PRICING AND POSITIONING OF REMANUFACTURED PRODUCTS USING A NESTED LOGIT MODEL

by

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to my family

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ABSTRACT

PRICING AND POSITIONING OF REMANUFACTURED PRODUCTS USING A NESTED LOGIT MODEL

Product line selection and pricing are two critical strategic decisions for a manufacturing firm in terms of profitability. Remanufacturing can also significantly contribute to the profitability of a firm by reusing the used products by capturing the residual value in them rather than discarding them. In this work, we consider these two concepts together and focus on selecting optimal products within a set of candidate remanufactured products to be offered by their existing counterparts in the same market as well as setting the optimal prices of existing products and their remanufactured versions simultaneously. We formulate a mixed-integer nonlinear programming model with a cost constraint in the aim of maximizing the profit function. The customer choice is assumed to be probabilistic in which the purchase probability is proportional to an increasing monotonic function of the utility which is a linear function of product quality and price for the product, and substitution is considered by a nested structure. A base model and an extension of it are proposed. In the extended model the optimal quality level of an offered remanufactured product is tried to be determined by taking its unit production cost of quality as another decision variable in the model. The resulting model is solved by decomposing it into two sub-problems. The pricing sub-problem is solved by a modified and restarted simplex search procedure whereas the product line selection problem is solved via complete enumeration. Some computational studies are made to identify the situations under which selling remanufactured products together with their existing counterparts is profitable.

ÖZET

KÜMELENMİŞ MÜŞTERİ MODELİ KULLANARAK YENİDEN İMAL EDİLMİŞ ÜRÜNLERİN FİYATLANDIRILMASI VE KONUMLANDIRILMASI

Ürün gamı seçimi ve fiyatlandırılması bir imalat firmasının karlılığı açısından iki kritik stratejik karardır. Ayrıca yeniden imalat kullanılmış ürünleri ıskartaya ayırmak yerine onlarda artakalan değeri yakalayarak firmanın karlılığına önemli katkılar sağlayabilir. Bu çalışmada bu iki kavramı birlikte göz önüne alarak eski muadilleri ile birlikte aynı pazara sunmak üzere yeniden imal edilmiş ürünler kümesinden en iyi ürünleri seçerken aynı zamanda tüm ürünlerin fiyatlarının belirlenmesi üzerinde durulmuştur. Kar fonksiyonunu en büyüklemek amacıyla maliyet kısıtlı karışık tamsayılı doğrusal olmayan programlama modeli formüle ettik. Müşteri tercihi olasılıklı olup, bir ürünün faydasının onun fiyatı ve kalitesinin doğrusal bir fonksiyonu olduğu ve bir ürünün seçilme olasılığının o ürünün faydasının monoton artan fonksiyonuyla doğru orantılı olduğu varsayılmaktadır ve yerine kullanma kümeleme yapısıyla göz önüne alınmıştır. Bir temel model ve bunun bir eklentisi sunulmaktadır. Genişletilmiş modelde sunulan yeniden imal edilen ürünün birim üretim maliyeti modelde bir diğer karar değişkeni olarak kullanılarak bu ürünün en iyi kalite seviyesi belirlenmeye çalışılmaktadır. Elde edilen model iki yan probleme parçalanarak çözülmüştür. Fiyatlandırma problemi değiştirilmiş ve yeniden başlatılmış bir simpleks arama prosedürüyle çözülürken ürün gamı seçimi tam sayım yöntemiyle çözülmüştür. Yeniden imal edilen ürünlerin önceden varolan muadilleriyle birlikte satılmasının karlı olduğu durumların belirlenmesi için bazı sayısal çalışmalar yapılmıştır.

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LIST OF SYMBOLS /ABBREVATIONS

a, b, c	Alternatives (products) in the choice set $C(N)$
a_{jt}	Binary digit represents whether alternative j owns attribute t
A, B, C_1, C_2	Subsets of the set of alternatives in the choice set C
c_j	Variable cost of product j
c_{hj}	Unit production cost of product j in nest H_h
C	The choice set of alternatives
C_h	Set of products in nest C_h
C_t	Set of alternatives that have attribute t
d	Total population of individuals facing the same choice set C
d_i	Size of customer segment i
$\mathbf{d}$	Direction vector used to form an initial simplex
f_{hj}	Fixed cost of introducing remanufactured product j in nest H_h
h, r	Index for nests
H	Set of nests
H_h	Set of products in nest H_h
i	Index for customer segments
IV_h^i	Inclusive value of nest H_h to customer segment i
j, k	Index for products
K_{hj}^i	Scalar cost coefficient of product j in nest H_h to customer segment i
m	Number of nests other than nest 0 (i.e. outside group)
m_h	Number of products in nest H_h
M	Set of customer segments
M_a	Market space of alternative a
N	Set of products including the competitors'
N_C	Set of competitors' products
N_E	Set of manufactured (existing) products

N_R	Candidate set of remanufactured products
N_0	Outside group representing no-purchase alternative
p_h^i	Marginal probability that segment i purchases a product within nest H_h
p_{hj}^i	Probability that segment i purchases product j in nest H_h
$p_{j\|h}^i$	Conditional probability that segment i purchases product j given that nest H_h is chosen
p_{01}^i	Probability that segment i does not purchase anything
$P(a\|C)$	Probability of choosing alternative a from the choice set C
Q_{hj}^i	Perceived quality of product j in nest H_h for a customer in segment i
r,t	Index for attributes related to alternatives in the choice set C
$\Re^T$	Attribute space of T attributes
S_j	Selling price of product j
S_{hj}	Selling price of product j in nest H_h
S	Selling price vector
T	Number of attributes related to alternatives in the choice set C
u_t	Utility function value for an attribute t
$U(a)$	Utility function value of an alternative a
U_h	Marginal utility component related to nest H_h in the utility function
U_j	Utility function value of alternative (product) j
$U_{j\|h}$	Conditional utility component related to alternative j in nest H_h in the utility function
U_{ij}	Utility of customer i for product j
U_{hj}^i	Random utility of product j in nest H_h for a customer in segment i
U_{01}^i	Utility of no-purchase alternative related to segment i
$U_j(y)$	Utility function value of alternative j for consumer y
v_h	Marginal random component related to nest H_h in the utility function
$v_{j\|h}$	Conditional random component related to nest H_h in the utility function

V_h	Representative utility depends on variables that describe nest H_h
V_j	Known part of utility (representative utility) for alternative (product) j
$V^i_{j\|h}$	Representative utility of product j in nest H_h for a customer in segment i
$V^i_{1\|0}$	Representative utility of no-purchase alternative related to segment i
w	Index for decision-maker
x_{ij}	Binary variable showing whether segment i chooses product j
X^i_{hj}	Amount of product j in nest H_h that is sold to segment i
X^i_{01}	Amount of people in segment i that does not purchase anything
y	Consumer's address (ideal point) in attribute space $\Re^T$
Y_j	Binary variable denoting whether the product j in nest H_h is offered or not
Y_{hj}	Binary variable denoting whether product j is offered
z_j	Address of alternative j in attribute space $\Re^T$

α	Reflection coefficient in simplex search
α^i_p	Coefficient of selling price for segment i
β	Contraction coefficient in simplex search
β^i_q	Coefficient of perceived quality for segment i
χ	Shrinkage coefficient in simplex search
ε^i_{hj}	Random utility term of product j in nest H_h for a customer in segment i
γ	Euler's constant
γ_e	Expansion coefficient in simplex search
λ_h	Upper level scale parameter related to nest H_h
μ^i_h	Lower level scale parameter related to nest H_h for a customer in segment i
Π	Profit function
υ_{ij}	Value of product j chosen by customer i to the seller

CNL	Cross-nested logit
CPU	Central processing unit

EBA	Elimination by aspects
GEV	Generalized extreme value
IIA	Independence of irrelevant alternatives
MINLP	Mixed-integer nonlinear programming
MNL	Multinomial logit
MNM	Modified Nelder and Mead
NLP	Nonlinear programming
NL	Nested logit
NM	Nelder and Mead
NNNL	Non-normalized nested logit
OEM	Original equipment manufacturer
REVLOG	European working group on reverse logistics
RL	Reverse logistics
RMNM	Restarted and modified Nelder and Mead
RU	Random utility
RUM	Random utility maximization
UMNL	Utility maximization nested logit

1. INTRODUCTION

In today's competitive business environment, product line selection and pricing decisions of similar products are critical for firms to remain profitable. It is a common belief that increasing product variety also increases the market power of a firm, and accordingly its profit margin and market share. However, there is a conflicting effect of product variety on revenue and cost, and firms should consider this effect in product line decisions.

Many manufacturing firms offer a broad range of products to respond to different tastes of customers and capture the potential demand from multiple customer segments. Nonetheless, the decision of broadening the product line, i.e. offering a variety of differentiated products is a critical issue for many firms. Such extensive differentiation is prevented by some factors. On the demand side, introduction of additional products may result in a decrease on the demand of the existing products, which further may reduce the total profit. Namely, the new product may satisfy the needs of some customers in the market better, and it may prevent these customers from, purchasing other products, which can result in a reduction in the sales volume, revenue, or market share of a more profitable (higher-margin) product. This phenomenon is called cannibalization. Therefore, a firm may prefer to sell a smaller number of products considering that the products it offers will cannibalize each other. Furthermore, a firm would bear some additional costs due to the introduction of a new product and the amount of these fixed costs may be greater than the additional profit gained by that product, which will in turn make the introduction of this product unprofitable. Hence, firms must pay attention to the revenue and cost implications of product line decisions because it directly affects the firm's profitability.

On the other hand, the number of manufacturers that are paying attention to the remanufacturing as a production strategy has been increasing recently for various reasons such as environmental issues, legislative pressure, and the possibility of incremental profit. Remanufacturing is a recovery process which adds value to the collected used products after replacing worn-out components by new ones (Thierry *et al.* 1995). Since remanufactured products are often valued less than their brand-new counterparts by the customers, their prices need to be lower than those of manufactured products.

The aim of this thesis is to determine the conditions under which it is a better strategy to offer the remanufactured products together with their existing versions in the product line, where the market consists of various homogeneous customer segments. The main concern is to explore the profitability of the remanufactured products. In our study, customer choice is modeled using the nested logit (NL) model to capture some correlations between alternatives since a remanufactured product can be viewed as a close substitute of its brand-new version, and cannibalizes its sales more than the other products offered in the same market. We define the observed utility of a product as a linear function of its quality and price, and the purchase probability is proportional to an increasing monotonic function of the utility for the product. We assume that customers are able to distinguish between new and remanufactured products. Customers are allowed to choose a product from those offered in the market, but they have also the option of nothing to purchase. If a customer makes a purchase decision, he/she purchases at most one unit of product. Some of the competitors' products are included in the model to monitor the competitive effects. We consider variable unit production costs of all products together with the fixed cost of introducing a remanufactured product.

We propose a base model and an extension of it. In the base model, we try to determine which products should be included in the product line, and how should the prices of these products be set to maximize the profit. After determining the optimal product line composition, we try to find an answer to the question of how to position the remanufactured product that is included in the final product line, i.e. to what quality level the remanufactured product should be restored.

We propose a mixed-integer nonlinear programming model. The solution procedure is based on partitioning the problem into two sub-problems which are pricing problem and product line selection problem. For fixed binary decision variables used in the selection of a product mix, the pricing problem is solved using a restarted and modified simplex search method. Complete enumeration is used to solve the product line selection problem, in which the aim is to determine the optimal set of products to be manufactured searching through the space of binary product selection variables.

This thesis is organized as follows. Chapter 2 explains the fundamental concepts about discrete choice models. Chapter 3 includes a review of the literature about the product line selection, pricing, and remanufacturing. Chapter 4 presents complete information about the proposed model with a detailed problem description, model formulations, and assumptions. Some of the optimality conditions and properties of the model is also provided. Chapter 5 explains the solution procedures for both the pricing and the product line selection problems. In Chapter 6, sensitivity analyses with respect to some model parameters, which may have an effect on optimal prices of the products in the product line, are given. Furthermore, the conditions that affect the optimal quality level (i.e. unit production cost) of a remanufactured product are discussed in this chapter. Finally, conclusions and suggestions for future studies are mentioned.

2. BACKGROUND ON DISCRETE CHOICE MODELS

Since consumer preference is represented by a consumer choice model in this study, some survey was made about consumer choice models.

In discrete choice situation, the choice decision of an alternative from a set of mutually exclusive alternatives is based on the relative attractiveness of these competing alternatives. Discrete choice models state the decision-makers' choices among the set of alternatives, called the choice set. The decision-maker is assumed to be an individual and the characteristics like gender, age and education of the individual must be included in the model to explain the heterogeneity of preferences among decision-makers. Each alternative in the choice set is composed of a set of attributes which affect the decision maker's decision. The decision maker chooses an alternative from the finite choice set after evaluating the attributes of the alternatives through a decision rule. Most models use the value utility, which is assumed to represent the decision-maker's preference for an alternative in the utility theory and the decision-maker chooses the alternative from the choice set with the highest utility. The utility that a decision-maker w obtains from an alternative j is denoted by U_{wj}, but the subscript w will not be used in the rest of the study for clarity.

Discrete choice models are differentiated by the assumptions about the decision rules used by the decision-maker. This section is devoted to the discrete choice models which are deterministic or probabilistic. In the neoclassical economic theory, the decision-maker has a utility function to rank the alternatives in a consistent manner and chooses the alternative that is ranked first. Therefore, the choice process is deterministic. On the other hand, the complexity of human behavior requires including a probabilistic dimension in the decision rule to represent heterogeneity of preference among a population of consumers. Thus, some specific families of models can be derived according to the assumptions about the source of uncertainty.

The Luce model or the Tversky's "elimination by aspects" approach assume a deterministic utility and a stochastic decision rule whereas Random Utility Models assume deterministic decision rules, where utilities are represented by random variables to capture uncertainty.

2.1. Neoclassical Economic Theory and Preference Relations

Consider a decision-maker who has to choose a single alternative from a finite choice set of alternatives C. This does not mean the decision-maker is forced to choose one of the alternatives in the choice set. The outside alternative, which corresponds to the option of choosing nothing, can be added to the choice set C. Neoclassical economic theory assumes that the decision-maker has perfect discriminatory power and unlimited information processing capacity to rank the alternatives in a well-defined and consistent manner. Thus, the decision-maker can determine the best choice for him/her and will repeat this choice under identical circumstances (Anderson *et al.*, 1992).

A choice between two alternatives corresponds to an expression of preference of one alternative to another and the mathematical construction that formalizes this notion of choice is a preference relation (Mahajan and van Ryzin, 2003).

The neoclassical economic theory assumes that each decision-maker is able to compare two alternatives a and b in the choice set C, using a preference-indifference operator $\succeq$, which indicates a binary relation on the choice set C, and if $a \succeq b$, the decision-maker either prefers a to b, or is indifferent (Bierlaire, 2004). Thus, the decision-maker's decision rule is deterministic.

The preference-indifference operator is supposed to have some properties such as reflexivity, transitivity, and comparability. The reflexivity property implies that any alternative in the choice set is as good as itself, i.e. $a \succeq a$. The second property, transitivity, tells that if $a \succeq b$ and $b \succeq c$, then $a \succeq c$ for any a, b, and c that are the three alternatives in the choice set. Finally, the comparability property indicates that all pair-wise comparisons can be made between the alternatives within the choice set, namely if a and b are the two alternatives in the choice set, either $a \succeq b$ or $b \succeq a$ is true.

Since the choice set is finite, the existence of an alternative which is preferred to the all other alternatives is guaranteed, i.e.

$$\exists\, a^* \text{ s.t. } a^* \succeq a, \quad \forall a \in C. \tag{2.1}$$

As a result of the properties mentioned above, preference relations are related to utility maximization and there exists a deterministic utility function $U : C \to \Re$ such that

$$a \succeq b \Leftrightarrow U(a) \geq U(b) \quad \forall a, b \in C. \tag{2.2}$$

Therefore the alternative a^* may be identified as

$$a^* = \arg\max_{a \in C} U(a). \tag{2.3}$$

To sum up, when a preference relation exists, selecting an alternative from any choice set is equivalent to choosing the alternative with the maximum utility which is determined by a suitably defined utility function.

The utility notion is very important in the context of discrete choice models but the assumptions of neoclassical economic theory present strong limitations for real world applications. Thus, it is desirable to have a probabilistic model of choice.

2.2. Mechanisms for Generating Preference Relations

The existence of a utility function suggests two generic approaches for modeling choice: constructing preference relations directly, called attribute models or constructing utilities and then applying utility maximization, called utility models (Mahajan and van Ryzin, 2003).

2.2.1. Attribute Models

In attribute models, preference relations are constructed directly. These models are based on modeling attributes of each alternative and ranking alternatives through a decision rule considering their attributes. Lexicographic and address models are two examples for the attribute models.

2.2.1.1. Lexicographic Models. In a lexicographic rule, a product is composed of some binary attributes, i.e. the product has the attribute or not. The consumer ranks all attributes and discards the alternatives which do not have the most important attribute. If more than one alternative exists after the elimination, the remaining alternatives are evaluated according to the next most important attribute. This rule generates a preference relation among the alternatives, and these preferences can be directly linked to the existence of the attributes in each alternative; however, the model implies the strict domination of attributes on each other (Mahajan and van Ryzin, 2003).

Since there exists a preference relation, a utility function can be constructed that give the same choices as the lexicographic model. Suppose there is a choice set with n alternatives that have T attributes, represented by t and the attributes are ranked, that highest-valued attribute is represented by 1 and the lowest by T. Let a_{jt}, $j = 1,\ldots,n$ and $t = 1,\ldots,T$, be binary digits represent whether alternative j owns the attribute t. Then, the utility can be stated as a binary number with the function $U_j = a_{j1}a_{j2}\ldots a_{jt}$. The maximization of this utility function for each alternative gives the same choices as the lexicographic model.

2.2.1.2. Address Models. Address models relaxes the restriction that some attributes strictly dominate others and link attributes to preference. In these kinds of models, the alternatives are represented as points in the attribute space. Assuming a choice set with n alternatives that have T real-valued attributes, alternatives can be represented as n points, $z_1,\ldots,z_n$, in the attribute space $\Re^T$. Each consumer has an address in the attribute space, $y \in \Re^T$, which indicates his/her ideal point, namely the most preferred product specification. The consumer evaluates the distance between the point of alternative and his/her ideal point, and chooses the closest one. The distance can be defined by Euclidean distance or by some other metrics, $\rho(z_j, y)$ on $\Re^T \times \Re^T$, and these distances construct a preference relation.

The utility function for consumer y that generates the same choices can be formulated as $U_j(y) = x - \rho(z_j, y)$ where x is an arbitrary number.

2.2.2. Utility Models

Utility models simply determine the utility values of each alternative $U_1, \ldots, U_n$ directly. For different customer segments, a different utility vector can be specified. There is not a big difference between attribute and utility models. In utility models, a function relating attribute values to utilities is determined directly. An example of a utility model is widely used in transportation choice models in which the utility function is expressed as a linear function of an alternative's attributes.

2.3. Probabilistic Mechanisms for Generating Preference Relations

Probabilistic models are useful for the following reasons. One of them is to represent unobservable heterogeneity of preference among the consumers since all consumers do not have the same preference relations. A second reason may be the decision-maker's inadequate observation of all relevant variables affecting the decision process. Another reason can be the lack of information that leads the analyst (or modeler) to ascribe probabilistic decision rules to decision-makers. For all these reasons, the uncertainty must be modeled using randomizations. This randomization can be based on preferences or choice behavior of the consumers since they may not behave consistent with a well-defined preference relation and they may exhibit some probabilistic tendency to make a choice.

2.3.1. Attribute Mechanisms with Randomization

The randomized versions of the lexicographic and address models are presented in this subsection.

<u>2.3.1.1. Lexicographic: The Tversky Model.</u> In Tversky's model the utility of different alternatives is deterministic, but the choice process is probabilistic which means that the decision-maker does not necessarily choose the alternative with highest utility. This model can be viewed as a randomization of the deterministic lexicographic rule. The choice of an alternative can be viewed as a stochastic process in which alternatives are successively eliminated until only one of them is left (Tversky, 1972).

The model of elimination by aspects (EBA) proposed by Tversky (1972) assumes that each alternative can be identified by a list of binary attributes (characteristics) so that each alternative either possesses or does not possess the attribute. If there are some attributes that are not binary by definition, Tversky suggests representing them by threshold values. Each attribute has a positive utility value which expresses the importance of the attribute to the decision-maker. The process of selection is as follows:

(i) Attributes that are common to all alternatives are deleted.

(ii) One of the remaining attributes is selected as the criterion for eliminating alternatives with probability proportional to a specified deterministic utility value of the attribute.

(iii) All the alternatives that do not have the selected attribute are removed from the choice set.

(iv) If a single alternative remains, it is chosen. If more than one alternative remain, a new attribute is determined and the above process continues until no elimination is possible. If several alternatives are left after the elimination process, then they are chosen with equal probability.

Obviously, different sequences of elimination can lead up to the choice of a particular alternative. Therefore, the probability of choosing a particular alternative is defined as the sum of probabilities of all sequences of choices which ends up with this alternative. Since the sequence of attributes chosen as elimination criterion of alternatives is random, Tversky's model can be considered as a randomization of the lexicographic rule.

If we let u_t be a nonnegative function that gives the utility value for each attribute t and $C_t \in C$ be the set of alternatives that have the attribute $t, t = 1, \ldots, T^{'}$ where $T^{'}$ is the number of characteristics left after the elimination of the attributes common to the alternatives in the choice set C, the choice probability of an alternative $a \in C$ in Tversky's EBA model is defined as

$$P(a \mid C) = \sum_{t=1}^{T^{'}} \frac{u_t}{\sum_{r=1}^{T^{'}} u_r} P(a \mid C_t). \tag{2.4}$$

In (2.4), the probability of choosing an attribute as an elimination criterion is given as the ratio of the utility of the attribute under consideration to the sum of utilities of all remaining attributes.

The EBA model can reflect the complex patterns of alternatives that have different degrees of similarity to the model, i.e. it does not impose the restriction independence of irrelevant alternatives (IIA). According to Twersky, the introduction of an additional alternative hurts similar alternatives more than dissimilar ones. Although this is an advantage, the binary structure of the attributes neglects the role of the absolute values of the attributes in the choice of an alternative. Furthermore, the large number of the alternatives in the choice set makes the number of possible elimination sequences to be taken into account grow rapidly and the choice process becomes very difficult.

2.3.1.2. Address Models. In the randomized address models, the only difference is that the location of the consumer's ideal point y is associated by a probability density, $f(y), y \in \Re^T$ in the attribute space. Therefore, the utility for each alternative a can be stated as

$$U_a(y) = x - \tau \| z_a - y \| \tag{2.5}$$

where x is the utility of the consumer's ideal point y and τ is used to represent the disutility because of deviations from this ideal point. The market space of alternative a is formulated as

$$M_a = \{y \in \Re^T : U_a(y) \geq U_j(y),\ a, j = 1, \ldots, n\}. \tag{2.6}$$

Then, the choice probability of alternative a is given by

$$P(a \mid C) = \int_{M_a} f(y)dy\,. \tag{2.7}$$

2.3.2. Random Utility Models

In random utility models the decision-maker is assumed to have a perfect discrimination capability but the problem analyst (or modeler) is assumed to have incomplete information, so the uncertainty should be considered. According to Manski (1977), there are four different sources of uncertainty. These are non-observable variations in individual utilities, non-observable individual characteristics, measurement errors, and functional misspecification of utility. The utility is modeled as a random variable to take this uncertainty into account.

Let $U_j, j = 1, \ldots, n$ denote the utility that a decision-maker obtains from an alternative j in the choice set C. However, the analyst cannot observe the decision-maker's utility exactly and observes only some attributes of the alternatives faced by the decision-maker, denoted by d_j, and some characteristics of the decision-maker, denoted by s, which are called the explanatory variables. The analyst can specify a function that relates these observed factors to the decision-maker's utility, which is the deterministic part of the utility. This function is the known part of utility and denoted by $V_j = V(d_j, s), \forall j \in C$. It is called the observable or representative or measured utility. If we define an appropriate vector valued function that defines a new vector of attributes from both d_j and s, $\mathbf{x}_j = H(d_j, s)$, then we have $V_j = V(\mathbf{x}_j)$ (Ben-Akiva and Bierlaire, 1999). The specification of the representative utility function is general but usually it is chosen as linear in parameters, i.e. additive, $V_j = \sum_t \beta_t x_{jt}$. Therefore, the representative utility is specified by the parameters β_t. Several computer packages are available for estimation of the logit models with additive representative utility, which will be mentioned subsequently.

Since the analyst cannot observe all aspects of utility, then $V_j \neq U_j$. Therefore the utility is decomposed as $U_j = V_j + \varepsilon_j$, where ε_j is the random term that captures the uncertainty. Let $f(\boldsymbol{\varepsilon})$ denote the joint density of the random vector $\boldsymbol{\varepsilon} = \langle \varepsilon_1, \ldots, \varepsilon_n \rangle$. The analyst can calculate the probabilities for different alternatives of a decision-maker. The probability that a decision-maker chooses alternative a is

$$P(a \mid C) = P(U_a > U_j, \forall j \neq a) = P\left(U_a = \max_{j=1,\ldots,n} U_j\right) \tag{2.8}$$

This probability is a cumulative probability distribution and it depends on the joint distribution of the random components ε_j. Using the density related to ε_j, this probability can be written as

$$\begin{aligned} P(a \mid C) &= P(V_a + \varepsilon_a > V_j + \varepsilon_j, \forall j \neq a) \\ &= P(\varepsilon_j - \varepsilon_a < V_a - V_j, \forall j \neq a) = \int_{\varepsilon} I(\varepsilon_j - \varepsilon_a < V_a - V_j, \forall j \neq a) f(\varepsilon)\, d\varepsilon \end{aligned} \tag{2.9}$$

where $I(.)$ is the indicator function, which is one if the expression in parentheses is true, zero otherwise. Different assumptions about the distribution of the random components, namely different specifications of the density $f(\boldsymbol{\varepsilon})$ yield different random utility models. For the model to be useful, the integral in (2.9) should take a closed-form expression. Logit is the most widely used model in terms of its tractability because the choice probabilities take a closed-form.

Discrete choice models give information about individual decision-makers. Although psychologists are interested in individual choices, economists and the problem analysts are usually interested in aggregate demands. Therefore, the total population of individuals d facing the same choice set C can be divided into a certain number of subpopulations of various sizes called segments, denoted by d_i, $i = 1,\ldots,I$. Each segment is homogeneous within itself with respect to certain observable socioeconomic factors like age, education, and profession. In the following sections, the index i is used to represent the segment when necessary. If the analyst knows the total number of people in each segment, then the aggregate expected demand for each alternative j in segment i can be calculated as

$$X_j^i = d_i P_i\left(j \mid C\right). \tag{2.10}$$

Then, the expected demand for alternative j is stated as

$$X_j = \sum_{i=1}^{I} X_j^i, \quad i = 1,\ldots,I \quad j = 1,\ldots,n. \tag{2.11}$$

<u>2.3.2.1. The Multinomial Logit Model.</u> The logit model was first introduced for binary choice where the error terms of the utility functions have a logistic cumulative distribution. It is generalized to more than two alternatives known as the Multinomial Logit (MNL) model. In this model, the random components of the utility functions are independent and identically Gumbel distributed (or Type I extreme value or double exponential). Therefore, each unobserved component of utility, ε_j, is distributed with cumulative distribution function

$$F\left(\varepsilon_j\right) = \exp\left(-e^{-(\mu\varepsilon_j+\gamma)}\right), \mu > 0. \tag{2.12}$$

Then, the density function corresponding to (2.12) is given by

$$f\left(\varepsilon_j\right) = \mu e^{-(\varepsilon_j\mu+\gamma)} \exp\left(-e^{-(\varepsilon_j\mu+\gamma)}\right). \tag{2.13}$$

where μ is a scale parameter, which is positive. The mean and variance are zero and $\frac{\mu^2\pi^2}{6}$, where $\gamma \cong 0.5772$ is Euler's constant.

A useful property of Gumbel distribution is that it is closed under maximization. Namely, the distribution of the maximum of n independent Gumbel random variables with the same scale parameter μ is also a Gumbel random variable.

The probability that a decision-maker chooses alternative a is

$$P(a \mid C) = P(V_a + \varepsilon_a > V_j + \varepsilon_j, \forall j \neq a) = P(\varepsilon_j < \varepsilon_a + V_a - V_j, \forall j \neq a). \qquad (2.14)$$

If ε_a is considered given, the cumulative distribution over all $j \neq a$ is

$$P(a \mid C, \varepsilon_a \ is\ given) = \prod_{j \neq a} \exp\left(-e^{-(\mu(\varepsilon_a + V_a - V_j) + \gamma)}\right). \qquad (2.15)$$

Since ε_a is not given, thus the choice probability requires the integral of $P(a \mid C, \varepsilon_a \ is\ given)$ over all values of ε_a weighted by its density as

$$P(a \mid C) = \int_{s=-\infty}^{\infty} \left(\prod_{j \neq a} \exp\left[-e^{-(\mu(s + V_a - V_j) + \gamma)}\right] \right) \mu e^{-(\mu s + \gamma)} \exp\left[-e^{-(\mu s + \gamma)}\right] ds \qquad (2.16)$$

where s is ε_a. Noting that $V_a - V_a = 0$ and then collecting terms in the exponent of e, we have,

$$\begin{aligned} P(a \mid C) &= \int_{s=-\infty}^{\infty} \left(\prod_{j} \exp\left[-e^{-(\mu(s + V_a - V_j) + \gamma)}\right] \right) \mu e^{-(\mu s + \gamma)} ds \\ &= \int_{s=-\infty}^{\infty} \exp\left(-\sum_j e^{-(\mu(s + V_a - V_j) + \gamma)} \right) \mu e^{-(\mu s + \gamma)} ds \\ &= \int_{s=-\infty}^{\infty} \exp\left(-e^{-(\mu s + \gamma)} \sum_j e^{-(\mu V_a - \mu V_j)} \right) \mu e^{-(\mu s + \gamma)} ds \qquad (2.17) \end{aligned}$$

Using a variable change, i.e. $t = \exp[-(\mu s + \gamma)]$ and $dt = -\mu \exp[-(\mu s + \gamma)]$, and noting the limiting cases (when s approaches infinity, t approaches zero, and when s approaches negative infinity, t becomes infinitely large.), the closed form of $P(a \mid C)$ can be calculated as

$$
\begin{aligned}
P(a \mid C) &= \int_{\infty}^{0} \exp\left(-t \sum_{j} e^{-(\mu V_a - \mu V_j)}\right)(-dt) \\
&= \int_{0}^{\infty} \exp\left(-t \sum_{j} e^{-(\mu V_a - \mu V_j)}\right) dt \\
&= \left. \frac{\exp\left(-t \sum_{j} e^{-(\mu V_a - \mu V_j)}\right)}{-\sum_{j} e^{-(\mu V_a - \mu V_j)}} \right|_{0}^{\infty} \\
&= \frac{1}{\sum_{j} e^{-(\mu V_a - \mu V_j)}} = \frac{e^{\mu V_a}}{\sum_{j} e^{\mu V_j}}
\end{aligned}
\tag{2.18}
$$

The logit probability of an alternative is never exactly zero and the choice probabilities of all alternatives sum to one as the denominator in (2.18) normalizes each choice probability. If the representative utility V_a, has a unique maximum and $\mu \to \infty$, the variance of the random terms approaches to zero and the MNL reduces to a deterministic model. Specifically,

$$
\lim_{\mu \to \infty} P(a \mid C) = \begin{cases} 1 & \text{if } V_a = \max_{j \in C} V_j \\ 0 & \text{otherwise} \end{cases} . \tag{2.19}
$$

Conversely, if $\mu \to 0$, the variance of the random terms approaches to infinity and the representative part of the utility function, V_a becomes negligible,

$$
\lim_{\mu \to 0} P(a \mid C) = \frac{1}{|C|}, \quad a \in C . \tag{2.20}
$$

The relation between the logit probability and representative utility has some important implications to understand the effect of changes in explanatory variables on choice probabilities. Their relation is sigmoid or S-shaped. Thus, if the representative utility of an alternative is very low or very high compared to others, a small increase in the utility affects its choice probability very little. When the choice probability of an alternative is close to 0.5, the increase in the representative utility has the greatest effect on the probability, i.e. a small increase in the utility causes a large change in the probability.

In the MNL model presented above, the consumers are constrained to purchase one unit of their most preferred alternative from the products offered in the market. This restriction can be relaxed by introducing an outside alternative representing no-purchase case. The utility of the outside alternative is given as $U_0 = V_0 + \varepsilon_0$ and the choice probabilities are calculated by considering the no-purchase alternative. The value of V_0 is usually equal to zero, which means that the consumer obtains no utility from that alternative. When $V_0 \rightarrow -\infty$, the outside alternative is not kept in the model at all.

Although MNL is a useful and widely used model of choice, it possesses some limitations (McFadden, 1974). When there is an improvement in the attributes of an alternative, its choice probability increases, which necessarily causes a decrease in other alternatives' probabilities. The substitution patterns among alternatives are very important in many situations. The MNL assumes proportional substitution patterns. This issue can be seen as a restriction on the ratios of probabilities, which is known as the independence of irrelevant alternatives (IIA). This property says that the ratio of choice probabilities of any two alternatives is independent of the choice set containing these alternatives. For any choice set $C_1 \subseteq C$ and $C_2 \subseteq C$, and for any alternatives a and b in both C_1 and C_2, the ratio of logit probabilities are

$$\frac{P(a \mid C_1)}{P(b \mid C_1)} = \frac{P(a \mid C_2)}{P(b \mid C_2)} \tag{2.21}$$

Equation (2.21) says that the ratio does not depend on any alternatives other than a and b. While IIA property can be realistic in some situations, it is not if the choice set contains similar alternatives that their utilities share some unobserved attributes, adding a new alternative reduces the choice probability of similar alternatives more than dissimilar alternatives. The famous red bus/blue bus paradox illustrates this limitation well (Debreu, 1960). A traveler is supposed to choose a mode of transportation from the choice set which contains a car and a blue bus. To simplify the statement, assume that they both have the same representative utility. In this case, the choice probabilities of the alternatives appear to be equal, i.e. $P(car\ is\ chosen) = P(blue\ bus\ is\ chosen) = 1/2$. Now suppose that a new red bus is introduced, which is exactly like the blue bus in all respects and the traveler considers them to be identical. Then, the MNL predicts the choice probabilities as $P(car\ is\ chosen) = P(blue\ bus\ is\ chosen) = P(blue\ bus\ is\ chosen) = 1/3$. In both cases, the ratio of probabilities, $\dfrac{P(car\ is\ chosen)}{P(blue\ bus\ is\ chosen)} = 1$, is the same whether or not another alternative exists. It is more realistic, however, that the probability of choosing the car to remain the same and the probability of taking a bus to be equally shared between two buses after the second bus is introduced. The MNL is not consistent with this intuitive result. In choice problems, this situation arises due to correlated random utilities. Thus, the MNL model should be used for the choice sets that contain equally dissimilar alternatives or the representative utility of the alternatives should be specified very carefully capturing all sources of correlation over alternatives. Some variations of the MNL model are introduced to avoid the IIA restriction.

2.3.2.2. The Nested Logit Model. Usually it is not possible for the analyst to capture all sources of correlation among alternatives. Therefore, the unobserved portions of utility (random utilities or error terms) are correlated and IIA does not hold. The Nested Logit (NL) model, first proposed by Ben-Akiva (1973), was derived as an extension of the MNL model to capture some correlations among alternatives, i.e. to overcome the IIA restriction. The NL model is appropriate when the set of alternatives can be grouped in subsets, called nests, such that similar alternatives are collected in a group. Therefore, NL model is based on partitioning the choice set C into m non-overlapping nests C_h that each nest contains the similar alternatives having several characteristics in common such that

$$C = \bigcup_{h=1}^{m} C_h \text{ and } C_h \cap C_{h'} = \varnothing \quad \forall h \neq h' \tag{2.22}$$

In terms of IIA limitation, the following two properties hold for the NL model:

(i) IIA holds within each nest, namely the ratio of choice probabilities for any two alternatives that are in the same nest is not affected by the attributes or existence of all other alternatives.

(ii) For any two alternatives that are in different nests, IIA does not hold in general, in other words, the ratio of probabilities can be affected by the attributes of other alternatives in the two nests.

Substitution patterns can be explained better with a tree diagram. The tree diagram of the nesting structure and the sequential choice procedure for the NL model is given in Figure 2.1.

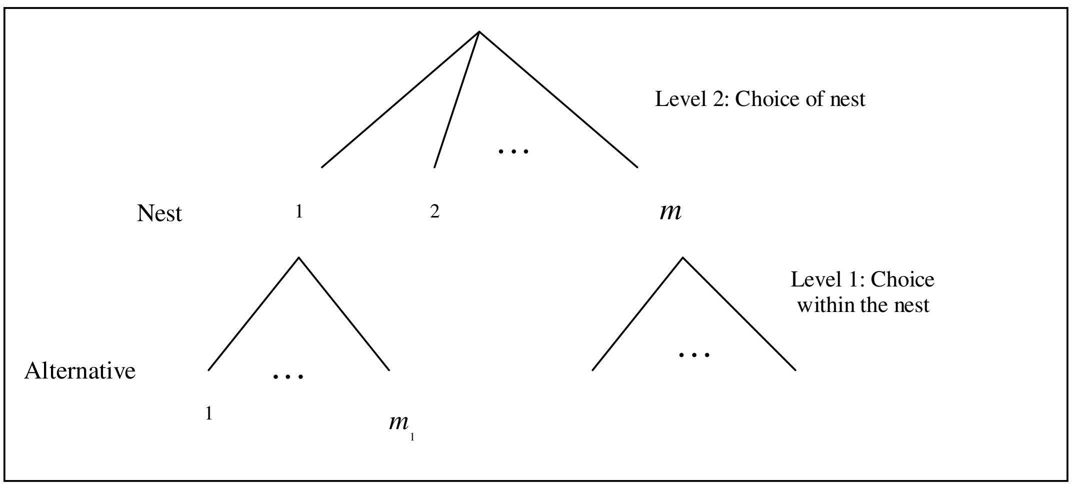

Figure 2.1. Tree diagram for the NL model

The tree consists of branches in Level 2 for the nests of alternatives and each of the branches contains twigs for the alternatives within the nest in Level 1. There is proportional substitution across twigs within a branch but not across branches. The decision-maker first chooses the nest with a certain probability and then selects the alternative according to a probability depends on the utility of that alternative.

The random utility U_j that a consumer obtains from alternative j within nest C_h is composed of a term specific to the alternative and a term associated with the nest, which both consists of a deterministic part V, observed by the analyst and a stochastic part v whose value is not observed by the analyst. Then, the random utility is written as

$$U_j = U_h + U_{j|h} = (V_h + v_h) + (V_{j|h} + v_{j|h}), \quad j \in C_h \tag{2.23}$$

In equation (2.23), U_h is the marginal utility component from Level 2 and its deterministic part V_h depends only on variables that describe nest C_h, and they are common to all alternatives within each nest whereas they differ over other nests. $U_{j|h}$ is the conditional utility component from Level 1 and its deterministic part $V_{j|h}$ depends on variables that describe alternative j and they differ over alternatives within nest C_h.

The error terms v_h and $v_{j|h}$, coming from the upper and lower levels respectively, are assumed to be independent of each other. The error terms $v_{j|h}$ are assumed to be independent and identically Gumbel distributed with nest-specific scale parameter μ_h which can be different for each nest. This can be considered as an interpretation of the correlation among the alternatives' errors within nest C_h (Heiss, 2002). The compound error terms ε_j are the sum of two stochastic error terms, v_h and $v_{j|h}$. They are distributed such that the sum of U_h and $U^*_{j|h}$, the maximum of $U_{j|h}$, is Gumbel distributed with scale parameter λ_h (Ben-Akiva and Lerman, 1985). The nested logit probability $P(a \mid C)$ for the alternative $a \in C_h$ can be written as the product of two standard logit probabilities. These are the choice probability of an alternative within nest C_h, called marginal choice probability, and the probability that alternative a is chosen given that an alternative in C_h is chosen, called conditional choice probability as:

$$P(a \mid C) = P(C_h \mid C) P(a \mid C_h) \tag{2.24}$$

where

$$P(C_h \mid C) = \frac{\exp\left(\lambda_h V_h + \dfrac{\lambda_h}{\mu_h} IV_h\right)}{\sum_{r=1}^{m} \exp\left(\lambda_r V_r + \dfrac{\lambda_r}{\mu_r} IV_r\right)}, \tag{2.25}$$

$$P(a \mid C_h) = \frac{\exp(\mu_h V_{a|h})}{\sum_{j \in C_h} \exp(\mu_h V_{j|h})}, \tag{2.26}$$

and

$$IV_h = \ln \sum_{j \in C_h} \exp(\mu_h V_{j|h}) \tag{2.27}$$

If there is only one alternative in a nest, then the conditional probability is equal to one. The quantity IV_h, called the inclusive value, inclusive utility or log-sum term, can be considered as a linkage between the upper and lower level models and it brings information from the lower model to the upper model. In particular, IV_h is the log of the denominator of the lower model and has an important implication. $\frac{1}{\mu_h} IV_h$ is interpreted as the expected maximum utility that decision-maker obtains from the choice among alternatives in nest C_h and the coefficient μ_h of IV_h in the upper model is often called the log-sum coefficient. Therefore, we can say that the inclusive value acts as an explanatory variable in the upper model and the probability of choosing nest C_h depends on the expected utility that the decision-maker receives from that nest. This expected utility consists of the utility that he/she receives from the nest, which is V_h, plus the expected utility that he/she receives by choosing the best alternative in the nest, which is $\frac{1}{\mu_h} IV_h$.

We can show that IIA holds within each nest of alternatives but not across nests by using the nested logit probability formula of the alternatives. For two alternatives $a \in C_1$ and $b \in C_2$, the ratio of probabilities is as follows:

$$\frac{P(a \mid C_1)}{P(b \mid C_2)} = \frac{\exp(\mu_1 V_{a|1}) \exp\left(\lambda_1 V_1 + \frac{\lambda_1}{\mu_1} IV_1\right) \sum_{j \in C_2} \exp(\mu_2 V_{j|2})}{\exp(\mu_2 V_{b|2}) \exp\left(\lambda_2 V_2 + \frac{\lambda_2}{\mu_2} IV_2\right) \sum_{j \in C_1} \exp(\mu_1 V_{j|1})}. \tag{2.28}$$

If $C_1 = C_2 = C$, i.e. a and b in the same nest and $\mu_1 = \mu_2 = \mu$, the terms related to nests cancel out and we have

$$\frac{P(a \mid C)}{P(b \mid C)} = \frac{\exp(\mu V_a)}{\exp(\mu V_b)}. \tag{2.29}$$

The ratio in (2.29) is independent of all other alternatives but when the alternatives are in different nests, the ratio of probabilities depends on the attributes of all alternatives in the nests that include a and b as it can be seen in equation (2.28).

The scale parameters μ and λ describe the variances of the unobservable errors and they reflect the degree of independence (correlation) in unobserved utility among the alternatives in nest C_h. In other words, the error terms of alternatives within a nest are correlated. The variances related to the error terms can be stated as

$$\operatorname{var}(v_{j|h}) = \frac{\pi^2}{6\mu_h^2} \qquad \operatorname{var}(\varepsilon_j) = \operatorname{var}(v_h + v_{j|h}^*) = \frac{\pi^2}{6\lambda_h^2} \tag{2.30}$$

The compound random utility component ε_j consists of variance components both from the lower level and the upper level. Thus the variances on the lower level must be smaller than those on the upper level and the scale parameters should satisfy the following condition (Silberhorn *et al.*, 2008):

$$\lambda_h < \mu_h \tag{2.31}$$

The covariance between the utility of two alternatives a and b in nest C_h is

$$\operatorname{cov}\left(U_a, U_b\right) = \begin{cases} \operatorname{var}(v_h) & \text{if } a \text{ and } b \in C_h \\ 0 & \text{otherwise} \end{cases} \tag{2.32}$$

and the correlation is

$$\operatorname{corr}\left(U_a, U_b\right) = \begin{cases} 1 - \frac{\lambda_h^2}{\mu_h^2} & \text{if } a \text{ and } b \in C_h \\ 0 & \text{otherwise} \end{cases} \tag{2.33}$$

Therefore, since the correlation cannot be negative, the inequality $0 \leq \lambda_h / \mu_h \leq 1$ must hold. If the ratio λ_h / μ_h is equal to one, then the correlation between the utility of two alternatives is equal to zero. In fact, only their ratio is meaningful and it is not possible to interpret them separately. To arbitrarily constrain one of them to a specific value, usually to one, is common in practice.

The introduction of an outside alternative (no-purchase) is also possible in the NL model as it was mentioned in the MNL model. It is accepted as a separate nest of only one alternative (no-purchase), namely the outside group and all probability calculations take this group into consideration.

Two different NL model specifications have been researched so far. The non-normalized nested logit (NNNL) model and the utility maximization nested logit (UMNL) model have some different properties which affect the estimation results. The elementary NNNL model by Daly (1998) is not consistent with utility maximization theory when any coefficients are common across nests, namely the coefficient of an explanatory variable is the same for the alternatives in different nests (Koppelman and Wen, 1998). On the other hand, the UMNL model derived from McFadden's generalized extreme value (GEV) (McFadden, 1978; McFadden, 1981) is consistent with the utility maximization theory.

Due to the identification problems, one of the scale parameters in the UMNL specification needs to be normalized to one (Silberhorn, 2008). RU1 UMNL model is constructed by normalization on the lower Level 1 $(\mu_h = \mu_m = 1)$ whereas normalization on the upper Level 2 $(\lambda_h = \lambda_m = 1)$ results in the RU2 UMNL model (Silberhorn, 2008).

Each alternative's choice probability must not change when adding a constant term to each alternative's deterministic utility component to be consistent with the utility maximization theory (Koppelman and Wen, 1998). In the NNNL model, $(\mu_h = \mu_m = \mu)$ must hold to be consistent with utility maximization theory. In the RU1 UMNL specification, consistency can only be reached by imposing the restriction $(\lambda_h = \lambda_m = \lambda)$ whereas in RU2 UMNL specification consistency with the utility maximization theory is satisfied without imposing any restrictions.

We can interpret the meaning of parameters λ_h and μ_h in an NL model specification by considering the correlation formulae (2.33). For example, in the RU2 UMNL specification, since $(\lambda_h = \lambda_m = 1)$ equality holds the value of μ_h must be equal or greater than one as it is given in the following inequality

$$\mu_h \geq 1. \tag{2.34}$$

A lower value of μ_h indicates greater independence and less correlation between the alternatives in nest C_h. A value of $\mu_h = 1$ means complete independence within nest C_h (no correlation). When $\mu_h = 1$ for all h indicating no correlation among all alternatives in all nests, the NL model reduces to the MNL model.

The parameters of an NL model can be estimated either by using some simultaneous standard maximum likelihood estimation techniques or sequential estimation. Some software packages are available for estimating an NL model by maximum likelihood.

In the NL model, the correlation across nests cannot be captured. Thus, if it is not possible to partition the alternatives into well separated nests to reflect their correlation, the NL model is not appropriate (Ben-Akiva and Bierlaire, 1999).

Sometimes it becomes inappropriate to accept an alternative as a member of only one nest. To overcome this restriction, a model, called Cross-nested Logit (CNL) model has been specified with overlapping nest. This model is a direct extension of the NL model and each alternative may belong to more than one nest in this model.

<u>2.3.2.3. The Generalized Extreme Value Family.</u> McFadden (1978) developed a process to generate generalized extreme value (GEV) models. This family consists of a large family of models that include the MNL and the NL models. The process allows the researcher developing new GEV models which best meet his/her research requirements.

Let n be the number of alternatives in the choice set C. Consider a non-negative differentiable function G defined on R_+^n with following properties:

(i) G is homogenous of degree $\mu > 0$. Actually, McFadden's original formulation $\mu = 1$ was generalized to $\mu > 0$ by Ben-Akiva and François (1983).

(ii) $\lim_{x_j \to \infty} G(x_1, \ldots, x_j, \ldots, x_n) = \infty, \quad j = 1, \ldots, n$.

(iii) The partial derivatives of G with respect to k distinct variables x_j exist and they are non-negative if k is odd, and non-positive if k is even, that is, for any distinct $j_1, \ldots, j_k \in (1, \ldots, n)$ we have

$$(-1)^k \frac{\partial^k G}{\partial_{x_{j_1}} \ldots \partial_{x_{j_k}}}(x) \leq 0 \;\forall x \in R_+^n . \qquad (2.35)$$

If the function G meets the properties listed above, then a discrete choice model can be based on it. Then, the probability of choosing alternative j within the choice set C can be defined as

$$P(j \mid C) = \frac{e^{V_j} \dfrac{\partial G}{\partial e^{V_j}}\left(e^{V_1}, \ldots, e^{V_n}\right)}{\mu G\left(e^{V_1}, \ldots, e^{V_n}\right)} \qquad (2.36)$$

The MNL model and the NL model are GEV models with

$$G(x) = \sum_{j=1}^{n} x_j^{\mu} \qquad (2.37)$$

for the MNL model,

$$G(x)=\sum_{h=1}^{M}\left(\sum_{j\in C_h} x_j^{\mu_h}\right)^{\frac{\lambda_h}{\mu_h}} \tag{2.38}$$

for the NL model.

2.3.2.4. The Probit Model. The MNL model has some limitations. That are it cannot represent random taste variation, it exhibits the IIA property which results in restrictive substitution patterns, and it cannot be used with the panel data when unobserved factors are correlated over time for each decision-maker (Train, 2003). Although GEV models relax the IIA property, other limitations remain. However, Probit model overcomes all these limitations.

In the Probit model, the unobserved (random) components of utility are assumed to be jointly normal distributed with non-zero covariance. Although the model is more realistic for many real world applications since it captures the all sources of correlation between alternatives explicitly, it is limited in terms of its applicability because there is no closed-form expression for the integral in (2.9) and it becomes intractable when the number of alternatives increases.

2.4. Choice Axiomatized by Probabilities: Luce Model

According to the behavioral theorists in contrast to the economists, a model that allows for imperfect discriminatory power and limited information processing capability, i.e. bounded rationality is more realistic (Mahajan and van Ryzin, 2003). Namely, a decision-maker may not give the same choice decision between two alternatives under the same conditions, which indicates the inconsistencies in human behavior. The Luce model is based on an assumption on the choice probabilities which is known as the choice axiom which gives the relationship between the choice probabilities defined on different subsets of the set of all alternatives C. For any $A \subseteq C$ and $B \subseteq C$ such that $A \subseteq B$, the choice axiom can be stated as

(i) If for given $a \in A$, $P(a \mid \{a,b\}) \neq 0,1$ for all $b \in B$, then the probability of choosing a from B is equal to the product of the probability that A is chosen from B and the probability that a is chosen from the subset A:

$$P(a \mid B) = P(A \mid B)P(a \mid A) \tag{2.39}$$

(ii) If there is an alternative b which is always preferred to the alternative a, i.e. $P(a \mid \{a,b\}) = 0$ for some $a,b \in B$, then for all $A \subseteq B$, removing a from the choice set B does not change the choice probability of any other alternative in B:

$$P(A \mid B) = P(A - \{a\} \mid B - \{a\}) \tag{2.40}$$

Luce has shown that (2.39) is satisfied if and only if there is a positive real-valued function U defined on A, unique up to multiplication by a positive constant, the following equation holds:

$$P(a \mid A) = \frac{U(a)}{\sum_{b \in A} U(b)} \tag{2.41}$$

The function $U(a)$ can be interpreted as deterministic utility of alternative a. We see in (2.41) that the probability that a be chosen is increasing in its utility and is decreasing in the utility of other alternatives.

If we equate $U(a)$ with $\exp(\mu U(a))$ and define it as V_a, the choice probabilities in (2.41) become identical in form to the MNL model in (2.18).

The Luce model also exhibits the restrictive property of IIA, which is explained in subsection 2.3.2.1 in detail, like the MNL model. Although the model has some important limitations, it provides a tractable probabilistic structure and it can be viewed as an approximation of complex decision-making procedures.

3. LITERATURE REVIEW ON PRODUCT LINE SELECTION, PRICING, AND REMANUFACTURING

Determining the right products to offer in the market is of crucial importance and it is one of the most critical strategic decisions for a firm. Firms often attempt to offer a variety of products to satisfy the diversified consumer tastes effectively, gain competitive advantage, and increase their profits. However, this product proliferation of product lines generally increases the incremental costs of the firms due to the new product designs, production and initial investments required, and it also creates revenue losses due to the market cannibalization. Therefore, product line design or selection decisions must be made focusing on profit. In the literature, there is a great amount of studies about product line design or selection.

A firm must understand the structure of the market when designing a new product line or extending an existing one. Then, it is important to identify homogenous customer segments in which the customers have distinct preferences.

Product line design problems deal with the conceptual design of the products to be offered whereas product line selection focuses on the selection of a subset of potential product profiles. Most product line design models use a framework where products are represented by points in a multi-attribute attribute space. In this common attribute space each customer has an ideal point (product). The preferences of a particular customer are assumed to be inversely related to the distance of a product's location from the customer's ideal point. Namely, the customer chooses the product that is closest to his/her ideal point which is a deterministic approach. The randomized version of these models is also available. These approaches are mentioned in Chapter 2 under the name Address Models. The main focus of these studies is to identify a new optimal product location in the attribute space which is an attractive specification of product attributes where the selling prices may be considered as an attribute. Since we are not dealing with a product line design problem, the literature about it is not extensively stated. However, some studies that are closely related to the product line selection are mentioned.

In product line selection models, each customer values a product which is represented by utility and customer preferences are expressed by these utilities. There are two main approaches to determine the utilities, which are compositional and decompositional models. In compositional models, parameter estimations are explicitly done by customers. The two-stage, self-explicated model is a kind of these models. Customers rate the desirability of all possible attribute levels for each attribute in the first stage, and then they rate the weight, i.e. the importance of each attribute. The utility of a product is then calculated as the sum of the weighted attribute levels. In decompositional models, parameter values are estimated according to the responses of customer to a set of alternative product specifications, which is called stimuli. Conjoint analysis is the most widely used model of this type. Firstly, the overall evaluation of customers over a set of stimuli which are designed before by using some type of factorial structure is measured. Then, these evaluations of customers are decomposed into separate and compatible utilities using one of several preference models such as vector model, ideal point model, or parth-woth model (Green and Srinivasan, 1978). Then, the utility of a product can be obtained by summing up the preference parameters found by using one of these models.

The choice of the customers can be modeled using different assumptions on the decision rules of the customers after determining the utilities of the alternative products. If a model with deterministic choice rule is utilized, then each customer chooses the product with highest utility or surplus according to his/her evaluations, which is the difference between the reservation price of a customer (the highest price a customer would be willing to pay) and the selling price of that product. This type of choice is called max-utility choice. In probabilistic case, the probability of choosing a product for a customer in a particular segment is implicitly or explicitly defined as a fraction of that customer segment purchases this particular product (Yano and Dobson, 1998). In probabilistic choice models, either the utilities are expressed as random variables, and choice probabilities are formed according to the assumption on the distribution of the random terms in utility, or the utilities are deterministic whereas the decision rule is probabilistic.

It is a well-known fact that customers are willing to pay more for a product that better matches their desires. Thus, a wider product variety increases the chance that a customer would find a product that is closer to his/her desires. However, this situation brings about the cannibalization of demand for more profitable products with higher prices by some other cheaper products. Therefore, the well-known phenomenon cannibalization may negatively affect the profit due to the demand losses of more profitable products. Hence, selection and pricing of a product line cannot be separated, and they should be done simultaneously.

In most of the studies in the literature about product line selection and pricing problems the market is divided into a number of homogeneous segments of various sizes. Namely, an ideal product or utility assigned to a segment, which are the same to all customers in that particular segment. Customers are usually allowed to choose just one product, which is composed of different combinations of attribute levels, from a finite set of the products offered, and in some studies also an outside alternative is defined allowing customers nothing to purchase. Utility values of products for each segment, segment sizes, variable cost of products, and sometimes fixed costs of introduction are used as inputs of the model. A common approach for solving the problem is to divide it into sub-problems as pricing and product line selection, and then solving them sequentially.

The literature review mainly consists of the studies about product line selection and pricing problem. Moreover, some necessary information about remanufacturing which is related to our work is given.

3.1. Product Line Selection and Pricing Problem

The aim is to maximize either profit or consumer welfare in most of the recent studies about this area.

Product line design and pricing problem is studied by Mussa and Rosen (1978) and Moorthy (1984) which is closely related to the problem of product line selection and pricing. Mussa and Rosen assume that the market is composed of an infinite number of segments, and the utility of a segment for a product is a linear function of the quality of that product. They find the optimal set of products and their prices. In Moorthy's study, customers are grouped into a finite number of segments, the utility of which is a non-decreasing deterministic function of the product's quality, and each segment chooses the product with the highest surplus. Moorthy focuses on the monopolist's problem of selecting a subset of products under cannibalization and pricing each product so as to maximize profit.

In Green and Krieger's study (1985), choosing a subset of B products from a set of C product alternatives is of interest under max-utility choice. Each customer is assumed to choose at most one product which is either his/her status quo or one of the offered products. Two different problems were formulated which are buyer's and seller's welfare problems. In buyer's welfare problem, the total utility that the buyers obtain is aimed to be maximized whereas in seller's problem the total value to the seller is maximized. The model fixes the number of products B to be offered but does not consider prices or costs. The buyer's welfare problem is formulated as follows:

$$\max \quad \sum_{i\in M}\sum_{j\in C} U_{ij}x_{ij} \tag{3.1}$$

$$s.t. \quad \sum_{j\in C} Y_j \le B \tag{3.2}$$

$$\sum_{j\in C} x_{ij} = 1 \qquad \text{for } i \in M \tag{3.3}$$

$$0 \le x_{ij} \le Y_j \le 1 \qquad \text{for } i \in M,\ j \in C \tag{3.4}$$

$$Y_j = \begin{cases} 1 & \text{if the product is offered} \\ 0 & \text{otherwise} \end{cases} \qquad \text{for } j \in C \tag{3.5}$$

$$x_{ij} = \begin{cases} 1 & \text{if the the customer chooses product } j \\ 0 & \text{otherwise} \end{cases} \qquad \text{for } i \in M,\ j \in C \tag{3.6}$$

The seller's welfare problem is formulated as follows:

$$\max \quad \sum_{i\in M}\sum_{j\in C} \upsilon_{ij}x_{ij} \tag{3.7}$$

$$s.t. \quad x_{ij} \le Y_j \qquad \text{for } i \in M,\ j \in C \tag{3.8}$$

$$\sum_{j\in C} Y_j \le B+1 \tag{3.9}$$

$$Y_j = \begin{cases} 1 & \text{if the product is offered} \\ 0 & \text{otherwise} \end{cases} \qquad \text{for } j \in C \tag{3.10}$$

$$x_{ij} = \begin{cases} 1 \text{ if } U_{ij}Y_j \geq U_{ij}Y_l,\ l \in C \\ 0 \text{ otherwise} \end{cases} \qquad \text{for } i \in M,\ j \in C \tag{3.11}$$

$$Y_0 = 1 \tag{3.12}$$

where U_{ij} is the utility of customer i for product j, and υ_{ij} is the value to the seller if product j is chosen by customer i. M denotes the set of customers. Due to the inappropriateness of using complete enumeration for finding the optimal solution to the proposed combinatorial problem, they suggest using some heuristics. They suggest a greedy heuristic, an interchange heuristic, and Lagrangian relaxation for solving the buyer's problem, and a greedy heuristic for the seller's problem. They state that these heuristics are designed to provide good solutions (not necessarily optimal) to the product line selection problem. An example to illustrate the application of proposed heuristics is also presented.

McBride and Zufryden (1988) extend the study of Green and Krieger (1985) by using some integer programming techniques to the product line selection problem. They consider the problem of the seller's welfare, and the difference is just in the formulation of constraint (3.11) which is

$$Y_jU_{ij} \geq Y_lU_{ij} - \Gamma(1 - x_{ij}) \quad \text{for } i \in M,\ j,l \in C, \text{and } l \neq j \tag{3.13}$$

where Γ is an arbitrarily large number which is used to relax or make the constraint binding. This constraint ensures that if a customer chooses product j (i.e. if $x_{ij} = 1$) and if j is offered in the product line, then the utility of that product is greater than or equal to the utility of other products offered in the product line. If the customer does not choose product j, then the constraint becomes redundant due to the presence of Γ.

The authors state that the combinatorial nature of constraint (3.13) requires a large number of such constraints to be considered. They are interested in solving the original problem and a simplified case of it where $\upsilon_{ij} = \upsilon_i$, namely the value to the seller does not depend on which product is chosen by the customer. In this case, the problem is simplified by relaxing constraint (3.13), and the problem has a similar formulation as the buyer's welfare problem that is stated by Green and Krieger (1985) without constraint (3.13). With this simplification, the problem becomes much easier to solve, and given the optimal solution of the simplified problem, the original problem can also be solved by updating constraint (3.13) with $Y_l + x_{ij} \leq 1$. They suggest using integer programming methods to solve the problems. Optimal solutions for small instances are also reported for both problems.

Dobson and Kalish (1988) develop a model similar to Green and Krieger's (1985) in which the aim is to choose the number of the products to be offered, and they extend the model by taking prices and fixed costs together with the variable costs into consideration as well as a no-purchase option is also offered. The selling prices of the products offered are also the decision variables. The aim is to maximize profit (or welfare) under the max-utility choice rule. The problem formulation is as follows:

$$\text{Profit:} \quad \max \quad \sum_{i\in M}\sum_{j\in C} d_i(S_j - c_j)x_{ij} - \sum_{j\in C} f_j Y_j \tag{3.14}$$

$$\text{Welfare:} \quad \max \quad \sum_{i\in M}\sum_{j\in C} d_i(U_{ij} - c_j)x_{ij} - \sum_{j\in C} f_j Y_j \tag{3.15}$$

$$s.t. \quad \sum_{j\in C} x_{ij} = 1 \quad \text{for } i \in M \tag{3.16}$$

$$x_{ij} \leq Y_j \quad \text{for } i \in M, j \in C \tag{3.17}$$

$$\sum_{l\in C} (U_{il} - S_l)x_{il} \geq (U_{ij} - S_j)Y_j \quad \text{for } i \in M, j \in C \tag{3.18}$$

$$S_0 = 0 \tag{3.19}$$

$$x_{ij}, Y_j = 0,1 \quad \text{for } i \in M, j \in C \tag{3.20}$$

where S_j is the selling price of product j, c_j is the variable cost of product j, d_i is the size of segment i, and f_j is the fixed cost for product j.

Constraint (3.18) states the max-utility choice rule which ensures that customers in a particular segment choose the product provides them the maximum utility. The heuristic they propose for the profit maximizing problem decomposes the problem into an assignment sub-problem where the segments are assigned to the products, and a pricing sub-problem where the prices are set according to the results of the assignment sub-problem. They realize that the dual of the pricing sub-problem can be decomposed into a set of shortest path problems on a network, and for a given assignment of segments the pricing problem can be solved very quickly and efficiently. In stage one; the goal is to find a good feasible assignment of segments to products, and choose the subset of products to consider. The subset of products to consider and a feasible assignment of segments to products which are the findings of the first stage serve as input to the second stage, and the prices that maximize the profit consistent with this assignment are found in the second stage. At each iteration, only one segment is reassigned, and the maximum profit prices are solved again. This procedure continues until no further profit improvement is possible.

For the welfare maximizing problem, the heuristic starts with assigning each segment to the product with maximum welfare. Then, at each iteration one product is deleted systematically, and the resulting new total welfare is computed and recorded with the related subset by again assigning the segments to the remaining products according to the maximum welfare criterion. After deleting all products, the subset with the largest welfare is selected.

Dobson and Kalish (1993) extend their earlier work in where the objective is again to maximize either profit or total welfare. The model formulation is almost the same as in their earlier work mentioned above. Although the problem is NP-complete, they show that the welfare problem is equivalent to the uncapacitated plant location problem, which can be solved (almost always to optimality) using the greedy interchange heuristic. They suggest a faster new greedy heuristic which is slightly better than alternative heuristics for the profit problem.

In recent years, more complex cost structures are considered together with product line selection and pricing problems. Raman and Chhajed (1995) provide a model in which the number of products to be produced, their exact specifications, and the process by which these products are manufactured to reach these specifications are of concern. A decomposition-based solution procedure which iterates between the product design and process selection is developed.

Hanson and Martin (1996) propose an efficient heuristic for maximizing the profit of a firm by setting the prices of a given set of products that are offered where the consumer choice is modeled with the MNL model. Their version of the choice model allows a positive probability to a product with negative surplus that will be bought by a particular segment. They are interested in pricing rather than determining the products to be offered. They state that the resulting profit function is generally non-concave. They propose a new path following procedure which finds a path of prices from the global optimum of a related concave, logit profit function, to the global optimum of the non-concave objective function.

Chen and Hausman (2000) propose a product line selection and pricing model so as to maximize expected profit. Their model differs from Hanson and Martin's in that there are constraints on the number of candidate products that will be offered (i.e. there is a lower and an upper bound for that number), and that prices can only take on a set of discrete values. They set lower and upper bounds for the prices of the products. Integrality constraints for the discrete price choices can be relaxed due to the special structure of the problem and the resulting nonlinear problem can be solved by using standard nonlinear programming software for the problems of realistic size. Nonetheless, the proposed model suffers from the limitations of the IIA property because of the usage of the MNL choice model.

Aydin and Ryan (2000) consider the product line selection and pricing problem from a retailer's perspective in three steps. Their assumptions are similar to Chen and Hausman's. Customer choice is modeled using the MNL model. They work with only one customer segment and they do not impose constraints on the number of the products to be offered. In the first step which is called incremental problem the retailer owns $n-1$ products with fixed prices and considering adding the n^{th} problem of given quality. For this problem, they show that there exists a minimum price above which it is always profitable to offer the new product. In the second step, the optimal prices of the n models are determined simultaneously. They prove that there is a unique profit margin (i.e. the difference between selling price and unit variable cost) that optimizes the profit, and the profit margins of all products are equal at optimality. In the last step, the problem is to determine optimal set of products in the product line that are selected among a fixed set of potential products, and their optimal prices. They show that the optimal product line consists of all products in the absence of the fixed costs. However, the increase in the profit function becomes smaller as the number of the products offered increases.

Kraus and Yano (2003) develop heuristic algorithms to the product line selection, i.e. determining the set of products to offer, and pricing problems in the aim of maximizing total profit which is total revenue minus total variable cost under a share-of-surplus choice model in which the fraction of customer segment that chooses a product is defined as the ratio of the surplus from this particular product to the segment's total surplus of all offered products. In the model customers buy a product if and only if its surplus is positive for them. The non-concave, mixed-integer optimization problem is decomposed into sub-problems, one for each feasible combination of binary variables. Selection of the set of products in the product line is handled by means of simulated annealing, and the pricing problem is solved by a steepest ascent type of procedure. They state that the procedures they propose perform extremely well.

Hopp and Xu (2005) address a similar model of product line selection and pricing problem to Aydin and Ryan's (2000) model using a Bayesian logit model. They show the impacts of degree of modularity in design on the optimal product line length, price, and market share, and they take extend the model of Aydin and Ryan by including the fixed product development costs. They assume that the firm is risk sensitive with an exponential utility function.

They state that the variety offered in the product line increases by reducing the cost of introducing new products via modularity in design, which leads to increases in market share in a multiple-segment market and increased price markups in a single-segment market. In addition, they indicate that the optimal price markups of the n products are positive and equal, which is a similar result obtained by Anderson *et al.* (1992) for a nested-logit model and Aydin and Ryan for a logit model. Nevertheless, they show that if the firm is risk-adverse, it may be optimal to decrease the variety to be offered which results in a lower price margin in a single-segment market by a reduction in the production costs.

3.2. Remanufacturing

Reverse logistics (RL) has been receiving a growing attention within the context of sustainable development for its profitability due to recovering the remaining value of used products. Furthermore legislations and directives, consumer awareness and social responsibilities are some other reasons for the growing popularity of RL disposal (Pokharel and Mutha, 2009). The European Working Group on Reverse Logistics, REVLOG, defines reverse logistics as:

"The process of planning, implementing and controlling backward flows of raw materials, in process inventory, packaging and finished goods, from a manufacturing, distribution or use point, to a point of recovery or point of proper disposal (Pokharel and Mutha, 2009)."

Waste management, material recovery (recycling), parts recovery or product recovery through remanufacturing (value-added recovery) is of interest in RL (Pokharel and Mutha, 2009). The operations that are made to recover the residual value of used products by reusing components that are still working well are generally referred as product recovery which is an element of RL and encompasses collection, transportation, inspection and sorting, inventory management, and production planning and scheduling of returned products, which in turn determines the cost of recovered products. Once used products are returned to a firm which is involved in product recovery or recoverable manufacturing, it should apply one or more of the following activities depending on the condition of the returned products: repair, refurbishing, remanufacturing, cannibalization, and recycling (Thierry *et al.*, 1995). Refurbishing brings the used products up to a specified quality level which is less than that of a new product by fixing or replacing critical components. In repairing operations, the quality level again is less than the new product. On the other hand, the remanufacturing activity consists of disassembling used products, inspecting, cleaning, repairing some of the components, and replacing their worn-out or outdated components by new ones to carry them back to their original specifications, i.e. they are brought back to "as good as new" condition by restoring the quality level of used products to that of new products. This means that the quality level of a remanufactured product is at least as high as the quality level of its brand-new counterpart.

The remanufacturing activities may be performed by an Original Equipment Manufacturer (OEM), or by a contractor. At the remanufacturing facility, the returned used products are subjected to detailed inspection to decide whether they can be remanufactured or have to be disposed of. If the used products are not appropriate for remanufacturing, then they should be disposed of by the company.

Remanufacturing is utilized in many industries including photocopiers, telecommunication equipment, office furniture, tires, steel, commercial aircraft, computers, automobiles, chemicals, appliances, and medical items. Some of the companies that have practiced remanufacturing include Delphi, Xerox, General Motors, Hewlett-Packard, DuPont, BMW, Kodak, and Caterpillar.

As was mentioned, both environmental and economic goals are of concern in RL, and make product recovery systems attractive for many companies. The environmental laws are tightened in many countries of the world which force manufacturers to design remanufacturable products that reusing them or their components are possible. The other goal is related to economic issues. Remanufacturing can reduce the unit cost of production by reutilizing the components. Companies that are involved in remanufacturing are estimated to save 40 to 60 per cent of the cost of manufacturing a completely new product while requiring only 20 per cent of effort (Dowlatshahi, 2000). For example, Xerox Europe obtained savings over $80 million by means of implementing an end-of-life equipment take-back and reprocessing program in 1997 (Maslennikova and Foley, 2000). However, the cost of remanufacturing increases with the quality of remanufactured product to be restored (Fleischmann *et al.*, 1997).

Besides its benefits, remanufacturing has some inherent problems. One of the problems faced by companies is taking back sufficient amount of used products before the end of their useful life to gain some revenue by remanufacturing them. Another concern is the uncertainty in the quantity, timing, and quality of returned products. A more important issue is that remanufactured products are valued less than the new products by customers even if they are brought back to the same quality level as the new products. Customers' quality perception of remanufactured products can be changed by providing the same satisfaction warranty to them as the one given to the new products (Maslennikova and

Foley, 2000). Pricing strategy of remanufactured products is also an important mechanism for the market penetration particularly when these products are given the same warranties as the new ones. Since the remanufactured product is cheaper to manufacture and there might be more than one quality level of the remanufactured products, reselling them at different discounted prices may create a demand opportunity for them in secondary markets. This situation increases the attention paid to revenue management in remanufacturing which focuses on setting the prices of remanufactured products of different quality levels to maximize the total revenue. However, pricing the remanufactured products is a complex and difficult issue. Furthermore, there may be a competition between the remanufactured product and new product in the market. Thus, the investment on product recovery should be made carefully. Nonetheless, if a firm which is confronting intense competition and low profit margins uses remanufacturing effectively, it can gain a competitive advantage in its industry.

The literature related to remanufacturing is mainly about disassembly, coordination, supply chain, inventory, pricing the remanufactured products and competition issues (Pokharel and Mutha, 2009). In most of the papers, price, demand rate, and remanufacturability level are assumed to be known, and customers cannot differentiate new and remanufactured products with respect to their quality.

Mahumder and Groenevelt (2001) propose a two-period game-theoretic model to study the competition between an OEM and a local remanufacturer, which affects the supply of used products and the price of the remanufactured products. In the first period, the OEM is the only player and it sets the manufactured quantity which determines the available amount of used products for remanufacturing in the second period. In the second period the local remanufacturer acts as a second player in the game. They find the Nash equilibrium of the price and quantity for both competitors in different cases.

Ferguson and Toktay (2006) also studied the competition between a monopolist OEM and an entrant of third-party remanufacturer (external cannibalization), and discuss strategies to deter entry of other remanufacturers as well as they analyze the competition between new and remanufactured products (internal cannibalization) where they try to identify the conditions under which they should not remanufacture in a two-period model.

They try to monitor the impact of fixed and variable costs on recovery strategies. They find how the remanufactured product should be priced in relation to the firm's existing product. They conclude that the choice of remanufacturing should be considered as an OEM's competitive strategy and they state the cases, under which remanufacturing is advantageous.

Another work considers the competition is the work of Ferrer and Swaminathan (2006). They analyze the profitability of offering remanufactured product along with the new products in the monopoly and duopoly scenarios, and identify some managerial insights to find effective policies for the product lines in the existence of remanufacturing. They find the Nash equilibrium of the price and quantity for different planning horizons while exploring the effect of some parameters in the equilibrium for remanufacturing operations.

Guide *et al.* (2003) consider prices of remanufactured products as decision variables, and develops an economic analysis for finding the optimal acquisition prices of used phones, and optimal selling prices for remanufactured products for a cellular phone remanufacturer to maximize the profit where the firm can influence the quality, quantity, and timing of product returns by offering price incentives to customers depending on the quality of the retuned products. The company remanufactures the gathered used phones of different quality levels to a single quality level, and sells them at a certain price. They develop a heuristic to obtain the optimal solution, and carry out sensitivity analysis to provide managerial insights.

There exist some other studies which are developed to determine the optimal selling price for remanufactured products. The work of Debo *et al.* (2005) is relevant to our work in the sense that they focus on selling both new and remanufactured products by an OEM in a market where customers are heterogeneous in their willingness-to-pay and they can differentiate between the new and the remanufactured products. They study the joint pricing of new and remanufactured products, and production technology selection which determines the level of remanufacturability to maximize the profitability of the OEM where competition with third-party remanufacturers may exist. Customer preference is explained under max-utility choice model. Fixed cost is also included in the model. The monopolist decides on the remanufacturability level of the new products and segments the market between new and remanufactured products. They explore how the characteristics of the market and cost structure affect the pricing and remanufacturability level decisions. They state that high production costs of single-use product, low remanufacturing costs and low incremental costs to make a single-use product remanufacturable are the main technological drivers which make remanufacturing strategy profitable. Moreover, the additional cost to make a product remanufacturable may be valuable if enough customers value the remanufactured product highly. In addition, optimal remanufacturability level is determined by the joint effects of the customer profile and the fixed cost.

Varayasan and Ryan (2006) also consider determining the proportion of the returned products to refurbish and their optimal selling price for an electronics manufacturer in a competitive market for new products by modeling the sale, return, refurbishment, and resale processes in an open queuing network for profit maximization. The demand model is similar to the model of Debo *et al.* (2005). The price of the new products is not a decision variable. They state that introducing refurbished product to the market can be a profitable strategy even when they cannibalize the demand of new products. Furthermore, the optimal quantity of refurbished products to be offered differs according to the situations.

Mitra (2007) studies the pricing of remanufactured and refurbished products for remanufacturing firms in developing countries to maximize the total revenue where demand depends on the availability of discarded products and price, not quality.

Esenduran (2004) studied a very similar problem to ours under a gravity-based model for customer choice in her thesis. However, there is some opposition to the used gravity model since it suffers from the IIA limitation. Therefore, in our work we try to overcome this restriction.

4. THE MODEL

In this chapter, the problem descriptions of a base model and an extension of it are given. The related assumptions, formulations, and properties of each model are detailed.

4.1. The Base Model

We develop a model for a product line of an original equipment manufacturer (OEM) which has remanufacturing capability. The current product line of the OEM contains only the manufactured products. We may call them the existing products and denote by N_E. The OEM may extend its product line by remanufacturing and selling used products from a predetermined candidate set N_R. We also assume that there is a set of competitors' products denoted by N_C. In our model, customers are not restricted to purchase a product from the products offered in the market, namely they have the option of purchasing nothing. This approach seems more realistic since some customers may not prefer purchasing any products offered in the market for reasons such as high prices and/or relatively low quality values. No-purchase alternative can be represented by an outside group, and it can be denoted by N_0. We let N denote the set of all products, i.e. the choice set among which customers make their choices, as $N = N_0 \cup N_E \cup N_R \cup N_C$.

The market consists of a certain number of customer segments of various sizes which are determined by the customer's preferences and the set of these segments is denoted by M. Each segment is homogeneous within itself, which means all customers within a segment have the same quality perception for a certain product in the product line, and the quality perception for the same product in another segment differs. The size (i.e. the number of people included) of each segment is assumed to be known and denoted by d_i, $i = 1, \ldots, I$.

Customer choice is modeled using a probabilistic decision rule, the NL model. It is used to capture some correlations between alternatives. The NL model is chosen to overcome the restrictive limitation of the MNL model, i.e., the IIA property mentioned in Chapter 2. The particular specification of RU2 UMNL model is preferred in our model since its consistency with utility maximization theory is satisfied without any restriction.

The error terms of products are correlated with each other within a nest and uncorrelated in different nests. Since an existing product and its remanufactured version may share unobservable utility components, it is reasonable to group these products into the same nest. Thus, we propose a two-level nested model for customer choice the structure of which is shown in Figure 3.1. The choice set N is partitioned into $m+1$ nests, H_h where $H = \bigcup_{h=0}^{m} H_h$. $h=0$ is used to represent the nest with no-purchase alternative. An existing product and, if offered, its remanufactured version constitute a nest while the product of each competitor acts as one nest. In addition, there is a nest for the outside group called Nest 0.

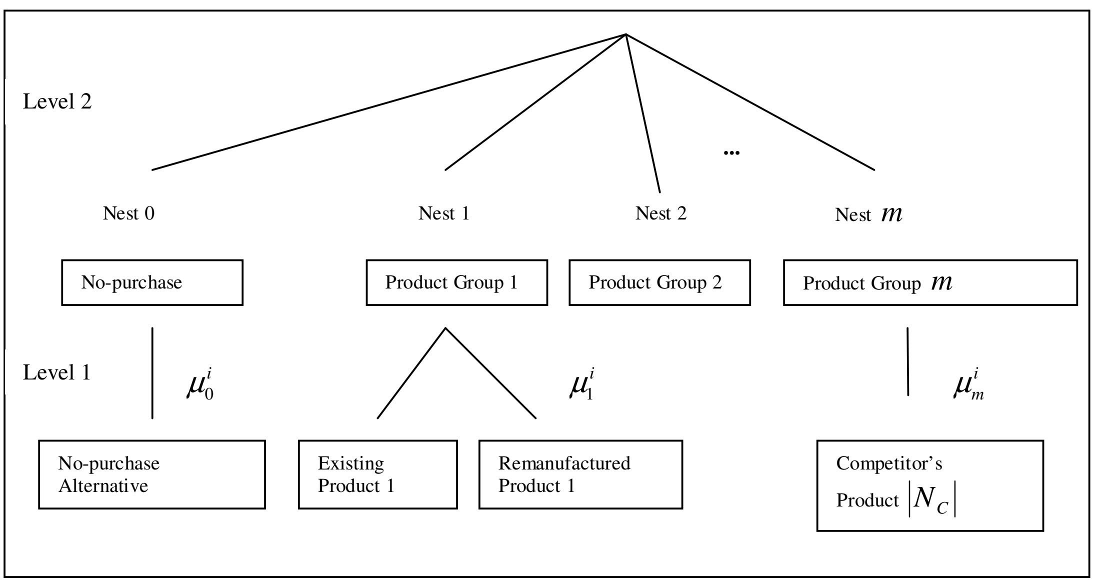

Figure 3.1. The nested structure of our consumer choice model

In our work, we suppose there are $m+1$ product groups, indexed by $h = 0,\ldots,m$, each of which consists of m_h products. The representative utility function of a product j in nest H_h for a customer in segment i, $V_{j|h}^i$ depends only on variables that describe product j and is expressed as a linear function of the product's attributes (the explanatory variables). It includes the perceived quality value of the product, Q^i_{hj}, and the selling price of this product, S_{hj}, as the explanatory variables because customers usually make their purchase decisions based on price and quality. We assume that quality is a measurable variable which involves some product attributes such as performance, styling etc. that are observable to the customer. The value of a product's quality is assumed to be known to the OEM for each segment and customers have sufficient information to evaluate the product quality. We call it as perceived quality because the quality perception for an alternative differs in different segments since some customers may be willing to pay more for some features while the others not. In addition, the quality perception differs for the new manufactured products and their remanufactured versions. We assume that customers have complete information about all products offered in the market, and their selling prices to be able to compare the products in terms of their quality values and their selling prices, and to choose the one with the best price-quality relationship according to their valuations. Moreover, if a customer makes a purchase decision, he/she buys only one unit of a product type from the products offered in the market or buys nothing. It should be noted that price discrimination is not allowed, so that if a product is offered then any segment can buy it at the same price.

Since the proposed model is designed for a single period, competitors do not react to the policy of the OEM. It is important for an OEM to distinguish its products from its competitors' in marketing (Moorthy, 1988). Therefore, some of the competitors' products that are important to the OEM are taken into account in our model to gain competitive advantage in the pricing problem and to monitor the competitive effects. Competitors' selling price and quality values are assumed to be known and fixed.

The representative utility function is increasing in the product's perceived quality and decreasing in the product's selling price. This assumption seems realistic since customers always prefer a high-quality product to a low-quality product or less expensive to more expensive when everything else is kept the same. In this work, customers are supposed to distinguish between the new products and their remanufactured versions since the remanufactured products are sold with a label on them in many countries and they are usually perceived as lower quality items than their manufactured versions. Therefore, the perceived quality values of remanufactured products are expected to be less than their new versions. Moreover, the unit cost of remanufacturing is less than the unit cost of producing a new product, and hence, the selling price of the remanufactured product is expected to be less than the selling price of its new version. Since the remanufactured products are cheaper to manufacture and their prices are lower, this situation creates an opportunity to sell the remanufactured products to customers in low-preference segments. Selling price of a product is assumed to be the same in all segments. The random utility that a customer in segment i obtains from product j within nest H_h, U^i_{hj} can be expressed as the sum of the representative utility $V^i_{j|h}$ and the random utility term ε^i_{hj},

$$U^i_{hj} = V^i_{j|h} + \varepsilon^i_{hj} = \beta^i_q Q^i_{hj} - \alpha^i_p S_{hj} + \varepsilon^i_{hj} \tag{4.1}$$

where β^i_q and α^i_p are the parameters to be estimated for each segment i and they identify the relative effects of the perceived quality (the subscript q represents quality), and the selling price (the subscript p represents price) on customer preference, respectively. The utility related to no-purchase alternative is $U^i_{01} = \varepsilon^i_{01}$, which means $V^i_{1|0} = 0$ for all segments.

We can represent the probability p^i_{hj} that for a customer in segment i purchases product j within nest H_h, as

$$p^i_{hj} = p^i_h p^i_{j|h} \quad i \in M, j \in H_h \cap N, h \in H \tag{4.2}$$

where

$$p^i_h = \frac{\exp\left(\frac{1}{\mu^i_h} IV^i_h\right)}{\sum_{r=0}^{m} \exp\left(\frac{1}{\mu^i_r} IV^i_r\right)} = \frac{\exp\left(\frac{1}{\mu^i_h} IV^i_h\right)}{\sum_{r=1}^{m} \exp\left(\frac{1}{\mu^i_r} IV^i_r\right) + 1} \quad i \in M, h \in H, \tag{4.3}$$

$$p^i_{j|h} = \frac{\exp\left(\mu^i_h V^i_{j|h}\right)}{\sum_{k=1}^{m_h} \exp\left(\mu^i_h V^i_{k|h}\right)} \quad i \in M, j \in H_h \cap N, h \in H, \tag{4.4}$$

and

$$IV^i_h = \ln \sum_{k=1}^{m_h} \exp\left(\mu^i_h V^i_{k|h}\right) \quad i \in M, j \in H_h \cap N, h \in H \tag{4.5}$$

Then, the probability that a customer in segment i does not purchase any product in the market is given by:

$$p^i_{01} = \frac{1}{\sum_{r=1}^{m} \exp(\frac{1}{\mu^i_r} IV^i_r) + 1} \quad i \in M \tag{4.6}$$

In the formulas above, μ^i_h is the scale parameter related to nest H_h for segment i, and it can be used as an indication of correlation among products in this nest. It should be greater than or equal to one since we use RU2 UMNL specification to be consistent with the utility maximization theory.

Let Y_{hj} be a binary variable which is one if candidate product j in nest $H_h \cap N_R$ is offered in the final product line and zero otherwise. Since existing products and competitors' products are already being sold in the market, and there is always a no-purchase alternative, we set $Y_{hj} = 1$ for $j \in H_h \cap (N_0 \cup N_E \cup N_C)$.

If a product is offered (i.e. $Y_{hj} = 1$), then there exists a positive purchase probability for that product. This probability is used to calculate the amount of product j within nest $H_h \cap (N_E \cup N_R \cup N_C)$ sold to segment i denoted by X^i_{hj}. Since the total number of people in each segment i, d_i, is assumed to be known, then the aggregate expected demand for each product $j \in H_h \cap (N_E \cup N_R \cup N_C)$ for segment i can be calculated as

$$X_{hj}^{i} = d_i p_{hj}^{i} \quad i \in M, j \in H_h \cap (N_E \cup N_R \cup N_C), h \in H \tag{4.7}$$

$X_{01}^{i} = d_i p_{01}^{i}$ gives the amount people in segment i who do not purchase anything.

We define the marginal profit of a product j in nest $H_h \cap (N_E \cup N_R)$ as the difference between the selling price and the unit cost of the product (i.e. $S_{hj} - c_{hj}$) where c_{hj} is the unit variable production cost of product j in nest H_h, and it is assumed to be known for existing products and their remanufactured versions. We assume that the OEM is expected to care about the total profits resulting from the whole product line. A negative marginal profit of a product in the optimal solution would be an indication that the product incurs a loss. Hence, we add the constraint $S_{hj} \geq c_{hj}$ to our model.

The problem is formulated as a mixed-integer nonlinear program. The objective of maximizing the total profit can be written as

$$\max \Pi = \sum_{i \in M} \sum_{h=0}^{m} \sum_{j=1}^{m_h} X_{hj}^{i} \left(S_{hj} - c_{hj}\right) - \sum_{\substack{j=1 \\ j \in H_h \cap N_R}}^{|N_R|} f_{hj} Y_{hj} , \tag{4.8}$$

where f_{hj} is the fixed cost of introducing a remanufactured product j in the product line, and it is assumed to be known for all remanufactured products. Existing products do not have fixed costs since they are already being manufactured. The first term of the objective function is the total revenue due to the sales revenue minus total variable cost where the second term represents the total fixed cost of introducing the remanufactured products. We assume that we already have enough items (products) on hand to satisfy demand.

After giving all the ingredients related to the problem, the complete formulation of the proposed model can be given as

$$\max \Pi = \sum_{i \in M} \sum_{h=0}^{m} \sum_{j=1}^{m_h} X_{hj}^{i} \left(S_{hj} - c_{hj}\right) - \sum_{\substack{j=1 \\ j \in H_h \cap N_R}}^{|N_R|} f_{hj} Y_{hj} \tag{4.9}$$

$$\text{s.t. } p^i_{hj} = p^i_h p^i_{j|h} \qquad i \in M, j \in H_h \cap N, h \in H \tag{4.10}$$

$$X^i_{hj} = d_i p^i_{hj} \qquad i \in M, j \in H_h \cap N, h \in H \tag{4.11}$$

$$\mathrm{S_{hj}} \geq c_{hj} \qquad i \in M, j \in H_h \cap (N_E \cup N_R), h \in H \tag{4.12}$$

$$Y_{hj} = 1 \qquad j \in H_h \cap (N_0 \cup N_E \cup N_C), h \in H \tag{4.13}$$

$$Y_{hj} \in \{0,1\} \qquad j \in H_h \cap N_R, h \in H \tag{4.14}$$

Here

$$p^i_h = \frac{\exp\left(\frac{1}{\mu^i_h} IV^i_h\right)}{\sum_{r=1}^{m} \exp\left(\frac{1}{\mu^i_r} IV^i_r\right) + 1} \quad i \in M, h \in H, \tag{4.15}$$

$$p^i_{j|h} = \frac{\exp\left(\mu^i_h V^i_{j|h}\right) Y_{hj}}{\sum_{k=1}^{m_h} \exp\left(\mu^i_h V^i_{k|h}\right) Y_{hk}} \quad i \in M, j \in H_h \cap N, h \in H, \tag{4.16}$$

$$IV^i_h = \ln \sum_{k=1}^{m_h} \exp\left(\mu^i_h V^i_{k|h}\right) Y_{hk} \quad i \in M, j \in H_h \cap N, h \in H, \tag{4.17}$$

$$V^i_{j|h} = \beta^i_q Q^i_{hj} - \alpha^i_p S_{hj} \quad i \in M, j \in H_h \cap (N_E \cup N_R \cup N_C), h \in H, \tag{4.18}$$

$$V^i_{1|0} = 0 \quad i \in M, \tag{4.19}$$

and

$$N = N_0 \cup N_E \cup N_R \cup N_C. \tag{4.20}$$

The model presented above is a profit maximization model. The binary variables indicating whether a remanufactured product is offered in the product line or not, and the selling price values related to the existing products, and their remanufactured versions that are included in the product line are the decision variables. The solution of the model will help the OEM to determine the remanufactured products that should be offered in the product line and the selling prices of the remanufactured products as well as the existing ones.

4.1.1. Optimality conditions

Proposition 3.1. Let $N' = N_0 \cup N_E \cup N'_R \cup N_C$ be the set of products offered in the market which consists of only one customer segment with no-purchase alternative. Then, there exists an optimal marginal profit for any product $a \in H_s \cap (N_E \cup N'_R)$ which occurs at

$$(S_{sa} - c_{sa}) = \frac{1}{\alpha_p \mu_s} + \sum_{k=1}^{m_s} (S_{sk} - c_{sk}) p_{k|s} (1 - \frac{1}{\mu_s}) + \frac{\sum_{h=0}^{m} \sum_{j=1}^{m_h} (S_{hj} - c_{hj}) p_h p_{j|h}}{\mu_s} \tag{4.21}$$

and it is positive and equal for all products that are in the same nest.

Proof. We show the calculations for only one customer segment for the sake of clarity. If we write p^i_{hj} in an explicit form in constraint (4.11) using the expression in (4.10), we get

$$X^i_{hj} = d_i p^i_h p^i_{j|h} . \tag{4.22}$$

By putting the resulting equality for X^i_{hj} in the objective function we can write the objective function as

$$\Pi(S,Y) = \sum_{i \in M} \sum_{h=0}^{m} \sum_{j=1}^{m_h} d_i p^i_h p^i_{j|h} \left(S_{hj} - c_{hj} \right) - \sum_{\substack{j=1 \\ j \in H_h \cap N_R}}^{|N_R|} f_{hj} Y_{hj} . \tag{4.23}$$

We are interested in the derivative of the objective function with respect to the price S_{sa} of product a in nest H_s ,. The second part of the objective function related to the fixed costs does not affect the derivative since it acts as a constant and its derivative is zero with respect to the price, thus this part is omitted in the calculations. We may partition the objective function into two parts and write it as the sum of two functions. Π_1 represents the profit coming from the nest that contains product a and Π_2 represents the profit resulting from the other nests.

$$\begin{aligned}\Pi &= \Pi_1 + \Pi_2 \\ &= \sum_{\substack{k=1 \\ a\in H_s}}^{m_s} dp_s\, p_{k|s}\left(S_{sk} - c_{sk}\right) + \sum_{\substack{h=0 \\ h\neq s}}^{m} \sum_{j=1}^{m_h} dp_h\, p_{j|h}\left(S_{hj} - c_{hj}\right)\cdot\end{aligned} \tag{4.24}$$

To take the partial derivative of Π_1 with respect to S_{sa} we first calculate the following derivatives

$$\frac{\partial p_{a|s}}{\partial S_{sa}} = \alpha_p \mu_s p_{a|s}(p_{a|s} - 1)\,, \tag{4.25}$$

$$\frac{\partial p_{b|s}}{\partial S_{sa}} = \alpha_p \mu_s p_{a|s} p_{b|s} \quad b \in H_s\,, \tag{4.26}$$

$$\frac{\partial p_s}{\partial S_{sa}} = \alpha_p p_{a|s} p_s (p_s - 1) \quad . \tag{4.27}$$

Using (4.25), (4.26) and (4.27) the partial derivative of Π_1 with respect to S_{sa} is

$$\frac{\partial \Pi_1}{\partial S_{sa}} = dp_s p_{a|s}\left[\alpha_p \sum_{k=1}^{m_s} (S_{sk} - c_{sk}) p_{k|s}(p_s - 1 + \mu_s) - \alpha_p \mu_s (S_{sa} - c_{sa}) + 1\right]. \tag{4.28}$$

The partial derivative of the second part of the objective function gives

$$\frac{\partial \Pi_2}{\partial S_{sa}} = dp_s p_{a|s}\left[\alpha_p \sum_{\substack{h=0 \\ h\neq s}}^{m} \sum_{j=1}^{m_h} (S_{hj} - c_{hj}) p_h p_{j|h}\right]. \tag{4.29}$$

It is obvious that the derivative given in (4.29) is positive since $S_{hj} - c_{hj} \geq 0$ is guaranteed by constraint (4.12) and all other terms in the equation are already positive. This means an increase in the price of a particular product (for example product a) would cause an increase in the profit due to the other products in different nests. This expected increase in Π_2 would stem from the decrease in the demand of product a. Therefore, an increase in the price of a particular product causes an increase in the demand of other products in different nests, and accordingly results in an increase in the profit due to other products, i.e. Π_2.

(4.28) and (4.29) can be summed up to have the overall derivative of objective function as

$$\frac{\partial \Pi}{\partial S_{sa}} = \frac{\partial \Pi_1}{\partial S_{sa}} + \frac{\partial \Pi_2}{\partial S_{sa}} = dp_s p_{a|s} \left[\begin{array}{l} \alpha_p \sum_{k=1}^{m_s} (S_{sk} - c_{sk}) p_{k|s} (\mu_s - 1) - \alpha_p \mu_s (S_{sa} - c_{sa}) + 1 \\ + \alpha_p \sum_{h=0}^{m} \sum_{j=1}^{m_h} (S_{hj} - c_{hj}) p_h p_{j|h} \end{array} \right] \quad (4.30)$$

We see from (4.30) that there exists a finite optimum because the derivative is not always positive since we subtract the positive term $\alpha_p \mu_s (S_{sa} - c_{sa})$ from the remaining positive terms. According to the first order optimality conditions, if a point is local minimum of a function, then its gradient is zero at that point. Equating (4.30) to zero and solving for $(S_{sa} - c_{sa})$ (the marginal profit of product a) results in the following equation:

$$(S_{sa} - c_{sa}) = \frac{1}{\alpha_p \mu_s} + \sum_{k=1}^{m_s} (S_{sk} - c_{sk}) p_{k|s} (1 - \frac{1}{\mu_s}) + \frac{\sum_{h=0}^{m} \sum_{j=1}^{m_h} (S_{hj} - c_{hj}) p_h p_{j|h}}{\mu_s} \quad (4.31)$$

We see that the marginal profit of a product is positive since $S_{hj} - c_{hj} \geq 0$ holds by constraint (4.12) for all j, all probability values (p_h and $p_{j|h}$ for all j and h) are positive and $\mu_s \geq 1$. Moreover, the marginal profit is the same for all products that are in the same nest. This result is consistent with the findings of Aydin and Ryan (2000) and Anderson *et al.* (1992). They state that the marginal profits of all products are equal at optimality using the MNL model.

We should consider the effect of cannibalization when a new product is added to the product line. Suppose the product line is extended by adding a product while the selling prices of the existing products are kept the same. Since a new product is offered, a positive purchase probability is created for this product. This means the choice set is expanded, and this new product will cannibalize the purchase probabilities of the existing products and no-purchase alternative since the sum of all purchase probabilities over the choice set is equal to one. Therefore, we can conclude that when a new product is introduced to the market, the purchase probability and accordingly the demand for each existing product and the number of customers that do not purchase anything decreases.

Let us take this analysis one step further to investigate the effect of fixed cost. Suppose the existing product line is expanded by the introduction of a new product where its selling price is a decision variable. In the analysis, the selling prices of the existing products are fixed.

Consider an optimal product line with n products with their optimal prices. If an additional product a is added to the product line, the demand of existing products decreases as it was stated above. Hence, it is expected that the contribution of the existing products to the total profit will decrease after the introduction of product a. The change in the total profit due to existing products can be stated as

$$\begin{aligned}\Delta\Pi_E = \Pi_E(\text{without } a) - \Pi_E(\text{with } a) = &\sum_{i\in M}\sum_{h=0}^{m}\sum_{j=1}^{m_h} d_i p_{hj}^i \left(S_{hj} - c_{hj}\right) \\ &- \sum_{i\in M}\sum_{h=0}^{m}\sum_{j=1}^{m_h} d_i q_{hj}^i \left(S_{hj} - c_{hj}\right)\end{aligned} \tag{4.32}$$

Here, $\Pi_E(\text{without } a)$ represents the total profit of existing products before product a is introduced, whereas $\Pi_E(\text{with } a)$ represents the profit contribution of existing products to the total profit after the inclusion of product a in the product line. The difference between them indicates the decrease in demand of existing products. The purchase probabilities related to existing products are denoted by p_{hj}^i (q_{hj}^i) before (after) the introduction of product a. Although the profit share of the existing products in total profit decreases after the addition of product a to nest H_h, there will be an increase in the total profit due to the additional profit comes from the new product. The additional profit results from product a can be given as

$$\Delta\Pi_a = \sum_{i \in M} d_i q_{hj}^i \left(S_{ha} - c_{ha} \right) \tag{4.33}$$

Therefore, to provide an increase in the total profit function due to the new product, the additional profit $\Delta\Pi_a$ due to the inclusion of product a should be greater than the decrease $\Delta\Pi_E$ in profit due to existing products. Namely,

$$\begin{aligned} \Delta\Pi_a &\geq \Delta\Pi_E \\ \sum_{i \in M} d_i q_{ha}^i \left(S_{ha} - c_{ha} \right) &\geq \sum_{i \in M} \sum_{h=0}^{m} \sum_{j=1}^{m_h} d_i p_{hj}^i \left(S_{hj} - c_{hj} \right) - \sum_{i \in M} \sum_{h=0}^{m} \sum_{j=1}^{m_h} d_i q_{hj}^i \left(S_{hj} - c_{hj} \right) \\ S_{ha} &\geq c_{ha} + \frac{\sum_{i \in M} \sum_{h=0}^{m} \sum_{j=1}^{m_h} d_i (p_{hj}^i - q_{hj}^i) \left(S_{hj} - c_{hj} \right)}{\sum_{i \in M} d_i q_{ha}^i} \end{aligned} \tag{4.34}$$

As we see, there is a lower bound on the price of the new product. It is profitable to introduce the new product to the product line with prices greater than that lower bound and the prices of the existing products do not change in the absence of fixed cost of introducing the new product.

If we consider the situation where all prices are left as decision variables, it is always more profitable to extend the product line with as many products as possible when there is no fixed cost of introduction. Namely, a new product always creates additional profit at its optimal price. We can call this additional profit as incremental profit and it is defined as the difference between the profits obtained when the pricing problem is solved including and excluding the product in the product line in the absence of the fixed cost. In their work, Aydin and Ryan (2000) show that the optimal solution is to carry as many products as possible within a product line in the absence of fixed costs providing each customer his ideal product. This situation can be stated with the following proposition.

Proposition 3.2. If the fixed cost of every candidate remanufactured product can be ignored, namely zero, then it is more profitable for the OEM to include all the candidate products in the product line.

Proof. In the absence of fixed cost of introducing the remanufactured products, the second term of the objective function related it in (4.9) can be omitted. Since $\mathrm{S}_{\mathrm{hj}} - c_{hj}$ is positive by Proposition 3.1 at optimality, the contribution of a remanufactured product to the objective function is always positive indicating that a remanufactured product without fixed cost is always profitable and should be included in the product line.

On the other hand, in the existence of fixed costs, a new product is included in the product line if and only if its incremental profit is greater than its fixed cost.

Therefore, the product line extension decision must be analyzed considering the trade-off between the incremental profit and fixed cost.

4.2. The Extended Model : Positioning the Remanufactured Product

In the extended model, we consider the problem of correctly positioning the remanufactured product that is offered in the optimal product line, namely finding out the optimal quality level of the remanufactured product and their price as well as the prices of the existing products so as to obtain the maximum profit value.

As was mentioned before, not all customers are alike in their preferences. A customer may prefer a refurbished product with a lower quality than its manufactured counterpart due to its relatively lower price whereas another one may want a recovered product with more additional features or improved performance. In the latter case used product goes through a remanufacturing process by using brand-new components or superior technology which increases the perceived quality of that product and the cost of remanufacturing. Hence, the optimal quality level to which used products has to be restored is of interest in the extended model. After determining the optimal product line composition with given quality values, if the optimal product line consists of one or more recovered products, then the analysis of positioning the one of the recovered products can be made.

In the literature, it is a common practice to assume that the marginal cost of producing a product increases with its quality (Mussa and Rosen, 1978; Moorthy, 1984; Moorthy, 1988; Choudharry *et al.*, 2005; Desai *et al.*, 2001; Kim and Chhajed, 2002; Balachander and Srinivasan, 1994) and the marginal cost for a product is convex in its quality. Consistent with the literature, we assume that cost of quality is convex and the unit production cost of a remanufactured product is stated as a function of its quality level:

$$c_{hj}\left(q_{hj}^{i}\right)=K_{hj}^{i}\left(Q_{hj}^{i}\right)^{2}\ i\in M,\ j\in H_{h}\cap N_{R}, h\in H \tag{4.35}$$

where K_{hj}^{i} is a scalar cost coefficient that recognizes differences in cost of producing unit quality for different remanufactured products and for different segments since quality perception differs in different segments. A difference in our cost model is that it catches the differences between the recovery process that are applied to products and the quality perception with respect to segments. Since different recovered products may go through different recovery processes that improve different quality type attributes, the increase in their unit production cost may not be at the same quantity since the effect of improvements in different quality attributes can cause different cost increases. This can be discriminated by using different cost coefficients related to different products. Furthermore, we are not working with real quality values, we use perceived quality values. Hence, these differences are caught with the differentiated constant K_{hj}^{i} which can vary with respect to products and segments. We can use the cost function $c_{hj}\left(Q_{hj}^{i}\right)$ and consider the unit production cost c_{hj} as another decision variable of the problem.

4.2.1. Optimality conditions

In this subsection, we are interested in determining the optimal quality value of a remanufactured product. We again show the calculations for only one customer segment for clarity.

Let c_{sa} denote the unit production cost of remanufactured product a in nest H_s. Replacing Q_{sa} with $\sqrt{\dfrac{c_{sa}}{K_{sa}}}$ in the representative utility of product a in equation (4.1), we get

$$V_{a|s} = \beta_q \sqrt{\frac{c_{sa}}{K_{sa}}} - \alpha_p S_{sa} \tag{4.36}$$

By putting this equation in the formulae of purchase probabilities in (4.23), we can examine the derivative of the objective function with respect to c_{sa}.

The derivatives that we use to take the partial derivative of Π_1 with respect to c_{sa} are

$$\frac{\partial p_{a|s}}{\partial c_{sa}} = \frac{\beta_q \mu_s}{2\sqrt{K_{sa} c_{sa}}} p_{a|s}(1 - p_{a|s}), \tag{4.37}$$

$$\frac{\partial p_{b|s}}{\partial c_{sa}} = -\frac{\beta_q \mu_s}{2\sqrt{K_{sa} c_{sa}}} p_{a|s} p_{b|s} \quad b \in H_s, \tag{4.38}$$

$$\frac{\partial p_s}{\partial c_{sa}} = \frac{\beta_q}{2\sqrt{K_{sa} c_{sa}}} p_{a|s} p_s (p_s - 1) \quad . \tag{4.39}$$

Using (4.37), (4.38) and (4.39) the partial derivative of Π_1 with respect to c_{sa} is calculated as

$$\frac{\partial \Pi_1}{\partial c_{sa}} = dp_s p_{a|s} \left[\begin{array}{l} \dfrac{\beta_q}{2\sqrt{K_{sa} c_{sa}}} \displaystyle\sum_{k=1}^{m_s} (S_{sk} - c_{sk}) p_{k|s} (1 - p_s - \mu_s) \\ + \dfrac{\beta_q \mu_s}{2\sqrt{K_{sa} c_{sa}}} (S_{sa} - c_{sa}) - 1 \end{array} \right]. \tag{4.40}$$

The partial derivative of the second part of the objective function is

$$\frac{\partial \Pi_2}{\partial S_{sa}} = dp_s p_{a|s} \left[-\frac{\beta_q}{2\sqrt{K_{sa} c_{sa}}} \sum_{\substack{h=0 \\ h \neq s}}^{m} \sum_{j=1}^{m_h} (S_{hj} - c_{hj}) p_h p_{j|h} \right]. \tag{4.41}$$

Summing up (4.40) and (4.41), the total derivative of the objective function becomes

$$\frac{\partial \Pi}{\partial S_{sa}} = dp_s p_{a|s} \begin{bmatrix} \frac{\beta_q}{2\sqrt{K_{sa} c_{sa}}} \sum_{k=1}^{m_s} (S_{sk} - c_{sk}) p_{k|s} (1 - p_s - \mu_s) + \frac{\beta_q \mu_s}{2\sqrt{K_{sa} c_{sa}}} (S_{sa} - c_{sa}) \\ -1 \\ -\frac{\beta_q}{2\sqrt{K_{sa} c_{sa}}} \sum_{\substack{h=0 \\ h \neq s}}^{m} \sum_{j=1}^{m_h} (S_{hj} - c_{hj}) p_h p_{j|h} \end{bmatrix} \tag{4.42}$$

Equating (4.42) to zero and solving for $(S_{sa} - c_{sa})$ results in the following equation:

$$(S_{sa} - c_{sa}) = \frac{2\sqrt{K_{sa} c_{sa}}}{\beta_q \mu_s} + \sum_{k=1}^{m_s} (S_{sk} - c_{sk}) p_{k|s} (1 - \frac{1}{\mu_s}) + \frac{\sum_{h=0}^{m} \sum_{j=1}^{m_h} (S_{hj} - c_{hj}) p_h p_{j|h}}{\mu_s} \tag{4.43}$$

Again, we see that the marginal profit for the remanufactured product a is positive and it is the same for all products that are in the same nest at optimality.

Replacing the marginal profit value in (4.43) with the optimal marginal profit given in (4.21) results in the following equality for c_{sa}:

$$c_{sa} = \frac{\beta_q^{\ 2}}{4\alpha_p^{\ 2} K_{sa}} \tag{4.44}$$

Note that there is an optimal cost value of a remanufactured product which satisfies the equality in (4.44).

5. SOLUTION PROCEDURE

In this chapter we explain the solution procedure for the resulting mixed-integer nonlinear programming (MINLP) problem developed in Chapter 4. The optimal solution of MINLP is hard to find. When the product line composition is fixed, i.e. a feasible zero-one assignment for Y_{hj}'s is given, the problem turns into a nonlinear programming problem (NLP). The following sections include the explanation for solving this continuous non-convex optimization problem. Then, using this solution procedure, optimal product line composition will be determined.

5.1. Solution Procedure for the Pricing Problem

For the optimization of unconstrained nonlinear objective functions usually some gradient methods are used if the function is differentiable. However, in many problems, either the derivatives are not in explicit form or they are in a very complex form that makes the usage of them inconvenient. In such cases, some direct search methods are used, some of which are Rosenbrock's method (Rosenbrock, 1960), the pattern search algorithm of Hooke and Jeeves (1961), and the simplex search algorithms of Spendley *et al.* (1962), and Nelder and Mead (1965). Hooke and Jeeves (1961) defines direct search as:

"We use the phrase "direct search" to describe sequential examination of trial solutions involving comparisons of each trial solution with the "best" obtained up to that time together with a strategy for determining (as a function of earlier results) what the next trial solution be. The phrase implies our preference, based on experience, for straightforward search strategies which employ no techniques of classical analysis except where there is demonstrable advantage in doing so."

Namely, in both methods an initial set of solutions are generated and a new candidate set of solutions is obtained at each iteration. This new candidate is accepted if it improves the objective function. In direct search methods, the risk of being trapped into a local optimum exists. Therefore, restarting with different initial points is a common practice to avoid this risk.

5.1.1. Pricing the Product Line with a Restarted and Modified Simplex Search

When we examine the complex structure of our problem, we see that it is not practical to use the gradient methods for the optimization of our objective function. Instead of using derivatives, it seems convenient to use one of the direct search methods. The popular Nelder and Mead (NM) simplex search method is utilized during the solution stage with an extension (Zhao *et al.*, 2009). Firstly, we give the steps of the NM method in Figure 5.1. Then, we will explain the extensions and modifications in detail.

The NL simplex search method originally developed for solving unconstrained problems. However, we have a constraint that the selling prices of the products must be grater than or equal to their unit cost. In computational analyses, the original method is used without considering the cost constraints and the results are examined whether the cost constraints satisfied or not. In all computational results, the cost constraints are always satisfied. Therefore, the cost constraints are handled without any modifications in the algorithm.

Simplex search uses a polyhedron with $n+1$ vertices for a problem with n variables. Each vertex of the simplex is represented by an n-dimensional vector. In the pricing problem, the objective is to determine the optimal prices of products for a fixed product line composition. Therefore each vertex is an n-dimensional price vector where n is the number of products in the product line. At each iteration, one of these vertices is dropped and a new vertex is created to form a new simplex. In the NM method, three candidate vertices along the line that joints the centroid of the n best vertices are considered, which are reflection, expansion and contraction. The iteration replaces the worst vertex by a better candidate vertex with a higher objective value to form a new simplex. Consequently, the simplex changes its shape during the iterations. If none of the candidate vertices deserves to replace the worst one, the n worst points are moved toward the best one shrinking the volume of the simplex. This procedure is continued until the termination criterion is met.

1. Creating initial simplex: Choose the points $\mathbf{S}_1, \mathbf{S}_2, \ldots, \mathbf{S}_{n+1}$ to form a simplex in E_n. Choose a reflection coefficient $\alpha > 0$, an expansion coefficient $\gamma_e > 1$, a contraction coefficient $0 < \beta < 1$, and a shrinkage coefficient $\chi > 0$. Go to Step 2.

2. Initialization: Let $\mathbf{S}_{\mathbf{max}}, \mathbf{S}_{\mathbf{min}} \in \{\mathbf{S}_1, \mathbf{S}_2, \ldots, \mathbf{S}_{n+1}\}$ be such that

$$\prod(\mathbf{S}_{\mathbf{max}}) = \max_{1 \le h \le n+1} \prod(\mathbf{S}_h) \text{ and } \prod(\mathbf{S}_{\mathbf{min}}) = \min_{1 \le h \le n+1} \prod(\mathbf{S}_h)$$

Let $\overline{\mathbf{S}} = \frac{1}{n} \sum_{\substack{h=1 \\ h \ne \min}}^{n+1} \mathbf{S}_h$. Go to Step 3.

3. Reflection: Let

$$\mathbf{S}_{\mathbf{r}} = \overline{\mathbf{S}} + \alpha(\overline{\mathbf{S}} - \mathbf{S}_{\mathbf{min}}).$$

If $\prod(\mathbf{S}_{\mathbf{r}}) > \prod(\mathbf{S}_{\mathbf{max}})$, go to Step 4. If $\prod(\mathbf{S}_{\mathbf{r}}) \le \prod(\mathbf{S}_{\mathbf{max}})$, but $\prod(\mathbf{S}_{\mathbf{r}}) \ge \min_{h, h \ne \min} \prod(\mathbf{S}_h)$, then replace $\mathbf{S}_{\mathbf{min}}$ by $\mathbf{S}_{\mathbf{r}}$ to form a new set of $n+1$ points, and go to Step 6. Otherwise go to Step 5.

4. Expansion: Let

$$\mathbf{S}_{\mathbf{e}} = \overline{\mathbf{S}} + \gamma_e(\mathbf{S}_{\mathbf{r}} - \overline{\mathbf{S}}).$$

Replace the point $\mathbf{S}_{\mathbf{min}}$ by $\mathbf{S}_{\mathbf{e}}$ if $\prod(\mathbf{S}_{\mathbf{r}}) < \prod(\mathbf{S}_{\mathbf{e}})$ and by $\mathbf{S}_{\mathbf{r}}$ if $\prod(\mathbf{S}_{\mathbf{r}}) \ge \prod(\mathbf{S}_{\mathbf{e}})$ to yield a new set of $n+1$ points. Go to Step 6.

5. Contraction: Let

$$\mathbf{S}_{\mathbf{c}} = \overline{\mathbf{S}} + \beta(\hat{\mathbf{S}}_{\mathbf{min}} - \overline{\mathbf{S}}),$$

where $\hat{\mathbf{S}}_{\mathbf{min}}$ is defined as $\prod(\hat{\mathbf{S}}_{\mathbf{min}}) = \max\{\prod(\mathbf{S}_{\mathbf{min}}), \prod(\mathbf{S}_{\mathbf{r}})\}$. If $\prod(\mathbf{S}_{\mathbf{c}}) \ge \prod(\hat{\mathbf{S}}_{\mathbf{min}})$, replace $\mathbf{S}_{\mathbf{min}}$ by $\mathbf{S}_{\mathbf{c}}$, and go to Step 6 . If $\prod(\mathbf{S}_{\mathbf{c}}) < \prod(\hat{\mathbf{S}}_{\mathbf{min}})$, replace $\mathbf{S}_h$ with $\mathbf{S}_h + \chi(\mathbf{S}_{\mathbf{max}} - \mathbf{S}_h)$ for $h = 1, \ldots, n+1$. Go to Step 2.

6. Termination: If $\left\{ \frac{1}{n+1} \sum_{h=1}^{n+1} \left[\prod(\mathbf{S}_h) - \prod(\overline{\mathbf{S}}) \right]^2 \right\}^{1/2} < \varepsilon$, then stop, $\mathbf{S}_{\mathbf{max}}$ is an optimal solution. Otherwise, go to Step 2.

Figure 5.1. The NL simplex search algorithm (Nelder and Mead, 1965)

In the NM method, we need to construct an initial simplex and we will define it before stating the modifications.

5.1.1.1. The Construction of the Initial Simplex. The initial simplex can be constructed by using the suggestion in Bazaraa *et al*. (1993). The first price vector is selected arbitrarily and the remaining vectors (vertices) of the simplex are generated using the direction vectors $\mathbf{d}$ as follows:

$$\mathbf{S}_{h+1} = \mathbf{S}_1 + \mathbf{d}_h \quad h = 1, \ldots, n. \tag{5.1}$$

Here, $\mathbf{d}_h$ is the direction vector the h^{th} component of which is equal to a and all other components of which are equal to b with

$$a = \frac{\varphi}{n\sqrt{2}}\left(\sqrt{n+1} + n - 1\right) \tag{5.2}$$

$$b = \frac{\varphi}{n\sqrt{2}}\left(\sqrt{n+1} - 1\right) \tag{5.3}$$

where φ is positive scalar. Since we try to form a simplex, we need to show that $n+1$ vertices, $\mathbf{S}_1, \mathbf{S}_2, \ldots, \mathbf{S}_{n+1}$ is suitable for the construction of the initial simplex if the method mentioned above is used.

If $n+1$ points form a simplex, then they should be affinely independent. Furthermore, if the points $\mathbf{S}_1, \mathbf{S}_2, \ldots, \mathbf{S}_{n+1}$ are affinely independent, then the points $\mathbf{S}_{h+1} - \mathbf{S}_1$ for $h = 1, \ldots, n$ are linearly independent. Thus, to show that the $n+1$ points form a simplex, we should show that $\mathbf{S}_{h+1} - \mathbf{S}_1 = \mathbf{S}_1 + \mathbf{d}_h - \mathbf{S}_1 = \mathbf{d}_h$ for all h are linearly independent. For the points to be linearly independent, the following equation must hold:

$$\sum_{h=1}^{n} \lambda_h \mathbf{d}_h = \mathbf{0} \tag{5.4}$$

with $\lambda_h = 0$ for all h. Here $\mathbf{0}$ is $n \times 1$ vector with all component are equal to zero. We can write equality (5.4) in a more explicit form for each component of the direction vector as

$$\lambda_m a + \sum_{\substack{h=1 \\ h \neq m}}^{n} \lambda_h b = 0 \quad m = 1,\ldots,n \,. \tag{5.5}$$

After replacing a and b by their definitions, the equation (5.5) becomes

$$\lambda_m \left[\frac{\varphi}{n\sqrt{2}} \left(\sqrt{n+1} + n - 1 \right) \right] + \sum_{\substack{h=1 \\ h \neq m}}^{n} \lambda_h \left[\frac{\varphi}{n\sqrt{2}} \left(\sqrt{n+1} - 1 \right) \right] = 0 \quad . \tag{5.6}$$

After arranging the terms, equation (5.6) can be written as

$$\lambda_m n + \left(\lambda_m + \sum_{\substack{h=1 \\ h \neq m}}^{n} \lambda_h \right) \left[\frac{\varphi}{n\sqrt{2}} \left(\sqrt{n+1} - 1 \right) \right] = 0 \tag{5.7}$$

which results in

$$\lambda_m n + \left(\sum_{h=1}^{n} \lambda_h \right) \left[\frac{\varphi}{n\sqrt{2}} \left(\sqrt{n+1} - 1 \right) \right] = 0 \tag{5.8}$$

and thus

$$\lambda_m = -\frac{1}{n} \left(\sum_{m=1}^{n} \lambda_m \right) \left[\frac{\varphi}{n\sqrt{2}} \left(\sqrt{n+1} - 1 \right) \right] \quad m = 1,\ldots,n \,. \tag{5.9}$$

From (5.9) it is seen that λ's are equal. Therefore, $\sum_{m=1}^{n} \lambda_m$ can be replaced by $n\lambda_m$. Then by using equality (5.8)

$$n\lambda_m\left[1+\frac{\varphi}{n\sqrt{2}}\left(\sqrt{n+1}-1\right)\right]=0 \tag{5.10}$$

is obtained. Since $n \geq 2$ and $\varphi > 0$, $\lambda_m = 0$.

We constructed the initial simplex by using the method explained above during our solution stage. There are some other methods to generate the initial simplex. For example, Humhrey and Wilson (2000) use the following approach to form the simplex:

$$\mathbf{S}_{h+1} = \mathbf{S}_1 + d\mathbf{e}_h \quad h = 1,\ldots,n. \tag{5.11}$$

Here, $\mathbf{e}_h$ is the unit vector with one in the h^{th} component and zero elsewhere. Again $\mathbf{S}_1$ is chosen arbitrarily and d is defined as:

$$d = \begin{cases} \max\{\tau|S_{1j}| : j = 1,\ldots,n & \text{if } \mathbf{S}_1 \neq 0 \\ 1 & \text{otherwise} \end{cases} \tag{5.12}$$

where τ is a step size parameter.

We repeat the simplex runs at least five times with different values of φ to avoid the risk of being trapped into a local optimum. In our computational experiments, $\{2,4,8,10,20\}$ are used as initial φ values and the best solution among the results of these five runs is selected. The reflection (α), expansion (γ_e), contraction (β), and shrinkage (χ) coefficients are initially set to the following values, $\alpha = 1, \gamma_e = 2, \beta = 0.5,$ and $\chi = 0.5$ as suggested by Nelder and Mead (1965).

Finally, in our computational experiments $\varepsilon = 10^{-4}$ is chosen to stop the algorithm.

5.1.1.2. Modifications on Simplex Runs. Some modifications of the NM method proposed by Zhao *et al*. (2009) are used for the pricing problem to improve the algorithm's efficiency. They propose a simple modification on the shrinking step. They suggest moving the $n+1$ vertices one by one starting from the worst one until an improvement is obtained instead of moving all vertices at once in the direction of the best vertex, and call this method Modified Nelder and Mead (MNM). Namely, they move the worst vertex with the minimum objective function value. If there is no improvement in the next iteration, the second worst one of the original $n+1$ vertices is moved and the procedure continues until a vertex with an objective value better than that one of a vertex with the worst value is obtained (Zhao *et al.*, 2009). If all vertices in the original simplex are shrunk, it turns into the classical algorithm of Nelder and Mead. They state the advantages of this modification as follows (Zhao *et al.*, 2009):

(i) The main reason for divergence of the NM method is caused by the classical shrinking step since the simplex greatly reduces its volume.

(ii) The chance of missing the local minimum is larger with the classical shrinking step. In this case, more function evaluations are needed to reach the local minimum if the NM converges.

They also suggest a heap data structure for storing and updating the vertices of the simplex for reducing the complexity of one NM iteration, and for computational efficiency. Nonetheless, we did not use this data structure in our work since the computational times were quite reasonable.

Moreover, they suggest a revised simplex search procedure just like the one Humphrey and Wilson (2000) proposed in their study. In the study of Humphrey and Wilson, the first local optimum is found using the classical NM method, and it is restarted by constructing an initial simplex around the solution found in the previous phase. Namely, the first vertex of the initial simplex is equated to the resulting best price vector of the previous run and initial simplex is constructed around this point. They also suggest decreasing the step size of the initial simplex geometrically and increasing the shrinkage coefficient linearly over successive phases. Finally, they state the best of ending values for the three phases as the final optimum. However, Zhao *et al.* (2009) keeps the size (or volume) of the simplex unchanged in all phases, i.e. the step size and the shrinkage coefficient is not changed. They call this procedure restarted and modified Nelder and

Mead simplex search method (RMNM). We also worked with different initial step sizes as it was mentioned in the previous subsection even if they do not suggest this modification. They finish the procedure when the solution of the current phase is not better than the best one obtained in the previous phase.

5.2. Solution Procedure for the Product Line Selection Problem

For the product line selection problem, using complete enumeration is a simple approach. Another common approach is to use some kind of heuristic methods.

5.2.1. Product Line Selection with Complete Enumeration

Despite its inefficiency for the solutions of combinatorial problems in general, it can be used for our problem since the number of candidate products is quite small. In fact, the situation is similar in real world, i.e. the number of products to be introduced in the product line is relatively small in almost all real world applications.

For a problem with n remanufactured candidate products with fixed costs, the simplex search procedure has to be called for each of the 2^n different product line compositions. If the fixed cost related to some candidate remanufactured products is zero, they are included in the optimal product line since the products with zero fixed cost are always included in the product line at optimality as stated in Proposition 3.2. Hence, the binary decision variables Y_{hj} for these products are set to one and they are not included in the enumeration process.

In our computational experiments, computational times were quite reasonable since we worked with two candidate remanufactured products.

If the number of possible product line compositions is too large, a heuristic based on genetic algorithms can be used to search through the product spaces to find the best product line composition encoding them in binary form as individuals or chromosomes.

6. COMPUTATIONAL RESULTS

In this chapter, computational results for the base model and the extended model are given. Different parameter values are used in experiments to see and interpret their effects on the solutions. In the base model, optimal prices are determined and the possible effects of the parameters on the solutions are observed. In the extended model, optimal quality levels are also determined for a remanufactured product in addition to the prices of all products in the product line.

Since the run times are quite short, CPU times are not listed. All experiments with different parameter values are repeated at least five times and the one with the maximum objective value is taken as a result. The results will be given for one customer segment and five products (two existing products, their remanufactured versions and the competitor's product) for simplicity.

The sample problems are solved with the simplex search procedure. The algorithm is coded in Microsoft Visual Studio 2005 Visual C# environment and it is run for each product line composition. To be able to make comparisons between the results, same experiments are solved with Excel Solver, too. The results do not differ for small sized problems in both.

6.1. Computational Results for the Base Model

There are some parameters of the problem which may have considerable effects on the optimal solution. These are the parameters β_q and α_p related to quality and the selling price on customer preference. Furthermore, the scale parameter μ_h should also be considered. They all should be estimated from real data in applications.

Some parameter values related to quality, cost, and fixed cost of products for the sample problem are presented in Table 6.1.

Table 6.1. Parameter values for the sample problem

Products	E_1	E_2	R_1	R_2	C_1
Quality (Q)	3.6	4.2	2.34	2.42	3.85
Cost (c)	15	20	6	7	-
Fixed Cost (f)	-	-	300	700	-

E_1 and E_2 denote the products that the OEM is currently selling in the market whereas C_1 denotes the competitor's product. OEM wants to extend its product line with the remanufactured versions R_1 and R_2 of the existing products E_1 and E_2. Therefore, the customers make their choice decisions among five products as well as they have an option of not purchasing anything. We see that $Q_{E_1} < Q_{E_2}$, which designates that the customers' perception of quality is lower for the existing product E_1 than E_2, and this situation is identical for the remanufactured products, i.e. $Q_{R_1} < Q_{R_2}$. Furthermore, the unit production cost values of R_1 and R_2 are relatively low compared to their existing counterparts E_1 and E_2. The segment size is 3000. The price of the competitor's product is taken to be 22. Since an existing product and its remanufactured version are similar alternatives and there is some correlation among them, they are collected in a nest. Thus there are four nests in our sample problem. The four nests contain the outside group which indicates the option of purchasing nothing; E_1 and R_1; E_2 and R_2; C_1 respectively. Since there are two candidate products ($R = 2$) to extend the product line, the possible product line compositions are $2^2 = 4$. Therefore, all experiments are repeated four times. The nesting structure of the sample problem is shown in Figure 6.1.

Lower level scale parameter values, μ_h, $h = 0,\ldots,3$, which describe the variances of the unobservable errors within the nests should be equal to one or greater than one to be consistent with the utility maximization theory as mentioned in equation (2.31) since we use the RU2 UMNL model. As an existing product and its remanufactured version share unobservable utility components and are correlated, the parameters μ_1 and μ_2 which reflect this correlation should be greater than one. Thus, both of them are chosen to be equal and equated to two arbitrarily, i.e. $\mu_1 = \mu_2 = 2$ in order to avoid the effects of these parameters on the results. Since there is only one element in no-purchase nest and the third nest, the scale parameters associated with these nests are equated to one, i.e. $\mu_0 = \mu_3 = 1$.

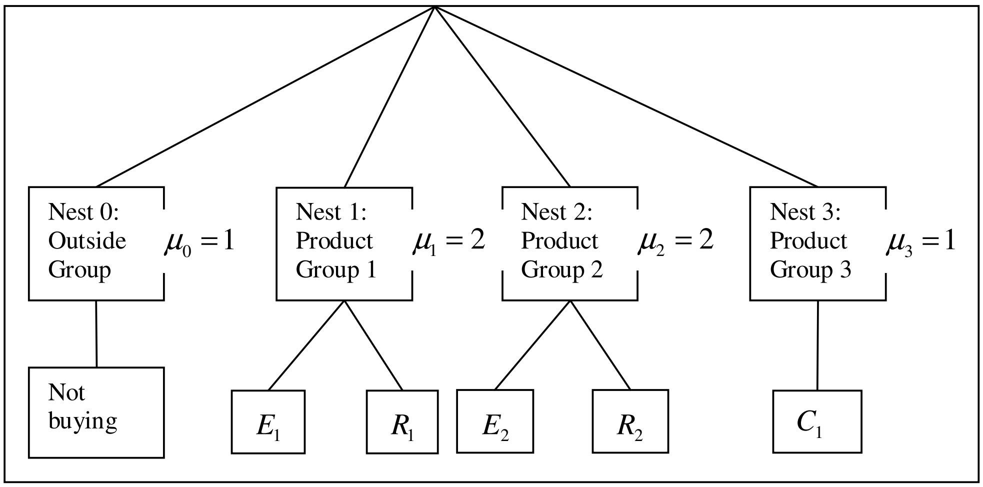

Figure 6.1. The nesting structure of the sample problem

6.1.1. The Effect of Changes in α_p and β_q

The effects of the changes in α_p and β_q on the optimal profit are figured out when all other parameters are kept the same. Since α_p and β_q are the parameters of price and quality respectively in the representative utility component of the random utility function of a product, some arbitrary values are used for the analysis. In the real world applications, the parameter values, α_p and β_q would be expected to take place in an interval that are not making the utility value of the competitor too negative, because if the competitor's utility value becomes too negative, the probability of the competitor's product to be chosen would converge to zero. Since the competitor's price and quality values are assumed to be known and it is taken as a reference point for competitive advantage in the pricing problem, we must consider this situation, but in order to monitor the effect of these parameters on the optimal results, we examined a wider interval that may sometimes make the competitor's utility negative.

The effect of a change in α_p is analyzed first. β_q is kept fixed at seven during the analysis. To have a better understanding, we first make the analysis in the absence of the competitor's product because the behavior of the objective function differs according to the competitor's product. The optimal product line compositions with optimal prices and profits for some different α_p values in the existence of the competitor's product are listed in Table 6.2. The behavior of the optimal profit, for the values of α_p between 0.2 and 2 is illustrated in Figure 6.2 while competitor's product is not taken into account. In Figure 6.3, computational results are provided when the competitor's product is included.

Table 6.2. Optimal compositions, prices, and profits for different α_p values when $\beta_q = 7$

α_p	Optimal Product Line				Optimal Profit
	E_1	E_2	R_1	R_2	
0.4	21.95	26.95	-	-	13342.65
0.6	20.25	25.25	-	-	10742.54
0.8	19.58	24.58	-	-	10001.52
1	19.65	24.65	10.65	-	10640.52
1.2	20.7	25.7	11.7	12.7	13599.54
1.4	19.49	24.49	10.5	-	11044.32
1.6	18.32	23.33	9.32	-	7794.785
1.8	17.43	22.43	8.43	-	5310.36
2	16.74	21.74	7.74	-	3411.18

We observe that in all cases $S_{E_1} < S_{E_2}$ and $S_{R_1} < S_{R_2}$ when all candidate products are included in the optimal product line, which justifies that higher quality items can be sold at higher prices. Also $S_{R_1} < S_{E_1}$ and $S_{R_2} < S_{E_2}$, which leads to the conclusion that remanufactured versions of the products are priced always less than their new versions.

We state in Proposition 3.1 that marginal profits of the products that are in the same nest are positive and equal, i.e. $(S_{1E_1} - c_{1E_1}) = (S_{1R_1} - c_{1R_1})$ and $(S_{2E_2} - c_{2E_2}) = (S_{2R_2} - c_{2R_2})$ should hold at optimality when the optimal product line consists of all products. For example when $\alpha_p = 1.2$, the optimal product line includes all products, and marginal profits for the products in the same nest are equal. Even marginal profits are the same for all products that are included in the optimal product line, i.e. $(S_{1E_1} - c_{1E_1}) = (S_{1R_1} - c_{1R_1}) = 5.7 = (S_{2E_2} - c_{2E_2}) = (S_{2R_2} - c_{2R_2})$.

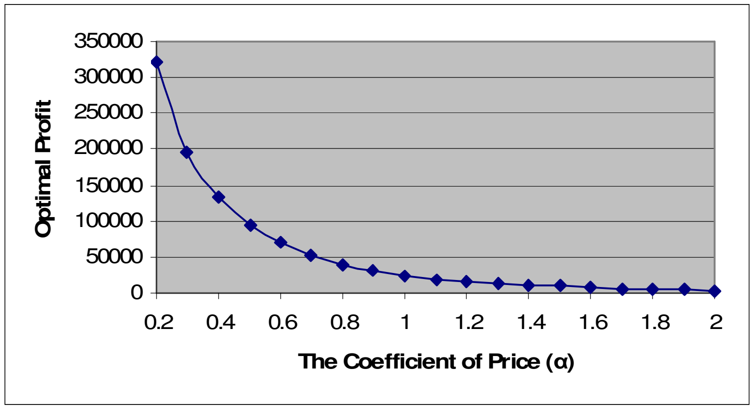

Figure 6.2. The effect of α_p on the optimal profit in the absence of the competitor's product

We see in Figure 6.2 that the optimal profit decreases with α_p as expected since the relative importance of the price increases with α_p, and we cannot charge high prices to the products offered in the product line. Due to the decrease in the selling prices of the products, the optimal profit decreases in α_p and it converges to zero because customers become more sensitive to the price, and prefer purchasing nothing. However, there are some differences in the prices of the products when the competitor's product is taken into consideration. Extensive explanation about the optimal prices and product line compositions will be given for the case with the inclusion of the competitor's product below.

Let us examine the behavior of the optimal profit in Figure 6.3 when the competitor's product is included in the analysis. When small α_p values are being used, the effect of price on the purchasing decisions is relatively low and the prices start with high values and decreases as α_p increases. Until α_p is one, it is optimal not to extend the product line at all, namely the optimal product line consists of the products E_1 and E_2, because the incremental profit of the candidate products, which is defined as the incremental profit due to the inclusion of a product in the product line, cannot compensate their fixed costs. The relatively low utility values of R_1 and R_2 compared to E_1 and E_2 causes this low incremental profit because purchasing probabilities, which directly determine the demand values, stay quite small as a result of low utility values. When α_p reaches one, it becomes profitable to include R_1 in the product line, because its incremental profit overcomes its fixed cost. When α_p is increased from one to 1.1, R_2 also becomes profitable and the optimal product line includes all the products. In the interval between 1.1 and 1.3, the utility values of R_1 and R_2 becomes higher. As a result, their contribution to the optimal profit increases and the fixed costs are compensated whereas the contribution of E_1 and E_2 decreases as a result of higher price effect on the choice decisions. Since the costs and the prices of E_1 and E_2 are relatively higher than those of R_1 and R_2, the customers do not want to buy these products. Instead, they prefer cheaper products, like R_1 and R_2. For the α_p values higher than 1.3, the effect of price on the choice decisions becomes more significant. Therefore, the prices of the products occur at a value closer to their corresponding costs. In these α_p values, the product line is composed of E_1, E_2 and R_1. Since the costs of E_1 and E_2 are quite high, they become unprofitable and their purchase probabilities become quite smaller, but they are kept in the optimal product line composition because they do not have the fixed cost of introduction as they are the existing products. Thus, the great amount of the profit comes from R_1. If α_p is continued to be increased, the profit would converge to zero, because customers would become more price-sensitive and they would prefer buying nothing. Optimal product line compositions are observed to be the same in the absence of the competitor's product.

When the competitor's product is not included in the analysis, the optimal prices and the profit keep their decreasing with α_p as expected. An interesting point to consider is that there is an increase in the optimal prices of the products and the profit for α_p values between 0.9 and 1.2 which is different from the situation in the absence of the competitor's product. This situation can be explained by the existence of an attractive outside alternative, namely the competitor's product. In this case, for some α_p values, the optimal profit can be increased by increasing the price of the products a small amount. The increase in the prices of all products results in demand loss for some products and demand increase for some other products. As a result, the products with relatively higher utilities steal some demand from the competitor's product and other products in the product line. Therefore, the profit increases.

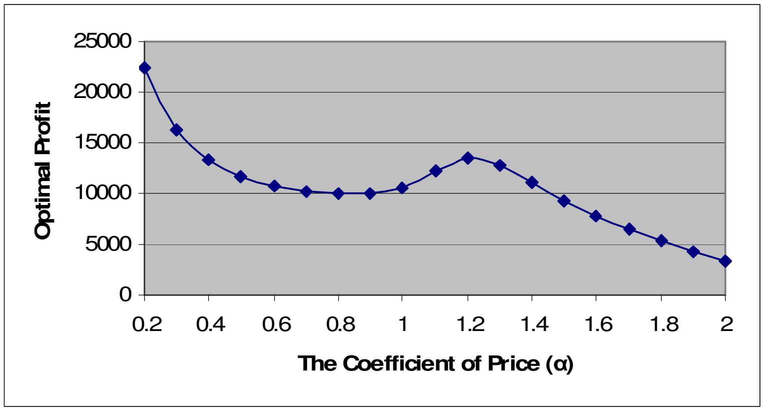

Figure 6.3. The effect of α_p on the optimal profit when the competitor's product is taken into account

To have comparable prices with respect to the competitor, it is expected that if the OEM has a product quality value which is close to competitor's quality, the price of this product should not be very different from the competitor's price. An α_p value between 0.7 and 1.7 seems to be more appropriate. Different problems with different sizes give similar results. Therefore, we focus on the problem with $\alpha_p = 1$ in the rest of the analyses because the optimal prices of existing products E_1 and E_2 are relatively close to the price of the competitor.

The effect of β_q on the optimal profit when holding α_p fixed at one is shown in Figure 6.4 in the absence of the competitor's product and in Figure 6.5 when it is kept in the analysis.

We see in Figure 6.4 that the optimal profit increases with β_q due to increase in importance of the quality relative to the price. Therefore, selling prices of the products increases with β_q. Similar to the case where the effect of α_p analyzed, there are some differences in the prices of the products when the competitor's product is taken into consideration.

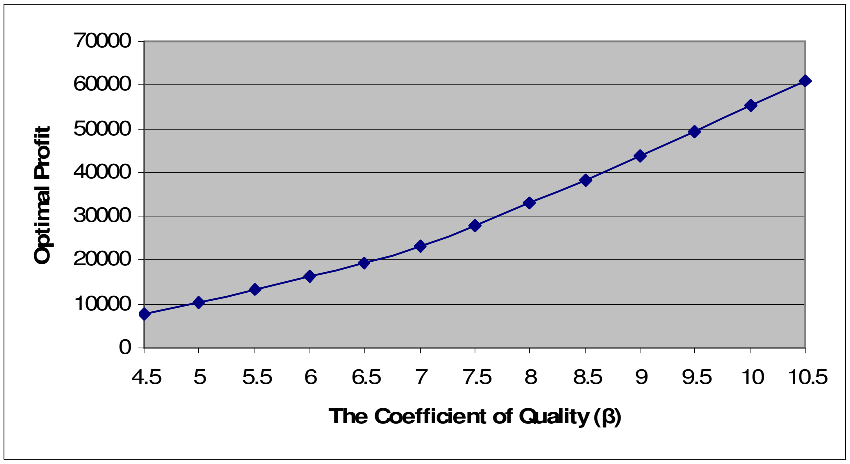

Figure 6.4. The effect of β_q on the optimal profit in the absence of the competitor's product

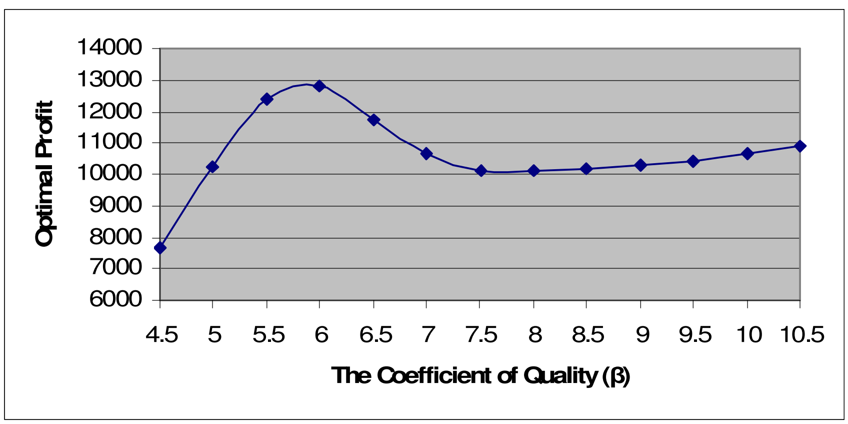

Figure 6.5. The effect of β_q on the optimal profit when the competitor's product is taken into account

In the existence of the competitor's product, the prices of the products in the optimal product line and profit increase with β_q up to six. For the β_q values between six and eight the competitor becomes a more powerful actor and steals some demand from other products in the product line. Thus, to reduce the amount of stolen demand, the prices of products in the optimal product line start to decrease. As a result, profit also decreases.

After the β_q value of eight prices and profit start to increase again and keeps their increasing.

The results show that the values of α_p and β_q have a significant effect on the product line composition and optimal prices. A comparative study with respect to the changes in α_p and β_q for different product types can give useful insights. In the following sections we hold β_q constant at seven and change α_p when necessary to see the general behaviors. Unless otherwise stated, $\alpha_p = 1$.

6.1.2. The Effect of Changes in μ

As was stated before, the lower level scale parameters μ_h can be considered as an indication of the correlation among the alternatives' errors. A higher μ_h value means the products in that nest are more similar. Therefore, the lower the nest-specific scale parameter μ_h, the greater is the variety (the differentiated products) provided in the nest. This is because decreasing μ_h reduces cannibalization since the products in the same nest diverge from each other and does not become close substitutes.

To monitor the effect of μ_h on the optimal product line composition with optimal profit and optimal prices, we discard R_2 from the choice set and consider whether to include R_1 in the product line or not. With $\alpha_p = 0.9$ and $\beta_q = 7$, the change of optimal profit with respect to μ_1 is given in Figure 6.6.

Until μ_1 reaches 1.4, it is optimal to keep R_1 in the product line. For the values greater than 1.4, it becomes unprofitable to include R_1 in the product line because E_1 and R_1 are getting closer substitutes to each other and customers prefer the one with higher utility value from the products that are in the same nest. In this case the product with higher utility is E_1 and the purchase probability related to it increases while the probability of R_1 decreases with increasing μ_1. Therefore, while μ_1 is increasing, the incremental profit of R_1 is reduced due to the decrease in its purchase probability and its fixed cost is not compensated after 1.4.

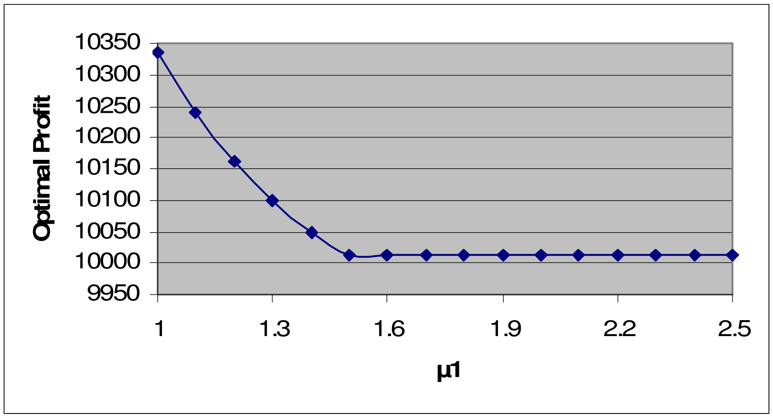

Figure 6.6. The effect of μ_1 on optimal profit

In our model there is a no-purchase alternative and competitor's product all of which can be considered as outside alternatives. Since some customers choose not to purchase one of the products offered by the OEM, two component effects are effective while μ_1 is increasing. For any given set of prices, a lower μ_1 leads to more evenly distributed customers across the products since we offer more differentiated products. Second, demand (the purchase probability) is less sensitive to prices as subjective factors in choice become more important to customer evaluations of a given alternative. Therefore, if the outside alternative is relatively attractive, here it is the competitor's product, both effects work in favor of the differentiated product, and prices and profit rise necessarily. While μ_1 is increasing, the effects act in the opposite way. Namely, customers become more price-sensitive since we offer more similar products and prices and profit decrease. This result is in accordance with the derivative information given in equation (4.21). μ_1 and μ_2 are equated to two, i.e. $\mu_1 = \mu_2 = 2$ in the rest of the analyses as before.

6.1.3. The Effect of Changes in the Fixed Cost

The fixed cost values have no impact on pricing decisions since they act as constants for a given product line composition and only reduce the profit if the candidate product is included in the product line. On the other hand, they have a direct effect on the solution of the product line selection problem.

To see this effect, the incremental profit value of the candidate products will be examined. The incremental profit is the difference between the profits obtained when the pricing problem is solved including and excluding the product in the product line in the absence of the fixed cost. As pointed at before, inclusion of a new product in the product line in the absence of fixed costs increases the profit since its incremental profit is always positive. This incremental profit value can be interpreted as the fixed cost that changes the product inclusion decision, which means the product inclusion is obviously profitable if its fixed cost is less than that calculated incremental profit value.

It should be noted that the incremental profit depends on the current product line. This means that two incremental profit values can be calculated for two different product lines. Therefore, this quantity cannot be used to determine the optimal product line composition. We can only check whether the inclusion of a new product is profitable or not.

For the sample problem with $\alpha_p = 1$ and $\beta_q = 7$, candidate product R_1 is included in the optimal product line, whereas R_2 is not. The profit values of the current optimal product line in which R_2 is included and the best in which R_2 is not included without fixed cost of R_2 are given in Table 6.3.

Table 6.3. Optimal profit values for the product lines with and without R_2 in the absence of R_2's fixed cost

Product line composition	E_1, E_2, R_1 and C_1	$E_1, E_2, R_{1,} R_2$ and C_1
Optimal Profit	10,640.52	11,111.83

The difference between the profits obtained for these two product line compositions, namely the incremental profit of R_2 is 471.31, which gives the fixed cost for which we are indifferent between the two product line compositions. We observe that it is less than the fixed cost of introducing R_2, 700. If the fixed cost value of R_2 was less than 471.31, R_2 would be offered as a remanufactured product in the product line. Figure 6.7 illustrates that if the fixed cost of introducing R_2 is more than this breakeven value, then its inclusion in the product line becomes unprofitable and the optimal profit value stays same. Similar analysis can be done for different product line compositions.

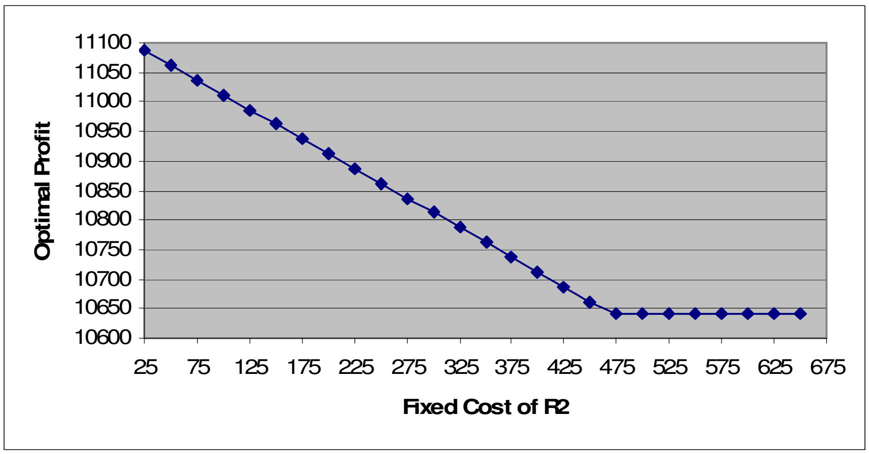

Figure 6.7. Change in the optimal profit for the sample problem when the fixed cost of R_2 increases

6.1.4. The Effect of Changes in the Segment Size

The effect of the segment size should also be analyzed. Since there is one segment in the sample problem, the segment size has no effect on pricing decision and accordingly on the purchase probabilities. If the segment size is increased gradually, the profit and the incremental profit of each product become greater due to the increase in the number of customers that make purchasing decisions. Thus, a product that is not included in the optimal product line for smaller segment sizes will be included in the product line as the segment size increases, because the incremental profit of the product will ultimately compensate its fixed cost.

To show the effect of increase in the segment size, the optimal product line compositions and their related optimal profit values for the sample problem with $\alpha_p = 1$ and $\beta_q = 7$ are given in Table 6.4. In the second column of the table, “1” implies that a product is in the product line whereas "0" implies it is not. When the segment size is smaller than 4500, it is observed that R_2 is not in the optimal product line composition. However, from this point on, R_2 is also included in the product line and producing it becomes profitable.

Table 6.4. Optimal product line compositions and optimal profits for different segment sizes

Segment Size	**Optimal Product Line** $(E_1, E_2, R_1, R_2, C_1)$	**Optimal Profit**
3000	(1,1,1,0,1)	10640,52
3400	(1,1,1,0,1)	12099,25
3800	(1,1,1,0,1)	13557,99
4200	(1,1,1,0,1)	15016,72
4500	(1,1,1,1,1)	16117,75
4600	(1,1,1,1,1)	16498,14
5000	(1,1,1,1,1)	18019,72
5400	(1,1,1,1,1)	19541,3

6.2. Computational Results for the Extended Model : Positioning the Remanufactured Product

After determining the candidate products that should be included in the product line, we continue with the investigation of the optimal quality level of the remanufactured product. We assume that the cost of quality is convex and the unit production cost of a remanufactured product is stated as a function of its quality level, that is $c_{hj}\left(Q_{hj}^i\right) = K_{hj}^i\left(Q_{hj}^i\right)^2 \; i \in M, j \in H_h \cap N_R, h \in H$. The superscripts related to the segments can be ignored since we are working with only on one segment.

In this section, we investigate the effects of model parameters on the remanufactured product's optimal position for the sample problem introduced in the previous section. Recall that, for the sample problem with $\alpha_p = 1$ and $\beta_q = 7$, candidate product R_1 is included in the optimal product line, whereas R_2 is not. Before making the analysis with the cost function, we show that the optimal profit decreases as the unit production cost of the remanufactured product increases. On the other hand, the optimal profit increases with the quality level of the remanufactured product. Figure 6.8 shows the behavior of the optimal profit with respect to the unit production cost of R_1, i.e. c_{R_1}. As c_{R_1} increases, the optimal profit decreases up to a level and stays the same from that point on because the production cost increase of R_1 while keeping the quality same, decreases the choice probability of R_1 and its incremental profit converges to zero after a threshold c_{R_1} value above which producing R_1 becomes unprofitable

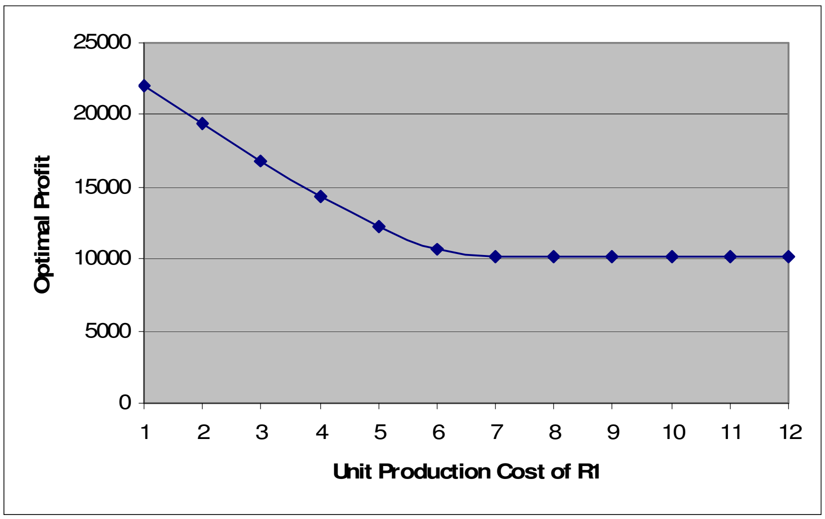

Figure 6.8. Change in the optimal profit for the sample problem with the increase in the unit production cost of $R_1(c_{R_1})$

Figure 6.9. Change in the optimal profit for the sample problem with the increase in the unit production quality of $R_1(Q_{R_1})$

In Figure 6.9, we observe the effect of the increase in the quality level of R_1 on the optimal profit. Up to a quality level of R_1 that makes producing R_1 profitable, the optimal profit keeps its value unchanged because relatively low quality values with respect to the constant production cost value makes R_1 unprofitable. After this level, the optimal profit increases as a result of high quality values at a relatively low cost value.

If we turn our attention back to find out the optimal quality-cost pair of R_1 with the usage of the cost function that gives the maximum profit, both of the quality and cost values increase in this case. Thus, we can suppose that the consequences of this scenario would result due to the combined effects observed in Figure 6.8 and 6.9. Figure 6.10 exhibits the change in the optimal profit for $c_{R_1} \in [1,25]$ with $K_{R_1} = 1.1$.

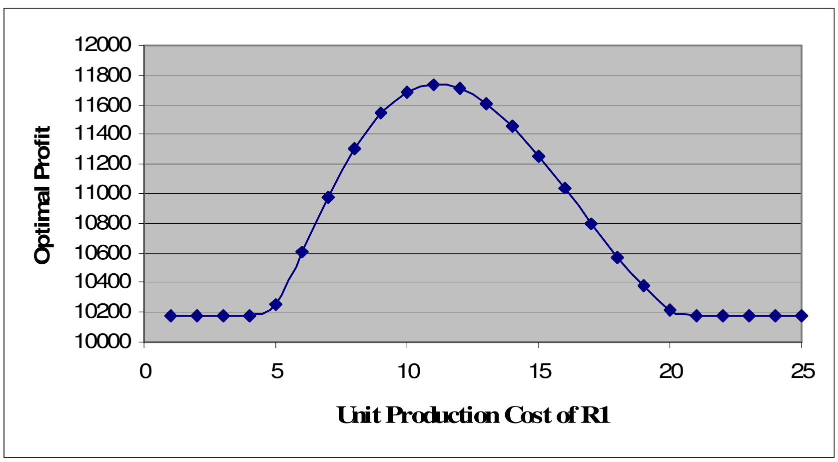

Figure 6.10. Optimal positioning of R_1 for the sample problem

We observe that the optimal profit remains the same until $c_{R_1} = 5$ and producing R_1 is unprofitable because low quality levels, and accordingly low probability values keep its incremental profit at low values, and as a result its fixed cost cannot be compensated. From this point on, the optimal profit increases with the unit production cost until it reaches a maximum value after which it starts to decrease. Restoring a used product to a higher quality level increases its purchase probability which directly lead to an increased optimal profit. Until the unit cost reaches a threshold value, this effect is more effective, i.e. dominant. Since an increase in the unit cost reduces the additional profit of the remanufactured product, for unit cost values greater than this threshold value, the effect of the increased unit cost dominates the effect of increased quality level and the optimal profit starts to decrease. The mentioned threshold value of unit cost is the optimal unit production cost that gives the maximum optimal profit. For the sample problem, the maximum profit is obtained at the unit production cost, 11.136 of R_1. For the unit cost values greater than 20 the optimal profit remains the same because the incremental profit of R_1 does not compensate its fixed cost anymore, thus it is not profitable to manufacture R_1 anymore. Since fixed cost affects only the optimal product line composition, we may examine the optimal profit curves for each product without fixed costs which are shown in Figure 6.11 to indicate the reason of the main behavior of the optimal profit curve presented in Figure 6.10.

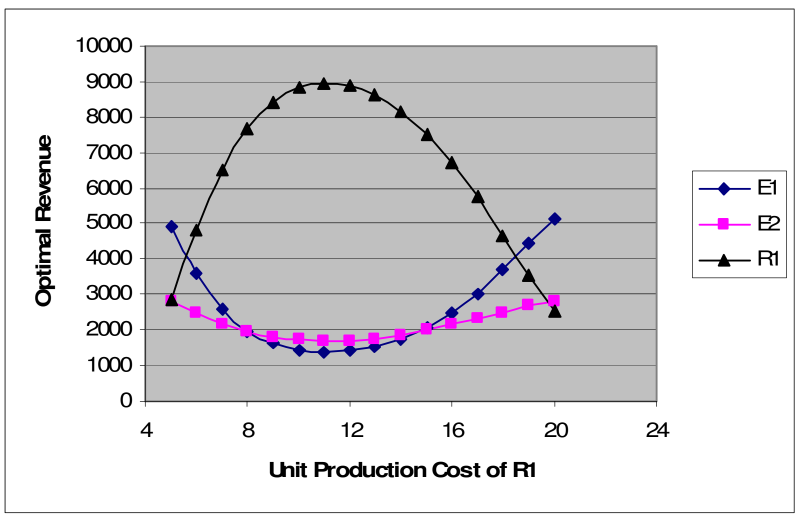

Figure 6.11. Optimal profit of the products without fixed cost of R_1 as a function of c_{R_1}

We observe that optimal profit values from E_1 and E_2 are decreasing until a certain unit cost value of R_1, which is the optimal unit cost (quality level) of R_1 with maximum optimal profit, and then starts increasing whereas the optimal profit of R_1 exhibits the opposite behavior, namely it increases first but start decreasing gradually. We can say that the increase in the quality level of the remanufactured product up to a threshold value increases its purchase probability and its profit. An increase in the probability of one alternative necessitates decreasing the probability of others. Thus, the purchase probability of E_1 and E_2 decreases up to this point, and as a result their profits decrease. For the values greater than this threshold value the probability and profit of R_1 start decreasing whereas the others' increase. Another interesting result to consider is that the profit of E_1 is affected more from the increases and decreases in the profit of R_1. We observe sharper increases or decreases in the profits of E_1 and R_1 more than in E_2's. This can be explained by the nesting structure of the similar alternatives. Since E_1 and R_1 may be accepted as similar products, an increase or a decrease in the purchase probability (or as a result its profit) of one directly and mostly affects the other's. Consequently, we can attribute the behavior of the optimal profit curve to the optimal profit curve of R_1.

6.2.1. The Effect of Changes in K

The previous problems are solved for $K_{R_1} = 1.1$. We examine the effect of coefficient K_{R_1} used in the cost function of remanufactured product R_1 on the optimal profit in this subsection. Optimal profit curves for different K_{R_1} values are presented in Figure 6.12.

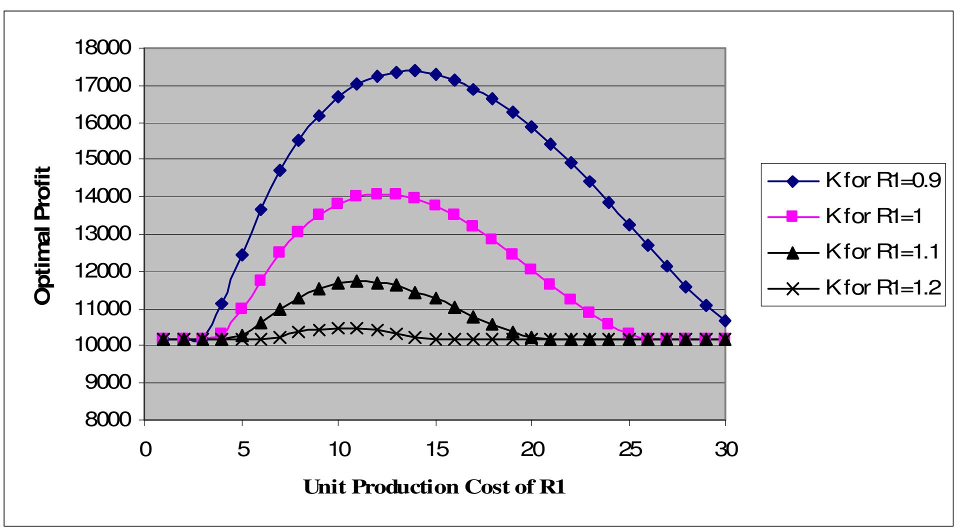

Figure 6.12. The effect of K_{R_1} on the optimal profit while positioning R_1

We observe that as K_{R_1} increases, the optimal unit cost corresponding to the maximum profit appears to be lower. This result is intuitive since higher K_{R_1} necessitates more unit production cost of R_1 to restore the same quality level. Furthermore, the unit cost value after which producing R_1 is not profitable appears at lower unit cost values with higher K_{R_1} values. For higher K_{R_1} values increasing the quality level the same amount becomes more costly and as a result, the increase in the purchase probability of R_1 corresponding to the quality level increase is affected less and it does not compensate the decrease on the incremental revenue of the remanufactured product due to the increase in the unit cost value of it. Therefore, the fixed cost cannot be compensated, and the unit cost that it is not optimal to include R_1 in the product line becomes smaller.

6.2.2. The Effect of Changes in the Fixed Cost

The effect of fixed cost of introducing R_1 on the optimal profit is presented in Figure 6.13 for the sample problem. As the fixed cost increases, the value of unit cost at which producing R_1 is not profitable anymore becomes smaller. However, the optimal unit cost of R_1 does not change because increasing fixed cost only shifts the original optimal profit curve downwards as much as the total increase in the fixed cost of R_1.

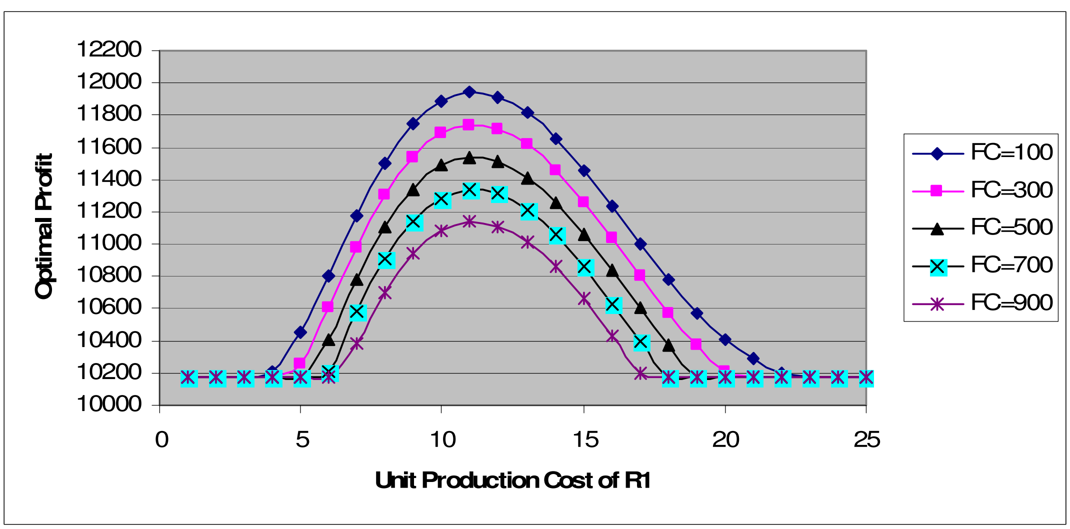

Figure 6.13. The effect of fixed cost of R_1 on the optimal profit while positioning R_1

6.2.3. The Effect of Changes in the Selling Price and Quality of Competitor's Product

Two other factors that may have an effect on the optimal unit production cost of the remanufactured product are the selling price and quality of competitor's product. The effect of these factors on the optimal profit is presented in Figure 6.14 and Figure 6.15 for the sample problem. The optimal profit increases with the price of the competitor's product. This is intuitive since the competitor's price is a reference point for determining the prices of other products in the product line offered by the OEM and increasing it will allow us assigning higher prices for these products, which accordingly increases the profit. However, the optimal unit production cost of the remanufactured product does not change with the price of the competitor's product. This result is in accordance with derivative information in equation (4.44) since the optimal unit production cost of the remanufactured product does not depend on the price of the competitor's product. Moreover, the effect of fixed cost is observed earlier for lower prices of the competitor's product, i.e. the unit cost value from which on the remanufactured product cannot compensate its fixed cost is smaller when the selling price of the competitor's product is lower.

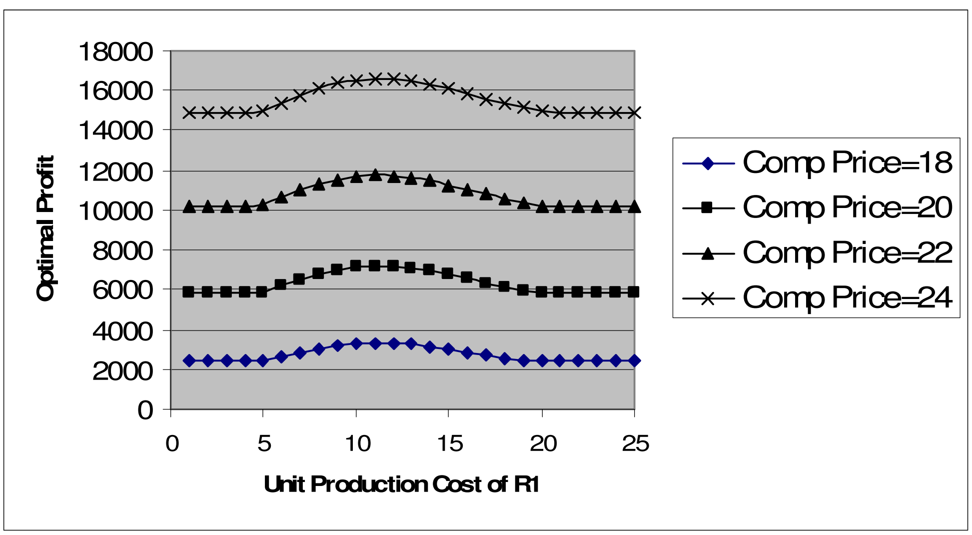

Figure 6.14. The effect of the price of the competitor's product on the optimal profit while positioning R_1

Similar interpretation can be made about the effect of quality of the competitor's product. The optimal profit decreases with the quality of the competitor's product. This increase in the quality of competitor's product makes it more attractive and the purchase probabilities of the products by the OEM decrease, which accordingly decreases the profit. Again, the optimal unit production cost of the remanufactured product does not change with the price of the competitor's product since the derivative in equation (4.44) does not depend on the quality of the competitor's product. The effect of fixed cost is observed earlier for higher quality values of the competitor's product.

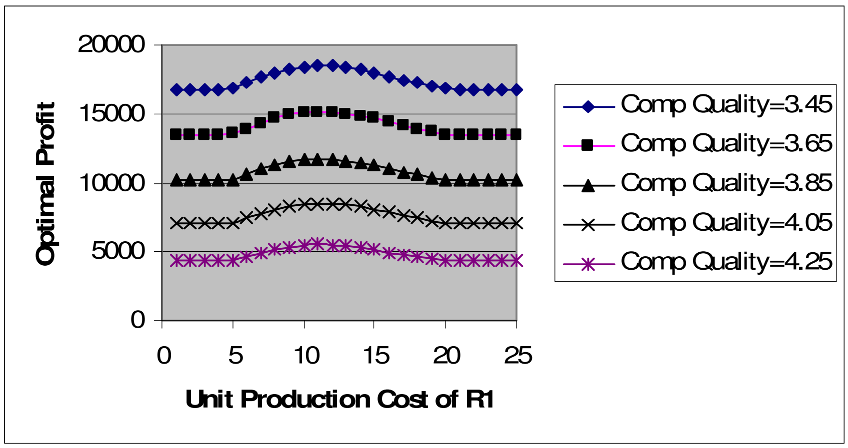

Figure 6.15. The effect of the quality of the competitor's product on the optimal profit while positioning R_1

7. CONCLUSIONS

In this thesis, we considered the problem of extending a product line with remanufactured products of existing ones, and jointly pricing of existing and remanufactured products from the perspective of an OEM. They are differentiated by their unit production costs and quality levels. We assume that the market consists of a number of homogenous customer segments in each of which the quality perception of a particular product offered in the market is the same for the customers in the segment. Customer choice is modeled using the NL model to reflect the effects of substitutability of existing and remanufactured products to the analyses and customers purchases at most one of the products of just one unit. Quality level and price of a product are the determinants that are taken into consideration by customers to make their purchase decisions. The objective is to determine the remanufactured products to be offered in the product line as well as the selling prices of existing and offered remanufactured products so as to maximize the OEM's profit in the presence of fixed costs and variable production costs. Moreover, there is a competitor and customers prefer a product from the products offered in the market as well as they have the option of nothing to purchase. An extension of the base model is also proposed in which the optimal quality level of the offered remanufactured products to be restored is of concern.

We decompose the problems into two sub-problems as product line selection and pricing. The product line selection problem is solved by complete enumeration and the pricing problem is solved using a restarted and modified simplex search method.

Computational results are obtained for sample problems, and the effects of fixed cost, segment size, and some model parameters are observed. Computational results revealed that it is optimal to include all candidate remanufactured products in the product line in the absence of fixed costs. Nevertheless, when there is fixed cost of introducing a remanufactured product, it is not offered in the product line if its incremental profit cannot compensate its fixed cost. The effect of fixed cost on the optimal product line composition diminishes by the increase in segment size which increases the number of offered remanufactured products. The first order optimality conditions for the base model show that the marginal profit is the same for all products that are in the same nest and it is positive.

Under the assumption that the unit production cost of a remanufactured product is stated as a quadratic function of its quality level, we observed that the optimal unit production cost (accordingly quality level) of a remanufactured product is not affected by the price or quality of the competitor’s product.

A future research direction could be the inclusion of availability restriction of the returned products in the model. We proposed a single-period model. However, the amount of available used products depends on the sales of remanufacturable products in the previous periods. Therefore, a multi-period model including the uncertainty of returned products can be studied as another future work.

REFERENCES

Anderson, S. P., A. de Palma and J.-F. Thisse, 1992, *Discrete Choice Theory of Product Differentiation*, the MIT Press, Cambridge, Massachusetts.

Aydin, G. and J. K. Ryan, 2000, "Product Line Selection and Pricing Under the Multinomial Logit Choice Model", *Working Paper*, Stanford University, Stanford, CA.

Balachander, S. and K. Srinivasan, 1994, "Selection of Product Line Qualities and Prices to Signal Competitive Advantage", *Management Science*, Vol. 40, No. 7, pp. 824-841.

Bazaraa, M. S., D. S. Hanif and C. M. Shetty, 1993, *Nonlinear Programming: Theory and Algorithms*, John Wiley and Sons Inc., USA.

Ben-Akiva, M. E., 1973, *Structure of Passenger Travel Demand Models*, Ph.D. Thesis, Massachusetts Institute of Technology.

Ben-Akiva, M. E. and M. Bierlaire, 1999, "Discrete Choice Methods and Their Applications in Short Term Travel Decisions", in R. Hall (ed.), *The Handbook of Transportation Science*, pp. 5-33, Kluwer Academic Publishers, Dordrecht, The Netherlands.

Ben-Akiva, M. E. and B. François, 1983, "μ Homogenous Generalized Extreme Value Model", *Working Paper*, Department of Civil Engineering, MIT, Cambridge, Ma.

Ben-Akiva, M. E. and S. R. Lerman, 1985, *Discrete Choice Analysis: Theory and Application to Travel Demand,* the MIT Press, Cambridge, Massachusetts.

Bierlaire, M., *Discrete Choice Models*, 2004, http://roso.epfl.ch/mbi/papers/discretechoice/paper.html.

Chen, K. D. and W. H. Hausman, 2000, “Technical Note: Mathematical Properties of the Optimal Product Line Selection Problem Using Choice-Based Conjoint Analysis”, *Management Science*, Vol. 46, No. 2, pp. 327-332.

Choudhary, V., A. Ghose, T. Mukhopadhyay and U. Rajan, 2005, “Personalized Pricing and Quality Differentiation”, *Management Science*, Vol. 51, No. 7, pp. 1120-1130.

Daly A., 1987, “Estimating ‘Tree’ Logit Models”, *Transportation Research B*, Vol. 21, No. 4, pp. 251-267.

Debo, L., B. Toktay and L. N. van Wassenhove, 2005, “Market Segmentation and Production Technology Selection for Remanufacturable Products”, *Management Science*, Vol. 51, No. 8, pp. 1193-1205.

Debreu, G., 1960, “Review of R.D. Luce Individual Choice Behavior”, *American Economic Review*, Vol. 50, pp. 186-188.

Desai, P., S. Kekre, S. Radhakrishnan and K. Srinivasan, 2001, “Product Differentiation and Commonality in Design: Balancing Revenue and Cost Drivers”, *Management Science*, Vol. 47, No. 1, pp. 37-51.

Dobson, G. and S. Kalish, 1988, “Positioning and Pricing a Product Line”, *Marketing Science*, Vol. 7, No. 2, pp. 107-125.

Dobson, G. and S. Kalish, 1993, “Heuristics for Pricing and Positioning a Product Line Using Conjoint and Cost Data”, *Management Science*, Vol. 39, No. 2, pp. 160-175.

Dowlatshahi, S., 2000, “Developing a Theory of Reverse Logistics”, *Interfaces*, Vol. 30, No. 3, pp. 143-155.

Esenduran, G., 2004, *Product Line Selection and Pricing with Availability Restrictions on Remanufacturing*, M.S. Thesis, Boğaziçi University.

Ferguson, M. and B. Toktay, 2006, "The Effect of Competition on Recovery Strategies", *Production and Operations Management*, Vol. 15, No. 3, pp. 351-368.

Ferrer, G. and J. Swaminathan, 2006, "Managing New and Remanufactured Products", *Management Science*, Vol. 52, No. 1, pp. 15-26.

Fleischmann, M., J. M. Bloemhof-Ruwaard, R. Dekker, E. Lean, J. A. E. E. Nunen and L. N. V. Wassenhove, 1997, "Quantitative Models for Reverse Logistics: A Review", *European Journal of Operations Research*, Vol. 103, pp. 1-17.

Green, P. E. and A. M. Kreiger, 1985, "Models and Heuristics for Product Line Selection", *Marketing Science*, Vol. 4, No. 1, pp. 1-19.

Green, P. E. and V. Srinivasan, 1978, "Conjoint Analysis in Consumer Research: Issues and Outlook", *Journal of Consumer Research*, Vol. 5, No. 2, pp. 103-123.

Guide Jr, V. D. R., H. Teunter, L. N. van Wassenhove, 2003, "Matching Demand and Supply to Maximize Profits from Remanufacturing", *Manufacturing and Service Operations Management*, Vol. 5, No. 4, pp. 303-316.

Hanson, W. and K. Martin, 1996, "Optimizing Multinomial Logit Profit Functions", *Management Science*, Vol. 42, No. 7, pp. 992-1003.

Heiss, F., 2002, "Structurel Choice Analysis with Nested Logit Models", *The Stata Journal*, Vol. 2, No. 3, pp. 227-252.

Hooke, R. and T. A. Jeeves, 1961, "Direct Search Solution of Numerical and Statistical Problems", *Journal of the ACM*, Vol. 8, No. 2, pp. 212-229.

Hopp, W. and X. Xu, 2005, "Product Line Selection and Pricing with Modularity in Design", *Manufacturing and Service Operations Management*, Vol. 7, No. 3, pp. 172-187.

Humphrey, D. G. and J. R. Wilson, 2000, "A Revised Simplex Search Procedure for Stochastic Simulation Response Surface Optimization", *INFORMS Journal of Computing*, Vol. 12, No. 4, pp. 272-283.

Kim, K. and D. Chhajed, 2002, "Product Design with Multiple Quality-Type Attributes", *Management Science*, Vol. 48, No. 11, pp. 1502-1511.

Koppelman F. S. and C. H. Wen, 1998, "Nested Logit Models: Which Are You Using?", *Transportation Research Record*, Vol. 1645, No. 1. pp. 1-7.

Kraus, U. G. and C. A. Yano, 2003, "Product Line Selection and Pricing Under a Share-of-Surplus Choice Model", *European Journal of Operations Research*, Vol. 150, pp. 653-671.

Mahajan, S. and G. J. van Ryzin, 2003, "Retail Inventories and Consumer Choice", in S. Tayur, R. Ganeshan and M. Magazine (eds.), *Quantitative Models for Supply Chain Management*, pp. 491-553, Kluwer Academic Publishers, Massachusetts.

Majumder, P. and H. Groenevelt, 2001, "Competition in Remanufacturing", *Production and Operations Management*, Vol. 10, No. 2, pp. 125-141.

Manski, C. F., 1977, "The Structure of Random Utility Models", *Theory and Decision*, Vol. 8, No. 3, pp. 229-254.

Maslennikova, I. and D. Foley, 2000, "Xerox's Approach to Sustainability", *Interfaces*, Vol. 30, No. 3, pp. 226-233.

McBride, R. D. and F. S. Zufryden, 1988, "An Integer Programming Approach to the Optimal Product Line Selection Problem", *Marketing Science*, Vol. 7, No. 2, pp. 126-140.

McFadden, D., 1974, “Conditional Logit Analysis of Qualitative Choice Behavior”, in P. Zarembka (ed.), *Frontiers in Econometrics*, pp. 105-142, Academic Press, New York.

McFadden, D., 1978, “Modeling the Choice of Residential Location”, in A. Karlqvist, L. Lundqvist, F. Snickars and J. W. Weibull (eds.), *Spatial Interaction Theory and Planning Models*, pp. 75-96, North-Holland, Amsterdam.

McFadden, D., 1981, “Econometric Models of Probabilistic Choice”, in C. F. Manski and D. McFadden (eds.), *Structural Analysis of Discrete Data with Econometric Applications*, pp. 198-272, the MIT Press, Cambridge.

Mitra, S., 2007, “Revenue Management for Remanufactured Products”, *Omega*, Vol. 35, pp. 553-562.

Moorthy, K. S., 1984, “Market Segmentation, Self-Selection, and Product Line Design”, *Marketing Science*, Vol. 3, No. 4, pp. 288-305.

Moorthy, K. S., 1988, “Product and Price Competition in a Duopoly”, *Marketing Science*, Vol. 7, No. 2, pp. 141-168.

Mussa, M. and S. Rosen, 1978, “Monopoly and Product Quality”, *Journal of Economic Theory*, Vol. 18, No. 2, pp. 301-317.

Nelder, J. A. and R. Mead, 1965, “A Simple Method for Function Minimization”, *The Computer Journal*, Vol. 7, No. 4, pp. 308-313.

Pokharel, S. and A. Mutha, 2009, “Perspectives in Reverse Logistics: A Review”, *Resources, Conservation and Recycling*, Vol. 53, pp. 175-182.

Raman, N. and D. Chhajed, 1995, “Simultaneous Determination of Product Attributes and Prices, and Production Processes in the Product Line Design”, *Journal of Operational Management*, Vol. 12, pp. 187-204.

Rosenbrock, H. H., 1960, “An Automatic Method for Finding the Greatest or Least Value of a Function”, *The Computer Journal*, Vol. 3, No. 3, pp. 175-184.

Silberhorn, N., Y. Boztuğ and L. Hildebrandt, 2008, “Estimation with the Nested Logit Model: Specifications and Software Particularities”, *OR Spectrum*, Vol. 30, No. 4, pp. 635-653.

Spendley, W., G. R. Hext and F. R. Himsworth, 1962, “Sequential Application of Simplex Designs in Optimization and Evolutionary Operation”, *Technometrics*, Vol. 4, pp. 441-461.

Thierry, M., M. Salomon, J. van Nunen and L. N. van Wassenhove, 1995, “Strategic Issues in Product Recovery Management”, *California Management Review*, Vol. 37, No. 2, pp. 114-135.

Train, K. E., 2003, 1992, *Discrete Choice Methods with Simulation*, Cambridge University Press, Cambridge, United Kingdom.

Tversky, A. , 1972, “Elimination by Aspects: A Theory of Choice”, *Psychological Review*, Vol. 79, No. 4, pp. 281-299.

Vorasayan, J. and S. Ryan, 2006, “Optimal Price and Quantity of Refurbished Products”, *Production and Operations Management*, Vol. 15, No. 3, pp. 369-383.

Yano, C. and G. Dobson, 1998, “Profit Optimizing Product Line Design, Selection and Pricing with Manufacturing Cost Considerations: A Survey”, in T. Ho and C. Tang (eds.), *Product Variety Management, Research Advances*, pp. 146-173, Kluwer International Series in Operations Research and Management Science.

Zhao, Q. Z., D. Urosević, N. Mladenović and P. Hansen, 2009, “A Restarted and Modified Simplex Search for Unconstrained Optimization”, *Computers and Operations Research*, Vol. 36, pp. 3263-3271.