DISTRIBUTED CONSENSUS BASED ADAPTIVE RAMP METERING ALGORITHMS IN FREEWAY SYSTEMS

by

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ABSTRACT

DISTRIBUTED CONSENSUS BASED ADAPTIVE RAMP METERING ALGORITHMS IN FREEWAY SYSTEMS

In recent years, traffic management has the attention of many researchers due to the public's increasing demand for fast, efficient, and convenient means of travel. In the literature, there has been a vast amount of research and improvement on traffic management control in freeways in various forms such as optimization, consensus protocols, and nonlinear control. The objective of this thesis is to develop a distributed consensus based ramp metering algorithm with the coordination of the traffic network. The applications of the distributed consensus algorithms in the literature indicate the importance of design and analysis of consensus protocols with which the agents in the systems achieve a common objective by exchanging information. Contrary to existing studies on coordinated ramp metering, centralized and decentralized on-ramp flow control algorithms are proposed without assigning priorities to on-ramps with bottlenecks at the upstream cells. Subsequently, the on-ramp flow control algorithm is improved with a consensus based density control algorithm which provides a smooth traffic density in the traffic infrastructure. At the final step, the mainstream inflow control is utilized in order to achieve the decided on-ramp flow and the traffic density. The averaging based and minimum consensus protocols are used to control the mainstream inflow. The performance and the convergence speed analysis of the proposed algorithms are evaluated with numerical simulations.

ÖZET

ANAYOLLAR İÇİN DAĞITIK ONAYLAŞIM TABANLI UYARLAMALI ANAYOL KATILIM ALGORİTMALARI

Son yıllarda, trafik yönetimi halkın hızlı, verimli ve rahat seyahat etme yollarına olan artan talebi nedeniyle birçok araştırmanın konusu olmuştur. Literatürde, trafik yönetimi kontrolü için optimizasyon, onaylaşım protokolleri ve doğrusal olmayan kontrol gibi çeşitli algoritmalar ile çok sayıda araştırma ve geliştirme yapılmıştır. Bu tezin amacı, trafik altyapısı kapasitesine ilişkin olarak dağıtık onaylaşım tabanlı anayol katılım algoritması geliştirmektir. Literatürdeki dağıtık onaylaşım protokolleri üzerine çalışmalar, etmenlerin arasında bilgi alışverişi yaparak belirli bir ilgi alanı değişkeni üzerindeki onaylaşım ve protokollerin tasarım ve analizinin önemini göstermektedir. Koordineli rampa ölçümü konusundaki mevcut çalışmaların aksine, bağlı olduğu anayol hücresinin sıkışıklık durumuna göre öncelik sağlamayan merkezi ve merkezi olmayan anayol katılım algoritmaları önerilmiştir. Anayol katılım algoritmaları, trafik akışını stabil bir şekilde sağlamak için trafik yoğunluğu kontrol algoritmaları ile birleştirilmiştir. Algoritmanın son adımı olarak karar verilen anayol katılım oranı ve trafik yoğunluğunu elde etmek için anayol trafik akış kontrolü önerilmiştir. Ortalama onaylaşım ve minimum onaylaşım algoritmaları anayol trafik akış kontrolü için kullanılmış, sayısal analizler simulasyonlar ile desteklenmiştir.

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A	The adjacency matrix
a_{ij}	The (i, j)-th element of the matrix A
a_m	The parameter of the fundamental diagram
$\mathcal{E}$	The set of edges
$\mathcal{G}$	A time-invariant graph
K_r	The regulator parameter of ALINEA control
L	The Laplacian matrix
l_i	The length of cell i
l_i^r	The on-ramp length of cell i
$l_i^q(k)$	The queue length at the on-ramp of cell i
$l_{max,i}^q$	The maximum on-ramp queue capacity of cell i
$\mathcal{N}_i$	The neighbor set of agent i
N	The number of cells in the freeway
$N_i(t)$	The maximum number of vehicles that can be present in cell i at time t.
n	The number of agents in the network
$n_i(t)$	The cell occupancy at time t
$o_{in}(k)$	The measured upstream motorway occupancy
o_{crit}	The critical upstream motorway occupancy
$\hat{o}$	The desired upstream motorway occupancy
$p_i(t)$	The number of vehicle in cell i at time t
R	The set of real numbers
$r(k)$	The on-ramp flow
$r_i^d(k)$	The flow demand of the on-ramp at cell i
r_{min}	The minimum on-ramp flow
r_{max}	The maximum on-ramp flow
$\mathcal{V}$	The set of vertices
v	The mean speed of vehicles at the freeway

v_0	The speed of vehicles at the maximum flow and the critical density
v_f	The free flow speed of vehicles at the freeway
$Q_i(t)$	The maximum number of vehicles that can flow into cell i
q	The flow of vehicles at the freeway
q_{cap}	The downstream motorway capacity
q_{max}	The maximum flow of vehicles at the freeway or the capacity of the freeway
$q_{in,i}(k)$	The inflow into cell i
$q_{out,i}(k)$	The outflow from cell i
x	The spatial coordinate in the direction of traffic flow
$x_i(t)$	The the state value of agent i
x_i^a	The maximum allowable mainstream flow in cell i with free flow speed, v_f
$x_{max,i}$	The maximum mainstream flow in cell i
$x_{min,i}$	The minimum mainstream flow in cell i
w_{ij}	The non-negative weighting coefficient
$W(t)$	The system matrix with the weighting coefficients $w_{ij}(t)$
t	The time

Δk	The sampling period
γ	The amount of capacity in freeway system
γ^d	The total desired on-ramp flow
κ	The constant for METANET traffic model
λ_i	The number of lanes on the freeway in cell i
λ_r	The number of lanes on the on-ramp
ρ	The density of the freeway
ρ_i^a	The maximum allowable density of cell i with free flow speed, v_f
ρ_{cr}	The critical density of the freeway
$\rho_{max,i}$	The maximum density of cell i
$\rho_{min,i}$	The minimum density of cell i

ρ_j	The jam density of the freeway
$\rho(T)$	Modulus of matrix T
σ	Switching signal
τ	The time constant
$\theta(t)$	The heading of the agent i
υ	The anticipation constant

LIST OF ACRONYMS/ABBREVIATIONS

2D	Two Dimensional
3D	Three Dimensional
CTM	Cell Transmission Model
DC	Demand - Capacity Strategy
FD	Fundamental Diagram
HERO	Heuristic ramp metering coordination
OCC	Percent - Occupancy Strategy
SWARM	System Wide Adaptive Ramp Metering

1. INTRODUCTION

Traffic management is an important topic that has been studied over the years. There has been a vast amount of research and improvement on traffic management control in freeways in various forms such as optimization, consensus, and nonlinear control. Among these problems, the problem of accomplishing a typical value in a distributed manner, specifically consensus has been investigated [1–10].

Over the last half-century, traffic demand on freeways and connected ramps to them are increased due to public demand, growing population, and increased vehicle ownership. The consistently growing demand, length of queue, and formation of traffic jam on freeways result in an increase in total travel time, emission pollutant release, and decrease in overall traffic safety. In consideration of this demand change, traffic management problem should also overcome the longer delay time and more critical safety problems as stated in [11]. The objective of control problems is to maintain optimal utilization of the freeway capacity without causing any traffic jam instead of increasing the capacity of the infrastructure. Traffic management on freeway entrance and leaving ramps known as ramp management is defined in [12] as an application to regulate the number of vehicles entering or leaving the freeway by using control devices, such as traffic lights, signing, and gates in order to maintain the optimal capacity, prevent the congestion, and achieve traffic safety objectives.

Ramp metering was introduced in the 1960s to manage traffic flow entering the freeways as an output of several types of research and studies performed on the supply-demand relationship in the form of demand and capacity of the freeway. In 1963, the first ramp meters were deployed on Eisenhower Expressway at Chicago and ramp meters were manually controlled by allowing one vehicle to enter at a predetermined rate, and time with a direction of traffic officers [13]. Today, ramp metering technology is improved and evolved. The authors of [14] investigate the results of with and without ramp metering cases in the Twin Cities. Performance measurements of the research consist of seven parameters such as; accessibility, equity, productivity, consumer surplus, travel time variation, and demand responses. It is found that ramp meters have positive effect on traffic safety and provide improvement on freeway traffic flow. However, total travel time improvement is observed only in long distance trips.

Papageorgiou and Kotsialos [15] overview the ramp metering algorithms and explain the reasoning behind the substantial improvement of traffic conditions on the freeway when a ramp metering algorithm is applied. It is emphasized that the implementation cost of the ramp metering system is a relatively low price to pay when it is compared to the expected and proven benefits in free flow, traffic safety, environmental pollution, and total travel time.

In [16], the main advantages of ramp metering are defined as reduction of accidents and consequently less congested roads, and smooth freeway traffic flow. Ramp metering breaks up vehicle platoons formed at the on-ramp; therefore, merging to freeway occurs more efficiently. Fuel consumption and release of emission pollutants are also reduced. In [17], besides the advantages of ramp metering, the disadvantages are also mentioned. Traffic diversion is one of the disadvantages because of the route change demand to avoid the ramp meter. Secondly, ramp meter can not prevent congestion; however, congestion location may be shifted to another location in the freeway system. Therefore, location change can cause an equity problem between the drivers. Most importantly, if the ramp metering algorithm is not adjusted and installed properly by considering all possible scenarios, traffic in the freeway may have a worse condition than without the ramp metering case.

In [18], the ramp metering control is divided into two groups according to the real time data usage and the ramp metering response. The first group is fixed-time control which does not require the real time measurement of the traffic conditions. It is based on historical demand data. The second group is defined as traffic responsive control. On the contrary to the first group, traffic responsive control needs real time measurements of the traffic conditions. Therefore, it requires additive signaling adjustments. In addition to classification based on data collection, the ramp metering is also divided into three sub-systems according to the installation type. If the traffic flow into the freeway is controlled via only one ramp meter, it is defined as local ramp metering which is also named as isolated systems. In isolated systems, each ramp meter manages its traffic flow without considering other ramp meters and their allowed flow. Another group is coordinated ramp metering or system-wide in which traffic flow is controlled with several ramp meters and metering rate of the on-ramps are determined by considering the traffic conditions. The last group is integrated systems, which uses several ramp metering installations and other traffic control applications such as route guidance and signal timings.

1.1. Motivation of the Thesis

In this thesis, coordinated ramp metering with traffic responsive control is investigated. The main goal of this thesis is to improve the demand - capacity algorithm by adding density estimation with a consensus algorithm and determine the ramp metering rate by achieving consensus between the on-ramps. The study presented in [8] is improved by adding the maximum allowable capacity at the on-ramps. The upper bound on the traffic capacity demand is explicitly investigated. In this way, the freeway instability and the congested conditions are not allowed. In the presence of continuous demand coming from the on-ramp, the maximum and optimum capacity both on freeway and on-ramps are investigated; therefore, the demand at the on-ramps is updated with an online algorithm. In addition to the improvement on the algorithm in [8], the centralized on-ramp control is also provided. After coordinating the on-ramp flow, the density control is performed by using consensus based algorithms which are based on centralized and decentralized approaches. Therefore, the density requirement on each cell is updated and the density distribution in each cell is arranged. By this way, the on-ramp flow control and the density control are achieved with a consensus algorithm instead of solving nonlinear equations.

1.2. Contribution of the Thesis

The main contributions of this thesis are summarized as follows:

- Three novel algorithms to be used in the traffic control of on-ramp, density, and mainstream inflow of freeway systems are proposed. The proposed control algorithms are inspired by consensus algorithms.
- The on-ramp flow control is performed with centralized and decentralized approaches. The performance of the proposed algorithm is further verified by numerical simulations.
- The density estimation and control are introduced in addition to the on-ramp flow control. The centralized and the decentralized approaches used in power distribution are modified and applied to traffic density control.
- The performance of the mainstream inflow control algorithm is investigated by using three different weighting matrices. The algorithm is shown to achieve convergence to a common value and the convergence speed depends on the weighting matrix selection.

1.3. Organization of the Thesis

The remainder of this thesis is organized as follows. In Chapter 2, mathematical notation and graph theory related information that will be used throughout the thesis are given. Then, distributed consensus protocols in continuous and discrete time are reviewed. Chapter 2 also states the distributed average consensus with different weighting algorithms. In Chapter 3, traffic flow theory and its evolution to different traffic models are investigated and presented. Also existing ramp metering methods are explained. Further in this chapter, adaptive metering techniques are examined in detail. Chapter 4 provides on-ramp meter algorithms based on distributed consensus. The problem of ramp metering control is analyzed with both centralized and decentralized approaches. In Chapter 5, the traffic density control and coordination are discussed and illustrated with an example. Chapter 6 introduces the mainstream inflow control algorithm with consensus based approaches to achieve the density and the on-ramp flow. Finally, there are some concluding remarks and outcomes of this thesis in Chapter 7.

2. OVERVIEW OF DISTRIBUTED CONSENSUS

In this introductory chapter, the basic concepts of graph theory which includes the definitions of the adjacency matrix, the Laplacian, connectivity and the spanning tree are reviewed. After presenting the consensus algorithm in the continuous and the discrete time, the average consensus, maximum, and minimum consensus are introduced.

2.1. Review of Graph Theory

A graph is represented as $\mathcal{G} = (\mathcal{V}, \mathcal{E})$ where $\mathcal{V} = \{v_1, v_2, ..., v_n\}$ is a set of vertices indexed by the set $I = \{1, 2, ..., n\}$ and $\mathcal{E} \subseteq \mathcal{V}$ x $\mathcal{V}$ corresponds to a set of edges representing the links between two vertices. More specifically, if node j receives information from node i, it means that (v_i, v_j) exists.

The graph is said to be undirected if $(v_i, v_j) \in \mathcal{E}$ also implies $(v_j, v_i) \in \mathcal{E}$. In a directed graph, the information flow direction is important. The information flow can be explained as (v_i, v_j) refers to information from node i to node j and there is an edge from v_i to v_j. When two vertices are connected by an edge, the vertices are considered as endpoints and the edge is defined as incident to the vertices. If vertices are not connected to any other edges, they are not defined as endpoints. If vertex v_i is the endpoint of the same edge with vertex v_j, vertices v_i and v_j are said to be adjacent to each other. For an undirected graph, the neighbors of vertex v_i are defined as the adjacent vertices of the vertex v_i. For a directed graph, the neighbors of vertex v_i are given by a set $\mathcal{N}_i$ which is defined as $\mathcal{N}_i = \{v_j : (v_i, v_j) \in \mathcal{E}\}$.

Connectivity of a graph is an essential concept. An undirected graph is said to be connected when there is a path between every pair of vertices. If the graph has multiple disconnected vertices and edges, then the graph is said to be disconnected. For an undirected graph, connectivity implies the existence of a spanning tree [20].

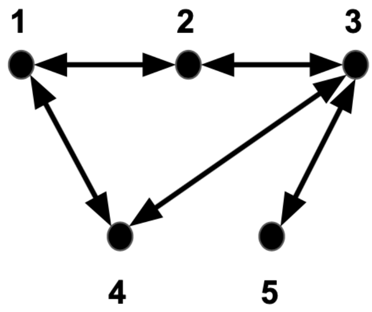

Figure 2.1. An undirected graph with 5 vertices and 10 edges.

The information transferred between the vertices of a graph is described by the adjacency and Laplacian matrices.

Definition 2.1. *The adjacency matrix $A = [a_{ij}]$ of a graph $\mathcal{G}$ has elements which are defined as*

$$a_{ij} = \begin{cases} 1, & \text{if } (v_i, v_j) \in \mathcal{E} \text{ and } i \neq j \\ 0, & \text{otherwise} \end{cases}$$

The adjacency matrix, A, is a non-negative matrix.

Definition 2.2. *The Laplacian $L = [l_{ij}]$ of a graph $\mathcal{G}$ is defined as*

$$l_{ij} = \begin{cases} \sum_{i=1, i\neq j}^{n} a_{ij}, & \text{if } j = i \\ -a_{ij}, & \text{if } j \neq i \end{cases}$$

where a_{ij}'s are the elements of the adjacency matrix.

By Definitions 2.1 and 2.2, if the graph $\mathcal{G}$ is undirected, it means that A and L are symmetric matrices, i.e, $A = A^T$ and $L = L^T$.

Example 2.1. *Consider the undirected graph illustrated in Figure 2.1. The adjacency and Laplacian matrices associated with the graph are given by*

$$A = \begin{pmatrix} 0 & 1 & 0 & 1 & 0 \\ 1 & 0 & 1 & 0 & 0 \\ 0 & 1 & 0 & 1 & 1 \\ 1 & 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 \end{pmatrix}, \quad L = \begin{pmatrix} 2 & -1 & 0 & -1 & 0 \\ -1 & 2 & -1 & 0 & 0 \\ 0 & -1 & 3 & -1 & -1 \\ -1 & 0 & -1 & 2 & 0 \\ 0 & 0 & -1 & 0 & 1 \end{pmatrix}$$

2.2. Distributed Consensus Protocol

The consensus problem is defined as an agreement among a group of agents for a single value by exchanging information between the agents. The consensus problem is one of the fundamental problems in the control of multi-agent systems.

The objective of consensus is to agree on a common value between the agents such as

$$\lim_{t\to\infty} \mathbf{x}(\mathbf{t}) = c\mathbf{1} \tag{2.1}$$

where c denotes the common value of the agents and $\mathbf{1}$ is the vector of all ones as $\mathbf{1} = [1, ..., 1]^T$.

The consensus problem is investigated in continuous and discrete time according to the model of the agents dynamics.

2.2.1. Continuous Time Consensus Algorithm

Given a system with node dynamics $\dot{x}_i(t) = u_i(t)$, a simple continous time consensus algorithm is given in [20] as follows:

$$\dot{x}_i(t) = \sum_{j \in N_i} a_{ij} \left(x_j(t) - x_i(t) \right), \ i = 1, 2, ..., n \tag{2.2}$$

where N_i is the set of neighbors and $a_{ij}(t)$ is the (i, j)-th entry of the corresponding network matrix at time t. By using the Laplacian, (2.2) can be expressed in matrix form as

$$\dot{\mathbf{x}}(t) = -\mathbf{L}\mathbf{x}(t). \tag{2.3}$$

One of the important properties of the Laplacian matrix for the analysis of a consensus algorithm is the zero sum of its rows. Since the sum of each row of Laplacian, $\mathbf{L}$, is zero, $\mathbf{L}$ has always a zero eigenvalue with an eigenvector $\mathbf{1} = [1, ..., 1]^T$.

2.2.2. Discrete Time Consensus Algorithm

In [21], a discrete-time model is introduced for autonomous agents by Vicsek et.al. In the Vicsek's model, heading of each agent is updated based on a local rule which is the average of the agent's own heading plus the headings of neighbors. Neighbors of an agent are defined as agents which are inside a circle centered at the agent's position with a pre-specified radius. In [22], a linearized version of Vicsek's model is given by

$$\theta_i(t+1) = \frac{\theta_i(t) + \sum_{j \in \mathcal{N}(t)} \theta_j(t)}{1 + n_i} \tag{2.4}$$

where $\theta_i(t)$ is the heading of agent i, $\mathcal{N}_i$ is the set of neighbors of agent i, and n_i is the number of agents in $\mathcal{N}_i$.

The Vicsek's model is a special form of a more general discrete time distributed consensus algorithm expressed as

$$x_i(t+1) = w_{ii}(t)x_i(t) + \sum_{j \in \mathcal{N}_i} w_{ij}(t)x_j(t) \tag{2.5}$$

where $x_i(t)$ is the state value of agent i, $w_{ij}(t)$ is the non-negative weighting coefficient, and $\mathcal{N}_i$ is the set of neighbors. The update rule for the n agent network can be represented in matrix form as

$$x(t+1) = W(t)x(t) \tag{2.6}$$

where $x(t) = [x_1(t), ..., x_n(t)]^T \in \mathbb{R}^n$ is the vector containing state values of the agents and $W(t) \in \mathbb{R}^{n \times n}$ is the system matrix with the weighting coefficients $w_{ij}(t)$. Throughout this thesis, the following assumptions will be adapted [21–25].

Assumption 2.1. *(Neighbor Weighting) Information coming from the neighbors and other agents is defined as*

$$w_{ij}(t) = \begin{cases} \delta, & \textit{if } (v_i, v_j) \in \mathcal{E} \\ 0, & \textit{if} (v_i, v_j) \notin \mathcal{E} \end{cases}$$

where δ is a positive quantity which satisfies $\delta \in (0, 1]$.

Assumption 2.2. *(Total Weights) The weights, $w_{ij}(t)$ should sum up to 1 such that*

$$\sum_{j=1}^{n} w_{ij}(t) = 1, \ \forall i \in I, \forall t \in N.$$

The system in (2.6) is said to achieve consensus if there exists a common value, c, such that (2.1) is satisfied. We further say that average consensus is achieved if

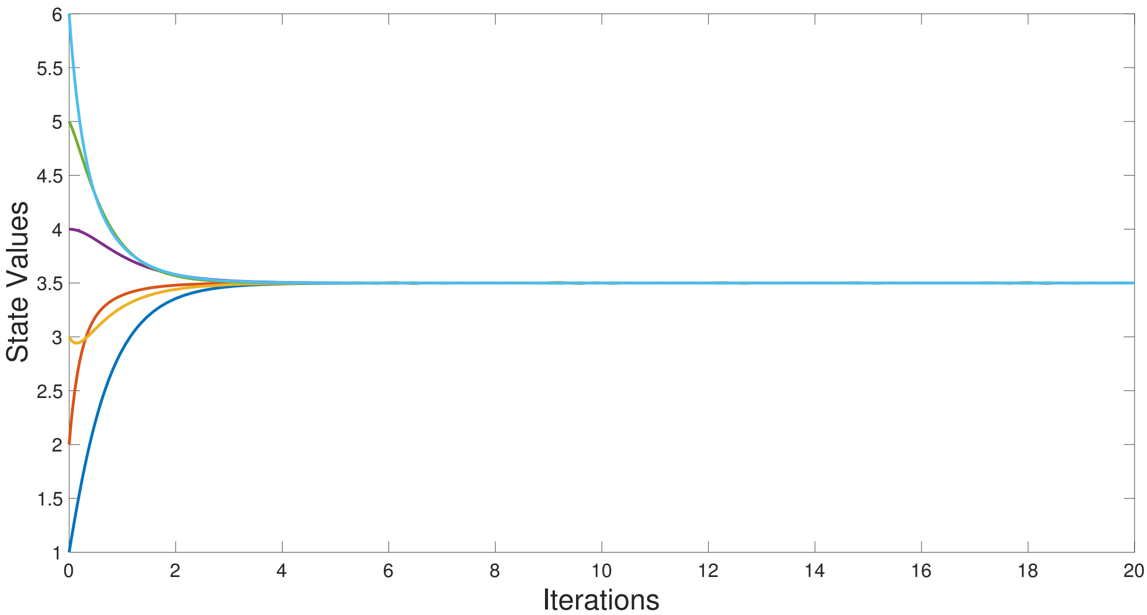

Figure 2.2. Consensus algorithm example

consensus value is equal to the average of the initial conditions such that

$$c = \frac{1}{n}\sum_{i=1}^{n} x_i(0). \tag{2.7}$$

Figure 2.2 shows the state changes for 6 agents ensuring the objective defined in (2.7). The agents start from a random initial state given as $x(0) = [1, 2, 3, 4, 5, 6]^T$ and adjust their state values based on the information received from other agents to achieve consensus.

2.3. Weighting Matrix Selection

In [26] and [27], the problem of finding the proper weighting matrix and the linear iterations which can provide the fastest convergence in a distributed consensus protocol are investigated. For a given multi-agent system, determining the weighting matrix that yields the fastest convergence is not an easy process to handle. In this section, three most used heuristic methods in the literature to construct weighting matrices are investigated.

2.3.1. The Maximum Degree Weights

The maximum degree weighting is based on global knowledge on the number of neighbours of the agents. The weighting matrix is given in [26] as

$$w_{ij}(t) = \begin{cases} \frac{1}{\mathcal{N}_{max}}, & \text{if } (v_i, v_j) \in \mathcal{E}(t) \\ \frac{\mathcal{N}_i}{\mathcal{N}_{max}}, & \text{if } v_i = v_j \\ 0, & \text{otherwise} \end{cases} \tag{2.8}$$

where $\mathcal{N}_{max}$ is the maximum value of the degree of vertices. Because of the global knowledge requirement, the maximum degree model may be unfeasible for large scale multi-agent systems.

2.3.2. The Local Degree Weights

The local degree weighting method uses the information coming from the neighbors of the agents and the weight matrix is defined in [27] as

$$w_{ij}(t) = \begin{cases} \frac{1}{\max\{\mathcal{N}_i, \mathcal{N}_j\}}, & \text{if } (v_i, v_j) \in \mathcal{E}(t) \\ 1 - \sum_{j \in \mathcal{N}_i} w_{ij}(t), & \text{if } v_i = v_j \\ 0, & \text{otherwise} \end{cases} \tag{2.9}$$

2.3.3. The Metropolis-Hasting Weights

The Metropolis-Hasting weights are improved according to the nearest neighbor rule proposed in [21] is given by

$$w_{ij}(t) = \begin{cases} \frac{1}{1+\mathcal{N}_i}, & \text{if } (v_i, v_j) \in \mathcal{E}(t) \text{ or } v_i = v_j \\ 0, & \text{otherwise} \end{cases} \tag{2.10}$$

However, the nearest neighbor rule will not preserve the average of the initial values [22]. Therefore, Metropolis-Hasting weights are defined in [28] as

$$w_{ij}(t) = \begin{cases} \frac{1}{1+\max\{\mathcal{N}_i,\mathcal{N}_j\}}, & \text{if } (v_i, v_j) \in \mathcal{E}(t) \\ 1 - \sum_{j\in\mathcal{N}_i} w_{ij}(t), & \text{if } v_i = v_j \\ 0, & \text{otherwise} \end{cases} \tag{2.11}$$

2.4. Maximum and Minimum Consensus Algorithms

The main objective of the maximum or minimum consensus algorithm is to find common value, c in (2.1), according to the maximum and minimum of the initial states.

2.4.1. Maximum Consensus Algorithm

The main objective of the maximum consensus is to achieve an agreement on the largest value of the initial states. One of the most common problem, where the maximum consensus is used, is to make a decision on the minimum time for a meeting. In that problem, each agent needs a minimum time to reach the meeting point. The only possible common value for all of the agents is to get an agreement on the largest value of all the possible meeting times. A maximum consensus algorithm is given in [29] by

$$x_i(t+1) = \max_{j\in\mathcal{N}_i}\{x_j(t)\} \text{ for } i = 1, 2, ..., n. \tag{2.12}$$

For a given directed graph, $\mathcal{G}$, maximum consensus said to be achieved if there exists an integer l such that

$$\begin{aligned} x_i(t) &= x_j(t) \\ &= \max\{x_1(0), ..., x_n(0)\} \text{ for } t \geq l,\ i, j = 1, 2, ..., n. \end{aligned} \tag{2.13}$$

If (2.13) holds for all initial conditions of $\mathbf{x}(\mathbf{0})$, then it is said that strong max-consensus is achieved. If (2.13) is achieved only for a subset of all possible initial states, $\mathbf{x}(\mathbf{0})$, then the consensus is defined as weak max-consensus.

2.4.2. Minimum Consensus Algorithm

The definition of the minimum consensus is similar to the maximum consensus algorithm. A minimum consensus algorithm is defined as

$$x_i(t+1) = \min_{j \in \mathcal{N}_i}\{x_j(t)\} \text{ for } i = 1, 2, ..., n. \tag{2.14}$$

For a given directed graph, $\mathcal{G}$, minimum consensus said to be achieved if there exists an integer l such that

$$\begin{aligned} x_i(t) &= x_j(t) \\ &= \min\{x_1(0), ..., x_n(0)\} \text{ for } t \geq l,\ i, j = 1, 2, ..., n. \end{aligned} \tag{2.15}$$

If (2.15) is true for all $\mathbf{x}(\mathbf{0})$, then strong min-consensus is said to be achieved. If (2.15) is achieved only for a subset of all possible initial states, $\mathbf{x}(\mathbf{0})$, then the consensus is referred as weak min-consensus.

2.5. Summary of the Chapter

The problem considered in this thesis, namely the on-ramp flow control and the density control, will be formulated with a graph notation; therefore, some fundamentals of graph theory are reviewed in this chapter. The proposed algorithms in the proceeding chapters are inspired by the consensus algorithms which are introduced briefly and illustrated with an example in this chapter.

3. TRAFFIC FLOW THEORY

In this chapter, traffic flow theory which is important in designing a ramp metering algorithm is explained. Then, several types of traffic flow models are discussed and the most suitable traffic model for this thesis is introduced. Next in proceeding sections, existing ramp metering control algorithms and applications are presented.

3.1. Overview of Traffic Flow Theory

The objective of traffic flow theory is to understand the relationship and interactions between vehicles, drivers, and the infrastructure in a mathematical way. In 1930s, the scientific studies on traffic flow began with the application of probability theory by Greenshields et.al. in [30]. Traffic flow theory is described with three main variables; flow, density, and speed:

- Flow, q, is the total number of vehicles that are passing a certain point during a defined time period, (veh/h).
- Density, ρ, is the total number of vehicles that are occupying a certain space during a defined time period, (veh/km).
- Speed, v, is the travel rate of vehicles, (km/h).

In [30], the relationship between the speed and the traffic density is investigated by using the photographic measurement of the three main variables. A linear relationship between the speed and the density is proposed as shown in Figure 3.1. In order to express the density in vehicles per hour, Greenshields used an equation, ($density \times speed$), which describes the "traffic flow" in (veh/h). The linear relationship between the speed and the density in Figure 3.1 is converted to a parabolic relationship between the speed and the traffic flow as shown in Figure 3.2 and the diagram is called as "q-v diagram". In the q-v diagram, it is seen that there are two different regions such that same traffic flow indicates two different speeds. Therefore, the traffic flow is classified in two different states such as stable (or uncongested) and unstable (or congested).

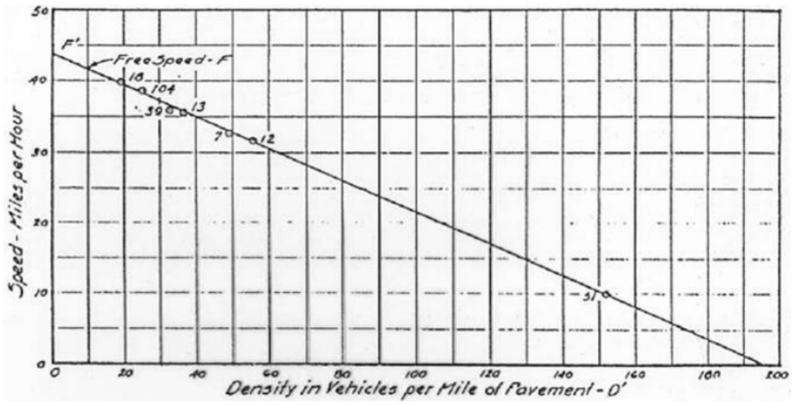

Figure 3.1. Speed and density relation in [30]

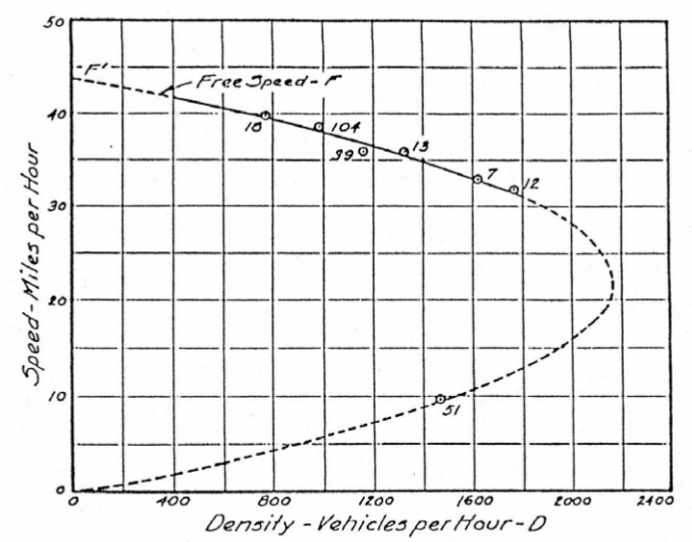

Figure 3.2. Speed and traffic flow relation as q-v diagram in [30]

The fundamental equation of traffic flow which Greenshields uses to describe the flow is given in by

$$q = \rho v. \tag{3.1}$$

From (3.1), it is seen that if a relationship between any of the two variables of the traffic flow is known, then the relationship of the third variable can be established and controlled. The relationship between the flow, q, and the density, ρ, is defined as "fundamental diagram of traffic theory" in [31]. The fundamental diagram is important in understanding the traffic characteristics of a freeway.

Since Greenshields assumed a linear relationship between the speed and the density, the equation is derived as

$$v = v_f - \frac{v_f}{\rho_j}\rho \tag{3.2}$$

where v is the mean speed at the density ρ, v_f is defined as a free flow speed, and ρ_j is the jam density that corresponds to the maximum density in which there is no space between the vehicles. In (3.2), it is clearly seen that if the density becomes zero, the mean speed will approach to the free flow speed. By using (3.1) and (3.2), the relationship between the flow and the density can be derived as

$$q = v_f\rho - \frac{v_f}{\rho_j}\rho^2. \tag{3.3}$$

From (3.3), it is seen that flow will reach its maximum value and then it will start to decrease. When the density reaches a maximum value, ρ_j, flow will be zero. The maximum value of flow, q_{max}, is referred to the capacity of the freeway and the density corresponding to q_{max} is defined as the critical density, ρ_{cr}.

Similarly, the relationship between the flow and the speed is obtained by substituting $\rho = \frac{q}{v}$ into (3.3) as

$$q = \rho_j v - \frac{\rho_j}{v_f}v^2. \tag{3.4}$$

The relationship in (3.4) has a parabolic characteristics as shown in Figure 3.2.

The boundary conditions of the traffic model are the jam density, ρ_j, the maximum flow, q_{max}, and the free flow speed, v_f. Since the relationship between the density and the flow also has a parabolic relationship, the density at the maximum flow can be derived by differentiating (3.3) with respect to the density and equating it to zero such as

$$\frac{\partial q}{\partial \rho} = v_f - \frac{v_f}{\rho_j} 2\rho = 0. \tag{3.5}$$

Since the critical density is defined as the density at the maximum capacity, the critical density is equal to the half of the jam density as

$$\rho_{cr} = \frac{\rho_j}{2}. \tag{3.6}$$

The maximum capacity, q_{max}, can be derived by using (3.3), (3.4), and (3.5) as

$$q_{max} = v_f \rho_{cr} - \frac{v_f}{\rho_j} \rho_{cr}^2 = \frac{v_f \rho_j}{4}. \tag{3.7}$$

Furthermore, the speed at the maximum capacity can be found by substituting the critical density into (3.2) as

$$v_0 = v_f - \frac{v_f}{\rho_j} \rho_{cr} = \frac{v_f}{2}. \tag{3.8}$$

In [32], Pipes proposed a new model for the speed by introducing a new parameter, a, to form a more generalized modeling approach based on the Greenshields traffic model. In the Pipes model, the speed and the density relation is modified as

$$v = v_f \left(1 - \left(\frac{\rho}{\rho_j} \right)^a \right). \tag{3.9}$$

Equation (3.9) indicates that if a is set to one, the Pipes model gives the same results with the Greenshields model. By varying the values of a, the traffic model can be modified according to the traffic infrastructure.

In [33] and [34], the traffic model is improved with the help of fluid dynamics differential equations by considering traffic characteristics as a one dimensional compressible fluid. The model, named as the "LWR Model", is based on the conservation law on the vehicles on the freeway. As in the previous works, the traffic flow is related to the traffic density. By assuming that the traffic flow direction is from left to right, the continuity equation of the LWR model can be expressed as

$$\frac{\partial \rho(x,t)}{\partial t} + \frac{\partial q(x,t)}{\partial x} = 0 \tag{3.10}$$

where x is the spatial coordinate in the direction of the traffic flow and t is the time. In (3.10), there are two unknown variables, the density, $\rho(x,t)$, and the flow, $q(x,t)$, which depend on the x and t . Since in the LWR model, the flow, $q(x,t)$, is assumed to be related to the density, $\rho(x,t)$, (3.10) has only one unknown variable, $\rho(x,t)$, and can be rewritten as

$$\frac{\partial \rho(x,t)}{\partial t} + \frac{\partial q(\rho(x,t))}{\partial x} = 0. \tag{3.11}$$

In [35], Greenberg et.al. defines a logarithmic relationship between the speed and the density by using the hydrodynamic analogy in the LWR model as

$$v = v_0 \ln \frac{\rho_j}{\rho}. \tag{3.12}$$

The major drawback of (3.12) is that the speed will tend to infinity when the density is zero. In order to overcome the limitation of (3.12) at the lower densities, an exponential model is proposed in [36] by Underwood et.al. as

$$v = v_f \exp\left(-\frac{\rho}{\rho_{cr}}\right). \tag{3.13}$$

In this model, the speed becomes zero only if the density reaches infinity, which creates the major drawback of this model.

In [38], a discrete time differential equation of traffic model, i.e., the cell transmission model, is developed in the form of difference equations. The freeway is divided into homogeneous road sections which are referred to cells. The length of the sections, denoted by $l_i(t)$, is determined by the distance travelled by a vehicle under the uncongested state with the free flow speed in a sample time. There are two variables, $N_i(t)$ and $Q_i(t)$, related to each cell defined as follows:

- $N_i(t)$ = the maximum number of vehicles that can be present in cell i at time t such that

$$N_i(t) = \rho_i(t) l_i(t). \tag{3.14}$$

- $Q_i(t)$ = the maximum number of vehicles that can flow into cell i (also called as the capacity of cell i).

The recursive relationship of the cell transmission model is defined as

$$p_i(t+1) = p_{i-1}(t) \text{ where } i = 1, ..., N \tag{3.15}$$

where $p_i(t)$ is the number of vehicles in cell i at time t and N is the number of cells. In the derivation of (3.15), it is assumed that all vehicles in cell i can transfer to cell $i+1$, however when the flow from cell i to $i+1$ exceeds the capacity of cell $i+1$, the use of (3.15) is not reasonable. Therefore, the flow adjustment of cell transmission model is formed as

$$n_i(t+1) = n_i(t) + q_i(t) - q_{i+1}(t) \tag{3.16}$$

where $n_i(t)$ is the cell occupancy at time t, and $q_i(t)$ is the inflow from cell $i-1$ to i at time t. The flow $q_i(t)$ is defined as

$$q_i(t) = \min[n_{i-1}(t), Q_i(t), N_i(t) - n_i(t)] \tag{3.17}$$

where $n_{i-1}(t)$ is the number of vehicles in cell $i-1$ at time t, $Q_i(t)$ is the capacity flow into i for time interval $(t-1, t]$, and $N_i(t) - n_i(t)$ is the amount of empty space in cell i at time t.

The discretization of the LWR model based on the cell transmission model with time step Δk is obtained as

$$\rho_i(k+1) = \rho_i(k) + \frac{\Delta k}{l_i \lambda_i}[q_{in,i}(k) - q_{out,i}(k)] \tag{3.18}$$

where $\rho_i(k)$ is the traffic density in cell i in time period k, l_i is the length of the cell i, λ_i is the number of lanes in cell i, $q_{in,i}(k)$ is the inflow into cell i, and $q_{out,i}(k)$ is the outflow from cell i.

In [39], the METANET traffic model in which the speed of the vehicle is a function of the traffic density at a location x, is developed and simulated. The main difference between the previous defined models and the METANET model is the definition of the vehicle speed. The speed is defined as a function of the density by Greenshields, Pipes, Greenberg, and Underwood in (3.2), (3.9), and (3.13). However, in the METANET model, the speed formulation is proposed as

$$\begin{aligned} v_i(k+1) =& v_i(k) + \frac{\Delta k}{\tau}(V[\rho_i(k)] - v_i(k)) + \frac{\Delta k}{l_i}(v_{i-1}(k) - v_i(k))v_i(k) \\ &- \frac{\upsilon \Delta k}{\tau l_i} \frac{\rho_{i+1}(k) - \rho_i(k)}{\rho_i(k) + \kappa} \end{aligned} \tag{3.19}$$

where τ is the time constant, υ is the anticipation constant, κ is the constant which are same for cell links and represent the characteristics of the freeway, and $V[\rho_i(k)]$ is free flow speed defined as

$$V[\rho_i(k)] = v_f \exp\left[-\frac{1}{a_m}\left(\frac{\rho_i(k)}{\rho_{cr}(k)})\right)^{a_m}\right] \tag{3.20}$$

where a_m is the parameter of the fundamental diagram.

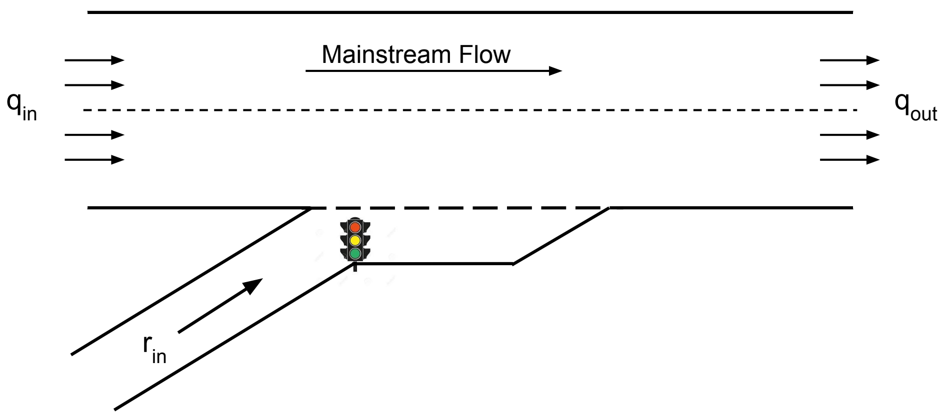

Figure 3.3. Ramp metering illustration

3.2. Ramp Metering Algorithms

In Chapter 3.1, the traffic models in the literature are introduced and the fundamental equation to derive a proper traffic model is provided. In this section, the ramp meter control algorithms are provided.

Ramp metering is the operation of controlling the traffic flow entering a freeway by using traffic signals on the ramps, as shown in Figure 3.3. As stated in Chapter 1, there are three main types of installing a ramp meter control such as local, coordinated, and integrated. The main logic behind all types is to prevent the freeway from bottlenecks formed because of the exceeding capacity of the freeway. In local ramp metering, the control algorithm is applied only to the on-ramp, where the bottleneck occurs upstream cell of the on-ramp. On the other hand, in coordinated ramp metering, several ramp meters on the freeway are controlled. Finally, integrated ramp meter control combines the ramp meters with other traffic information measurements, such as variable speed limits, route guidance, and signal timings. In the rest of this section, a summary of the ramp meter control algorithms is provided.

3.2.1. Local Ramp Metering

A local ramp metering control algorithm can improve the current traffic situation by determining the inflow from the on-ramp to the freeway in the existence of a bottleneck. In [40], it is stated that traffic condition measurements in the vicinity of the related on-ramp are used to regulate the local ramp metering strategies. Some of the important local ramp metering control techniques are discussed below.

3.2.1.1. Demand-Capacity Strategy. In [41], Demand-Capacity (DC) control is defined as a feedforward traffic sensitive algorithm. The DC algorithm coordinates the ramp meter flow according to the freeway traffic condition of the upstream of the on-ramp. In the DC algorithm, the ramp flow is equal to the difference between the downstream capacity and the upstream flow. The DC strategy is formulated in [42] as

$$r(k) = \begin{cases} q_{cap} - q_{in}(k-1), & \text{if } o_{in}(k-1) \leq o_{crit} \\ r_{min}, & \text{otherwise} \end{cases} \tag{3.21}$$

where $r(k)$ is the on-ramp flow in veh/h, q_{cap} is the specified capacity of the freeway downstream of the on-ramp in veh/h, $o_{in}(k-1)$ is the upstream motorway occupancy during the previous time period in percentages, o_{crit} is the critical motorway occupancy, and r_{min} is the pre-specified minimum on-ramp flow. The DC algorithm limits the on-ramp flow to the minimum on-ramp flow, r_{min}, if the occupancy of the upstream cell exceeds the critical level.

3.2.1.2. Percent-Occupancy Strategy. The Percent-Occupancy ramp metering strategy, also known as OCC, assumes a linear relationship between the flow and the occupancy. In [40], the upstream freeway flow is calculated from the fundamental diagram of the traffic model as

$$q_{in}(k) = \frac{v_f o_{in}(k-1)}{g} \tag{3.22}$$

where v_f is the free flow speed, $o_{in}(k-1)$ is the last measured upstream freeway occupancy, and g is a factor depending on vehicle length and effective detector length. By using (3.21) and (3.22), the ramp metering rate of OCC strategy is given by

$$r(k) = q_{cap} - \frac{v_f}{g} o_{in}(k-1). \tag{3.23}$$

3.2.1.3. RWS Strategy. The RWS strategy uses the speed of the traffic on the freeway in addition to the flow and the on-ramp capacity. In [43], the RWS metering rate is obtained as

$$r(k) = q_{cap} - I(k-1) \tag{3.24}$$

where q_{cap} is the pre-specified capacity of the upstream freeway of the on-ramp and $I(k-1)$ is the upstream freeway flow. The next calculation of the on-ramp flow in RWS system is performed according to a cycling time defined as

$$t = \frac{\lambda_r \times 3600}{r(k)} \tag{3.25}$$

where λ_r is the number of lanes on the on-ramp.

3.2.1.4. ALINEA. In [44], ALINEA is proposed as an occupancy based feedback control strategy for the on-ramp control. ALINEA control strategy is based on the difference between the ideal and the current occupancy of the motorway as

$$r(k) = r(k-1) + K_r(\hat{o} - o_{in}(k-1)) \tag{3.26}$$

where K_r is a regulator parameter, $\hat{o}$ is the desired downstream occupancy, and $o_{in}(k-1)$ is the measured downstream occupancy.

3.2.2. Coordinated Ramp Metering

The main objective of coordinated ramp metering algorithms is to assign a metering rate to each ramp meter by considering the total benefit of the whole traffic infrastructure.

3.2.2.1. Heuristic Ramp Metering Coordination (HERO). In [45], the HERO methodology is defined as an extended version of ALINEA local ramp metering control and a generic software that can implement the HERO coordination scheme is proposed as in Figure 3.4. The ALINEA regulators calculate the ramp metering rate according to the ALINEA strategy in (3.26). However, the local ALINEA regulators are controlled by the HERO algorithm. The HERO uses a master-slave structure based on the location of the bottleneck to manage the on-ramp metering rates. The HERO assigns the master role to the downstream on-ramp where the bottleneck occurs and the slave roles to upstream on-ramps. In this way, HERO uses the capacity of slave on-ramps for the master ramp. On the other hand, the critical density can be estimated and updated with the estimation module involving a Kalman filter.

3.2.2.2. System Wide Adaptive Ramp Metering (SWARM). The objective of SWARM is to achieve the traffic density below the pre-defined critical density for each segment bounded by the two bottlenecks and contains on-ramps and off-ramps. In [46], the working principle of the SWARM algorithm is defined as a prediction of future states by using a Kalman filter. The densities for the bottleneck regions are predicted with a regression and a Kalman filter. In the SWARM algorithm, $Target\,Density$, $Excess\,Density$, and $Volume\,Reduction$ are calculated. The metering rate is defined by using $Volume\,Reduction$. The adjusting parameter, T_{crit}, is calculated to identify the present and future traffic density by using previous traffic data as shown in Figure 3.5.

The $Excess\,Density$ is defined as the difference between the predicted density and the pre-specified threshold density. In general, the threshold density corresponds to

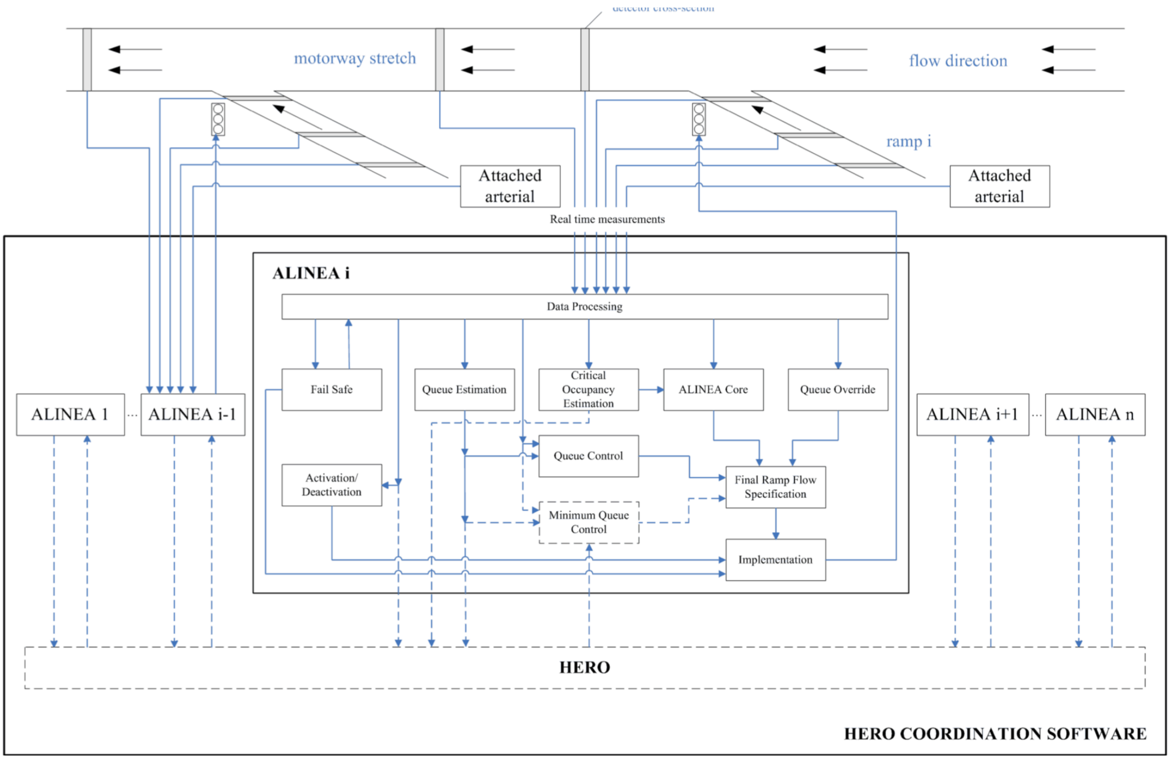

Figure 3.4. Structure of the HERO coordination software in [45]

the jam density at the bottleneck region. Therefore, the $Target\,Density$ is calculated from the $Excess\,Density$ to avoid any congestion in T_{crit} as

$$\begin{aligned} \rho_{TARGET} &= \rho_{CURRENT} - \frac{1}{T_{crit}} \times \rho_{EXCESS} \\ &= (\rho_{LOCAL} - \rho_{TARGET}) \times \lambda \times l_{distance} \end{aligned} \tag{3.27}$$

where ρ_{TARGET} is the target density which is below the critical density, $\rho_{CURRENT}$ is the current density, ρ_{EXCESS} is the excess density which is calculated to achieve target density in next cycles, ρ_{LOCAL} is the local density, λ is the number of lanes, and $l_{distance}$ is the distance to next cell.

The metering rates of the on-ramps are decided by using the $Volume\,Reduction$ rate by distributing the reduced volume to upstream on-ramps as

$$\text{Vol}_R = (\rho_{LOCAL} - \rho_{TARGET}) \times \lambda \times l_{distance} \tag{3.28}$$

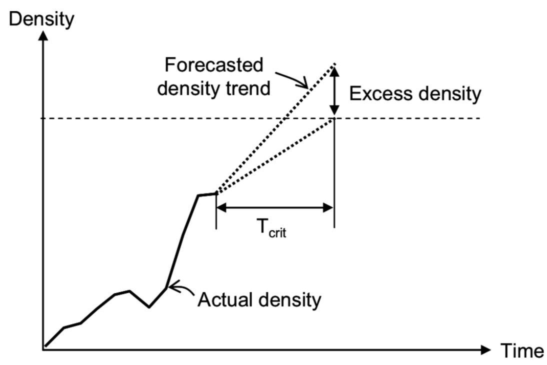

Figure 3.5. Forecasting theory of SWARM in [46]

where Vol_R is the reduction in volume.

3.2.2.3. METALINE. The METALINE algorithm is an extension of the ALINEA algorithm by adding the proportional and integral feedback control strategy [47]. The metering rate of each on-ramp is computed from the available occupancy rate by using the deviation between the current and critical occupancy as

$$\vec{r}(k) = \vec{r}(k-1) - K_1\left(\vec{o}(k) - \vec{o}(k-1)\right) - K_2\left(\vec{O}(k) - \vec{O}^d\right) \tag{3.29}$$

where $\vec{r}(k) \in R^m$ is the vector of metering rates for the m controlled on-ramps, $\vec{o}(k) \in R^m$ is the vector of measured occupancy, and $\vec{O}(k)$, $\vec{O}^d \in R^m$ is the measured and desired occupancy downstream of m controlled on-ramps. The main challenge for the METALINE is to calculate the proportional and integral gain matrices, K_1 and K_2.

3.2.2.4. Zone Algorithm. In [18], the working principle of the Zone algorithm is defined as controlling the metering sections of the traffic network. Traffic network is divided into metering zones and the flow in each zone is coordinated by the Zone algorithm to find a balance between the entering and leaving flow in each zone and in complete traffic network.

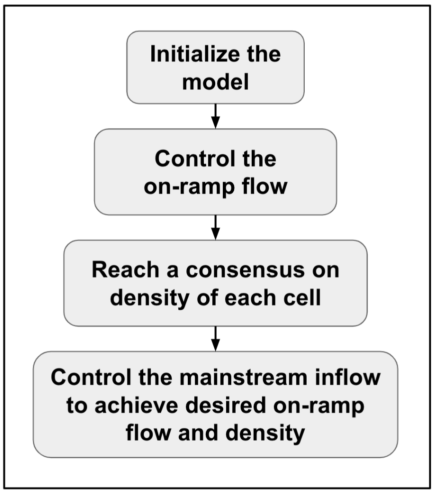

Figure 3.6. Control algorithm steps

The maximum flow value of the on-ramps can be calculated by letting available space to zero and the metering rate is derived based on the ramp factor, $(M+F)$ such that

$$\begin{aligned} M+F =&(x+B)-(A+U) \\ r(k) =&f_r(M+F) \end{aligned} \tag{3.30}$$

where M is the total volume of metered on-ramps, F is the total metered freeway to freeway volume, X is the total volume of off-ramps, B is the bottleneck capacity of downstream zones, A is the mainstream volume of the upstream zones, U is the total volume of non-metered ramps, and f_r is the ramp factor.

3.3. Summary of the Chapter

In this chapter, traffic flow theory is reviewed. The relationships between the density, flow, and the speed are introduced and several techniques to model traffic infrastructure are described. In addition to the traffic flow theory, the existing ramp metering algorithms are presented. In this thesis, the main problem is to control the freeway traffic network with consensus based algorithms, therefore the on-ramp flow control, the density control, and the mainstream inflow control are performed. The proposed algorithm is performed in three steps as shown in Figure 3.6. The coordination and control of the on-ramp flow is addressed by both decentralized and centralized approaches to control the flow without causing any traffic jam. In the density control, the proposed algorithms are utilized to obtain the same traffic density on each cell. Finally, the mainstream inflow control algorithm is performed to ensure achieving the desired traffic density in each cell by using consensus algorithms.

4. DISTRIBUTED CONSENSUS BASED ON-RAMP FLOW CONTROL

In this chapter, coordination and control of the freeway traffic network with a consensus based ramp metering algorithm is considered. The metering rate of on-ramps is coordinated according to the capacity of each cell. Firstly, the decentralized algorithm for on-ramp flow control presented by Kim et.al. in [8] is discussed. Secondly, the proposed centralized and decentralized algorithms are presented and simulated. The contribution of this chapter is to provide a coordinated ramp metering algorithm for each on-ramp without giving a priority to the presence of bottleneck on the freeway.

4.1. On-Ramp Flow Control Algorithm by Kim et.al. [8]

In [8], the problem statement is to coordinate the on-ramp flow and control the mainstream inflow for achieving the pre-specified traffic density for each cell. The traffic model is based on the cellular transmission model. The freeway consists of a finite number of homogeneous road cells and the traffic density of each cell depicted in Figure 4.1 is represented as

$$\rho_i(k) = \rho_i(k-1) + \frac{\Delta k}{l_i \lambda_i}\left(q_{i-1,i}(k) - q_{i,i+1}(k) + r_i(k)\right) \tag{4.1}$$

where $\rho_i(k)$ is the traffic density per lane at cell i in $(veh/km - lane)$, $q_{i-1,i}(k)$ is the mainstream inflow from cell $i-1$ to cell i in (veh/h), $q_{i,i+1}(k)$ is the mainstream outflow from cell i to cell $i+1$ in (veh/h), $r_i(k)$ is the on-ramp flow entering cell i in (veh/h), λ_i is the number of lane at cell i in $(lane)$, l_i is the length of cell i in (km), and Δk is the sampling time in (h).

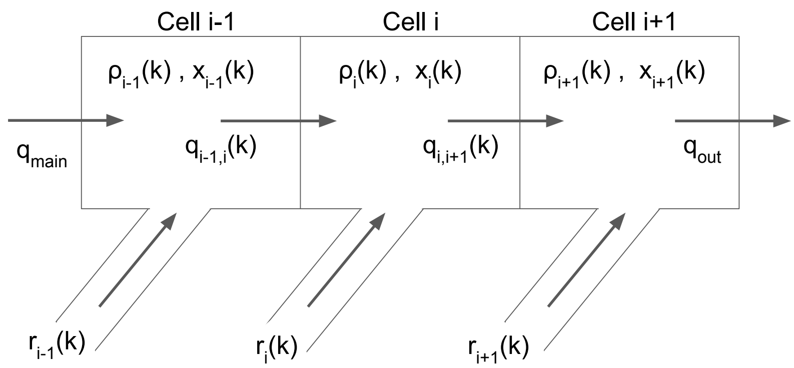

Figure 4.1. A freeway traffic network

The total occupancy created by inflows in cell i is introduced as a variable, denoted by $x_i(t)$ to the traffic density equation and defined as the mainstream flow

$$x_i(k) = x_i(k-1) + r_i(k). \tag{4.2}$$

The maximum mainstream inflow to cell i from cell $i-1$ can be found by using the definition of $x_i(k)$ such that

$$q_{i-1,i}(k) = x_i(k) - x_{i-1}(k). \tag{4.3}$$

The derivation of (4.3) is similar to (3.15). As in (3.15), the use of (4.3) is not reasonable, because cell i may not have enough capacity to accept the desired flow request coming from cell $i-1$. Therefore, the maximum inflow should be determined by the occupancy difference and the allowable capacity in the cell.

The traffic density equation, (4.1), can be modified by substituting (4.2) as

$$\rho_i(k) = \frac{\Delta k}{l_i \lambda_i} x_i(k) + \frac{\Delta k}{l_i \lambda_i} \left(q_{i-1,i}(k) - q_{i,i+1}(k) \right). \tag{4.4}$$

The problem of coordinating the on-ramp flow to achieve desired density in each cell is considered with Assumptions 4.1 - 4.4 below.

Assumption 4.1. *The traffic density in each cell has density constraints as*

$$\rho_{min,i} \leq \rho_i(k) \leq \rho_{max,i}. \tag{4.5}$$

Then the corresponding mainstream flow has also constraints as

$$x_{min,i} \leq x_i(k) \leq x_{max,i}. \tag{4.6}$$

Assumption 4.2. *The mainstream inflow on each cell is lower bounded as*

$$q_{min,i} \leq q_{i,i+1}(k). \tag{4.7}$$

Assumption 4.3. *The desired traffic density, ρ_i^d, and the mainstream flow, x_i^d, are provided.*

Assumption 4.4. *The traffic network is represented with an undirected and connected graph.*

In the on-ramp flow coordination, the desired mainstream flow should satisfy

$$\sum_{i=1}^{n} x_{min,i} \leq \sum_{i=1}^{n} x_i^d \leq \sum_{i=1}^{n} x_{max,i}. \tag{4.8}$$

The on-ramp flow has the following constraints:

$$r_{min,i}(k) \leq r_i(k) \leq r_{max,i}(k). \tag{4.9}$$

The maximum and the minimum limit of the on-ramp flow are defined according to the mainstream flow capacity and the mainstream flow at the previous state such that

$$r_{min,i}(k) = x_{min,i} - x_i(k-1),\ r_{max,i}(k) = x_{max,i} - x_i(k-1). \tag{4.10}$$

The on-ramp flow is coordinated in [8] to satisfy (4.8) and (4.9) with

$$r_i(k) = r_{min,i}(k) + \frac{r_{max,i}(k) - r_{min,i}(k)}{\sum_{j=1}^{n}\left(r_{max,j}(k) - r_{min,j}(k)\right)} \times \sum_{j=1}^{n}\left(x_j^d - x_j(k-1) - r_{min,j}(k)\right) \tag{4.11}$$

where x_i^d is the desired mainstream flow for cell i. It is seen in (4.11) that the global information transfer between the on-ramps is needed to control the on-ramp flow because of the summation terms, $\sum_{j=1}^{n}\left(r_{max,j}(k) - r_{min,i}(k)\right)$ and $\sum_{j=1}^{n}\left(x_j^d - x_i(k-1) - r_{min,i}(k)\right)$.

In [8], the decentralized version of (4.11) is implemented as

$$r_i(k) = r_{min,i}(k) + \left(\frac{r_{max,i}(k) - r_{min,i}(k)}{\mu_i(m)}\right) \times \pi_i(m) \tag{4.12}$$

where $\mu_i(m)$ and $\pi_i(m)$ are the reached consensus values after m iterations which are defined by

$$\begin{aligned}
\pi_i(k+1) &= \frac{1}{1+|\mathcal{N}_i|}\pi_i(k) + \sum_{j\in\mathcal{N}_i}\frac{1}{1+|\mathcal{N}_j|}\pi_j(k),\\
\pi_i(0) &= x_i^d - x_i(k-1) - r_{min,i}(k),\\
\mu_i(k+1) &= \frac{1}{1+|\mathcal{N}_i|}\mu_i(k) + \sum_{j\in\mathcal{N}_i}\frac{1}{1+|\mathcal{N}_j|}\mu_j(k),\\
\mu_i(0) &= r_{max,i}(k) - r_{min,i}(k),
\end{aligned} \tag{4.13}$$

where $|\mathcal{N}_i|$ is the degree of the node i.

In [8], the above algorithm is shown to converge under Assumptions 4.1 - 4.4. Simulations of the decentralized on-ramp flow control algorithm are performed with the randomly generated desired traffic densities for each cell under the pre-specified traffic density constraints given in Table 4.1. The desired traffic densities of each cell is given by $\rho^{des} = [80, 140, 60, 120, 150, 110]^T\ veh/km - lane$ and the initial state vector is taken as

$$\rho(0) = [100, 90, 80, 110, 95, 120]^T\ veh/km - lane.$$

Table 4.1. The traffic density and the mainstream flow constraints of each cell

	Mainstream flow, x	**Traffic density, ρ**
Cell 1	[1000, 6000]	[20, 120]
Cell 2	[2000, 8000]	[40, 160]
Cell 3	[2000, 6000]	[40, 120]
Cell 4	[1000, 6000]	[20, 120]
Cell 5	[2000, 8000]	[20, 120]
Cell 6	[1000, 8000]	[20, 160]

The on-ramp flow given in (4.12) is illustrated and shown in Figure 4.2. It is clearly seen that the algorithm proposed by Kim et.al. allows the negative values in the on-ramp flow coordination. The negative values correspond to the outflow from the on-ramps and it causes a weakness in the algorithm.

4.2. On-Ramp Flow Control Problem Setup

In this chapter, two on-ramp flow control algorithms are proposed with centralized and decentralized approaches. In both algorithms, in addition to Assumptions 4.1 - 4.4, the assumptions given below are used.

Assumption 4.5. *The traffic density constraints defined in the Assumption 4.1 is preserved, however the maximum density, $\rho_{max,i}$, is defined to be equal to the jam density and the minimum density is defined as zero to avoid negative results in the density such as*

$$\rho_{max,i} = \rho_i^j(k) \text{ and } \rho_{min,i} = 0. \tag{4.14}$$

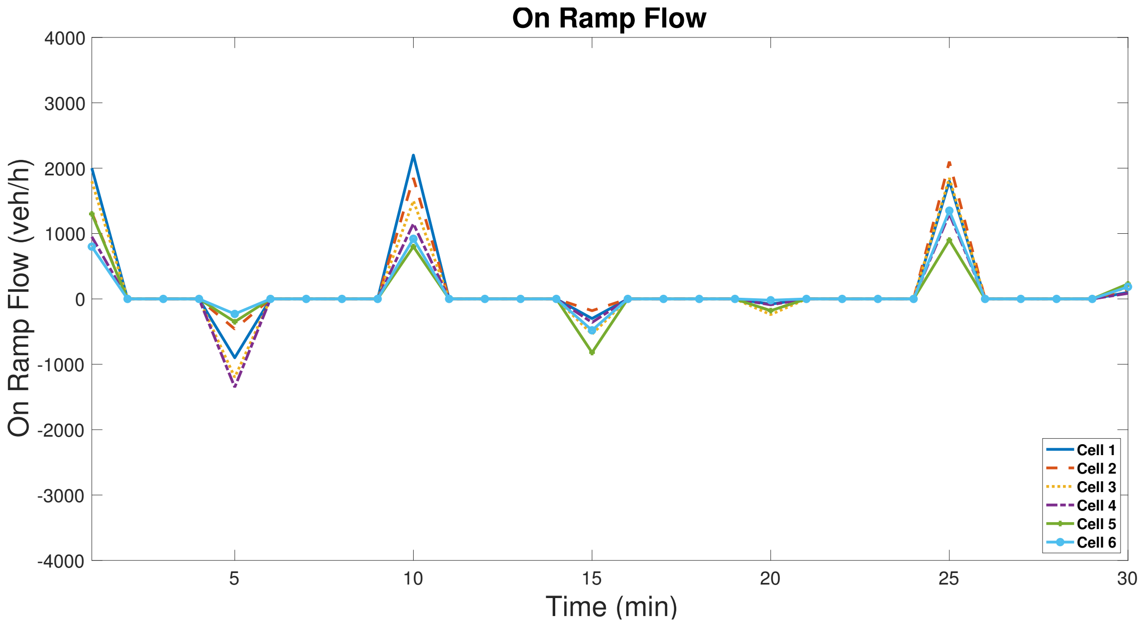

Figure 4.2. The on-ramp flow in [8]

Assumption 4.6. *The vehicles in the traffic network have the same length denoted with V_l. The safety distance between the vehicles, S_l, is defined as the half of the vehicle length.*

Assumption 4.7. *In the traffic network, the vehicle speed is limited by the maximum allowable speed limitation in the highways as*

$$v_i(k) \leq 120\ km/h. \tag{4.15}$$

Assumption 4.8. *Each on-ramp has a pre-specified flow demand, $r_i^d(k)$.*

Under Assumption 4.1, the maximum and the minimum of the mainstream flow are defined as

$$x_{min,i} = \frac{l_i \lambda_i}{\Delta k} \rho_{min,i},\ x_{max,i} = \frac{l_i \lambda_i}{\Delta k} \rho_{max,i}. \tag{4.16}$$

The jam density of each cell in the traffic network is calculated as

$$\rho_i^j = \frac{l_i \lambda_i}{V_l} \tag{4.17}$$

where V_l is the vehicle length. The maximum allowable density of the traffic network with the free flow speed, ρ_i^a, is defined as

$$\rho_i^a = \frac{l_i \lambda_i}{V_l + S_l} \tag{4.18}$$

where S_l is the safety distance between the vehicles.

In the on-ramp flow control, each cell is allowed to achieve the maximum density with the free flow speed, ρ_i^a. The coordination of the on-ramp flow is performed by using ρ_i^a and then the reached value is modified according to the demand capacity of each on-ramp, $r_i^d(k)$. In addition to the traffic network equations in [8], the queue length occurring at the on-ramps is formulated as

$$l_i^q(k) = l_i^q(k-1) + \Delta k \left(r_i^d(k) - r_i(k) \right) \tag{4.19}$$

where $l_{q,i}(k)$ is the queue length at the on-ramp of cell i. The maximum on-ramp queue capacity is formulated according to the on-ramp length, l_i^r, the number of lanes at the on-ramp, λ_i^r, and the vehicle length as

$$l_{max,i}^q = \frac{l_i^r \lambda_i^r}{V_l}. \tag{4.20}$$

Remark 4.1. *Under Assumption 4.8, the on-ramp demand, $r_i^d(k)$, is limited if the on-ramp queue is greater than the half of the queue length capacity of the on-ramp.*

4.3. Proposed Centralized Approach for On-Ramp Flow Control

In [48], distributed algorithms for distributed energy resources are discussed. Since the traffic flow can be considered as an energy distribution and energy transfer between the cells by the flow transfusion, the algorithms can be modified according to the traffic flow model. In the centralized approach, the traffic network consists of n nodes having a leader node. The global information about the total amount of available capacity in each cell is transferred to the leader node; therefore, the global information is provided to each cell by information exchange between the neighboring cells. The total amount of capacity information that the leader node has is given by

$$\gamma = \sum_{i=1}^{n} \left(x_i^a - x_i(k-1)\right), \; x_i^a = \frac{l_i \lambda_i}{\Delta K} \rho_i^a \tag{4.21}$$

where γ is equal to the total desired on-ramp flow, $\gamma^d = \gamma$.

The main objective in the centralized on-ramp flow control is to distribute the total desired on-ramp flow between the on-ramps to satisfy

$$\sum_{i=1}^{n} r_i(k) = \gamma^d. \tag{4.22}$$

In this thesis, the communication is achieved in every time step because the desired on-ramp flow is defined with the available capacity at each cell. Therefore, the following Assumption 4.9 is utilized.

Assumption 4.9. *When the leading node sends the initial information, the communication between the leader node and the remaining nodes will be achieved if the desired on-ramp flow changes.*

In order to achieve the centralized on-ramp flow control, the iterative algorithm, proposed in [49], is used. The iterative algorithm updates the value at node i based on the current value of node i and the current value of the neighboring nodes of node i.

The centralized on-ramp flow coordination is given by

$$r_i(k) = \pi_i(m) \tag{4.23}$$

where $\pi_i(k)$ is value of the proposed centralized algorithm at node i and $\pi_i(m)$ is the reached consensus value after m iterations whose iterations are described by

$$\begin{aligned} \pi_i(k+1) &= \frac{1}{1+|\mathcal{N}_i|}\pi_i(k) + \sum_{j\in\mathcal{N}_i} \frac{1}{1+|\mathcal{N}_j|}\pi_j(k), \\ \pi_i(0) &= \frac{\gamma^d}{n}, \end{aligned} \tag{4.24}$$

where $|\mathcal{N}_i|$ is the degree of the node i. The desired total demand, γ^d, is defined as

$$\gamma^d = \sum_{i=1}^{n} \gamma_i(k) \tag{4.25}$$

where $\gamma_i(k)$ can be calculated by

$$\gamma_i(k) = \begin{cases} x_{max,i} - x_i(k-1), & \text{if } x_{max,i} - x_i(k-1) \leq r_i^d(k) \\ r_i^d(k), & \text{if } x_{max,i} - x_i(k-1) > r_i^d(k) \end{cases} \tag{4.26}$$

The iterative algorithm proposed in (4.24) is based on splitting equally the current value of the total available capacity among the current node i and the neighboring nodes.

4.4. Proposed Decentralized Approach for On-Ramp Flow Control

In the centralized approach, the capacity constraints on the mainstream flow and the boundary conditions of the on-ramp flow are not considered. The leader node has the information about the maximum flow demand of the mainstream and splits the flow into the on-ramps. However, in the decentralized approach, each node transfers its current status information to the neighboring nodes.

The boundary conditions for each on-ramp are defined in (4.10) without considering the on-ramp flow demand. Therefore the achieved on-ramp flows can be greater than the on-ramp capacity and can cause a negative queue length. The minimum and maximum flow that each on-ramp can provide are defined as

$$\begin{aligned} r_{min,i}(k) &= x_{min,i} - x_i(k-1), \\ r_{max,i}(k) &= \begin{cases} x_{max,i} - x_i(k-1), & \text{if } x_{max,i} - x_i(k-1) \leq r_i^d(k) \\ r_i^d(k), & \text{if } x_{max,i} - x_i(k-1) > r_i^d(k) \end{cases} \end{aligned} \tag{4.27}$$

The decentralized on-ramp flow control algorithm defined by Kim et.al. in [8] is used to coordinate the flow of each on-ramp. The proposed decentralized algorithm can be summarized as follows:

$$\begin{aligned} r_i(k) &= r_{min,i}(k) + \left(\frac{r_{max,i}(k) - r_{min,i}(k)}{\mu_i(m)}\right) \times \pi_i(m), \\ \pi_i(k+1) &= \frac{1}{1+|\mathcal{N}_i|}\pi_i(k) + \sum_{j \in \mathcal{N}_i} \frac{1}{1+|\mathcal{N}_j|}\pi_j(k), \\ \pi_i(0) &= \begin{cases} x_i^d - x_i(k-1) - r_{min,i}(k), & \text{if } x_i^d - x_i(k-1) \leq r_i^d(k) \\ r_i^d(k) - r_{min,i}(k), & \text{if } x_i^d - x_i(k-1) > r_i^d(k) \end{cases} \\ \mu_i(k+1) &= \frac{1}{1+|\mathcal{N}_i|}\mu_i(k) + \sum_{j \in \mathcal{N}_i} \frac{1}{1+|\mathcal{N}_j|}\mu_j(k), \\ \mu_i(0) &= r_{max,i}(k) - r_{min,i}(k). \end{aligned} \tag{4.28}$$

Recall that the algorithm by Kim et.al. in [8] does not take the effect of on-ramp demand and capacity. Additionally, the algorithm by Kim et.al. allows the negative flow for the on-ramps which creates inconsistency between the on-ramp flow direction and the resulting flow direction.

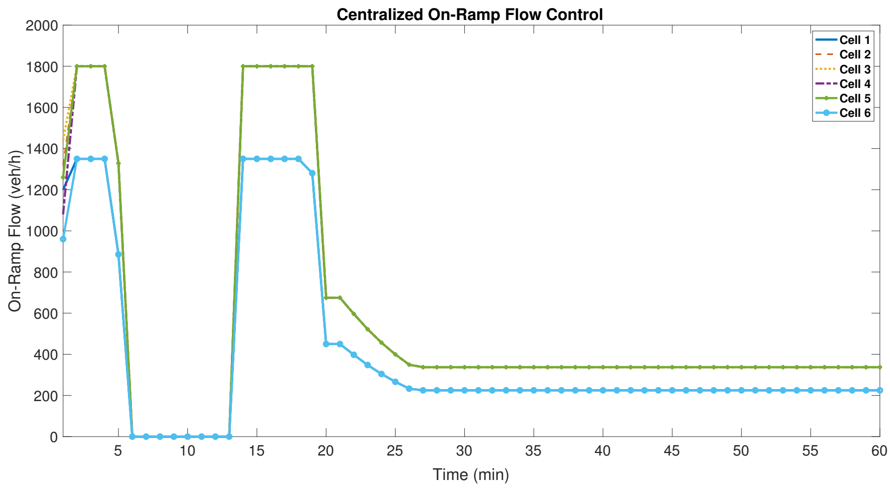

Figure 4.3. Centralized on-ramp flow control

Remark 4.2. *The main contribution of our algorithm is to use modified initial conditions for $\pi_i(0)$ and $\mu_i(0)$ which ensure that decision of the on-ramp flow is made according to the on-ramp demand, capacity and queue length in (4.28).*

4.5. Numerical Evaluation of On-Ramp Flow Control

Consider the traffic network with n cells with equal lengths as shown in Figure 4.1 with a randomly selected initial state vector, $\rho(0) = [\rho_1(0), \rho_2(0), ..., \rho_6(0)]^T$, the constant mainstream flow into the first cell, q_{main}, and the free flow speed, v_f. The variables are chosen as $n = 6\ cells$, $\rho(0) = [100, 90, 80, 110, 95, 120]^T\ veh/km - lane$, $q_{main} = 4800\ veh/h$, $v_f = 120\ km/h$, $l_i = 1\ km$, and $\lambda_i = 1\ lane$.

Remark 4.3. *The main difference between the proposed centralized and decentralized approaches is the construction of the iteration algorithms. In the centralized approach, the main objective is to coordinate the on-ramp flow according to the total available capacity in the traffic network. The initial state of the iteration algorithm is defined to split the total available capacity between the cells. However, in the decentralized approach, the initial states of the iteration algorithm are based on the on-ramp flow capacity, the on-ramp demand, and the available capacity in the upstream cell.*

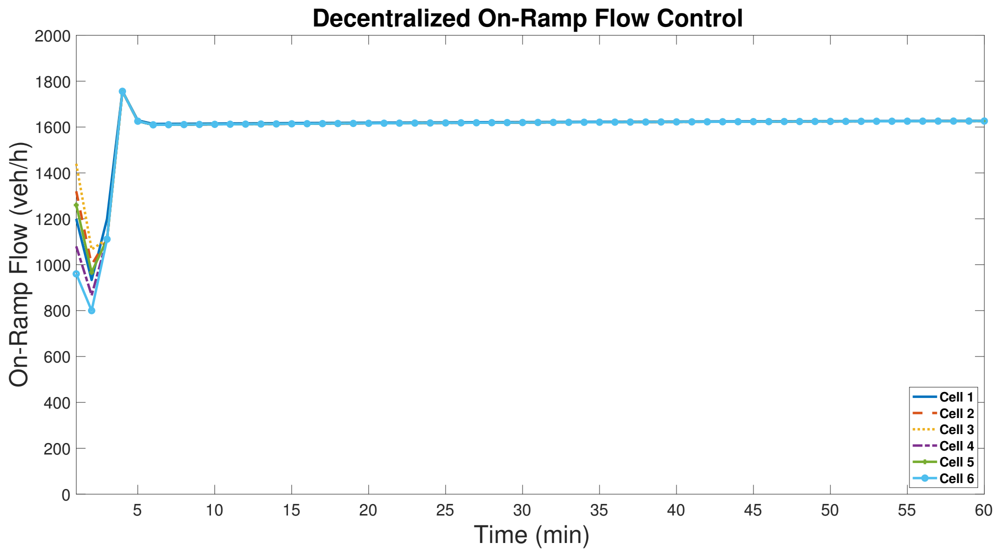

Figure 4.4. Decentralized on-ramp flow control

In the proposed centralized approach, it is seen that the on-ramp flow coordination satisfies the on-ramp demand for a limited time. However, it causes zero metering rate to allow the elimination of the occupancy. Besides this inconsistent result, the centralized control algorithm provide and maintain the non-zero metering rates as shown in Figure 4.3.

In the decentralized on-ramp flow control algorithm, the minimum and maximum capacity constraints are introduced. Therefore, each cell decides its final value under these constraints and the metering rate is controlled by the maximum capacity as depicted in Figure 4.4. In addition to the metering rate, the on-ramp queue length is shown in Figure 4.5.

By comparing the results of these two approaches, it is seen that the decentralized approach in the ramp flow control provides more consistent metering rates. Additionally, higher metering rates correspond to less queue length at on-ramps. Therefore, in the proceeding sections, the decentralized on-ramp control is used to illustrate the proposed control algorithms for density and mainstream control.

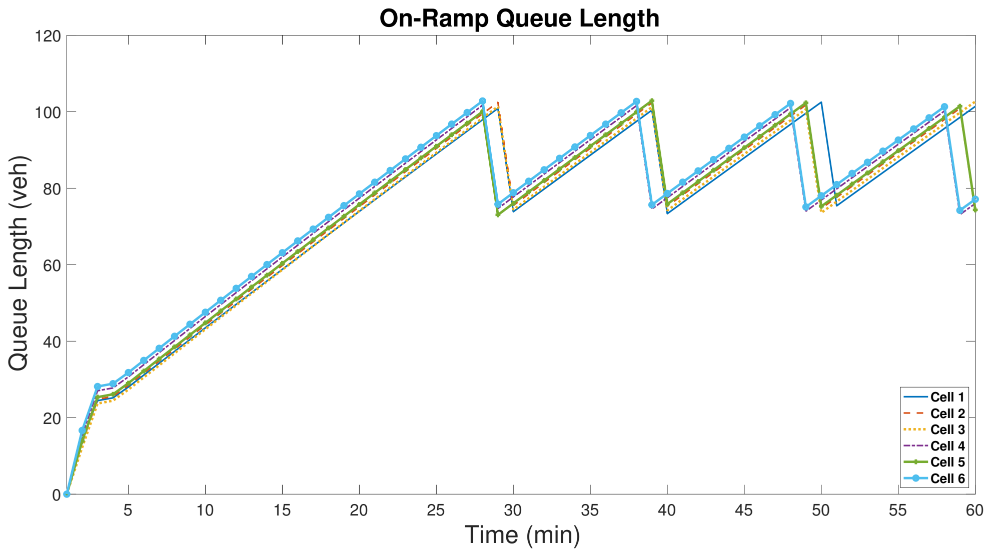

Figure 4.5. Queue length at the on-ramps

4.6. Summary of the Chapter

In this chapter, the traffic model of the freeway system used throughout this thesis is addressed. The coordination of the on-ramp flow proposed in [8] is discussed and illustrated with an example. In order to improve the results provided by Kim et.al., a decentralized algorithm is introduced with additional necessary conditions. In addition to the proposed decentralized algorithm, the centralized on-ramp flow control is presented. The numerical analyses for both the centralized and the decentralized approaches are compared and the most suitable algorithm for on-ramp coordination is selected.

5. DISTRIBUTED CONSENSUS BASED DENSITY CONTROL

In this chapter, centralized and decentralized control algorithms are proposed to estimate the possible traffic density in each cell. The contribution of this chapter is to provide density estimation algorithms according to the allowable capacity at each cell. The update rule for the density is based on using the node capacity and the capacity of the neighboring nodes.

The main objective of the density control is to achieve the same traffic density, which is below the jam density in each cell. In this way, each cell will reach the consensus value about their traffic conditions and the total density in the traffic network will be distributed among the cells.

The density control algorithm is improved by using the power generation algorithm, which is introduced in [50]. In distributed power systems, the fundamental requirement is achieving the supply-demand balance. Since the traffic flow model is studied under the fluid dynamics, if the desired traffic density is considered as the demand, the flow corresponds to the supply in the power systems. The main problem is to achieve the traffic density coordination under the desired net traffic density which can be obtained with the determined on-ramp flow in Chapter 4.2. The traffic density control is formulated with Assumption 4.1 which defines the constraints on the traffic density, $\rho_i(k)$ and the mainstream flow, $x_i(k)$.

5.1. Centralized Consensus Based Density Control

In the centralized approach, the information of the obtained density after the on-ramp flow coordination is transferred to the leader node. The total obtained traffic density within the traffic network after the on-ramp flow coordination is given by

$$\rho_D(k) = \sum_{i=1}^{n} \rho_i^{'}(k) \tag{5.1}$$

where $\rho_i^{'}(k)$ is the obtained traffic density at each cell such that

$$\rho_i^{'}(k) = \frac{\Delta k}{l_i \lambda_i} x_i(k-1) + r_i(k). \tag{5.2}$$

The density coordination problem in (5.1) can be expressed as determination of the desired traffic density in each cell after the on-ramp control. Therefore, the total obtained traffic density in traffic network is derived as

$$\rho_D(k) = \sum_{i=1}^{n} \rho_i^{d}(k). \tag{5.3}$$

Under Assumption 4.1, the constraints on the total traffic density are formulated as

$$\begin{aligned} &\rho_{min,tot} = \sum_{i=1}^{n} \rho_{min,i}, \ \rho_{max,tot} = \sum_{i=1}^{n} \rho_{max,i}, \\ &\rho_{min,tot} \le \rho_D(k) \le \rho_{max,tot}. \end{aligned} \tag{5.4}$$

The desired traffic density for each cell is given by

$$\rho_i^{d}(k) = \rho_{min,i} + \frac{\rho_{max,i} - \rho_{min,i}}{\rho_{max,tot} - \rho_{min,tot}} \times \left(\rho_D(k) - \rho_{min,tot}\right). \tag{5.5}$$

In [50], the desired net energy for the distributed power systems is formulated by achieving the supply-demand balance. In order to design the desired traffic density, the desired net energy formulation is adjusted for traffic network as in (5.5). The iterative algorithm proposed in [50] is used to achieve centralized density control.

The desired density at each cell is coordinated with centralized control as

$$\rho_i^d(k) = \rho_{min,i} + \left(\frac{\rho_{max,i} - \rho_{min,i}}{\mu_i(m)}\right) \times \pi_i(m) \tag{5.6}$$

where $\mu_i(m)$ and $\pi_i(m)$ are the reached consensus values after m iterations and the iteration value of each node is defined as

$$\begin{aligned}
\pi_i(k+1) =& \ \frac{1}{1+|\mathcal{N}_i|}\pi_i(k) + \sum_{j\in\mathcal{N}_i} \frac{1}{1+|\mathcal{N}_j|}\pi_j(k), \\
\pi_i(0) =& \begin{cases} \rho_D(k) - \rho_{min,i}, & \text{if i is the leading node} \\ -\rho_{min,i}, & \text{otherwise} \end{cases} \\
\mu_i(k+1) =& \ \frac{1}{1+|\mathcal{N}_i|}\mu_i(k) + \sum_{j\in\mathcal{N}_i} \frac{1}{1+|\mathcal{N}_j|}\mu_j(k), \\
\mu_i(0) =& \ \rho_{max,i}(k) - \rho_{min,i}(k).
\end{aligned} \tag{5.7}$$

where $|\mathcal{N}_i|$ is the degree of the node i.

Equation (5.6) is the consensus based centralized density control approach of (5.5).

5.2. Decentralized Consensus Based Density Control

In the centralized approach, the density control is achieved by calculating the total desired traffic density, and the leading node has the complete information within the traffic network. In decentralized approach, the traffic density will be controlled by the information flow between only the neighboring cells.

The desired traffic density for each cell is formulated as

$$\rho_i^d(k) = \rho_{min,i} + \frac{\rho_{max,i} - \rho_{min,i}}{\sum_{j=1}^{n}\left(\rho_{max,j} - \rho_{min,j}\right)} \times \sum_{j=1}^{n}\left(\rho_j^{'}(k) - \rho_{min,j}\right). \tag{5.8}$$

The decentralized approach for (5.8) is designed by using the iterative algorithm as in the centralized approach. The desired traffic density in each cell is obtained by using the consensus scheme

$$\rho_i^d(k) = \rho_{min,i} + \left(\frac{\rho_{max,i} - \rho_{min,i}}{\mu_i(m)}\right) \times \pi_i(m) \tag{5.9}$$

where $\mu_i(m)$ and $\pi_i(m)$ are the reached consensus values after m iterations and the iteration value of each node is defined as

$$\begin{aligned} \pi_i(k+1) &= \frac{1}{1+|\mathcal{N}_i|}\pi_i(k) + \sum_{j\in\mathcal{N}_i} \frac{1}{1+|\mathcal{N}_j|}\pi_j(k), \\ \pi_i(0) &= \rho_i^{'}(k) - \rho_{min,i}, \\ \mu_i(k+1) &= \frac{1}{1+|\mathcal{N}_i|}\mu_i(k) + \sum_{j\in\mathcal{N}_i} \frac{1}{1+|\mathcal{N}_j|}\mu_j(k), \\ \mu_i(0) &= \rho_{max,i}(k) - \rho_{min,i}(k). \end{aligned} \tag{5.10}$$

Equation (5.9) is the consensus based decentralized control approach of (5.8).

Remark 5.1. *The difference between the proposed centralized and decentralized density control algorithms is the definitions of $\pi_i(0)$ and $\mu_i(0)$. As seen in (5.7), the initial condition of $\pi_i(0)$ is defined according to the information of the total obtained traffic density after the on-ramp flow control. However, in (5.10), the initial condition is calculated by the difference between the obtained traffic density of the cell and the minimum traffic density.*

5.3. Numerical Evaluation of Density Control

In the simulations of the density control, the traffic network model shown in Figure 4.1 is used with the parameters defined in Chapter 4.5.

The on-ramp flow coordination with both centralized and decentralized control is shown in Figures 4.3 and 4.4. Since the on-ramp flow control is not affected from

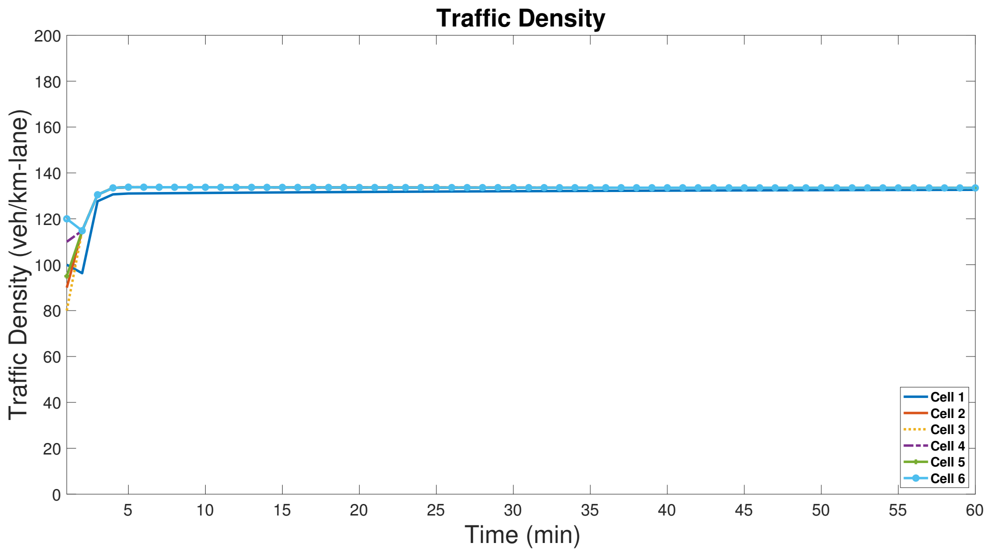

Figure 5.1. Traffic density with centralized density control

the changes in density control algorithm, the next step in the algorithm is to control traffic density. Figures 5.1 and 5.2 show the results of centralized and decentralized control. In the centralized approach, the leader node has the density information of all nodes and the consensus is reached by distributing the total demand in density between the cells. In the decentralized approach, each node has the information from its neighboring cells. In the traffic network configuration, the cells except the first and the last cell have two neighbors and the density is arranged according to the density of neighboring nodes. Since the middle nodes have the same number of neighboring nodes, the distribution of the density is the same with the centralized approach. However, the main difference between the centralized and decentralized approaches is the constraints on the requirement to have whole information in one leader node.

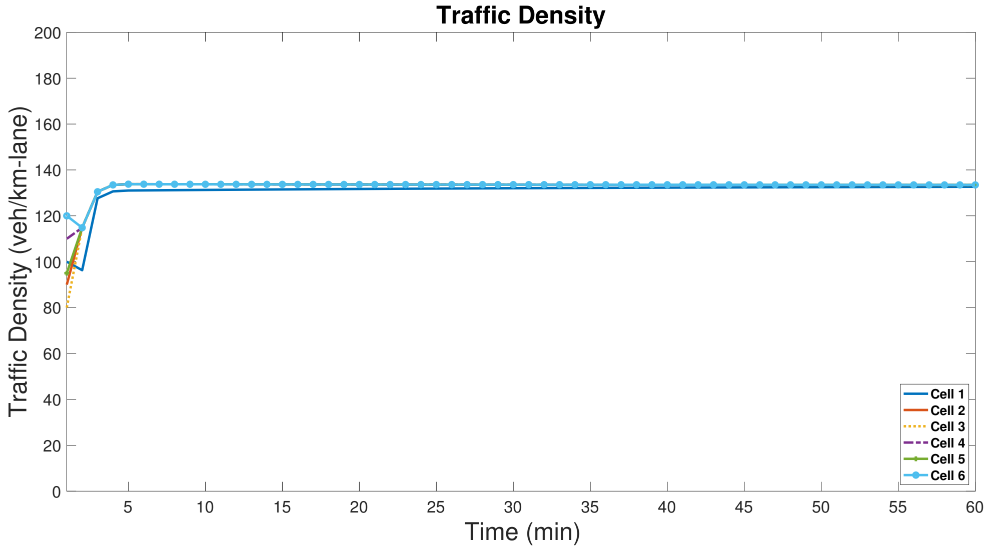

Figure 5.2. Traffic density with decentralized density control

5.4. Summary of the Chapter

In this chapter, the proposed density control algorithms are introduced. The density control algorithms are performed to achieve a consensus value on the density of each cell after the decision on the metering rates. The algorithms given in this chapter are based on the centralized and decentralized consensus. From the numerical analysis of the both algorithms, it is observed that the centralized and the decentralized control algorithms provide the same results.

6. MAINSTREAM INFLOW CONTROL

In this chapter, mainstream inflow control is discussed to achieve the desired on-ramp flow and the traffic density. In Chapter 5, the traffic density control is proposed both with centralized and decentralized approaches. In order to achieve the decided traffic density, the mainstream inflow should be coordinated with the following assumption.

Assumption 6.1. *There is a pre-specified or known mainstream flow,* $q_{main}(k)$ *into the first cell of the traffic network.*

Under Assumptions 4.7 and 6.1, the mainstream inflow between the cells is coordinated according to the elimination of the pre-specified mainstream inflow, $q_{main}(k)$, with the maximum vehicle speed constraint.

6.1. Coordination of Mainstream Inflow Control

Mainstream inflow coordination is based on the elimination of the difference between the obtained traffic density after the on-ramp flow control, $\rho_i(k)$, and the desired traffic density, ρ_i^d. The main objective is to achieve the desired traffic density defined in Chapter 5 by arranging the inflow between cells.

The difference between the traffic density and the desired traffic density is defined as the coordination error, $\rho_{e,i}(k)$. The coordination error is derived as

$$\rho_{e,i}(k) = \rho_i(k) - \rho_i^d(k) \tag{6.1}$$

where $\rho_i^d(k)$ is achieved with (5.6) or (5.9) and $\rho_i(k)$ is the obtained density such that

$$\rho_i(k) = \frac{\Delta k}{l_i \lambda_i} x_i^{'}(k) + \frac{\Delta k}{l_i \lambda_i} \left(q_{i-1,i}(k) - q_{i,i+1}(k) \right). \tag{6.2}$$

Here, $x_i^{'}(k)$ is the mainstream flow after the on-ramp flow coordination and formulated by using $\rho_i^{'}(k)$ in (5.2) as

$$x_i^{'}(k) = \frac{l_i \lambda_i}{\Delta k} \rho_i^{'}(k). \tag{6.3}$$

Then, the new coordination error is given by

$$\rho_{e,i}^{'}(k) = \frac{\Delta k}{l_i \lambda_i} x_i^{'}(k) - \rho_i^d. \tag{6.4}$$

In order to eliminate the effect of the term $\frac{\Delta k}{l_i \lambda_i}$, the auxiliary mainstream inflow is defined as

$$\begin{aligned} q_{i-1,i}^{'}(k) &= \frac{\Delta k}{l_i \lambda_i} q_{i-1,i}(k), \\ q_{i,i+1}^{'}(k) &= \frac{\Delta k}{l_i \lambda_i} q_{i,i+1}(k). \end{aligned} \tag{6.5}$$

Equation (6.1) can be rearranged by substituting (6.4) and (6.5) as

$$\rho_{e,i}(k) = \rho_{e,i}^{'}(k) + \left(q_{i-1,i}^{'}(k) - q_{i,i+1}^{'}(k) \right). \tag{6.6}$$

Since the new coordination error, $\rho_{e,i}^{'}(k)$ is defined as the difference between the mainstream flow, $x_i(k)$ and the desired density, ρ_i^d, its value can be calculated after the on-ramp flow coordination because the mainstream flow is defined as $x_i(k) = x_i(k-1) + r_i(k)$ in (4.2). The only way to achieve the desired density at each cell is to coordinate mainstream inflow, $q_{i-1,i}(k)$ according to the coordination error such that

$$\begin{aligned} \rho_{e,i}(k) = \rho_{e,i}^{'}(k) + \left(q_{i-1,i}^{'}(k) - q_{i,i+1}^{'}(k) \right) = 0 \\ q_{i-1,i}^{'}(k) - q_{i,i+1}^{'}(k) = -\rho_{e,i}^{'}(k). \end{aligned} \tag{6.7}$$

In [8], the auxiliary mainstream inflow without considering the direction of the traffic flow is designed as

$$q'_{j,i}(k) = h_{ij}(m),\ j \in \mathcal{N}_i \tag{6.8}$$

where $h_{ij}(m)$ is the reached consensus value after m iterations whose iterations are described by

$$\begin{aligned} &h_{ij}(k+1) = h_{ij}(k) + a_{ij}\left(g_j(k) - g_i(k)\right) \\ &h_{ij}(0) = 0 \\ &g_i(k+1) = g_i(k) + \sum_{j \in \mathcal{N}_i} a_{ij}\left(g_j(k) - g_i(k)\right) \\ &g_i(0) = \rho'_{e,i}(k) \end{aligned} \tag{6.9}$$

where a_{ij} is the Metropolis-Hasting weight in (2.11).

6.2. Traffic Flow Direction Coordination

In (6.8), the resulting auxiliary mainstream inflow, $q'_{j,i}(k)$, does not depend on the traffic flow direction which means that the traffic flow from cell i to cell $i-1$ is allowed to achieve the desired density in each cell. Since the traffic flow has restriction on the direction, from cell i to cell $i+1$, the result of (6.8) should be updated according to the direction. Additionally, there is an inflow coming to the first cell from the freeway; therefore, the inflow between cells should be at least equal to the incoming flow in order to extinguish the inflow and use the available capacity for the on-ramp flows.

In [8], the mainstream inflow is designed to be compatible with the flow direction such as

$$q_{i,i+1}(k) = \begin{cases} \frac{l_i \lambda_i}{\Delta k}\left(\alpha'\right), & \text{if } i = n \\ \frac{l_i \lambda_i}{\Delta k}\left(\alpha' + q'_{i,i+1}(k)\right), & \text{if } i = 1, ..., n-1 \end{cases} \tag{6.10}$$

where $\alpha^{'}$ is determined by the min-consensus algorithm in [29] such that

$$\alpha^{'} = \min \alpha \text{ s.t } \min_{i} \frac{l_i \lambda_i}{\Delta k} \left(\alpha^{'} + q^{'}_{i,i+1}(k) \right) \geq q_{main}(k) \tag{6.11}$$

where $q_{main}(k)$ is the mainstream incoming flow.

6.3. Weight Matrices and Convergence Speed Analysis

The working principle of the averaging based consensus algorithms is based on the update rule such that the nodes update their current states according to the states of the neighboring nodes and the inner state from the last iteration as stated in Chapter 2.2.2. Therefore, as an alternative to Metropolis-Hasting weights used in [8], maximum degree model and local degree model can be used to design mainstream inflow which could affect the convergence speed of the algorithm.

The convergence speed of the consensus algorithm (2.5) with the different weight matrices is derived by using the second largest eigenvalue modulus as

$$\mu(W) = \max_{i=2,\ldots,n} |\lambda_i(W)| = \max\left(\lambda_2(W), ..., \lambda_n(W)\right) \tag{6.12}$$

The weight matrix, W, can be expressed in Jordan form as

$$W = TJT^{-1} \tag{6.13}$$

Therefore, the k-th power of the weight matrix is derived by using (6.13) as

$$W^k = Q_{11}1^k + \sum_{i=2}^{q} \sum_{l=0}^{m_i-1} Q_{il} k^l \lambda_i^k \tag{6.14}$$

where q is the number of distinct eigenvalues of W, m_i is the multiplicity of the eigenvalue, λ_i, and Q_{ij} is the scaling matrix.

The state vector is expressed as

$$\mathbf{x}(\mathbf{k}) = W^k\mathbf{x}(\mathbf{0}) = Q_{11}\mathbf{x}(\mathbf{0}) + \sum_{i=2}^{q}\sum_{l=0}^{m_i-1}\left(Q_{il}\mathbf{x}(\mathbf{0})\right)k^l\lambda_i^k \tag{6.15}$$

It is seen that (6.15) converges faster if the second largest eigenvalue modulus, $\mu(W)$, is smaller. Therefore, the weighting algorithm is selected according to the resulting second largest eigenvalue modulus.

6.4. Numerical Evaluation of Mainstream Inflow Control

In this section, the mainstream inflow control with different approaches on averaging weighting algorithm are reviewed such as the Metropolis-Hasting weights, the maximum degree model, and the local degree model. Since these models are using for determining the inflow rate between the cells without considering the traffic flow direction, the negative flow between the cells can occur and it is adjusted with minimum consensus algorithm.

The weight matrices of the different approaches are constructed by using (2.8), (2.9), and (2.11) as

$$W_{max} = W_{lc} = \begin{pmatrix} 1/2 & 1/2 & 0 & 0 & 0 & 0 \\ 0 & 1/2 & 1/2 & 0 & 0 & 0 \\ 0 & 0 & 1/2 & 1/2 & 0 & 0 \\ 0 & 0 & 0 & 1/2 & 1/2 & 0 \\ 0 & 0 & 0 & 0 & 1/2 & 1/2 \\ 0 & 0 & 0 & 0 & 0 & 1 \end{pmatrix},$$

$$W_{mh} = \begin{pmatrix} 2/3 & 1/3 & 0 & 0 & 0 & 0 \\ 0 & 2/3 & 1/3 & 0 & 0 & 0 \\ 0 & 0 & 2/3 & 1/3 & 0 & 0 \\ 0 & 0 & 0 & 2/3 & 1/3 & 0 \\ 0 & 0 & 0 & 0 & 2/3 & 1/3 \\ 0 & 0 & 0 & 0 & 0 & 1 \end{pmatrix}$$

where W_{max} is the maximum degree weight, W_{lc} is the local degree weight, and W_{mh} is the Metropolis-Hasting weight. The second largest eigenvalue modulus of the weight matrices are $\mu(W_{max}) = 0.500$, $\mu(W_{lc}) = 0.500$, and $\mu(W_{mh}) = 0.6667$ respectively. The convergence speed equality between the maximum degree and the local degree weight is expected, because the weight matrix of both model is identical. Since $\mu(W_{max}) = \mu(W_{lc}) \leq \mu(W_{mh})$, the maximum degree and the local degree systems will converge faster. Since, the maximum degree algorithm needs global information about the neighbor number of each cell, the local degree model is used in this thesis.

The mainstream inflow is obtained by using the minimum consensus algorithm (6.11). Since the decided density values are the same in both the centralized and the decentralized algorithm, the obtained mainstream inflow control is performed with the decentralized density control. From the results shown in Figure 6.1, it is seen that the mainstream inflow has reached the consensus value to meet the requirements.

The speed of the vehicles in each cell is calculated by assuming a linear relationship between the speed and density as in (3.2). Since the vehicle speed is introduced as a constraint on the mainstream inflow control, the resulting speeds are limited with maximum allowable vehicle speed as shown in Figure 6.2.

To illustrate the significance of Assumption 4.7 on the proposed control algorithms, the mainstream inflow control is utilized without considering the limitation on the vehicle speed. As shown in Figure 6.3, the vehicle speed can take negative values without the vehicle speed constraint and after some point all of the vehicles are stopped; therefore, the traffic density of each cell reaches the jam density except the

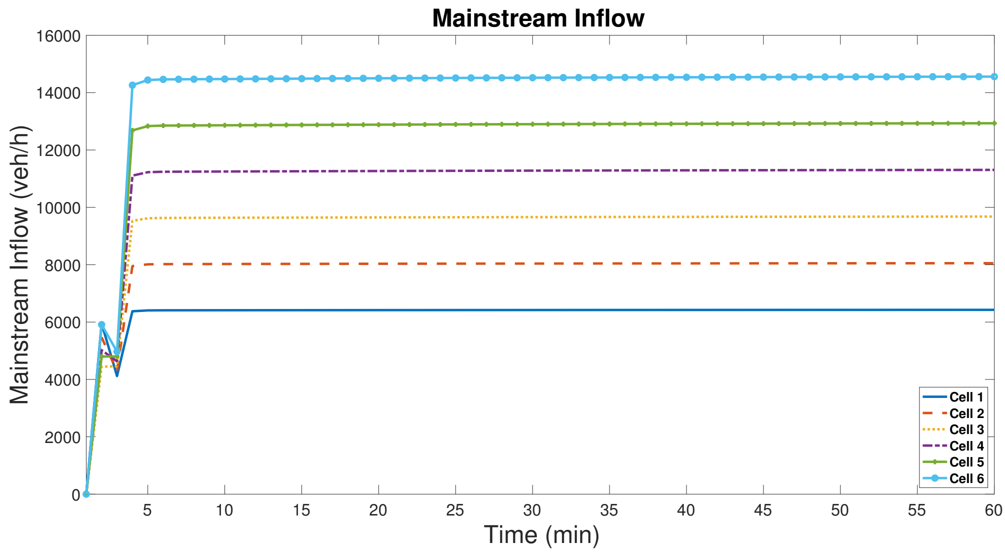

Figure 6.1. Mainstream inflow of each cell

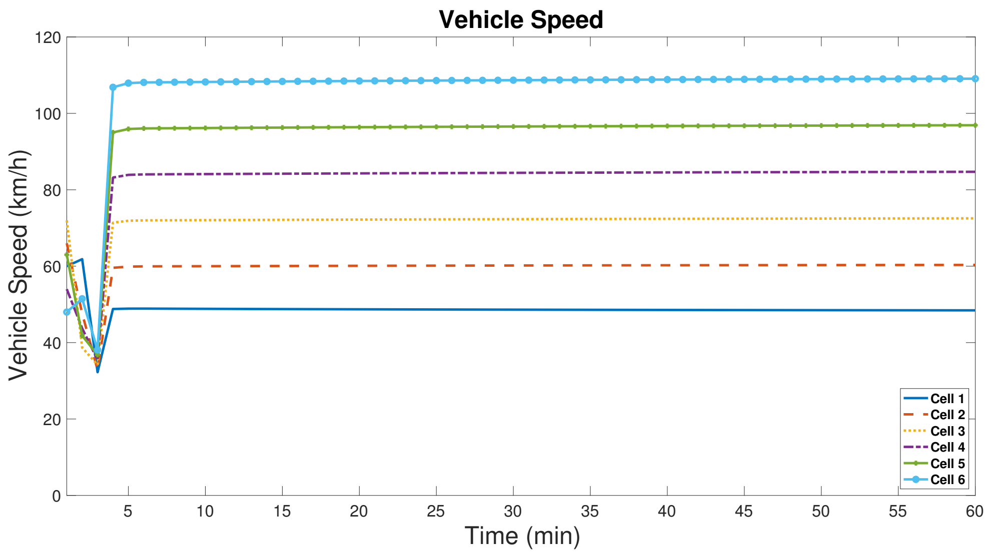

Figure 6.2. Vehicle speed in each cell

last cell as shown in Figure 6.4. The reason of the negative density in the last cell is an expected result since there is not any inflow to the last cell, however the outflow still occurs. As a concluding remark, Assumption 4.7 is an essential requirement for the algorithm.

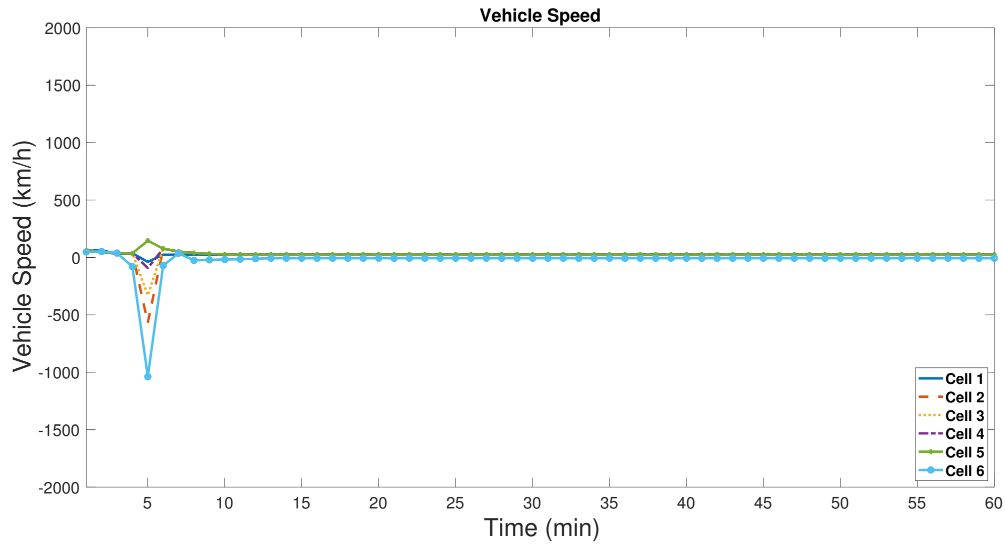

Figure 6.3. Vehicle speed without Assumption 4.7

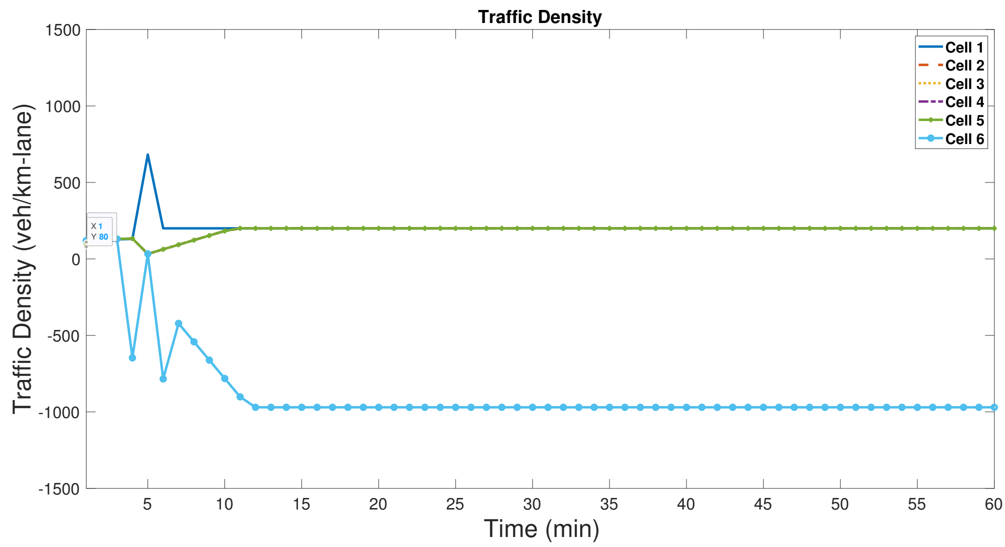

Figure 6.4. Traffic density without Assumption 4.7

6.5. Summary of the Chapter

In this chapter, the mainstream inflow control algorithm is introduced. Firstly, the proposed algorithm is designed without considering the traffic flow direction. The algorithm is performed with different averaging weight matrices. Furthermore, the analyses in this section show the convergence speed of the proposed weighting matrices. Secondly, the mainstream inflow is arranged according to the traffic flow direction by utilizing the minimum consensus algorithm under the vehicle speed constraint. Finally, the numerical analyses are provided by simulations.

7. CONCLUSION

Inspired by a variety of applications in traffic control, this thesis has studied the consensus problem for the ramp meter control and the density control in traffic network by using a traffic model in discrete-time. The objective of this thesis is to provide consensus based centralized/decentralized ramp metering algorithms with the density and the mainstream inflow coordination. In the introductory chapters of this thesis, basic graph theoretical concepts related to the networks are reviewed and the mathematical theory of the discrete-time consensus algorithm is stated clearly. Additionally, the traffic flow theory and several different approaches for the ramp meter control are introduced.

An important contribution of this thesis is to provide a consensus based coordinated ramp meter algorithm with centralized and decentralized approaches. In Chapter 4, the coordination and control of the on-ramp flow with a consensus based ramp metering algorithm are presented. The algorithm has been investigated with the maximum allowable density for each cell. The decentralized on-ramp control algorithm proposed by Kim et.al. is improved by introducing the boundary conditions and the centralized approach for the on-ramp flow control is developed. Both the centralized and decentralized approaches for the on-ramp flow are illustrated with numerical examples.

Another contribution is the proposed density algorithm in Chapter 5 where the main objective of this chapter is to coordinate the density of each cell after the on-ramp flow control. The proposed decentralized and centralized approaches are based on power distribution and modified for the traffic flow. Even though the centralized and decentralized algorithms provide similar results for the density control, the centralized approach requires whole capacity information of the traffic network. Therefore, the decentralized approach is more suitable for the real-life applications.

Mainstream inflow control is performed in Chapter 6 to achieve the desired traffic density obtained in Chapter 5. The main contribution of Chapter 6 is to provide a convergence speed comparison between the proposed weighting matrices. The minimum consensus algorithm is used to convert the obtained flow results according to the traffic flow direction.

The proposed algorithms in this thesis provide consensus based on-ramp metering adjustments for freeway systems. The coordination of the traffic density allows a smooth traffic density distribution between the cells of the infrastructure. The most important advantage of the proposed algorithms is to provide a simpler control that can be adopted any network based structure.

7.1. Future Work

There are extensions of this thesis that are interesting and challenging for future studies and these can be stated as follows:

- In this thesis, the relationship between the speed and the density is assumed as linear and the speed control is not included. Another direction for research can be to extend the results by achieving a consensus value for the speed in each cell to reduce acceleration and deceleration rate of the vehicles.
- Since the mainstream inflow to the first cell is assumed as a constant, the effect of the changing flow can be investigated.
- The fuel consumption rate can be introduced in the mainstream inflow control in addition to the vehicle speed constraint.
- The algorithms presented in this thesis are evaluated under static communication. However, in most of the real-world applications the interaction among the agents in the network has some communication delays. Therefore, the results on the traffic network can be investigated in the case of time-dependent communication.
- The on-ramp control algorithm can be investigated with the occurrence of a bottleneck in any cell and the density control algorithm can be modified in order to eliminate the occuring density instead of achieving the same density in each cell.
- The safety distance can be defined according to the vehicle speed.

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