A TWO-STAGE JOINT INVENTORY AND PRICING GAME WITH CAPACITY COMMITMENT

by

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In loving memory of my father,

İsmail Bayramoğlu (1951-2015)

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ABSTRACT

A TWO-STAGE JOINT INVENTORY AND PRICING GAME WITH CAPACITY COMMITMENT

In this thesis a two-stage capacity, inventory, and pricing game is studied. Two competing vendors make a capacity commitment to invest in the first stage. This capacity limits how much they can produce in order to meet random demand in the second stage, where they decide on quantities and prices simultaneously. The equilibria are analyzed for both stages of the game. The interaction between first stage capacity decisions and second stage joint quantity-pricing decisions is explored. The recommended pricing policies are determined for both players. A computational method is pursued in the numerical analysis of how the first stage capacity decisions impact the second stage equilibria.

ÖZET

KAPASİTE TAAHHÜTLÜ İKİ SEVİYELİ ENVANTER VE FİYATLANDIRMA OYUNU

Bu tezde iki seviyeli bir kapasite, üretim ve fiyatlandırma oyunu incelenmektedir. İki satıcı birinci dönemde aynı anda belirli bir kapasite yatırımı yaparlar. Bu kapasite ikinci dönemde karşılaşacakları rastgele talebi karşılamak için üretebilecekleri miktarı kısıtlar. İkinci dönemde ise satıcılar talebi karşılarken fiyatlandırma ve üretim kararı vermektedirler. Bu oyunun iki dönem için dengeleri incelenmektedir. Birinci dönem kapasite kararlarının ikinci dönem üretim ve fiyatlandırma kararları ile etkileşimi incelenmekte ve oyunculara en uygun fiyatlandırma politikaları önerilmektedir. Birinci dönemde verilen kapasite kararlarının ikinci dönem denge noktaları üzerindeki etkileri hesaplamalı olarak da incelenmektedir.

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LIST OF SYMBOLS

a	Intercept of demand function where prices are zero
b	Price elasticity of demand with respect to p_i
c	Price elasticity of demand with respect to p_j
C	Exponential demand function parameter
D_i	Demand function vendor i
K_i	Capacity investment cost parameter of vendor i
l_i	Minimum support parameter of ϵ_i
L_i	Deterministic demand function of vendor i
p_i	Price variable of vendor i
q_i	Production quantity variable of vendor i
Q_i	Capacity variable of vendor i
u_i	Maximum support parameter of ϵ_i
w_i	Production cost parameter of vendor i
y_i	Safety stock variable of vendor i
ϵ_i	Uniform distribution random variable of vendor i
λ_i	Lagrange multiplier of vendor i
Λ_i	Lagrangian of vendor i
π_i^k	Profit function of vendor i in stage k

LIST OF ACRONYMS/ABBREVIATIONS

2D	Two dimensional
3D	Three dimensional
KKT	Karush-Kuhn-Tucker
NE	Nash Equilibrium

1. INTRODUCTION

Inventory and price competition has been a popular subject of both economics and operations research. The subject draws attention due to the fact that it describes a vendor's decision making strategy in handling a common problem that has applications in the practical world, integrating operations and marketing.

Models where a capacity commitment is made followed by pricing appears in agriculture, Internet service providing, and energy networks. In all of these sectors a significant irreversible investment is necessary before engaging in price competition.

A variety of models have been studied since Edgeworth (1925) introduced the capacity constraint into Bertrand's (1883) model of price competition. While the model describes a simple game setting, it has been extended to many different models which describe more complex variants.

Kreps and Scheinkman (1983) carried Edgeworth's (1925) duopoly into a two-stage game, where in the first stage the firms install capacities, then observe a deterministic demand before choosing prices in the second stage. They show that the full game has a unique equilibrium, equivalent to the Cournot outcome. Davidson and Deneckere (1986) later argued that Kreps and Scheinkman's (1983) result depends on their selected rationing rule, and that the result is generally not true.

Reynolds and Wilson (2000) have extended Kreps and Scheinkman's (1983) model of capacity-price game with uncertain demand and analyzed the effect of demand variation. Reynolds and Wilson's (2000) main finding is that beyond a certain level of demand variation, a symmetric pure strategy NE for capacities does not exist. This model with demand uncertainty is further analyzed by Lepore (2012). He has generalized the results where a sub-game perfect equilibrium exists that corresponds to the Cournot outcome. Frutos and Fabra (2011) have also investigated the role of demand uncertainty, but in a fixed sized market. They also analyze the effect of the timing of demand uncertainty, comparing the cases there demand is realized before and after pricing decisions are made. They have concluded that the timing of demand certainty does not alter the nature of equilibrium capacity decisions.

The papers mentioned up until now are various models of the Edgeworth-Bertrand model, which describe a two-stage capacity and pricing game between two vendors. We adopt their perspective of long-run capacity and short-run pricing decision to form our model. What we do not adopt from these models is that they consider demand functions that take the total market demand and distribute it between competitors. Our demand model does not assume a fixed total market size.

In the classical newsvendor problem, price is only a parameter. Whitin (1955) has made a major contribution to the model by including price as a decision variable. He uses a price-dependent demand model and a sequential method to find the optimal price by expressing inventory in terms of price. Petruzzi and Dada (1999) also analyzed a model where inventory and price are set simultaneously in a single stage game for a single vendor. They used two different demand models - additive and multiplicative. They argue that while the additive demand model has a constant variance with respect to the price, the multiplicative demand has a constant coefficient of variation. We adopt their additive model $D(p, \epsilon) = y(p) + \epsilon$ and extend it to a competitive case with two vendors.

Bernstein and Federgruen (2005) and Zhao and Atkins (2008) both consider a single stage competition over price and inventory between N vendors. While Bernstein and Federgruen (2005) use a multiplicative stochastic demand function, Zhao and Atkins (2008) use an additive demand model. Zhao and Atkins (2008) have described a method to show quasi-concavity to prove the existence of NE, and developed sufficient conditions for its uniqueness. Their additive demand model is the competitive version of Petruzzi and Dada's (1999). They assume demand substitution between vendors, however we ignore demand substitution, despite homogeneous products, for the sake of simplicity.

Petruzzi and Dada (1999), Zhao and Atkins (2008), and Bernstein and Federgruen (2005) have studied the single stage game of production and pricing. This helps us characterize our second stage game only. As for the two-stage game, Van Mieghem and Dada (1999) consider such a model in capacity, inventory, and pricing decisions. They analyze how the timing of these decisions influence the investment decision and the firm value under demand uncertainty.

Our work is positioned as a combination of the literature expressed so far. The purpose of this work is to analyze the case of two competing firms over two consecutive periods, where a capacity commitment is made and followed by pricing and inventory decisions, before demand is realized. The market size is price-dependent, and the demand function is additive, having both a deterministic and a stochastic component. We use two different deterministic demand structures - linear and exponential, both adopted from Petruzzi and Dada (1999). Our second stage problem is a subset of Zhao and Atkins' (2008) model, while the first stage problem is similar to the Edgeworth-Bertrand model.

Our motivation in selection of this subject is to analyze the impact of a capacity commitment on the second stage equilibrium, and whether a sub-game perfect NE exists under a capacity constraint. We are also looking for a unique equilibrium for the full game with capacity decisions. In this context we investigate how a small firm with limited capacity would act against a larger firm with higher capacity. Moreover we analyze how a change in the capacity installment cost per unit of production affects the equilibrium. We also formulate the Stackelberg game for the first stage and compare the best responses of the leader and the follower. For these purposes, we first characterize the second stage equilibria and then carry this information onto the first stage. We begin by approaching the problem analytically and later carry on with computational analysis to display numerical and graphical results.

The rest of the thesis is organized as follows. Chapter 2 describes the model for each stage and formulates the equilibrium, Chapter 3 analyzes several computational cases and the properties of the equilibrium, and Chapter 4 concludes the results.

2. MODEL

We consider two vendors $i = 1, 2$ in the same market, selling similar products. Customers compare prices before purchasing. In case of a stock-out, customers decide not to buy, i.e. they do not try another vendor.

The vendors invest in capacities, Q_i, and decide on retail prices, p_i, and production quantities, q_i. The game is realized in the course of two stages. In the first stage the vendors simultaneously choose production capacities, Q_i, investing at a cost of K_i per unit of capacity. Afterwards the capacity decisions are made available to both vendors. In the second stage the companies engage in price, p_i, and quantity, q_i, competition, while constrained by the capacities they chose in the first stage. They endure a production cost, w_i, per unit of product. All decisions are made before the demand is observed.

The two-stage model is represented in Figure 2.1.

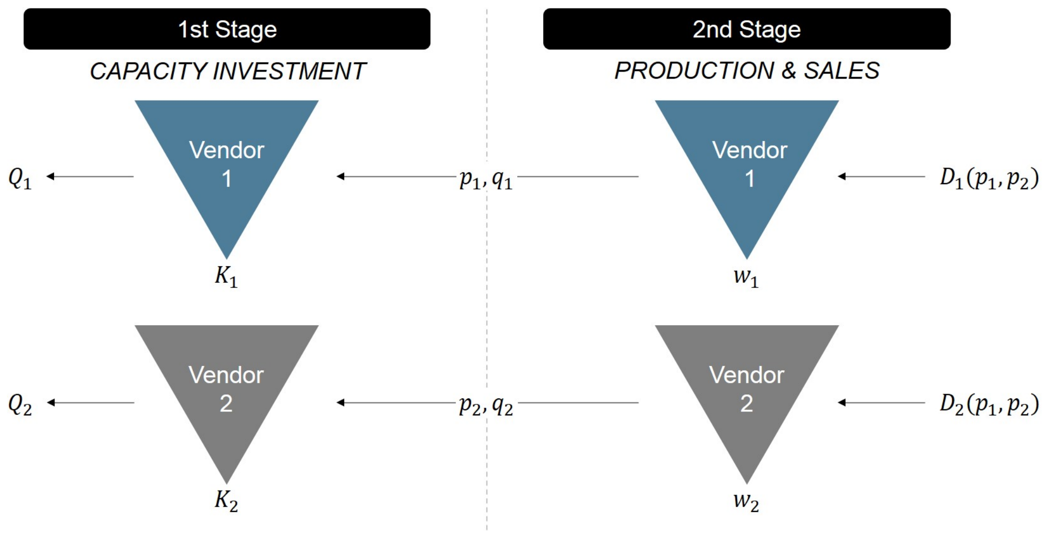


Figure 2.1. The model in two stages.

We first characterize the second stage problem under given capacities, Q_i, and then carry this information onto the first stage to find the sub-game perfect NE in pure strategies.

2.1. The Second Stage Problem

In the second stage, the vendors choose retail prices, p_i, and production quantities, q_i, before the demand is observed. The capacity decisions, Q_i, from the first stage are taken as given.

The second stage problem for player i is:

$$\begin{aligned} \underset{p_i, q_i}{\text{maximize}} \quad & \overline{\pi}_i^2(\vec{p}, q_i \mid Q_i) \\ \text{subject to} \quad & q_i \leq Q_i \end{aligned} \tag{2.1}$$

where $\vec{p} = (p_1, p_2)$ and the profit function is:

$$\overline{\pi}_i^2(\vec{p}, q_i \mid Q_i) = -w_i q_i + p_i \mathrm{E}[\min(D_i(\vec{p}), q_i] \tag{2.2}$$

The first term represents the total production cost. The second term represents the total expected sales revenue as a result of the chosen price and quantity. The only cost item here is the production cost. In our model, without losing generality, we ignore holding and shortage costs.

Following Petruzzi and Dada (1999), we assume that the demand function has the additive form:

$$D_i(\vec{p}) = L_i(\vec{p}) + \epsilon_i \tag{2.3}$$

The first part, $L_i(\vec{p})$, represents the deterministic component of the demand function and demonstrates the price competition between the vendors. It is a function of retail prices only, which is non-increasing in p_i, i.e. $L_i^{(i)}(\vec{p}) \triangleq \partial L_i(\vec{p})/\partial p_i < 0$ and non-decreasing in p_j, i.e. $L_i^{(j)}(\vec{p}) \triangleq \partial L_i(\vec{p})/\partial p_j > 0$, $i \neq j$. The second part, ϵ_i, represents the random component, where ϵ_i is a random variable with mean zero and standard deviation σ_i.

The amount produced by each vendor accounts for both the deterministic demand and the safety stock, y_i, the extra inventory kept to mitigate the risk of stock-out in case of demand uncertainty.

$$q_i = L_i(\vec{p}) + y_i \tag{2.4}$$

The profit function to be maximized can be rewritten with the new variable, y_i, as:

$$\pi_i^2(\vec{p}, y_i \mid Q_i) = (p_i - w_i)L_i(\vec{p}) - w_i y_i + p_i \mathrm{E}[\min(\epsilon_i, y_i)] \tag{2.5}$$

The first term represents the deterministic profit. The second term is the cost of producing safety stock. The third term represents the expected revenue out of safety stock. Note that $\overline{\pi}_i^2(\vec{p}, q_i \mid Q_i)$ is transformed into $\pi_i^2(\vec{p}, y_i \mid Q_i)$ by substituting q_i. We are able to make this transformation, because the game with decision variables (p_i, q_i) is equivalent to that with (p_i, y_i), implying that if (p_i^*, q_i^*) is an equilibrium in the former, then $(p_i^*, L_i(\vec{p}^*) + y_i^*)$ is an equilibrium in the latter (Zhao and Atkins, 2008). We characterize the equilibrium solution in Section 2.3.

2.2. The First Stage Problem

In the first stage, the companies decide on the capacity Q_i that they will invest in.

The first stage problem is:

$$\underset{Q_i}{\text{maximize}} \quad \pi_i^1(\vec{Q}) \tag{2.6}$$

The profit function is:

$$\pi_i^1(\vec{Q}) = -K_i Q_i + \begin{pmatrix} \underset{p_i, y_i}{\text{maximize}} & \pi_i^2(\vec{p}, y_i \mid Q_i) \\ \text{subject to} & y_i + L_i(\vec{p}) \leq Q_i \end{pmatrix} \tag{2.7}$$

The first term represents the total capacity investment. The second term represents the maximum profit expected in the second stage for a given p_j and Q_j. Using the sub-game perfect NE concept, we substitute the NE prices $p_i^*(\vec{Q})$ and $p_j^*(\vec{Q})$ in Equation (2.7).

2.3. Existence of Pure Strategy Nash Equilibria in the Second Stage

According to Cachon and Netessine (2004), at least one symmetric pure strategy NE exists in a symmetric game where "for each player the strategy space is compact and convex and the payoff function is continuous and quasi-concave with respect to each player's own strategy". In our case, we allow a symmetric game, while assuming $w_i \leq p_i \leq p_i^{max}$ and $y_i^{min} \leq y_i \leq y_i^{max}$ to ensure a compact strategy set. We make sure that both p_i^{max} and y_i^{max} are sufficiently large that they do not constrain the players or the mathematical optima (Zhao and Atkins, 2008). We approach the constrained problem after understanding the unconstrained problem.

2.3.1. The Unconstrained Equilibrium

We already know that the unconstrained second stage profit function is jointly quasi-concave with respect to p_i and y_i by Zhao and Atkins (2008). The solution to the optimal safety stock of vendor i which maximizes expected profit is the critical ratio given as:

$$y_i^*(p_i) = F^{-1}\left(\frac{p_i - w_i}{p_i}\right) \tag{2.8}$$

If we substitute this quantity into π_i^2 we obtain $\widetilde{\pi}_i^2(\vec{p})$ as follows:

$$\widetilde{\pi}_i^2(\vec{p}) = (p_i - w_i)\left(L_i(\vec{p}) + F^{-1}\left(\frac{p_i - w_i}{p_i}\right) + l_i\right) \tag{2.9}$$

where l_i is the lower bound on the random variable of demand distribution. Zhao and Atkins (2008) propose conditions under which this function is quasi-concave on p_i, which is sufficient for the existence of a NE in the unconstrained case.

2.3.2. The Constrained Equilibrium

We need to show the effects of the capacity decision given earlier on the equilibrium behavior.

We start by approaching the problem analytically, looking for a second stage pure strategy NE, (p_i^*, y_i^*), where Q_i and Q_j are given. We write down the Karush-Kuhn-Tucker necessary conditions for the second stage problem. The second stage problem is:

$$\begin{aligned} \underset{p_i, y_i}{\text{maximize}} \quad & \pi_i^2(\vec{p}, y_i \mid Q_i) \\ \text{subject to} \quad & y_i + L_i(\vec{p}) \leq Q_i \end{aligned} \tag{2.10}$$

The Lagrangian of vendor i is defined as:

$$\Lambda_i(\vec{p}, y_i, \lambda_i) = \pi_i^2(\vec{p}, y_i \mid Q_i) + \lambda_i(y_i + L_i(\vec{p}) - Q_i) \tag{2.11}$$

where λ_i is the Lagrange multiplier.

The necessary KKT conditions for the problem stated in Equation (2.10) are:

- *Stationarity*

$$\frac{\partial \pi_i^2(\vec{p}, y_i)}{\partial y_i} = \lambda_i \tag{2.12}$$

$$\frac{\partial \pi_i^2(\vec{p}, y_i)}{\partial p_i} = \lambda_i \frac{\partial L_i(\vec{p})}{p_i} \tag{2.13}$$

- *Complementary slackness*

$$\lambda_i(y_i + L_i(\vec{p}) - Q_i) = 0 \tag{2.14}$$

- *Primal feasibility*

$$y_i + L_i(\vec{p}) - Q_i \leq 0 \tag{2.15}$$

- *Dual feasibility*

$$\lambda_i \geq 0 \tag{2.16}$$

These KKT conditions imply two cases where a solution exists, summarized below.

(i) *Inactive constraint*: If $\lambda_i^* = 0$ then the capacity constraint is redundant and the solution is the same as that of the unconstrained case, and we have:

$$\Lambda_i(\vec{p}^*, y_i^*, \lambda_i^*) = \pi_i^2(\vec{p}^*, y_i^* \mid Q_i)$$

Stationarity conditions imply that $\nabla_{p_i, y_i} \pi_i^2(\vec{p}^*, y_i^*) = 0$.

Primal feasibility condition implies that (p_i^*, y_i^*) is a feasible point.

Then the solution is equal to the unconstrained safety stock defined in Equation (2.8) and its corresponding price.

(ii) *Active constraint*: If $y_i^* + L_i(\vec{p}) = Q_i$ then we have:

$$\Lambda_i(\vec{p}^*, y_i^*, \lambda_i^*) = \pi_i^2(\vec{p}^*, y_i^* \mid Q_i) + \lambda_i^*(y_i^* + L_i(\vec{p}^*) - Q_i)$$

Under the assumption that $\epsilon_i \sim U[l_i, u_i]$

$$y_i^*(p_i, \lambda_i) = Q_i - L_i(\vec{p}) = \left(\frac{p_i - w_i + \lambda_i}{p_i}\right)(u_i - l_i) + l_i$$

Note that $y_i^*(p_i, \lambda_i)$ is increasing in λ_i. Then we can solve for

$$\lambda_i^* = p_i \frac{Q_i - L_i(\vec{p}) - l_i}{u_i - l_i} - (p_i - w_i)$$

If this $\lambda_i^* \geq 0$ and we can find p_i^* from

$$\frac{\partial \pi_i^2(\vec{p}, y_i)}{\partial p_i} = \lambda_i^* \frac{\partial L_i(\vec{p})}{\partial p_i}$$

then this is also a solution.

Note that in both cases the safety stock is independent of $p_j,\ i \neq j$, as a consequence of the additive demand assumption.

We use this method in our computations in Chapter 3.

3. COMPUTATIONAL RESULTS

Recall the demand function given in Equation (2.3). For the computational study, we assume that ϵ_i is a uniformly distributed random variable on an interval, such that $\epsilon_i \sim U[l_i, u_i]$.

Demand is assumed to be symmetric for each player, i.e. the same parameters apply for all. We assume there is no demand substitution.

We examine two different cases for the deterministic demand component $L_i(\vec{p})$. We take it to be linear in one case and exponential in the other.

In both the linear and the exponential case, we assume the uniform distribution parameters to be $\epsilon_i \sim U[-10, 10]$. The stochastic component signifies how much safety stock should be produced, and since it has negative support, it means that actual sales can be lower than what the deterministic demand function suggested.

3.1. Computational Method

We execute the optimization problem in MATLAB, using the fmincon function to solve the second stage problem as given in Equation (2.1). fmincon implements an interior point algorithm. In our algorithm, the optimal price p_i is calculated for each capacity option Q_i and for every possible price p_j that the competitor might choose. Hence a matrix for each player is created, which displays the player's strategy with respect to its competitor. Each element of the matrix is a price value in compliance with the KKT conditions, which is a best response price, since it contains all the information within. An example for this matrix is provided in Appendix A, displaying the best price response of vendor i at selected Q_i and p_j values. Using these best response price matrices, it is possible to calculate the NE, equilibrium capacity decisions, and the price and quantity that will be chosen if a capacity commitment is already made.

$$p_{i_{m,n}} = \quad Q_i \downarrow \overset{\displaystyle p_j \; \longrightarrow}{\begin{bmatrix} p_i(Q_{i_1}, p_{j_1}) & \cdots & p_i(Q_{i_1}, p_{j_n}) \\ \vdots & \ddots & \vdots \\ p_i(Q_{i_m}, p_{j_1}) & \cdots & p_i(Q_{i_m}, p_{j_n}) \end{bmatrix}}$$

Figure 3.1. The best response price matrix for player i.

3.2. Linear Deterministic Demand Case

The linear case is represented by:

$$L_i(\vec{p})_L = a - bp_i + cp_j \tag{3.1}$$

where a, b, c are parameters such that $a > 0,\ b > 0,\ c > 0$. We let $b > c$ in order to reflect a higher price sensitivity on the vendor's own price, compared to the competitor's. We take the demand parameters as $a = 1000,\ b = 30,\ c = 10$ and production cost as $w_i = 1$. A step size of 1 is used in the calculations unless stated otherwise.

3.2.1. The effect of capacity constraint on the second stage sub-game

We already know there is a solution to the unconstrained second stage problem by Zhao and Atkins (2008), stated in Equation (2.8). The best response price curves for the unconstrained problem are represented by straight lines in Figure 3.2. The second stage best response curves under capacity constraint are represented by dashed lines in the same figure.

The second stage best response curves are increasing functions due to our assumptions on the demand function. Notice how the capacity constraint causes the best response curves to bend and break at a certain point, marked by circles. The breakage point of the respective player corresponds to where its capacity constraint becomes active. Upon facing a capacity constraint while responding to an increasing p_j, p_i begins reacting more. Because the vendor can no longer increase quantity, in

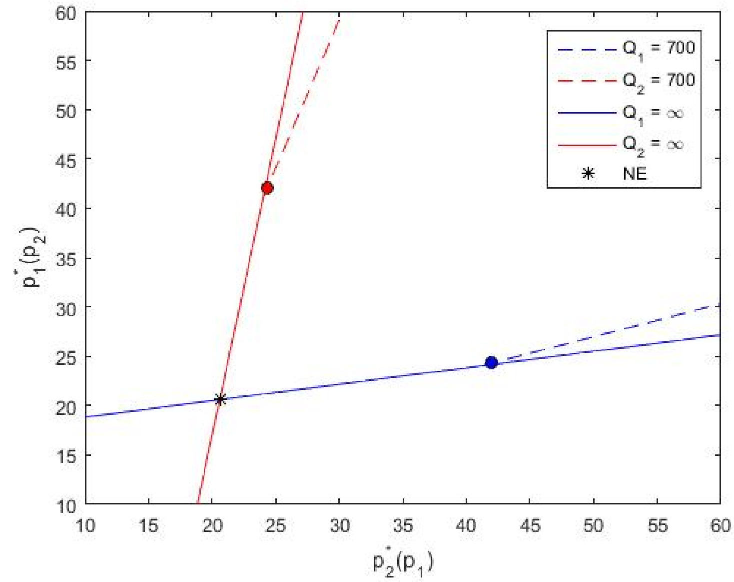


Figure 3.2. The effect of capacity constraint on the second stage sub-game.

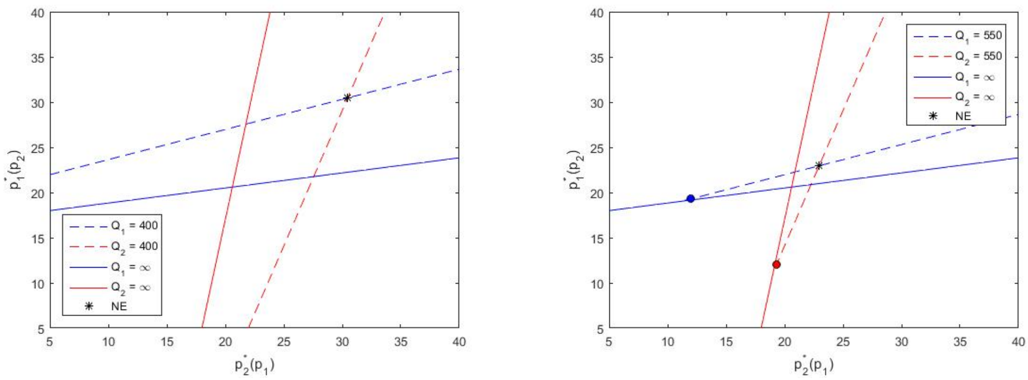
order to stay competitive it chooses to respond with a higher price than before.

Table 3.1. The second stage NE under different capacity constraints.

	(p_1^*, p_2^*)	(q_1^*, q_2^*)	(π_1, π_2)
$Q_1 = Q_2 = \infty$	(20.6, 20.6)	(597, 597)	(11,515, 11,515)
$Q_1 = Q_2 = 700$	(20.6, 20.6)	(597, 597)	(11,515, 11,515)
$Q_1 = Q_2 = 550$	(23.0, 23.0)	(550, 550)	(11,866, 11,866)
$Q_1 = Q_2 = 400$	(30.5, 30.5)	(400, 400)	(11,502, 11,502)
$Q_1 = 400, Q_2 = \infty$	(27.6, 21.8)	(400, 632)	(10,370, 12,920)
$Q_1 = 580, Q_2 = \infty$	(21.2, 20.7)	(580, 600)	(11,525, 11,634)

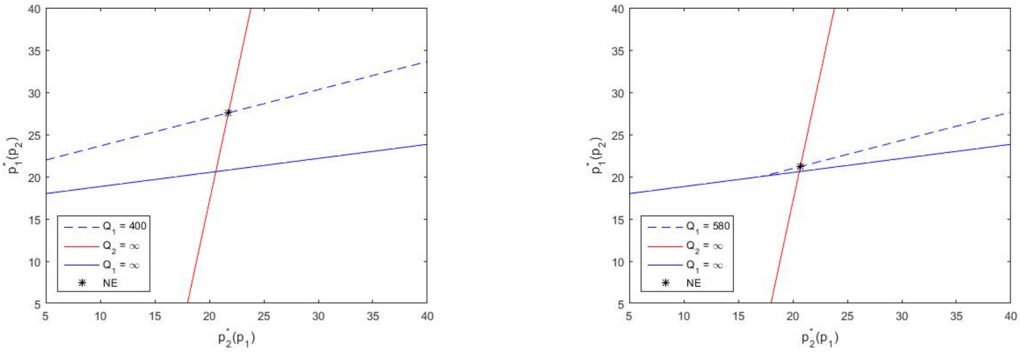
A sufficiently large capacity constraint does not shift the NE as seen in Figure 3.2, giving exactly the same result as the unconstrained case, thus the constraint is redundant at high capacity values. The decision variables at equilibrium can be found in Table 3.1 for different capacity constraints.

Now let's see the effects of a more limited capacity. In Figure 3.3 (a), the limited capacity is causing the best response curves to intersect elsewhere, hence shifting the NE to a point with higher prices. This leads to a lower profit despite higher prices for both companies, as seen in Table 3.1 (row 4). This result is expected, since the profits are lower than the unconstrained case. However, it is also possible to obtain a higher profit under certain capacity constraints, as in the case of Figure 3.3 (b), in contrast to the decreasing production quantities (Table 3.1, row 3).



(a) Constraint leads to lower profit. (b) Constraint leads to higher profit.
Figure 3.3. The effect of players' capacity constraints on their profits.

When we consider a case with asymmetric capacities, e.g. where one vendor is unconstrained and the other has limited capacity, we can see how this asymmetry affects the NE in Figure 3.4. The constraint on a vendor's capacity is always in favor of the unconstrained vendor. In Figure 3.4 (a), the constrained vendor increases its price to make up for the loss in profit caused by a lower production quantity. The unconstrained vendor responds by increasing its price as well, counting on the fact that its demand has increased. They reach a NE where the constrained player makes less profit and the unconstrained player makes higher profit than the unconstrained symmetric game described in Figure 3.2. But not all cases end up in a similar way. In Figure 3.4 (b) we observe that a vendor's capacity constraint is helping both vendors make higher profits. This happens because of the quasi-concave structure of the profit function, which is further analyzed in Section 3.2.2.



(a) Constrained vendor makes lower profit. (b) Both vendors make higher profits.

Figure 3.4. The effect of a player's capacity constraint on their profits.

3.2.2. The first stage dynamics when vendors can decide on capacities

The first stage game is played when the vendors are allowed to choose their capacity before the second stage game takes place. Figure 3.5 illustrates the first stage best response capacity curves. Each data point, i.e. (Q_1,Q_2) pair, corresponds to a (p_1^*,p_2^*) NE pair. A sample of best response price values at selected Q_i and Q_j are given in Appendix A. We observe a unique (Q_1^*, Q_2^*) NE in the first stage game.

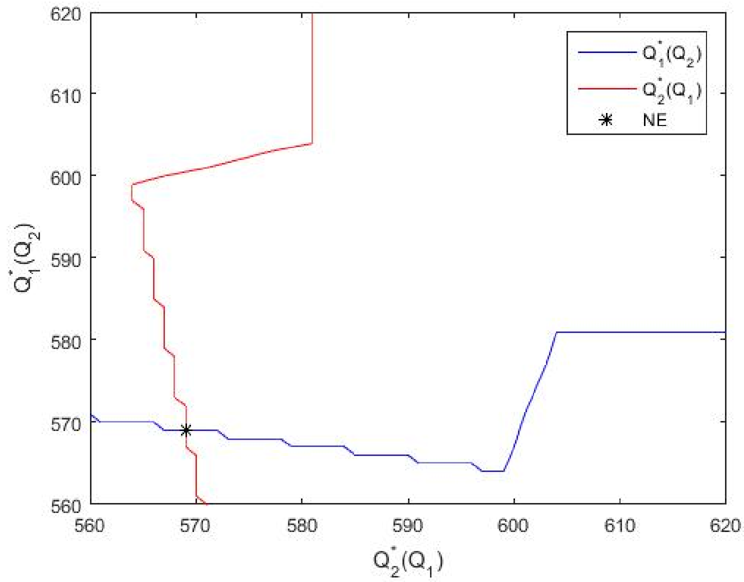
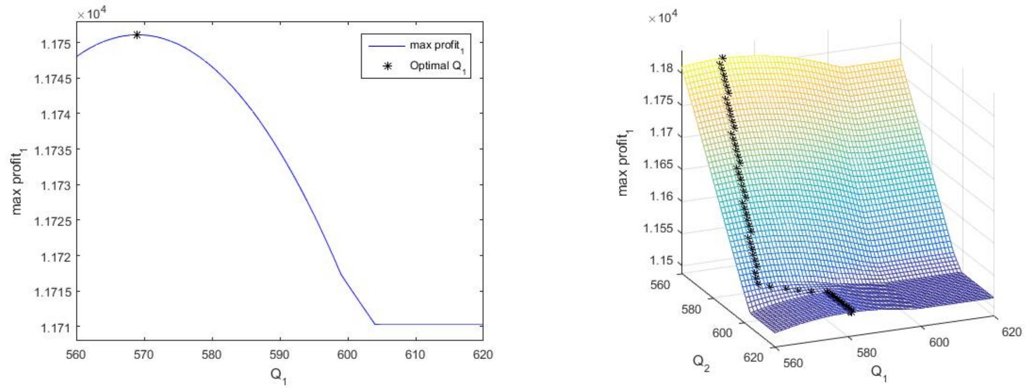


Figure 3.5. Capacity best response curves.

Contrary to the second stage price best response curves, the first stage capacity curves are decreasing functions up until a certain point, and then are stabilized at a specific value. The value at which the function stabilizes seems to be higher than other capacity values in Figure 3.5. However, this is not necessarily true, since this figure omits the lower capacity values, where the function value is indeed higher than the aforementioned stable part of the curve. Note that during the decrease before reaching the stable section, the (p_1^*, p_2^*) NE pair is also decreasing. The reason for this decrease can be explained as follows. An increase in the vendor's capacity will allow a higher production quantity, thus a lower price. The competitor will respond to this by lowering its price as well. Because of the structure of the demand function, the demand will decrease, hence a smaller capacity will be sufficient. The stabilization that comes after a certain point is where an increase in the vendor's capacity is no longer affecting the competitor's response, and the (p_1^*, p_2^*) NE pair after this point is also constant.

The sudden increase before stabilization in Figure 3.5 happens within a short range of capacity. The reason for this phenomenon is that the (p_1^*, p_2^*) NE pairs at these data points lead to an increasing demand only within this range.



(a) The 2D profile when $Q_2 = 569$. (b) 3D view.

Figure 3.6. The maximum profit function of a vendor in response to capacities.

The maximum profit graphs in Figure 3.6 display the quasi-concave behavior of the maximum profit function. This function indicates the maximum profit the vendor can attain for a given capacity constraint, by choosing the optimal price and quantity at that capacity. Figure 3.6 (a), plotted for the constrained optimal $Q_2 = 569$ (for numerical details see Table 3.2, row 1), shows how the vendor will react in response to a given capacity of the competitor Q_j by varying its capacity decision Q_i. This graph visualizes the first stage profit function in Equation (2.7).

When we compare the constrained NE in Figure 3.5 to the unconstrained NE in Figure 3.2, we observe how the capacity decision changes the equilibrium. When vendors are given a chance to choose any capacity, they prefer a capacity lower than their unconstrained optimal production quantity in the second stage. They choose to produce less while increasing prices, and eventually reach a higher profit than than the unconstrained game, which is a result of the quasi-concave structure of the profit function.

Notice that Figure 3.6 (a) has a peak, marked by an asterisk denoting the optimal Q_i, i.e. the vendor's decision if capacity can be chosen. If the vendor is constrained by a very low capacity, i.e. a value less than the optimal capacity, it falls onto the left side of the peak. In this area, the vendor chooses to produce as much as its capacity. When given a capacity range to choose from within this area, the vendor chooses the highest capacity due to its expectation of a higher profit (Table 3.2, row 2).

Table 3.2. Vendor 1's response to $Q_2 = 569$ under different capacity ranges.

Given Q_1 range	q_1^*	p_1^*	π_1
569	569	22.0	11,750
500 - 550	550	22.7	11,737
580 - 600	580	21.6	11,747
600	600	20.9	11,716

If it is constrained by a sufficiently high capacity, it will leave the capacity constraint redundant. Then the vendor falls onto the very right side of the curve, where the function is straight and linear, and not affected by increasing capacity any further. In this area, the vendor chooses to produce as much as the unconstrained optimal and leaves excess capacity, where excess capacity is defined as $Q_i - q_i > 0$.

However, when constrained by a value in between the peak and the unconstrained straight area, the vendor chooses to produce as much as its capacity constraint, and adjusts its price accordingly. To understand this behavior better, compare the cases in Table 3.2 between rows 3 and 4. When given a capacity range within this area, the vendor chooses the lowest capacity due to its expectation of a higher profit (Table 3.2, row 3). It consumes its full capacity during production. The same applies to the case in Table 3.2 (row 4). In either case, the vendor does not go with the option of producing at the level of Table 3.2 (row 1) and leaving some excess capacity. Instead it chooses to adjust its price and produces up to its full capacity.

Figure 3.6 (b) visualizes this function in 3D, showing the effect of competitor's capacity decision on maximum profit. The maximum profit obtained at each Q_j is marked with an asterisk. The graph in Figure 3.6 (a) is a profile from this graph at $Q_2 = 569$.

3.2.3. When capacity investment has a cost

All the cases we have analyzed so far have ignored the capacity investment cost K. In Figure 3.7 we introduce this cost and gradually increase it, since a gradual increase is sufficient to display its influence on the profit. In this figure we notice that the maximum profit curve shifts down, as intuitively expected, and quasi-concavity is not affected. This supports the fact that a unique NE still exists.

The maximum point shifts to the left, meaning that the peak and hence the best response is obtained at a smaller Q_i. This situation also alters the first stage NE. Although this cost does not have a direct influence on the second stage game, through decreasing the best response capacities, the second stage best response prices are also affected. These numerical impacts can be seen in Table 3.3.

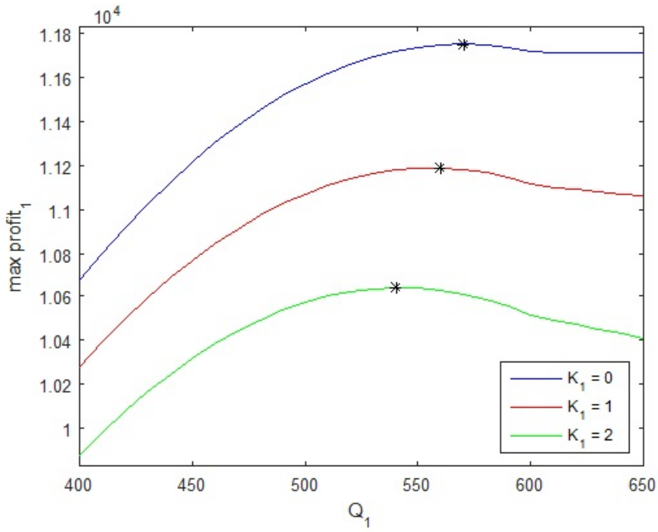


Figure 3.7. Maximum profit curves at different capacity costs.

Table 3.3. The full game NE of vendor i at various capacity investment costs.

	Q_i^*	p_i^*	π_i
$K = 0$	569	22.0	11,750
$K = 1$	555	23.0	11,250
$K = 2$	545	23.2	10,800

3.2.4. The first stage dynamics when a vendor moves first

For this case where vendors engage in a Stackelberg competition, we assume that vendor i is the leader and vendor j is the follower. Vendor i moves first into choosing a capacity and vendor j chooses its capacity after observing the leader's decision. After their successive decisions on capacities in the first stage, they go through with their pricing decisions in the second stage simultaneously.

The first stage problem for the follower is defined as:

$$\underset{Q_j}{\text{maximize}} \quad \pi_j^1(Q_j \mid Q_i) \tag{3.2}$$

while for the leader it is:

$$\underset{Q_i}{\text{maximize}} \quad \pi_i^1(Q_i, Q_j(Q_i)) \tag{3.3}$$

The second stage game for each vendor remains the same, as is described in Equation (2.10).

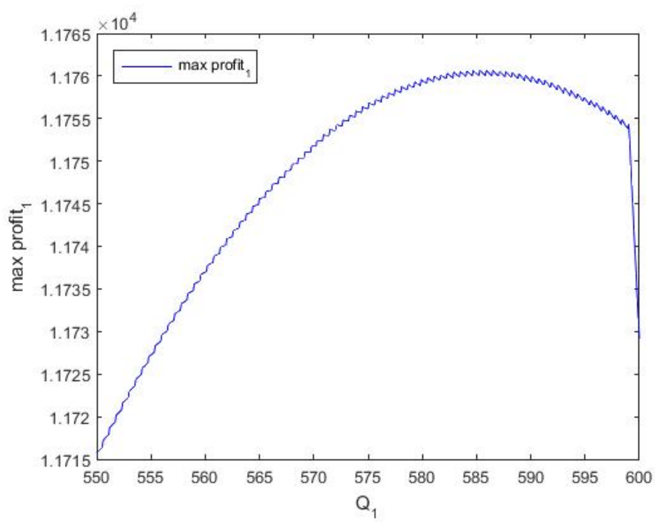


Figure 3.8. Maximum profit curve of the leader in the Stackelberg game.

Figure 3.8 displays the quasi-concave maximum profit curve for the leader. The equilibrium values can be found in Table 3.4. For this case only, the selected step size is 0.1.

Table 3.4. The sub-game NE in a Stackelberg game.

	Q^*	p^*	π
Leader	585	21.4	11,761
Follower	566	21.9	11,639

The leader has the advantage in this game, achieving a higher profit than the follower by acting first to decide on capacity. Comparing this result to the simultaneous game in Table 3.3 (row 1), we observe that the leader selects a higher capacity than before, while the follower chooses a smaller one. Also, the price equilibrium of the second stage game is at lower values than the case mentioned. Although the values did not change substantially, there is a significant shift in the equilibrium.

3.3. Exponential Deterministic Demand Case

The exponential case is represented by:

$$L_i(\vec{p})_E = C^{a-bp_i+cp_j} \tag{3.4}$$

where a, b, c, C are parameters such that $a > 0,\ b > 0,\ c > 0,\ C > 1$. We again let $b > c$ in order to reflect a higher price sensitivity on the vendor's own price, compared to the competitor's.

We take the demand parameters as $a = 100,\ b = 2,\ c = 1,\ C = 1.1$ and production cost as $w_i = 1$. This section reflects a different scale of market due to the selected demand parameters. A step size of 1 is used in the calculations unless stated otherwise.

3.3.1. The effect of capacity constraint on the second stage sub-game

Similar to the case analyzed in Section 3.2.1 for linear deterministic demand, the second stage best response functions break at a certain point, which creates a NE different from that of the unconstrained case. The unconstrained best response function appears to be a straight line as seen in Figure 3.9, however they are not. This is due to the fact that at this scale, an increase in these functions are observed at much higher values.

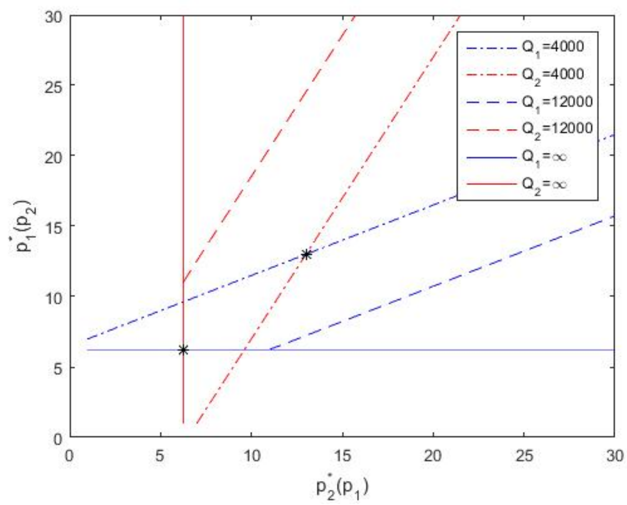


Figure 3.9. The effect of capaciy constraint on the second stage sub-game.

Table 3.5 demonstrates how the capacity constraint affects the NE of the second stage. In this table we observe qualitatively the same behavior as in the linear case. A sufficiently high capacity does not change the sub-game NE, but a lower capacity increases the equilibrium prices. Lowering capacity may cause an increase in the profits, as seen in Table 3.5 (row 3).

The sensitivity is also different for the exponential case. While the linear model was more sensitive to price, the exponential model is less sensitive. This causes the capacity constraint to be less influential on the equilibrium prices, but more effective on the quantities.

Table 3.5. The second stage NE under different capacity constraints.

	(p_1^*, p_2^*)	(q_1^*, q_2^*)	(π_1, π_2)
$Q_1 = Q_2 = \infty$	(6.3, 6.3)	(7000, 7000)	(39,850, 39,850)
$Q_1 = Q_2 = 12000$	(6.3, 6.3)	(7000, 7000)	(39,850, 39,850)
$Q_1 = Q_2 = 4000$	(13.0, 13.0)	(4000, 4000)	(47,891, 47,891)

3.3.2. The first stage dynamics when vendors can decide on capacities

The first stage dynamics of the exponential demand case is also similar to that of the linear demand. In Figure 3.10 we observe a unique (Q_1^*, Q_2^*), which corresponds to a slightly higher price equilibrium, and results in much higher profits.

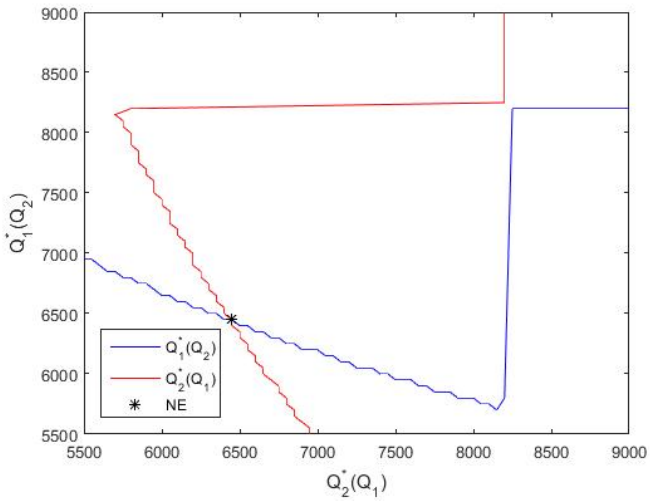


Figure 3.10. Capacity best response curves.

The NE at constrained optimal is given in Table 3.6.

Table 3.6. The first stage NE when vendors can decide on capacities.

(Q_1^*, Q_2^*)	(p_1^*, p_2^*)	(π_1, π_2)
(6450, 6450)	(8.0, 8.0)	(44,950, 44,950)

4. CONCLUSION

In this thesis, the competition between two firms has been analyzed across two periods through their capacity, price, and inventory decisions. In the first period, the firms choose capacities to invest in, which limits their production quantities in the following term. In the second period, they simultaneously choose prices and quantities before observing the demand. We assume a price-elastic demand function in the additive form, composed of a deterministic and a stochastic part. Competition arises due to demand being responsive to the prices of both firms. The cost and demand parameters are assumed symmetric for each player.

We first considered a linear deterministic demand and analyzed the following cases: an unconstrained game, the second stage games under various capacity constraints, the first stage game where capacity is chosen, asymmetric capacity constraints, and various investment cost parameters. Then we replicated these results for an exponential deterministic demand function. We used a computational analysis to obtain the results.

In our computations, we were able to find a second stage sub-game perfect NE under all feasible capacity values. The symmetric games produced symmetric results in each case.

Linear and exponential deterministic demands produced qualitatively similar results. The two cases were evaluated at different scales, having different demand functions and parameters, which gave rise to different equilibrium prices and profits as well as different sensitivities. The linear case was more sensitive to prices, while in the exponential case the profits were less responsive to changes in price. This led to the vendors adjusting mainly the prices in the former case and quantities in the latter.

At a sufficiently high value of given capacity, although the equilibrium results were equivalent, the price best response functions were not, because they break at a certain point.

At a lower value of given capacity, the constraint becomes active and this alters the price equilibrium. Upon decreasing the capacity constraint, the price equilibrium values increase indefinitely. Either symmetric or asymmetric, a decrease in the capacity constraint of one vendor increases the equilibrium prices of both vendors.

Excess capacity is incurred only in the case of a sufficiently high capacity constraint, where the vendor chooses to produce just as much as the unconstrained optimal quantity. In cases where constraint is lower and symmetric, the vendor uses up all of its capacity. For asymmetric constraints, capacity consumption of each vendor depends on the equilibrium determined by those capacities. In a case with a high degree of asymmetry, a large firm will produce at an optimal level less than its capacity, while a smaller firm will consume its full capacity.

In the case where vendors can choose capacities, the first stage game appears to have a unique (Q_i^*, Q_j^*). The capacity best response functions are non-linear.

Although advantageous, being the first-mover does not bring a vast amount of additional revenue to the leader. A sequential game in the first stage slightly shifts the equilibrium from where it was in the simultaneous game. While the leader gains a price advantage over the follower, the sequential competition also lets the follower choose a lower price than the simultaneous game.

The maximum profit function appears to be quasi-concave independent of the value of investment cost, K. Hence the calculated NE, (p_i^*, p_j^*), are always unique. We assumed a constant investment cost, K, in all cases. We started analyzing the cases with zero investment cost, in order to leave out its effects on the characteristics of the game and equilibria. In the absence of this cost, we observed that after a certain level of capacity decision, the maximum profit function, i.e. the maximum profit that will be attained in the second stage sub-game, will stabilize and give the same equilibrium results. Increasing this cost decreased the profits as expected, and shifted the full game

NE. At higher investment costs, we observed that the best response capacity drops, while the best response price increases.

The results presented in this thesis can be extended in many ways. The computational results can be extended for a multiplicative demand model and the impact of demand variation on equilibrium can be analyzed. An analytical solution can be investigated in order to generalize our findings. The capacity investment cost can be taken as a function of quantity, rather than a constant. Analysis of asymmetry also can be extended to cases of asymmetric cost and demand models.

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APPENDIX A: SAMPLA DATA SETS

Table A.1. The best response price matrix of vendor i for linear deterministic demand case at selected values of Q_i and p_j.

	$p_j = 10$	$p_j = 15$	$p_j = 20$	$p_j = 25$	$p_j = 30$	$p_j = 35$	$p_j = 40$
$Q_i = 560$	18.8	20.0	21.6	23.3	25.0	26.6	28.3
$Q_i = 565$	18.8	19.8	21.5	23.1	24.8	26.5	28.1
$Q_i = 570$	18.8	19.7	21.3	23.0	24.6	26.3	28.0
$Q_i = 575$	18.8	19.7	21.1	22.8	24.5	26.1	27.8
$Q_i = 580$	18.8	19.7	21.0	22.6	24.3	26.0	27.6
$Q_i = 585$	18.8	19.7	20.8	22.5	24.1	25.8	27.5
$Q_i = 590$	18.8	19.7	20.6	22.3	24.0	25.6	27.3
$Q_i = 595$	18.8	19.7	20.5	22.1	23.8	25.5	27.1
$Q_i = 600$	18.8	19.7	20.5	22.0	23.6	25.3	27.0
$Q_i = 605$	18.8	19.7	20.5	21.8	23.5	25.1	26.8
$Q_i = 610$	18.8	19.7	20.5	21.6	23.3	25.0	26.6

Table A.1 displays a sample set of best response price data calculated for vendor i. Each row on its own represents a second stage best response price curve of the vendor at a given Q_i, i.e. $p_i(p_j \mid Q_i)$. A similar matrix is calculated for vendor j as well. The cases in Section 3.2.1 where the capacity constraint is given are calculated by picking the relevant rows from each vendor's matrix and intersecting them. This gives the second stage equilibrium prices at each (Q_i, Q_j) pair and profits are calculated using this information. Equilibrium prices for vendor i are shown in Table Table A.2.

The cases in Section 3.2.2 where vendors can decide on capacities are calculated by Table A.2. For every given Q_j, i.e. every column of this table, the price leading to the highest profit and the corresponding Q_i are determined, which gives the best response capacity curves. Similar calculations are done for both vendors, the intersection of best response capacity curves produces the first stage equilibrium.

Table A.2. The best response price matrix of vendor i for linear deterministic demand case at selected values of Q_i and Q_j.

	$Q_j = 560$	$Q_j = 565$	$Q_j = 570$	$Q_j = 575$	$Q_j = 580$	$Q_j = 585$
$Q_i = 560$	22.0	22.0	21.9	21.9	21.8	21.7
$Q_i = 565$	21.9	21.8	21.7	21.7	21.6	21.5
$Q_i = 570$	21.7	21.6	21.5	21.5	21.4	21.4
$Q_i = 575$	21.5	21.4	21.4	21.3	21.2	21.2
$Q_i = 580$	21.3	21.2	21.2	21.1	21.0	21.0
$Q_i = 585$	21.1	21.0	21.0	20.9	20.9	20.8

Table A.3 shows the calculation steps for the Stackelberg game in Section 3.2.4. At every Q_i of the leader, the best response capacity of the follower, $Q_j(Q_i)$, is calculated based on Table A.2. The price equilibria under each capacity pair and the profits are determined. The highest profit and the corresponding variables give the results to the Stackelberg game.

Table A.3. The Stackelberg game capacities, best response prices, and profits.

Q_i	570	575	580	585	590	595	600
$Q_j(Q_i)$	569	568	567	566	566	565	567
p_i^*	22.0	21.8	21.6	21.4	21.3	21.1	20.9
p_j^*	22.0	22.0	21.9	21.9	21.9	21.8	21.7
$\pi_i(Q_i, Q_j(Q_i))$	11751	11757	11761	11763	11756	11755	11731