EMPIRICAL ANALYSES ON PORTFOLIO OPTIMIZATION STRATEGIES

by

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ABSTRACT

EMPIRICAL ANALYSES ON PORTFOLIO OPTIMIZATION STRATEGIES

In this study, the success of the application of control engineering approaches to the financial portfolio construction problem are investigated empirically. Often, the aim of the investor is to maximize the returns while keeping the risks at minimum possible level. In our work, the investor's problem is formulated mathematically by using Modern Portfolio Theory. The risk and return parameters in the formulation are estimated by using historical data. The investment strategy of an investor is characterized by three variables. The first of these variables is the risk tolerance of the investor. In our work, the investors are divided into three groups according to their risk appetites: Risk Avoiders, Controlled Risk Takers and Adventurers. The second variable is named as the investment horizon. The investment horizon denotes the future time at which the investor hopes to get a return. The third variable determines the length of the historical data that is used for the estimation of the risk and return parameters. By changing the values of these variables inside a nested for loop structure, various investment scenarios are simulated to decide on the most useful investment strategy. The empirically best investment strategy is then applied onto a dynamically updated portfolio and the performance of the portfolio is compared to a specified benchmark. In all analyses and simulations, the data from Istanbul Stock Exchange is used. The ISE30 index is chosen as the benchmark. A comprehensive software program with a graphical user interface is developed using MATLAB for simulations and analyses.

ÖZET

PORTFÖY OPTİMİZASYON STRATEJİLERİ ÜZERİNE AMPİRİK ANALİZLER

Bu çalışmada kontrol mühendisliği yaklaşımlarının finansal portföy oluşturmadaki başarısı ampirik olarak incelenmektedir. Çoğu zaman, yatırımcıların amacı, yatırımın getirisini maksimize ederken yatırım üzerindeki riskleri minimum seviyede tutmaktır. Çalışmamızda Modern Portföy Teorisi kullanılarak yatırımcının problemi matematiksel olarak formüle edilmektedir. Formülasyondaki risk ve getiri parametreleri geçmiş finansal veriler kullanılarak kestirilmektedir. Yatırımcının yatırım stratejisi üç değişken ile karakterize edilmektedir. Bu değişkenlerden birincisi yatırımcının risk toleransıdır. Çalışmamızda, yatırımcılar risk iştahlarına göre üç gruba ayrılmaktadır: Riskten Kaçınanlar, Kontrollü Olarak Risk Alanlar ve Maceracılar. İkinci değişken yatırım ufku olarak adlandırılmaktadır. Yatırım ufku yatırımcının kazanç elde etmeyi ümit ettiği ileri bir tarihi belirtir. Üçüncü değişken risk ve getiri parametrelerinin kestiriminde kullanılan geçmiş finansal verinin uzunluğunu belirler. En kullanışlı yatırım stratejisine karar verebilmek adına, bu değişkenlerin değerlerinin içice yuvalanmış bir 'for' döngüsü içerisinde değiştirilmesi ile birçok yatırım stratejisi için benzeştirme yapılmaktadır. Ampirik olarak gözlemlenen en iyi yatırım stratejisi dinamik bir portföy üzerine uygulanmakta ve bu portföyün performansı belirlenen kıstas ile karşılaştırılmaktadır. Bütün analiz ve benzetimlerde İstanbul Menkul Kıymetler Borsası verileri kullanılmaktadır. IMKB30 endeksi kıstas olarak seçilmiştir. Benzetim ve analizler için MATLAB kullanılarak kapsamlı bir yazılım ve grafik ara yüz geliştirilmiştir.

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LIST OF ACRONYMS/ABBREVIATIONS

CAPM	Capital Asset Pricing Model
CVaR	Conditional Value at Risk
FTF	Future Time Frame
GDP	Gross Domestic Product
ID	Investment Date
ISE	Istanbul Stock Exchange
MPT	Modern Portfolio Theory
PMPT	Post Modern Portfolio Theory
PTF	Past Time Frame
QP	Quadratic Programming
QS	Quarterly Success
PS	Periodic Success
RT	Risk Tolerance
TL	Turkish Lira
VaR	Value at Risk
YS	Yearly Success

1. INTRODUCTION

The universe of finance has been re-defined in the last twenty years by two dominant factors. The first factor is the increased complexity in the behavior of financial instruments. The mathematical approaches to the financial problems are combined with the vast computing power and giving start to the discipline of the financial engineering. The second factor is the globalization current. The finance sector has benefited from the advances in the communication technology at a maximum level. As a result, the markets all around the world became more connected to each other, expanding the scopes of all types of financial analyses.

The booms of a series of economic crises across the globe for the last ten years have highlighted the importance of financial planning. The public awareness on the financial issues has also increased. The tension that is faced by governments and corporations has increased the demand for financial research and qualified workforce. The difficult problems that arose in financial markets have attracted the researchers from a variety of fields such as physics, mathematics, statistics and engineering.

The term financialization is used when referring to the dominance of financial markets over the traditional industrial economy. Financialization describes an economic process that attempts to convert all exchanged values into financial instruments. An interesting fact which emphasizes the significance of the financialization concept is that the total financial turnover in the United States economy corresponds to 5000% percent of the country's GDP [1, 2].

Financialization is assumed to be the driving force of growth in developing countries. Turkey is an emerging market and there is an increasing demand for financial research from investment companies, banks and government institutions. A solid knowledge pool on the dynamics of financial markets is vital for the Turkish finance industry.

Although the field of finance has been subject to a major evolution in the last several decades, the concern of the investor is the same: 'To get more for less'. However the decision process of any investor for reaching that simple goal is far from being straightforward due to the complex structured financial markets of today. A sharpened sense of risk awareness is one key factor for success. Another key factor is the sound evaluation of the tolerances against the faced risks.

Risk tolerances of investors vary widely due to the diversity in their budgets, return expectations, lifestyles and so on. Disappointing scenarios for the investor could be prevented only by the correct identification of the investor's risk tolerance.

Harry Markowitz introduced a mathematical model for portfolio optimization problem in 1952, in his seminal paper "Portfolio Selection". Markowitz's theory is also known as Modern Portfolio Theory. Modern Portfolio Theory suggests the explicit recognition of risk. The theory aims to minimize the faced risk for a given expected return or maximize the expected return for a given level of risk by exploring through a unique set of portfolio combinations [3].

Modern Portfolio Theory provides a mathematical formulation for the diversification concept in financial investments. The theory models an asset's return by a random variable. Standard deviation of this random variable is used to measure the risk associated with the relevant asset. The risk over a set of stocks can be calculated by using individual assets' standard deviations and the correlations among these assets. The risk of the portfolio can be lowered by careful adjustment of the included asset proportions.

The mathematical model of the problem leads to the formulation of the "efficient frontier", a special curve with expected return in one axis and standard deviation on the other. Efficient frontier matches every possible expected return with lowest possible risk (standard deviation) [4].

Modern Portfolio Theory has become a milestone for the mathematical analysis of financial investments. The theory is often being referred to as "the classical Markowitz model" in the literature, because there are several extended versions of the theory.

William F. Sharpe's diagonal model is an extension of original Markowitz model, but is computationally simpler. Sharpe's idea was to split the risk associated with each asset into two components. These are systematic risk and asset specific risk. Systematic risk models the common risk factor over all the assets on which the investor has interest. An indicator of market's general condition could be used for estimating the systematic risk. Sharpe has used S&P 500 index in his work as the common factor of S&P stock returns. For that reason, the name single index model is also used to refer to Sharpe's diagonal model. The asset specific risk or unsystematic risk is independent across assets. The unsystematic risk is used to explain the variation between the behavior of the accepted common factor and the behavior of the asset. The overall risk of an asset is given by the sum of asset specific risk and systematic risk [5].

Using the diagonal model as a basis, Sharpe himself and several other researchers (Treynor, Lintner and Mossin) independently formulated the Capital Asset Pricing Model (CAPM). The CAPM analyzes the equilibrium conditions on a market where investors are investing only in efficient portfolios. The expected returns of assets are modeled with reference to the risk free return of the market. A government bond or an interest account can be considered as examples of risk free investments [6, 7]. (Today in 2012, due to the recent financial crisis in European economies, there are serious concerns about the feasibility of riskless investment assumptions.)

In Modern Portfolio Theory (MPT), the risk of an asset is measured by the standard deviation of the asset's return. The risk is the total variability of the return around the expectation. Possible upside and downside variations are involved equally in the calculation of the standard deviation. Therefore, in MPT the excessive profits and excessive losses are penalized in the same way. Post Modern Portfolio Theory (PMPT) modifies the symmetric risk concept in MPT. In PMPT the penalization of upside and downside variations are not the same. The respective penalty weights of surprising gains and surprising losses are determined by the investor's preference [8].

In 1991, Black and Litterman formulated a portfolio optimization strategy by combining Markowitz's Portfolio Theory and Sharpe's Capital Asset Pricing Model. In the previous approaches the market equilibrium was set by the consolidation of respective efficient sets of portfolios that the investors face. Black and Litterman analyzed the situation going backwards from the market equilibrium. The optimization of global portfolios (portfolios that consist of different currencies and assets from various national markets) is investigated in their work [9].

The portfolio management problem deals with the allocation of money into different assets with the objective to maximize the total wealth at some future time. The problem can be described by an open loop system since the investment decisions do not affect future asset returns [10].

The analysis on the system is conducted over four main aspects:

- The Control Problem
- The System Identification Problem
- The Model Reduction Problem
- The Verification Problem

The control problem is about deciding the best input to the system under control. The best input in this context refers to the input whose associated output is "the best" according to a pre-defined objective function. An objective function is a function that provides a metric for the ranking of the results in terms of their desirability.

The decisions that affect the formulation of the portfolio constitute 'the control problem' question of the portfolio management system.

The identification of the system implies making assumptions on the mapping between the decisions and their consequences. Given the input and the output data pairs, we are to decide on a model that mimics the system behavior. A model is necessarily an imperfect representation of the system. Once the model is set, it serves as the basis for predicting possible outcomes of various input combinations

For all natural phenomena, the process of trade-off is inevitable. As we improve the accuracy of models, the complexity we face also grows. Hence, one must search for ways to reduce the complexity while retaining as much accuracy as possible. The model reduction problem aims to simplify a model down to a practical simulation level with minimum loss in accuracy.

In portfolio optimization setup, MPT and its successors are used for dealing with the system identification and the model reduction problems.

The performance of the complete solution is monitored in the verification step. Practically, verification process does not aim to prove that a system will always work; instead, verification tries to locate the points of failure. The modern world benefits from the approximation of many real systems. As a result, verification has become vital for both industry and research.

In portfolio management systems, the performances of the algorithms are usually compared with specified benchmarks for verification purposes.

In this work, the success of the application of control engineering approaches to the financial portfolio management problem is investigated empirically. The problem is formulated mathematically by using MPT. Past financial data is used to estimate the input parameters of MPT framework.

A software program is developed in MATLAB to perform the financial analyses that are in the scope of the thesis and possible future work. The software program offers a graphical interface with which the users can analyze the past data, construct portfolios and run simulations.

The closing prices of stocks in Istanbul Stock Exchange (ISE) 30 index are used as past data.

The optimal strategy for managing the portfolio is decided by an empirical analysis and the performance of the chosen strategy is evaluated with respect to the performance of the ISE 30 index.

2. FINANCIAL RISK CONCEPT AND MEASURES

Financial markets are becoming more sophisticated in pricing, isolating, repackaging and transferring the risks. Many practical applications (risk limit calculations, trader performance reviews, portfolio optimization and etc.) depend on the measurement of the risk. Without a sound definition of the concept, it is unclear what such measurements reflect [11].

2.1. The Concept of Financial Risk

Two essential concepts need to be clarified before attempting to define the risk. These are uncertainty and exposure. Uncertainty is a state of not knowing whether an argument is true or false. When there is uncertainty regarding to a process, it is not possible to exactly describe the existing state and future outcomes. Probability is often used as a metric of uncertainty. Exposure defines the critical material consequences of an argument. The criticality of the outcomes is subjective and is directly related to one's perception. Utility is often used as a metric of exposure [11].

Uncertainty and exposure are two basic components of risk. Risk can be defined generally as exposure to an uncertain argument. Here are some examples of situations that involve risk:

- launching a new business
- heart surgery
- trading stocks
- sky diving

Two elements are common for all of the situations above. The first is that the outcomes of these situations are important for people. This relates to the exposure component of the risk. The second is that no one knows the outcomes of these situations for certain and this relates to the uncertainty component of the risk.

In the financial context, the risk denotes the uncertainty associated with an investment's return (both positive and negative returns should be considered).

The risk of a portfolio of financial assets consists of systematic risk and unsystematic (specific) risk [12].

Systematic risk represents the portion of the risk that cannot be reduced by diversification. It is also named as market risk or un-diversifiable risk. Systematic risk is the vulnerability to the events that affect the market in a broad sense. For instance events like earthquakes and major weather catastrophes have economy-wide consequences [13].

The systematic risk is the inherent risk that is present in the market and it cannot be eliminated by any clever portfolio choice. However the exposures of different industries to the systematic risk are not the same. For example, the effects of a financial crisis will differ for the banking and healthcare sectors.

Unlike systematic risk, specific risk can be diversified away. Specific risk defines the risk that is associated with an individual asset. Specific risk is also named as unsystematic risk or idiosyncratic risk. Unsystematic risk is the vulnerability to the events that affect only specific businesses or industries.

Unsystematic risk can be reduced or eliminated by a cleverly diversified portfolio. An example of a specific risk for a technology company might be the possible challenge over one of its patents.

2.2. Standard Deviation as a Measure of Risk

Generally, a measure is defined as a mapping from a set of random variables to the real numbers. The interpretation of the assigned numbers is named as a metric. Measures are employed to quantify metrics. Risk measures are used to quantify risk metrics. Some examples of risk metrics are volatility, beta and convexity. There are different ways to calculate a single risk metric. Each different way represents a different risk measure [14].

Standard deviation is a measure of volatility. Volatility is a risk metric that is used to interpret the variations in the price of a financial asset. The more an asset's price (or return) varies from the average, the more volatile the asset becomes [15].

In Figure 2.1, the values of two different portfolios over ten months are illustrated. Both portfolios increase in value from 7 Turkish Liras (TL) to 11 Turkish Liras. However their volatilities are clearly different.

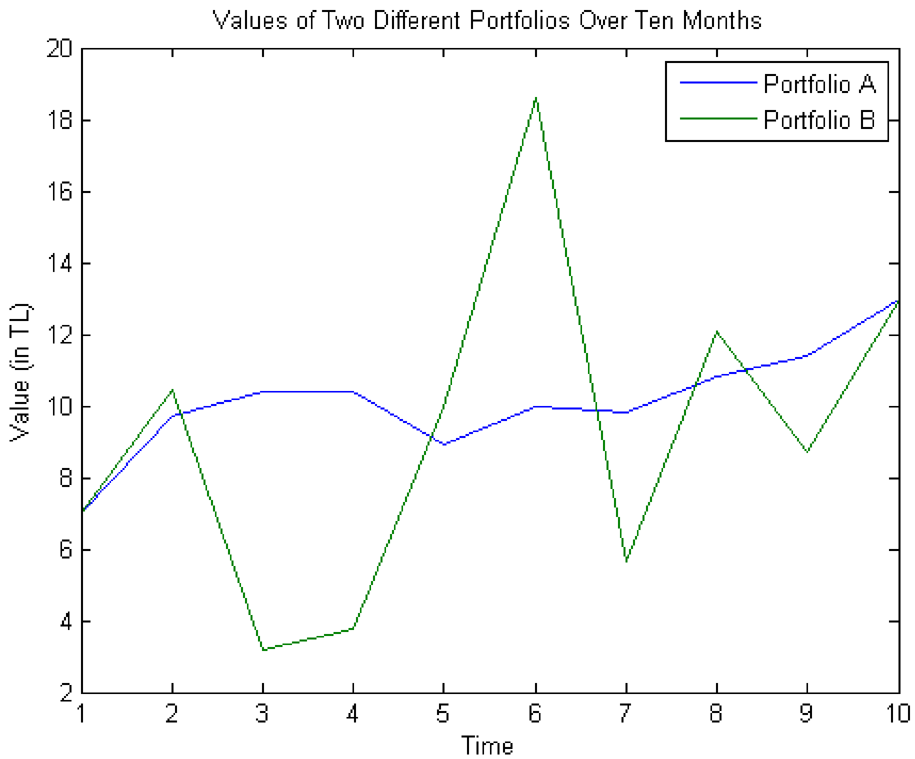

Figure 2.1. Portfolio B is more volatile than Portfolio A.

Standard deviation is a specific case of the more generalized risk measure concept named 'deviation risk measures'. A function is a deviation risk measure if it satisfies the conditions of shift-invariance, normalization, positive homogeneity, sub-linearity and positivity [16].

For a function $D : L^2 \rightarrow [0, +\infty]$ where L^2 is the L2 space of random portfolio returns, the mathematical definitions of these properties are given below:

- Shift Invariance : $D(X + r) = D(X)$ for any $r \in \mathbb{R}$
- Normalization : $D(0) = 0$
- Positive Homogeneity : $D(\lambda X) = \lambda D(X)$ for any $X \in L^2$ and $\lambda > 0$
- Sub-Linearity : $D(X + Y) \leq D(X) + D(Y)$ for any $X, Y \in L^2$
- Positivity : $D(X) > 0$ for all non-constant X, and $D(X) = 0$ for any constant X

Standard deviation is one of the simplest measures for risk. One of the two common shortcomings of the standard deviation as a risk measure is that it penalizes the marginal profits and the marginal losses in the same way. Possible upside and downside variations are involved equally in the calculation of the standard deviation. The other common drawback of the standard deviation is that it does not adequately account for the phenomenon of “fat tails” in loss distributions.

Value at Risk (VaR) and Conditional Value at Risk (also known as Expected Shortfall) are examples of more sophisticated risk measures. The common special characteristic of VaR and CVaR is that they both mark a boundary between normal events and extreme events. VaR and CVaR are used practically by many risk management companies [17, 18].

To keep the focus of the discussion on the control engineering aspects, standard deviation is employed as the measure of risk for the analyses that are done in the scope of this thesis.

3. MODERN PORTFOLIO THEORY

Modern Portfolio Theory (MPT) attempts to maximize the expected return of a portfolio for a given amount of risk, or equivalently minimize the amount of risk for a given level of expected return, by exploring various combinations of asset proportions.

MPT provides a mathematical formulation for the diversification concept in financial investments. MPT analyzes the changes in the price of an asset relative to the changes in the prices of the other assets. The theory aims to reduce the total risk on the return of the portfolio by combining different assets whose returns are not perfectly positively correlated [19].

Basic ideas and assumptions of the theory can be listed as follows:

- MPT models the asset returns as elliptically distributed random variables.
- The risk is defined as the standard deviation of the return.
- Portfolios are modeled as weighted combinations of assets.
- MPT assumes that the investors are rational and the markets are efficient.

According to MPT, investing is a tradeoff between the expected return and the risk. For a given amount of risk, MPT describes how to select a portfolio with the highest possible expected return. Alternatively, for a given expected return, MPT describes how to select a portfolio with the lowest possible risk. MPT explains mathematically how to find the best possible diversification strategy [20].

The remaining parts of this chapter mainly follow [21].

3.1. Expected Return and Standard Deviation

In the presence of risk, the outcome of any action is not known with certainty. Outcomes are usually represented by frequency distribution functions. A frequency

Table 3.1. An Example Frequency Distribution Function.

Return	**Probability**	**Event**
12%	$\frac{1}{3}$	1
9%	$\frac{1}{3}$	2
6%	$\frac{1}{3}$	1

distribution function is a listing of all possible outcomes along with the probability of the occurrence of each. Table 3.1 shows an example of a frequency distribution function. The investment has three possible returns. If event 1 occurs, the investor receives a return of 12%; if event 2 occurs, he receives 9% and if event 3 occurs, he receives 6%.

The possibilities for real assets are sufficiently numerous that developing a table like Table 3.1 for each asset becomes a very complex task. In general, two basic measures are used for capturing the characteristics of a frequency function. The first one is used for measuring the average value of the frequency function and the second one is used for measuring the dispersion around the average value.

3.1.1. Determining the Average Outcome

The average outcome is determined by calculating the expected value of the frequency function.

Symbol R_{ij} denotes the jth possible outcome for the return on asset i. P_{ij} is the probability of the jth return on the ith asset and M is the total number of possible events.

$$E[R_i] = \sum_{j=1}^{M} P_{ij} R_{ij} \qquad (3.1)$$

3.1.2. A Measure of Dispersion

Having a measure of the average return by itself is not sufficient. It is of critical importance to have some measure to see how much the outcomes differ from the average. The dispersion from the average is measured by standard deviation. The formula for the standard deviation of the return on the ith asset is given in Equation 3.2.

$$\sigma_i = \sqrt{\sum_{j=1}^{M} [P_{ij}(R_{ij} - E[R_i])^2]} \tag{3.2}$$

In general, the summary statistics mean and standard deviation (or equivalently mean and variance) are used instead of the full frequency function. In our representation, R_{ij} are assumed to be normally distributed random variables and a random variable with a normal distribution is fully described by its mean and variance.

3.2. Characteristics of Portfolios in General

The return of a portfolio of assets is simply a weighted average of the return on the individual assets. The weight applied to each return is the fraction of the portfolio invested in the related asset.

Let R_{Pj} represent the jth possible return on the portfolio. X_i is the fraction of the investor's funds invested in the ith asset and the total number of assets is N. The expression for the return is given by Equation 3.3.

$$R_{Pj} = \sum_{i=1}^{N} (X_i R_{ij}) \tag{3.3}$$

The expected return of the portfolio becomes:

$$E[R_P] = E\left[\sum_{i=1}^{N}(X_i R_{ij})\right] = \sum_{i=1}^{N} E[X_i R_{ij}] = \sum_{i=1}^{N}(X_i E[R_{ij}]) \tag{3.4}$$

From Equation 3.4, we see that the expected return of a portfolio is a weighted average of the expected returns of individual assets.

The risk of a portfolio is different from a simple average of the risk of the individual assets in the portfolio.

Let σ_P represent the standard deviation of the portfolio return. The expression for the standard deviation of the portfolio return is given by Equation 3.5.

$$\sigma_P = \sqrt{\sum_{j=1}^{N}(X_j \sigma_j)^2 + \sum_{j=1}^{N}\sum_{\substack{k=1\\k\neq j}}^{N}(X_j X_k \sigma_{jk})} \tag{3.5}$$

The formula for the variance of the portfolio return consists of two parts:

- Variance Part:

$$\sum_{j=1}^{N}(X_j \sigma_j)^2 \tag{3.6}$$

- Covariance Part:

$$\sum_{j=1}^{N}\sum_{\substack{k=1\\k\neq j}}^{N}(X_j X_k \sigma_{jk}) \tag{3.7}$$

By careful choice of the assets and careful adjustment of the asset weights, the contribution of the covariance part to the total variance of the portfolio return can be lowered.

3.3. Efficient Frontier

In theory, an investor can consider all possible combinations of assets one by one as his/her investment portfolio. Such a strategy would require an infinite number of possibilities to be considered. If we could find a set of portfolios that

- offered a bigger return for the same risk
- offered a lower risk for the same return

we would have identified the portfolios in which the rational investor has interest. All other portfolios could be ignored.

If all the possibilities in the risk-return space were plotted, the resulting diagram would look like Figure 3.1.

Examine the portfolios A and B in Figure 3.1. Portfolio B would be preferred by all rational investors to portfolio A because it offers a higher return with the same level of risk. One can also see that portfolio C would be preferable to Portfolio A because it offers less risk at the same level of return. At this point in the analysis, there are no portfolios that can dominate portfolio B or portfolio C. It is obvious that an efficient set of portfolios cannot include points from the interior region of the shaded area.

Examine the point D in Figure 3.1. Point D can be eliminated from consideration by the existence of portfolio E. Portfolio E has more return for the same risk. The same logic holds for every other portfolio as we move up the outer shell from point D to point C. Portfolio C cannot be eliminated since there is no portfolio in our set that has less risk for the same return or more return for the same risk. Point C represents global minimum risk portfolio in our investment universe.

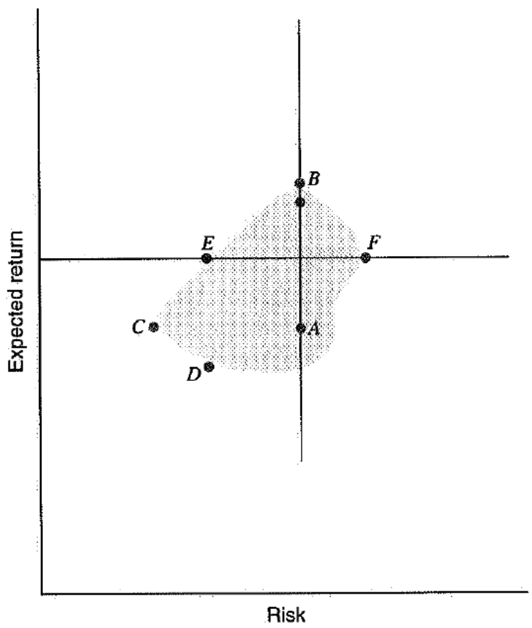

Figure 3.1. Risk and Return Possibilites for Various Portfolios (Each point in the risk-return space represents a portfolio).

Examine the point F in Figure 3.1. There is no need to consider portfolio F because portfolio E has less risk for the same return. As we move up the outer shell curve from point F, all portfolios are dominated until we come to portfolio B. Portfolio B cannot be eliminated since there is no portfolio in our set that has the same return and less risk or the same risk and more return than portfolio B. Point B represents the portfolio that offers the highest expected return in our investment universe. The portfolio with the highest expected return usually consists of single asset.

The efficient set consists of the envelope curve of all the portfolios that lie between global minimum risk portfolio (point C for Figure 3.1) and the maximum return portfolio (point B for Figure 3.1). This set of points is called the efficient frontier. Figure 3.2 illustrates an example efficient frontier graph.

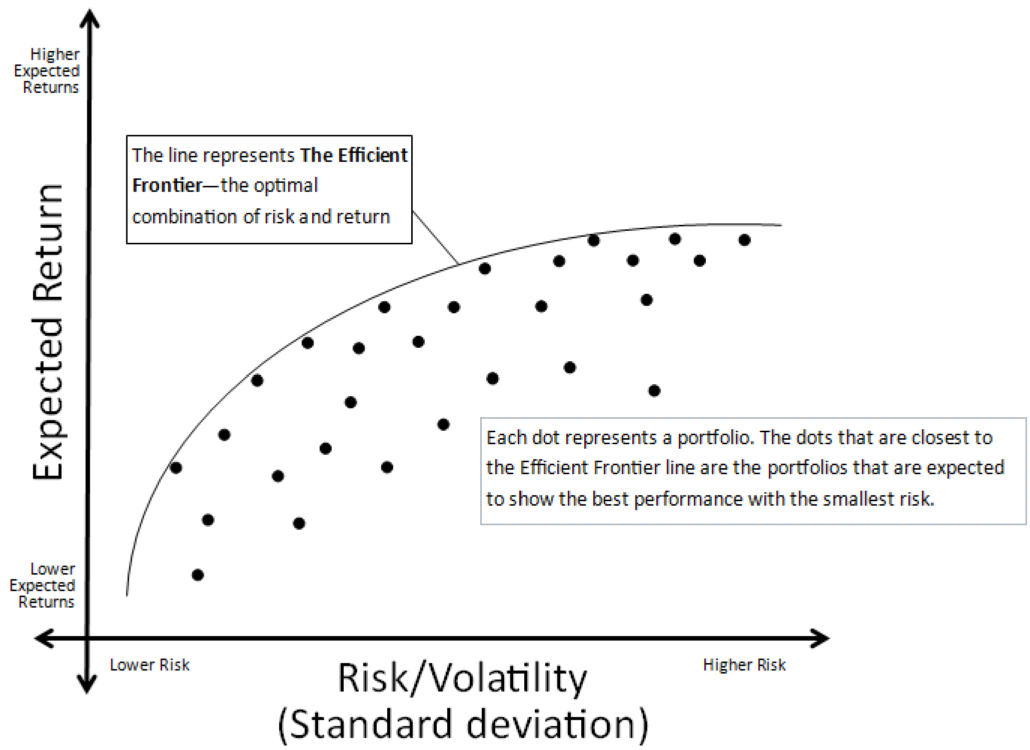

Figure 3.2. The Efficient Frontier.

4. PORTFOLIO OPTIMIZATION

The construction of the efficient frontier requires the optimal portfolios to be sorted out from all possible portfolio combinations. The portfolios on the efficient frontier can be obtained by solving a quadratic programming problem.

4.1. Mathematical Model of the Problem

The mathematical representation of the problem results in a constrained optimization problem [4].

The portfolio optimization problem is:

$$\text{minimize} \qquad \sum_{i=1}^{N}\sum_{j=1}^{N} w_i w_j \sigma_{ij} \tag{4.1}$$

subject to the following constratins:

$$\sum_{i=1}^{N} w_i \mu_i = R^* \tag{4.2}$$

$$\sum_{i=1}^{N} w_i = 1 \tag{4.3}$$

$$0 \leq w_i \leq 1, i = 1, ..., N \tag{4.4}$$

Where,

N is the number of assets available.

μ_i is the expected return of asset i $(i = 1, ..., N)$.

σ_{ij} is the covariance between assets i and j $(i = 1, ..., N;\ j = 1, ..., N)$.

R^* is the desired expected return.

w_i is the decision variable that represents the proportion $(0 \leq w_i \leq 1)$ held of asset i $(i = 1, ..., N)$.

Equation 4.1 minimizes the total risk associated with portfolio. Equation 4.2 ensures that the portfolio has the desired expected return, which is R^*. Equation 4.3 makes sure that the proportions add to one. Equation 4.4 ensures that there is no short selling of any stocks.

The mathematical formulation above is a quadratic (non-linear) programming problem.There are many computationally effective algorithms available in the literature for solving quadratic programming (QP) problems. Thanks to these algorithms, the generation of the efficient frontier for any particular portfolio set becomes a straightforward process. The efficient frontier can be obtained by solving the described QP for varying values of R^*.

An alternative formulation of the problem can be presented by the introduction of the weighting parameter λ $(0 \leq \lambda \leq 1)$:

$$\text{minimize} \qquad \lambda\left[\sum_{i=1}^{N}\sum_{j=1}^{N} w_i w_j \sigma_{ij}\right] - (1-\lambda)\left[\sum_{i=1}^{N} w_i \mu_i\right] \tag{4.5}$$

subject to the constratins given by Equation 4.3 and Equation 4.4.

For Equation 4.5, the case $\lambda = 0$ implies maximization of the expected return (irrespective of the involved risk). In this case the optimal portfolio will consist of only a single asset, the asset with the highest expected return. The case $\lambda = 1$ implies the minimization of the portfolio risk. The optimal portfolio for the minimum risk solution will in general contain a number of assets. The values of λ that satisfies the condition $0 < \lambda < 1$ represent the tradeoff between the risk and the return. Hence λ can be seen as a parameter that quantifies risk aversion.

In this study, the QP problem that is defined by Equation 4.5, Equation 4.3 and Equation 4.4 is solved by using MATLAB Optimization Toolbox. [22]

A trust region based algorithm is employed for the solution of the constrained optimization problem [23, 24].

4.2. Empirical Analysis Framework

An empirical framework is setup for the investigation of the effects of various investment decisions. The framework is concerned with the execution of two main exercises:

- Estimation of the characteristics of the return distribution.
- Grouping of the investors in terms of their risk tolerances.

For estimating the parameters of the overall return distribution, we start with the assumption that the daily closing prices of the assets follow a geometric random walk process [25].

Let

$\mathbf{S}^d$ be the vector containing the daily closing prices of N assets on date d,

$\mathbf{R}$ represent the multivariate random vector of the asset returns,

$\mathbf{L}$ represent the multivariate random vector of the asset log-returns,

The geometric random walk process of the prices can be described by Equation 4.6 and Equation 4.7. Equation 4.7 indicates that the asset returns follow a multivariate log-normal distribution.

$$\mathbf{S}^d = \mathbf{S}^{d-1}\mathbf{R}_{daily} \tag{4.6}$$

$$\mathbf{R}_{daily} \sim ln\mathcal{N}(\mathbf{A}_{daily}, \mathbf{B}_{daily}) \tag{4.7}$$

The logarithms of the daily closing prices follow a simple normal random walk which can be decribed by Equation 4.8, Equation 4.9 and Equation 4.10. Equation 4.10 indicates that the asset log-returns follow a multivariate normal distribution.

$$\mathbf{R}_{daily} = \exp(\mathbf{L}_{daily}) \tag{4.8}$$

$$\log(\mathbf{S}^{d}) = \log(\mathbf{S}^{d-1}) + \mathbf{L}_{daily} \tag{4.9}$$

$$\mathbf{L}_{daily} \sim \mathcal{N}(\boldsymbol{\mu}_{daily}, \boldsymbol{\Sigma}_{daily}) \tag{4.10}$$

The parameters of the distributions in Equation 4.7 and Equation 4.10 are not directly observable by the investor.

The parameters of the multivariate log-return distribution (Equation 4.10) are estimated $(\hat{\boldsymbol{\mu}}, \hat{\boldsymbol{\Sigma}})$. Transformations are applied respectively on the estimated parameters of the multivariate log-return distribution to obtain the parameters of the regular multivariate return distribution [26].

The transformations for obtaining the vector of expected returns and the covariance matrix of returns are given by Equation 4.11 and Equation 4.12.

$$\hat{\mathbf{A}} = \exp\left(\hat{\boldsymbol{\mu}} + \frac{Diag\left(\hat{\boldsymbol{\Sigma}}\right)}{2}\right) \tag{4.11}$$

$Diag\left(\hat{\boldsymbol{\Sigma}}\right)$ represents the vector that is composed of the diagonal entries of the covariance matrix of log-returns.

$$\hat{b}_{ij} = \exp\left(\hat{\sigma}_{ij} - 1\right)\exp\left(\hat{\mu}_i + \hat{\mu}_j + \frac{\hat{\sigma}_{ii} + \hat{\sigma}_{jj}}{2}\right) \tag{4.12}$$

$\hat{b}_{ij}$ represents the entry of the estimated covariance matrix of **returns** at the ith column and jth row.$\hat{\sigma}_{ij}$ represents the entry of the estimated covariance matrix of **log-returns** at the ith column and jth row.

Historical data is used for estimating the parameters of the multivariate log-return distribution.

$\mathbf{L}^{d}_{daily}$ is the $1 \times N$ vector of the daliy log returns on date d.

4.2.1. Estimation of Expected Log-Return

The average vector of the daily log-returns is used as a bias free estimator for the expected value of the vector of daily log-returns.D is the total number of trading days.

$$\hat{\boldsymbol{\mu}}_{daily} = \frac{\sum_{d=1}^{D} \mathbf{L}^{d}_{daily}}{D} \tag{4.13}$$

The estimations for the expected values of the weekly,monthly and yearly log-returns are given by Equation 4.14, Equation 4.15 and Equation 4.16.[1]

$$\hat{\boldsymbol{\mu}}_{weekly} = \hat{\boldsymbol{\mu}}_{daily} \times 5 \tag{4.14}$$

[1]The number of trading days in a month is assumed to be 22 and the number of trading days in a year is taken as 258.

$$\hat{\boldsymbol{\mu}}_{monthly} = \hat{\boldsymbol{\mu}}_{daily} \times 22 \tag{4.15}$$

$$\hat{\boldsymbol{\mu}}_{yearly} = \hat{\boldsymbol{\mu}}_{daily} \times 258 \tag{4.16}$$

The general formula for the estimation of the expected log-return over d trading days is:

$$\hat{\boldsymbol{\mu}}_{ddays} = \hat{\boldsymbol{\mu}}_{daily} \times d \tag{4.17}$$

4.2.2. Estimation of Covariance Matrix of Log-Returns

For the estimation of the covariance matrix of daily log-returns, the matrix of the daily log-returns is formed. The matrix of the daily log-returns is given by Equation 4.18. l_i^d represents the daily log return of the ith asset on date d.

$$\mathbf{M}_{daily\,logreturns} = \begin{bmatrix} \vdots \\ \mathbf{L}_{daily}^{d-1} \\ \mathbf{L}_{daily}^{d} \\ \mathbf{L}_{daily}^{d+1} \\ \vdots \end{bmatrix} = \begin{bmatrix} \ddots & \vdots & \vdots & \vdots & \iddots \\ \dots & l_{i-1}^{d-1} & l_i^{d-1} & l_{i+1}^{d-1} & \vdots \\ \dots & l_{i-1}^{d} & l_i^{d} & l_{i+1}^{d} & \vdots \\ \dots & l_{i-1}^{d+1} & l_i^{d+1} & l_{i+1}^{d+1} & \vdots \\ \iddots & \dots & \dots & \dots & \ddots \end{bmatrix} \tag{4.18}$$

The covariance matrix of the daily log-returns is estimated as:

$$\hat{\mathbf{\Sigma}}_{daily} = E\left[\left(\mathbf{M}_{daily\,logreturns} - E[\mathbf{M}_{daily\,logreturns}]\right)\left(\mathbf{M}_{daily\,logreturns} - E[\mathbf{M}_{daily\,logreturns}]\right)^T\right] \tag{4.19}$$

$$\hat{\mathbf{\Sigma}}_{daily} = \begin{bmatrix} \ddots & \vdots & \vdots & \vdots & \iddots \\ \dots & \hat{\sigma}^d_{(i-1)(j-1)} & \hat{\sigma}^d_{(i-1)(j)} & \hat{\sigma}^d_{(i-1)(j+1)} & \vdots \\ \dots & \hat{\sigma}^d_{(i)(j-1)} & \hat{\sigma}^d_{(i)(j)} & \hat{\sigma}^d_{(i)(j+1)} & \vdots \\ \dots & \hat{\sigma}^d_{(i+1)(j-1)} & \hat{\sigma}^d_{(i+1)(j)} & \hat{\sigma}^d_{(i+1)(j+1)} & \vdots \\ \iddots & \dots & \dots & \dots & \ddots \end{bmatrix} \tag{4.20}$$

Again, $\hat{\sigma}^d_{ij}$ represents the entry of the estimated covariance matrix of daily log-returns at the ith column and jth row.

The estimations for the weekly, monthly and yearly covariances of the log-returns of the assets i and j are given by Equation 4.21, Equation 4.22 and Equation 4.23.

$$\hat{\sigma}^{weekly}_{ij} = \hat{\sigma}^{daily}_{ij} \times \sqrt{5} \tag{4.21}$$

$$\hat{\sigma}^{monthly}_{ij} = \hat{\sigma}^{daily}_{ij} \times \sqrt{22} \tag{4.22}$$

$$\hat{\sigma}^{yearly}_{ij} = \hat{\sigma}^{daily}_{ij} \times \sqrt{258} \tag{4.23}$$

The general formula for the estimation of the covariance of the log-returns of the assets i and j over d trading days is:

$$\hat{\sigma}_{ij}^{d\,days} = \hat{\sigma}_{ij}^{daily} \times \sqrt{d} \tag{4.24}$$

4.2.3. Risk Grouping of Investors and Portfolios

The investors are grouped into three categories according to their risk tolerances:

- Risk Avoiders
- Contolled Risk Takers
- Adventurers

For our analyses, we have chosen three different portfolios on the efficient frontier are marked for respective investor types (Figure 4.1). The risk levels of the portfolios are set by the adjustment of the weighting parameter λ in Equation 4.5.

- Low Risk Portfolio : The portfolio choice for the risk avoiders. ($\lambda = 0.9$)
- Med Risk Portfolio : The portfolio choice for the contolled risk takers. ($\lambda = 0.5$)
- High Risk Portfolio : The portfolio choice for the adventurers. ($\lambda = 0.1$)

4.2.4. Analysis of Investment Scenarios

Using the historical data, various investment scenarios are simulated. The variables that are used in the simulations are as listed follows:

- Risk Tolerance (RT): The risk tolerance of the investor.
- Past Time Frame (PTF): The length of the historical data that is used for esti-

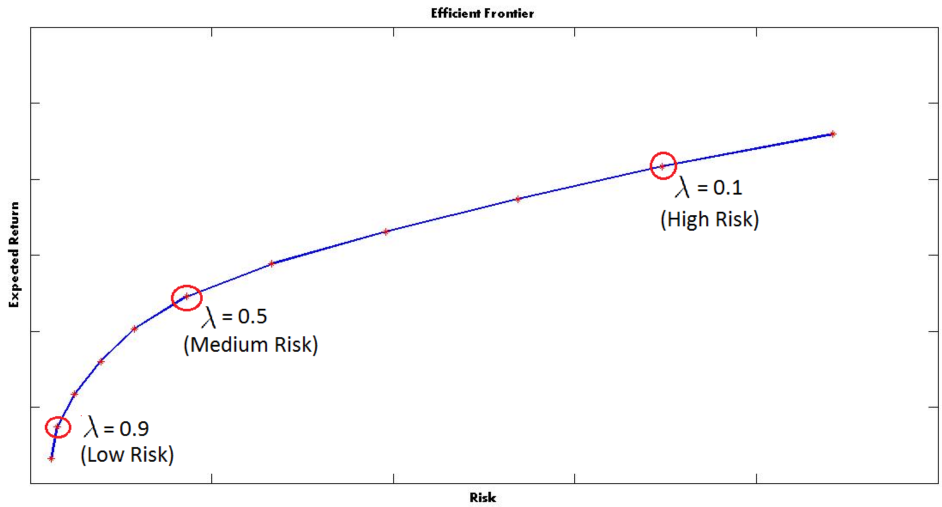

Figure 4.1. The Portfolios for Different Risk Levels (Using Equation 4.5).

mating the parameters of the multivariate return distribution.

- Future Time Frame (FTF): The investment horizon
- Investment Date (ID): The starting date of investment (the reference date for PTF and FTF)

A multi-dimensional analysis is done by changing the values of the simulation variables inside a nested for loop structrue (Figure 4.2). For every ID, a three dimensional matrix of portfolio returns is obtained (RT, PTF and FTF). The 3-D matrix can be divided into three return tables for low, medium and high risk portfolios (Figure 4.3).

The average matrix of the three dimensional matrices (average over the variable ID) is calculated to determine the most useful investment plan on the average.

```
for Investment Date do
  for Risk Tolerance do
    for Past Time Frame do
      for Future Time Frame do
        Estimate the Parameters of the Log-Return Distribution
        Convert the Parameters of Log-Return Distribution to Return Parameters
        Form the Efficient Frontier
        Find the Optimal Portfolio for the Given Level of Risk
        Build the Portfolio with Optimal Weights
        Calculate the Actual Return of the Optimal Portfolio (Using Future Data)
      end for
    end for
  end for
end for
```

Figure 4.2. Multi-Dimensional Analysis on Investment Strategies.

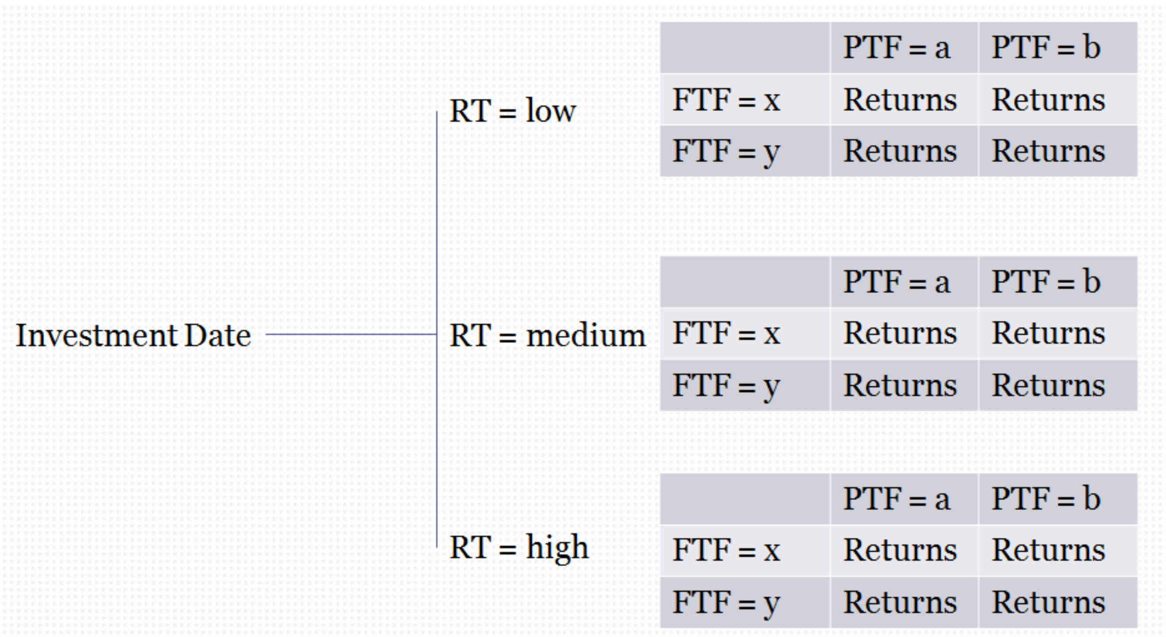

Figure 4.3. Expansion of The Matrix of Simulated Returns.

5. ANALYSIS SOFTWARE AND DATA

The empirical analyses are done using the historical data from Istanbul Stock Exchange (ISE). The investment universe is limited to the stocks that are listed in the ISE30 index. The historical data at hand, covers the closing prices of the stocks between the dates 01 Jan 2005 and 31 Dec 2010. ISE30 list is updated periodically by Istanbul Stock Exchange, only the data for 26 stocks which were on the list in every year between 2005 and 2011 is used for simplicity. The investment universe contains the stocks of the following companies:

- AKBNK: Akbank
- ARCLK: Arcelik
- DENIZ: Denizbank
- DOAS: Dogus Automotive Service
- DOHOL: Dogan Holding
- DYHOL: Dogan Media Holding
- EREG: Eregli Iron and Steel
- FINBN: Finansbank
- GARAN: Garanti Bank
- HURGZ: Hurriyet Media
- ISCTR: Is Bankasi
- ISGYO: Is GYO
- KCHOL: Koc Holding
- PETKM: Petkim
- PTOFS: Petrol Office
- SAHOL: Sabanci Holding
- SISE: Sisecam
- SKBNK: Sekerbank
- TCELL: Turkcell
- THYAO: Turkish Airlines
- TOASO: Tofas

- TSKB: Industrial Development Bank of Turkey
- TUPRS: Turkish Petrolium Rafineries
- ULKER: Ulker
- VESTL: Vestel
- YKBNK: Yapi Kredi Bank

The raw data that is provided by ISE is processed. Sevreal missing entries (27 closing prices among 6 x 258 x 26 closing prices) are interpolated using cubic spline interpolation method. In the mathematical field of numerical analysis, spline interpolation is a form of interpolation where the interpolant is a special type of piecewise polynomial called a spline. A third order polynomial is used as the interpolant for cubic spline [27, 28].

A MATLAB software together with a graphical user interface is developed for empirical analyses (Figure 5.1).

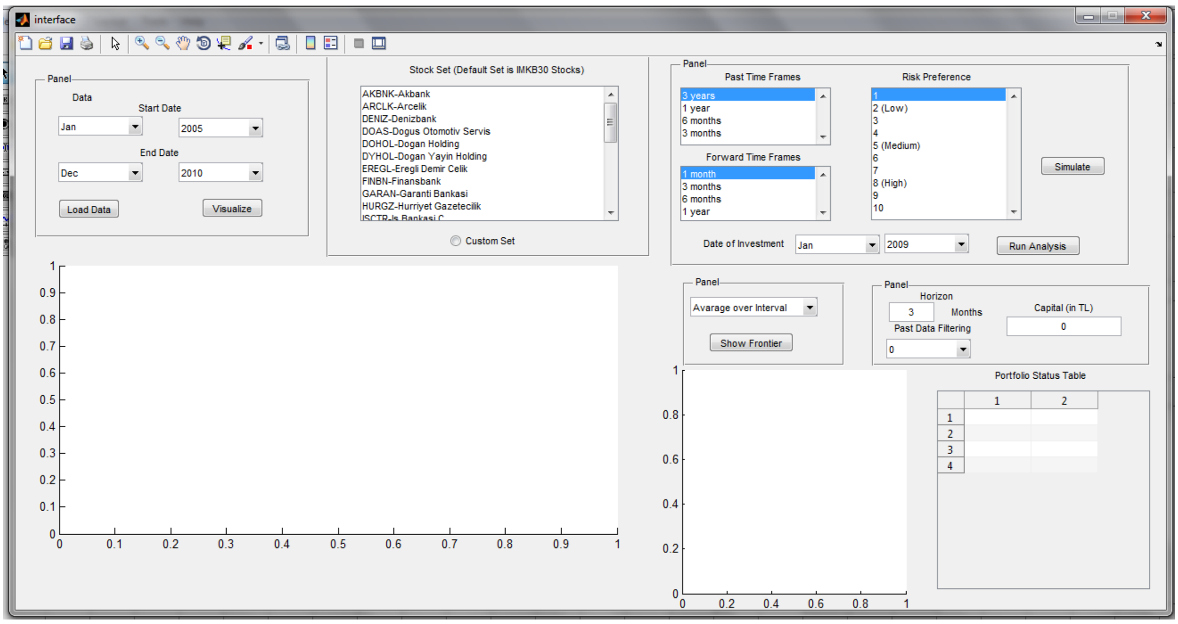

Figure 5.1. An Example View of Analysis Software.

The software enables the users to:

- Set a number of different values for the length of historical data (PTF)
- Set a number of different values for the investment horiozn (FTF)
- Analyze various risk choices on the efficient frontier (RT)
- See all possible portfolio proportions for the portfolios in the efficient frontier
- Invest at different past times (ID)
- Plot historical data

The software has a modular structure. Other data and different measures of risk can be integrated easily into the software for further research. More information about the sofware can be found in Appendix A.

6. RESULTS

6.1. Estimation of Covariance Matrix of Log-Returns for ISE30 Stocks

The estimation of the covariance matrix of the log-returns for ISE30 stocks requires the estimation of $\frac{(26x26)}{2} = 338$ parameters.[2] Since the number of the parameters to be estimated is large, a large amount of historical data must be used for statistical stability. On the other hand, a smaller amount of data can be used for the estimation of the expected log-returns (26 parameters). For this reason, the PTF parameter (which represents the length of the historical data that is used for the estimation) is handled differently for the estimation of the expected value of the log-returns and the covariance matrix of log-returns.

- PTF_{Σ} represents the length of the historical data that is used for the estimation of the covariance matrix of log-returns.
- PTF_{μ} represents the length of the historical data that is used for the estimation of the expected log-returns.

For all of the analyses in the thesis PTF_{Σ} is taken as two years constant.

6.2. Annualized Returns

In the analyses, the returns for different investment horizons are compared with each other. For example, 6% return of an investment with a maturity of three months is to be compared with the 10% return of another investment with a maturity of six motnths. To enable this type of comparisons all returns are annualized compoundly. The formula for annualizing the returns is given by Equation 6.1. The annual returns

[2] ISE30 list is updated periodically by Istanbul Stock Exchange, only the 26 stocks which were on the list in every year between 2005 and 2011 are used.

for the example case become $1.06^{\frac{12}{3}} - 1 = 26\%$ and $1.10^{\frac{12}{6}} - 1 = 21\%$.

$$Annual\ Return = \left((1 + Return)^{\left(\frac{12}{FTF\ in\ Months}\right)} - 1\right).100\% \tag{6.1}$$

6.3. Sweeping of Simulation Variables

- Investment Dates (ID): The return simulations are done for 24 different investment dates. The investments are done on the last trading day of every month between Jan 2008 and Dec 2009.
- Risk Tolerances (RT): The return simulations are repeated for all risk levels.
  - (i) Low Risk
  - (ii) Medium Risk
  - (iii) High Risk
- Past Time Frames (PTF_μ): In the return simulations, the following values are set for the length of historical data that is used for estimation of the expected log-returns:
  - (i) 3 years
  - (ii) 2 years
  - (iii) 1 year
  - (iv) 9 months
  - (v) 6 months
  - (vi) 3 months
- Future Risk Frames (FTF): In the return simulations, the following values are set as the investment horizon.
  - (i) 1 year
  - (ii) 9 months
  - (iii) 6 months
  - (iv) 3 months
  - (v) 2 months
  - (vi) 1 months

The average matrix of the 24 three dimensional matrices is calculated and an average return table is formed respectively for each risk level. The average return tables for low, medium and high risk level portfolios are given in Table 6.1, Table 6.2 and Table 6.3.

From the average return tables we see that the best results on the average is observed when $PTF_\mu = 9\,months$ and $FTF = 1\,month$ for all risk levels.

Table 6.1. Summary of the Periodic Successes of Low Risk Level Dynamic Portfolios with Different Update Rates.

Average of All Tables	$PTF_\mu = 3\,months$	$6\,months$	$9\,months$	$1\,year$	$2\,years$	$3\,years$
$FTF_\mu = 1\,month$	87%	88%	94%	87%	88%	92%
$2\,months$	46%	45%	44%	44%	45%	46%
$3\,months$	37%	36%	33%	34%	34%	34%
$6\,months$	48%	48%	51%	50%	43%	44%
$9\,months$	64%	68%	68%	69%	57%	60%
$1\,year$	55%	57%	57%	55%	47%	50%

Table 6.2. Summary of the Periodic Successes of Low Risk Level Dynamic Portfolios with Different Update Rates.

Average of All Tables	$PTF_\mu = 3\,months$	$6\,months$	$9\,months$	$1\,year$	$2\,years$	$3\,years$
$FTF_\mu = 1\,month$	59%	71%	82%	56%	58%	71%
$2\,months$	38%	34%	23%	23%	29%	33%
$3\,months$	35%	25%	11%	12%	16%	18%
$6\,months$	33%	24%	25%	24%	19%	23%
$9\,months$	46%	39%	40%	42%	25%	32%
$1\,year$	42%	33%	36%	29%	24%	29%

6.4. Performance Against the Benchmark

The performance of the (empirically) best investment strategy is tested against the performance of ISE30 index by creating a dynamic virtual portfolio.

- The portfolio is updated every month. (FTF = 1 month)

Table 6.3. Summary of the Periodic Successes of Low Risk Level Dynamic Portfolios with Different Update Rates.

Average of All Tables	$PTF_{\mu} = 3\,months$	$6\,months$	$9\,months$	$1\,year$	$2\,years$	$3\,years$
$FTF_{\mu} = 1\,month$	62%	65%	113%	68%	35%	27%
$2\,months$	39%	26%	13%	13%	3%	7%
$3\,months$	37%	19%	-5%	-8%	-9%	-7%
$6\,months$	18%	3%	-4%	0%	-4%	-1%
$9\,months$	28%	17%	10%	15%	2%	6%
$1\,year$	29%	18%	16%	7%	4%	10%

- 9 months of historical data is used for the estimation of the parameters of the return distribution.
- The investments start in Jan 2007. (48 months until Dec 2010)
- Trading costs (consisting of the buy/sell spread and the fees) and dividend payments are ignored

The performance of the dynamic portfolio is compared with the performace of ISE30 index. The performances of the high,medium and low risk level portfolios are plotted together with the performance of ISE30 index in Figure 6.1, Figure 6.2 and Figure 6.3.

In Figure 6.4, the performance of ISE30 index and the performances of low,medium and high risk level portfolios are all shown together. The effect of the financial crisis in 2008 is clearly visible at all risk levels.

High risk level portfolios contain only a few number of stocks (sometimes only two stocks) and a major decrease in the price of one of the stocks in the portfolio means a major decrease in the value of the portfolio. Therefore high risk portfolios are very sensitive to financial crises. The performance of the high risk level dynamic portfolio is above the ISE30 index until the middle of year 2008 (Figure 6.1). The value of the portfolio decreases very sharply during the financial crisis in 2008 and the losses can not be recovered for the rest of the investment period.

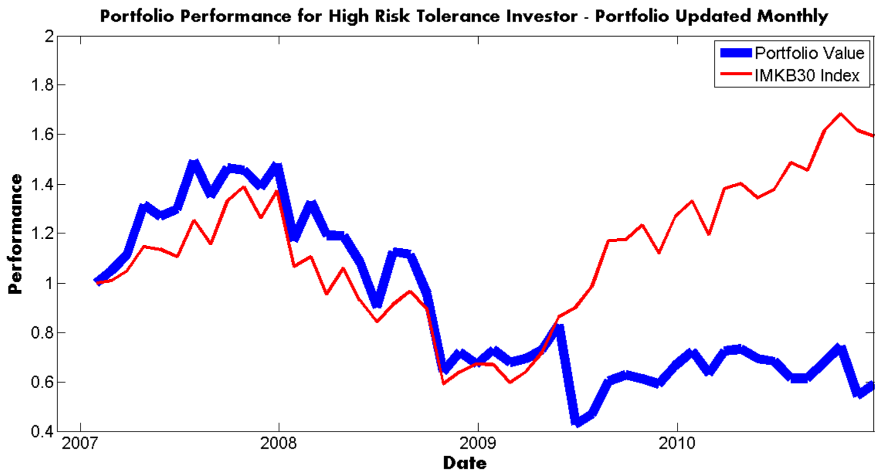

Figure 6.1. The Performance of Montly Updated High Risk Level Dynamic Portfolio Against ISE30 Index.

The performance of the medium risk level dynamic portfolio follows the performance of the ISE30 index closely until year 2010. The performance of the index in year 2010 dominates the performance of the medium risk level dynamic portfolio. At the end of the investment period, the return of the medium risk level dynamic portfolio is around 5% whereas the return of the index is 60% (Figure 6.2).

Low risk level portfolios contain a number of stocks and are more resistant to the shocks in the market. The low risk level dynamic portfolio has been able recover from the 2008 financial crisis and achieve a return of 40% at the end of the investment period (Figure 6.3).

The update rate of the proportions of the stocks in the portfolio is increased to achieve better results. For all risk levels, the return simulations are repeated for the porfolio update rates of

- 15 days,
- 7 days,
- 1 day.

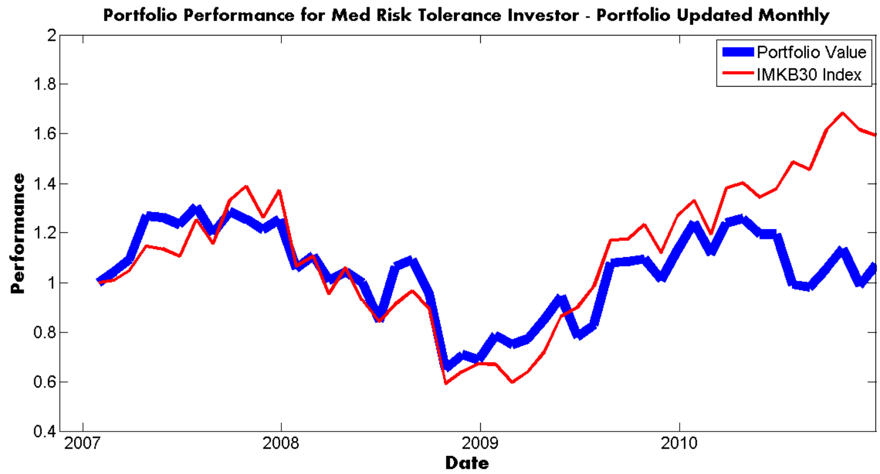

Figure 6.2. The Performance of Montly Updated Medium Risk Level Dynamic Portfolio Against ISE30 Index.

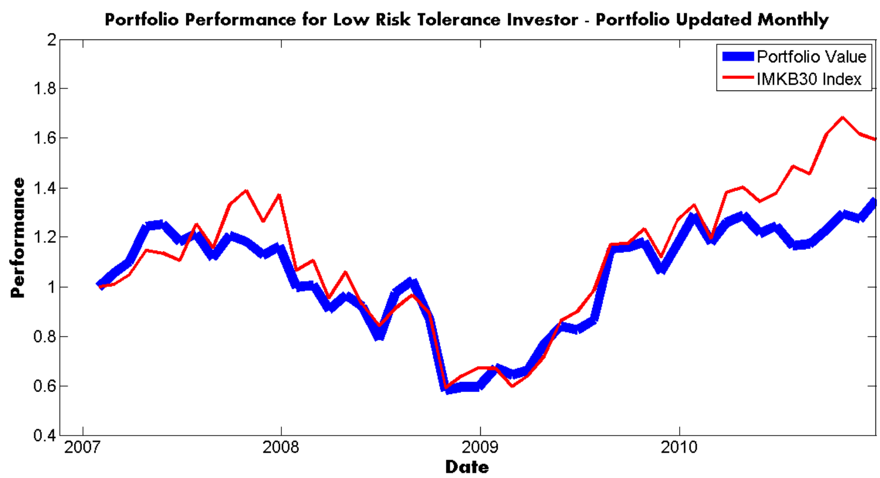

Figure 6.3. The Performance of Montly Updated Low Risk Level Dynamic Portfolio Against ISE30 Index.

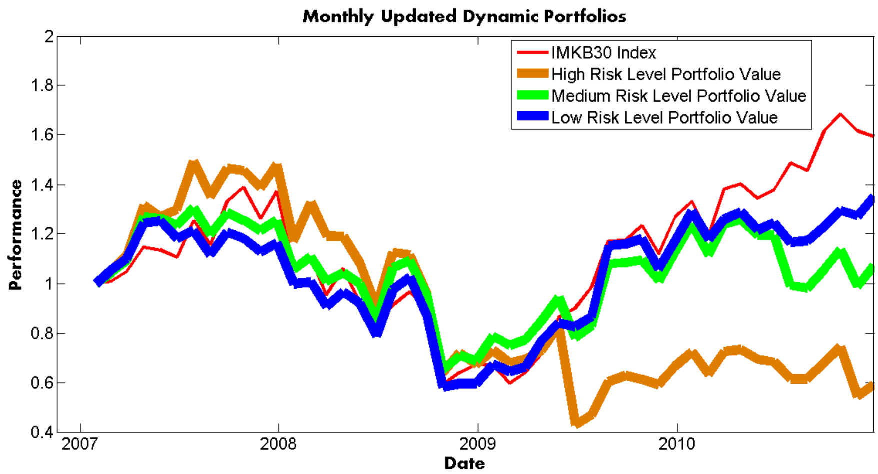

Figure 6.4. The Performances of Monthly Updated Portfolios for Different Risk Levels Against ISE30 Index.

while keeping $PTF_{\mu} = 9\, months$ and $FTF = 1\, month$.

The performances of the high,medium and low risk level portfolios that are updated every 15 days are shown in Figure 6.5, Figure 6.6 and Figure 6.7. In Figure 6.8, the performance of ISE30 index and the performances of low,medium and high risk level portfolios that are updated every 15 days are all shown together.

The performances of the high,medium and low risk level portfolios that are updated weekly are shown in Figure 6.9, Figure 6.10 and Figure 6.11. In Figure 6.12, the performance of ISE30 index and the performances of low,medium and high risk level portfolios that are updated weekly are all shown together.

At the end of the investment period the weekly updated medium risk level dynamic portfolio yields a return of 70% (10% more than the return of the index).

The performances of the high,medium and low risk level portfolios that are updated daily are shown in Figure 6.13, Figure 6.14 and Figure 6.15. In Figure 6.16, the performance of ISE30 index and the performances of low,medium and high risk level portfolios that are updated daily are all shown together. The daily updated dynamic

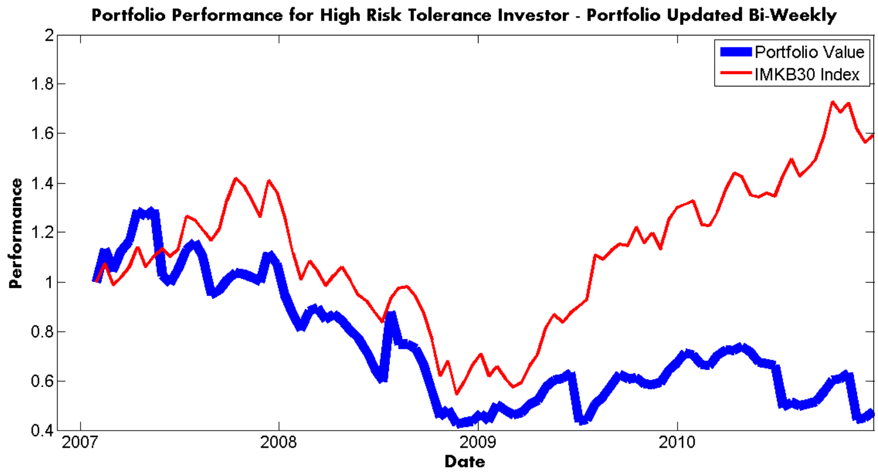

Figure 6.5. The Performance of Bi-Weekly Updated High Risk Level Dynamic Portfolio Against ISE30 Index.

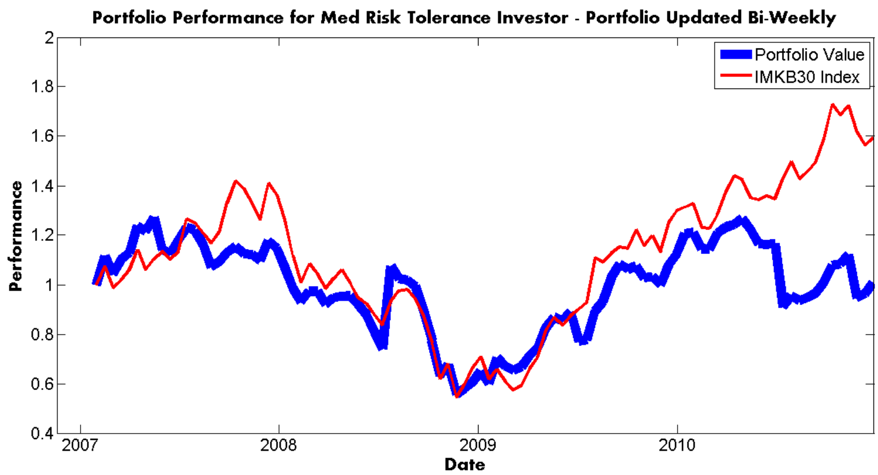

Figure 6.6. The Performance of Bi-Weekly Updated Medium Risk Level Dynamic Portfolio Against ISE30 Index.

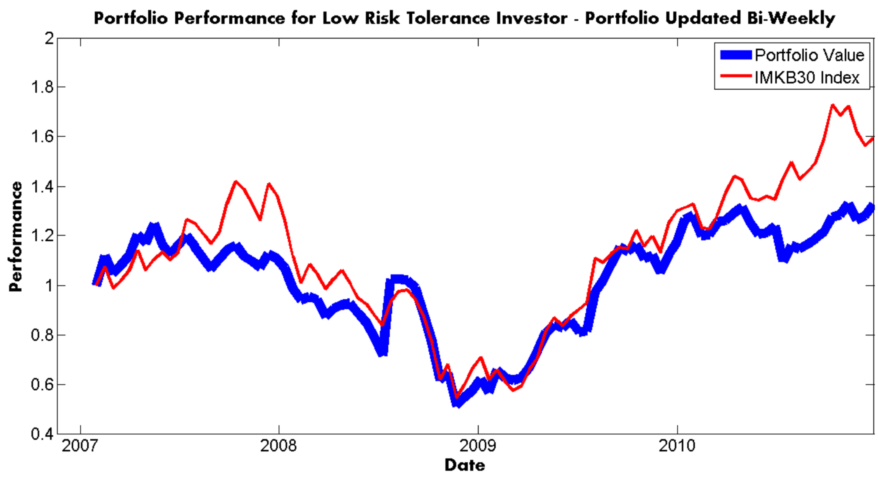

Figure 6.7. The Performance of Bi-Weekly Updated Low Risk Level Dynamic Portfolio Against ISE30 Index.

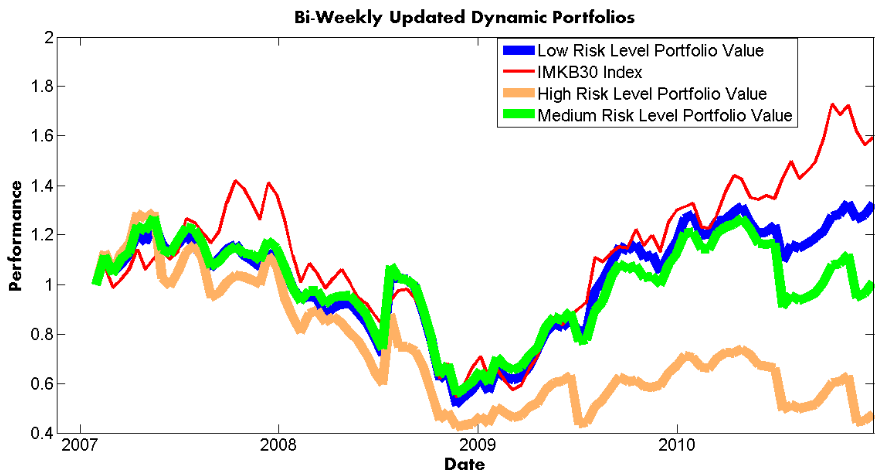

Figure 6.8. The Performances of Bi-Weekly Updated Portfolios for Different Risk Levels Against ISE30 Index.

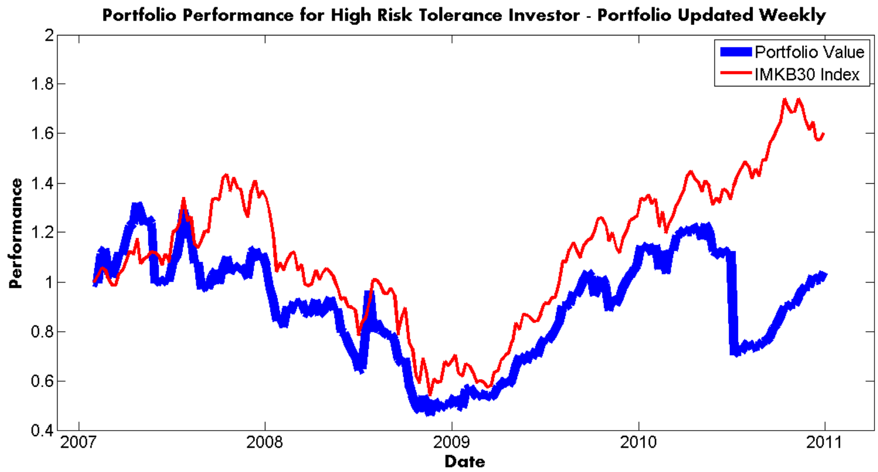

Figure 6.9. The Performance of Weekly Updated High Risk Level Dynamic Portfolio Against ISE30 Index.

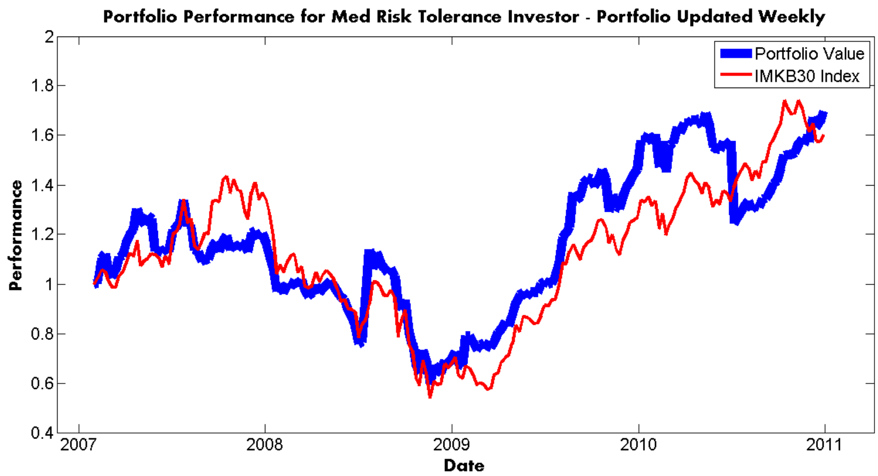

Figure 6.10. The Performance of Weekly Updated Medium Risk Level Dynamic Portfolio Against ISE30 Index.

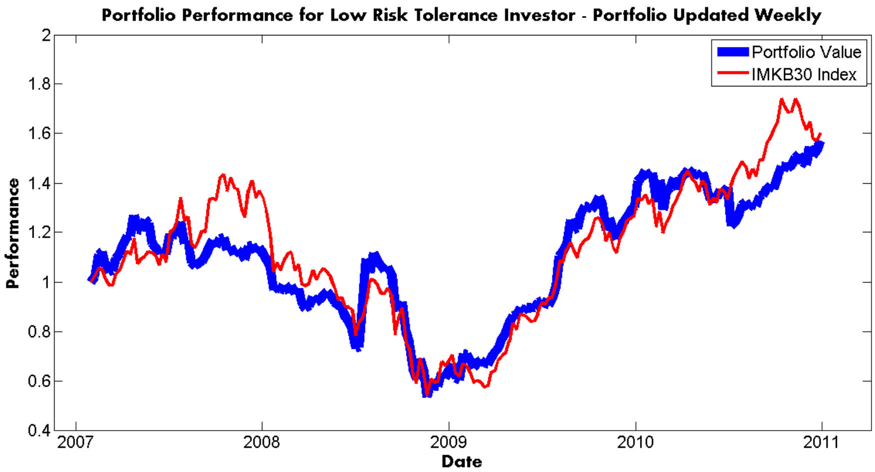

Figure 6.11. The Performance of Weekly Updated Low Risk Level Dynamic Portfolio Against ISE30 Index.

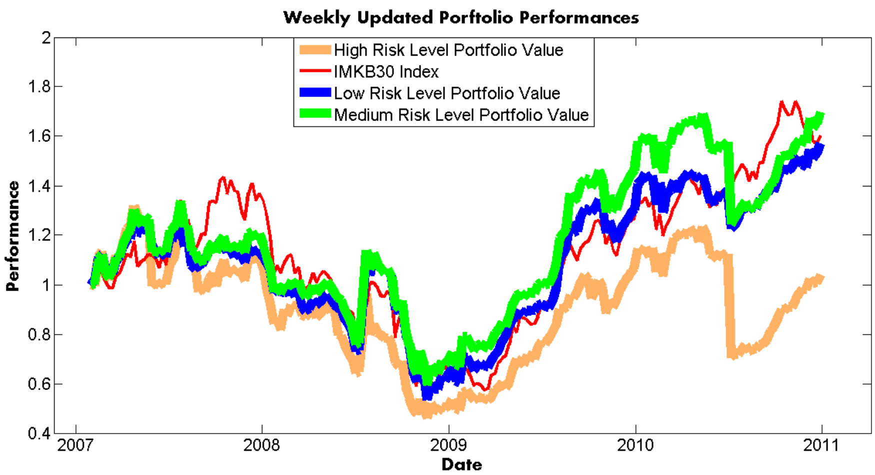

Figure 6.12. The Performances of Weekly Updated Portfolios for Different Risk Levels Against ISE30 Index.

portfolios perform better than the index at all risk levels.

The daily updated high risk level dynamic portfolio yields a return of 2000%. However one must keep in mind that the trading costs are ignored. When the dynamic portfolio is updated daily, the effect of the trading costs might become dominant.

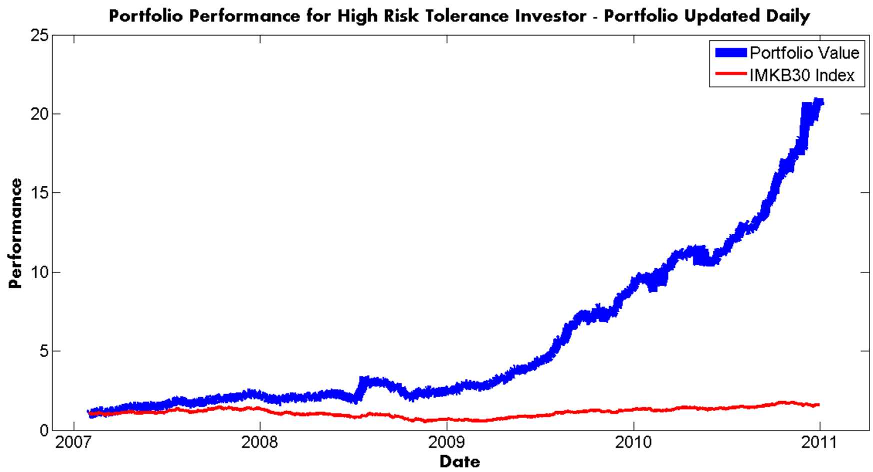

Figure 6.13. The Performance of Daily Updated High Risk Level Dynamic Portfolio Against ISE30 Index.

The portfolio performances for different update rates are shown together respectively for each risk level in Figure 6.17, Figure 6.18 and Figure 6.19.

In the finance industry, it is common to measure the performance of a fund on a quarterly or a yearly basis. In the thesis, the following formula is used to measure the quarterly and yearly success (QS and YS) of the dynamic portfolios:

$$Periodic\, Success\, (PS)\, of\, the\, Portfolio = \frac{Period\, Return\, of\, the\, Portfolio}{Period\, Return\, of\, the\, Benchmark} \quad (6.2)$$

The case $PS > 1$ means that the portfolio has a higher return for the given year than the benchmark. The case $PS < 1$ means that performance of the portfolio is

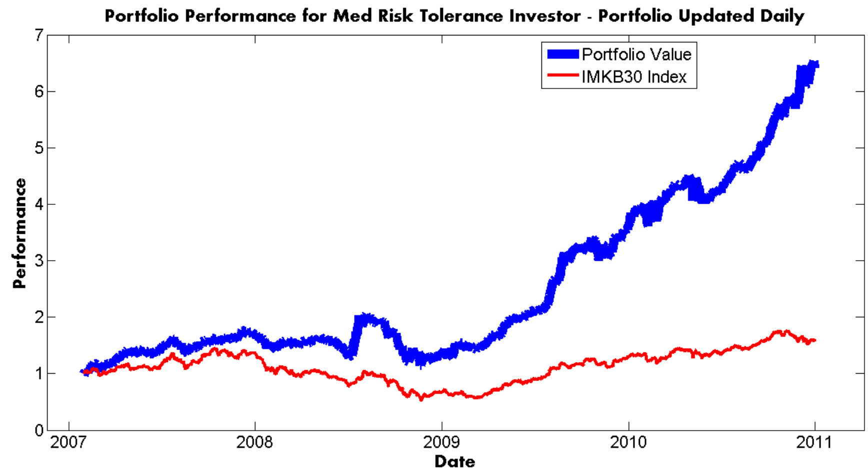

Figure 6.14. The Performance of Daily Updated Medium Risk Level Dynamic Portfolio Against ISE30 Index.

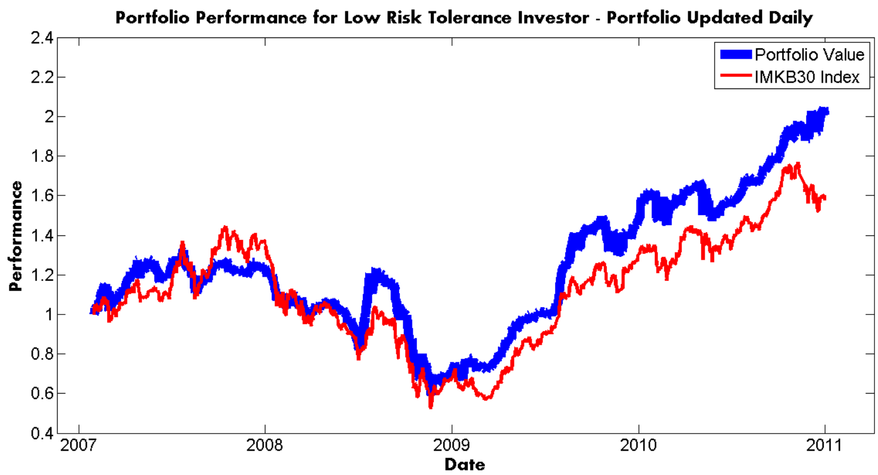

Figure 6.15. The Performance of Daily Updated Low Risk Level Dynamic Portfolio Against ISE30 Index.

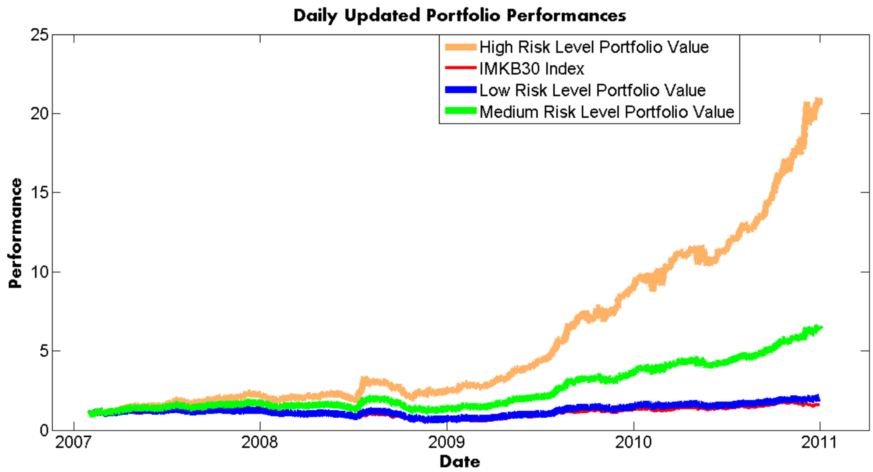

Figure 6.16. The Performances of Daily Updated Portfolios for Different Risk Levels Against ISE30 Index.

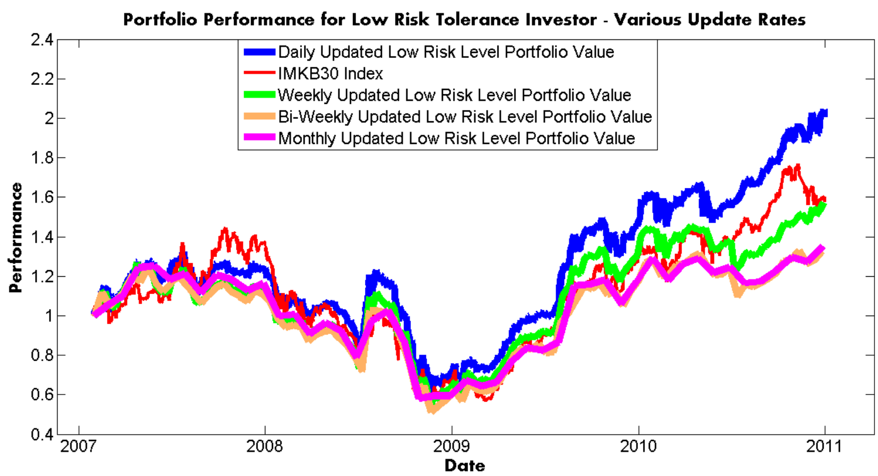

Figure 6.17. The Performances of Low Risk Level Portfolios for Various Update Rates.

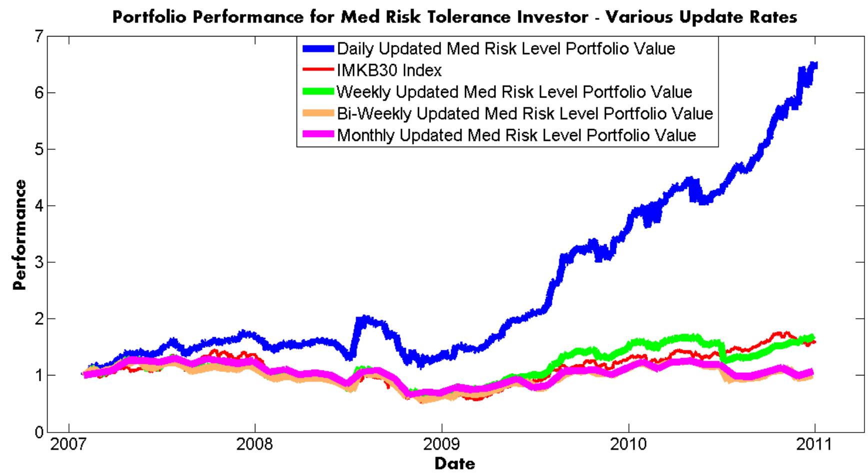

Figure 6.18. The Performances of Medium Risk Level Portfolios for Various Update Rates.

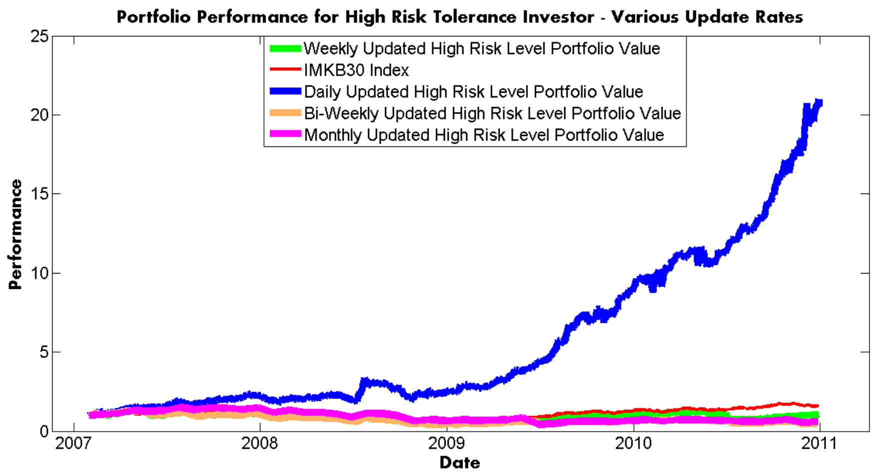

Figure 6.19. The Performances of High Risk Level Portfolios for Various Update Rates.

below the benchmark for that year.

The quarterly and yearly successes of the low, medium and high risk level dynamic portfolios with various update rates are summarized in Table 6.4, Table 6.5 and Table 6.6.

Table 6.4. Summary of the Periodic Successes of Low Risk Level Dynamic Portfolios with Different Update Rates.

Year / **Update Rate**	2007	2008	2009	2010	Avg. YS	# of years YS > 1	Avg. QS	# of quarters (2007-2011) QS > 1
Monthly Update	0.936	1.003	0.988	0.848	0.944	1	0.944	5
Bi-Weekly Update	0.960	0.991	0.966	0.832	1.002	0	0.876	2
Weekly Update	0.930	0.987	1.077	0.981	0.994	1	0.930	3
Daily Update	1.012	1.072	1.191	1.254	1.132	4	1.104	14

Table 6.5. Summary of the Periodic Successes of Medium Risk Level Dynamic Portfolios with Different Update Rates.

Year / **Update Rate**	2007	2008	2009	2010	Avg. YS	# of years YS > 1	Avg. QS	# of quarters (2007-2011) QS > 1
Monthly Update	0.990	1.175	0.931	0.672	0.942	1	0.970	7
Bi-Weekly Update	0.884	0.971	0.928	0.632	0.854	0	0.869	3
Weekly Update	0.933	1.083	1.193	1.059	1.067	3	0.967	5
Daily Update	1.280	1.984	2.663	4.063	2.498	4	2.236	16

Table 6.6. Summary of the Periodic Successes of High Risk Level Dynamic Portfolios with Different Update Rates.

Year / **Update Rate**	2007	2008	2009	2010	Avg. YS	# of years YS > 1	Avg. QS	# of quarters (2007-2011) QS > 1
Monthly Update	1.099	1.089	0.541	0.370	0.775	2	0.862	9
Bi-Weekly Update	0.860	0.707	0.547	0.298	0.603	0	0.685	2
Weekly Update	0.887	0.794	0.848	0.649	0.795	0	0.724	2
Daily Update	1.637	3.741	6.406	13.022	6.202	4	4.826	16

7. CONCLUSION AND FUTURE WORK

The improvements in the performances of the dynamic portfolios that are observed with the increased portfolio update frequencies imply two main ideas.

- The Value of Information : The results show that the increased portfolio update rates provide higher returns. Updating the portfolio proportions more frequently means that the investor is making better use of the information in the market. If the portfolio manager can have more information about the market conditions he or she can improve the return performance of the portfolio.
- Micro-efficiency of the Market : The stock price estimations are done using the historical data on the market. With the increased portfolio update rates,the optimization algorithm is supplied with new closing prices more often. The results show that updating the portfolio proportions more frequently (e.g. using every new closing price on the market) results in higher returns for the investor. This idea implies that the prices in the market reflect sufficient information. In other words, the market is micro-efficient.

It has been argued that the stock market is micro efficient but not macro inefficient. That is, the efficient markets hypothesis works much better for individual stocks than it does for the aggregate stock market [29]. The financial crises occur due to the inconsistencies at the macro level. The effects of the financial crises in 2008 are clearly visible in all return simulations.

The investment strategies that are developed empirically provide returns that are quite close to the benchmark. However there is a large potential for improvement in the results (See Section 7.1).

The development of the MATLAB Software and Graphical Interface constitutes a major part of the effort in this thesis. The software has a modular structure and will be used in the next projects for further research.

7.1. Future Work

The following improvements are planned for the next projects:

- Employment of more sophisticated risk measures (especially VaR and CVaR). By using better risk measures the investor can state his or her investment goal more clearly.
- Use of different return distributions.
- Use of copulas between asset returns to better model the correlations.
- Addition of the trading costs into the simulations.
- Addition of the dividend payments into the simulations.
- Filtering of the historical data for more accurate estimation of the return distribution parameters. The results might be improved by putting more emphasis on the recent data.
- Expansion of the investment universe. Different financial instruments such as bonds, commodities, precious metals and different currencies can be added to the investment pool.
- Expansion of the analysis framework to include intraday trading.
- Performing of similar studies for other stock exchanges.

Further analysis can be performed on some of the empirical results:

- Analysis of the anomaly about the poor performance of the bi-weekly updated dynamic portfolios
- Analysis for a larger set of portfolio update rates (e.g.2,3,4,....,27,28,29 days)

APPENDIX A: Analysis Software User Manual

A software program is developed in MATLAB to perform the financial analyses that are in the scope of the thesis and possible future work. The software program offers a graphical interface from which the users can:

- Load and visualize the historical data
- Do analysis on a single investment scenario
- Do multi-dimensional analysis for different investment scenarios
- Compare performance with the benchmark

A sample view of the graphical interface is shown in Figure A.1.

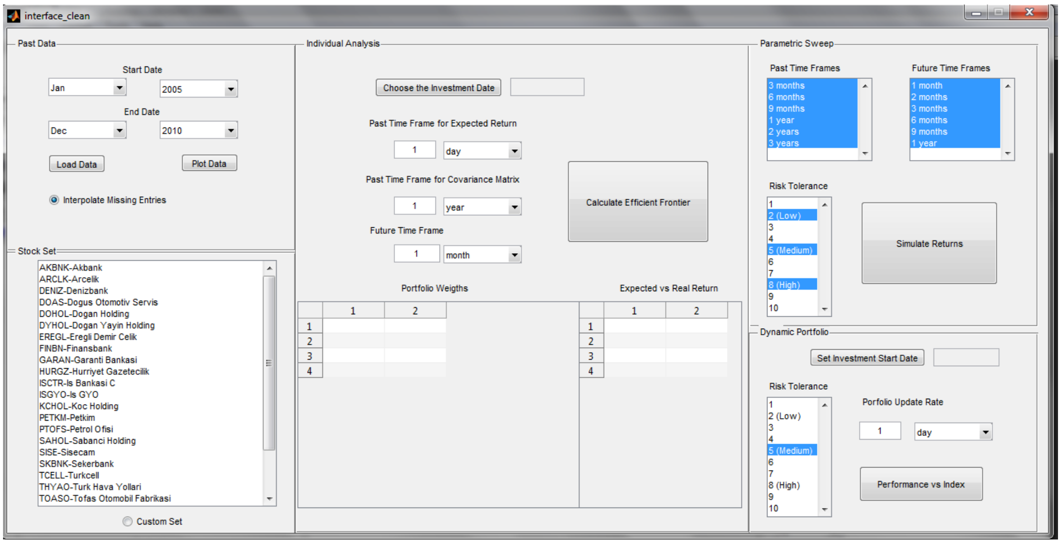

Figure A.1. A Sample View from the Graphical User Interface.

A.1. Loading and Visualizing Historical Data

The user is required to set the time limits of the historical data which will be used by the software to perform various analyses. The panel named ‘Past Data’ is used for loading and visualizing the historical data. The panel is located on the upper left part of the graphical interface window (Figure A.2). The user can set the starting and the ending time of the historical data using the pop-up menus on the panel (Figure A.3).

Once the time limits of the historical data are chosen the data can be loaded into the software by clicking the button ‘Load Data’. In the default settings, the software will load the historical data for all of the ISE30 stocks for the given time interval. If the user is interested in only a subset of the stocks, then the user should click to the radio button ‘Custom Set’ on the bottom left of the screen (in the panel named ”Stock Set”) and make choices from the stock list (Figure A.4) before clicking the ‘Load Data’ button.

In the default settings, the missing data entries are interpolated, the user can turn this feature off by using the radio button “Interpolate Missing Entries”.

The user can plot the historical prices into a new figure by clicking on the button ”Plot Data”. An example plot of historical prices is shown in Figure A.5.

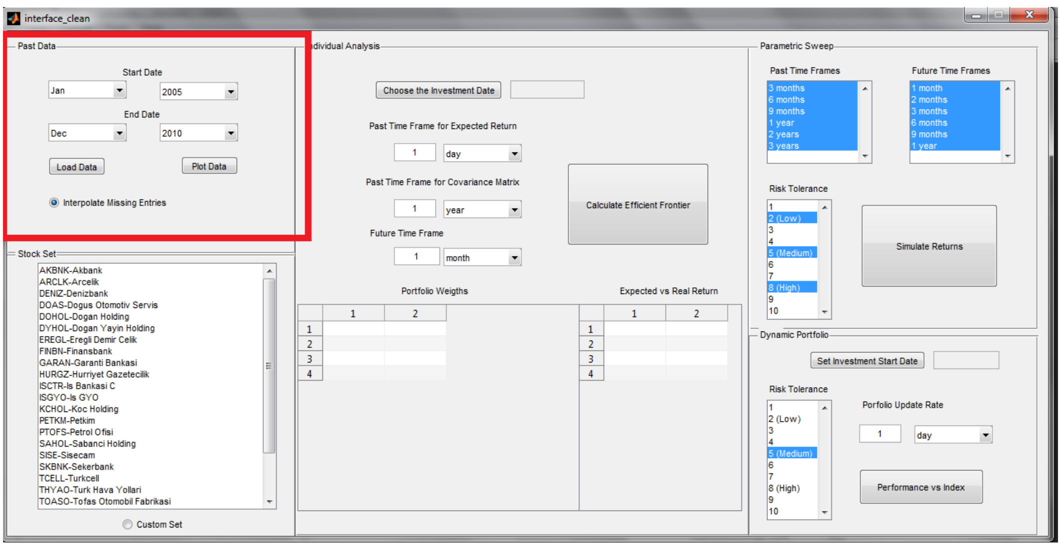

Figure A.2. Panel ‘Past Data’: The panel is located on the upper left part. The user can load and visualize the historical data from the panel.

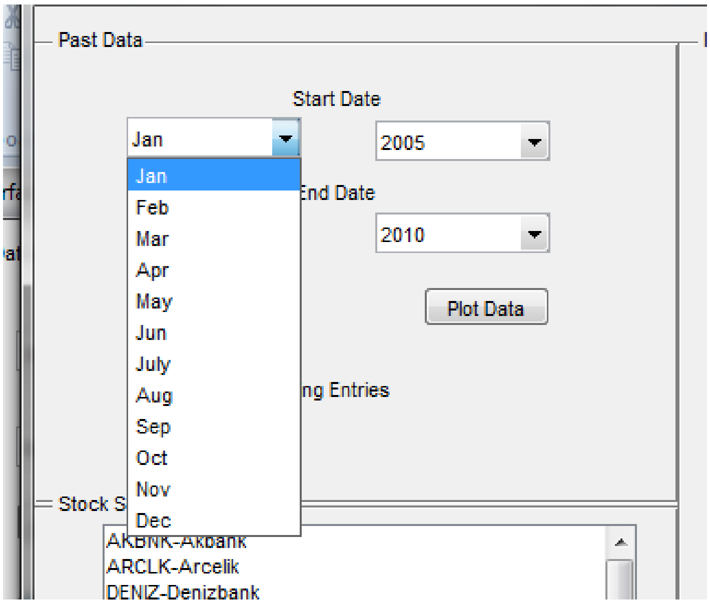

Figure A.3. Pop-Up Menus for Starting and Ending Dates of Historical Data.

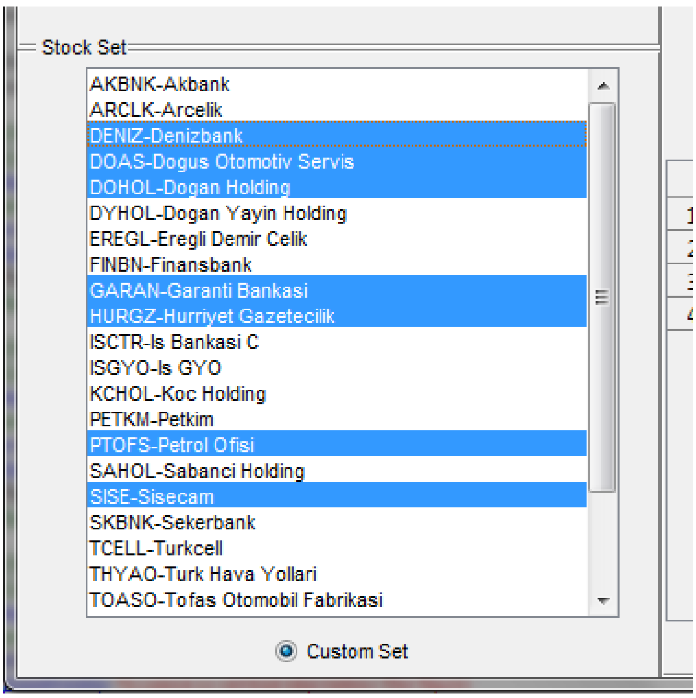

Figure A.4. Custom Selection of the Investment Pool.

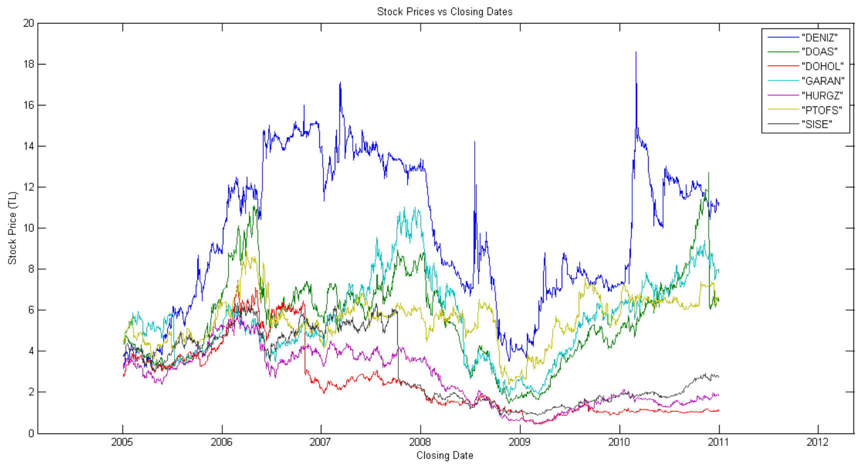

Figure A.5. Custom Selection of the Investment Pool.

A.2. Analysis on a Single Investment

The user can analyze the return of a single investment scenario by using the 'Individual Analysis Panel' (Figure A.6). By clicking the button "Choose Investment Date", the user can set the investment date (Figure A.7). The values for $PTF_{expected\,return}$, $PTF_{covariance\,matrix}$ and FTF are entered by using the data entry boxes on the panel. When the button "Calculate Efficient Frontier" is clicked, the software calculates the efficient frontier and plots it on a new figure (Figure A.8).

The asset proportions for each portfolio on the efficient frontier curve are shown in the data table at the bottom of the panel (Figure A.9). The true returns of all efficient portfolios are shown together with their expected (estimated) values (Figure A.10).

A.3. Multi-Dimensional Analysis for Different Investment Scenarios

Various investment scenarios can be simulated using the "Parametric Sweep" panel (Figure A.11). The button "Simulate Returns" simulates returns by using the set of values that are entered by the user. The software is adjusted to simulate 24

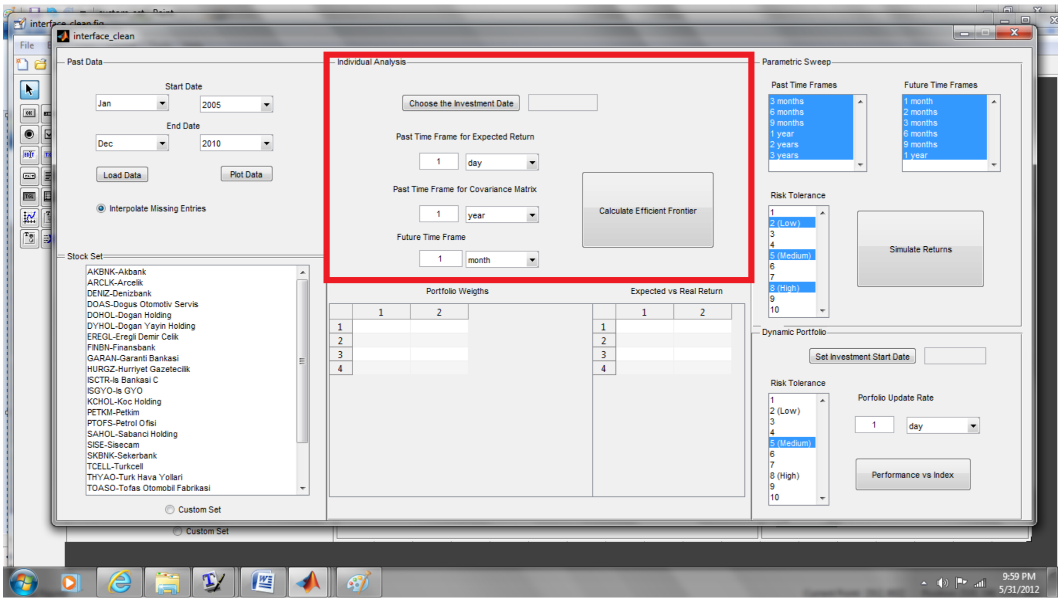

Figure A.6. Panel 'Single Analysis': The return of a single investment scenario can be analyzed by using the panel The user can calculates the efficient frontier and plot it on a new figure.

investments. The investments are done every month between Jan 2008 and Dec 2009.[3] The outputs are printed into respective excel files for each risk level.

A.4. Performance against Benchmark

The panel named "Dynamic Portfolio" is used for evaluating the performance of the chosen investment strategy (Figure A.12).The best strategy that is observed from the return simulations suggests having an investment horizon of 1 month and using 9 months of historical data for estimation of the parameters of the return distribution.[4] By clicking the button "Set Investment Start Date" the user can set the starting date of the investment period. The investment period will cover the time between the start date that is entered by the user and the latest available closing date at hand. The user can set his/her risk tolerance by using the list-box on the panel. The user can

[3]The mentioned numbers and dates are just variables for the system and can be adjusted accordingly for future work.

[4]Again, these number are just variables for the software and can be adjusted accordingly for future work.

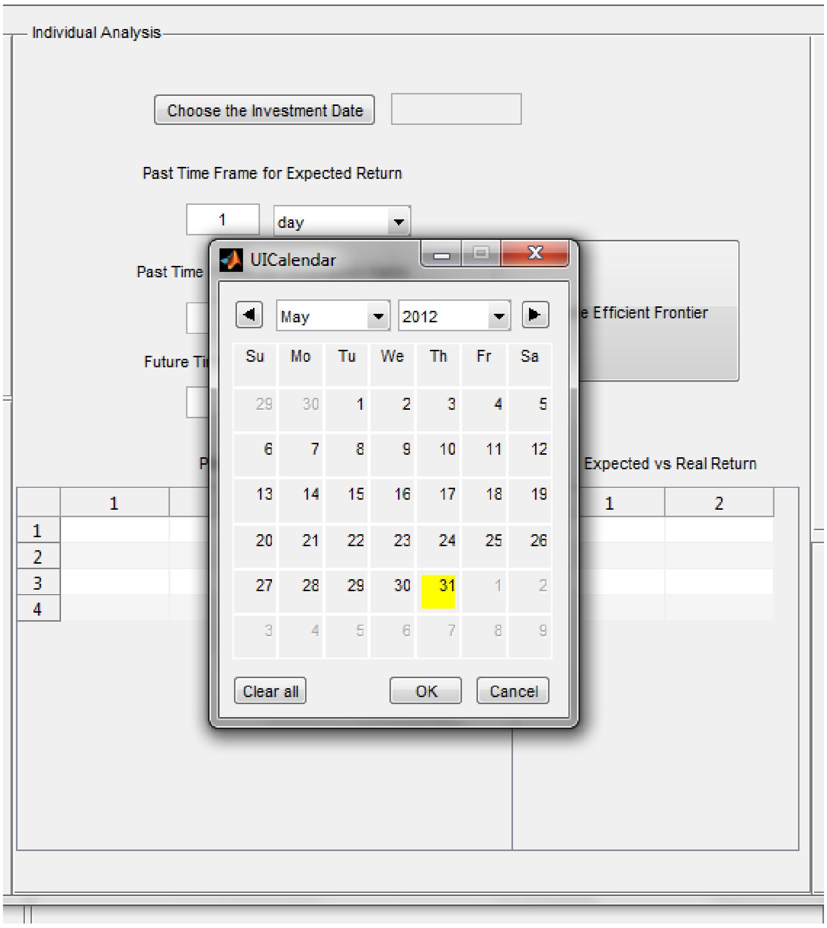

Figure A.7. Set the Investment Date for Single Investment.

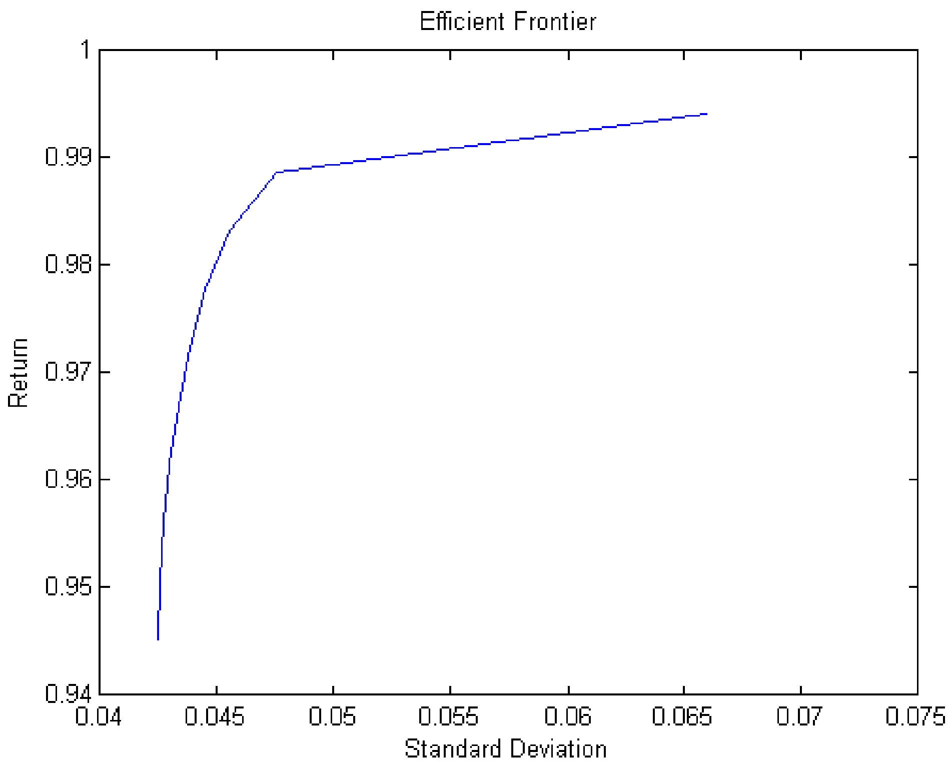

Figure A.8. The Efficient Frontier is Plotted on a New Figure

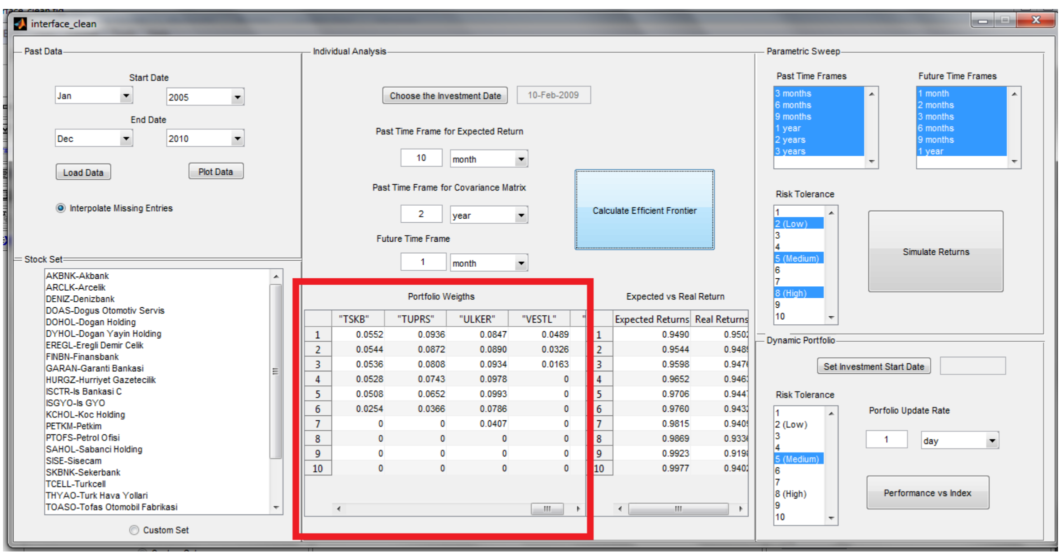

Figure A.9. Table 'Portfolio Weights': The asset proportions for each portfolio on the efficient frontier curve are listed.

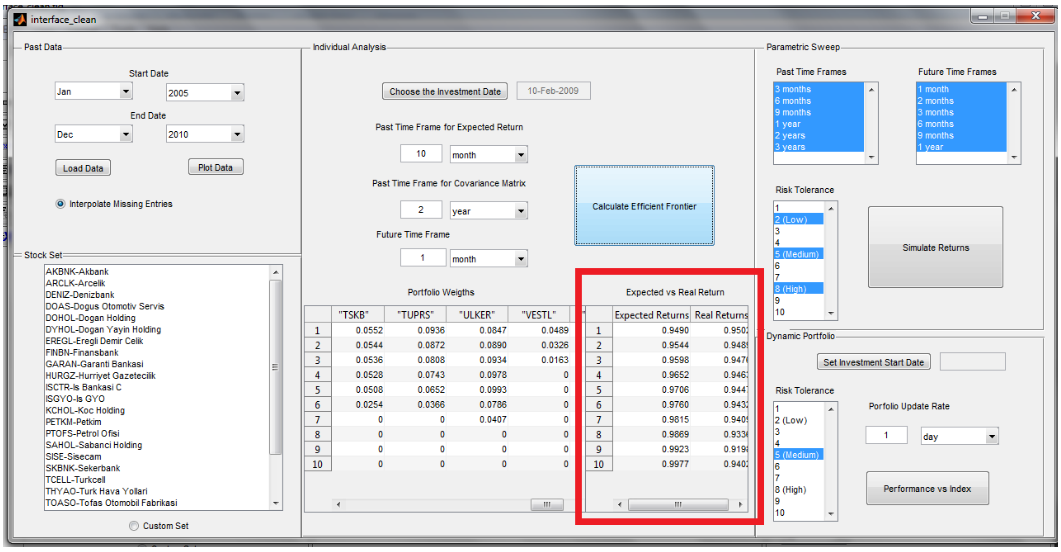

Figure A.10. Table ‘Expected Returns vs. Real Returns’: The true returns of efficient portfolios are shown together with their estimated values.

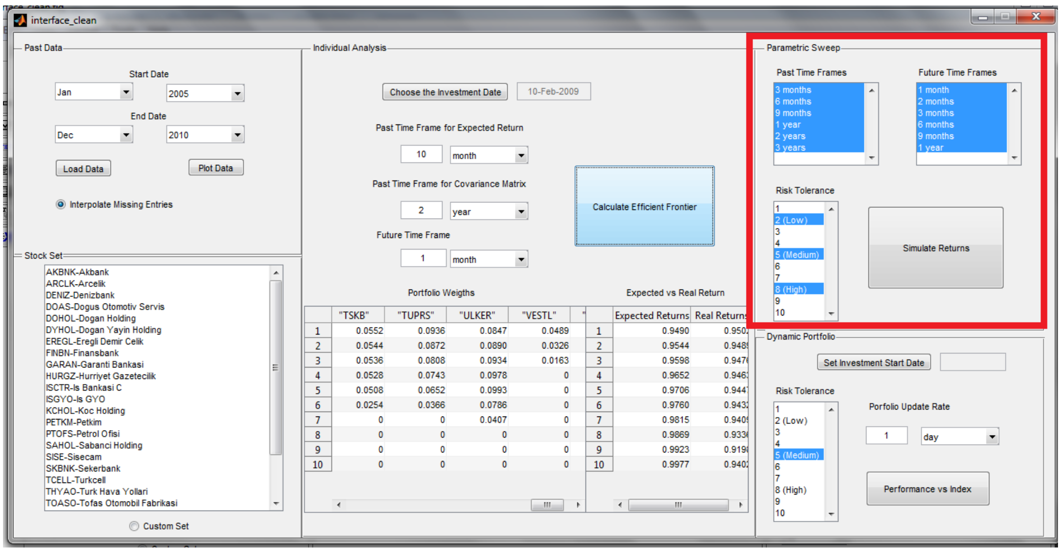

Figure A.11. Panel ‘Parametric Sweep’: Various investment scenarios can be simulated. The results are written into excel files.

set the portfolio update rate by using the data entry boxes on the panel. The button 'Performance vs. Index' compares the performance of the dynamic portfolio with the performance of the ISE30 index and plots the results on a new figure (Figure A.13).

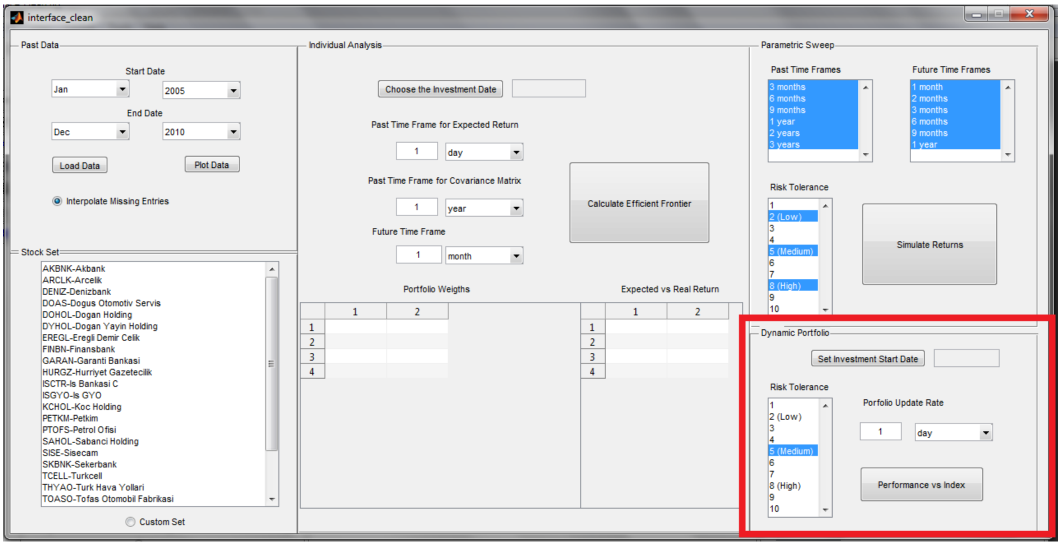

Figure A.12. Panel 'Dynamic Portfolio': The performance of a dynamically updated portfolio can be evaluated from the panel. The results are plotted into a new figure.

A.5. Function Library

The functions that are used in the software and their brief descriptions are given below.

- **open_past_data**: Loads the historical data into MATLAB.
- **get_IMKB30_data**: Imports the related ISE data into MATLAB
- **get_IMKB30_index**: Imports ISE30 Index data from database
- **get_first_date_of_past_data**: Returns starting year and starting month of the past data
- **get_last_date_of_past_data**: Returns ending year and ending month of the past data
- **plot_stock_prices_versus_closing_dates**: Plots the stock prices vs. closing dates on a new window.

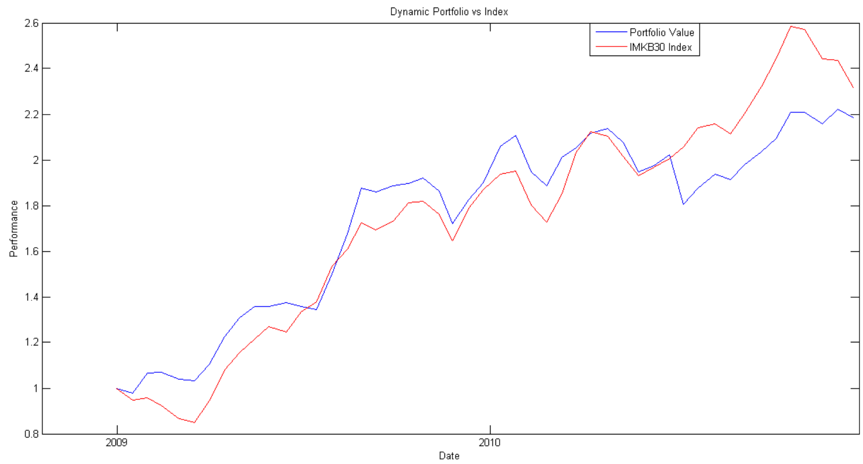

Figure A.13. Porfolio Performance against the Performance of the Benchmark

- **ia_calculate_efficient_frontier**: Calculates the efficient frontier for individual analysis
- **enable_custom_selection_of_IMKB30_stocks**: Enables switch function for investor's custom selection of IMKB30 stocks
- **enable_interpolation_of_missing_entries_in_past_data**: Enables switch function for the interpolation of missing entries in past data
- **estimate_return_distribution_parameters**: Estimates the return distribution parameters
- **calculate_return_parameters**: Converts (normal) log-return parameters to (log-normal) return parameters
- **calculate_real_return_of_the_portfolio**: Calculates the real return of the portfolio
- **annualize_returns**: Annualizes the return values for various maturities
- **simulate_returns**: Simulates the returns for varying values of FTF, PTF and Risk Tolerance
- **returnSIMULATION_iter**: Performs the operations for a single iteration of return simulation
- **initialize_excel_file_for_returns**: Exports the simulation results into an excel file

- **write_return_table_into_excel**: Writes the return data into excel file
- **simulate_dynamic_portfolio_performance**: Simulates the dynamic portfolio performance
- **zoomAdaptiveDateTicks**: Enables the date axis of plots to adapt to zooming

More information about these functions can be obtained from the commentaries inside the function codes.

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