

SIMULATION OF SEPARATELY
EXCITED CHOPPER CONTROLLED
DC MACHINE

by

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This thesis has been approved by:

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(Thesis Supervisor) Submitted to the Faculty of the
School of Engineering in Partial Fulfillment

of the Requirements for the Degree of

MASTER OF SCIENCE

in

ELECTRICAL ENGINEERING

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ACKNOWLEDGEMENTS

I would like to express my gratitude to Dr. Okyay Kaynak, thesis supervisor, for his encouragement and guidance in accomplishment of this work.

Thanks are also due to Dr. Ömer Cerid and Dr. Selahattin Kuru, for their useful critiques, interest and helpful suggestions.

ABSTRACT

In this thesis, a novel approach for the simulation of a chopper-fed dc machine, is presented. Instead of the general technique of combining the Kirchhoff's voltage, current and branch characteristic equations, which would result in an equation of the form $\dot{\underline{x}} = \underline{f}(\underline{x}, t)$, the branch currents are evaluated in terms of node voltages by applying Kirchhoff's voltage and current equations, and they are immediately converted to algebraic equations by trapezoidal formula. Later on, the nonlinearities are considered, and thyristor and diodes are linearized by piecewise linear models. In this way, breaking down the circuit operation into different modes, which causes laborious analytical manipulation or requires long computer time, is avoided. Also, a method of obtaining the complete set of equations for any network, only by observing the topology of the circuit, is developed.

ÖZET

Bu tezde, bir kıyıcı ile beslenen bir doğru akım makinasının sayısal benzetimi için yeni bir yöntem önerilmiş ve uygulanmıştır. Bilinen bir genel teknik olan, Kirchhoff'un voltaj, akım ve eleman tanım bağıntılarını birleştirip $\dot{x}=f(x,t)$ şeklinde bir denklem elde etmek yerine; eleman akımları, Kirchhoff'un voltaj ve akım bağıntılarını uygulayarak, düğüm voltajları cinsinden yazılmış ve hemen 'trapezoid formülü' ile cebirsel bağıntılara dönüştürülmüştür. Daha sonra, devredeki diyod ve tristör gibi doğrusal olmayan elemanlar parçalı doğrusal yaklaşıklama yöntemi ile doğrusallaştırılmıştır. Bu şekilde, büyük analitik işlemleri ve uzun bilgi sayar zamanı gerektiren, devre işlevini çeşitli kiplere ayrıştırma yönteminden kaçınılmıştır. Ayrıca, herhangi bir devre için, sadece devrenin yapısına bakarak, devre denklemlerini elde etmek için kurallar geliştirilmiştir.

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CHAPTER 1

INTRODUCTION

A new era in electric power conversion began with the development of the silicon controlled rectifier (SCR) in late 1957. From the first 16 ampere average current rated unit, an entirely new family of semiconductor devices was created. Thyristors are now available with current ratings from milli-amperes to over a thousand amperes and with voltage ratings up to 2,000 volts.

The first thyristor applications involved simple phase control for lamp dimmers, heat controls, and ac motor voltage controls. As a wider range of triacs became available, they replaced the SCR in most of these applications. More advanced phase control circuits were pursued next including reversing dc motor drives with inversion to the ac system for regenerative braking of up to 6000 hp armature controlled motors for large mill drives. A great variety of sophisticated static power conversion equipment is now being produced, ranging from small appliance controls to variable frequency 100 hp ac drives for a wide range of industrial applications.

In an electrical power control system, the power converter is the critical link for matching the source to the

load. The characteristics of both the source and the load and often the electrical and ambient environment determine the duty on the converter. It falls upon the circuit designer to choose the proper circuit arrangement for the specific application. However, he is often constrained by the capability of the available power switching devices. Recently, this device capability has become the major factor in determining the size and efficiency of a converter circuit, hence the optimum arrangement for the required duty. The most recent developments in thyristor characteristics have been to make this limiting factor in power system design less restrictive.

Thyristors are found to be very reliable working in the harsh conditions of a machine environment, thus there is a decrease in the maintenance costs. They can increase the efficiency of a machine installation. For example, the efficiency of the control and drive gear of railway equipment is increased from sixty seven percent to ninety five percent when changed from conventional resistance control to thyristor control. Thyristors lend themselves very readily to mass production using the well proven technology of semiconductor transistor and diode manufacture, thus they are relatively cheap.

The thyristor is basically a switch, that can be switched on by the application of a gate pulse of a few milliamps current and a very short time in duration (provided the anode cathode voltage is positive and greater than one volt in magnitude). When on, the current through the thyristor is

determined only by the parameters of the external circuit, and an approximately constant voltage drop (one volt) is observed across the device. The thyristor can be turned off by reducing the current through it below its latching value (usually about one hundred milliamps) whereupon it returns to its blocking state. The circuitry required to turn the device on is quite simple but that required for turn off can lead to an increase in the control circuitry.

In thyristor control applications, harmonic generation and torque pulsation due to switching action of the thyristor, unwanted turn-on due to excessive rates of change of applied voltage and fatal breakdown of the thyristor due to transient voltage surges of the supply present themselves as possible sources of malfunction. In designing the control system, special considerations must be given to these aspects and compensation measures must be taken.

It is difficult to design an efficient control circuitry and at the same time have enough safety margin for the possible sources of malfunction mentioned above. In most cases, the designer adopts a trial and error approach or overrates the components in the circuitry. On the other hand, if the exact behaviour of the circuitry is known, then an optimum design can be made. This is why there is a great need to simulate an installation so that the performance of the installation can be ascertained without the danger of damaging very expensive equipment.

In this thesis, the result of work carried out for the digital simulation of separately excited chopper controlled dc machine is reported. After the introduction of Chapter 1, some basic chopper circuits are introduced in Chapter 2, and the methods for commutation are discussed, dc machine dynamics, transfer functions and block diagrams are presented and the equivalent circuit of a separately excited dc machine is derived. In Chapter 3, the merits of analogue and digital simulations are compared briefly, and general techniques of digital simulation, specially the simulation of electrical machines are discussed, and the general formulation of circuit equations which is used in this thesis, is given. In Chapter 4, the circuit equations of chopper controlled dc motor are formulated, and some other details of the simulation are explained. In the final chapter, Chapter 5, the results are presented and discussed. A full derivation of circuit equations of separately excited chopper controlled dc machine is given in the Appendix.

CHAPTER 2

CHOPPERS AND DC MACHINE DYNAMICS

2-1. INTRODUCTION

For the control of the voltage applied to the armature of a dc machine, chopper circuits are frequently used.

The supply voltage is varied in average value by the regular switching on and off of a thyristor. The voltage seen by the machine is thus made equal to the supply voltage for a fixed time and then to zero for a further time (the voltage is chopped). Therefore by varying the ratio of on-time to off-time the average voltage seen by the machine can be varied from zero voltage (no on-time) to full supply voltage (no off-time). There are however practical limits to the minimum on-time and minimum off-time set by the physics of the thyristor.

In Fig.2-1, the equivalent circuit of a chopper controlled dc machine is shown. If a switch is connected between a dc source and a load as in the figure, it is possible to energize or de-energize the load, i.e. the dc machine, by simply closing or opening the switch. The voltage at the load terminals appears as in the figure, if the switch is open and closed periodically.

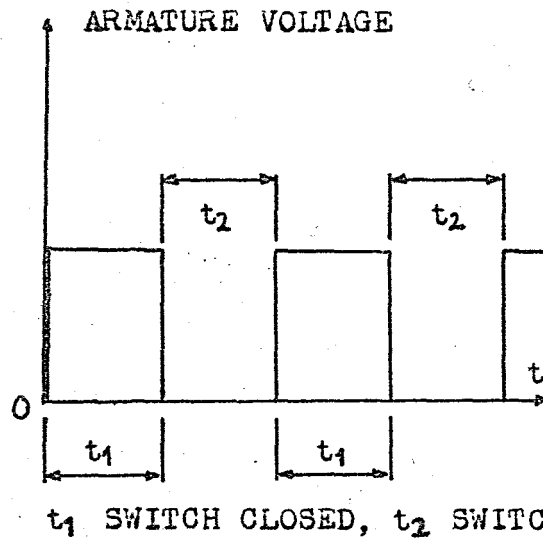
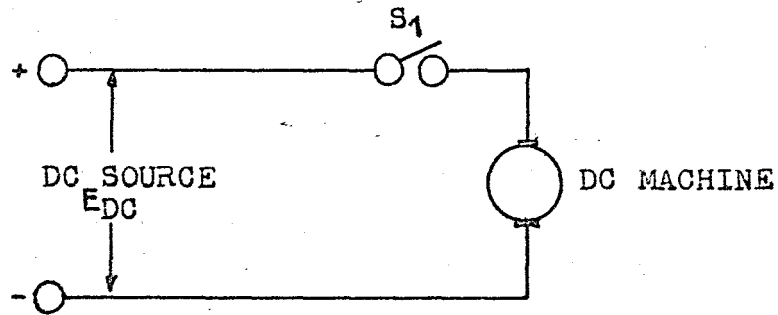


Fig. 2-1. The Equivalent Circuit of a Chopper Controlled DC Machine

The average voltage to the load is $E_{DC}/2$, if t_1 is made equal to t_2 . This voltage can be varied by maintaining t_1 constant and varying t_2 (sometimes called pulsed rate modulation) or by holding t_1+t_2 constant and varying t_1 (sometimes called pulse width modulation), or by varying both t_1 and t_2 . It is easy to see how this system can be used to decrease the average voltage to the load, thus serving the purpose of a dc step-down transformer.

2-2. CHOPPERS

In a basic chopper circuit the thyristor is turned on by the conventional way (i.e. a gate pulse and forward bias), but the turnoff is by forced commutation, that is the current through the thyristor is forced below the latching value so as to commute (switch-off) the thyristor; the circuits that perform this forcing down of current are known as commutation circuits. The turnoff of the current carrying SCR is achieved by applying an adequate reverse voltage for a certain time across the SCR, and by commutating its current to another branch while it is reversed biased. Inductive loads, however, result in complicated turnoff problems, on account of their stored energy. Either the turnoff capacitor must be enlarged to absorb this energy itself (load-related parallel capacitor turnoff) or an alternative path must be provided for inductive load current. When the load current is unidirectional, as with choppers, a freewheeling diode forms an ideal alternative path. Invertors, having an alternating load current, require more complex return-current diode paths to provide for both directions of load current.

Briefly, for a thyristor to be turned off correctly, the following must take place (3) :

- (a) The thyristor current must be reduced to zero.
- (b) A reverse voltage must appear across the thyristor for a time longer than its turnoff time.

(c) The subsequent reapplication of forward voltage must be at a rate less than its dV/dt rating.

(d) An alternative path must be provided for the load current where inductive.

The precharged capacitor performs action (a) by diverting the load current from the thyristor to itself. It performs action (a), (b) and (c) by being connected effectively in parallel with the thyristor, so that its stored voltage reverse-biases the thyristor; it has sufficient capacitance to carry the load current for longer than the turnoff time before becoming discharged, and it likewise limits the rate of rise of forward voltage during recharging. The way in which the circuit performs action (d) introduces a fundamentally important distinction between two forms of parallel capacitor turnoff for inductive loads. If, after turnoff, the load current is transferred away from the capacitor to a constant-voltage circuit before the load current decays to zero, the circuit employs 'impulse commutation', a fast turnoff process, which is thyristor-related (8). The commutating, or turnoff, capacitor is relatively small, governed as it is by the thyristor turnoff time, and it carries the load current for a relatively short period, typically 50-100 μ s. If the turnoff capacitor carries the load current unaided until the load current has been reduced to zero, the capacitor is much larger in order to absorb all the load energy for a tolerable rise of voltage. The reverse-voltage time thus greatly exceeds the turnoff time, and the commutation process is very much slower;

it is load-related since the relationship exists between stored load energy and capacitor energy. The distinction between thyristor-related impulse capacitor turnoff and load-related capacitor turnoff arises only for inductive loads. A resistive load does not store energy; therefore, the provision of the alternative path (d) above is unnecessary. The circuit can be equally viewed as using a thyristor-related or load related turnoff technique.

There are many types of commutation circuits (3,4,8,9,10), but they basically fall into two categories.

(1) Switched capacitor turnoff circuits:

The precharged capacitor is switched directly across the main thyristor I to be turned off by a second thyristor II in the basic circuit of Fig.2-2.

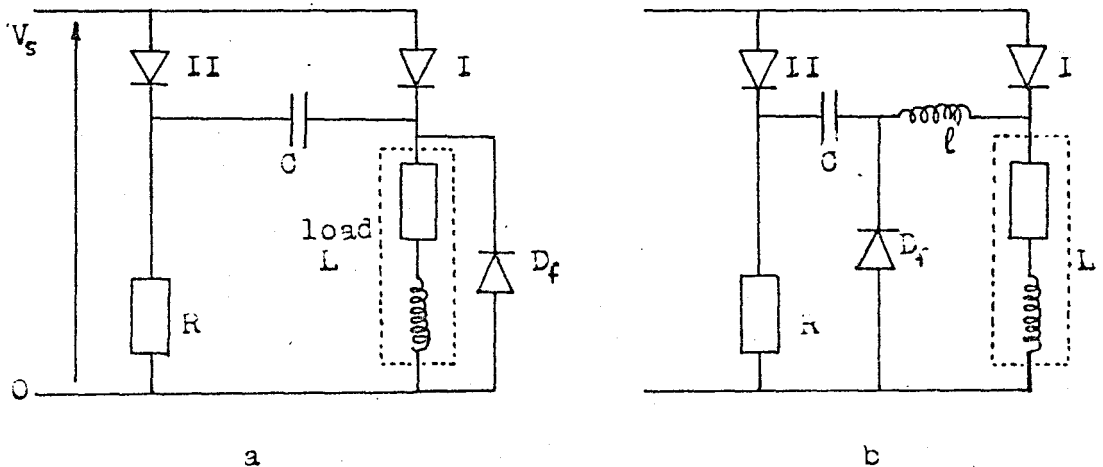


Fig.2-2. Switched Capacitor turnoff circuits

Thyristor I conducts the load current keeping the right-hand plate of C at the potential $+V_s$. The left hand-plate potential is reduced to zero as the capacitor charges through R. If, when C is fully charged, thyristor II is fired, the left hand-plate of C will be raised to $+V_s$, and hence its right-hand plate to a potential $+2V_s$. The cathode of thyristor I is thus more positive than its anode; it is reverse-biased, and hence ceases conduction. The load current now flows from supply through the second thyristor II and C, discharging C. While C discharges from V_s to zero, thyristor I is reverse-biased; this is called the reverse-voltage time t_{RV} , which must be greater than the turnoff time of thyristor I. Thus,

$$C = I_L t_{RV} / V_s \quad \text{where } t_{RV} > t_{off} \quad (2.1)$$

Eqn.2.1 is based on a constant load current (capacitor current) during the period t_{RV} , resulting in a linear discharge of C, thus simplifying the general integral equation

$$C = 1/V_s \int_0^{t_{RV}} i_L \cdot dt \quad (2.2)$$

During the reverse-voltage time, while thyristor I has been reverse-biased, the load L supports $2V_s$ initially, decreasing to V_s when C is discharged. The load current must increase somewhat during the reverse-voltage time, owing to the additional voltage across it. After C has been discharged, the load current continues to flow through it, charging it in the reverse direction until the load voltage is zero. From the firing of thyristor II until the load voltage falls to zero, the path thyristor II-C has been the alternative path for the load current. From this

time onwards, it is usual for the load current to transfer to a diode freewheeling path D_f in parallel with the load. Without this, the energy stored in the load would continue to charge the capacitor to a voltage in excess of the economic rating for thyristor I. An essential feature in any capacitor turnoff circuit is the recharging of the capacitor ready for the next turnoff process for thyristor I. In the basic circuit of Fig.2-2.a, this is performed by firing thyristor I again. A reverse process turns off thyristor II, after which C discharges and recharges to the original polarity. This basic technique is used in most chopper and inverter circuits and is sometimes referred to as impulse commutation (8). In practice, two thyristors and a capacitor should not be connected in a loop as they are in Fig.2-2.a, because each will experience high di/dt when fired into the reverse (carrier-storage) conduction of the other. Likewise, when thyristor I is fired while D_f is conducting a freewheeling current, thyristor I will experience high di/dt while D_f conducts transiently in the reverse direction. Inductor(s), i.e. l in Fig.2-2.b, must be included in these two paths to limit di/dt and their presence introduces a measure of resonant behaviour based on the principles of Resonant Capacitor Turnoff circuits. More advanced techniques in this type of commutation will be discussed in Chapter 4 and 5.

(ii) Resonant capacitor turnoff circuits:

Resonant turnoff does not require a second thyristor.

Instead, the firing of the load thyristor initiates a resonant reversal of the capacitor voltage, followed automatically by a thyristor current reduction and the application of a reverse voltage to the thyristor. Fig.2-3. illustrates the basic circuit

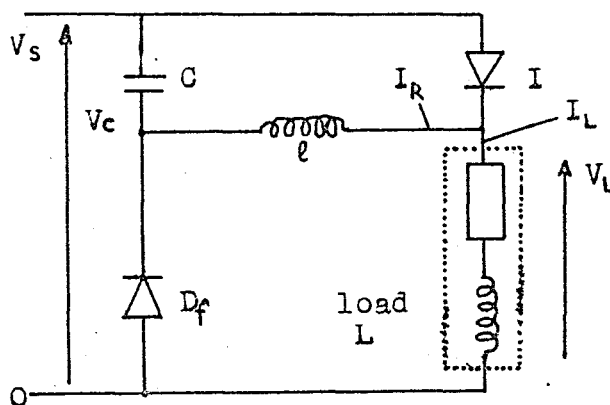


Fig.2-3. Resonant capacitor turnoff circuit

in which l, C and thyristor I form the resonant circuit, and losses are neglected (8). Initially thyristor I is off and C is charged through L (load) and l with its lower plate at zero potential.

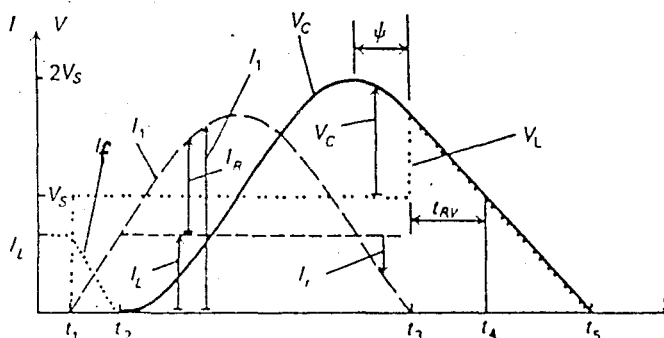


Fig.2-4. Voltage and current waveforms of resonant capacitor turnoff circuit, in Fig.2-3.

When thyristor I is fired at t_1 , any load current has been free-wheeling through D_f and ℓ decays as the current in thyristor I builds up, their sum being the load current (assumed constant). When all the load current has transferred to thyristor I, D_f ceases conduction at t_2 (Fig.2-4.). The resonant circuit ℓ C (and thyristor I) carries a resonant current I_R charging the lower plate of C positively until it reaches a potential $+2V_S$ after a halfcycle. The next halfcycle of resonant oscillation now begins, during which the resonant current opposes the load current in thyristor I. Provided that the peak resonant current exceeds the load current, the thyristor current is reduced to zero at t_3 . At t_3 , the constant load current, now flowing through C and ℓ produces no voltage drop across ℓ so that the remaining voltage V_{c3} (F) reverse-biases thyristor I. The reverse voltage across thyristor I persists until t_4 when the load current has linearly discharged C, and $V_{\ell c} = V_S$, after which C recharges with its lower plate negative until its voltage reaches V_S , whereupon D_f conducts and freewheels the load current. The waveforms illustrating the above description are given in Fig.2-4., in which the resonant current I_R has a raised zero at the level I_L , and the capacitor voltage has a raised zero at $+V_S$ (3).

There are very many other commutation circuits, for further details, one should refer references 3,4,8,9,10.

2-3. DC MACHINE DYNAMICS

2-3-1. DYNAMIC EQUATIONS

The electromechanical coupling terms are the magnetic torque T , and the generated voltage e_a , for a dc machine are:

$$T = K_a \cdot \Phi_d \cdot i_a \quad (2.3)$$

$$e_a = K_a \cdot \Phi_d \cdot \omega_m \quad (2.4)$$

where $K_a = \frac{P \cdot Z_a}{2 \pi a}$, which is a constant fixed by the design

of the winding, and the symbols are defined as:

Φ_d : direct axis air gap flux per pole

P : number of poles

Z_a : total number of conductors in the armature winding

a : number of parallel paths through the winding

i_a : the current in the external armature circuit

ω_m : mechanical speed

These equations together with the differential equation of motion of the mechanical system, the volt-ampere equations for the armature and field circuits, and the magnetization curve describe the system performance.

Consider first the ideal machine shown in Fig.2-5. , with one field winding and negligible magnetic saturation. The direct-axis air-gap flux Φ_d is then linearly proportional to the field current i_f , and equations 2.3 and 2.4 can be expressed as:

$$T = k_f \cdot i_f \cdot i_a \quad (2.5)$$

$$e_a = k_f \cdot i_f \cdot \omega_m \quad (2.6)$$

where k_f is a constant. With the brushes in the quadrature axis the mutual inductance between the field and armature circuits is zero, just as it would be for two coils whose axes are perpendicular. The voltage equation for the field circuit then is

$$v_f = L_{ff} \cdot p i_f + R_f \cdot i_f \quad (2.7)$$

where v_f , i_f , R_f and L_{ff} are the terminal voltage, current, resistance, and self-inductance of the field circuit, respectively, and p is the derivative operator d/dt . For motor, reference directions are indicated in Fig.2-5. The voltage equation for the armature circuit

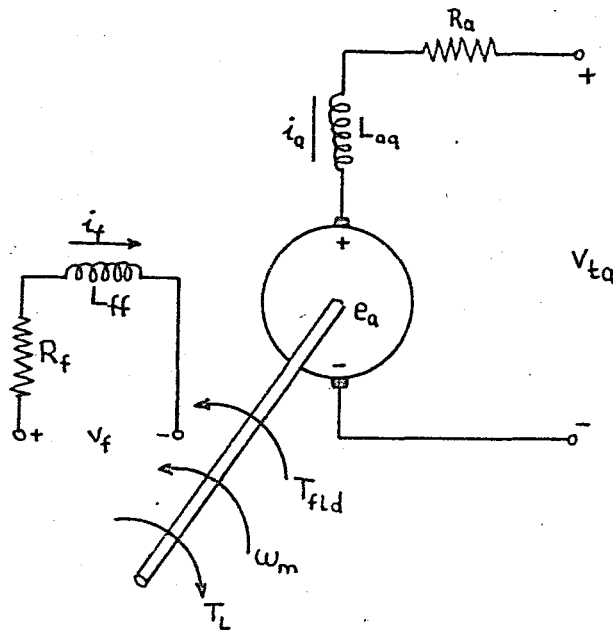


Fig.2-5 Schematic representation of a dc machine

$$is \quad v_{ta} = e_a + L_{aq} \cdot p i_a + R_a \cdot i_a \quad (2.8)$$

$$v_{ta} = k_f \cdot i_f \omega_m + L_{aq} \cdot p i_a + R_a \cdot i_a \quad (2.9)$$

where v_{ta} , i_a , R_a , and L_{aq} are the terminal voltage, current,

resistance and self inductance of the armature circuit, respectively. The subscript q is used with the inductance because the axis of the armature mmf is along the quadrature-axis. The inductance L_{qq} includes the effect of any quadrature-axis stator windings in series with the armature, such as interpoles and pole-face compensating windings used on large machines to improve commutation. For a motor the dynamic equation for the mechanical system is

$$T = k_f \cdot i_f \cdot i_a = J p \omega_m + T_L \quad (2.10)$$

where J is the moment of inertia and T_L is the mechanical load torque opposing rotation.

Energy storage is associated with the magnetic fields produced by the field and armature currents and with the kinetic energy of the rotating parts. The state of a physical system can be described in terms of its stored energy. Accordingly, the field and armature currents and the speed are state variables. Equations 2.7 through 2.10 are first-order differential equations containing product nonlinearities $i_f \omega_m$ and $i_f \cdot i_a$ of these state variables. These equations together with the Kirchhoff-law equations for the circuits connected to the field and armature terminals and the torque-speed characteristics of the mechanical system connected to the shaft, determine the system performance. Their application to specific cases will be described in the following section.

2-3-2. TRANSFER FUNCTIONS AND BLOCK DIAGRAMS OF DC MACHINES

The most difficult obstacle to overcome in analysis of dc machines is the inclusion of magnetic saturation. Linear analyses omitting saturation serve two useful purposes, however. First, by virtue of the relatively simple linear differential equations which may then be written, a fuller appreciation of other factors affecting transient performance is made possible, and an approximate picture of the events is gained. Second, for those system problems involving complex combinations of machines and other equipment, dynamic system studies are made possible which otherwise would be practically prohibitive without resort to a computer.

(a) DC generators. Linear analysis

Consider the generator of Fig.2-0, and assume that operation is restricted to linear portion of the magnetization curve of Fig.2-7.

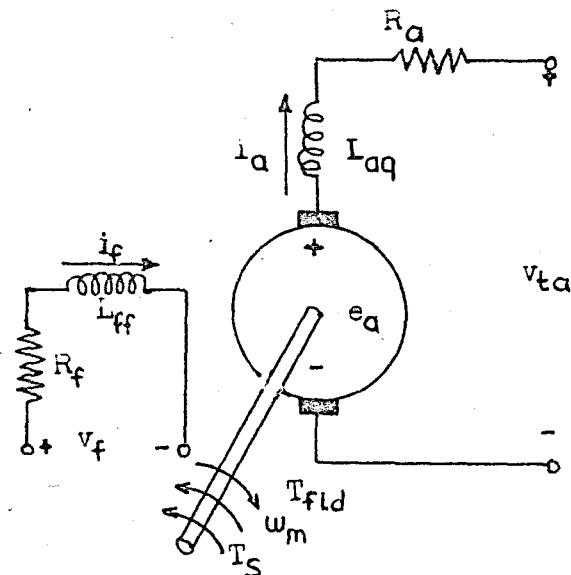


Fig.2-0. Schematic representation of a dc generator

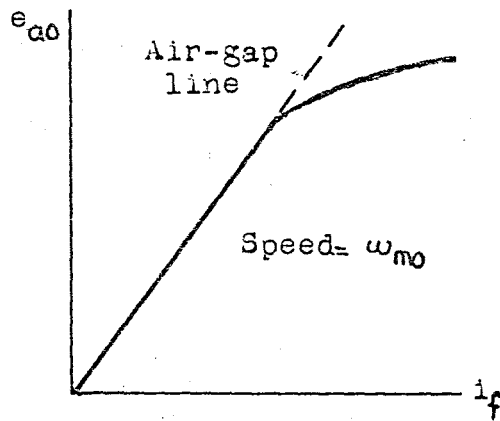


Fig.2-7. Magnetization curve

The inductance of the field winding then is constant, and the voltage equation for the field circuit is

$$v_f = R_f \cdot i_f + L_{ff} \cdot p i_f = R_f (1 + \tau_f p) i_f \quad (2.11)$$

where $\tau_f = L_{ff} / R_f$ is the time constant of the field circuit. At magnetization-curve speed ω_{m0} and with operation restricted to the linear range, the armature emf e_{a0} is

$$e_{a0} = K_g i_f \quad (2.12)$$

where K_g is the slope of the air-gap line at speed ω_{m0} .

Rearrangement of Eq. 2.11, in state-variable form gives

$$p i_f = 1/\tau_f (v_f / R_f - i_f) \quad (2.13)$$

The block diagram with the integrator $1/p$ in the forward path is shown in Fig. 2-8. Multiplication of the output i_f by K_g then gives the generated emf e_{a0} at magnetization-curve speed. Since generated emf is proportional to speed, the emf e_a at any other speed ω_m is

$$e_a = e_{a0} \omega_m / \omega_{m0} \quad (2.14)$$

as shown by the multiplier in the output of Fig. 2-8, the

corresponding transfer function in complex variables is obtained by replacing the derivative operator p in Eq. 2.11 by the complex frequency s . The variables $E_{ao}(s)$ and $V_f(s)$ then are the complex

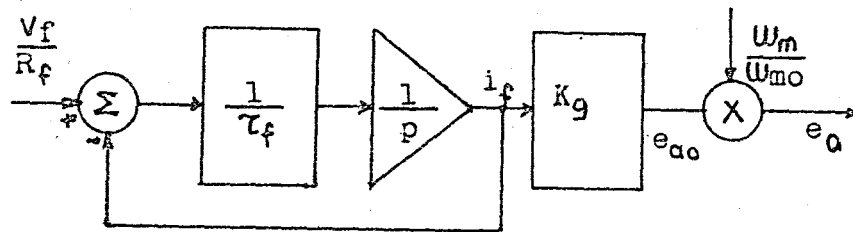


Fig.2-8. Block diagram of Eq. 2.13

amplitudes of the corresponding time variables, and the equations become algebraic equations in s ; thus

$$\frac{E_{ao}(s)}{V_f(s)} = \frac{K_g I_f}{V_f} = \frac{K_g/R_f}{1-\tau_f s} \quad (2.15)$$

as shown in Fig.2-9

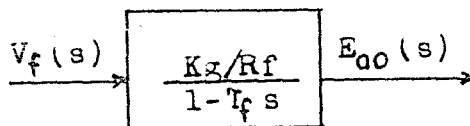


Fig.2-9. Block diagram of Eq. 2.15

The armature current i_a is determined by the generated emf e_a and the electrical circuits connected to the armature terminals. The magnetic torque T is then determined by the direct-axis flux and armature currents, as in Eq. 2.3.

(b) Separately excited dc motors

Direct-current motors are often used in applications requiring precise control of speed and torque output over a wide range. One of the common ways of control is the use of separately excited motor with constant field excitation. The speed is controlled by variation of voltage applied to the armature terminals. The analysis then involves the electrical transients in the armature circuit and the dynamics of the mechanical load driven by the motor.

A separately excited motor is shown in Fig. 2-10. The

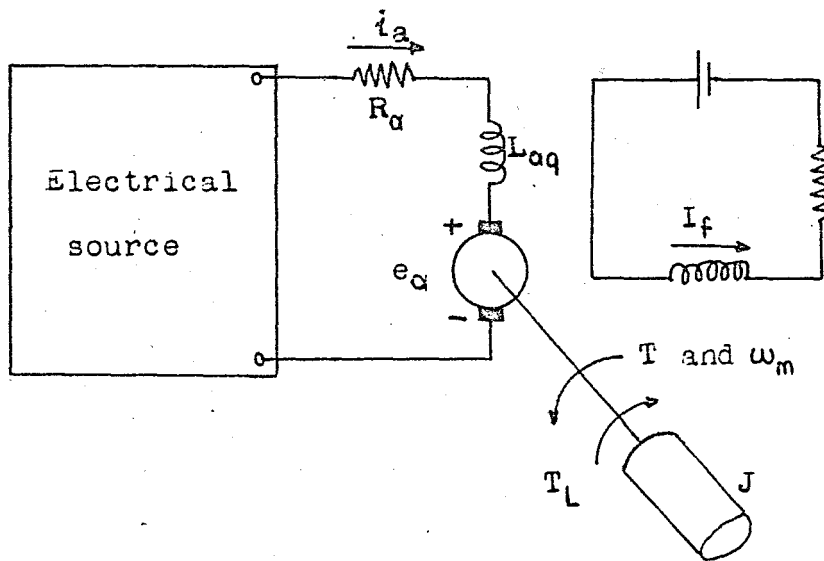


Fig. 2-10. Schematic diagram of a separately excited dc motor

source may be either a solid-state controller, or a conventional resistance control. At constant motor field current I_f the magnetic torque and generated emf are

$$T = K\phi \cdot i_a \quad (\text{newton-meters}) \quad (2.16)$$

$$e_a = K\phi \cdot \omega_m \quad (\text{volts}) \quad (2.17)$$

where $K\phi = k_f \cdot I_f$ is a constant. In terms of the magnetization curve

$$K\phi = e_{a0} / \omega_{m0} \quad (2.18)$$

with e_{a0} the generated emf corresponding to the field current I_f at the speed ω_{m0} rad/sec. In mks units the constant $K\phi$ in newton-meters per ampere (Eq.2.16) equals the constant $K\phi$ in volt-seconds per radian (Eq.2.17). The response of the motor to changes in source voltage and the effects of the load torque will now be investigated.

From Eq.2.8, after rearrangement of the terms and division by R_a , the differential equation for the armature current i_a is

$$\frac{L_a}{R_a} \cdot p i_a = T_a \cdot p i_a = \frac{V_s - e_a}{R_a} - i_a \quad (2.19)$$

where V_s is the source voltage, e_a is the back emf (Eq.2.17), R_a , and L_a include the series resistance and inductance of the source and armature circuit, and $T_a = L_a/R_a$ is the electrical time constant of the armature circuit. The magnetic torque T is given by Eq.2.16, and from Eq.2.10 the acceleration is

$$p\omega_m = \frac{T}{J} - \frac{T_L}{J} = \frac{K\phi i_a}{J} - \frac{T_L}{J} \quad (2.20)$$

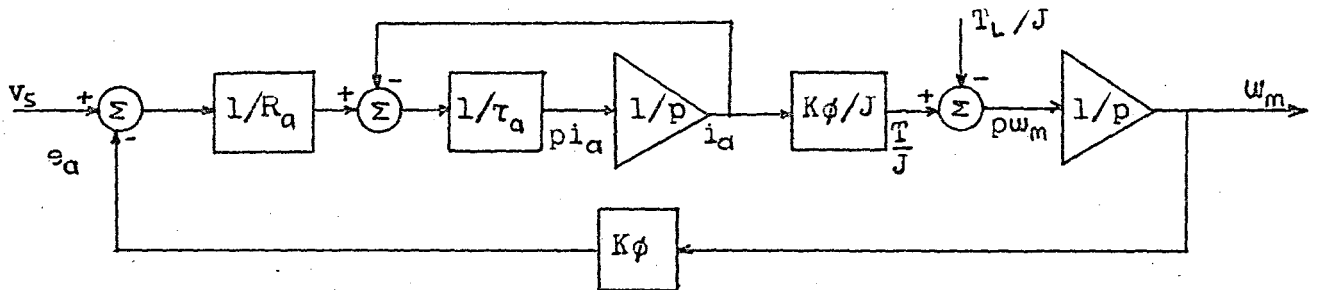
where J is the moment of inertia including that of the load, and T_L is the load torque opposing rotation.

The block diagram representing Eqs.2.16, 2.20 is shown

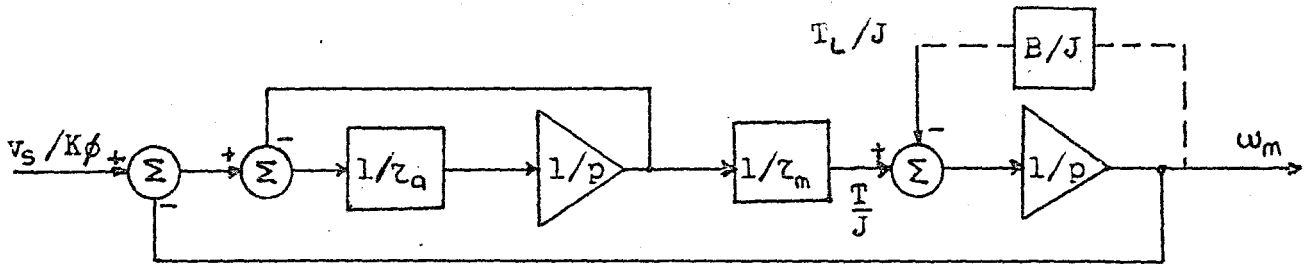
in Fig.2-11-a in terms of the state variables i_a and ω_m with V_s and T_L/J as inputs. Division of the input v_s by $K\phi$ and combination of constants in the forward path yields the simpler form shown in Fig.2-11-b where

$$\tau_m = \frac{JR_a}{K\phi^2} \tag{2.21}$$

is the 'inertia time constant'. Physically interpreted, $V_s/K\phi$ is the steady-state no load speed corresponding to a constant dc input voltage V_{dc} .



(a)



(b)

Fig.2-11. Block diagrams of Eqs. 2.16 to 2.20 for a separately excited dc motor

In general the load torque is a function of speed. It is sometimes assumed that the load torque is proportional to the

speed ; thus

$$T_L = B\omega_m \quad \text{or} \quad T_L/J = B\omega_m/J \quad (2.22)$$

where B is the slope of the torque-speed curve at the operating point and may be assumed to be constant for small changes. The parameters J/B is the 'load time constant τ_L ' describing the rate at which the motor coasts when its armature circuit is open, and B/J is the corresponding damping factor. It varies over a wide range from no load to full load but its effect usually is not very important with integral-horsepower motors. The effect of the load damping is shown in Fig.2-11-b by the feed back B/J around the second integrator.

The block diagram in terms of complex frequency s is shown in Fig.2-12, where now Ω_m, V_s and T_L are the complex amplitudes of the corresponding time variables. The functional notation 'of s ' will be omitted for the sake of simplicity with the understanding that capital letters such as Ω_m and V_s are complex amplitudes.

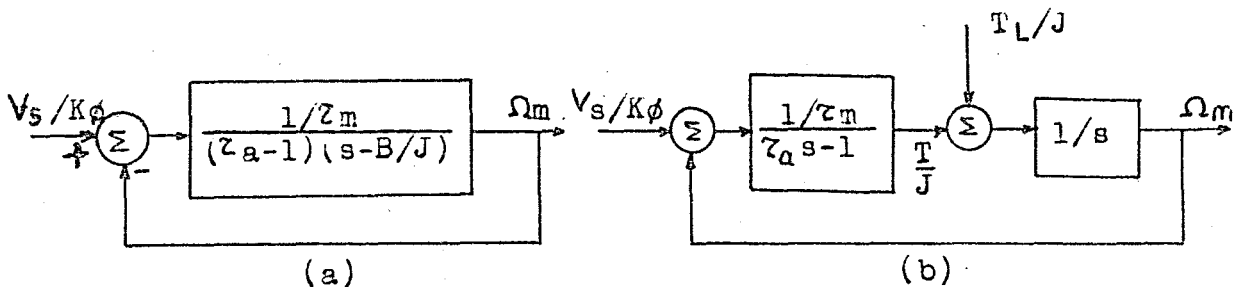


Fig.2-12. Block diagrams of separately excited dc motor in terms of complex variables

The first integrator in Fig.2-11-b becomes the algebraic term $1/(\tau_\alpha s+1)$ in Fig.2-12. The second integrator in Fig.2-11-b with damping B/J becomes the algebraic term $1/(s+B/J)$ in Fig.2-12-a. The block diagram with mechanical damping neglected and T_L assumed to be independent variable is shown in Fig.2-12-b. The transfer function relating speed to input voltage, found by elimination of the negative feedback in Fig.2-12-a, is

$$\frac{\Omega_m}{V_s/K\phi} = \frac{1}{\tau_m(\tau_\alpha s+1)(s+B/J)+1} \quad (2.23)$$

With mechanical damping neglected

$$\frac{\Omega_m}{V_s/K\phi} = \frac{1}{\tau_m s(\tau_\alpha+1)+1} \quad (2.24)$$

and the transfer function relating speed to load torque then is

$$\frac{\Omega_m}{T_L} = \frac{-R_a}{K\phi^2} \frac{1}{\tau_m s(\tau_\alpha s+1)+1} \quad (2.25)$$

An analog electrical circuit is shown in Fig.2-13 in which the inertia is represented by an equivalent capacitance C_{eq} and the damping by a shunt conductance G_{eq} . The torque equation for the mechanical system can be expressed as

$$T = K\phi \cdot i_a = J \cdot p\omega_m + B\omega_m \quad (2.26)$$

Division by $K\phi$ and substitution of $\omega_m = e_a/K\phi$ in the result gives

$$i_a = \frac{J}{K\phi^2} \frac{de_a}{dt} + \frac{B}{K\phi^2} \cdot e_a \quad (2.27)$$

which is identical to the node equation for the C_{eq} G_{eq} circuit provided that

$$C_{eq} = J/K\phi^2 \quad \text{and} \quad G_{eq} = B/K\phi^2 \quad (2.28)$$

In terms of the analog circuit the time constants τ_m and τ_L are

$$\tau_m = R_a C_{eq} \quad \text{and} \quad \tau_L = C_{eq}/G_{eq} \quad (2.29)$$

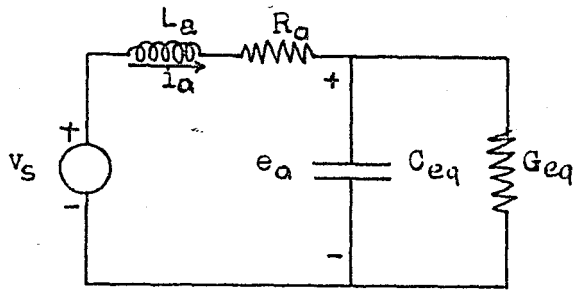


Fig.2-13. Electrical circuit equivalent of separately excited dc machine

The natural frequencies s of the system are given by the poles of the transfer function, Eq.2.23, or the roots of the equation,

$$(s+1/\tau_a)(s+B/J)+1/\tau_a\tau_m = 0 \quad (2.30)$$

$$s^2+(1/\tau_a+B/J)s+1/\tau_a(1/\tau_m+B/J) = 0 \quad (2.31)$$

Comparison with the standard form for a second-order equation,

$$s^2+2\alpha s+\omega_n^2 = 0$$

shows that the undamped natural frequency ω_n is

$$\omega_n = \sqrt{1/\tau_a(1/\tau_m+B/J)} \quad (2.32)$$

and the damping factor α is

$$\alpha = 1/2(1/\tau_a+B/J) \quad (2.33)$$

The relative damping factor or damping ratio ζ is,

$$\zeta = \alpha/\omega_n \quad (2.34)$$

The roots are given by the well-known solution,

$$s_1, s_2 = -\zeta\omega_n \pm \omega_n \sqrt{\zeta^2-1} = -\zeta\omega_n \pm j\omega_n \sqrt{1-\zeta^2} \quad (2.35)$$

where the first form, $\zeta > 1$, gives two exponential terms with negative real exponents, and the second form $\zeta < 1$, gives a damped sinusoid. The mechanical load usually has only a small effect on

ω_n , and α , although of course it does affect the steady-state speed. If B/J is neglected

$$\omega_n = \sqrt{1/\tau_\alpha \tau_m} \quad (2.36)$$

$$\alpha = 1/2\tau_\alpha \quad (2.37)$$

$$\zeta = 1/2\sqrt{\tau_m/\tau_\alpha} \quad (2.38)$$

It should be noted that for transient conditions and especially for large accelerations, T_m is much larger than T_L , therefore T_L can be neglected. This means that in the model of separately excited dc motor, G_{eq} can be considered as zero, greatly simplifying the model.

CHAPTER 3

GENERAL TECHNIQUES OF SIMULATION

3-1. INTRODUCTION

As was stated in Chapter 1, it is important to be able to predict the performance of a thyristor installation so that an optimum and efficient design can be made. The prediction can be made by a simulation of which there are two types.

3-2. ANALOGUE SIMULATION

In this type of simulation, the physical equations of the installation are represented by electronic circuitry. The variables of the physical system are represented by some other variables, (for example voltage), that can be controlled. Thus the simulation is obtained by manipulating the representative variable in accordance with the equations of the system. For example the equations from 2.16 to 2.20 which describes the behaviour of a separately excited dc motor is simulated in Fig.2-11-a in terms of the state variables i_a and ω_m with v_s and T_L/J as inputs. The common components for an analogue simulation are summer, integrator, inverter, constant multiplier and variable multiplier, which are readily available in the form of electronic integrated circuits.

Analogue simulations have the great advantage in that the simulation gives a physical 'feel' to the problem (i.e. the problems are solved in real time). In our example of the simulation of separately excited dc motor (in Fig.2-11-a), by putting a voltmeter in the output circuit one can actually see how the output (mechanical speed ω_m , or armature current i_a) behaves for any input(s) (V_s and T_L/J). The accuracy of the simulation is limited by the accuracy with which you can measure the representative variable.

However, digital simulations, where the physical equations of the installation are solved numerically, are much more flexible, in that more complex equations can be solved without the corresponding increase in the complexity of the simulation. The accuracy of the simulation is governed only by the accuracy of the particular numerical method being used, and the accuracy to which one wants to obtain the solution, and of course the assumptions made in the formulating the equations. Because a great deal of 'number crunching' is involved in digital simulations, an invaluable tool to use is a digital computer. With the growing popularity of the digital computer during the past ten years, the digital simulation has also become more popular. But digital simulations do not have the physical 'feel' of working with real time as analogue simulations have.

The aim of this thesis is the simulation of separately excited chopper controlled dc machine. Although, one can do this

by analogue simulation, if the output waveform of the chopper can be generated and be fed into the input of Fig.2-11-a, the mechanical speed ω_m and the armature current i_a can easily be obtained as outputs, but the variables in the chopper, which are much more interesting, can not be accessed. Therefore the system has to be simulated as a whole, which is possible only by digital simulation.

3-3. DIGITAL SIMULATION

3-3-1. GENERAL

Since the late 1950's, the state variable approach has also become popular for analyzing large-scale electronic circuits with the aid of a digital computer. In the state variable approach, a linear time invariant network is characterized by two equations of the following forms:

$$\dot{\underline{x}} = \underline{A}\underline{x} + \underline{B}\underline{u} \quad (3.1)$$

and
$$\underline{y} = \underline{C}\underline{x} + \underline{D}\underline{u} + (\underline{D}_1\underline{u} + \dots) \quad (3.2)$$

where

\underline{u} : $m \times 1$ vector representing the m inputs (independent sources)

\underline{y} : $p \times 1$ vector representing the p outputs (voltages and/or currents of interest)

\underline{x} : $n \times 1$ vector consisting of a set of n independent auxiliary variables

$\underline{A}, \underline{B}, \underline{C}, \underline{D}, \underline{D}_1$: constant, real matrices of appropriate

dimensions; note that \underline{A} is always square matrix of order n

The first equation, Eq.3.1, is a set of n first-order differential equations (coupled in general), usually referred to as the 'state equation' in normal form. The set of auxiliary variables $x_1, x_2, x_3, \dots, x_n$ are called state variables, and $\underline{x} = (x_1, x_2, \dots, x_n)^t$ is called the state vector. To find the time-domain response we may apply any of the solution techniques. After $\underline{x}(t)$ is found from Eq.3-1, it is a trivial matter to obtain $\underline{y}(t)$ from Eq.3-2.

Any lumped network obeys three basic laws: Kirchhoff's voltage law, Kirchhoff's current law, and elements law (branch characteristics). If, from these three types of constraints, we can succeed in reducing the equations to a system of linearly independent first-order equations

$$\dot{\underline{x}} = \underline{f}(\underline{x}, t) \quad (3.3)$$

where \underline{x} is the set of n independent, auxiliary variables, we say that the normal-form equations for the network exists. In this case, the entries $x_1, x_2, x_3, \dots, x_n$ are called the state variables and n is defined to be the order of complexity of the network.

It is known in elementary differential equations that the most general solution of Eq.3.1 must contain n arbitrary constants, to be determined by n initial conditions. Usually, but not always, these n initial conditions are the values of x_1, x_2, \dots, x_n at $t=0$. Then the order of complexity n of a network is equal to the number of independent initial conditions that

can be, must be specified in terms of electrical variables in order to have the complete solution of $\underline{x}(t)$, and hence $\underline{y}(t)$. The computer formulation of state equations for RLCM networks and for linear active networks are given in reference 7.

Given a dynamic linear network characterized by Eq.3.1 and 3.2, the initial state $\underline{x}(t_0)$ and the input $\underline{u}(t)$ for $t \geq t_0$, we wish to find the output $\underline{y}(t)$ for $t \geq t_0$. Since the solution is to be obtained with digital techniques, both $\underline{x}(t)$ and $\underline{y}(t)$ will be determined only for some discrete values of t , for example,

$$t = t_0, t_0 + T, t_0 + 2T, \dots$$

where T is the chosen 'time step'. The input $\underline{u}(t)$ may be expressed explicitly as functions of t or may be available only in the form of sample data.

3-3-2 SOLUTIONS OF THE STATE EQUATIONS

(i) Analytical solution:

The most general solution of equation 3.1 is (7)

$$\underline{x}(t) = e^{\underline{A}t} \int_{t_0}^t e^{-\underline{A}\tau} \underline{B}\underline{u}(\tau) d\tau - e^{\underline{A}(t-t_0)} \underline{x}(t_0) \quad (3.4)$$

and putting Eq.3.4 in Eq.3.2, we obtain

$$\underline{y}(t) = \underline{C} e^{\underline{A}(t-t_0)} \underline{x}(t_0) + \left[\underline{C} e^{\underline{A}t} \int_{t_0}^t e^{-\underline{A}\tau} \underline{B} \underline{u}(\tau) d\tau + \underline{D} \underline{u}(t) + \underline{D}_1 \dot{\underline{u}}(t) + \dots \right] \quad (3.5)$$

The first term in Eq.3.5 is the 'natural response' or 'zero-input response', the second term is the 'forced response' or 'zero-state response'.

(ii) Digital solution:

Although Eq 3.4 is the exact solution of Eq 3.1, it is not in a form suitable for digital processing. With a digital computer, we can only calculate $\underline{x}(t)$ for some discrete values of t . Usually, we calculate $\underline{x}(t)$ for $t=kT$, where k is an integer and T is a suitably chosen time interval. Since the input $\underline{u}(kT)$ is assumed to be known for all k , what we need is an equation relating $\underline{x}[(k+1)T]$ to $\underline{u}(kT)$ and $\underline{x}(kT)$. Such an equation is a special case for a difference equation. Once the difference equation is obtained, $\underline{x}(kT)$ may be calculated successively for all k . In Eq 3.4, let $t_0=kT$ and $t=(k+1)T$, we have

$$\underline{x}[(k+1)T] = e^{\underline{A}T} \underline{x}(kT) + e^{\underline{A}(k+1)T} \int_{kT}^{(k+1)T} e^{-\underline{A}\tau} \underline{B} \underline{u}(\tau) d\tau \quad (3.6)$$

If $\underline{u}(t)$ is approximated to piecewise constant curve, such that

$$\underline{u}(t) = \underline{u}(kT) \quad \text{for} \quad kT \leq t < (k+1)T \quad k=0,1,2,\dots$$

and as, $\int_0^t e^{\underline{A}\tau} d\tau = \underline{A}^{-1} (e^{\underline{A}t} - 1) = (e^{\underline{A}t} - 1) \underline{A}^{-1}$ where $\det \underline{A} \neq 0$

$$\begin{aligned} \int_{kT}^{(k+1)T} e^{-\underline{A}\tau} \underline{B} \underline{u}(\tau) d\tau &= \int_{kT}^{(k+1)T} e^{-\underline{A}\tau} d\tau \underline{B} \underline{u}(kT) \\ &= \left[-e^{-\underline{A}\tau} \right]_{kT}^{(k+1)T} \underline{A}^{-1} \underline{B} \underline{u}(kT) \\ &= \left[-e^{-\underline{A}(k+1)T} + e^{-\underline{A}kT} \right] \underline{A}^{-1} \underline{B} \underline{u}(kT) \end{aligned}$$

Therefore, Eq 3.6 becomes

$$\underline{x}[(k+1)T] = e^{\underline{A}T} \underline{x}(kT) + \left[e^{\underline{A}T} - 1 \right] \underline{A}^{-1} \underline{B} \underline{u}(kT) \quad (3.7)$$

which is the desired difference equation.

The algorithms for formulating the equations of motion of large class of dynamic nonlinear networks into the 'normal form' of Eq.3.3 are available in reference 7. Generally, an analytical solution does not exist for Eq.3.3 but there are many numerical algorithms, which are based on two approaches: the Taylor series expansion approach and the polynomial approximation approach. Algorithms based on the Taylor series expansion approach are usually called Runge-Kutta algorithms. Those based on the polynomial approximation approach are usually called numerical integration algorithms. Any book on numerical analysis will give explanations and examples of these and many other methods.

3-4. THE DIGITAL SIMULATION OF ELECTRICAL MACHINES

Many methods of digital simulation are being developed at the moment for application to electrical machines. In our case, the dc machines are easily modelled, as in section 2-3-2, by linear lumped circuit, which is a linear circuit by itself; but the difficulty has been found in the representation of switching of thyristors, which makes the control circuitry both nonlinear and time varying. However, the nonlinearity is not so strong, it can easily be linearized and time variation can be controlled. In most of the methods the thyristors are either represented by perfect switches or piecewise linear models. Therefore, the circuit, which is linearized can be modelled by Eq.3.1 and Eq.3.2 for each mode of operation, and the solution can be obtained by Eq.3.4 or Eq.3.7; having obtained a solution from one mode, and calculating

the final values (to become the initial values for the next mode of operation). Obviously, depending on the number of diodes and thyristors and the complexity of their models, the number of modes increase. But, these methods look as if it would require a great amount of computer time to solve a specific complicated analytical equation for each mode of operation, and there may be many modes of operation in each cycle of operation. For example Mellit and Rashid (1) tackled the problem by breaking down the circuit operation into different modes, and the procedure above was followed. Their method also treats the thyristors as perfect switches and it ignores the parts of the circuit that plays a minor role (like snubber circuits) in the circuits operation.

In this thesis, a novel approach for the simulation of the chopper-fed dc machine is used. Instead of general technique of combining the Kichhoff's voltage, current and branch characteristic equations, which would end up with Eq.3.3 ; the branch currents are evaluated in terms of node voltages by applying voltage and current equations, and immediately converted to algebraic equations by trapezoidal formula. Later on the nonlinearities are considered, and thyristor and diodes are linearized by piecewise linear models. In this way, breaking down the circuit operation into different modes, which causes laborious analytical manipulation or requires long computer time, is avoided. In this method , it is a trival exercise to write down the complete set of equations for any network, just observing the topology of the circuit, as it will be explained in the next section.

3-5. THE GENERAL FORMULATION OF CIRCUIT EQUATIONS

To formulate the equations of the circuit, firstly the expressions describing the behaviour of the circuit elements are obtained by trapezoidal rule, which enables the calculation of the new value of a variable (for example, voltage or current) from the knowledge of the previous value of the variable. Secondly, branch currents are obtained in terms of node voltages, and Kirchhoff's current law equations and input current equation(s) are obtained. In order to be able to combine Kirchhoff's current equations and input current equation(s), imaginary currents are injected to every node, for convenience.

To find the voltage current expressions for resistors, capacitors and inductors, it will be assumed, for any time varying function of voltage across an element, there is a corresponding time varying function of current through the element as shown in Fig.3.1. Let V_{n-1} and i_{n-1} be the values of voltage (across the element) and current (through the element) before the time increment h , and V_n and i_n be the values of voltage (across the element) and

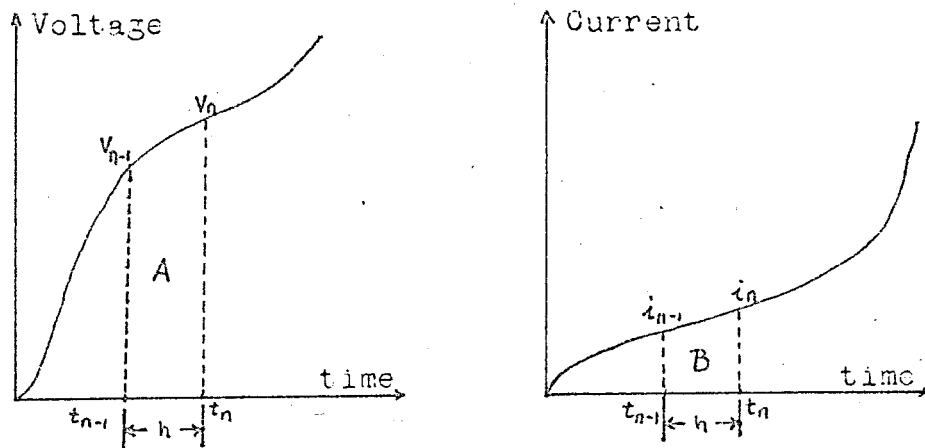


Fig.3-1 (a) Voltage versus time

(b) Current versus time, curves

current (through the element) after the increment of time. Also t_{n-1} and t_n are the times before and after the time increment.

(a) Consider a resistor:

In general $i=V/R$

At the n-th iteration

$$i_n = V_n/R \tag{3.8}$$

(b) Consider an inductor:

In general $V=L di/dt$

$$i = 1/L \int_{-\infty}^t V dt$$

At the n-th iteration

$$\begin{aligned} i_n &= 1/L \int_{-\infty}^{t_n} V dt \\ &= 1/L \int_{t_{n-1}}^{t_n} V dt + i_{n-1} \end{aligned}$$

$\int_{t_{n-1}}^{t_n} V dt$ is the area A in Fig.3-1-a, and can be approximated

to $(V_n + V_{n-1}) h/2$, therefore,

$$i_n = i_{n-1} + h/2L (V_n + V_{n-1}) \tag{3.9}$$

(c) Consider a capacitor:

In general $V = 1/C \int_{-\infty}^t i dt$

At the n-th iteration

$$\begin{aligned} V_n &= 1/C \int_{-\infty}^{t_n} i dt \\ &= 1/C \int_{t_{n-1}}^{t_n} i dt + V_{n-1} \end{aligned}$$

Similarly $\int_{t_{n-1}}^{t_n} i dt$ is the area B in Fig.3-1-b, and can be approximated to $(i_n + i_{n-1}) h/2$

$$V_n - V_{n-1} = h/2C (i_n + i_{n-1})$$

$$i_n = -i_{n-1} + 2C/h (V_n - V_{n-1}) \tag{3.10}$$

The Eq.3.8 gives the value of resistive branch current in terms of the node voltages where resistance R is not necessarily a constant, it may be voltage or current dependent, as in the models of thyristors and diodes. On the other hand Eq.'s 3.9 and 3.10 enable us to calculate the new value of a dynamic branch currents from knowledge of the node voltages and previous value of current. So, differential equations for dynamic branches are converted to algebraic relations, at the very beginning. The next step will be to obtain Kirchhoff's current law equations and input current equation(s), which can be best illustrated with reference to a simple RLC circuit.

Consider the circuit in Fig.3-2 and label junctions between the elements, and inject imaginary currents (i_a , i_b , i_c) at each junction.

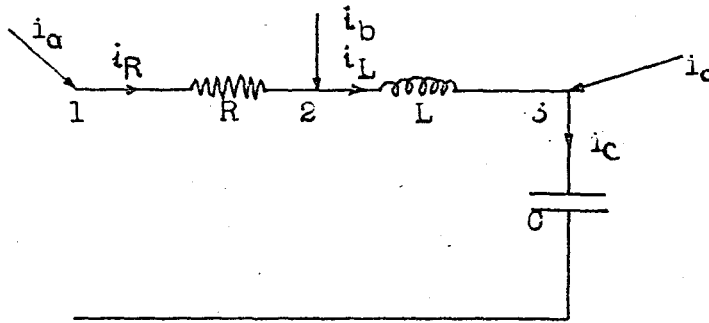


Fig.3-2 A simple RLC circuit

From Eq.'s 3.8, 3.9, and 3.10, at the n -th iteration

$$i_{Rn} = 1/R(V_{1n} - V_{2n}) \quad (3.11)$$

$$i_{Ln} = i_{Ln-1} + h/2L [(V_{2n} - V_{3n}) + (V_{2n-1} - V_{3n-1})] \quad (3.12)$$

$$i_{Cn} = -i_{Cn-1} + 2C/h (V_{3n} - V_{3n-1}) \quad (3.13)$$

The imaginary injected currents, where i_a is the input current equation and i_b , i_c are the Kirchoff's current equations, can be formed from the above (Eq.'s 3.11, 3.12, 3.13) branch currents as follows :

$$i_a = i_R = 1/R(V_{1n} - V_{2n}) \quad (3.14)$$

$$i_b = i_L - i_R = i_{Ln-1} + h/2L \left[(V_{2n} - V_{3n}) + (V_{2n-1} - V_{3n-1}) \right] - 1/R (V_{1n} - V_{2n}) \quad (3.15)$$

$$i_c = i_C - i_L = -i_{Cn-1} + 2C/h(V_{3n} - V_{3n-1}) - i_{Ln-1} - h/2L \left[(V_{2n} - V_{3n}) + (V_{2n-1} - V_{3n-1}) \right] \quad (3.16)$$

Then, re-arranging the equations, the following matrices can be set up.

$$\begin{bmatrix} i_a \\ i_b \\ i_c \end{bmatrix} = \begin{bmatrix} 0 \\ i_{Ln-1} \\ -i_{Cn-1} - i_{Ln-1} \end{bmatrix} + \begin{bmatrix} 1/R & -1/R & 0 \\ -1/R & h/2L + 1/R & -h/2L \\ 0 & -h/2L & h/2L + 2C/h \end{bmatrix} \begin{bmatrix} V_{1n} \\ V_{2n} \\ V_{3n} \end{bmatrix}$$

$$\begin{bmatrix} 0 & 0 & 0 \\ 0 & h/2L & -h/2L \\ 0 & -h/2L & h/2L - 2C/h \end{bmatrix} \begin{bmatrix} V_{1n-1} \\ V_{2n-1} \\ V_{3n-1} \end{bmatrix} \quad (3.17)$$

which is of the form

$$[i_n] = [i_{n-1}] + [A] [V_n] + [B] [V_{n-1}] \quad (3.18)$$

The elements of the matrix $[i_{n-1}]$ are known as the 'corrector currents'. We know that all the elements of the matrix $[i_n]$ are zero except for i_a (the input current). So we can partition the matrices of Eq.3.17 as shown, so that these known zero currents

(Kirchhoff's current equations) will yield a a solution for the junction voltages at the n-th iteration.

$$\begin{bmatrix} i_a \\ i_b \\ i_c \end{bmatrix} \begin{bmatrix} 0 \\ i_{Ln-1} \\ -i_{Cn-1} - i_{Ln-1} \end{bmatrix} \begin{bmatrix} 1/R & -1/R & 0 \\ -1/R & 1/R - h/2L & -h/2L \\ 0 & -h/2L & h/2L - 2C/h \end{bmatrix} \begin{bmatrix} V_{1n} \\ V_{2n} \\ V_{3n} \end{bmatrix} \\
 + \begin{bmatrix} 0 & 0 & 0 \\ 0 & h/2L & -h/2L \\ 0 & -h/2L & h/2L - 2C/h \end{bmatrix} \begin{bmatrix} V_{1n-1} \\ V_{2n-1} \\ V_{3n-1} \end{bmatrix} \quad (3.18)$$

Naming these submatrices

$$\begin{bmatrix} i_{an} \\ i_{pn} \end{bmatrix} = \begin{bmatrix} i_{an-1} \\ i_{pn-1} \end{bmatrix} + \begin{bmatrix} A_{aa} & A_{ap} \\ A_{pa} & A_{pp} \end{bmatrix} \begin{bmatrix} V_{an} \\ V_{pn} \end{bmatrix} + \begin{bmatrix} B_{aa} & B_{ap} \\ B_{pa} & B_{pp} \end{bmatrix} \begin{bmatrix} V_{an-1} \\ V_{pn-1} \end{bmatrix} \quad (3.20)$$

Multiplying out these matrices

$$[i_{pn}] = [i_{pn-1}] + [A_{pa}] [V_{an}] + [A_{pp}] [V_{pn}] + [B_{pa}] [V_{an-1}] + [B_{pp}] [V_{pn-1}] = 0$$

Re-arranging

$$[V_{pn}] = -[A_{pp}]^{-1} \left[[i_{pn-1}] + [A_{pa}] [V_{an}] + [B_{pa}] [V_{an-1}] + [B_{pp}] [V_{pn-1}] \right] \quad (3.21)$$

Thus the new values of node voltages can be calculated from knowledge of the previous values and matrices. The input current(s) can be found from the equation.

$$[i_{an}] = [i_{an-1}] + [A_{aa}] [V_{an}] + [A_{ap}] [V_{pn}] + [B_{aa}] [V_{an-1}] + [B_{ap}] [V_{pn-1}] \quad (3.22)$$

Although, there is quite a lot of manipulation involved in the derivation of these matrices, especially for a more complex network, the Eq.3.18 can be written down by inspection of the circuit, the rules being:

- 1- The value of an element of the A matrix on the principal diagonal (A_{jj}) is the sum of the admittances connected to that junction.
- 2- The value of an off diagonal element (A_{jk}) of the A matrix, is the negated sum of the admittances connected between junctions j and k.
- 3- The value of the elements of the B matrix are the same as the A matrix, but the resistance terms are zero and the capacitance terms are negated.
- 4- The admittance of an inductor is $h/2L$, the admittance of a capacitor is $2C/h$, and conductance of a resistor is $1/R$, where h is the time increment.

Also, it should be noted that A and B matrices are symmetrical.

- 5- In general, the corrector currents $[i_{n-1}]$ are found by forming

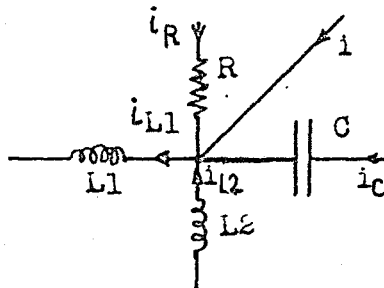


Fig.3-3. A general node with injected current i .

the injected currents from the branch currents. The resistive current contributes zero to the corrector current, the inductive current contributes it's previous value to the corrector current, and capacitive current contributes it's previous value negated to the corrector current. For example, if we consider a node as shown in Fig.3-3, the node equation will be : $i = i_{L1} - i_{L2} - i_R - i_C$ and the value of the corrector current is $i_{L1(n-1)} - i_{L2(n-1)} + i_{C(n-1)}$

6- Generally the following conventions are used:

- a) The junction(s) with the non-zero current shall be labelled with 'A' junction, the other junctions shall be numbered from '1'.
- b) The flow of positive current is defined by directions on the circuit diagram.

CHAPTER 4

THE SIMULATION

4-1. THE CIRCUIT

The aim of this thesis is the simulation of separately excited chopper controlled dc machine. The circuit and all the

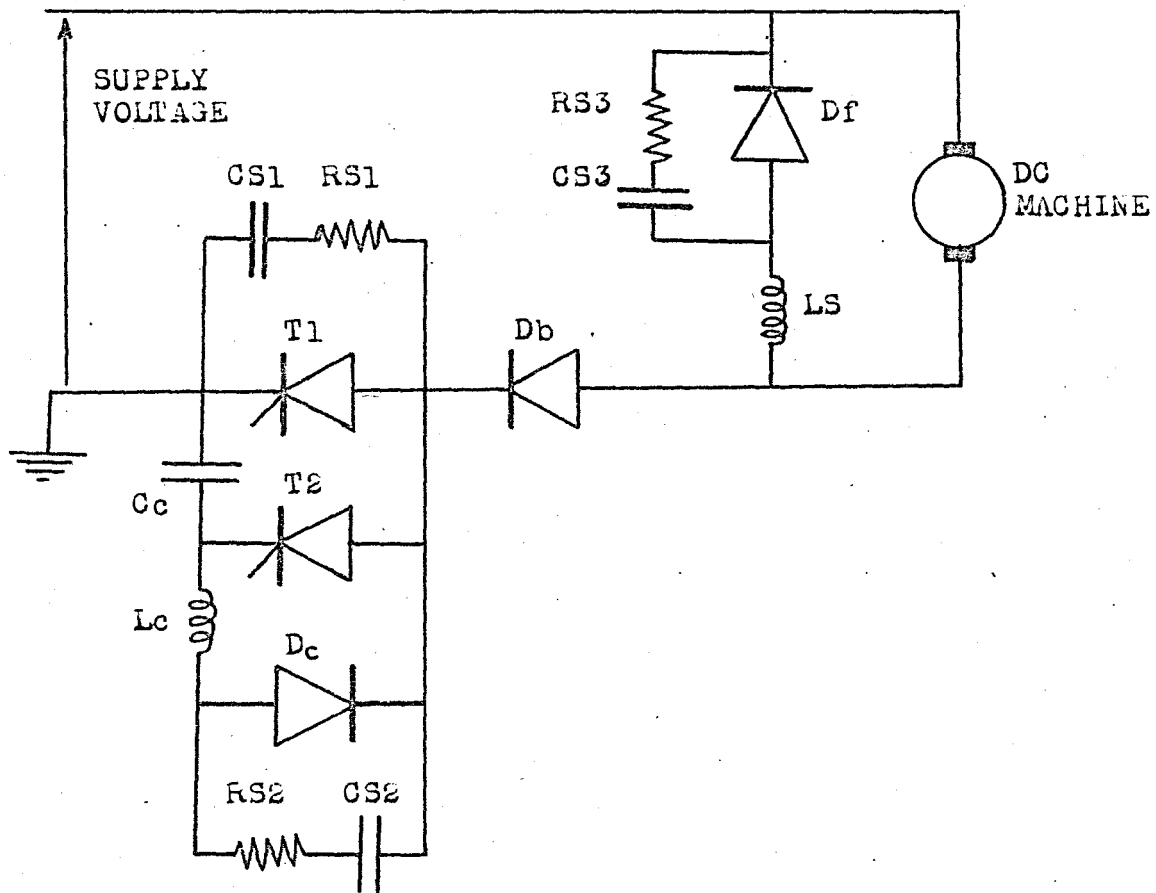


Fig.4-1. The armature circuit diagram of separately excited chopper controlled dc machine

element values are as used in an actual chopper circuit feeding a 10 hp dc machine. The machine has a separately excited field winding, and the chopper is a switched capacitor type circuit, which is developed from the circuit explained in section 2-2. The simulation includes the thyristor commutation circuit, and all the suppressor elements to limit excessive rates of changes of voltage and current. As the machine has a separately excited field winding, it gives a field of constant flux; therefore the field circuit is ignored as long as constant $K\phi$ is considered, and the armature circuit which will be simulated is shown on Fig.4-1. Also the firing logic circuitry is taken into account by the logical statements of the program, so it can be ignored in the armature circuit. The inertia constant J takes into account the load inertia as well as the machine inertia. T_1 and T_2 are the main and auxiliary thyristors, respectively, and D_f is the freewheeling diode. Elements with the suffix 'C', 'S', and 'M' are the elements of the commutation, suppressor and the representation of the dc machine, respectively. The inductor L_S is for the suppression of excessive rate of change of current through the freewheeling diode D_f .

4-2. THE CIRCUIT OPERATION

Initially, the commutation capacitance C_c should be setted to supply voltage negative at the ground end, which can easily be done by firing T_2 . Firing T_1 initiates the on period, and firing T_2 terminates the on period. When the main thyristor

T1 is fired, the supply voltage appears across the armature terminals and secondly, as the loop Cc, Lc, Dc, and T1 is closed, a half period sinusoidal oscillation occurs between the elements Cc and Lc, which reverses the voltage across the commutation capacitance Cc (positive at the ground end). This polarity of charge on Cc is essential to turn off the main thyristor T1. When the auxiliary thyristor T2 is fired, the commutation capacitance Cc takes over the armature current through the auxiliary thyristor T2, and also a negative voltage appears across the main thyristor T1; to be able to turn off T1, the voltage across Cc should stay negative at least for a time greater than the turn-off time of the main thyristor T1, as explained in section 2-2, even for the maximum value of armature current. Then Cc charges up to the supply voltage value (negative at the ground end), the current through the Cc, T2 path drops to zero, so T2 turns off too. During rest of the off period both T1 and T2 are non-conducting. When we fire the main thyristor T1 again, next cycle begins, with the application of supply voltage to the armature terminals, and closing the loop Cc, Lc, Dc and T1, which reverses the voltage across the commutation capacitance Cc (as explained above). Obviously, the freewheeling diode Df takes over the armature current for the time when armature voltage tends to negative values . The rest of the details will be discussed in Chapter 5 together with the results and graphs.

4-3. REPRESENTATION OF THE DIODES AND THYRISTORS IN THE SIMULATION

4-3-1. MODELLING OF DIODES AND THYRISTORS

Both the diodes and thyristors are represented by piecewise-linear models, as shown in Fig.4-2. When the diode

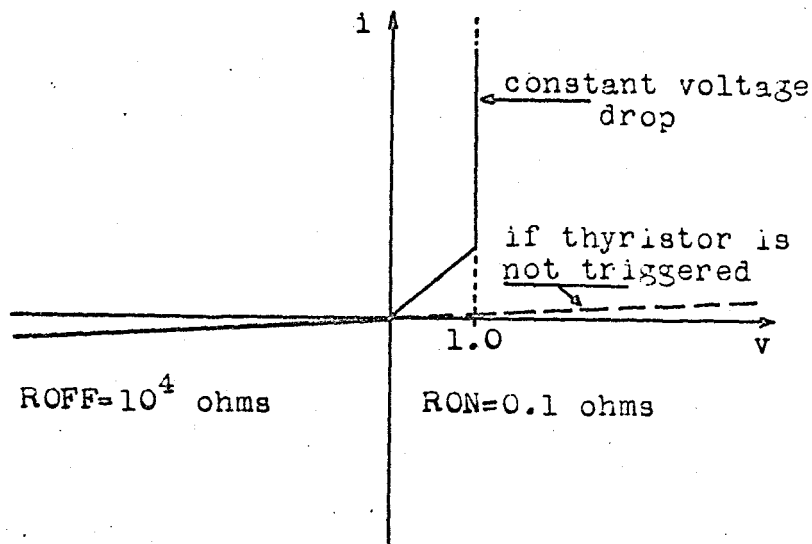


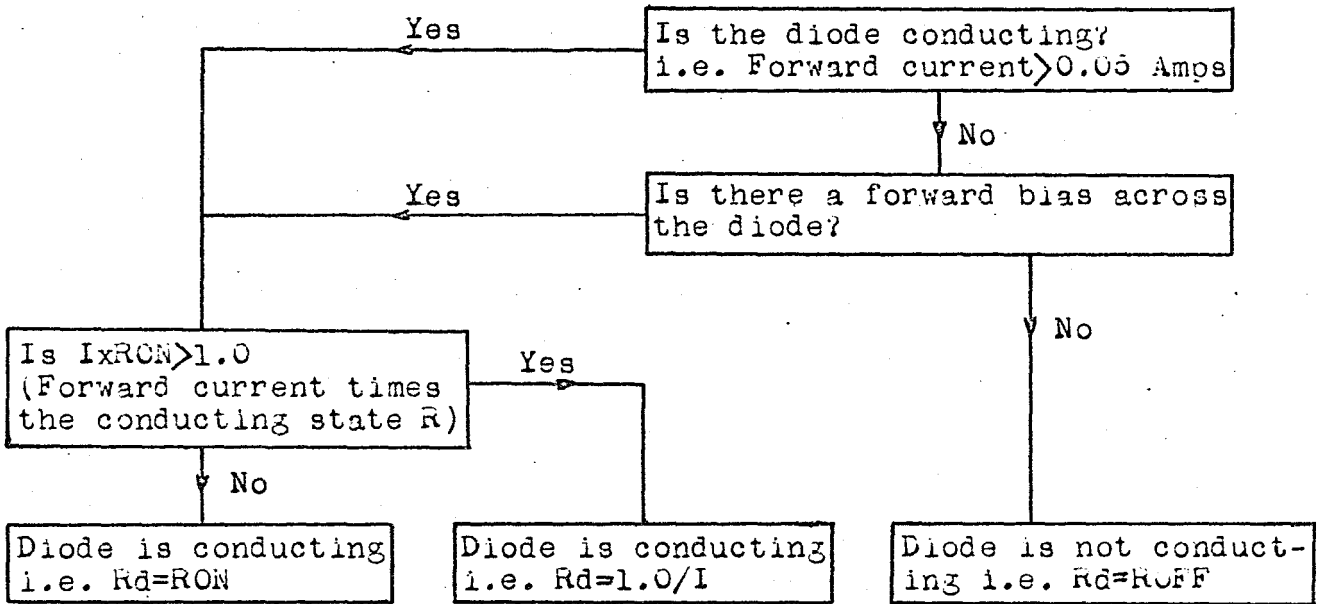
Fig.4-2. Simplified diode and thyristor characteristics in the v-i plane (piecewise-linear model)

(or thyristor) is in the conducting state, it is replaced by a low value resistance (0.1 ohms), provided the voltage across it is below one volt. If the voltage across the element reaches the value one volt, it is replaced by a resistance of a value $1.0/1$ ohms ($R_d = v/i$, where v is one volt), and when the diode (or thyristor) is in the non-conducting state, it is replaced by a high value resistance (10 Kilo ohms). It is quite realistic, as a $v/10$ mA leakage current flows through the diode (or thyristor) in non-conducting state.

4-3-2. DETERMINATION OF THE CONDUCTION STATES OF THE DIODES AND THYRISTORS

(a) The diode.

The following flow diagram is derived from the logic conditions for diode conduction.



The subroutine was called 'DTEST' and was as follows:

```
SUBROUTINE DTEST (V, I, R, ROFF, RON)
C V IS THE VOLTAGE ACROSS THE DIODE
C I IS THE FORWARD CURRENT THROUGH THE DIODE
C R IS THE RESISTANCE OF THE DIODE
C ROFF IS THE RESISTANCE OF THE DIODE IN THE NON-CONDUCTING STATE
C RON IS THE RESISTANCE OF THE DIODE IN THE CONDUCTING STATE
REAL I
IF (I.GT.0.05) GO TO 10
IF (V.GT.0.0) GO TO 10
R=ROFF
GO TO 20
10 R=RON
IF ((I*RON).GT.1.0) R=1.0/I
20 RETURN
END
```

(b) The thyristor.

A very simple logic is used to determine the thyristors' triggering mode, which is summarized as follows:

Consider the first cycle of operation in Fig.4-3, where V_a is the supply voltage, t_2 is the periodic time and t_1 is the main thyristor's on time, which is equal to periodic time times the time ratio.

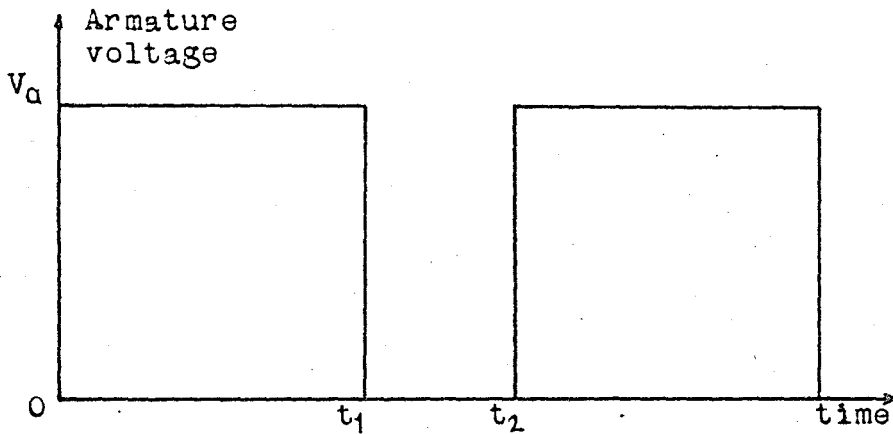


Fig.4-3 Armature voltage versus time

If at time t

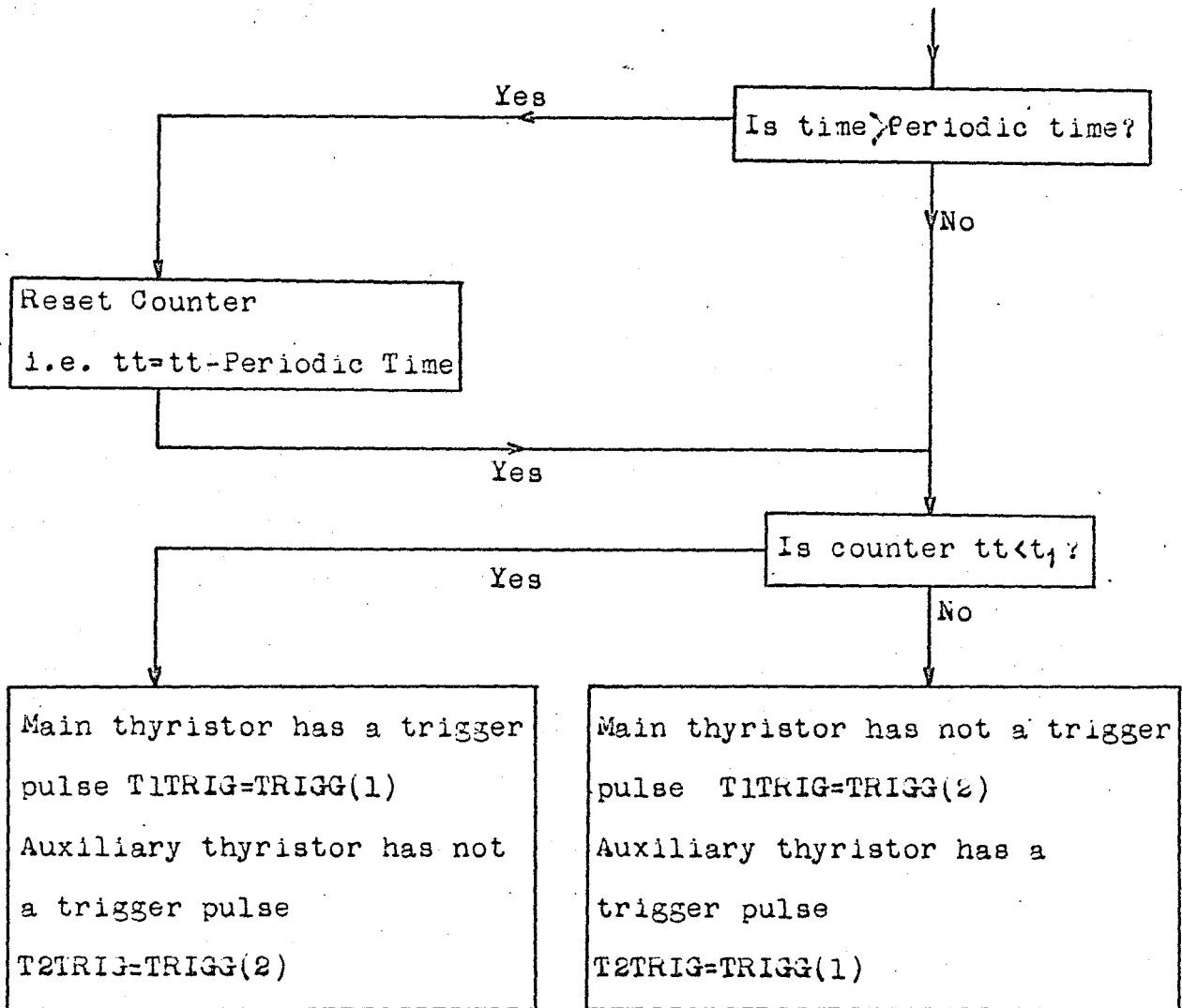
(i) $0 \leq t < t_1$ there is a trigger pulse present on the main thyristor.

there is not a trigger pulse present on the auxiliary thyristor.

(ii) $t_1 < t \leq t_2$ there is not a trigger pulse present on the main thyristor.

there is a trigger pulse present on the auxiliary thyristor.

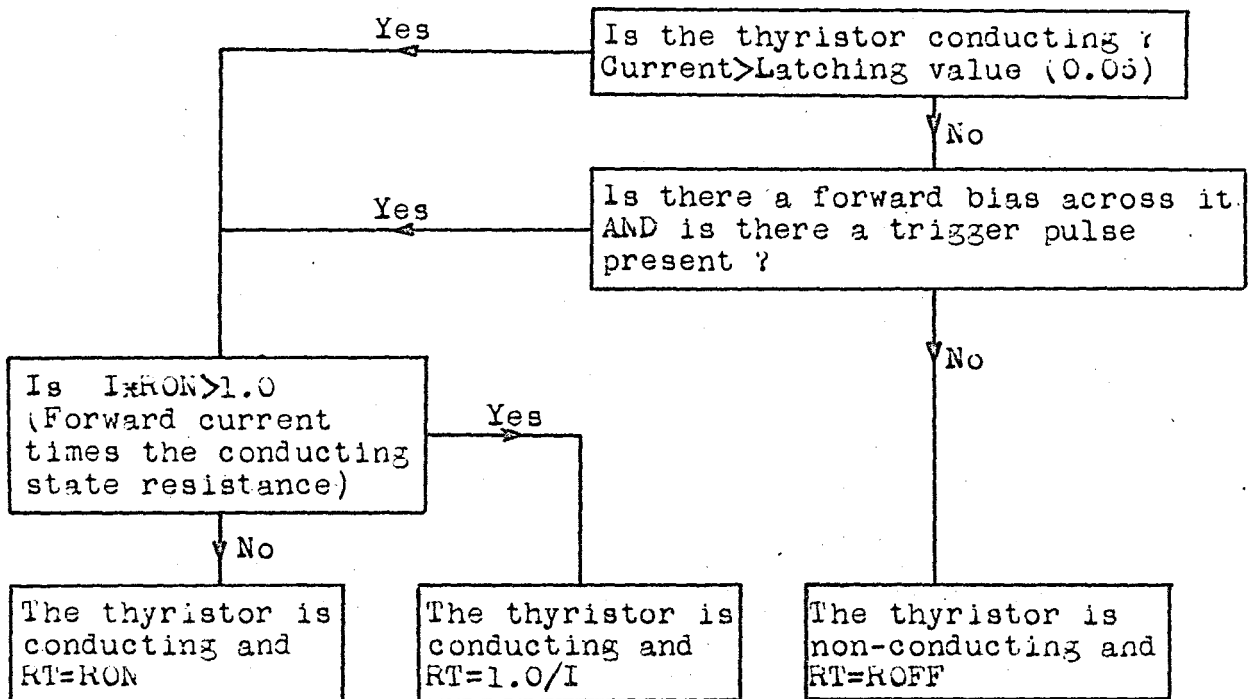
The flow diagram in the general case is



This flow diagram was programmed as

```
IF(TT.GT.PT) TT=TT-PT
IF(TT.LT.(PT*TR)) GO TO 10
T1TRIG=TRIGG(2)
T2TRIG=TRIGG(1)
GO TO 20
10 T1TRIG=TRIGG(1)
T2TRIG=TRIGG(2)
20 CONTINUE
```

The following flow diagram is obtained from the logic conditions of the thyristor.



This was programmed as a subroutine called SORTES, which was

```

SUBROUTINE SORTES (V,I,T,TRIGG,R,ROFF,RON)
C V IS THE VOLTAGE ACROSS THE THYRISTOR
C I IS THE CURRENT THROUGH THE THYRISTOR
C TRIGG IS AN ARRAY OF TWO ELEMENTS
C TRIGG(1)=1.0 AND REPRESENTS A TRIGGER PULSE BEING PRESENT
C TRIGG(2)=0.0 AND REPRESENTS A TRIGGER PULSE BEING ABSENT
C T IS THE STATE OF THE TRIGGER SIGNAL (EITHER 1.0 OR 0.0)
C R IS THE RESISTANCE OF THE THYRISTOR
C ROFF IS THE RESISTANCE OF THE THYRISTOR IN THE NON CONDUCTING STATE
C RON IS THE RESISTANCE OF THE THYRISTOR IN THE CONDUCTING STATE
DIMENSION TRIGG(2)
REAL I
IF (I.GT.0.05) GO TO 10
IF ((V.GT.0.0).AND.(T.EQ.TRIGG(1))) GO TO 10
R=ROFF
GO TO 20
10 R=RON
IF ((IxRON).GT. 1.0).R=1.0/I
20 RETURN
END
  
```

4-4. THE EQUIVALENT CIRCUIT

The equivalent circuit diagram, shown in Fig.4-4 is obtained from Fig.4-1 following the steps given below:

- (i) The armature of the dc machine is replaced by its model which is derived in section 2-3-2 and on Fig 2-13, where R_M and L_M are the armature resistance and inductance, respectively, and C_M has the value of $Jl/K\phi^2$; G_{eq} is ignored as the model is for transient conditions, where the acceleration is very

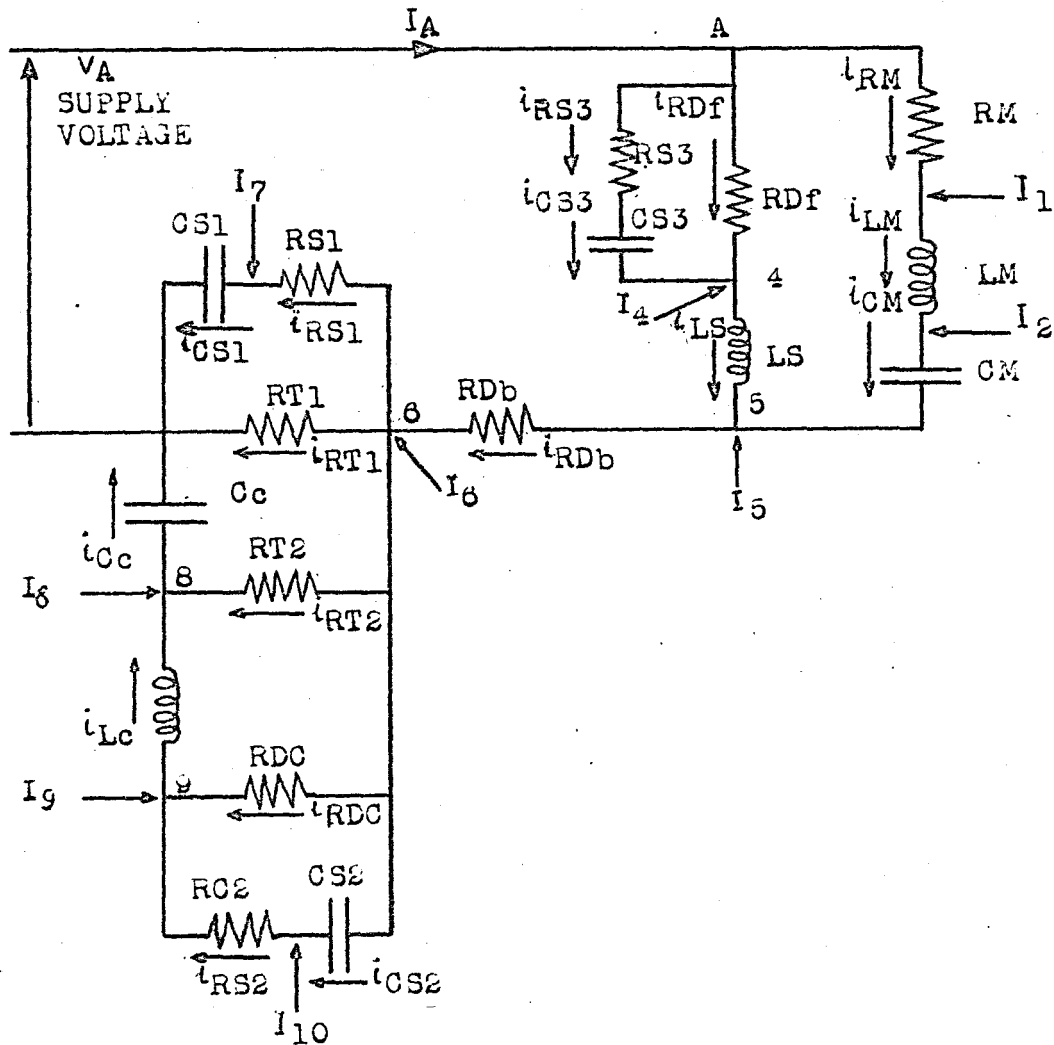


Fig.4-4. The equivalent circuit diagram of Fig.4-1.

high, and T_m is much larger than T_L . (T_L can be ignored). As the machine is separately excited, it gives a field of constant flux, therefore the field circuit is ignored as long as constant K_f is considered.

(ii) The diodes and the thyristors are replaced by variable resistances, $RT1$, $RT2$, Rdf , RDb , RDC for thyristors $T1$, $T2$, and the diodes Df , Db , Dc , and their values are evaluated by the corresponding subroutines $SCRTES$, $DTEST$, in each step (after every time increment of h).

Additionally, a branch current is assigned to every branch, generally in the direction of armature current (clock-wise direction), and the injected imaginary currents with a subscript of its node number to every node.

4-5. FORMULATION OF THE CIRCUIT EQUATIONS.

As explained in section 3-5, the general circuit equations are in the form of Eq 3.18 and all the rules given at the end of section 3-5, are applicable for the derivation of circuit equations of Fig.4-4. However, it should be noted that, the \underline{A} matrix for Fig,4-4 is no more a constant matrix, as it contains diode and thyristor resistances, which are re-evaluated in every step. Also the equations to be solved are exactly in the same form as in the Eq.'s 3.21 and 3.22 .

The \underline{A} and \underline{B} matrices are obtained following the rules 1-4 given at the end of section 3-5. For example, the entries

to $A(\delta, \delta)$ are the sum of the admittances connected to node δ , i.e. $1/RDb + 1/RS1 + 1/RT1 + 1/RT2 + 1/RDc + 2CS2/h$, and the entry to $B(\delta, \delta)$ is exactly the same as $A(\delta, \delta)$, but resistance terms are zero and capacitance terms are negated, i.e. $-2CS2/h$.

The corrector currents are obtained following the rule 5 given at the end of section 3-5. For example, for node δ , $I_{\delta} = i_{Cc} - i_{RT2} - i_{Lc}$ and, as the resistive current contributes zero to the corrector current, and inductive current contributes its previous value to the corrector current, the capacitive current contributes its previous value negated to the corrector current, the column 8 of the corrector current vector will be, $-i_{Lc(n-1)} - i_{Cc(n-1)}$.

Although in this section, A, B matrices and corrector current vector are written up by following the rules in section 3-5, a full derivation of these matrices are given in the Appendix.

The corrector currents
(for the circuit in Fig 4-4)

A	0
1	$i_{LM(n-1)}$
2	$-i_{CM(n-1)} - i_{LM(n-1)}$
3	$-i_{CS3(n-1)}$
4	$i_{LS(n-1)} + i_{CS3(n-1)}$
5	$i_{CM(n-1)} - i_{LS(n-1)}$
6	$i_{CS2(n-1)}$
7	$-i_{CS1(n-1)}$
8	$-i_{Cc(n-1)} - i_{Lc(n-1)}$
9	$i_{Lc(n-1)}$
10	$i_{CS2(n-1)}$

The A Matrix

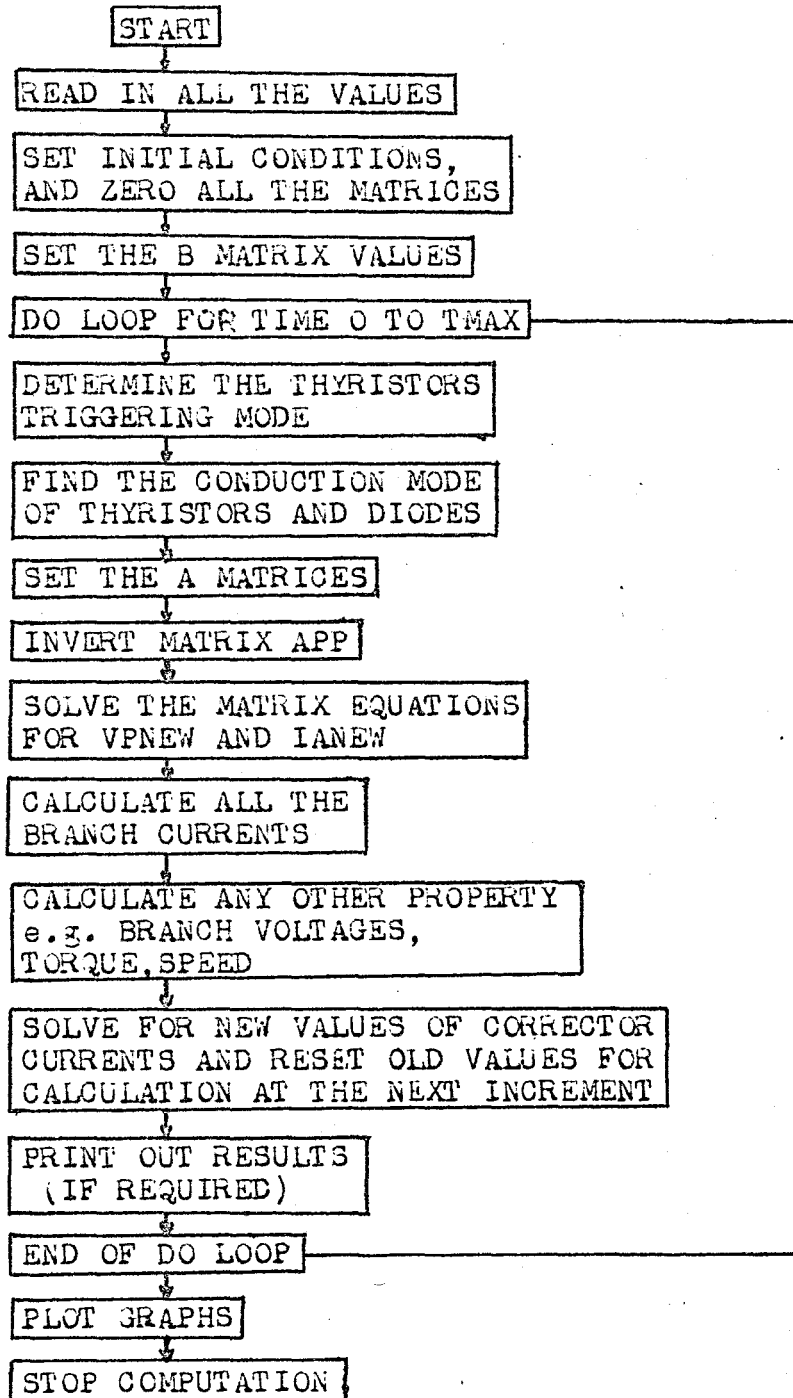
	A	1	2	3	4	5	6	7	8	9	10
A	$1/RM$ $+1/RS3$ $+1/RDf$	$-1/RM$		$-1/RS3$	$-1/RDf$						
1	$-1/RM$	$1/RM$ $+h/2LM$	$-h/2LM$								
2		$-h/2LM$	$h/2LM$ $+2CM/h$			$-2CM/h$					
3	$-1/RS3$			$1/RS3$ $+2CS3/h$	$-2CS3/h$						
4	$-1/RDf$			$-2CS3/h$	$2C3/h$ $+1/RDf$ $+h/2LS$	$-h/2LS$					
5			$-2CM/h$		$-h/2LS$	$h/2LS$ $+2CM/h$ $+1/RDb$	$-1/RDb$				
6						$-1/RDb$	$1/RS1$ $+2CS1/h$	$-1/RS1$	$-1/RT2$	$-1/RDC$	$-2CS2/h$
7							$-1/RS1$	$1/RS1$ $+2CS1/h$			
8							$-1/RT2$		$2Cc/h$ $+1/RT2$ $+h/2Lc$	$-h/2Lc$	
9							$-1/RDC$		$-h/2Lc$	$h/2Lc$ $+1/RDC$ $+1/RS2$	$-1/RS2$
10							$-2CS2/h$			$-1/RS2$	$1/RS2$ $+2CS2/h$

The B Matrix

	A	1	2	3	4	5	6	7	8	9	10
A											
1		$h/2LM$	$-h/2LM$								
2		$-h/2LM$	$h/2LM$ $-2CM/h$			$2CM/h$					
3				$-2CS3/h$	$2CS3/h$						
4				$2CS3/h$	$h/2LS$ $-2CS3/h$	$-h/2LS$					
5			$2CM/h$		$-h/2LS$	$h/2LS$ $-2CM/h$					
6							$-2CS2/h$				$2CS2/h$
7								$-2CS1/h$			
8									$h/2Lc$ $-2Cc/h$	$-h/2Lc$	
9									$-h/2Lc$	$h/2Lc$	
10							$2CS2/h$				$-2CS2/h$

4-6. THE COMPUTER PROGRAM

The general procedure of the computer program for the solution of any problem is best described by a flow diagram given below:



Most of the details of the computer program and the subroutines are explained in the preceding sections, and the rest is self explanatory. The complete computer program is given on the following pages.

AFT'1, IS
FTN'ARI

*08/19/82-13:31

```
1. C THIS PROGRAM SIMULATES THE SEPERATLEY EXCITED CHOPPER CONTROLLED
2. C DC MACHINE
3. C
4. REAL IPNEW(10), IAOLD, IANEW, IRM, ILM, ICM, IPS3, ICS3, IRDF, ILS, IRDB,
5. *IRS1, ICS1, IRT1, IPT2, IRDC, ICS2, IPS2, ILC, ICC, LM, LC, LS, J1, KPHI, ILMOLD
6. *, ICMOLD, ICS3OL, ILSOLD, ICS1OL, ICCOLD, ILCOLD, ICS2OL, IPOLD(10),
7. *IAPP(10,10)
8. DIMENSION X1(110), Y1(110), Y2(110), Y3(110), Y4(110), Y5(110), Y6(110),
9. *Y7(110), Y8(110), Y9(110), Y10(110), Y11(110), Y12(110), Y13(110), Y14(11
10. *0), Y15(110), Y16(110), Y17(110), Y18(110), Y19(110)
11. DIMENSION APP(10,10), VPNEW(10), VPOLD(10), TRIGG(2)
12. DIMENSION YMIN(19), YMAX(19)
13. COMMON AAA, APA(10,1), AAP(1,10), BAA, BPA(10,1), BAP(1,10), BPP(10,10),
14. *VA
15. C SET THE MATRICIES AND THE INITIAL CONDITIONS AT 0.0
16. DATA IAOLD, ILMOLD, ICMOLD, ICS3OL, ILSOLD, ICS1OL, ICS2OL, ILCOLD,
17. *ICCOLD, WOLD, E/11*0.0/
18. DATA TRIGG/1.0, 0.0/
19. DATA T, IANEW, VM, IRM, IRDF, IRDB, IRDC, ICC, IRT1, IRT2, SPEED/11*0.0/
20. DATA RT1, RT2, RDF, RDB, RDC/5*10000.0/
21. DATA YMIN/19*1.0E30/
22. DATA YMAX/19*-1.0E30/
23.
24. READ(5,100) RS1, RS2, RS3, CS1, CS2, CS3
25. 100 FORMAT(6E9.2)
26. READ(5,110) RM, LM, CC, LC, LS, RON, ROFF
27. 110 FORMAT(7E9.2)
28. READ(5,120) VA, J1, KPHI, H, TP, F, TMAX, II
29. 120 FORMAT(7E9.2, I3)
30. WRITE(6,200) RS1, RS2, RS3, CS1, CS2, CS3, RM, LM, CC, LC, LS, RON, ROFF
31. 200 FORMAT(1H1, 10X, 31HTHE VALUES OF THE ELEMENTS ARE, //, 11X, 4HRS1=, E9.
32. *2, /, 11X, 4HRS2=, E9.2, /, 11X, 4HRS3=, E9.2, /, 11X, 4HCS1=, E9.2, /, 11X, 4HCS
33. *2=, E9.2, /, 11X, 4HCS3=, E9.2, /, 12X, 3HPM=, E9.2, /, 12X, 3HLM=, E9.2, /, 12X,
```

```

34.      *3HCC=,EQ.2,/,12X,3HLC=,EQ.2,/,12X,3HLS=,EQ.2,/,11X,4HRONE=,EQ.2,/,1
35.      *OX,5HROFF=,EQ.2)
36.      WRITE(6,210) VA, J1,KPHI,H,TP,F,TMAX,N
37. 210   FORMAT(1H,12X,3HVA=,EQ.2,/,12X,3HJ1=,EQ.2,/,10X,5HKPHI=,EQ.2,/,13
38.      *X,2HH=,EQ.2,/,12X,3HTR=,EQ.2,/,13X,2HF=,EQ.2,/,10X,5HTMAX=,EQ.2,/,
39.      *13X,2HN=,I2)
40.      M=1
41.      N1=0
42.      AAA=0.0
43.      BAA=0.0
44.      DO 11 I=1,N
45.      DO 22 J=1,N
46.      APP(I,J)=0.0
47. 22    BPP(I,J)=0.0
48.      AAP(I,I)=0.0
49.      APA(I,1)=0.0
50.      BAP(I,I)=0.0
51.      BPA(I,1)=0.0
52.      VPOLD(I)=0.0
53.      VPNEW(I)=0.0
54. 11    IPOLD(I)=0.0
55.      CM=J1/(KPHI**2)
56.      PT=1.0/F
57.      PI=3.1416
58.      TORQUE=0.0
59.      VPOLD(8)=250.0
60.  C    SET THE B MATRICES
61.      BPP(1,1)=H/(2.0*LM)
62.      BPP(1,2)=-BPP(1,1)
63.      BPP(2,1)=-BPP(1,1)
64.      BPP(2,2)=H/(2.0*LM)-2.0*CM/H
65.      BPP(2,5)=2.0*CM/H
66.      BPP(3,3)=-2.0*CS3/H
67.      BPP(3,4)=-BPP(3,3)
68.      BPP(4,3)=-BPP(3,3)

```

1
2
2
1
1
1
1
1
1
1

```

69.      BPP(4,4)=-2.0*CS3/H+H/(2.0*LS)
70.      BPP(4,5)=-H/(2.0*LS)
71.      BPP(5,2)=BPP(2,5)
72.      BPP(5,4)=BPP(4,5)
73.      BPP(5,5)=H/(2.0*LS)-2.0*CM/H
74.      BPP(6,6)=-2.0*CS2/H
75.      BPP(6,10)=-BPP(6,6)
76.      BPP(10,6)=BPP(6,10)
77.      BPP(7,7)=-2.0*CS1/H
78.      BPP(8,8)=-2.0*CC/H+H/(2.0*LC)
79.      BPP(8,9)=-H/(2.0*LC)
80.      BPP(9,8)=BPP(8,9)
81.      BPP(9,9)=H/(2.0*LC)
82.      BPP(10,10)=-2.0*CS2/H
83.      WRITE(6,230)
84.      230  FORMAT(////)
85.      C
86.      231  FORMAT(1H ,6X, ,***, , , T          VM          IANEW          IRM          IRDF
87.      *      ILS          IRS1          IPT1          , , , , , 9X, , , IRT2          IRDC          IRS2
88.      *ICC          VPNEW(3)          VRM          VLM          VCM          , , , , , 9X, , , VRDF          VRDR
89.      *      VLS          VRT1          VRT2          VRDC          VLC          E          , , , )
90.      C      DO LOOP TO CALCULATE AT A GIVEN TIME T
91.      TT=0.0
92.      JMAX=IFIX(TMAX/H)
93.      TMAX=JMAX+1
94.      L3=1
95.      L2=4
96.      DO 1 L1=1,TMAX
97.      L2=L2+1
98.      T=FLOAT(L1)*H
99.      IF(TT.GE.PT) N1=0
100.     IF(TT.GE.PT) TT=TT-PT
101.     IF(TT.LT.(PT+TR)) GO TO 10
102.     TTRIG=TRIGG(2)
103.     GO TO 20

```

```

1
1
1
1
1
1
1

```

```

1 104. 10 T1TRIG=TRIGG(1)
1 105. 20 IF(IT.GE.(PT+TR)) GO TO 30
1 106. T2TRIG=TRIGG(2)
1 107. GO TO 40
1 108. 30 T2TRIG=TRIGG(1)
1 109. 40 CONTINUE
1 110. C FIND THE CONDUCTING MODE OF THE THYRISTORS & DIODES
1 111. C TEST ON THYRISTOR 1
1 112. IRSET=2
1 113. VT1=VPOLD(6)
1 114. R1=RT1
1 115. CALL SCRTES (VT1,IRT1,T1TRIG,TRIGG,RT1,ROFF,RON,M)
1 116. IF(RT1.NE.R1) IRSET=1
1 117. C TEST ON THYRISTOR 2
1 118. VT2=VPOLD(6)-VPOLD(8)
1 119. R2=RT2
1 120. CALL SCRTES (VT2,IRT2,T2TRIG,TRIGG,RT2,ROFF,RON,M)
1 121. IF(RT2.NE.R2) IRSET=1
1 122. C TEST ON THE DIODES
1 123. VDF=VPOLD(4)-VA
1 124. DI=-TRDF
1 125. R3=RDF
1 126. CALL DTEST(VDF,DI,RDF,ROFF,RON,M)
1 127. IF(RDF.NE.R3) IRSET=1
1 128. VDB=VPOLD(5)-VPOLD(6)
1 129. R4=RDB
1 130. CALL DTEST(VDB,IPDB,RDB,ROFF,RON,M)
1 131. IF(RDB.NE.R4) IRSET=1
1 132. VDC=VPOLD(2)-VPOLD(6)
1 133. DI1=-IPDC
1 134. R5=RDC
1 135. CALL DTEST(VDC,DI1,RDC,ROFF,RON,M)
1 136. IF(RDC.NE.R5) IRSET=1
1 137. IF(II.EQ.2) RDC=ROFF
1 138. IF(IRSET.EQ.2) GO TO 50

```

```

1 139. GO TO 50
1 140. C WRITE(6,222) PT1,RT2,RDF,RDB,RDC
1 141. 222 FORMAT(1H0,10X,5F12.3)
1 142. 50 CONTINUE
1 143. IF(M.GT.1.AND.IRSET.EQ.2) GO TO 70
1 144. IF(H1.EQ.0) GO TO 81
1 145. R0 H1=2
1 146. R1 DUM=0.
1 147. C SET THE A MATRICES
1 148. AAA=1.0/RM+1.0/PDF+1.0/RS3
1 149. AAP(1,1)=-1.0/RM
1 150. AAP(1,3)=-1.0/RS3
1 151. AAP(1,4)=-1.0/RDF
1 152. APA(1,1)=AAP(1,1)
1 153. APA(3,1)=AAP(1,3)
1 154. APA(4,1)=AAP(1,4)
1 155. APP(1,1)=1.0/RM+H/(2.0*LM)
1 156. APP(1,2)=-H/(2.0*LM)
1 157. APP(2,1)=APP(1,2)
1 158. APP(2,2)=H/(2.0*LM)+2.0*CM/H
1 159. APP(2,5)=-2.0*CM/H
1 160. APP(3,3)=1.0/RS3+2.0*CS3/H
1 161. APP(3,4)=-2.0*CS3/H
1 162. APP(4,3)=APP(3,4)
1 163. APP(4,4)=1.0/RDF+2.0*CS3/H+H/(2.0*LS)
1 164. APP(4,5)=-H/(2.0*LS)
1 165. APP(5,2)=APP(2,5)
1 166. APP(5,4)=APP(4,5)
1 167. APP(5,5)=H/(2.0*LS)+2.0*CM/H+1.0/PDB
1 168. APP(5,6)=-1.0/PDB
1 169. APP(6,5)=APP(5,6)
1 170. APP(6,6)=1.0/RDB+1.0/RS1+1.0/RT1+1.0/RT2+1.0/RDC+2.0*CS1/H
1 171. APP(6,7)=-1.0/RS1
1 172. APP(6,8)=-1.0/RT2
1 173. APP(6,9)=-1.0/RDC

```

```

1 174. APP(6,10)=-2.0*CS2/H
1 175. APP(7,6)=APP(6,7)
1 176. APP(7,7)=1.0/RS1+2.0*CS1/H
1 177. APP(8,6)=APP(6,8)
1 178. APP(8,8)=2.0*CC/H+1.0/RT2+H/(2.0*LC)
1 179. APP(8,9)=-H/(2.0*LC)
1 180. APP(9,6)=APP(6,9)
1 181. APP(9,8)=APP(8,9)
1 182. APP(9,9)=H/(2.0*LC)+1.0/RDC+1.0/RS2
1 183. APP(9,10)=-1.0/RS2
1 184. APP(10,6)=APP(6,10)
1 185. APP(10,9)=APP(9,10)
1 186. APP(10,10)=1.0/RS2+2.0*CS2/H
1 187. C CALCULATE THE INVERSE OF APP=IAPP
1 188. CALL MI(IAPP,APP,N)
1 189. 70 DUM=0.
1 190. RIDC=IRDC
1 191. C SOLVE THE EQUATION FOR VPNEW & IANEW
1 192. CALL VP1A(VPNEW,VPOLD,IANEW,IAOLD,N,IPOLD,IAPP)
1 193. C CALCULATE ALL THE BRANCH CURRENTS
1 194. IRDC=(VPNEW(6)-VPNEW(9))/RDC
1 195. IF(IRDC.GT.0.0.AND.RIDC.LT.0.0) N1=1
1 196. IF(N1.EQ.1) RDC=ROFF
1 197. IF(N1.EQ.1) GO TO 80
1 198. IRM=(VA-VPNEW(1))/RM
1 199. ILM=ILMOLD+H/(2.0*LM)*((VPNEW(1)-VPNEW(2))+(VPOLD(1)-VPOLD(2)))
1 200. ICM=-ICMOLD+2.0*CM/H*((VPNEW(2)-VPNEW(5))-(VPOLD(2)-VPOLD(5)))
1 201. ICM=ILM
1 202. IRDF=(VA-VPNEW(4))/RDF
1 203. IRS3=(VA-VPNEW(3))/RS3
1 204. ICS3=-ICS3OL+2.0*CS3/H*((VPNEW(3)-VPNEW(4))-(VPOLD(3)-VPOLD(4)))
1 205. ILS=ILSOLD+H/(2.0*LS)*((VPNEW(4)-VPNEW(5))+(VPOLD(4)-VPOLD(5)))
1 206. IRDR=(VPNEW(5)-VPNEW(6))/RDR
1 207. IRS1=(VPNEW(6)-VPNEW(7))/RS1
1 208. ICS1=-ICS1OL+2.0*CS1/H*(VPNEW(7)-VPOLD(7))

```

```

1 209. ICC=-ICCOLD+2.0*CC/H*(VPNEW(8)-VPOLD(8))
1 210. ILC=ILCOLD+H/(2.0*LC)*(VPNEW(9)-VPNEW(8))+(VPOLD(9)-VPOLD(8))
1 211. IRT1=VPNEW(6)/RT1
1 212. IRT2=(VPNEW(6)-VPNEW(8))/RT2
1 213. IRS2=(VPNEW(10)-VPNEW(9))/RS2
1 214. ICS2=-ICS2OL+2.0*CS2/H*(VPNEW(6)-VPNEW(10))-(VPOLD(6)-VPOLD(10))
1 215. *)
1 216. VM=VA-VPNEW(5)
1 217. E=E+H/(2*CM)*(ILMOLD+ILM)
1 218. VRM=VA-VPNEW(1)
1 219. VLM=VPNEW(1)-VPNEW(2)
1 220. VCM=VPNEW(2)-VPNEW(5)
1 221. VLS=VPNEW(4)-VPNEW(5)
1 222. VRDF=VA-VPNEW(4)
1 223. VRDB=VPNEW(5)-VPNEW(6)
1 224. VRT1=VPNEW(6)
1 225. VRT2=VPNEW(6)-VPNEW(8)
1 226. VRDC=VPNEW(6)-VPNEW(9)
1 227. VLC=VPNEW(9)-VPNEW(8)
1 228. VCC=VPNEW(8)
1 229. VCS1=VPNEW(7)
1 230. VLS=VPNEW(4)-VPNEW(5)
1 231. C CALCULATE THE SPEED
1 232. W=E/KPHI
1 233. SPEED=60.0*W/(2.0*PI)
1 234. TORQUE=KPHI*IRM
1 235. IF((L1/2*2).NE.L1) GO TO 90
1 236. C WRITE(6,240) T,VM,IANEW,IRM,TRDF,ILS,IRS1,IRT1
1 237. 240 FORMAT(1H,6X, ,**, ,8F9.2)
1 238. C WRITE(6,250)IRT2,IPDC,IRS2,ICC,VPNEW(8),VRM,VLM,VCM
1 239. C WRITE(6,250)ILC,SPEED,IRS3,IPDR,ICM,ILM
1 240. C WRITE(6,250)VRDF,VRDB,VLS,VRT1,VRT2,VRDC,VLC,E
1 241. 250 FORMAT(1H ,9X,8F9.2)
1 242. 90 DUM=0.
1 243. C CALCULATE NEW VALUES OF CORRECTOR CURRENTS

```

```

1      244.      IAOLD=0.0
1      245.      IPOLD(1)=ILM
1      246.      IPOLD(2)=-ICM-ILM
1      247.      IPOLD(3)=-ICS3
1      248.      IPOLD(4)=ILS+ICS3
1      249.      IPOLD(5)=-ILS+ICM
1      250.      IPOLD(6)=-ICS2
1      251.      IPOLD(7)=-ICS1
1      252.      IPOLD(8)=-ILC-ICC
1      253.      IPOLD(9)=ILC
1      254.      IPOLD(10)=ICS2
1      255.      ILMOLD=ILM
1      256.      ICMOLD=ICM
1      257.      ILSOLD=ILS
1      258.      ICS10L =ICS1
1      259.      ICS20L =ICS2
1      260.      ICS30L =ICS3
1      261.      ILCOLD=ILC
1      262.      ICCOLD=ICC
1      263.      DO 4 L4=1,11
2      264.      4 VPOLD(L4)=VPNEW(L4)
1      265.      WOLD=W
1      266.      M=M+1
1      267.      C CALCULATE THE MINIMUM AND MAXIMUM VALUES OF THE VARIABLES PLOTTED
1      268.      IF(VM.LT.YMIN(1)) YMIN(1)=VM
1      269.      IF(E.LT.YMIN(2)) YMIN(2)=E
1      270.      IF(IPM.LT.YMIN(3)) YMIN(3)=IPM
1      271.      IF(ILS.LT.YMIN(4)) YMIN(4)=ILS
1      272.      IF(IANNEW.LT.YMIN(5)) YMIN(5)=IANNEW
1      273.      IF(IPT1.LT.YMIN(6)) YMIN(6)=IPT1
1      274.      IF(IRT2.LT.YMIN(7)) YMIN(7)=IRT2
1      275.      IF(ICC.LT.YMIN(8)) YMIN(8)=ICC
1      276.      IF(ILC.LT.YMIN(9)) YMIN(9)=ILC
1      277.      IF(IRS1.LT.YMIN(10)) YMIN(10)=IRS1
1      278.      IF(VCS1.LT.YMIN(11)) YMIN(11)=VCS1

```

```

1 279. IF (VRDF.LT.YMIN(12)) YMIN(12)=VRDF
1 280. IF (VLS.LT.YMIN(13)) YMIN(13)=VLS
1 281. IF (VRT1.LT.YMIN(14)) YMIN(14)=VRT1
1 282. IF (VRT2.LT.YMIN(15)) YMIN(15)=VRT2
1 283. IF (VRDC.LT.YMIN(16)) YMIN(16)=VRDC
1 284. IF (VCC.LT.YMIN(17)) YMIN(17)=VCC
1 285. IF (SPEED.LT.YMIN(18)) YMIN(18)=SPEED
1 286. IF (TORQUE.LT.YMIN(19)) YMIN(19)=TORQUE
1 287. IF (VM.GT.YMAX(1)) YMAX(1)=VM
1 288. IF (E.GT.YMAX(2)) YMAX(2)=E
1 289. IF (IPM.GT.YMAX(3)) YMAX(3)=IPM
1 290. IF (IILS.GT.YMAX(4)) YMAX(4)=IILS
1 291. IF (IANFW.GT.YMAX(5)) YMAX(5)=IANFW
1 292. IF (IPT1.GT.YMAX(6)) YMAX(6)=IPT1
1 293. IF (IPT2.GT.YMAX(7)) YMAX(7)=IPT2
1 294. IF (ICC.GT.YMAX(8)) YMAX(8)=ICC
1 295. IF (ILC.GT.YMAX(9)) YMAX(9)=ILC
1 296. IF (IPS1.GT.YMAX(10)) YMAX(10)=IPS1
1 297. IF (VCS1.GT.YMAX(11)) YMAX(11)=VCS1
1 298. IF (VRDF.GT.YMAX(12)) YMAX(12)=VRDF
1 299. IF (VLS.GT.YMAX(13)) YMAX(13)=VLS
1 300. IF (VRT1.GT.YMAX(14)) YMAX(14)=VRT1
1 301. IF (VRT2.GT.YMAX(15)) YMAX(15)=VRT2
1 302. IF (VRDC.GT.YMAX(16)) YMAX(16)=VRDC
1 303. IF (VCC.GT.YMAX(17)) YMAX(17)=VCC
1 304. IF (SPEED.GT.YMAX(18)) YMAX(18)=SPEED
1 305. IF (TORQUE.GT.YMAX(19)) YMAX(19)=TORQUE
1 306. IF (L2.NE.5) GO TO 1
1 307. X1(L3)=T+1.0E03
1 308. Y1(L3)=VM
1 309. Y2(L3)=E
1 310. Y3(L3)=IPM
1 311. Y4(L3)=IILS
1 312. Y5(L3)=IANFW
1 313. Y6(L3)=IPT1

```

```

1      314.      Y7(L3)=RT2
1      315.      Y8(L3)=ICC
1      316.      Y9(L3)=ILC
1      317.      Y10(L3)=IRS1
1      318.      Y11(L3)=VCS1
1      319.      Y12(L3)=VRDF
1      320.      Y13(L3)=VLS
1      321.      Y14(L3)=VRT1
1      322.      Y15(L3)=VRT2
1      323.      Y16(L3)=VRDC
1      324.      Y17(L3)=VCC
1      325.      Y18(L3)=SPEED
1      326.      Y19(L3)=TORQUE
1      327.      L2=L2-5
1      328.      L3=L3+1
1      329.      1      TT=TT+H
330.      WRITE(6,500)
331.      500      FORMAT(1H,13X,,VM      E      TRM      ILS      IANEW      IPT1
332.      *      IRT2,,///)
333.      WRITE(6,501) (YMIN(I),I=1,7)
334.      501      FORMAT(6X,,MIH,,7E9.2,///)
335.      WRITE(6,502) (YMAX(I),I=1,7)
336.      502      FORMAT(6X,,MAX,,7E9.2,///)
337.      WRITE(6,503)
338.      503      FORMAT(1H,9X,, ICC      ILC      IRS1      VCS1      VRDF      VLS
339.      *      VRT1,,///)
340.      WRITE(6,504) (YMIN(I),I=8,14)
341.      504      FORMAT(6X,,MIH,,7E9.2,///)
342.      WRITE(6,505) (YMAX(I),I=8,14)
343.      505      FORMAT(6X,,MAX,,7E9.2,///)
344.      WRITE(6,506)
345.      506      FORMAT(1H,9X,, VRT2      VRDC      VCC      SPEED      TORQUE,,///)
346.      WRITE(6,507) (YMIN(I),I=15,19)
347.      507      FORMAT(6X,,MIH,,5E9.2,///)
348.      WRITE(6,508) (YMAX(I),I=15,19)

```

1
0
1

```
349. 508 FORMAT(6Y,,MAX,,SEP.2,///)
350. CALL GRAPH4(8.,8.,100,X1,Y1)
351. CALL GRAPH4(8.,8.,100,X1,Y2)
352. CALL GRAPH4(8.,8.,100,X1,Y3)
353. CALL GRAPH4(8.,8.,100,X1,Y4)
354. CALL GRAPH4(8.,8.,100,X1,Y5)
355. CALL GRAPH4(8.,8.,100,X1,Y6)
356. CALL GRAPH4(8.,8.,100,X1,Y7)
357. CALL GRAPH4(8.,8.,100,X1,Y8)
358. CALL GRAPH4(8.,8.,100,X1,Y9)
359. CALL GRAPH4(8.,8.,100,X1,Y10)
360. CALL GRAPH4(8.,8.,100,X1,Y11)
361. CALL GRAPH4(8.,8.,100,X1,Y12)
362. CALL GRAPH4(8.,8.,100,X1,Y13)
363. CALL GRAPH4(8.,8.,100,X1,Y14)
364. CALL GRAPH4(8.,8.,100,X1,Y15)
365. CALL GRAPH4(8.,8.,100,X1,Y16)
366. CALL GRAPH4(8.,8.,100,X1,Y17)
367. CALL GRAPH4(8.,8.,100,X1,Y18)
368. CALL GRAPH4(8.,8.,100,X1,Y19)
369.
370. END
371.
```

```
372. SUBROUTINE SCPTES (V,I,T,TRIGG,R,ROFF,RON,M)
373. C V IS THE VOLTAGE ACROSS THE THYRISTOR
374. C I IS THE CURRENT THROUGH THE THYRISTOR
375. C TRIGG IS AN ARRAY OF TWO ELEMENTS
```

```

376. C TRIGG(1)=1.0 REPRESENTS A TRIGGER PULSE BEING PRESENT
377. C TRIGG(2)=0.0 REPRESENTS A TRIGGER PULSE BEING ABSENT
378. C T IS THE STATE OF THE TRIGGER SIGNAL (EITHER 1.0 OR 0.0)
379. C R IS THE RESISTANCE OF THE THYRISTOR
380. C ROFF IS THE RESISTANCE OF THE THYRISTOR IN THE NON-CONDUCTING STATE
381. C RON IS THE RESISTANCE OF THE THYRISTOR IN THE CONDUCTING STATE
382.     DIMENSION TRIGG(2)
383.     REAL I
384.     IF(I.GT.0.05) GO TO 10
385.     IF((V.GT.0.0).AND.(T.EQ.TRIGG(1))) GO TO 10
386.     R=ROFF
387.     GO TO 20
388. 10    R=RON
389. C     IF((I*RON).GT.1.0) R=1.0/I
390.     IF(M.GT.1) R=1./I
391. 20    RETURN
392.     END
393.

```

```

394.     SUBROUTINE DTEST(V,I,R,ROFF,RON,M)
395. C V IS THE VOLTAGE ACROSS THE DIODE
396. C I IS THE FORWARD CURRENT THROUGH THE DIODE
397. C R IS THE RESISTANCE OF THE DIODE
398. C ROFF IS THE RESISTANCE OF THE DIODE IN THE NON-CONDUCTING STATE
399. C RON IS THE RESISTANCE OF THE DIODE IN THE CONDUCTING STATE
400.     REAL I
401.     IF(I.GT.0.05) GO TO 10
402.     IF(V.GT.0.0) GO TO 10

```

```

403.      P=ROFF
404.      GO TO 20
405.      10  R=RON
406.      C   IF((I*RON).GT.1.0) R=1.0/I
407.      IF(M.GT.1) R=1./I
408.      20  RETURN
409.      END
410.

```

```

1 411.      SUBROUTINE MI(E,F,N)
2 412.      DIMENSION E(10,10),F(10,10),IJ(10)
1 413.      DO 10 I=1,N
1 414.      DO 1 J=1,N
1 415.      1  E(I,J)=F(I,J)
1 416.      10  IJ(I)=0
1 417.      ITER=0
1 418.      12  ITER=ITER+1
1 419.      IF(ITER.GT.11) GO TO 100
1 420.      CNV=0.0
1 421.      DO 11 J=1,N
1 422.      IF(IJ(J).GT.0) GO TO 11
1 423.      TEST=ABS(E(J,J))
1 424.      IF(TEST.LT.CNV) GO TO 11
1 425.      CNV=TEST
1 426.      K=J
1 427.      11  CONTINUE
1 428.      IJ(K)=1
1 429.      E(K,K)=1.0/E(K,K)

```

```

1 430.      DO 3 I=1,N
2 431.      DO 3 J=1,N
2 432.      IF(I-K) 4,3,4
2 433.      IF(J-K) 5,3,5
2 434.      5  E(I,J)=E(I,J)-F(I,K)*E(K,K)*F(K,J)
2 435.      3  CONTINUE
436.      DO 6 I=1,N
1 437.      IF(I-K) 7,6,7
1 438.      7  E(I,K)=E(I,K)*E(K,K)
1 439.      E(K,I)=-E(K,I)*E(K,K)
1 440.      6  CONTINUE
441.      GO TO 12
442.      100 RETURN
443.      END
444.

```

```

445.      SUBROUTINE VP1A(VPNEW,VPOLD,IANEW,IAOLD,N,IPOLD,IAPP)
446.      C THIS SUBROUTINE CALCULATES THE VALUES OF VPNEW&IANEW
447.      REAL VPNEW(10),VPOLD(10),IANEW,IAOLD,IPOLD(10),IAPP(10,10)
448.      COMMON AAA,APA(10,1),AAP(1,10),BAA,BPA(10,1),BAP(1,10),BPP(10,10),
449.      *VA
450.      DIMENSION DUMMY(10),BRAC(10)
451.      C TO CALCULATE THE MATRIX EQUATION -(APP)**-1((IPOLD)+(APA)(VA)+
452.      C (BPA)VA)+B(BPP)(VPOLD))
453.      C PERFORM THE MATRIX MULTIPLICATION(BPP)*(VPOLD)=(DUMMY)
454.      DO 1 I=1,N
1 455.      DUMMY(I)=0.0
1 456.      DO 2 J=1,N

```

```

2      457.  2      DUMMY(I)=DUMMY(I)+BRP(I,J)*VPOLD(J)
1      458.  1      CONTINUE
1      459.  C      ADD THE TERMS IN THE BRACKETS=(BRAC)
          460.      DO 3 I=1,N
1      461.  3      BRAC(I)=IPOLD(I)+APA(I,1)*VA+RPA(I,1)*VA+DUMMY(I)
1      462.  C      MULTIPLY THE SUM OF THE BRACKETS AND MINUS THE INVERSE OF APP
          463.      DO 4 I=1,N
1      464.      VPNEW(I)=0.0
1      465.      DO 5 J=1,N
2      466.  5      VPNEW(I)=VPNEW(I)+IAPP(I,J)*BRAC(J)
1      467.  4      VPNEW(I)=-VPNEW(I)
1      468.  C      CALCULATE THE EQUATION FOR IANEW
          469.      DUMMY1=0.0
          470.      DUMMY2=0.0
          471.      DO 6 I=1,N
1      472.      DUMMY1=DUMMY1+AAP(1,I)*VPNEW(I)
1      473.  6      DUMMY2=DUMMY2+BAP(1,I)*VPOLD(I)
          474.      IANEW=IAOLD+AAA*VA+BAA*VA+DUMMY1+DUMMY2
          475.      RETURN
          476.      END

```

END FTN 1589 IBANK 3256 DBANK 143 COMMON
 ΔMAP,I
 MAP 30R1 S74T11 08/19/82 13:32:51

ADDRESS LIMITS 001000 050676 20415 IBANK WORDS DECIMAL
 051000 063341 5346 DBANK WORDS DECIMAL
 STARTING ADDRESS 045612

SEGMENT BOUNDS 001000 050676 051000 063341

CHAPTER 5

RESULTS AND DISCUSSION

5-1. INTRODUCTION

The computer program can supply the following outputs:

- (i) The thyristor and diode resistances, namely $RT1$, $RT2$, RDF , RDB , RDC can be printed (by the statement `WRITE(6,222)`), when at least one of them change.
- (ii) The values of the variables which are evaluated after each time increment of 'h', can be tabulated, and their extramum values can be calculated.
- (iii) All the variables in the circuit can be plotted automatically by the SUBROUTINE GRAPH4.

The graphs of most of the variables are presented together with their detailed explanations, in the next section. Unfortunately, there is no continous graph plotter in our Computer Centre, and it is almost impossible to hand-plot as there are too many points. Therefore, the only alternative was to plot the graphs on the line printer, some of them may not be very accurate, as only the 20% of the points evaluated, are printed and the variation of curves are discrete. The tables of the thyristor, diode resistances and the value of the variables, are not presented,

since they take very large space but provide little information.

The program was run up to 2×10^{-3} seconds at the highest operating frequency (1000 Hz) in order to have commutation cycles comparable to a complete period. The time ratio was chosen as 0.5 to give equal on and off time intervals. There are 400 steps, as the time increment h is set to 5.0×10^{-6} seconds. For

$k \times 10^{-3} \leq t < (k+1/2) \times 10^{-3}$ sec. are 'on', and for

$(k+1/2) \times 10^{-3} \leq t < (k+1) \times 10^{-3}$ sec. are 'off' time intervals

where $k=0,1,2,3, \dots$

5-2. DISCUSSION OF RESULTS

The simulation is started from time zero, at zero state condition, which means all the variables are zero, except the voltage across the commutation capacitance C_c , which is made equal to the supply voltage negative at the ground end ($V_{POLD}(\delta) = 250.0V$), this would in practice be done by firing the auxiliary thyristor T_2 just before starting the operation.

The graphs obtained by the computer, for the variables $V_M, E, I_{RM}, I_{LS}, I_A, I_{RT1}, I_{RT2}, I_{CC}, I_{LC}, I_{RS1}, V_{CS1}, V_{RDF}, V_{RT1}, V_{RT2}, V_{RDC}, V_{CC}, SPEED$ and $TORQUE$, and they are presented at the end of this section. For convenience, the computer variables (such as $V_M, E, I_{RM}, \dots, SPEED$) in capital letters designate the complete waveform in the relevant time interval which is defined at the beginning of each sub-section, and lower case letters

with the suffix in capital letters (such as, $v_M(t)$, $e(t)$, $i_{RM}(t)$, $i_{LS}(t)$, ..., $v_{RDF}(t)$) designate the value of the corresponding variable at time t . T1, T2, Df, Dc, Db, RS1, CS1, Ls, Cc, Lc represent the main thyristor, auxiliary thyristor, freewheeling diode, commutation diode, series diode, suppressor resistance and capacitance of the first snubber network, freewheeling inductance, commutation capacitance and inductance, respectively (as shown on Fig.4-1). Also, graphs plotted are defined as follows (the directions are shown on Fig.4-4) :

VM [$v_M(t)$] : armature voltage, i.e. $v_A(t) - v_5(t)$

E [$e(t)$] : the back emf voltage, i.e. $v_2(t) - v_5(t)$

IRM [$i_{RM}(t)$] : armature current

ILS [$i_{LS}(t)$] : current through the inductor Ls, which is approximately equal to the freewheeling diode current

IA [$i_A(t)$] : supply current

IRT1 [$i_{RT1}(t)$] : the current through the main thyristor

IRT2 [$i_{RT2}(t)$] : the current through the auxiliary thyristor

ICC [$i_{CC}(t)$] : the commutation capacitance current

ILC [$i_{LC}(t)$] : the commutation inductance current

IRS1 [$i_{RS1}(t)$] : the current of the first snubber circuit

VCS1 [$v_{CS1}(t)$] : voltage of the first snubber capacitance, $v_7(t)$

VRDF [$v_{RDF}(t)$] : voltage of the freewheeling diode, $v_A(t) - v_4(t)$

VRT1 [$v_{RT1}(t)$] : voltage of the main thyristor, i.e. $v_0(t)$

VRT2 [$v_{RT2}(t)$] : voltage of the auxiliary thyristor $v_0(t) - v_8(t)$

VRDC [$v_{RDC}(t)$] : voltage of the commutation diode $v_9(t) - v_8(t)$

SPEED : rotor speed in rpm

TORQUE : mechanical torque in newton-meters

For each of the four time intervals, the simulation results are discussed below. The reader should refer to the relevant graphs as the discussion is presented.

5-2-1. THE FIRST ON-TIME INTERVAL ($0.0 \leq t < 0.5 \times 10^{-3}$ sec.)

During this time interval, T1 is triggered and T2 is not triggered. T1, being forward biased, starts conduction and stays in conduction state and T2 is in off state throughout this time interval.

VM rises to supply voltage immediately, as T1 is conducting. Therefore,

$$v_M(t) = 250.0 \text{ V for } 0.0 \leq t < 0.5 \times 10^{-3} \text{ sec.}$$

IRM rises as the sum of the two exponential functions, from zero value. The characteristic equation of the armature circuit, is given in Eq.2.30, where B can be ignored as explained in section 4-4, $\tau_a = LM/RM = 20 \times 10^{-3} / 0.5 = 4.0 \times 10^{-2}$ sec. and $\tau_m = 0.5 \times 1.0 = 0.5$ sec.. Therefore the characteristic equation is

$$s^2 - 25s - 50 = 0$$

Its roots are

$$s_1 = -22.807 \quad s_2 = -2.192$$

Then

$$i_{RM}(t) = A_1 e^{-22.8t} + A_2 e^{-2.19t}$$

with the initial conditions: $i_{RM}(0) = 0.0A$

$$\left. \frac{di_{RM}}{dt} \right|_{t=0} = 1.25 \times 10^4 \text{ A/sec.}$$

gives $A_1 = -606.5 \quad A_2 = 606.5$

so $i_{RM}(t) = -0.006.5 e^{-22.8t} + 0.006.5 e^{-2.19t}$
for $0.0 \leq t \leq 0.5 \times 10^{-3}$ sec.

and $i_{RM}(0.5 \times 10^{-3}) = 0.21$ A

i.e. by the end of this time interval i_{RM} rises to the value of 0.21 A, which is approximately equal to the value obtained by the simulation .

E is the voltage on CM , and it can be calculated as

$$e(t) = 1/CM \int_0^t i_{RM}(t) dt = 1.0 (26.6 e^{-22.8t} - 276 e^{-2.19t}) \Big|_0^t$$

$$e(t) = 250.3 + 26.6 e^{-22.8t} - 276 e^{-2.19t}$$

for $0.0 \leq t < 0.5 \times 10^{-3}$ sec.

and $e(0.5 \times 10^{-3}) = 1.521 \times 10^{-3}$ V

i.e. by the end of this time interval E rises to the value of 1.521×10^{-3} V, which is approximately equal to the value obtained by the simulation.

D_f is reverse biased, as V_M is equal to 250.0 V; so the current through D_f and inductor L_S , i_{LS} is zero, that is

$$i_{LS}(t) = 0.0 \quad \text{for} \quad 0.0 \leq t < 0.5 \times 10^{-3} \text{ sec.}$$

i_A is equal to i_{RM} , because i_{LS} is zero, that is

$$i_A(t) = i_{RM}(t) \quad \text{for} \quad 0.0 \leq t < 0.5 \times 10^{-3} \text{ sec.}$$

V_{CC} has been setted to 250.0 V ($V_{POLD}(c) = 250.0$) before this time interval. A sinusoidal oscillation occurs in the loop $C_c, L_c, D_c,$ and T_1 for a half period ($= \pi \sqrt{L_c C_c}$), when T_1 starts conduction at time zero.

$$v_{CC}(t) = A \sin(\omega t + \phi) \quad \omega = 1/\sqrt{L_c C_c}$$

and $L_c = 50 \times 10^{-6} \text{ H}$ $C_c = 10 \times 10^{-9} \text{ F}$ $\omega = 4.472 \times 10^4 \text{ rad/sec.}$

as $v_{CC}(0) = 250.0 \text{ V}$

$$v_{CC}(t) = 250 \sin(4.472 \times 10^4 t + \phi) \quad \text{where } \phi = \pi/2 \text{ rad.}$$

or simply

$$v_{CC}(t) = 250 \cos(4.472 \times 10^4 t) \quad \text{for } 0.0 \leq t \leq 7.025 \times 10^{-5} \text{ sec.}$$

and $i_{CC}(t) = C_c \, dv_{CC}(t)/dt$

$$i_{CC}(t) = -111.8 \sin(4.472 \times 10^4 t) \quad \text{for } 0.0 \leq t \leq 7.025 \times 10^{-5} \text{ sec.}$$

Oscillation stops at $t = 7.025 \times 10^{-5} \text{ sec.}$, because the diode D_c does not let the next cycle which is in opposite direction.

Therefore

$$v_{CC}(t) = -250.0 \text{ V} \quad \text{for } 7.025 \times 10^{-5} \leq t < 0.5 \times 10^{-3} \text{ sec.}$$

$$i_{CC}(t) = 0.0 \text{ A} \quad \text{for } 7.025 \times 10^{-5} \leq t < 0.5 \times 10^{-3} \text{ sec.}$$

The oscillation current ($= i_{CC}(t)$) is observed on the graphs I_{RT1} , I_{LC} , I_{CC} .

$$i_{RT1}(t) = -i_{CC}(t) = 111.8 \sin(4.472 \times 10^4 t) \quad \text{for } 0.0 \leq t < 7.025 \times 10^{-5} \text{ sec.}$$

$$i_{RT1}(t) = i_{RM}(t) = -606.5 e^{-22.8t} + 606.5 e^{-2.19t} \quad \text{for } 7.025 \times 10^{-5} \leq t < 0.5 \times 10^{-3} \text{ sec.}$$

Also

$$i_{LC}(t) = i_{CC}(t) = -111.8 \sin(4.472 \times 10^4 t) \quad \text{for } 0.0 \leq t < 7.025 \times 10^{-5} \text{ sec.}$$

$$i_{LC}(t) = 0.0 \text{ A} \quad \text{for } 7.025 \times 10^{-5} \leq t < 0.5 \times 10^{-3} \text{ sec.}$$

I_{RT2} is zero for this time interval, since $T2$ is not triggered and not conducting.

$$i_{RT2}(t) = 0.0 \text{ A} \quad \text{for } 0.0 \leq t < 0.5 \times 10^{-3} \text{ sec.}$$

V_{RT1} is zero, as T1 is in conduction state throughout this interval.

$$v_{RT1}(t) = 0.0 \text{ V} \quad \text{for } 0.0 \leq t < 0.5 \times 10^{-3} \text{ sec.}$$

and as a result of this no current through its snubber circuit (IRS1) and no voltage across the suppressor capacitance (VCS1), should be observed as seen on the graphs of V_{RT1} , IRS1, VCS1.

V_{RDF} is equal to V_M , because Df is reverse biased, that is ILS is zero as explained before

$$V_{RDF}(t) = v_M(t) = 250.0 \text{ V} \quad \text{for } 0.0 \leq t < 0.5 \times 10^{-3} \text{ sec.}$$

V_{RT2} is equal to the negative of VCC, because

$V_{RT2} = v_o - v_g$ and $v_o = 0.0 \text{ V}$ as T1 is conducting, so $V_{RT2} = -v_g = -V_{CC}$

$$V_{RT2}(t) = -250 \cos(4.472 \times 10^4 t)$$

$$\text{for } 0.0 \leq t < 7.025 \times 10^{-5} \text{ sec}$$

$$V_{RT2}(t) = 250.0 \text{ V} \quad \text{for } 7.025 \times 10^{-5} \leq t < 0.5 \times 10^{-3} \text{ sec.}$$

V_{RDC} is equal to zero for the time, it carries the oscillation current ($= i_{OC}(t)$), and than is equal to V_{RT2} , i.e.

$$V_{RDC}(t) = 0.0 \quad \text{for } 0.0 \leq t < 7.025 \times 10^{-5} \text{ sec.}$$

$$V_{RDC}(t) = V_{RT2}(t) = 250.0 \text{ V} \quad \text{for } 7.025 \times 10^{-5} \leq t < 0.5 \times 10^{-3} \text{ sec.}$$

$SPEED = \frac{60}{2\pi} \times \frac{E}{K\phi}$, which means the SPEED (in rpm) is directly proportional to back emf voltage E for all time intervals.

$TORQUE = K\phi \times I_{RM}$, which means the mechanical torque (in newton-meters) is directly proportional to the armature current I_{RM} , for all time intervals.

5-2-2. THE FIRST OFF-TIME INTERVAL ($0.5 \times 10^{-3} \ll t < 1.0 \times 10^{-3}$ sec.)

T1 is not triggered and T2 is triggered. T2 starts conduction, as it is forward biased ($v_{RT2}(0.5 \times 10^{-3}) = 250.0$ V) and triggered.

As soon as T2 starts conduction ($t = 0.5 \times 10^{-3}$), VCC appears across T1, and reverse biases. Therefore T1 ceases conduction, and the path Cc-T2 takes over the armature current IRM. Voltage at node 6

$$v_6(t) = -250.0 + 1/Cc \int_{0.5 \times 10^{-3}}^t i_{RM}(t) dt$$

As mentioned before, the power of the dc machine is 10 H.P., for which the armature current can not exceed 30 A, when fed from a 250.0 V supply. On the other hand, even if IRM is as high as 50 A, $v_6(t)$ stays negative for longer than the turn-off time of T1 (typically 50-100 μ sec.).

The equation for the outer loop which is made out of Cc, T2, Db, CM, LM, RM, and supply voltage VA, will be:

$$v_{CC}(t) + e(t) + v_{LM}(t) + v_{RM}(t) = v_A(t)$$

$$-250.0 + 1/Cc \int_{0.5 \times 10^{-3}}^t i_{RM}(t) dt + e(0.5 \times 10^{-3}) + 1/CM \int_{0.5 \times 10^{-3}}^t i_{RM}(t) dt + LM di_{RM}(t)/dt + RM i_{RM}(t) = 250.0$$

The characteristic equation will be:

$$RM + LM s + \frac{1}{CM s} + \frac{1}{Cc s} = 0 \quad , \text{ which gives}$$

$$s^2 + 25 s + 2.0 \times 10^{-7} = 0$$

Its roots are:

$$s_1 \approx -25.0 \quad s_2 \approx 0.0$$

$$i_{RM}(t) = A_1 e^{-25 t} + A_2$$

with the initial conditions

$$i_{RM}(0.5 \times 10^{-3}) = 6.21 \text{ A} \quad \left. \frac{di_{RM}(t)}{dt} \right|_{t=0.5 \times 10^{-3}} = 2.4845 \times 10^4 \text{ A/sec.}$$

gives $A_1 = -1014.5$ $A_2 = 1008.1$

$$i_{RM}(t) = -1014.5 e^{-25 t} + 1008.1$$

$$v_M(t) = v_A(t) - v_C(t) = 250.0 - (-250.0 + 1/Cc \int_{0.5 \times 10^{-3}}^t (-1014.5 e^{-25t} + 1008.1) dt)$$

$$v_M(t) = 500 - 10^5 (40.58 e^{-25t} + 1008.1t) \Big|_{0.5 \times 10^{-3}}^t$$

$$v_M(t) = 4.05849 \times 10^6 - 4.058 \times 10^6 e^{-25t} - 1.0081 \times 10^6 t$$

But it is quite difficult to find the time when v_M drops to zero ($v_M(t)=0$), that is why $v_M(0.7 \times 10^{-3})$ should be evaluated and compared with the value obtained by the simulation

$$v_M(0.7 \times 10^{-3}) = 330 \text{ V which is very close to the value seen}$$

on the graph of v_M . But

$$i_{RM}(1.0 \times 10^{-3}) = 18.6 \text{ A which is quite high than the}$$

value seen on the graph of i_{RM} .

The back emf voltage E can be calculated as :

$$e(t) = e(0.5 \times 10^{-3}) + 1/CM \int_{0.5 \times 10^{-3}}^t i_{RM}(t) dt$$

But, more simply, $e(1.0 \times 10^{-3}) = e(0.5 \times 10^{-3}) + \Delta Q/CM$, where ΔQ is the total charge carried by $i_{RM}(t)$ for $0.5 \times 10^{-3} \leq t < 1.0 \times 10^{-3}$ sec.

$$\Delta Q = Cc \Delta V = 10^{-5} \times 500 = 5.0 \times 10^{-3} \text{ C}$$

$$e(1.0 \times 10^{-3}) = 1.021 \times 10^{-3} + 5.0 \times 10^{-3} = 6.021 \times 10^{-3} \text{ V}$$

which is exactly equal to the value seen on the graph of E .

Current through the freewheeling diode D_f and inductor L_s is zero for $v_M(t) > 0.0$. As shown on the graph of V_M , V_M drops to zero value, at the end of this time interval, and a spike of 3.5 A is observed on ILS, which is equal to the freewheeling diode current.

The supply current I_A is the sum of the armature current I_{RM} and freewheeling diode current I_{LS} i.e. $i_A(t) = i_{RM}(t) + i_{LS}(t)$. I_{LS} is zero up to 1.0×10^{-3} sec, therefore

$$i_A(t) = i_{RM}(t) \quad \text{for } 0.5 \times 10^{-3} \leq t < 1.0 \times 10^{-3} \text{ sec. ,}$$

but $i_{LS}(1.0 \times 10^{-3}) = -3.5 \text{ A}$, so

$$i_A(1.0 \times 10^{-3}) = 11.5 - 3.5 = 8.0 \text{ A} , \text{ as seen on the graph}$$

of I_A .

As mentioned above, as soon as T_2 starts conduction at $t = 0.5 \times 10^{-3}$, V_{CC} , which is negative for at least the turn-off time of T_1 for any armature current, appears across T_1 and reverse biases it. Therefore T_1 ceases conduction till the time it is triggered again ($t = 1.0 \times 10^{-3}$ sec.), I_{RT1} is therefore zero throughout the interval, i.e. ,

$$i_{RT1}(t) = 0.0 \quad \text{for } 0.5 \times 10^{-3} \leq t < 1.0 \times 10^{-3} \text{ sec.}$$

C_c and T_2 take over the armature current, from the time T_2 starts conduction, to the time V_M drops to zero. In this time interval, V_M drops to zero at $t = 1.0 \times 10^{-3}$ sec.. Therefore

$i_{CC}(t) = i_{RT2}(t) = i_{RM}(t)$ for $0.5 \times 10^{-3} \leq t < 1.0 \times 10^{-3}$ sec.,
as seen on the graphs ICC and IRT2 .

The current through Lc and Dc is zero, because oscillation occurs on the beginnings of on time intervals, that is T1 just starts conduction.

$$i_{LC}(t) = 0.0 \quad \text{for} \quad 0.5 \times 10^{-3} \leq t < 1.0 \times 10^{-3} \text{ sec. .}$$

As soon as T2 starts conduction at $t = 0.5 \times 10^{-3}$ sec.

$$v_{RT2}(t) = v_{\delta}(t) = v_{CC}(t) = -250.0 + 1/Cc \int_{0.5 \times 10^{-3}}^t i_{RM}(t) dt .$$

That is VRT2 jumps to -250.0 (on the graph of VRT2 it is seen as -200.0 V) than rises to +250.0 V as VCC rises to +250.0 V by the end of this off time interval.

The function of the snubber circuit (RS1-CS1) is to limit the change of voltage across T1. When T2 starts conduction at $t = 0.5 \times 10^{-3}$ the voltage of node δ , is expected to jump -250.0 V. But at the same time

$$i_{RS1}(t) = \frac{-250.0}{RS1} e^{-(t-1.0 \times 10^{-3})/RS1 CS1} \\ = -7.575 e^{-3.03 \times 10^5 (t-1.0 \times 10^{-3})}$$

Then VRT1 rises almost linearly, a constant current through the snubber circuit, is observed (see graph IRS1) and the voltage across CS1 follows VRT1 .

As explained before, current through Df and inductor Ls is zero in this time interval, i.e. Df is reversed biased,

and there is no voltage drop across the inductor L_s ,

$$v_{RDF}(t) = v_A(t) \quad \text{for } 0.5 \times 10^{-3} \leq t < 1.0 \times 10^{-3} \text{ sec.}$$

T2 is conducting throughout this interval, therefore,

$$v_{RT2}(t) = 0.0 \text{ V} \quad \text{for } 0.5 \times 10^{-3} \leq t < 1.0 \times 10^{-3} \text{ sec.}$$

As ILC (and VLC) is zero throughout the interval

$$v_{RDC}(t) = v_{RT2}(t) \quad \text{for } 0.5 \times 10^{-3} \leq t < 1.0 \times 10^{-3} \text{ sec.}$$

5-2-3. THE SECOND ON-TIME INTERVAL ($1.0 \times 10^{-3} \leq t < 1.5 \times 10^{-3}$ sec.)

T1 is triggered, and T2 is not triggered. T1 starts conduction, since it is triggered and forward biased ($v_{RT1}(1.0 \times 10^{-3}) = 250.0 \text{ V}$), and it stays in the conduction state throughout this interval.

The supply voltage (250.0 V) appears across the armature terminals, i.e.

$$v_M(t) = 250.0 \text{ V} \quad \text{for } 1.0 \times 10^{-3} \leq t < 1.5 \times 10^{-3} \text{ sec.}$$

i_{RM} rises as the sum of the two exponential functions from $i_{RM}(1.0 \times 10^{-3}) = 11.5 \text{ A}$

$$i_{RM}(t) = A_1 e^{-22.3 t} + A_2 e^{-2.19 t} \quad \text{with the initial}$$

conditions

$$i_{RM}(1.0 \times 10^{-3}) = 11.5 \text{ A} \quad \left. \frac{di_{RM}(t)}{dt} \right|_{t=1.0 \times 10^{-3}} = 1.2212 \times 10^4 \text{ A/sec.}$$

give $A_1 = -007.75$ $A_2 = 005.25$ so,

$$i_{RM}(t) = -007.75 e^{-22.3t} + 005.25 e^{-2.19t} \quad \text{for } 1.0 \times 10^{-3} \leq t < 1.5 \times 10^{-3} \text{ sec.}$$

by the end of this time interval i_{RM} reaches the value of 16 A
 $i_{RM}(1.5 \times 10^{-3}) = 15.98$ A, which is approximately equal to the
 value seen on the graph of i_{RM} ..

E is the voltage on CM, and it can be calculated as:

$$e(t) = e(1.0 \times 10^{-3}) + 1/CM \int_{1.5 \times 10^{-3}}^t i_{RM}(t) dt$$

$$6.521 \times 10^{-3} + \frac{1}{1.5 \times 10^{-3}} \int_{1.5 \times 10^{-3}}^t (-607.75 e^{-22.8t} + 605.28 e^{-2.19t}) dt$$

$$e(t) = 249.71 + 20.05 e^{-22.8t} - 270.38 e^{-2.19t}$$

and by the end of this time interval E reaches to the value
 $e(1.5 \times 10^{-3}) = 1.5034 \times 10^{-2}$ V, which is almost equal to the value
 obtained by the simulation.

The freewheeling diode current is obviously zero
 throughout the on time interval, as shown on the graph of i_{LS} ,
 since D_f is reverse biased ($v_M(t) = v_{RDF}(t) = 250.0$ V)

i_A is equal to the armature current i_{RM} , since the
 freewheeling diode current i_{RDF} (or i_{LS}) is zero

$$i_A(t) = i_{RM}(t) \quad \text{for} \quad 1.0 \times 10^{-3} \leq t < 1.5 \times 10^{-3} \text{ sec.}$$

When T_1 starts conduction at $t = 1.0 \times 10^{-3}$ sec, a sinusoidal
 oscillation occurs in the loop L_c, C_c, D_c , and T_1 for a half period
 ($= \pi \sqrt{L_c C_c}$). Therefore,

$$i_{R11}(t) = -i_{CC}(t) + i_{RM}(t)$$

$$= 111.8 \sin(4.47 \times 10^4 (t - 1.0 \times 10^{-3})) + 11.5$$

$$\text{for} \quad 1.0 \times 10^{-3} \leq t < 1.0702 \times 10^{-3} \text{ sec.}$$

$$i_{RT1}(t) = i_{RM}(t) = -607.75 e^{-22.5t} + 605.28 e^{-2.19t}$$

$$\text{for } 1.0702 \times 10^{-3} \leq t < 1.5 \times 10^{-3} \text{ sec.}$$

Also this oscillation current starting at $t = 1.0 \times 10^{-3}$ sec., is observed on the graphs of ILC and ICC .

$$i_{CC}(t) = i_{LC}(t) = -111.8 \sin(4.47 \times 10^4 (t - 1 \times 10^{-3}))$$

$$\text{for } 1.0 \times 10^{-3} \leq t < 1.0702 \times 10^{-3} \text{ sec.}$$

$$i_{CC}(t) = i_{LC}(t) = 0.0 \quad \text{for } 1.0702 \times 10^{-3} \leq t < 1.5 \times 10^{-3} \text{ sec.}$$

T2 is not conducting (as it is not triggered), therefore i_{RT2} is zero for the on time interval.

$$i_{RT2}(t) = 0.0 \quad \text{for } 1.0 \times 10^{-3} \leq t < 1.5 \times 10^{-3} \text{ sec.}$$

v_{RT1} is zero, as T1 is conducting, therefore no current through the snubber circuit and no voltage across CS1 is observed (see the graphs, v_{RT2} , i_{CS1} and v_{CS1}).

Df is reversed biased and v_{RDF} is equal to the armature voltage v_M throughout this interval.

$$v_{RDF}(t) = v_M(t) \quad \text{for } 1.0 \times 10^{-3} \leq t < 1.5 \times 10^{-3} \text{ sec.}$$

A sinusoidal oscillation occurs in the loop L_c , C_c , D_c , and T1 for a half period ($= \pi \sqrt{L_c C_c}$), when T1 starts conduction at $t = 1.0 \times 10^{-3}$ sec. Therefore

$$v_{CC}(t) = 250 \cos(4.47 \times 10^4 (t - 1.0 \times 10^{-3}))$$

$$\text{for } 1.0 \times 10^{-3} \leq t < 1.0702 \times 10^{-3} \text{ sec.}$$

$$v_{CC}(t) = -250.0 \text{ V} \quad \text{for } 1.0702 \times 10^{-3} \leq t < 1.5 \times 10^{-3} \text{ sec.}$$

$$v_{RT2}(t) = v_o(t) - v_g(t) = v_o(t) - v_{CC}(t) = -v_{CC}(t)$$

as T1 conducting $v_o(t) = 0.0$ V, therefore

$$v_{RT2}(t) = -250.0 \cos(4.47 \times 10^4 (t - 1.0 \times 10^{-3}))$$

for $1.0 \times 10^{-3} \leq t < 1.0702 \times 10^{-3}$ sec.

$$v_{RT2}(t) = 250.0 \text{ V for } 1.0702 \times 10^{-3} \leq t < 1.5 \times 10^{-3} \text{ sec.}$$

The diode Dc conducts for $1.0 \times 10^{-3} \leq t < 1.0702 \times 10^{-3}$ sec., as it carries the oscillation current, while $v_{RDC}(t) = 0.0$ then

$$v_{RDC}(t) = v_{RT2}(t) = 250.0 \text{ V for } 1.0702 \times 10^{-3} \leq t < 1.5 \times 10^{-3} \text{ sec.}$$

since $v_{LC}(t) = 0.0$ V.

5-2-4. THE SECOND OFF TIME INTERVAL ($1.5 \times 10^{-3} \leq t < 2.0 \times 10^{-3}$ sec.)

T1 is not triggered, and T2 is triggered. T2 starts conduction, as as it is triggered and forward biased

$$(v_{RT2}(1.5 \times 10^{-3}) = 250.0 \text{ V})$$

As soon as, T2 starts conduction ($t = 1.5 \times 10^{-3}$ sec.), VCC appears across T1, and reverse biases it. Therefore, T1 ceases conduction, and Cc-T2 takes over the armature current i_{RM} . From the IA graph, it is seen that, $i_{RM}(1.5 \times 10^{-3}) = 18$ A and

$$i_{RM}(t) = 18.0 \text{ A for } 1.7 \times 10^{-3} \leq t < 2.0 \times 10^{-3} \text{ sec.}$$

If we assume a constant value for i_{RM} such that

$$i_{RM}(t) = 18.0 \text{ for } 1.5 \times 10^{-3} \leq t < 1.7 \times 10^{-3} \text{ sec.}$$

$v_M(t)$ can be obtained as

$$\begin{aligned} v_M(t) &= v_A(t) - v_o(t) = 250.0 - (-250.0 + 1/Cc \int_{1.5 \times 10^{-3}}^t i_{RM}(t) dt) \\ &= 500.0 - 10^5 (18 t) \Big|_{1.5 \times 10^{-3}}^t \end{aligned}$$

$$v_M(t) = 3200.0 - 18 \times 10^5 t \quad \text{for } 1.5 \times 10^{-3} \leq t < 1.778 \times 10^{-3} \text{ sec.}$$

$$v_M(t) = 0.0 \text{ V} \quad \text{for } 1.778 \times 10^{-3} \leq t < 2.0 \times 10^{-3} \text{ sec.}$$

But, with reference to graph of VM, $v_M(1.5 \times 10^{-3}) = 320.0 \text{ V}$, which must have been 500.0 V and VM drops to zero at $t = 1.7 \times 10^{-3} \text{ sec}$.

This error is probably due to the high value of the time increment h, or to the fact that only one point is printed out of every five points and the peak can thus be missed .

The back emf voltage E, can be calculated as :

$$e(t) = e(1.5 \times 10^{-3}) + 1/CM \int_{1.5 \times 10^{-3}}^t 19 dt$$

$$= 1.3054 \times 10^{-2} + (19t) \Big|_{1.5 \times 10^{-3}}^t$$

$$e(t) = -1.544 \times 10^{-2} + 19t \quad \text{for } 1.5 \times 10^{-3} \leq t < 2.0 \times 10^{-3}$$

so E reaches the value $e(2.0 \times 10^{-3})$ by the end of this time interval, $e(2.0 \times 10^{-3}) = 2.25 \times 10^{-2} \text{ V}$, which is almost equal to the value obtained by the simulation.

After VM drops to zero value, the freewheeling diode takes over the armature current, T2 ceases conduction. As seen on the graph of ILS (= IRDF).

$$i_{LS}(t) = 0.0 \text{ A} \quad \text{for } 1.5 \times 10^{-3} \leq t < 1.778 \times 10^{-3} \text{ sec.}$$

$$i_{LS}(t) = -15 \text{ A} \quad \text{for } 1.778 \times 10^{-3} \leq t < 2.0 \times 10^{-3} \text{ sec.}$$

Similarly the supply current IA is equal to the armature current for the time $v_M(t) > 0.0$, then IA drops to 1.5 A .

$$i_A(t) = i_{RM}(t) \quad \text{for } 1.5 \times 10^{-3} \leq t < 1.778 \times 10^{-3} \text{ sec.}$$

$$i_A(t) = 1.5 \text{ A} \quad \text{for } 1.778 \times 10^{-3} \leq t < 2.0 \times 10^{-3} \text{ sec.}$$

There is a small error in the simulation, because after VM drops to zero, all the armature current should be transferred to freewheeling diode path and supply current should drop to zero.

T1 is not conducting in this time interval, and

$$i_{RT1}(t) = 0.0 \text{ A} \quad \text{for } 1.5 \times 10^{-3} \leq t < 2.0 \times 10^{-3} \text{ sec}$$

T2 and Cc, carries the armature current before Df takes over it, therefore

$$i_{RT2}(t) = i_{CC}(t) = i_{RM}(t) \quad \text{for } 1.5 \times 10^{-3} \leq t < 1.728 \times 10^{-3} \text{ sec.}$$

$$i_{RT2}(t) = i_{CC}(t) = 0.0 \text{ A} \quad \text{for } 1.778 \times 10^{-3} \leq t < 2.0 \times 10^{-3} \text{ sec.}$$

This can be observed on the graphs of IRT2 and ICC.

There is no current through Lc, i.e. ,

$$i_{LC}(t) = 0.0 \text{ A} \quad \text{for } 1.5 \times 10^{-3} \leq t < 2.0 \times 10^{-3} \text{ sec.}$$

$$v_{RT1}(t) = v_g(t) + v_{RT2}(t)$$

for $1.5 \times 10^{-3} \leq t \leq \text{time T2 ceases conduction } (1.045 \times 10^{-3} \text{ sec})$

But, $v_{RT2}(t) = 0.0 \text{ V}$, since it is conducting

$$v_{RT1}(t) = v_g(t) = v_{CC}(t) = -250 + 1/Cc \int_{1.5 \times 10^{-3}}^t i_{RM}(t) dt$$

$$= -250 + 10^5 (19t) \Big|_{1.5 \times 10^{-3}}^t$$

$$= -3100 + 19 \times 10^5 t \quad \text{for } 1.5 \times 10^{-3} \leq t < 1.045 \times 10^{-3} \text{ sec.}$$

that is $v_{RT1}(1.5 \times 10^{-3}) = -250.0 \text{ V}$, but this jump is not observed on the graph v_{RT1} , it may be because of it happens in a very short time (almost discontinuous) then

$$v_{RT1}(t) = 250.0 \text{ V} \quad \text{for } 1.045 \times 10^{-3} \leq t < 2.0 \times 10^{-3} \text{ sec.}$$

(refer to the graph of v_{RT1})

During the time ($1.5 \times 10^{-3} \leq t < 1.645 \times 10^{-3}$ sec.) V_{RT1} rises from -250.0 V to $+250.0$ V, the current in the snubber circuit (I_{RS1}) is observed, and obviously V_{CS1} follows V_{RT1} (see the graphs I_{RS1} and V_{CS1}).

V_{RDF} is equal to V_M for the time $v_M(t) > 0.0$, as it is reversed biased, than drops to zero, as it takes over the armature current I_{RM} .

$$V_{RDF}(t) = v_M(t) \quad \text{for} \quad 1.5 \times 10^{-3} \leq t < 1.778 \times 10^{-3} \text{ sec.}$$

$$V_{RDF}(t) = 0.0 \text{ V} \quad \text{for} \quad 1.778 \times 10^{-3} \leq t < 2.0 \times 10^{-3} \text{ sec.}$$

V_{RT2} is zero while it is conducting for $1.5 \times 10^{-3} \leq t < 1.778 \times 10^{-3}$ sec, then drops below zero, as node o goes below zero value. Therefore,

$$V_{RT2}(t) = 0.0 \text{ V} \quad \text{for} \quad 1.5 \times 10^{-3} \leq t < 1.778 \times 10^{-3} \text{ sec.}$$

$$V_{RT2}(t) = -30.0 \text{ V} \quad \text{for} \quad 1.778 \times 10^{-3} \leq t < 2.0 \times 10^{-3} \text{ sec.}$$

V_{RDC} is equal to V_{RT2} throughout this interval, because $i_{LC}(t) = 0.0$ $v_{LC}(t) = Lc \frac{di_{LC}(t)}{dt} = 0.0$ V, therefore

$$V_{RDC}(t) = 0.0 \text{ V} \quad \text{for} \quad 1.5 \times 10^{-3} \leq t < 1.778 \times 10^{-3} \text{ sec.}$$

$$V_{RDC}(t) = -30.0 \text{ V} \quad \text{for} \quad 1.778 \times 10^{-3} \leq t < 2.0 \times 10^{-3} \text{ sec.}$$

V_{CC} charges from $v_{CC}(1.5 \times 10^{-3}) = -250.0$ V to $v_{CC}(1.645 \times 10^{-3}) =$

$$v_{CC}(t) = -250 + 1/Cc \int_{1.5 \times 10^{-3}}^t i_{RM}(t) dt \quad 250.0 \text{ V}$$

$$= -250.0 + 10^5 (19 t) \Big|_{1.5 \times 10^{-3}}^t$$

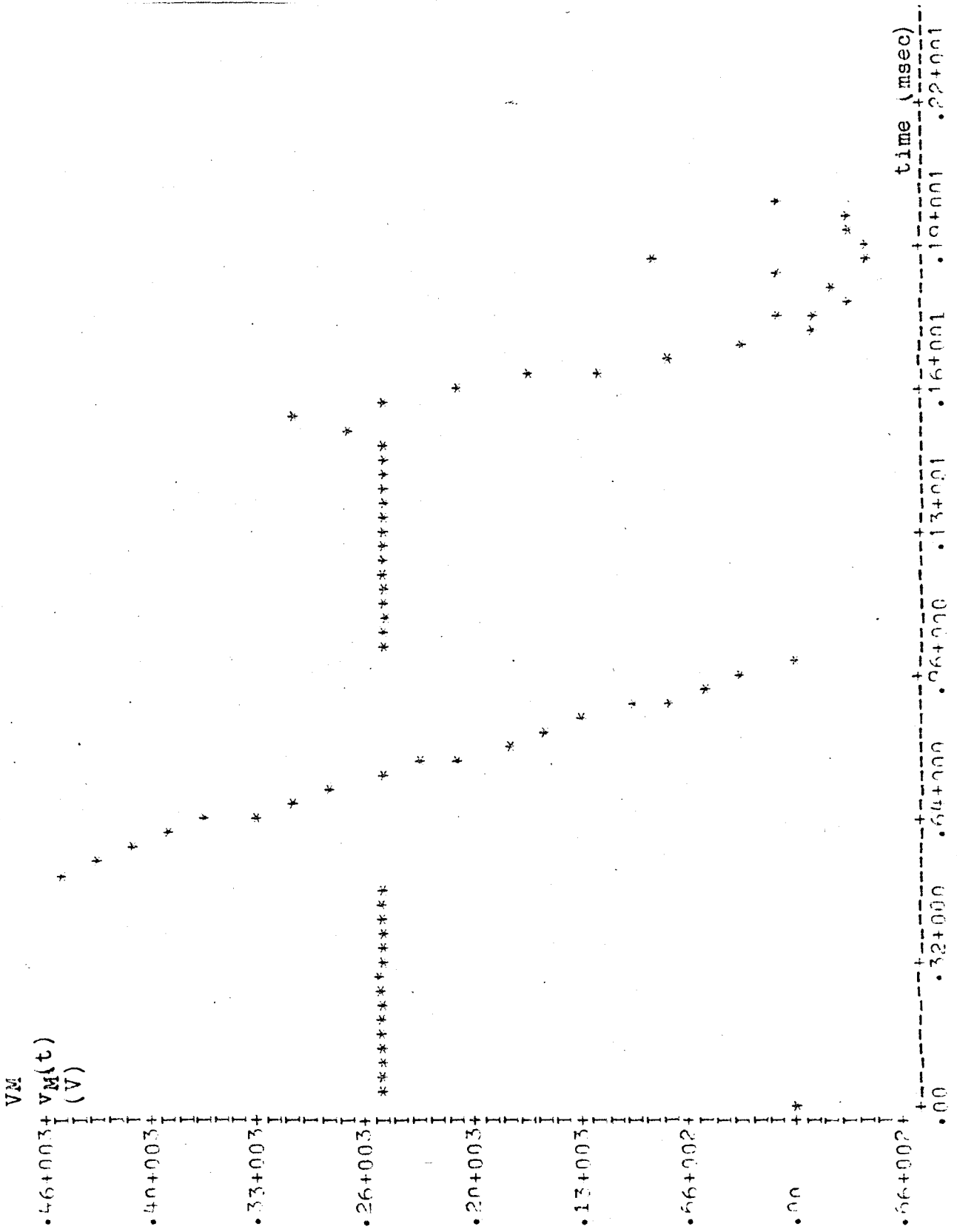
$$v_{CC}(t) = -3100 + 19 \times 10^5 t \quad \text{for} \quad 1.5 \times 10^{-3} \leq t < 1.645 \times 10^{-3} \text{ sec.}$$

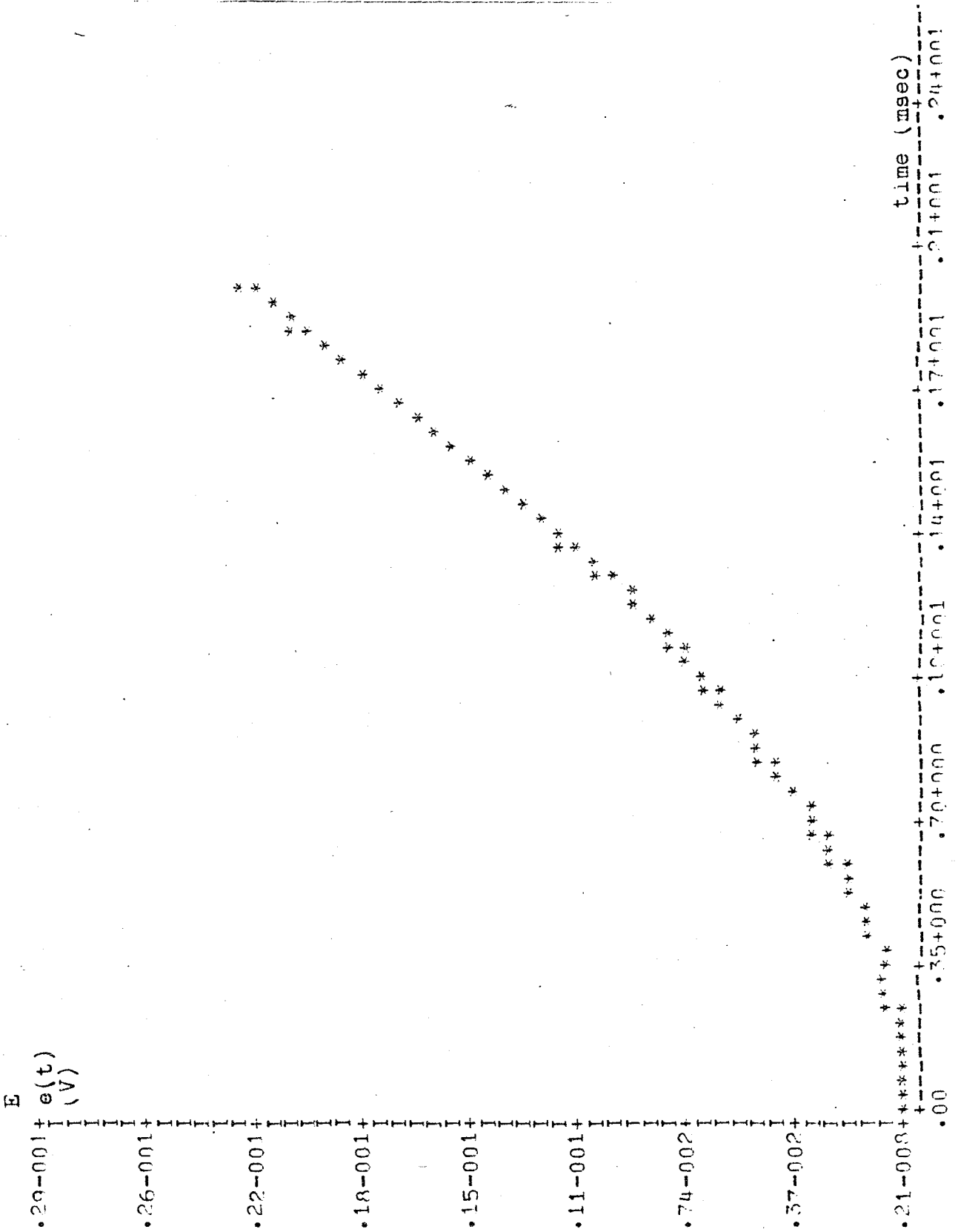
$$v_{CC}(t) = 250.0 \text{ V} \quad \text{for} \quad 1.645 \times 10^{-3} \leq t < 2.0 \times 10^{-3} \text{ sec.}$$

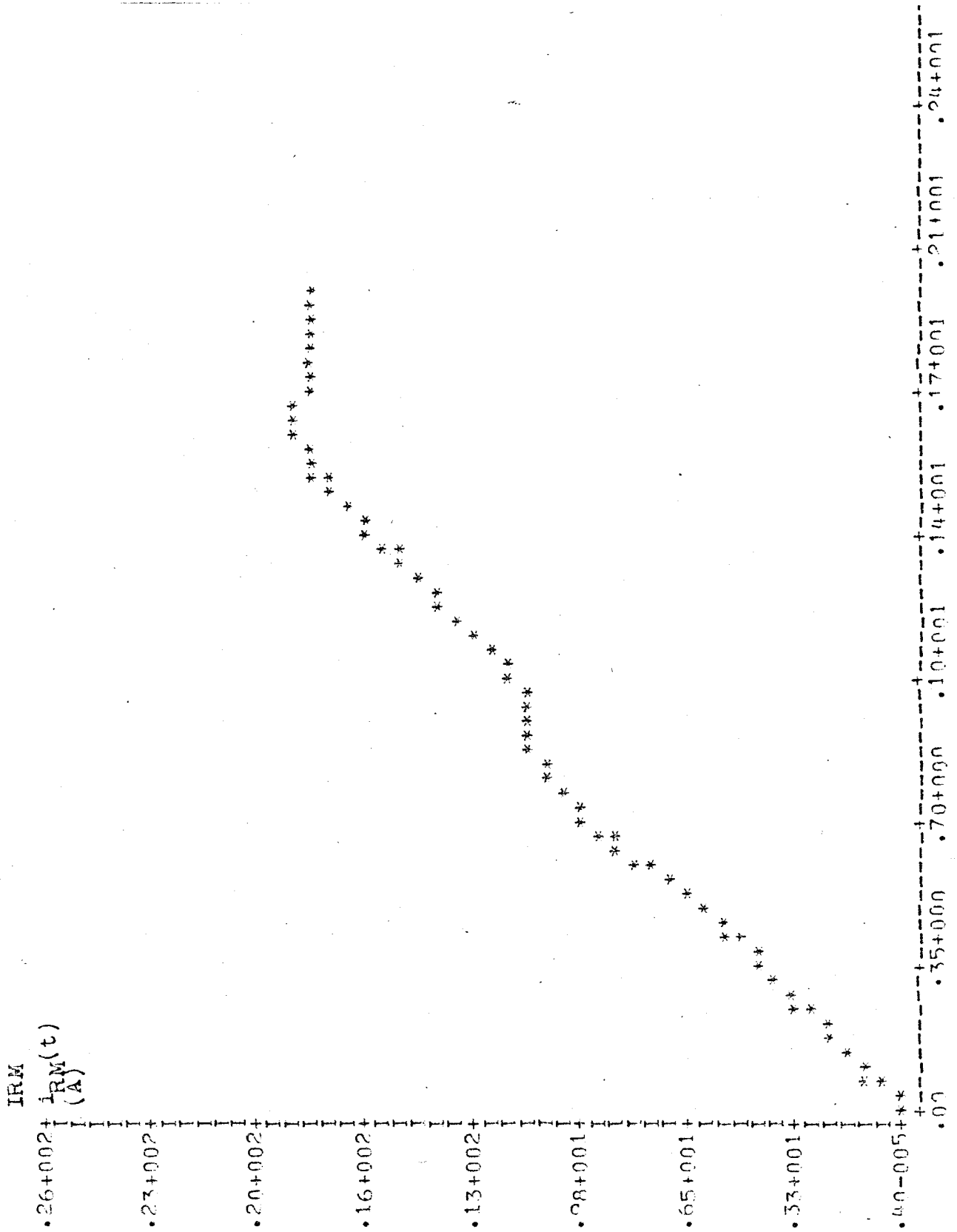
THE VALUES OF THE ELEMENTS ARE:

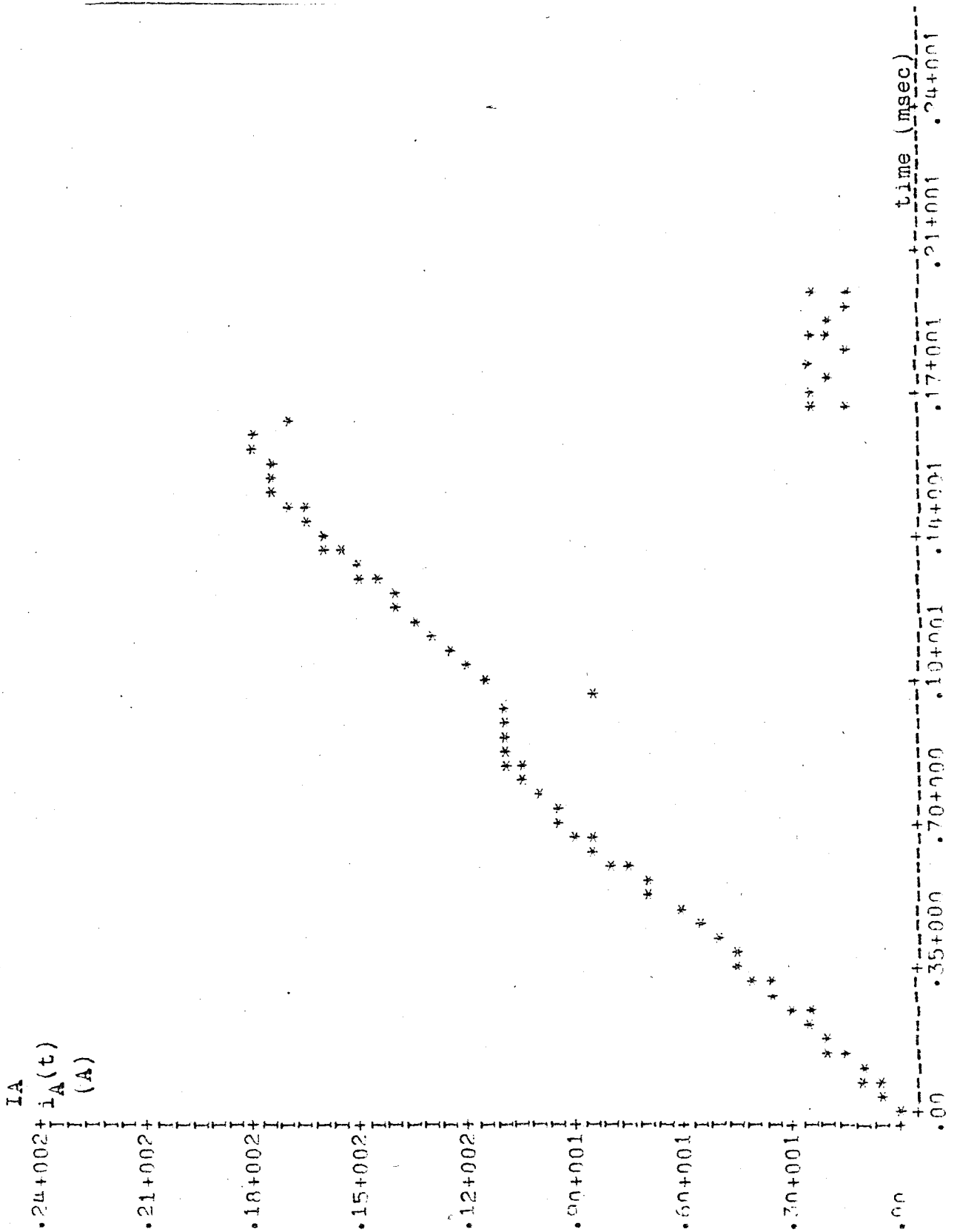
RS1= .33+002
RS2= .33+002
RS3= .33+002
CS1= .10-006
CS2= .10-006
CS3= .20-006
RM= .50+000
LM= .20-001
CC= .10-004
LC= .50-004
LS= .20-004
ROFF= .10+000
ROFF= .10+005
VA= .25+003
J1= .40+001
KPHI= .20+001
H= .50-005
TR= .50+000
F= .10+004
TMAX= .20-002
N=10

	VM	E	IRM	ILS	IANEW	IRT1	IRT2
MIN	-.72+002	.22-007	.89-002	-.17+002	.60+000	-.17-001	-.25-001
MAX	.42+003	.22-001	.18+002	.36+001	.18+002	.13+003	.17+002
	ICC	ILC	IRS1	VCS1	VRDF	VLS	VPT1
MIN	-.11+003	-.11+003	-.45-001	-.17+003	-.21+002	-.98+002	-.17+007
MAX	.17+002	.50+000	.37+001	.28+003	.43+003	.18+002	.28+003
	VPT2	VRDC	VCC	SPEED	TORQUE		
MIN	-.25+003	-.21+003	-.26+003	.11-006	.18-001		
MAX	.27+003	.28+003	.28+003	.11+000	.36+002		

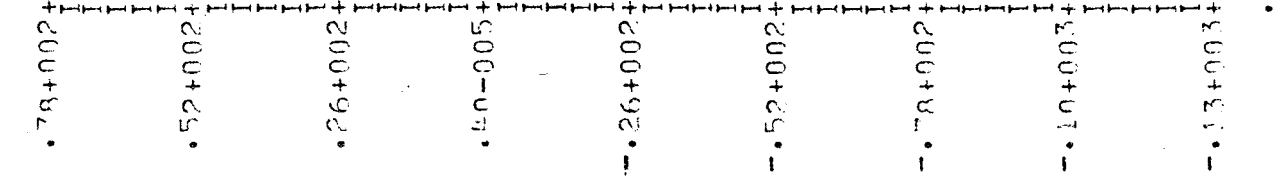






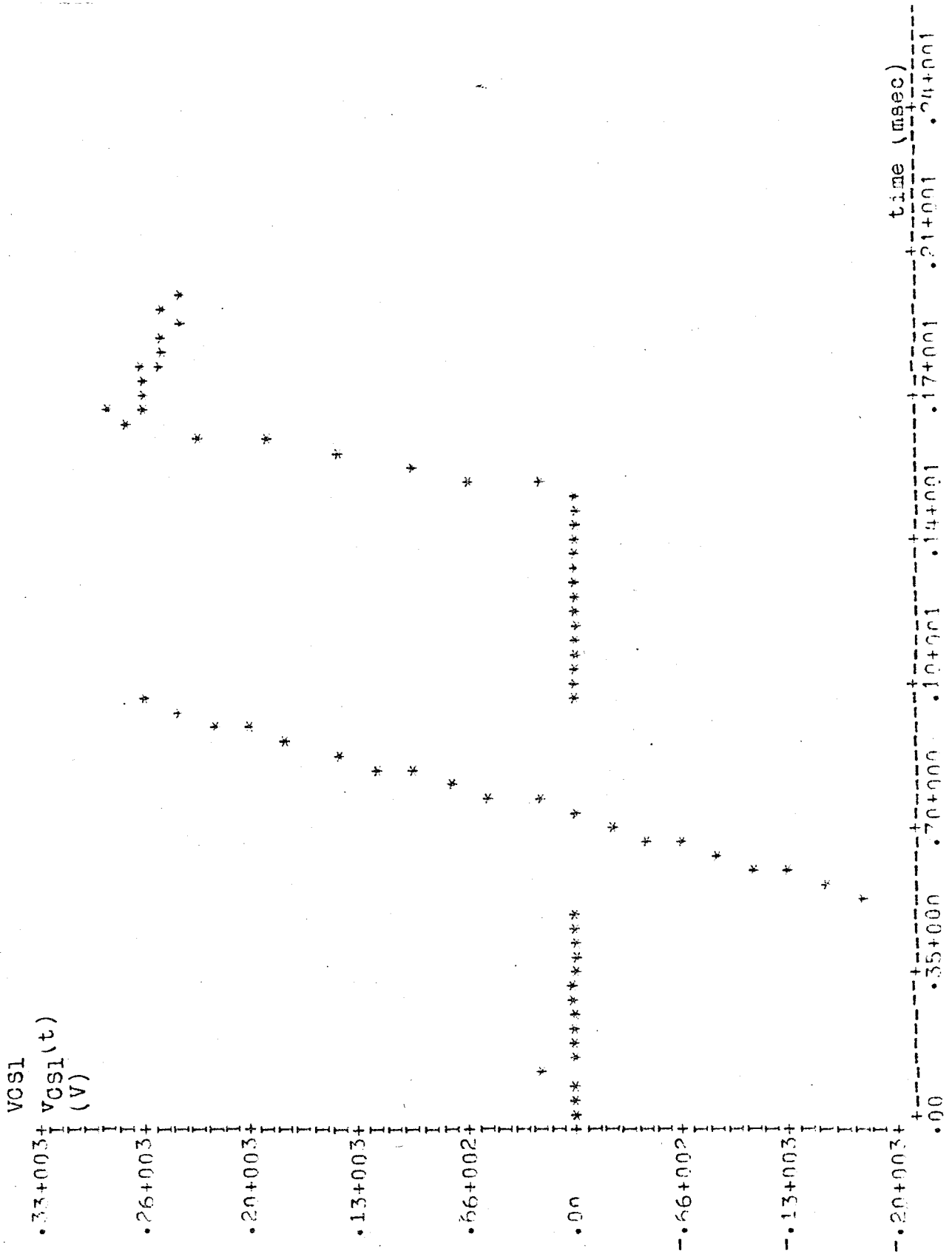


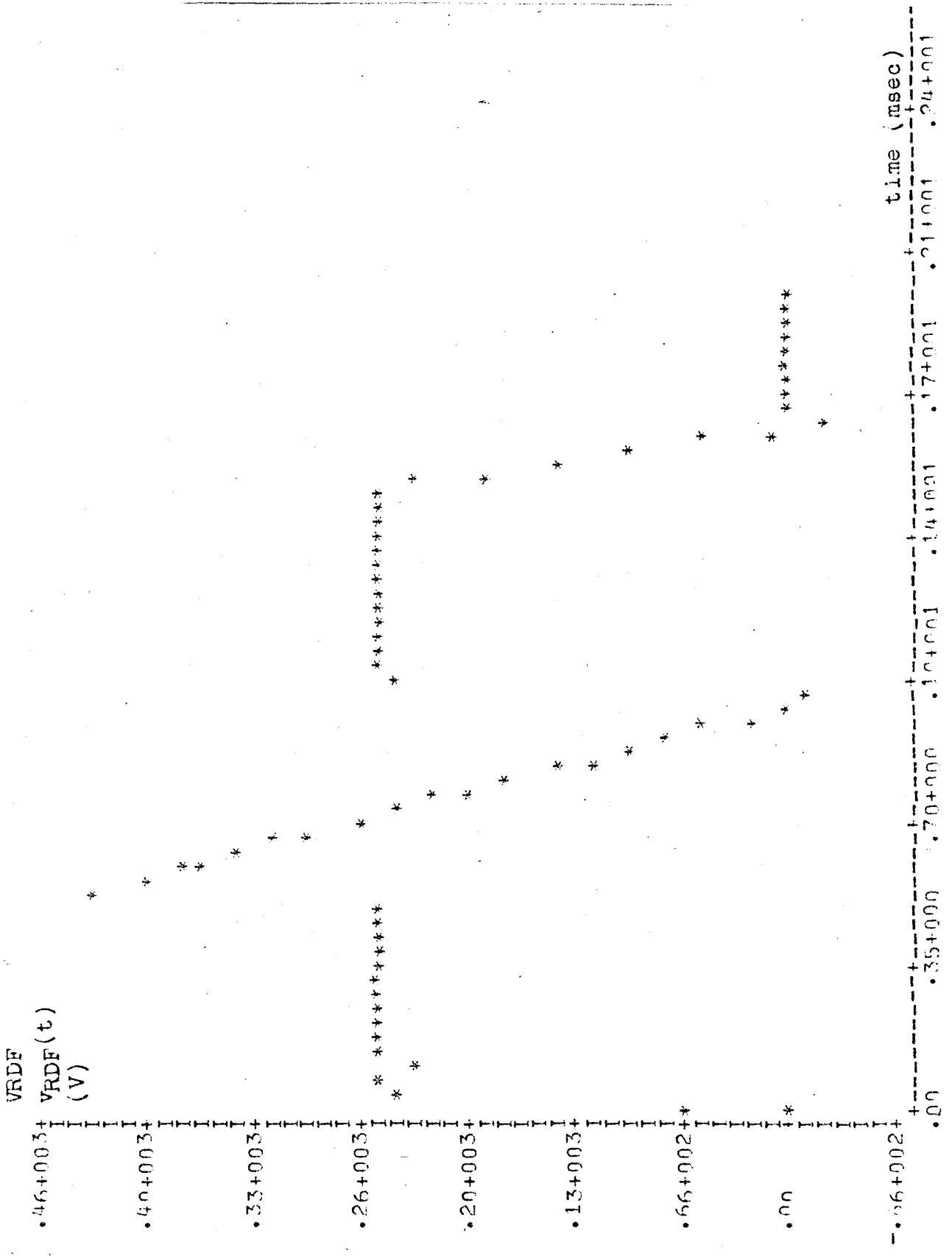
ICC
ICC (t)
(A)



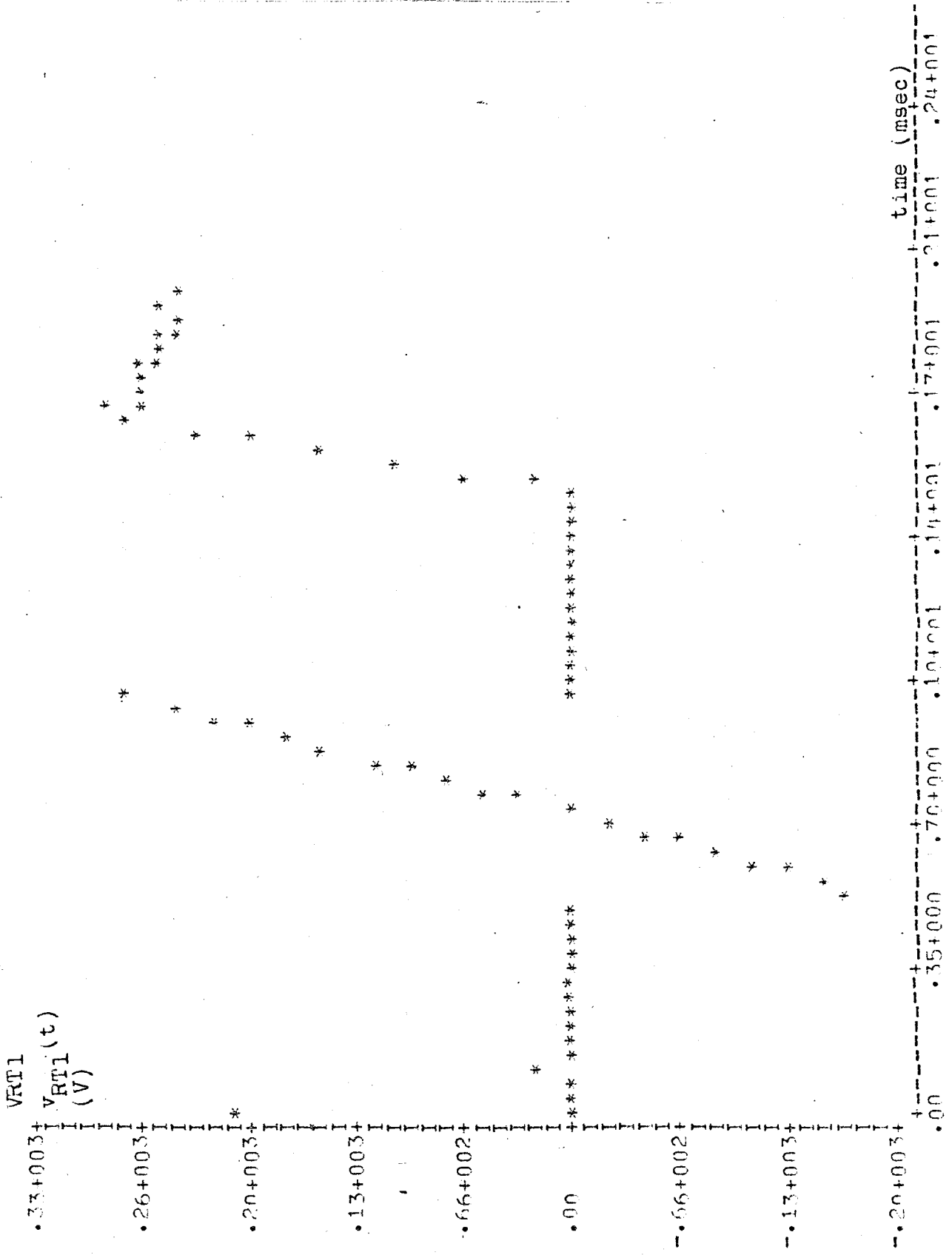
time (msec)

0.00 0.03 0.05 0.06 0.07 0.08 0.10 0.14 0.17 0.21 0.24



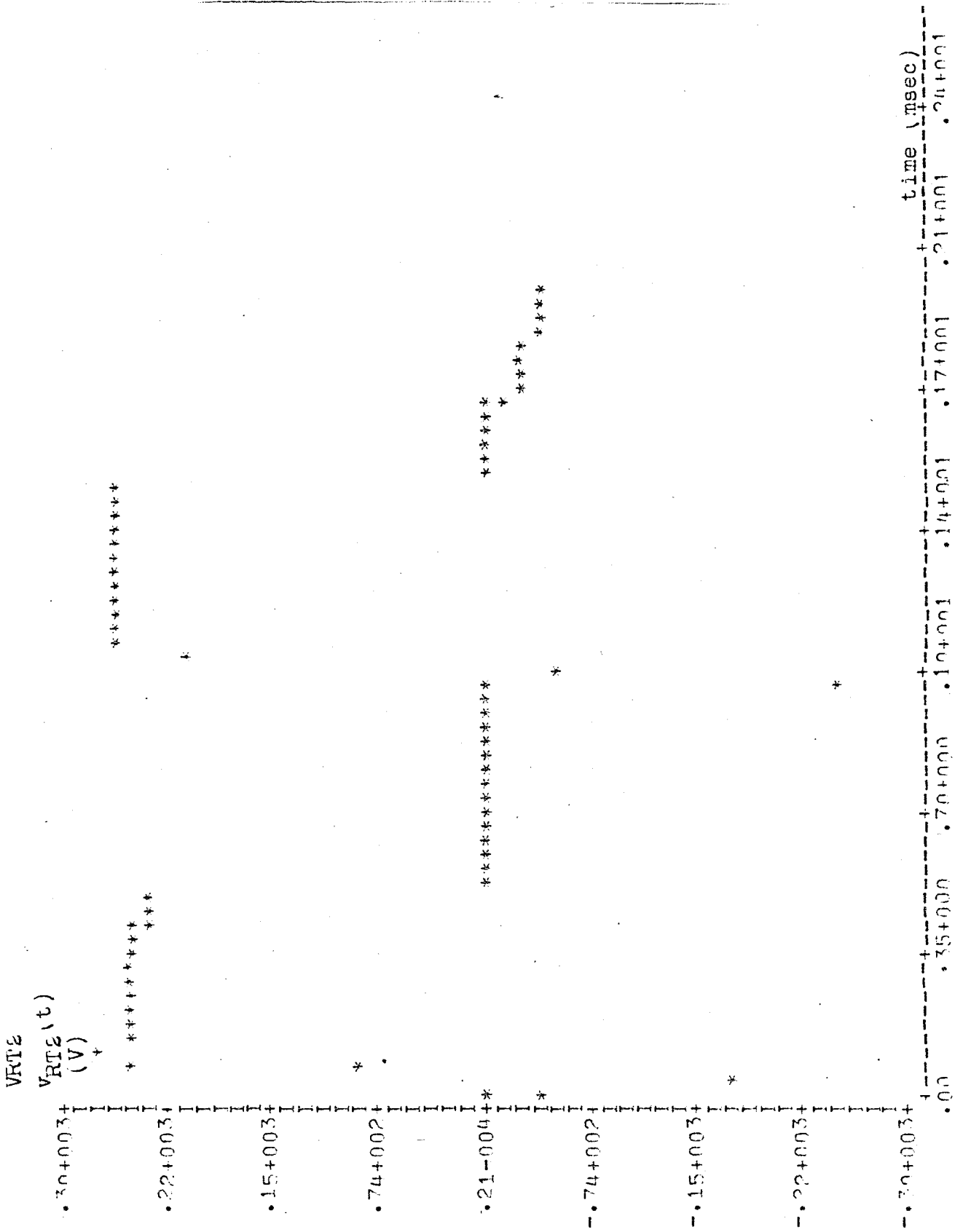


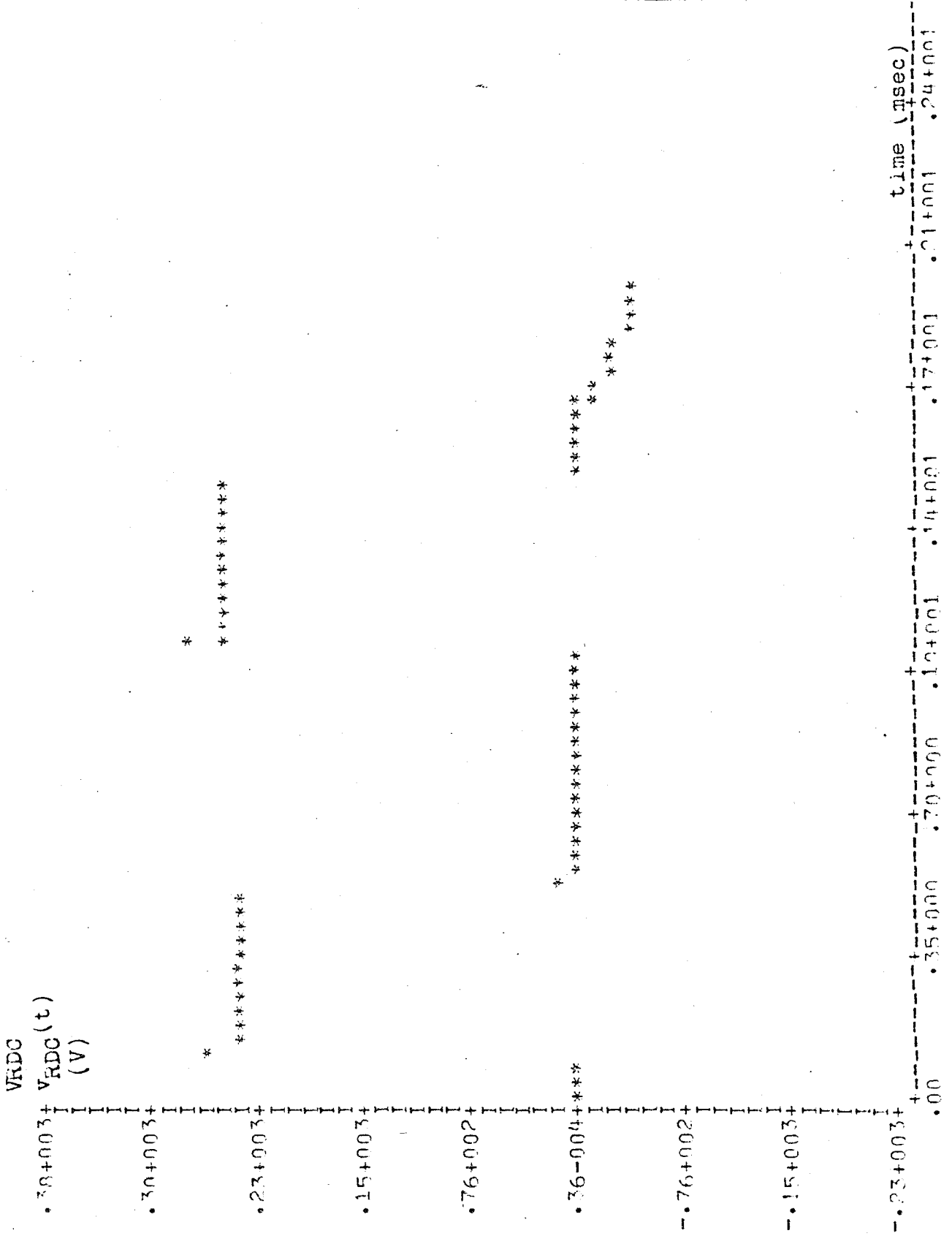
VRT1
VRT1 (t)
(V)

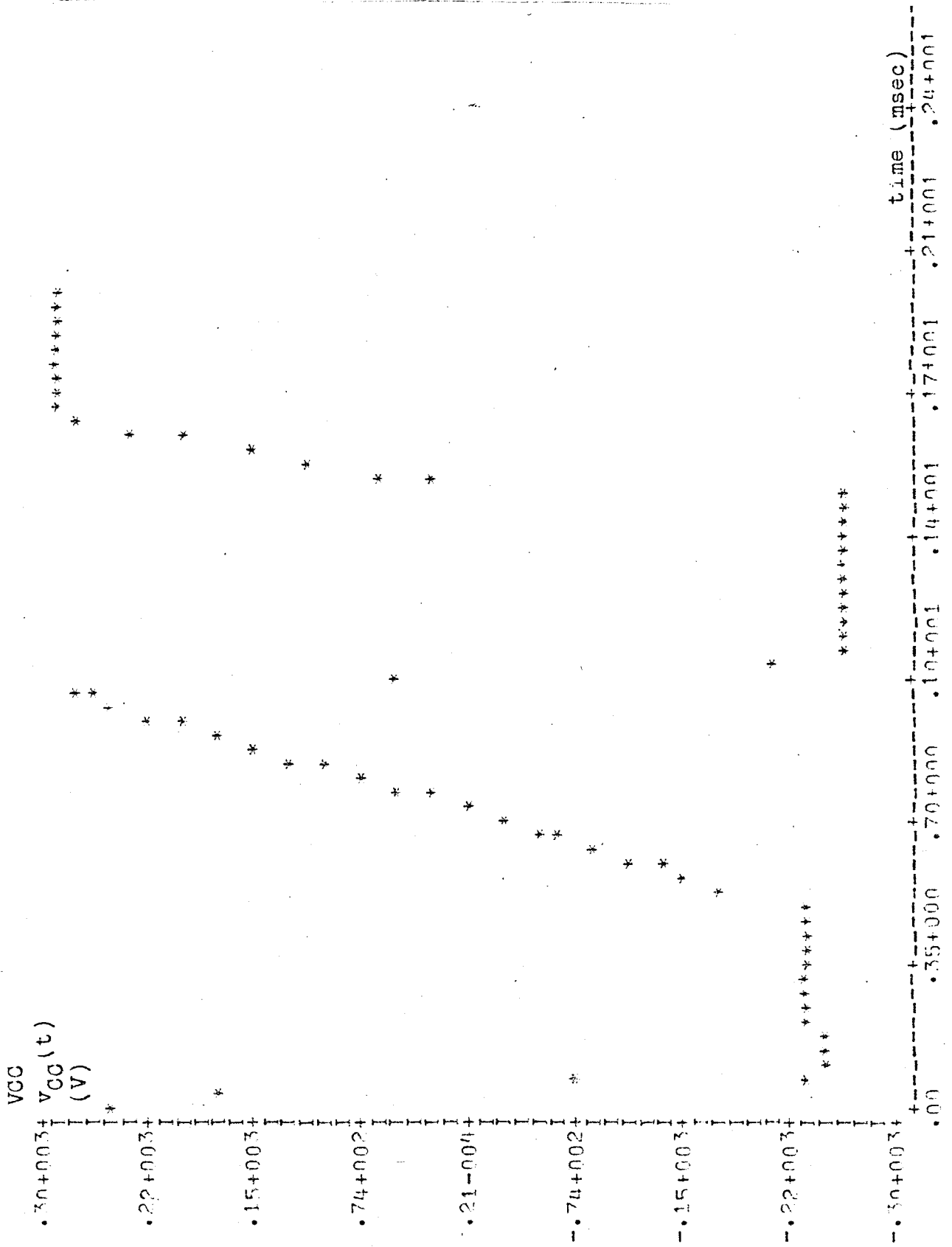


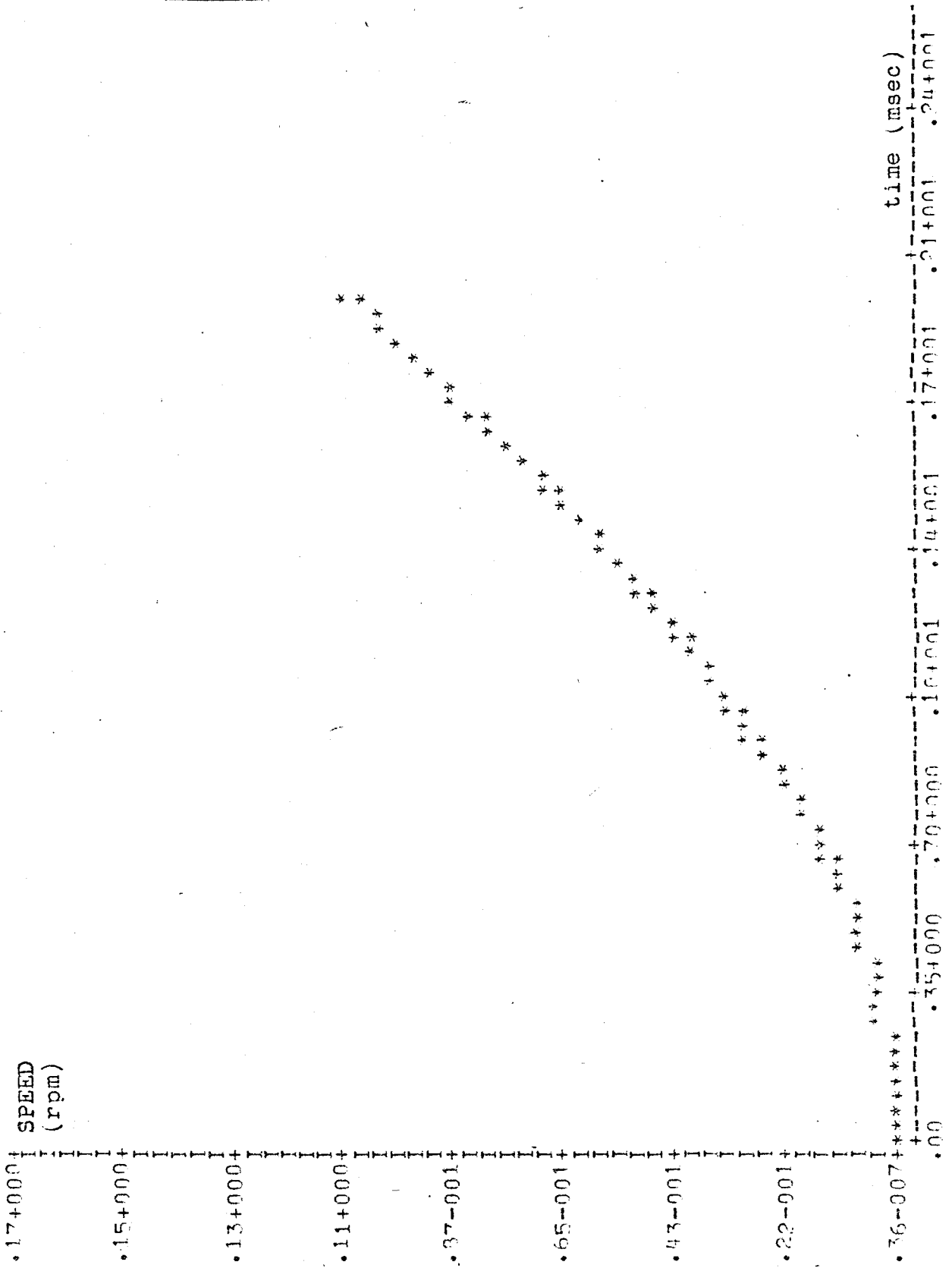
time (msec)

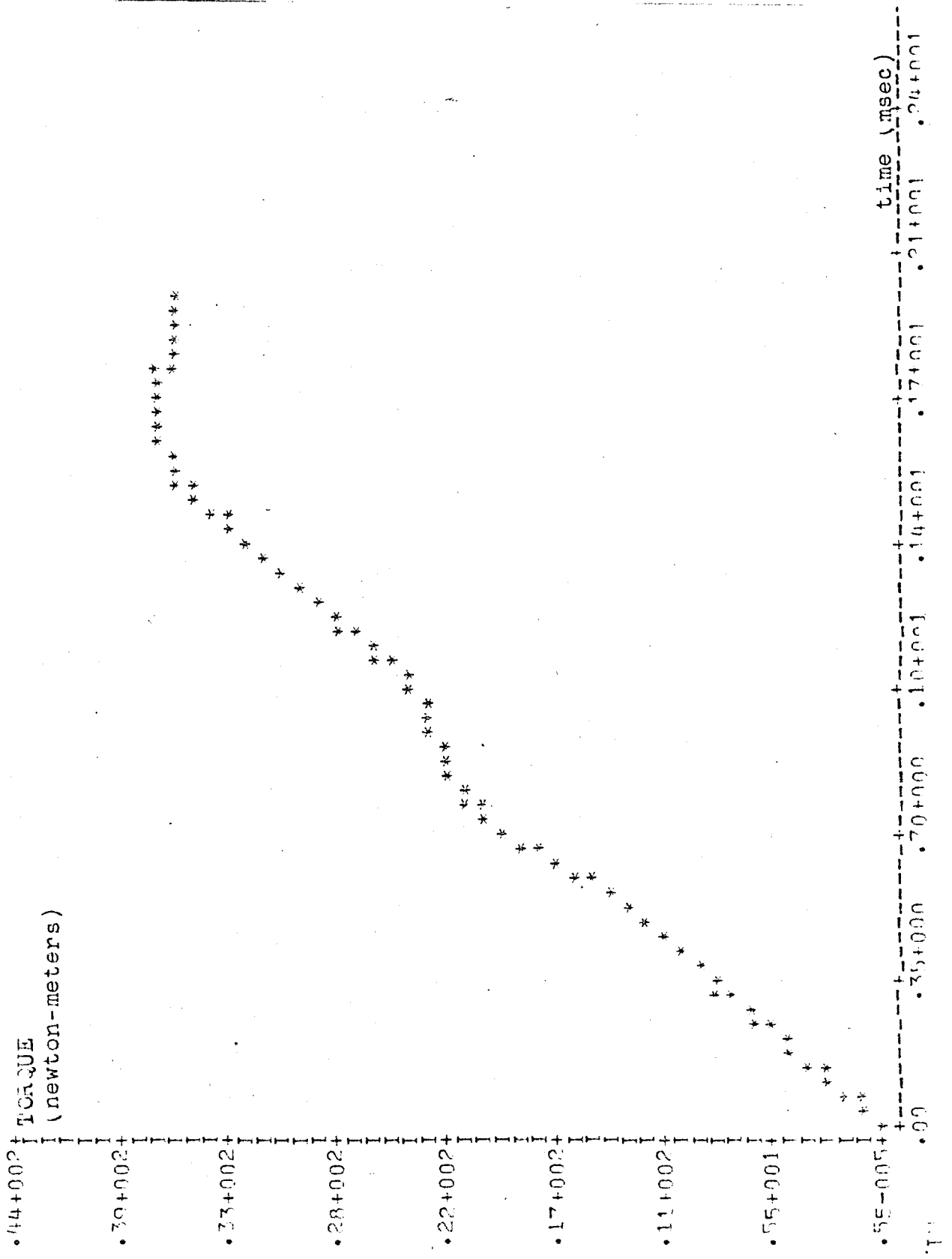
0.00 0.35+000 0.70+000 1.10+001 1.49+001 1.79+001 0.21+001 0.24+001











5-3. CONCLUSIONS

In the preceding four sub-sections, the circuitry simulated is analysed analytically and the values obtained for various circuit parameters are compared with those of the simulation results. It is seen that analytical values and the simulation results agree very closely. In some instances, there are small discrepancies, the probable reasons of which are explained. It is thus shown that the novel approach used for the simulation is very effective in the prediction of the circuit behaviour.

In the next section, some suggestions are made to overcome the shortcomings of the simulation.

5-4. SUGGESTION FOR FURTHER WORK

1- As it is seen on the results, the simulation is not very accurate for extreme rapid changes of the variables. This must be due to very different time constants of the circuit, i.e. the system poles are far apart from each other. Therefore, a moderate time increment of h , gives satisfactory results for times during which the variables do not change very rapidly. On the other hand, if the time increment is reduced to a very small value, it may give good results even for rapid changes but in this case machine truncation error increase tremendously. Therefore, a variable time increment algorithm should be applied. A large time increment could be used for times when variables

change smoothly, and it should be reduced to a small value when rapid changes are expected. In this way, the total number of steps will not be so high, as to result in a high value of machine error.

2- The program can be modified in such a way that the matrices A, B and corrector current vector can be generated with in program, for a given topology. But this may not be easy, because the matrix A contains the resistances of the thyristor and diodes which are variable.

3- The method applied for the simulation of thyristor-diode installation allows us to use much more accurate thyristor and diode models (instead of piecewise linear models) without a corresponding increase in the complexity of the circuit simulation.

4- The method can be applied for other and more complicated thyristor diode installations.

5- The program can be modified for the optimization of components such as snubber circuits which are used in the network. However before this, modification proposed in (1) must be applied.

APPENDIX

DERIVATION OF CIRCUIT EQUATIONS FOR THE ARMATURE CIRCUIT OF SEPARATELY EXCITED CHOPPER CONTROLLED DC MACHINE

In section 4-5, the equations for the circuit in Fig 4-4, namely A and B matrices and the corrector current vector, have been obtained following the rules given in section 3-5. A full derivation of the equations for this circuit is presented in this Appendix.

The branch currents in terms of the previous and current node voltages (V_{xn} and V_{xn-1}), can be calculated, using the formulas 3.8, 3.9, 3.10, as follows :

$$i_{AHn} = \frac{1}{R_H} (V_A - V_{1n})$$

$$i_{LHn} = i_{LHn-1} + \frac{h}{2LM} \left\{ (V_{1n} - V_{2n}) + (V_{1n-1} - V_{2n-1}) \right\}$$

$$i_{CMn} = -i_{CMn-1} + \frac{2CM}{h} \left\{ (V_{2n} - V_{5n}) - (V_{2n-1} - V_{5n-1}) \right\}$$

$$i_{Rdfn} = \frac{1}{R_{Df}} (V_A - V_{4n})$$

$$i_{RS3n} = \frac{1}{R_{S3}} (V_A - V_{3n})$$

$$i_{CS3n} = -i_{CS3n-1} + \frac{2CS3}{h} \left\{ (V_{3n} - V_{4n}) - (V_{3n-1} - V_{4n-1}) \right\}$$

$$i_{LSn} = i_{LSn-1} + \frac{h}{2LS} \left\{ (V_{4n} - V_{5n}) + (V_{4n-1} - V_{5n-1}) \right\}$$

$$i_{RDbn} = \frac{1}{R_{Db}} (V_{5n} - V_{6n})$$

$$i_{RS1n} = \frac{1}{RS1} (V_{6n} - V_{7n})$$

$$i_{CS1n} = -i_{CS1n-1} + \frac{2CS1}{h} (V_{7n} - V_{7n-1})$$

$$i_{RT1n} = \frac{1}{RT1} (V_{6n})$$

$$i_{RT2n} = \frac{1}{RT2} (V_{6n} - V_{8n})$$

$$i_{CCn} = -i_{CCn-1} + \frac{2CC}{h} (V_{8n} - V_{8n-1})$$

$$i_{LCn} = i_{LCn-1} + \frac{h}{2LC} \left\{ (V_{9n} - V_{8n}) + (V_{9n-1} - V_{8n-1}) \right\}$$

$$i_{RDCn} = \frac{1}{RDC} (V_{6n} - V_{9n})$$

$$i_{RS2n} = \frac{1}{RS2} (V_{10n} - V_{9n})$$

$$i_{CS2n} = -i_{CS2n-1} + \frac{2CS2}{h} \left\{ (V_{6n} - V_{10n}) - (V_{6n-1} - V_{10n-1}) \right\}$$

Then, the input current (I_A) and the imaginary injected currents ($I_1, I_2, I_3, I_4, \dots, I_{10}$) can be calculated from the above branch currents as follows :

$$I_A = i_{RM} + i_{RDF} + i_{RS3} = \frac{1}{RH} V_A + \frac{1}{RS3} V_A + \frac{1}{RDF} V_A - \frac{1}{RM} V_{1n} - \frac{1}{RS3} V_{3n} - \frac{1}{RDF} V_{4n}$$

$$I_1 = i_{LM} - i_{RM} = -\frac{1}{RH} V_A + \frac{1}{RH} V_{1n} + \frac{h}{2LM} V_{1n} - \frac{h}{2LM} V_{2n} + \frac{h}{2LM} V_{4n-1} - \frac{h}{2LM} V_{2n-1} + i_{LMn-1}$$

$$I_2 = i_{CM} - i_{LM} = -\frac{h}{2LM} V_{1n} + \frac{h}{2LM} V_{2n} + \frac{2CM}{h} V_{2n} - \frac{2CM}{h} V_{5n} - \frac{h}{2LM} V_{4n-1} + \frac{h}{2LM} V_{2n-1} - \frac{2CM}{h} V_{2n-1} \\ + \frac{2CM}{h} V_{5n-1} - i_{CMn-1} - i_{LMn-1}$$

$$I_3 = i_{CS3} - i_{RS3} = -\frac{1}{RS3} V_A + \frac{1}{RS3} V_{3n} + \frac{2CS3}{h} V_{3n} - \frac{2CS3}{h} V_{4n} - \frac{2CS3}{h} V_{3n-1} + \frac{2CS3}{h} V_{4n-1} - i_{CS3n-1}$$

$$I_4 = i_{LS} - i_{RDF} - i_{CS3} = -\frac{1}{RDF} V_A - \frac{2CS3}{h} V_{3n} + \frac{2CS3}{h} V_{4n} + \frac{1}{RDF} V_{4n} + \frac{h}{2LS} V_{4n} - \frac{h}{2LS} V_{5n} + \frac{2CS3}{h} V_{4n-1} \\ + \frac{h}{2LS} V_{4n-1} - \frac{2CS3}{h} V_{4n-1} - \frac{h}{2LS} V_{5n-1} + i_{LSn-1} + i_{CS3n-1}$$

$$I_5 = i_{RDB} - i_{LS} - i_{CM} = -\frac{2CM}{h} V_{2n} - \frac{h}{2LS} V_{4n} + \frac{h}{2LS} V_{5n} + \frac{2CM}{h} V_{5n} + \frac{1}{RDB} V_{5n} - \frac{1}{RDB} V_{6n} \\ + \frac{2CM}{h} V_{2n-1} - \frac{h}{2LS} V_{4n-1} + \frac{h}{2LS} V_{5n-1} - \frac{2CM}{h} V_{5n-1} + i_{CMn-1} - i_{LSn-1}$$

$$I_6 = i_{RS1} + i_{RT1} + i_{RT2} + i_{RDC} + i_{CS2} - i_{RDB} = -\frac{1}{RDB} V_{5n} + \frac{1}{RS1} V_{6n} + \frac{1}{RT1} V_{6n} + \frac{1}{RT2} V_{6n} + \frac{1}{RDC} V_{6n} \\ + \frac{2CS2}{h} V_{6n} + \frac{1}{RDB} V_{6n} - \frac{1}{RS1} V_{7n} - \frac{1}{RT2} V_{8n} - \frac{1}{RDC} V_{9n} - \frac{2CS2}{h} V_{10n} - \frac{2CS2}{h} V_{6n-1} + \frac{2CS2}{h} V_{10n} \\ - i_{CS2n-1}$$

$$I_7 = i_{CS1} - i_{RS1} = -\frac{1}{RS1} V_{6n} + \frac{1}{RS1} V_{7n} + \frac{2CS1}{h} V_{7n} - \frac{2CS1}{h} V_{7n-1} - i_{CS1n-1}$$

$$I_8 = i_{CC} - i_{RT2} - i_{LC} = -\frac{1}{RT2} V_{6n} + \frac{2Cc}{h} V_{8n} + \frac{1}{RT2} V_{8n} + \frac{h}{2LC} V_{8n} - \frac{h}{2LC} V_{9n} + \frac{h}{2LC} V_{8n-1} \\ - \frac{2Cc}{h} V_{8n-1} - \frac{h}{2LC} V_{9n-1} - i_{CCn-1} - i_{LCn-1}$$

$$I_9 = i_{LC} - i_{RDC} - i_{RS2} = -\frac{1}{RDC} V_{6n} - \frac{h}{2LC} V_{8n} + \frac{h}{2LC} V_{9n} + \frac{1}{RDC} V_{9n} + \frac{1}{RS2} V_{9n} - \frac{1}{RS2} V_{10n} - \frac{h}{2LC} V_{8n-1} \\ + \frac{h}{2LC} V_{9n-1} + i_{LCn-1}$$

$$I_{10} = i_{RS2} - i_{CS2} = -\frac{2CS2}{h} V_{6n} - \frac{1}{RS2} V_{9n} + \frac{1}{RS2} V_{10n} + \frac{2CS2}{h} V_{10n} + \frac{2CS2}{h} V_{6n-1} - \frac{2CS2}{h} V_{10n-1} \\ + i_{CS2n-1}$$

To obtain A and B matrices and the corrector current vector, we can put the above equations into the form of Eq.3.18, as given below

$$\begin{bmatrix} I_A \\ I_1 \\ I_2 \\ I_3 \\ I_4 \\ I_5 \\ I_6 \\ I_7 \\ I_8 \\ I_9 \\ I_{10} \end{bmatrix} = \begin{bmatrix} 0 \\ i_{LMn-1} \\ -i_{CMn-1} - i_{LMn-1} \\ -i_{CS3n-1} \\ i_{LSn-1} - i_{CS3n-1} \\ i_{CMn-1} - i_{LSn-1} \\ -i_{CS2n-1} \\ -i_{CS1n-1} \\ -i_{CCn-1} - i_{LCn-1} \\ i_{LCn-1} \\ i_{CS2n-1} \end{bmatrix} + \begin{bmatrix} \\ \\ \\ \\ \\ \\ \\ \\ \\ \\ \\ \end{bmatrix} A \begin{bmatrix} V_A \\ V_{1n} \\ V_{2n} \\ V_{3n} \\ V_{4n} \\ V_{5n} \\ V_{6n} \\ V_{7n} \\ V_{8n} \\ V_{9n} \\ V_{10n} \end{bmatrix} + \begin{bmatrix} \\ \\ \\ \\ \\ \\ \\ \\ \\ \\ \\ \end{bmatrix} B \begin{bmatrix} V_A \\ V_{1n-1} \\ V_{2n-1} \\ V_{3n-1} \\ V_{4n-1} \\ V_{5n-1} \\ V_{6n-1} \\ V_{7n-1} \\ V_{8n-1} \\ V_{9n-1} \\ V_{10n-1} \end{bmatrix}$$

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