DISCRETE TIME
VARIABLE STRUCTURE SYSTEMS

by

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ABSTRACT

The design principles of Variable Structure Systems (VSS) sliding mode controllers in continuous time, achieving plant parameter insensitivity and disturbance rejection, are studied. The continuous VSS theory is extended to the discrete time domain. New design methods in Discrete Variable Structure Systems (DVSS) by which it is possible to reach the switching planes in a stepwise fashion are developed. Also an adaptation mechanism counteracting noise and plant parameter variations, not requiring any information about the noise statistics, is introduced.

Both regulation and tracking problems for discrete time control systems are considered and suitable algorithms are developed.

The feasibility of the control algorithm is verified through simulations on a digital computer.

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DEĞİŞKEN YAPILI DİZGELER

ÖZETÇE

Bu çalışmada değişken yapılı dizgelerle, parametre değer değişmelerine ve bozan etkenlere karşı duyarsızlık sağlayan kayma-kipi denetimci tasarımı tanıtılmaktadır. Sürekli zaman denetim sorunları için önerilmiş bulunan tasarım ilkeleri ayrık zaman denetim dizgelerini kapsayacak biçimde bu araştırmada geliştirilmektedir.

Dizgenin durum vektörünün anahtarlama düzlemine bir dizi adımlar biçiminde ulaşacak şekilde ayrık zaman uzayında yeni tasarım yöntemleri geliştirilmiş bulunmaktadır. Bunun yanısıra istatistik bilgilere gereksinim duymaksızın, dizge parametre değerlerindeki değişmelere ve bozan etkenlere karşı bir uyarlama yöntemi önerilmektedir.

Düzengeleme ve izleme sorunları ayrık zaman denetim dizgelerinde tartışılmakta ve uygun algoritmalar geliştirilmektedir.

Ayrıca anahtarlama düzleminin en küçüklemesi yolu ile en iyileme sağlanmaktadır.

Denetim algoritmalarının gerçeklenebilirliği bilgisayar benzetimleri ile doğrulanmaktadır.

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LIST OF SYMBOLS

α	:	Controller parameter
β	:	Controller parameter
$\underline{\underline{A}}_M$	:	Model Matrix
$\underline{\underline{A}}_p$	:	Plant Matrix
DVSS	:	Discrete variable structure system
$f(k)$	:	Disturbance
$\underline{\underline{G}}$	:	Switching hyperplane
σ	:	Switching plane
u_{eq}	:	Equivalent control
u_s	:	Stepwise control
u_{sa}	:	Stepwise adaptive control
ψ	:	Controller parameters vector
δ	:	Additional term in the controller
r	:	Reference trajectory
ζ	:	Step vector
ℓ	:	Step number
$\eta(k)$	:	Measured value of the switching plane
$\underline{\xi}$	:	Step vector

I. INTRODUCTION

In optimal control theory, the linear state regulator design procedures such as eigenvalue placement (pole placement) or quadratic minimization, etc.. Optimal Control with state variable feedback where a performance index is minimized requires the solutions of algebraic or sometimes differential matrix Riccatti equations which are not suitable for hand calculations [1] . Although the theory is general and completely suitable for machine computation, the controller parameters are evaluated off-line and a fixed structure controller is implemented. This creates problems whenever plant parameter variation and disturbances, are present, in which case adaptation procedures requiring noise statistics, accurate modelling and parameter identification have to be applied.

In variable structure systems, the control is allowed to change its structure and consequently the controller is not a fixed controller. The idea of changing the structure of the system is a natural one and early utilization can be found in [2] - [3]. A reward for introducing the additional complexity of changing the structure of the system is the possibility to combine useful properties of each one of the structures [4]. Moreover, a variable structure system can posses new properties not present in any of the structures used.

The salient feature of VSS is the so-called sliding mode. While in sliding mode, the system remains insensitive to plant parameter variations and disturbances. The design of a variable structure control which includes sliding mode doesn't require accurate modelling and parameter identification ; it is sufficient to know only the bounds of the model parameters.

Linear model-following control (LMFC) is an efficient control method that avoids the difficulty of specifying a performance index which is usually encountered in the application of optimal control to multivariable control systems. The model that specifies the design objective is part of the system. However, LMFC systems are inadequate when there are large parameter variations or disturbances. This has led to the development of the so called adaptive model-following control system (AMFC) [5]. The stability conditions in AMFC guarantee that the error goes to zero as time tends to infinity, however not offering any direct quantitative control over the transient. The design method proposed by Young [6] provides a systematic and effective procedure for specifying the transient response of the error. The control is discontinuous on a number of switching hyperplanes. During the sliding mode which exists on the intersection of the hyperplanes the system becomes more insensitive to system parameter variations and noise disturbances.

In Chapter 2 and 3, the design methods of continuous time VSS with sliding modes is discussed and the control hierarchy method in multivariable control is studied.

In Chapter 4, continuous time model following VSS is explained and its advantages are introduced.

In Chapter 5, the continuous time VSS theory is extended into the discrete time domain and the controller design method guaranteeing insensitivity to plant parameter variations and external disturbances is established.

In Chapter 6, a new linear and adaptive discrete time controller is formulated. In this method, the step number or vector to reach the switching hyperplanes is set a priori. The adaptation procedure is applied by measuring the value of the switching plane, and no information about the noise statistics is necessary.

In Chapter 7, the stepwise control method is extended into the model folowing discrete time case and finally in Chapter 8 optimality is discussed by the minimization of the switching hyperplane.

II. VARIABLE STRUCTURE SYSTEMS AND SLIDING MODE

In the linear state regulator design, the structure of the state feedback is fixed as

$$u = \underline{k}^T \underline{x} \text{ for } \dot{\underline{x}} = \underline{\underline{A}}\underline{x} + \underline{b}u$$

where the constant parameters $\underline{k}$ are chosen according to various design procedures such as eigenvalue placement or quadratic performance index minimization. In variable structure systems, the control is allowed to change its structure, that is, to switch at any instant from one to another member of a set of possible continuous functions of state.

Variable-structure systems offer the control designer new possibilities for improving the quality of control in comparision with fixed-structure systems. In fact, VSS may have transients which are quite unattainable in fixed-structure systems. This includes the possibility of synthesizing high-quality stable VSS which combine unstable structures in a certain scheme so that the resulting system behaviour is stable.

As an illustration, we consider a VSS controlling a conservative plant by switching the sign of the feedback.

The system (Fig 1.a) with negative feedback loop has phase plane trajectories with elliptic structure. However, if the plant is included in a positive feedback (Fig 1.b), it is a periodically unstable system having trajectories with hyperbolic structures with asymptotes $\sigma_1 = x_2 + a_1x_1$, $\sigma_2 = x_2 - a_1x_1$.

Neither of systems (a) or (b) is satisfactory as far as the quality of the transient is concerned for the fact that they are unstable. Neverthless, certain parts of the phase trajectories of both systems are quite satisfactory. For example, the error of system (a) decreases rapidly in the first quadrant of the phase plane ($x_1x_2>0$) and the system (b) has a good phase trajectory in the fourth quadrant.

If we divide the phase plane into four regions as in Figure 1.c and include negative feedback in region I and III and positive feedback in region II and IV, the system under consideration is globally asymptotically stable and the transient is either aperiodic or involves at most one overshoot. Then we have a stable system which

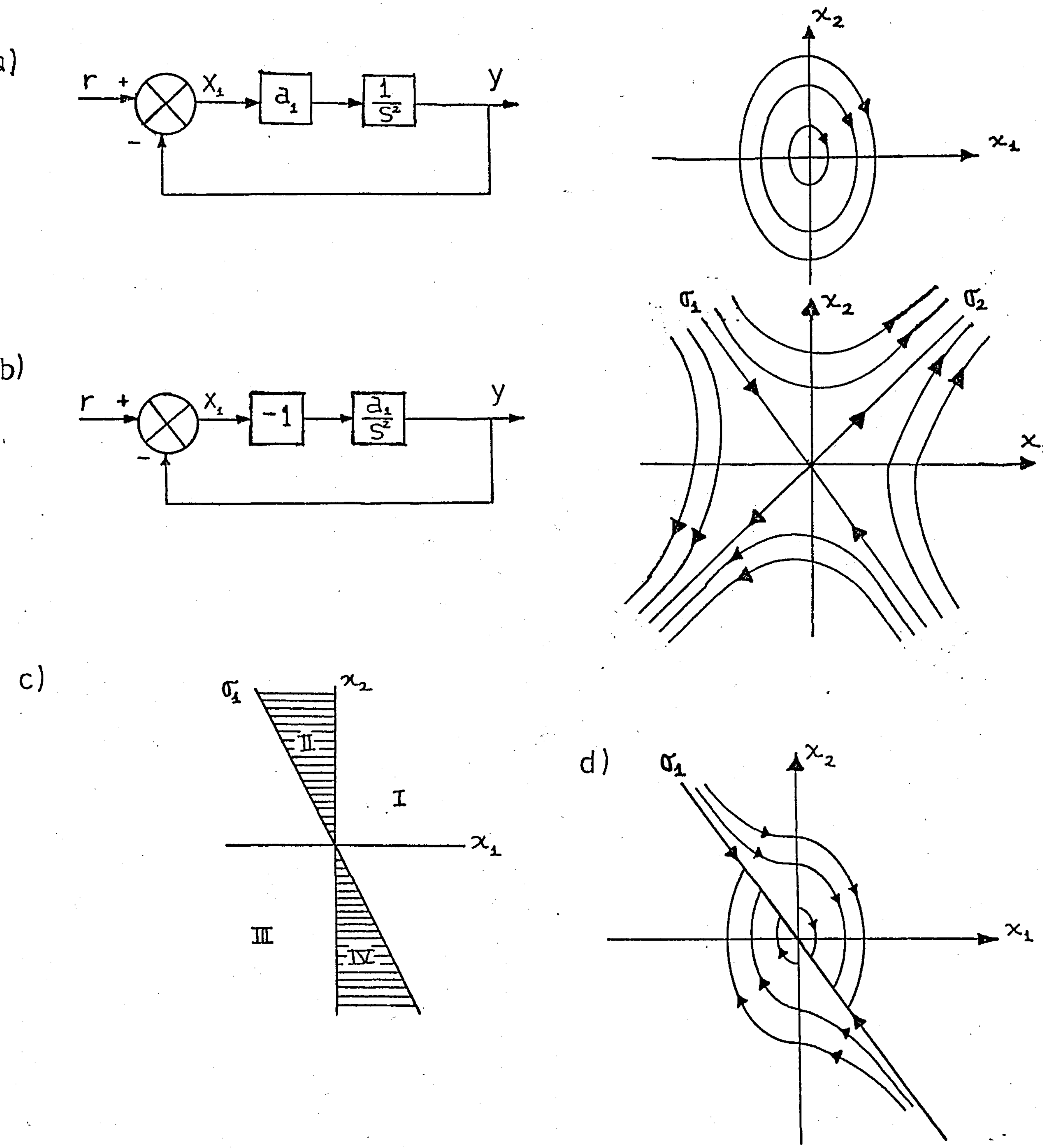
a)
r
+
−
X1
a1
1/S²
y
x2
x1
b)
r
+
−
X1
−1
a1/S²
y
σ1
σ2
x2
x1
c)
σ1
x2
x1
I
II
III
IV
d)
σ1
x2
x1

FIGURE 1

is synthesized from two unstable systems.

The above example shouldn't give the impression that VSS are synthesized solely on the basis of unstable structures. High quality VSS may also be designed from structuraly stable systems and a single VSS may incorporate both stable and unstable structures. In such cases, application of the variable structure principle yields a significantly superior transient in comparison with each of the component stable structures. As an example, consider the system in Figure 2. The system with configuration (a) has a transient of long duration which doesn't satisfy speed requirements, however there is no overshoot in the system. On the other hand, the configuration of (b) with a local switch open results in a conservative unstable system, although oscillatory, whose inital response is relatively fast. To construct aVSS, at the first stage of the transient, when the absolute value of the error is large, we open the local feedback path until the error is sufficiently small in absolute value, then we close the path. This will eliminate overshoot in the system and give a fast response.

In the VSS with the phase portrait of Figure 1.d, if the structure doesnt't change at the precise instant when the representative point crosses the asymptote $\sigma_1 = x_2 + a_1 x_1 = 0$, say owing to the effect of noise, the motion of the system requires special investigation. The

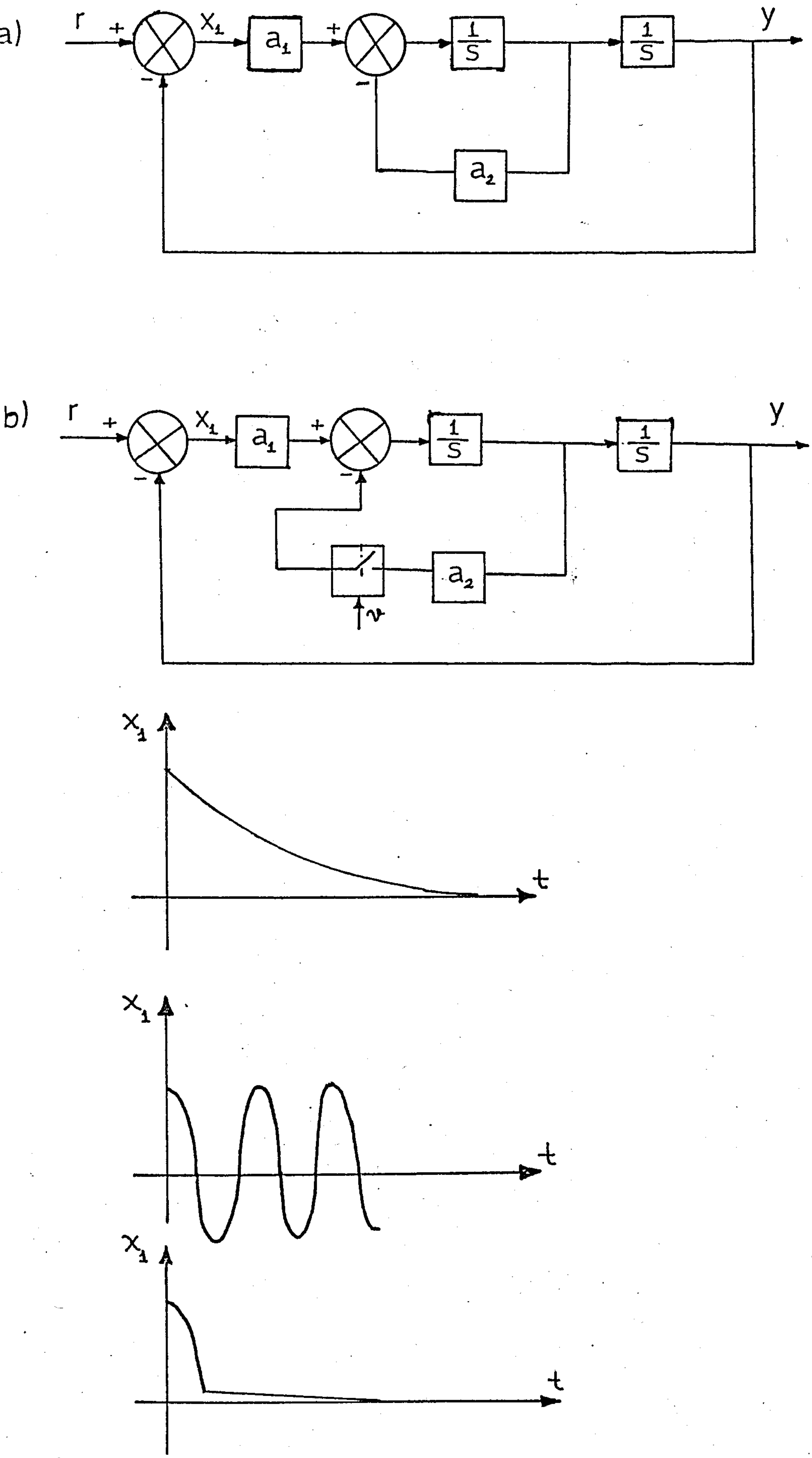

FIGURE 2

result of this investigation was the discovery of an important advantage of VSS : the possibility of synthesizing systems which are insensitive to external disturbances and variation of the plant parameters within wide ranges.

For the system in Fig 1.a and b, the asymptote $\sigma = x_2 + a_1x_1 = 0$ acts as a switching line. In the general case, a switching line need not be an asymptote ; it might be a straight line $\sigma = x_2 + cx_1 = 0$ where $0<c<\infty$. The phase plane is again divided into four regions as in Figure 3.a.

We first consider the case $c > a_1$. The asymptote $x_2 + a_1x_1 = 0$ lies entirely in the union of regions I and III. Therefore, the representative point (RP) starting out from region I and reaching the line $\sigma = 0$ at the instant the structure switches from elliptic to hyperbolic continues to move in region IV along an arc of a hyperbola that deviates from the asymptote (Fig 3.b). Consequently, when at a certain time the RP reaches the ordinate axis, the structure is again switched to elliptic and the process then repeats itself periodically performing oscillations. According to the contracting mapping principle, these oscillations will be damped.

We consider now the case $0<c< a_1$. Suppose that at time $t_0>0$ the representative point starts out in region I. When the RP reaches the line $\sigma = x_2 + cx_1 = 0$, the

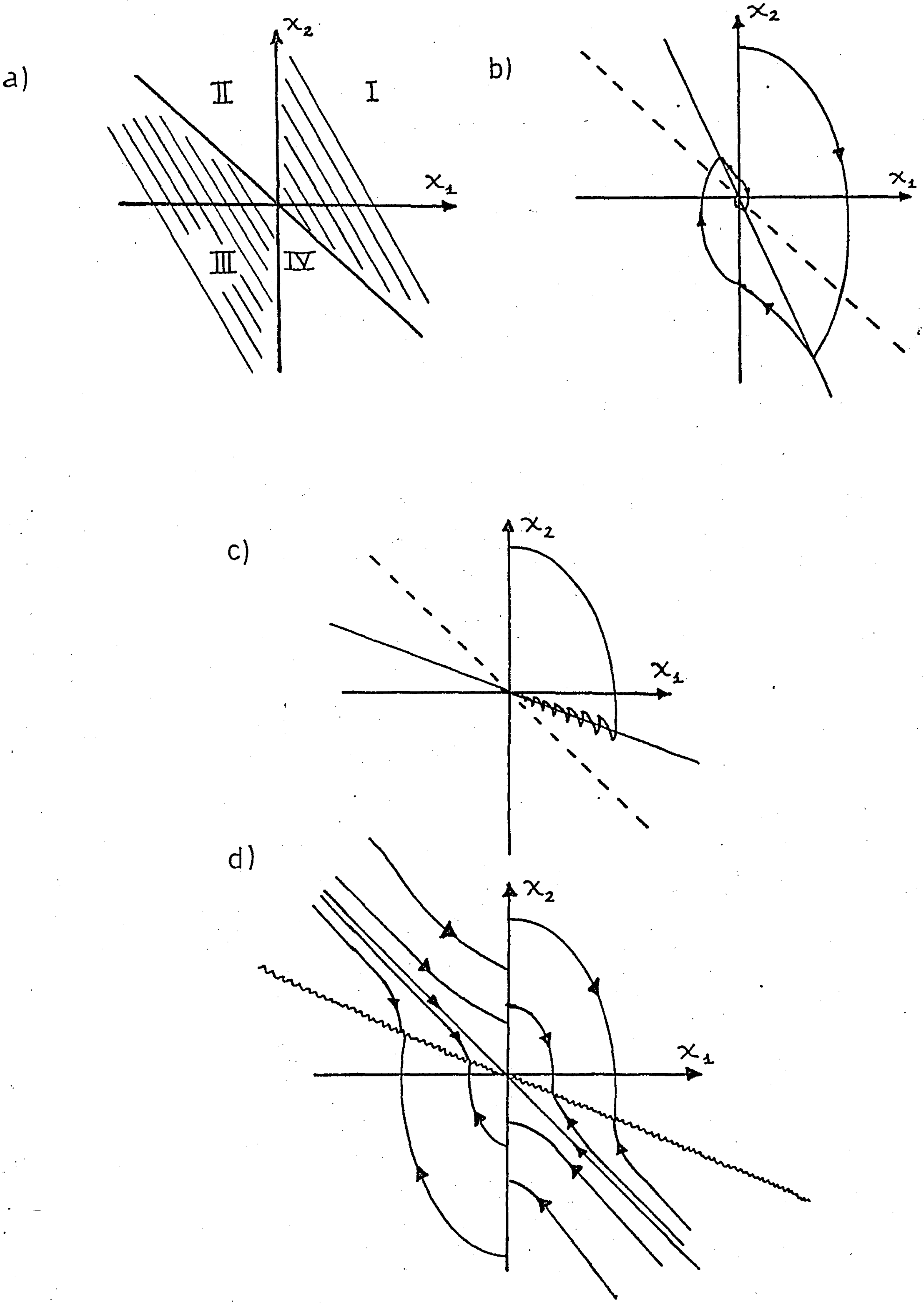

FIGURE 3

structure of the system switches to the hyperbolic. But at the points of the line $\sigma = 0$ the hyperbolic trajectories point into region I so that the RP must quickly leave region IV and return to region I. However, it cannot remain there since the phase trajectories immediately return the RP to region IV, then it is again expelled into region I and so on. Thank's to the topology of hyperbolic and elliptic trajectories, the RP will reach the origin along the line σ.

Region I : $x_1 \geqslant 0,\ x_2 + cx_1 > 0$

Region II : $x_1 < 0,\ x_2 + cx_1 \geqslant 0$

Region III : $x_1 \leqslant 0,\ x_2 + cx_1 < 0$

Regino IV : $x_1 > \ ,\ x_2 + cx_1 \leqslant 0$

If the switching frequency is very high, the RP performs oscillations of fairly small amplitude about $\sigma = 0$. Then the motion of the system will be damped by the differential equation.

$$\dot{x}_1 + cx_1 = 0 \qquad (1)$$

The motion approximated by the above equation is known as a sliding regime or SLIDING MODE. The solution of the above differential equation is given by ;

$$x_1(t) = x_1(to)e^{-c(t-to)}$$

where to is the time at which the system enters the sliding mode. Since c>0, the motion is asymptotically stable.

Sliding regimes have an important property : The corresponding motion of the system is independent of changes in the plant parameters and of external disturbances

In the specific VSS just considered, the performance of the VS system is independent of the gain a_1. This follows from Equation (1) which doesn't involve the parameter a_1 explicitly. Changing the gain a_1 will only change the slope of the phase velocity vectors relative to the line $\sigma = 0$, but these vectors will again point in opposite directions. Consequently, the system will again move in a sliding regime on the same straight line $\sigma = 0$.

Suppose now that the VSS experiences an external disturbance f(t). The forced motion of the VSS is described by the system.

$$\dot{x}_1 = x_2$$
$$\dot{x}_2 = \pm a_1 x_1 - f(t)$$

The disturbance f(t) undoubtely distorts the phase trajectories. However, if these trajectories point in opposite directions in the neighborhood of $\sigma = 0$, the system will again move in a sliding regime and its motion will be invariant with respect to f(t).

Unfortunately, for higher order systems it is not possible to make a sketch and see the behaviour of the phase-plane trajectories. Because of this reason an easy decision cannot be made about when to switch in order to have a stable system. However, there are mathematical ways to settle this problem.

Consider the general system of differential equations

$$\frac{dx_i}{d_t} = f_i(x_1, x_2, \dots x_n, t) \; ; \; (i = 1, \dots n) \quad (2)$$

Let's assume that the right hand members of these equations are discontinuous on a certain hypersurface $\sigma(x_1, x_2, \dots x_n) = 0$ in the phase space $\mathcal{H}(x_1, \dots x_n)$ in such a way that the left and right-hand limits of the functions $f_i(x_1, \dots x_n, t)$; $i = 1, \dots n$, exist as the RP approaches $\sigma = 0$ from either side.

$$\lim_{\sigma \to 0^-} f_i(x_1, \dots x_n, t) = f_i^-(x_1, \dots x_n, t) \qquad (3)$$

$$\lim_{\sigma \to +0} f_i(x_1, \ldots x_n, t) = f_i^+(x_1, \ldots x_n, t) \qquad (4)$$

The derivative of the function σ along the trajectories of the system (2) is

$$\frac{d\sigma}{dt} = \sum_{i=1}^{n} \frac{\partial \sigma}{\partial x_i} \cdot \frac{dx_i}{dt} = \sum_{i=1}^{n} \frac{\partial \sigma}{\partial x_i} \cdot f_i = (\underline{f}.\text{grad } \sigma)$$

where $\underline{f}$ is a vector with components $f_1, f_2, \ldots f_n$. By (2), (3), (4) the following limits exist ;

$$\lim_{\sigma \to -0} \frac{d\sigma}{dt} = (f^- \cdot \text{grad } \sigma) \qquad (5)$$

$$\lim_{\sigma \to +0} \frac{d\sigma}{dt} = (f^+ \cdot \text{grad } \sigma) \qquad (6)$$

At each point of $\sigma = 0$, the sign of the limits (5) and (6) may stand in several relations. Among these, the most interesting one is the following relation, as it corresponds to an ideal sliding regime on the hypersurface $\sigma(x_1, x_2, \ldots x_n) = 0$

$$\lim_{\sigma \to +0} \frac{d\sigma}{dt} \leqslant 0 \leqslant \lim_{\sigma \to -0} \frac{d\sigma}{dt}$$

The equivalent inequality is ;

$$\lim_{\sigma \to 0} \sigma \frac{d\sigma}{dt} < 0 \qquad (7)$$

Inequality (7) may also be written

$$\lim_{\sigma \to 0} \frac{d(\sigma^2)}{dt} < 0 \qquad (8)$$

In fact, the above inequality (8) suggests a necessary condition for system (2) to have a Lyapunov function of the form

$$V(x_1, x_2, \ldots x_n) = \left[\sigma(x_1, x_2, \ldots x_n)\right]^2 \qquad (9)$$

The function (9) is positive semidefinite, Furtermore, since the derivative of V is required to be negative semidefinite in the neighborhood of the hypersurface, then V is a nonincreasing function near $\sigma = 0$ leading to a conditionally stable system relative to the manifold $\sigma(x_1, x_2, \ldots x_n) = 0$.

III. SLIDING MODE CONTROL

3.1. SLIDING MODE EQUATIONS

a. Scalar Control

A single-input single output (SISO) system is described by the equations given below

$$\dot{\underline{x}}(t) = \underline{\underline{A}}\underline{x}(t) + \underline{b}u(t) \qquad (1)$$

$$y(t) = \underline{\underline{C}}\underline{x}(t) \qquad (2)$$

where $\underline{\underline{A}}$ is (nxn) matrix and $\underline{b}^T = \begin{bmatrix} 0 & 0 & \ldots & b_n \end{bmatrix}$. For simplicity, the states are assumed to be accessible so that no observer design is involved in the analysis.

Suppose we have found a control such that the states move on the switching plane ;

$$S(t) = \sum_{i=1}^{n} c_i x_i(t) = 0$$

where $c_n = 1$ and c_i = constant, $i = 1, \ldots n$

By the above assumption, the last state can be expressed as a linear combination of the remaining (n-1) states.

If x_n is substitued into the original system Equation (1), the following new system equations are obtained, which are called the sliding mode equations.

$$\dot{x}_i(t) = \sum_{j=1}^{n-1} (a_{ij} - c_j a_{in}) x_j(t) \quad ; \; i=1, \ldots n-1 \quad (3)$$

If the system is in canonical form with a disturbance being added into the system, the equations become :

$$\dot{x}_i(t) = x_{i+1}(t)$$

$$\dot{x}_n(t) = - \sum_{i=1}^{n} a_i x_i(t) - u(t) - f(t)$$

setting

$$s(t) = \sum_{i=1}^{n} c_i x_i(t) = 0 \; ; \; c_n = 1$$

We get the following sliding mode equations which are insensitive to plant parameters and external disturbances

$$\begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \\ \cdot \\ \cdot \\ \cdot \\ \cdot \\ \dot{x}_{n-1} \end{bmatrix} = \begin{bmatrix} 0 & 1 & 0 & \dots & 0 \\ 0 & 0 & 1 & \dots & 0 \\ \dots & \dots & \dots & \dots & \dots \\ 0 & \dots & \dots & \dots & 1 \\ -c_1 & -c_2 & \dots & \dots & -c_{n-1} \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ \cdot \\ \cdot \\ \cdot \\ x_{n-1} \end{bmatrix} \quad (4)$$

Now, it is possible to have a new stable system by adjusting the parameters c_i's so that all eigenvalues of the new system matrix have negative real parts. In other words, the zeroes of the characteristic polynomial

$$|sI - A'| = s^{n-1} + c_1 s^{n-2} + \dots + c_{n-2} s + c_{n-1}$$

have negative real parts.

Instead of making substitutions mentioned above, when $\underline{b}$ is a general vector, the sliding mode equations can be obtained from the so called equivalent control method.

Again, we assume that we have found a control such that $s(t) = 0$ is achieved and there is no deviation from $s(t)$, that is $\dot{s}(t) = 0$

$$\dot{\underline{x}}(t) = \underline{f}(x,t) + \underline{b}u(t) \quad (5)$$

$$S(t) = \underline{C}^T \underline{x}(t) \quad (6)$$

$$\dot{S}(t) = \underline{C}^T\left[\underline{f}(x,t) + \underline{b}u(t)\right] \tag{7}$$

Setting $\dot{s}(t) = 0$ and solving Eq.7 for $u_{eq}(t)$, we obtain

$$u_{eq}(t) = -(\underline{C}^T\underline{b})^{-1}\,\underline{C}^T\underline{f}(x,t) \tag{8}$$

The Equation (8) tells us that for sliding to occur, $(\underline{C}^T\underline{b})^{-1}$ should exist.

As a simple illustration, let's take the following plant into consideration.

$$\dot{\underline{x}}(t) = \underline{\underline{A}}\underline{x}(t) + \underline{b}u(t)$$

where

$$\underline{b}^T = \left[b_1 b_2 \;\dots\; b_n\right]$$

$$S(t) = \underline{C}^T\underline{x}(t)$$

$$u_{eq}(t) = -(\underline{C}^T\underline{b})^{-1}\,\underline{C}^T\underline{\underline{A}}\underline{x}(t)$$

Substituting $u_{eq}(t)$ into the original system equation, the new system becomes

$$\dot{\underline{x}}(t) = \left[\underline{\underline{I}} - \underline{b}(\underline{C}^T\underline{b})^{-1}\underline{C}^T\right]\underline{\underline{A}}\underline{x}(t)$$

with

$$x_n(t) = -\sum_{i=1}^{n-1} c_i x_i(t)$$

Making the necessary manipulations, we come up with the following (n-1)x1 reduced order sliding mode equations.

$$\dot{\underline{x}}^1(t) = \underline{\underline{A}}^1 \underline{x}^1(t)$$

where $a_{ij}^1 = a_{ij} - a_{in}c_j - b_i(\underline{c}^T\underline{b})^{-1}\left[\underline{c}^T\underline{a}^j - c_j(\underline{c}^T\underline{a}^n)\right]$

$i = 1, \ldots n-1 \; , \; j = 1, \ldots n-1$

a^j is the $j^{\underline{th}}$ column vector of the matrix $\underline{\underline{A}}$.

b. Multivariable control

In multivariable control, it isn't possible to organize sliding regime by utilizing only one switching hyperplane. We have to use switching hyperplanes as many as the control inputs. Then, the aim is the simultaneous existence of a siliding regime on several switching hyperplanes. We shall show that in multivariable VSS of a general type, suitable choice of the control law yields a sliding regime simultaneously on several switching hyperplanes and as a result one can stabilize plants of a general type even with variable parameters.

Consider the system

$$\underline{\dot{x}}(t) = \underline{\underline{A}}\underline{x}(t) + \underline{\underline{B}}\underline{u}(t)$$

$$(n x n)\,(n x 1)\,(n x m)\,(m x 1)$$

Let u(t) be a vector control chosen in such a way that the system has a sliding regime simultaneously on m switching hyperplanes $s_1, s_2, \ldots s_m$;

$$s_j = \sum_{i=1}^{n} c_{ji} x_i$$

$$j = 1, \ldots m$$

$$i = 1, \ldots n$$

where

$$c_{jn} = 1$$

In vector notation, this is simply

$$\underline{S} = \underline{\underline{C}}\,\underline{x}$$

$$m x 1 \quad m x n \quad n x 1$$

When there exist a siliding regime simultaneously on all m switching hyperplanes.

$$\frac{d\underline{s}}{dt} = \underline{0}$$

$$\frac{d\underline{s}}{dt} = \underline{\underline{C}}\dot{\underline{x}}(t) = \underline{\underline{C}}\left[\underline{\underline{A}}\underline{x}(t) + \underline{\underline{B}}\underline{u}(t)\right] = 0$$

Then, we find $\underline{u}_{eq}(t) = -(\underline{\underline{C}}\underline{\underline{B}})^{-1}\underline{\underline{C}}\underline{\underline{A}}\underline{x}(t)$

This control input achieves sliding mode in systems where there is no parameter variations in the system matrices A and B. In general the above control u_{eq} is not the actual control applied to the plant. It is only instrumental in finding the sliding mode equations in the most general cases where variations in the plant parameters are allowed. If we substitute this control into the original system equations supposing that $(\underline{\underline{C}}\underline{\underline{B}})^{-1}$ exists, we obtain the sliding mode equations as follows.

$$\dot{\underline{x}}(t) = \left[\underline{\underline{I}}_{n} - \underline{\underline{B}}(\underline{\underline{C}}\underline{\underline{B}})^{-1}\underline{\underline{C}}\right]\underline{\underline{A}}\underline{x}(t) \tag{9}$$

The above state equations appear to be of order (nx1), however this is not the case, because due to the sliding $\underline{s}(t) = \underline{\underline{C}}\underline{x}(t) = 0$ is achieved, and therefore m of the state variables can be expressed in terms of the remaining (n-m) state variables. Obviously system (9) can be reduced to an (n-m)x1 dimensional system of equations.

$$s_1 = \sum_{i=1}^{n-1} c_{1i}x_i + x_n = 0$$

$$s_2 = \sum_{i=1}^{n-1} c_{2i}x_i + x_n = 0$$

$$\vdots$$

$$s_m = \sum_{i=1}^{n-1} c_{mi} x_i + x_n = 0$$

However, the substitution is cumbersome in the above case. It becomes easier if we select $\underline{\underline{C}}$ as a matrix of the following form.

$$\underline{\underline{C}} = \begin{bmatrix} c_{11} & c_{12} & \cdots\cdots\cdots\cdots c_{1,n-1} & & 1 \\ c_{21} & c_{22} & \cdots\cdots c_{2,n-2} & 1 & 0 \\ \cdots\cdots\cdots\cdots\cdots\cdots & 1 & & 0 & 0 \\ \cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots \\ c_{m1} \cdots c_{m,n-m} & 1 & 0 & \cdots\cdots\cdots & 0 \end{bmatrix}$$

Then ;

$$\begin{bmatrix} x_n \\ x_{n-1} \\ \vdots \\ x_{n-m+1} \end{bmatrix} = \begin{bmatrix} -\sum_{i=1}^{n-1} c_{1i} x_i \\ -\sum_{i=1}^{n-2} c_{2i} x_i \\ \vdots \\ -\sum_{i=1}^{n-m} c_{mi} x_i \end{bmatrix}$$

The substitution of x_{n-j} $(j = 0, 1, \ldots m-1)$ into $\dot{\underline{x}} = \left[\underline{\underline{I}}_n - \underline{\underline{B}}(\underline{\underline{C}}\underline{\underline{B}})^{-1}\underline{\underline{C}} \right] \underline{\underline{A}}\underline{x}$ will reduce the original system equations to a system of order $(n-m)x1$ of the following form.

$$\underset{(n-m)x1}{\dot{\underline{x}}(t)} = \underset{(n-m)x(n-m)}{\underline{\underline{A}}_m \underline{x}(t)}$$

The components of $\underline{\underline{A}}m$ can be expressed through recursive relations in terms of the elements of $\underline{\underline{A}}$ and C_{ij}'s. Then the stability of the new system is achieved by adjusting C_{ij}'s.

Moreover, there is a transformation that can facilitate the above procedure.

In the system

$$\dot{\underline{x}}(t) = \underline{\underline{A}}\underline{x}(t) + \underline{\underline{B}}\underline{u}(t) \tag{10}$$

$$\underline{s}(t) = \underline{\underline{C}}\underline{x}(t)$$

consider the transformation $\underline{q}(t) = \underline{\underline{M}}\underline{x}(t)$ as two successive transformations, that is

Denote

$$\underline{\underline{MAM}}^{-1} = \left[\begin{array}{c|c} \underline{\underline{A}}_{11} & \underline{\underline{A}}_{12} \\ \hline \underline{\underline{A}}_{21} & \underline{\underline{A}}_{22} \end{array}\right]$$

where

$$\left.\begin{array}{ll} \underline{\underline{A}}_{11} & (n-m)x(n-m) \\ \underline{\underline{A}}_{12} & (n-m)x(m) \\ \underline{\underline{A}}_{21} & (m)x(n-m) \\ \underline{\underline{A}}_{22} & (m)x(n-m) \end{array}\right\} \text{matrices}$$

with

$$\underline{s}(t) = \underline{\underline{C}}\,\underline{\underline{M}}^{-1}\underline{q}(t) = \underline{\underline{C}}_1\underline{q}_1 + \underline{\underline{C}}_2\,\underline{q}_2$$

Setting $\dot{\underline{s}}(t) = 0$, $\underline{u}_{eq}(t)$ is obtained as follows.

$$\underline{u}_{eq}(t) = -(\underline{\underline{C}}_2\underline{\underline{B}}_1)^{-1}\Big[(\underline{\underline{C}}_1\underline{\underline{A}}_{11} + \underline{\underline{C}}_2\underline{\underline{A}}_{21})\underline{q}_1 + (\underline{\underline{C}}_1\underline{\underline{A}}_{12} + \underline{\underline{C}}_2\underline{\underline{A}}_{22})\underline{q}_2\Big]$$

For sliding to occur, $(C_2B_1)^{-1}$ should exist.

The equivalent control system becomes

$$\dot{\underline{q}}_1(t) = \underline{\underline{A}}_{11}\underline{q}_1(t) + \underline{\underline{A}}_{12}\underline{q}_2(t)$$

$$\dot{\underline{q}}_2(t) = \underline{\underline{A}}_{21}\underline{q}_1(t) + \underline{\underline{A}}_{22}\underline{q}_2(t) + \underline{\underline{B}}_1\underline{u}_{eq}(t)$$

with

$$\underline{s}(t) = \underline{\underline{C}}_1\underline{q}_1(t) + \underline{\underline{C}}_2\underline{q}_2(t) = 0$$

$$\underline{q}_2(t) = -\underline{\underline{C}}_2^{-1}\underline{\underline{C}}_1\underline{q}_1(t)$$

Thus, the (n-m)th order equivalent system becomes

$$\dot{\underline{q}}(t) = (\underline{\underline{A}}_{11}-\underline{\underline{A}}_{12}\underline{\underline{C}}_2{}^{-1}\underline{\underline{C}}_1)\underline{q}_1(t)$$

3.2. SLIDING MODE CONTROL CONDITIONS

a. Scalar Control

So far we have assumed that we have found a control such that sliding regime is attained that is $\underline{s}(t) = 0$ is achieved. Now, the problem is how to find this specific control.

First, a control of the following form is proposed.

$$u(t) = \begin{cases} u^{+}(x,t) & \text{if } s(x) \geq 0 \\ u^{-}(x,t) & \text{if } s(x) < 0 \end{cases}$$

for the single-input plant

$$\dot{\underline{x}}(t) = \underline{\underline{A}}\,\underline{x}(t) + \underline{b}u(t)$$

For sliding to exist

$$s\dot{s} < 0$$

must be satisfied.

$$s(t) = \underline{c}^{T}\underline{x}(t), \quad \dot{s}(t) = \underline{c}^{T}\dot{\underline{x}}(t)$$

It follows directly from Eq.11 that

$$s\left[\underline{c}^T\underline{\underline{A}}\underline{x}(t) + \underline{c}^T\underline{b}u^+(x(t))\right] < 0$$

$$s\left[\underline{c}^T\underline{\underline{A}}\underline{x}(t) + \underline{c}^T\underline{b}u^-(x(t))\right] < 0$$

This leads to the following inequalities for u^+ and u^-

$$\underline{c}^T\underline{b} > 0 \qquad u^+ < -(\underline{c}^T\underline{b})^{-1}\underline{c}^T\underline{\underline{A}}\underline{x}(t)$$

$$\underline{c}^T\underline{b} < 0 \qquad -u^+ < -(\underline{c}^T\underline{b})^{-1}\underline{c}^T\underline{\underline{A}}\underline{x}(t)$$

$$\underline{c}^T\underline{b} > 0 \qquad u^- > -(\underline{c}^T\underline{b})^{-1}\underline{c}^T\underline{\underline{A}}\underline{x}(t)$$

$$\underline{c}^T\underline{b} < 0 \qquad -u^- > -u(\underline{c}^T\underline{b})^{-1}\underline{c}^T\underline{\underline{A}}\underline{x}(t)$$

In a closed form

$$\left[\mathrm{sgn}(\underline{c}^T\underline{b})\right] u^+ < -|\underline{c}^T\underline{b}|^{-1}\underline{c}^T\underline{\underline{A}}\underline{x}(t)$$

$$\left[\mathrm{sgn}(\underline{c}^T\underline{b})\right] u^- > -|\underline{c}^T\underline{b}|^{-1}\underline{c}^T\underline{\underline{A}}\underline{x}(t)$$

u(t) is selected as a function of the states

$$u(t) = -\sum_{i=1}^{n} \psi_i x_i$$

with

$$\psi_i = \begin{cases} \alpha_i & x_i s \geqslant 0 \\ \beta_i & x_i s < 0 \end{cases}$$

where α_i and β_i are constants to be determined. In our case, $\dot{s}(t) = \sum_{i=1}^{n} \underline{c}^T \underline{a}^i x_i - \underline{c}^T \underline{b} \sum_{i=1} \psi_i x_i$. By using Eq 11 and Eq 12, we come up with the following sliding mode conditions.

$$\text{sgn}(\underline{c}^T\underline{b})\alpha_i > |\underline{c}^T\underline{b}|\underline{c}^T\underline{a}^i$$

$$\text{sgn}(\underline{c}^T\underline{b})\beta_i < |\underline{c}^T\underline{b}|\underline{c}^T\underline{a}^i$$

where $\underline{a}^i$ is the i th column vector of $\underline{\underline{A}}$.

If the parameters of the plant are time varying the condition (13) should be modified. i.e.

$$\text{sgn}(\underline{c}^T\underline{b})\alpha_i > \max_t |\underline{c}^T\underline{b}|^{-1}\underline{c}^T\underline{a}^i(t)$$

$$\text{sgn}(\underline{c}^T\underline{b})\beta_i < \min_t |\underline{c}^T\underline{b}|^{-1}\underline{c}^T\underline{a}^i(t)$$

In order to have a simpler controller, some of the states may not be switched. It is interesting to investigate what happens in this case.

$$u(t) = - \sum_{i=1}^{k} \psi_i x_i \qquad k \leqslant n$$

In this case,

$$\dot{s}(t) = \sum_{i=1}^{k} \left[\underline{c}^T \underline{a}^i - c_i (\underline{c}^T \underline{a}^n) - (\underline{c}^T \underline{b}) \psi_i \right] x_i(t)$$

$$+ \sum_{i=k+1}^{n-1} \left[\underline{c}^T \underline{a}^i - \underline{c}_i (\underline{c}^T \underline{a}^n) \right] x_i(t)$$

For $s\dot{s}<0$ to be satisfied, the following inequalities and constraints should be stisfied.

$$\text{sgn}(\underline{c}^T \underline{b}) \alpha_i > (\underline{c}^T \underline{a}^i - c_i (\underline{c}^T \underline{a}^n))$$

$$\text{sgn}(\underline{c}^T \underline{b}) \beta_i < (\underline{c}^T \underline{a}^i - c_i (\underline{c}^T \underline{a}^n))$$

$$i = 1, \ldots k$$

The additional constraints are

$$\frac{\underline{c}^T \underline{a}^i}{c_i} = \underline{c}^T \underline{a}^n, \; i = k+1, \ldots n-1$$

For such systems, c_i's cannot be shosen arbitrarily because these coefficients should satisfy the constraints. In such cases, it is desirable to follow the procedure outlined below.

Let

$$C_{n-1} - a_n = r$$

$$C_{n-1} = r + a_n$$

$$C_{n-2} - a_{n-1} = C_{n-1}(C_{n-1} - a_n)$$

$$C_{n-2} - a_{n-1} = C_{n-1}r$$

$$C_{n-2} = (r + a_n)r + a_{n-1}$$

$$C_{n-2} = r^2 + ra_n + a_{n-1}$$

$$C_{n-j} = r^j + r^{j-1}a_n + \ldots + ra_{n-j} + a_{n-j+1}$$

The general formula is

$$C_i = r^{n-i} + a_n r^{n-i-1} + \ldots + a_i r + a_{i-1}$$

$$i = k + 1, \ldots n-1$$

The last constraint for $i = n$ is an equality

$$C_{n-1} - a_n = C_n(C_{n-1} - a_n) \; ; \; C_n = 1$$

Usually a small term δ is added into the control in order to counteract disturbances resulting from parameter uncertainties and the additive noise in the system input.

$$u(t) = - \sum_{i=1}^{k} \psi_i x_i - \delta$$

$$\delta = \begin{cases} \delta_u & s > 0 \\ -\delta_u & s < 0 \end{cases}$$

b. Multivariable Control

The problem in multivariable systems is to find a control vector $\underline{u}(t)$ such that a sliding regime is achieved simultaneously on m switching hyperplanes. The so called "control hierarchy method" ensures sliding on m switching hyperplanes simultaneously for the following general type of a plant.

$$\dot{\underline{x}}(t) = \underline{\underline{A}}\underline{x}(t) + \underline{\underline{B}}\underline{u}(t)$$

The procedure of this method is as follows.

Step 1 : First, a hierarchy of switching hyperplanes is selected. By a hierarchy of switching hyperplanes we mean that sliding mode occurs earlier on those switching hyperplanes which are higher in the hierarchy.

Suppose the hierarchy $s_1 \rightarrow s_2 \rightarrow \ldots \rightarrow s_m$ is assumed. The arrow points in the direction of decreasing priority.

Step 2 : Let $i = m$

Step 3 : Suppose sliding mode occurs on the surfaces $s_j = 0$, $j = 1, \ldots i-1$

Solve for the equivalent control $\underline{u}_{eq}^{i-1}$ of the variable structure control $(\underline{u}^{i-1})^T = (u_1, \ldots u_{i-1})$ as a function of u_i and $(\underline{u}^{i+1})^T = (u_{i+1}, \ldots u_m)$ from the algebraic equations $\dot{s}_j = 0$, $j = 1, \ldots i-1$ Note that $\underline{u}^{i+1}$ is known since u_ℓ^+ , u_ℓ^-, $\ell = i+1, \ldots m$, have been determined previously.

Step 4 : For the surface $s_i = 0$, find $u_i^+(x)$ and $u_i^-(x)$ such that $s_i \dot{s}_i < 0$.

Usually a control for u(t) of the following form is proposed.

$$u_j(x) = - \sum_{i=1}^{n} (\alpha_j^i |x_i| + \delta_j) \operatorname{sgn}(s_j)$$

where

$$u_j(x) = \begin{cases} u_j^+(x) & \text{if } s_j \geqslant 0 \\ u_j^-(x) & \text{if } s_j < 0 \end{cases}$$

$j = 1, \ldots m$

The inequality $s_i \dot{s}_i < 0$ guarantees that the state space trajectories of the system move towards the surface $s_i = 0$ along the intersections of the surfaces $s_j = 0$, $j = 1, \ldots i-1$ and slides on it after reaching it.

While satisfying $s_i \dot{s}_i < 0$, the max or min $\underline{u}^{i+1}$ should be taken into consideration. That is

$$u_i^+(x) < - \underset{\underline{u}^{i+1}}{\text{min or max}} \left(f(\underline{x}, \underline{u}_{eq}^{i-1}, \underline{u}^{i+1})\right)$$

$$u_i^-(x) > - \underset{\underline{u}^{i+1}}{\text{max or min}} \left(f(\underline{x}, \underline{u}_{eq}^{i-1}, \underline{u}^{i+1})\right)$$

Step 4 : Let $i = i-1$ if $i > 0$ go to Step 3 Else Stop.

As it is indicated by this procedure, initially sliding mode occurs on the switching plane $s_1 = 0$ and then on the intersection of the switching planes $s_1 = 0$ and $s_2 = 0$ and so on until sliding mode occurs on the intersection of all switching hyperplanes and we say that sliding mode occurs on $\underline{s}(t) = 0$.

Note that $s_i \dot{s}_i < 0$ has the same form as in the design of single input VSS. This fact reveals the basic idea behind the hierarchy of control method which is to replace the multiinput problem by a sequence of single-input problems.

Now, let's see what happens to the control vector if we change the hierarchy for a fourth order system with two control inputs.

$$\dot{\underline{x}}(t) = \underline{\underline{A}}x + \underline{\underline{B}}\underline{u}$$

$$\underline{u}^T = \left[u_1 u_2\right]$$

Since we have two control inputs, two switching hyperplanes have to be selected.

$$s_1 = \sum_{i=1}^{n} c_{1i}x_i$$

$$s_2 = \sum_{i=1}^{n-1} c_{2i}x_i \; ; \; c_{14} = 1 \quad c_{23} = 1$$

First select the hieararchy $s_1 \rightarrow s_2$

According to the control hierarchy method, it is assumed that sliding has already occured on $s_1 = 0$

Solving $\dot{s}_1 = 0$ for $u_{1eq}(t)$

$$u_{1eq}(t) = \frac{1}{\sum_{i=1}^{n} b_{i1}} \left[- \sum_{i=1}^{n} c_{1i}((\underline{a}^i)^T \underline{x}) - \sum_{i=1}^{n} b_{i2}u_2 \right]$$

Now, the condition $s_2\dot{s}_2 < 0$ must be satisfied for u_2 if u_{1eq} is substituted for u_1, the inequality $s_2\dot{s}_2 < 0$ gives

$$u_2^+(x) < \underbrace{-\frac{1}{\Pi}\left[\sum_{i=1}^{n-1} c_{2i}((\underline{a}^i)^T\underline{x}) - \frac{\sum_{i=1}^{n-1} b_{i1}}{\sum_{i=1}^{n} b_{i1}}\left(\sum_{i=1}^{n} c_{1i}((\underline{a}^i))^T\underline{x}\right]\right.}_{f_1}$$

$$u_2^-(x) > -f_1$$

where

$$\Pi = \left(\sum_{i=1}^{n-1} b_{i2}\right)\left(1 - \frac{\sum_{i=1}^{n-1} b_{i1}}{\sum_{i=1}^{n} b_{i1}}\right)$$

Now that $u_2^+(x)$ and $u_2^-(x)$ have been determined, the inequality $s_1\dot{s}_1 < 0$ should be satisfied.

Following similar steps as in the above case

$$\underset{\substack{\text{max or min} \\ u_2}}{u_1^+(x)} < -\frac{1}{\sum_{i=1}^{n} b_{i1}} \underbrace{\left(\sum_{i=1}^{n} c_{1i}((\underline{a}^i)^T \underline{x}) + \sum_{i=1}^{n} b_{i2}u_2 \right)}_{f_2}$$

$$\underset{\substack{\text{min or max} \\ u_2}}{u_1^-(x)} > -f_2$$

Selecting now the hierarchy $s_2 \to s_1$, we assume that sliding has occured first on s_2 requiring $\dot{s}_2 = 0$.

$$u_{2eq} = \frac{1}{\sum_{i=1}^{n-1} b_{i2}} \left[- \sum_{i=1}^{n-1} c_{2i}(\underline{a}^i)^T \underline{x} - \sum_{i=1}^{n} b_{i1}u_1 \right]$$

$$s_1 \dot{s}_1 < 0$$

$$u_1^+(x) \quad - \frac{1}{\Gamma} \left[\sum_{i=1}^{n} c_{1i}((a^i)^T \underline{x}) - \right.$$

$$\underbrace{\left. \frac{\sum_{i=1}^{n} b_{i2}}{\sum_{i=1}^{n-1} b_{i2}} \sum_{i=1}^{n-1} c_{2i}((a^i)^T \underline{x}) \right]}_{f_3}$$

$$u_1^-(x) > - f_3$$

where

$$\Gamma = \left(\sum_{i=1}^{n} b_{i1} \right) \left(1 - \frac{\sum_{i=1}^{n} b_{i2}}{\sum_{i=1}^{n-1} b_{i2}}\right)$$

with $u_1^+(x)$, $u_1^-(x)$ determined in this manner, $s_2 \cdot \dot{s}_2 < 0$ must be satisfied.

$$\underset{\substack{\text{max or min} \\ u_1}}{u_2^+(x)} < - \frac{1}{\sum_{i=1}^{n-1} b_{i2}} \left[\sum_{i=1}^{n-1} c_{2i}((\underline{a}^i)^T \underline{x}) + \sum_{i=1}^{n-1} b_{i1} u_1 \right]$$

$$f_4$$

$$\underset{\substack{\text{min or max} \\ u_1}}{u_2^-(x)} > - f_4$$

It is easily observed that if the hierarchy is changed the control vector changes. Besides, the bounds on the controller parameters differ from each other. While selecting the hierarchy, the inital conditions have an important role. If some of the selected switching hyperplanes is zero initially due to inital conditions, it is useful to give these switching planes higher hierarchy than the other switching planes since sliding occures initally on these planes.

3.3. The effect of disturbances on VSS system

$$\dot{\underline{x}}(t) = \underline{\underline{A}}\underline{x}(t) + \underline{\underline{B}}\underline{u}(t) + \underline{h}(x,t)$$

where the vector $\underline{h}$ represents disturbances and parameter variations. In the sliding mode

$$\underline{u}_{eq}(t) = -(\underline{\underline{C}}\underline{\underline{B}})^{-1}\underline{\underline{C}}(\underline{\underline{A}}\underline{x} + \underline{h})$$

and

$$\dot{\underline{x}}(t) = \left[\underline{\underline{I}} - \underline{\underline{B}}(\underline{\underline{C}}\underline{\underline{B}})^{-1}\underline{\underline{C}}\right](\underline{\underline{A}}\underline{x} + \underline{h})$$

For total disturbance rejection $\underline{\underline{C}}$ must be chosen such that

$$(\underline{\underline{I}} - \underline{\underline{B}}(\underline{\underline{C}}\underline{\underline{B}})^{-1}\underline{\underline{C}})\underline{h} = 0$$

requiring rank $\left[\underline{\underline{B}} : \underline{h}\right]$ = rank $\underline{\underline{B}}$

This gives

$$\dot{\underline{x}}(t) = \left[\underline{\underline{I}} - \underline{\underline{B}}(\underline{\underline{C}}\underline{\underline{B}})^{-1}\underline{\underline{C}}\right]\underline{\underline{A}}\underline{x}(t)$$

The desired motion is achieved by adjusting the coefficients of $\underline{\underline{C}}$.

IV. VARIABLE STRUCTURE MODEL FOLLOWING CONTROL SYSTEMS

4.1. Model Following VSS

In model following systems, the plant is controlled in such a way that its dynamic behaviour approximates that of a specified model. The model is part of the control system and it specifies the design objectives. The adaptive controller should force the error between the model and the plant to zero as time tends to infinity.

$$\dot{\underline{x}}_p(t) = \underline{\underline{A}}_p(t)\underline{x}_p(t) + \underline{\underline{B}}_p(t)\underline{u}(t)$$

$$\dot{\underline{x}}_m(t) = \underline{A}_m(t)\underline{x}_m(t) + \underline{B}_m(t)\underline{r}(t)$$

We assume that $(\underline{\underline{A}}_p, \underline{\underline{B}}_p)$, $(\underline{\underline{A}}_m, \underline{\underline{B}}_m)$ are stabilazible and $\underline{x}_p$ is accessible.

$$\underline{e}(t) = \underline{x}_m(t) - \underline{x}_p(t)$$

$$\dot{\underline{e}}(t) = \dot{\underline{x}}_m(t) - \dot{\underline{x}}_p(t)$$

$$\dot{\underline{e}}(t) = \underline{\underline{A}}_M \underline{e}(t) + (\underline{\underline{A}}_M - \underline{\underline{A}}_p)\underline{x}_p + \underline{\underline{B}}_M \underline{r}(t) - \underline{\underline{B}}_p u(t)$$

For the perfect model following, as it has been shown in Ref. 5.

$$\text{rank } \underline{\underline{B}}_p = \text{rank}(\underline{\underline{B}}_p : \underline{\underline{B}}_M) = \text{rank}(\underline{\underline{B}}_p : \underline{\underline{A}}_M - \underline{\underline{A}}_p)$$

Throughout the study, we assume that perfect model following conditions are satisfied.

Variable structure control has the form

$$u_i(t) = \begin{cases} u_i^+(\underline{x}_p, \underline{e}, \underline{r}) & s_i(\underline{e}) \geq 0 \\ u_i^-(\underline{x}_p, \underline{e}, \underline{r}) & s_i(\underline{e}) < 0 \end{cases}$$

In the most general case ;

$$u_i(t) = -\left[\psi_i^T \underline{q}(t)\right] + \delta_i \qquad i = 1, 2, \ldots m$$

$$\underline{q}^T(t) = \left[\underline{e}^T : \underline{x}_p^T : \underline{r}^T\right]$$

where the jth component of ψ_i vector is given by

$$\psi_{ij} = \begin{cases} \alpha_{ij} & q_j s_i(\underline{e}) \geq 0 \\ \beta_{ij} & q_j s_i(\underline{e}) < 0 \end{cases}$$

and

$$\delta_i = \begin{cases} \delta_i^+ & s_i(\underline{e}) \geq 0 \\ \delta_i^- & s_i(\underline{e}) < 0 \end{cases}$$

$i = 1, \ldots m$

$j = 1, 2, \ldots 2n + \ell$

where ℓ represents the number of references to be tracked.

In multivariable case, it is preferable to select a control of the following form.

$$u_i(t) = \{-\left[\psi_i^T |\underline{q}(t)|\right] + \delta_i\} \operatorname{sgn}(s_i(\underline{e}))$$

Define

$$\underline{s}(e) = \underline{\underline{C}}\ \underline{e}$$

In the sliding mode $\underline{s}(e) = 0$, the dynamic behaviour is

$$\dot{\underline{s}}(e) = \underline{\underline{C}}\,\dot{\underline{e}}(t)$$

$$\underline{u}_{eq}(t) = -(\underline{\underline{C}}\underline{\underline{B}}_p)^{-1}\underline{\underline{C}}\left[\underline{\underline{A}}_M\underline{e}(t) + (\underline{\underline{A}}_M-\underline{\underline{A}}_p)\underline{x}_p(t) + \underline{\underline{B}}_M r(t)\right]$$

Substitution into the original system

$$\dot{\underline{e}}(t) = \left[\underline{\underline{I}}-\underline{\underline{B}}_p(\underline{\underline{C}}\underline{\underline{B}}_p)^{-1}\underline{\underline{C}}\right]\left[\underline{\underline{A}}_M\underline{e}(t) + (\underline{\underline{A}}_M-\underline{\underline{A}}_p)\underline{x}_p(t) + \underline{\underline{B}}_M r(t)\right]$$

For the perfect model following case, we obtain

$$\dot{\underline{e}}(t) = \left[\underline{\underline{I}} - \underline{\underline{B}}_p(\underline{\underline{C}}\underline{\underline{B}}_p)^{-1}\underline{\underline{C}}\right]\underline{\underline{A}}_M\underline{e}(t)$$

If the same transformation explained in the previous chapter is made, we get an expression for the (n-m)th order equivalent system.

$$\dot{\underline{e}}_1(t) = (\underline{\underline{A}}_{11} - \underline{\underline{A}}_{12}\underline{\underline{C}}_2^{-1}\underline{\underline{C}}_1)\underline{e}_1(t)$$

Then the eigenvalues of the above equation can be placed arbitrarily in the complex plane to ensure stable motion giving $\lim_{t\to\infty} \underline{e}(t) = \underline{0}$.

4.2. Model Matching

For a single input system, a new method is proposed which forces an nth order system to follow the dynamics of a desired (n-1)th order model without taking the error space into consideration. The controller used is much simpler than the controller utilized in model following VSS. The response is better in this case since the desired model order is decreased by one.

Consider the plant given below

$$\dot{\underline{x}}(t) = \underline{\underline{A}}\underline{x}(t) + \underline{b}u(t) + \underline{d}f(t) \tag{1}$$

with a switching plane

$$s(t) = \underline{c}^T\underline{x}(t), \; C_n = 1 \tag{2}$$

where

$$\underline{b} = \begin{bmatrix} 0 & 0 & \dots & b_n \end{bmatrix}$$

$$\underline{d} = \begin{bmatrix} 0 & 0 & \dots & d_n \end{bmatrix}$$

$\underline{\underline{A}}$ is assumed to be in canonical form.

If a control is found such that

$$s(t) = b'r(t)$$

Then, the new system becomes

$$\underset{(n-1)x1}{\dot{\underline{x}}(t)} = \begin{bmatrix} 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ \cdots & \cdots & \cdots & \cdots \\ -c_1 & -c_2 & & -c_{n-1} \end{bmatrix} \underline{x}(t) + \begin{bmatrix} 0 \\ 0 \\ \vdots \\ b' \end{bmatrix} \dot{r}(t) \qquad (3)$$

This is actually a reduced order model which the plant will follow if $s(t) = b'r(t)$ is achieved.

A new switching plane

$$s'(t) = s(t) - b'r(t)$$

is selected.

Setting $s'(t) = 0$ is the same as setting $s(t) = b'r(t)$. This is guaranteed by a control satisfying the following inequality.

$$s'(t)\ \dot{s}'(t) < 0$$

As easily seen, the above control differs from MFVSS system and is simpler because we don't deal with the error space. The system behaviour is again insensitive to plant parameters and external disturbances.

V. DISCRETE TIME VARIABLE STRUCTURE SYSTEMS

5.1. Sliding Mode Equations

A single input-single output discrete time system is described by the equations given below.

$$\underline{x}(k+1) = \underline{\underline{A}}\underline{x}(k) + \underline{b}u(k) + \underline{d}f(k) \qquad (1)$$

where u(k) is the control and f(k) is the disturbance added into the system.

$$\underline{b}^T = \begin{bmatrix} 0 0 \ldots b_n \end{bmatrix}, \quad \underline{d}^T = \begin{bmatrix} 0 0 \ldots d_n \end{bmatrix}$$

Suppose we have found a control such that the states move on the switching plane.

$$s(k) = \sum_{i=1}^{n} c_i x_i(k) = 0 \qquad (2)$$

where $c_n = 1$, c_i = constant $i = 1, \ldots n-1$ From Eq.(2)

$$x_n(k) = - \sum_{i=1}^{n-1} c_i x_i(k)$$

Substituting Eq.(2) into Eq.(1), the following new reduced order system equations are obtained.

$$x_i(k+1) = \sum_{j=1}^{n-1} (a_{ij} - c_j a_{in}) x_j(k)$$

$i = 1, \ldots n-1$

$j = 1, \ldots n-1$

As easily seen, the above equations are insensitive to external disturbances. If the plant matrix were in canonical form, the new system equations would be insensitive to plant parameters and external disturbances.

Now, since the system is a discrete time system, the reduced order system is stable if the absolute values of the eigenvalues of the new system matrix are less than one. This is simply achieved by adjusting the coefficients of the switching plane.

In a plant of a general type, the sliding mode equations can be obtained from the so called equivalent control method.

When $s(k) = 0$ is achieved, it has to be maintained. This is possible with a control input which is the solution of $s(k+1) - s(k)=0$ after having achieved $s(k) = 0$, i.e. $s(k+1) = 0$ with $s(k) = 0$.

This kind of control mechanism is suitable for adaptation purposes which is not possible in continuous time case. The adaptation procedure will be described in subsequent chapters.

Consider the discrete time system given below.

$$\underline{x}(k+1) = \underline{\underline{A}}\underline{x}(k) + \underline{\underline{B}}\underline{u} \tag{3}$$

$$\underline{s}(k) = \underline{\underline{C}}\underline{x}(k)$$

where

A(nxn) matrix ; $\underline{x}$(nx1) vector
B(nxm) matrix ; $\underline{u}$(mx1) vector
$\underline{\underline{C}}$(mxn) matrix ; $\underline{s}$(mx1) vector

From

$$\underline{s}(k+1) = 0$$

$$\underline{u}_{eq}(k) = -(\underline{\underline{C}}\underline{\underline{B}})^{-1}\underline{\underline{C}}\underline{\underline{A}}\underline{x}(k)$$

The substitution into Eq.(3) gives the reduced order sliding mode equations.

$$\underline{x}(k+1) = \left[\underline{\underline{I}} - \underline{\underline{B}}(\underline{\underline{C}}\underline{\underline{B}})^{-1}\underline{\underline{C}}\right]\underline{\underline{A}}\underline{x}(k) \tag{5}$$

with $\underline{s}(k) = \underline{0}$

The Equation (5) becomes (n-m) dimensional after s(k) = 0 is substituted. The same transformation used in continuous time case facilitates the sliding mode equations. i.e.

$$\underset{(n-m)x1}{\underline{x}_{1n}(k+1)} = \left[\underline{\underline{A}}_{11} - \underline{\underline{A}}_{12}\underline{\underline{C}}_{2}^{-1}\underline{\underline{C}}_{1}\right]\underline{x}_{1n}(k) \tag{6}$$

where

$$\underset{(nx1)}{\underline{x}_n} = \underset{(nxn)}{\underline{\underline{M}}}\underset{(nx1)}{\underline{x}(k)} \; ; \; \underline{x}_n = \begin{bmatrix} \underline{x}_{1n} \\ --- \\ \underline{x}_{2n} \end{bmatrix}$$

M is a product of elemantary transformations on $\underline{\underline{B}}$ and $\underline{\underline{C}}$ such that

$$\underline{\underline{M}}\underline{\underline{B}} = \begin{bmatrix} \underline{\underline{0}} \\ \cdots \\ \underline{\underline{B}}_1 \end{bmatrix} , \; \underline{\underline{C}}\underline{\underline{M}}^{-1} = \left[\underline{\underline{C}}_1 \; \underline{\underline{C}}_2\right]$$

where $\underline{\underline{B}}_1$ and $\underline{\underline{C}}_2$ are (mxm), $\underline{\underline{C}}_1$ is (n-m)xm matrices

$$\underline{\underline{M}}\underline{\underline{A}}\underline{\underline{M}}^{-1} = \left[\begin{array}{c|c} \underline{\underline{A}}_{11} & \underline{\underline{A}}_{12} \\ \hline \underline{\underline{A}}_{21} & \underline{\underline{A}}_{22} \end{array}\right]$$

The equivalent system becomes

$$\underline{x}_{1n}(k+1) = \underline{\underline{A}}_{11}\underline{x}_{1n}(k) + \underline{\underline{A}}_{12}\underline{x}_{2n}(k) \qquad (7)$$

$$\underline{x}_{2n}(k+1) = \underline{\underline{A}}_{21}\underline{x}_{1n}(k)\ \underline{\underline{A}}_{22}\underline{x}_{2n}(k) + \underline{\underline{B}}_{1}\underline{u}_{eq}(k)$$

when

$$\underline{s}(k) = \underline{\underline{C}}_1\underline{x}_{1n}(k) + \underline{\underline{C}}_2\underline{x}_{2n}(k) = 0$$

is achieved then Eq.(7) takes the form of Eq(6).

5.2 Sliding Mode Control Conditions

If a control is found such that

$\left[s(k+1) - s(k)\right] . s(k) < 0$ is satisfied then the states will hit the switching hyperplane from any inital conditions and will chatter around it. As a result we say that $s(k) = 0$ is achieved.

A control input of the following form

$$u(k) = \begin{cases} u^+(\underline{x}(k)) & s(k) \geqslant 0 \\ u^-(\underline{x}(k)) & s(k) < 0 \end{cases}$$

is proposed for the single input plant

$$\underline{x}(k+1) = \underline{\underline{A}}x(k) + \underline{b}u(k)$$

$$s(k) = \underline{c}^T\underline{x}(k)$$

For sliding to exist

$$s(k)\cdot\left[s(k+1) - s(k)\right] < 0$$

must be satisfied. i.e.

$$\left[\text{sgn}(\underline{c}^T\underline{b})\right] u^+(\underline{x}(k)) < -|\underline{c}^T\underline{b}|^{-1}\underline{c}^T\left[\underline{\underline{A}}-\underline{\underline{I}}\right]\underline{x}(k)$$

$$\left[\text{sgn}(\underline{c}^T\underline{b})\right] u^-(\underline{x}(k)) < -|\underline{c}^T\underline{b}|^{-1}\underline{c}^T\left[\underline{\underline{A}}-\underline{\underline{I}}\right]\underline{x}(k)$$

The above conditions are different from those obtained for the continuous time case.

For a simple illustration, if we consider a plant in canonical form as given below.

$$\underline{x}(k+1) = \underline{\underline{A}}\underline{x}(k) + \underline{b}u(k)$$

$$\underline{b} = \left[0 0 \ldots b_n \right]$$

where b_n is assumed to be positive, and if we let

$$u(k) = -\sum_{i=1}^{n} \psi_i x_i(k)$$

where

$$\psi_i = \begin{cases} \alpha_i & x_i(k)s(k) > 0 \\ \beta_i & x_i(k)s(k) < 0 \end{cases}$$

we can determine the controller parameters through

$$s(k+1) - s(k) = \sum_{i=1}^{n-1} c_i x_{i+1}(k) - \sum_{i=1}^{n} a_i x_i(k) -$$

$$b_n \sum_{i=1}^{n} \psi_i x_i(k) - \sum_{i=1}^{n} c_i x_i(k)$$

After some simplification, we obtain

$$s(k+1) - s(k) = \sum_{i=2}^{n} (c_{i-1} - a_i - b_n \psi_i - c_i) x_i(k)$$

$$+ (-a_1 - b_n \psi_1 - c_1) x_1(k)$$

$$s(k)\cdot\left[s(k+1) - s(k)\right] < 0 \text{ must be satisfied.}$$

if $x_i(k)s(k) > 0$, $\psi_i = \alpha_i$

Then

$$\alpha_i > \frac{1}{b_n}(c_{i-1} - a_i - c_i), \ i = 2, \ldots n$$

$$\alpha_1 > \frac{1}{b_n}(-a_1 - c_1)$$

if $x_i(k)\ s(k) < 0$; $\psi_i = \beta_i$

Then

$$\beta_i < \frac{1}{b_n}(c_{i-1} - a_i - c_i), \ i = 2, \ldots n$$

$$\beta_1 < \frac{1}{b_n}(-a_1 - c_1)$$

5.3. Multivariable Control

For multivariable control, a control hierarchy method similar to the continuous case is proposed.

Step 1. Suppose the hierarchy

$$s_1(k) \rightarrow s_2(k) \rightarrow \ldots \rightarrow s_m(k)$$

is imposed.

Step 2. Let $i = m$

Step 3. Suppose sliding has already occured on the surfaces

$$s_j(k) = 0, \; j = 1, \ldots i-1$$

Step 4. For the surface $s_i = 0$ find u_i^+ and u_i^-

$$u_i(x(k)) = \begin{cases} u_i^+(\underline{x}(k)) & s_i(k) > 0 \\ u_i^-(\underline{x}(k)) & s_i(k) < 0 \end{cases}$$

such that

$$s_i(k)\left[s_i(k+1) - s_i(k)\right] < 0$$

Since it is assumed that sliding has already occured on the surfaces $s_j(k) = 0$, $j = 1, \ldots i-1$, an equivalent control is substitued for the value of $u_j(k)$ which is the solution of $s_j(k+1) = 0$ with $s_j(k) = 0$ After that for the remaining u_ℓ, $\ell = i+1, \ldots m$, the evaluated values of u_ℓ^+ or u_ℓ^- are substituted since they are already determined for $\ell < m$

Step 5. Let $i = i-1$ if $i > 0$ Go to Step 3 else stop.

VI. STEPWISE ADAPTIVE DVS CONTROL

In DVSS, the method that is proposed in Chapter 5 gives a control such that $s(k)\cdot\left[s(k+1) - s(k)\right] < 0$ is satisfied. With this kind of control, the states are expected to slide and chatter around the switching plane after a certain number of steps, which is dependent on the controller parameters. The exact step number at which the states reach the switching hyperplane cannot be set a priori. In this chapter, a new control method is proposed by which it is possible to set the step number at which $\underline{s}(k) = 0$ is desired to be reached and then maintained.

A single-input single-output (SISO) discrete system is described by the equations given below.

$$\underline{x}(k+1) = \underline{\underline{A}}\underline{x}(k) + \underline{b}u(k) \tag{1}$$

with a switching hyperplane

$$s(k) = \sum_{i=1}^{n} c_i x_i(k) \quad , \quad c_n = 1 \tag{2}$$

The design principles and the reduced order sliding mode equations are the same as explained in Chapter 5. However, the proposed method of finding the control input achieving the desired motion is different.

First a step value ζ is selected as ;

$$\zeta = \frac{s(0)}{\ell} \tag{3}$$

where ℓ represents the desired step number to reach the switching hyperplane.

$$s(k) = \underline{c}^T \underline{x}(k)$$

$$s(k+1) = \underline{c}^T \left[\underline{\underline{A}}\,\underline{x}(k) + \underline{b}u(k) \right]$$

then, the inequality below

$$s(k+1) = s(k) - \zeta \tag{4}$$

simply tells us that for every step the value of s(k) will decrease by ζ.

Now, let's find a control input for the plant (1), which will decrease the value of s(k) by ζ.

$$s(k+1) = s(k) - \zeta$$

$$\underline{c}^T \left[\underline{\underline{A}}\underline{x}(k) + \underline{b}u_s(k)\right] = \underline{c}^T\underline{x}(k) - \zeta$$

$$\underline{c}^T\underline{b}u_s(k) = \underline{c}^T\underline{x}(k) - \underline{c}^T\underline{\underline{A}}\underline{x}(k) - \zeta$$

$$u_s(k) = (\underline{c}^T\underline{b})^{-1}\left[\underline{c}^T(\underline{\underline{I}}-\underline{\underline{A}})\underline{x}(k) - \zeta\right] \qquad (5)$$

From Eq.4, it is clear that the above control $u_s(k)$ wil make the value of $s(k)$ to be zero at the end of ℓ steps since we select $\zeta = \frac{s(0)}{\ell}$.

After ℓ steps, $s(\ell)=0$ is achieved, and then the control should be changed such that $s(k)=0$ for $k > \ell$.

This is achieved simply by setting $\zeta = 0$ for $k > \ell$. In fact, this control is the so called equivalent control in DVSS.

$$u_{eq}(k) = (\underline{c}^T\underline{b})^{-1} \; \underline{c}^T(\underline{\underline{I}}-\underline{\underline{A}})\underline{x}(k)$$

The proposed control can be summarized in Figure 6.1.

As easily seen, $u_s(k)$ and $u_{eq}(k)$ rely upon the plant parameters and therefore parameter variations may affect the control adversely. In addition to that, if there is noise added into the system, the above controls cannot maintain $s(k) = 0$. Because after ℓ steps $s(\ell) = \varepsilon(\ell)$ due to the distarbances instead of being $s(\ell) = 0$. Then, $u_{eq}(k)$ will try to maintain $s(k) = \varepsilon(k)$ for $k > \ell$ which cannot regulate the states.

In this case, the sliding mode equations become ;

$$\underline{x}_n(k+1) = \underline{\underline{A}}'(a_i, c_i)x_n(k) + \varepsilon(k)$$

provided that $\underline{b}^T = \begin{bmatrix} 00 & \ldots & b_n \end{bmatrix}$. For a general $\underline{b}$ vector case the corresponding expression can be found as explained in the previous chapters.

Although the new system above is stable since c_i's are selected such that $\underline{\underline{A}}'$ is a stable matrix, the states will not go to zero as $k \to \infty$, instead $\underline{x}(k) \to \underline{q}$ which is difficult to find because $\varepsilon(k)$ is not constant and changes at every step.

The problem can be solved by having an adaptive control for $k > \ell$ of the following form.

$$s(k+1) = s(k) - \eta(k) \qquad (6)$$

where

$$\eta(k) = s_m(k) = \text{measured value of } s(k)$$

$$s(k) = \underline{c}^T \underline{x}(k)$$

From Eq.6, we find an adaptive control ;

$$u_a(k) = (\underline{c}^T \underline{b})^{-1} \left[\underline{c}^T (\underline{\underline{I}} - \underline{\underline{A}}) \underline{x}(k) - \eta(k) \right] \quad \text{for } k > \ell$$

It is easily seen that if $\eta(k) \to 0$; $u_a(k) \to u_{eq}(k)$

The adaptation in the above formulation is performed for $k > \ell$. For $k > \ell$ the step value ζ can be updated for every step so that the cumulative error resulting from the deviation from $s(k) = 0$ due to noise and external disturbances will be much smaller. The method of updating ζ is as given below.

$$\zeta(\ell) = \frac{s(k)}{\ell}$$

where

$$\ell = N, \ldots 1$$
$$k = 0, \ldots N-1$$

As easily seen, ζ is updated for every step. Then, the control input $u_{sa}(k)$ for $k < N$ becomes

$$u_{sa}(k) = (\underline{c}^T \underline{b})^{-1} \left[\underline{c}^T (\underline{\underline{I}} - \underline{\underline{A}}) \underline{x}(k) - \zeta(\ell) \right]$$

The adaptation mechanism is summarized in Figure 6.2.

As already stated, in multivariable control switching hyperplanes as many as the control inputs are selected. The control hierarchy method doesn't give us any definite step number at which states reach the switching hyperplanes. Besides, the hyperplanes are reached in a hierarchical order. Similar to the scalar case, a multivariable control by which the hyperplanes can be reached at desired steps which can be determined beforehand as desired is proposed. Then, the switching hyperplanes can be reached either at the same time or in a hierarchical order.

Consider the system of a general type

$$\underline{x}(k+1) = \underline{\underline{A}}\underline{x}(k) + \underline{\underline{B}}\underline{u}(k) \tag{7}$$

with

$$\underline{s}(k) = \underline{\underline{G}}\underline{x}(k)$$

First, suppose that it is desired to reach $s_i(k) = 0$, $i = 1, \dots m$ at the same time. In this case, a step vector is selected as ;

$$\underline{\zeta} = \begin{bmatrix} \zeta_1 \\ \zeta_2 \\ \vdots \\ \zeta_m \end{bmatrix} = \frac{\underline{s}(0)}{\ell}$$

where ℓ represents the desired number of steps to reach the switching hyperplanes and $\zeta_i = \dfrac{s_i(0)}{\ell}$; $i = 1, \dots m$

Now, the aim is to reach the switching hyperplanes simultaneously such that $s_i(k) = 0$, $i = 1, \dots m$, is achieved at the same time and maintained.

The control vector performing this can be formulated as ;

$$\underline{s}(k+1) = s(k) - \underline{\zeta}$$

$$\underline{u}_s(k) = -\ (\underline{\underline{G}}\underline{\underline{B}})^{-1}\underline{\underline{G}}(\underline{\underline{A}}-\underline{\underline{I}})\underline{x}(k) - (\underline{\underline{G}}\underline{\underline{B}})^{-1}\underline{\zeta}$$

After ℓ steps $\underline{s}(k) = 0$ and it has to be maintained so we set $\underline{\zeta} = \underline{0}$ and apply

$$\underline{u}_{eq}(k) = -(\underline{\underline{G}}\underline{\underline{B}})^{-1}\underline{\underline{G}}(\underline{\underline{A}}-\underline{\underline{I}})\underline{x}(k) \qquad k > \ell$$

If it is desired to reach the switching hyperplanes in different times, the $\underline{\zeta}$ vector at the inital time becomes

$$\zeta_i = \frac{s_i(0)}{\ell_i} \quad i = 1, \dots m \tag{9}$$

In this case, the hierarchy has an effect on the value of $\underline{\zeta}$ such that its value changes at every time one of the switching hyperplanes is reached. Suppose the following hierarchy is assumed.

$$s_1 \rightarrow s_2 \rightarrow \ldots \rightarrow s_m$$

then (10)

$$\ell_m > \ell_{m-1} > \ldots > \ell_1$$

After ℓ_i steps s_i becomes zero then ζ_i has to be set to zero, otherwise $s_i = 0$ cannot be maintained. That's why the value of $\underline{\zeta}$ vector is changed at the instants one of the switching hyperplanes is reached.

As it is seen, the problem becomes more difficult. This can be seen in Figure 6.4 as compared to the case where $\underline{s}(k) = 0$ is achieved at the same time in Figure 6.3.

If there are disturbances in the system, a similar adaptation procedure as in the scalar case can be followed.

Suppose that it is desired to come to the switching hyperplanes at the same time. Then,

$$\underline{u}_s(k) = -(\underline{\underline{G}}\underline{\underline{B}})^{-1}\underline{\underline{G}}(\underline{\underline{A}}-\underline{\underline{I}})\underline{x}(k) - (\underline{\underline{G}}\underline{\underline{B}})^{-1}\underline{\zeta}$$

is applied for ℓ steps.

After ℓ steps, s(k) = 0 is supposed to be achieved. However, due to disturbances this condition may not be satisfied. Then the following adaptation procedure is followed.

$$\underline{s}(k+1) = s(k) - \underline{\eta}_m(k)$$

$$\eta_m(k) = \begin{bmatrix} s_{1m}(k) \\ s_{2m}(k) \\ \vdots \\ s_{mm}(k) \end{bmatrix}$$

$$\underline{u}_a(k) = -(\underline{\underline{G}}\underline{\underline{B}})^{-1}\underline{\underline{G}}(\underline{\underline{A}}-\underline{\underline{I}})\underline{x}(k) - (\underline{\underline{G}}\underline{\underline{B}})^{-1}\eta_m(k) \qquad k > \ell$$

An adaptation for $\underline{\zeta}$ similar to the scalar case can be formulated as follows.

$$\underline{\zeta} = \frac{\underline{s}(k)}{\ell}$$

where

$$k = 0, \ldots N-1$$

$$\ell = N, \ldots 1$$

The control vector becomes

$$\underline{u}_{sa}(k) = -(\underline{\underline{G}}\underline{\underline{B}})^{-1}\underline{\underline{G}}(\underline{\underline{A}}-\underline{\underline{I}})\underline{x}(k) - (\underline{\underline{G}}\underline{\underline{B}})^{-1}\underline{\zeta}(\ell) \quad k < N$$

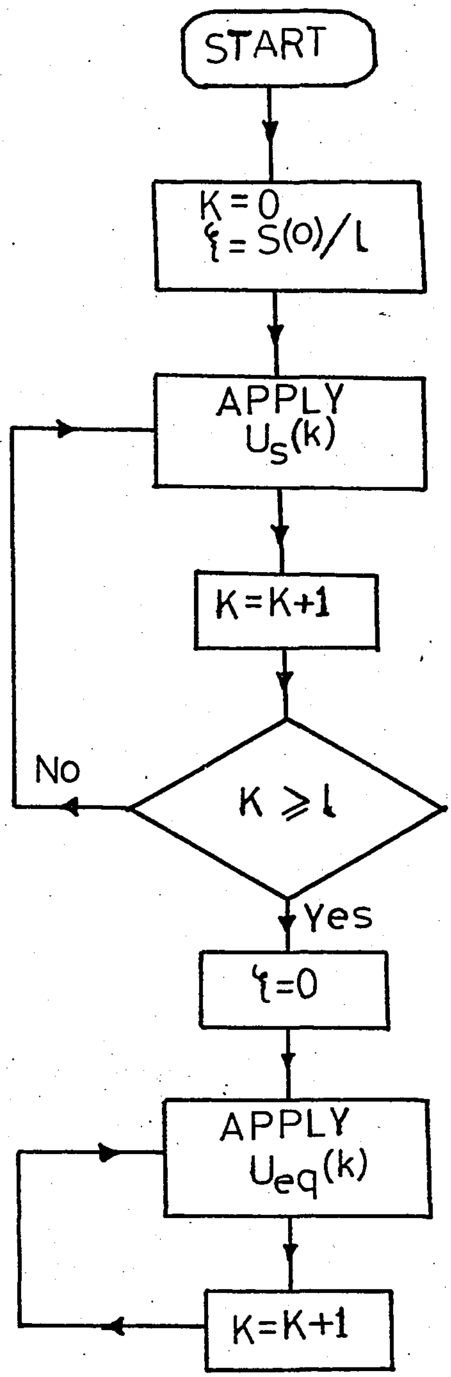

FIGURE 6.1 Stepwise DVSS Control Algorithm

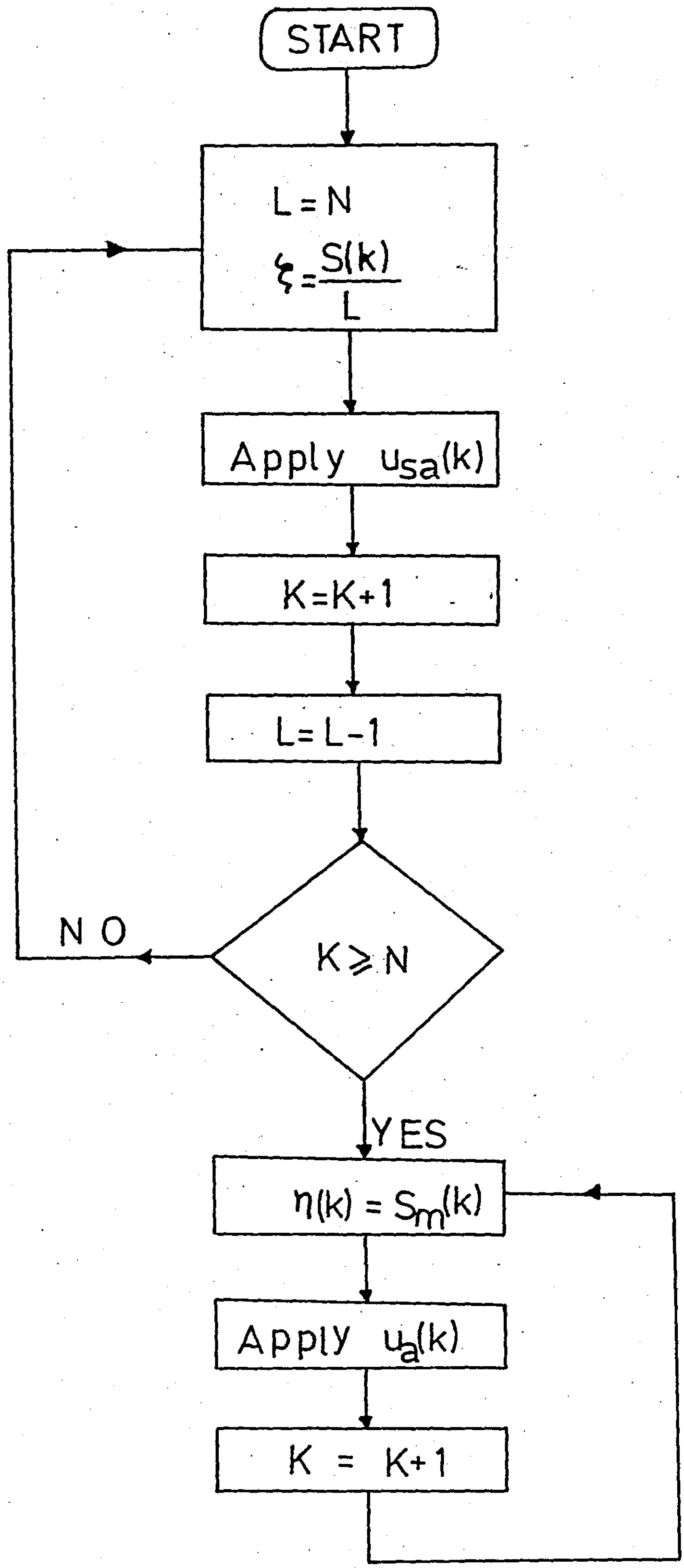

FIGURE 6.2 Adaptation mechanism of Stepwise DVSS Control

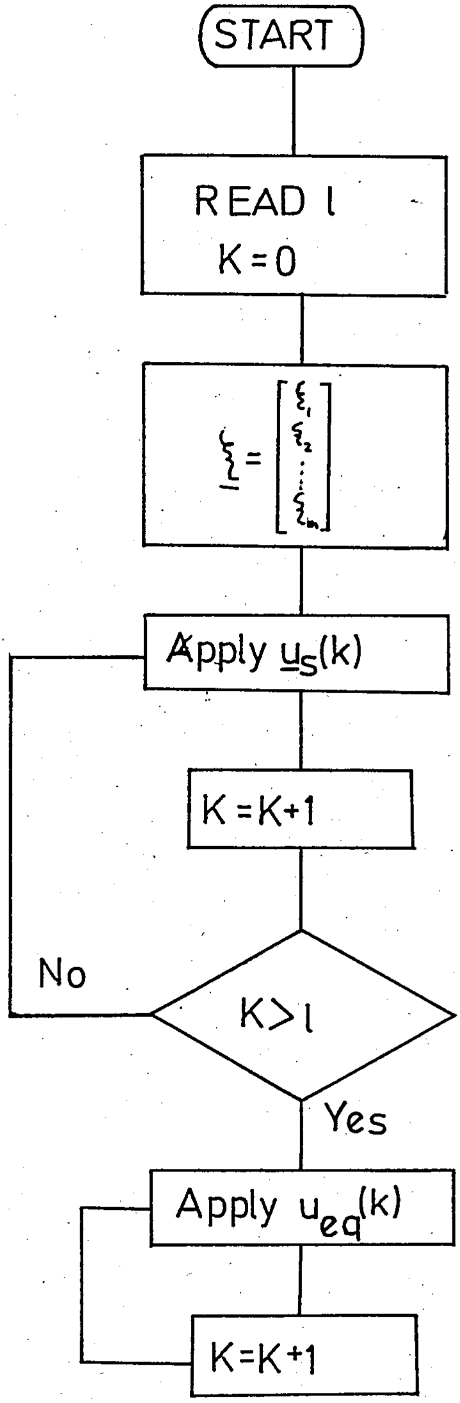

FIGURE 6.3 Stepwise multivariable control algorithm

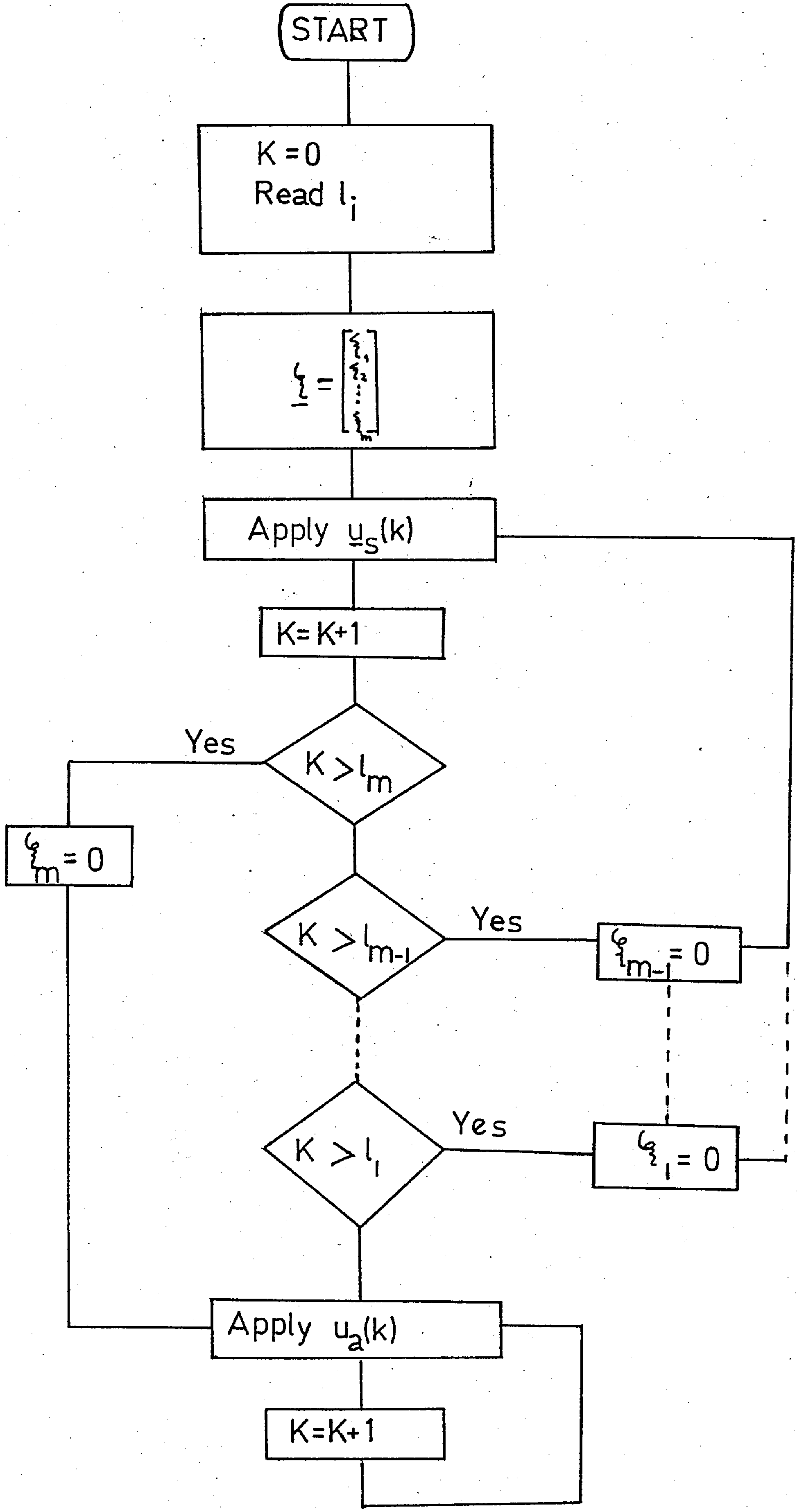

FIGURE 6.4 Multivariable stepwise control algorithm

VII. DISCRETE TIME
VSS
MODEL FOLLOWING

In model following, a model is selected which specifies the design objectives. The controller is designed such that the error between the plant and model states goes to zero as $k \to \infty$.

The plant and the model are described by the equations given below.

$$\underline{x}_p(k+1) = \underline{\underline{A}}_p \underline{x}_p(k) + \underline{\underline{B}}_p \underline{u}(k) \tag{1}$$

$$\underline{x}_m(k+1) = \underline{\underline{A}}_m \underline{x}_m(k) + \underline{\underline{B}}_m \underline{r}(k) \tag{2}$$

$$\underline{e}(k) = \underline{x}_m(k) - \underline{x}_p(k) \tag{3}$$

Then ;

$$\underline{e}(k+1) = \underline{\underline{A}}_m \underline{e}(k) + (\underline{\underline{A}}_m - \underline{\underline{A}}_p)\underline{x}_p(k) + \underline{\underline{B}}_m \underline{r}(k) - \underline{\underline{B}}_p \underline{u}(k) \tag{4}$$

A switching hyperplane as a function of the error space is defined as follows.

$$\underline{s}(\underline{e}(k)) = \underline{\underline{G}}\underline{e}(k)$$

In the sliding mode $\underline{s}(\underline{e}(k))$ is desired to be zero and then maintained. i.e.

$$\underline{s}_e(k+1) - \underline{s}_e(k) = 0 \qquad k > \ell \tag{5}$$

$$k = 0, \ldots N$$

where ℓ is the desired number of steps to reach $\underline{s}_e(k) = 0$

From Eq. 5,

$$\underline{u}_{eq}(k) = \left[(\underline{\underline{G}}\underline{\underline{B}}_p)^{-1}\underline{\underline{G}}\right]\left[(\underline{\underline{A}}_m-\underline{\underline{I}})\underline{e}(k) + (\underline{\underline{A}}_m-\underline{\underline{A}}_p)\underline{x}_p(k) + \underline{\underline{B}}_m\underline{r}(k)\right]$$

with

$$\underline{s}_e(k) = \underline{\underline{G}}\underline{e}(k) = 0$$

Upon substitution into Eq.4, we obtain

$$\underline{e}(k+1) = \left[\underline{\underline{I}}-\underline{\underline{B}}_p(\underline{\underline{G}}\underline{\underline{B}}_p)^{-1}\underline{\underline{G}}\right]\left[\underline{\underline{A}}_m\underline{e}(k) + (\underline{\underline{A}}_m-\underline{\underline{A}}_p)\underline{x}_p(k) + \underline{\underline{B}}_m\underline{r}(k)\right] \tag{6}$$

For the perfect model following

$$\text{rank}(\underline{\underline{B}}_p) = \text{rank}(\underline{\underline{B}}_p : \underline{\underline{B}}_m) = \text{rank}(\underline{\underline{B}}_p : (\underline{\underline{A}}_m - \underline{\underline{A}}_p))$$

as it has been explained in reference 5.

If the perfect model following conditions exist, then Eq.6 is reduced into the from given below.

$$\underline{e}(k+1) = \left[\underline{\underline{I}} - \underline{\underline{B}}_p (\underline{\underline{G}}\underline{\underline{B}}_p)^{-1} \underline{\underline{G}} \right] \underline{\underline{A}}_m \underline{e}(k)$$

with

$$\underline{s}_e(k) = \underline{\underline{G}}\underline{e}(k) = 0$$

The design problem is to adjust the $\underline{\underline{G}}$ matrix such that the system in Eq.7 is stable. i.e. $\lim_{k\to\infty} e(k) = 0$

The control input can be found by the same idea introduced in the previous chapter.

First, a step vector is selected

$$\underline{\S} = \frac{\underline{s}_e(0)}{\ell}$$

If there is noise added into the system, $\underline{\S}$ should be updated for every step $k < \ell$. i.e.

$$\underline{\S}(\ell) = \frac{\underline{s}_e(k)}{\ell}$$

where

$$\ell = N, \ldots 1$$

$$k = 0, \ldots N-1$$

Then the inequality below

$$\underline{s}_e(k+1) = \underline{s}_e(k) - \S(\ell)$$

gives

$$\underline{u}_{se}(k) = (\underline{\underline{G}}\,\underline{\underline{B}}_p)^{-1}\underline{\underline{G}}\Big[(\underline{\underline{A}}_m - \underline{\underline{I}})\underline{e}(k) + (\underline{\underline{A}}_m - \underline{\underline{A}}_p)\underline{x}_p(k)$$

$$+ \underline{\underline{B}}_m\underline{r}(k)\Big] - (\underline{\underline{G}}\,\underline{\underline{B}}_p)^{-1}\underline{\S}(\ell) \qquad k < \ell$$

After $\ell=N$ steps $\underline{s}_e(k) = 0$, then it has to be maintained. i.e.

$$\underline{s}_e(k+1) - \underline{s}_e(k) = 0$$

Again, in order to counteract the disturbances into the system, the following adaptation is utilized.

$$\underline{s}_e(k+1) = \underline{s}_e(k) - \underline{\eta}_{em}(k) \qquad (8)$$

where

$\underline{\eta}_e(k) = \underline{s}_{em}(k)$ = measured value of $\underline{s}_e(k)$

It follows directly from (8) that

$$\underline{u}_{se}(k) = (\underline{\underline{G}}\,\underline{\underline{B}}_p)^{-1}\underline{\underline{G}}\left[(\underline{\underline{A}}_m-\underline{\underline{I}})\underline{e}(k) + (\underline{\underline{A}}_m-\underline{\underline{A}}_p)\underline{x}_p(k) + \underline{\underline{B}}_m\underline{r}(k)\right]$$

$$- (\underline{\underline{G}}\,\underline{\underline{B}}_p)^{-1}\underline{\eta}_e(k) \qquad k > \ell$$

VIII. MINIMIZATION OF THE SWITCHING HYPERPLANE

Consider the plant

$$\underline{x}(k+1) = \underline{\underline{A}}\underline{x}(k) + \underline{\underline{B}}\underline{u}(k) \tag{1}$$

with the switching hyperplane

$$\underline{s}(k) = \underline{\underline{G}}\underline{x}(k)$$

where $\underline{s}$ is (mx1) vector and $\underline{\underline{G}}$ is (mxn) matrix.

Define the performance index

$$J_N = \sum_{i=1}^{N} \left(\underline{s}^T(i)\underline{s}(i) + \underline{u}^T(i-1)\underline{\underline{R}}(i-1)u(i-1) \right) \tag{2}$$

We begin by defining V_N to be the minimum value of the performance measure J_N in Eq.2.

$$V_N = \min_{u(0)u(1)\ \ldots\ u(N-1)} \sum_{i=1}^{N} \Big(\underline{s}^T(i)\underline{s}(i) + \underline{u}^T(i-1)\underline{\underline{R}}(i-1)\underline{u}(i-1) \Big) \tag{3}$$

Using the principle of optimality, we proceed by starting with the last stage of control in our problem.

$$V_1 = \min_{u(N-1)} \left(\underline{s}^T(N)\underline{s}(N) + \underline{u}^T(N-1)\underline{\underline{R}}(N-1)u(N-1) \right) \tag{4}$$

where

$$\underline{s}(N) = \underline{\underline{G}}\underline{x}(N) \tag{5}$$

$$\underline{x}(N) = \underline{\underline{A}}\underline{x}(N-1) + \underline{\underline{B}}\underline{u}(N-1)$$

Upon substitution in Eq.5, we obtain

$$\underline{s}(N) = \underline{\underline{G}}\left[\underline{\underline{A}}\underline{x}(N-1) + \underline{\underline{B}}\underline{u}(N-1)\right]$$

Then, V_1 becomes

$$V_1 = \min_{\underline{u}(N-1)} \Big[(\underline{\underline{A}}\underline{x}(N-1) + \underline{\underline{B}}\underline{u}(N-1))^T \underline{\underline{G}}^T \underline{\underline{G}} (\underline{\underline{A}}\underline{x}(N-1) + \underline{\underline{B}}\underline{u}(n-1))$$

$$+ \underline{u}^T(N-1)\underline{\underline{R}}(N-1)\underline{u}(N-1) \Big]$$

If we drop the time argument for simplicity

$$V_1 = \min_{\underline{u}(N-1)} \Big[\underline{x}^T \underline{\underline{A}}^T \underline{\underline{G}}^T \underline{\underline{G}}\underline{\underline{A}}\underline{x} + \underline{x}^T \underline{\underline{A}}^T \underline{\underline{G}}^T \underline{\underline{G}}\underline{\underline{B}}\underline{u} + \underline{u}^T \underline{\underline{B}}^T \underline{\underline{G}}^T \underline{\underline{G}}\underline{\underline{A}}\underline{x}$$

$$+ \underline{u}^T \underline{\underline{B}}^T \underline{\underline{G}}^T \underline{\underline{G}}\underline{\underline{B}}\underline{u} + \underline{u}^T \underline{\underline{R}}\underline{u} \Big]$$

Denote $G^TG = Q$, it is easily seen that Q is automatically a positive semidefinite symmetric matrix.

$$V_1 = \min_{u(N-1)} \left[x^TA^TQAx + x^TA^TQBu + u^TB^TQAx + u^T(B^TQB + R)u \right] \quad (6)$$

since Q is symmetric

$$(u^TB^TQAx)^T = x^TA^TQ^TBu = x^TA^TQBu$$

Then, the third term in Eq.6 is the transpoze of the second term. Since both are scalars, the two terms are equal.

Therefore, we write

$$V_1 = \min_{u(N-1)} \left[x^TA^TQAx + 2x^TA^TQBu + u^T(B^TQB + R)u \right] \quad (7)$$

We obtain the minimum in Eq.7 by setting the gradient of the terms with respect to u equal to zero. Then, we have

$$2x^TA^TQB + 2u^T(B^TQB \quad R) = 0$$

Solving for u we see that

$$u(N-1) = -\left[B^TQB + R \right]^{-1} B^TQAx(N-1)$$

As it is seen, if R is selected as a positive definite matrix, the resulting control law is physically realizable and additionally is linear and involves feedback of the current state.

We define

$$\underline{\underline{L}}(N-1) = -\left[\underline{\underline{B}}^T \underline{\underline{Q}}\underline{\underline{B}} + \underline{\underline{R}}\right]^{-1} \underline{\underline{B}}^T \underline{\underline{Q}}\underline{\underline{A}}$$

Then,

$$\underline{u}(N-1) = \underline{\underline{L}}(N-1)\underline{x}(N-1)$$

As the reader will readily recall, in the discrete time optimal regulator problem, the following performance measure is selected. (See optimal control by Meditch)

$$J_N = \sum_{i=1}^{N} \left[\underline{x}^T(i)\underline{\underline{C}}(i)\underline{x}(i) + \underline{u}^T(i-1)\underline{\underline{R}}(i-1)\underline{u}(i-1)\right] \quad (8)$$

For the plant in Eq.1, if we evaluate V_N for the above J_N, it becomes

$$V_N = \min_{\underline{u}(N-1)} \left[\underline{x}^T \underline{\underline{A}}^T \underline{\underline{C}}\underline{\underline{A}}\underline{x} + 2\underline{x}^T \underline{\underline{A}}^T \underline{\underline{C}}\underline{\underline{B}}\underline{u} + \underline{u}^T (\underline{\underline{B}}^T \underline{\underline{Q}}\underline{\underline{B}} + \underline{\underline{R}})\underline{u}\right]$$

This is the same as Eq.6 with the exception that $\underline{\underline{C}}$ is replaced by $\underline{\underline{Q}}$ which is the product of the switching hyperplane matrix by its transpoze. i.e. $\underline{\underline{G}}\underline{\underline{G}}^T$. The design approach is different in our case. Rather than selecting a $\underline{\underline{C}}$ being at least positive semidefinite and selecting a performance index as in Eq.8 which is to regulate the states, we attempt to minimize the switching plane resulting in a new system having properties as explained in the previous chapters and we regulate the states at the same time.

If we continue to derive the performance measure for the N-stages, we get the following equations.

$$\underline{u}(k) = \underline{\underline{L}}(k)\underline{x}(k)$$

$$\underline{\underline{L}}(k) = -\left[\underline{\underline{B}}^T\underline{\underline{W}}(k+1)\underline{\underline{B}} + \underline{\underline{R}}\right]\underline{\underline{B}}^T\underline{\underline{W}}(k+1)\underline{\underline{A}}$$

$$\underline{\underline{W}}(k) = \underline{\underline{A}}^T\underline{\underline{W}}(k+1)\underline{\underline{A}} + \underline{\underline{A}}^T\underline{\underline{W}}(k+1)\underline{\underline{B}}L(k) + Q(k)$$

for k = N-1, N-2 ... 0

Where

$$\underline{\underline{W}}(N) = \underline{\underline{Q}}(N)$$

$\underline{\underline{B}}^T\underline{\underline{W}}(k+1)\underline{\underline{B}} + \underline{\underline{R}}$ is required to be positive definite for all k.

IX. SIMULATION STUDIES

9.1 Simulation Results for CVSS (Chapter 3)

a. Scalar Control

A continuous time unstable system

$$\dot{\underline{x}}(t) = \begin{bmatrix} 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ 12 & 16 & 1 & -4 \end{bmatrix} \underline{x}(t) + \begin{bmatrix} 0 \\ 0 \\ 0 \\ 20 \end{bmatrix} u(t) + \begin{bmatrix} 0 \\ 0 \\ 0 \\ 1 \end{bmatrix} f(t)$$

having eigenvalues

$$\underline{\lambda} = \begin{bmatrix} 2 & -1 & -2 & -3 \end{bmatrix}^T$$

is simulated.

First the switching plane

$$s_1 = \begin{bmatrix} 12 & 19 & 8 & 1 \end{bmatrix} \underline{x}(k) \qquad (1)$$

is selected which results in new system eigenvalues that are found to be

$$\underline{\lambda}_1 = \begin{bmatrix}-1 & -3 & -4\end{bmatrix}^T$$

selecting another switching plane given by

$$s_2 = \begin{bmatrix}6 & 11 & 6 & 1\end{bmatrix}\underline{x}(k) \qquad (2)$$

the new system eigenvalues become

$$\lambda_2 = \begin{bmatrix}-1 & -2 & -3\end{bmatrix}^T$$

and when the performance of the two resulting new systems are compared, it is seen that the system behaviour with the switching plane in Eq.1 is faster than the behaviour with the switching plane in Eq.2 This verifies the design of pole placement in VSS. Different switching planes are selected and it is observed that if it is desired to speed up the response of the system, switching planes having larger coefficients must be chosen which in turn require large controller parameters and as a result a large control effort is needed. Therefore, a compromise between the speed and the control has to be made. Much of the control effort is used at the inital time to force the states to come to the switching plane no matter whether the system is stable or unstable.

The most favorable aspect of VSS is that in case additive noise is present in the system and the parameters are varied, the response of the system doesn't change as it has been verified by simulation studies.

b. Multivariable Control With Control Hierarchy Method

The Following Plant

$$\underline{x}(t) = \begin{bmatrix} 0 & 1 & 0 & 0 \\ 2 & 1 & 3 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 2 & 3 & 3 \end{bmatrix} \underline{x}(t) + \begin{bmatrix} 0 & 0 \\ 1 & 2 \\ 0 & 0 \\ 2 & 1 \end{bmatrix} \begin{bmatrix} u_1 \\ u_2 \end{bmatrix}$$

with switching planes

$$s_1 = 2x_1 + x_2$$

$$s_2 = 2x_3 + x_4$$

is simulated. The aim is to regulate the states x_1 and x_3 with inital conditions.

$$x(0) = \begin{bmatrix} 2 \\ 0 \\ 2 \\ 0 \end{bmatrix}$$

First, the hierarchy $s_1 \rightarrow s_2$ is imposed and for the controller

$$u_j(t) = \left[- \sum_{i=1}^{4} \alpha_{ij} \left| x_i(t) \right| \right] \mathrm{sgn}(s_j)$$

$i = 1, \ldots 4$

$j = 1,2$

the following controller parameters

$$\underline{\alpha} = \begin{bmatrix} 7 & 8 & 8 & 7 \\ -2 & -2 & -3 & -3 \end{bmatrix}$$

are used. Then, the hierarchy is rewersed. i.e. $s_2 \rightarrow s_1$ and the controller parameters in this case are

$$\underline{\alpha} = \begin{bmatrix} -2 & -1 & -2 & -4 \\ 5 & 5 & 8 & 14 \end{bmatrix}$$

Observing the simulation results shown in Figure 9.1., we see that response speed changes when the hierarchy is changed.

Furthermore it is observed that when the hierarchy is changed the roles of the control inputs are interchanged, although the values are not exactly the same due to the different parameter values determining the control inputs as derived in detail in Chapter 3. As mathematically proved in chapter 3, when the hierarchy is changed, the bounds for the controller parameters are different in each case.

Simulation results have also verified that in case the hierarchy is $s_1 \rightarrow s_2$, s_1 converges faster, while in the reverse order s_2 converges.

9.2. Simulation Results for Model Following VSS (Chapter 4)

a. Model Following

An unstable plant given below

$$\dot{\underline{x}}(t) = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ -4 & 4 & 1 \end{bmatrix} \underline{x}(t) + \begin{bmatrix} 0 \\ 0 \\ 10 \end{bmatrix} u(t) + \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix} f(t) \quad (3)$$

is desired to follow the model described by

$$x_m(t) = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ -6 & -11 & -6 \end{bmatrix} x_m(t) + \begin{bmatrix} 0 \\ 0 \\ 6 \end{bmatrix} r(t) \quad (4)$$

Inital conditions are $\underline{x}_p = \begin{bmatrix} -1 \\ 0 \\ 0 \end{bmatrix} \quad \underline{x}_m = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$

A switching plane

$$\underline{s}(t) = \begin{bmatrix} 12 & 7 & 1 \end{bmatrix} \underline{e}(t)$$

is selected. The controller parameters used in the simulation study are

$$\underline{\alpha}^T = \begin{bmatrix} -2 & -2 & -2 & -2 & -1 & -1 & -3 \end{bmatrix}$$
$$\underline{\beta}^T = \begin{bmatrix} 2 & 2 & 3 & 3 & 7 & 5 & 1 \end{bmatrix} \quad (5)$$

It is observed that whether noise is applied or not, the system behaviour is almost the same. The simulation results where noise is added into the system are tabulated in Table 9.2

b. Model Matching with VSS

In model matching as explained in Chapter 4, a plant is controlled by a reduced order model without using model following control theory.

The same plant described in Eq.3 is simulated using the same switching plane as a function of the plant states, not being the function of the error states as in the model following case.

$$s(t) = \begin{bmatrix} 12 & 7 & 1 \end{bmatrix} x_p(t)$$

A new switching plane

$$s'(t) = s(t) - 12\, r(t)$$

is selected where r(t) is the reference trajectory. If s'(t) = 0, then the reduced order system equations are in the form given below.

$$\dot{\underline{x}}_p(t) = \begin{bmatrix} 0 & 1 \\ -12 & -7 \end{bmatrix} \begin{bmatrix} x_{p1} \\ x_{p2} \end{bmatrix} + \begin{bmatrix} 0 \\ 12 \end{bmatrix} r(t) \qquad (6)$$

The controller parameters are

$$\underline{\alpha}^T = \begin{bmatrix} -3 & -5 & -5 \end{bmatrix}$$

$$\underline{\beta}^T = \begin{bmatrix} 4 & 3 & 3 \end{bmatrix}$$

It is seen that the controller is simpler than the controller used in model following cases. Moreover, in the two cases x_{p1} is desired to track r(t), in model matching case the speed with which x_{p1} tracks r(t) is higher than in the model following cases since the plant is forced to behave as the system in Eq.6 which is a reduced order model. In cases where r(t) has a sudden change as shown in Figure 9.2, the deviation of s(t) from zero becomes large and at that point a larger control value is needed as compared to the model following case, since it is necessary to make $s(t) = 0$ again.

9.3. Simulation Results for Discrete Time VSS (Chapter 5)

a. Scalar Control

A discrete time system given below

$$\underline{x}(k+1) = \begin{bmatrix} 1.00 & 0.1 & 0 & 0 \\ 0 & 1.0 & 0.1 & 0 \\ 0 & 0 & 1.0 & 0.1 \\ 7.20 & 5.40 & 0.1 & 0.4 \end{bmatrix} \underline{x}(k) + \begin{bmatrix} 0 \\ 0 \\ 0 \\ 2 \end{bmatrix} u(k)$$

$$+ \begin{bmatrix} 0 \\ 0 \\ 0 \\ 1 \end{bmatrix} f(k)$$

with a switching plane

$$s(k) = \begin{bmatrix} 60 & 47 & 12 & 1 \end{bmatrix} \underline{x}(k)$$

is simulated. The controller parameters satisfying $\left[s(k+1) - s(k)\right] . s(k) < 0$ are evaluated and the following conditions for α and β are found

$$\underline{\alpha}^T = \begin{bmatrix} 3.6 & 5.7 & 2.2 & 0.3 \end{bmatrix}$$

$$\underline{\beta}^T = \begin{bmatrix} 3.6 & 5.7 & 2.2 & 0.3 \end{bmatrix}$$

In the simulation study, the results of which are shown in Figure 9.3.a, the following values for α and β are used.

$$\underline{\alpha} = \begin{bmatrix} 3.67 \\ 6.00 \\ 2.50 \\ 0.40 \end{bmatrix} \quad \underline{\beta} = \begin{bmatrix} -0.1 \\ -0.1 \\ -0.1 \\ -0.1 \end{bmatrix}$$

It is observed that if large values of α and β are used, the response speed becomes faster. However, very large values may result in large deviation from $s(k) = 0$ and instability may occur. Therefore, while selecting the parameters values very large numerical values are not advisable. By large values, we mean that, if for example for a parameter α the condition is $\alpha > 3$, values much greater than 3 shouldn't be used.

Furthermore, one should be careful while selecting switching planes having large coefficients because initally they may take very large values and consequently it may take much time to reach them, resulting in slow system response.

The noise in Fig 9.3.a. is added into the system and as it is seen in the same figure the response is almost the same although no additional term for counteracting the noise is used and the same controller parameters are preserverd. It is seen that in the noisy case the control input changes polarity at different instants from the noiseless case control input.

b. Multivariable DVSS

The plant given below

$$\underline{x}(k+1) = \begin{bmatrix} 1.00 & 0,1 & 0 \\ 0 & 1.0 & 0,1 \\ 0.6 & -0.1 & 0.6 \end{bmatrix} x(k) + \begin{bmatrix} 0 & 0 \\ 0.1 & 0 \\ 0,2 & 1 \end{bmatrix} \begin{bmatrix} u_1 \\ u_2 \end{bmatrix}$$

$$+ \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix} f(k)$$

with a switching hyperplane

$$\underline{s}(k) = \begin{bmatrix} 2 & 3 & 1 \\ 6 & 1 & 0 \end{bmatrix} \underline{x}(k)$$

is simulated. The following controller parameters are used.

$$\underline{\underline{\alpha}}^T = \begin{bmatrix} 0 & 7 & 2 \\ 0.7 & 1.10 & 0 \end{bmatrix}$$

$$\underline{\underline{\beta}}^T = \begin{bmatrix} 0 & -0.2 & -0.2 \\ 0 & -3.0 & -0.7 \end{bmatrix}$$

The simulation results are shown in Figure 9.3.b. Although not shown in the figure, if the hierarchy is changed, the response speed of the system changes. Moreover, selection of switching hyperplanes resulting in new reduced order system equations having faster eigenvalues increases the response speed of the system. The noise added into the system doesn't effect the stability of the system behaviour even with the same controller parameters used in the noiseless case.

9.4. Simulation Results for Stepwise Adaptive DVS Control (Chapter 6)

a. Scalar Control

A discrete time system

$$\underline{x}(k+1) = \begin{bmatrix} 1,00 & 0,1 & 0 & 0 \\ 0 & 1 & 0,1 & 0 \\ 0 & 0 & 1 & 0,1 \\ 1,2 & 1,6 & 0,1 & 0,6 \end{bmatrix} \underline{x}(k) + \begin{bmatrix} 0 \\ 0 \\ 0 \\ 2 \end{bmatrix} u(k) + \begin{bmatrix} 0 \\ 0 \\ 0 \\ 1 \end{bmatrix} f(k)$$

with a switching plane

$$s(k) = \begin{bmatrix} 60 & 47 & 12 & 1 \end{bmatrix} \underline{x}(k)$$

is considered. It is desired to bring s(k) = 0 in 6 steps. The initial conditions are

$$\underline{x}(0) = \begin{bmatrix} 1 \\ 0 \\ 0 \\ 0 \end{bmatrix}$$

therefore it is easily found that $\zeta = 10$. The control vector is

$$u_s(k) = \begin{bmatrix} -0.6 & -3.8 & -2.4 & -0.4 \end{bmatrix} \underline{x}(k) - 5$$

$$u_a(k) = \begin{bmatrix} -0.6 & -3.8 & -2.4 & -0.4 \end{bmatrix} \underline{x}(k) - 0.5s_m(k)$$

In the first example, noise isn't added into the system i.e. f(k) = 0. The simulation results are shown in Figure 9.4.a. Although the design procedure does not require any particular noise model or noise statistics, in this simulation a Gaussian random noise shown in Figure 9.4.b is added into the system and the adaptation procedure is applied. It is observed that the system behaviour is almost the same with the noiseless case. The only change is in the control input.

Besides, although not shown in the figures if the step value ζ is increased with the objective to reach the switching plane earlier, the response of the system becomes faster whereas the control input increases slightly.

b. Multivariable Control

A discrete time system

$$\underline{x}(k+1) = \begin{bmatrix} 1 & 0,1 & 0 & 0 \\ 0 & 1 & 0,1 & 0 \\ 0 & 0 & 1 & 0,1 \\ 1,2 & 1,6 & 0,1 & 0,6 \end{bmatrix} x(k) + \begin{bmatrix} 0 & 0 \\ 0 & 0 \\ 0,1 & 0 \\ 0,2 & 1 \end{bmatrix} \begin{bmatrix} u_1 \\ u_2 \end{bmatrix} + \begin{bmatrix} 0 \\ 0 \\ 0 \\ 1 \end{bmatrix} f(k)$$

with inital conditions $x(0) = \begin{bmatrix} 1 \\ 0 \\ 0 \\ 0 \end{bmatrix}$ is simulated. The switching hyperpalene and control vector are

$$s(k) = \begin{bmatrix} 60 & 47 & 12 & 1 \\ 35 & 12 & 1 & 0 \end{bmatrix} \underline{x}(k)$$

$$\underline{u}(k) \quad \begin{bmatrix} 0 & -35 & -12 & -1 \\ -1.2 & 41.4 & 12 & 0,6 \end{bmatrix} x(k) - \begin{bmatrix} 35 \\ 43 \end{bmatrix}$$

The above control has a step vector $\zeta = \begin{bmatrix} 6 \\ 3,5 \end{bmatrix}$ which means that $\underline{s}(k) = 0$ is desired to be achieved in 10 steps.

In the first simulation example shown in Figure 9.4.c no noise is added into the system. In Figure 9.4.d, the results for the noisy case are shown, In that case the control vector is

$$\underline{u}(k) = \begin{bmatrix} 0 & -35 & -12 & -1 \\ -1,2 & 41.4 & 12 & 0.6 \end{bmatrix} \underline{x}(k) - \begin{bmatrix} 0 & 10 \\ 1 & -14 \end{bmatrix} \begin{bmatrix} s_{1m}(k) \\ s_{2m}(k) \end{bmatrix}$$

It is seen that the system behaviour is almost the same in the two cases. By the adaptation procedure every time the cumulative error is eliminated, except in the interval from k to k+1. This results in a small deviation in s(k) from zero. This small deviation doesn't effect the state behaviour.

Another interesting point that should be mentioned is that if it is desired to come to the switching planes in different step numbers, it is observed that the response is different in each case.

Although not shown, a noise vector $\underline{d}^T = \begin{bmatrix} 0 & 0 & 1 & 1 \end{bmatrix}$ is used, and again the simulation shows that the stability of the system isn't effected.

If it is observed carefully in DVSS, although the adaptation is inherent, the deviation of $\underline{s}(k)$ from zero is much larger than the deviation in Stepwise DVS. This is because in the stepwise DVS we control the step size.

9.5. Simulation Results for Model Following DVSS (Chapter 7)

The discrete time system

$$\underline{x}_p(k+1) = \begin{bmatrix} 1{,}0 & 0{,}1 & 0 \\ 0 & 1{,}0 & 0{,}1 \\ -0{,}4 & 0{,}4 & 1{,}1 \end{bmatrix} \underline{x}_p(k) + \begin{bmatrix} 0 \\ 0 \\ 0{,}1 \end{bmatrix} u(k) \quad \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix} f(k)$$

is desired to follow a model described by

$$\underline{x}_m(k+1) = \begin{bmatrix} 1{,}0 & 0{,}1 & 0 \\ 0 & 1 & 0{,}1 \\ -0{,}6 & -1{,}1 & 0{,}4 \end{bmatrix} \underline{x}_m(k) + \begin{bmatrix} 0 \\ 0 \\ 0{,}6 \end{bmatrix} r(k)$$

The initial conditions are

$$\underline{x}_p = \begin{bmatrix} -1 \\ 0 \\ 0 \end{bmatrix} \quad \underline{x}_m = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

A switching plane

$$s(k) = \begin{bmatrix} 35 & 12 & 1 \end{bmatrix} \underline{x}(k)$$

is selected. At the end of 7 steps, $s(k) = 0$

is desired to be reached. i.e. $\zeta_e = 5$.

The simulation results where $f(k) = 0$ are shown in Table 9.5. In the case where noise is added into the system, although there is little deterioration as compared to the noiseless case, the convergence of the plant states to the model states is perfect. The success of this adaptation is quite remarkable considering the particular extreme case simulated, where the additive noise to the system has values as large as the state values.

9.6. Simulation Results for the Minimization of the Switching Hyperplane (Chapter 8)

The following discrete time plant

$$\underline{x}(k+1) = \begin{bmatrix} 1.0 & 0,1 & 0 \\ 0 & 1 & 0,1 \\ 0.6 & -0.1 & 0.6 \end{bmatrix} \underline{x}(k) + \begin{bmatrix} 0 \\ 0 \\ 0.2 \end{bmatrix} u(k)$$

is to be regulated by a control input minimizing

$$J = \sum_{i=1}^{30} (s^T(i)\ s(i) + u^T R u)$$

where $R = 0.25$

In the simulation study performed, several switching planes are selected.It is observed that the smaller the eigenvalues of the new system, the faster the regulation time is, verifying our design approach.

In order to check the validity of the design procedure a switching plane $s = \begin{bmatrix} -12 & 1 & 1 \end{bmatrix} x(k)$ is selected which results in an unstable reduced order system with eigenvalues $\lambda_1 = 1.1$ and $\lambda_2 = 1.2$. It is observed that although s = 0 is reached, the state value increases as expected and reaches the value 25.16. These results suggest that some care should be given to avoid condidate switching planes which may result in unstable systems which can not be regulated. In Table 9.6 the simulation results are shown where $s(k) = \begin{bmatrix} 12 & 7 & 1 \end{bmatrix} \underline{x}(k)$

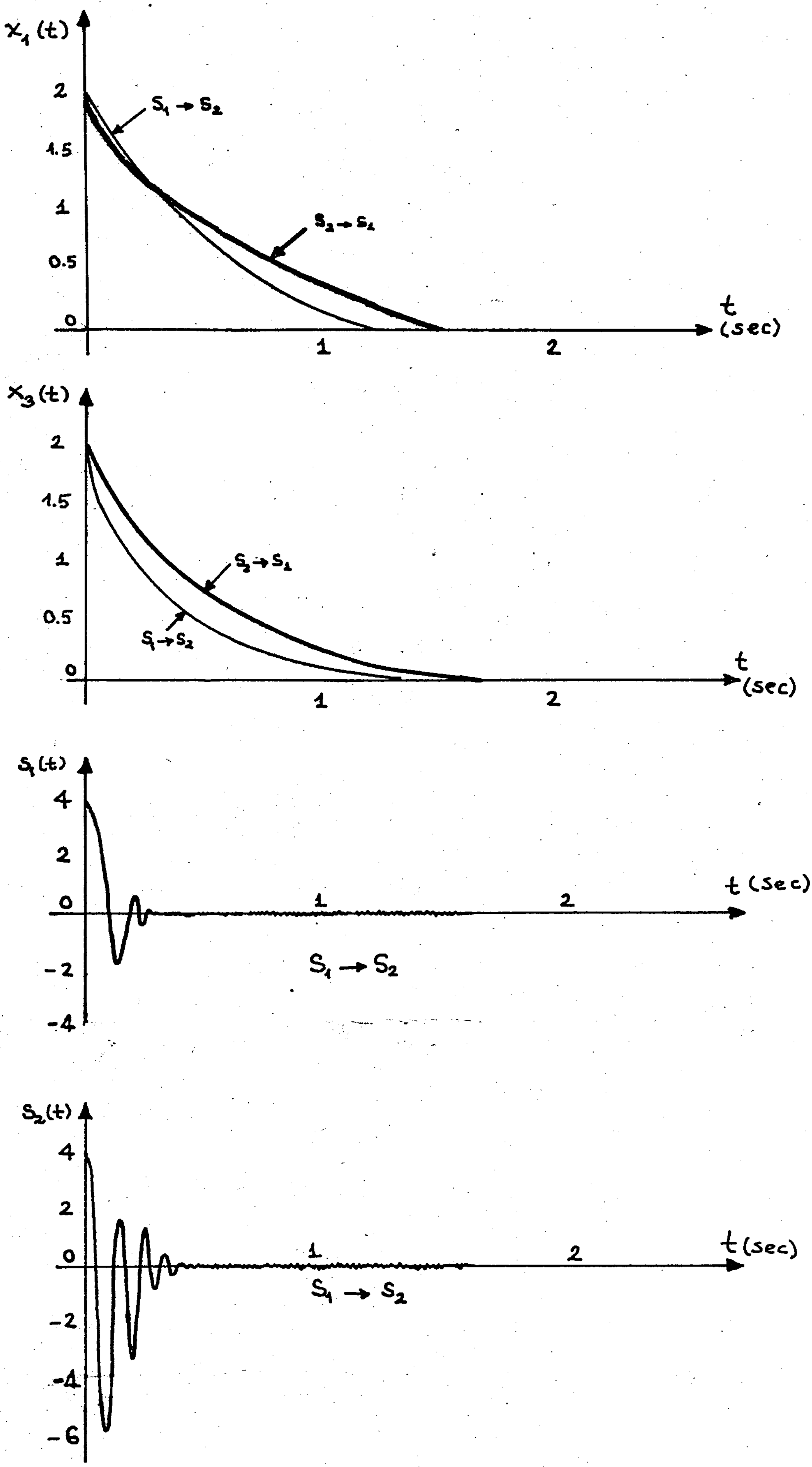

FIGURE 9.1 Simulation results for continuous time VSS control hierarchy

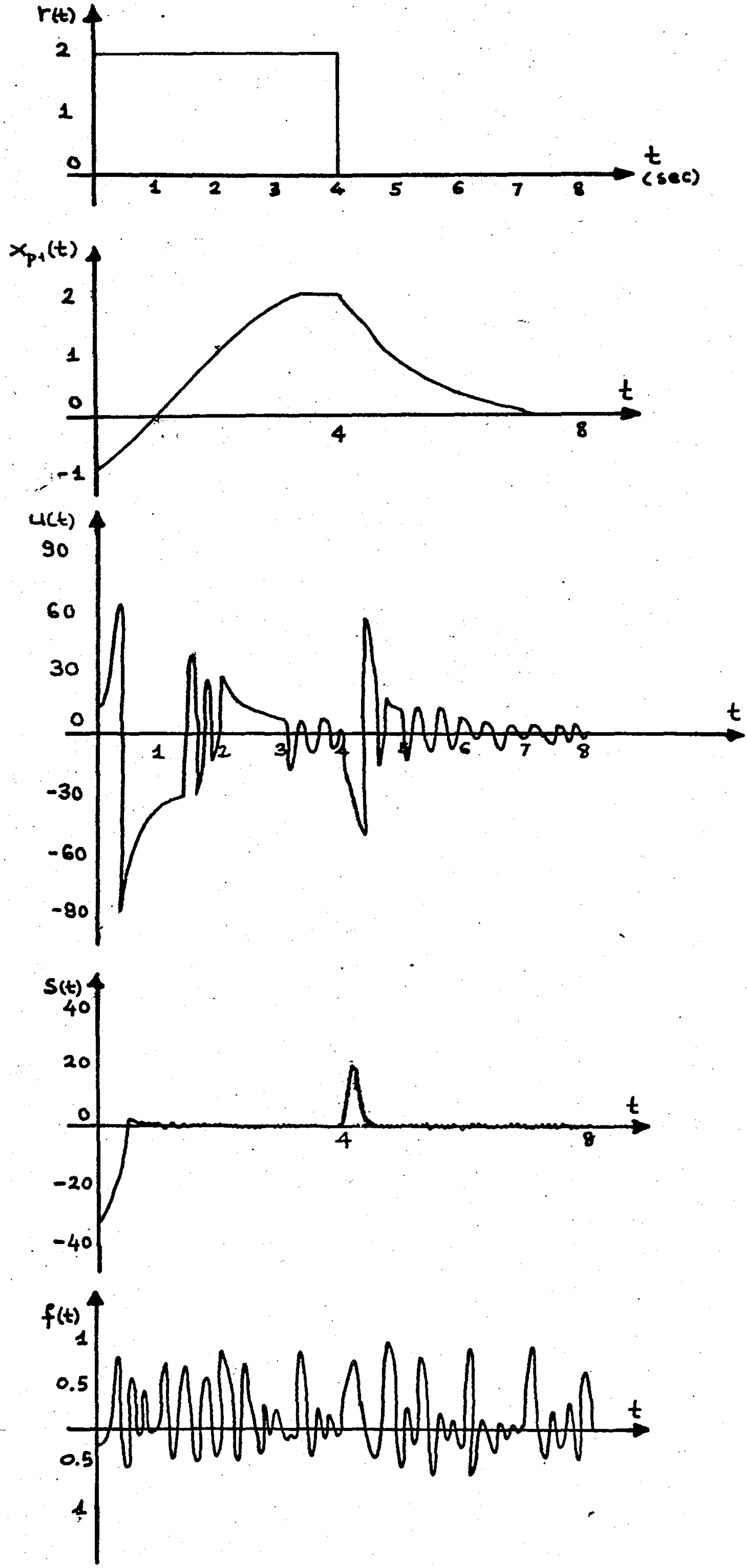

FIGURE 9.2 Simulation results for model matching

MODEL	PLANT	ERROR	NOISE	REFERENCE	SWITCHING PLANE
0.000	-1.000	1.000	-.227	2.000	12.000
.012	-.887	.899	-.085	2.000	4.559
.072	-.611	.682	.918	2.000	.191
.184	-.291	.474	-.360	2.000	1.134
.334	.030	.304	.743	2.000	-1.207
.505	.324	.181	-.184	2.000	-1.152
.682	.581	.102	.948	2.000	-.021
.855	.804	.051	-.193	2.000	-1.683
1.017	.998	.019	.602	2.000	.027
1.163	1.165	-.002	-.129	2.000	-1.005
1.293	1.307	-.015	.866	2.000	-1.014
1.406	1.429	-.023	-.043	2.000	-.972
1.504	1.532	-.029	-.096	2.000	.036
1.587	1.620	-.033	-.295	2.000	.252
1.657	1.693	-.036	.749	2.000	-1.409
1.716	1.755	-.039	-.466	2.000	.044
1.765	1.806	-.041	-.430	2.000	-1.420
1.806	1.849	-.042	.511	2.000	.063
1.840	1.884	-.044	.537	2.000	-1.349
1.869	1.914	-.046	.730	2.000	1.306
1.892	1.938	-.046	-.305	2.000	.041
1.901	1.944	-.043	-.241	0.000	.793
1.860	1.895	-.035	-.120	0.000	-.185
1.763	1.791	-.028	-.406	0.000	-1.103
1.625	1.648	-.023	-.064	0.000	-1.179
1.463	1.484	-.021	.664	0.000	.015
1.293	1.316	-.023	-.228	0.000	-1.017
1.126	1.154	-.028	.508	0.000	-.932
.969	1.000	-.031	.973	0.000	-.780
.826	.857	-.031	.986	0.000	-.525
.698	.724	-.026	-.408	0.000	-.230

TABLE 9.2 Simulation results for model following VSS

.587	.602	-.015	.933	0.000	.626
.491	.498	-.007	-.128	0.000	.457
.409	.411	-.002	-.118	0.000	.039
.340	.339	.001	-.028	0.000	-.051
.281	.279	.003	.844	0.000	.318
.233	.229	.004	-.403	0.000	.184
.192	.188	.004	.900	0.000	-.006
.158	.154	.004	-.071	0.000	.168
.130	.127	.003	-.215	0.000	.010

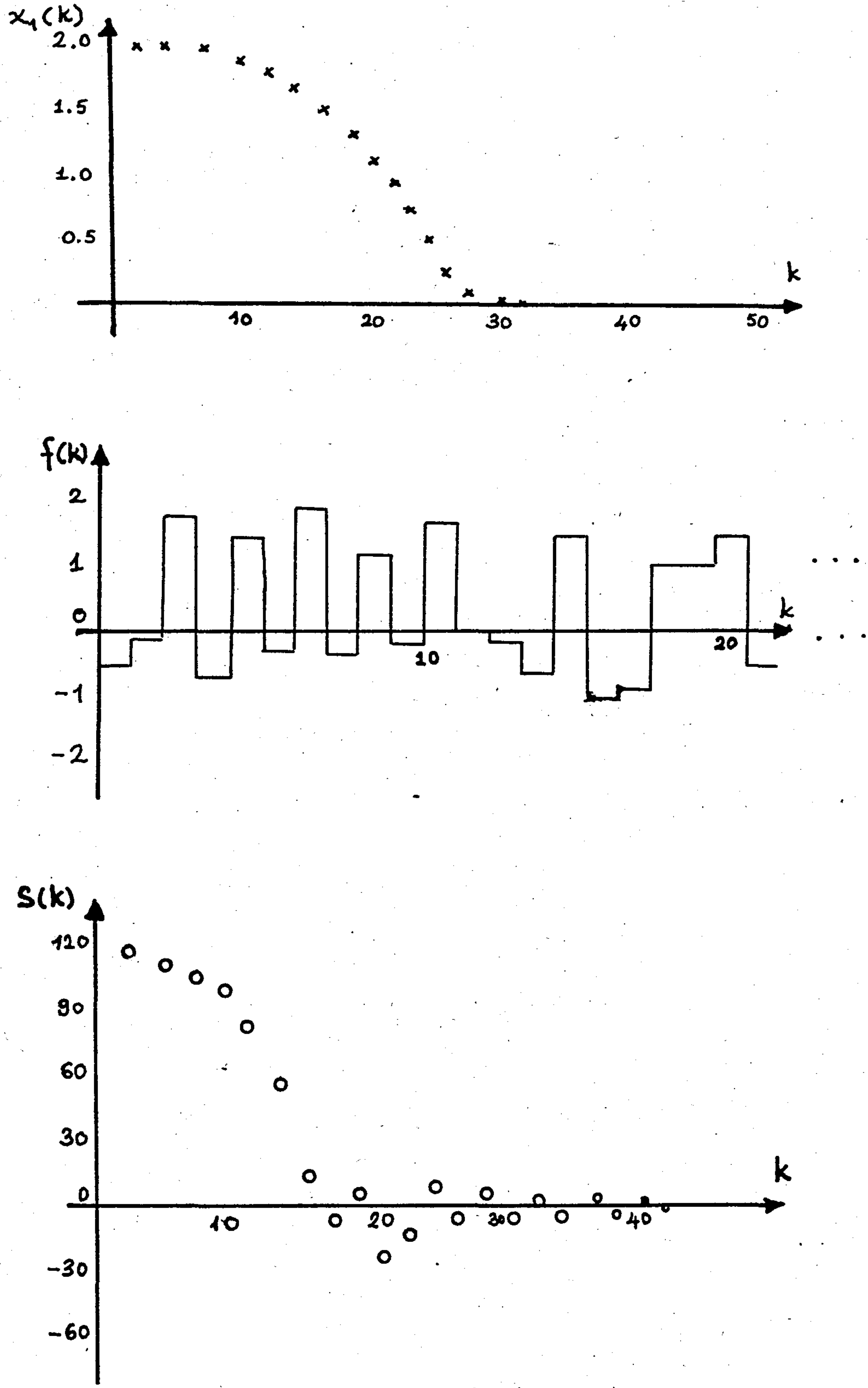

FIGURE 9.3.a Simulation results for DVSS

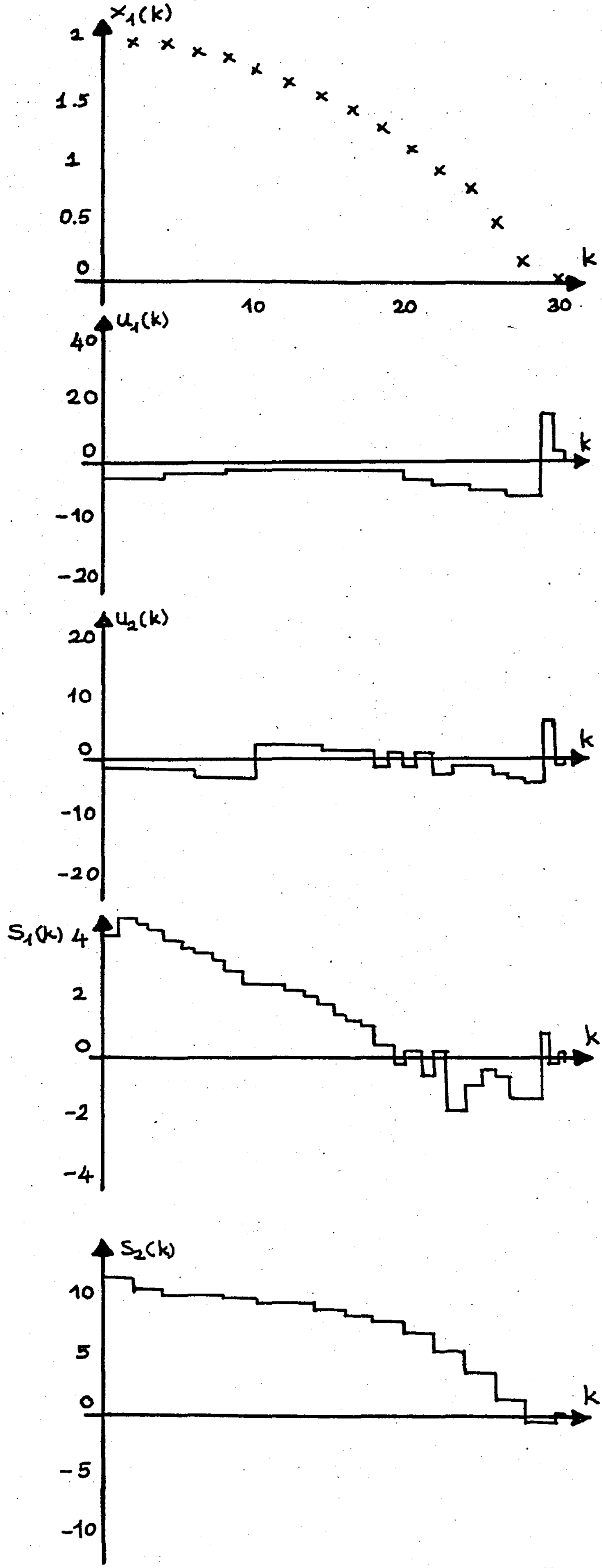

FIGURE 9.3.b Simulation results for multivariable DVSS

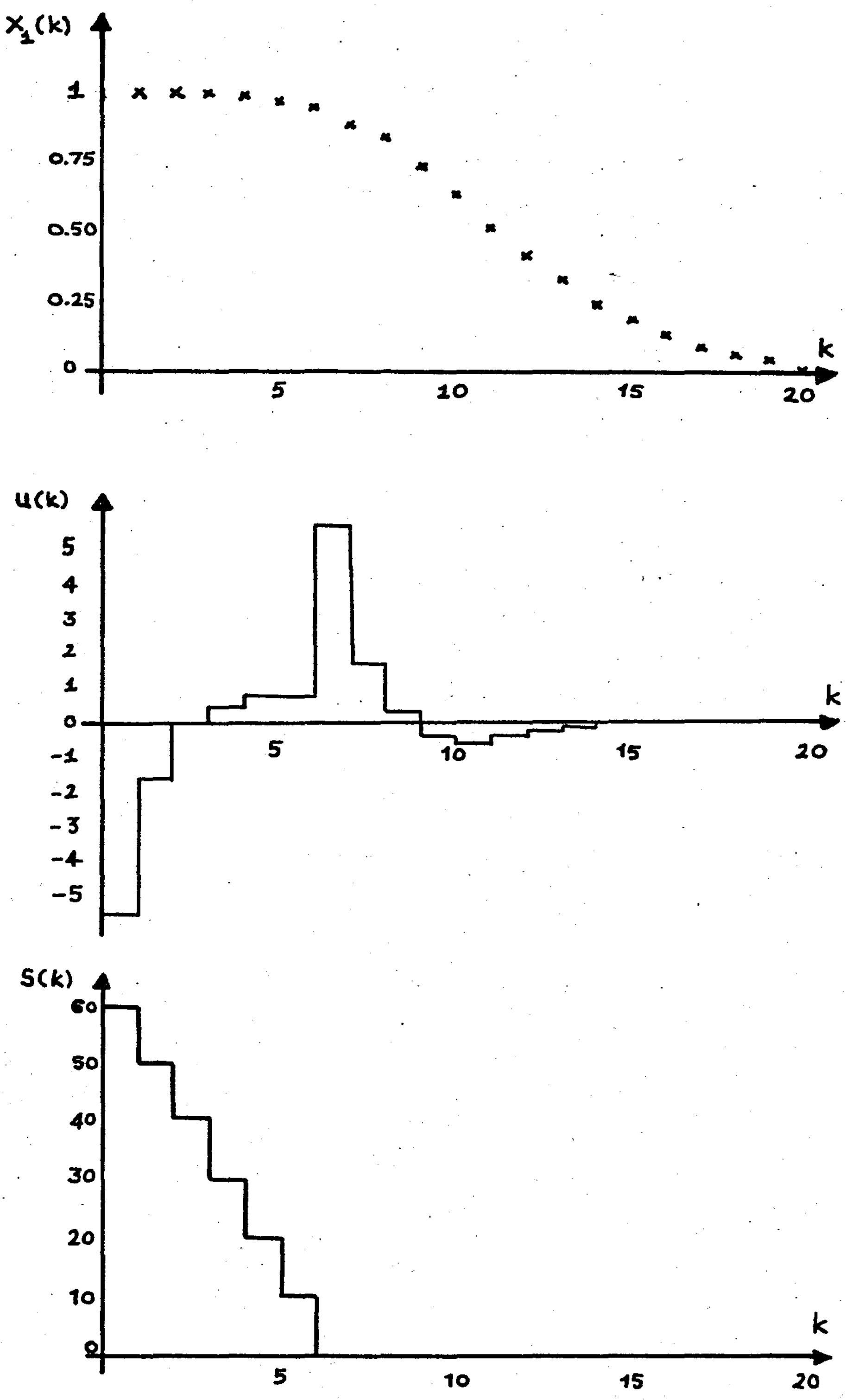

FIGURE 9.4.a Simulation results for stepwise DVSS

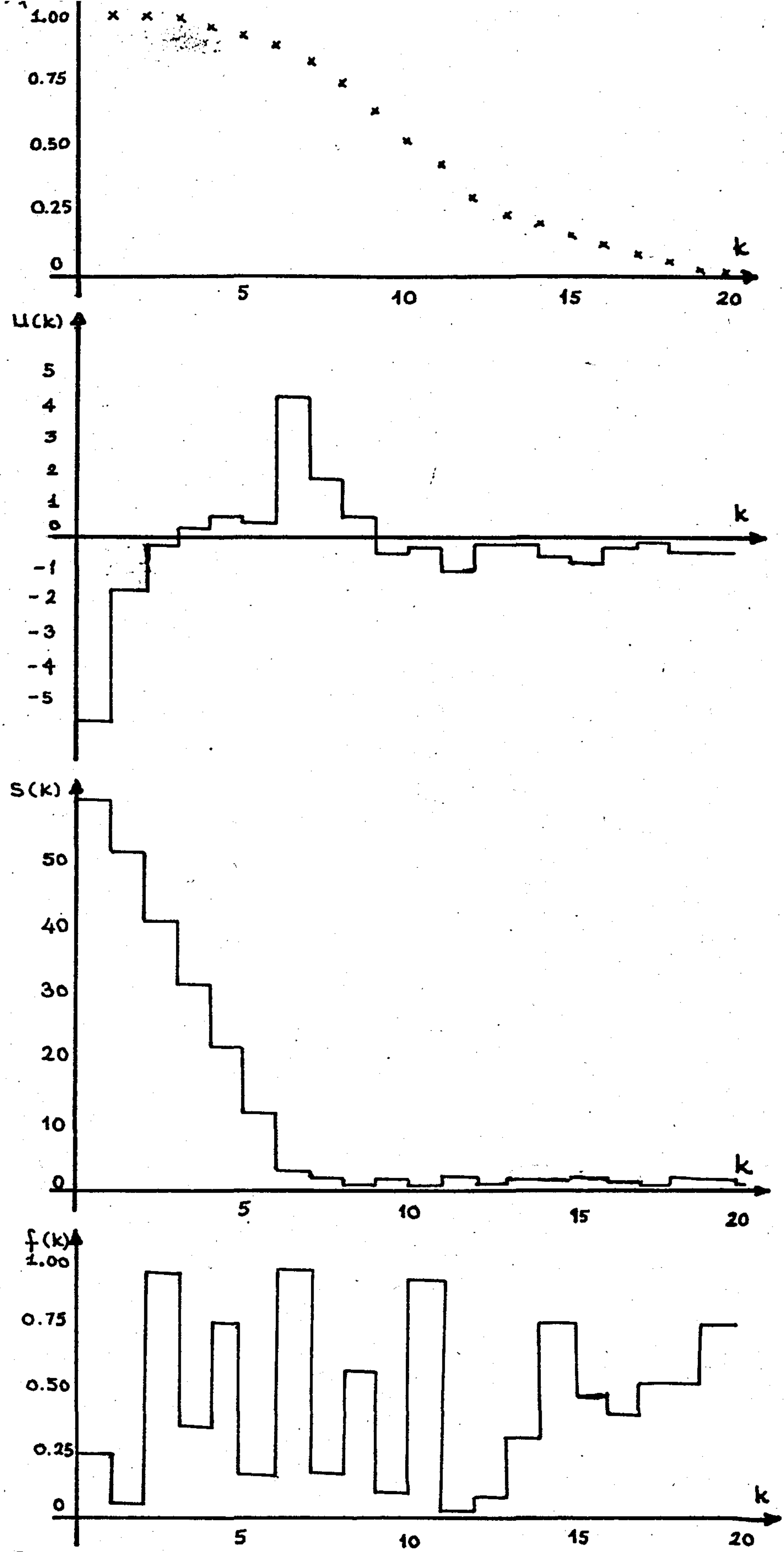

FIGURE 9.4.b Simulation results for stepwise adaptive DVSS

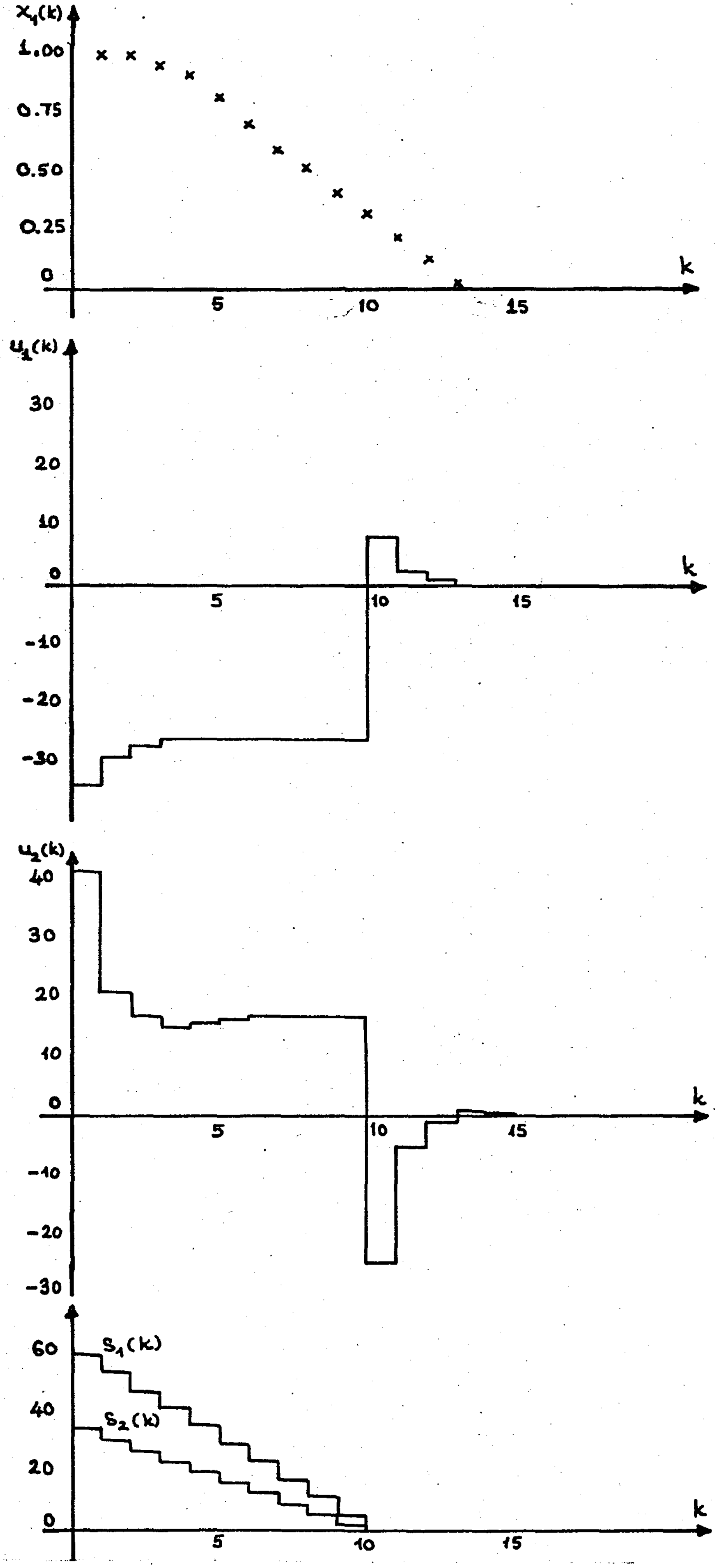

FIGURE 9.4.c Simulation results for multivariable stepwise

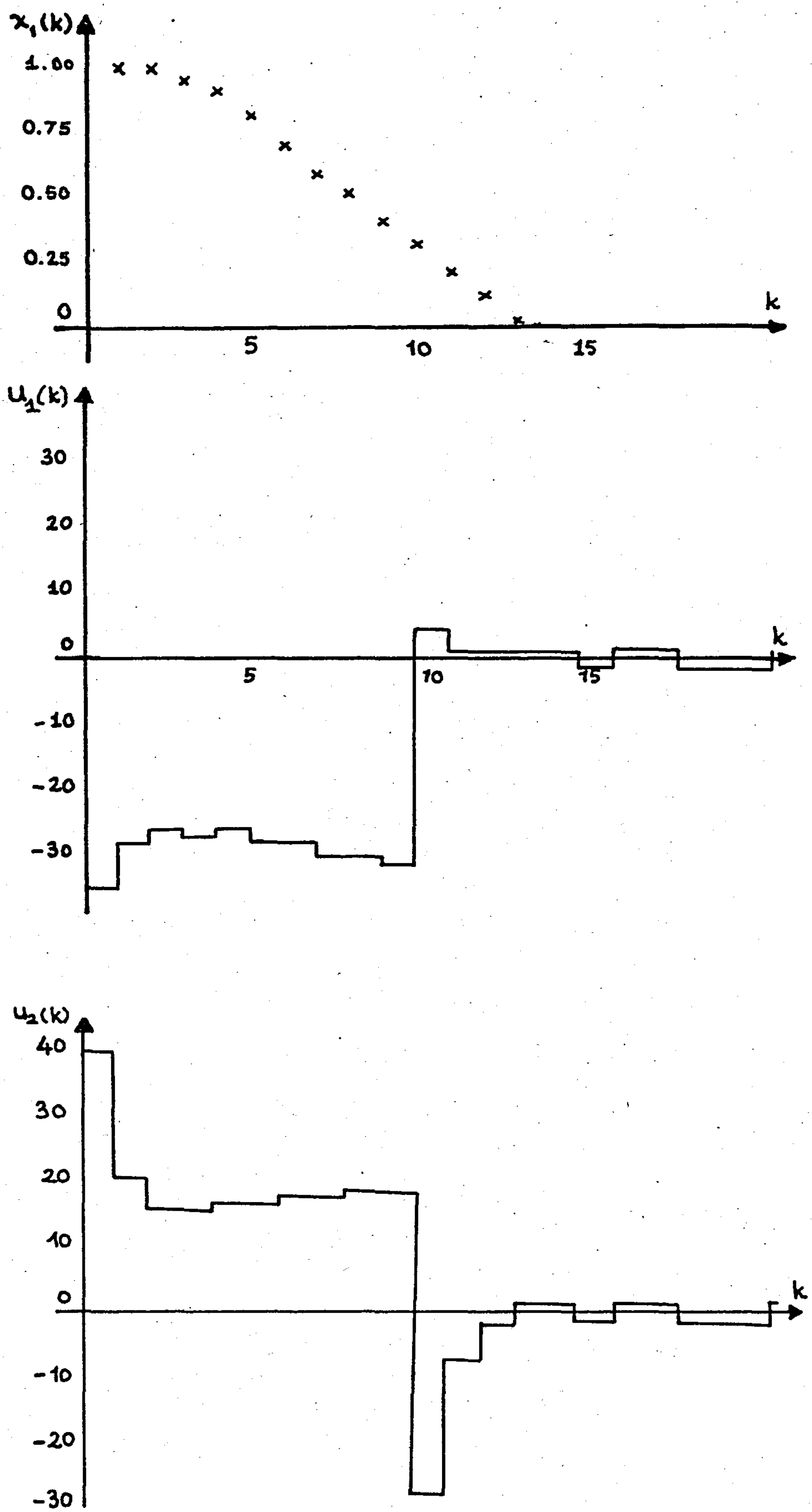

FIGURE 9.4.d Simulation results for multivariable adaptive stepwise DVSS

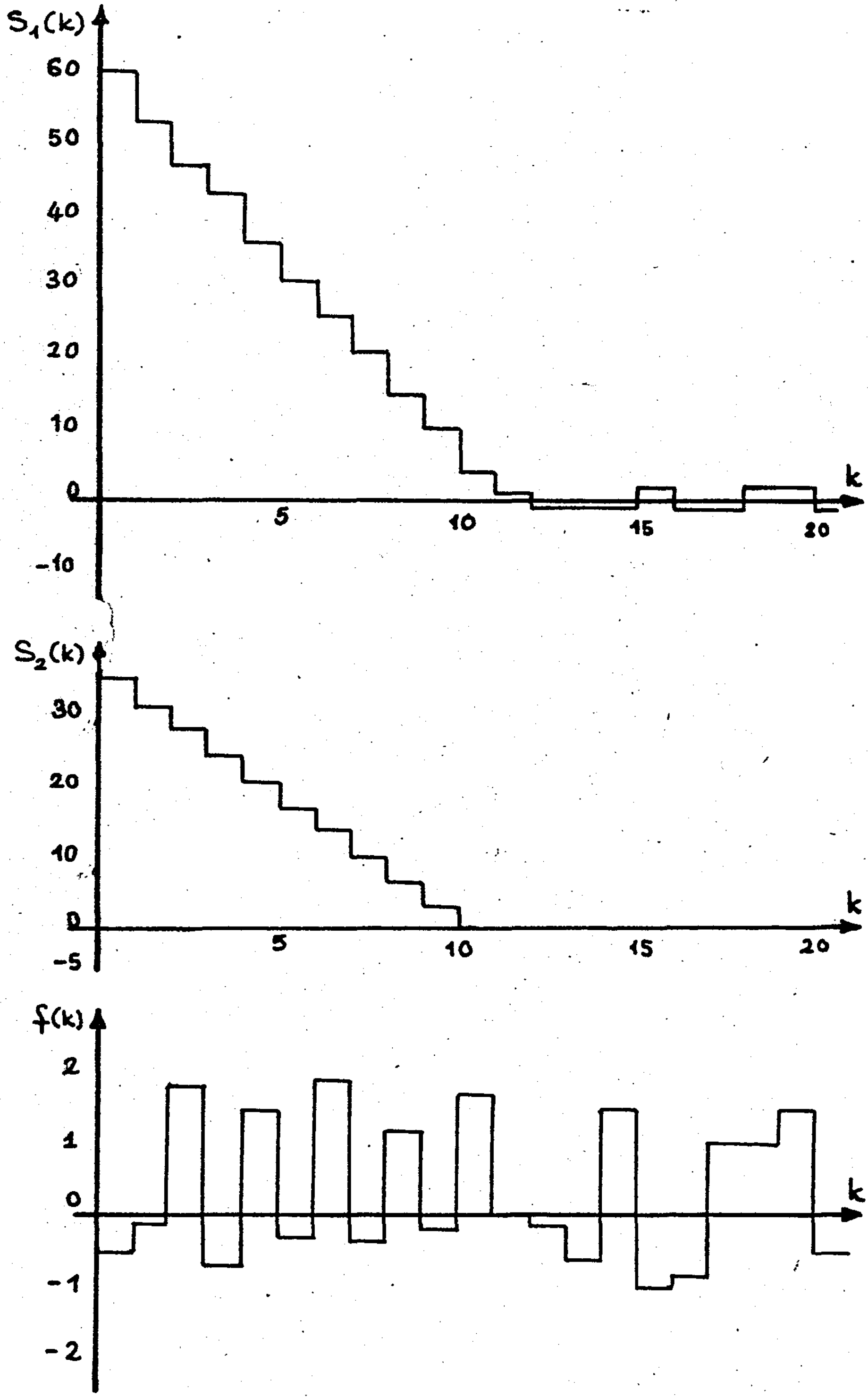

FIGURE 9.4.d Simulation results for multivariable adaptive stepwise DVSS

0.000	-1.000	52.[illegible]	35.000
0.000	-1.000	-17.200	30.000
0.000	-1.000	-26.280	25.000
.006	-.944	-23.012	20.000
.020	-.840	-18.783	15.000
.044	-.702	-15.575	10.000
.074	-.543	-13.293	5.000
.112	-.370	-61.580	.000
.154	-.189	4.822	.000
.200	-.002	12.560	.000
.248	.138	8.958	.000
.296	.239	5.073	.000
.345	.315	2.654	.000
.393	.378	1.438	.000
.439	.431	.954	.000
.483	.479	.865	.000
.525	.523	.972	.000
.565	.564	1.166	.000
.602	.602	1.392	.000
.637	.637	1.622	.000
.669	.669	1.843	.000
.699	.699	2.049	.000
.726	.726	2.239	.000
.752	.752	2.412	.000
.775	.775	2.569	.000
.796	.796	2.711	.000
.815	.815	2.839	.000
.833	.833	2.955	.000
.849	.849	3.059	.000
.863	.863	3.153	.000
.877	.877	3.237	.000
.888	.888	3.314	.000
.899	.899	3.382	.000

U(1)= -63.09	S*(1)= 34.00	X(1, 1)= 3.00	S(1,1)= -21.0
U(2)= -34.37	S*(2)= 25.18	X(1, 2)= 3.00	S(2,1)= -21.0
U(3)= -16.17	S*(3)= 16.37	X(1, 3)= 3.00	S(3,1)= -21.0
U(4)= -5.56	S*(4)= 10.77	X(1, 4)= 2.89	S(4,1)= -21.0
U(5)= .60	S*(5)= 6.50	X(1, 5)= 2.67	S(5,1)= -21.0
U(6)= 5.43	S*(6)= 4.26	X(1, 6)= 2.36	S(6,1)= -21.0
U(7)= 4.43	S*(7)= 1.29	X(1, 7)= 2.01	S(7,1)= -21.0
U(8)= 4.35	S*(8)= .67	X(1, 8)= 1.66	S(8,1)= -21.0
U(9)= 3.80	S*(9)= .05	X(1, 9)= 1.33	S(9,1)= -21.0
U(10)= 3.07	S*(10)= -.24	X(1,10)= 1.04	S(10,1)= -21.0
U(11)= 2.20	S*(11)= -.35	X(1,11)= .80	S(11,1)= -21.0
U(12)= 1.71	S*(12)= -.40	X(1,12)= .59	S(12,1)= -21.0
U(13)= 1.20	S*(13)= -.36	X(1,13)= .43	S(13,1)= -21.0
U(14)= .80	S*(14)= -.27	X(1,14)= .31	S(14,1)= -21.0
U(15)= .51	S*(15)= -.23	X(1,15)= .22	S(15,1)= -21.0
U(16)= .31	S*(16)= -.17	X(1,16)= .15	S(16,1)= -21.0
U(17)= .17	S*(17)= -.12	X(1,17)= .10	S(17,1)= -21.0
U(18)= .08	S*(18)= -.08	X(1,18)= .07	S(18,1)= -21.0
U(19)= .03	S*(19)= -.05	X(1,19)= .04	S(19,1)= -21.0
U(20)= .00	S*(20)= -.03	X(1,20)= .03	S(20,1)= -21.0
U(21)= -.01	S*(21)= -.02	X(1,21)= .02	S(21,1)= -21.0
U(22)= -.02	S*(22)= -.01	X(1,22)= .01	S(22,1)= -20.9
U(23)= -.02	S*(23)= -.00	X(1,23)= .01	S(23,1)= -20.9
U(24)= -.02	S*(24)= -.00	X(1,24)= .00	S(24,1)= -21.0
U(25)= -.01	S*(25)= .00	X(1,25)= .00	S(25,1)= -21.0
U(26)= -.01	S*(26)= .00	X(1,26)= .00	S(26,1)= -21.0
U(27)= -.01	S*(27)= .00	X(1,27)= .00	S(27,1)= -20.8
U(28)= -.01	S*(28)= .00	X(1,28)= .00	S(28,1)= -19.5
U(29)= -.00	S*(29)= .00	X(1,29)= .00	S(29,1)= -15.8
U(30)= -.00	S*(30)= .00	X(1,30)= .00	S(30,1)= -8.6

TABLE 9.6 Simulation results for the minimization of the switching hyperplane

CONCLUSIONS

The use of variable structure systems theory for the establishment of a sliding mode in a control system is effective in lowering the system sensitivity with respect to changes in the parameters of the system and the external disturbances. It should be remembered that complete invariance is achieved only when the system is in a sliding mode. The duration of the transient up to the hitting of the sliding plane should therefore be kept as small as possible. This has been made possible by the stepwise discrete variable structure controller by which the step number to reach the switching hyperplanes is set a priori.

The design technique is straightforward and requires little computational effort. Most designs can be carried out without any computer assistance. Besides, the design procedure doesn't require an exact knowledge of the system parameters. However, the control algorithm requires a precise knowledge of almost all of the states, because the switching plane value which is a function of the states has to be known so that decisions can be made on the controller parameters in discrete time variable structure systems and adaptation can become possible in stepwise DVSS.

Another aspect of the sliding mode is the inevitable chattering on the sliding plane. In order to minimize this, the system sensors should be designed carefully, not to allow too big deviations of the representative point of the system from the sliding plane. This has also been ensured by the adaptation procedure in stepwise DVS.

At the initial times before hitting occurs, increasing the values of the components of the step vector results in a better response speed and the insensitivity to plant parameter variations becomes better. As already mentioned, this is made possible by the stepwise DVS controller. Since no control on the stepsize is involved in DVS controller, this improvement of the insensitivity can be made possible by increasing the values of the controller parameters. However, after hitting the switching plane large parameters result in large deviations from the switching plane which in turn result in parameter sensitivity. Therefore, it is advisable to use stepwise DVS controller until the state values decrease to a predetermined small number and then DVS controller is applied.

The simulation studies have shown that in DVSS more control effort is used as compared to the stepwise DVS controller. This is because in stepwise DVS, the stepsize is controlled and adaptation is made according to the deviation from the switching plane so that the control effort is used optimally.

It is also advisable to use stepwise DVS controller in multivariable control since the step vector values can easily be adjusted such that the switching planes can be reached at the same time or in a hierarchical order. This isn't possible by DVS controller since in DVS a hierarchy is imposed and according to this hierarchy, the design procedure is carried out.

The most favorable aspect of DVSS and stepwise DVSS controller is that they are on line, real time and adaptive procedures and can easily be applied to both regulation and tracking problems.

If the simulation results of VSS is compared with optimal control, it is seen that more control effort is needed in VSS. This shouldn't create any problem since the control effort is obtained by using state feedback.

Moreover, the plant parameter and external disturbance insensitivity fails in optimal linear regulator design.

In such cases, the use of Kalman Estimator is needed which is a more sophisticated control algorithm as compared to the VSS control algorithm.

In time optimal control problems, switching hypersurfaces are required to be reached as in VSS system. However, in time optimal control, an analytical expression showing the behovior of the phase trajectories for different control inputs has to be known, which is generally difficult to obtain for higher order systems (n 3). In VSS, a it is sufficient to select an arbitrary switching hyperplane and adjust the coefficients of the switching hyperplane such that the resulting system behaviour is stable. This is made possible by simple substitutions without requiring knowledge about the behaviour of the phase trajectories.

Although the adaptation introduced in stepwise DVS controller works very nicely, it suffers in the interval between k and k 1. For further research, it is advisable to design a controller predicting the value of the switching plane for the next interval so that s(k) = 0 is perfectly achieved. As a matter of fact, the deviation of the switching plane value from zero is an information. It is advisable to study whether it can be possible to make identification by using this information.

The establishment of discrete variable structure controller opens up a new horizon to the digital control field which should be looked into in more detail.

```
      PROGRAM DER(ZEYNO,FERRUH,TAPE5=ZEYNO,TAPE6=FERRUH)
      DIMENSION ALF(10),BE(10),ALFL(10),ALFS(10),BEL(10),BES(10)
      DIMENSION ALFA1(10),ALFA2(10),ALFA3(10)
      DIMENSION BETA1(10),BETA2(10),BETA3(10),SE(10),EN(10)
      DIMENSION C(10),C1(10),C2(10),C3(10),T(500),U3(500)
      DIMENSION V1(10),V2(10),V3(10),V4(10),Y(10)
      COMMON/AHMET/X(5,1500),D(10),GN(1000)
      COMMON/ALI/S(1500),U(1500)
      COMMON/AYHAN/A(10,10),B(10)
      READ(5,*)N,K,H,ISET,BOUND,ADD
      READ(5,*)DEL,DEL1,DEL2,DEL3
      READ(5,*)IP,IN,IEXT,ISW,IDV,ILS
C     SWITCHING PLANE PARAMETERS
      DO 1 I=1,N
      READ(5,*)ALF(I),BE(I),ALFL(I),ALFS(I),BEL(I),BES(I)
    1 CONTINUE
      DO 7 I=1,N
      READ(5,*)ALFA1(I),ALFA2(I),ALFA3(I),BETA1(I),BETA2(I),BETA3(I)
    7 CONTINUE
      DO 8 I=1,N
      READ(5,*)C(I),C1(I),C2(I),C3(I)
    8 CONTINUE
C     OUTPUT  VECTOR &INITAL VALUES
      DO 2 I=1,N
      READ(5,*)B(I),D(I),X(I,1),SE(I),EN(I)
    2 CONTINUE
C     SYSTEM MATRIX
      READ(5,91)((A(I,J),J=1,N),I=1,N)
   91 FORMAT(4F6.3)
      IF(IP.EQ.0) GO TO 100
C     RANDOM NOISE GENERATION
      IS=456
      CALL RANSET(IS)
      DO 3 I=1,ISET
      GN(I)=RANF()
      IF(GN(I)-0.5)467,467,468
  467 GN(I)=-1*GN(I)
      GO TO 3
  468 GN(I)=GN(I)
    3 CONTINUE
  100 IF(ISW.EQ.1)GO TO 200
      IF(ILS.EQ.1)GO TO 300
  800 CALL DUYGU(DEL,BE,ALF,N,K,C)
      IF(IDV.EQ.1)GO TO 500
      GO TO  400
  300 CALL DUYGU(DEL,BEL,ALFL,N,K,C)
      UV=U(K)
      IF(ABS(UV).LT.BOUND) GO TO 400
      CALL DUYGU(DEL,BES,ALFS,N,K,C)
      GO TO 400
  200 CALL DUYGU(DEL3,BETA3,ALFA3,N,K,C3)
      UV=U(K)
      IF(ABS(UV).LT.BOUUD) GO TO 400
      CALL DUYGU(DEL2,BETA2,ALFA2,N,K,C2)
      UV=U(K)
      IF(ABS(UV).LT.BOUND) GO TO 400
      CALL DUYGU(DEL1,BETA1,ALFA1,N,K,C1)
  400 DO 10 I=1,N
      V1(I)=0.
      DO 11 J=1,N
      V1(I)=V1(I)+A(I,J)*X(J,K)
   11 CONTINUE
   10 CONTINUE
      DO 12 I=1,N
      V1(I)=V1(I)+B(I)*U(K)+D(I)*GN(K)
```

```
 12 CONTINUE
    HL=0.5
    DO 13  I=1,N
    Y(I)=X(I,K)+HL*H*V1(I)
 13 CONTINUE
    CALL RUNGA(V2,U,HL,H,N,Y)
    HL=1.
    CALL RUNGA(V3,U,HL,H,N,Y)
    CALL RUNGA(V4,U,HL,H,N,Y)
    DO 14 I=1,N
    X(I,K+1)=X(I,K)+(H/6.)*(V1(I)+2.*V2(I)+2.*V3(I)+V4(I))
 14 CONTINUE
    GO TO 600
500 DO  15 I=1,N
    X(I,K+1)=0.
    DO 16  J=1,N
    X(I,K+1)=X(I,K+1)+X(J,K)*A(I,J)
 16 CONTINUE
 15 CONTINUE
    DO 18 I=1,N
    X(I,K+1)=X(I,K+1)+B(I)*U(K)
 18 CONTINUE
600 K=K+1
    IF(K.GT.ISET) GO TO 700
    IF(ISW.EQ.1)  GO TO 200
    IF(ILS.EQ.1)  GO TO 300
    GO TO 800
700 WRITE(6,391)
    WRITE(6,322)((A(I,J),J=1,N),I=1,N)
322 FORMAT(15X,4F7.2,/)
    WRITE(6,392)
    WRITE(6,395)(B(I),I=1,N)
    WRITE(6,592)(SE(I),I=1,N)
592 FORMAT(15X,' SYSTEM EIGEN VALUES ',//,15X,3F7.2)
    WRITE(6,593)(EN(I),I=1,N-1)
593 FORMAT(15X,' NEW SYSTEM EIGEN VALUES ',//,15X,2F7.2)
    WRITE(6,393)
    IF(ISW.EQ.1) GO TO 1000
    GO TO 1100
1000 WRITE(6,395)(C1(I),I=1,N)
    WRITE(6,395)(C2(I),I=1,N)
    WRITE(6,395)(C3(I),I=1,N)
    GO TO 1600
1100 WRITE(6,395)(C(I),I=1,N)
1600 WRITE(6,394)
    IF(ISW.EQ.1)GO TO 1200
    WRITE(6,395)(ALF(I),I=1,N)
    WRITE(6,395)(BE(I),I=1,N)
    GO TO 1700
1200 WRITE(6,395)(ALFA1(I),I=1,N)
    WRITE(6,395)(ALFA2(I),I=1,N)
    WRITE(6,395)(ALFA3(I),I=1,N)
    WRITE(6,395)(BETA1(I),I=1,N)
    WRITE(6,395)(BETA2(I),I=1,N)
    WRITE(6,395)(BETA3(I),I=1,N)
391 FORMAT(15X,'SYSTEM MATRIX A;',/)
392 FORMAT(15X,'MATRIX B;',/)
393 FORMAT(15X,'SWITCHING PLANE PARAMETERS',/)
394 FORMAT(15X,'VSS    CONTROL    PARAMETERS',/)
395 FORMAT(15X,F7.2,2X,F7.2,2X,F7.2,2X,F7.2,/)
1700 WRITE(6,396)
396 FORMAT(15X,'THE  EVALUATED VALUES OF X1 ',/)
    WRITE(6,333)(X(1,I),I=1,ISET,10)
    WRITE(6,397)
397 FORMAT(15X,'THE  EVALUATED VALUES OF U  ',/)
    WRITE(6,333)(U(I),I=1,ISET,10)
```

```
      WRITE(6,398)
 398  FORMAT(15X,'THE  EVALUATED VALUES OF S  ',/)
      WRITE(6,333)(S(I),I=1,ISET,10)
      WRITE(6,598)
 598  FORMAT(15X,'NOISE ADDED INTO THE SYSTEM ',/)
      WRITE(6,333)(GN(I),I=1,ISET,10)
 333  FORMAT(15X,/,10(10F7.2,/))
      XAX=10
      YAX=13
      N=200
      J=0
      DO  1003 I=1,ISET,10
      J=J+1
      U3(J)=X(1,I)
 1003 CONTINUE
      DO 1004 I=1,J
      T(I)=I
 1004 CONTINUE
      WRITE (6,9001)
 9001 FORMAT(15X,'  THE PLOT OF THE STATE X1 ',/)
      CALL PLOT(XAX,YAX,100,U3,T,'*')
C     CALCULATION OF THE TOTAL CONTROL EFFORT
      SUM=0.
      DO  9010 I=1,ISET
      SUM=SUM+ABS(U(I))*0.01
 9010 CONTINUE
      WRITE(6,9011) SUM
 9011 FORMAT(15X,' TOTAL WEIGHTED CONTROL EFFORT =',F10.5,/)
C     CALCULATION OF MAX AND MIN CONTROL EFFORT
C     INITIALIZE LOOP INDEX AND GREAT
      I=2
      GREAT=U(1)
 9012 IF(GREAT.LT.U(I)) GREAT=U(I)
      I=I+1
      IF(I.LT.ISET) GO TO 9012
      WRITE(6,9013) GREAT
 9013 FORMAT(15X,' MAXIMUM POSITIVE VALUE OF U=',F10.5,/)
      I=2
      SMALL=U(1)
 9014 IF(SMALL.GT.U(I)) SMALL=U(I)
      I=I+1
      IF(I.LT.ISET) GO TO 9014
      WRITE(6,9015) SMALL
 9015 FORMAT(15X,' MAXIMUM NEGATIVE VALUE OF U=',F10.5,/)
      STOP
      END
      SUBROUTINE RUNGA (T,US,HL,H,NA,YI)
      COMMON/AHMET/X(5,1500),D(10),GN(1000)
      COMMON/AYHAN/A(10,10),B(10)
      DIMENSION T(10),YI(10),US(1500)
      DO 101 I=1,NA
      T(I)=0.
      DO 102 J=1,NA
      T(I)=T(I)+A(I,J)*YI(J)
 102  CONTINUE
 101  CONTINUE
      DO 202 I=1,NA
      T(I)=T(I)+B(I)*US(K)+D(I)*GN(K)
 202  CONTINUE
      DO 103 I=1,NA
      YI(I)=X(I,K)+HL*H*T(I)
 103  CONTINUE
      RETURN
      END
      SUBROUTINE DUYGU(TEL,TEB,TEF,N,K,CR)
      COMMON/ALI/Y(1500),YU(1500)
```

```
      COMMON/AMECT/TEX(5,1500)
      DIMENSION CR(10),TAL(10),TEB(10),TEF(10)
      V(K)=0.
      DO 121 I=1,N
      V(K)=V(K)+CR(I)*TEX(1,K)
  121 CONTINUE
      DO 122 I=1,N
      P=V(K)*TEX(I,K)
      IF(P)260,260,265
  260 TAL(I)=TEB(I)
      GO TO 122
  265 TAL(I)=TEF(I)
  122 CONTINUE
      IF(V(K).LT.0.)GO TO 123
      DELFA=-1.*TEL
      GO TO 124
  123 DELFA=TEL
  124 YU(K)=0.
      DO 125 I=1,N
      YU(K)=YU(K)+TAL(I)*TEX(I,K)
  125 CONTINUE
      YU(K)=YU(K)+DELFA
      RETURN
      END
      SUBROUTINE PLOT(XAX,YAX,NP,X,Y,DOT)
      DIMENSION X(0:NP),Y(0:NP),XNUM(0:15),YNUM(0:120)
      CHARACTER*1 P(0:480,0:112),DOT,HXY(2)
      CHARACTER*10 BOTLIN(15)
      CHARACTER*5 HLS(2)
      CHARACTER*3 HMM(2)
      DATA SXMAX,SYMAX,XNEAR,YNEAR/11.,45.,0.,0./
      DATA HXY/'X','Y'/,HLS/'LARGE','SMALL'/,HMM/'MAX','MIN'/
      DATA P,BOTLIN/54353*' ',15*'---------+'/
      IF(XAX.GT.SXMAX) THEN
      WRITE(6,90) HXY(1),HLS(1),SXMAX,HMM(1)
      XAX=SXMAX
      GO TO 40
      END IF
      IF(XAX.LT.3.) THEN
      XAX=3.
      WRITE(6,90) HXY(1),HLS(2),XAX,HMM(2)
      END IF
   40 IF(YAX.GT.SYMAX) THEN
      WRITE(6,90) HXY(2),HLS(1),SYMAX,HMM(1)
      YAX=SYMAX
      GO TO 41
      END IF
      IF(YAX.LT.1.5) THEN
      YAX=1.5
      WRITE(6,90) HXY(2),HLS(2),YAX,HMM(2)
      END IF
   41 XX=XAX*10.-1.
      YY=YAX*10.-1.
      XMIN=X(0)
      YMIN=Y(0)
      XMAX=X(0)
      YMAX=Y(0)
      DO 17 I=1,NP-1
      IF(X(I).LT.XMIN) XMIN=X(I)
      IF(X(I).GT.XMAX) XMAX=X(I)
      IF(Y(I).LT.YMIN) YMIN=Y(I)
      IF(Y(I).GT.YMAX) YMAX=Y(I)
   17 CONTINUE
      IXND=XX/10+1
      IYND=YY/6+1
      CALL SCALE(XX,XMAX,XMIN,IXND,12.,10.,SFX,XNUM,XNEAR,E1)
```

```
      CALL SCALE(YY,YMAX,YMIN,IYNO,8.,6.,SFY,YNUM,YNEAR,F2)
      DO 6 I=0,NP-1
      X(I)=X(I)/SFX+F1+XNEAR/SFX+0.5
      Y(I)=Y(I)/SFY+F2+YNEAR/SFY+0.5
      JXP=IFIX(X(I))
      IYP=IFIX(Y(I))
    6 P(IYP,JXP)=DOT
      WRITE(6,94)
      NPRX=IXNO*10
      NPRY=IYNO*6
      WRITE(6,83)(XNUM(I),I=0,IXNO)
      WRITE(6,82)(BOTLIN(I),I=1,IXNO)
      K=0
      L=6
      DO 8 I=0,NPRY
      IF(L.EQ.7) THEN
      L=1
      K=K+1
      END IF
      IF(L.EQ.6) THEN
      WRITE(6,80) YNUM(K),(P(I,J),J=0,NPRX)
      ELSE
      WRITE(6,81) (P(I,J),J=0,NPRX)
      END IF
    8 L=L+1
      RETURN
   94 FORMAT(1H ////1H )
   90 FORMAT(//1X,'*WARNING* SCALE FACTOR GIVEN FOR ',A1,
     &          'AXIS IS TOO      ',A5/11X,'IT IS  ',F4.1,'(',A3,'
     &          )   ASSUMED.')
   80 FORMAT(2X,E8.2,'+',115A1)
   81 FORMAT(10X,'I',115A1)
   82 FORMAT(11X,'+',11A10)
   83 FORMAT(7X,12E10.2)
      END
      SUBROUTINE SCALE(TT,TMAX,TMIN,INO,C1,C2,SFT,TNUM,TNEAR,F)
      DIMENSION TNUM(0:120)
      IF(TMIN.GE.0.) THEN
      SFT=TMAX/TT
      F=0.0
      DO 2 I=0,INO
    2 TNUM(I)=C2*I*SFT
      RETURN
      ELSE
      SFT=(TMAX-TMIN)/(TT-C1)
      F=ABS(TMIN)/SFT+C1
      DO 3 I=0,INO
    3 TNUM(I)=C2*I*SFT-F*SFT
      IF(TMAX.GT.0.0) THEN
      DO 4 I=0,INO
      IF(TNUM(I).GT.0.0) THEN
      TNEAR=TNUM(I-1)
      DO 5 K=0,INO
    5 TNUM(K)=TNUM(K)-TNEAR
      RETURN
      END IF
    4 CONTINUE
      END IF
      END IF
      RETURN
      END
16.09.10.UCLP, AA, P03    ,        0.358KLNS.
```

APPENDIX B

```
      PROGRAM FER(EPAR,NOMA,TAPE5=EPAR,TAPE6=NOMA)
C     MODEL FALLOWING VARIABLE STRUCTURE SYSTEM SIMULATION STUDY
      DIMENSION ALF(10),BE(10),C(10),S(1000),U(1000),AL(10),REF(100
      DIMENSION V1(10),V2(10),V3(10),V4(10),ER(10),Y(10),ERROR(1000
      COMMON/ALI/XM(5,1000),AM(10,10),BR(10)
      COMMON/ANI/XP(5,1000),AP(10,10),BP(10),GN(1000),D(10)
      DIMENSION TAP(150),TAM(150),TAE(150),BAL(150)
      K=1
      NR=7
      READ(5,*)N,H,R,ISET
C     INITAL CONDITIONS
      READ(5,*)(XP(I,1),I=1,N)
      READ(5,*)(XM(I,1),I=1,N)
C     SWITCHING PLANE PARAMETERS
      READ(5,*)(C(I),I=1,N)
C     VSS  CONTROL PARAMETERS
      READ(5,*)(ALF(I),I=1,NR)
      READ(5,*)(BE(I),I=1,NR)
C     INPUT  VECTORS
      READ(5,*)(BP(I),I=1,N)
      READ(5,*)(BR(I),I=1,N)
      READ(5,*)(D(I),I=1,N)
C     SYSTEM & MODEL MATRICES
      READ(5,*)((AP(I,J),J=1,N),I=1,N)
      READ(5,*)((AM(I,J),J=1,N),I=1,N)
C     REFERENCE  INPUT
      REF(1)=2.
      DO  400 I=1,200
      REF(I+1)=REF(I)
  400 CONTINUE
      DO 410 I=201,400
      REF(I+1)=REF(I)
  410 CONTINUE
      DO 420 I=401,800
      REF(I+1)=0.
  420 CONTINUE
C     RANDOM NOISE GENERATION
      IS=456
      CALL RANSET(IS)
      DO 430 I=1,ISET
      GN(I)=RANF()
      IF(GN(I)-0.5)467,467,468
  467 GN(I)=-1*GN(I)
      GO TO 430
  468 GN(I)=GN(I)
  430 CONTINUE
C     VARIABLE  STRUCTURE CONTROLLER
   70 R=REF(K)
      DO 2 I=1,N
      ER(I)=XM(I,K)-XP(I,K)
    2 CONTINUE
      ERROR(K)=XM(1,K)-XP(1,K)
      S(K)=0.
      DO 3 I=1,N
      S(K)=S(K)+C(I)*ER(I)
    3 CONTINUE
      DO 4 I=1,N
      Z=S(K)*ER(I)
      IF(Z)20,20,25
   20 AL(I)=BE(I)
      GO TO 4
   25 AL(I)=ALF(I)
    4 CONTINUE
      DO 7 I=1,N
      L=I+N
```

```
      IF(Z)30,30,35
   30 AL(L)=BE(L)
      GO TO 7
   35 AL(L)=ALF(L)
    7 CONTINUE
      NR=2*N+1
      Z=R*S(K)
      IF(Z)60,60,65
   60 AL(NR)=BE(NR)
      GO TO 9
   65 AL(NR)=ALF(NR)
    9 U(K)=0.
      DO 10 I=1,N
      J=I+N
      U(K)=U(K)-(AL(I)*ER(I)+AL(J)*XP(I,K))
   10 CONTINUE
      U(K)=U(K)-(AL(NR)*R)
      DO 11 I=1,N
      V1(I)=0.
      DO 12 J=1,N
      V1(I)=V1(I)+AM(I,J)*XM(J,K)
   12 CONTINUE
   11 CONTINUE
      DO 13 I=1,N
      V1(I)=V1(I)+BR(I)*R
   13 CONTINUE
      HL=0.5
      DO 14 I=1,N
      Y(I)=XM(I,K)+HL*H*V1(I)
   14 CONTINUE
      CALL MOD(V2,R,HL,H,N,Y,K)
      HL=1.
      CALL MOD(V3,R,HL,H,N,Y,K)
      CALL MOD(V4,R,HL,H,N,Y,K)
      DO  15 I=1,N
      XM(I,K+1)=XM(I,K)+(H/6.)*(V1(I)+2.*V2(I)+2.*V3(I)+V4(I))
   15 CONTINUE
      DO 16 I=1,N
      V1(I)=0.
      DO 17 J=1,N
      V1(I)=V1(I)+AP(I,J)*XP(J,K)
   17 CONTINUE
   16 CONTINUE
      HL=0.5
      DO 18 I=1,N
      V1(I)=V1(I)+BP(I)*U(K)
   18 CONTINUE
      DO 440  I=1,N
      V1(I)=V1(I)+D(I)*GN(K)
  440 CONTINUE
      DO 118 I=1,N
      Y(I)=XP(I,K)+HL*H*V1(I)
  118 CONTINUE
      US=U(K)
      CALL RUNGA(V2,US,HL,H,N,Y,K)
      HL=1.
      CALL RUNGA(V3,US,HL,H,N,Y,K)
      CALL RUNGA(V4,US,HL,H,N,Y,K)
      DO 19 I=1,N
      XP(I,K+1)=XP(I,K)+(H/6.)*(V1(I)+2.*V2(I)+2.*V3(I)+V4(I))
   19 CONTINUE
      K=K+1
      IF(K.LT.ISET) GO TO 70
      WRITE(6,779)
      WRITE(6,778)(U(I),I=1,ISET)
  778 FORMAT(15X,/,10(13E7.2,/))
```

```
 779 FORMAT(15X,' THE EVALUATED VALUES OF VSS CONTROL ;',//)
     JK=0.
     DO 501 I=1,ISET,20
     JK=JK+1
     S(JK)=S(I)
     XP(1,JK)=XP(1,I)
     XM(1,JK)=XM(1,I)
     ERROR(JK)=ERROR(I)
     REF(JK)=REF(I)
 501 CONTINUE
     WRITE(6,100)N,H
 100 FORMAT(15X,' ORDER  =',I4,'INTEGR',F6.3,/)
     WRITE(6,101)
     WRITE(6,108)((AP(I,J),J=1,N),I=1,N)
     WRITE(6,102)
     WRITE(6,108)((AM(I,J),J=1,N),I=1,N)
     WRITE(6,103)
     WRITE(6,108)(BP(I),I=1,N)
     WRITE(6,104)
     WRITE(6,108)(BR(I),I=1,N)
     WRITE(6,105)
     WRITE(6,108)(C(I),I=1,N)
     WRITE(6,106)
     WRITE(6,908)(ALF(I),I=1,NR)
     WRITE(6,107)
     WRITE(6,908)(BE(I),I=1,NR)
 908 FORMAT(7F5.2)
 101 FORMAT(15X,'SYSTEM MATRIX AP=',/)
 102 FORMAT(15X,'MODEL  MATRIX AM=',/)
 103 FORMAT(15X,'OUTPUT MATRIX BP=',/)
 104 FORMAT(15X,'OUTPUT MATRIX BR=',/)
 105 FORMAT(15X,'SWITCHING PLANE PARAMETERS=',/)
 106 FORMAT(15X,'VSS   CONTROL   PARAMETERS ALF=',/)
 107 FORMAT(15X,'VSS   CONTROL   PARAMETERS BE=',/)
 108 FORMAT(15X,3F12.4,//)
     WRITE(6,109)
 109 FORMAT(12X,'MODEL',12X,'PLANT',12X,'ERROR',
    *12X,'SWITCHING PLANE',5X,'REFERENCE',5X,'NOISE',/)
     WRITE(6,110)
 110 FORMAT(10X,70('*'),/)
     DO 311 I=1,JK
     WRITE(6,111)XM(1,I),XP(1,I),ERROR(I),S(I),REF(I),GN(I)
 111 FORMAT(10X,F7.3,9X,F7.3,15X,F7.3,14X,F7.3,10X,F7.3,
    *10X,F5.3,/)
 311 CONTINUE
     XAX=7
     YAX=13
     DO 5001 I=1,JK
     BAL(I)=I
5001 CONTINUE
     WRITE(6,5002)
5002 FORMAT(15X,' THE PLOT OF THE ERROR STATE ;',//)
     CALL PLOT(XAX,YAX,100,ERROR,BAL,'*')
     STOP
     END
     SUBROUTINE MOD(T,R,HL,H,NA,AY,K)
     COMMON/ALI/XM(5,1000),AM(10,10),BR(10)
     DIMENSION T(10),AY(10)
     DO 200 I=1,NA
     T(I)=0.
     DO 201 J=1,NA
     T(I)=T(I)+AM(I,J)*AY(J)
 201 CONTINUE
 200 CONTINUE
     DO 202 I=1,NA
     T(I)=T(I)+BR(I)*R
```

```
202 CONTINUE
    DO 203 I=1,NA
    AY(I)=XM(I,K)+HL*H*T(I)
203 CONTINUE
    RETURN
    END
    SUBROUTINE RUNGA(TR,US,HL,H,NA,AZ,K)
    COMMON/ANI/XP(5,1000),AP(10,10),BP(10),GN(1000),D(10)
    DIMENSION TR(10),AZ(10)
    DO 200 I=1,NA
    TR(I)=0.
    DO 201 J=1,NA
    TR(I)=TR(I)+AP(I,J)*AZ(J)
201 CONTINUE
200 CONTINUE
    DO 202 I=1,NA
    TR(I)=TR(I)+BP(I)*US +D(I)*GN(K)
202 CONTINUE
    DO 203 I=1,NA
    AZ(I)=XP(I,K)+HL*H*TR(I)
203 CONTINUE
    RETURN
    END
    SUBROUTINE PLOT(XAX,YAX,NP,X,Y,DOT)
    DIMENSION X(0:NP),Y(0:NP),XNUM(0:15),YNUM(0:120)
    CHARACTER*1 P(0:480,0:112),DOT,HXY(2)
    CHARACTER*10 BOTLIN(15)
    CHARACTER*5 HLS(2)
    CHARACTER*3 HMM(2)
    DATA SXMAX,SYMAX,XNEAR,YNEAR/11.,45.,0.,0./
    DATA HXY/'X','Y'/,HLS/'LARGE','SMALL'/,HMM/'MAX','MIN'/
    DATA P,BOTLIN/54353*' ',15*'---------+'/
    IF(XAX.GT.SXMAX) THEN
    WRITE(6,90) HXY(1),HLS(1),SXMAX,HMM(1)
    XAX=SXMAX
    GO TO 40
    END IF
    IF(XAX.LT.3.) THEN
    XAX=3.
    WRITE(6,90) HXY(1),HLS(2),XAX,HMM(2)
    END IF
 40 IF(YAX.GT.SYMAX) THEN
    WRITE(6,90) HXY(2),HLS(1),SYMAX,HMM(1)
    YAX=SYMAX
    GO TO 41
    END IF
    IF(YAX.LT.1.5) THEN
    YAX=1.5
    WRITE(6,90) HXY(2),HLS(2),YAX,HMM(2)
    END IF
 41 XX=XAX*10.-1.
    YY=YAX*10.-1.
    XMIN=X(0)
    YMIN=Y(0)
    XMAX=X(0)
    YMAX=Y(0)
    DO 17 I=1,NP-1
    IF(X(I).LT.XMIN) XMIN=X(I)
    IF(X(I).GT.XMAX) XMAX=X(I)
    IF(Y(I).LT.YMIN) YMIN=Y(I)
    IF(Y(I).GT.YMAX) YMAX=Y(I)
 17 CONTINUE
    IXNO=XX/10+1
    IYNO=YY/6+1
    CALL SCALE(XX,XMAX,XMIN,IXNO,12.,10.,SFX,XNUM,XNEAR,F1)
    CALL SCALE(YY,YMAX,YMIN,IYNO,8.,6.,SFY,YNUM,YNEAR,F2)
```

```
      DO 6 I=0,NP-1
      X(I)=X(I)/SFX+F1+XNEAR/SFX+0.5
      Y(I)=Y(I)/SFY+F2+YNEAR/SFY+0.5
      JXP=IFIX(X(I))
      IYP=IFIX(Y(I))
    6 P(IYP,JXP)=DOT
      WRITE(6,94)
      NPRX=IXNO*10
      NPRY=IYNO*6
      WRITE(6,83)(XNUM(I),I=0,IXNO)
      WRITE(6,82)(BOTLIN(I),I=1,IXNO)
      K=0
      L=6
      DO 8 I=0,NPRY
      IF(L.EQ.7) THEN
      L=1
      K=K+1
      END IF
      IF(L.EQ.6) THEN
      WRITE(6,80) YNUM(K),(P(I,J),J=0,NPRX)
      ELSE
      WRITE(6,81) (P(I,J),J=0,NPRX)
      END IF
    8 L=L+1
      RETURN
   94 FORMAT(1H ////1H )
   90 FORMAT(//1X,'*WARNING* SCALE FACTOR GIVEN FOR ',A1,
     &       'AXIS IS TOO     ',A5/11X,'IT IS  ',F4.1,'(',A3,'
     &       )  ASSUMED.')
   80 FORMAT(2X,E8.2,'+',115A1)
   81 FORMAT(10X,'I',115A1)
   82 FORMAT(11X,'+',11A10)
   83 FORMAT(7X,12E10.2)
      END
      SUBROUTINE SCALE(TT,TMAX,TMIN,INO,C1,C2,SFT,TNUM,TNEAR,F)
      DIMENSION TNUM(0:120)
      IF(TMIN.GE.0.) THEN
      SFT=TMAX/TT
      F=0.0
      DO 2 I=0,INO
    2 TNUM(I)=C2*I*SFT
      RETURN
      ELSE
      SFT=(TMAX-TMIN)/(TT-C1)
      F=ABS(TMIN)/SFT+C1
      DO 3 I=0,INO
    3 TNUM(I)=C2*I*SFT-F*SFT
      IF(TMAX.GT.0.0) THEN
      DO 4 I=0,INO
      IF(TNUM(I).GT.0.0) THEN
      TNEAR=TNUM(I-1)
      DO 5 K=0,INO
    5 TNUM(K)=TNUM(K)-TNEAR
      RETURN
      END IF
    4 CONTINUE
      END IF
      END IF
      RETURN
      END
 14.15.42.UCLP, AA, P03     ,        0.358KLNS.
```

APPENDIX C

```
      PROGRAM FERRI(AL,VER,TAPE5=AL,TAPE6=VER)
      DIMENSION X(4,400),SW(400),S(400,4),B(5),R(4,4)
      DIMENSION A(5,5),BT(5,5),Q(5,5),W(400,4,4)
      DIMENSION U(400),AT(5,5),ATW(5,5),ATWA(5,5),BTW(5,5)
      DIMENSION BTWA(5,5),ES(5,5),ATWBS(5,5)
      MUK=1
      DATA M,AR,ISET,N,IFINAL/1,0.25,31,3,31/
      DATA E1,E2/0.9,1.2/
      DATA C1,C2,C3/-2.,-1.,1./
      DATA B(1),B(2),B(3)/0.,0.,0.2/
      DATA X(1,1),X(2,1),X(3,1)/3.,0.,0./
      DATA A(1,1),A(1,2),A(1,3)/1.,0.1,0./
      DATA A(2,1),A(2,2),A(2,3)/0.,1.,0.1/
      DATA A(3,1),A(3,2),A(3,3)/0.6,-0.1,0.6/
      DATA Q(1,1),Q(1,2),Q(1,3)/10.,+0.,0./
      DATA Q(2,1),Q(2,2),Q(2,3)/0.,11.,0./
      DATA Q(3,1),Q(3,2),Q(3,3)/0.,0.,11./
C     W(N)=Q(N)=ST(N)*S(N)
      DO 2 I=1,N
      DO 3 J=1,N
      W(ISET,I,J)=Q(I,J)
    3 CONTINUE
    2 CONTINUE
C     TRANSPOZE OF MATRIX A
      DO 7 I=1,N
      DO 8 J=1,N
      AT(I,J)=A(J,I)
    8 CONTINUE
    7 CONTINUE
C     AT*W(K+1)
   70 DO 9 I=1,N
      DO 10 J=1,N
      ATW(I,J)=0.
      DO 11 L=1,N
      ATW(I,J)=ATW(I,J)+AT(I,L)*W(ISET,L,J)
   11 CONTINUE
   10 CONTINUE
    9 CONTINUE
C     (AT*W(+1)*A
      DO 12 I=1,N
      DO 13 J=1,N
      ATWA(I,J)=0.
      DO 14 L=1,N
      ATWA(I,J)=ATWA(I,J)+ATW(I,L)*A(L,J)
   14 CONTINUE
   13 CONTINUE
   12 CONTINUE
C     BT*W(K+1)
      DO 15 I=1,N
      DO 16 J=1,N
      BTW(I,J)=0.
      DO 17 L=1,N
      BTW(I,J)=BTW(I,J)+B(L)*W(ISET,L,J)
   17 CONTINUE
   16 CONTINUE
   15 CONTINUE
C     BTW*B
      DO 18 I=1,N
      BTWB=0.
      DO 19 J=1,N
      BTWB=BTWB+BTW(I,J)*B(J)
   19 CONTINUE
   18 CONTINUE
      RA=BTWB+AR
      BTW=(1/C( )
```

```
      DO 20 I=1,N
      DO 21 J=1,N
      BTWA(I,J)=0.
      DO 22 L=1,N
      BTWA(I,J)=BTWA(I,J)+BTW(I,L)*A(L,J)
   22 CONTINUE
   21 CONTINUE
   20 CONTINUE
      ISET=ISET-1
C     S(K)
      DO 24 J=1,N
      S(ISET,J)=-1.*RIN*BTWA(1,J)
   24 CONTINUE
C     B*S
      DO 25 I=1,N
      DO 26 J=1,N
      BS(I,J)=B(I)*S(ISET,J)
   26 CONTINUE
   25 CONTINUE
C     ATW*BS
      DO 28 I=1,N
      DO 29 J=1,N
      ATWBS(I,J)=0.
      DO 30 L=1,N
      ATWBS(I,J)=ATW(I,L)*BS(L,J)
   30 CONTINUE
   29 CONTINUE
   28 CONTINUE
C     NEW W(K)
      DO 31 I=1,N
      DO 32 J=1,N
      W(ISET,I,J)=ATWA(I,J)+ATWBS(I,J)+Q(I,J)
   32 CONTINUE
   31 CONTINUE
      IF(ISET.GT.1) GO TO 70
      K=1
C     CONTROL U(ISET)
   71 U(K)=0.
      DO 35 I=1,N
      U(K)=U(K)+S(K,I)*X(I,K)
   35 CONTINUE
C     STATE VALUES
      DO 36 I=1,N
      X(I,K+1)=0.
      DO 37 J=1,N
      X(I,K+1)=X(I,K+1)+A(I,J)*X(J,K)
   37 CONTINUE
   36 CONTINUE
      DO 50 I=1,N
      X(I,K+1)=X(I,K+1)+B(I)*U(K)
   50 CONTINUE
      SW(K)=C1*X(1,K)+C2*X(2,K)+C3*X(3,K)
      K=K+1
      IF(K.LT.IFINAL)GO TO 71
      WRITE(6,502) AR
  502 FORMAT(15X,'MATRIX R',/,15X,'R(1,1)=',F7.2,///)
      IF(NUK.EQ.1) GO TO 505
      WRITE(6,500)C1,C2,C3
  500 FORMAT(15X,'SWITCHING PLANE PARAMETERS',/,15X,
     $'C1=',F8.4,3X,'C2=',F8.4,3X,'C3=',F8.4,///)
      WRITE(6,501)E1,E2
  501 FORMAT(15X,'NEW  SYSTEM  EIGEN VALUES',/,15X,
     $'E1=',F8.4,3X,'E2=',F8.4,///)
  505 DO 171 I=1,N
      WRITE(6,130) (I,J,A(I,J),J=1,N)
  171 CONTINUE
```

```
 130  FORMAT(20X,'A(',I1,',',I1,')=',F5.2,3X,'A(',I1,',',I1,')=
     $',F5.2,3X,'A(',I1,',',I1,')=',F5.2,/)
      DO 172 I=1,N
      WRITE(6,131) (I,J,AT(I,J),J=1,N)
 172  CONTINUE
 131  FORMAT(20X,'AT(',I1,',',I1,')=',F5.2,3X,'AT(',I1,',',I1,')=
     $',F5.2,3X,'AT(',I1,',',I1,')=',F5.2,/)
      DO 173 I=1,N
      WRITE(6,132) (I,J,Q(I,J),J=1,N)
 173  CONTINUE
 132  FORMAT(20X,'Q(',I1,',',I1,')=',F7.2,3X,'Q(',I1,',',I1,')=
     $',F7.2,3X,'Q(',I1,',',I1,')=',F7.2,/)
      DO 60 I=1,(IFINAL-1)
      WRITE(6,100) I,U(I),I,SW(I),I,X(1,I),I,S(I,1)
  60  CONTINUE
 100  FORMAT(5X,'U(',I2,')=',F7.2,5X,'SW(',I2,')=',F7.2,
     $5X,'X(1,',I2,')=',F7.2,5X,'S(',I2,',','1)=',F7.2,/)
      STOP
      END
16.34.56.UCLP, AA, P04     ,          0.185KLNS.
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