rupture, this surface of rupture does not form and Coulombe's conditions prevail. The actual form of the surface of rupture is difficult to determine for either the active or the passive conditions, but it has been determined for certain special conditions. However, the trace of the curved portion of a surface of rupture can be assumed to be an arc of a circle without introducing significant errors. So the method which makes use of circular arcs will be considered here.

a) Active Pressures

When the outer surface of rupture is obstructed by the wall or when the formation of this surface is not obstructed by the wall but still this surface does not form because sliding occurs on the back of the wall, the friction circle method may be used to advantage. The circular surface of rupture which is appropriate for both of these conditions is illustrated in Fig. 16. It extends from the heel of the wall to the bottom of the zone of cracking, and is located by trial.

In Fig. 16, it is assumed that the location of the circular arc which corresponds to the surface of rupture is known. Its center is at O in a) and its radius is r. It is assumed that a crack forms along BB' . The resultant pressure on AB

is assumed to be the same as the pressure on AB'. The pressure on AB' is determined as follows:

1) Compute the depth h_o of the crack M'M' by the formula:

$$h_o = \frac{2c}{\gamma} \tan(45° + \phi/2)$$

2) Determine the weight W and the centroid of the wedge of soil AM'M'B'.

3) Compute the adhesion along the back of the wall. $C' = c' \cdot \overline{AB}$. As has been stated, this force is often omitted.

4) Compute the magnitude and direction of the resultant cohesion on the arc AM. $C = cL_c$, where L_c = length of chord AM, and C acts parallel to the chord.

5) Compute the line of action of the resultant cohesion, C.

$$cL_c m = cl_a r$$

where l_a = length of arc AM

r = radius of arc AM

m = distance of the line of action of C from the center of the arc. From the above equation $m = \frac{l_a}{L_c} r$

6) Determine the point of application of P", the resultant of the normal stress and the friction along AB. This can be located approximately by drawing a vertical line

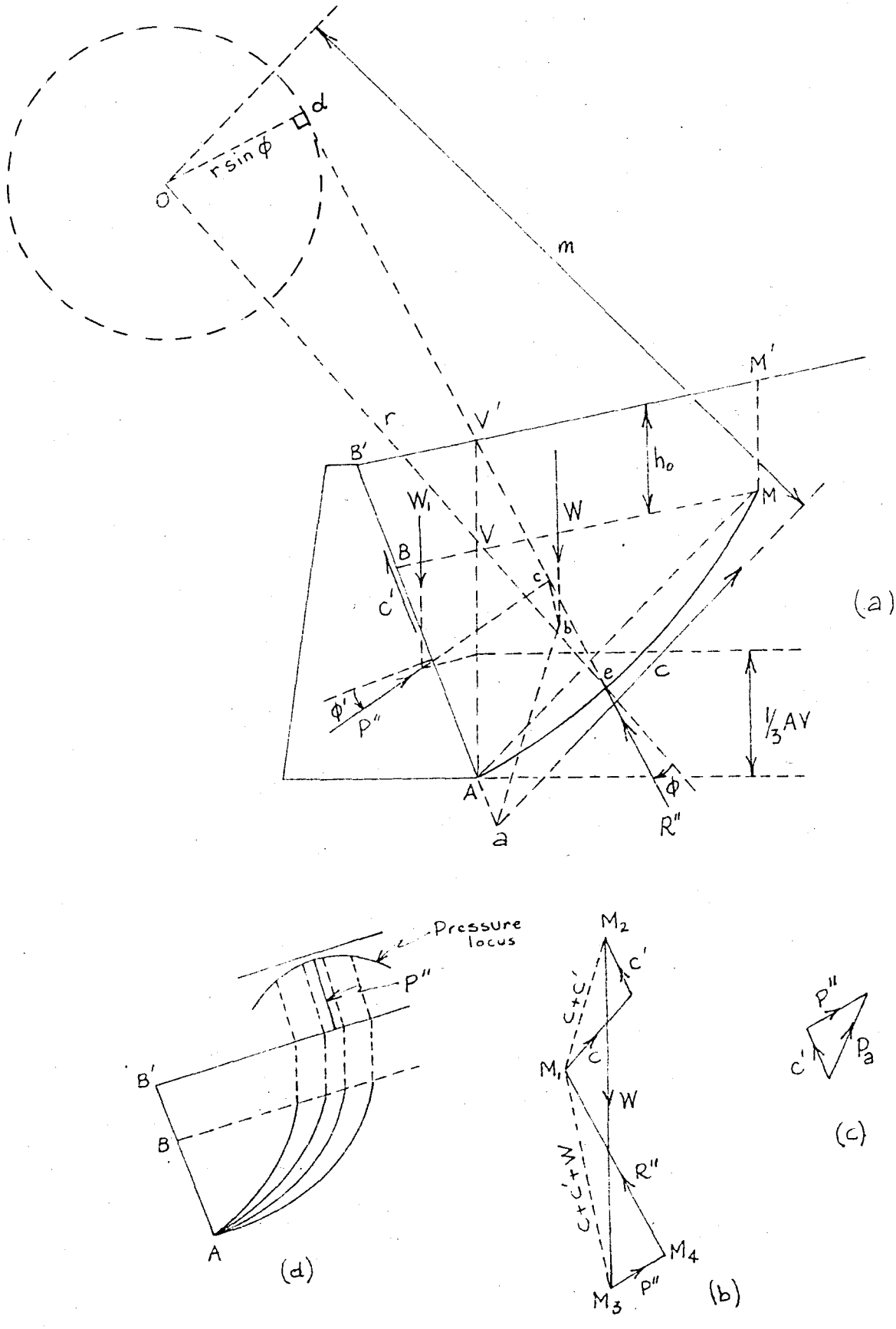


Fig. 16 - Active pressure by friction circle method for cohesive soils.

through the centroid of the triangle $AB'V'$ until it intersects a line through the lower third point of AV and parallel to the ground surface. The resultant P" may be assumed to act through this point at angle ϕ' below the normal to AB, as shown in (a)

7) Lay off the forces W, C', and C as shown in(b)

8) Determine the magnitude and direction of the resultant of C and C' by drawing M_1M_2 in b). Determine the line of action of this resultant by drawing a line parallel to M_1M_2 in b) through the intersection of the lines of action of these forces at point a in a) and extend until it intersects the line of action of W at point b.

9) Determine the magnitude and direction of the resultant of W, C' and C by drawing M_1M_3 in b). Determine the line of action of this resultant by drawing a line parallel to M_1M_3 through point b in a) and extending it until it intersects the line of action of P" at point c.

10) Draw a circle in a) with its center at O and a radius equal to $r \sin\phi$. Draw a line through point c tangent to this circle at point d. By geometry, this line makes an angle ϕ with the radius of the surface of rupture at point e where the line intersects this surface. The line de is the line of action of the resultant force R" on the surface of rupture AM.

11) Through M_1 in b) draw a line parallel to de in a) and a line through M_3 parallel to P" in a). These lines meet at M_4 and cause the force polygon to close. The magnitude and direction of P" is M_3M_4 and of R" is M_1M_4.

12) Determine P_a, the active pressure on AB', draw the force triangle in c) in which P_a is the resultant of P" and C'.

In the preceding procedure for determining P_a it is assumed that the location of the surface of rupture is known. Actually it is not known. It is therefore necessary to repeat the procedure for several trial surfaces of rupture with different centers and different radii. As in the trial wedge method, the trial surface which results in the maximum value of P" is the surface of rupture. The maximum value is determined by the procedure illustrated in Fig. 16 d)

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V) LOGARITHMIC SPIRAL METHOD

This method was first presented by Karl Terzaghi under the name of "the general wedge theory". Here the shape of the surface of sliding is assumed to be a logarithmic spiral with the polar equation:

$$r = r_o e^{\alpha \tan \phi} \qquad (3)$$

where r = length of a vector through the center of the spiral.

r_o = length of arbitrarily selected reference vector.

e = base of natural logarithms.

α = angle between r and r_o

ϕ = angle of internal friction

Fig. 17 shows such a spiral according to eq. (3). It has the property that a vector passing through the center of the spiral and any arbitrary point on the spiral, intersects the normal to this point at an angle equal to the angle ϕ of internal friction. If, therefore, the surface of rupture Bc in Fig. 17 is assumed to be a portion of a logarithmic spiral, the reactions on all its elements at the moment of failure must pass through a common point, namely, the center of the spiral. The determination of the active pressure P_A can thus be made from the weight of the wedge ABC.

The general wedge theory assumes that the frictional resistance is fully mobilized over the entire surface of sliding. As a consequence the reaction dF on each element

of the sliding surface intersects the normal to this surface at an angle ϕ . Hence all the reactions dF and the resultant reaction F must pass through the center of the spiral, O. If the point of application and direction of lateral pressure are now known, then it is seen that its magnitude can be found by taking moments about the center of the spiral

$$P_A = W \frac{p}{m}$$

where p and m are the distances from the spiral center to the vectors W representing the weight of the wedge and the vector P_A representing the lateral pressure, respectively. Several spirals are assumed to intersect the horizontal surface of the bank at different distances from the rim of the cut, A. The resulting active earth pressures are plotted to form a curve, the maximum ordinate of which, OO_1, will represent the active earth pressure on surface AB.

The general wedge theory is based on the empirical facts that the surface of sliding intersects the horizontal surface of the bank at a right angle and that the center of the lateral pressure on the timbering of a cut, with a depth h in sand, is somewhere between an elevation of 0.5 h and 0.6 h above the bottom of the cut. This ratio has been designated by n in Fig. 17 and must be known or assumed . Likewise the angle of internal friction ϕ must also be known or assumed.

THESIS
ROBERT COLLEGE GRADUATE SCHOOL
BEBEK, ISTANBUL

An assumption of complete absence of friction between the wall and the earth will be on the safe side. The magnitude of W can be taken, with sufficient accuracy, as

$$W = \frac{2}{3} AB \times AC$$

and its distance from the bank as $0.4\ AC$.

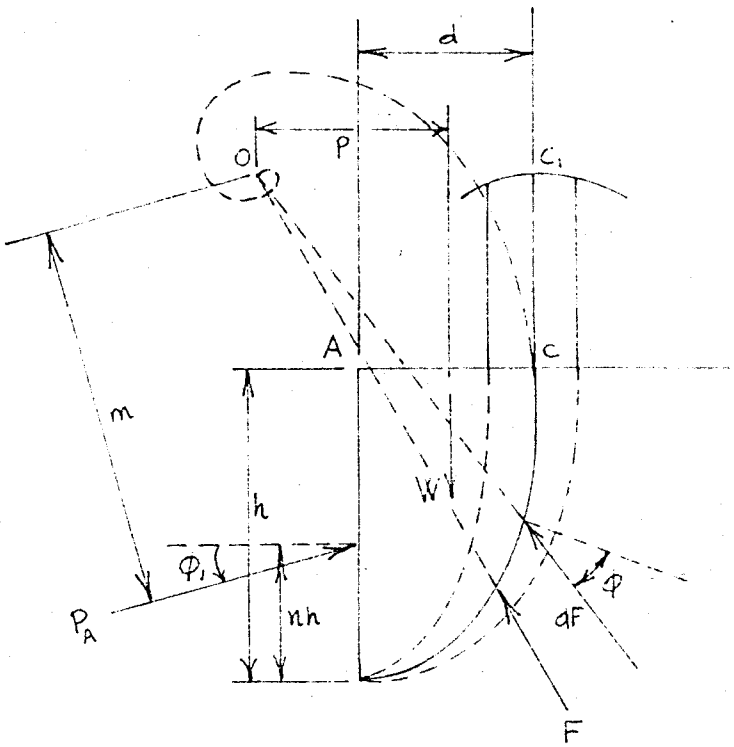


Fig. 17 - General Wedge Theory

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VI) J. BRINCH HANSEN'S METHOD

a) Assumptions

The earth may consist of gravel sand, silt, clay or any other material encountered in the foundation technique (except solid rock). Also such materials as for exemple, grain, cemont, coal and ore may be included in the general term "earth". The essential requirement is that the material obeys Coulomb's law, at least approximately. This law states that a shearing stress τ , to which corresponds a normal stress, σ , is limited by the condition:

$$\tau = c + \sigma \tan \phi \qquad (4)$$

where c (the cohesion) and ϕ (the angle of internal friction are assumed to be constants for the material in question.

Equation (4) is principally valid for dry earth. In order to arrive at simple results, it is used for submerged or moist earth too.

The earth may be stratified, but within each separato layer it is assumed to be homogeneous and isotropic.

The shearing stress acting between a structure (here generally called a wall) and the earth is, in analogy with Coulomb's Law, assumed to be limited by the condition:

$$f \leq a + e \tan \delta$$

THESIS
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BEBEK, ISTANBUL

where e and f are the normal and tangential components respectively of the unit earth pressure. a (the adhesion) cannot exceed c , and δ (the angle of wall friction) cannot exceed ϕ . The maximum values of a and δ , which will occur in case of a relative tangential movement between wall and earth, are assumed to be constants for the wall and earth in question. If no such movement takes place lower values of a and δ can occur.

In the special case $a = 0$, $\delta = 0$ the wall is called perfectly smooth, and in the other special case $a = c$, $\delta = \phi$ it is called perfectly rough. Actual walls will generally approximate rather closely the latter case, but nevertheless, it is often preferred to assume the walls to be smooth, this being simpler and usually on the safe side.

With regard to the movements of the wall, it is assumed that these are great enough to produce a state of rupture in the earth, but, on the other hand, they are small in comparison with the height of the wall. The wall itself may, or may not, be in a state of failure. In the latter case a limited number of yield hinges are supposed to develop.

In many cases the movement of the wall in its own plane can be assumed negligible in comparison with its movements perpendicular to this plane. This is usually the case when

the wall is founded on the firm bottom, or on piles, or is driven into firm ground. In such cases the wall is said to make normal movements only (e.g. normal rotation). In certain other cases (e.g. anchor slabs and cellular coffer-dams) the tangential movements are not negligible, as the wall may be lifted in the state of failure. For perfectly smooth walls this problem is of no importance.

Elastic deformations of earth or wall are assumed to be small of a higher order than the plastic deformations, and these are supposed to be small of a higher order than the height of the wall.

One result of this is that an elastic earth zone, the interior of which is not in the state of failure, can be assumed to move as a rigid body. Another consequence is that the wall, in which either no cross section at all, or a finite number of sections, are in a plastic state, will move either as one rigid body or as a finite number of rigid parts connected by links (yield hinges). Each such part rotates in the state of failure about a certain rotation center.

It is assumed that the earth is incompressible, i.e. that even in a plastic zone the volume of the earth does not change through the deformation. This assumption is made in

most plasticity theories and is very accurate for saturated clay, whereas for sand it is only correct after the shear strain has reached a certain minimum value. In cases of progressive failure it is a very good approximation, even for sand.

Problems with states of plain strain are considered only.

b) Main Principles

An earth pressure problem involves, generally, a number of given forces and dimensions as well as some unknown ones, which should be determined by the calculation. Incidentally, unknown dimensions can seldom be determined directly, and it is therefore usually necessary to start the calculation with estimated values of the unknown dimensions and correct them later if necessary.

In the calculation the state of failure is considered. On the other hand the structure should be designed in such a way that there is a certain safety against failure. This is achieved by carrying out the calculation, not with the actual loads and shear strengths, but with the actual loads multiplied by certain safety factors, and with the actual shear strengths divided by other safety factors.

THESIS
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The first step of the calculation consists in determining or choosing the type of movement to be performed by the structure in the state of failure. In some cases the movement can be chosen so as to obtain the maximum efficiency. In other cases the movement is practically given (e.g. a retaining wall must tilt forward, rotating about a point below its foot).

For more complicated structures more than one state of failure may be possible. In such cases any one of the possible states of failure can be chosen as the basis of the design, it being mainly a matter of economy and convenience as to which is preferred.

When the type of movement of the structure in the state of failure has been determined, the possible figures of rupture in the adjacent earth masses must be investigated. Only such figures of rupture should be considered, which imply movements of the earth masses compatible with those of the structure.

The next step is to calculate the earth pressures acting between the structure and the adjacent earth masses. These pressures will depend on the assumed rupture-figures in the earth, and they can be calculated if we know the internal stresses in the lines of rupture.

This knowledge is obtained by two different means. First, we use Kötter's equation which determines the stresses at any point of a curved rupture-line, provided that they are known at one point of the line. Second, we use a special boundary condition, which is devoloped in such a way that the most reliable results are obtained.

When the internal stresses in the rupture-lines are known we can calculate the earth pressures by means of the statical conditions of equilibrium for the different earth zones bounded by rupture-lines, ground surfaces, walls or other boundaries. For each separate zone we have 3 conditions of equilibrium.

Often more than one figure of rupture may satisfy the conditions. The final calculation must then be based on the most critical rupture-figure, i.e., the one for which the work done by the earth pressure acting upon the earth is a minimum.

When the earth pressures on the structure have been determined, we must finally consider the equilibrium of the structure proper. This gives another 3 statical conditions, but in some cases only 2 of these are necessary to solve the main problem.

As already mentioned, the original problem will usually

present one or more unknown quantities. The assumed movement of the structure may involve additional unknown quantities. Finally, the assumed rupture-figures in the earth will add a new set of unknown quantities.

If the problem is to be soluble the total number of unknown quantities must equal the total number of conditions (statical, kinematical, and geometrical). However, this can actually be obtained in most practical earth pressure problems.

c) Stresses in Rupture-Circle

(General Case)

We consider an infinitely small earth element in Fig. 18 and use the polar coordinates r and v, the angle v being measured from the vertical. By projection of all forces (the indicated stresses multiplied by the corresponding side lengths) on the radius and the tangent respectively the following equations are obtained:

$$r\frac{\partial \sigma_r}{\partial r} + \frac{\partial \tau_{rv}}{\partial v} + \sigma_r - \sigma_v - \gamma r \cos v = 0 \qquad (5)$$

$$r\frac{\partial \tau_{rv}}{\partial r} + \frac{\partial \sigma_v}{\partial v} + 2\tau_{rv} + \gamma r \sin v = 0 \qquad (6)$$

where γ = unit weight of earth.

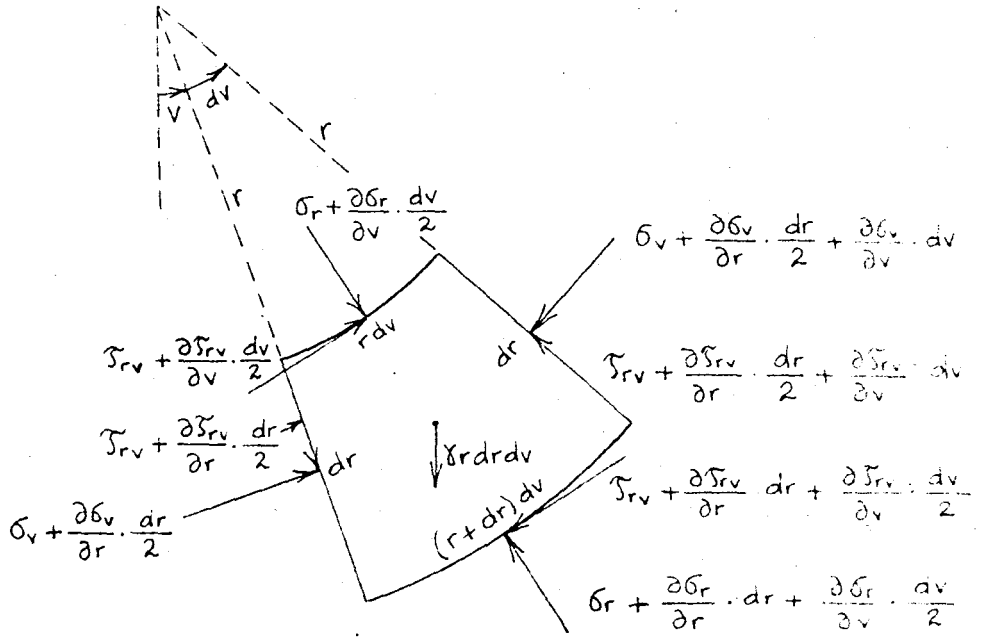


Fig. 10 - Small earth element in rupture-circle.

We shall now investigate the stresses in a rupture-circle with its center in the pole of the coordinate system. Inserting the stresses σ_r and τ_{rv} in Coulomb's Law (eq. (4)) we have:

$$\tau_{rv} - \mu \sigma_r \leq c \qquad \text{where } \mu = \tan \varphi \tag{7}$$

The function indicated on the left side of eq. (7) must have a constant value in the rupture line proper and, with respect to the variation in the radial direction, it must either be constant (zone-rupture) or attain a maximum value at the rupture line (line-rupture). In both cases we have:

$$\frac{\partial (\tau_{rv} - \mu \sigma_r)}{\partial r} = 0 \tag{8}$$

$$\frac{\partial (\tau_{rv} - \mu \sigma_r)}{\partial v} = 0 \tag{9}$$

Multiplying eq. (5) with $\mu = \tan \phi$, subtracting it from eq. (6) and using eq. (8) we get:

$$\frac{\partial(\sigma_v - \mu \tau_{rv})}{\partial v} + \mu(\sigma_v - \sigma_r) + 2\tau_{rv} + \gamma r \sec\phi \sin(v+\phi) = 0 \qquad (10)$$

For the sake of simplicity we shall in the following omit the subscripts on the stresses σ and τ in the rupture-line proper. For other normal stresses we get by means of Mohr's circle in Fig. 19:

$$\sigma_v = \sigma + 2\mu\tau \qquad (11)$$

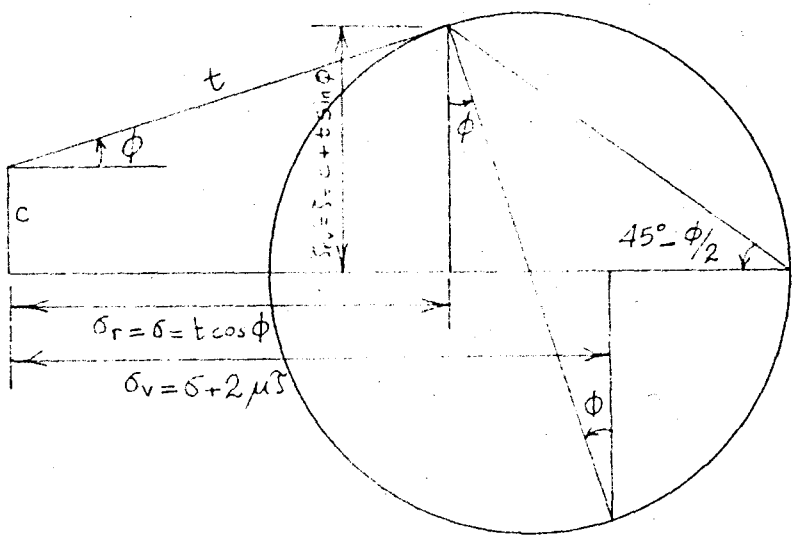


Fig. 19 - Mohr's circle for stresses

Using eq. (9) and eq. (11) to eliminate the normal stresses from eq. (10) we find the following differential equation in τ alone, valid for the rupture-line proper:

$$\frac{d\tau}{dv} + 2\mu\tau + \gamma r \sin\phi \sin(v+\phi) = 0 \qquad (12)$$

This is Kötter's equation for the variation of the shear stress in a rupture-line. Kötter derived it for cohesionless earth, and Jáky showed that it is also valid for earth with cohesion.

The solution of eq. (12) is, for r = constant (e.g. circle):

$$\tau = K e^{-2\mu v} + \gamma r \sin\phi \cos\psi \cos(v+\phi+\psi) \qquad (13)$$

where K is a constant stress and ψ a constant angle defined by:

$$\psi = \arctan 2\mu$$

If v' and τ' are corresponding values, then eq. (13) gives:

$$K = e^{2\mu v'}\left[\tau' - \gamma r \sin\phi \cos\psi \cos(v'+\phi+\psi)\right] \qquad (14)$$

Instead of the stresses σ and τ in the rupture-line we shall now introduce the cohesion c and the remaining stress t which makes an angle ϕ with the normal. From Mohr's circle (Fig. 19) it will be seen that:

$$\sigma = t\cos\phi \qquad (15)$$

$$\tau = c + t\sin\phi \qquad (16)$$

In Fig. 20, we consider now a rupture-circle with a radius r, a chord-length, k , a central angle 2α and

THESIS
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angle β between chord and horizon. The point at which the boundary stresses are known is called the starting point (') and the other end the finishing point (") . β Should be assumed positive when the starting point is located above the finishing point. We have then:

$$V' = \beta - \alpha \qquad (17)$$

$$V'' = \beta + \alpha \qquad (18)$$

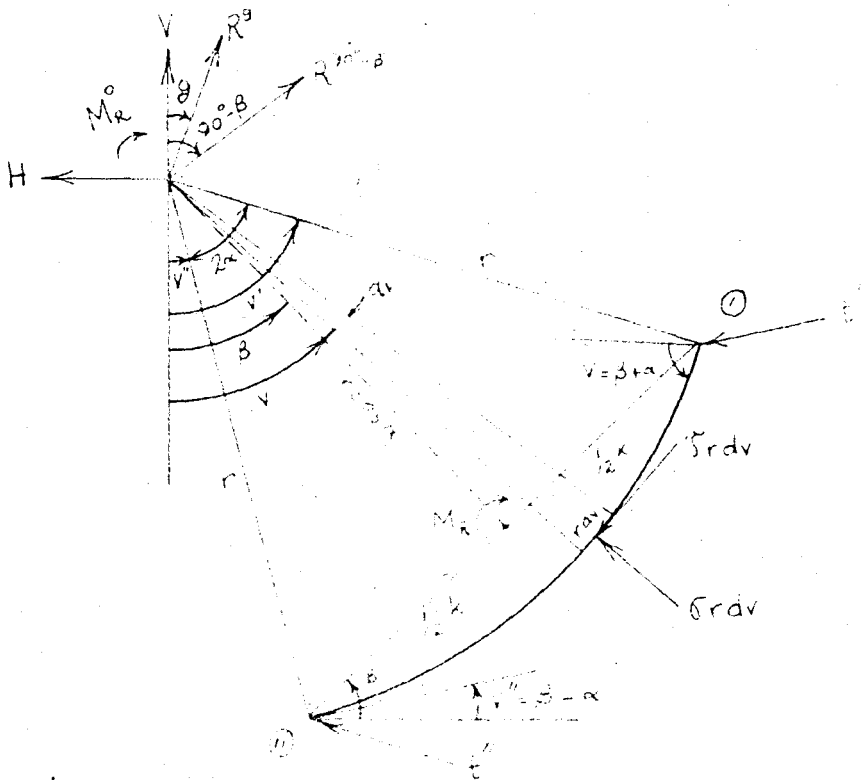


Fig. 20 - Resultants of stresses in rupture-circle

Using eq. (13) for the finishing point, inserting K from eq. (14), eliminating τ' and τ'' by means of eq. (16)

and finally using eqs. (17) and (18) we get for the stress, t":

$$t'' = \gamma r \cos\psi \left[\cos(\beta - \alpha + \phi + \psi) - s\cos(\beta + \alpha + \phi + \psi)\right] + st' + \frac{c(s-1)}{\sin\phi} \qquad (19)$$

where $s = e^{4\mu\alpha}$ (e = basis of natural logarithm)

or

$$t'' = \gamma k \left(t^{\gamma_x} \sin\beta + t^{\gamma_y} \cos\beta\right) + \left(t' + \frac{c}{\sin\phi}\right) s - \frac{c}{\sin\phi} \qquad (19a)$$

where:

$$t^{\gamma_x} = \frac{\cos\psi}{2\sin\alpha}\left[s\sin(\psi + \phi + \alpha) - \sin(\psi + \phi - \alpha)\right]$$

$$-t^{\gamma_y} = \frac{\cos\psi}{2\sin\alpha}\left[s\cos(\psi + \phi + \alpha) - \cos(\psi + \phi - \alpha)\right]$$

Next, we shall determine the resultants of the stresses acting in the rupture-circle on the moving earth mass outside this circle in Fig. 20. On each element $r\,dv$ of the circle acts a shear stress τ, given by eq. (13) and a normal stress σ, which can be found by means of Mohr's circle in Fig. 19 :

$$\sigma = (\tau - c)\cot\phi \qquad (20)$$

The resulting force in a direction making an angle g with the vertical is:

$$R^g = \int_{v''}^{v'} \left[\sigma\cos(v+g) - \tau\sin(v+g)\right] r\,dv \qquad (21)$$

Eliminating σ by means of eq. (20), inserting τ from eq. (13), carrying out the integrating, inserting K from eq. (14), eliminating τ' by means of eq. (16) and, finally using eqs. (17) & (18) we can find:

$$R^g = \frac{1}{2}\gamma r^2 \cos^2\psi\Big[\sin(2\psi-g) + 2\alpha\sec\psi\cos(\psi-g) - s\sin(2\alpha+2\psi-g)$$

$$+ \sin(2\beta+2\phi+g)\Big\{s - \sec\psi\cos(2\alpha-\psi)\Big\}\Big] + \Big(t'+\frac{c}{\sin\phi}\Big) r\cos\psi$$

$$\Big[\sin(\beta+\alpha+\phi-\psi+g) - s\sin(\beta-\alpha+\phi-\psi+g)\Big] - 2cr\cot\phi\sin\alpha\cos(\beta+g) \qquad (22)$$

The vertical component, V is obtained with $g = 0$, and the horizontal component, H with $g = -90^\circ$. Then we have:

$$V = \gamma k^2\Big[V^{\gamma z} + V^{\gamma x}\sin(2\beta+2\phi)\Big] - ck\cot\phi\cos\beta$$

$$+ \Big(t'+\frac{c}{\sin\phi}\Big)k\Big[V^{tx}\sin\beta + V^{ty}\cos\beta\Big] \qquad (23)$$

$$H = \gamma k^2\Big[H^{\gamma z} + H^{\gamma y}\cos(2\beta+2\phi)\Big] - ck\cot\phi\sin\beta$$

$$+ \Big(t'+\frac{c}{\sin\phi}\Big)k\Big[H^{tx}\sin\beta + H^{ty}\cos\beta\Big] \qquad (24)$$

where:

$$-V^{tx} = H^{ty} = \frac{\cos\psi}{2\sin\alpha}\Big[s\cos(\psi-\phi+\alpha) - \cos(\psi-\phi-\alpha)\Big]$$

$$V^{ty} = H^{tx} = \frac{\cos\psi}{2\sin\alpha}\Big[s\sin(\psi-\phi+\alpha) - \sin(\psi-\phi-\alpha)\Big]$$

$$-H^{\gamma_y} = V^{\gamma_x} = \frac{\cos^2\psi}{8\sin^2\alpha}\left[s - \sec\psi\cos(\psi - 2\alpha)\right]$$

$$V^{\gamma_z} = \frac{\cos^2\psi}{8\sin^2\alpha}\left[\sin 2\psi - s\sin(2\psi + 2\alpha) + 2\alpha\right]$$

$$H^{\gamma_z} = \frac{\cos^2\psi}{8\sin^2\alpha}\left[\cos 2\psi - s\cos(2\psi + 2\alpha) - 2\alpha + \tan\psi\right]$$

The moment about the center of the circle is:

$$M_R^\circ = \int_{V''}^{V'} \gamma r^2 dv \qquad (25)$$

Inserting γ from eq. (13), carrying out the integration, inserting K from eq. (14) eliminating γ' by means of eq. (16) and, finally, using eqs. (17) & (18) we can find:

$$M_R^\circ = \gamma r^3 \cos\psi\left[2\sin\phi\sin\alpha\cos(\beta+\phi+\psi) - \frac{1}{2}(s-1)\cos\phi\right.$$

$$\left.\cos(\beta+\alpha+\phi+\psi)\right] + \left(t' + \frac{c}{\sin\phi}\right)\frac{1}{2}r^2(s-1)\cos\phi \qquad (26)$$

The moment about the middle of the chord is (Fig. 20):

$$M_R = M_R^\circ + R^{90^\circ-\beta}\, r\cos\alpha \qquad (27)$$

Inserting eqs. (26) & (22) with $g = 90^\circ - \beta$ in eq. (27) we get:

$$M_R = \gamma k^3\left(M_R^{\gamma_x}\sin\beta + M_R^{\gamma_y}\cos\beta\right) + \left(t' + \frac{c}{\sin\phi}\right)k^2 M_R^t \qquad (28)$$

where:

$$M_R^t = \frac{\cos\psi \cot\alpha}{4\sin\alpha}\left[\cos(\psi-\phi-\alpha) - s\cos(\psi-\phi+\alpha) + \frac{1}{2}(s-1)\cos\phi\sec\psi\sec\alpha\right]$$

$$M_R^{\gamma x} = \frac{\cos\psi\cot\alpha}{16\sin^2\alpha}\left[2\alpha\cos\psi - 4\sin\phi\sin(\psi+\phi)\tan\alpha + \sin 2\phi\cos(\psi-2\alpha) + \cos\psi\sin 2\psi - \sin(\psi+\phi+\alpha)\left\{2s\cos\psi\cos(\psi-\phi+\alpha) - (s-1)\cos\phi\sec\alpha\right\}\right]$$

$$M_R^{\gamma y} = \frac{\cos\psi\cot\alpha}{16\sin^2\alpha}\left[2\alpha\sin\psi + 4\sin\phi\cos(\psi+\phi)\tan\alpha - \cos 2\phi\cos(\psi-2\alpha) - \cos\psi\cos 2\psi + \cos(\psi+\phi+\alpha)\left\{2s\cos\psi\cos(\psi-\phi+\alpha) - (s-1)\cos\phi\sec\alpha\right\}\right]$$

From Figs. 18 and 20 it will be seen that γ is assumed positive when, acting upon the moving earth mass outside the rupture-line, it is directed from the starting point (') towards the finishing point ("), i.e. towards the foot of the wall. The corresponding pressure in the rupture-line is called passive, whereas the pressure corresponding to a negative γ is called active.

The preceding formulas are valid in all cases if:

c, ϕ, ψ and μ are assumed <u>positive</u> for <u>passive</u> pressure,

c, ϕ, ψ and μ are assumed <u>negative</u> for <u>active</u> pressure,

α and r are assumed <u>positive</u> for a <u>concave</u> circle,

α and r are assumed <u>negative</u> for <u>convex</u> circle,

β is assumed <u>positive</u>, when the chord <u>falls</u> towards the wall.

β is assumed <u>negative</u>, when the chord <u>rises</u> towards the wall.

For a straight rupture-line ($\alpha \to 0$) the following formulas can be derived:

$$t'' = \gamma k \sin(\beta+\phi) + t'$$

$$R^g = \frac{1}{2}\gamma k^2 \sin(\beta+\phi)\cos(\beta+\phi+g) + kt'\cos(\beta+\phi+g) - ck\sin(\beta+g)$$

$$V = \frac{1}{4}\gamma k^2 \sin(2\beta+2\phi) + kt'\cos(\beta+\phi) - ck\sin\beta$$

$$H = \frac{1}{2}\gamma k^2 \sin^2(\beta+\phi) + kt'\sin(\beta+\phi) + ck\cos\beta$$

$$M_R = \frac{1}{12}\gamma k^3 \cos\phi \sin(\beta+\phi)$$

As usual, ϕ and c should be assumed positive for passive pressure and negative for active pressure.

d) Factors of Safety

In the design of earth-retaining structures by means of a theory of rupture it is necessary to introduce certain safety factors for the following reasons:

1) The actual loads may deviate from the specified or calculated values.

2) The actual strengths of the soils and the structural materials may deviate from the estimated or measured values.

3) The calculation as such involves a number of unavoidable inaccuracies, mainly due to simplifications, approximations, etc.

In order to take these facts into consideration the calculation is carried out with the specified loads multiplied by certain safety factors, and with the measured shear strengths of the soils divided by other safety factors. Finally, the structures are designed for the calculated forces and moments with allowable stresses equal to the measured strengths of the materials divided by suitable safety factors.

With regard to the loads, the most important ones, such as, the weights of the earth masses, can be determined with great accuracy by means of very simple tests. Therefore, it is proposed that no special safety factor be applied to these weights, that is, denoting $\underline{n}$ for safety factor, $n_\gamma = 1$. This is also preferable from practical point of view. For the weight of the structures proper it is likewise proposed that $n_\gamma = 1$ be assumed.

As regards the surcharges $\underline{P}$ these are, of course, rather uncertain. On the other hand, it seems to be current practice to specify a surcharge so high that it will actually rarely, if ever, occur over any considerable area. For this reason it is proposed to use $n_p = 1$, but it will present no difficulties at all if $n_p \neq 1$ is used.

Water pressures can usually be calculated very accurately, but the fixing of the water levels may involve some inaccuracy. Therefore, a certain safety factor should be applied to water pressures, and it is proposed to use $n_w = 1.2$.

Suitable safety factors must also be applied to other exterior loads such as wave pressures, wind pressures, impact forces from berthing ships, etc.

The actual shear strength of the soil under the prevailing conditions should always be determined by proper soil mechanics test. It will usually be found that the internal friction of cohesionless materials (sand and gravel) can be determined with less uncertainty than the cohesion (undrained shear strength) of frictionless materials (fully saturated clay). Therefore, it seems logical to apply asmaller safety factor to $\mu = \tan\phi$ than to c. The following values are proposed:

$$n_\mu = 1.25 \qquad n_c = 1.50$$

The proposed value of $n_\mu = 1.25$ corresponds to the ratio between tan 36° and tan 30°. Therefore, if the actual friction angle of a soil is 36° , the calculations should be made with $\phi = 30°$.

The ultimate strengths of the structural materials should be determined experimentally, but deviations from the measured values must, of course, be expected, and these deviations are greater for concrete and timber than for steel. Therefore, a greater safety factor must be applied to concrete and timber than to steel. On the other hand, all these safety factors can be smaller than those usually specified for such materials, because we have already applied certain safety factors to the shear strength of the soil and to some of the exterior loads.

For steel, we find the allowable bending or tensile stress by dividing the yield stress by a safety factor n_s . For concrete, we must divide the compressive strength by a safety factor n_c . Finally, for timber, the bending strength should be divided by n_t . The following values are proposed:

$n_s = 1.2$ $\qquad$ $n_c = 2.5$ $\qquad$ $n_t = 2.5$

It is, of course, also possible to design the structure in the conventional way with the usual allowable stresses, but in that case the forces and moments found by our calculations should first be divided by about 1.3.

e) Main Steps in the Theory

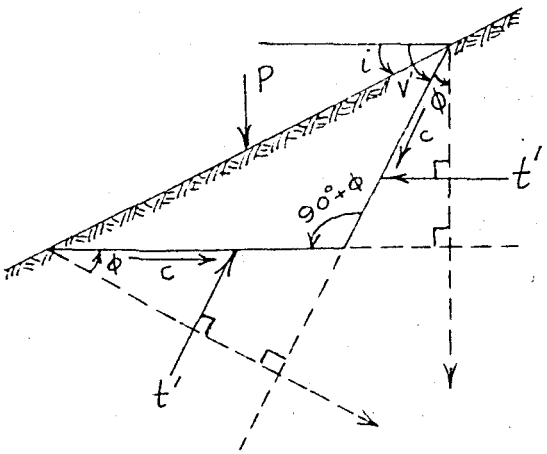
The main steps in the theory are briefly as follows:

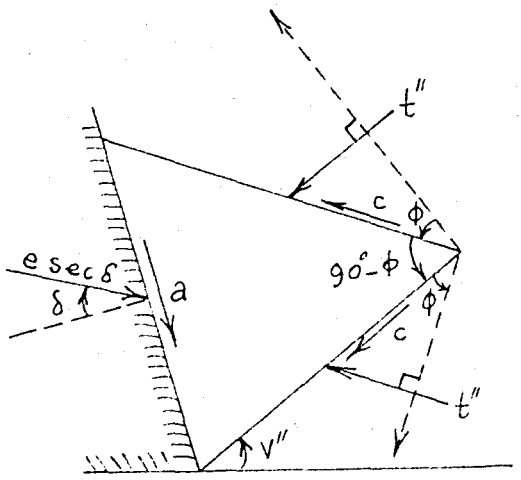
(1) The normal and shear stresses acting on a soil element together with its weight, Fig. 18, are resolved radially and tangentially. Using Coulomb's equation $\tau = c + \sigma \tan \phi$, the normal stresses may be eliminated to give Kötter's equation eq. (12).

(2) The radius, r of the slip line is variable and unknown. Therefore, the actual line is replaced by a number of circular slip lines and/or straight line, so that over each part of the line r is constant. Integrating Kötter's equation for constant r and putting in the limiting resultant stresses t' and t" at each end of the part of the slip line considered, together with the angle 2α through which the line has turned, Fig. 20, a relation, eq. (19 a), between the resultant stresses t', t" at each end of the line is obtained. Resolving horizontally vertically and taking moments, the resultant forces and moment of the normal and shear stresses round the slip line are obtained in general eqs. (23), (24) & (28). These equations contain 14 dimensionless coefficients, which vary with ϕ and α and which have so far been tabulated for $\phi = 0^\circ$ and 30° in the book by Brinch Hansen, "Earth Pressure Calculation" 1953.

(3) The equilibrium of small earth wedges at the soil surface and adjacent the wall respectively, Fig. 21 (a) and (b), are considered. By resolving the forces in two dotted directions

for each wedge we have two equations. At the surface there are two unknowns only, the resultant stress t' and the slip line direction v' , so that each is determined. At the wall there are three unknowns, since the earth pressure e is unknown also. Provided the chosen slip line





assumption:

$$\frac{a}{c} = \frac{\tan\delta}{\tan\phi}$$

e = unit normal earth pressure

a) at ground surface
unknowns: v' & t'

b) at wall unknowns:
v" , t" & e

Fig. 21 - Small earth elements at ground surface and at wall.

satisfies v' and t', then using Kötter's equation t" and e are obtained at the wall. When the chosen slip line cannot satisfy the direction v' at the surface it is necessary to consider the equilibrium of the earth mass between the wall and the slip line as a whole. For this purpose general expressions are developed for the force and moment due to the weight of the earth mass, which can then be applied to a composite slip line

by dividing the slipping mass into separate sectors.

These are the only data required to solve zone ruptures resulting behind stiff masonry walls capable of outward tilting. For smooth vertical walls with level fill the slip line direction at the wall and the surface are the same and prove that a straight slip line is a solution. This gives the Bell formula for cohesive soil, expressed generally as $e = \lambda \gamma d + \rho p + \mathcal{H} c$ where p is surcharge pressure. For rough walls and more complex problems it is simplest to calculate the three components of earth pressure separately, assuming in turn $\gamma = 0$, $p = 0$; $\gamma = 0$, $c = 0$; $c = 0$, $p = 0$ to obtain the coefficients $\mathcal{H}$, ρ and λ respectively. The coefficients $\mathcal{H}$ and ρ are obtained by transferring the stresses from the surface to the wall, while the coefficient λ is obtained more accurately by considering the equilibrium of the earth mass. The coefficients obtained in this way are compared with those given in the Code and in Tables by Caquot and Kerisel. For all practical purposes the values are the same, so that the general theory by Hansen does not have any advantage. However, it may be extended to cases of line rupture when the wall rotates about a point above the base as for the pinned stiff wall.

f) Solution of Retaining Wall Problems by Brinch Hansen's Method

1) General

We consider the general case of a retaining wall with an inclined back which makes an angle j with the vertical in Fig. 22. The ground has a sloping surface, making an angle i with the horizontal, and is loaded by a vertical surcharge p per unit area of the sloping surface. i and j should be given their proper signs; in Fig. 23 they are both positive.

For a retaining wall founded directly on the ground, in order to mobilize the necessary friction, its base must move away from the fill, and, in order to resist the overturning moment, the toe must settle more than the heel. Consequently, the rotation center must be located somewhere below the base in the state of failure. The same will usually be the case, if the wall is founded on piles. This means that the rupture-figure in the backfill will usually be of the type P or, in special cases, type R.

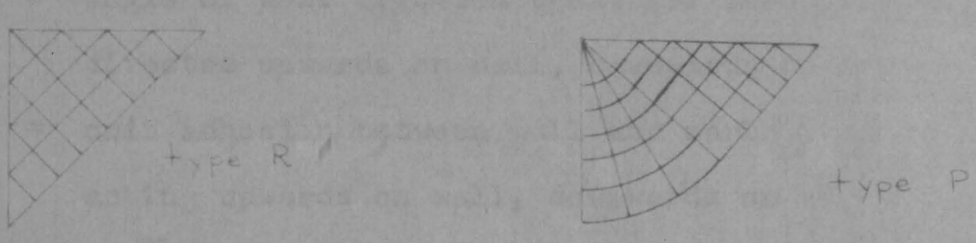


Fig. 22 - Zone -Ruptures

When the pressure is <u>active</u>, we have the following expressions for the earth pressures on the back of the wall, for homogeneous earth:

$$e = \gamma d \lambda + p\rho + c\mathcal{H} \qquad f = e \tan\delta + a \qquad (29) - (30)$$

$$E = \frac{1}{2}\gamma h^2 \lambda + ph\rho + ch\mathcal{H} \qquad F = E \tan\delta + ha \qquad (31) - (32)$$

$$E_z = \frac{1}{6}\gamma h^3 \lambda + \frac{1}{2} ph^2 \rho + \frac{1}{2} ch^2 \mathcal{H} \qquad (33)$$

where e = normal component of unit earth pressure on wall (positive when producing compression)

γ = Unit weight of earth

d = depth below ground surface, measured along the wall (positive downwards)

$\lambda, \rho, \mathcal{H}$ = dimensionless constants

p = unit vertical surcharge, per unit area of sloping surface (positive downwards)

c = unit cohesion of the earth

f = tangential component of unit earth pressure on wall (positive when acting upwards on wall, downwards on earth)

δ = angle of wall friction (positive when the pressure is directed upwards on wall, downwards on earth).

a = unit adhesion between wall and earth (positive when acting upwards on wall, downwards on earth).

E = Normal component of total earth pressure on wall (positive when producing compression)

h = height of wall, measured along wall.

F = Tangential component of total earth pressure on wall (positive when acting upwards on wall, downwards on earth).

Z = distance from the foot of the wall to earth pressure center (positive upwards).

According to eq. (29), negative pressures may be found on the upper part of the wall to a depth d_o, which is determined by putting $e = 0$ in eq. (29)

$$d_o = -\frac{P\rho + c\mathcal{H}}{\gamma\lambda} \qquad (34)$$

In order to be on the safe side, it is recommended that such negative pressures be disregarded, in which case we find (for honogeneous earth):

$$E = \frac{1}{2}\gamma\lambda\,(h - d_o)^2 \qquad (35)$$

$$z = \frac{1}{3}\,(h - d_o) \qquad (36)$$

$$F = E\tan\delta + a\,(h - d_o) \qquad (37)$$

In the formulas (29)→(37), the cohesion c should be assumed positive, whereas $\mathcal{H}$, δ and a are negative. For smooth wall we must put $\delta = a = 0$, and for a rough wall $\delta = -|\phi|$ and $a = -|c|$. For an imperfectly rough

wall intermediate values may be used.

In cohesive earth, the tensile stresses in the upper layers may cause the formation of open cracks in the earth or between the wall and the earth. If surface water enters such a crack, the wall will be subjected to water pressures, for which it is usually not designed. In order to avoid this, suitable drainage should be provided behind the wall, or the ground surface should be covered with an impervious layer.

In rational design, the foundation of the wall also should be investigated for the state of failure, and in that case the earth pressures on the wall should be calculated with values of ϕ and c, to which appropriate safety factors have been applied.

2) Sloping Surface and Inclined Back

In the general case of $i \neq j \neq 0$ the procedure is to determine first V' and V'' by means of eqs. (38)→(41):

$$c\cos\phi\sin i\sin(2v'+\phi-i)+(p\sin\phi+c\cos\phi\cos i)\cos(2v'+\phi-i)+p\sin i=0 \qquad (38)$$

Equation (38) is of the type;

$$X\sin 2u + Y\cos 2u + Z = 0 \qquad (39)$$

the solution of which is:

$$\tan u = \frac{X \pm \sqrt{X^2+Y^2-Z^2}}{Y-Z} \qquad (40)$$

$$\cos(2v''+\phi+\delta-2j) = \frac{\sin\delta}{\sin\phi} \qquad (41)$$

The constants ρ, $\mathcal{H}$ and λ are then calculated by means of eqs. (42) → (44) :

$$\rho = \frac{\cos\delta \sin(v'+\phi)\cos(v''-j)}{\sin(v'-i)\cos(v''+\phi+\delta-j)}\, e^{2\mu(v'-v'')} \qquad (42)$$

$$\mathcal{H} = \left[\rho \frac{\sin(v'+\phi-i)}{\sin(v'+\phi)} - 1\right] \cot|\phi| \qquad (43)$$

$$\lambda = \rho \cos(j-i) \qquad (44)$$

In the special case of frictionless earth ($\phi = \delta = 0$), the eqs. (45) → (48) are used :

$$\cos(2v'-2i) = -\frac{P}{c}\sin i \qquad (45)$$

$$\cos(2v''-2j) = \frac{a}{c} \qquad (46)$$

$$\rho_o = \frac{\sin v'}{\sin(v'-i)} \qquad (47)$$

$$\pm \mathcal{H}_o = 2(v'-v'') + \cot(v'-i) + \sin(2v''-2j) \qquad (48)$$

In the other special case of cohesionless earth ($c = a = 0$), the eqs. (49) , (50) together with eqs. (42) → (44) are used:

$$\cos(2v'+\phi-i) = -\frac{\sin i}{\sin\phi} \qquad (49)$$

$$\cos(2v''+\phi+\delta-2j) = \frac{\sin\delta}{\sin\phi} \qquad (50)$$

When the pressure is active, negative values of μ, ϕ, δ, a, c should be inserted in all these formulas and in eq. (48) the lower one of the double signs should be used.

Example

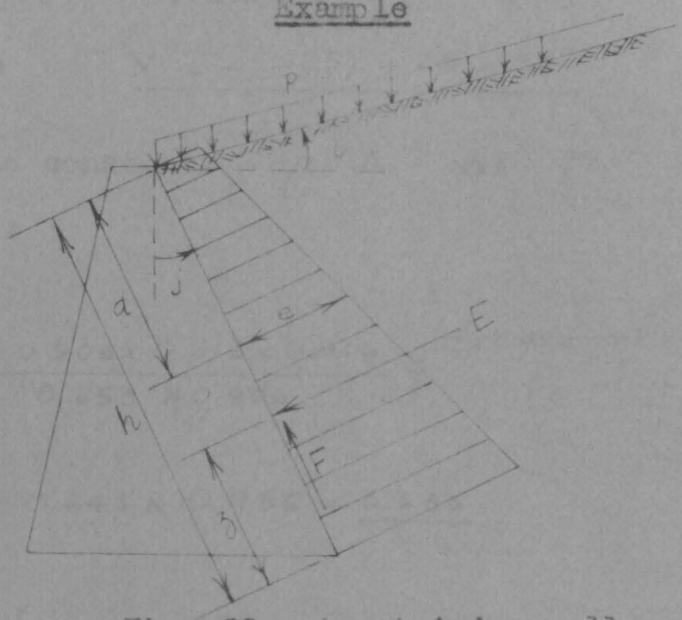


Fig. 23 - A retaining wall

In Fig. 23 an angle-shaped retaining wall is considered. The wall is assumed to be rough. The fill is clayey sand with a friction angle of 25° and a cohesion of 0.5 t/m^2. The given quantities are

$$h = 5 \text{ m} \qquad i = 15^\circ \qquad j = 25^\circ \qquad p = 2 \text{ t/m}^2$$

$$\gamma = 1.8 \text{ t/m}^3 \qquad \phi = \delta = -25^\circ \qquad c = a = -0.5 \text{ t/m}^2$$

Solution: Equation (38) gives:

$$-0.5 \times 0.906 \times 0.259 \sin(2v' - 25^\circ - 15^\circ) + 2 \times 0.259$$
$$+ (-2 \times 0.422 - 0.5 \times 0.906 \times 0.965) \cos(2v' - 25^\circ - 15^\circ) =$$

$$= -0.117 \sin(2v' - 40°) - 1.281 \cos(2v' - 40°) + 0.518 = 0$$

By means of eqs. (39) & (40) we find $\underline{v' = 55.8°}$. For a perfectly rough wall eq. (41) is reduced to:

$$v_r'' = -\phi + j$$

which gives $\underline{v'' = -(-25) + 25 = 50°}$

To find constants ρ, λ and $\mathcal{H}$ we use eqs. (42) → (44):

$$\rho = \frac{0.906 \times 0.512 \times 0.906}{0.653 \times 0.906} e^{-2 \times 0.466 \times 5.8 \times \frac{\pi}{180}} = \underline{0.645}$$

$$\lambda = 0.645 \times 0.985 = \underline{0.635}$$

$$\mathcal{H} = \left[\frac{0.645 \times 0.273}{0.512} - 1\right] \times 2.14 = \underline{-1.40}$$

the corresponding unit earth pressures are found from eqs. (29) & (30) (with c positive):

$$e = 1.8 \times d \times 0.635 + 2 \times 0.645 - 0.5 \times 1.40 = 1.14\,d + 0.59 \quad (t/m^2)$$

$$f = -(1.14d + 0.59) \times 0.466 - 0.5 = -0.53d - 0.77 \quad (t/m^2)$$

Total earth pressures and distance z are found from eqs. (31) → (33):

$E = \underline{17.2\ t/m}$ $F = \underline{-10.5\ t/m}$ $z = \underline{1.81\ m}$

For compression, a graphical extreme-calculation using Coulomb's method gives $\underline{E = 17.6 \text{ t/m}}$

3) Horizontal Surface and Vertical Back

For the case of $i = j = 0$, the constants $\rho_s = \lambda_s$ (smooth wall) are given by eq. (51) :

$$\rho_s = \lambda_s = \tan^2 (45^\circ + \phi/2) \qquad (51)$$

on the other hand ρ_r and λ_r (rough wall) are given by eqs. (52) & (53) respectively:

$$\rho_r = e^{\mu(\pi/2 + \phi)} \cos\phi \tan (45^\circ + \phi/2) \qquad (52)$$

$$\lambda_r = \rho_r + 0.007 \left(e^{9 \sin\phi} - 1 \right) \qquad (53)$$

For active pressure, negative values of ϕ must be used in these formulas. $\mathcal{H}$ can be calculated by eq. (54)

$$\mathcal{H} = (\rho - 1) \cot |\phi| \qquad (54)$$

For the critical height h_c of a vertical bank, we have eq. (55) :

$$h_c = \frac{1}{\gamma} \left[4c \tan (45^\circ + \phi/2) - 2p \right] \qquad (55)$$

in which positive values of ϕ and c should be used.

In the special case of <u>frictionless earth</u> ($\phi = 0$) for

<u>active pressures</u> we have $\rho = \lambda = 1$ and $\mathcal{H}_s = -2$ or $\mathcal{H}_r = -2.57$. For critical height, h_c , we have eq. (56):

$$h_c = \frac{1}{\gamma}(4c - 2p) \qquad (56)$$

where c should be assumed positive.

g) Summary of Hansen's Method

The fundamental basis of the new method is a connection, established by Hansen, between two known methods, which may be termed the extreme-method and the equilibrium-method respectively. In both these methods a state of rupture in the earth is considered, and an earth wedge is investigated, which is bounded by the ground surface, the wall and a rupture-line through the lowest point of the wall.

In the extreme-method a statical condition of equilibrium for the said earth wedge is set up in such a way, that it does not contain the unknown internal forces in the rupture-line. The earth pressure on the wall is then determined by an extreme-condition which states that the active earth pressure should be a maximum, or the passive a minimum. This method necessitates the assumption that the rupture-line is a logarithmic spiral (Rendulic), in special cases a circle (Fellanius) or a straight line (Coulomb).

In the equilibrium-method no extreme-condition is employed, but the earth pressure is determined by means of all three statical conditions of equilibrium for the above mentioned earth wedge. This presumes that the internal forces in the rupture-line are known, which is effected by means of the so-called Kötter's equation in connection with a boundary condition at the ground surface. However, so far, it has only been possible to indicate this boundary condition in special cases, such as, when the rupture-line meets the ground surface at a certain angle, or when the ground surface is unloaded and the earth cohesionless.

In order to judge the applicability and limitations of these methods it is necessary to consider, first, the figures of rupture which may develop in the earth. Hansen distinguishes between the following three different types:

1) Zone-ruptures, in which the stresses at any point within a certain area satisfy the condition of failure.

2) Line-ruptures, in which only the stresses at the points on a certain curve satisfy the condition of failure.

3) Composite ruptures, which consist of more than one rupture-zone or rupture-line.

Many rupture-figures consist thus exclusively or partly of a single rupture-line separating two zones, in which the state of rupture is not attained. Disregarding the elastic

deformations, and assuming incompressibility, such a single rupture-line must, for kinematical reasons, be a circle, the center of which is located on a normal to the wall through its rotation center. Consequently, this rupture-figure cannot usually be dealt with by means of the extreme-method, which requires the use of logarithmic spirals, and the indefinite boundary condition excludes normally the employment of the equilibrium method.

Hansen has succeeded in overcoming these difficulties by proving that the two methods will give identical results, when a logarithmic spiral is used as a rupture-line in both, and when a certain boundary condition is used in the equilibrium-method. This boundary condition is namely also, as a very good approximation, applicable to the circular rupture-line which must be used in the case of a given rotation center. In this way Hansen has made possible a general application of the equilibrium-method.

For the design or calculation of a certain earth retaining structure it is necessary to determine first the type of movement to be performed by the structure in the state of failure. Disregarding the elastic deformations, the structure must either move as one rigid body, or as a finite number of rigid parts connected by yield hinges. A further investigation seems to show that, to a certain degree, it is possible to choose freely the type of movement on which to base the calculation.

the calculation.

Corresponding to the chosen type of movement for the structure such figures of rupture in the adjacent earth masses are then investigated, which imply movements and deformations of earth compatible with those of the structure. The corresponding earth pressures on the structure are determined by means of the equilibrium-method in its general form. If more than one figure of rupture satisfies the kinematical conditions, the design of the structure must be based on the rupture-figure, for which the work done by the earth pressure (acting upon the earth) is a minimum.

Finally, the dimensions of the structure are so determined that the statical equilibrium conditions for the structure proper are satisfied and the allowable stresses not exceeded. As in the whole calculation the state of failure considered, it is necessary to divide the actual shear strengths of the soils by suitable safety factors before carrying out the calculation. Corresponding allowable stresses are obtained when the ultimate strength of the structural materials are divided by other safety factors.

VII) CANTILEVER AND COUNTERFORT WALLS

The backs of cantilever and counterfort walls depart so widely from plane surfaces that neither Rankine's nor Coulomb's conditions apply exactly to walls of this type, but Rankine's method can usually be used without significant error. $\beta = \frac{1}{2}(90° - \phi) - \frac{1}{2}\left(\frac{\sin i}{\sin \phi} - i\right)$

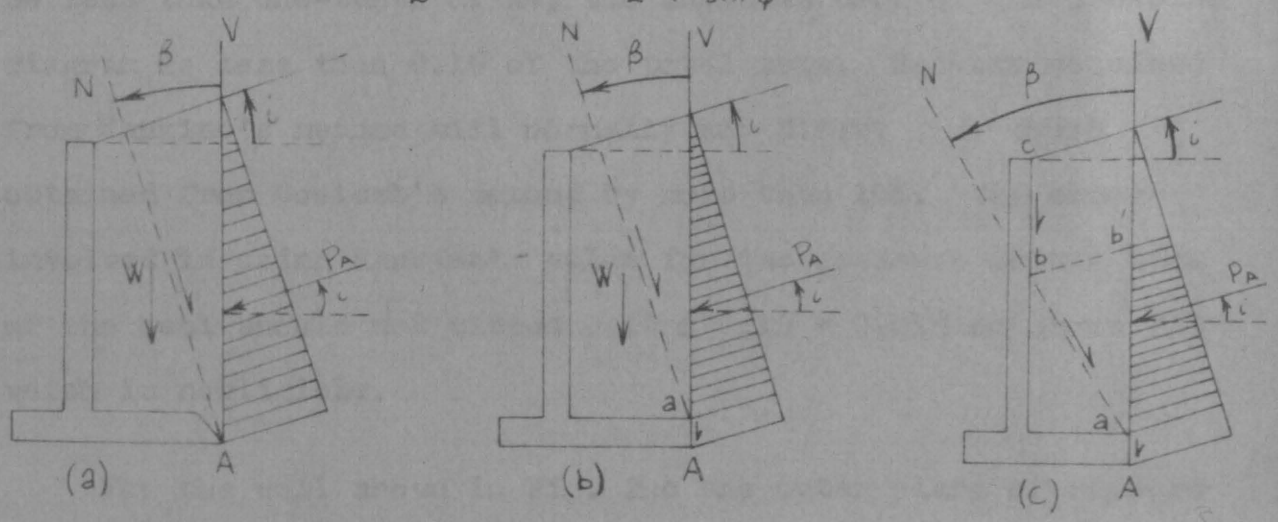


Fig. 24 - Resultant pressure against cantilever wall

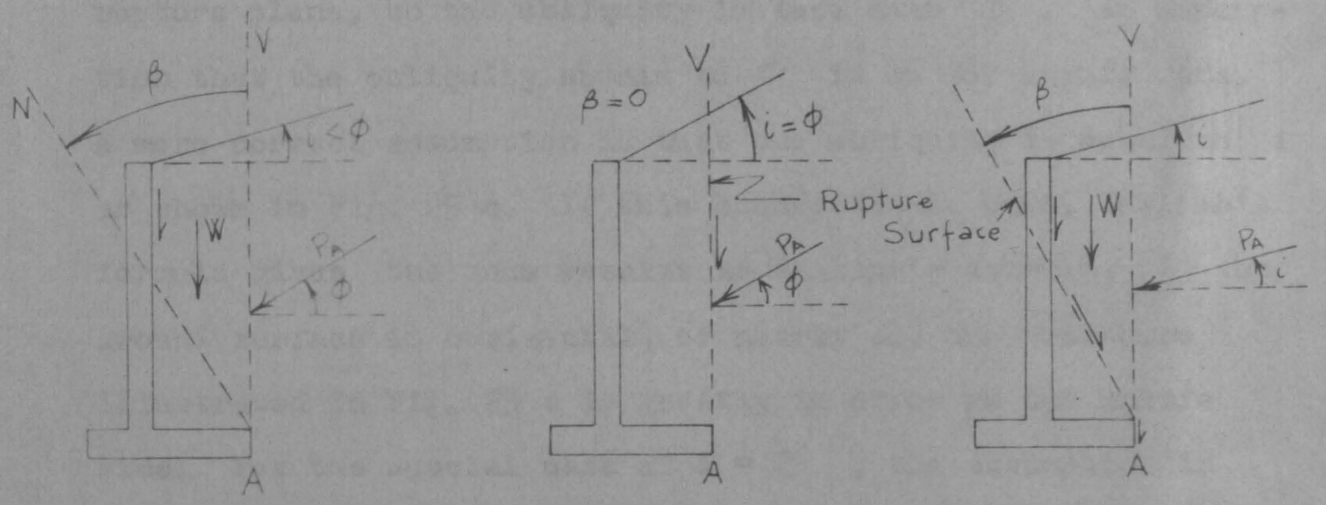
The cantilever wall, shown in Fig. 24a has such proportions that the outer plane of rupture can form without being obstructed. The resultant pressure against the back of the wall is therefore determined correctly by using Rankine's formula to compute P_A, the resultant pressure against the vertical plane AV, and combining with the weight W of the block of soil between the vertical plane and the back of the wall.

For the wall shown in Fig. 24b, soil slides along Aa, the back of the heel, as well as the rupture surface N_a.

Coulomb's method may be applied, with negligible error, to that portion of the pressure against the heel, and Rankine's method to the portion above the heel. The unshaded lower portion of the pressure diagram showing the Rankine pressure against the vertical plane is therefore in error by the small difference between Rankine's and Coulomb's values. Since A_a will normally be less than one-tenth of AV, the unshaded area of the pressure diagram is less than 0.19 of the total area. Results obtained from Rankine's method will normally not differ from those obtained from Coulomb's method by more than 10%. The error involved in using Rankine's value for the pressure on the back of the heel should not exceed $0.10 \times 0.19 = 0.019$ or about 2%, which is negligible.

For the wall shown in Fig. 24c the outer plane of rupture strikes the back of the vertical stem so that the soil slides on the wall surfaces cb and aA as well as on the rupture surface ba. Coulomb's method may be applied to the two unshaded portions of the pressure diagram. The length b'V will rarely be as large as four-tenths of AV. The upper unshaded area will therefore rarely exceed 16% of the total area of the pressure diagram. Thus unshaded area and that in the lower part of the pressure diagram, equal about 35% of the total area of the pressure diagram. Again assuming that the results obtained from Rankine's theory differ by less than 10% from those obtained from Coulomb's theory, the error involved in using Rankine's

value for the pressure on the upper portion of the vertical stem and the back of the heel should not exceed 0.10 x 0.35 = 0.035 or 3.5%, which is also negligible.



Incorrect and unsafe

(a) i less than ϕ

Correct

(b) i equal to ϕ

Sufficiently Accurate for Design

(c) i has any value

Fig. 25 - Incorrect, correct and deisgn procedures for cantilever walls.

A very common but incorrect procedure for computing the active pressure against cantilever and counterfort retaining walls is illustrated in Fig. 25 a. According to this procedure, the pressure is computed against a vertical plane AV through the heel, as in the procedure using Rankine's formula. However, the vertical pressure against the vertical plane is computed by Coulomb's formula, assuming the obliquity of this pressure to be ϕ . This assumption is correct if the ground surface is plane and slopes upward from the top of the wall at an inclination of $i = \phi$, as shown in Fig. 25 b. For

that case, the outer plane of rupture is vertical, so the obliquity of the resultant pressure is ϕ . For all other cases, however, the vertical plane through the heel is not a rupture plane, so the obliquity is less than ϕ . An assumption that the obliquity equals to ϕ is on the unsafe side. A more correct assumption is that the obliquity is equal to i as shown in Fig. 25 c. If this assumption is made, Coulomb's formula gives the same results as Rankine's formula. If the ground surface is horizontal, or nearly so, the procedure illustrated in Fig. 25 a is greatly in error on the unsafe side. For the special case of $i = \phi$, the assumption in Fig. 25 c involves no error if the back of the stem is vertical. Rankine's method can be used with the same reasoning for counterfort walls.

VIII) DISCUSSION

Coulomb's method is the result of a purely theoretical study. It is an extreme-method where earth pressure problems are solved by means of a single equilibrium condition in connection with a maximum or minimum condition. Coulomb considered the limit equilibrium of the entire wedge of soil behind a retaining wall. Accordingly, the limit value of the earth pressure at failure of the soil was expressed by him in terms of the total pressure and not of unit pressures. It is a method for calculating zone-ruptures. Its severest limitation is that it uses straight rupture-lines only. For active pressures this involves unimportant deviations, but for passive pressures it often leads to considerable errors. For the calculation of line-ruptures Coulomb's method cannot be used at all, because it neither allows the determination of the pressure center nor that of the rotation center. As long as we consider only, a rupture-line through the foot of the wall, Coulomb's theory does not provide us with any means of determining the earth pressure distribution. Actually, Coulomb's method must be considered as an approximate means of calculating a zone-rupture; in this case the actual earth pressure distribution is hydrostatical, but the actual rupture-line is usually not straight. In spite of this theoretical inconsistency, it has been found that active earth pressures calculated by means of Coulomb's method are only a few percent smaller than the correct values. On the other hand, passive

earth pressures as found by Coulomb's method may exceed the correct values by 50% or more.

Consequently, Coulomb's method is most suitable in the case of active zone ruptures. Such ruptures occur in the back-fill of retaining walls, and for these structures the method is, therefore, a very relisble means of design and calculation.

Coulomb's method is true for purely granular, cohesionless soils and rigid walls only. Although this is the case, it is particularly notable because with some restrictions its results are to be considered valid to this day. Coulomb's analysis was limited to plane surfaces of failure, in order to simplify the mathematics involved.

Coulomb's theory does not determine the amount of deformation which is necessary in order to fully mobilize the shearing resistance of the soil and to transform the soil from at-rest state to an active or to a passive state. The theory assumes that the latter two limit conditions are reached after a certain amount of lateral motion has taken place. Coulomb does not specify this amount.

Coulomb's theory is valid only under the condition that the upper edge of the lateral support of the soil yields in a horizontal direction to or beyond a certain critical distance.

Coulomb knew about soil cohesion and considered it in his

calculations. The friction between the wall and the backfill did not enter into Coulomb's analysis and was introduced in calculations only some time later.

One of the main deficiencies in Coulomb's theory is that, in general, it does not satisfy the static equilibrium condition occuring in nature. The three forces (weight of the sliding wedge, its reaction, and earth pressure) acting on the sliding wedge generally do not meet in a common point when sliding surface is assumed, as in Coulomb's theory, to be planar. The theory is effective in proportion to the reliability of the corresponding constants to be used in the formulas. These soil constants should be determined as accurately as possible in the soil mechanics laboratory.

For Coulomb's formula to apply, the first and either, but not both, of the second and third of the following conditions must apply:

1) The back of the wall must be plane or so nearly plane that an equivalent plane surface may be substituted for the real surface for purposes of computation.

2) The wall does not interfere with the formation of the outer plane of rupture but sliding occurs along the back of the wall because the obliquity of the resultant pressure is equal to or greater than ϕ_1, or some assumed value of δ which is less than ϕ_1.

3) The wall interferes with the formation of the outer plane of rupture, so sliding occurs along the back of the wall and an outer plane of rupture does not develop.

Coulomb's method for the passive state of stress also assumes that soil is compressed sufficiently to develop that state of stress. It assumes that the soil slides along a solid pressure surface, rather than a surface through the soil, and that the obliquity of the resultant pressure on the surface has some given value δ which is equal to or less than ϕ. Coulomb's method for passive pressures gives the resultant pressure against the pressure surface directly. In deriving Coulomb's formula for the resultant passive pressure, it is assumed that the surface of rupture is plane. For most cases involving passive pressures encountered in retaining wall design, the ground surface is horizontal, the pressure surface may be assumed to be vertical, and δ may be assumed to equal zero. Passive pressures computed on this basis are on the safe side.

a) SUMMARY OF RANKINE'S AND COULOMB'S CONDITIONS

The conditions which have been explained for the application of Rankine's and Coulomb's methods may be summarized as follows:

1) If the back of the wall is a plane surface or a surface

which can be assumed to be plane without introducing significant errors, either method may apply depending upon which of the following conditions prevails:

a) If the outer plane of rupture, as determined by Rankine's method, cannot form, Coulomb's conditions prevail.

b) If the outer plane of rupture can form, Coulomb's conditions prevail if the obliquity of the pressure on the plane, as computed by Rankine's method, is equal to or greater than ϕ_1, and Rankine's conditions prevail if the obliquity is equal to or less than ϕ_1.

2) If the back of the wall cannot be assumed to be a plane surface, Coulomb's conditions cannot prevail but Rankine's conditions prevail if the outer plane of rupture can form without being obstructed by the wall.

In general, solid gravity walls with the usual proportions satisfy Coulomb's conditions, especially if ϕ_1, is assumed to be equal to or less than $2/3\,\phi$.

Cantilever and counterfort walls do not satisfy the assumptions of either theory but walls with the usual proportions can be assumed to satisfy Rankine's conditions without introducing significant errors in the computed pressure against the back of the wall.

OTHER METHODS

b) Trial wedge method - It consists of simple graphic procedures by means of which the active and passive pressures computed by either Coulomb's or Rankine's formulas can be obtained for cohesionless soils. The pressures for all of the conditions to which these formulas apply, and many other to which they don't apply, can be computed by trial wedge method. The only limitation in the method is that the inner surface of rupture must be plane or so nearly plane that assuming a plane surface does not introduce significant errors. This condition is satisfied when computing active pressures, but not usually when computing passive pressures. No other method for earth pressure computation need be used in the design of retaining walls or other structures if the active state of stress exists in the backfill. Trial wedge method can also be applied to many cases where the soils are cohesive and, therefore, are not susceptible of solution by the usual forms of Coulomb's and Rankine's formulas, which apply to cohesionless soils.

As explained before, Coulomb's method is based on the assumption that the surface of rupture is plane when it is actually curved along its lower portion. Investigations have shown that the theoretical error in Coulomb's formula for active pressure due to assuming a plane surface of rupture is insignificant. If the ground surface is plane, the magnitude of the theoretical error in the trial wedge method due to assuming a

plane surface of rupture is the same as in Coulomb's formula because the two procedures lead to the same result.

The theoretical errors involved in assuming a plane surface of rupture for cases where Coulomb's conditions apply but his formula does not apply, such as irregular ground surfaces and nonuniform surcharge loads, have not been investigated but are probably not important.

When Rankine's conditions are fulfilled, there is no theoretical error. The magnitude of the theoretical error in any solution based on Rankine's conditions depends upon the extent to which these conditions are not fulfilled, and this error is unimportant.

The important differences between actual and computed earth pressures are due to the wide differences between the properties of actual soils and these of the ideal soils used in the computation procedures and the lack of agreement with assumed deformation conditions.

Trial wedge method when used for computing passive pressures in cohesive soils gives theoretically correct results only when the ground surface is horizontal, the pressure surface vertical, and the obliquity of the pressure assumed to be zero. For other cases, the magnitude of the error is determined by the extent to which these conditions are not fulfilled .

c) Friction Circle Method - In this method the surface of rupture is not plane when Coulomb's conditions prevail. The error involved when it is assumed to be plane is not significant when computing active pressures but it may be so large and on the unsafe side when computing passive pressures. When the wall interferes with the development of an outer plane of rupture passing through the heel, the surface of rupture is a combination of a plane and a curved surface. It is obvious that the friction circle method has a rather laborious procedure and it is not very useful, because the trial wedge method, Coulomb's formula, and Rebhann's construction all give results which are not significantly in error for active pressures even though they assume a plane surface of rupture. This surface may be considered as including an arc with an infinite radius. In the friction circle method, it is assumed that the resultant pressure R will make an angle ϕ with the normal to the surface of rupture at its point of intersection. This is direction of radius at that point. It is true that, at each point along the surface of rupture, the obliquity of the pressure is ϕ, but it was shown that the circle to which R is tangent is slightly larger than the friction circle. However, the error is unimportant and is on the safe side.

This method satisfies the condition of equilibrium which requires that the sum of the moments of a system of forces, about any point must equal zero. Although not usually useful

for purposes of computation, this method provides a procedure for evaluating the error in other less precise methods and illustrates very well the basic principles involved in earth pressure computations.

In general, the friction circle method is the most effective procedure for computing the passive pressures when the ground surface is horizontal. For the special cases in which the ground surface is horizontal, the pressure surface vertical, and the pressure assumed to be normal to the pressure surface, then Rankine's conditions prevail and the corresponding surface of rupture is plane.

d) J. Brinch Hansen's Method - The purpose of Hansen is to introduce a new general method for the calculation of earth pressures. The method covers a wider field of possible rupture-figures than these previously known, and this enables the choice of rupture-figures compatible with the movements of the considered structures in the state of failure. The corresponding calculations have been made possible, partly by means of the well-known Kötter's equation, and partly by means of a connection, established by Hansen, between two known but up to now unrelated methods of calculation.

However, earth pressure calculation has not been made simpler by the introduction of Hansen's method, but this is the price one has to pay for greater accuracy. Still most of the calculation methods developed by Hansen are relatively simple as compared

with many of the methods in common use for the calculation of other structures.

With regard to economy, it is evident that smaller factors of safety can be allowed when more exact calculation methods are available. Therefore, Hansen's method, when employed in combination with suitable safety factors, should be able to effect considerable savings as compared with conventional design methods.

Hansen has applied his method to the majority of plane earth pressure problems encountered in practice, such as, retaining walls, anchor slabs, free sheet walls, fixed sheet walls, braced walls, double sheet walls and cellular cofferdams. Incidentally, the method is also applicable to stability and foundation problems.

The proposed method is applicable to earth with internal friction or cohesion or both, to walls with any inclination and roughness, and to ground surfaces with any slope and surcharge. The effects of hydrostatic or hydrodynamic water pressures can be taken into account, at least approximately, and the same applies also in the case of stratified earth.

The practical calculations are facilitated considerably by the use of the tables and graphs given by Hansen in his book called "Earth Pressure Calculation" 1953. The tables enable an easy and quick calculation of the internal forces in a rupture-circle, but only for a friction angle of either 0^{o} or 30^{o}. By

means of the graphs it is possible to determine easily the earth pressure on a vertical wall with horizontal ground surface, for any friction angle, and for any given location of the rotation center for the wall.

Due to the wide range of application of the new method Hansen has only been able to check its results against practical experience in a few instances. However, the connection established between the new method, which is an equilibrium method, and the extreme-method should be a safeguard against serious errors, as it is a well-known fact that extreme-methods will produce very reliable results when used properly.

Consequently, provided that Hansen's methods are used with appropriate caution, it is to be expected that they will result in sufficiently safe and rather economical structures.

e) Original Contributions by Brinch Hansen - The most important of Hansen's contributions to the subject of earth pressure calculation are the following:

1) A connection has been established between the extreme-method and the equilibrium-method, leading to a certain boundary condition by means of which a general application of the equilibrium-method has been made possible.

2) The principles have been developed for a general earth pressure theory, based mainly on the equilibrium-method, and making use of Kötter's equation as well as the above mentioned

boundary conditions.

3) A set of tables has been prepared by means of which the internal forces in a rupture-circle can be calculated quickly and easily for $\phi = 0^\circ$ or $\phi = 30^\circ$.

4) A systematic analysis has been made of differential rupture-figures, especially the so-called composite rupture, which have hardly received any attention before. A convenient system of denominations for rupture-figures has also been proposed.

5) An analysis has been made of different states of failure which may occur in earth pressure and stability investigations.

6) General methods have been indicated for calculation of the earth pressures corresponding to the most important figures of rupture. They include a new approximate method for the calculation of zone-ruptures.

7) A set of graphs has been prepared, by means of which it is possible to calculate the earth pressure on a vertical wall with horizontal ground surface, rotating about any given point on the wall proper or its extensions. The graphs may be used for any friction angle, and for smooth as well as rough walls.

8) Although the earth pressure distribution is indeter-

minate, except for zone-ruptures, a simple, tentative earth pressure diagram has been propsoed. Moreover, this diagram has been used to take into account, in an approximate but simple way, the effects of water pressures and stratified earth.

9) For the design of retaining walls new formulas have been developed which are more accurate than Coulomb's and usually on the safe side.

10) For the design of anchor slabs a new method has been indicated which takes into account the fact that inclination of the passive earth pressure is limited by the equilibrium conditions.

11) For free sheet walls, anchored sheet walls and fixed sheet walls, completely new design methods have been developed.

12) For the design of braced walls in cuts a simple approximate method has been proposed.

13) Completely new methods have been developed for the stability investigation of cellular cofferdams, on rock as well as in earth.

14) A simple extreme-method has been indicated for the determination of the necessary distance between an anchored sheet wall and its anchor slab.

15) A number of small-scale model tests have been carried out in order to study the different rupture-figures occuring in sand by the rotation of a rigid vertical wall about different points.

PART III

COMPARISON OF RANKINE'S AND COULOMB'S METHODS

A) RANKINE'S METHOD FOR VERTICAL WALLS

(Cohesion, c = 0)

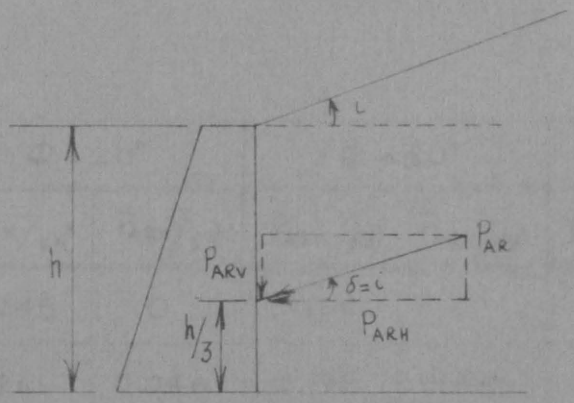


Fig. 26 - A retaining wall with vertical back.

Rankine's formula for active pressure is the following:

$$P_{AR} = \frac{1}{2}\gamma h^2 \left[\cos i \; \frac{\cos i - \sqrt{\cos^2 i - \cos^2 \phi}}{\cos i + \sqrt{\cos^2 i - \cos^2 \phi}} \right] \qquad \delta = i$$

$P_{ARH} = P_{AR} \cos i$ = Horizontal component of Rankine's active pressure.

$P_{ARV} = P_{AR} \sin i$ = Vertical component of Rankine's active pressure.

$$\frac{P_{ARH}}{\gamma h^2} = \frac{1}{2} \cos^2 i \; \frac{\cos i - \sqrt{\cos^2 i - \cos^2 \phi}}{\cos i + \sqrt{\cos^2 i - \cos^2 \phi}}$$

$$\frac{P_{ARV}}{\gamma h^2} = \frac{1}{4} \sin 2i \; \frac{\cos i - \sqrt{\cos^2 i - \cos^2 \phi}}{\cos i + \sqrt{\cos^2 i - \cos^2 \phi}}$$

	$\phi = 20°$		$\phi = 30°$		$\phi = 40°$	
i	$P_{ARH}/\gamma h^2$	$P_{ARV}/\gamma h^2$	$P_{ARH}/\gamma h^2$	$P_{ARV}/\gamma h^2$	$P_{ARH}/\gamma h^2$	$P_{ARV}/\gamma h^2$
0°	0.245	0	0.166	0	0.109	0
10°	0.261	0.046	0.172	0.030	0.111	0.020
20°	0.441	0.161	0.195	0.071	0.118	0.043
30°	—	—	0.375	0.217	0.136	0.079
40°	—	—	—	—	0.293	0.246

Table 1 – Horizontal and Vertical components of Rankine's active pressure for vertical walls, and different values of i & ϕ (for all δ's).

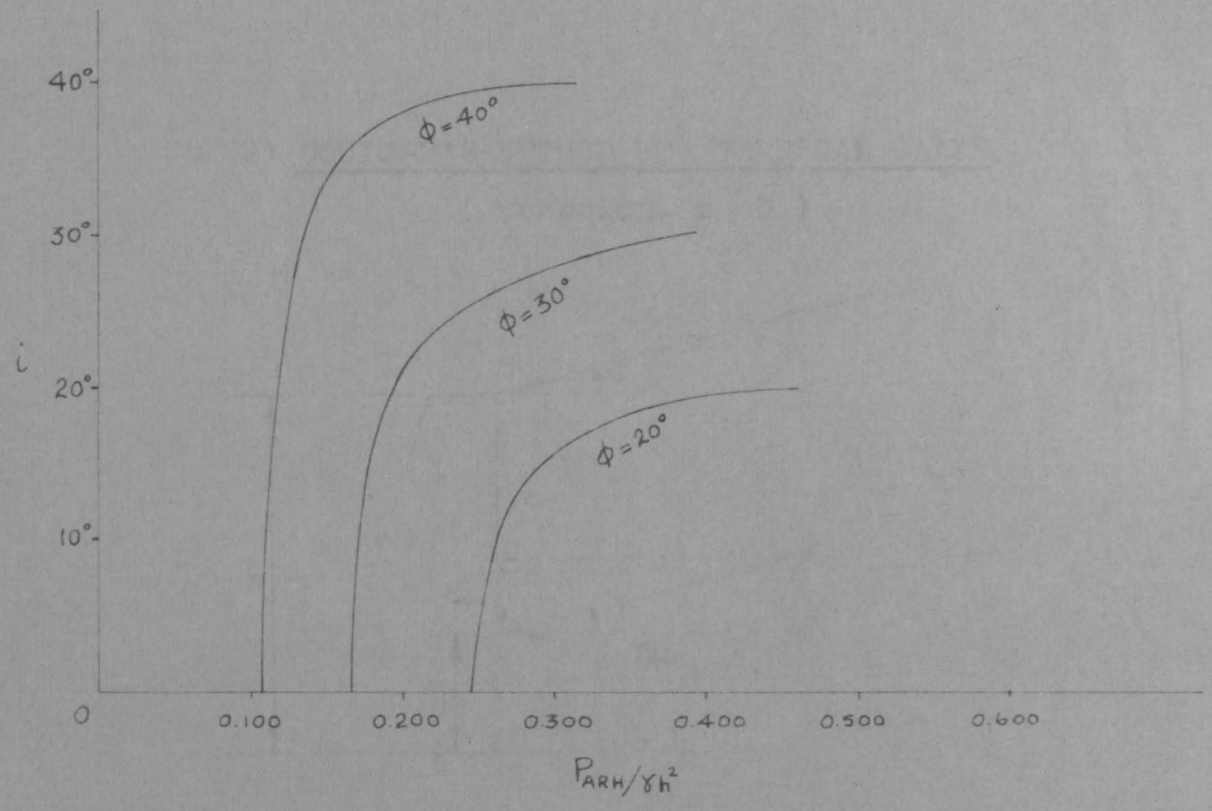


Fig. 27 - Angle i vs. $P_{ARH}/\gamma h^2$ graph for $\phi = 20°, 30°, 40°$

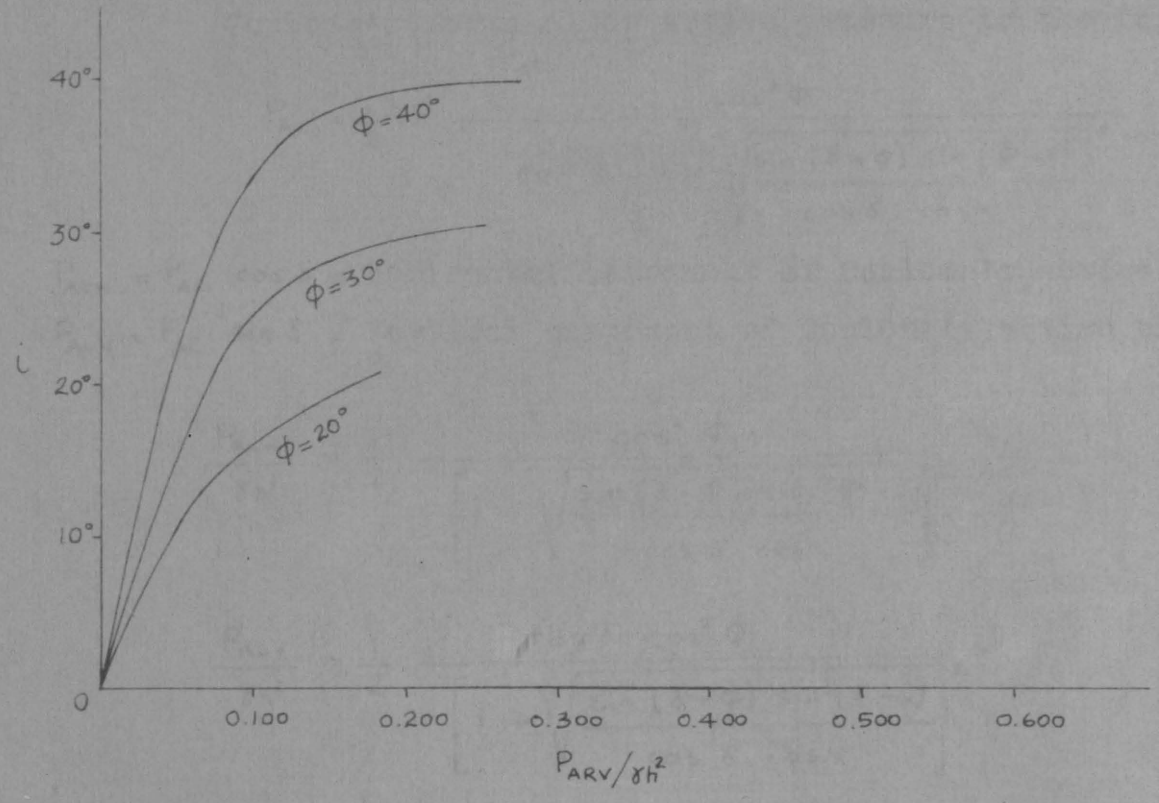


Fig. 28 - Angle i vs. $P_{ARV}/\gamma h^2$ graph for $\phi = 20°, 30°, 40°$

B) COULOMB'S METHOD FOR VERTICAL WALLS

(Cohesion, c = 0)

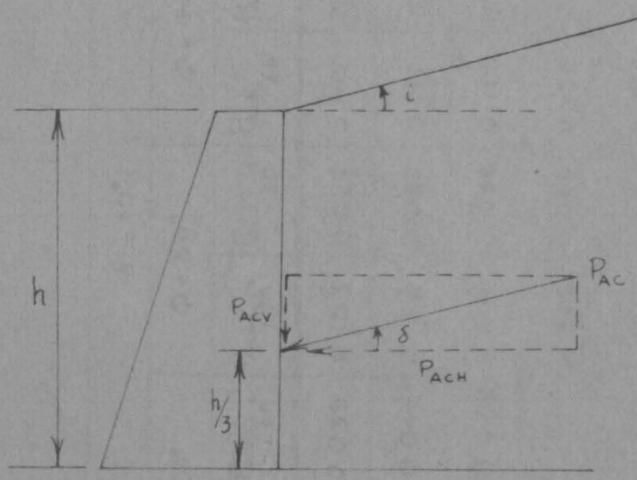


Fig. 29- A retaining wall with vertical back.

Coulomb's formula for active pressure is the following:

$$P_{AC} = \frac{1}{2}\gamma h^2 \frac{\cos^2\phi}{\cos\delta\left[1+\sqrt{\frac{\sin(\delta+\phi)\sin(\phi-i)}{\cos\delta\cdot\cos i}}\right]^2}$$

$P_{ACH} = P_{AC}\cos\delta$ = Horizontal component of Coulomb's active pressure

$P_{ACV} = P_{AC}\sin\delta$ = Vertical component of Coulomb's active pressure

$$\frac{P_{ACH}}{\gamma h^2} = \frac{1}{2}\frac{\cos^2\phi}{\left[1+\sqrt{\frac{\sin(\delta+\phi)\sin(\phi-i)}{\cos\delta\cdot\cos i}}\right]^2}$$

$$\frac{P_{ACV}}{\gamma h^2} = \frac{1}{2}\frac{\tan\delta\cdot\cos^2\phi}{\left[1+\sqrt{\frac{\sin(\delta+\phi)\sin(\phi-i)}{\cos\delta\cdot\cos i}}\right]^2}$$

	$\delta = 0°$						$\delta = 10°$					
	$\Phi = 20°$		$\Phi = 30°$		$\Phi = 40°$		$\Phi = 20°$		$\Phi = 30°$		$\Phi = 40°$	
i	$P_{ACH}/\gamma h^2$	$P_{ACV}/\gamma h^2$	$P_{ACH}/\gamma h^2$	$P_{ACV}/\gamma h^2$	$P_{ACH}/\gamma h^2$	$P_{ACV}/\gamma h^2$	$P_{ACH}/\gamma h^2$	$P_{ACV}/\gamma h^2$	$P_{ACH}/\gamma h^2$	$P_{ACV}/\gamma h^2$	$P_{ACH}/\gamma h^2$	$P_{ACV}/\gamma h^2$
0°	0.245	0	0.166	0	0.109	0	0.221	0.039	0.153	0.027	0.102	0.018
10°	0.285	0	0.188	0	0.119	0	0.261	0.046	0.172	0.030	0.111	0.020
20°	0.442	0	0.220	0	0.133	0	0.442	0.078	0.206	0.036	0.125	0.022
30°	—	—	0.375	0	0.159	0	—	—	0.375	0.066	0.151	0.026
40°	—	—	—	—	0.294	0	—	—	—	—	0.294	0.051

Table 2 – Horizontal and vertical components of Coulomb's active pressure for vertical walls and different values of i, Φ, δ

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	$\delta = 20°$					
	$\Phi = 20°$		$\Phi = 30°$		$\Phi = 40°$	
i	$P_{ACH}/\gamma h^2$	$P_{ACV}/\gamma h^2$	$P_{ACH}/\gamma h^2$	$P_{ACV}/\gamma h^2$	$P_{ACH}/\gamma h^2$	$P_{ACV}/\gamma h^2$
0°	0.201	0.073	0.140	0.051	0.094	0.034
10°	0.243	0.088	0.160	0.058	0.104	0.037
20°	0.441	0.161	0.195	0.071	0.118	0.043
30°	—	—	0.375	0.137	0.144	0.052
40°	—	—	—	—	0.294	0.107

Table 3 - Horizontal and vertical components of Coulomb's active pressure for vertical walls and different values of i, Φ, δ

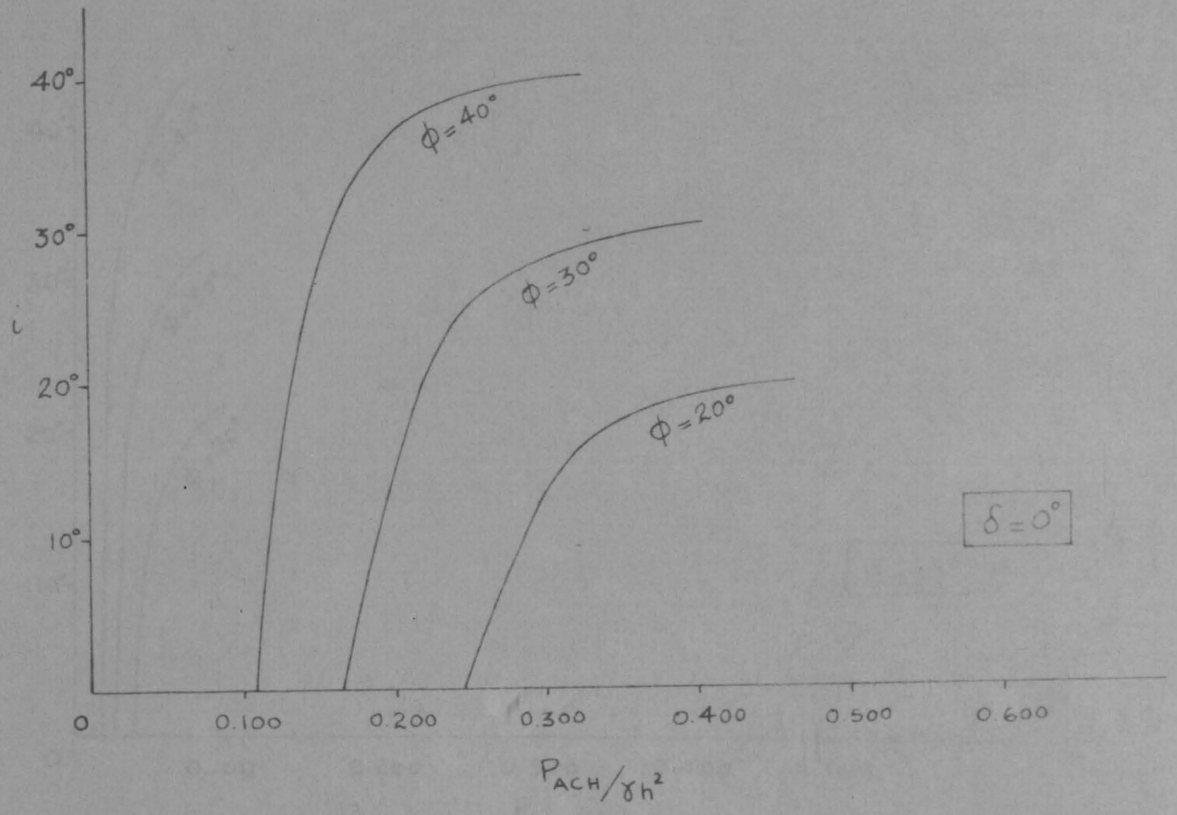


Fig. 30 - Angle i vs. $P_{ACH}/\gamma h^2$ graph for $\delta = 0°$, $\Phi = 20°, 30°, 40°$

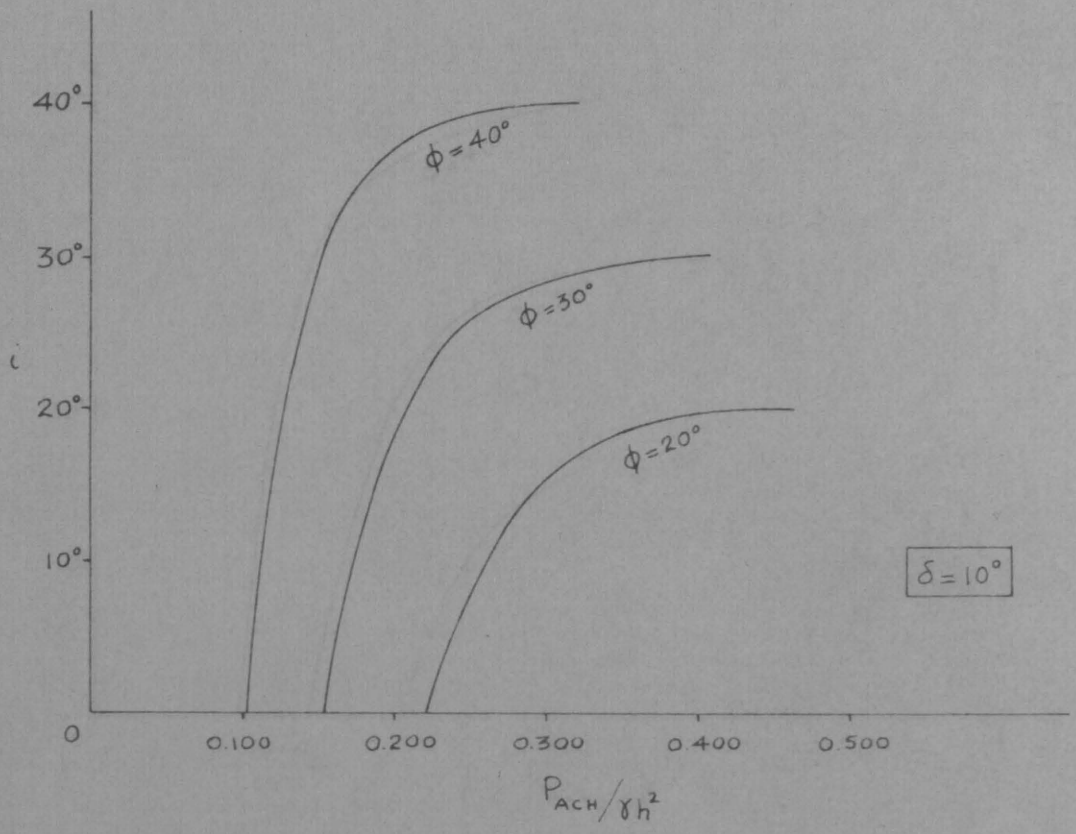


Fig. 31 – Angle i vs. $P_{ACH}/\gamma h^2$ graph for $\delta=10°$, $\phi=20°, 30°, 40°$

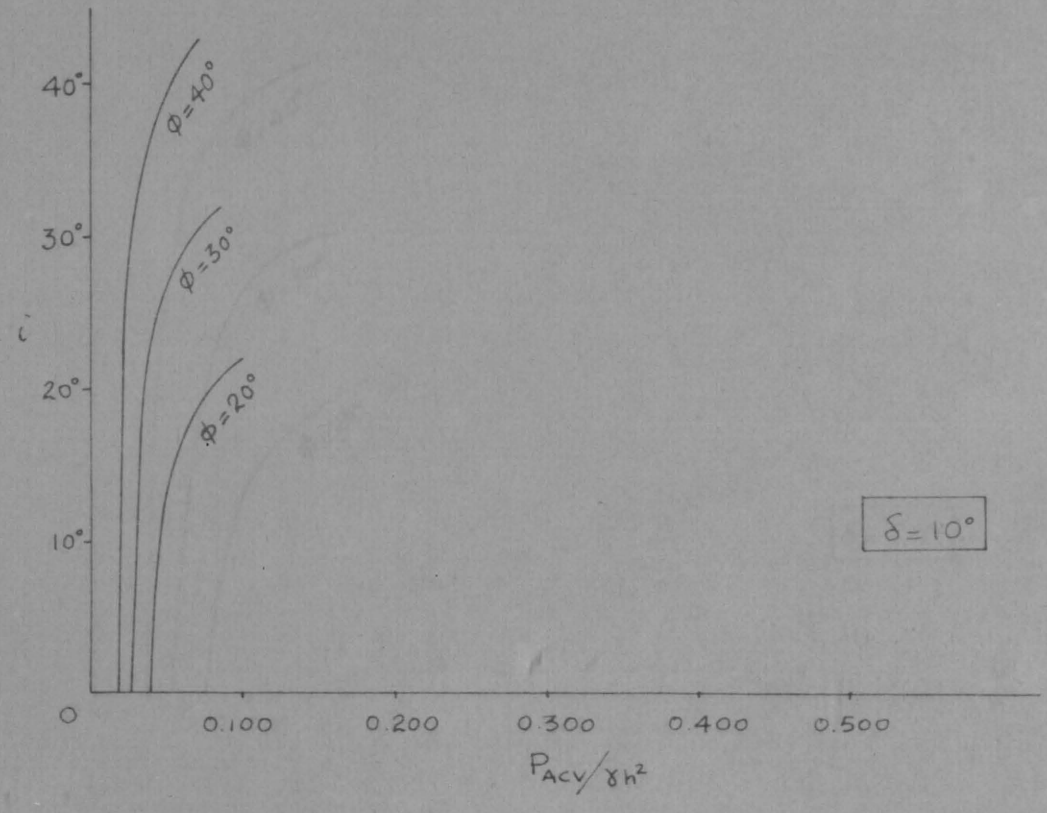


Fig. 32 – Angle i vs. $P_{ACV}/\gamma h^2$ graph for $\delta=10°$, $\phi=20°, 30°, 40°$

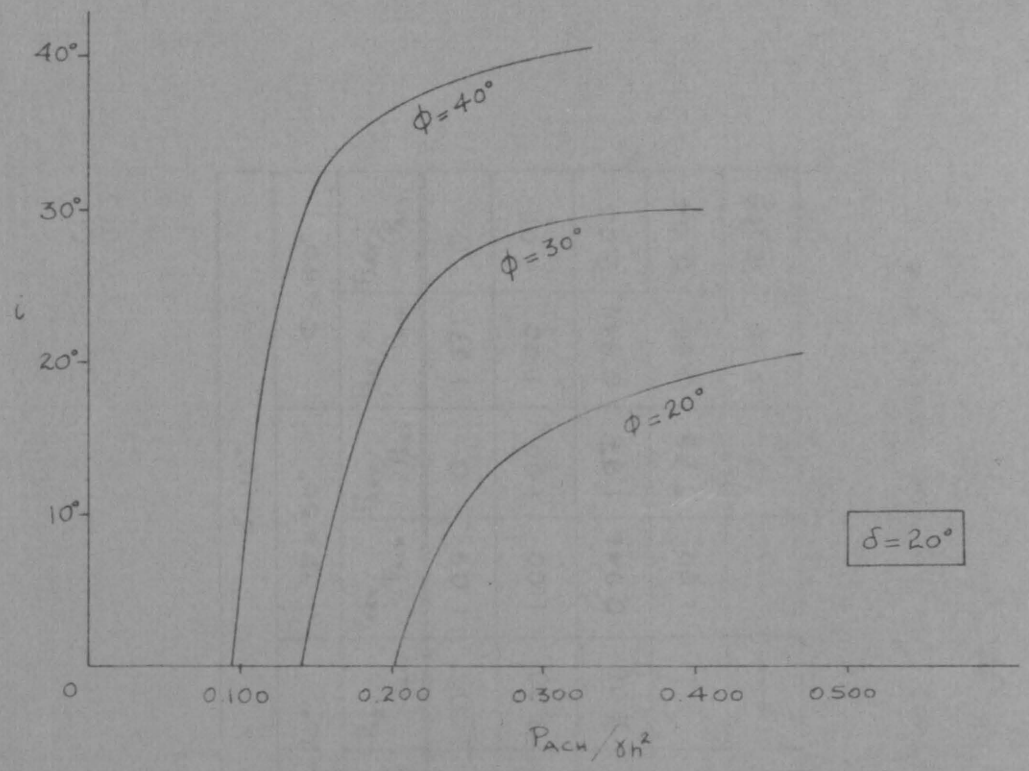


Fig. 33 - Angle i vs. $P_{ACH}/\gamma h^2$ graph for $\delta=20°$, $\phi=20°, 30°, 40°$

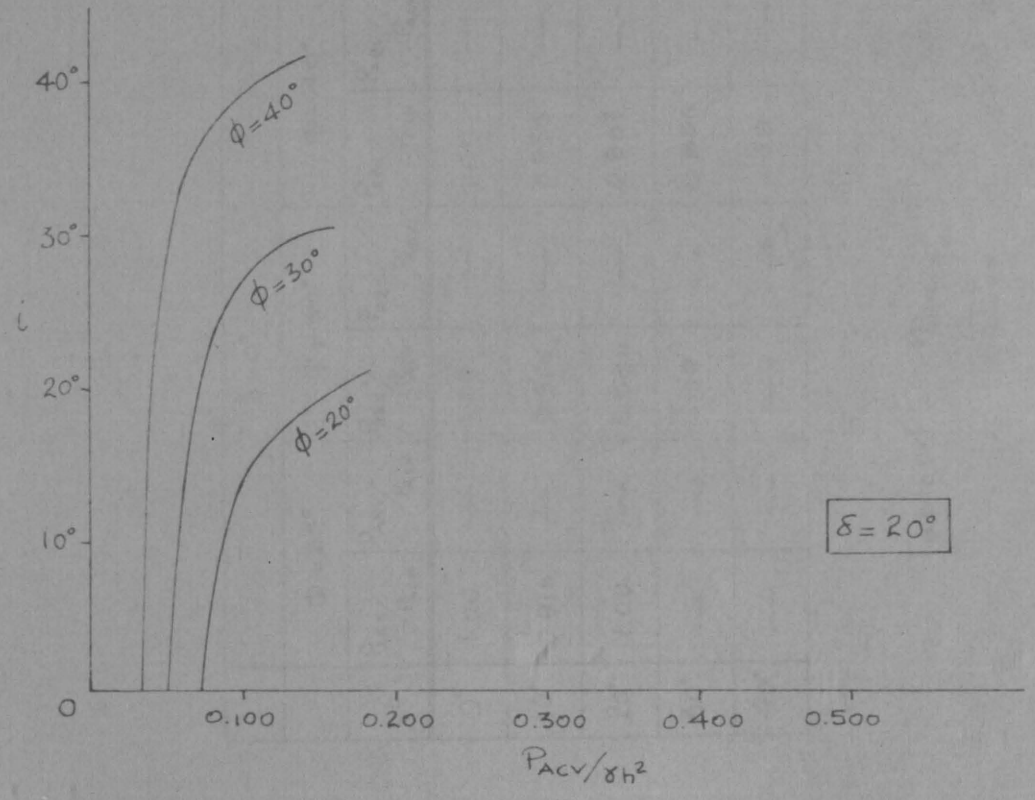


Fig. 34 - Angle i vs. $P_{Acv}/\gamma h^2$ graph for $\delta=20°$, $\phi=20°, 30°, 40°$

	$\delta = 0°$						$\delta = 10°$					
	$\Phi = 20°$		$\Phi = 30°$		$\Phi = 40°$		$\Phi = 20°$		$\Phi = 30°$		$\Phi = 40°$	
i	P_{ARH}/P_{ACH}	P_{ARV}/P_{ACV}	P_{ARH}/P_{ACH}	P_{ARV}/P_{ACV}	P_{ARH}/P_{ACH}	P_{ARV}/P_{ACV}	P_{ARH}/P_{ACH}	P_{ARV}/P_{ACV}	P_{ARH}/P_{ACH}	P_{ARV}/P_{ACV}	P_{ARH}/P_{ACH}	P_{ARV}/P_{ACV}
0°	1.00	—	1.00	—	1.00	—	1.109	0	1.09	0	1.07	0
10°	0.916	—	0.915	—	0.933	—	1.00	1.00	1.00	1.00	1.00	1.00
20°	1.00	—	0.886	—	0.887	—	1.00	2.06	0.946	1.97	0.944	2.00
30°	—	—	1.00	—	0.855	—	—	—	1.00	3.29	0.901	3.04
40°	—	—	—	—	1.00	—	—	—	—	—	1.00	4.83

Table 4 - P_{ARH}/P_{ACH} and P_{ARV}/P_{ACV} ratios for vertical walls and values of $\delta = 0°, 10°$, $\Phi = 20°, 30°, 40°$

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	$\delta = 20°$					
	$\Phi = 20°$		$\Phi = 30°$		$\Phi = 40°$	
c	P_{ARH}/P_{ACH}	P_{ARV}/P_{ACV}	P_{ARH}/P_{ACH}	P_{ARV}/P_{ACV}	P_{ARH}/P_{ACH}	P_{ARV}/P_{ACV}
0°	1.22	0	1.19	0	1.16	0
10°	1.07	0.523	1.08	0.517	1.07	0.54
20°	1.00	1.00	1.00	1.00	1.00	1.00
30°	—	—	1.00	1.58	0.94	1.52
40°	—	—	—	—	1.00	2.30

Table 5 – P_{ARH}/P_{ACH} and P_{ARV}/P_{ACV} ratios for vertical walls and values of $\delta = 20°$, $\Phi = 20°, 30°, 40°$.

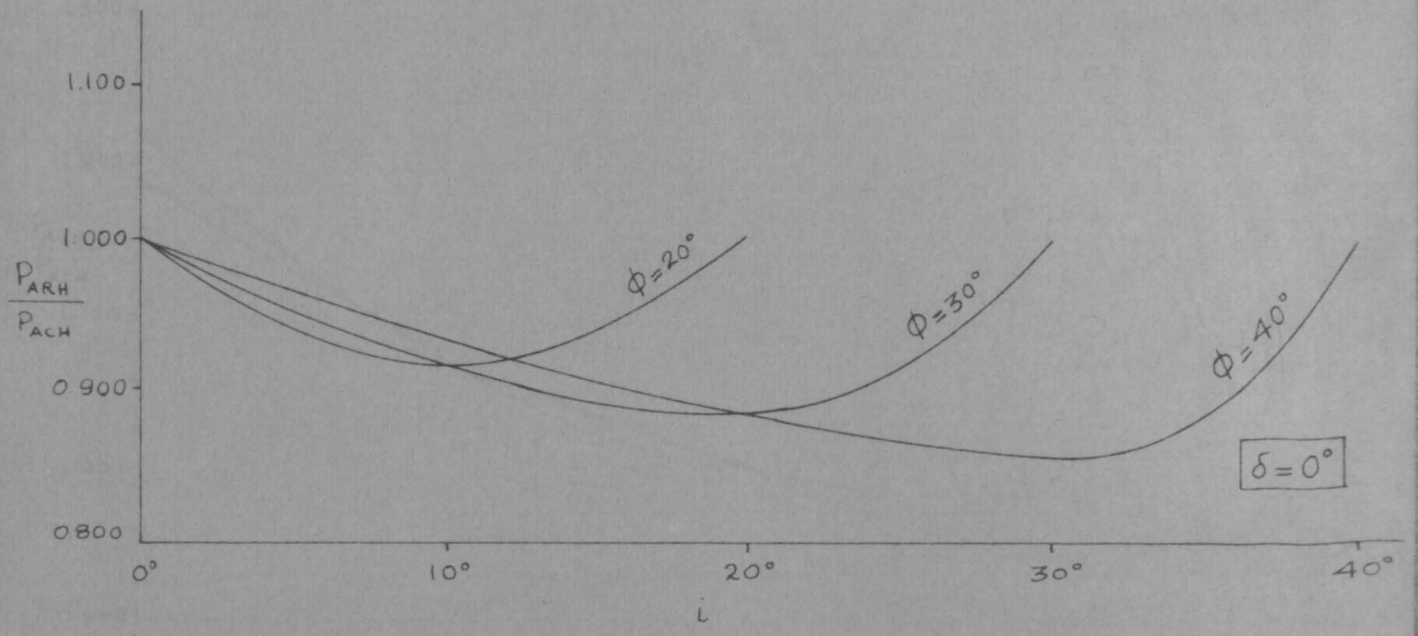


Fig. 35 – $\frac{P_{ARH}}{P_{ACH}}$ ratio vs. angle i graph for $\delta=0°$, $\phi=20°, 30°, 40°$

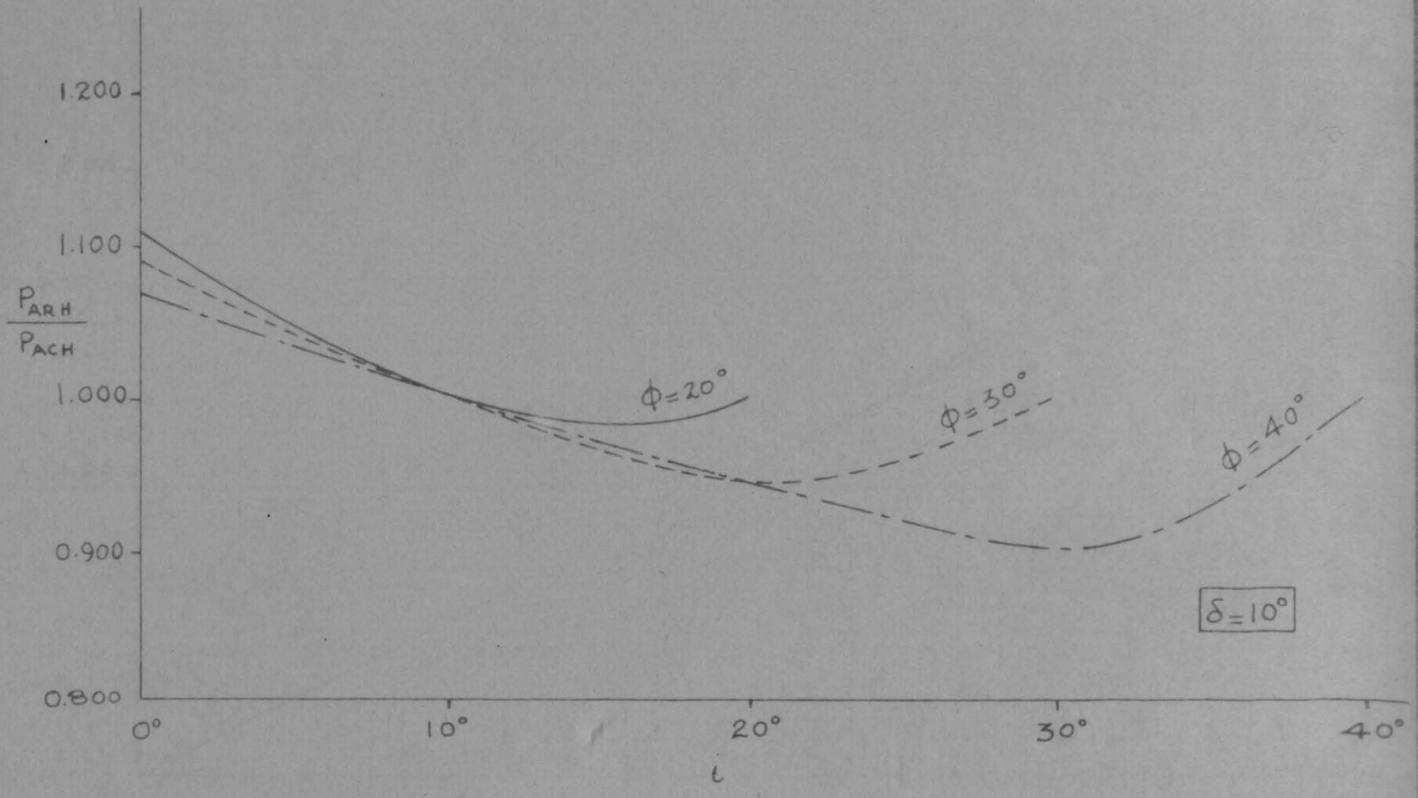


Fig. 36 – $\frac{P_{ARH}}{P_{ACH}}$ ratio vs. angle i graph for $\delta=10°$, $\phi=20°, 30°, 40°$

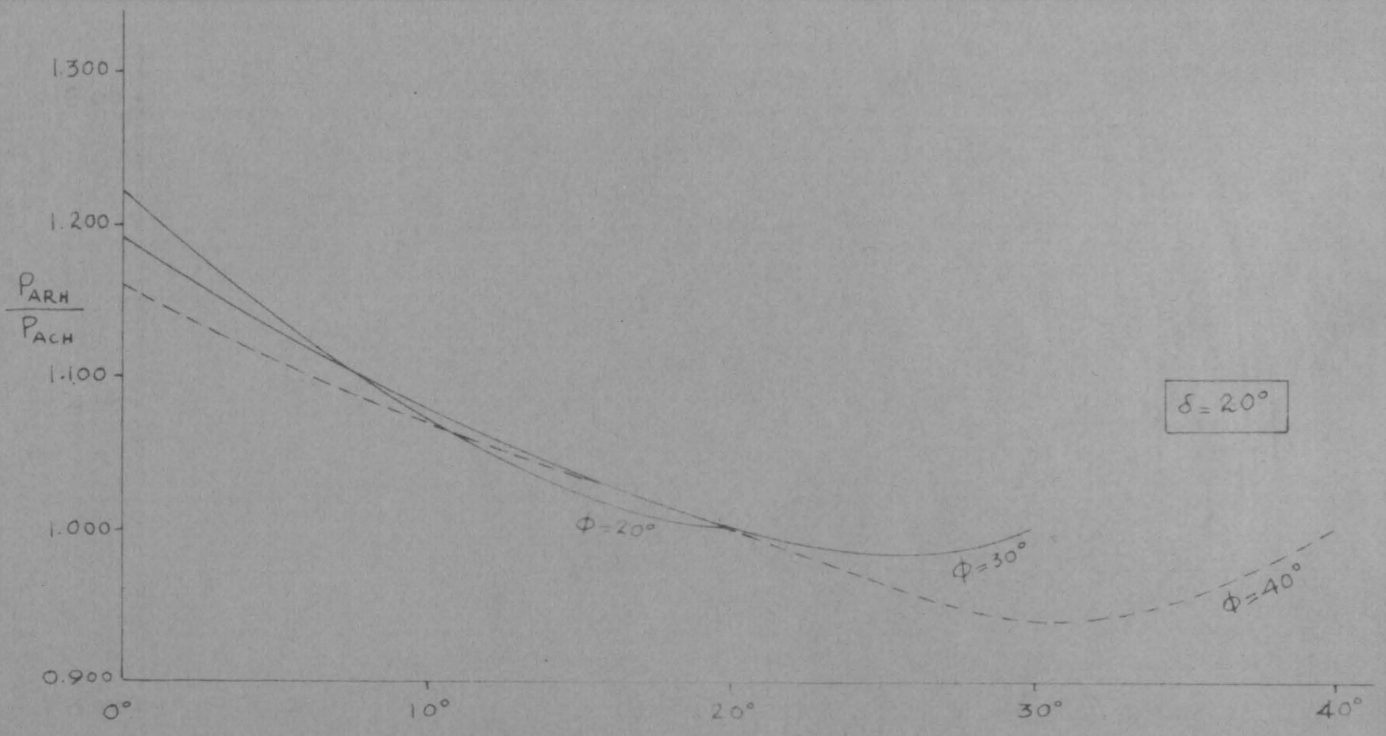


Fig. 37 – $\frac{P_{ARH}}{P_{ACH}}$ ratio vs angle i graph for $\delta = 20°$, $\Phi = 20°, 30°, 40°$

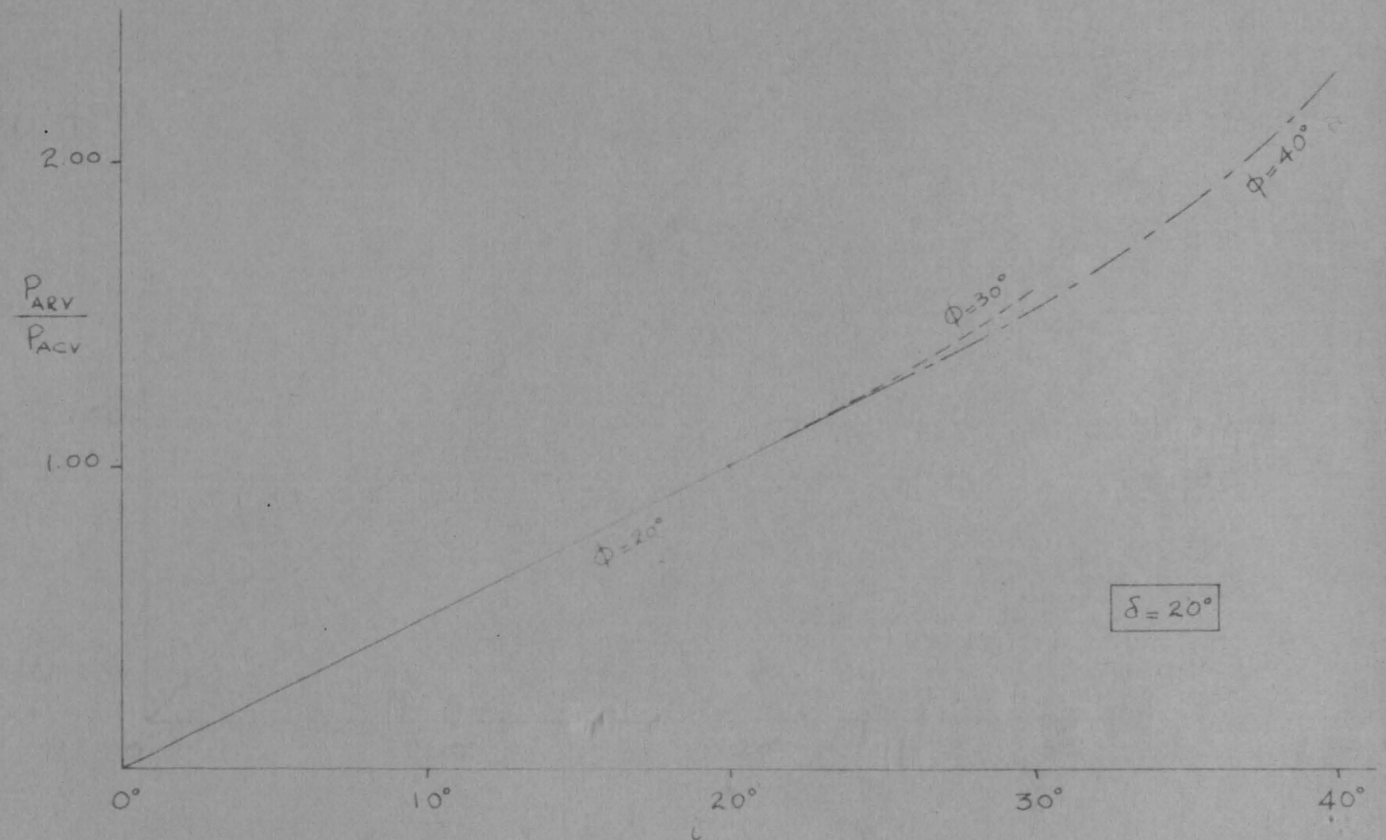


Fig. 38 – $\frac{P_{ARV}}{P_{ACV}}$ ratio vs angle i graph for $\delta = 20°$, $\Phi = 20°, 30°, 40°$

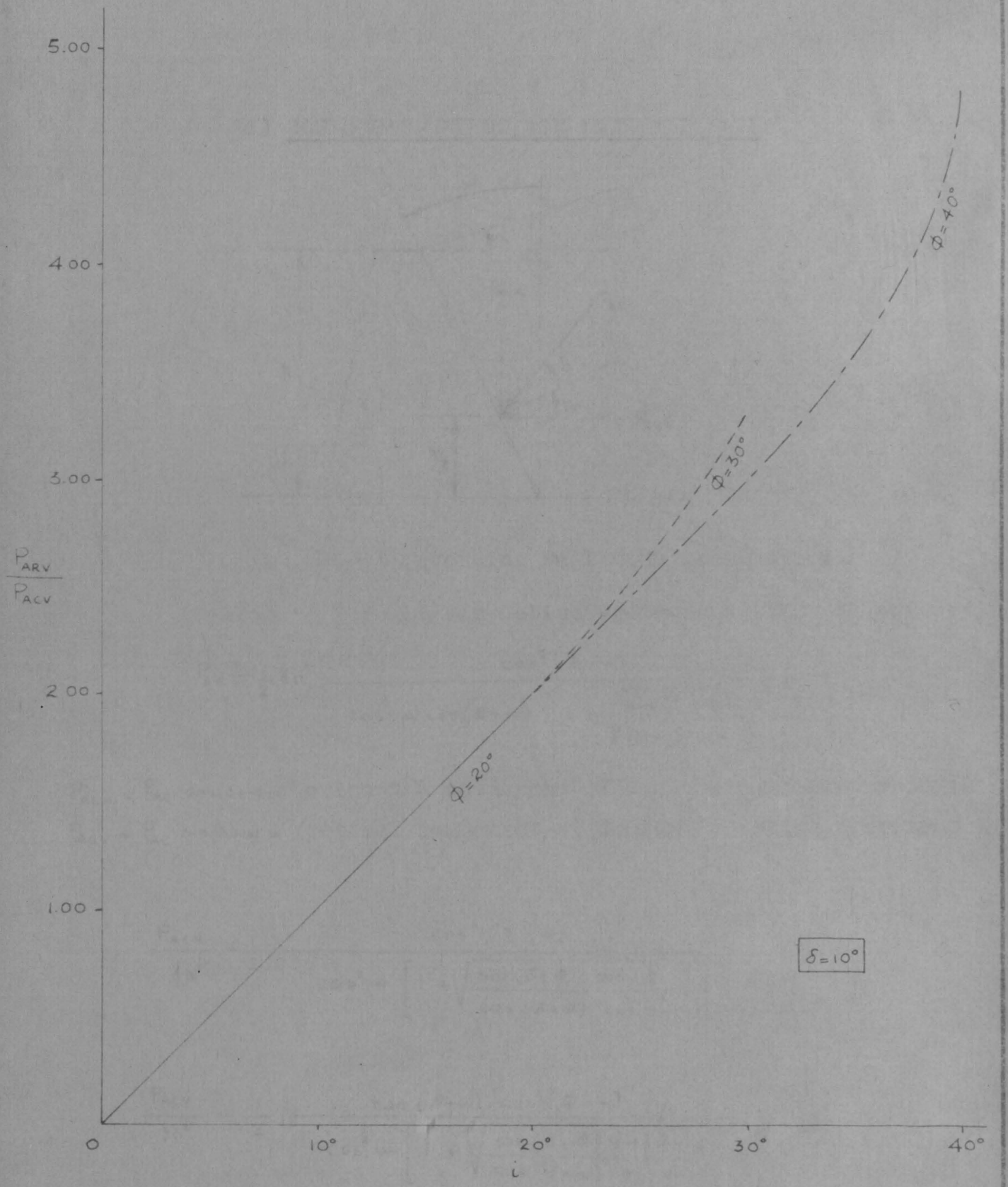


Fig 39 - $\frac{P_{ARV}}{P_{ACV}}$ ratio vs. angle i graph for $\delta=10°$, $\phi=20°, 30°, 40°$

C) COULOMB'S METHOD FOR INCLINED WALL

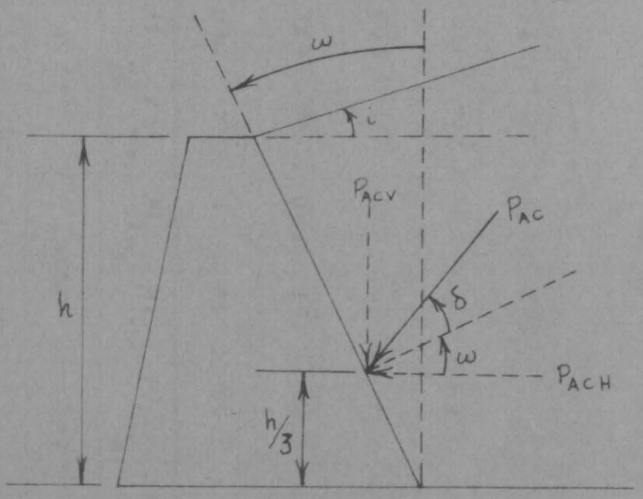


Fig. 40- A retaining wall with inclined back.

Coulomb's formula for active pressure is the following:

$$P_{AC} = \frac{1}{2}\gamma h^2 \frac{\cos^2(\phi-\omega)}{\cos^2\omega\cos(\delta+\omega)\left[1+\sqrt{\frac{\sin(\delta+\phi)\sin(\phi-i)}{\cos(\delta+\omega)\cos(\omega-i)}}\right]^2}$$

$P_{ACH} = P_{AC}\cos(\delta+\omega)$ = Horizontal component of Coulomb's active pressure

$P_{ACV} = P_{AC}\sin(\delta+\omega)$ = Vertical component of Coulomb's active pressure

$$\frac{P_{ACH}}{\gamma h^2} = \frac{1}{2}\frac{\cos^2(\phi-\omega)}{\cos^2\omega\left[1+\sqrt{\frac{\sin(\delta+\phi)\sin(\phi-i)}{\cos(\delta+\omega)\cos(\omega-i)}}\right]^2}$$

$$\frac{P_{ACV}}{\gamma h^2} = \frac{1}{2}\frac{\tan(\delta+\omega)\cos^2(\phi-\omega)}{\cos^2\omega\left[1+\sqrt{\frac{\sin(\delta+\phi)\sin(\phi-i)}{\cos(\delta+\omega)\cos(\omega-i)}}\right]^2}$$

	$\delta = 0°$																	
	$\Phi = 20°$						$\Phi = 30°$						$\Phi = 40°$					
	$\omega = 10°$		$\omega = 20°$		$\omega = 30°$		$\omega = 10°$		$\omega = 20°$		$\omega = 30°$		$\omega = 10°$		$\omega = 20°$		$\omega = 30°$	
c	$P_{ACH}/\gamma h^2$	$P_{ACV}/\gamma h^2$	$P_{ACH}/\gamma h^2$	$P_{ACV}/\gamma h^2$	$P_{ACH}/\gamma h^2$	$P_{ACV}/\gamma h^2$	$P_{ACH}/\gamma h^2$	$P_{ACV}/\gamma h^2$	$P_{ACH}/\gamma h^2$	$P_{ACV}/\gamma h^2$	$P_{ACH}/\gamma h^2$	$P_{ACV}/\gamma h^2$	$P_{ACH}/\gamma h^2$	$P_{ACV}/\gamma h^2$	$P_{ACH}/\gamma h^2$	$P_{ACV}/\gamma h^2$	$P_{ACH}/\gamma h^2$	$P_{ACV}/\gamma h^2$
0°	0.276	0.049	0.304	0.111	0.332	0.192	0.200	0.035	0.235	0.086	0.268	0.155	0.142	0.025	0.176	0.064	0.213	0.123
10°	0.322	0.057	0.361	0.131	0.400	0.231	0.227	0.040	0.268	0.098	0.313	0.181	0.157	0.028	0.198	0.072	0.243	0.140
20°	0.500	0.088	0.566	0.206	0.646	0.373	0.270	0.047	0.322	0.117	0.383	0.221	0.177	0.031	0.227	0.083	0.285	0.164
30°	—	—	—	—	—	—	0.455	0.080	0.549	0.200	0.666	0.384	0.213	0.037	0.275	0.100	0.350	0.202
40°	—	—	—	—	—	—	—	—	—	—	—	—	0.387	0.068	0.500	0.182	0.646	0.373

Table 6 – Horizontal and vertical components of Coulombs active pressure for inclined walls and different value of c ω Φ δ

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	$\delta = 10°$																	
	$\Phi = 20°$						$\Phi = 30°$						$\Phi = 40°$					
	$\omega = 10°$		$\omega = 20°$		$\omega = 30°$		$\omega = 10°$		$\omega = 20°$		$\omega = 30°$		$\omega = 10°$		$\omega = 20°$		$\omega = 30°$	
i	$P_{ACH}/\gamma h^2$	$P_{ACV}/\gamma h^2$	$P_{ACH}/\gamma h^2$	$P_{ACV}/\gamma h^2$	$P_{ACH}/\gamma h^2$	$P_{ACV}/\gamma h^2$	$P_{ACH}/\gamma h^2$	$P_{ACV}/\gamma h^2$	$P_{ACH}/\gamma h^2$	$P_{ACV}/\gamma h^2$	$P_{ACH}/\gamma h^2$	$P_{ACV}/\gamma h^2$	$P_{ACH}/\gamma h^2$	$P_{ACV}/\gamma h^2$	$P_{ACH}/\gamma h^2$	$P_{ACV}/\gamma h^2$	$P_{ACH}/\gamma h^2$	$P_{ACV}/\gamma h^2$
0°	0.245	0.089	0.266	0.153	0.285	0.239	0.179	0.065	0.207	0.120	0.232	0.195	0.128	0.047	0.158	0.091	0.187	0.157
10°	0.294	0.107	0.325	0.188	0.356	0.298	0.207	0.075	0.242	0.140	0.276	0.232	0.144	0.052	0.179	0.103	0.216	0.181
20°	0.500	0.182	0.566	0.327	0.646	0.542	0.250	0.091	0.297	0.171	0.348	0.292	0.165	0.060	0.208	0.120	0.255	0.214
30°	—	—	—	—	—	—	0.455	0.166	0.549	0.317	0.667	0.560	0.201	0.073	0.257	0.148	0.323	0.271
40°	—	—	—	—	—	—	—	—	—	—	—	—	0.386	0.141	0.500	0.289	0.646	0.542

Table 7 - Horizontal and vertical components of Coulomb's active pressure for inclined walls and different values of i, ω, Φ, δ.

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	$\delta = 20°$																	
	$\Phi = 20°$						$\Phi = 30°$						$\Phi = 40°$					
	$\omega = 10°$		$\omega = 20°$		$\omega = 30°$		$\omega = 10°$		$\omega = 20°$		$\omega = 30°$		$\omega = 10°$		$\omega = 20°$		$\omega = 30°$	
i	$P_{ACH}/\gamma h^2$	$P_{ACV}/\gamma h^2$	$P_{ACH}/\gamma h^2$	$P_{ACV}/\gamma h^2$	$P_{ACH}/\gamma h^2$	$P_{ACV}/\gamma h^2$	$P_{ACH}/\gamma h^2$	$P_{ACV}/\gamma h^2$	$P_{ACH}/\gamma h^2$	$P_{ACV}/\gamma h^2$	$P_{ACH}/\gamma h^2$	$P_{ACV}/\gamma h^2$	$P_{ACH}/\gamma h^2$	$P_{ACV}/\gamma h^2$	$P_{ACH}/\gamma h^2$	$P_{ACV}/\gamma h^2$	$P_{ACH}/\gamma h^2$	$P_{ACV}/\gamma h^2$
0°	0.220	0.127	0.236	0.198	0.244	0.290	0.163	0.094	0.184	0.154	0.199	0.237	0.119	0.069	0.142	0.120	0.162	0.193
10°	0.272	0.157	0.295	0.247	0.317	0.378	0.190	0.109	0.217	0.182	0.243	0.289	0.133	0.077	0.162	0.136	0.190	0.227
20°	0.500	0.289	0.566	0.475	0.646	0.770	0.235	0.136	0.275	0.231	0.314	0.374	0.154	0.089	0.191	0.160	0.228	0.272
30°	—	—	—	—	—	—	0.455	0.263	0.549	0.461	0.667	0.795	0.190	0.110	0.238	0.200	0.294	0.350
40°	—	—	—	—	—	—	—	—	—	—	—	—	0.387	0.223	0.500	0.420	0.646	0.770

Table 8 – Horizontal and vertical components of Coulomb's active pressure for inclined walls and different values of i, ω, Φ, δ.

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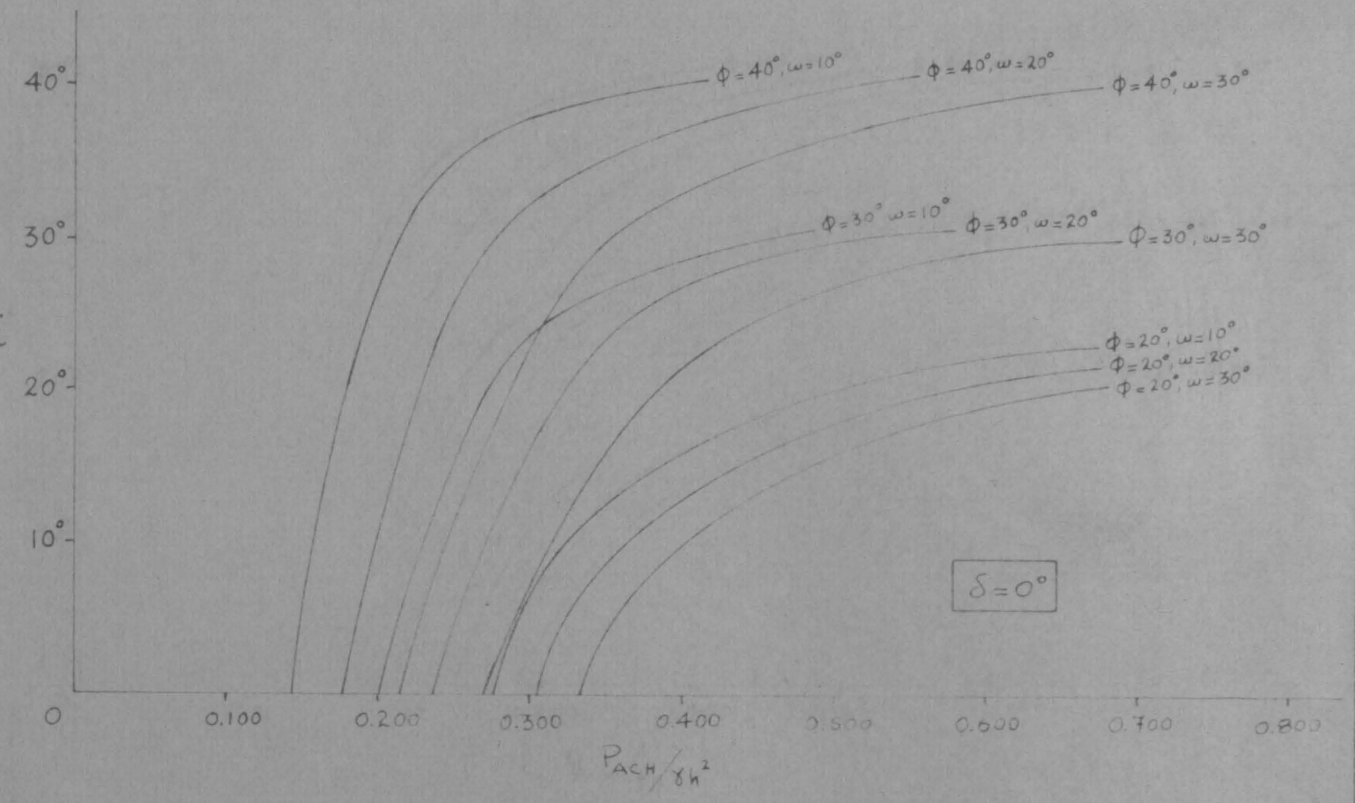


Fig. 41 – Angle i vs $P_{ACH}/\gamma h^2$ graph for $\delta=0°$

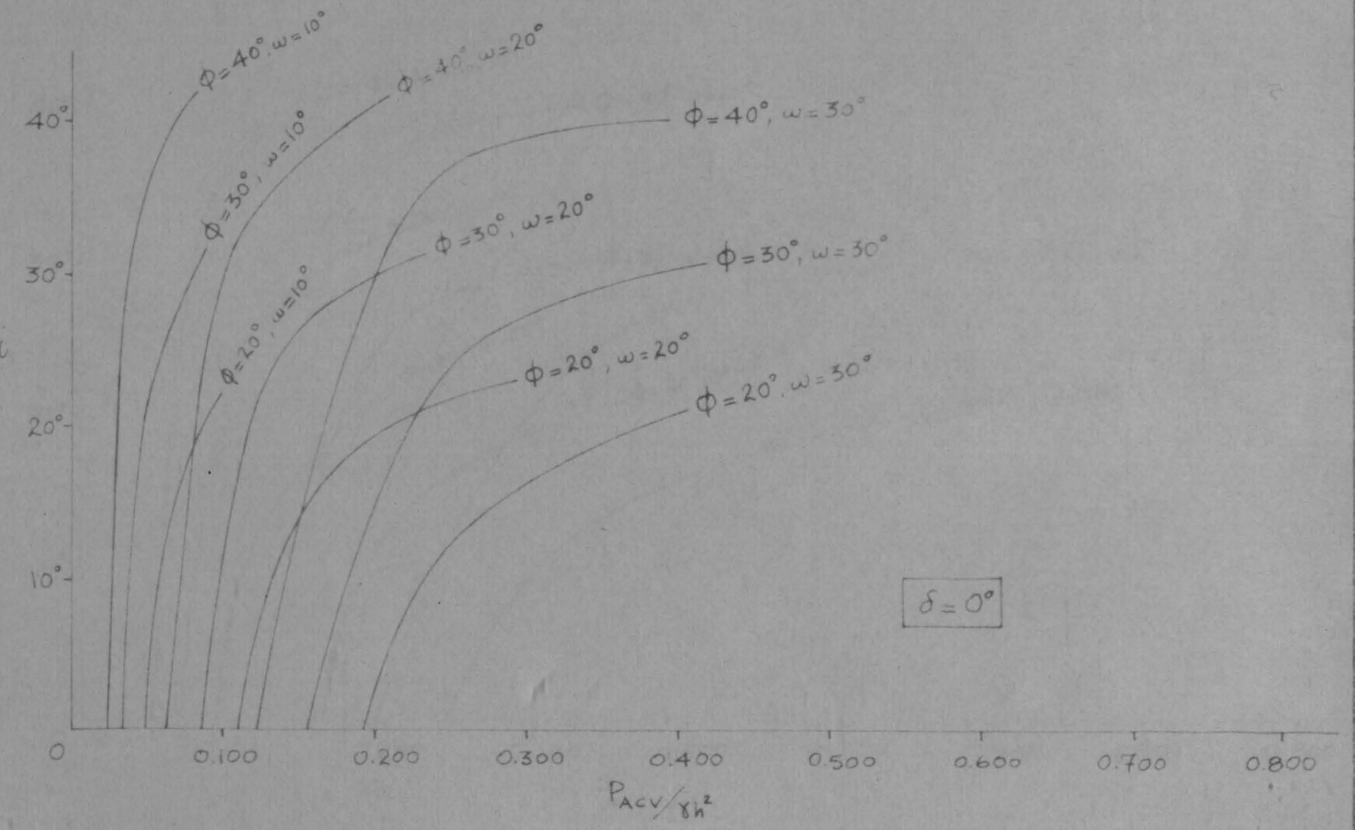


Fig. 42 – Angle i vs. $P_{ACV}/\gamma h^2$ graph for $\delta=0°$

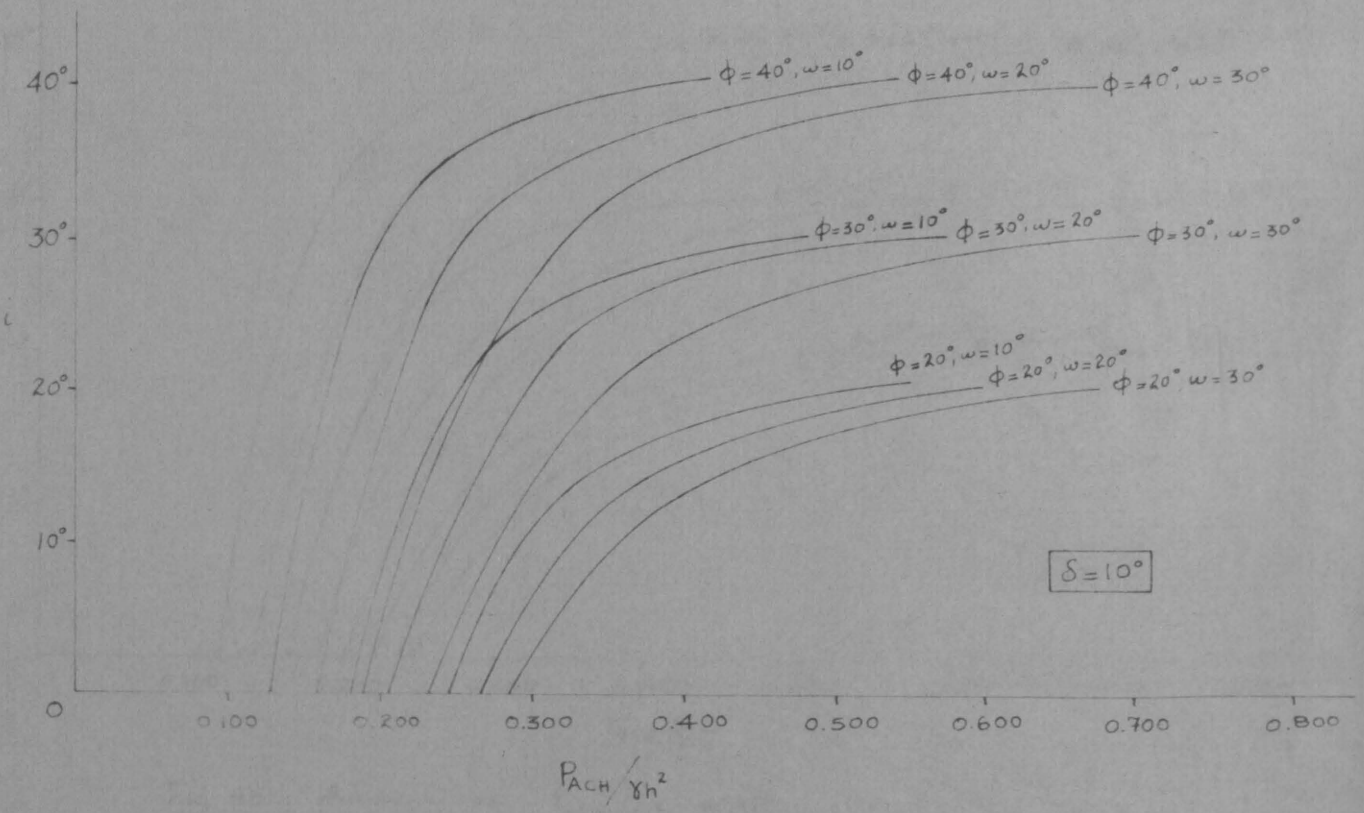


Fig. 43 - Angle i vs. $P_{ACH}/\gamma h^2$ graph for $\delta=10°$

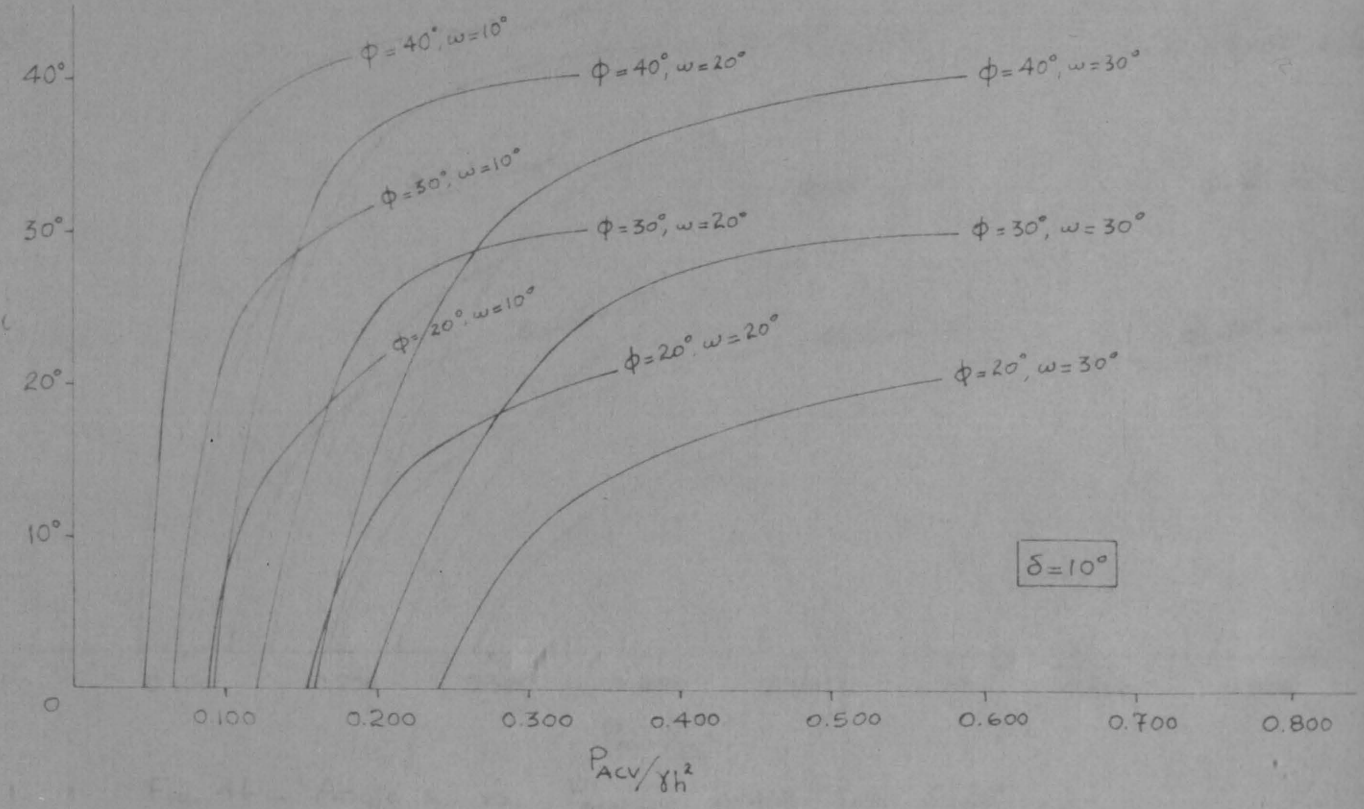


Fig. 44 - Angle i vs. $P_{ACV}/\gamma h^2$ graph for $\delta=10°$

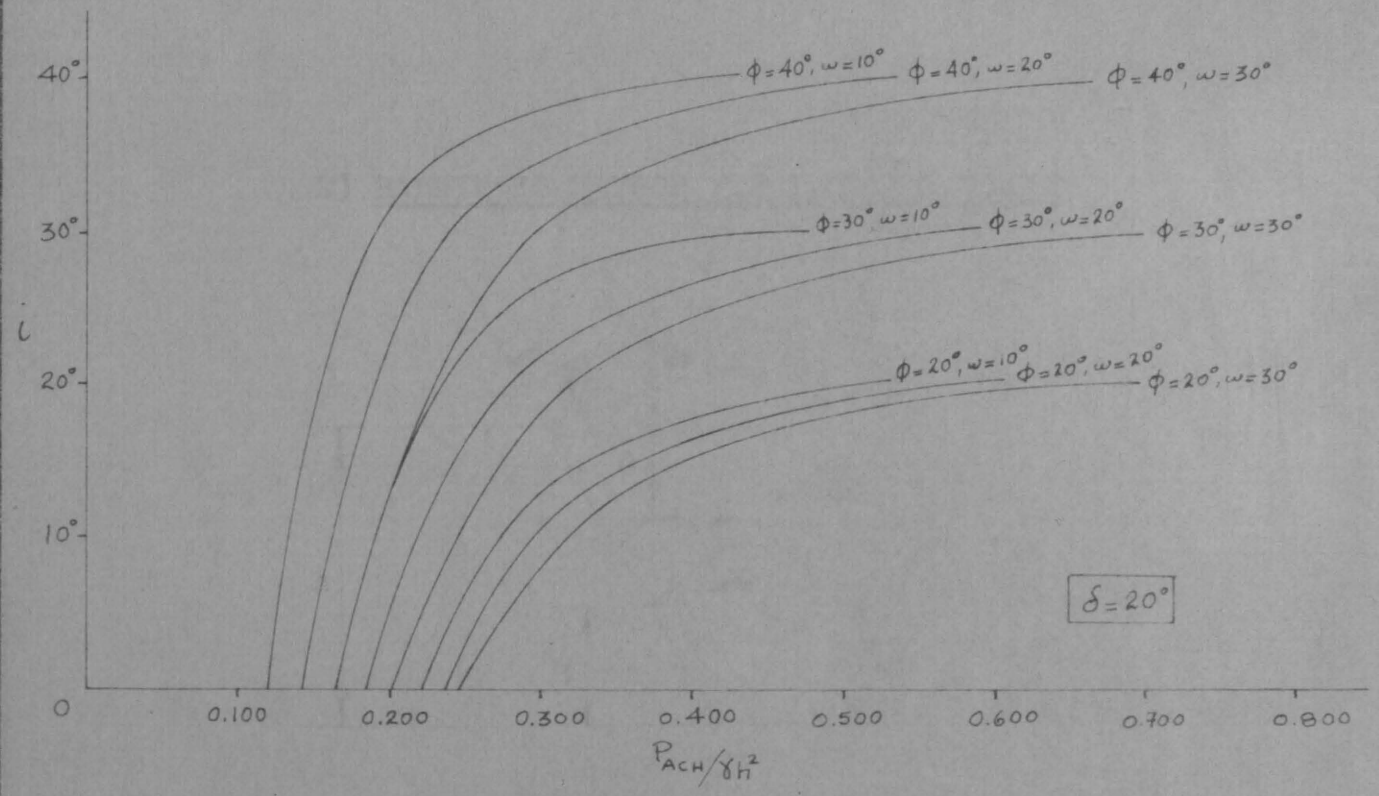


Fig. 45 – Angle i vs. $P_{ACH}/\gamma h^2$ graph for $\delta = 20°$

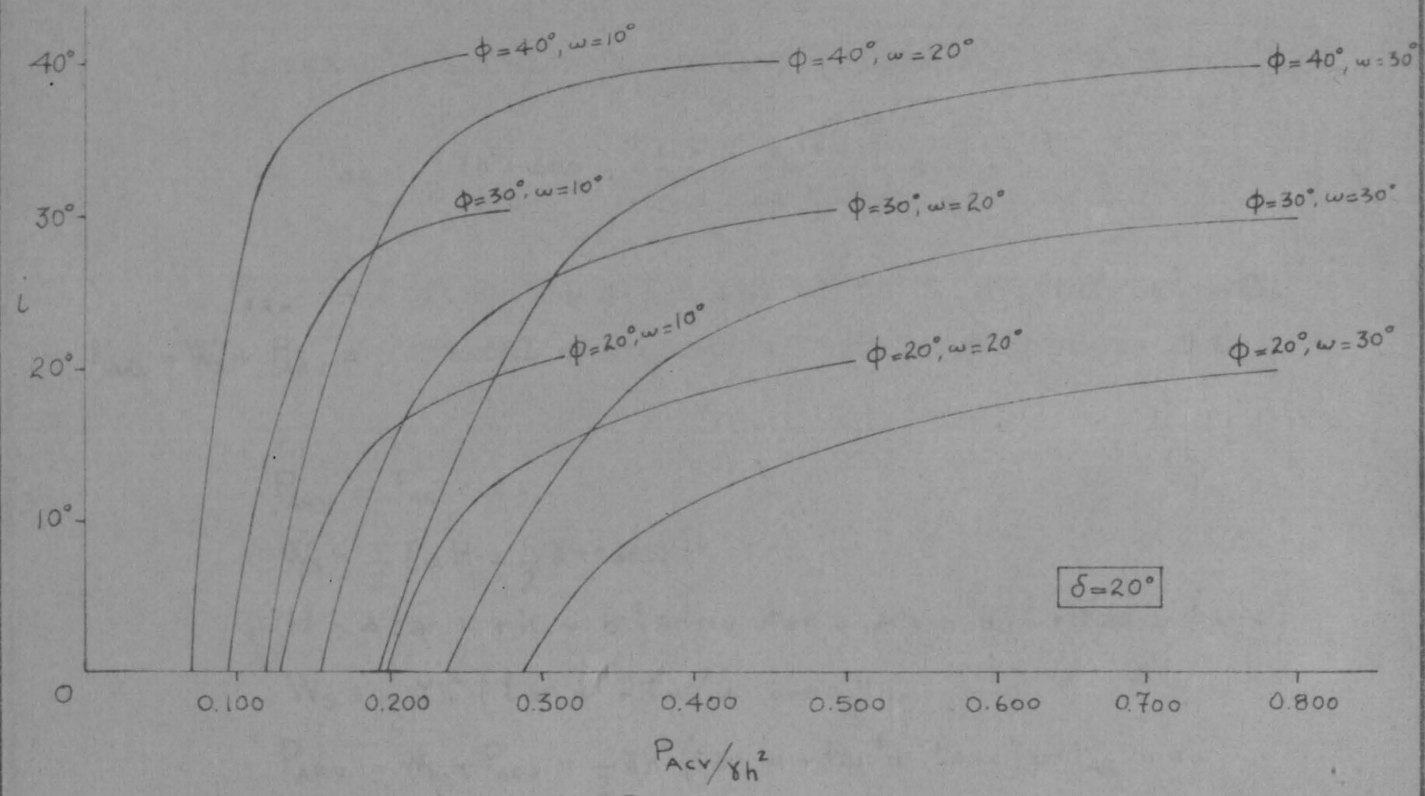


Fig. 46 – Angle i vs. $P_{ACV}/\gamma h^2$ graph for $\delta = 20°$

D) RANKINE'S METHOD FOR INCLINED WALLS

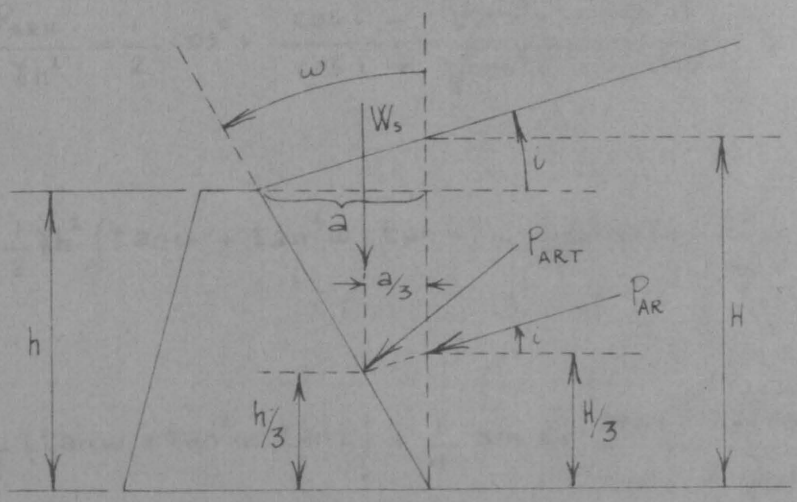


Fig. 47 - A retaining wall with inclined back.

Rankine's formula for active pressure is the following:

$$P_{AR} = \frac{1}{2}\gamma h^2 \left[\cos i \, \frac{\cos i - \sqrt{\cos^2 i - \cos^2 \phi}}{\cos i + \sqrt{\cos^2 i - \cos^2 \phi}} \right]$$

P_{ARH} = Horizontal component of total pressure on wall

$P'_{ARV} = W_s + P_{ARV}$ = Vertical component of total pressure on wall

$$P_{ARH} = P_{AR} \cos i$$

$$W_s = \frac{1}{2}\gamma a H = \frac{1}{2}\gamma h \tan\omega \, H$$

$$H = a \tan i + h = h \tan\omega \cdot \tan i + h = h(1 + \tan\omega \tan i)$$

$$\therefore \; W_s = \frac{1}{2}\gamma h^2 (\tan\omega + \tan^2\omega \cdot \tan i)$$

$$P'_{ARV} = W_s + P_{ARV} = \frac{1}{2}\gamma h^2 (\tan\omega + \tan^2\omega \cdot \tan i) + P_{AR} \sin i$$

$$\frac{P_{ARH}}{\gamma h^2} = \frac{1}{2}\cos^2 i \;\frac{\cos i - \sqrt{\cos^2 i - \cos^2\phi}}{\cos i + \sqrt{\cos^2 i - \cos^2\phi}}$$

$$P'_{ARV} = \frac{1}{2}\gamma h^2\left(\tan\omega + \tan^2\omega \cdot \tan i\right) + \frac{1}{4}\gamma h^2 \sin 2i \;\frac{\cos i - \sqrt{\cos^2 i - \cos^2\phi}}{\cos i + \sqrt{\cos^2 i - \cos^2\phi}}$$

$$\frac{P'_{ARV}}{\gamma h^2} = \frac{1}{2}\left(\tan\omega + \tan^2\omega \cdot \tan i\right) + \frac{1}{4}\sin 2i \;\frac{\cos i - \sqrt{\cos^2 i - \cos^2\phi}}{\cos i + \sqrt{\cos^2 i - \cos^2\phi}}$$

	$\phi = 20°$						$\phi = 30°$						$\phi = 40°$					
	$\omega = 10°$		$\omega = 20°$		$\omega = 30°$		$\omega = 10°$		$\omega = 20°$		$\omega = 30°$		$\omega = 10°$		$\omega = 20°$		$\omega = 30°$	
i	$P_{ARH}/\gamma h^2$	$P'_{ARV}/\gamma h^2$	$P_{ARH}/\gamma h^2$	$P'_{ARV}/\gamma h^2$	$P_{ARH}/\gamma h^2$	$P'_{ARV}/\gamma h^2$	$P_{ARH}/\gamma h^2$	$P'_{ARV}/\gamma h^2$	$P_{ARH}/\gamma h^2$	$P'_{ARV}/\gamma h^2$	$P_{ARH}/\gamma h^2$	$P'_{ARV}/\gamma h^2$	$P_{ARH}/\gamma h^2$	$P'_{ARV}/\gamma h^2$	$P_{ARH}/\gamma h^2$	$P'_{ARV}/\gamma h^2$	$P_{ARH}/\gamma h^2$	$P'_{ARV}/\gamma h^2$
0°	0.245	0.088	0.245	0.182	0.245	0.289	0.166	0.088	0.166	0.182	0.166	0.289	0.109	0.088	0.109	0.182	0.109	0.289
10°	0.262	0.136	0.262	0.240	0.262	0.364	0.173	0.121	0.173	0.225	0.173	0.349	0.111	0.110	0.111	0.214	0.111	0.338
20°	0.442	0.255	0.442	0.367	0.442	0.510	0.195	0.165	0.195	0.277	0.195	0.420	0.118	0.137	0.118	0.249	0.118	0.392
30°	—	—	—	—	—	—	0.375	0.314	0.375	0.438	0.375	0.602	0.137	0.176	0.137	0.300	0.137	0.464
40°	—	—	—	—	—	—	—	—	—	—	—	—	0.294	0.347	0.294	0.484	0.294	0.674

Table 9 – Horizontal and vertical components of Rankine's active pressure for inclined walls and different values of i, ω, ϕ (for all δ's)

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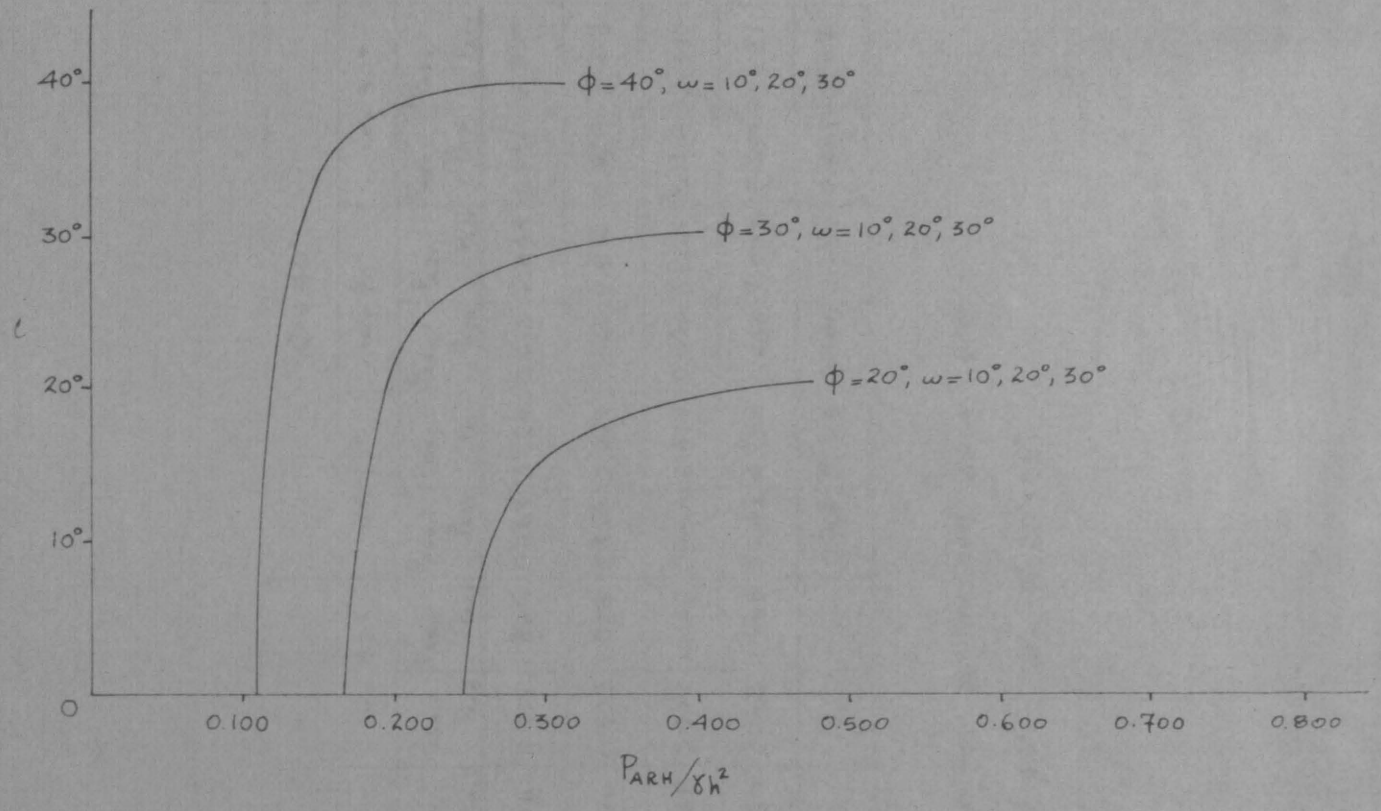


Fig. 48 – Angle i vs. $P_{ARH}/\gamma h^2$ graph for all δ's

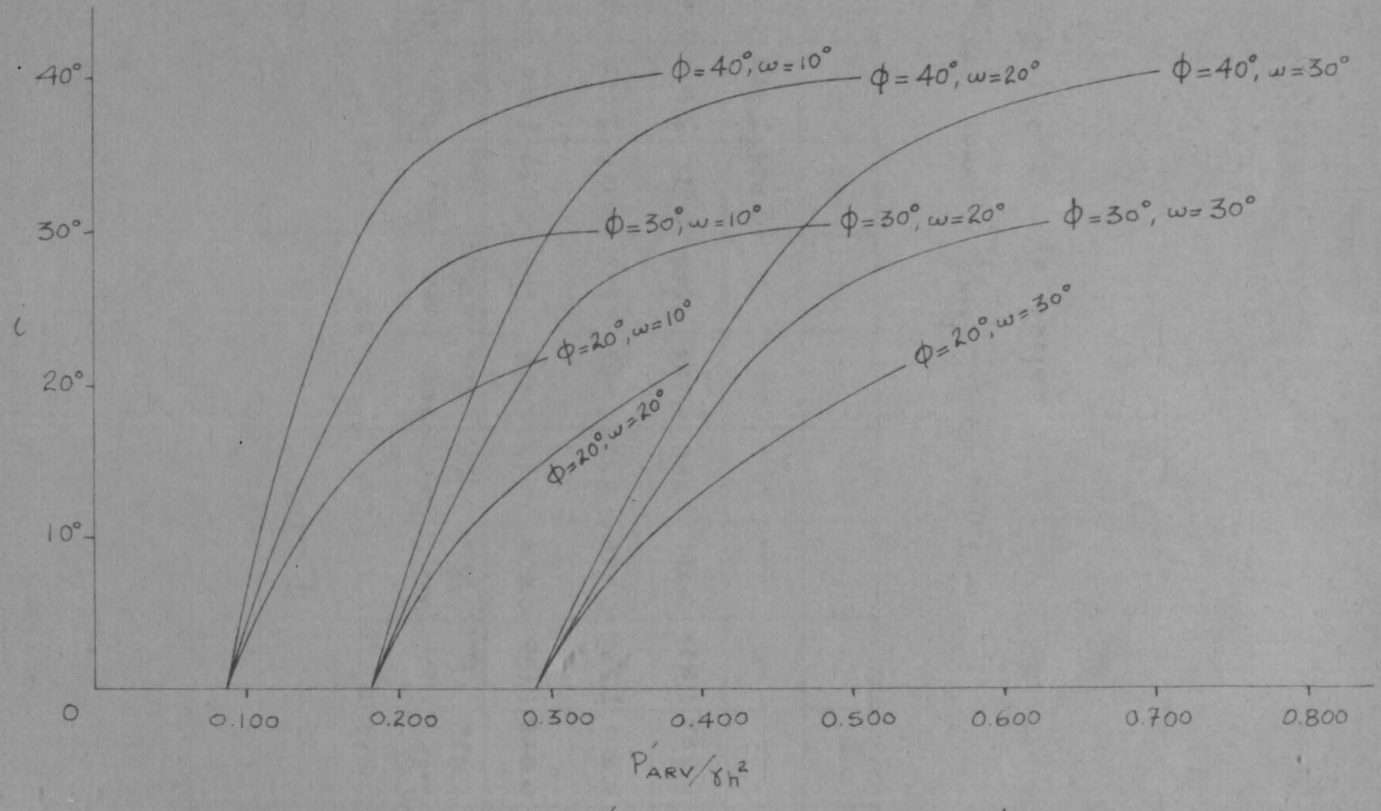


Fig. 49 – Angle i vs. $P'_{ARV}/\gamma h^2$ graph for all δ's

	$\delta = 0°$																	
	$\Phi = 20°$						$\Phi = 30°$						$\Phi = 40°$					
	$\omega = 10°$		$\omega = 20°$		$\omega = 30°$		$\omega = 10°$		$\omega = 20°$		$\omega = 30°$		$\omega = 10°$		$\omega = 20°$		$\omega = 30°$	
i	P_{ARH}/P_{ACH}	P'_{ARV}/P_{ACV}	P_{ARH}/P_{ACH}	P'_{ARV}/P_{ACV}	P_{ARH}/P_{ACH}	P'_{ARV}/P_{ACV}	P_{ARH}/P_{ACH}	P'_{ARV}/P_{ACV}	P_{ARH}/P_{ACH}	P'_{ARV}/P_{ACV}	P_{ARH}/P_{ACH}	P'_{ARV}/P_{ACV}	P_{ARH}/P_{ACH}	P'_{ARV}/P_{ACV}	P_{ARH}/P_{ACH}	P'_{ARV}/P_{ACV}	P_{ARH}/P_{ACH}	P'_{ARV}/P_{ACV}
0°	0.888	1.796	0.806	1.640	0.738	1.505	0.830	2.514	0.706	2.116	0.619	1.865	0.768	3.520	0.619	2.844	0.512	2.350
10°	0.814	2.386	0.726	1.832	0.655	1.576	0.762	3.025	0.646	2.296	0.553	1.928	0.707	3.929	0.561	2.972	0.457	2.414
20°	0.884	2.898	0.781	1.782	0.684	1.367	0.722	3.511	0.606	2.368	0.509	1.905	0.667	4.419	0.520	3.000	0.414	2.390
30°	—	—	—	—	—	—	0.824	3.925	0.683	2.190	0.563	1.568	0.643	4.757	0.498	3.000	0.391	2.297
40°	—	—	—	—	—	—	—	—	—	—	—	—	0.760	5.103	0.588	2.659	0.455	1.807

Table 10 - P_{ARH}/P_{ACH} and P'_{ARV}/P_{ACV} ratios for inclined walls and values of $\delta = 0°$, $\Phi = 20°, 30°, 40°$, $\omega = 10°, 20°, 30°$

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	$\delta = 10°$																	
	$\phi = 20°$						$\phi = 30°$						$\phi = 40°$					
	$\omega = 10°$		$\omega = 20°$		$\omega = 30°$		$\omega = 10°$		$\omega = 20°$		$\omega = 30°$		$\omega = 10°$		$\omega = 20°$		$\omega = 30°$	
i	P_{ARH}/P_{ACH}	P'_{ARV}/P_{ACV}	P_{ARH}/P_{ACH}	P'_{ARV}/P_{ACV}	P_{ARH}/P_{ACH}	P'_{ARV}/P_{ACV}	P_{ARH}/P_{ACH}	P'_{ARV}/P_{ACV}	P_{ARH}/P_{ACH}	P'_{ARV}/P_{ACV}	P_{ARH}/P_{ACH}	P'_{ARV}/P_{ACV}	P_{ARH}/P_{ACH}	P'_{ARV}/P_{ACV}	P_{ARH}/P_{ACH}	P'_{ARV}/P_{ACV}	P_{ARH}/P_{ACH}	P'_{ARV}/P_{ACV}
0°	1.000	0.988	0.921	1.190	0.859	1.209	0.927	1.354	0.802	1.516	0.716	1.482	0.852	1.872	0.689	2.000	0.583	1.840
10°	0.891	1.271	0.806	1.277	0.736	1.221	0.836	1.613	0.715	1.607	0.627	1.504	0.771	2.115	0.620	2.077	0.514	1.867
20°	0.884	1.401	0.781	1.122	0.684	0.941	0.780	1.813	0.657	1.619	0.560	1.438	0.715	2.283	0.567	2.075	0.463	1.832
30°	—	—	—	—	—	—	0.824	1.892	0.683	1.382	0.562	1.075	0.682	2.412	0.533	2.027	0.424	1.712
40°	—	—	—	—	—	—	—	—	—	—	—	—	0.762	2.461	0.588	1.675	0.455	1.243

Table II – P_{ARH}/P_{ACH} and P'_{ARV}/P_{ACV} ratios for inclined walls and values of $\delta = 10°$, $\phi = 20°, 30°, 40°$, $\omega = 10°, 20°, 30°$

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	$\delta = 20°$																	
	$\phi = 20°$						$\phi = 30°$						$\phi = 40°$					
	$\omega = 10°$		$\omega = 20°$		$\omega = 30°$		$\omega = 10°$		$\omega = 20°$		$\omega = 30°$		$\omega = 10°$		$\omega = 20°$		$\omega = 30°$	
i	P_{ARH}/P_{ACH}	P'_{ARV}/P_{ACV}	P_{ARH}/P_{ACH}	P'_{ARV}/P_{ACV}	P_{ARH}/P_{ACH}	P'_{ARV}/P_{ACV}	P_{ARH}/P_{ACH}	P'_{ARV}/P_{ACV}	P_{ARH}/P_{ACH}	P'_{ARV}/P_{ACV}	P_{ARH}/P_{ACH}	P'_{ARV}/P_{ACV}	P_{ARH}/P_{ACH}	P'_{ARV}/P_{ACV}	P_{ARH}/P_{ACH}	P'_{ARV}/P_{ACV}	P_{ARH}/P_{ACH}	P'_{ARV}/P_{ACV}
0°	1.113	0.693	1.038	0.919	1.000	0.996	1.018	0.936	0.902	1.182	0.834	1.219	0.916	1.275	0.767	1.516	0.673	1.497
10°	0.963	0.866	0.888	0.972	0.826	0.963	0.912	1.110	0.797	1.236	0.712	1.207	0.835	1.429	0.685	1.574	0.584	1.489
20°	0.884	0.882	0.781	0.773	0.684	0.662	0.830	1.213	0.709	1.199	0.621	1.123	0.766	1.539	0.618	1.556	0.518	1.441
30°	—	—	—	—	—	—	0.824	1.194	0.683	0.950	0.562	0.757	0.721	1.600	0.575	1.500	0.466	1.326
40°	—	—	—	—	—	—	—	—	—	—	—	—	0.759	1.556	0.588	1.152	0.455	0.875

Table 12 - P_{ARH}/P_{ACH} and P'_{ARV}/P_{ACV} ratios for inclined walls and values of $\delta = 20°$, $\phi = 20°, 30°, 40°$, $\omega = 10°, 20°, 30°$.

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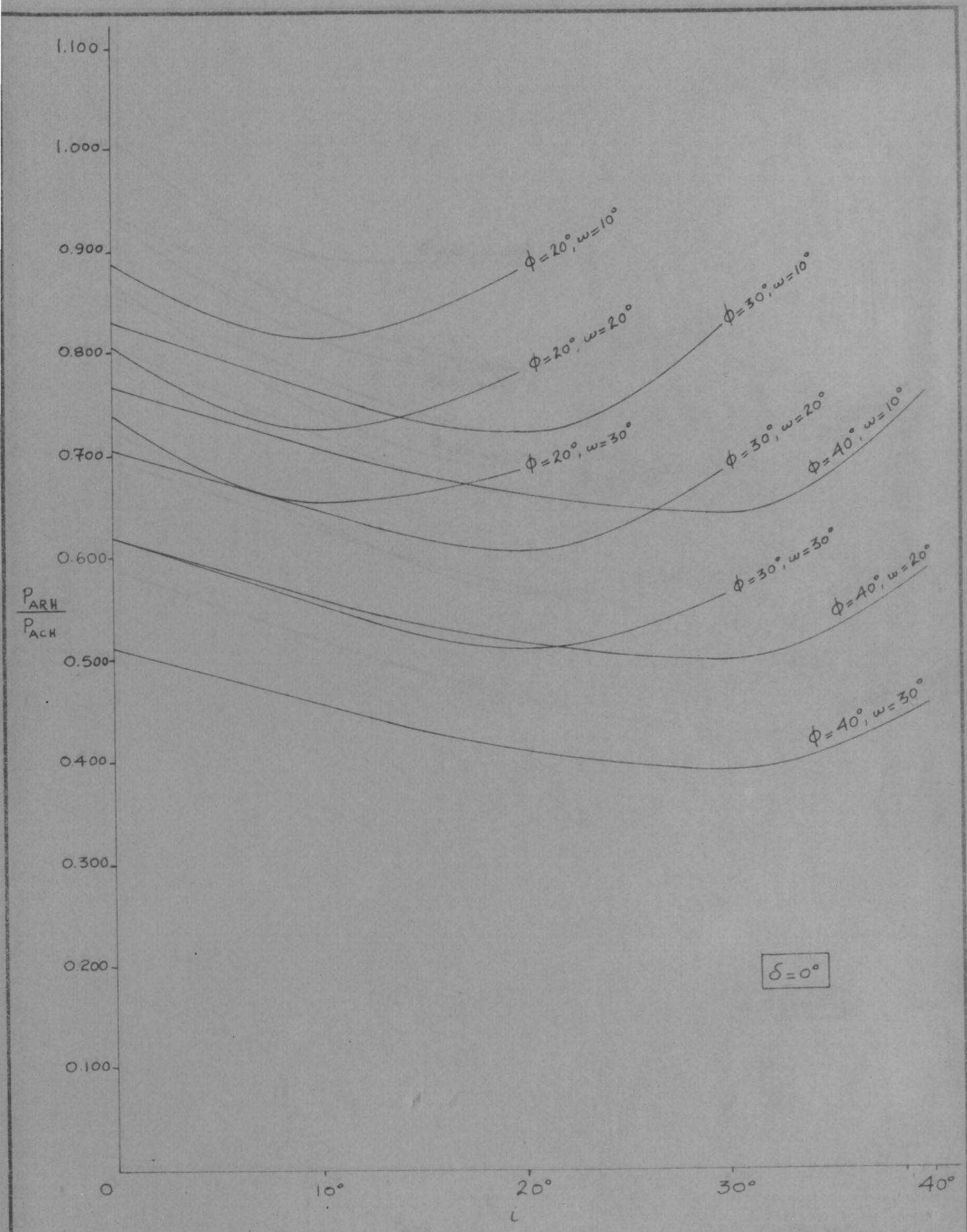


Fig. 50 – $\frac{P_{ARH}}{P_{ACH}}$ ratio vs. angle i graph for $\delta=0°$

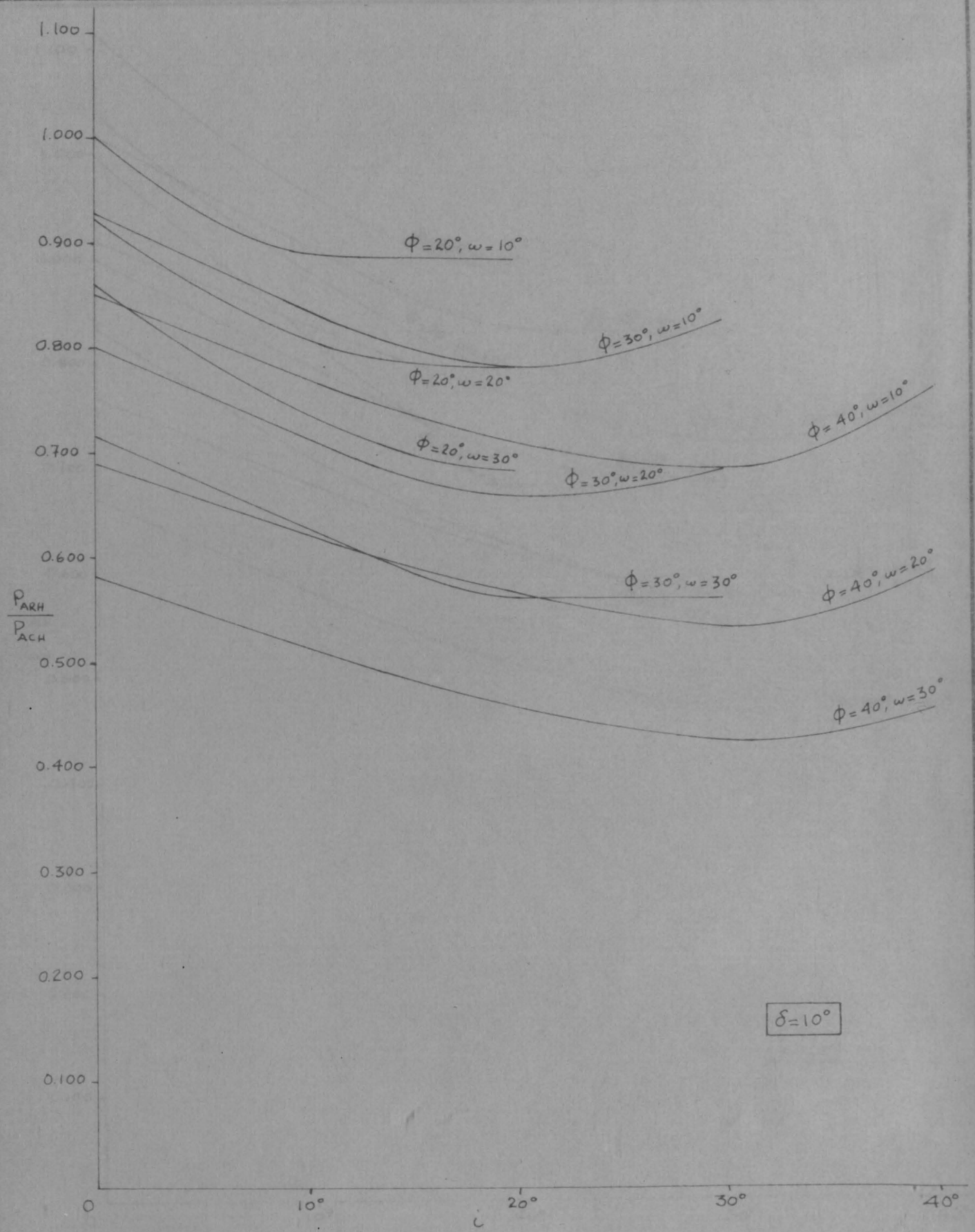


Fig. 51 – $\frac{P_{ARH}}{P_{ACH}}$ ratio vs. angle i graph for $\delta=10°$

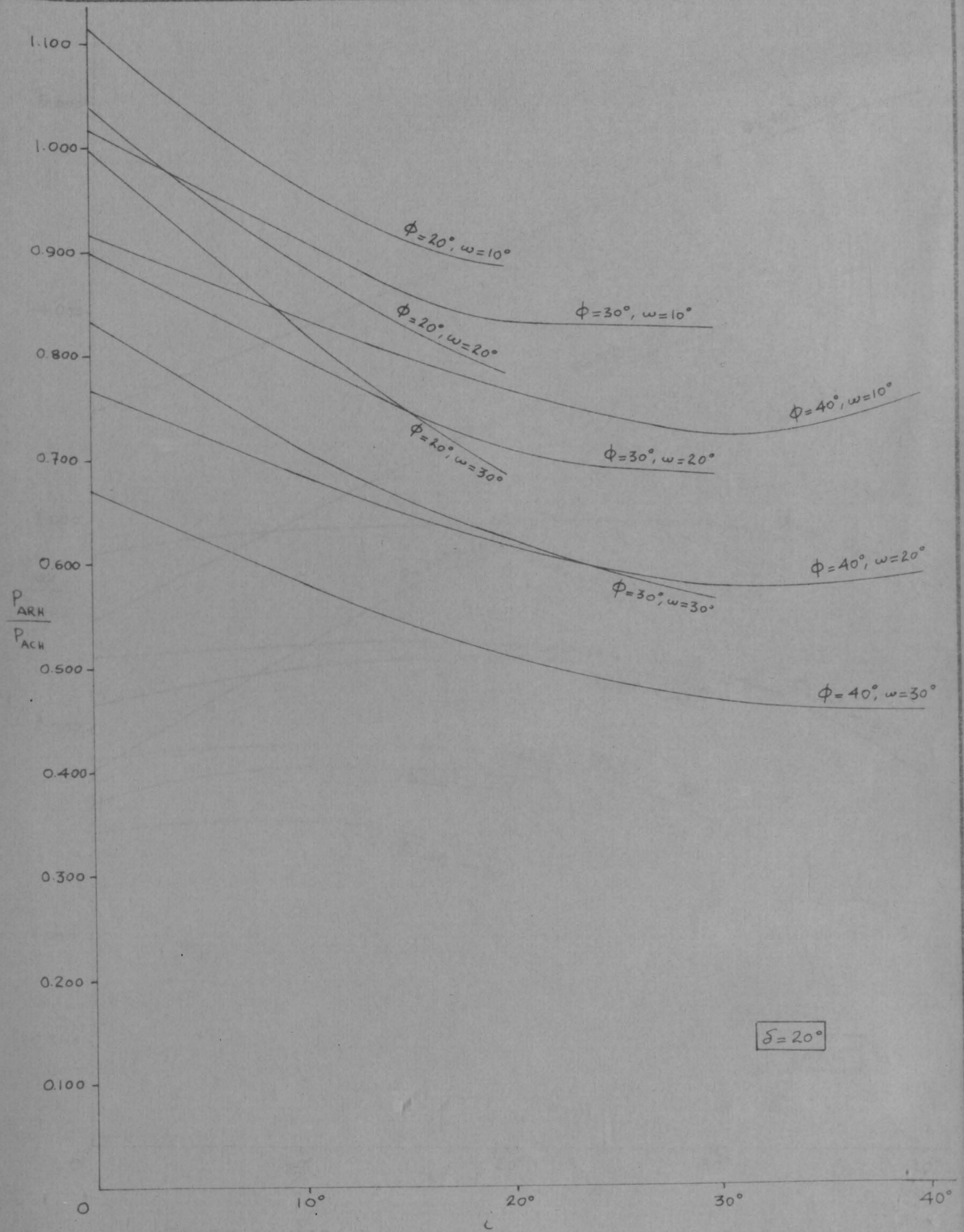


Fig. 52 - $\frac{P_{ARH}}{P_{ACH}}$ ratio vs angle i graph for $\delta=20°$

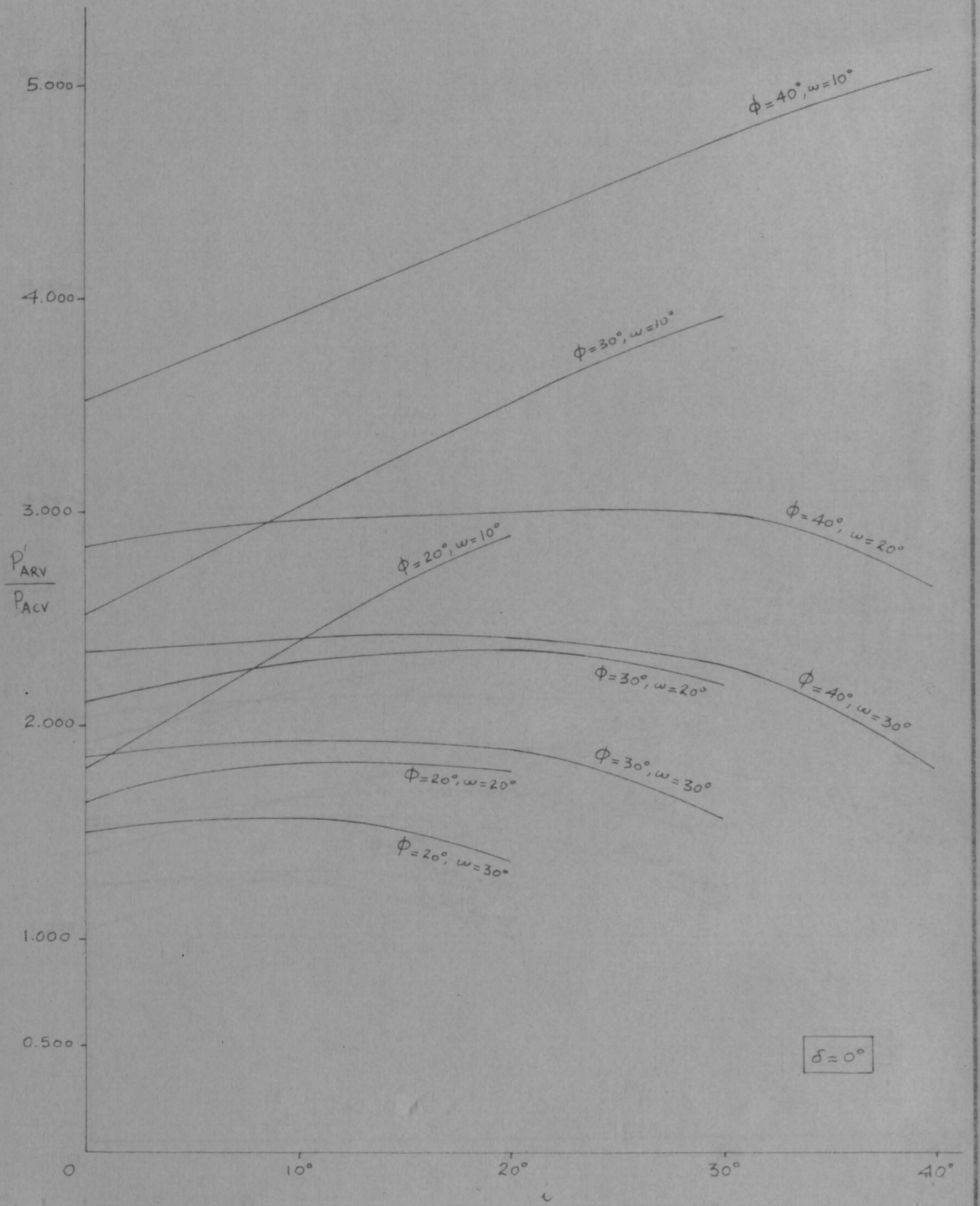


Fig. 53 - $\frac{P'_{ARV}}{P_{ACV}}$ ratio vs. angle i graph for $\delta=0°$

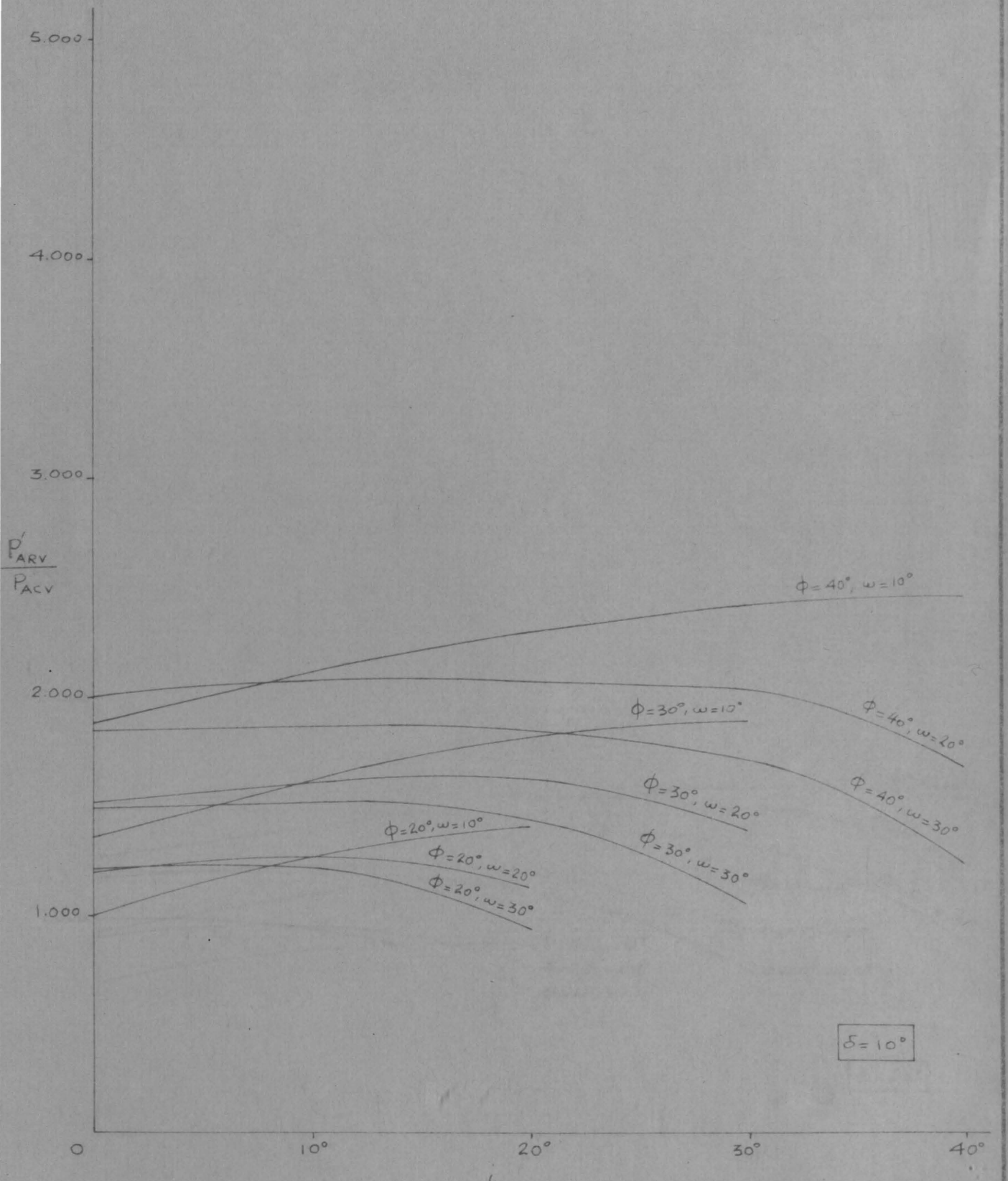


Fig. 54 - $\frac{P'_{ARV}}{P_{ACV}}$ ratio vs. angle i graph for $\delta=10°$

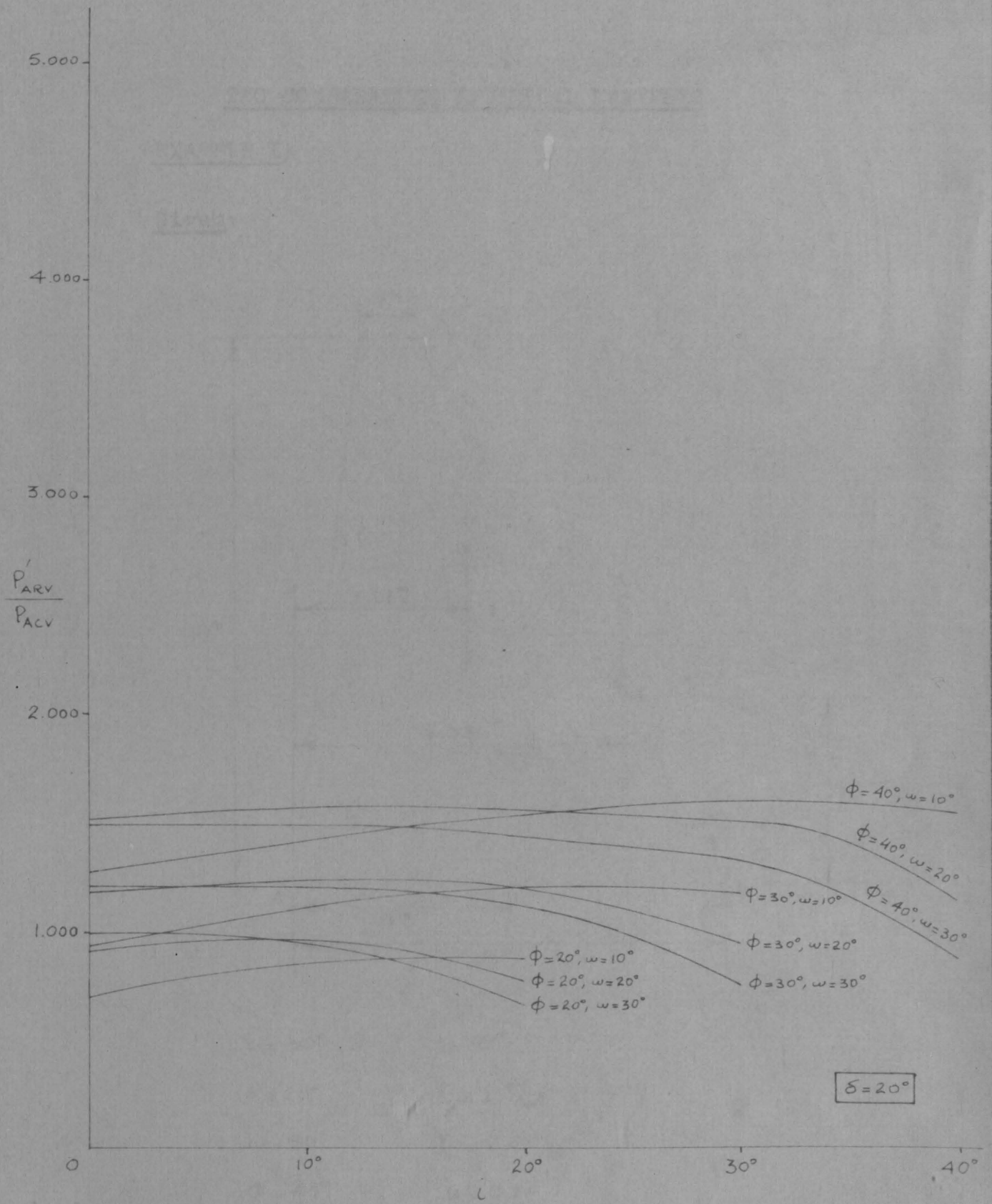
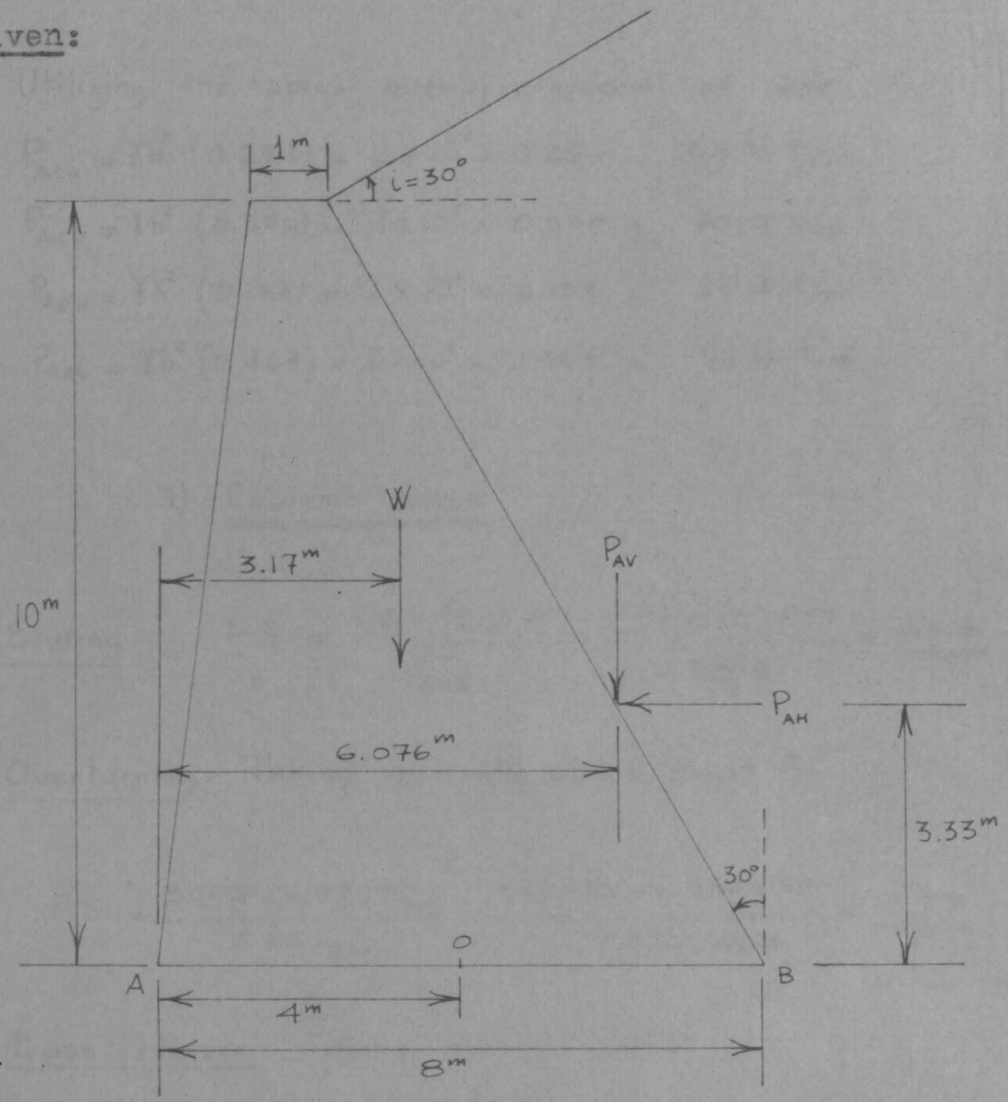


Fig. 55 – $\frac{P'_{ARV}}{P_{ACV}}$ ratio vs. angle i graph for $\delta = 20°$

TWO COMPARATIVE NUMERICAL EXAMPLES

EXAMPLE I)

Given:



$i = 30°$ $\quad h = 10^m$

$\delta = 20°$ $\quad \gamma_s = 2\ t/m^3$

$\omega = 30°$ $\quad \gamma_c = 2.2\ t/m^3$

$\phi = 40°$ $\quad \mu = 0.55$

Solution: Taking unit width

$$W = \frac{10(1+8)}{2} \times 2.2 = 99\ t/m$$

Utilizing the tables already prepared we have:

$P_{ACH} = \gamma h^2 (0.294) = 2 \times 10^2 \times 0.294 = 58.8\ t/m$

$P_{ACV} = \gamma h^2 (0.350) = 2 \times 10^2 \times 0.350 = 70.0\ t/m$

$P_{ARH} = \gamma h^2 (0.137) = 2 \times 10^2 \times 0.137 = 27.4\ t/m$

$P'_{ARV} = \gamma h^2 (0.464) = 2 \times 10^2 \times 0.464 = 92.8\ t/m$

a) Coulomb Case

Sliding: $F.S. = \frac{(W + P_{ACV})\mu}{P_{ACH}} = \frac{(99+70)0.55}{58.8} = \underline{1.58}$

Overturning: Taking moments about point A

$$F.S. = \frac{3.17W + 6.076 P_{ACV}}{3.33 P_{ACH}} = \frac{3.17 \times 99 + 6.076 \times 70}{3.33 \times 58.8} = \underline{3.77}$$

Base Pressure: Taking moments about point O

$$M_o = 0.83 W + 3.33 P_{ACH} - 2.076 P_{ACV}$$

$$M_o = 0.83 \times 99 + 3.33 \times 58.8 - 2.076 \times 70.0 = 133.2\ t\text{-}m$$

maximum soil pressure occurs at point A, and minimum at point B.

$$\frac{I}{c} = \frac{bd^2}{6} \qquad b = 1^m \quad d = 8^m \qquad \therefore \frac{I}{c} = \frac{1 \times 8^2}{6} = 10.65$$

$$P_{max} = \frac{W + P_{ACV}}{A} + \frac{M_o c}{I}$$

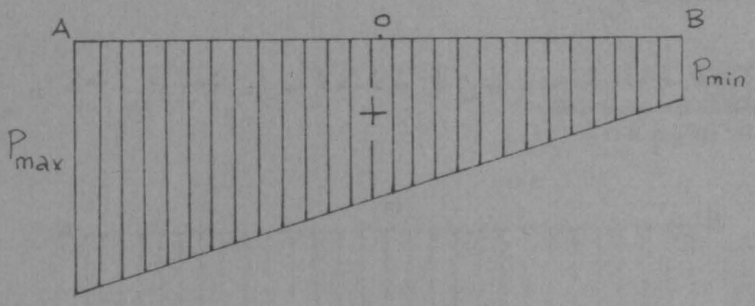
$$P_{max} = \frac{99 + 70}{8 \times 1} + \frac{133.2}{10.65} = 21.1 + 12.5 = +33.6 \ t/m^2$$

$$= \underline{+3.36 \ kg/cm^2} \ (\text{compression})$$

(at point A)

$$P_{min} = 21.1 - 12.5 = +8.6 \ t/m^2 = \underline{+0.86 \ kg/cm^2} \ (\text{compression})$$

(at point B)



b) Rankine Case

Sliding: $F.S. = \frac{(W + P'_{ARV})\mu}{P_{ARH}} = \frac{(99 + 92.8)0.55}{27.4} = \underline{3.85}$

Overturning: Taking moments about point A

$$F.S. = \frac{3.17W + 6.076 P'_{ARV}}{3.33 P_{ARH}} = \frac{3.17 \times 99 + 6.076 \times 92.8}{3.33 \times 27.4} = \underline{9.63}$$

Base Pressure: Taking moments about point O

$$M_o = 0.83W + 3.33 P_{ARH} - 2.076 P'_{ARV}$$

$$M_o = 0.83 \times 99 + 3.33 \times 27.4 - 2.076 \times 92.8 = -18.5 \text{ t-m}$$

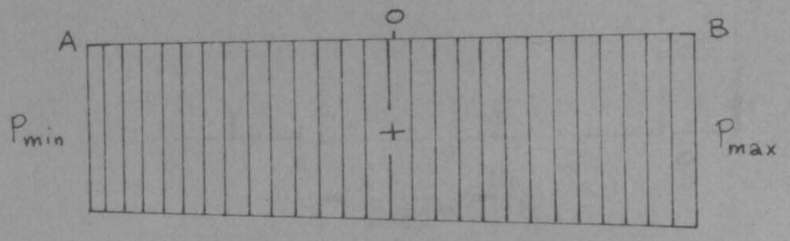
Since we have negative moment then maximum soil pressure occurs at point B, and minimum at point A.

$$P_{max} = \frac{W + P'_{ARV}}{A} - \frac{M_o c}{I}$$

$$P_{max} = \frac{99 + 92.8}{8 \times 1} - \frac{(-18.5)}{10.65} = 24 + 1.74 = +25.74 \text{ t/m}^2$$

$$= \underline{+2.574 \text{ kg/cm}^2} \text{ (compression)}$$
(at point B)

$$P_{min} = 24 - 1.74 = +22.26 \text{ t/m}^2 = \underline{+2.226 \text{ kg/cm}^2} \text{ (compression)}$$
(at point A)



EXAMPLE II)

Given:

$i = 0°$

$\delta = 20°$

$\omega = 0°$

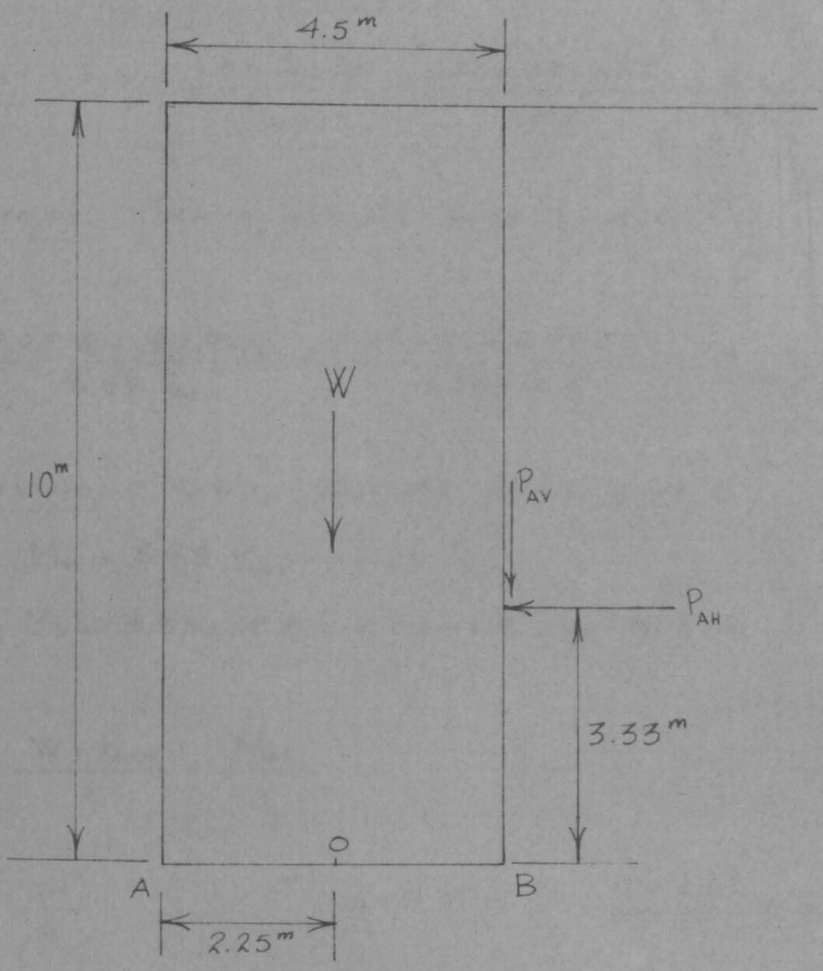
$\phi = 40°$

$h = 10^m$

$\gamma_s = 2\ t/m^3$

$\gamma_c = 2.2\ t/m^3$

$\mu = 0.55$



Solution: Taking unit width

$$W = 10 \times 4.5 \times 2.2 = 99\ t/m$$

Utilizing the tables already prepared we have:

$$P_{ACH} = \gamma h^2 (0.094) = 2 \times 10^2 \times 0.094 = 18.8\ t/m$$

$$P_{ACV} = \gamma h^2 (0.034) = 2 \times 10^2 \times 0.034 = 6.8\ t/m$$

$$P_{ARH} = \gamma h^2 (0.109) = 2 \times 10^2 \times 0.109 = 21.8\ t/m$$

$$P_{ARV} = \gamma h^2 (0) = 0$$

a) Coulomb Case

Sliding: $F.S. = \dfrac{(W + P_{ACV})\,\mu}{P_{ACH}} = \dfrac{(99 + 6.8)\,0.55}{18.8} = \underline{5.63}$

Overturning: Taking moments about point A

$$F.S. = \frac{2.25\,W + 4.5\,P_{ACV}}{3.33\,P_{ACH}} = \frac{2.25 \times 99 + 4.5 \times 6.8}{3.33 \times 18.8} = \underline{4.05}$$

Base Pressure: Taking moments about point O

$$M_o = 3.33\,P_{ACH} - 2.25\,P_{ACV}$$

$$M_o = 3.33 \times 18.8 - 2.25 \times 6.8 = 47.3 \text{ t-m}$$

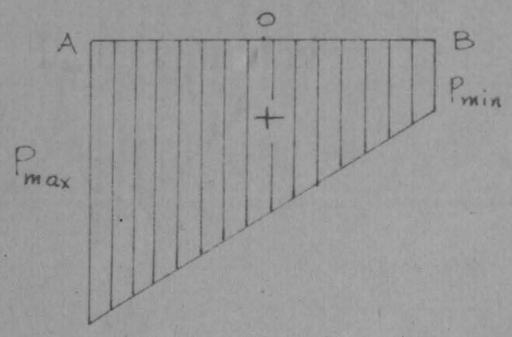
$$P_{max} = \frac{W + P_{ACV}}{A} + \frac{M_o c}{I}$$

$$\frac{I}{c} = \frac{bd^2}{6} \qquad b = 1^m \quad d = 4.5^m \quad \frac{I}{c} = \frac{1 \times 4.5^2}{6} = 3.38$$

$$P_{max} = \frac{99 + 6.8}{4.5 \times 1} + \frac{47.3}{3.38} = 23.5 + 14 = +37.5 \text{ t/m}^2$$

$$= \underline{+3.75 \text{ kg/cm}^2} \text{ (compression)}$$
(at point A)

$$P_{min} = 23.5 - 14 = +9.5 \text{ t/m}^2 = \underline{+0.95 \text{ kg/cm}^2} \text{ (compression)}$$
(at point B)



b) Rankine Case

Sliding: $F.S. = \frac{(W + P_{ARV})\mu}{P_{ARH}} = \frac{(99+0)\,0.55}{21.8} = \underline{2.5}$

Overturning: Taking moments about point A

$$F.S. = \frac{2.25\,W + 4.5\,P_{ARV}}{3.33\,P_{ARH}} = \frac{2.25 \times 99 + 4.5 \times 0}{3.33 \times 21.8} = \underline{3.07}$$

Base Pressure: Taking moments about point O

$$M_o = 3.33\,P_{ARH} - 2.25\,P_{ARV}$$

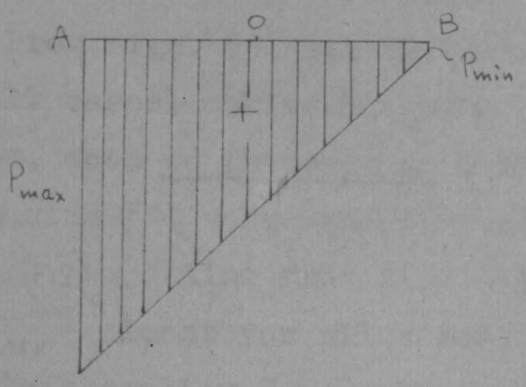
$$M_o = 3.33 \times 21.8 = 72.6 \text{ t-m}$$

$$P_{max} = \frac{W + P_{ARV}}{A} + \frac{M_o\,c}{I}$$

$$P_{max} = \frac{99+0}{4.5 \times 1} + \frac{72.6}{3.38} = 22 + 21.4 = +43.4 \text{ t/m}^2$$

$$= \underline{+4.34 \text{ kg/cm}^2} \text{ (compression)}$$
(at point A)

$$P_{min} = 22 - 21.4 = +0.6 \text{ t/m}^2 = \underline{+0.06 \text{ kg/cm}^2} \text{ (compression)}$$
(at point B)



CONCLUSION

In all the previous calculations only cohesionless soil was considered.

A retaining wall is checked for the following points:

1) Stability against sliding failure.
2) Stability against overturning failure.
3) Stability against excessive base pressure.
4) Overall stability.

We shall consider the first three of the above checks, since overall stability is independent of the calculation method. To begin with, we assume that Coulomb's method gives correct results, and it is a safe method in all cases of retaining wall design.

1) Stability against sliding failure - One way to write factor of safety expression against sliding is as follows:

$$(F.S.)_C = \frac{(W + P_{ACV})\,\mu}{P_{ACH}} \qquad \text{(Coulomb Case)}$$

$$(F.S.)_R = \frac{(W + P'_{ARV})\,\mu}{P_{ARH}} \qquad \text{(Rankine Case)}$$

where μ = coefficient of friction at the base.
W = weight of wall.

From figs. 50, 51, 52 we see that $P_{ARH} < P_{ACH}$ almost for all cases, making $(F.S.)_R > (F.S.)_C$. We obtain larger P_{ACH} values, when we increase ω, and decrease δ. Thus increasing $(F.S.)_R$ value and increasing the difference $(F.S.)_R - (F.S.)_C$. Also from figs. 53, 54, 55 we see that, $P'_{ARV} > P_{ACV}$ almost for all cases, making again $(F.S.)_R > (F.S.)_C$. We obtain smaller P'_{ARV} values when we increase ω, and

increase δ , thus decreasing $(F.S.)_R$ value, and decreasing the difference $(F.S.)_R - (F.S.)_C$. The result will be always $(F.S.)_R > (F.S.)_C$ Since Rankine's method gives greater safety factors it is on the unsafe side. Eventually we can conclude that, for very small and very large values of ω , Rankine's method gives even safer results than Coulomb's method. But, for in between values of ω , Rankine's method gives results which are almost three times on the unsafe side over the results obtained from Coulomb's method. The above argument is valid for these medium values of ω .

2) Stability against overturning failure - Factor of safety against overturning can be shown as :

$$(F.S.)_C = \frac{C_1 W + C_2 P_{ACV}}{C_3 P_{ACH}} \qquad \text{(Coulomb Case)}$$

$$(F.S.)_R = \frac{C_1 W + C_2 P'_{ARV}}{C_3 P_{ARH}} \qquad \text{(Rankine Case)}$$

The same argument can be drawn for overturning failure as in sliding failure, since formulas for factor of safeties are essentially the same. Thus, here also Rankine's method gives greater safety factors. So, we are on the unsafe side again by using Rankine's method for medium values of ω . For very small and very large values of ω , Rankine's method is even safer than Coulomb's method, with respect to overturning failure.

3) Stability against excessive base pressure -

$$(P)_C = \frac{W + P_{ACV}}{A} + \frac{(C_1 W + C_2 P_{ACH} - C_3 P_{ACV})c}{I} \qquad \text{(Coulomb Case)}$$

$$(P)_R = \frac{W + P'_{ARV}}{A} + \frac{(C_1 W + C_2 P_{ARH} - C_3 P'_{ARV})c}{I} \qquad \text{(Rankine Case)}$$

From the above general formulas it is impossible to draw any conclusion about the effect of Rankine's and Coulomb's methods on base pressures. But from many examples solved, and from the two special examples already given before, we can say that for greater values of ω , it is unsafe to use Rankine's method with respect to base pressures since it gives smaller base pressure values than Coulomb's. But, for very small values of ω , Rankine's method again, is even safer than Coulomb's method, since we obtain larger base pressures in this case.

SYMBOLS

a = unit adhesion between wall and earth.

c = unit cohesive strength or resistance of soil.

c' = unit adhesive strength or resistance of soil along a wall surface.

e = normal component of unit earth pressure on wall.

f = tangential component of unit earth pressure on wall.

h = height of wall.

h_o = vertical distance from ground surface of a cohesive soil to a point of zero stress.

H = horizontal component of internal forces in rupture-line.

i = angle of inclination of ground slope.

M^o = moment about center of rupture-circle.

n = F.S. = factor of safety.

$P_a = P_A$ = active earth pressure.

P_p = passive earth pressure.

R = resultant active earth pressure on an inner rupture surface.

R' = resultant passive earth pressure on an inner rupture surface.

r = radius of rupture arc or slip circle.

R^g = component, in a direction making angle g with vertical, of internal forces in rupture line.

s = shear stress, shear strength.

$t = \sigma \sec\varphi$ = resulting stress, exclusive of cohesion, c, in rupture line.

V = vertical component of internal forces in rupture line.

W = weight of soil.

z_o = vertical distance from ground surface of a cohesive soil to a point of zero stress.

α = inner angle of rupture for Rankine's active state of stress.

β = outer angle of rupture for Rankine's active state of stress.

γ = unit weight of soil.

δ = angle of obliquity of active earth pressure.

δ = angle of friction between soil and concrete.

τ = shear stress, shear strength.

σ = normal stress.

ϕ = angle of internal friction or angle of shearing resistance of soil.

$\phi' = \phi_1$ = angle of friction between soil and concrete.

ω = angle between a pressure surface, such as back of a wall and the vertical.

ρ = angle of rupture.

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