A COMPARATIVE ANALYSIS OF DIFFERENT EXPECTATION MODELS FOR THE EL FAROL BAR PROBLEM

by

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ABSTRACT

A COMPARATIVE ANALYSIS OF DIFFERENT EXPECTATION MODELS FOR THE EL FAROL BAR PROBLEM

The El Farol Bar Problem is first discussed by W. B. Arthur (1994) and he presents this problem to introduce a new field that he names as "complexity economics". The El Farol Bar Problem is widely used in the literature, especially in congestion and coordination studies. In this study, we model the El Farol Bar Problem using the agent-based modelling methodology and Python computer language. We create different agent types with distinctive expectation models. The emergent behaviors of different agent types are compared with respect to several performance measures. After several experiments with different agent types, we reach two important conclusions: The heterogeneity of the decisions is the key factor in obtaining low standard deviation of attendance values and the assumption of knowing the bar capacity value is crucial for a good performance. Agents who make expectations randomly generate the highest heterogeneity in the attendance values, which is consistent with the findings in the literature. In this thesis, we also introduce agents who use exponential smoothing method in forming expectations. They create low heterogeneity in decisions and a poor performance compared to other agents. Nevertheless, the exponential smoothing method works well in learning the capacity value. Accordingly, we introduce an agent type that combines random attendance expectations with the exponential smoothing method in estimating the capacity. When the bar capacity is unknown, this agent type produces mean attendance values gravitating towards the bar capacity ensuring the heterogeneity in the decisions. Lastly, we develop Yasarcan-Çetiner agents that do not use expectation models, but a hysteresis structure in decision-making. Although, they do not have explicitly coded capacity learning mechanism in their algorithms, they still learn the bar capacity as a swarm according to their emergent collective behavior.

ÖZET

EL FAROL BAR PROBLEMİ'NDE FARKLI BEKLENTİ MODELLERİNİN KARŞILAŞTIRMALI ANALİZİ

El Farol Bar Problemi ilk olarak W. B. Arthur (1994) tarafından ortaya atılmıştır ve bu problem "kompleksite iktisadı" olarak isimlendirilen yeni bir alanı tanıtmak için sunulmuştur. El Farol Bar Problemi özellikle tıkanıklık ve koordinasyon problemleri çalışmalarında sıklıkla kullanılmıştır. Bu çalışmada El Farol Bar Problemi'ni ajan temelli benzetim metodolojisi ve Python bilgisayar dili yardımıyla modelledik. Farklı beklenti modelleri ile çeşitli ajan türleri oluşturarak ajan türlerinin ortaya çıkardığı davranışları performans ölçütlerine bağlı olarak karşılaştırdık. Farklı ajan türleri ile yapılan deneyler sonucunda iki önemli sonuca ulaştık: Kararların heterojenliği ajanların başarısında kilit rol oynamaktadır ve bar kapasitesinin ajanlar tarafından bilindiği varsayımı bu problemde iyi bir performansın ortaya çıkması için zorunludur. Literatürdeki sonuçlar ile paralel olarak, beklenti değerlerini rassal bir şekilde oluşturan ajanlar katılım değerleri açısından en yüksek heterojenliği oluşturmaktadır. Bu tez çalışmasında, beklenti değerlerini üstel düzeltme yöntemi ile oluşturan ajanlar modelleyerek literatüre sunduk. Bu ajanlar, diğer ajan türleriyle kıyaslandığında kararlarda düşük heterojenlik nedeniyle kötü bir performans sergilemektedirler ancak üstel düzeltme yöntemi kapasiteyi öğrenme konusunda gayet iyi çalışmaktadır. Buna bağlı olarak, üstel düzeltme yöntemiyle kapasiteyi öğrenen ve katılım tahminini rassal bir şekilde yapan yeni bir ajan türü oluşturduk. Bar kapasitesinin bilinmediği durumlarda bu ajan türü ortalama katılımın bar kapasitesine doğru yaklaşan ve kararlarda heterojenliği sağlayan bir davranış üretmektedir. Son olarak, karar verme süreçlerinde beklenti modelleri yerine histerezis yapısı kullanan Yasarcan-Çetiner ajanlarını modelledik. Bu ajan tipinde bar kapasitesini öğrenmek için hiçbir özel algoritma bulunmamasına rağmen Yasarcan-Çetiner ajanları, ortaya çıkan kolektif davranışları sonucunda bar kapasitesini bir sürü olarak öğrenmektedir.

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LIST OF SYMBOLS

i	Counter variable for the agents
j	Counter variable for the strategies
k	Number of strategies of each agent
m	Total number of strategies
n	Total number of agents
s	Counter variable for number of weeks
α_i	Smoothing parameter for Type 3 Agents
λ	Smoothing parameter for error term
σ	Standard deviation
μ	Mean

LIST OF ACRONYMS/ABBREVIATIONS

EFBP	El Farol Bar Problem
MAD	Mean Absolute Deviation

1. INTRODUCTION

W. B. Arthur (1994) introduces the El Farol Bar Problem (EFBP) in his article "Bounded Rationality and Inductive Reasoning". The EFBP has an agent-based structure in which independent agents make decisions according to their expectation models. He discusses that the EFBP forces agents to use inductive reasoning. If deductive reasoning were valid for agents in the EFBP, they would all use the same exact expectation model; either all will attend to the bar at the same time or, simultaneously, all will choose not to attend. Accordingly, agents will be forced to differ their own expectation models breaking up "commonality of expectations". As a "deductively rational solution" to the problem is not possible, they have to use different expectation models. Therefore, the agents in the EFBP can only be defined as "boundedly rational" (Arthur, 1994).

The EFBP contributes the literature of bounded rationality. The term "bounded rationality" can be defined as the rational choice of a decision maker within the limitations of both knowledge and computational capacity (Simon, 1990). The EFBP is inspired by a real bar located in Santa Fe, New Mexico and Arthur uses the problem to show that perfect information is not available for decision makers in such a decision environment. When the information is limited due to nature of the problem, decision makers should use inductive reasoning to make decisions. After introducing these ideas, Arthur criticizes the assumptions of conventional economic theory. Arthur claims that conventional economics is based on perfectly rational agents and static equilibriums that are analytically obtained (Arthur, 1994). However, according to Arthur's opinion, the economy should be analyzed as "an evolving system rather than a deterministic and mechanistic nature" (Arthur, 1999).

Arthur does not carry out different experiments on the EFBP. He only uses the agent-based model to show the importance of bounded rationality and inductive reasoning aiming to introduce a new field, "complexity economics". However, there are many studies that are curious about the behavioral analysis of the EFBP and the possible applications of the EFBP for the solution of different congestion and coordination problems.

Currently, there are many studies based on the EFBP and models inspired by the EFBP. The applications of the EFBP in the literature is not limited to economy studies; social network analysis (Epstein and Bazza, 2011; Chen and Gostoli, 2017), marine terminal gate congestion analysis (Sharif *et al.*, 2011), internet congestion and coordination failure (Bell and Sethares, 1999), congestion externalities (Zambrano, 2004), electric energy balancing (Radziszewska *et al.*, 2014), human motivations and consequences (Cross *et al.*, 2005), emergence of heterogeneity (Edmods, 1998), roadway routing problem (Adler and Blue, 2002), influenza vaccination model (Breban *et al.*, 2006), terrorist organizations (Hausken *et al.*, 2013), Kolkata restaurant problem (Chakrabarti, 2007), and minority game (Challet and Zhang, 1997; Petrova *et al.*, 2010; Baccan *et al.*, 2014).

The utility function and its different characteristics in the EFBP is not examined by Arthur. Challet and Zhang (1997) introduces minority game as a simplified version of the EFBP and they examine the analytical results of the utility function (i.e., payoff function) (Challet and Zhang, 1998). These studies aim to analyze the collective behavior and the utility of the group in an environment that has boundedly rational agents.

The EFBP is modelled in many different ways as Arthur (1994) does not provide the coding details of the EFBP. Garofalo (2006) and Wilensky and Rand (2015) choose to model the EFBP in NetLogo. Most of the following studies use these models to analyze the EFBP to carry out analyses with respect to different research questions (Papakonstantinou, 2006; Ponsiglione *et al.*, 2015). Ponsiglione *et al.* (2015) and Rand and Stonedahl (2007) focus on the fairness and efficiency issues in the EFBP. Moreover, St Luce and Sayama (2020) analyze the changing strategies and agent types in their experiments.

The aim of this thesis is to analyze an important and highly discussed problem, namely the El Farol Bar Problem, and to develop a deeper understanding on this complex adaptive system. This thesis focuses on creating different agent types by modelling their expectations in different ways.. We will study the emergent behaviours resulting from using different agent types. As a result of simulation experiments, we will try to identify the dynamic characteristics crated by the different agent types. We will also compare these different agent types with respect to several performance measures. We hope to determine the agent type that will relatively be successful in terms of performance measures and explain the reasons behind this success.

The thesis consist of 12 chapters. Introduction (Chapter 1) and Literature Review (Chapter 2) give information about the nature of the EFBP. Problem Description is given in Chapter 3 and the methodology used in this thesis is given in Chapter 4. In Model Overview (Chapter 5), we develop the generic flowchart of the algorithm for the EFBP. In Three Different Expectation Models (Chapter 6) we modify the generic flowchart for the different algorithms of the different agent types. We discuss the performance measures in detail in Chapter 7. In Chapter 8, experiments and results for the three different expectation models are reported. In Chapter 9, a discussion on the results of the experiments is carried out and further experiments are provided to give depth to the discussion. We introduce a new agent type in Type 4 Agents: Yasarcan-Çetiner Agents (Chapter 10). Lastly, Conclusion and Future Work are presented in Chapter 11 and 12.

2. LITERATURE REVIEW

We categorize the studies on the EFBP into five main areas: the studies that focus on complexity economics, the payoff/utility of the agents, the EFBP applications, minority game and its extensions, and the modelling of the EFBP. Although we speak of all these areas throughout this study, this thesis can be categorized under the modelling of the EFBP literature according to our categorization.

2.1. Complexity Economics

The conventional economic theories are not valid for the EFBP as the assumption of perfectly rational agents and deductive reasoning do not produce meaningful dynamic behavior. If deductive reasoning were valid for agents in the EFBP, they would all use the same exact expectation model; either all will attend to the bar at the same time or, simultaneously, all will choose not to attend. The agents can only be boundedly rational in Arthur's opinion due to the nature of the economical system. He sees economy as a complex evolving system where it always evolves instead of staying in equilibrium (Arthur, 1994; 1999).

The appearance of Arthur (1999) in journal Science is perceived as a sign that "complexity theory" started to gain importance (Manson, 2001). In his Science paper, Arthur discusses the limited circumstances under which conventional economic theory that is based on equilibrium approach can be considered as valid. He also discusses the out-of-equilibrium cases that require "a more general theory", namely, "complexity economics". The recent article "Foundation of Complexity Economics" written by Arthur (2021) confirms that the interest in the "complexity economics" is still popular. Starting with Arthur's papers in 1994 and 1999, various of studies contribute on the field complexity and specially the "complexity economics" (Arthur, 1995; Manson, 2001; Foxon *et al.*, 2013; Elsner, 2017). The EFBP is a pioneer problem that is used to introduce the field of "complexity economics" and it leads to new congestion and coordination problems. The importance of EFBP is also mentioned in the literature.

Casti (1996) declares that the EFBP is the "most important problem in complex systems theory" in his article "Seeing the Light at El Farol". Casti (1996) claims that the EFBP has the key components that can be observed in complex adaptive systems. One of the key components is the size of the interacting population of agents in his opinion. If the size is too big or too small, interesting emergent behavior cannot be observed. Another key component is that the agents are "intelligent and adaptive". In Casti (1996), last key component is that information is "local" as the agents do not have access to the individual decisions of other agents.

2.2. Payoff Function Formulation

Even though Arthur does not discuss the individual or collective payoff in the EFBP, many studies focus on the payoff function. One of the earliest research about the payoff in the EFBP is conducted by Challet and Zhang (1997) who also introduce the Minority Game as a version of the EFBP. In the article "Emergence of Cooperation and Organization in an Evolutionary Game", they study the utility in both the EFBP and the Minority Game, and interpret the results to understand the emergence of cooperation (Challet and Zhang, 1997). There are other studies that focus on the payoff of both the individuals and the population. Arthur (1994) does not specify any formulation for the payoff in his original article; he just states that if the bar capacity is exceeded, the attendees do not spend an enjoyable night. Nevertheless, we can see that utility functions or payoff values are formulated in different ways. Edmonds (1998) defines the individual utility with the values 0.4 for attending the bar at a crowded day, 0.5 for staying home, and 0.6 for attending the bar at an uncrowded day. Mishra *et al.* (1998) follow a similar logic for the three states of actions (attending the bar at a crowded day, staying home, and attending the bar at an uncrowded day) with different values such as -0.5, 0, and 0.5 respectively. Another example of different formulation of the individual payoff can be found in De Cara *et al.* (2008), Sellers *et al.* (2020) and St Luce and Sayama (2020). All the three studies use the same formulation for the payoff which is the "correct decision" approach. The correct decision gets the value of one and the wrong decision gets zero as a payoff. In other words, the agents get the payoff of one, if they attend the bar at an uncrowded day or stay home when the bar is crowded; on the other hand, the agents get the payoff of zero if they attend the bar at a crowded day or stay home when the bar is uncrowded. Sellers et al. (2020) support their formulation with the argument that if the agents gain utility only for attending the bar at uncrowded days, then the strategy of "always go" would be optimal, so staying home on crowded days should also be included in the formulation. We decide to follow the logic of De Cara *et al.* (2008), Sellers *et al.*, (2020) and St Luce and Sayama (2020) which gives a "Reward" value for each "correct decision". So, the agents in our simulation models gain the reward value of one for each correct decision, on the other hand they get the reward value of zero if they make a wrong decision.

Chen and Gostoli (2016) study the sensitivity analysis for different number of agents. Farago *et al.* (2002) use no-regret learning and Q-learning algorithms and compare the results in terms of utility. Furthermore, Cross *et al.* (2005) examine the effect of regret. Moreover, the sharp decrease in the payoff is also discussed in the literature; namely, when 60 people attend to the bar, all agents become "happy", however if 61 people attend to the bar, all 61 agents become "unhappy". This assumption is kept for almost all studies that we encountered in the literature, but we will also discuss this assumption briefly in Chapter 12: Future Research. Lastly, there are some key terms that are not used by Arthur, but we see in different studies. We are going to use the same terminology to be consistent with the literature and give detailed information within the relevant chapters. For example, the terms "happy" and "unhappy" are commonly used for the state of agents at attending an uncrowded or crowded bar (Coen and Riolo, 2001; Rand and Stonedahl, 2007; St Luce and Sayama, 2020). Also, the term "Reward" is used to count the days for each agent that the agent attends the bar, and the bar is uncrowded (Wilensky and Rand, 2015).

2.3. EFBP Applications

After Arthur introduced the EFBP, the EFBP has been seen as a good example to examine the coordination and congestion problems. Many studies take the EFBP to analyze different congestion problems from different fields. For example, Zambrano

(2004) studies the EFBP in his article "The Interplay Between Analytics and Computation in The Study of Congestion Externalities: The Case of The El Farol Problem" using both analytical and computational approaches. As a result of game theoretical analysis, he concludes that there is no pure strategy Nash Equilibrium for the "prediction game". However, the aggregate attendance can converge to a threshold value (60 in the EFBP), which is parallel with Arthur's findings (1994). Zambrano also discovers that the EFBP can be utilized for congestion problems such as traffic congestions and internet congestions externalities. Moreover, his paper argues that both analytical and computational approaches can be used as complementary methods instead of two competing methods.

In another example; Hausken *et al.* (2015) show the coordination difficulties in planning terrorist attacks with a game that they develop based on the EFBP aiming to discover counterterrorist strategies. For this purpose, different policies for coordination between ideologues and mercenaries (frontline fighters) are compared. The mercenary agents choose whether to join a terrorist attack or not, which corresponds to going to the bar or not. Hausken *et al.* (2015) claim that disrupting the communication between terrorist cells can be effective strategy for counterterrorist policies.

Adler and Blue (2002) present a conceptual model for a route guidance system comparing with the EFBP. Their aim is to find an optimal vehicle routing and scheduling by considering the cooperation between network operators and drivers. They claim that the EFBP is a standard model for studying network control and rational learning. Chen and Gostoli (2015) analyze the self-coordination mechanisms in using public resources. In their article "Coordination in the El Farol Bar Problem: The Role of Social Preferences and Social Networks", they try to find an optimal utilization of public resources in terms of efficiency by considering social preferences and social networks, and they use the EFBP as a self-coordination mechanism that can inspire their study. To conclude, they state that two elements which are not considered frequently on conventional EFBP literature can be helpful for the self-coordination mechanism in using public resources. These two elements are the social preferences and social networks.

Edmonds (1998) extends the EFBP by implementing a communication and learning process to the agents. He shows that if the global communication is allowed, it causes the prevention of emergence of the heterogeneity in terms of agents' preferences. Sharif *et al.* (2011) apply the EFBP model to the terminal gate congestion problem in the article "Application of El Farol Model for Managing Marine Terminal Gate Congestion". They state that the truck queuing at marine terminal gates is identified as a critical emission source, and the ways to eliminate this queuing are searched. This study uses the modified EFBP to examine the marine terminal gate congestion. As a result, they reach similar findings with Arthur that if the different depot agents differ their strategies, the congestion at marine terminal gates can be lowered.

2.4. Minority Game

The EFBP is classified as a congestion game which is a special case of potential games according to some researchers. (Whitehead, 2008). Since each agent's decisions and utility are determined by other agent's decisions and the strategies can be expressed by a potential function (i.e., payoff function), the problem can be classified under potential and congestion games.

Challet and Zhang (1997) introduce a game with n players inspired by the EFBP and they called it "Minority Game" (Petrova *et al.*, 2010). In Minority Game, each player or computer simulated agent chooses between two options (i.e., makes a binary decision). The total number for each decision option is obtained. The option with the least total number becomes the winning option and the agents/players who choose that option, receive a better payoff value. Minority Game is classified as an evolutionary game, and can be seen as "a simplified version of El Farol Bar Problem" (Challet and Zhang, 1997; Baccan *et al.*, 2014). Many studies are conducted using Minority Game to examine the coordination as well as the EFBP and its extended versions (Zhang, 1998; Lustosa and Cajueаria, 2009; Galib and Moser, 2011; Galstyan *et al.*, 2003). Challet and Zhang (1998) analyze the Minority Game using both agent-based simulation and analytical approach in their article "On the Minority Game: Analytical and Numerical Studies". They conclude that an analytical solution of the model is difficult to reach, however the strategies and their consequences in terms of individual and global payoff function can be analyzed.

The EFBP also leads to a new problem such as Kolkata Restaurant problem which is introduced in the article "Kolkata Restaurant Problem as a generalized El Farol Bar Problem" (Chakrabarti, 2007). Kolkata Restaurant problem is "a generalization of the EFBP" (Chakrabarti *et al.*, 2009). Chakrabarti (2007) extends the problem adding the chance of attending multiple restaurants options instead of one single bar in Arthur's case. He analyzes the problem's dynamics for n different restaurants using agent-based simulation method. After this research, Chakrabarti *et al.* (2009) study this problem in terms of resource utilization.

Minority Game is also analyzed by other researchers as a model of resource allocation. Some characteristics of Minority Game are studied such as memory size of agents and communication between agents (Remondino and Cappelini, 2004; Lustosa and Cajueearia, 2009; Cavagna, 1999). Moreover, the Minority Game is used to examine market mechanism, congestion problems and resource allocation problems just like the EFBP (Zhang, 1998; Galib and Moser, 2011; Galstyan *et al.*, 2003). Galstyan *et al.* (2003) consider the Minority Game as a version of congestion games. Galib and Moser (2011) uses the Minority Game as a model to study the road traffic optimization. Cavagne (2021) finds out that the dynamics of the Minority Game are completely independent from the memory size of the agents, and the only important information is that agents have the same information whether the information is true or false.

2.5. Modelling the EFBP

There are different experiments and various models on the EFBP in the literature. One common agent-based simulation environment that is used to model EFBP is NetLogo. Garofalo (2006) explains how he implemented the EFBP on the NetLogo environment in his article "Modeling the 'El Farol Bar Problem' in NetLogo". He discusses the assumptions of Arthur, the predictors working mechanism, and parameters of the model. While constructing the model in NetLogo, he tries to stick to Arthur's original assumptions and information shared in the article "Bounded Rationality and Inductive Reasoning". Although we are not going to use the NetLogo language and environment to create our model, which we will discuss the reasons in the Chapter 4, the Garofalo's article will be one of the references for our model.

Another important article about modelling the EFBP is written by Fogel *et al.* (1999) with the title "Inductive Reasoning and Bounded Rationality Reconsidered". Fogel *et al.* (1999) take the Arthur's challenge which is to enrich the predictors with genetic algorithms instead of using a "bag of strategies", so that the agents will use "more intelligent" predictors. They create their model by using a genetic algorithm that helps to eliminate ineffective prediction models. Surprisingly, they find the following result unlike the Arthur's expectation: "The mean attendance for the steady state is 56.3155 with a standard deviation of 1.0456" (Fogel *et al.*, 1999). This finding leads Rand and Stonedahl (2007) to study the relationship between the "computational power" and attendance at the bar in the EFBP. They use the NetLogo model (Wilensky and Rand, 2015) with the formulation of Fogel *et al.* (1999) for different levels of computational power and their results support Fogel *et al.* (1999)'s findings; "As we increase the computational power of the agent, the average attendance at the bar decreases." (Rand and Stonedahl, 2007). Furthermore, they analyze the efficiency with suggesting the term "societal benefit". Ponsiglione *et al.* (2015) also conduct simulation experiments about efficiency and fairness in the EFBP using four different models. As a brief result, they state that efficiency does not guarantee the fairness and the fairness can be observed only in the random choices case.

St Luce and Sayama (2020) study the agents with different decision strategies and the strategy switching dynamics in the EFBP. They create six different decision strategies which are "Random", "Enthusiastic", "Last Correct", "Last Incorrect", "Always" and "Never". We are not getting into details of this analysis in this chapter of the thesis, but the idea of investigating different decision strategies or different agent types in the same environment inspired us to suggest further experiments in Chapter 12.

3. PROBLEM DESCRIPTION

The EFBP is modelled and studied from various perspectives in the literature. In this thesis, we want to conduct an analysis on the EFBP by creating different expectation models. The comparative analysis of the expectation models with respect to the performance measures allows us to make inferences about this complex adaptive system. We will discuss the characteristics of the EFBP with the help of the different expectation models. Furthermore, we aim to determine which agent type will be more successful with respect to different performance measures and explain the reasons behind this success.

First, we discuss the original model with its assumptions and then rebuild the model of the EFBP with different agent types. The assumptions should be clearly identified, and the performance measures should be well defined in order to see the dynamics of the problem. The agent types help us to reach some conclusions and suggest a "solution" for the problem. By comparing different agent types who have different expectation models, we can analyze the dynamics of the problem in a more detailed way.

3.1. The El Farol Bar Problem

Arthur (1994) models the EFBP as an agent-based simulation model. Even though Arthur does not specify some parameters and formulations of the problem (Garofalo, 2006; Rand and Stonedahl, 2007), there are plenty of information in the article "Inductive Reasoning and Bounded Rationality" to have an understanding on the nature of the problem. For the missing parameter values on Arthur (1994) we will use Garofalo's (2006) assumptions and make our own assumptions when it is necessary while building our agent-based models.

Arthur assumes n people who decide whether to go to the bar or not each week, where n is 100. The bar has a certain capacity, and the bar capacity is taken as the 60% of the whole population, namely, 60. Agents decide according to their expected number of people who will attend the bar on that week, whether to go to the bar or to stay home. If the expectation of an agent is less than 60 people, the agent decides to go to the bar, otherwise (if the expectation is greater than or equal to 60 people) the agent stays home. There is no communication between agents and the experience they had in the previous weeks does not affect the decision of the agents. The only decision parameter is the weekly expectation value for each agent. Nevertheless, Arthur (1994) states that "the only information available is the numbers who came in past weeks" and we can claim that agents decide like a "statistician". In this case, deductive reasoning fails to create a good utilization of the bar because of the limited information in the nature of the problem. In other words, if the agents use the same information (i.e., attendance values) and same expectation models as deductive reasoning suggests, the expectations of the agents do not differ. Consequently, "if all believe few will go, all will go" and "if all believe most will go, nobody will go" (Arthur, 1994).

Because of the ill-defined nature of the problem from the deductive reasoning perspective, Arthur decides to assign different set of strategies to each agent randomly. Each strategy corresponds to a predictor. He assumes that each agent has "k" predictors for the weekly expectations and the "active predictor" determines the weekly expectation value for the given week. The active predictor is the most accurate predictor which is found according to the accuracy of the predictors with respect to the new attendance value.

Arthur gives some examples of the "bag of strategies" which he uses in his model. However, he does not specify the value of "k", namely, the number of predictors each agent has. Also, he specify neither the total number of strategy nor any algorithm including the memory size of the agents. In this case, we refer to Garofalo's parameters (2006) and see that he takes the value of k as 6, the memory size as 20 weeks, and the total number of strategies as 200. Arthur calls the set of strategies an "alphabet soup" in which the predictors are assigned to each agent randomly. He also claims that the useless predictors do not cause any muddy behavior because the useless parameters do not become the active predictor and they do not determine the agents' expectations and decisions. He even brings up a challenge for the readers to experiment with "more intelligent" predictors obtained from more sophisticated ways such as genetic algorithms. He argues that the system behavior will not change in this case in terms of the convergence of the mean attendance. His results show that the mean attendance always converges to 60 and he claims that the agents "self-organize" in this environment. Arthur explains this result with the following sentences:

> *"To get some understanding of how this happens, suppose that 70 percent of their predictors forecasted above 60 for a longish time. Then on average only 30 people would show up; but this would validate predictors that forecasted close to 30 and invalidate the above 60 predictors, restoring the 'ecological' balance among predictions, so to speak. Eventually the 40-60 percent combination would assert itself."* (Arthur, 1994)

3.2. Arthur's Assumptions

We have to present the original assumptions of Arthur (1994) about the EFBP in order to understand the nature of the problem and to create agent types who have different expectation models. The original assumptions are given below.

- The population does not change (no migration, no births, no deaths)
- Everyone is willing to attend the bar each week. (Everyone is assumed to be an adult)
- The bar capacity is fixed, and the value of the bar capacity is 60.
- The bar capacity is known by the agents.
- There is no communication between the agents. The agents make their decisions without knowing the attendance for that week.
- The agents learn each week's attendance as soon as it is realized. There is no delay on learning the weekly attendance.
- Agents make their decisions according to their individual expectations.
- "Choices are unaffected by previous visits." (Arthur, 1994)

- Agents use “k” sets of predictors. (strategies)
- Bar capacity can be exceeded.
- If the attendance is greater than the bar capacity all attendees become “unhappy”.
- If the attendance is less than or equal to the bar capacity all attendees become “happy”.

3.3. Research Questions

In this thesis, we seek answers to the following questions:

- Are the strategies that are used by Arthur meaningful?
- Which expectation model produces better performance values?
- What is the effect of the availability of the knowledge from environment by the agents such as the bar capacity value and the weekly attendance values?

There is no exact solution or optimum solution for the EFBP (Challet and Zhang, 1997). This situation is caused by the nature of the problem, and it is necessary to study the problem further to understand this phenomenon.

4. METHODOLOGY

The EFBP can be constructed as an agent-based model and almost all studies in the literature used agent-based modelling as the main methodology. Our primary approach is also agent-based modelling and simulation, which is consistent with the literature. We use statistical analysis to compare performances of the different expectation models, in other words, different agent types.

We develop our own agent-based simulation instead of using the existing models in the literature. There are couple of reasons that we choose to have our own model. First of all, we do not prefer any standardized software for simulation such as NetLogo that is commonly used on modelling the EFBP in the literature. Nevertheless, we choose to run our simulation models using the computer language Python because it is more flexible in terms of adding different modifications and reporting variables. With the help of Python interface, we manage to adapt many modifications as will be explained in the following chapters. Moreover, Python enables us to code different performance measures as we see fit. Furthermore, for the adaptive learning process of boundedly rational agents we prefer to use the technique of exponential smoothing which is commonly used in System Dynamics literature (Sterman, 1987).

5. MODEL OVERVIEW

This chapter consist of two sections; the flowchart and code. The flowchart presents the generic algorithm for the model and the code shows the coding details of the model in Python environment.

5.1. The Flowchart

We develop the generic flowchart of the algorithm for the EFBP. The flowchart of the model shows the agents' decision mechanism, loops, and reporting variables. The generic flowchart is given in Figure 5.1.

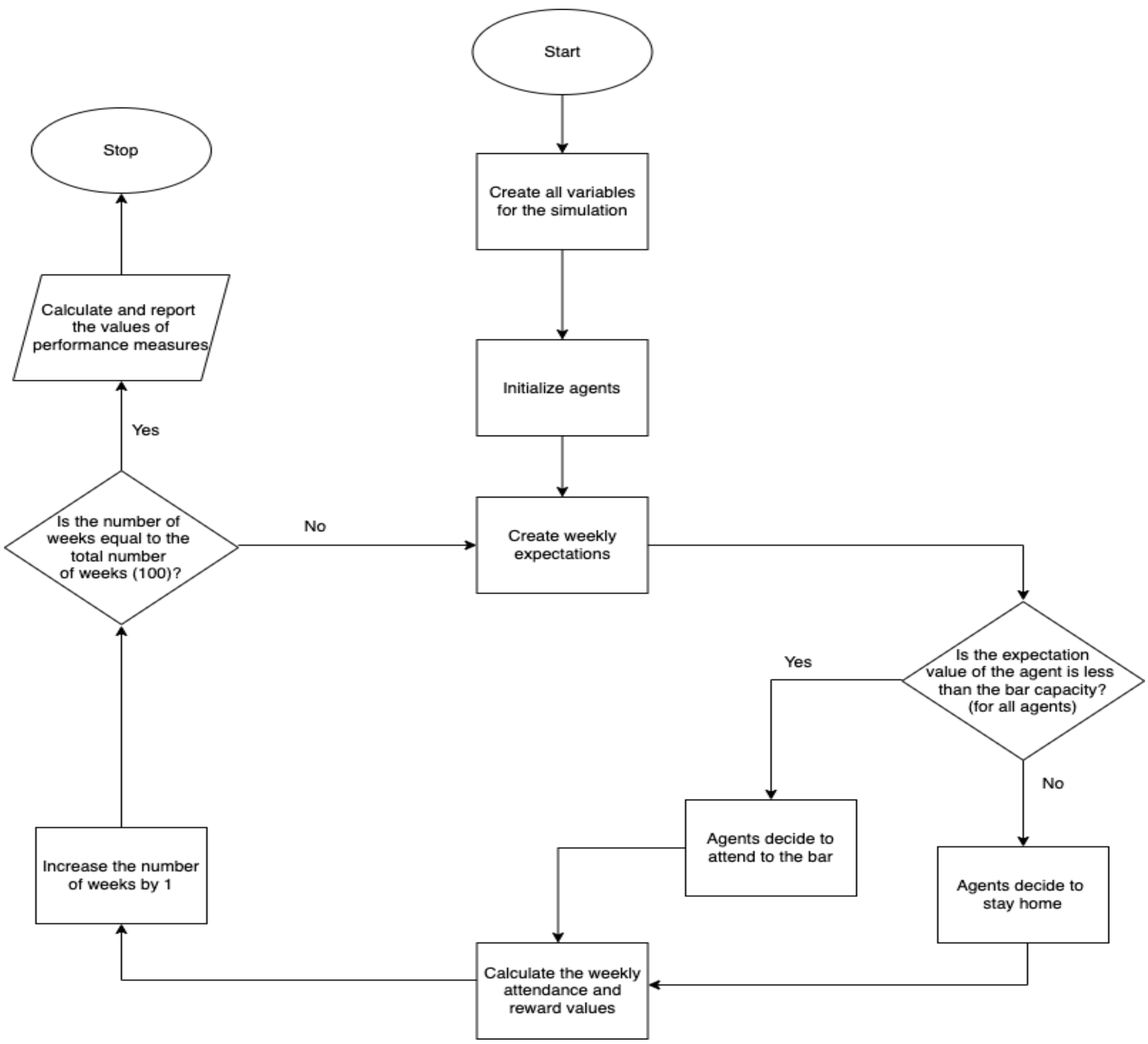


Figure 5.1. Flowchart of the model.

5.2. The Code

We use Python language to create the agent-based simulation model as we mentioned in Chapter 4. First, we import the libraries that we need. The library "numpy" is used for creating vectors and matrices, the library "random" is used for creating random numbers, the library "statistics" is used for the statistical analysis, and the library "matplotlib.pyplot" is used for drawing plots.

After we have imported the libraries, we define the function called elfarol that is dependent on two variables: Number of agents (n) and number of weeks (Weeks). n and Weeks are both taken as 100, but they can be adjusted for different experiment designs. For the counter variables of Weeks and n, the symbols "s" and "i" will be used respectively.

The function elfarol contains the matrices which have two indices such as n and Weeks. First, we create the "Expectation" matrix which determines the weekly expectation about the attendance on the bar for each agent. The expectation formulation is different for our three different types of agents (random expectations, Arthur/Garofalo bag of strategies, agents with adaptive learning) which we will introduce in the following chapter.

Then, "Decisions" matrix is created. It indicates the weekly decisions whether attend or not to the bar for each agent. Every element of the Decisions matrix is binary; if an agent decides to go to the bar on that week, the value in the Decisions matrix becomes one, if the agent stays home, the value is zero.

The weekly attendance is saved in "Attendance" matrix which is only dependent on Weeks. Unlike the Expectation and Decisions matrices, the Attendance matrix is not dependent on the number of agents (n). The weekly attendance is calculated by the sum of the number of agents who decide to attend the bar on that week. The equation of the weekly attendance values can be written as

$$Attendance_s = \sum_{i=1}^{n} Decisions_{s,i} \quad where \quad s \in \{1, 2, ..., 100\}. \tag{5.1}$$

After the attendance is found, the state of agents is determined in terms of correct decision. The agents who make the correct decisions gain the reward value of one. Otherwise, they get zero as the reward value. In other words, if the weekly attendance is less than or equal to the bar capacity the agents who attended to the bar are rewarded and if the weekly attendance is greater than the bar capacity, the agents who stayed home are rewarded. On the other hand, the agents who attended a crowded bar or stayed home when the bar is uncrowded get a reward value of zero. The individual reward values are calculated on weekly basis, and they are summed up for each agent at the end of the simulation. The equation of the reward values by agents can be written as

$$Reward_i = \sum_{s=1}^{TotalWeeks} CorrectDecisions_{s,i} \quad where \quad i \in \{1, 2, ..., n\}. \tag{5.2}$$

6. THREE DIFFERENT EXPECTATION MODELS

We create three different expectation models for the agents in the EFBP. First expectation model is random expectations which we call Type 1 Agents. The second expectation model is Arthur/Garofalo's mix expectation model (bag of strategies) which we call Type 2 Agents. Lastly, we will create adaptive learning expectation model using exponential smoothing technique and we call these agents as Type 3 Agents. The detailed information and expectation formulations are given in the following sections.

6.1. Algorithm for Type 1 Agents

Type 1 Agents have individual random expectation models. They simply create their expectations randomly without using any strategy for each week. A random number is assigned to each agent for each week to determine the agents' weekly expectation. The random number is uniformly distributed between 0 and 99 so that the expected number of people for an agent on the given week can be any integer between 0 and 99 randomly. The probability of each value is the same, namely, 0.01. The distribution of expectation values can be expressed as $Expectation_{s,i} \sim$ DISCRETE U[0, 99].

The attending mechanism works that way; the simulation generates random number between zero and one for each agent. Then, the random number is multiplied by n (where n is 100) to create the expectation value between 0 and 99. We take the integer part of the result of this multiplication as the expectation value. If the expectation of agent is less than the bar capacity, the agent decides to attend the bar. If the expectation of agent is greater or equal to the bar capacity the agent does not attend the bar. The flowchart of the algorithm for Type 1 Agents is given in Figure 6.1.

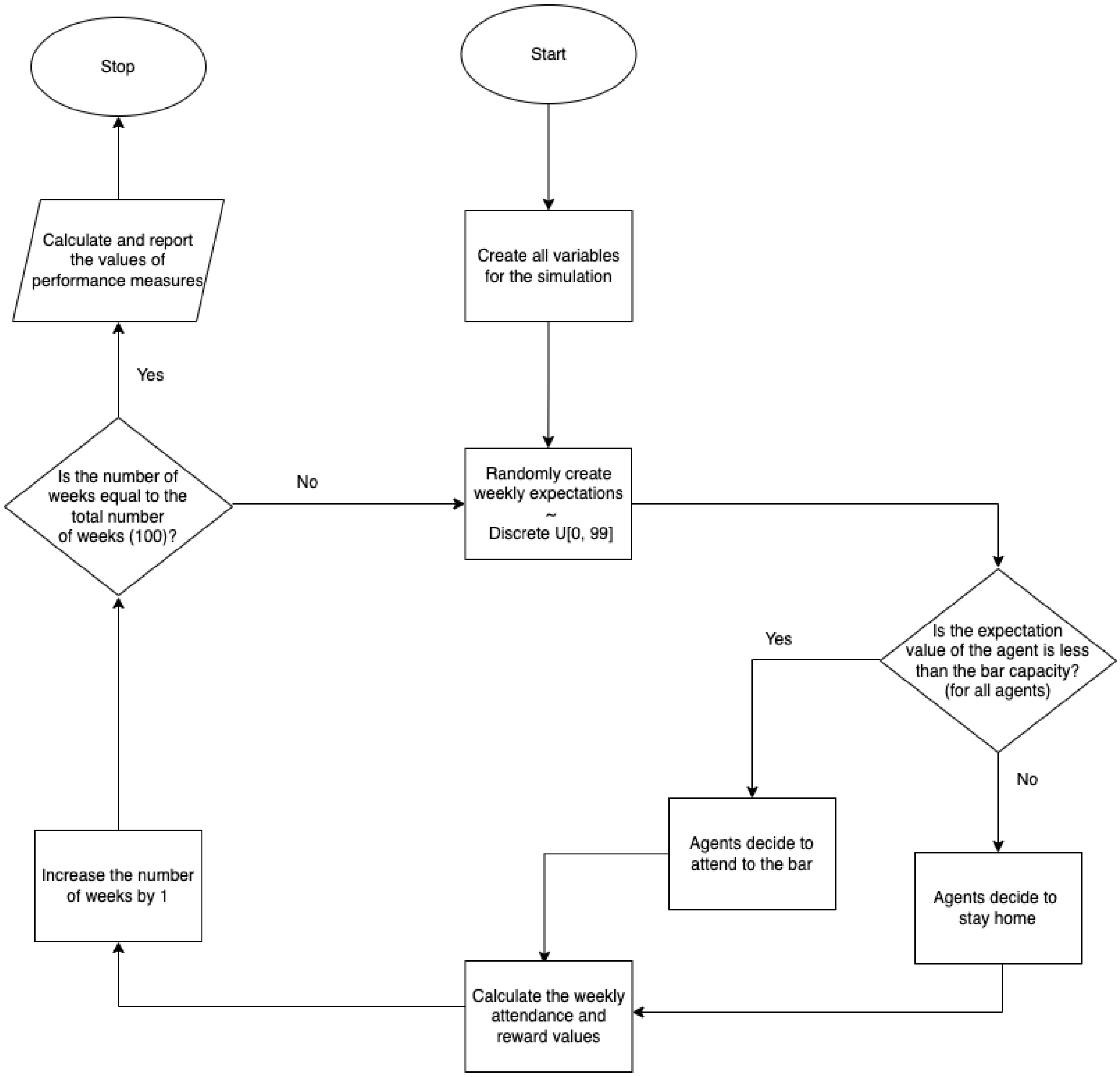


Figure 6.1. Flowchart of the algorithm for Type 1 Agents.

6.2. Algorithm for Extension of Type 1 Agents

We also want to use the extended version of random expectation model. In this case, we try to imitate Arthur's (1994) strategy switching mechanism with random expectations. As Arthur created some strategy sets in his model, we also create m strategies where m is 200 as Garofalo (2006) states. But unlike the Arthur's case, the strategies do not contain any "calculated" expectation values. These m strategies create random expectation values for each week, and these random expectation values are uniformly distributed between 0 and 99 like Type 1 Agents.

The k of these m strategies are assigned to each agent in Arthur's model. Our model is more flexible in terms of assigned strategies. We create a variable to determine the maximum number of strategy that each agent can have. Moreover, we can assign different number of strategies to each agent with the minimum number of two. As we stated earlier, Arthur does not specify the exact value of k, and we will take the Garofalo's assumption as a reference value to make comparison with our model.

After the strategies are assigned, the expectation value for the agents in each week is determined by the "active predictor". The active predictor is updated after each week according to the error term. The error term is calculated using new attendance value with an exponential smoothing formula (Ponsiglione *et al.*, 2015). The active predictor becomes the most accurate one in terms of lowest error term among the assigned strategies. The error term equation for each strategy can be expressed as

$$Error_{s,j} = (1 - \lambda) * Error_{s-1,j} + \lambda * |Strategy_{s,j} - Attendance_s| \tag{6.1}$$

where j is the counter variable for number of strategies.

The decision mechanism of going to the bar remains the same. If the expectation of agent which is taken from active predictor's expectation value is less than the bar capacity, the agent decides to attend the bar. If the expectation of agent is greater or equal to the bar capacity the agent does not attend the bar. The reason that we create the extended version of random expectation is to discuss about the strategies that Arthur creates. The flowchart of the algorithm for Type 1 Agents Extended is given in Figure 6.2.

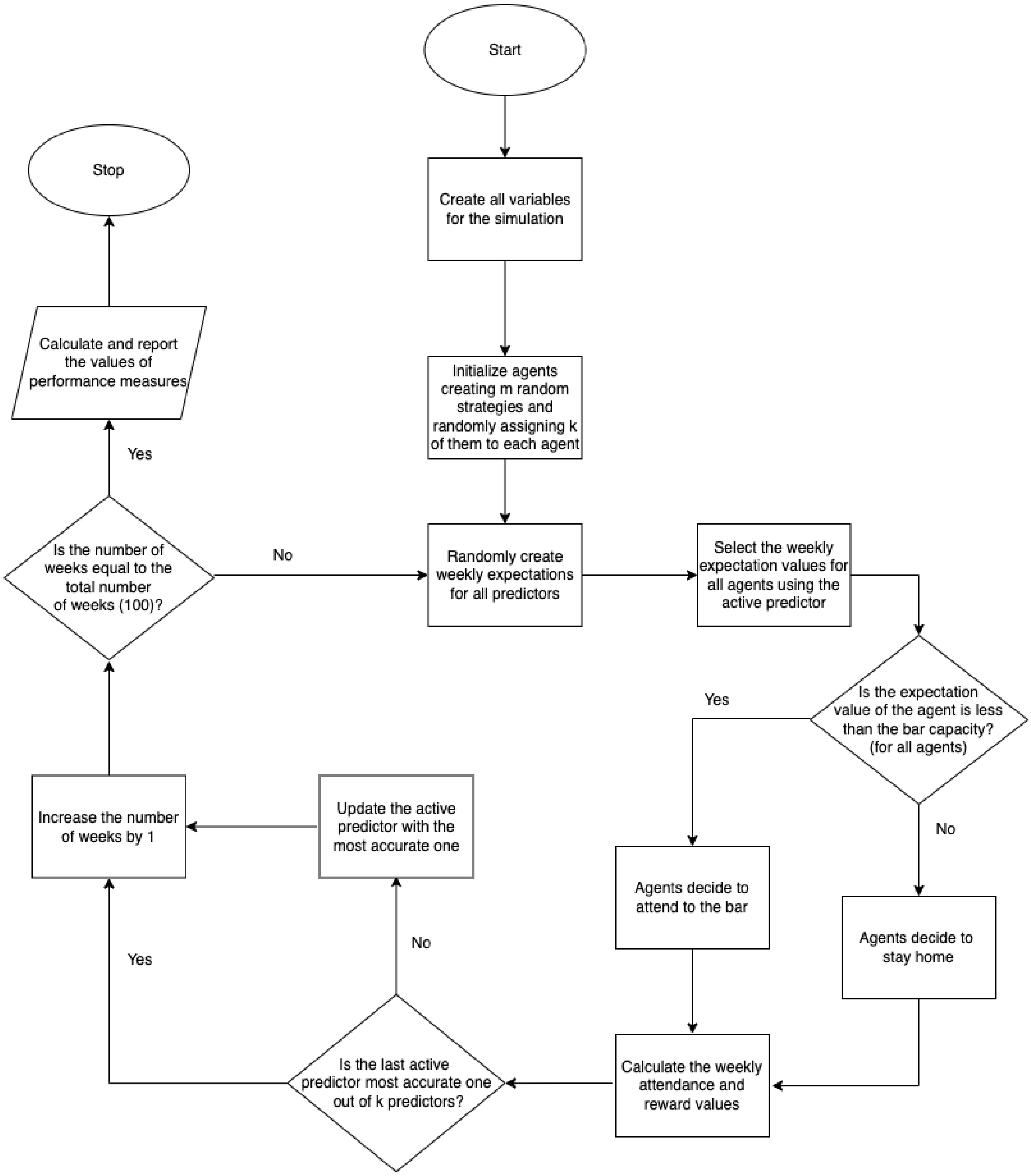


Figure 6.2. Flowchart of the algorithm for Type 1 Agents extended.

6.3. Algorithm for Type 2 Agents

We want to create Arthur/Garofalo's agents with our code. We use the same decision-making mechanism with the extended version of random expectation. However, in this case we do not create the strategies with random expectations, instead we use some "calculated predictors", in other words, "bag of strategies". Arthur (1994) gives very limited information about his predictors. He just presents few examples which include tit-for-tat, mirror image, moving average, trends, and period cycles. It is not obvious that which parameters and how many strategies are used in the article (Arhur, 1994). So, we use the strategies given by Garofalo (2006) and Ponsiglione *et al.* (2015) which include more detailed formulation and the memory size for the agents. The memory size is given as 20 weeks and the strategies consist of trends, moving averages, fixed rules and tit-for-tat. Moreover, if the history of the attendance information is not enough for prediction, the predictor uses random expectation (Garofalo, 2006). For example, a predictor takes the five weeks moving average to predict the expectation for the given week, but if the simulation is on the third week, then the predictor uses random expectation.

For Type 2 Agents in our simulation, we use the following strategies with memory size 20 and total number of strategies 200; tit-for-tat, mirror image of tit-for-tat, fixed rules, trends, mirror image of trends, moving averages, pessimist and optimist predictors. These 200 strategies consist of the combination of 10 different prediction method and 20 weeks memory size for each of the prediction.

After each week, the new attendance value is updated, and the error term is calculated. According to the error term of each predictor, the active predictor for each agent is determined. The error calculation and choosing the active predictor mechanism is the same with the extended random expectation model. The flowchart of the algorithm for Type 2 Agents is given in Figure 6.3.

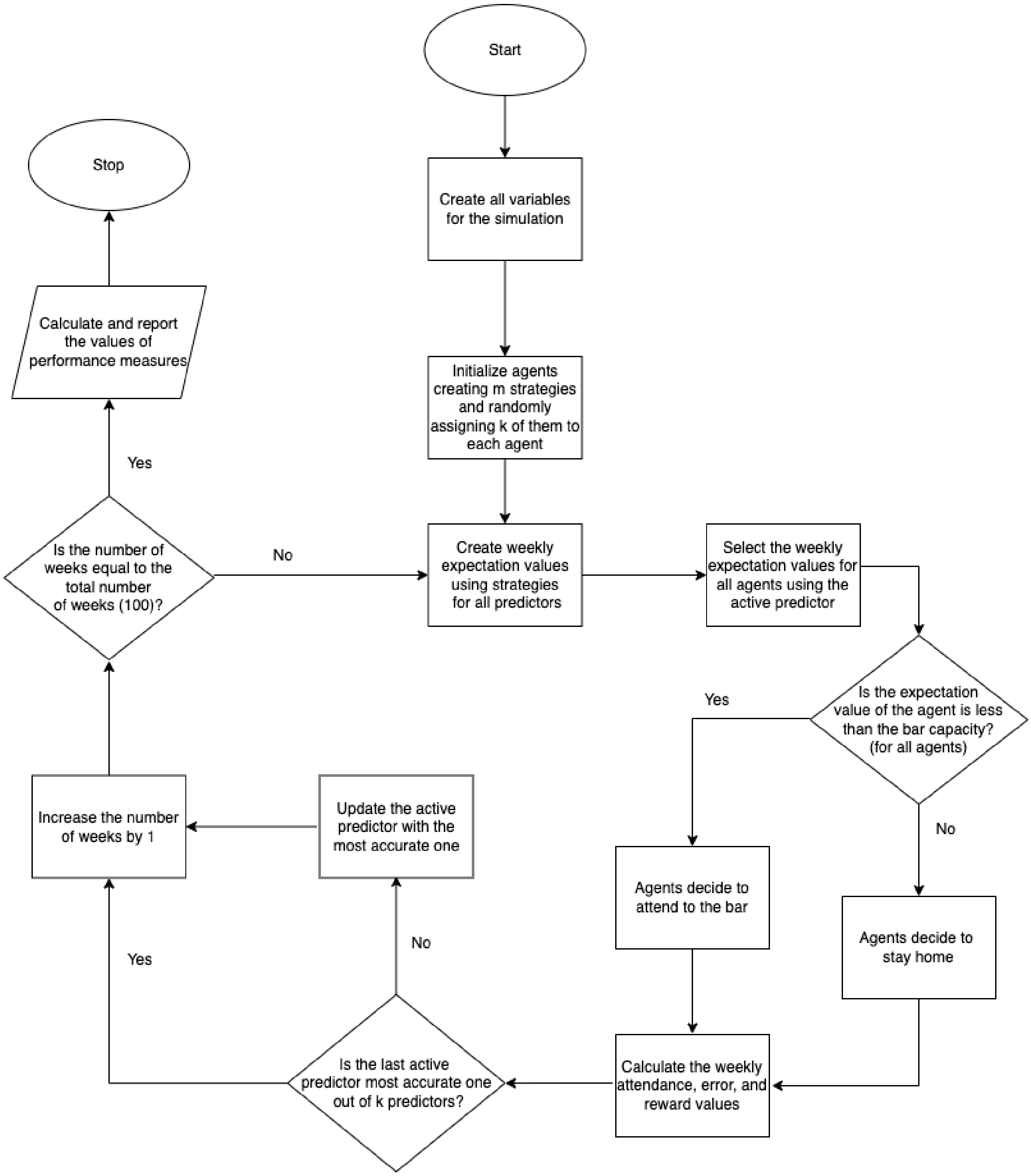


Figure 6.3. Flowchart of the algorithm for Type 2 Agents.

6.4. Algorithm for Type 3 Agents

Our third expectation model creates boundedly rational agents with adaptive learning. Various learning mechanisms are used to experiment with the EFBP. However, there are no studies in the EFBP literature using the exponential smoothing. We call it Type 3 Agents and this agent type is our contribution to the EFBP literature.

Sterman (1987) claims that exponential smoothing is a good way to model adaptive learning. In other words, he describes an inductive method to reflect behavioral expectations of humans in a dynamic simulation model, namely exponential smoothing (Sterman, 1987).

In order to create Type 3 Agents, we first create individual alpha values (α_i) for each agent. The alpha value indicates the weight of the new information in the exponential smoothing formulation, and it is called smoothing parameter. The alpha values are randomly distributed between 0.1 and 0.3 as suggested in the literature. The distribution of alpha values can be expressed as $\alpha_i \sim$ U[0.1, 0.3).

If the alpha value is close to 0.3, the weight of the new attendance information is more significant on creating the new expectation value. On the other hand, if the alpha value is close to 0.1, the new attendance value has a lower contribution on creating the new expectation value and the previous expectation value has a higher weight on the calculation. The initial expectation values are created randomly. The equation for expectation calculation with exponential smoothing can be expressed as

$$Expectation_{s,j} = (1 - \alpha_i) * Expecation_{s-1,j} + \alpha_i * Attendance_{s-1}. \tag{6.2}$$

After each week, the new attendance value is updated, and the expectation values are created using individual alpha values. The individual alpha values make each agents' expectation model to differ. If all agents used the same alpha value, they would have the same expectation model. Thus, it would cause the dynamic behavior that would emerge from deductive reasoning as it is shown in Figure 6.4. Arthur (1994) expresses this case as "if all believe few will go, all will go" and "if all believe most will go, nobody will go".

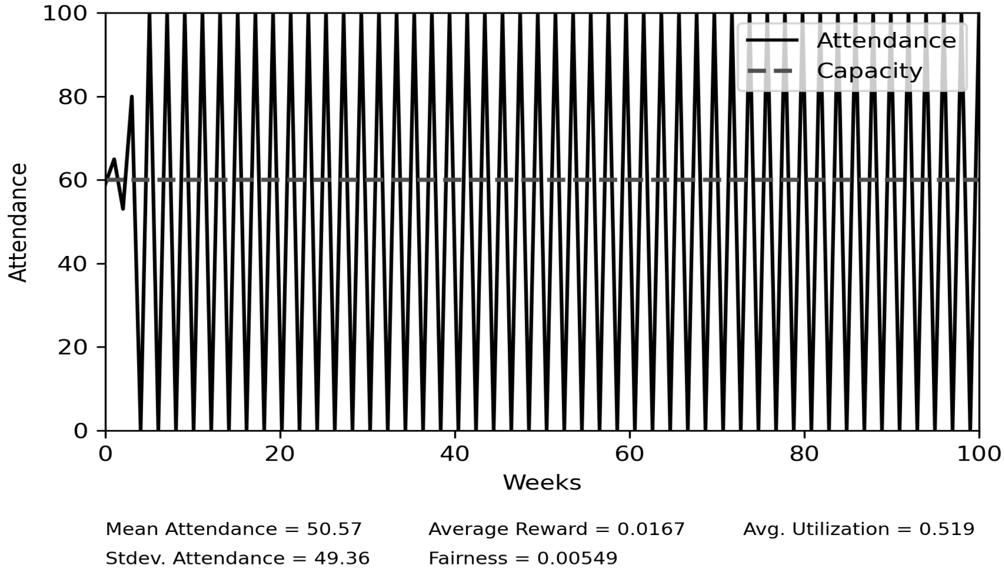


Figure 6.4. Type 3 Agents with same alpha value.

The flowchart of the algorithm for Type 3 Agents is given in Figure 6.5.

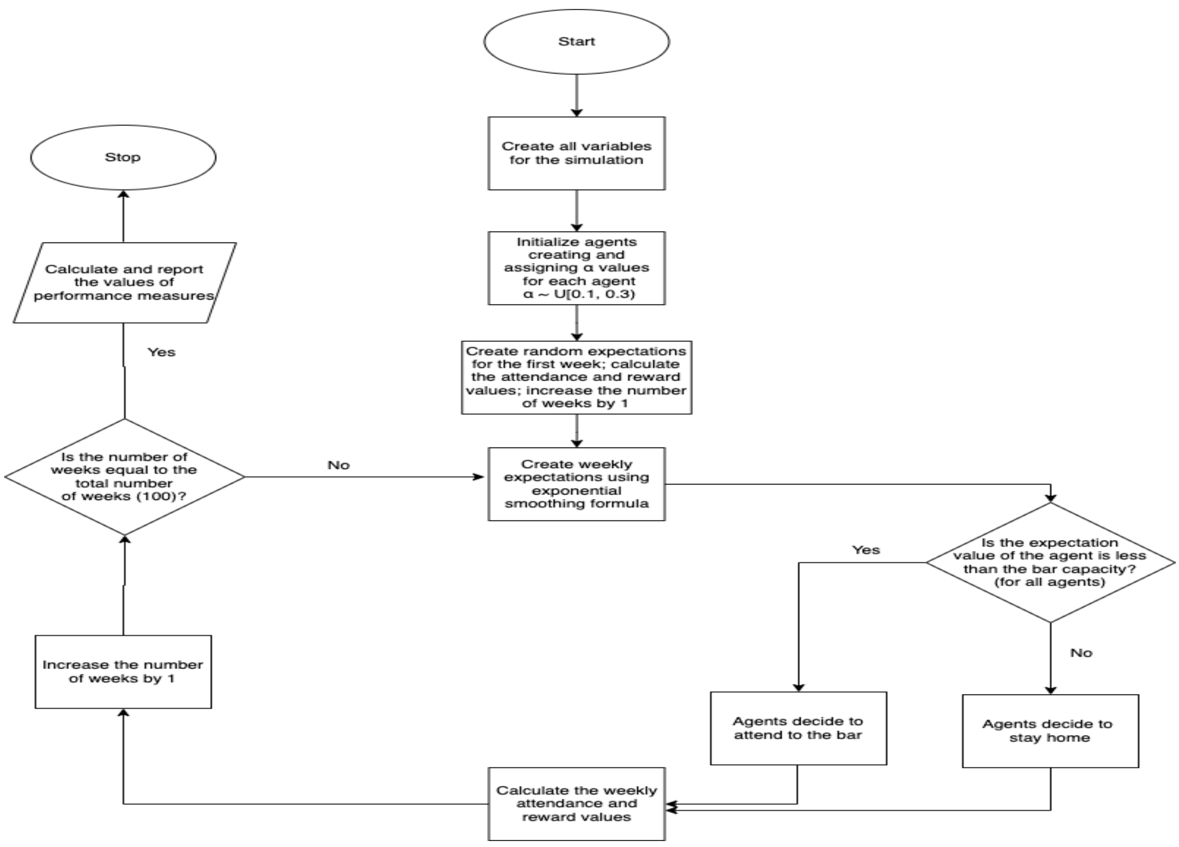


Figure 6.5. Flowchart of the algorithm for Type 3 Agents.

7. PERFORMANCE MEASURES

We conduct the statistical analysis using performance measures. The performance measures are introduced in this chapter, and the detailed formulations are given with the related equations. The performance measures that we will use for the comparative results for different agent types are mean attendance, standard deviation of attendance, reward, fairness, and bar utilization.

7.1. Mean Attendance

The mean attendance is the one of the most significant performance measures in the EFBP. The weekly attendance values are calculated by counting the number of agents who decide to attend the bar according to their expectation model. During each simulation period, which is 100 weeks for the most cases, the mean of the weekly attendance values is calculated to obtain the mean attendance. The weekly attendance and mean attendance equations can be expressed as

$$Attendance_s = \sum_{i=0}^{n} Decisions_{s,i} \quad where \quad s \in \{1, 2, ...100\} \tag{7.1}$$

$$MeanAttendance = \frac{\sum_{s=1}^{TotalWeeks} Attendance_s}{TotalWeeks}. \tag{7.2}$$

Arthur (1994) states that mean attendance always converges to 60 which is the bar capacity value. So, we expect to see that the mean attendance values also converge to 60 in the long term for our simulation models. Note that the mean attendance values are continuous, and the range is between 0 and 100. If nobody attends to the bar for 100 weeks, the mean attendance becomes 0, and if all agents attend the bar each week, the mean attendance becomes 100.

7.2. Standard Deviation of Attendance

We introduce the mean attendance as a performance measure in the EFBP. Arthur (1994) builds his arguments based on the mean attendance statistics. However, mean attendance values are not sufficient to comment on the different expectation models in the EFBP. The standard deviation of attendance values is also used to compare different models' performances in the literature (Fogel *et al.*, 1999; Garofalo, 2006; Rand and Stonedahl, 2007). We also obtain the standard deviation of attendance values for each simulation model and compare them to see which model performs better in terms of performance measures. The equation of the standard deviation of attendance can be written as

$$\sigma = \sqrt{\frac{\sum_{s=1}^{TotalWeeks}(Attendance_s - MeanAttendance)^2}{TotalWeeks - 1}}. \tag{7.3}$$

In the EFBP, high standard deviation causes a worse performing expectation model. So, we aim for a low standard deviation with convergence of the mean attendance to the bar capacity value. The standard deviation can vary between 0 and 50. For the extreme cases; if all attendance values are the same, we obtain 0 as standard deviation of attendance, and if the attendance values become minimum and maximum values (0 and 100) for all the consecutive weeks, the standard deviation of attendance becomes 50. These are the minimum and maximum values for the standard deviation of the attendance values.

7.3. Reward

The term reward is used by Wilensky and Rand (2015) while creating the NetLogo model for the EFBP. They count the enjoyable nights at the bar for each agent and call this value as reward. In other words, if an agent attends the bar and the bar is not crowded on that day, the agent gains a reward which has a payoff value of one. For all the other cases (i.e., staying home, attending the bar when it is crowded), the agents get no reward, namely, zero payoff. We create a similar performance measure and use the same term as "reward" for consistency. However, we modify the assumption and create a "correct decision reward". In other words, the agents are rewarded according to the number of correct decisions they made. The correct decision is defined as follows; if an agent attends the bar when the bar is uncrowded or if the agent stays home when the bar is crowded. In these two cases, the agent is rewarded with a reward value of one. On the other side, if the agent makes a "wrong decision"; attends the bar when it is crowded or stays home when the bar is uncrowded, the agent gets a reward value of zero.

After the individual reward values are calculated for each agent, the average reward of the model can be found. The average reward is calculated simply as the mean of the reward values by agents. The equations for the individual reward values for each agent and the average reward can be expressed as

$$Reward_i = \sum_{s=1}^{TotalWeeks} CorrectDecisions_{s,i} \quad where \quad i \in \{1, 2, ..., n\} \tag{7.4}$$

$$Avg.Reward = \frac{\sum_{i=1}^{n} Reward_i}{n}. \tag{7.5}$$

7.4. Fairness

The reward values can be taken as a performance measure in terms of efficiency in the EFBP: There is another performance measure in the EFBP called "fairness". The agents may utilize the bar in an efficient way, but is the bar utilization really fair in this case? To answer this question, we have to obtain the distribution of the reward values. Ponsiglione *et al.* (2015) seek the answer to this question in their article "The Fairness/Efficiency Issue Explored Through El Farol Bar Model". They use "Gini coefficient" to analyze the EFBP with four different models in terms of fairness and conclude that "no deliberate strategy beats purely random choices". They also claim that an efficient utilization of the bar does not mean a fair usage of the resources.

Although, we are inspired by the Ponsiglione *et al.* (2015) about examining the fairness issue in our model, we will not use the same performance measure which is the Gini coefficient. Instead, we define a new performance variable called "Fairness" to calculate the general fairness of the expectation model because the Gini coefficient measures the fairness of the distribution. However, we use Mean Absolute Deviation (MAD) as a measure of the deviation in the reward values. The MAD value will give us the information about the fairness of the expectation model. MAD is calculated by the absolute difference of each reward value and the mean reward value, and the result is divided by the total number of agents (n). The fairness value can be between zero and one, and a value closer to the zero indicates a more fair distribution of the reward values by agents. The equation of fairness can be written as

$$Fairness = \frac{\sum_{i=1}^{n} |Reward_i - Avg.Reward|}{n}. \tag{7.6}$$

7.5. Bar Utilization

Bar utilization is a performance measure that we add to see the utilization of the resources from a different perspective. The bar utilization shows what proportion of the bar capacity is used. It is calculated each week by dividing attendance values with the capacity values. The handicap of this performance measure is that if the bar is overcrowded for a week, the bar utilization value becomes maximum. However, crowdedness is not a desirable case in the EFBP. On the other hand, bar utilization measure is beneficial to distinguish between the real good performances and the cases where the standard deviation is low, but the bar capacity is not efficiently used. The bar utilization can be between zero and one. If the weekly attendance value is greater than or equal the bar capacity, the weekly bar utilization becomes one, and if the weekly attendance value is less than the bar capacity, the weekly bar utilization is the ratio of weekly attendance to the bar capacity. The weekly bar utilization and bar utilization equations can be expressed as

$$WeeklyBarUtilization_s = \left\{ \begin{array}{ll} 1, & \text{if } Attendance_s > BarCapacity \\ \frac{Attendance_s}{BarCapacity}, & \text{if } Attendance_s \leq BarCapacity \end{array} \right\} \tag{7.7}$$

$$BarUtilization = \frac{\sum_{s=1}^{TotalWeeks} WeeklyBarUtilization_s}{TotalWeeks}. \tag{7.8}$$

7.6. Deviation of Attendance from Capacity

An additional performance measure is necessary when the bar capacity is unknown in the EFBP. The unknown bar capacity case is discussed in Chapter 9. When the bar capacity is unknown for the agents in the EFBP, the performance measures such as standard deviation of attendance, reward, and bar utilization are unable to detect a poor performance. We have to introduce a performance measure which can distinguish between the agents who can learn the actual bar capacity when the bar capacity is unknown. We will call this performance measure as Deviation of Attendance from Capacity and use "Deviation A-C" as an abbreviation. This performance measure is equal to standard deviation of attendance for the experiments with known bar capacity values because the mean attendance converges to the bar capacity. Therefore, we do not need this performance measure for the cases in which agents know the actual bar capacity as an assumption. However, if the bar capacity is not known by the agents, this performance measures indicates a poor performance in terms of learning the actual bar capacity. In the equation of deviation of attendance from capacity, the capacity acts as the desired mean attendance value. The equation of this performance measure can be expressed as

$$DeviationA - C = \sqrt{\frac{\sum_{s=1}^{TotalWeeks} (Attendance_s - Capacity_s)^2}{TotalWeeks - 1}}. \tag{7.9}$$

8. EXPERIMENTS AND RESULTS FOR THE THREE DIFFERENT EXPECTATION MODELS

In this chapter, we present the experiment results of the three different agent types. However, we need a reference frame which will be the Arthur's (1994) results in the EFBP to compare the results with respect to the performance measures. The experimental design and the comparative results are given in the following sections starting with Arthur's results.

8.1. Arthur's Results

We introduced five different performance measures for the EFBP. These performance measures provide a basis to compare the expectation models' outputs with each other as well as the models in the literature. In terms of these performance measures, we are seeking for a model with a converging mean attendance to the bar capacity (60 in the regular case), a low standard deviation, a high reward value (close to 0.48 when the bar capacity is 60), a fairness value close to zero, and bar utilization value close to one.

We introduce the three different expectation models for the EFBP. Arthur's original model and the experiment results of the model in the article "Inductive Reasoning and Bounded Rationality" will be our reference to comment on the behaviors of our expectation models. So, we take the weekly attendance values from the article and plot the Arthur's result. The plot for the Arthur's results is given in Figure 8.1.

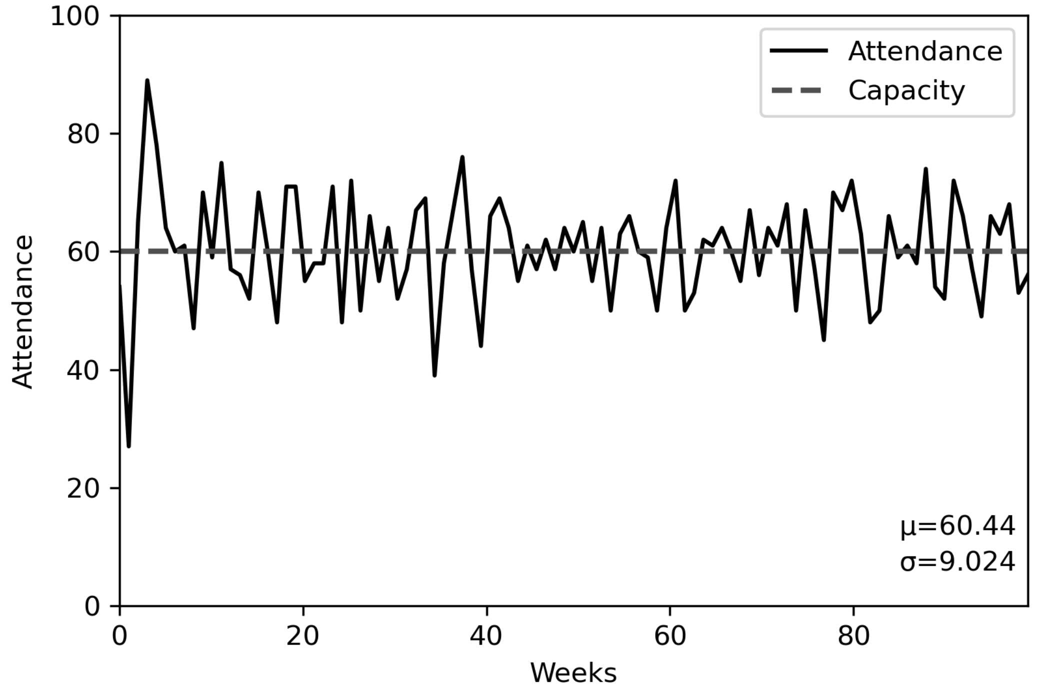


Figure 8.1. Arthur's EFBP results.

Then, we design the experiments for the three different expectation models, namely, Type 1 Agents with random expectations and its extended version; Type 2 Agents using Arthur and Garofalo's bag of strategies; Type 3 Agents with exponentially smoothed expectations. For the experiments, first we define all the parameters and set them accordingly. The number of weeks and the number of agents are 100 for all simulations (Weeks = 100 and n = 100). We also use fixed seed in order to avoid any undesirable effect of randomness. The fixed seeds help us to compare the results. There are other parameters such as the number of strategies for each agents for Type 2 Agents which is shown with the symbol "k" in the original article (Arthur, 1994), the number of total strategies (m), the weight for calculating the error term on each strategy (λ), and individual alpha values (α_i) for each agent which indicate the weight of new attendance information for calculating the new expectation value in the exponential smoothing formulation for Type 3 Agents. We will discuss these parameters in detail in the relevant part for each experiment.

8.2. Results for Type 1 Agents

Type 1 Agents create their expectations randomly between 0 and 99. All 100 values of expectations are equally likely. The agents decide to go to the bar according to their individual random expectation value. The parameters for Type 1 Agents are given below and the simulation result which shows the weekly attendance values is shown in Figure 8.2.

Parameters:

- Weeks = 100
- n = 100
- Random seed = 1

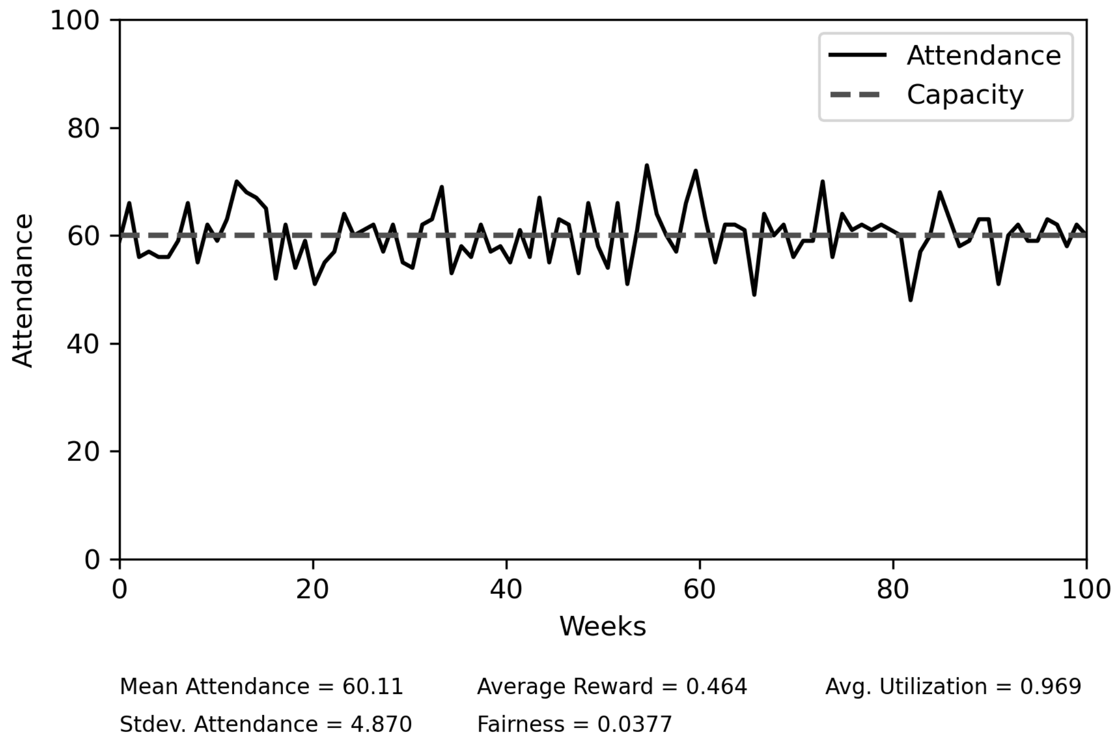


Figure 8.2. Type 1 Agents.

The mean attendance and the standard deviation of the attendance values are equal to 60.52 and 5.098 respectively. As we compare Type 1 Agents with the reference experiment of Arthur (1994), we find out that the mean attendance values are very close to each other, and they both converge to 60 which is the bar capacity value in the long term. However, the standard deviation of Type 1 Agents is less than the Arthur's result. It can be also understood by the behavior of attendance plot. The output of Type 1 Agents fluctuates around the value 60 with a lower standard deviation in terms of weekly attendance values. So, we can conclude that random expectational agents use the bar more efficiently comparing with the Arthur's agents which have "bag of strategies" to create their expectations. On the other hand, the mean attendance converges to the same value which is the bar capacity for both cases.

8.3. Results for Type 1 Agents Extended

After we encounter such a result on comparing the random expectational agents and Arthur's case, we become curious about the answer to the following question: What would happen if the agents used a similar strategy mechanism with Arthur (1994) which contains random expectations? To answer this question, we formulate the extended version of Type 1 Agents. In this case, m strategies are formed and k of them randomly assigned to each agent where m is 200 and k is 6 as Garofalo (2006) states. However, all these 200 strategies just create different random expectations. The error calculation and strategy switching mechanism is the same with Type 2 Agents.

The result of this experiment, which is given in Figure 8.3, surprises us because the output of the simulation is very similar with the Arthur's findings in terms of statistical values. The mean attendance is 59.81 and converges to 60 in long term. The standard deviation is 8.568 which is a very close value to Arthur's case (9.024). As a result, we claim that there is no clear evidence that the agents with random expectational strategies (Type 1 extended) behave differently than the Arthur's agents with bag of strategies.

Parameters:

- Weeks = 100
- n = 100
- Random seed = 1
- m = 200
- k = 6
- $\lambda = 0.2$

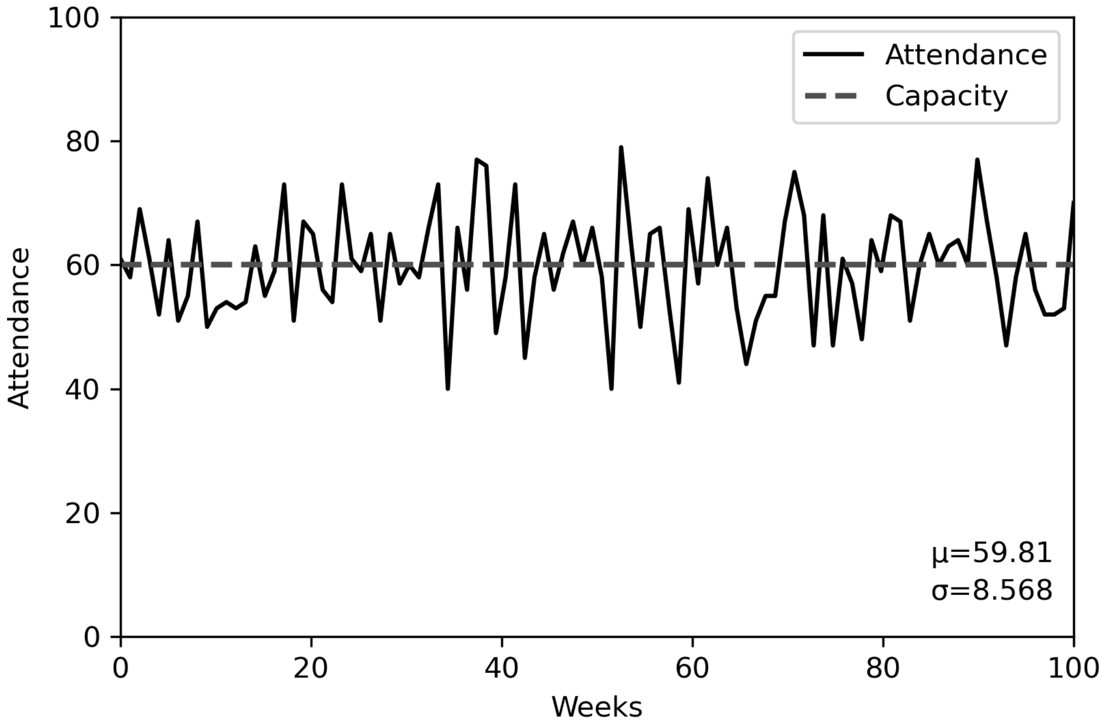


Figure 8.3. Type 1 Agents Extended.

After this surprising result, we want to investigate this extended model further. We adjust the k value of the model which indicates the number of strategies for each agent. We set the value of k as 20 and run the simulation model. The output of the simulation model in Figure 8.4 gives us a higher standard deviation which is 13.408. We find out that as the value of k increases (when the total number of strategies is constant), the standard deviation also increases. The reason for this increase is the increase of the number of commonly used predictors. The common predictors decrease the heterogeneity and make the agents decide with the same expectation models. This situation causes standard deviation to rise and avoid the efficient use of the bar.

Parameters:

- Weeks = 100
- n = 100
- Random seed = 1
- m = 200
- k = 20
- $\lambda = 0.2$

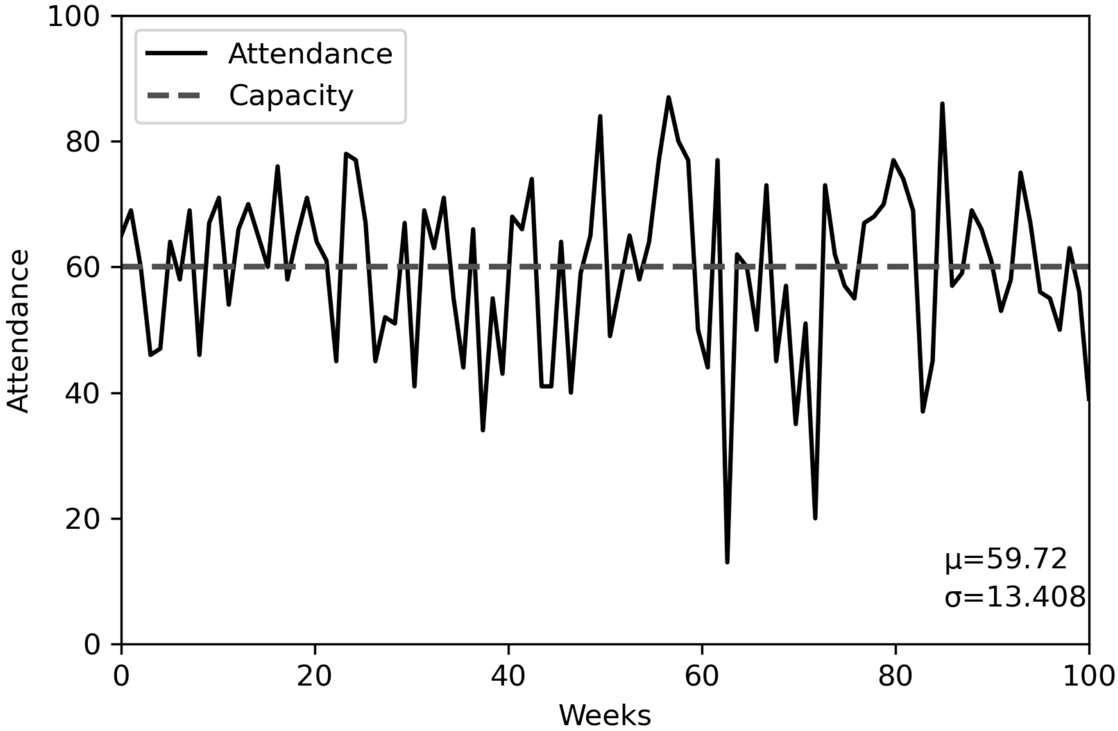


Figure 8.4. Type 1 Extended with k=20.

8.4. Results for Type 2 Agents

We want to mimic the Arthur's model, but as we earlier stated we had to use the parameters of Garofalo (2006). We use the bag of strategies that Arthur and Garofalo state to create the expectations, in other words we use the same predictors with Arthur and Garofalo to create Type 2 agents. There are total 200 strategies ($m = 200$) and each agent can have 6 strategies (k=6). The error term for switching strategy is calculated with an exponential smoothing formula where the weight of correction is 0.2 (λ=0.2).

The simulation result in Figure 8.5 shows that we obtain a similar result with the Arthur's result. If we analyze the two plots (Figure 8.1 and Figure 8.5) we cannot find any significant difference. Moreover, our findings are consistent with Garofalo's results as well. His EFBP model in NetLogo gives the result of 60.01 for mean attendance and standard deviation of 8.73 for 100 weeks (Garofalo, 2006).

Parameters:

- Weeks = 100
- n = 100
- Random seed = 1
- m = 200
- k = 6
- $\lambda = 0.2$

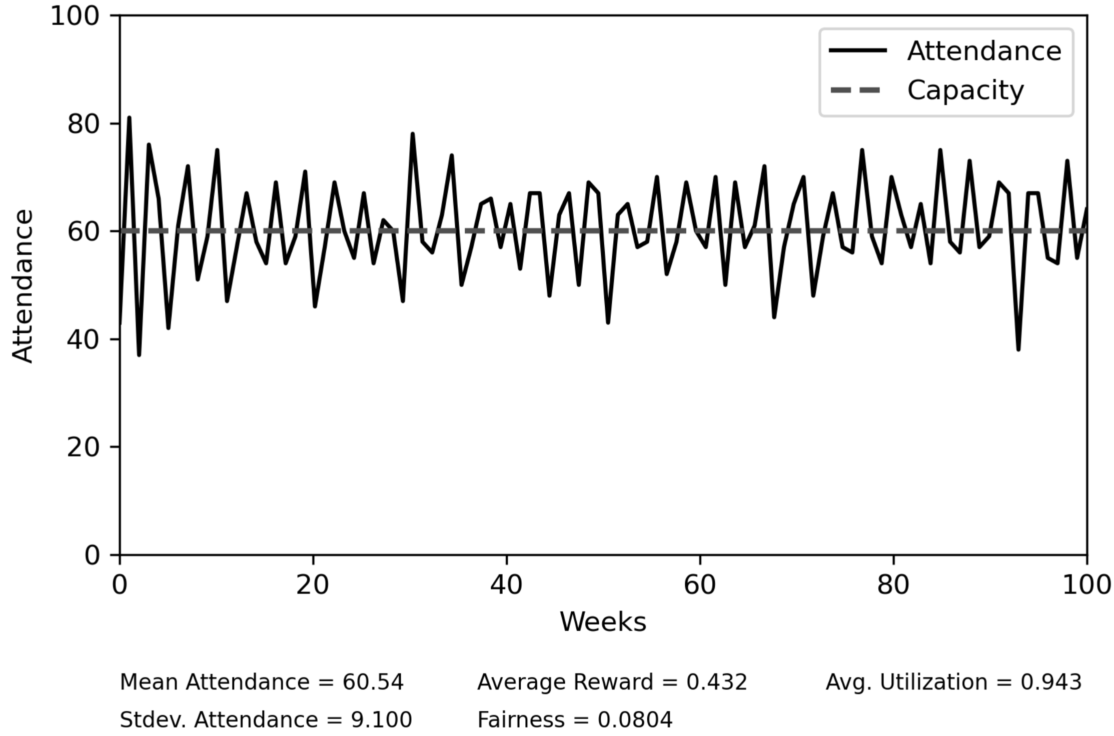


Figure 8.5. Type 2 Agents.

8.5. Results for Type 3 Agents

Our third expectation model contains boundedly rational agents with adaptive learning. The agents create their weekly expectation values using an exponential smoothing formula on new attendance values. Each agent has an individual alpha value which indicates the weight of the new attendance value on the expectation function. The alpha value is randomly determined between 0.1 and 0.3. If all the agents used the same alpha values, they would all have one single expectation model which cause all agents to attend or not to attend the bar at the same time (Figure 6.4). Although each agent has an individual expectation formulation with different alpha values we obtain again "commonality of expectations" in a slightly less density. Note that we also tried to assign alpha values between theoretical limits (0-1). We do not observe any significant difference in the behavior regardless of the change on alpha values. The simulation result for Type 3 Agents is given in Figure 8.6.

Parameters:

- Weeks = 100
- n = 100
- Random seed = 1
- α_i= U[0.1, 0.3)

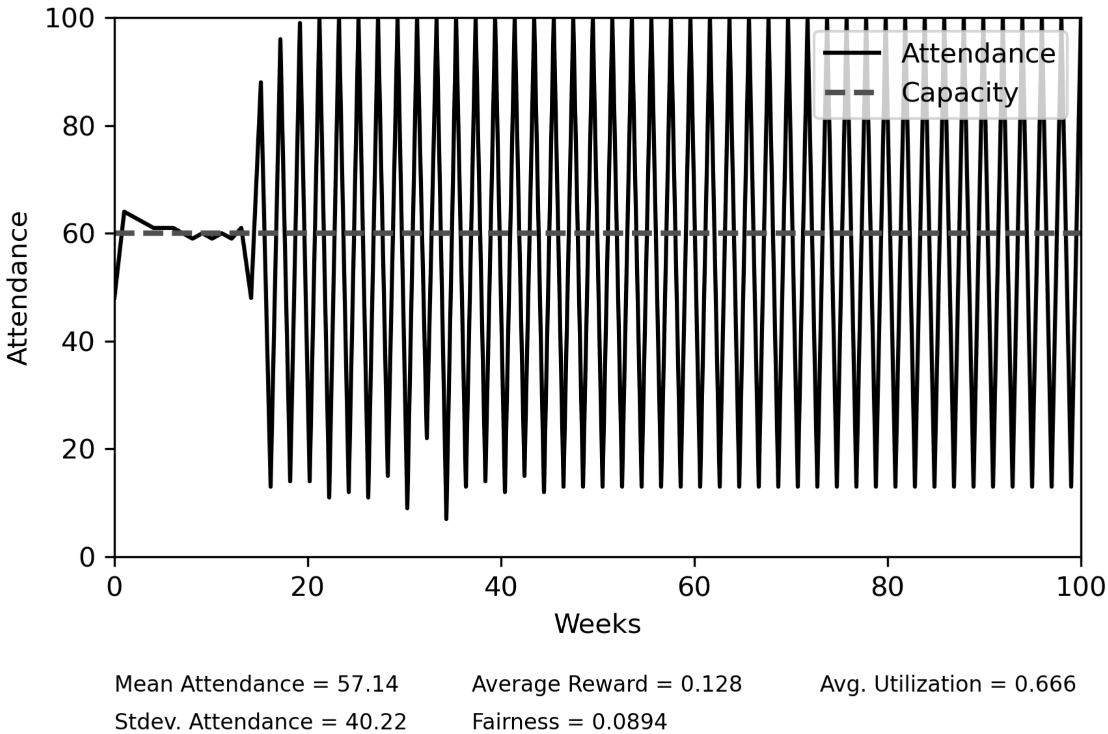


Figure 8.6. Type 3 Agents.

Figure 8.6 shows that not all but most of the agents give the same decision on the same weeks. The mean attendance is 57.14 and the standard deviation is very high, 40.220, which indicates that the performance of Type 3 Agents is very poor in terms of utilization of the bar. We see that the Type 3 Agents behave very similar to the "deductive reasoning agents" after a transition phase. Arthur claimed that if the agents in the EFBP used deductive reasoning, in other words the same expectation models, the decisions would not differ, and they all choose to attend or not to attend the bar at the same time. In this case, Type 3 Agents behavior looks very similar to this decision-making mechanism and it causes a poor performance in the EFBP.

8.6. Experimental Design and Comparative Results

First, we introduced the algorithms for three different agent types, and then we examined the results of each agent type under certain parameters. It is known that one single simulation run is not sufficient to conduct statistical analysis. Therefore, we run the simulation models using 100 random seeds and conduct a statistical analysis in terms of performance measures. Rerunning the simulation models with different random seeds prevents any undesirable effect of randomness. While we are examining each agent type separately, we only focused on the mean attendance and standard deviation of mean attendance values as we stated earlier. 100 simulation outputs with different random seeds allows us to discuss about the other performance measures such as reward, fairness, and bar utilization.

Table 8.1. Comparative results for three types of agents.

Performance Measures (Range)	Type 1 Agents	Type 2 Agents	Type 3 Agents
Mean Attendance (0-100)	59.89	60.31	57.07
Standard Deviation (0-50)	5.020	8.560	41.08
Avg. Reward (0-0.6)	0.4686	0.4328	0.1245
Fairness (0-0.48)	0.03924	0.07903	0.08509
Bar Utilization (0-1)	0.9658	0.9456	0.6591

As we examine the comparison table (Table 8.1) for the agent types, we directly recognize that Type 1 Agents perform better with respect to every performance measure. The only exception is the mean attendance which converges to the bar capacity for Type 1 and Type 2 Agents. Other than that, Type 1 Agents have the lowest standard deviation, highest average reward and bar utilization, and lowest fairness value which indicates a fair use of the bar by agents. In addition, we can see that Type 3 Agents' performance is far worse than the other agent types. The mean attendance of Type 3 Agents does not even converge to the bar capacity. Although each agent in the model use a different expectation model, the mean attendance does not converge to 60 on the contrary to Arthur's argument. We can claim that the behavior of Type 3 Agents "converges" to the deductive reasoning behavior (Figure 8.6).

9. DISCUSSION ON THE THREE EXPECTATION MODELS

We present the comparative results for the three different agent types. The comparison of each agent type with the original EFBP gives us some insight about the nature of this complex adaptive system. In this chapter, we will conduct a discussion on the characteristics of the EFBP in light of the comparative results that we obtained in the experiments.

9.1. The Effect of the Availability of the Information

We want to discuss the original EFBP model created by Arthur (1994) comparing with our three different expectation models. Our first model includes random expectational agents which we call Type 1 Agents. We see that the standard deviation of weekly attendance values for Type 1 Agents is less than the Arthur's result (1994), 4.870 and 9.024 respectively. It shows that the Type 1 Agents have a better performance in terms of utilization of the bar because the mean attendance converges to 60 in both cases. This finding is consistent with the results in the literature (Ponsiglione *et al.*, 2015).

If we want to interpret this result, we see that Type 1 Agents do not use the information of weekly attendance for creating their expectations. They only know the capacity of the bar. They create random expectation values and make their decisions according to the comparison between their expectations and the bar capacity. This mechanism causes a better performance comparing with Arthur's agents which use the weekly attendance values to create their expectations. So, we can conclude that using information from the environment or knowing the weekly attendance does not improve the population's performance in the EFBP.

9.2. The Heterogeneity

We want to examine this statement further. For this purpose, we create the extended version of Type 1 Agents. These agents also make random expectations, but each agent has certain number of strategy. All strategies create different random expectations for each week and the most accurate strategy for each agent becomes the active predictor of this agent. The agent uses the active predictor to determine the weekly expectation value. This mechanism is very similar to the expectation formulation mechanism which is used in the original article by Arthur (1994). The only difference is that the Arthur's agents use the information of weekly attendance to update their expectations with a specific formulation such as moving average, mirror images or trend calculation. On the other hand, the agents in the Type 1 extended model use the predictors which create random expectation values.

We obtain similar outputs for the Type 1 extended version and the original Arthur's model. Note that the number of agents is 100, total number of weeks is 100, and the number of strategies per agent is 6. The mean attendance values for these two models are 59.81 and 60.44 respectively, and they both converge to the 60 in the long run. Moreover, the standard deviations of weekly attendance values are 8.568 and 9.024 respectively. These similar system behaviors on the Figure 8.1 and Figure 8.3 and the similar statistical results lead us to ask the following question: Are the Arthur's strategies meaningful? We think that Arthur is aware of this question, and he even gives the answer in his article.

"It might be objected that I lumbered the agents in these experiments with fixed sets of clunky predictive models. If they could form more open-ended, intelligent predictions, different behavior might emerge. One could certainly test this using a more sophisticated procedure, say, genetic programming (John Koza, 1992). This continually generates new hypotheses, new predictive expressions, that adapt 'intelligently' and often become more complicated as time progresses. However, I would be surprised if this changes the above results in any qualitative way." (Arthur, 1994)

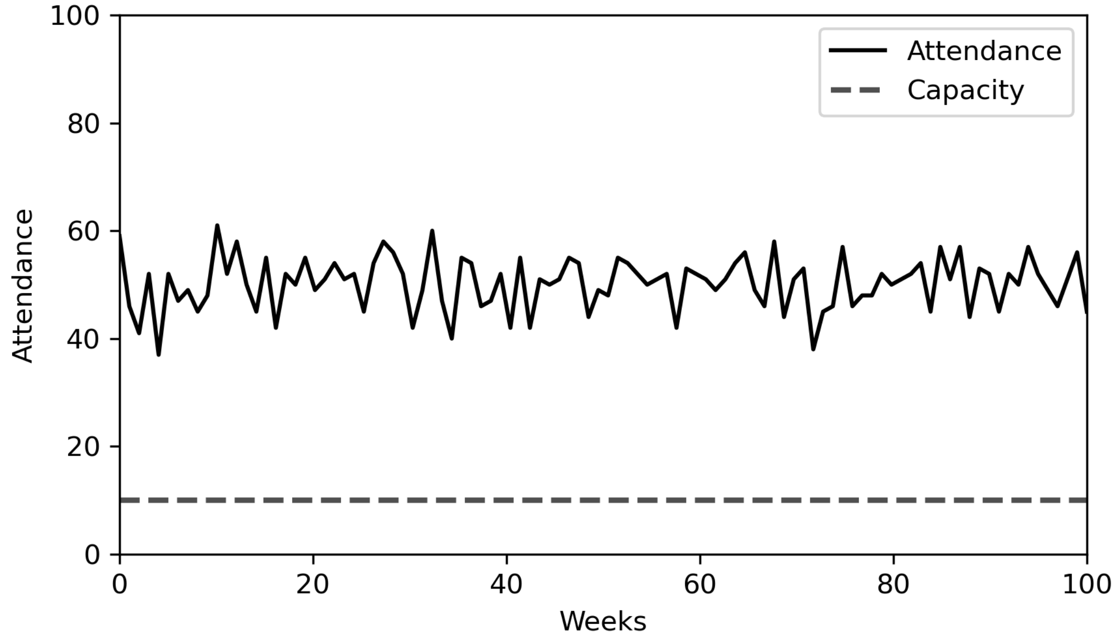
Despite of this explanation of Arthur, we have a different opinion with Arthur. Arthur is right to come up with a challenge which includes to experiment the EFBP with “more intelligent predictors”. He claims that the behavior dynamics would not differ in terms of convergence of the mean attendance. However, this statement makes clear a hidden assumption. The strategies that Arthur use for his model are neither significant nor meaningful. Our Type 1 extended model which includes completely random predictors create similar dynamics as Arthur stated. The key point is the heterogeneity of the expectations in this case and Arthur does not intend to create meaningful or real life strategies, but the strategies are used to create a heterogeneity in the expectation models. As a result of this heterogeneity, the decisions differ and the model create a co-evolving system around the capacity point. Also, the parameter of number of strategies per agent (k) affects the standard deviation value for the Type 1 extended model. It supports our claim about the heterogeneity because as k increases in our model, the number of agents who uses the same active predictor increases when the total number of strategies (m) is constant. It causes less heterogeneity for the population and increase the standard deviation (Figure 8.4). We will also focus this heterogeneity issue while discussing on Type 3 Agents.

Type 2 Agents model is created with the following Arthur's and Garofalo's assumptions. We use the certain parameters such as n = 100, Weeks = 100, m = 200, k = 6 and $\lambda = 0.2$. We assign six different strategies to each agent out of 200 total strategies which include moving averages, trends, fixed rules and tit-for-tat. As we take the Arthur's and Garofalo's assumptions to create our model, we expect to see similar dynamics and statistical results with their results. Our simulation result shows that we obtain very similar dynamics with Figure 8.1. The mean attendance and standard deviation of attendance values are 60.54 and 9.100. These results are consistent with the Garofalo's NetLogo model's outputs as well which has mean attendance value of 60.01 and the standard deviation of 8.73 for 100 weeks (Garofalo, 2006). The difference in the standard deviation is caused by the strategy types and randomness.

We also create a model which we do not encounter on the literature of the EFBP. Type 3 Agents are boundedly rational agents with adaptive learning which use exponential smoothing formulation to create their expectations. Sterman (1987) claims that exponential smoothing formulation is a suitable method which can model an adaptive learning process. If we analyze the simulation result of Type 3 Agents, we see that they perform far worse than the Type 1, Type 2 and the Arthur's agents with mean attendance of 57.14 and standard deviation of 40.220. As it can be seen from Figure 8.6, the deviation of the attendance is very high, and the bar is not used efficiently all weeks within a time period of 100 weeks. This behavior dynamics arises although each agent has a different alpha value which indicates the correction weight for the expectation value. As a consequence, we argue that any common knowledge used in the EFBP decreases the performance in the EFBP as we can see in Type 2 and Type 3 agents and the key term of the EFBP behavior dynamics is heterogeneity.

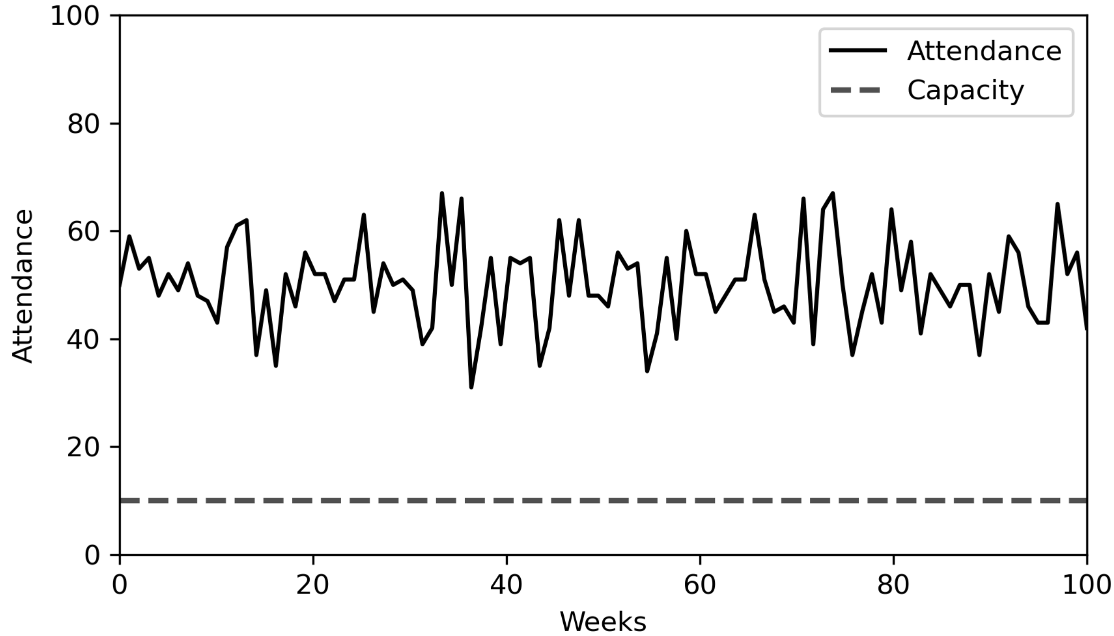
9.3. Experiments with an Unknown Constant Capacity

We see that random expectational agents have a better performance in utilization of the bar with a lower standard deviation. Moreover, we can claim that knowing and using the attendance values for creating new expectations does not improve the performance of the agents as we see in Type 2 and Type 3 Agents. We find out that the only important characteristics in the utilization of the bar is the heterogeneity. However, there is a hidden assumption which we want to study in detail. All three different types of agents and the agents in the original EFBP know the capacity and they make their decision according to this threshold. At this point a question arises: What would happen if agents do not know the bar capacity and they have to learn the capacity? We design two different experiments to answer this question. First, we conduct the experiments with an unknown constant capacity. In this case, we have the same three different types of agents, but they do not know the bar capacity and they assume that the capacity is 50. However, the actual bar capacity is not 50, but 10 in the experiments. Different bar capacity values can be selected to experiment with, but a single example is sufficient to show that any type of agents do not learn the actual bar capacity as it can be seen in Figure 9.1, 9.2, and 9.3.



Mean Attendance = 50.23 Average Reward = 0.498 Avg. Utilization = 1
Stdev. Attendance = 4.919 Fairness = 0.0422 Deviation A-C = 40.73

Figure 9.1. Type 1 Agents experiment with an unknown constant capacity.



Mean Attendance = 50.22 Average Reward = 0.498 Avg. Utilization = 1
Stdev. Attendance = 7.988 Fairness = 0.173 Deviation A-C = 41.20

Figure 9.2. Type 2 Agents experiment with an unknown constant capacity.

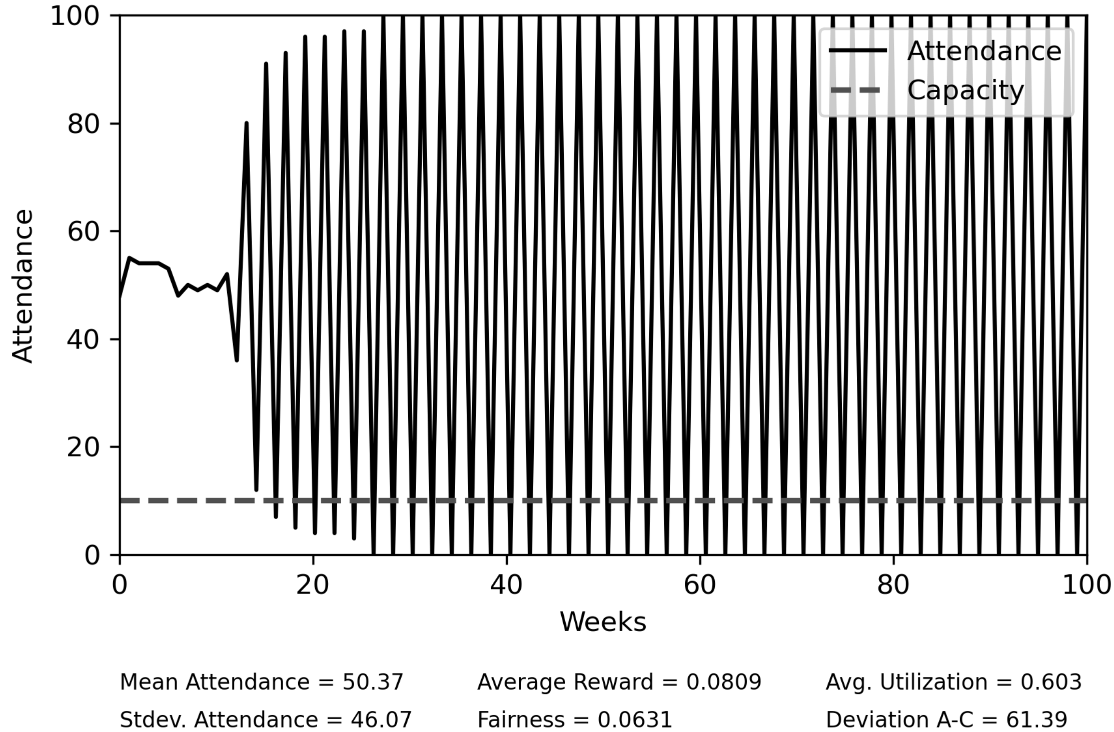


Figure 9.3. Type 3 Agents experiment with an unknown constant capacity.

As a result of this experiment, we see that our three different types of agents do not learn the bar capacity at all, and they all perform poorly. For example, when the actual bar capacity is 10, the mean attendance for Type 1 Agents is 50.48, for Type 2 Agents 49.0, and Type 3 Agents 49.75 (Figure 9.1, 9.2, and 9.3). On the other hand, the last performance measure, which is the deviation of attendance from capacity, proves the poor performance of these three agent types. The values of deviation of attendance from capacity are 40.73, 41.20, and 61.39 for each agent type respectively. These values are very high for a deviation and it shows that if the agents do not know the bar capacity value as a hidden assumption, the current EFBP models fail. We realize that there is no relationship (i.e. feedback mechanism) between the attendance and the capacity, and it causes these models to fail.

9.4. Capacity Estimation

The results of the experiments with an unknown constant capacity show that a learning mechanism for the bar capacity is necessary if the bar capacity is not known by the agents or if it is fluctuating. After this observation, we design a new experiment with adding two different types of capacity learning mechanism for the three different types of agents: Random expectations for the capacity and adaptive learning expectations for the capacity with exponential smoothing. We combine the capacity expectation models with attendance expectation models and obtain six (two capacity expectation methods and three attendance expectations) different cases.

In the previous models we called type 1 for random expectation, type 2 for Arthur/Garofalo bag of strategies, and type 3 for adaptive learning with exponential smoothing. We will keep this naming for six new agent types. The first number in the agent type name indicates the capacity estimation method and the second number indicates the attendance estimation method. We obtain six different types of agents and conduct experiment to see the effect of different capacity learning methods combined with attendance learning methods.

Table 9.1. Capacity and attendance estimation methods of new agent types.

New Agent Types	Capacity Estimation Method	Attendance Estimation Method
Type 1&1Agents	Random expectations	Random expectations
Type 1&2Agents	Random expectations	Bag of strategies
Type 1&3Agents	Random expectations	Exponential smoothing
Type 3&1Agents	Exponential smoothing	Random expectations
Type 3&2Agents	Exponential smoothing	Bag of strategies
Type 3&3Agents	Exponential smoothing	Exponential smoothing

We have designed six different types of agents which create capacity expectations and attendance expectations. In order to create them, we use a switch on the code which we call “Capacity Estimation Method”. If the value of Capacity Estimation Method is equal to zero, the agents do not use any estimation for the bar capacity, and it is assumed that all agents know the actual bar capacity value. If the value of Capacity Estimation Method is equal to one, the agents use random expectations for the capacity value. On the other hand, if the value of Capacity Estimation Method is equal to three, the agents use exponential smoothing formula to estimate the capacity value with a smoothing parameter 0.1. The working mechanism of capacity estimation method is given in Figure 9.4.

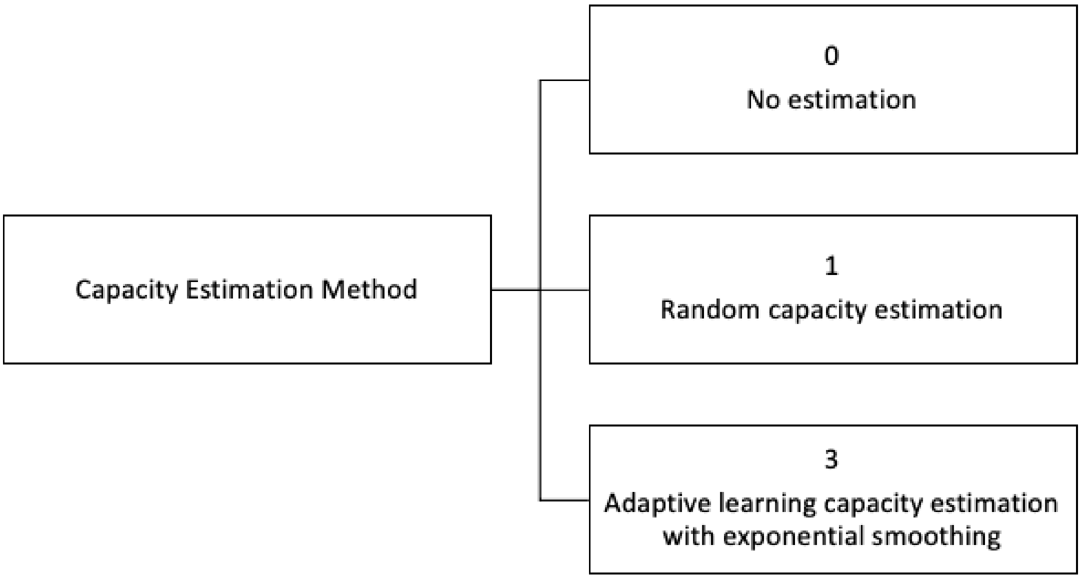


Figure 9.4. Capacity estimation method working mechanism.

The random expectations are created same way for the capacity estimation as it was for the attendance estimation of Type 1 Agents. On the other hand, the exponential smoothing method also works with a similar mechanism for the capacity estimation as it was for the attendance estimation of Type 3 Agents. However, the input for the equation is not attendance values, but the announced bar capacity value for each week. Note that bar capacity is unknown for all agents in these experiments and they adaptively learn the actual bar capacity by updating the assumed bar capacity using exponential smoothing technique. The initial assumption of agents for the bar capacity is 50 which is the half of the population.

After we created the capacity estimation method and six different types of agents accordingly, we design two different experiments to show the importance of learning the actual capacity. We repeat the unknown constant capacity experiments with our new agents who learn (or estimate) capacity value with two methods, namely, random capacity expectations and adaptive learning with exponential smoothing. As we realize that the assumption of knowing the actual value of the capacity is significant for the EFBP, we expect to see that the agents who learn the capacity with an exponential smoothing technique should perform better in terms of performance measures.

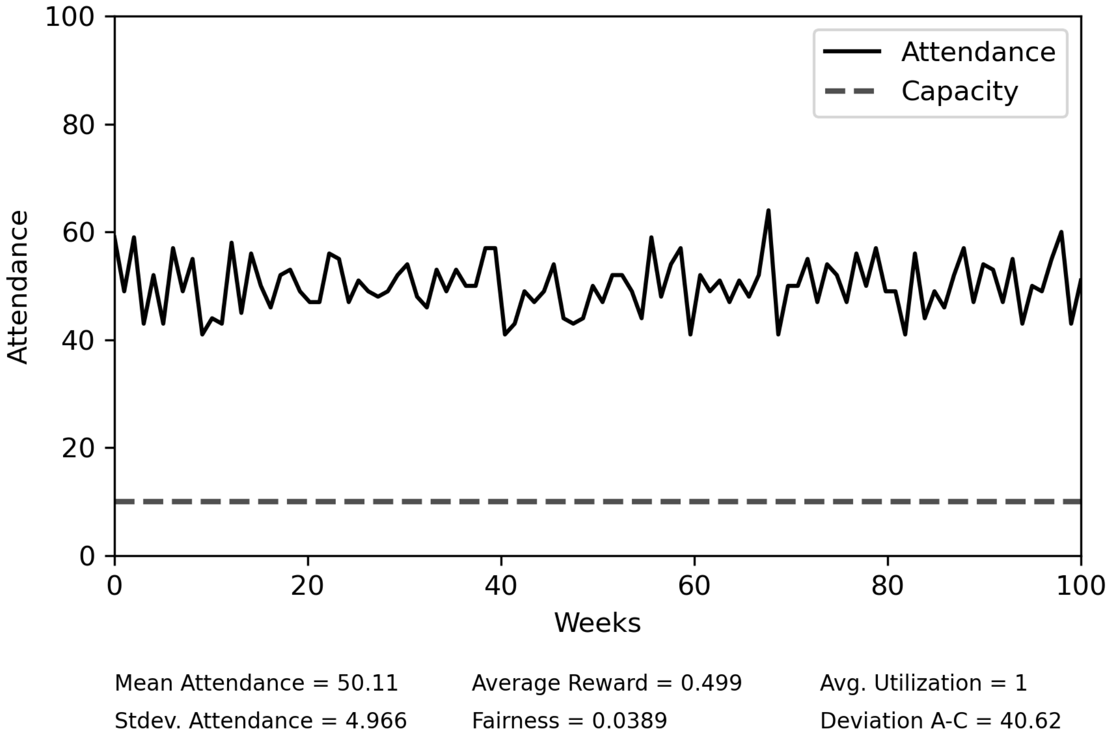


Figure 9.5. Type 1&1 Agents with the bar capacity=10.

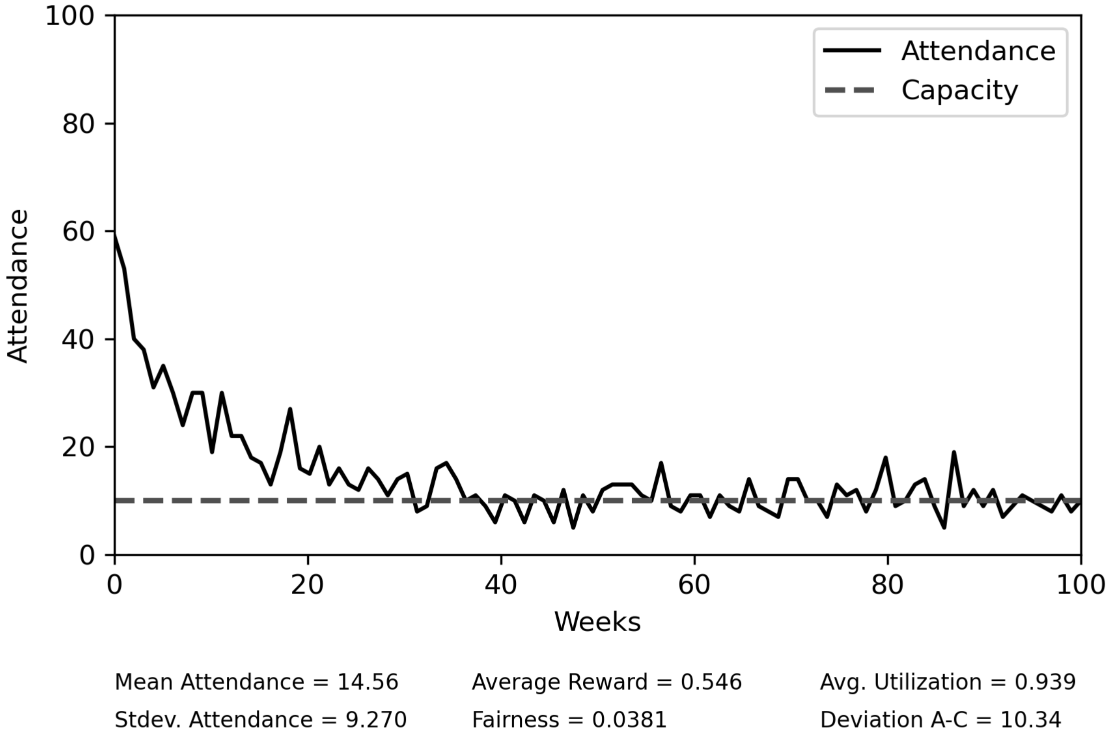


Figure 9.6. Type 3&1 Agents with the bar capacity=10.

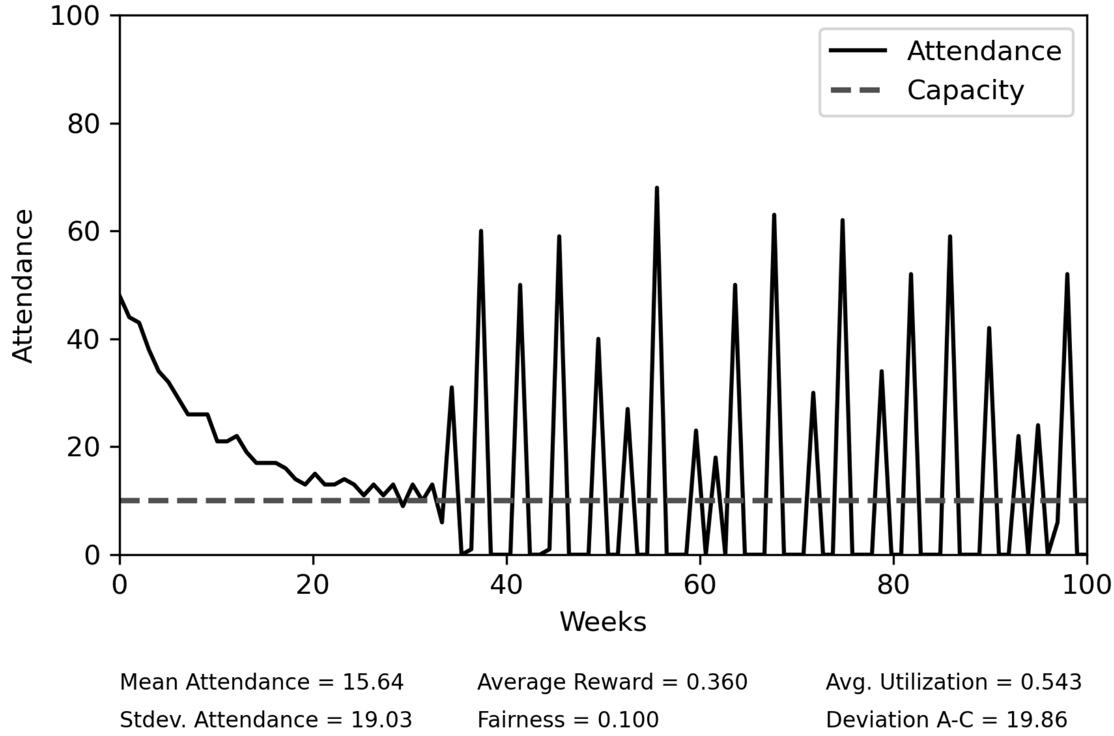


Figure 9.7. Type 3&3 Agents with the bar capacity=10.

The results are as we expected; the agents who use exponential smoothing formula to estimate the capacity value which are Type 3&1, 3&2 and 3&3 Agents produce mean attendance values that converges to the bar capacity. However, the agents who randomly guess the capacity Type 1&1, 1&2 and 1&3 Agents do not produce convergent mean attendance values (Figure 9.5 and 9.6). The agents who estimate the capacity with random expectations fail to find the actual bar capacity as it was in the experiments with an unknown constant capacity (Figure 9.5). On the other hand, the adaptive learning for the bar capacity expectation mechanism works well with a smoothing parameter of 0.1 (capacity estimation fraction). The agents who use adaptive learning for the bar capacity can learn the actual bar capacity after a time period (Figure 9.6 and 9.7) However, Type 3&1 agents have best performance out of these six different types of agents. The value of deviation of attendance from capacity for Type 3&1 Agents (10.34) is less than all of the other agent types. The reason is that they estimate a constant capacity value more accurately and they generate heterogeneity using random expectations for the attendance values. We can see this result clearly when we compare Figure 9.6 and 9.7. Furthermore, we conduct this experiment with the bar capacity value of 10. Different bar capacity values can be selected to challenge the Type 3&1 agents. For this purpose, we repeat the experiment with the bar capacity value of 90. We reach the same conclusion as we expected. Type 3&1 Agents can estimate the actual bar capacity after a time period as it can be seen in Figure 9.8 when the actual bar capacity is equal to 90.

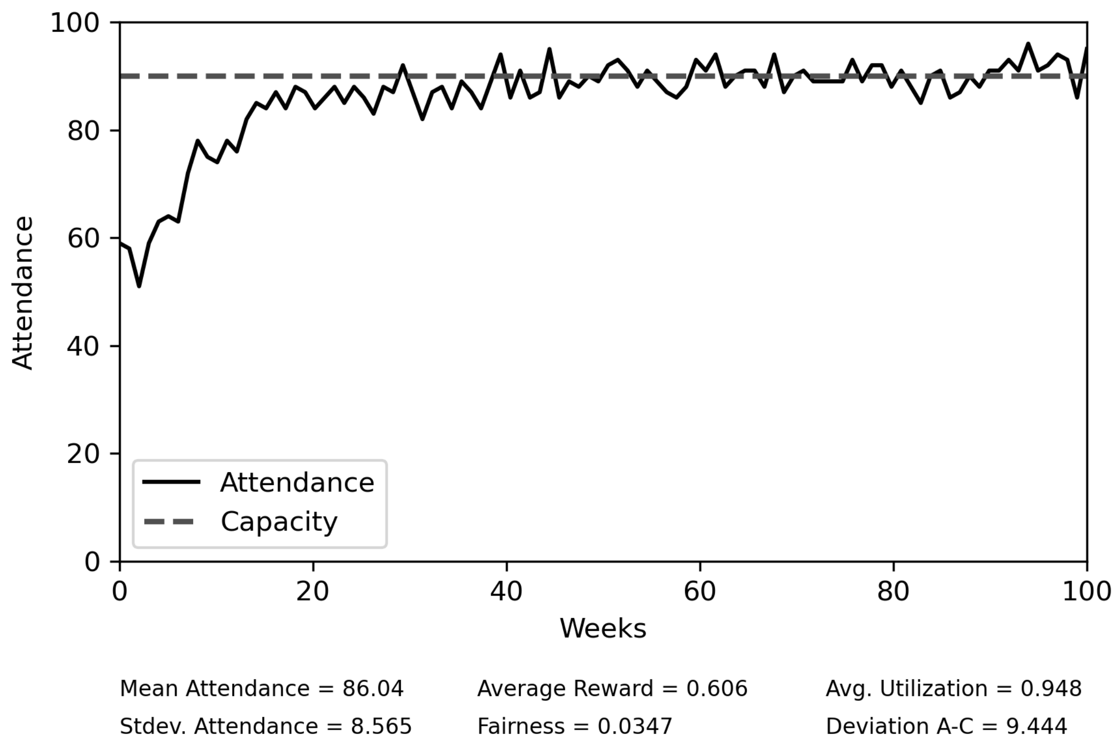


Figure 9.8. Type 3&1 Agents with the bar capacity=90.

After these experiments, we study the capacity issue further. In the first experiment, we use an unknown but constant capacity which has a value of 10 and 90. Then we asked the question what happens if the capacity is fluctuating around a stationary mean each week? Would the Type 3&1 agents be successful to estimate a fluctuating capacity value with a stationary mean. At this point, we refer to Sterman (1987) who claims that exponential smoothing is an adequate technique for an adaptive learning process, and it is very useful to estimate a variable with a stationary mean. For our second experiment, we use variable capacity and the distribution of the capacity values has a stationary mean. The distribution of the capacity values is uniform. The distribution for the bar capacity can be expressed as Bar Capacity $\sim$ U[70,90].

However, the range can be adjusted, and various experiments can be conducted. Our aim is to show the importance of learning capacity value in the EFBP, so extreme capacity values such as between 70 and 90 can be useful for our experimental design. In this case, capacity values are predetermined by a distribution.. We use a switch to determine the capacity distribution method with a similar mechanism in capacity estimation method. In this case, we create a switch called “Capacity Distribution Method” and it indicates the state of the bar capacity. If the Capacity Distribution Method is equal to zero, then the constant capacity formulation works, on the other hand if the Capacity Distribution Method is equal to one, then the fluctuating bar capacity mechanism works according to the given distribution which is U[70-90] in this experiment. The working mechanism of capacity distribution method is given in Figure 9.9.

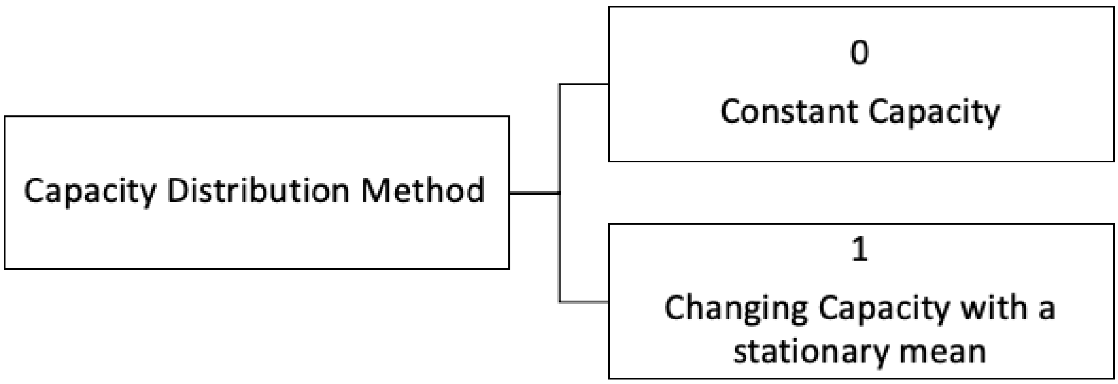
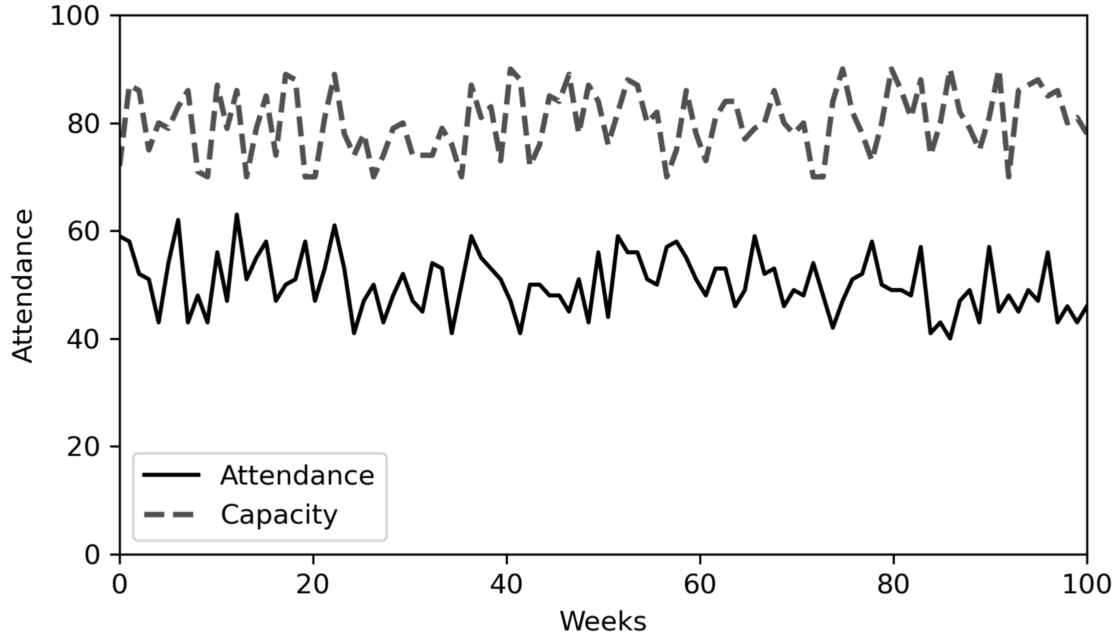
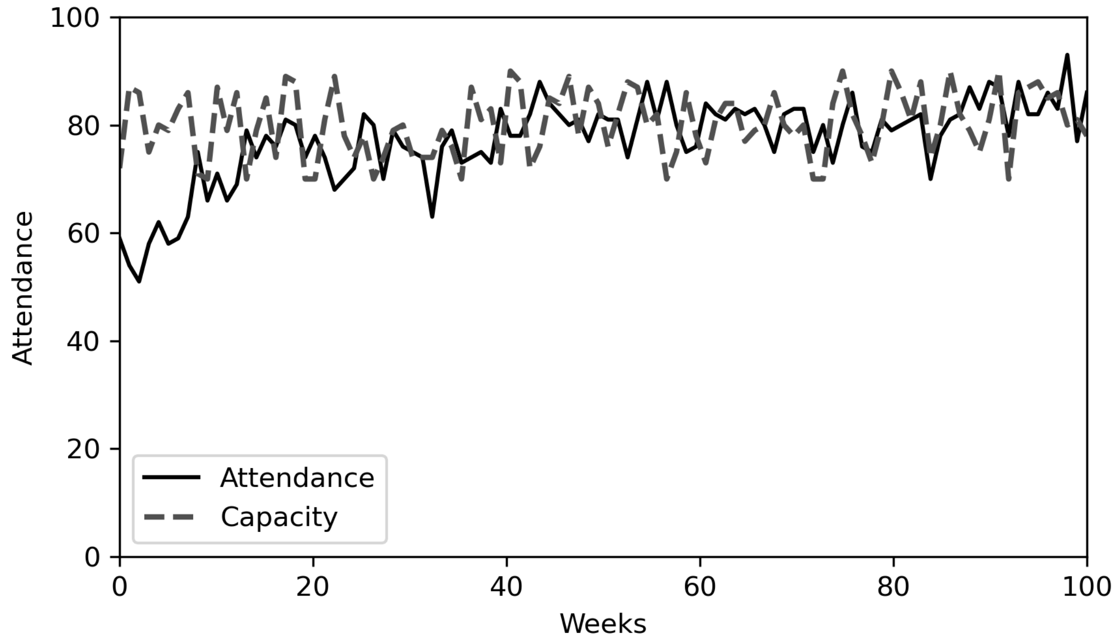


Figure 9.9. Capacity distribution method working mechanism.



Mean Attendance = 50.17 | Average Reward = 0.502 | Avg. Utilization = 0.629
Stdev. Attendance = 5.379 | Fairness = 0.0352 | Deviation A-C = 31.21

Figure 9.10. Type 1&1 Agents with the variable bar capacity.



Mean Attendance = 77.21 | Average Reward = 0.531 | Avg. Utilization = 0.937
Stdev. Attendance = 7.850 | Fairness = 0.0373 | Deviation A-C = 10.09

Figure 9.11. Type 3&1 Agents with the variable bar capacity.

The Type 3&1 agents become successful to use the bar efficiently when the bar capacity is fluctuating with a stationary mean as we can see in Figure 9.11. On the other hand, random expectations for the capacity fails again to estimate the actual bar capacity (Figure 9.10). All these experiments help us to reach one concrete conclusion: The efficient use of the bar in the EFBP depends on the knowing or learning the actual bar capacity and creating heterogeneity on the decision making process which can be provided by random expectations for the attendance. In other words, the agents have to follow this principle in order to achieve the goals in terms of performance measures: "Learn the capacity accurately and make attendance expectations randomly."

Lastly, a question arises; can we find a better strategy for the agents in terms of performance measures? After the analysis of the three different expectation models, we see that the best performance as a population is obtained by Type 1 Agents who use random expectations within these three expectation models. Moreover, the availability of the knowledge of the actual bar capacity is important for the agents' performances. We will study the question of "a better strategy" further in the following chapter and try to introduce a new agent type who performs better in the EFBP in terms of performance measures.

10. TYPE 4 AGENTS: YASARCAN-ÇETINER AGENTS

After analyzing the three expectation models, we reach to the conclusion that "the heterogeneity of the agent's decisions" and "the assumption that the capacity is known by the agents" are critical in obtaining a good performance in the EFBP. If the capacity is not known by the agents, capacity must be estimated with an adaptive learning approach. This conclusion is correct for agents who use expectation models in decision-making. Nonetheless, this conclusion that is based on our experimental results set an obstacle for us in developing a new type of agents that can outperform the others. My thesis supervisor, Dr. Yasarcan, is currently working on hysteresis causing structures (Yasarcan, 2022). Inspired by his ongoing work, he introduces agents that persist on keeping their current decisions as it is, for some time. This persistence creates hysteresis in the decision making processes of the agents. We code the algorithm introduced by Dr. Yasarcan and named the agents as Yasarcan-Çetiner agents.

10.1. Introducing a New Agent Type: Hysteresis

Yasarcan-Çetiner agents is our Type 4 Agents. According to our most current knowledge, Type 4 Agents are completely original and do not exist in the EFBP and EFBP related literatures. We seek a better strategy in terms of performance measures which can perform better than the random expectational agents (Type 1 Agents). At this point, my thesis advisor, Dr. Yasarcan, comes up with the idea to create a new type of agents that persist on their current decisions. These agents do not use expectation models in making decisions. We suggest that the agents can use the past experience and reward values to make their decisions with a hysteresis mechanism instead of expectation values.

The term "persistence" is used to refer to the maximum number of weeks that can be tolerated for keeping the decision as it is. In the EFBP, the agents have a binary decision either attend to the bar or stay home. Persistence here works as a threshold that indicates how many times an agent accepts a "wrong decision", in other words, getting a reward value of zero, when choosing to attend or to stay home. The agents switch their decisions only if they fail a certain number of weeks that is determined by the individual persistence values.

10.2. Algorithm for Type 4 Agents

The decision-making process in the EFBP is binary; attend to the bar or stay home. For our Type 4 Agents, we first create persistence thresholds for these two decisions. The unit of persistence threshold is weeks, and it indicates the threshold value for switching the decision of an agent either from one to zero or zero to one. We distribute two different values of persistence thresholds to each agent for both attending the bar decision and staying home decision. The persistence threshold values are between 1 and 10. There are no agents who has the same persistence threshold combination for the two decisions. However, an agent may have the same threshold value for attending to the bar or staying home. We present an example to show the working mechanism of the persistence threshold. Let us assume an agent has the persistence threshold for attending bar as three and the persistence threshold for staying home as one. On the first week, the decisions are made randomly. We assume that our agent attended to the bar on the first week and the agent gain a reward value of one because the bar capacity is not exceeded; this means that decision of the agent is correct. As the agent gain a positive reward value on the previous week, the agent keeps his decision and attends to the bar next week. If the bar is not overcrowded again, the agent does not reverse his decision because this decision is rewarded. On the other hand, if the bar is crowded and the agent gets reward value of zero, the consecutive weeks that the agent gets reward value of zero is counted. If the number of consecutive weeks that the agent gets zero reward value is equal to the persistence threshold for attending the bar, the agent reverses the decision and decides to stay home next week. In our example, the agent will attend the bar and receive a reward value of zero for up to three consecutive weeks because the persistence threshold for attending to the bar for that agent is three. On the other hand, once the agent reverses the decision and decide to stay home, the agent keeps this decision as long as he continues to receive a positive reward. However, if he stays home and the bar is not crowded for a week, he reverses his decision because the persistence threshold for staying home of the agent is one. If the agent gives the correct decision and gets rewarded, he keeps the current decision (attend to the bar or stay home) for the next week. This example of an agent's decision reversing mechanism is given in Table 10.1 and the flowchart that shows the decision-making algorithm for Type 4 Agents is given in Figure 10.1. Note that this agent has the persistence threshold for attending bar as three and the persistence threshold for staying home as one.

Table 10.1. An Agent's example decision reversing mechanism with hysteresis.

Week	Decision	Attendance	Reward
1	1	55	1
2	1	63	0
3	1	67	0
4	1	60	1
5	1	52	1
6	1	68	0
7	1	71	0
8	1	61	0
9	0	70	1
10	0	56	0
11	1	55	1
12	1	62	0

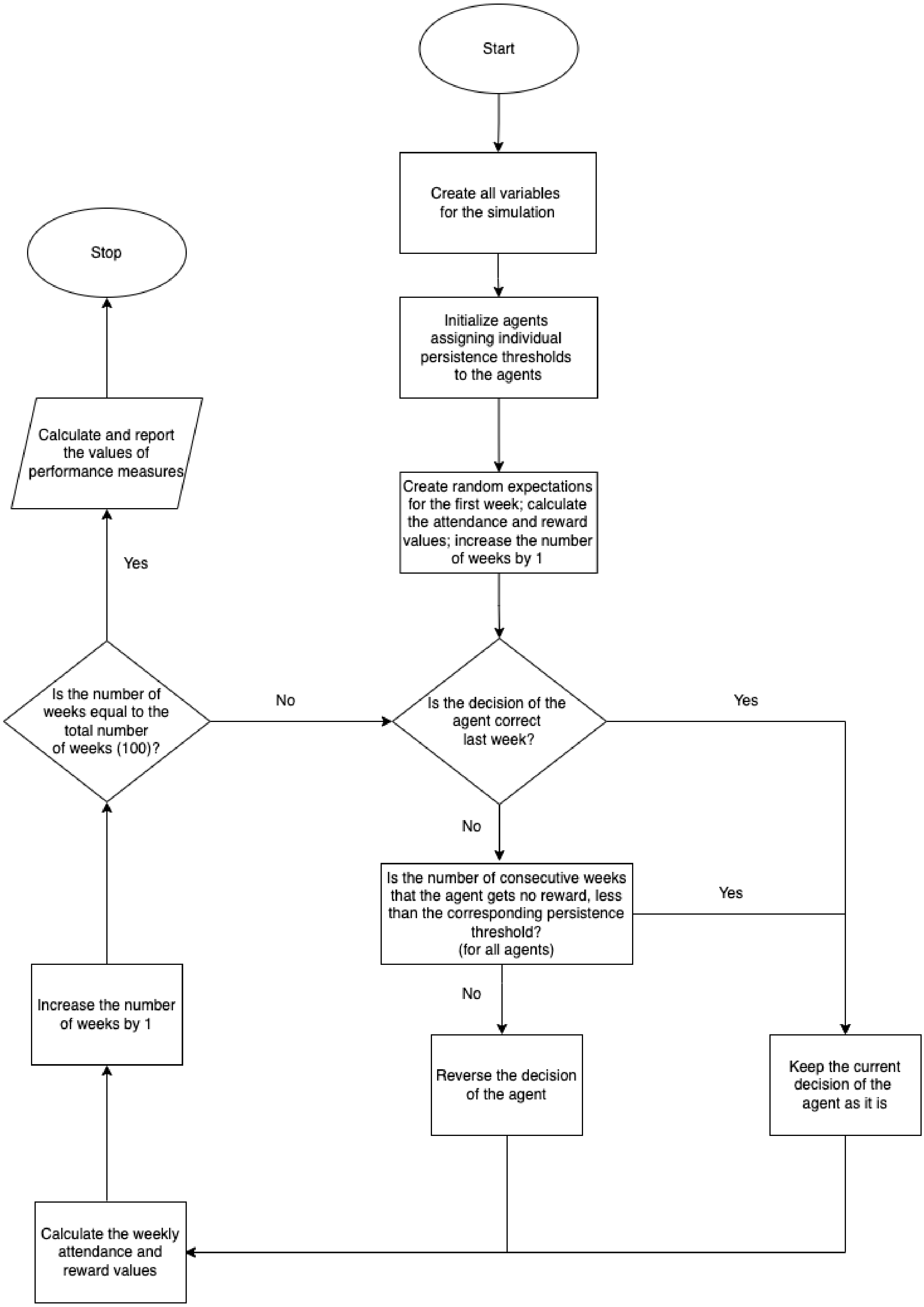


Figure 10.1. Flowchart for Type 4 Agents.

10.3. Experimental Results

The simulation result for Type 4 Agents produces the best values in the four out of five performance measures compared to the other three types of agents. After the initialization on the first week, agents find the bar capacity value and they attend to the bar with a very low standard deviation and the mean attendance value converging to 60. This agent behavior creates a very high performance which can be hardly improved. The simulation result is given in Figure 10.2.

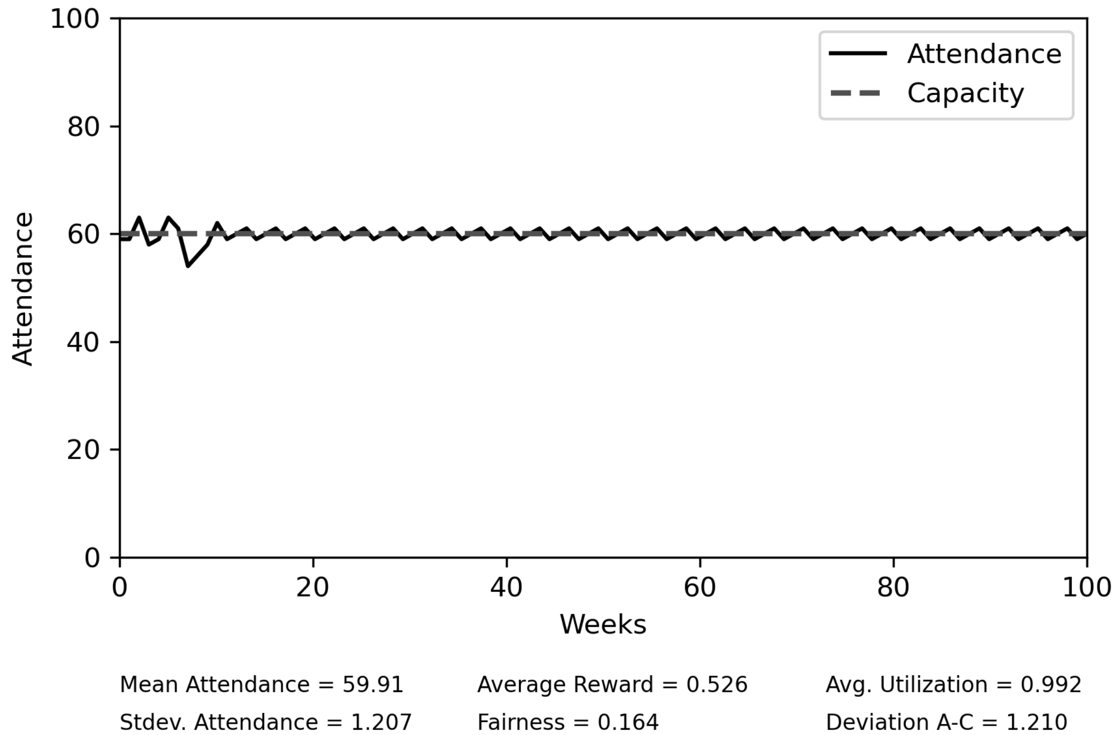


Figure 10.2. Results for Type 4 Agents.

As we can see from the Figure 10.2, the mean attendance is 59.91 for 100 weeks and converges to 60, the standard deviation is very low with a result of 1.210. Moreover, average reward and average bar utilization are higher than all other three types of agents with the values of 0.526 and 0.992 respectively. The only performance measure that Type 4 Agents are not better than the other three types of agents is fairness. The fairness value of Type 4 Agents is higher than the other models and it indicates that the reward distribution is less fair than the agents with random expectation models.

The assignment of persistence threshold values to the agents can be discussed. As we mentioned before, we distribute the persistence thresholds for both decisions individually so that each agent has a unique combination. The persistence thresholds could also be assigned randomly. We studied this case as well. However, we did not obtain any significantly different results.

After seeing the result for Type 4 Agents, the reader may remember the experiments with an unknown constant capacity and wonder if Type 4 Agents can learn the different capacity values. We take two simulation runs to study this issue with the bar capacity equal to 10 and 90.

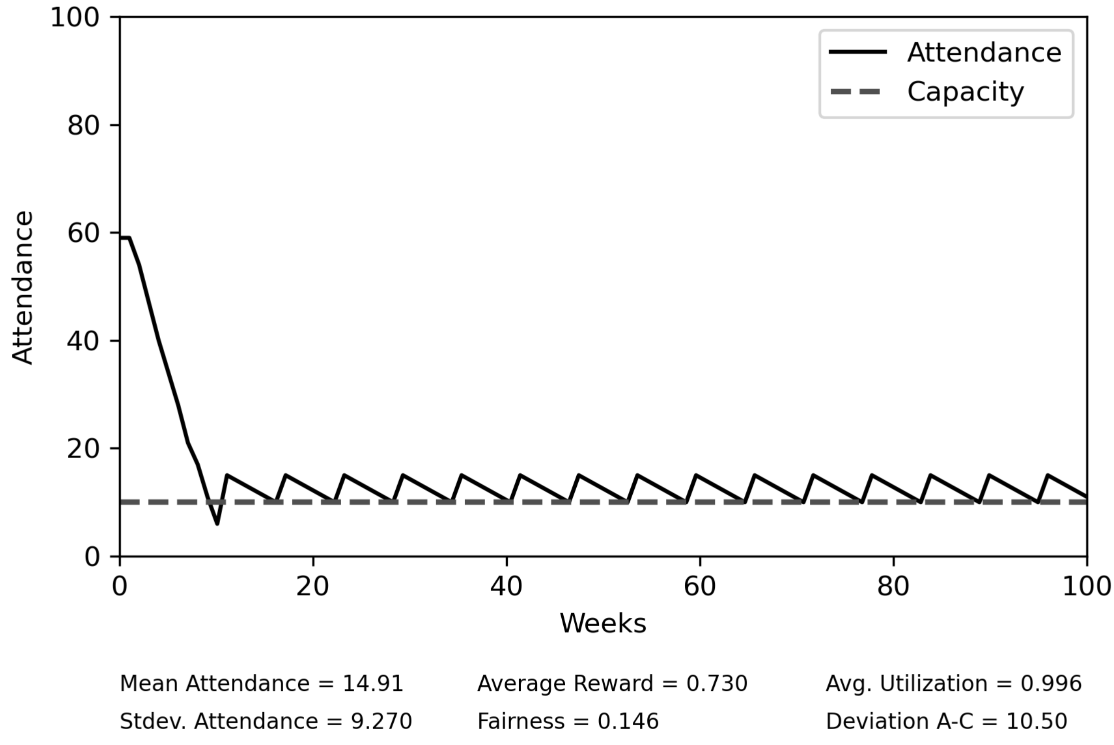


Figure 10.3. Results for Type 4 Agents with capacity=10.

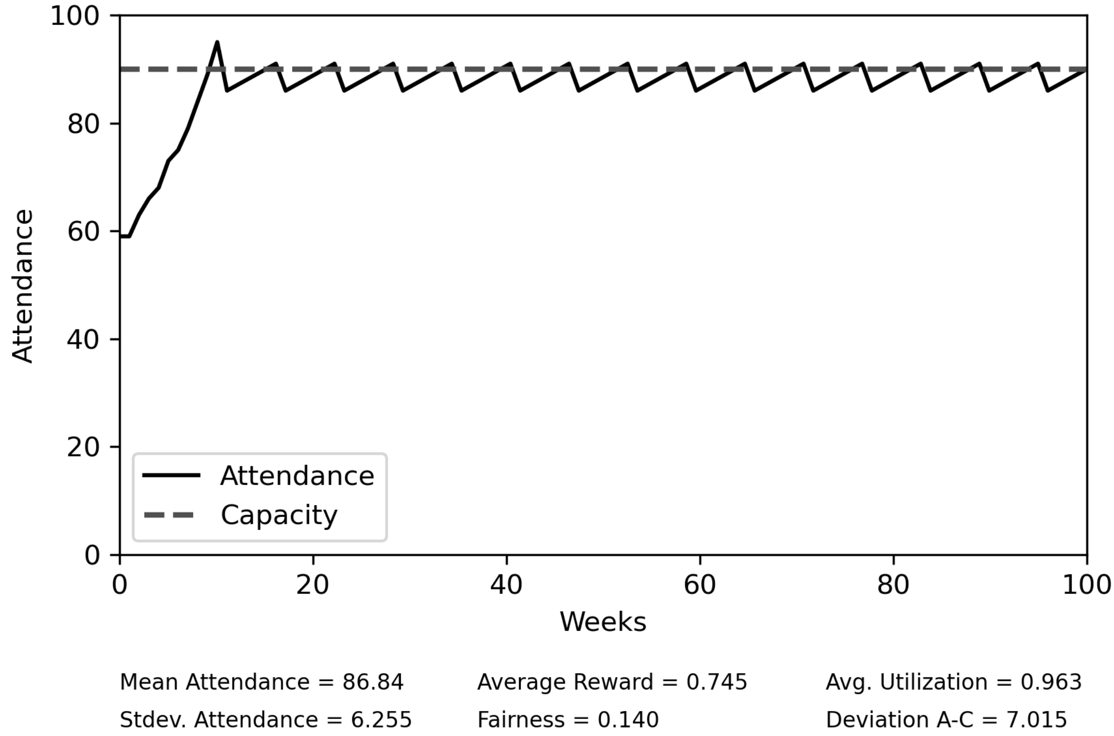


Figure 10.4. Results for Type 4 Agents with capacity=90.

As it can be seen from the simulation results in Figure 10.3 and 10.4, Type 4 Agents can learn the extreme bar capacity values and adapt them very quickly. Moreover, the performance measures for Type 4 Agents are very good even in extreme capacity values. The mean attendance almost converges to the bar capacity for both cases with the values of 14.91 and 86.84. The standard deviation of attendance may seem a bit high with the values of 9.270 and 6.260. However, it is caused by the transition phase in which agents learn the actual bar capacity. If we take a long simulation run such as 1000 weeks, the standard deviation of mean attendance decreases. The average reward and average bar utilization are also very high, which indicate the success of Type 4 Agents even under extreme bar capacity conditions. We want to point out that "learning the bar capacity" is not introduced in the algorithm of Yasarcan-Çetiner agents. On the contrary, individual agents do not know or even care about the bar capacity; Type 4 agents do not base their decisions explicitly on capacity and attendance values. Learning the capacity is an emergent result of the swarm of these agents.

10.4. Comparison for Type 4 Agents

We conduct a statistical analysis for Type 4 Agents. We take the simulation runs for 100 trials using a different random seed for each trial. Then, we calculate the performance measures for each agent type. As it can be seen in Table 10.2, the mean attendance of Type 4 Agents is 59.68 and the standard deviation is 1.911 which shows that Type 4 Agents have a better performance compared to the first three agents. The theoretical maximum limit for the average reward value is 0.6 and Type 4 Agents have the highest average reward. The bar utilization is another performance measure that Type 4 Agents have a great advantage on the other agent types with the value of 0.9881. It indicates a very high percentage of bar utilization. The only performance measure which Type 4 Agents perform worse than the others is the fairness. The fairness value of Type 4 Agents for 100 trials is 0.1548 and it is higher than the other three type of agents' result. The higher fairness value indicates that the distribution of reward values in Type 4 Agents are less fair. However, the range of fairness values is between 0 and 0.48, so the result of Type 4 Agents on fairness (0.1548) is not extremely poor.

Table 10.2. Comparative results for four types of agents.

Performance Measures (Range)	Type 1	Type 2	Type 3	Type 4
Mean Attendance (0-100)	59.89	60.31	57.07	59.68
Standard Deviation (0-50)	5.020	8.560	41.08	1.911
Avg. Reward (0-0.6)	0.4686	0.4328	0.1245	0.5208
Fairness (0-0.48)	0.03924	0.07903	0.08509	0.1548
Bar Utilization (0-1)	0.9658	0.9456	0.6591	0.9881

After we take the simulation runs for 100 trials, we also study the case with a single trial with a longer time horizon. The experiment is made with adjusting the number of weeks to 10000. In this case, the mean attendance for Type 4 Agents is exactly 60 and the standard deviation decreases to the value of 0.820. Moreover, the bar utilization becomes 0.994 which is still a comparatively successful result. The interesting issue on the longer simulation result is the fairness analysis. We obtain different fairness values for Type 4 Agents with different random seeds where the number of weeks is 10000. It means that fairness in the Type 4 Agents is dependent on the random seed. The random seed in Type 4 Agents only affects the first week's attendance value in which the agents make random expectations. So, we can conclude that the fairness in Type 4 Agents is dependent on the initial week's decisions.

11. CONCLUSION

In this thesis, firstly an analysis of the EFBP based on the three different expectation models is conducted. Three agent types are created, these agent types use random expectations, “bag of strategies”, and adaptive learning with exponential smoothing to create their expectations in the EFBP. We also introduce five performance measures to compare these three agent types. The performance measures are mean attendance, standard deviation of attendance, reward, fairness, and bar utilization.

As a result of the initial comparative analysis, we discover the importance of heterogeneity of the decisions in the EFBP. According to the results, Type 1 Agents (random expectations) are the most successful in terms of performance measures. Type 2 Agents and Type 3 Agents perform worse than the Type 1 Agents. These results show that any common knowledge that is used by the agents in the EFBP decreases the performance of the agents. In other words, we can conclude that using information from the environment such as knowing the weekly attendance and using them for weekly expectations does not improve the population’s performance in the EFBP.

Another issue related to the heterogeneity is the discussion of the strategies that Arthur (1994) used in his original EFBP model. Although Arthur does not give detailed information about formulations of the strategies, he gives some example strategies that predict the weekly attendance. We use the parameters that Garofalo (2006) suggests to create these strategies. We realize that Arthur does not discuss much about the forming of strategies because the strategies are not significant in the EFBP. We show this using the results of the extension of Type 1 Agents. We claim that the only function of the strategies that Arthur uses in his model is to create heterogeneity in the agents’ decisions. Furthermore, we find out that there is an important assumption in the EFBP, which is not explicitly explained in the literature. The agents know the bar capacity and make their decisions according to this value. We conduct experiments with an unknown constant capacity to show that without this assumption the agents’ performances worsens significantly. The bar capacity must either be known or learned by the agents in the EFBP. As a conclusion, we state that a good performance in the EFBP can be obtained by learning the bar capacity using an adaptive learning mechanism and creating expectations randomly to increase the heterogeneity (Type 3&1 Agents). The unknown capacity experiments and the dynamics obtained by Type 3&1 Agents show that if there are important inputs in decision-making, they must be estimated accurately. In the unknown bar capacity case, simply ensuring heterogeneity in the decisions of the agents would not be sufficient in producing results gravitating towards the capacity of the bar. Developing algorithms for the different agent types, introducing bag of strategies using random expectations, developing agents with adaptive learning mechanisms, comparing Arthur's strategies to random expectations, discovering the importance of the assumption that the capacity is known by the agents, studying the unknown capacity case, introducing agents that uses both adaptive learning and randomness in their decision processes are the original contributions of this thesis to the literature.

Lastly, we introduce a new agent type in this thesis, which we named as Yasarcan-Çetiner agents. Dr. Yasarcan, my thesis supervisor, suggests the algorithm for these agents inspired by his own unpublished work on hysteresis and, according to our knowledge, there is no other study in the literature reporting such agents. These agents do not use expectation models for their decisions, they have a hysteresis structure. Each agent persists on the current decision for a certain number of weeks according to the individual persistence thresholds for the two decision states. They only reverse their decision after a certain number of consecutive weeks that they get no reward value. We find out that Yasarcan-Çetiner agents, Type 4 Agents, perform better than the other three types of agents in terms of all performance measures except the fairness. These agents use the bar very efficiently with a low standard deviation of attendance.

Introducing Yasarcan-Çetiner agents is another original contribution of this thesis because they perform better than the other agents at the collective level. At the individual level, there is a fairness issue, which indicates that they do not all share the same total reward values. We also experimented with the unknown capacity case and reach to the conclusion that Yasarcan-Çetiner agents learn the capacity as a swarm.

This result is counterintuitive in the sense that Yasarcan-Çetiner agents do not have explicitly coded capacity learning mechanism in their algorithms; even when they collectively learn the capacity, individually they have no idea about it. Accordingly, the final conclusion of this thesis is that learning the bar capacity is an emergent collective behavior of the swarm of Yasarcan-Çetiner agents. In other words, their collective emergent intelligence is higher than their individual learning abilities; they are intelligent as a swarm.

12. FUTURE RESEARCH

After analyzing the dynamic behavior and performance measures obtained from the first three agent types (i.e., the three expectational models), we questioned the assumptions of the EFBP, which resulted in our findings about the importance of heterogeneity in decisions and the importance of the assumption that the capacity is known by the agents. This resulted in the unknown capacity experiments and, as a result, we introduced agents that uses both adaptive learning and randomness in their decision processes. Experimenting with the unknown capacity case was not the only idea; we identified many other potential experiments before we develop Yasarcan-Çetiner agents. These ideas are out-of-the scope of this thesis. Accordingly, we decided to present them as future research ideas.

Experiments can be carried out using adaptive bar capacity that responds the attendance values creating a feedback loop between the bar capacity and the attendance values. In order to obtain adaptive capacity, the bar must also be introduced as an agent that makes decisions about the capacity values. According to our most recent knowledge, this is also an original idea that does not exist in the current literature.

Another issue that can be studied is the payoff function formulation. The payoff function can be modified in the EFBP. With a closed-door assumption, when the number of agents who decide to attend the bar is greater than the bar capacity, the number of people which is equal to the bar capacity can get in the bar and the rest stays outside even they intend to attend the bar. The agents who stay outside the bar become unhappy and the agents who can get in the bar become happy.

A delay mechanism can be implemented for learning attendance values. The agents learn the actual attendance values with a certain delay and they make their expectations with the delayed attendance values. For the adaptive capacity case, a delay mechanism for learning the capacity value can also be added to the model. A "regret" mechanism can be added as a modification to the EFBP: The agents who become unhappy due to the crowdedness in the bar may have regret and decide not to attend the bar for a certain number of weeks.

Lastly, experiments with a mix of different agent types can be carried out. The mix agent types can be studied in terms of their group performance measures. Different types of agents can be mixed with different proportions. Moreover, these mixed agent types can be experimented with and analyzed under different assumptions.

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APPENDIX A: PYTHON CODE FOR THE SIMULATION MODEL

```
import numpy as np
import random as rd
matplotlib.pyplot as plt
import statistics as stat
import decimal
def to_sigfigs(value, sigfigs):
    return str(decimal.Context(prec=sigfigs).create_decimal(value))
def elfarol(n, Weeks):
    Expectation = np.zeros([Weeks, n]) # expectation of bar attendance by agent
    Decisions = np.zeros([Weeks, n], dtype = int) # attending bar decisions by agent
    Attendance = np.zeros([Weeks]) # total number of people going to the bar
    Happiness = np.zeros([Weeks, n], dtype = int) # happiness of individual agents
    Satisfaction = np.zeros([Weeks], dtype = int) # total happiness of agents
    Strategy = np.zeros([Weeks, m], dtype = int)
    Error = np.zeros([Weeks, m]) #error term for each strategy
    Max_number_Str = 6 #maximum number of strategy
    Str_Set = np.zeros([n, Max_number_Str], dtype = int) #the strategy set of agents
    Number_of_Str = np.zeros([n], dtype = int) #total number of strategies for agents
    Active_Str = np.zeros([Weeks, n], dtype = int) #active predictor of each agent
    Capacity = np.zeros([Weeks], dtype = int) # the adaptive capacity value
    IntendedCapacity = np.zeros([Weeks]) #intended capacity
    Correctness = np.zeros([Weeks, n], dtype = int) # correct decision payoff
    RewardCorrect = np.zeros([n]) # Reward 0/1 correct decision
    ExpectedCapacity = np.zeros([Weeks, n]) # capacity expectation matrix
    ExpectedCapacity1 = np.zeros([Weeks, n])
    DistCapacity = np.zeros([Weeks], dtype = int) # capacity distribution
    Type4Persist0 = np.zeros([n], dtype= int) # persistence of staying home
```

```
Type4Persist1 = np.zeros([n], dtype= int) # persistence of going to the bar
Type4Decisions = np.zeros([n], dtype= int) # Decisions of Type 4 agents
for i in range(n):
    Type4Persist0[i]=1+(i%10) #assignment of staying home persistence
    Type4Persist1[i]=1+((i/10)%10) #assignment of going to bar persistence
for s in range(Weeks): DistCapacity[s] = 70 + int(rd.random()*21)
for i in range (Type2agents):
    Number_of_Str[i+Type1agents] = Max_number_Str
    for k in range(Number_of_Str[i+Type1agents]):
        flag = 0
        while flag == 0:
            Str_Set[i+Type1agents, k] = int(rd.random()*m)
            if k == 0: flag = 1
            else:
                flag = 1
                for k1 in range (k):
                    if Str_Set[i+Type1agents, k] == Str_Set[i+
Type1agents, k1]: flag = 0
for j in range(20): Attendance[-1-j] = IniCap - 1
for s in range(Weeks):
    if CapacityDistMethod == 0:
        if s > 0:
            IntendedCapacity[s] = IntendedCapacity[s-1]+
caf * (Attendance[s-1]-IntendedCapacity[s-1])
            Capacity[s] = round(IntendedCapacity[s])
        else:
            Capacity[0] = IniCap # initial capacity of the bar
            IntendedCapacity[0] = IniCap
    else:
        Capacity[s] = DistCapacity[s]
    if Type1agents > 0:
```

```
            for i in range(Type1agents):
                Expectation[s,i] = int(rd.random()*n)
        if Type2agents > 0:
            for j in range(20):
                Strategy[s,j] = Attendance[s-1-j]
                Strategy[s,j+20] = n - Attendance[s-1-j]
                Strategy[s,j+40] = int(n * 0.02 * (j+5))
                Strategy[s,j+60] = int(n * 0.02 * (j+26))
                Strategy[s,j+80] = int(Attendance[s-1] + (Attendance[s-2] -
Attendance[s-1])*0.05* (j+1))
                Strategy[s,j+100] = int(n - (Attendance[s-1] + (Attendance[s-2]
- Attendance[s-1])*0.05* (j+1)))
                Strategy[s,j+120] = int(np.sum(Attendance[s-j-2:s-1])/(j+2))
                Strategy[s,j+140] = n - int(np.sum(Attendance[s-j-2:s-1])/(j+2))
                Strategy[s,j+160] = int(((j/20) * 0.2 + 0.8)*Attendance[s-1-j])
                Strategy[s,j+180]=int((((j+1)/20) *0.2 +1)*Attendance[s-1-j])
            for i in range(Type2agents):
                best_k = -1
                best_Error = 2*n
                if s==0: Active_Str[s, i+Type1agents]=Str_Set[i+Type1agents,0]
                else:
                    for k in range(Number_of_Str[i+Type1agents]):
                        if Error[s-1, Str_Set[i+Type1agents,k]] ¡ best_Error:
                            best_Error = Error[s-1, Str_Set[i+Type1agents,k]]
                            best_k = k

                    Active_Str[s, i+Type1agents] = Str_Set[i+Type1agents,
best_k]
            Expectation[s,i+Type1agents]=Strategy[s,Active_Str[s,i+Type1agents]]
        if Type3agents > 0:
            if s == 0:
```

```
for i in range(Type3agents):
    Expectation[0,i] = int(rd.random()*n)
    alpha[i] = 0.1 + (0.2 * rd.random())
else:
    for i in range(Type3agents):
        Expectation[s,i+Type1agents+Type2agents] = (1-alpha[i+
Type1agents+Type2agents]) * Expectation[s-1, i+Type1agents+Type2agents] +
alpha[i+Type1agents+Type2agents] * Attendance[s-1] #same as last week
if Type4agents > 0:
    if s == 0:
        for i in range(Type4agents):
            Expectation[0,i+Type1agents+Type2agents+Type3agents]
= int(rd.random()*n)
    else:
        for i in range(Type4agents):
            T40s = max(0, s - 1 - Type4Persist0[i])
            T40e = s-1
            T40d = T40e - T40s
            T41s = max(0, s - 1 - Type4Persist1[i])
            T41e = s-1
            T41d = T41e - T41s
            flag0 = 0
            flag1 = 0
            for ii in range(T40d):

                if Decisions[s-1-ii,i+Type1agents+Type2agents
+Type3agents] == 0:
                    if Correctness[s-1-ii,i+Type1agents+Type2agents
+Type3agents] == 0: flag0 += 1
            for ii in range(T41d):
                if Decisions[s-1-ii,i+Type1agents+Type2agents
```

```
+Type3agents] == 1:
                              if Correctness[s-1-ii,i+Type1agents+Type2agents
+Type3agents] == 0: flag1 += 1
                      if Decisions[s-1,i+Type1agents+Type2agents
+Type3agents] == 0:
                          if flag0 < Type4Persist0[i]: Type4Decisions[i] = 0
                          else: Type4Decisions[i] = 1
                      else:
                          if flag1 < Type4Persist1[i]: Type4Decisions[i] = 1
                          else: Type4Decisions[i] = 0
        if s == 0:
            for i in range(n):
                ExpectedCapacity[s,i] = int(rd.random()*(n+1))
                ExpectedCapacity1[s,i] = ExpectedCapacity[s,i]
        else:
            if CapacityEstimationMethod == 0:
                for i in range(n): ExpectedCapacity[s,i] = Capacity[s]
            elif CapacityEstimationMethod == 1:
                for i in range (n): ExpectedCapacity[s,i] = int(rd.random()*(n+1))
            elif CapacityEstimationMethod == 3:
                for i in range(n):
                    ExpectedCapacity1[s, i] = (1-CapEstPara) *
ExpectedCapacity1[s-1, i] + CapEstPara * Capacity[s-1]
                    ExpectedCapacity[s, i] = round(ExpectedCapacity1[s,i])
        if s == 0:
            for i in range(n):
                if round(Expectation[s, i]) < ExpectedCapacity[s, i]: Decisions[s,
i] = 1 # going to the bar decision
                else: Decisions[s, i] = 0 #not going to the bar decision
        else:
            for i in range (n-Type4agents):
```

if round(Expectation$[s, i]) < E$xpectedCapacity[s, i]: Decisions[s, i] = 1 # going to the bar decision

else: Decisions[s, i] = 0 #not going to the bar decision

for i in range(Type4agents):

Decisions[s,i+Type1agents+Type2agents+Type3agents] = Type4Decisions[i]

Attendance[s] = np.sum(Decisions[s])

if $Type2agents > 0$:

for j in range(m):

if s == 0: Error[s, j] = (1-l) * 0 + l * abs(Strategy[s, j] - Attendance[s])

else: Error[s, j] = (1-l) * Error[s-1, j] + l * abs(Strategy[s, j] - Attendance[s])

if $Attendance[s] <= Capacity[s]$: #capacity contraint

for i in range (n):

if Decisions[s, i] == 1:

Happiness[s, i] = 1 #being happy condition

Correctness[s,i] = 1 #correct decision

else:

Happiness[s, i] = 0

Correctness[s,i] = 0 #incorrect decision

else:

for i in range(n):

if Decisions[s,i] == 1:

Correctness[s,i] = 0 #incorrect decision

Happiness[s, i] = -1

else:

Correctness[s,i] = 1 #correct decision

Happiness[s, i] = 0

Satisfaction[s] = np.sum(Happiness[s])

for i in range(n):

```
        RewardC = np.sum(Correctness, axis=0)
        RewardCorrect[i] = RewardC[i] / Weeks
    TotUtilization = 0
    for s in range(Weeks):
        if Attendance[s] > Capacity[s]: TotUtilization += 1
        else: TotUtilization += Attendance[s] / Capacity[s]
    Utilization = TotUtilization/Weeks
    RewardCG = 0
    RewardCGtot = 0
    FairCG = 0
    FairCGtot = 0
    for i in range(n):
        RewardCGtot += RewardCorrect[i]
    RewardCG = RewardCGtot /n
    for i in range(n):
        FairCGtot += abs(RewardCorrect[i]-RewardCG)
    FairCG = FairCGtot /n
    return Attendance, Capacity, RewardCG, FairCG, Utilization
n = 100 # population
alpha = np.zeros([n])
Weeks = 100 # total number of weeks
rd.seed(1) # trials
caf = 0 #capacity adjustment fraction
l = 0.2
IniCap = 60 #initial capacity
m = 200 #total number of strategies
Type1ratio = 1 #ratio of Type 1 agents
Type2ratio = 0 #ratio of Type 2 agents
Type3ratio = 0 #ratio of Type 3 agents
Type4ratio = 0 #ratio of Type 4 agents
Type1agents = int(n*Type1ratio) #number of random expecctation agents
```

```
Type2agents = int(n*Type2ratio) #number of Arthur/Garofalo agents
Type3agents = int(n*Type3ratio) #number of adaptive learning agents
Type4agents = int(n*Type4ratio) #number of Yasarcan-Çetiner agents
CapacityEstimationMethod = 0 # Capacity estimation method
CapEstPara = 0.1 # capacity estimation smoothing parameter
CapacityDistMethod = 0 # Capacity Distribution Method
Attendance, Capacity, RewardCG, FairCG, Utilization = (elfarol(n, Weeks)) #recall-
ing elfarol function
MeanAtt = stat.mean(Attendance)
Std = round(stat.stdev(Attendance),2)
TimeArray = np.linspace(0, Weeks,num=Weeks)
fig100 = plt.figure(100)
plt.ylim(0, 100)
plt.xlim(0, Weeks)
plt.plot(TimeArray[0:], Attendance[0:], color = 'black', linestyle = 'solid', linewidth =
1.5, label= "Attendance")
plt.plot(TimeArray[0:], Capacity[0:], color = '#505050', linestyle = 'dashed', linewidth
= 2, label= "Capacity")
plt.legend()
plt.subplots_adjust(left=0.125, bottom=0.26, right=0.95, top=0.95, wspace=0)
plt.text(0, -25, "Mean Attendance = " +to_sigfigs(MeanAtt,4), dict(size=8))
plt.text(0, -32, "Stdev. Attendance = " +to_sigfigs(Std,4), dict(size=8))
plt.text(37, -25, "Average Reward = "+to_sigfigs(RewardCG,3), dict(size=8))
plt.text(37, -32, "Fairness = "+to_sigfigs(FairCG,3), dict(size=8))
plt.text(73, -25, "Avg. Utilization = "+to_sigfigs(Utilization,3), dict(size=8))
plt.ylabel("Attendance")
plt.xlabel("Weeks")
fig100.savefig("Type 4 Agents.png", dpi=300)
plt.close(fig100)
```