

DYNAMIC PRICING FOR FREE FLOATING CAR SHARING SYSTEMS

by

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ABSTRACT

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In recent years, the importance of sharing platforms has increased as a result of globalisation and networking. As the world has become more globalized, the sharing economy has spread, with people from all corners of the globe embracing collaborative consumption patterns. One of the emerging markets in the sharing economy is car sharing. Car sharing platforms have been studied in the literature as station-based and free-floating. Free-floating Car Sharing (FFCS) model is the most flexible among the car sharing models because customers are free to park vehicles anywhere in the service area. FFCS offers flexibility and convenience to its customers. However, this flexibility can create some problems such as uneven distribution of vehicles between stations, parking challenges and fleet management. Imbalanced distribution of vehicles leads to customer loss and dissatisfaction, which in turn leads to loss of profit for car sharing companies and damages long-term sustainability. In order to create customer loyalty to such a service, it is important that vehicles are available where customers are looking for them. In this thesis, combined vehicle relocation and trip pricing strategy is developed to solve imbalanced vehicle distribution and to increase the profitability of the companies. Mixed integer nonlinear mathematical model is formulated to solve this problem. Computational experiments are performed using both synthetic and real data. Due to the complexity of the model, the proposed model is tested with small-scale instances only. It is observed that by implementing a trip pricing strategy, a profit of \$128087.4 per day is achieved, whereas by applying a fixed pricing strategy without any relocation, the system is able to achieve a profit of only \$6947.9 per day. The improvement of the system is achieved by increasing the prices and decreasing the reasonable quantity of demand.

ÖZET

SERBEST DOLAŞAN ARAÇ PAYLAŞIMI SİSTEMLERİ İÇİN DİNAMİK FİYATLANDIRMA

Son yıllarda, küreselleşme ve ağ oluşturmmanın bir sonucu olarak paylaşım platformlarının önemi artmıştır. Küreselleşme paylaşım ekonomisinin yayılmasına ve dünyanın her köşesinden insanların iş birliğine dayalı tüketim modellerini benimsemesine yol açmıştır. Paylaşım ekonomisinde gelişmekte olan pazarlardan biri de araç paylaşımıdır. Araç paylaşım platformları literatürde istasyon tabanlı ve serbest dolaşımli olarak incelenmiştir. Serbest Dolaşan Araç Paylaşımı (SDAP) modeli araç paylaşım modelleri arasında en esnek olanıdır çünkü müşteriler araçlarını hizmet alanının herhangi bir yerine park edebilmektedir. SDAP müşterilerine esneklik ve kolaylık sunmaktadır. Ancak bu esneklik, araçların istasyonlar arasında dengesiz dağılımı, park zorlukları ve filo yönetimi gibi bazı sorunlar yaratabilir. Araçların dengesiz dağılımı müşteri kaybına ve memnuniyetsizliğine yol açarken, bu da araç paylaşım şirketleri için kâr kaybına neden olmakta ve uzun vadeli sürdürülebilirliğe zarar vermektedir. Böyle bir hizmette müşteri sadakati yaratmak için, araçların müşterilerin aradığı yerde mevcut olması önemlidir. Bu tezde, dengesiz araç dağılımını çözmek ve şirketlerin karlılığını artırmak için araç yer değiştirme ve yolculuk fiyatlandırma stratejisi geliştirilmiştir. Bu problemi çözmek için karma tamsayıli doğrusal olmayan matematiksel model formüle edilmiştir. Deneysel çalışmalar hem sentetik hem de gerçek veriler kullanılarak gerçekleştirilmiştir. Modelin karmaşıklığı nedeniyle, önerilen model yalnızca küçük ölçekli örneklerle test edilebilmiştir. Yolculuk fiyatlandırma stratejisi uygulandığında günlük 128087,4\$ kar elde edilirken, herhangi bir yer değiştirme olmaksızın sabit fiyatlandırma stratejisi uygulandığında sistemin günlük sadece 6947,9\$ kar elde edebildiği görülmüştür. Sistemin iyileştirilmesi, fiyatların artırılması ve makul talep miktarının azaltılmasıyla sağlanmaktadır.

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LIST OF SYMBOLS

d_{ij}^t	Demand from station i to station j in period t
D_{ij}^t	Potential demand from i to j at the beginning of period t
e	Epsilon
I	Set of stations
k_{ij}^t	Amount of satisfied demand from i to j in period t
l_{ij}^t	Amount of lost demand from station i to station j in period t
n_i^t	Number of available vehicles at station i
N	Total number of vehicles
p_{ij}^t	Rental price charged from station i to station j in period t
RC_{ij}^t	Relocation cost from station i to station j in period t
T	Set of time periods
TC_{ij}^t	Trip cost of rental from station i to station j in period t
x_{ij}^t	Number of vehicles that are relocated from station i to station j in period t
α_{ij}^t	Non-negative parameter representing the price elasticity
γ_{ij}^t	Relocation duration of a vehicle from i to j at period t
δ_{ij}^t	Rental duration of a vehicle from i to j at period t
Δ	Length of the period
λ	Eigenvalue of the hessian matrix
μ	Profit percentage
Π	Profit

LIST OF ACRONYMS/ABBREVIATIONS

EV	Electric Vehicle
FFCS	Free Floating Carsharing System
GV	General Vehicle
MINLP	Mixed Integer Nonlinear Programming
PAL	Price Adjustment Level
PM	Profit Maximization
UM	Utility Maximization

1. INTRODUCTION

In recent times, the increase of digitalization and globalization has led to transformative changes across various aspects of people's lives, influencing consumer purchasing behaviour, perceptions, and consumption patterns. Globalization has significantly expanded access to products and services from around the world. Also, it has transformed consumer preferences and resulted a shift in purchasing behaviour. Technology and global interconnectedness of the 21st century affect people's consumption behavior and enable people to have and easily access a greater choice of products. The interconnectedness fostered by globalization has exposed consumers to diverse cultures and lifestyles, prompting a re-evaluation of their consumption patterns and choices.

Simultaneously, the world's population continues to grow steadily. As our world gets more crowded, people need to be careful about how they use limited resources on the earth like water and food because there is not an endless supply. Our planet has finite resources and capacities. With more and more people around, we need to figure out how to allocate those resources between people such as water, land, and energy without running out. Because of this need, collaborative approaches and innovative strategies have emerged. Globalization and especially digitalization have made this process much easier.

One notable strategy that has emerged is the concept of servicizing models. Rothenberg [1] explained "servicizing" that suppliers change the focus of their business models from product sales to service sales. This means that consumers do not directly buy the product or service; instead, they have the capability to use the service or product which is offered by the companies. This mechanism that leads to a way of reutilization of resources among people redefines customer interaction, enabling access to services or products without ownership. Servicizing is a big part of the sharing economy that new emerging concept as a result of globalized world. Instead of owning things, it gives people the chance to use services or products.

Globalization has led to the spread of sharing economies and people from different corners of the world embrace collaborative consumption models. This enables a collective shift towards more sustainable and resource-conscious behaviour. Through sharing systems, consumers can meet their needs by sharing or renting instead of purchasing. On the other hand, sharing platforms are new emerging systems even today, most people may not want to use shared applications especially conventional/traditional generations. There are several reasons behind that such as privacy concerns, trust issue, cultural and generational differences and etc. Therefore, the sustainability of the application/system depends on the participation and loyalty of the users. In order for the application to be sustainable, it is very important to gain the trust of the user and eliminate their concerns related with the privacy and etc.

There are lots of different platforms as an example of sharing economy, home-sharing as Airbnb, ride-sharing services as Uber and Lyft, tool sharing like Letgo etc. Also, another example of sharing platforms is car sharing. Car rental companies can offer vehicles to consumers on a minute, hourly or daily basis for a small fee, making the need "affordable for everyone". It offers the consumer the feeling and comfort of being in your own car without purchasing a vehicle. Car sharing and ride sharing can sometimes be thought of as the same platforms, but they are different sharing platforms that are also studied in different research streams in the literature. In car sharing, a company offers vehicle fleet for users to utilize collectively within the predefined operational area and each user can rent a vehicle for own trip. On the other hand, ride sharing platforms, a vehicle can be rented by more than one user at the same time and trip charge is divided among them. Therefore, ridesharing and carsharing are address under different headings in the literature.

Car sharing has three different business models. Station-based, one-way, two-way and free-floating are the three different car sharing models that are studied in the literature and that operate in the real world. So, car sharing models can be categorized in a way that station based or not. Two way and one way car sharing models are station-based models which means that stations where vehicles can be picked up or dropped by users are predefined by the company. The difference between two models is whether

the station you pick up the car from and where you leave it are the same or not. For two-way car sharing model, users must drop the vehicle into the station where user pick up. ZipTrip is the example platform of two-way car sharing model in Turkey. Figure 1.1 and Figure 1.2 show the definition of two-way car sharing model in detail. As can be seen in Figure 1.1, Station 1 is pick up station for the given example and Figure 1.2 shows the drop off station, which is again Station 1. If a user picks up the car from Station 1, then he must leave the car at Station 1.

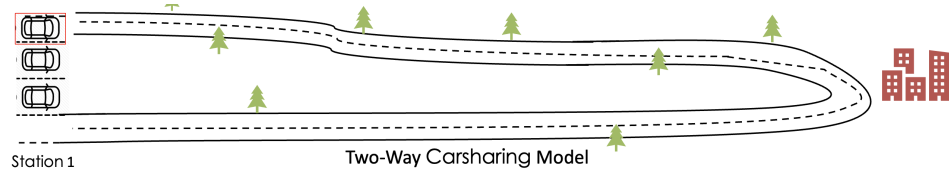


Figure 1.1. Two-way car sharing model simulation pick up.

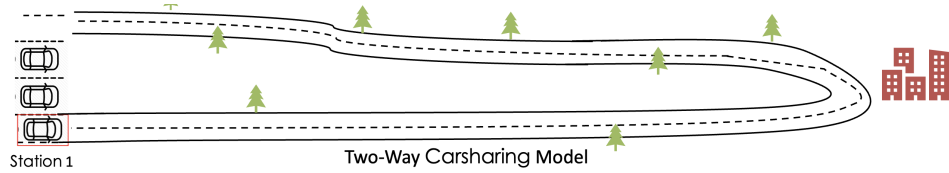


Figure 1.2. Two-way car sharing model simulation drop off.

Conversely, users can drop off vehicles to different stations if they use one-way car sharing platform. Zipcar is one of the examples of one-way car sharing model. System definition is illustrated in Figure 1.3 and Figure 1.4 in detail. There are two stations defined for the given example as Station 1 and Station 2. As can be seen from the Figure 1.3 and Figure 1.4, if a user picks up a vehicle from Station 1, he/she can leave the vehicle at Station 2 or any other stations. There is no restriction between the pick-up and drop off stations. Note that the stations are predefined by the company and known to the users.



Figure 1.3. One-way car sharing model simulation pick up.



Figure 1.4. One-way car sharing model simulation drop off.

On the other hand, free-floating car sharing model is the most flexible among them in terms of availability of service area. Company still defines an operational area, but it is not divided into stations. Users can drop off the vehicles anywhere in the operational area and can pick up vehicles from where the previous user left off. Thus, customers are not bounded by the stations. This business model provides great convenience for users if there is a parking space in terms of use. In the station-based model, the predefined stations can stay away from users' destination. For this reason, users may prefer substitute services such as taxi. GetirAraç, TiktTak, Martı, Binbin Scooter, Isbike, Tazı, Palm, Dost Scooter are some of the examples of free-floating car sharing and bike sharing applications in Turkey.

Free-floating car sharing (FFCS), which offers flexibility and convenience to its customers, is the most flexible business model between three models, but this flexibility can cause some problems such as imbalance vehicle distribution between stations, parking challenges, fleet management and maintenance. Imbalance of supply and demand is one of them and mostly studied in the literature. It is not only a problem for free-floating, but also for the other two car sharing models. Compared to vehicle distribution and availability between three models, it can be seen that, designated stations which ensures users can easily find vehicles at specific locations and improves the predictability of service availability. In contrast, vehicles may gather in certain popular

areas, leaving other areas underservice on free floating model, because users are free to park anywhere within a defined operating area. Vehicles sometimes get stuck in areas of lower individual mobility demand while needed in zones of higher demand [2] .

In this research, imbalanced vehicle distribution problem of car sharing models is studied. The dynamic pricing scheme, which is a competitive price-dependent demand model for free-floating business models is developed as a solution approach to the given problem. The rest of the study is organized as follows. Chapter 2 provides a comprehensive literature review on the topic. Chapter 3 provides details regarding the problem setting, basic assumptions, notation used and the approach, which is a mixed integer nonlinear model we have proposed to solve this problem. Then, in Chapter 4, the results and main foundations related to the results are discussed. Finally, Chapter 5 concludes the study and provides recommendations for future research.

2. LITERATURE REVIEW

Car sharing is a new emerging market in recent years, but it has rapid growth, and many studies expect that carsharing will be one of the popular travel modes in the following years [3–6]. Zhou *et al.* [7] forecasted that the carsharing industry is expected to grow to 11% – 25% of all trips in Australia, Indonesia, Malaysia, and Thailand by the year 2030. Therefore, it is one of the sectors that requires focus.

Research on carsharing systems design has focused primarily on three categories of decisions: strategic, tactical, and operational in the literature. Strategic decisions that are long term decisions include choices related with the station planning like determination of locations, quantities, and capacities. These decisions play a vital role in balancing the need to improve service capacity with the objective of minimizing fixed costs such as infrastructure construction. The tactical decisions that entail making informed choices regarding fleet size, staff sizing and deployment of vehicles among stations are mid-term decisions regarding the service [8–10]. Fleet sizing and deployment refers to decisions related to number of vehicles to be employed and how to allocate them among stations. Staff sizing includes decisions regarding the number of staffs to be hired to deal with the issue of vehicle imbalances among stations by implementing the vehicle relocation operations.

The short-term decisions are referred to operational decisions of the service. Vehicle relocation operations and trip pricing decisions are involved in operational decisions. These policies impact several critical business objectives, such as improving customer satisfaction, increasing efficiency, maximizing revenue, resolving vehicle imbalances, and ensuring competitiveness. When appropriate pricing policies and effective vehicle routing strategies are implemented, it ensures that the needs of customers can be met, and the profitability of the business can be increased by offering attractive offers to customers. Therefore, developing and managing these policies correctly is vital to a successful and sustainable carsharing business. Wu and Xu [11] reviewed most relevant studies in the literature. They classified the studies according to targeted decisions,

i.e., strategic, tactical, and operational in the optimization problem. As mentioned in Chapter 1, there are three types of car-sharing systems which are station based, one-way and two-way car sharing systems, and free-floating car sharing system. Therefore, the studies in the literature can be classified based on the car sharing model type and the targeted decisions concerned in the optimization process by the studies. Since the focus of this study is on vehicle relocation and trip pricing decisions, which are short-term operational decisions, the papers addressing these problems are reviewed.

Free-floating car sharing system which offers flexibility and convenience to its customers is the most flexible one among three business models, but this flexibility of the system can cause some problems such as imbalanced vehicle distribution among stations, parking challenges, fleet management and maintenance. Imbalance of supply and demand is one of them and mostly studied in the literature [2], [8], [12–14]. This is not only a problem for free-floating car sharing, but also for the other two car-sharing models. Compared to vehicle distribution and availability between three models, it can be seen that, designated stations which ensures users can easily find vehicles at specific locations and improves the predictability of service availability. In contrast, vehicles may gather in certain popular areas, leaving other areas underservice on free floating model, because users are free to park anywhere within a defined operating area. Vehicles sometimes get stuck in areas of lower individual mobility demand while needed in zones of higher demand [2]. There are limited studies in the literature for free-floating car sharing systems as it is an emerging model in recent years.

Different solution approaches for imbalanced demand and supply problem are studied in the literature and one of them is the relocation of cars between locations by users or operators [15–17]. The relocation of the vehicles is studied in the literature as operator-based, user-based and hybrid models. The main studies in the literature related to the vehicle relocation are as summarized in Table 2.1.

The studies are categorized according to the vehicle type (electric or general vehicle), the type of car sharing mode (one-way, two-way or free float), and who performs the operation (users or operators). Also, the main objective and proposed method of the

studies are given in the table. It is seen that the main focus of the studies is one-way car sharing system. Free-float and two-way car sharing system get relatively little attention compared to one-way car sharing system. Also, most studies focus on the operator-based relocation in their solution methods. In operator-based, certain number of operators are supplied by the company to relocate the cars with wage cost. On the other hand, in user-based system, vehicles are relocated by users with several incentives such as extra minutes for the next trip or price discounts. As can be seen from Table 2.1 and as stated in the study Wu and Xu [11], majority of the literature focus on operator-based relocation strategies for station-based model to solve this problem [8], [15], [17–20]. The operator-based relocation strategy represents a deterministic methodology as opposed to the user-based relocation strategy. This distinction results from the fact that a dedicated workforce is allocated by the company to manage this task and a predetermined number of employees are assigned to this role. As a result, the firm knows with certainty that the relocation will take place. On the other hand, user-based relocation is a probabilistic strategy that depends on the user’s will. User pattern can be traced and be predicted based on customer behaviour and this model does not include additional staffing costs. [21–24] are studied user-based relocation strategies for station-based car sharing models.

The main studies for vehicle relocation will be explained in detail. In this concept, Weikl and Bogenberger [16] divided operational area into 15 macroscopic and 478 microscopic zones to try to relocate the vehicles in an operator-based free-floating car sharing system. They developed an algorithm to find the optimal movement of vehicles between zones for both conventional and electric vehicles (EVs). Also, for the first time in the literature, the model was developed and tested in the real-world. Boyacı *et al.* [8] studied the one-way car sharing repositioning of EVs with operator-based and developed a multi-objective mathematical model for electric vehicles with reservations. They assumed that trip reservations are known in advance. Reservations or having prior knowledge of demand simplifies the problem-solving process because when demand is uncertain, a significant challenge arises, necessitating the resolution of demand estimation. In the case of directing vehicles based on wrong estimates, a company can face larger amounts of losses instead of running a profitable business. Herbawi *et al.* [25]

studied free-floating car sharing system and developed a genetic algorithm. Relocations in the study are conducted by the operators who share a shuttle. Their focus is optimizing the route of the shuttle.

Table 2.1. Main studies related with vehicle relocation.

Author	Vehicle (GV/EV)¹	Car Sharing System	Method	Objective	Operation
Jorge <i>et al.</i> [26]	GV	One-Way	MINLP, Local Search Algorithm	Maximize Total Profit	User-Based
Ren <i>et al.</i> [27]	EV	One-Way	MINLP	Maximize Total Profit	-
Gambella <i>et al.</i> [13]	EV	One-Way	MILP	Maximize System Profit	Operator-Based
Xu <i>et al.</i> [28]	EV	One-Way	MINLP, Outer-Approximation Algorithm	Maximize Total Profit	Operator-Based
Giogione and Viti [29]	GV	Two-Way	Agent Based Simulation	Profit and Utility Optimization	Operator-Based
Bruglieri <i>et al.</i> [15]	EV	One-Way	MILP, Heuristic Algorithm	Maximization Service Level	Operator-Based
Wang <i>et al.</i> [30]	EV	One-Way	MILP, Genetic Algorithm	Minimize Total Relocation Cost	User-Based

Table 2.1. Main studies related with vehicle relocation (cont.).

Clemente <i>et al.</i> [31]	EV & GV	One-Way and Two-Way	Decision Support System (Discrete Event Simulation and Particle Swarm Optimization)	Maximize Level of Service (LOS)	User-Based
Boyacı <i>et al.</i> [17]	EV	One-Way	Multi-objective MILP and Discrete Event Simulation Framework	Maximization Service Level, Minimization Cost	Operator-Based
Folksetad <i>et al.</i> [32]	EV	FFCS	MIP, Hybrid Genetic Search with Adaptive Diversity Control Algorithm	Minimize Cost	Operator-Based
Di Febbraro <i>et al.</i> [22]	GV	One-Way	Discrete Event Simulation	Maximize System Profit	User-Based
Huo <i>et al.</i> [33]	EV	One-Way	MINLP	Maximize System Profit	Operator-Based
Wang <i>et al.</i> [34]	GV	One-Way	MINLP	Maximize System Profit	User-Based

Table 2.1. Main studies related with vehicle relocation (cont.).

Wang and Ma [35]	EV	One-Way	MINLP, Genetic Algorithm	Minimum difference of desired vehicle inventory with high and low threshold	User-Based
Huang <i>et al.</i> [36]	GV	One-Way	Two-Stage Stochastic Programming, Gradient Search	Maximize System Profit	Operator-Based
Schiffer <i>et al.</i> [37]	GV	FFCS	MILP, Polynomial Algorithm	Maximize System Profit	User-Based
Bruglieri <i>et al.</i> [12]	EV	One-Way	Multi-objective MILP	Minimize Staff Size and Route Duration, Maximize relocation requests	Operator-Based
Wang <i>et al.</i> [38]	EV	One-Way	MILP	Maximize System Profit	Hybrid (Operator-User Based)
Yang <i>et al.</i> [39]	GV	One-Way	MILP, Decomposition Algorithm	Minimize Cost	Operator-Based

¹GV:General Vehicle, EV: Electric Vehicle

Another approach for solving the imbalance problem of supply and demand which exists in the literature is trip pricing [10], [26–28]. Trip pricing has received little attention compared to vehicle relocation in the literature. The studies in the literature related to trip pricing are summarised in the Table 2.2. The Table 2.2 shows that each study focuses on whether trip pricing, vehicle relocation or both. The studies are again categorised according to whether the vehicle type is electric or conventional, the type of car sharing mode, whether it is one-way, two-way or free float. These studies also consider elastic demand for trip pricing, so the demand function used in the study is also specified. It can be seen that linear or exponential demand functions are mostly used in the studies.

Table 2.2. Main studies related with the trip pricing.

Author	Vehicle	Car Sharing System	Dynamic Pricing	Vehicle Relocation	Elastic Demand	Demand Function
Jorge <i>et al.</i> [26]	GV	One-Way	✓	✓	✓	Linear
Ren <i>et al.</i> [27]	EV	One-Way	✓	✓	✓	Linear
Xu <i>et al.</i> [28]	EV	One-Way	✓	✓	✓	Exponential
Bao <i>et al.</i> [40]	EV	One-Way	✓	-	✓	Linear
Sakurama [41]		One-Way	✓	Station Relocation	✓	Nonlinear
Wang <i>et al.</i> [35]	EV	One-Way	✓	✓	✓	Nonlinear S-Curve
Huang <i>et al.</i> [42]	GV	One-Way	✓	-	✓	Linear
Xi <i>et al.</i> [43]	EV	One-Way	✓	-	✓	Linear
Huang <i>et al.</i> [36]	GV	One-Way	✓	✓	✓	Nonlinear

Soppert *et al.* [44] characterizes pricing mechanisms in three dimensions which are pricing basis, spatio-temporal pricing and state dependency. The first dimension of pricing is based on the duration of the rental and users are charged according to the time they use the vehicle. The operator may also offer package prices for long rental durations or monthly membership fees. Spatio-temporal pricing depends on the rental's duration and location of the origin and destination. In the literature, the studies focused on trip-based pricing, origin-based pricing and destination- pricing. When company sets their prices depend on the origin and destination, it is called trip-based pricing. Origin-based and destination-based pricing depend on the origin and destination, respectively. The third dimension is state dependence which is dynamic/differentiated pricing. Dynamic pricing adjusts prices based on the current state of the system. On the other hand, differentiated pricing/trip pricing do not depend on the current state of the system [45]. [10], [26–28] are categorised under the differentiated pricing. To the best of our knowledge, only 9 studies in the literature have addressed this issue.

These studies consider on the elasticity of demand. The elasticity of demand is a measure of how sensitive the quantity demanded is to changes in price. In order to design effective pricing schemes, manage fleet utilisation and remain competitive in the market, demand elasticity is an important factor for carsharing systems. Carsharing services can optimise their operations, increase user satisfaction, and maintain a sustainable business model by using information on how users respond to price changes. For this purpose, linear or nonlinear relationships between demand and price are considered and linear or nonlinear demand functions are used. The main idea behind this approach is by charging high prices for trips that increase imbalance and lowering prices for trips that help improve balance, controlling customers' behaviour to stabilize vehicle stocks and thus make carsharing systems more manageable and profitable.

The studies on trip pricing for car sharing models are explained in detail. Jorge *et al.* [26] studied trip-based pricing model for one way car sharing systems and developed mixed integer non-linear programming model (MINLP) to maximize system profit. It is the first study in the literature with optimized method to set a variable trip pricing. Due to nonlinearity of the MINLP, also iterative local search metaheuristic is proposed and

applied to the network with 75 stations. Linear elastic demand function with respect to the trip price is used. Also, elasticity of demand is assumed to be equal for any interval of price variation, applied to the unit price per trip not total price of a trip so elasticity not depend on duration of the trip. Based on their computational studies' results, trip pricing can be considered an effective method to improve the profitability of one-way carsharing systems. Xu *et al.* [28] formulate a mixed-integer non-linear and non-convex programming model to maximize the profit of operators by determining the electric vehicle (EV's) fleet size, trip pricing in one-way carsharing system by taking into account the vehicle relocation and personnel assignment. Compared to traditional gas vehicles, additional attributes, i.e., transportation and power stations, needs to be considered for EV sharing systems. They used nonlinear demand function to be able to better depict the elasticity of demand in reality. Due to complexity of the proposed model, commercial solvers could not solve it by optimality. Proposed model is converted to mixed-integer convex programming model by exploiting the special model structure, then used outer-approximation algorithm to solve it.

Ren *et al.* [27] contributed literature with dynamic trip pricing model for EV's sharing. Also, different from other models, they proposed time-varying price matrix which contains two price adjustment levels (PAL's) for each origin and destination station pairs. Besides price elasticity of demand, price elasticity of the time is considered, and travel time varies according to simple elastic behaviour. Due to complexity of problem, small problem instances like four stations and three hours cases could be solved. Huang *et al.* [10] considered the difference between the effectiveness of the user-based relocation model and operator-based relocation model for one-way electric car sharing systems by taking into account trip pricing. Nonlinear (exponential) demand elasticity function is used same as [28]. Due to complexity of model, an iterated local search is proposed as solution algorithm.

In conclusion, in the literature many different solution approaches are studied for solving imbalance problem of the car sharing systems. Vehicle relocation, trip pricing or combinations of two strategies are two of them. This research aims to construct a dynamic pricing scheme that is a competitive price-dependent demand model for free-

floating business models for the first time in the literature. A mixed-integer nonlinear programming model is proposed to solve this problem and tested with both synthetic and real data. To the best of our knowledge, this is the first study to simultaneously consider maximizing system profit and minimizing unused vehicles.

3. MODEL

This study focuses on the short-term operational decisions in car sharing systems, namely vehicle relocation and trip pricing. As mentioned previously, sustainable and successful car sharing is highly dependent on keeping a properly balanced distribution of the vehicles. Due to various factors such as customer patterns, peak hours, seasonal factors, operational challenges such as vehicle breakdowns, or fuelling issues, the system can easily become imbalanced. This means that there are not enough vehicles in places where there is high demand, and there is no demand in areas where there are plenty of vehicles. For example, at peak times such as morning and evening, many users want to access vehicles at the same time, causing imbalances, especially in areas where business centers are dense.

Different approaches have been proposed in the literature for the solution of the imbalance problem in car sharing systems, including dynamic pricing, user-based or operator-based vehicle relocation, and data analytic approaches to better predict demand. In this study, a combined dynamic pricing and relocation strategy is developed to deal with the problem.

3.1. Problem Setting

We assume that the service area of the car sharing company consists of $|I|$ stations, where each station is denoted by $i, j \in I$. An operational day is divided into equal $|T|$ time intervals and each period is denoted by $t \in T$. The price p_{ij}^t is charged for the trips from station i to station j starting at period t . This is the main decision variable of the mathematical model which aims to maximize the profit of the company. Other decisions include the vehicle inventory n_i^t at station i at period t , the served demand d_{ij}^t for trips from station i to station j starting at period t , and the vehicle relocations x_{ij}^t from station i to station j starting at period t . The total relocation cost RC_{ij}^t and trip cost TC_{ij}^t from station i to station j at period t are calculated based on the fuel consumption during the trip.

The following assumptions are made to keep the model as tractable as possible:

- The demand is price-elastic, and as the price of the trip increases, the demand decreases based on the inverse relationship between the demand and the price. The expected demand d_{ij}^t is estimated by the linear demand function $D_{ij}^t - \alpha_{ij}^t p_{ij}^t$, where D_{ij}^t is the potential demand, p_{ij}^t is the trip price, and α_{ij}^t is a given nonnegative parameter [26], [28], [46].
- There are sufficient empty spaces in the public parking area for vehicle returns.
- Rentals take place at the beginning of each period, whereas relocation operations occur at the end of each period. These operations have the purpose of balancing the system at the end of the period by moving the cars from their positions after the rentals to other locations.

For this problem, we formulate a mixed integer nonlinear mathematical model (MINLP) to optimize pricing and relocation decisions in a free-floating car-sharing system. The following notation is used throughout the study:

Table 3.1. Decision variables.

Decision Variables	Definition
p_{ij}^t	Rental price charged from station i to station j in period t
d_{ij}^t	Demand from station i to station j in period t
k_{ij}^t	Amount of satisfied demand from station i to station j in period t
l_{ij}^t	Amount of lost demand from station i to station j in period t
x_{ij}^t	Number of vehicles that are relocated from station i to station j in period t
n_i^t	Number of available vehicles at station i that can be rented at the beginning of period t

Table 3.2. Sets and parameters.

Sets	Definition
$i, j \in I$	set of stations
$t, \tau \in T$	set of time intervals
Parameters	Definition
Δ	Length of the period (e.g., 15 minutes)
N	Total number of vehicles
D_{ij}^t	Potential demand from station i to station j at the beginning of period t
α_{ij}^t	Nonnegative parameter representing the price elasticity
δ_{ij}^t	Rental duration of a vehicle from station i to station j at period t
γ_{ij}^t	Relocation duration of a vehicle from station i to station j at period t
RC_{ij}^t	Relocation cost from station i to station j in period t
TC_{ij}^t	Trip cost of rental from station i to station j in period t

3.2. Mathematical Model

An integer nonlinear programming (MINLP) model which is called Profit Maximization (PM) is formulated as

$$\max \sum_{i \in I} \sum_{j \in I \setminus i} \sum_{t \in T} (p_{ij}^t k_{ij}^t - TC_{ij}^t k_{ij}^t - RC_{ij}^t x_{ij}^t), \quad (3.1)$$

$$\text{subject to } d_{ij}^t = D_{ij}^t - \alpha_{ij}^t p_{ij}^t, \quad i, j \in I, t \in T, \quad (3.2)$$

$$d_{ij}^t = k_{ij}^t + l_{ij}^t, \quad i, j \in I, t \in T, \quad (3.3)$$

$$n_i^t = n_i^{t-1} - \sum_{j \in I \setminus i} (k_{ij}^t + x_{ij}^t) + \sum_{\tau=1}^t \sum_{j \in I \setminus i: \delta_{ji}^\tau \leq \Delta(t+1-\tau)} k_{ji}^\tau + \sum_{\tau=1}^t \sum_{j \in I \setminus i: \gamma_{ji}^\tau \leq \Delta(t+1-\tau)} x_{ji}^\tau, \quad i \in I, t \in T, \quad (3.4)$$

$$\sum_{j \in I \setminus i} k_{ij}^t \leq n_i^t, \quad i \in I, t \in T, \quad (3.5)$$

$$\sum_{j \in I \setminus i} x_{ij}^t \leq n_i^t - \sum_{j \in I \setminus i} k_{ij}^t + \sum_{j \in I \setminus i: \delta_{ji}^\tau \leq \Delta(t+1-\tau)} k_{ji}^\tau, \quad i \in I, t \in T, \quad (3.6)$$

$$\sum_{i \in I} n_i^0 \leq N_{ev}, \quad i \in I, \quad (3.7)$$

$$d_{ij}^t, x_{ij}^t, n_i^t, k_{ij}^t, l_{ij}^t, p_{ij}^t \in \mathbb{Z}^+. \quad (3.8)$$

Equation (3.1) maximizes the total profit of the car sharing company for a typical day. This is calculated as the total revenue earned from rentals less the vehicle relocation cost and fuel consumption cost of rentals. The rental revenues and trip costs are dependent on the origin-destination pair and period. Equations (3.2) and (3.3) are related to the demand. Equation (3.2) relates the amount of demand for the origin-destination pair and period to its price. Equation (3.3) states that the demand is equal to the sum of the satisfied demand and lost (unsatisfied) demand. This means that the demand can be lost due to the unavailability of vehicles at the origin station. Equation (3.4) calculates the number of available vehicles n_i^{t+1} at station i at the beginning of period $t + 1$. It is equal to the number of available vehicles at station i at the beginning of period t less the number of vehicles that are rented or relocated from station i plus vehicles that are returned or relocated to station i at period t . Note that the number of returned or relocated vehicles to station i is calculated considering the duration of the trip. If a user ends their journey within the period t , then the associated vehicle can be considered as contributing to the supply for the specified period t .

Equations (3.5) and (3.6) calculate the number of rentals and relocation operations in each period and station. Equation (3.5) states that the amount of satisfied demand from station i to other stations in period t can be at most equal to the number of available vehicles at station i at the beginning of period t . Equation (3.6) calculates the total number of relocation operations from station i to others at the end of period t . The number of available vehicles that have not been rented at the beginning of period t , i.e., $n_i^t - k_i^t$, plus the vehicles that return to station i during period t can be used for the relocation operations from station i to others at the end of period t . Equation (3.7) sets an upper bound on the fleet size. Limiting the fleet size makes the solution more realistic. Otherwise, if the fleet size is determined by the model, we may get a solution with an unaffordable fleet size with respect to the operator's budget. Equation (3.8)

defines the domains of the variables $d_{ij}^t, x_{ij}^t, n_i^t, k_{ij}^t, l_{ij}^t$ and p_{ij}^t where \mathbb{Z}^+ denotes the set of non-negative integers.

3.2.1. Demand Function

Different demand functions are used in the literature for relating realized demand and the price for a service [47]. In modeling car sharing systems, both linear and nonlinear demand functions are studied [10, 26–28]. Linear demand function is $d_{ij}^t = D_{ij}^t - \alpha_{ij}^t p_{ij}^t$ and exponential demand function is $d_{ij}^t = D_{ij}^t e^{-\alpha_{ij}^t p_{ij}^t}$. A nonlinear demand function may result in having a mixed integer non-linear non-convex programming model [28]. Therefore, we decide to continue with the linear demand function due to simplicity. With the goal of investigating the effect of the demand function on the results, we developed an unconstrained mathematical model, where all related costs are assumed to be zero and relocation is not taken into account. For the linear demand function, the model consists of the equation (3.9) which maximizes the firm's revenue by optimizing the prices. It is given as

$$\max_{\mathbf{p}} \sum_{i \in I} \sum_{j \in I \setminus i} \sum_{t \in T} p_{ij}^t (D_{ij}^t - \alpha_{ij}^t p_{ij}^t - l_{i,j}^t). \quad (3.9)$$

The first and second partial derivatives of the equation (3.9) with respect to p and l is given below. Some relaxations are necessary to be able to apply KKT conditions, so all decision variables are assumed to be positive variable instead of integer. Equations are written by ignoring the indices for the sake of simplicity. They are given as

$$\begin{aligned} f(p, l) &= Dp - \alpha p^2 - pl, \\ \frac{\partial f}{\partial p} &= D - 2p\alpha - l, \\ \frac{\partial^2 f}{\partial p^2} &= -2\alpha, \\ \frac{\partial^2 f}{\partial p \partial l} &= -1, \\ \frac{\partial f}{\partial l} &= -p, \\ \frac{\partial^2 f}{\partial l^2} &= 0, \\ \frac{\partial^2 f}{\partial l \partial p} &= -1. \end{aligned} \quad (3.10)$$

Hessian matrix can be written with respect to partial derivatives as

$$\begin{bmatrix} -2\alpha & -1 \\ -1 & 0 \end{bmatrix}. \quad (3.11)$$

For a function to be concave, hessian matrix must be negative definite. Eigenvalues of the hessian matrix are $\lambda_1 = -\alpha + \sqrt{\alpha^2 + 1}$ and $\lambda_2 = -\alpha - \sqrt{\alpha^2 + 1}$. Since not all eigenvalues are negative, concavity can not be guaranteed. Even with the linear demand function, an integer nonlinear programming model is obtained because of the terms in the objective function. Due to convexity, the global optimal solution may not be found by the global solvers in a reasonable time and got stuck in local optima. Furthermore, based on the preliminary computations on the proposed model above, it was observed that there is a need to balance the vehicle utilisation rates of the system. The service level of the system can reach up to a certain level. As a result of this, the utilization rate is decreased while the service level remains the same. This may be due to the fact that the objective function of the model focuses only on the supply side of the market, namely on its profitability. However, the demand for car rental is elastic and there are many substitutes such as taxis and so on. Charging high prices may lead to a loss of customers who choose other car sharing apps, public transportation or taxis instead. Considering only the profit of supply in the objective function without considering the price sensitivity of demand can lead to market failure. Therefore, another model is proposed in order to optimize the utilisation rates by minimising the difference between the empty vehicles and the vehicles on lease.

3.2.2. Modification of the Objective Function

We use the same notation as PM. The equation (3.18) is modified as the minimization of the difference between the empty vehicles and the rented vehicles. The new proposed model which is an integer nonlinear mathematical model is called Utility Maximization (UM). The objective function of the model is given as follows

$$\min \sum_{i \in I} \sum_{t \in T} (n_i^t - \sum_j k_{ij}^t). \quad (3.12)$$

Under the same constraints, the model reduces the price as low as possible and brings the demand to a level close to the potential demand D_{ij}^t . In fact, in cases where the

number of vehicles is greater than the potential demand, the price becomes zero, and the vehicles are utilized at the maximum rental level. This situation is a problem in terms of supply, and it is not possible for the company to bring the prices to these levels for the service to be sustainable. Therefore, the proposed solution is to combine the two models and develop an optimization model that considers the satisfaction on both sides of the market. This means that two models will be run sequentially. Under these conditions, an additional constraint, equation (3.19), is introduced into the model. It simply states that the total profit must exceed a given percent μ of the profit Π that can be obtained from PM. Π is the total profit as a result of running PM. μ is the parameter given as a percentage of the desired profit. Related constraint is given as follows

$$\sum_{t \in T} \sum_{i \in I} \sum_{j \in I \setminus i} (p_{ij}^t k_{ij}^t - TC_{ij}^t k_{ij}^t - RC_{ij}^t x_{ij}^t) \geq \mu \Pi. \quad (3.13)$$

To obtain the value of Π , we first solve PM with the profit maximization objective. Then, we solve UM with the minimization objective setting the minimum profit equal to μ percent of Π .

4. COMPUTATIONAL EXPERIMENTS

In this chapter, synthetic data are used to evaluate the efficiency of the solution approach. First, we generate small-scale problem instances to understand the complexity of solving the model. The number of stations is assumed to vary between 4 and 10, which are randomly chosen from the districts in Istanbul. The road distances between districts are taken from site of the Ministry of Transport and Infrastructure. The traveling cost between each station pair is calculated based on this distance.

The demand elasticity α_{ij}^t is taken as 0.10 [28]. The potential demand D_{ij}^t is set to 30 which is the same for each origin-destination pair and in each time interval. The length of an interval, Δ , is set to 60 minutes [28], and the model is run for four, six, and eight intervals. Hence, the model was tested for four, six, and eight hours. The rationale behind this choice is to see the effect of increasing the number of periods on the performance of the model. These parameters are determined after several runs of the model.

In this sharing network, there are different numbers of sharing vehicles travelling between stations. To see the effect of the number of vehicles in the fleet on demand and profitability, the number of vehicles available in the system starts at 50 and increases to 250 in the increments of 50 depending on the level of demand. The literature shows that there is a positive correlation between vehicle availability and frequency of the rentals [48]. Furthermore, Hardt and Bogenberger [49] states that availability is a prerequisite for the generation of rentals and that customers are likely to value high-availability systems for their reliability. Hence, we can deduce that the availability of vehicles and the frequency of rentals determine the success and efficiency of a car sharing system. Therefore, the model is solved with different number of vehicles in order to observe its effect on the efficiency of the system. Finally, it is assumed that the rental and relocation duration between each origin-destination pair does not exceed a time interval for the sake of simplicity.

To sum up, the model is run for 99 different instances by using the combinations of the above-mentioned parameters. The evaluation of the efficiency of a car sharing system involves the assessment of a number of metrics to ensure that it meets the objectives of both users and operators. Some of the commonly used metrics are vehicle utilisation rates, customer satisfaction, which can be measured by service levels, and financial metrics such as revenue and profitability.

4.1. Efficiency Metrics for Car Sharing Systems

- **Utilization Rate:** This metric provides a percentage that represents how many vehicles in the fleet are currently rented or in active use compared to the total number of vehicles available. A higher percentage indicates more effective utilization of the fleet, while a lower percentage suggests that a significant portion of the fleet is not currently rented out or in use. Car rental companies use this metric to assess the efficiency of their fleet management and to make decisions about adjusting fleet size, redistributing vehicles, or implementing marketing strategies to increase rental demand and utilization. Utilization rate is calculated by the formula $\sum_i \sum_j \sum_t k_{ij}^t / n_i^t$.
- **Service Level:** The service level represents the percentage of successful rentals compared to the total amount of potential demand. Service level is calculated by the formula $\sum_i \sum_j \sum_t k_{ij}^t / \sum_i \sum_j \sum_t D_{ij}^t$. This metric is an indicator of how well the car sharing system responds to the overall demand. A higher demand response rate means a better level of service in terms of providing users with the vehicles when they need them.

Each instance is labeled as follows: number of stations / the number of time intervals/ the number of vehicles. For example, 10/4t/250 represents an instance with 10 stations, 4 periods and 250 vehicles. As mentioned above, we generate 99 instances from each problem. The results of these instances are collected and presented in an aggregated way. Figure 4.1 shows the service level and vehicle utilisation rates of each corresponding instance. The color code shows the number of stations and the shape code indicates the number of vehicles in each station. Each graph is plotted according

to the number of periods. For example, the graph in the top right-hand corner shows the results of samples with four periods and the top left-hand corner shows the results of samples with six periods. The pattern in the results does not vary according to the number of stations. Therefore, only 4 random station numbers are used as an example when drawing the graphs. It has been observed that when 7 different station numbers are reported in the same graph, it creates a complex image due to 7 different colour codes and makes the interpretation difficult. Therefore, the results of the instances with 6, 7, 8 and 9 stations are shown in Figure 4.1.

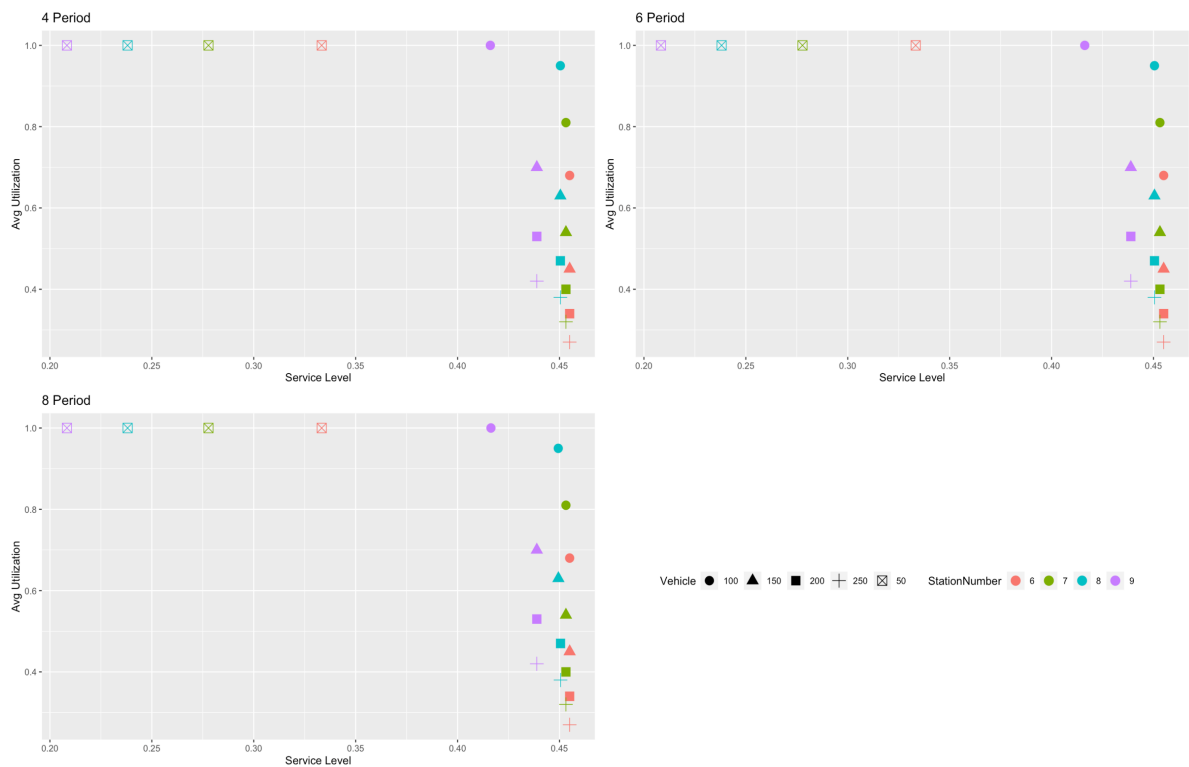


Figure 4.1. PM utilization rates vs service level.

Figure 4.2 illustrates the average profit per station per time period and vehicle utilization rates in each instance. Just as the service level converges to a certain point, the profitability also converges to a certain value. Since the service level does not exceed 45%, it is expected that the profit will remain constant despite the increase in the number of available vehicles.

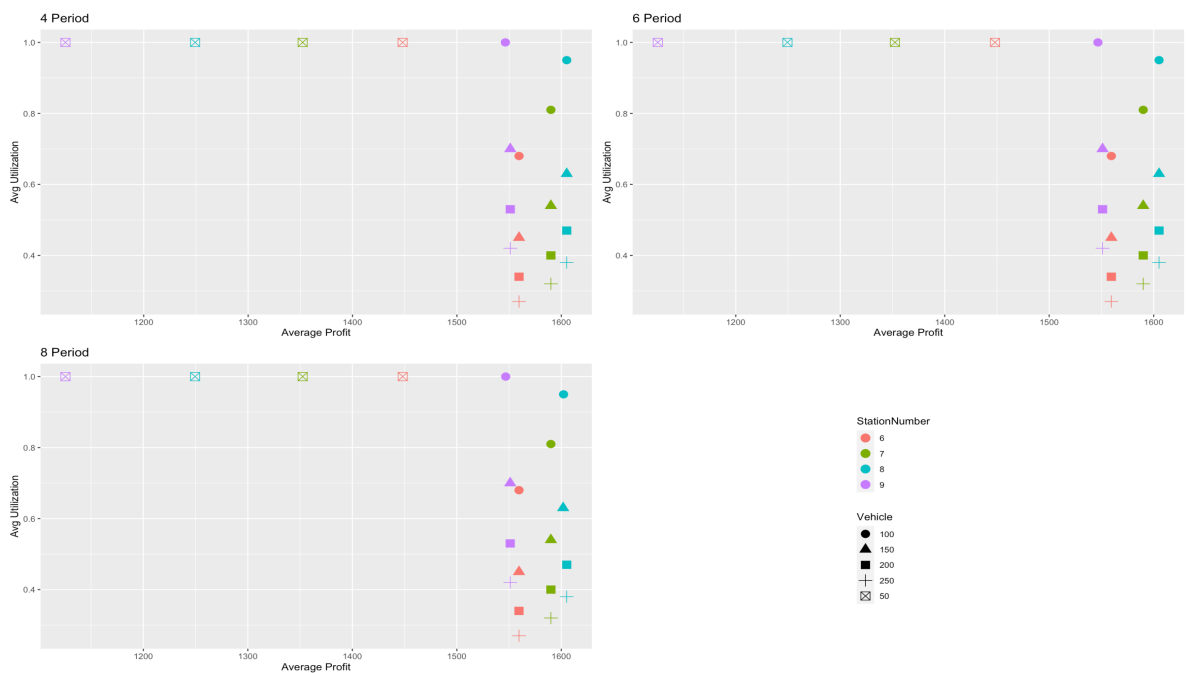


Figure 4.2. PM utilization rates vs average profit.

To better illustrate the results, one of the instances is selected and the metrics are drawn. The instance with the largest problem size was chosen, which has 10 stations and 8 periods. Figure 4.3 shows how the average number of rentals and the average price vary depending on the initial number of available vehicles at each station.

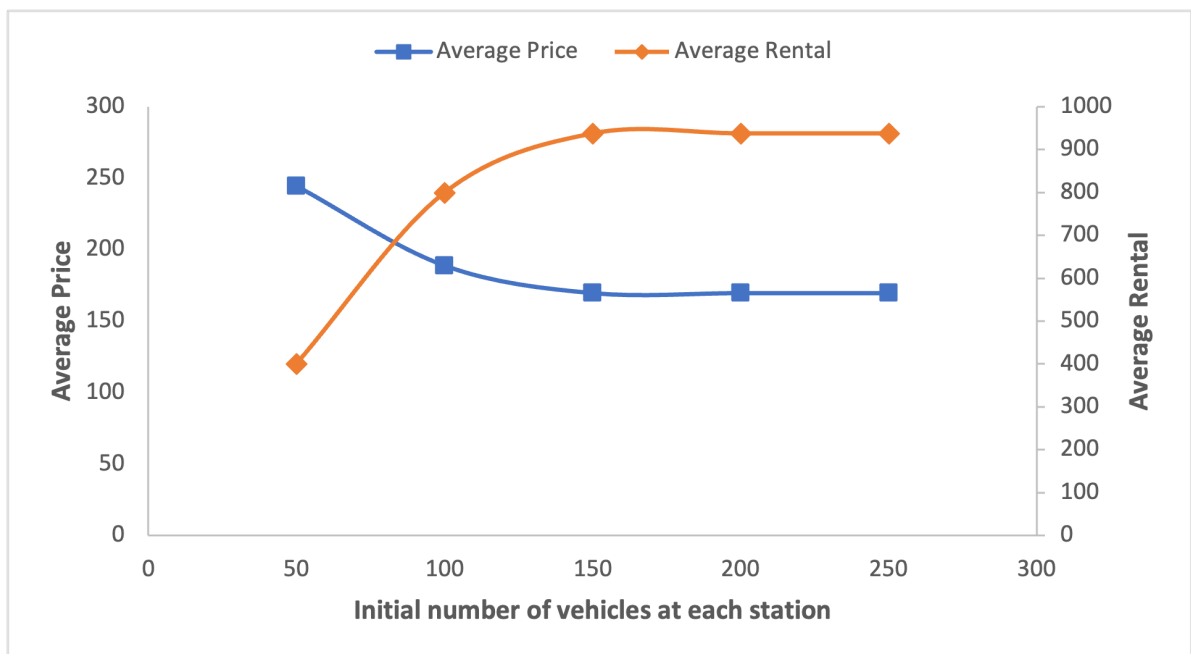


Figure 4.3. Change in the average rental and price.

Figure 4.4 displays how the service level and utilization rates change with the initial number of available vehicles at each station. As can be seen in Figure 4.4, the initial number of vehicles at each station is gradually increased but the service level and average rental converge to a point. The increase in the number of vehicles leads to a decrease in the utilization rate of the vehicles, while the level of service remains the same.

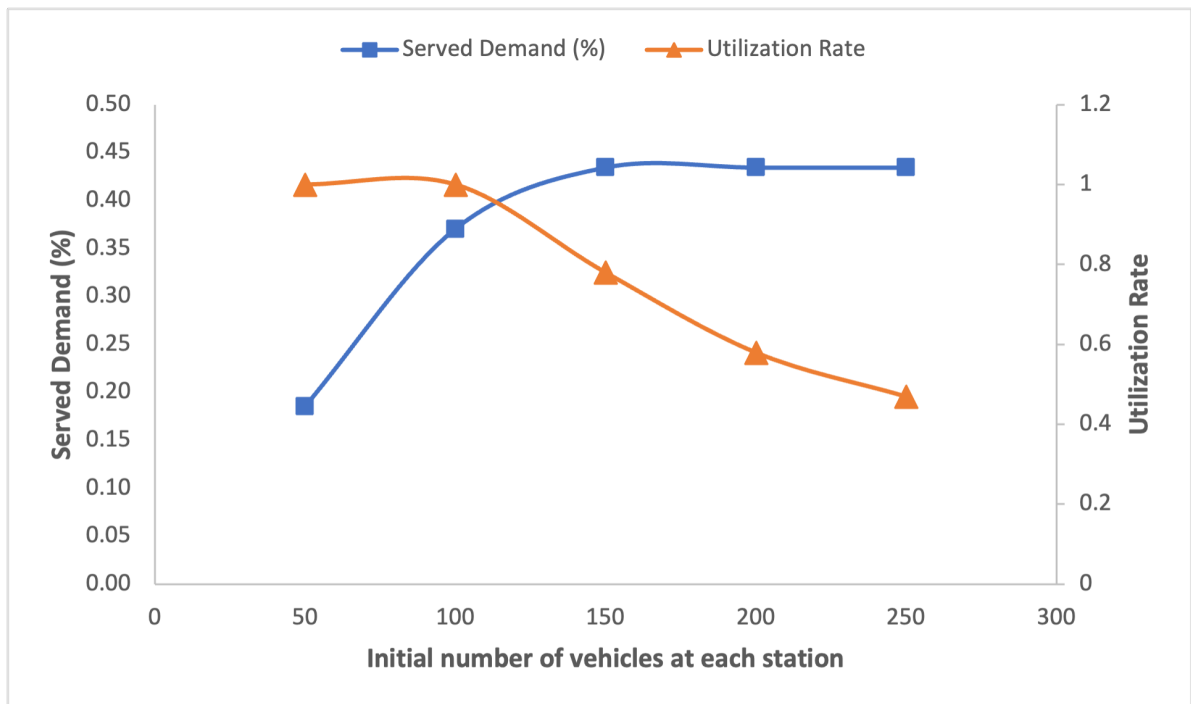


Figure 4.4. Service level and utilization rates.

The main inferences that can be drawn from the results are as follows:

- Regardless of the number of time periods and the number of stations, the increase in the number of vehicles does not lead to an increase in the service level. It can be seen that the increase in the number of vehicles does not lead to an increase in the service level, so resulting in low utilization rates. Since the demand is elastic with respect to price, prices have to be zero in order to satisfy all the potential demand. On the other hand, since the objective function is profit maximization, very low prices are far from optimal, and high prices lead to a reduction in the satisfied demand, therefore so service levels cannot exceed a certain level. Jorge *et al.* [26] reports results similar to ours. They conclude that if prices are unreasonably high,

demand falls to zero by elasticity. At zero demand, the model shuts down the service, which is interpreted as a zero-profit outcome.

- Since an increased number of vehicles does not affect the demand met, we observe that the utilization rate decreases while the service level remains the same. Also, the average profit remains the same as can be seen in Figure 4.2. We observe that the service level converges to 45% and no matter how much we increase the number of vehicles, it goes beyond this level. This service level may be compared with that of the unconstrained model. For unconstrained model, the optimal price level and the optimal quantity demanded depending on the price are as follows, $p^* = \frac{D}{2\alpha}$; $d^* = \frac{D}{2}$. We can see that the optimal quantity demanded that can occur in the unconstrained model is half of the potential demand. The optimal price level depends on the potential demand and price elasticity of demand. If the model were unconstrained, meeting half of the demand would be the optimal point.

Although high prices at this level seem like high profitability for the company, but it may cause problems in terms of sustainability in the long run as it will cause the low service level and low customer satisfaction. Given the dynamics of the carsharing system and the elasticity of demand, a holistic approach that considers both supply and demand is critical to a sustainable and successful business model. Maximising supply-side profitability can be perverse if it leads to pricing strategies that discourage customers and drive them to alternatives. A comprehensive model should address the price sensitivity of demand, given the potential for customers to switch to substitutes such as taxis or other carsharing platforms. A balanced objective function is proposed to reduce the risk of market failure. The objective of UM is the maximisation of the utilisation rate through the minimisation of the difference between the number of empty vehicles and the number of vehicles in use. This approach takes into account that low utilisation rates not only have an impact on profitability but are also a sign of operational inefficiency. Finding the balance between the profitability of the supply side and the sensitivity of the demand side is essential to ensure the long-term sustainability and success of the car sharing service in a competitive and dynamic market. Proposed model details were explained in Chapter 3.3.2.

According to the proposed approach, which is explained in Chapter 3.3.2, UM is run with the same parameters used to run PM. Related profit parameter which equals to objective function value of the PM is taken after each run. Only new parameter which will explain at least what percentage of the profit will be earned is defined as μ . It is chosen as 1, 0.9, 0.8, 0.7, 0.6 and 0.5. Also, same problem representation is used only the difference is μ which is added. Then, new instance representation becomes as number of stations/ time-period/ number of vehicles/the given percentage of lower bound (μ).

For better understanding, one of the instances is selected and run for UM and the graphs of the metrics were drawn for this example. The instance with the largest problem size is chosen which has 10 stations, 8 periods and 250 vehicles. Figure 4.5 shows the relationship between utilization rates, profit, and service level. On the primary vertical axis, utilization rate and service level are shown, and profit is given on the secondary vertical axis. μ is given on the horizontal axis. For example, it can be seen that when μ is 0.6, utilization rate is 77%, service level is 71% and profit is almost 750,000 dollars. It can be seen that the proposed model is able to increase service level and utilization rates by increasing the profit compared to fixed pricing strategy. Higher service levels and utilisation rates can be achieved by relaxing the constraint as μ decreases.

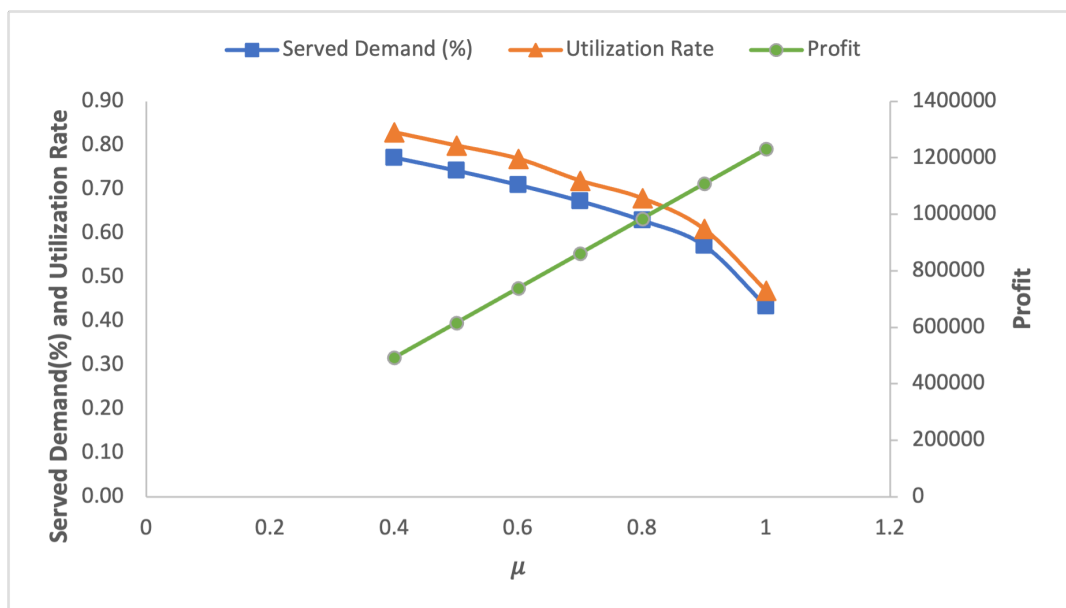


Figure 4.5. Results of UM.

Figure 4.6 shows how the average rentals and the average price change depending on the μ . Since a decrease in μ is a decrease in the profit to be earned, it is to be expected that there will be a decrease in average prices. We have seen that the highest profit can be made when $\mu = 1$, so the average price level is highest when $\mu = 1$. It can also be observed that when the average price drops, the average rental increases due to the inverse relationship between demand and price.

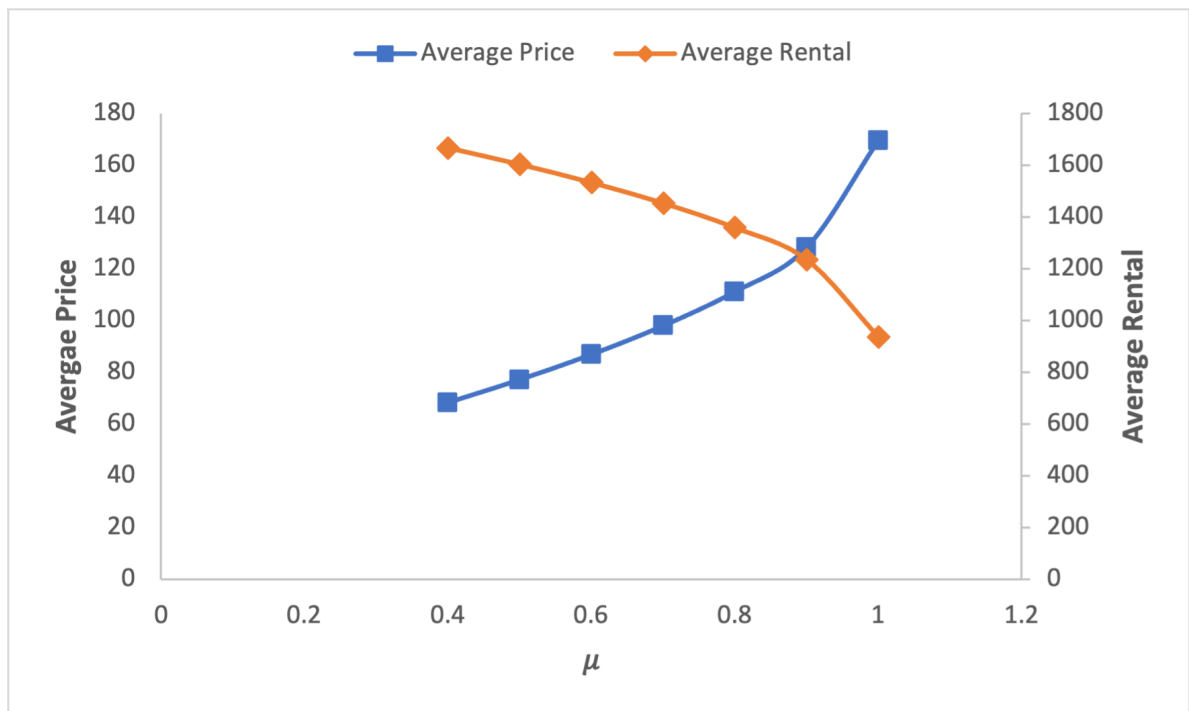


Figure 4.6. Change in the average price and rental of the UM.

Figure 4.7 and Figure 4.8 show the comparison of PM and UM. In Figure 4.7 and Figure 4.8, purple points are the results of PM and orange points are the results of UM. The instances with 10 stations, 8-time intervals 150, 200 and 250 vehicles are shown in Figure 4.7 and Figure 4.8. It can be seen from Figure 4.1 that the service level converges to a level of 45% after the number of vehicles is higher than 150. Therefore, UM is run for the instances with 150, 200 and 250 vehicles. Figure 4.7 shows the average utilization rates and average profit of each instance. Figure 4.8 also shows that service level and utilization rates of each instance. There is an intersection between two model results, which is the point where the given percentage of the lower bound in UM equals 1, because to achieve the same profit with PM, the price level must be the same. It is then observed that service levels and utilisation rates increase as the percentage of profit

required to be earned decreases. Depending on that average profit is also decreasing but is the expected because the prices are set to lower levels when we compare with PM. We see that service level reaches almost 80% which is relatively acceptable rate compare with PM. Also, utilization rates are at high levels for the instances with 150 and 200 vehicles, but service level can reach only 50% when utilization rate is 1 with 150 vehicles. This means that 150 vehicles are not enough to satisfy demand. For 250 vehicles, utilization rates are relatively low compared to the instances with 150 and 200 vehicles. For this case, 200 vehicles seem suitable to satisfy demand. Therefore, we can say more than 200 vehicles are not necessary and will create extra operating cost to company.

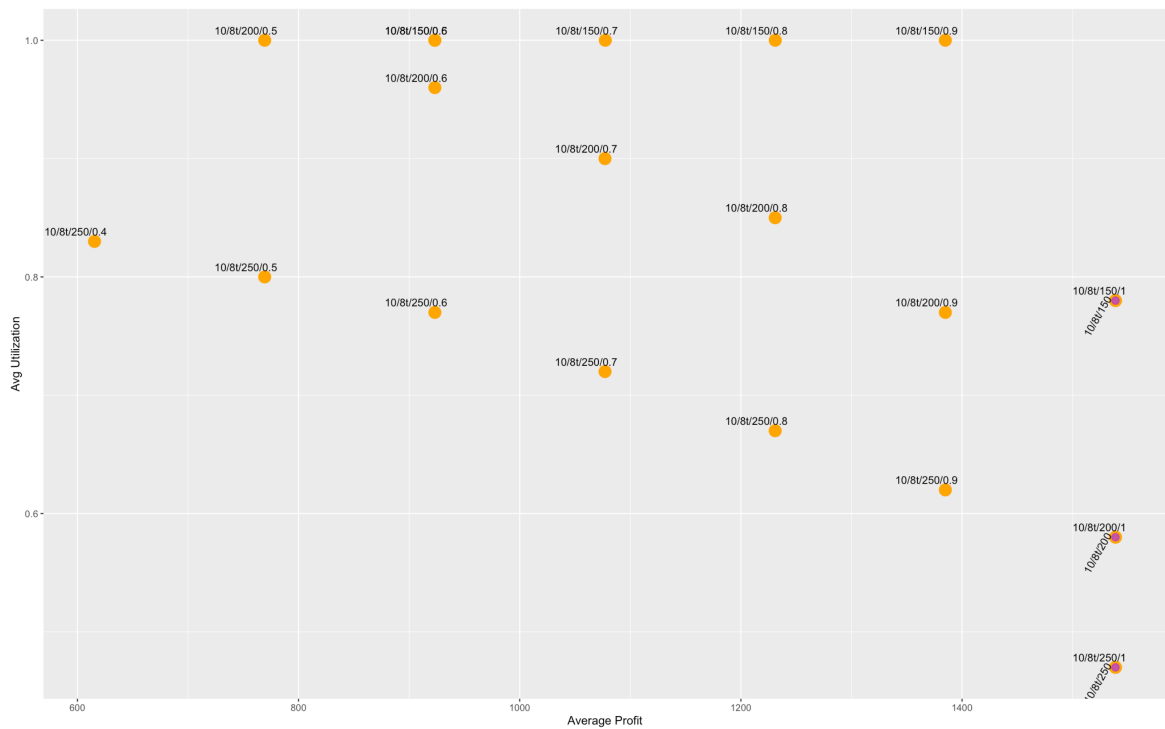


Figure 4.7. Average profit and utilization rates of the UM.

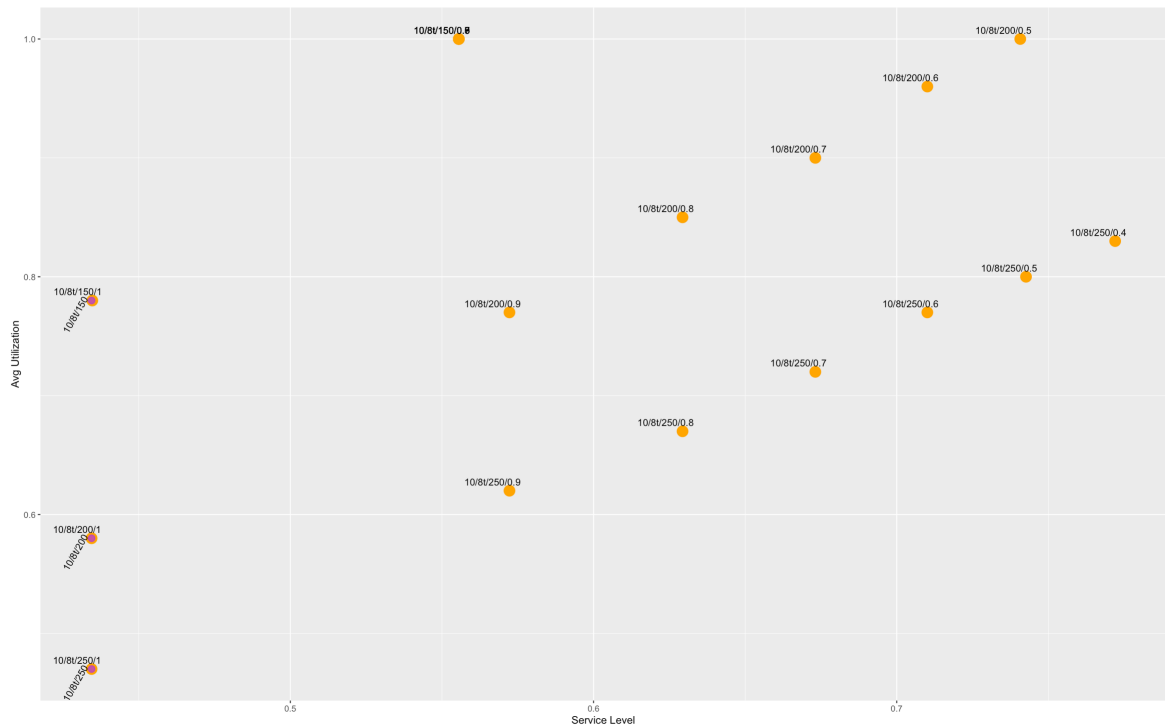


Figure 4.8. Service level and utilization rates of the UM.

4.2. Real Case Study

This section explains the application of the model to a real-life example. The proposed model is also tested using real, publicly available historical data. It is important to use real data rather than synthetic data to assess the robustness of the model. Real data can more accurately reflect how well the model performs in the real-world conditions. While synthetic data sets often have a degree of realism, they may not fully reflect the complexity and variability of the real world. Real data will also contain different distributions and it is important to be able to assess how well the model adapts to these variations. Therefore, the model has been implemented on the real bike sharing system which is the “Capital Bikeshare” and is operating in the city of Washington DC, US. The dataset contains details of the trip ID, pick-up station ID, drop-off station ID and latitude and longitude of the stations, trip duration, dates and trip cost. It has more than 700+ stations with 6000 bikes across 7 jurisdictions. We have processed the data relating to 1st quarter of 2023 trips.

Firstly, all the data is filtered by trip duration. Trips between 30 minutes and 180 minutes are considered for the case study. Long-duration trips are preferred to resemble the car-sharing system. Then, rental data is analysed to define the travel pattern of users. First, rental frequency of each time-period was analysed and to be able to show this histogram chart was preferred. Figure 4.9 shows a histogram chart of the hourly rental frequency. As can be seen from Figure 4.9, the distribution of demand in each period is not uniform. While at some times of the day there is a strong increase in demand, at other times it is close to zero.

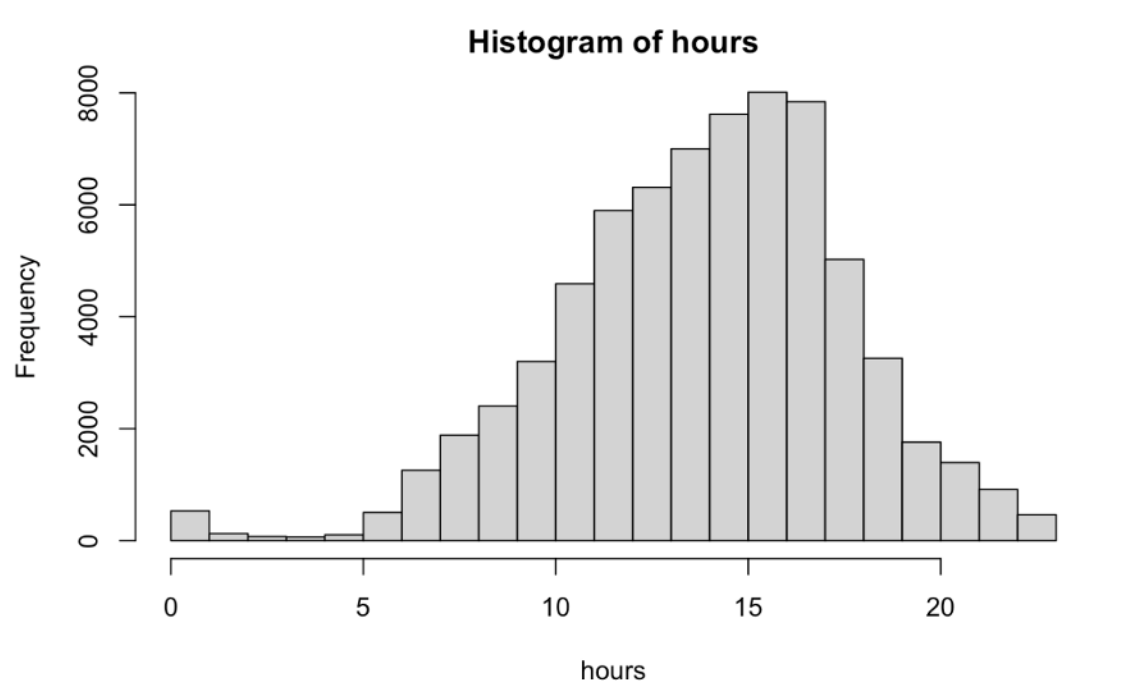


Figure 4.9. Histogram chart of the rentals.

After that, an analysis is carried out to show how the daily and hourly demands differed. As shown in Figure 4.10, the quantity of demand at 12.00-20.00 is significantly higher than other time periods, and there is a significant increase at the weekend compared to working days. Also, demand at 17.00-18.00 also peaks on weekdays, but at weekends demand peaks at noon.

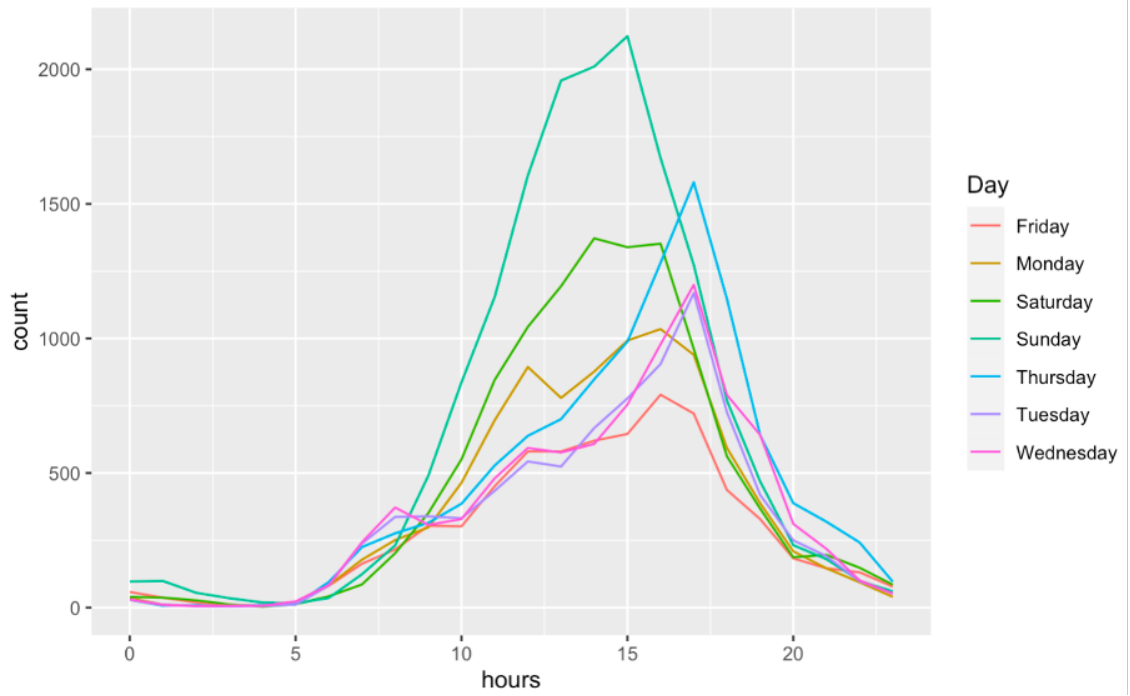


Figure 4.10. Travel patterns of the users.

Especially, the difference between weekend and weekday can be seen from Figure 4.11 below.

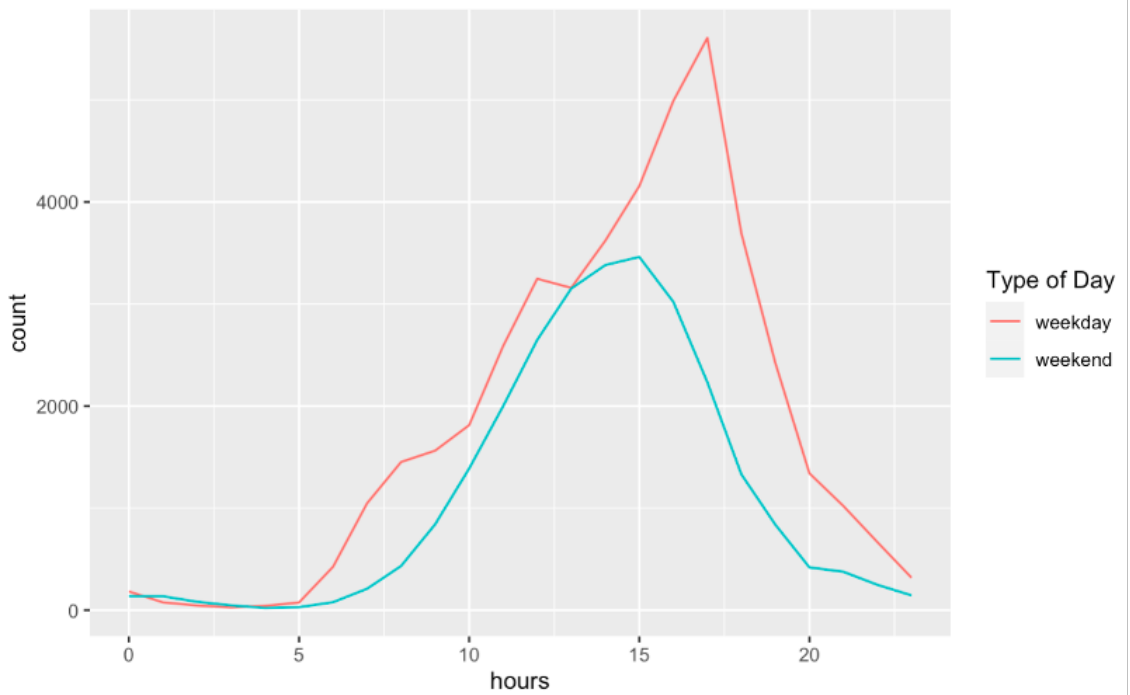


Figure 4.11. Weekend and weekday's travel patterns of the users.

If we summarise analyses results, there is pattern in travel behaviour based on the day and hour as we expected. Due to model complexity, not all the data was used, because even filtering by trip duration showed that there are more than 700 stations, increasing the size of the problem exponentially. Therefore, 8 stations with the highest number of rentals were selected for the case study. This is, of course, a simplification, but it is a general way of testing the suitability of a method in the literature [26]. The rental data with 8 stations is used to run the proposed model. The trip cost is set to 0.55\$ per kilometres between station i and j based on the previous study [50]. They assume that transportation costs are equal to 0.5 Euro per kilometres and independent of the time of the day for the bike sharing system. Since the pricing of the company is in dollars, the trip cost is converted in dollars according to the current dollar and euro exchange rate. As the data contains the coordinates of the stations, the distance between the stations is calculated using these coordinates. To the best of our knowledge, there are no studies in the literature that specifically address the calculation of the price elasticity of demand for car sharing. Therefore, the demand elasticity Q_{ij}^t is taken as 0.10 based on the reference [28]. The initial number of bikes at each station, set at 160, is relatively sufficient to meet the maximum potential demand. The reference price that is charged to users is 0.05\$ per minutes.

The proposed method is coded in GAMS Studio 38. All of the experiments were carried out on a computer equipped with a 2.81 GHz Intel Core i5-8400 processor and 8 GB of RAM, running the Windows 10 Pro operating system.

4.2.1. Results and Discussion

After implementing trip pricing strategy, profit of 128087.4 \$ per day is achieved. In contrast, by applying fixed pricing strategy without any relocation, the system is able to achieve a profit of only 6947.9 \$ per day. Table 4.1 shows the overall results of the system with and without trip pricing and relocation. It can be seen that the profit is indeed high compared to the fixed price strategy, but it is important to note that the results depend on the elasticity of demand. If the elasticity of demand is higher, the price cannot increase to this level and profits cannot be earned at this level. Utilization

rates for both cases almost equal to each other. On the other hand, service level is decreased to 83% due to decrease in served demand. Customer losses due to high prices are likely to occur, because the average price charged relatively higher than the reference price. While the reference price charged to users is 0.05\$ per minute, the average price charged to users after trip pricing is 0.20\$ per minute. However, it can be clearly seen that UM will offer a much more profitable system, as it will serve an acceptable level of demand under the assumption that the elasticity of demand is equal to 0.10. Trip pricing and relocation strategy delivers significantly better revenues by charging higher prices and taking into account the elasticity of demand.

Table 4.1. Overall results of the fixed pricing and trip pricing

	Utilization Rate	Served Demand	Service Level	Average Price \$/Min	Profit	Number of Vehicles	CPU
Fixed Pricing	0.34	2858	0.93	3.27	6947.962	160	0.032
UM	0.30	2585	0.84	10.9	128087.4	160	300

Figure 4.12 shows the number of vehicles in and out of each station and the difference between the trip pricing strategy and the unbalanced system of fixed pricing without relocation. Vehicle entries and exits at each station indicate whether the station has a surplus or shortage of vehicles. Vehicle surplus means that the number of vehicle entries in a station is higher than the number of vehicles exits. Vehicle shortage is the reverse.

Each graph represents a specific time interval. Since the day is divided into 8 equal time intervals, there are 8 separate graphs in the figure. The blue bars show the system before trip pricing and without relocation, the orange bars show the system with trip pricing. For the time intervals 1,2,3,4 and 8, there is no change to be observed in the system. Since the demand for these time intervals is comparatively lower than for others, there is no need to balance the system. On the other hand, it can be seen that the system is more balanced with trip pricing and relocation for time intervals 5, 6 and 7, because relocation is mainly needed for the rush hours. These are the peak periods which are shown in the previous Figure 4.10. It can clearly be seen that without relocation and trip pricing, there is a high surplus or shortage of vehicles at the stations during these time intervals. In particular, during time interval 7, which is the peak demand period, the excess or shortage of vehicles for each station is almost zero.

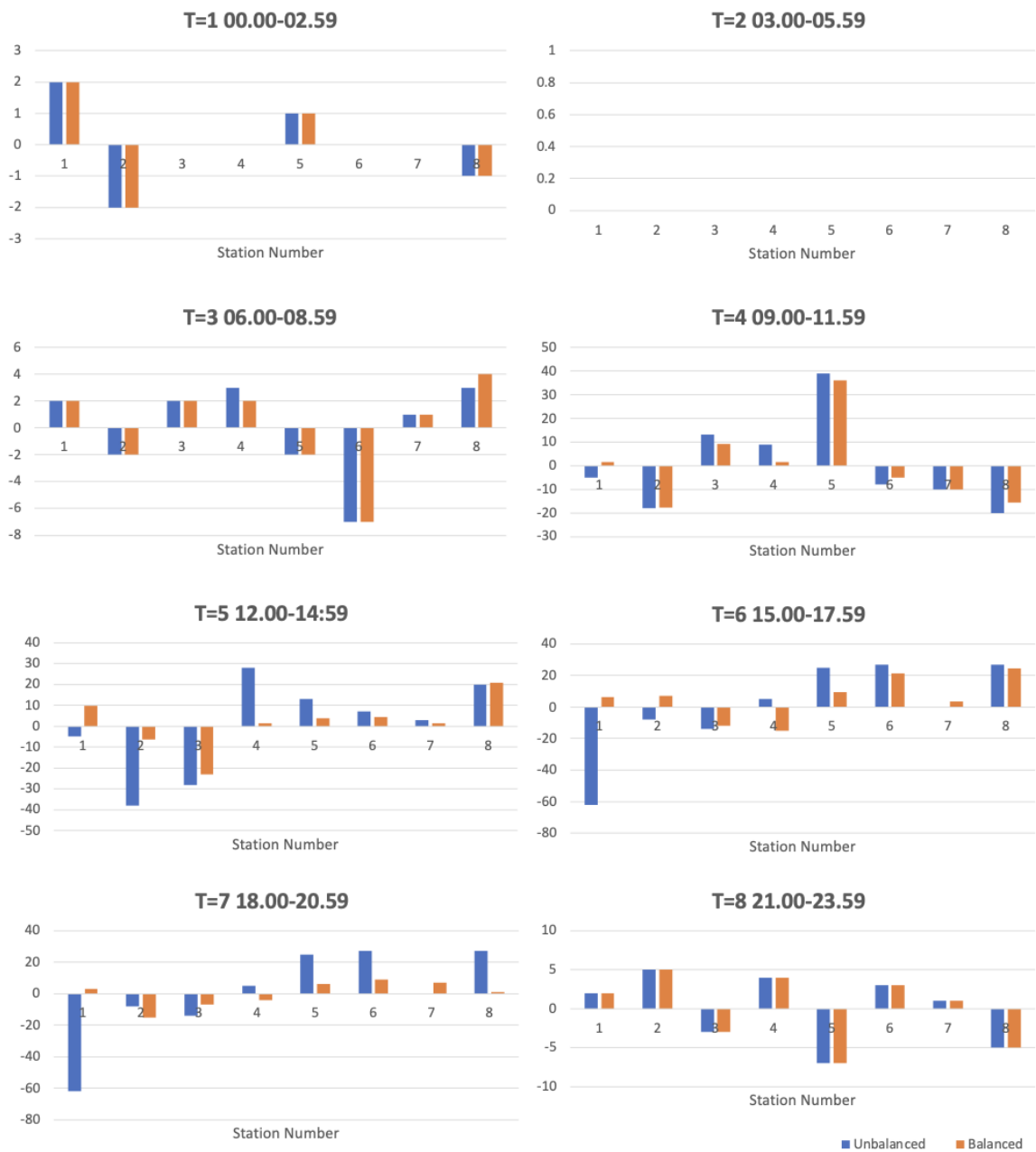


Figure 4.12. Vehicle stock in the stations with fixed pricing and trip pricing of minimization objective model (UM).

If PM is applied, the system is more balanced for all the time intervals. Figure 4.13 shows the number of vehicles in and out of each station. The graphs are drawn to show the difference between without trip pricing and profit maximization strategy. Also, Table 4.2 shows the overall results of three strategies.

Table 4.2. Overall results of the three models

	Utilization Rate	Served Demand	Service Level	Average Price \$/Min	Profit	Number of Vehicles	CPU
Fixed Pricing	0.34	2858	0.93	3.27	6947.962	160	0.032
UM	0.30	2585	0.84	10.9	128087.4	160	300
PM	0.18	1528	0.50	34.5	181982.4074	160	0.43

It is seen that the PM charge relatively higher prices than others. Utilization rate is only 18%, and due to high prices service level decreases up to 50%, and this situation creates a problem for sustainability of the business because of this UM is proposed. This is discussed in Chapter 3.3.1 in detail.

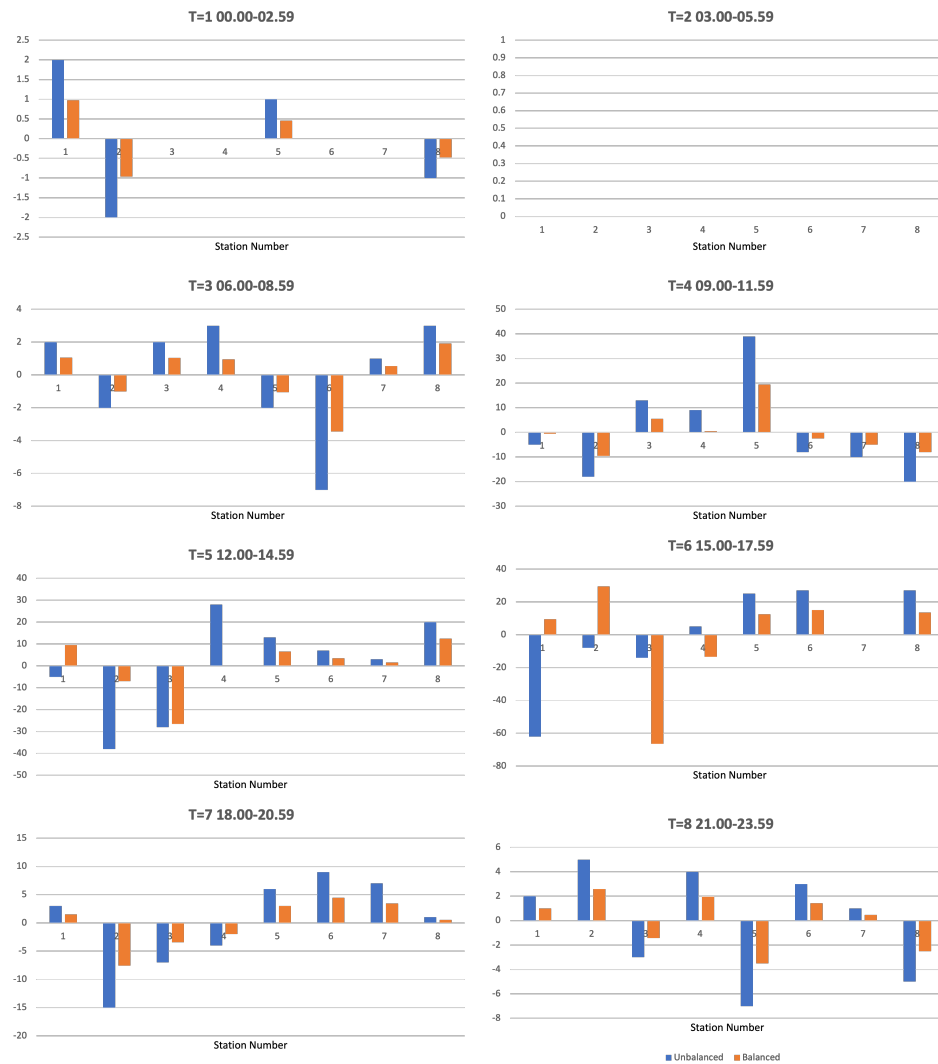


Figure 4.13. Vehicle stock in the stations with fixed pricing and trip pricing of profit maximization model (PM).

In conclusion, trip pricing offers increase in revenue and so more profitable business to carsharing companies. It should be noted that price elasticity of demand should be considered not to lead high loss of customers. Charging higher prices can lead to failure of the system. Also, it is clear that fixed pricing strategy results low profitable system compared to trip pricing due to increase in prices with considering elasticity. Therefore, the improvement in the system is achieved by increasing prices and decreasing reasonable quantity of the demand. Imbalance problem of the system is solved by applying both pricing and relocation strategy. The proposed model is tested with both synthetic data and real case. Due to the problem complexity is increasing exponentially, commercial solvers cannot be able to achieve to find global optimum for large-scale problems in a reasonable time. Therefore, the proposed model can be tested with only 8 stations and 8-time intervals. The number of stations and time intervals must only be large enough to allow the algorithm to solve the problem optimally in a reasonable time. Setting the number of stations too high may not be favourable, as the problem size increases exponentially with the number of stations and time intervals [26].

5. CONCLUSION

The free-floating car-sharing system offers customers the use of a car without ownership. In terms of parking spaces, it is also the most flexible of the three business models. Customers can pick up or drop off vehicles anywhere within the service area. On the other hand, one-way carsharing and two-way carsharing are station-based business models. Customers have to leave the car at these stations, but the location of the stations can be very far from the customer's destination. In the end, FFCS offers greater flexibility for users, but as a result, vehicle distribution management becomes more difficult for companies. There is high demand in areas where there is a shortage of vehicles and lower demand in areas where there is a surplus of vehicles. The availability of vehicles in the right location is key to maintaining customer loyalty and satisfaction. Therefore, the balance of vehicle distribution plays a crucial role in the profitability and sustainability of the business.

This paper discusses trip pricing and relocation strategies to balance the system. Trip pricing charges different fees for each origin-destination pair and each time interval. Experimental results show that trip pricing has the potential to increase revenue and make car sharing companies more profitable compared to a fixed pricing strategy. However, the price elasticity of demand must be taken into account to avoid losing customers. To address the relationship between price and demand, a linear elastic demand function is used. Thus, by increasing prices and decreasing reasonable demand, system improvement is achieved. The imbalance problem is solved by applying both pricing and relocation strategies. Both synthetic and real data are used to test the proposed model. Commercial solvers cannot find the global optimum for large-scale problems in a reasonable time as the complexity of the problem increases exponentially. Therefore, the number of stations and time intervals is limited to eight stations and time intervals. The number of stations and time intervals should only be sufficient to allow the algorithm to solve the problem optimally within a reasonable time.

For future work, it is necessary to develop a heuristic algorithm to test large-scale problems as companies have a significant number of stations. The algorithm's efficiency can be tested in this way. Also, clustering method can be used to group stations, in this way data aggregation can be done. Additionally, it is important to address the stochasticity of the demand to better understand the patterns of demand. In the end, the accuracy of the demand forecast has an impact on the performance of the model.

REFERENCES

1. Rothenberg, S., “Sustainability Through Servicizing”, *MIT Sloan Management Review*, Vol. 48, pp. 83–89, 2007.
2. Weikl, S. and K. Bogenberger, “Relocation Strategies and Algorithms for Free-floating Car Sharing Systems”, *IEEE Intelligent Transportation Systems Magazine*, Vol. 5, No. 4, pp. 100–111, 2013.
3. Shaheen, S. A., A. P. Cohen and J. D. Roberts, “Carsharing in North America: Market Growth, Current Developments, and Future Potential”, *Transportation Research Record*, Vol. 1986, No. 1, pp. 116–124, 2006.
4. Catalano, M., B. Lo Casto and M. Migliore, “Car Sharing Demand Estimation and Urban Transport Demand Modelling Using Stated Preference Techniques”, *European Transport*, Vol. 40, pp. 33–50, 2008.
5. Zheng, J., M. Scott, M. Rodriguez, W. Sierzechula, D. Platz, J. Y. Guo and T. M. Adams, “Carsharing in a University Community: Assessing Potential Demand and Distinct Market Characteristics”, *Transportation Research Record*, Vol. 2110, No. 1, pp. 18–26, 2009.
6. Shaheen, S. A. and A. P. Cohen, “Carsharing and Personal Vehicle Services: Worldwide Market Developments and Emerging Trends”, *International Journal of Sustainable Transportation*, Vol. 7, No. 1, pp. 5–34, 2013.
7. Zhou, F., Z. Zheng, J. Whitehead, R. Perrons, L. Page and S. Washington, “Projected Prevalence of Car-sharing in Four Asian-Pacific Countries in 2030: What the Experts Think”, *Transportation Research Part C: Emerging Technologies*, Vol. 84, pp. 158–177, 2017.
8. Boyacı, B., K. G. Zografos and N. Geroliminis, “An Optimization Framework for the Development of Efficient One-way Car-sharing Systems”, *European Journal of*

- Operational Research*, Vol. 240, No. 3, pp. 718–733, 2015.
9. Lu, M., Z. Chen and S. Shen, “Optimizing the Profitability and Quality of Service in Carshare Systems Under Demand Uncertainty”, *Manufacturing & Service Operations Management*, Vol. 20, No. 2, pp. 162–180, 2018.
 10. Huang, K., K. An and G. H. de Almeida Correia, “Planning Station Capacity and Fleet Size of One-way Electric Carsharing Systems with Continuous State of Charge Functions”, *European Journal of Operational Research*, Vol. 287, No. 3, pp. 1075–1091, 2020.
 11. Wu, T. and M. Xu, “Modeling and Optimization for Carsharing Services: A Literature Review”, *Multimodal Transportation*, Vol. 1, No. 3, p. 100028, 2022.
 12. Bruglieri, M., F. Pezzella and O. Pisacane, “A Two-phase Optimization Method for a Multiobjective Vehicle Relocation Problem in Electric Carsharing Systems”, *Journal of Combinatorial Optimization*, Vol. 36, pp. 162–193, 2018.
 13. Gambella, C., E. Malaguti, F. Masini and D. Vigo, “Optimizing Relocation Operations in Electric Car-sharing”, *Omega*, Vol. 81, pp. 234–245, 2018.
 14. Xu, M. and Q. Meng, “Fleet Sizing for One-way Electric Carsharing Services Considering Dynamic Vehicle Relocation and Nonlinear Charging Profile”, *Transportation Research Part B: Methodological*, Vol. 128, pp. 23–49, 2019.
 15. Bruglieri, M., A. Colorni and A. Luè, “The Relocation Problem for the One-way Electric Vehicle Sharing”, *Networks*, Vol. 64, No. 4, pp. 292–305, 2014.
 16. Weikl, S. and K. Bogenberger, “A Practice-ready Relocation Model for Free-floating Carsharing Systems with Electric Vehicles—Mesoscopic Approach and Field Trial Results”, *Transportation Research Part C: Emerging Technologies*, Vol. 57, pp. 206–223, 2015.
 17. Boyacı, B., K. G. Zografos and N. Geroliminis, “An Integrated Optimization-

- simulation Framework for Vehicle and Personnel Relocations of Electric Carsharing Systems with Reservations”, *Transportation Research Part B: Methodological*, Vol. 95, pp. 214–237, 2017.
18. Jorge, D., G. H. Correia and C. Barnhart, “Comparing Optimal Relocation Operations with Simulated Relocation Policies in One-way Carsharing Systems”, *IEEE Transactions on Intelligent Transportation Systems*, Vol. 15, No. 4, pp. 1667–1675, 2014.
 19. Bruglieri, M., F. Pezzella and O. Pisacane, “Heuristic Algorithms for the Operator-based Relocation Problem in One-way Electric Carsharing Systems”, *Discrete Optimization*, Vol. 23, pp. 56–80, 2017.
 20. Bruglieri, M., F. Pezzella and O. Pisacane, “An Adaptive Large Neighborhood Search for Relocating Vehicles in Electric Carsharing Services”, *Discrete Applied Mathematics*, Vol. 253, pp. 185–200, 2019.
 21. Barth, M., M. Todd and L. Xue, “User-based Vehicle Relocation Techniques for Multiple-Systems”, *Transportation Research Record*, Vol. 1887, pp. 137–144, 2004.
 22. Di Febbraro, A., N. Sacco and M. Saeednia, “One-way Carsharing: Solving the Relocation Problem”, *Transportation Research Record*, Vol. 2319, No. 1, pp. 113–120, 2012.
 23. Clemente, M., M. P. Fanti, A. M. Mangini and W. Ukovich, “The Vehicle Relocation Problem in Car Sharing Systems: Modeling and Simulation in a Petri Net Framework”, *Application and Theory of Petri Nets and Concurrency: 34th International Conference, PETRI NETS 2013, Milan, Italy, June 24-28, 2013. Proceedings 34*, pp. 250–269, Springer, 2013.
 24. Di Febbraro, A., N. Sacco and M. Saeednia, “One-way Car-sharing Profit Maximization by Means of User-based Vehicle Relocation”, *IEEE Transactions on Intelligent Transportation Systems*, Vol. 20, No. 2, pp. 628–641, 2018.

25. Herbawi, W., M. Knoll, M. Kaiser and W. Gruel, “An Evolutionary Algorithm for the Vehicle Relocation Problem in Free Floating Carsharing”, *2016 IEEE Congress on Evolutionary Computation (CEC)*, pp. 2873–2879, IEEE, 2016.
26. Jorge, D., G. Molnar and G. H. de Almeida Correia, “Trip Pricing of One-way Station-based Carsharing Networks with Zone and Time of Day Price Variations”, *Transportation Research Part B: Methodological*, Vol. 81, pp. 461–482, 2015.
27. Ren, S., F. Luo, L. Lin, S.-C. Hsu and X. I. Li, “A Novel Dynamic Pricing Scheme for a Large-scale Electric Vehicle Sharing Network Considering Vehicle Relocation and Vehicle-grid-integration”, *International Journal of Production Economics*, Vol. 218, pp. 339–351, 2019.
28. Xu, M., Q. Meng and Z. Liu, “Electric Vehicle Fleet Size and Trip Pricing for One-way Carsharing Services Considering Vehicle Relocation and Personnel Assignment”, *Transportation Research Part B: Methodological*, Vol. 111, pp. 60–82, 2018.
29. Giorgione, G. and F. Viti, “Profit and Utility Optimization Through Joint Dynamic Pricing and Vehicle Relocation in Carsharing Operations”, *Case Studies on Transport Policy*, Vol. 13, p. 101048, 2023.
30. Wang, N., Q. Liu, J. Guo and T. Fang, “A User-based Adaptive Joint Relocation Model Combining Electric Car-sharing and Bicycle-sharing”, *Transportmetrica B: Transport Dynamics*, Vol. 10, No. 1, pp. 1046–1069, 2022.
31. Clemente, M., M. P. Fanti, G. Iacobellis, M. Nolich and W. Ukovich, “A Decision Support System for User-based Vehicle Relocation in Car Sharing Systems”, *IEEE Transactions on Systems, Man, and Cybernetics: Systems*, Vol. 48, No. 8, pp. 1283–1296, 2017.
32. Folkestad, C. A., N. Hansen, K. Fagerholt, H. Andersson and G. Pantuso, “Optimal Charging and Repositioning of Electric Vehicles in a Free-floating Carsharing System”, *Computers & Operations Research*, Vol. 113, p. 104771, 2020.

33. Huo, X., X. Wu, M. Li, N. Zheng and G. Yu, “The Allocation Problem of Electric Car-sharing System: A Data-Driven Approach”, *Transportation Research Part D: Transport and Environment*, Vol. 78, p. 102192, 2020.
34. Wang, L., W. Ma, M. Wang and X. Qu, “Demand Control Model with Combinatorial Incentives and Surcharges for One-way Carsharing Operation”, *Transportation Research Part C: Emerging Technologies*, Vol. 125, p. 102999, 2021.
35. Wang, L. and W. Ma, “Pricing Approach to Balance Demands for One-way Car-sharing Systems”, *2019 IEEE Intelligent Transportation Systems Conference (ITSC)*, pp. 1697–1702, IEEE, 2019.
36. Huang, K., K. An, G. H. de Almeida Correia, J. Rich and W. Ma, “An Innovative Approach to Solve the Carsharing Demand-supply Imbalance Problem Under Demand Uncertainty”, *Transportation Research Part C: Emerging Technologies*, Vol. 132, p. 103369, 2021.
37. Schiffer, M., G. Hiermann, F. Rüdell and G. Walther, “A Polynomial-Time Algorithm for User-based Relocation in Free-floating Car Sharing Systems”, *Transportation Research Part B: Methodological*, Vol. 143, pp. 65–85, 2021.
38. Wang, N., J. Guo, X. Liu and Y. Liang, “Electric Vehicle Car-sharing Optimization Relocation Model Combining User Relocation and Staff Relocation”, *Transportation Letters*, Vol. 13, No. 4, pp. 315–326, 2021.
39. Yang, S., J. Wu, H. Sun, Y. Qu and T. Li, “Double-balanced Relocation Optimization of One-way Car-sharing System with Real-time Requests”, *Transportation Research Part C: Emerging Technologies*, Vol. 125, p. 103071, 2021.
40. Bao, L., R. Miao, B. Zhang, J. Zhou and H. Hu, “Research on Dynamic Pricing of Shared Electric Vehicles Based on System Utility Maximization”, *2021 IEEE International Conference on Industrial Engineering and Engineering Management (IEEM)*, pp. 1466–1470, IEEE, 2021.

41. Sakurama, K., “Optimal Control and Station Relocation of Vehicle-sharing Systems with Distributed Dynamic Pricing”, *IEEE Open Journal of Intelligent Transportation Systems*, Vol. 4, pp. 393–405, 2023.
42. Huang, C.-Y., Z.-P. Fan and C. Zhang, “Optimal Pricing and Pricing Policy Selection for a B2C Car-sharing Platform During Peak and Off-peak Hours”, *Information Sciences*, Vol. 604, pp. 197–209, 2022.
43. Xie, R., W. Wei, Q. Wu, T. Ding and S. Mei, “Optimal Service Pricing and Charging Scheduling of an Electric Vehicle Sharing System”, *IEEE Transactions on Vehicular Technology*, Vol. 69, No. 1, pp. 78–89, 2019.
44. Soppert, M., C. Steinhardt, C. Müller and J. Gönsch, “Differentiated Pricing of Shared Mobility Systems Considering Network Effects”, *Transportation Science*, Vol. 56, No. 5, pp. 1279–1303, 2022.
45. Agatz, N., A. M. Campbell, M. Fleischmann, J. Van Nunen and M. Savelsbergh, “Revenue Management Opportunities for Internet Retailers”, *Journal of Revenue and Pricing Management*, Vol. 12, pp. 128–138, 2013.
46. Huang, K., K. An, J. Rich and W. Ma, “Vehicle Relocation in One-way Station-based Electric Carsharing Systems: A Comparative Study of Operator-based and User-based Methods”, *Transportation Research Part E: Logistics and Transportation Review*, Vol. 142, p. 102081, 2020.
47. Huang, J., M. Leng and M. Parlar, “Demand Functions in Decision Modeling: A Comprehensive Survey and Research Directions”, *Decision Sciences*, Vol. 44, No. 3, pp. 557–609, 2013.
48. Ferrero, F., G. Perboli, M. Rosano and A. Vesco, “Car-sharing Services: An Annotated Review”, *Sustainable Cities and Society*, Vol. 37, pp. 501–518, 2018.
49. Hardt, C. and K. Bogenberger, “Empirical Analysis of Demand Patterns and Availability in Free-floating Carsharing Systems”, *2018 21st International Conference on*

Intelligent Transportation Systems (ITSC), pp. 1186–1193, IEEE, 2018.

50. Vogel, P., J. F. Ehmke and D. C. Mattfeld, “Cost-efficient Allocation of Bikes to Stations in Bike Sharing Systems”, *Computational Logistics: 8th International Conference, ICCL 2017, Southampton, UK, October 18-20, 2017, Proceedings 8*, pp. 498–512, Springer, 2017.
51. Bruglieri, M., A. Colorni and A. Lue, “The Vehicle Relocation Problem for the One-way Electric Vehicle Sharing: An Application to the Milan Case”, *Procedia-Social and Behavioral Sciences*, Vol. 111, pp. 18–27, 2014.