DESIGNING AN INVENTORY CONTROL SYSTEM FOR AN INDUSTRIAL EQUIPMENT DISTRIBUTOR COMPANY

by

Mehmet Genç

B.S., Industrial Engineering, Yıldız Technical University, 2003

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Zeynep, mavişim, besides the supports you gave during this study, your smile gave me more than you can imagine, thank you for everything…

ABSTRACT

DESIGNING AN INVENTORY CONTROL SYSTEM FOR AN INDUSTRIAL EQUIPMENT DISTRIBUTOR COMPANY

The purpose of this research is finding a practical solution to an inventory management problem of a distributor company which is facing performance and delivery problems. The solution needs to be practical so that business professionals can easily customize and adapt it to their company. It is accepted for inventory managers that, a close-optimum solution is sufficient except of exact optimal solutions. For this purpose, literature is searched to find a proper heuristic algorithm.

Silver (1974) proposes a practical procedure for selecting the order–up–to levels (S), can-order points (c) and must-order points (s) of a particular coordinated control system, namely (s, c, S) policy. Demand is Poisson and a fixed non-zero replenishment lead time is assumed. Heuristic is used to help on deciding the levels of the inventory control parameters. One category is selected among the product range to examine closely and ten most profitable items are selected again from this category. According to different customer service levels and selected item group, independent (s, S) and coordinated inventory control policies (s, c, S) are modeled and run in Arena Simulation Software. Furthermore, to improve the obtained results, alternative scenarios are developed and run in Arena.

Researches show that a certain parameter combination of joint order inventory control policy gives close-optimum solutions. It can be predict by obtained results that, improvements in customer service level will increase awareness of the potential customers for zero lead time and also increase annual sales income. At the end of the research, *an easy-practical decision taking tool* is developed to use in inventory management area.

ÖZET

SANAYİ EKİPMANLARI DİSTRİBÜTÖRLÜĞÜ YAPAN BİR FİRMA İÇİN ENVANTER KONTROL SİSTEMİ DİZAYNI

Bu çalışmada, dağıtım süreçlerinde performans problemleri yaşayan bir distribütör firmanın envanterini verimli bir şekilde yönetmek için pratik bir çözüm yolu geliştirilmiştir. Önerilen çözüm yolunun kolay uygulanabilir olması için, kesin optimal çözüm olmaktansa, optimale yakın sonuçlar vermesi yeterli olacaktır. Literatürde bu amaca uygun olacak heüristik algoritmalar üzerinde araştırma yapılmıştır.

Silver (1974), sipariş-tamamlama-seviyesi (S), ikincil-sipariş-verilebilir-seviye (c) ve mecburi-sipariş-verme-noktalarının (s) pratik bir prosedür yardımı ile belirlenmesi için, koordineli bir kontrol sistemi olan (s, c, S) politikasını önermektedir. Müşteri taleplerinin Poisson prosesine uygun olarak geldiği, termin süresinin sıfırdan farklı ve büyük olduğu kabul edilmiştir.

Ürün gamı içerisinden tek bir ürün kategorisi seçilmiş, ve seçilen bu ürün kategorisinden de en karlı 10 ürün Detaylı bir çalışma yapmak için ayrılmıştır. Seçilen bu ürünler için, bağımsız (s, S) ve koordineli (s, c, S) envanter kontrol politikaları Arena Simülasyon Yazılımı yardımı ile modellenmiş ve istenilen müşteri hizmet seviyelerine ulaşmayı temel alan koşumlar yaptırılmıştır. Bunun da ötesinde, elde edilen sonuçları daha da geliştirmek için alternatif senaryolar geliştirilmiş ve tekrar Arena'da koşumlar yaptırılmıştır.

Bu çalışmanın neticesinde, belirli bir parametre kombinasyonu ile koordineli sipariş veren envanter yönetim politikası optimale yakın sonuçlar verdiği görülmüştür. Müşteri hizmet seviyesinde elde edilen artış sonucunun yıllık satış gelirini arttırabileceği ve yine elde edilen neredeyse sıfır termin süresinin müşteri farkındalığı yaratıp, müşterilerin firmayı tercih sebeplerini arttırabileceği beklenebilir. Bu çalışmanın neticesinde, envanter yönetimi alanında kullanılabilecek *basit-pratik bir karar destek aracı* geliştirilmiştir.

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LIST OF SYMBOLS / ABBREVIATIONS

A	Fixed or header cost per replenishment (independent of the number of items involved), in €.
a	Variable Cost per item involved in a replenishment, in €.
c_i	Can order point of item i.
$\bar{I}_i$	Average on hand inventory level of item i, in pieces.
L	Replenishment lead time, in years.
N_i	Expected number of replenishments which involve item i per year.
NT_i	Expected number of replenishments *triggered* by item i per year.
n	Number of items in the family under consideration (i.e. a replenishment can include up to n different items).
P_1	Desired value of $PNSPRC$.
P_2	Desired value of FDS.
${p_w}^i(w_0)$	Probability for item i that the random variable w takes on the value w_0.
$p_{po}(x_0/z) = \frac{z^{x_0}\exp(-z)}{x_0!}$	Probability that a Poisson variable with parameter (mean) z takes on a value of x_0.
$p_{po\leq}(x_0/z)$	Probability that a Poisson variable with parameter z takes on a value less than less than or equal to x_0.
$\bar{Q}_i$	Average order quantity of item i, in pieces.
r	Inventory carrying charge, in €/€/yr.
S_i	Order-Up-to level (upper bound) of item i.
s_i	Must-order point (lower bound) of item i.
v_i	Standard unit cost of item i, in €/piece.
w	Available inventory level of an item when it is included in an order being placed.

$\hat{x}_L$	Forecast or expected demand over a replenishment lead time, in units.
σ_L	Standard deviation of errors of forecasts over a replenishment lead time, in units
λ_i	Poisson demand rate for item i, in pieces/yr.
$\lambda_i L$	Poisson demand rate (Lead Time Demand) for item i, in pieces/lead time.
EC_i	Expected relevant cost per unit time for item i, in €/yr.
EC	Expected relevant costs per unit time for all the items in the family, in €/yr.
ESPRC_i	Expected shortage per replenishment cycle of item i, in pieces.
FDS_i	Fraction of demand satisfied directly from shelf for item i. (Fill Rate)
PNSPRC_i	Probability of no shortage per replenishment cycle for item i.

1. INTRODUCTION

Distribution managers are coming face to face with the pressure of the market and high cost inventories. Also, global competition and the growing emphasis on customer satisfaction are underlying the need to improve customer service levels. In addition to these, the capital, space and obsolescence costs of holding inventories have increased the importance of the efficient management of inventory. As a result, inventory control and management throughout the supply chain are very important issues. The current work is confined to the operational issue of inventory control and management. Since holding of inventories in a supply chain can cost anywhere between 20 per cent and 40 per cent of the value of the product value, the effective management of inventory is critical in supply chain operations (Ballou, 1992).

Determining the right amount of inventory to hold without excesses or shortages has been the focus of many supply chain problems. Decisions about, when to re-order and in what quantities to order have to be made continuously. Companies hold inventory to balance supply and demand, to provide protection from uncertainties in demand. Inventory acts as a buffer between critical interfaces within the supply chain. Effective management of the supply chain is the key to the success of companies in terms of competition and profitability (Baganha, Pyke, & Ferrer, 1994).

The main concern of this study is designing an inventory control system for an industrial equipment distributor company which is representing a global producer in Turkey. The company faces with performance and delivery problems. It does not keep inventory on hand. Therefore, customers willing to buy goods give order to the distributor and have to wait until the lead time. Because of the long delivery terms, it is frequently seen that customers are hesitating to give order. This situation implies that often potential sales are lost. Therefore, there is a need to reestablish the company’s inventory management policy. The solution should be practical to be applied to the company.

It is obvious that there is a need to increase customer service level and hold products in stock. But, at this stage there are uncertainties. To clarify, this study should answer those questions i.e. how much will it cost to increase customer service level to a certain level? What benefits will the company gain if the company pays this cost?

The results obtained from this study can help the managers in their decision-making process of ordering product categories and determining the inventory level of each item. The impact of such decisions is tremendous in terms of cost reduction and providing high quality customer service.

The distributor company under the scope is “Leon Teknik Dış Ticaret”. It is one of the well-known providers in fluid-handling equipments industry in Turkey. Leon Teknik is representing “Crane Process Flow Technologies”, a Crane Co. Company, and distributing the Crane Brands (i.e. Saunders Diaphragm Valves, Centerline Butterfly Valves, Depa Positive Displacement Pumps, etc.) throughout Turkey since 2001. Further information about the Distributor Company and Provider Company will be given in Section 3.

This study is organized as follows: In Section 2, technical background of inventory management subject and concerning literature survey to find solutions are briefly discussed. In section 3, brief information about the global producer company, its products and sales network are given. Following subsections define the distributor company’s managerial problems about inventory management and set the objectives of the study. In Section 4, the problem environment is overviewed and the control mechanism of the suggested inventory control policy is defined. In the following subsections, the decision problem is mathematically stated and the initial control parameters (s, c, S) are computed. In Section 5, *Independent* and *Coordinated Control Policies* are modeled with Arena. And also, constructed models are tested and verified in this section. In Section 6, results of the simulations and their analyses are stated in detail. Finally, Chapter 7 concludes the thesis and states future research directions.

2. LITERATURE REVIEW

With the globalization of business, companies are outsourcing and distributing components and finished goods across the world. Customers want to have their products very rapidly and reliably. They don't care whether they are coming from another continent. They often want to look for other suppliers if the delivery becomes late. So, inventory management and supply chain management have become very critical to success in competition compared to old times (Silver, Pyke, & Peterson, 1998).

The question of how frequently the inventory status should be determined specifies the review interval. Review interval is the time that passes between two sequential times when the company knows the level of the inventory. The extreme case is continuous review in which the inventory level is always known. But in practical life, continuous information is not required. Each transaction (shipment, receipt, demand etc.) triggers an immediate update of the inventory level. This type of action is called transactions reporting (Silver, Pyke, & Peterson, 1998).

Products in the A category are defined to be those at the high end of level of importance. This means that the total costs of replenishment, carrying stock, and shortages with respect to these products are so high that they justify a more detailed control system than the items in category B and C. This means a high annual usage volume. Usage means retail customer demand as well as usage of spare parts in production, for example. Additionally, some special factors may require that an item be placed in the A category. For example, an item can be necessary to round out the product line. Another example is the high end items required for their prestige values. Poor customer service on such items may cause unwanted effects on customers that buy large amounts of other products. So the bottom-line is that trade-off between control system costs and inventory management costs are a very critical issue (Silver, Pyke, & Peterson, 1998).

Usually it makes sense to coordinate the control of various stock keeping units. It means that the different items are all stocked in the same location and they share a common supplier, or mode of transportation, or a common production facility (Wildeman, Frenk, & Dekker, 1997).

Balintfy (1964) was the first researcher who concerned with finding a way which evaluates and compares classes of multi-item inventory problems, where joint order of several items may save a part of the setup cost. A cost ratio and simple decision rule are determined for joint versus individual orders in specified cases. The comparisons call for the necessity of a new policy for reorder-point triggered random output multi-item systems. This policy, the “random joint order policy”, operates through the determination of a reorder range within which several items can be orders. The existence of an optimum reorder range is proved. He also proved in his studies that, the random joint order policy model is especially suitable for computer controlled inventory systems.

Ignall (1969) is concerned with finding the optimal stationary ordering policies for continuously reviewed two-product inventory system with joint setup costs. He shows that optimal control polices can be extremely complex even for two products. Therefore, simple polices like can-order may not be optimal.

There are frequent occasions where the coordination of replenishment orders for selected groups of items can lead to significant savings in the costs of replenishment. Silver (1974) is concerned with a practical procedure for selecting the order–up–to levels, can-order points and must-order points of a particular coordinated control system, namely (s, c, S) control system. Demand is Poisson and a fixed non-zero replenishment lead time is assumed. He found that cost comparisons with the best independent control strategy indicate substantial savings (averaging 18.8% over some 104 examples) are possible through coordination.

Although Silver’s (1974) paper was written 34 years ago, assumptions of some key points (i.e. Poisson demand, fixed non-zero replenishment lead time etc.) are still useful, and practical. Therefore, during this thesis study, Silver’s practical procedure is used to select the order–up–to levels, can-order points and must-order points.

For interested researchers, some further literature survey is given below about *coordinated control inventory control systems.*

Thompstone & Silver (1975) presented a reasonable procedure for determining the values of the various control variables of an (s, c, S) control system. With this paper subject of Silver's (1974) paper was extended. It was assumed that demand is compound Poisson and the replenishment lead time is of neglegible duration. Comparison with the best independent control strategy indicates that substantial savings (averaging 15.9% over some 64 examples) are possible through coordination of replenishments.

Federgruen, Groenevelt, & Tijms (1984) considered a continuous review multi-item inventory system with compound Poisson demand process; excess demands are backlogged and each replenishment requires a lead time. Moreover, there are holding and penaty costs. They presented an algorithm which searches for a simple coordinated control rule minimizing the long run average cost per unit time subject to a service level constraint per item on the fraction of demand satisfied directly from on hand inventory. Their algorithm is based on a heuristic decomposition procedure and specialized policy-iteration method to solve the single item subproblems generated by the decomposition procedure. The model applies to multi-location inventory system with simisliar cost structures for coordinated deliveries.

Wildeman, Frenk, & Dekker (1997) presented an alternative optimal approach based on global optimisation theroy. By appliyng Lipschitz optimisation one can find a solution with an arbitraliy small deviation from an optimal value. An efficient procedure is presented which uses a dynamic Lipschitsz constant and generates a solution in little time.

Kayış, Bilgiç, & Karabulut (2008) studied a two-item continuous-review inventory system. Demands for item 1 and item 2 occur at epochs generated by independet poisson processes. They modeled the system as a Semi-Markov decision process and as simple anumeriation algrorithme is proposed for its solution. The study show that previous formulations of the problem do not necessrily converge the best can-order policy by providing numerical examples.

3. PROBLEM DEFINITION

In this chapter, description of the producer company (CRANE Process Flow Technologies) and definition of the problem is given. In the first section, the current sales and distribution system of the producer company is described. Flow of finished goods and information in the overall system is explained. Then, the subsystem related to this thesis is specified and its boundaries are drawn. In Section 3.2, current problems in the inventory control and distribution system are described in detail. Conditions affecting the inventory control are analyzed and general characteristics of the current situation are determined. Finally, objectives to do this thesis study are depicted in Section 3.3.

3.1. Crane PFT and Its Existing Sales and Distribution System

Crane Co. is a diversified manufacturer of highly engineered industrial products in a number of focused niche markets with approximately 12,000 employees working together in five business segments across 25 countries. Crane generated 2008 net sales of $2.6 billion.

Crane was founded in 1855 by R.T. Crane in Chicago and it produced valves, fittings and specialty castings when it was first established. In 1860’s, as Northwest Manufacturing Co., Crane was supplying essential steam components to the railroads; the company entered the steam heating business. Crane Elevator Company began making hoists, elevators and steam engines. With the 1880’s Crane’s business has boomed, the company was operating four manufacturing facilities and employed over 1500 people. Branch houses extend the company’s business to the Western U.S. In 1885, Crane Co. was adopted as the Company name in 1900. Crane started to produce steel valves and fittings in its own steel foundries. With this achievement, Crane’s rigorous approaches to metallurgical science and material testing become the model for the valve and fitting industry.

Founder R.T. Crane has died in 1912 and his son succeeded as the Company’s president. Crane listed in the New York Stock Exchange in 1936. The Golden Gate Bridge opened with its railing material supplied by Crane Co. in the following year. In 1942, to meet wartime demand, Crane increased steel valve output capacity four-fold to 25,000 tons annually. The U.S. Navy requires from 1,500 to 15,000 valves per ship. After the war, in 1952, Crane supplied flow control products for aircrafts. Hydro-Aire, a manufacturer of highly developed precision aircraft products is acquired.

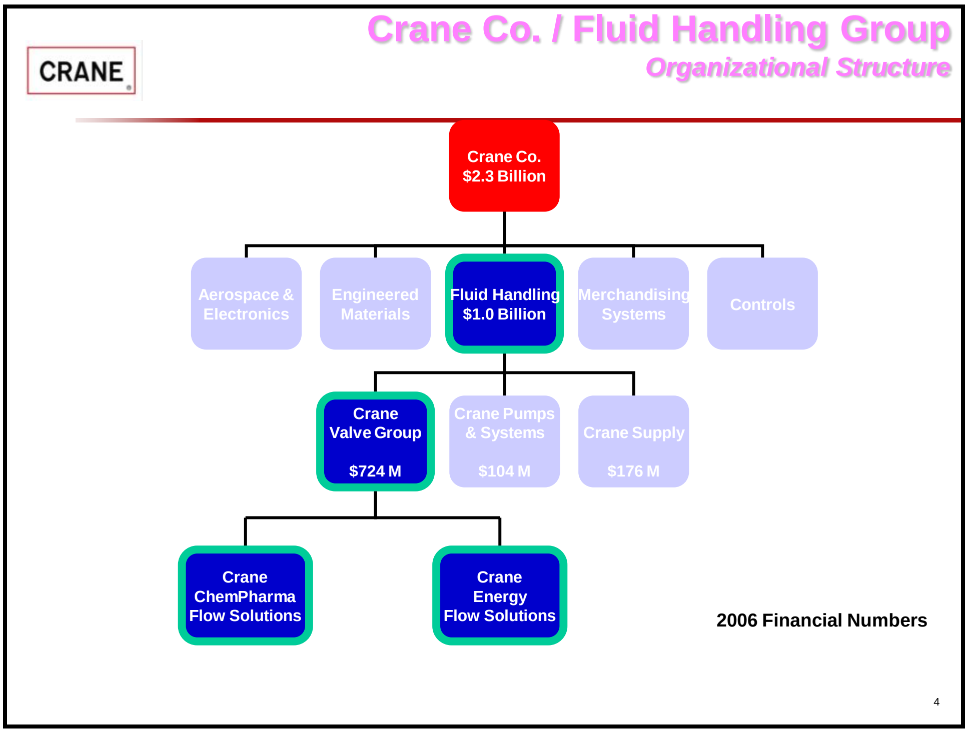
T.M. Evans was elected Chairman and CEO in 1959. A new management philosophy moved Crane to streamline the distribution network and concentrate on industrial manufacturing. With the 1960s, industrial expansion was furthered through domestic and international acquisitions such as Chapman Valve, Cochrane Corporation, Deming Company and Chempump. In year 1969 Crane pumps were on the moon. Hydro-Aire and Chempump pumps are used in the Gemini and Apollo Space programs. In 1970s, Crane was well-positioned in four major areas of concentration: fluid and pollution control, steel, building products and aircraft/aerospace. In 1980s, Crane exited the plumbing and plumbing distribution businesses in the U.S.

In 1984, R.S. Evans elected Chairman and CEO. With the 1990s, Evans leaded Crane as a growth company with strong acquisition program in areas of fluid handling, engineered materials, merchandising systems, aerospace and controls.

Eric C. Fast elected President and CEO in 2001. He continued strategic leadership of a collection of linked businesses to move Crane from a holding company to an operating company. In 2005 Crane celebrated its 150th anniversary.

In early 2000s: Foundation for future profitable growth was laid with the implementation of new Operational Excellence tools, greater development of intellectual capital, strategic linkages and strong customer focus. After 2005 the Company implemented Company-wide business system and realigned businesses in all segments to focus on customer solutions that create value and deliver results.

Crane Co. has five business segments. These are *Aerospace & Electronics, Engineered Materials, Merchandising Systems, Fluid Handling* and *Controls.* The Fluid Handling Group has the largest annual sales income compare to others. This group reached sales figures of, 937 Million USD and 1,005 Billion USD in year 2005 and 2006 and made 44 per cent of the all Crane Co. Group companies. (See Figure 3.1)



CRANE

Crane Co. / Fluid Handling Group

Organizational Structure

Crane Co.
$2.3 Billion

Aerospace & Electronics

Engineered Materials

Fluid Handling
$1.0 Billion

Merchandising Systems

Controls

Crane Valve Group
$724 M

Crane Pumps & Systems
$104 M

Crane Supply
$176 M

Crane ChemPharma Flow Solutions

Crane Energy Flow Solutions

2006 Financial Numbers

4

Figure 3.1. Annual sales figures of Crane Co. in 2006

The Fluid Handling Segment is Crane's largest business and thus an important engine for profit growth. But it has some major weaknesses. As stated above, after 2005 the Company implemented Company-wide business system to reorganize itself. Therefore the Fluid Handling group has also being reorganized, which still continues. The reasons that stimulated the company to change are listed below:

- Performance and delivery problems in almost every business
- Lack of customer focus
- Limited new product development
- Attracting and retaining top talent

Performance and delivery problems that Crane faces are the most important and damaging problem among all the other problems. Therefore, while selecting a topic for this thesis study our motto was finding a good solution to this damaging handicap.

The Fluid Handling Group of "Crane Co. Company" is holding many sub companies under its control. Some of these companies are established by Crane itself as the result of organic growth strategy and some are purchased companies. The frame company manages this business is called "CRANE Process Flow Technologies". The abbreviation "Crane-PFT" will be used in the further sections of this thesis.

The worldwide Supply Chain System of Crane PFT, is divided in three main geographical regions, as seen in Figure 3.2 The first region is *AMERICAS*, covering North & South America Continent. The second region is *EIMEA* which covers Europe, Middle East, and Africa and west part of Russia. And the last one is *Asia*. During this thesis study, the two irrelevant regions, *Americas* and *Asia*, will be ignored. So, the region that will be observed is *EIMEA*.

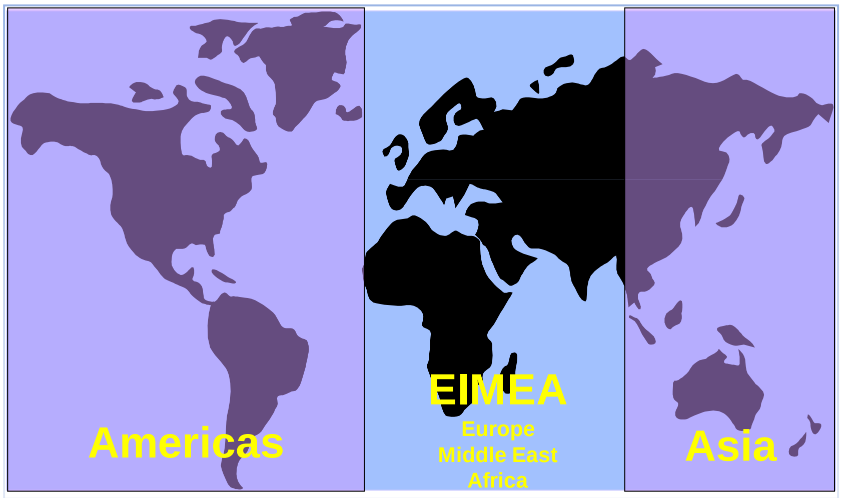


Figure 3.2. Sales actions of Crane fluid handling segment in 3 regions

In the *EIMEA* region, production is carried out in seven factories and then marketed over Europe, Middle East and Russia. Sales to African countries are negligible due to the very weak industrial growth in this continent. In Middle East and Russia Crane PFT have its own sales-marketing and service offices, one is in Moscow and the other is in Dubai, and supplying these sub regions from Europe. There are three factories in England, three in Germany and one in Northern Ireland which are depicted as *Primary Locations* in Figure 3.3. Also Crane PFT is outsourcing the production of its XOMOX Brand Valves to a

Hungarian Contract Manufacturer which is also shown in Figure 3.3 as *LCC*[1] *Manufacturing*.

Beside these production centers, Crane PFT has four service centers in: Barcelona/Spain, Mulhouse/France, Waalwijk/Netherlands, Anaköski/Finland and one Distribution Center in Wavre/Belgium.

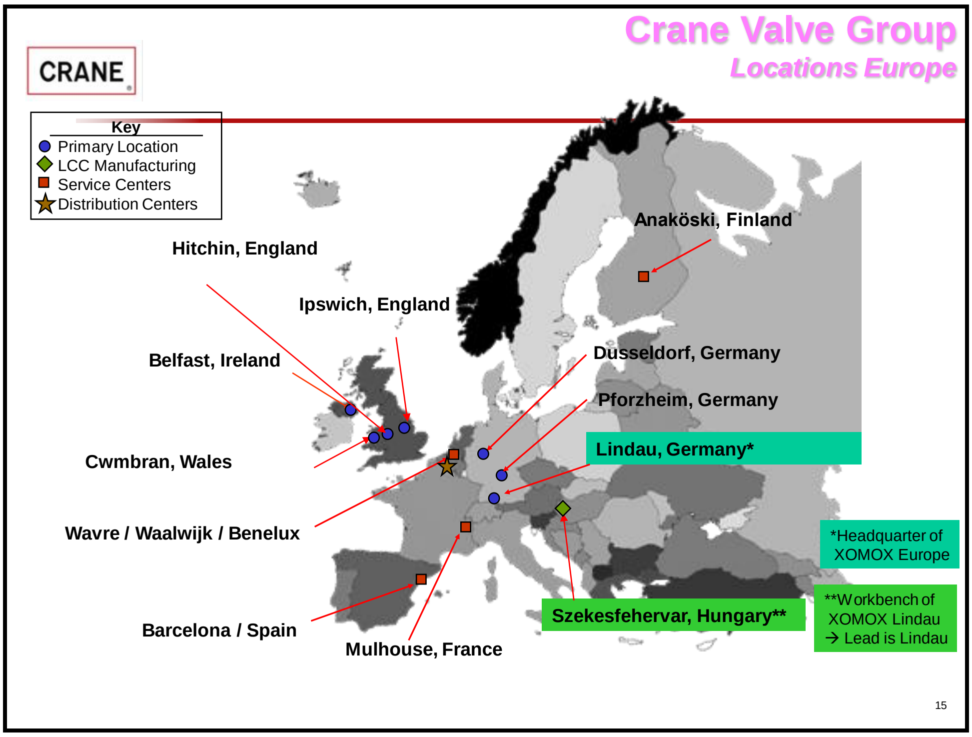


Figure 3.3. CRANE PFT locations in Europe

The main product distribution flow which will be analyzed here can be represented schematically as in Figure 3.4. District Distribution Center located in Belgium is very important element in this system and this center acts as buffer stocking point between EIMEA Region and Production Centers in Asia/Pacific and North America (Shown in Figure 3.5 and Figure 3.6) . The product types which are not produced in the EIMEA Region are transported by ships to this Distribution Center and stocked in here for the need of the customers in EIMEA Region. It is very complicated and not related with the thesis subject to explain which brand or what size of which products is produced in where. So,

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[1] Least Cost Country

due to this complication and especially for company privacy this subject will not be included into the study topics.

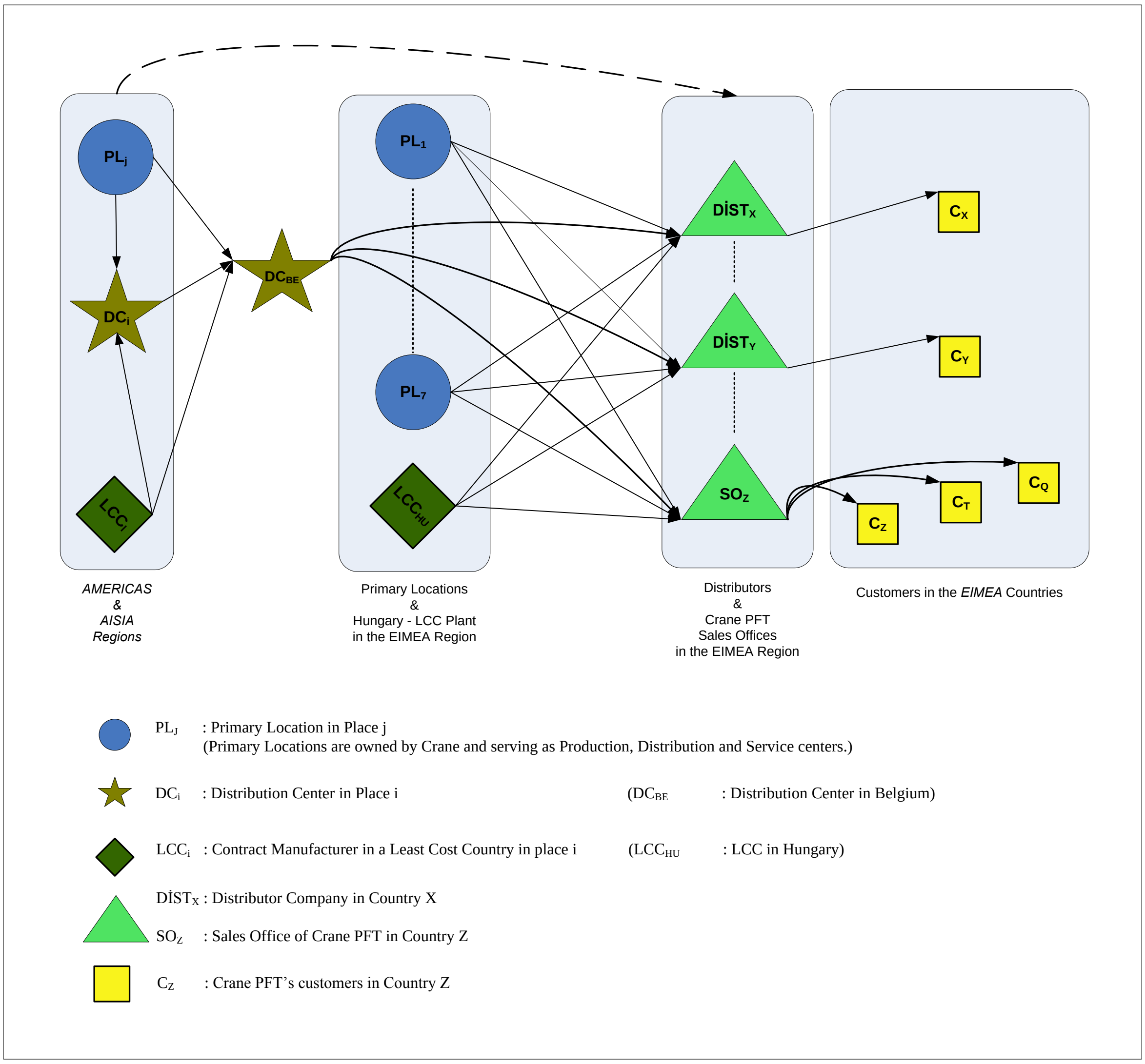


Figure 3.4. Distribution chain in EIMEA region

Production is carried out in seven different factories in Europe and then marketed over EIMEA Region and other regions. Almost 90 per cent of these products are consumed in the EIMEA Region and rest is distributed all over globe. These factories are owned by Crane PFT's itself and also serving as Distribution, Sales & After-Sales Service Centers, due to their multifunctional role they are called *Primary Locations* in Crane's Supply Chain System. The Primary Location in Dusseldorf is also the Head Quarter of EIMEA Region at the same time. Secondly; Wales/Cwmbran Primary Location is also servicing as Central Sales Office. The rest of the Sales & Distribution Organization of Crane PFT for the EIMEA Region is presented in Table F. 1. The operation of the Sales & Distribution system in *EIMEA Region* can be described as follows:

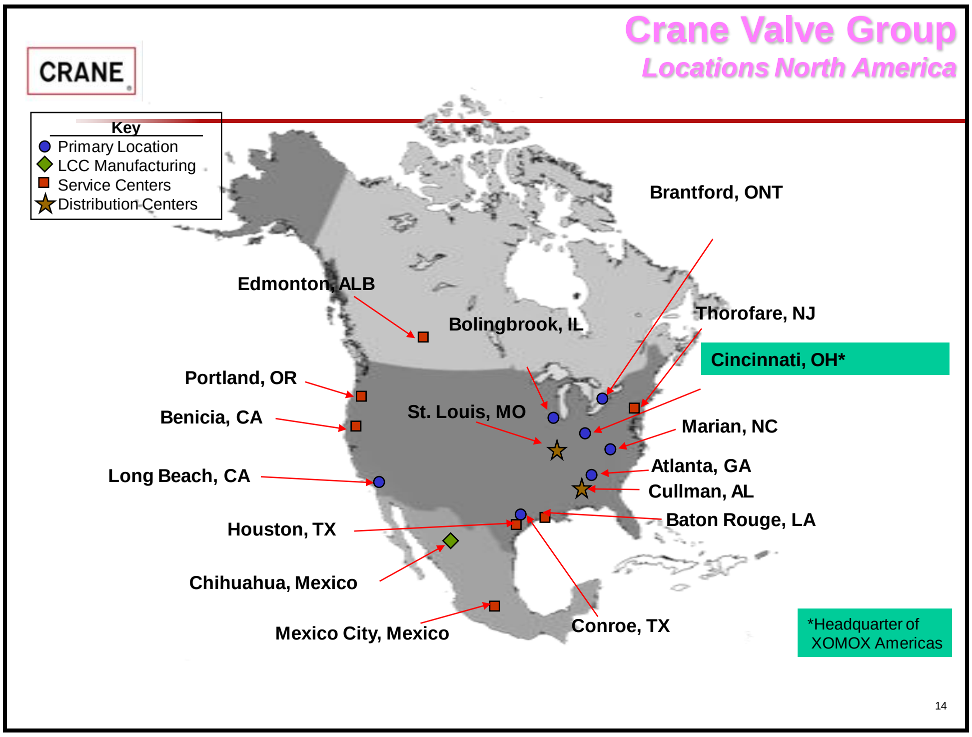
(i) A sales operation starts with the order of a customer to the distributor company in that country (In most cases it can assumed that there is a distributor in the country). In some countries Crane PFT has its own Sales Offices, if Crane does not have a distributor or a sales office in a country, then sales teams in the Head Quarter deal and directly sell to this customer.

(ii) If a distributor has sufficient inventory on hand then the customer’s demand would directly be supplied. But almost all distributors or sales offices carry very limited stocks for very few types of products or spare parts for key customers. So, most of the time a distributor supplies the demand not from its own stock but builds to order.

(iii) Total lead time to the end customer changes depending on the primary location’s circumstances. For example; while primary locations in Germany deliver finished goods in 6 or 8 weeks, Cwmbran Plant delivers approximately in 10 weeks. Additionally, if a requested item is not in stock in Belgium, an overseas replenishment, for example from China, may require at least 16 weeks. In such an overseas replenishment case, to save time, a shipment could be directly made to the country distributor instead of shipping to Belgium.

(iv) Before a customer gives an order to a distributor or sales office, distributor gives a written quotation to the customer which includes technical specifications of products, price and delivery terms.

It should also be noted that not all quotations result with an order. Customers might cancel the order due to long delivery term. This case happens very frequently.


CRANE

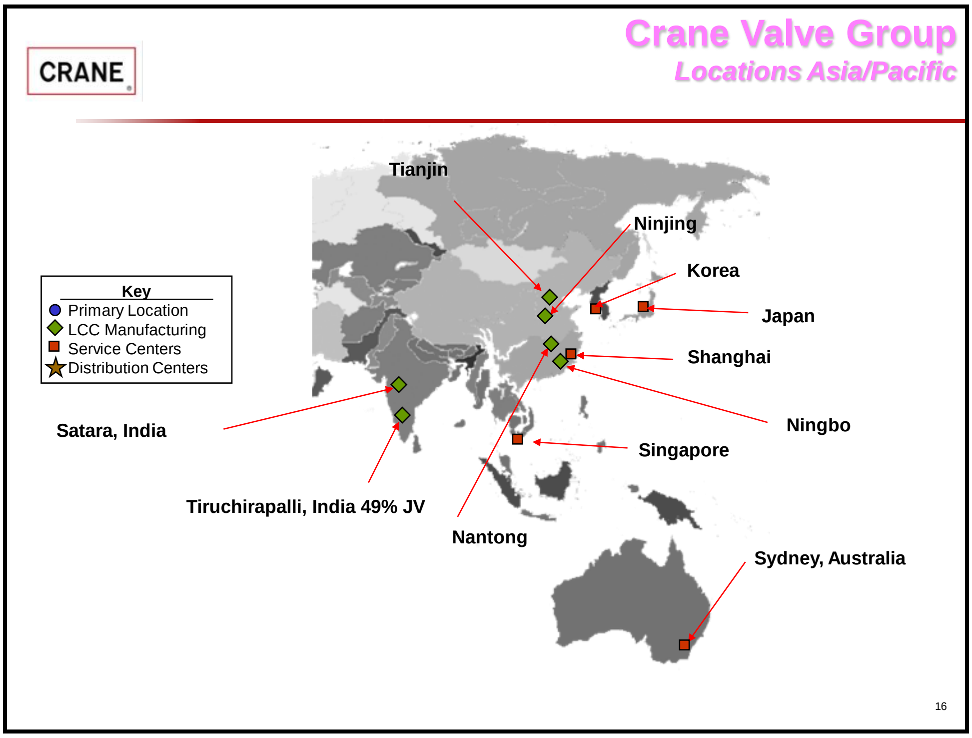
Crane Valve Group

Locations North America

Key
- ● Primary Location
- ◆ LCC Manufacturing
- ■ Service Centers
- ★ Distribution Centers

*Headquarter of XOMOX Americas

Figure 3.5. Crane PFT locations in North America


CRANE

Crane Valve Group

Locations Asia/Pacific

Key
- ● Primary Location
- ◆ LCC Manufacturing
- ■ Service Centers
- ★ Distribution Centers

Figure 3.6. Crane PFT locations in Asia / Pacific

3.2. Defining the Distributor Company’s Inventory Control Problem

It was also emphasized in introduction chapter that, the main concern of this thesis study is designing an inventory control system for a distributor company. But before defining the inventory problem which will be studied, the Global Sales & Distribution System of Crane PFT has been summarized in Section 3.1. It was also necessary to briefly characterize the activities of Crane PFT in the EIMEA Region. In following section, for simplicity, the subject will be restricted only with the Crane PFT’s Sales & Distribution activities throughout Turkish Market.

Leon Teknik Dış Ticaret is the distributor of Crane PFT for Turkey since 2001. Leon Teknik has an exclusive agreement for distributing all Crane Brand products throughout Turkey. Leon Teknik is importing Crane Products from mostly Crane’s Dusseldorf (Germany) and Cwmbran (U.K.) primary locations, and very rarely from other Crane Locations around globe.

Crane PFT has a very wide range of product categories and it would be very complicated to cover all of these categories as the subject of this thesis. Therefore, there should be a restriction instead of observing all product types and a narrowed sampling frame should be used. So, after this point forward, one product type will be selected and observed for simplicity.

Due to our experiences in the sector, the best suitable category that will be examined closely is DEPA AOD[2] Pumps (abbreviated as: DEPA Pumps). It would be better to give some brief explanations about DEPA Pumps. First of all, these pumps are being imported from Crane’s Dusseldorf Primary Location. The working principles of these pumps are same with a lively heart and widely preferred due to its easy maintenance, pumping efficiency and small space required dimensions. With its wide material range this pump can work against nearly all corrosive media types. Working principles of AOD Pumps are briefly shown in Figure 3.7.

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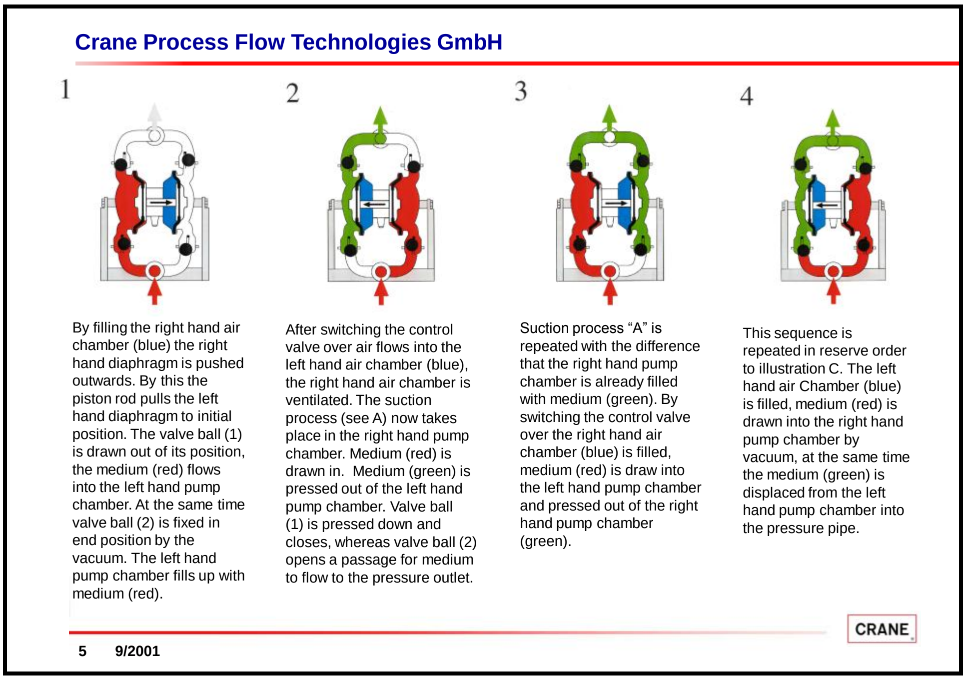
[2] Air Operated Diaphragm



Crane Process Flow Technologies GmbH

1 By filling the right hand air chamber (blue) the right hand diaphragm is pushed outwards. By this the piston rod pulls the left hand diaphragm to initial position. The valve ball (1) is drawn out of its position, the medium (red) flows into the left hand pump chamber. At the same time valve ball (2) is fixed in end position by the vacuum. The left hand pump chamber fills up with medium (red).

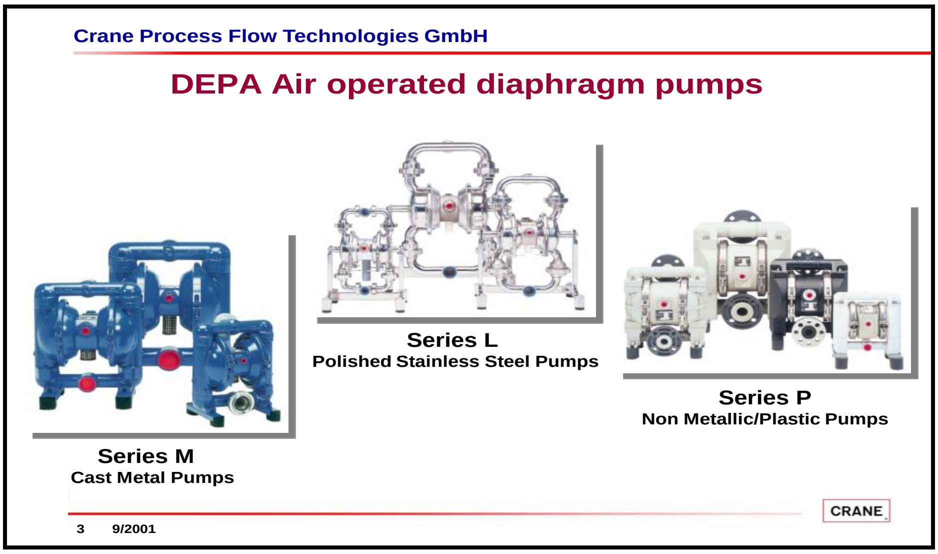
2 After switching the control valve over air flows into the left hand air chamber (blue), the right hand air chamber is ventilated. The suction process (see A) now takes place in the right hand pump chamber. Medium (red) is drawn in. Medium (green) is pressed out of the left hand pump chamber. Valve ball (1) is pressed down and closes, whereas valve ball (2) opens a passage for medium to flow to the pressure outlet.

3 Suction process "A" is repeated with the difference that the right hand pump chamber is already filled with medium (green). By switching the control valve over the right hand air chamber (blue) is filled, medium (red) is draw into the left hand pump chamber and pressed out of the right hand pump chamber (green).

4 This sequence is repeated in reserve order to illustration C. The left hand air Chamber (blue) is filled, medium (red) is drawn into the right hand pump chamber by vacuum, at the same time the medium (green) is displaced from the left hand pump chamber into the pressure pipe.

Figure 3.7. Working principles of AOD pumps

Application areas of DEPA Pumps in different industries and media types are briefly sketched in Figure 3.8 and 3.9. As may understood from the figure, there are many different types of materials are compulsorily used to produce these pumps which is the main source of our difficulties in the industry. Using different types of materials means different types of products and different type of SKU's[3] to manage.



Crane Process Flow Technologies GmbH

DEPA Air operated diaphragm pumps

Series M
Cast Metal Pumps

Series L
Polished Stainless Steel Pumps

Series P
Non Metallic/Plastic Pumps

Figure 3.8. Product ranges of DEPA AOD pumps

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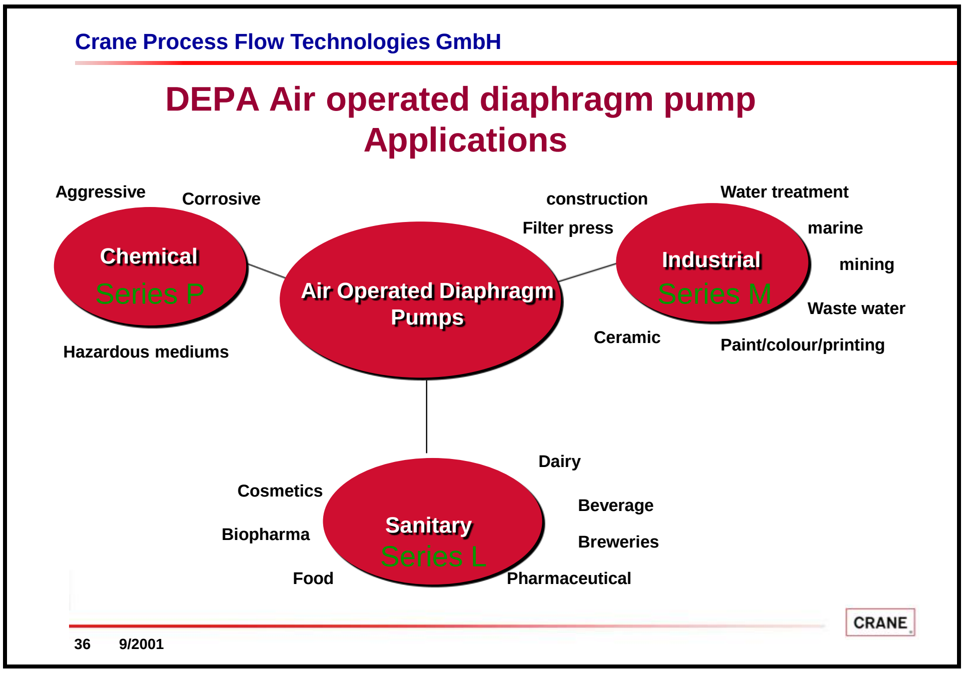
[3] Stock Keeping Unit



Crane Process Flow Technologies GmbH

DEPA Air operated diaphragm pump Applications

Air Operated Diaphragm Pumps

Chemical Series P: Aggressive, Corrosive, Hazardous mediums

Industrial Series M: construction, Filter press, Water treatment, marine, mining, Waste water, Ceramic, Paint/colour/printing

Sanitary Series L: Cosmetics, Biopharma, Food, Dairy, Beverage, Breweries, Pharmaceutical

36 9/2001

Figure 3.9. Application areas of DEPA AOD pumps

For identifying DEPA Pumps, six different terminologies are used for coding. These are; Pump Type, size, housing/control block material combination, diaphragm material, valve and valve seat materials. For each terminology some sub definitions are used. As briefly shown in Figure 3.10; there are nearly 6x6x14x8x8x12 = 387.072 different combinations to produce one DEPA Pump. This calculation is just for showing the size of combinations and off course it is not the real amount of SKU's in the product matrix. For instance, it is meaningless to produce a pump with Aluminum housing and PTFE diaphragms-seat-seat balls. The aim to use a PTFE material pump is exactly for highly Corrosive Medias but in the same pump structure Aluminum is the weakest material to choose for Corrosive Medias, even if this combination is produced, it may stand only one week and then the Aluminum housing would be wearied through. So, every combination in the chart is not produced and in practice there are approximately 100 different selling SKU's can be considered. During this thesis study, this huge product differentiation needs to be restricted again.

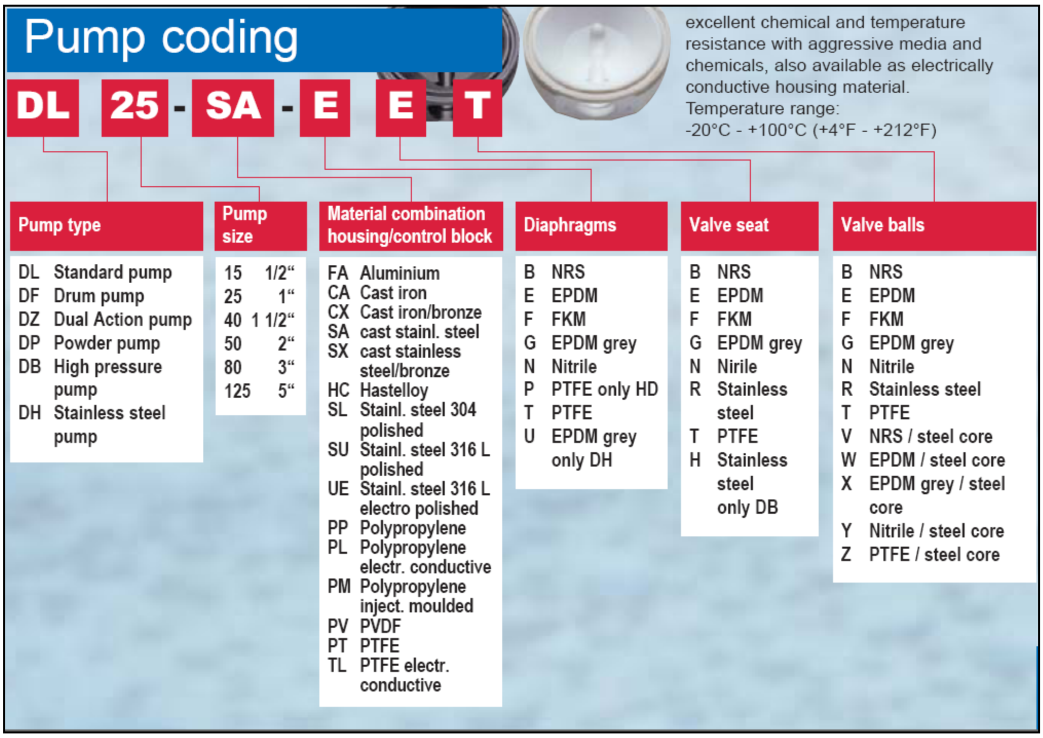


Figure 3.10. Product coding chart of DEPA pumps

Leon Teknik is keeping sales data of all sold products since its establishment. There is nearly four years long (47 months) accurate sales data of all product types for DEPA Pumps. This data are covering the period between January-2003 and November-2006. At first, DEPA Pump Sales has been selected from these sales data and summarized as in Table 3.1. For standard accurate computing and comparison, it is avoided to use end customer-selling prices. Instead, **2008 List Prices**[4] were given. Also the inflation effect is neglected for this period.

After refining sales data, it is found out that 34 different types of DEPA Pumps are sold in Turkish Market. For more focusing, it is decided to select the most profitable 10 items for sampling from these 34 different types. It is assumed that Leon Teknik is gaining the same profit ratio from each product type. So the product type that brings more sales income also brings more profit. In this respect, *DL40 SL GGG*[5] model DEPA Pump is the

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[4] List Prices are not including reduced distributor purchasing price, distributor profit or specific customer discounts.

[5] Standard Designed, $1\frac{1}{2}$ inch, Polished Stainless Steel Body, EPDM Grey working parts - Hygienic Air Operated Diaphragm Pump.

most profitable product type sold in Turkey. (The rest is given in Table 3.1 and product code of the selected 10 product types are printed in bold.)

Table 3.1. Sales figures of DEPA pumps (covering January-2003 and November-2006)

#	Product Code	Total Sold Quantity	Unit List Price (Euro)	Total Sales Income (Euro)
1	**DL40 SL GGG**	72	4,320	311,040
2	**DL25 SL GGG**	55	3,422	188,210
3	**DL50 FA NNN**	90	1,761	158,490
4	**DL25 FA NNN**	151	913	137,863
5	**DL25 SF GGG**	29	2,546	73,834
6	**DL25 PP TTT**	28	1,971	55,188
7	**DL40 FA NNN**	46	1,168	53,728
8	**DL80 FA NNN**	19	2,761	52,459
9	**DL40 PM EEE**	32	1,605	51,360
10	**DL40 SF GGG**	12	3,205	38,460
11	DL80 SA TTT	4	8,540	34,160
12	DL40 SA TTT	12	2,818	33,816
13	DL25 SA TTT	17	1,936	32,912
14	DL50 SL GGG	5	6,547	32,735
15	DL25 SL TTT	7	3,636	25,452
16	DL25 PP FFT	10	1,956	19,560
17	DL50 SA TTT	3	5,107	15,321
18	DL25 CA TTT	10	1,345	13,450
19	DL25 SA NNN	8	1,661	13,288
20	DL40 PP TTT	4	3,245	12,980
21	DL50 SF GGG	2	5,477	10,954
22	DL50 CA NNN	5	2,079	10,395
23	DL80 SF GGG	1	10,143	10,143
24	DL50 CX EEE	4	2,497	9,988
25	DL25 PM EEE	10	950	9,500
26	DL40 PM FFT	5	1,891	9,455
27	DL40 SL TTT	2	4,632	9,264
28	DL25 PM NNN	9	950	8,550
29	DL50 CA BBB	4	2,079	8,316
30	DL40 PM NNN	5	1,605	8,025
31	DL50 SL TTT	1	7,052	7,052
32	DL25 PM FFT	6	1,163	6,978
33	DL80 CA NNN	2	3,324	6,648
34	DL15 PM EEE	11	436	4,796

Currently, Leon Teknik is holding very limited stocks for emergency requests. Most of these stocked products are small size manual valves, small capacity DEPA pumps and some necessary spare parts for regular customers. Therefore, due to the limited stock keeping data, it will be neglected to observe previous periods' stock transactions.

All Crane products are highly durable, high quality and specific problem solver investment goods. The customers of these products are big production plants which are dealing with fluid processing in their main production processes or supporting processes all around the world. Crane's quality is the biggest advantage of Crane against its competitors but as indicated in Section 3.1: Crane has performance and delivery problems because of its structure and this is the biggest weakness it faces.

The situation is also the same in Turkey. Average delivery term for these pumps is between 6 to 9 weeks from Dusseldorf Plant. Longer delivery terms compare to local or other producers is a handicap and causes the loss of orders frequently. Table 3.2 shows the fraction of sales success of DEPA pumps by years. Average sales success is %31.91, it is known by experience that compared to other competitors in the sector this ratio is quite low. It can be assumed by this ratio that there is up to 70% lost sale volumes. By experience we assume that; 30% of this loss is related to long lead times, 30% is related with price competition and 10% for just because the customer is searching the market instead of being a serious buyer. These given ratios are just intuitive and we are not able to justify but one point is clear; shortening delivery terms is a very critical issue for Leon Teknik as a distributor of Crane PFT.

In general, delivery terms in the heavy industrial equipment industry are very long compared to other sectors. Due to this long delivery times, general application in the sector is, stocking and distributing industrial equipments from one central warehouse to all Turkey. Instead of using distinct regional distributing hubs, using a single distributor warehouse does not require to deal with the problem of designing complex sales-distribution networks. Therefore the only subject will be dealt is designing an inventory control system.

Table 3.2. Fraction of sales success for DEPA pumps

Years	**2002**	**2003**	**2004**	**2005**	**2006**	**2007**	**Total**
Sum of Quotation Value[6] (€)	696,944	1,235,441	1,510,385	1,385,627	2,403,608	1,531,984	**8,763,989**
Sum of Order Value[7] (€)	250,626	170,174	914,510	453,981	648,472	328,270	**2,766,032**
Fraction of Sales Success[8]	35.96%	13.77%	60.55%	32.76%	26.98%	21.43%	**31.91%**

3.3. Objective of the Thesis

This section draws a perspective for the solution which shortens the delivery lead time and increases customer service level. The selected method to solve the depicted problem designs a new inventory control system.

It should be stated at first that the main focus point of the study is not optimization. This is a practical study to solve a real life problem. The problem is: "*How much does it cost to increase customer service levels?"* Therefore, it is focused to find good ways to increase customer service levels while keeping the costs as low as possible. The objectives of this study are summarized as follows:

- To increase customer services level and reach targeted objectives.
- To keep inventory management cost as low as possible.

During the study, real data sets from previous years were used to clarify the case. Simulation is preferred as a tool to analyze the system and decide service levels. ARENA Software will be used during simulation studies. After the system is modeled, experiments that represent different situations are performed to see the effects of suggested inventory control system concerned. To see and analyze the effects of the new system, some

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[6] Sum of quatations given to Turkish Market for DEPA pumps.

[7] Sum of orders taken from Turkish Customers for DEPA pumps.

[8] $\text{Fraction of Sales Success} = 100 \times \frac{\text{Sum of Order Value}}{\text{Sum of Quotation Value}}$

performance measures are determined. These performance measures are the following ones:

- FDS (Fraction of demand satisfied directly from shelf): This is also called as fill rate. The fill rate is the fraction of customer demand that is met directly without backorder or lost sales. This measure is used usually when penalty and backordering costs cannot be determined.
- Inventory Holding Costs: The total inventory holding costs are determined for each product type. The most common convention of costing is to use the opportunity cost of the money invested. Formula is given in Equation 3.1. In expression, $\bar{I}$ is the average inventory in units (hence $\bar{I}v$ is the average inventory expressed in €'s), and r is the carrying charge, the cost in €'s of carrying one € of inventory for one year.

$$Holding\ Costs\ per\ year = \bar{I}vr \tag{3.1}$$

- Ordering Costs: The total ordering cost is calculated at the end of the simulation run time. It includes transportation costs also.
- Total Costs: Total cost consists of only inventory holding and ordering costs. The total costs that emerge at the end of the total simulation run time are also used as a quantitative performance measure.

4. MODELING & DEVELOPMENT OF A NEW CONTROL SYSTEM

A real life setting inspired the model. However, the model is a simplification of the real setting (both in business scope and actual parameters) to highlight the investigated issues. Care has been taken not to make the model oversimplified.

In this chapter the modeling phase and the development of a new inventory control system phases are detailed. In the first section, the problem depicted in Chapter 3 is summarized and frame of the problem environment is sketched. In section 4.2 "*Continuous Review Can-Order Inventory Control Policy"* is suggested as the solution policy to apply and the control mechanism of this suggested inventory control policy is described. In Section 4.3 and Section 4.4 assumptions of the model are given, input data are defined and inventory cost parameters are described. Finally, the decision problem is stated mathematically as the core of the solution procedure. Initial parameter computations are described in Section 4.5 and 4.6.

4.1. Problem Statement

The Production Plant at Dusseldorf produces ten different product types and these are supplied to ten distinct incoming customer streams, denoted by $i = 1, 2, 3, \ldots 10$ respectively (shown in Figure 4.1).

Before a customer gives an order to the distributor, distributor gives a written quotation to the customer which includes technical specifications of product/products, price and delivery terms. Then if the customer accepts these terms, it gives an order to the distributor.

If the distributor has sufficient inventory on hand then the customer's demand would be supplied directly from On Hand Inventory. But as described before, the distributor company holds very limited stocks on hand. So customer demand is backordered when there is insufficient inventory to satisfy the order. Backordered demand is satisfied immediately from the next shipment received.

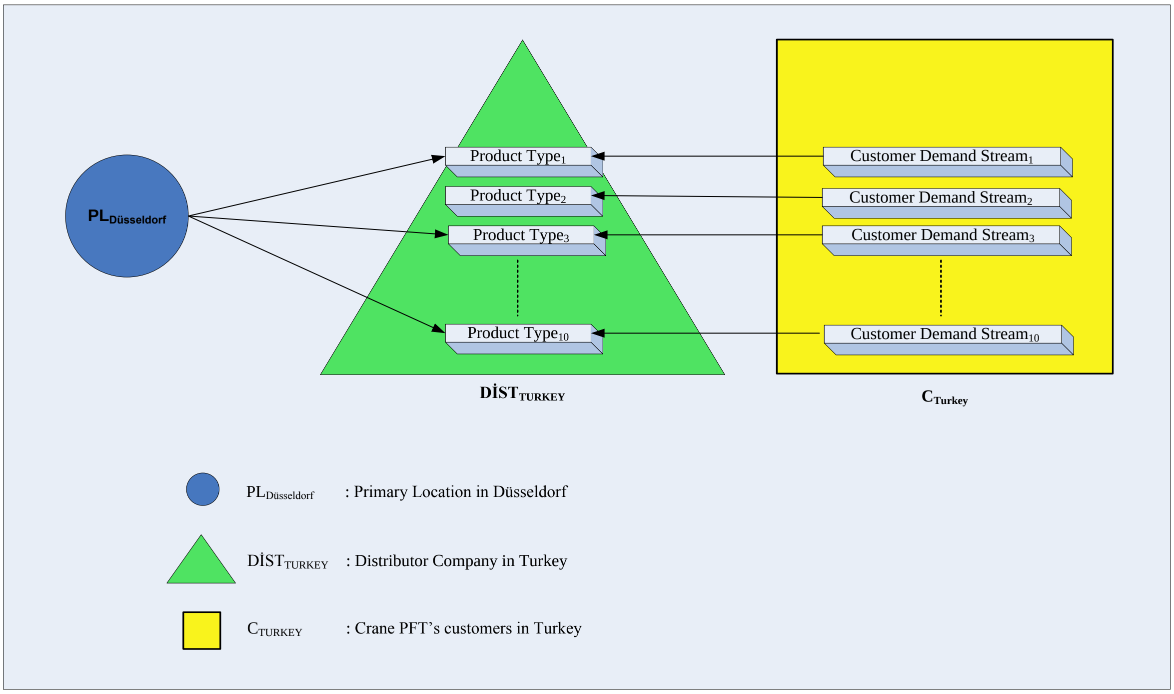

Figure 4.1. Inventory control system in Turkey

Lead times for all products are same because all the products are provided from Dusseldorf plant in this system. Therefore, it is thought that joint ordering inventory control policies will give better results for the overall performance of the supply chain. The description of the control mechanism is depicted in following section.

4.2. Control Mechanism of the Suggested Inventory Control Policy

Joint-Ordering policies are extensions of common base-stock policies to their multiple item case. For the continuous review joint-ordering (s, c, S) policies, a reorder point(s), an order-up-to point (S) and a can-order point (c) must be identified for each product type. Can-order point (c) is between reorder (s) and order-up-to (S) points. When the inventory position of any product type drops to its reorder point, all products with inventory position less than or equal to their can-order points are ordered up to their order-up-to points (Silver E. A., 1974).

In Figure 4.2 at time t_1, this item triggers replenishment. At time t_2 some other item in the group triggers replenishment, but this item is not included because its inventory position (available stock) is above the can-order point. At time t_3 some other item in the group triggers the third replenishment in which this item is now included. (For demonstration, lead time is assumed as equal to zero.)

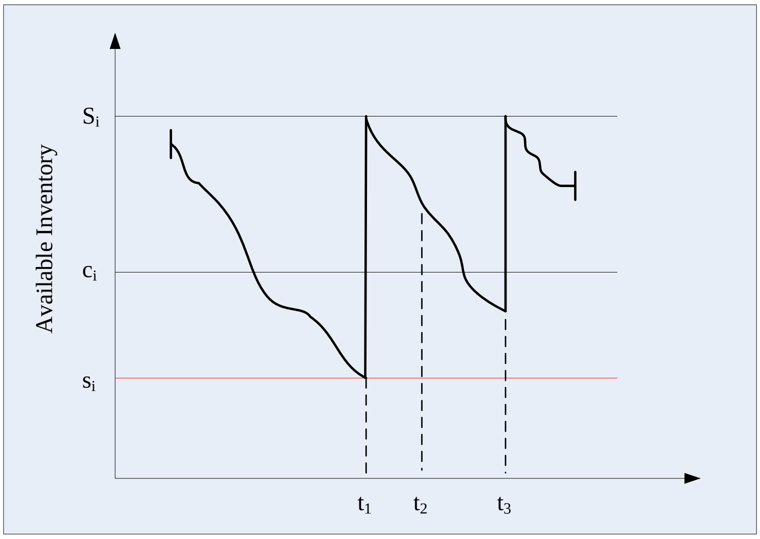

Figure 4.2. Behavior of an item under (*S*,*c*,*s*) control

When demand is probabilistic, it is useful to conceptually categorize inventories. For monitoring inventory levels, two different variables are used in this study. These are:

- I On Hand: Refers to On-hand Stock. This is stock that physically on the shelf. It can never be negative. This quantity is relevant in determining whether a particular customer demand is satisfied directly from the shelf.
- I Position: Refers to Inventory Position. This is sometimes called the available stock. The inventory position is defined by the relation:

  Inventory Position = On Hand + On Order – Back Orders

Details about these variables will be described in detail in Section 5.

4.3. Assumptions of the Model

We are dealing with a group of items where the cost of replenishing two or more items at the same time is less than the total cost of replenishing the same number in separate individual replenishments. To be more explicit, we assume that there is a fixed (or header) cost A associated with replenishment, as well as a variable cost a associated with each item involved in the replenishment. Therefore, the set-up cost of an order involving k item is $A + ka$ whereas the set-up cost of k replenishments, each involving a single item, is $k(A + a)$. There would be no difficulty in generalizing to the case where a become a_i, i.e. the line cost depends upon the specific item i (Silver E. A., 1974).

The Distributor Company does not have accurate data to estimate the probability distribution of the Replenishment Lead Time. But, it can be assumed by considering the current applications: In the real system, replenishment lead time is formed by the delay time of production process and international transportation. Domestic transportations, such as; from Leon Teknik's warehouse to end customer's warehouse, takes maximum 36 hours. Compare to long international delivery, this delay will be neglected. In current application, whenever a customer gives an order to the distributor, the distributor forwards this order info to the Production Plant at Dusseldorf and gives its order at the same day and files this transaction documents. And also, whenever a replenishment order arrives to a custom warehouse, the distributor retires the goods from customs and sends immediately to a customer nearly the same day. Documents of these transactions are also classified. So, by calculating the time difference between beginning and end of this process, we can calculate the average replenishment lead time. But, as explained before, these files are not accurate. Seen by some experimentations, we can assume the lead time variable as uniformly distributed between 6 to 9 weeks (36 to 54 days). Furthermore, its length does not depend upon which subset of the items of the family are involved in the replenishment.

During observation phase of the Sales Data, it is stated that nearly %85 of the customer demands are unit size. The reason for the given rate can be associated with the classification of these items. As stated earlier, these pumps are A-Class (low unit sales of a high-value) items which require high investment costs. Therefore, it is perceptible that customer transactions are mostly unit size for these pumps. During this study, the %15 of the multiple demand sized sales will be neglected and each Customer Demand is assumed to be of unit size for each product type.

It was also seen during the data collection phase that, the sales data are not proper enough to be fitted in a statistical process. Therefore, we need to make an assumption in case.

In section 3.2., the selection of the sample frame is described. Now, the selected product types are given again in table 4.1. Column-C of the table is showing the number of total sold quantities. These sold quantities can also be called as the number of customer demand arrivals in the time interval of 47 months. A random variable denoted with X is representing this number of outcomes occurred. As suggested in literature, experiments yielding numerical values of this X are called Poisson experiments. The number X of outcomes occurring during a Poisson experiment is called a Poisson Random variable, and its probability distribution is called the Poisson distribution (Walpole, Myers, Myers, & Ye, 2002).

Additionally, it is also stated for A-Class items that; if the expected demand over a replenishment lead time is below 10 units, then a discrete distribution, such as the Poisson is more appropriate based on extensive tests (Tayworth & L., 1997). In our case, the expected demand values over a replenishment lead time for all ten products are below 10 units as seen in Column-D of the Table 4.1. Therefore, it is assumed that, the arrival of customer demands are following a Poisson Process.

The expected demand over the replenishment lead time values are calculated with the following Equation 4.1:

$$\hat{x}_L = \frac{L}{Time\ \ Interval} \times X \tag{4.1}$$

For an illustrative purpose, the expected replenishment lead time demand of the first item is calculated as:

$$\hat{x}_L = \frac{7{,}5\ weeks}{47\ mont\ hs} \times 72 = 2{,}76.$$

In our model, λ is the expected number of outcomes per year. To find the λ value, average annual demands are calculated by $X \times \frac{12}{47}$. Then, λ values for all product types are given in Table 4.1.

Table 4.1. Selected product types

A	B	C	D	F	G
i	Product Code	X Total Sold Quantity[9]	$\hat{x}_L$ (Pieces/Lead Time)	λ_i (Pieces/yr.)	v_i (Euro /Piece)
1	DL40 SL GGG	72	2.76	18.383	4,320
2	DL25 SL GGG	55	2.11	14.042	3,422
3	DL50 FA NNN	90	3.45	22.978	1,761
4	DL25 FA NNN	151	5.78	38.553	913
5	DL25 SF GGG	29	1.11	7.404	2,546
6	DL25 PP TTT	28	1.07	7.149	1,971
7	DL40 FA NNN	46	1.76	11.744	1,168
8	DL80 FA NNN	19	0.73	4.851	2,761
9	DL40 PM EEE	32	1.23	8.170	1,605
10	DL40 SF GGG	12	0.46	3.063	3,205

The further assumptions and operating rules that the system is subjected are summarized again and given as follows:

- It has been assumed that, lead time is uniformly distributed between six to nine weeks (36 to 54 days).
- Customer Demands are assumed to be of unit size for a single product.
- Arrivals of customer demands are following Poisson Process. Where of course, each item have a different constant rate parameter given by λ_i.
- The unsatisfied demand is assumed to be backordered and satisfied immediately when replenishment arrives to the warehouse.
- The unit variable cost of an item does not depend on the size of the replenishment, i.e. there are no quantity discounts.

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[9] Covering the period of January-2003 and November-2006 / 47 Months Long

- Another assumption made, which is common in the modeling of inventory control problems, is that inventory carrying costs are proportional to the average inventory level.
- An additional point, which is not really an assumption but rather an accounting convenience, is that we shall say that the entire cost A is assigned to the item that triggers the replenishment.
- Rather than explicitly costing shortages we assume instead that the control parameters are selected so as to provide customer service at or above a prescribed level. For each item, service is defined in: Fraction of demand to be satisfied directly from shelf.
- Non working days are not included in computations and length of a year is assumed to be 50 weeks, and also equal to 300 working days. According to the explanation, following assumptions are made:

(i) 1 Year = 12 Months = 50 Weeks = 300 Days
(ii) 1 Month = 25 Days
(iii) 1 Week = 6 Days

4.4. Inventory Cost Parameters

Fundamentally, there are three categories of costs which occur in inventory systems. These are *ordering cost*, *inventory carrying cost* and *shortage/backordering cost*. However, it should be stated that it is quite difficult to represent mathematically all the cost components with near accuracy. Consequently, there is tendency to make some approximations when representing the costs in the mathematical models to be developed (Silver, Pyke, & Peterson, Costs and Other Important Factors, 1998).

The model used to solve the stated problem allows Backordering. Ordering and Inventory Carrying costs are realistic; we are able to quantify these costs. I.e. for every order given, company is paying money for transportation or for every item kept in stock company is tiding its money to the item. However, Backordering Cost is a totally different type of cost and it is unrealistic to include this cost in the Total Cost equation. Also in common practice, it is not possible, to charge a Backordering Cost to your supplier when a stockout occurs? The thing customer do in practice is, easily search to find another supplier. But most of all, there is no chance to quantify the Backordering Cost. Therefore, instead of charging Backorder Cost, the model will endeavor to reduce the probability of disservices.

4.4.1. Ordering Cost

In the inventory system being studied, the ordering cost has two components. First one is the *Fixed Cost* (A) which is incurred independently from the number of items and amounts. The second component of the ordering cost is, *Variable Cost per Item Involved In a Replenishment* (a). *Fixed Cost* contains the cost of placing an order such as communication and clerical work costs and a fixed charge paid to the transportation companies for each distribution. Among these cost terms, transportation or importation cost is the most important one. Therefore, the *Fixed Cost* is only associated with the importation costs.

In Table 4.2 there is a data set which depicts total importation costs and quantity of importations in a year, covering 2004 to 2006. In this study it is assumed, by dividing *Total Importation Costs* (column B) to the *Quantity of Importations* (column C) of the same year, an *Importation Cost per Operation* (column D) in a year can be found. By calculating the arithmetic mean of these *Importation Cost per Operation* for three years, *A* can assumed to be found in Equation 4.2 as:

$$A = \frac{(€507+€679+€738)}{3} \cong €640 \tag{4.2}$$

The *A* is regardless of the amount. In literature, this cost consists of the clerical work done to order a specific item, the cost of man-hour to place and receive that order, loading and unloading works for each lot and etc. But in the current study, it is again needed to make a practical assumption to calculate the a. Dividing the average of the *Total Importation Costs* to the sum of the *Annual Demand Rates* $(\sum \lambda_i)$ of the products, a can assumed to be found in Equation 4.3 as:

$$a = \frac{\sum Total\ \ Importation\ \ \ Costs\ /\ 3}{\sum_i^{10} \lambda_i} = \frac{(€8.115+€9.500+€11.803)/3}{19+15+23+39+8+8+12+5+9+4} \cong €70 \qquad (4.3)$$

Therefore, it is calculated that A is €640 and a is € 70.

Table 4.2. Total importation costs for DEPA pumps

A	B	C	D
Year	Total Importation Costs[10]	Quantity of Importation operations in a year	Importation Cost / Operation
2004	€ 8,115	16	€ 507
2005	€ 9,500	14	€ 679
2006	€ 11,803	16	€ 738

4.4.2. Inventory Carrying Cost

Inventory carrying costs are incurred as a result of holding inventories and increase in direct proportion to increases in inventory on-hand and the time for which inventoried item are held. The following components of the inventory carrying cost are the real out of pocket costs:

(i) Storage or warehouse rental costs
(ii) The costs of operating the warehouse such as light, heat, night watchmen etc.
(iii) Clerical and administrative costs
(iv) Insurance and taxes on inventory held
(v) Costs of depreciation, deterioration and obsolescence of inventory.

A cost which is frequently the most important is not direct out of pocket cost but rather an opportunity cost which would never appear on an accounting statement. As stated earlier, Leon Teknik is holding very limited stocks for emergency requests and it is not

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[10] Import Costs = Freight + Custom Expenses (excluding Customs Duties)
Freight: International Transport Costs
Custom Expenses: Bonded Warehousing + Portage Handling + Importing Services + Other

able to make observations to calculate inventory carrying costs. Therefore, the most common convention of costing[11] is to use

$$Carrying\ costs\ per\ year = \bar{I}vr \tag{4.4}$$

Where $\bar{I}$ is the average on hand inventory in units (hence $\bar{I}v$ is the average inventory expressed in €), and r is the carrying charge, the cost in € of carrying one € of inventory for one year (Silver, Pyke, & Peterson, Costs and Other Important Factors, 1998).

Average on hand inventory levels of each product will be obtained from the simulation and will be used for inventory carrying cost calculations. But before simulating the model, carrying charge and unit costs of each item has to be given. Unit costs were already given in Table 4.1 earlier. It is observed from the TCMB[12]'s website that average annual interest rates are around %10 since 2005. Therefore, it is decided to state carrying charge as in Equation 4.5. As a consequence, Table 4.3 is used to calculate the total inventory carrying cost.

$$r = 0{,}1\ €/€/yr \tag{4.5}$$

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[11] This method of costing is called cost of capital and incurred by having capital tied up in inventory rather than having it invested elsewhere. It reflects the lost earning power of capital and is equal to the largest rate of return which the system could obtain from alternative investments.

[12] Türkiye Cumhuriyeti Merkez Bankası (Central Bank of the Republic of Turkey)

Table 4.3. Carrying Costs

i	λ_i (pieces. / yr.)	v_i (€/pieces.)	$v_i \times r$ (€* yr. / pieces)
1	19	4,320	432
2	15	3,422	342
3	23	1,761	176
4	39	913	91
5	8	2,546	254
6	8	1,971	197
7	12	1,168	116
8	5	2,761	276
9	9	1,605	160
10	4	3,205	320

4.5. Mathematical Statement of the Decision Problem

In Section 4.5 and Section 4.6, we follow the method of the cited paper (Silver E. A., 1974) to mathematically state the problem and to compute sensible initial parameters.

Using the above assumptions and notation, the expected relevant cost per year for item i are given by Equation 4.6.

$$EC_i = \bar{I}_i v_i \, r + NT_i A + N_i a \tag{4.6}$$

The first term represents the inventory carrying costs and the other two the set-up charges allocated to item i. The total expected relevant costs per year of the group of items are given by

$$EC = \sum_{i=1}^{n} EC_i = \sum_{i=1}^{n} [\bar{I}_i v_i \, r + NT_i A + N_i a_i] \tag{4.7}$$

The service constraint takes one of two forms depending upon the measure of service used. These are:

1- PNSPRC: Probability of no shortage per replenishment cycle.
2- FDS: Fraction of demand satisfied directly from shelf.

It is vitally important to note here that; except of *Periodic Review,* during this study *Continuous Review* will be used for monitoring inventory levels. Therefore the second choice, FDS will be used to measure service quality. But it is compulsorily required to formulate and use PNSPRC as a base to compute $\bar{Q}_i$and s_i values.

4.5.1. Probability of no shortage per replenishment cycle

For item i we want[13]

$$PNSPRC_i \geq P_1 \quad (i = 1, 2, \dots, n) \tag{4.8}$$

If we let $U_i$be the event of no shortage of item i in a particular replenishment cycle, then

$$PNSPRC_i = p(U_i) = \sum_{w_0=s_i}^{c_i} p(U_i|\, w_0) p_w{}^i(w_0) \tag{4.9}$$

Now, a shortage occurs if and only if the lead time demand, which is Poisson with parameter $\lambda_i L$, is greater than the available inventory level of the item when the order is placed, i.e.

$$p(U_i|w_0) = p_{po\leq}(w_0|\lambda_i L) \tag{4.10}$$

Therefore Equation 4.8 becomes:

$$\sum_{w_0=s_i}^{c_i} p_{po\leq}(w_0|\lambda_i L)\, p_w{}^i(w_0) \geq P_1 \qquad (i = 1, 2, \dots, n) \tag{4.11}$$

---

[13] There would be no difficulty in handling the case where the desired probability was P_1, i.e. it depended on the specific item i.

4.5.2. Fraction of Demand Satisfied Directly From Shelf

The expected shortage of item i per replenishment cycle is

$$ESPRC_i \sum_{w_0=s_i}^{c_i}\left[\sum_{x_0=w_0+1}^{\infty}(x_0 - w_0)p_{po}\,(x_0|\lambda_i L)\right] p_w{}^i(w_0) \qquad (4.12)$$

The quantity in square brackets is the expected shortage in a cycle given that the item is ordered when its available stock is at the level w_0. Now the fraction of demand satisfied directly from shelf is, at least to a first approximation,

$$FDS_i = 1 - \frac{ESPRC_i}{\bar{Q}_i} \qquad (4.13)$$

Therefore, with the service measure we want using Equations 4.12 and 4.13.

$$1 - \frac{1}{\bar{Q}_i}\sum_{w_0=s_i}^{c_i}\sum_{x_0=w_0+1}^{\infty}(x_0 - w_0)p_{po}\,(x_0|\lambda_i L)\,p_w{}^i(w_0) \geq P_2 \qquad (4.14)$$

We can now state our problem mathematically as follows: Select the (S)'s, (c)'s and (s)'s (i.e. the values of $3n$ different variables) so as to minimize EC of Equation 4.7 subject to the n constraints of either Equation 4.11 or 4.14. The quantities $\bar{I}_i,\ NT_i,\ N_i,\ p_w{}^i(w_0)$ and $\bar{Q}_i$ depend upon one or more (most often, all) of the (S)'s, (c)'s and (s)'s in a rather complex fashion. It is clear that this is an extremely complicated optimization problem. But it should be stated here that, rather than optimization, the main focus of this thesis is finding a practical and sensible way to increase customer service levels with low costs.

4.6. Initial Parameter Computations

In the beginning of this section, it is important to state the main features of the mathematical solution procedure again: Instead of computing optimal (s, c, S) parameters, the main purpose will be computing sensible initial (s, c, S) parameters as the input of simulation phase. Starting the simulation with sensible initial parameters would bring us convenience and let us find good solutions.

In Section 4.6.1 of the study, independent control case will be considered at first to compute near-optimal (s, S) parameters by using the simple heuristic (defined by (Silver E. A., 1974)). Following, in section 4.6.2 coordinated control case will be considered. In the coordinated control case, the third parameter (c) will be computed with an approximation by using the near-optimal (s, S) pairs. Rather than optimizing with very complex computations of $3n$ different variables for 10 different product types, using the independent control case at first and then using an approximation to compute (s, c, S) parameters will bring convenience.

4.6.1. Independent Control Case

The case of independent control must be analyzed for purposes of comparison and computations of initial parameters. Independent control of item i is a special case of (s, c, S) control where we set $c_i = s_i$, i.e. the item is never ordered until it hits its must-order point. The order quantity is always of size $S_i - s_i$. Furthermore, no other item is ever included in replenishment; hence a set-up cost $(A + a)$ is incurred with replenishment.

For a single item i there are 3 decision variables, namely S_i, c_i and s_i. Recognizing that $c_i = s_i$, Equations 4.6, 4.11 and 4.14 become

$$\text{Minimize } EC_i = \left(\left(\left(\frac{S_i - s_i + 1}{2} \right) + \ (s_i - \lambda_i L) \right) v_i r + \frac{(A + a\ )\lambda_i}{S_i - s_i} \right) \tag{4.15}$$

Subject to

$$p_{po\le}(s_i | \lambda_i L) \ \ge P_1 \tag{4.16}$$

Or

$$1 - \frac{1}{S_i - s_i} \sum_{\mathrm{x}_0 = s_i + 1}^{\infty} (s_0 - s_i)\, p_{po}\,(x_0 | \lambda_i L) \ge P_2 \tag{4.17}$$

Strictly speaking, the pair (S_i, s_i) which minimizes Equation 4.15 while satisfying either Equation 4.16 or 4.17 should be found. However, it has been observed (see, e.g. Silver & Wilson, Cost Penalties of Simplified Procedures for Selecting Reorder Points and Order Quantities, 1972) that, under usual conditions, a much simpler sequential approach does almost as well from the standout of reducing replenishment and carrying costs. In this simple approach we first select the order quantity $\bar{Q}_i = S_i - s_i$ which minimizes EC_i, ignoring the constraints given with Equation 4.16 or 4.17. Then, given this value of $S_i - s_i$, the constraint is used to find the lowest allowable value of s_i. The first step leads to the usual economic order quantity expression

$$\bar{Q}_i = S_i - s_i = \sqrt{(2(A+a)\lambda_i / v_i r)} \tag{4.18}$$

In general this will not be an integer. Therefore, we test the two integer values of Q_i around $\hat{Q}_i$. The one giving the lowest value of EC_i in Equation 4.15 is used.

Equation 4.16 shows that, for the service measure being the probability of no shortage per cycle, the s_i value does not depend upon S_i; we solve for s_i using the cumulative distribution of the Poisson variable. For the other service measure, that from equation 4.17 it follows that the lowest s_i which satisfies the following inequality should be used as in Equation 4.19:

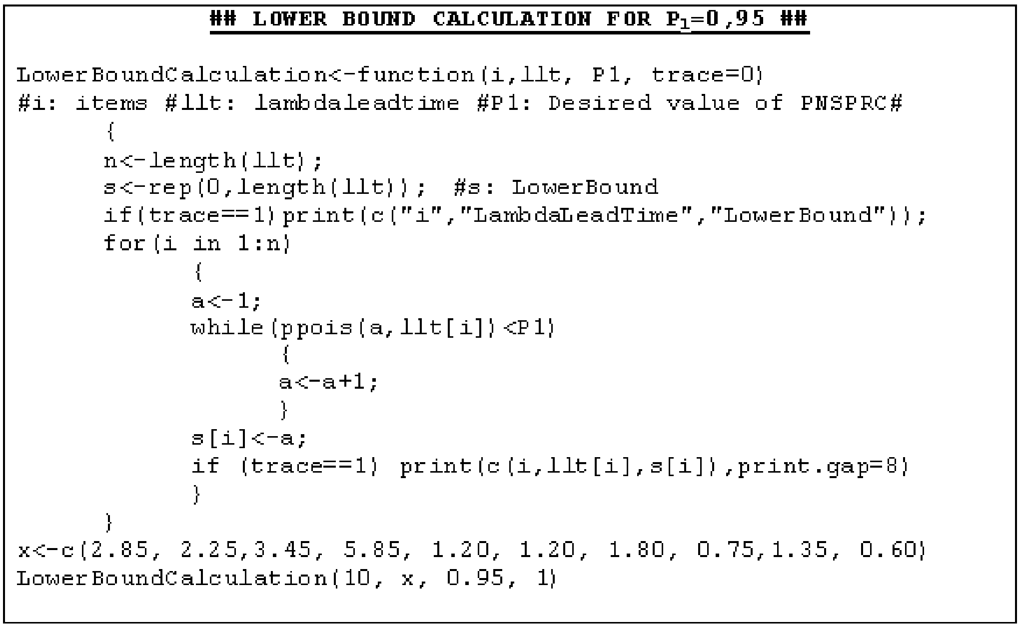
$$\lambda_i L - s_i - \lambda_i L p_{po\leq}(s_i - 1|\lambda_i L) + s_i p_{po\leq}(s_i|\lambda_i L) \leq Q_i(1 - P_2) \tag{4.19}$$

4.6.1.1. Computations for the Given P_1 Service Level. Consider the following case involving ten items:

Let consider, $P_1 = 0{,}95\,; A = €640; a = €70; r = 0{,}20\ €/€/year$ and finally for $L = 0{,}15$ year. As assumed before, the lead time is uniformly distributed between 6 to 9 weeks. For simplicity, during initial parameter calculations the mean of the distribution is taken.

We illustrate the determination of s_i using the first item. $\lambda_1 L = 19 \times 0{,}15 = 2{,}85$. Equation 4.16 says to select the smallest value of s_1 such that $p_{po\leq}(s_1|2{,}85) \geq 0{,}95$. Using R (Statistical Computing Program) code to compute s_i values which shown in Figure 4.3 we found $s_1 = 6$. The rest of the computed s_i values are given in Table 4.4.

Equation 4.18 is also coded with the R as shown in Figure 4.4, to compute the $\hat{Q}_i$ entries which are given in Table 4.4.

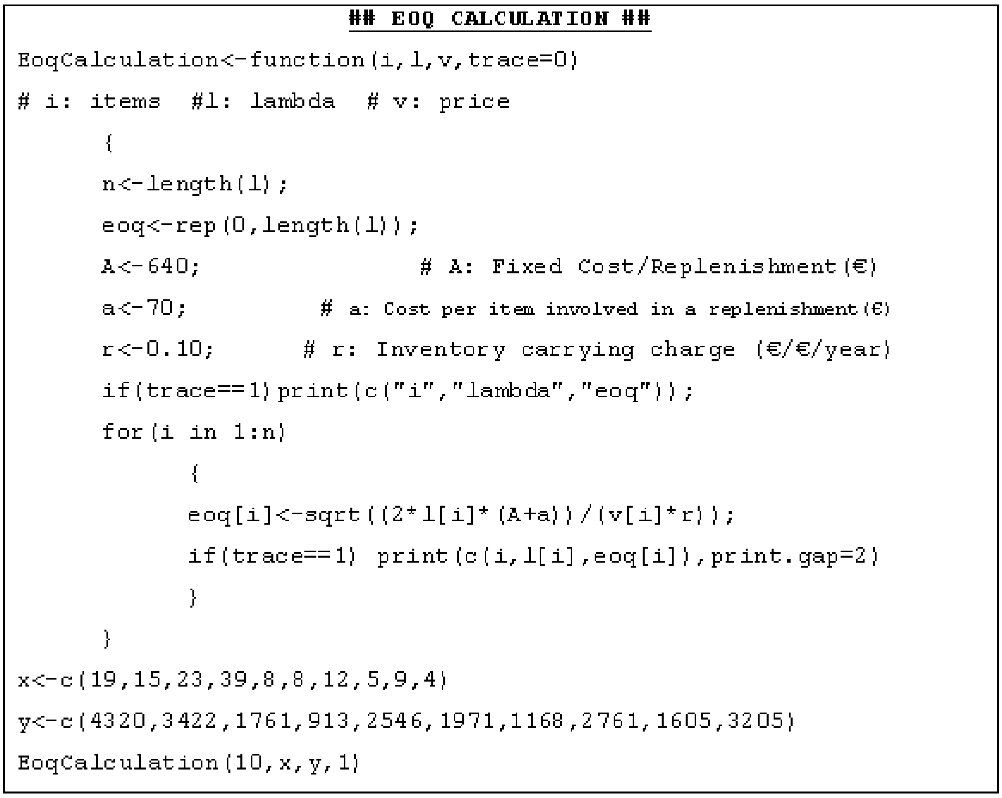


```
## LOWER BOUND CALCULATION FOR P1=0,95 ##

LowerBoundCalculation<-function(i,llt, P1, trace=0)
#i: items #llt: lambdaleadtime #P1: Desired value of PNSPRC#
        {
        n<-length(llt);
        s<-rep(0,length(llt)); #s: LowerBound
        if(trace==1)print(c("i","LambdaLeadTime","LowerBound"));
        for(i in 1:n)
                {
                a<-1;
                while(ppois(a,llt[i])<P1)
                        {
                        a<-a+1;
                        }
                s[i]<-a;
                if (trace==1) print(c(i,llt[i],s[i]),print.gap=8)
                }
        }
x<-c(2.85, 2.25,3.45, 5.85, 1.20, 1.20, 1.80, 0.75,1.35, 0.60)
LowerBoundCalculation(10, x, 0.95, 1)
```

Figure 4.3. R code for the lower bound calculation for the P_1 fraction

The value of $S_i - s_i$ and s_i are then substituted into Equation 4.15 to obtain the EC_i values shown in Table 4.4. The total annual expected cost per year is seen to be € 26.175. Actually, this theoretical calculation of expected cost is just for finding the best integer values of Q_i and will not be considered. With simulation applications, we are expecting to find lower costs for the independent case.



```
## EOQ CALCULATION ##
EoqCalculation<-function(i,l,v,trace=0)
# i: items  #l: lambda  # v: price
      {
      n<-length(l);
      eoq<-rep(0,length(l));
      A<-640;               # A: Fixed Cost/Replenishment(€)
      a<-70;           # a: Cost per item involved in a replenishment(€)
      r<-0.10;       # r: Inventory carrying charge (€/€/year)
      if(trace==1)print(c("i","lambda","eoq"));
      for(i in 1:n)
            {
            eoq[i]<-sqrt((2*l[i]*(A+a))/(v[i]*r));
            if(trace==1) print(c(i,l[i],eoq[i]),print.gap=2)
            }
      }
x<-c(19,15,23,39,8,8,12,5,9,4)
y<-c(4320,3422,1761,913,2546,1971,1168,2761,1605,3205)
EoqCalculation(10,x,y,1)
```

Figure 4.4. R code for EOQ calculation

Table 4.4. Results for $P_1=0.95$

i	$\lambda_i L$	s_i	$\hat{Q}_i$ **(Best Non Integer)**	Q_i **(Up Rounded Integer)**	EC_i **(For Up Rounded Integer)**	Q_i **(Down Rounded Integer)**	EC_i **(For Down Rounded Integer)**	$Q_i = S_i - s_i$ **(Best Integer Selection)**	EC_i **(For Best Integer)**
1	2.850	6	7.903	8	€ 4,991.05	7	€ 5,015.94	8	€ 4,991.05
2	2.250	5	7.890	8	€ 3,812.20	7	€ 3,831.28	8	€ 3,812.20
3	3.450	7	13.618	14	€ 3,112.33	13	€ 3,114.01	14	€ 3,112.33
4	5.850	10	24.629	25	€ 2,673.40	24	€ 2,673.90	25	€ 2,673.40
5	1.200	3	6.680	7	€ 2,288.11	6	€ 2,296.05	7	€ 2,288.11
6	1.200	3	7.592	8	€ 1,951.73	7	€ 1,954.61	8	€ 1,951.73
7	1.800	4	12.079	13	€ 1,729.94	12	€ 1,726.16	12	€ 1,726.16
8	0.750	2	5.071	6	€ 1,903.14	5	€ 1,883.43	5	€ 1,883.43
9	1.350	3	8.923	9	€ 1,777.33	8	€ 1,785.83	9	€ 1,777.33
10	0.600	2	4.210	5	€ 1,978.20	4	€ 1,959.95	4	€ 1,959.95
								$\sum EC_i=$ € 26,175.68	

4.6.1.2. Computations for the Given P2 Service Level. We use the same data as in previous section except that the service measure is now the fraction of demand satisfied directly from shelf. Suppose that the desired service level is 0.99 for all 10 items.

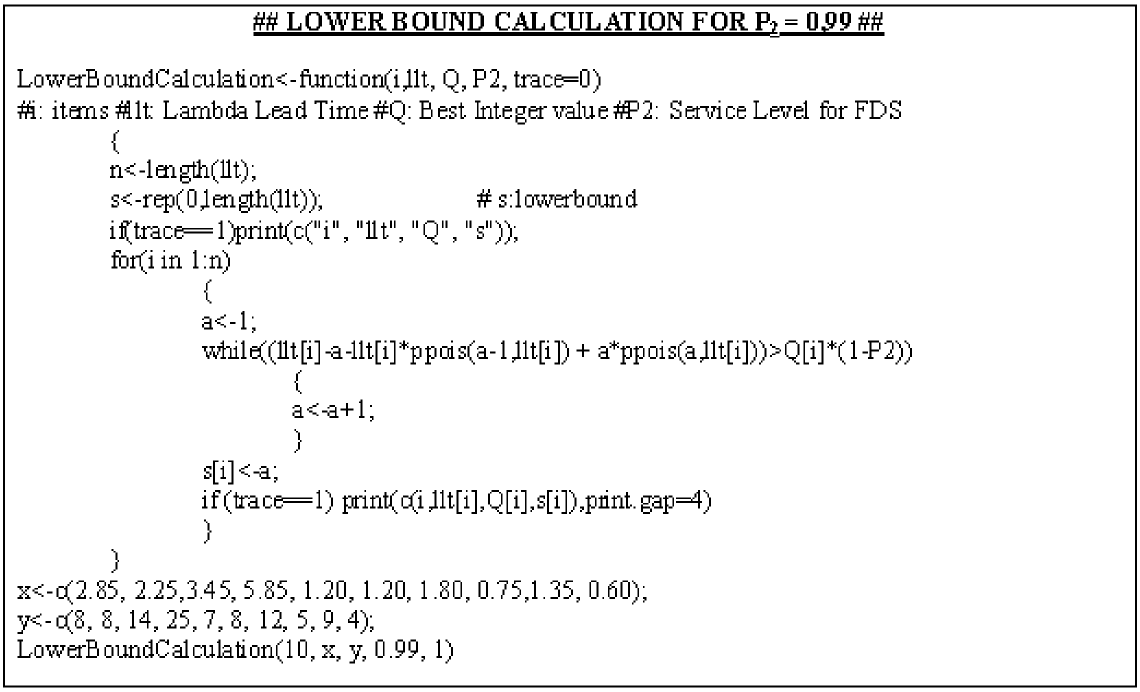
The best integer Q_i values are unaffected by the service measure. Hence they are the same as in Section 4.6.1.1.

The computation of s_i is again illustrated for item 1. We will use Equation 4.19 in computing. Equation 4.19 becomes:

$$2{,}85 - s_1 - 2{,}85 p_{po\leq}(s_1 - 1|2{,}85) + s_1 p_{po\leq}(s_1|2{,}85) \leq 8 \times (1 - 0{,}99) \quad (4.20)$$

The smallest $\boldsymbol{s_1}$ which satisfies this inequality is 6. The R code to compute $\boldsymbol{s_i}$ values is shown in Figure 4.5. All computed $\boldsymbol{s_i}$ values are shown in Table 4.5. The $\boldsymbol{EC_i}$ values calculated by substituting $(\boldsymbol{S_i - s_i})$ and $\boldsymbol{s_i}$ values into Equation 4.15 are again shown in Table 4.5.

We see that the total expected costs per year for the ten items are € 25,908.28. Again, this theoretical calculation of expected cost is just to find the (s, S) parameters.



```
## LOWER BOUND CALCULATION FOR P2 = 0,99 ##

LowerBoundCalculation<-function(i,llt, Q, P2, trace=0)
#i: items #llt: Lambda Lead Time #Q: Best Integer value #P2: Service Level for FDS
        {
        n<-length(llt);
        s<-rep(0,length(llt));                    # s:lowerbound
        if(trace==1)print(c("i", "llt", "Q", "s"));
        for(i in 1:n)
                {
                a<-1;
                while((llt[i]-a-llt[i]*ppois(a-1,llt[i]) + a*ppois(a,llt[i]))>Q[i]*(1-P2))
                        {
                        a<-a+1;
                        }
                s[i]<-a;
                if(trace==1) print(c(i,llt[i],Q[i],s[i]),print.gap=4)
                }
        }
x<-c(2.85, 2.25,3.45, 5.85, 1.20, 1.20, 1.80, 0.75,1.35, 0.60);
y<-c(8, 8, 14, 25, 7, 8, 12, 5, 9, 4);
LowerBoundCalculation(10, x, y, 0.99, 1)
```

Figure 4.5. R code for lower bound calculation for the P_2 fraction

Table 4.5. Results for $P_2 = 0.99$

i	$\lambda_i L$	$Q_i = S_i - s_i$ (Best Integer)	s_i	EC_i
1	2.850	8	6	€ 4,991.05
2	2.250	8	5	€ 3,812.20
3	3.450	14	6	€ 2,936.23
4	5.850	25	9	€ 2,582.10
5	1.200	7	3	€ 2,288.11
6	1.200	8	3	€ 1,951.73
7	1.800	12	4	€ 1,726.16
8	0.750	5	2	€ 1,883.43
9	1.350	9	3	€ 1,777.33
10	0.600	4	2	€ 1,959.95
			$\sum EC_i$ =	€ 25,908.28

4.6.2. Coordinated Control Case

In Section 4.6.1.2 s_i and Q_i values were given in Table 4.5. By substituting S_i with Q_i in Equation 4.18 all S_i values are calculated by

$$S_i = Q_i + s_i \tag{4.21}$$

For the coordinated control case, the third parameter (c) will be computed by an approximation. By using the calculated (s, S) pairs, (c) is correlated between these pairs. It is assumed that the can order point of each item is equal to:

$$c_i = s_i + 0{,}4 \times (S_i - s_i) \tag{4.22}$$

For desired values of FDS three different levels of P_2 are defined with 90 per cent, 95 per cent and 99 per cent. With the given desired values of P_2 customer service level, (s, c, S) parameters are computed by the following rules and explanations stated in previous sections (parameter computations with P_2=99 for only (s, S) pairs has already been illustrated in Section 4.6.1.2). The Results are summarized in Table 4.6.

Table 4.6. Results of initial parameter computations for P_2 service levels

i	P_2=0.90			P_2=0.95			P_2=0.99		
	s_i	c_i	S_i	s_i	c_i	S_i	s_i	c_i	S_i
1	3	7	11	4	8	12	6	10	14
2	2	6	10	3	7	11	5	9	13
3	3	9	17	4	10	18	6	12	20
4	4	14	29	6	16	31	9	19	34
5	1	4	8	2	5	9	3	6	10
6	1	5	9	2	6	10	3	7	11
7	1	6	13	2	7	14	4	9	16
8	1	3	6	1	3	6	2	4	7
9	1	5	10	2	6	11	3	7	12
10	1	3	5	1	3	5	2	4	6

5. MODELING AND SIMULATING THE PROBLEM WITH ARENA

To analyze the effects of inventory control policies under different conditions and situations, simulation models are developed in Arena Simulation Program. The basic time unit used in simulations is day. Each simulation is run for 6.000 days (20 years) with 100 replications and estimates the following statistics.

- Annual Total Ordering Costs
- Annual Total Holding Costs
- Annual Total Inventory Costs
- Fill Rate

In Section 5.1 of the study, actions of *Independent Control Policy* are implemented with an Arena model. Similarly in Section 5.2, *Coordinated Control Policy* is implemented with Arena. In Section 5.3, how do arena models collect statistics data is described. Finally, in Section 5.4 constructed Arena Models are verified.

5.1. Programming Independent Control Policy Model

The Arena Model of the *Independent Control Policy* has three main segments which are combined together; first one is *Customer Demand Generation*, the second is *Replenishments Management* and the last is *Inventory Management*.

The Model also has *Customer Demand* and *Order to Plant* entities. They are both circulating together and monitoring on-hand-inventory and inventory-position. These entities also decrement or increment inventory levels. *Order to Plant* entity also gives an order to plant when necessary.

The *Customer Demand Generation* segment is generating multiple types of customers and their demands. It also models the drawing down of on hand inventories. The *Replenishments Management* segment is initiating orders to production plant for each product type as necessary and increases the inventories of related items after replenishment arrivals. The *Inventory Management* segment has multiple inventory types with shared storage. Figure 5.1 is showing *Independent Control Policy Model*. We next examine the model logic of each segment.

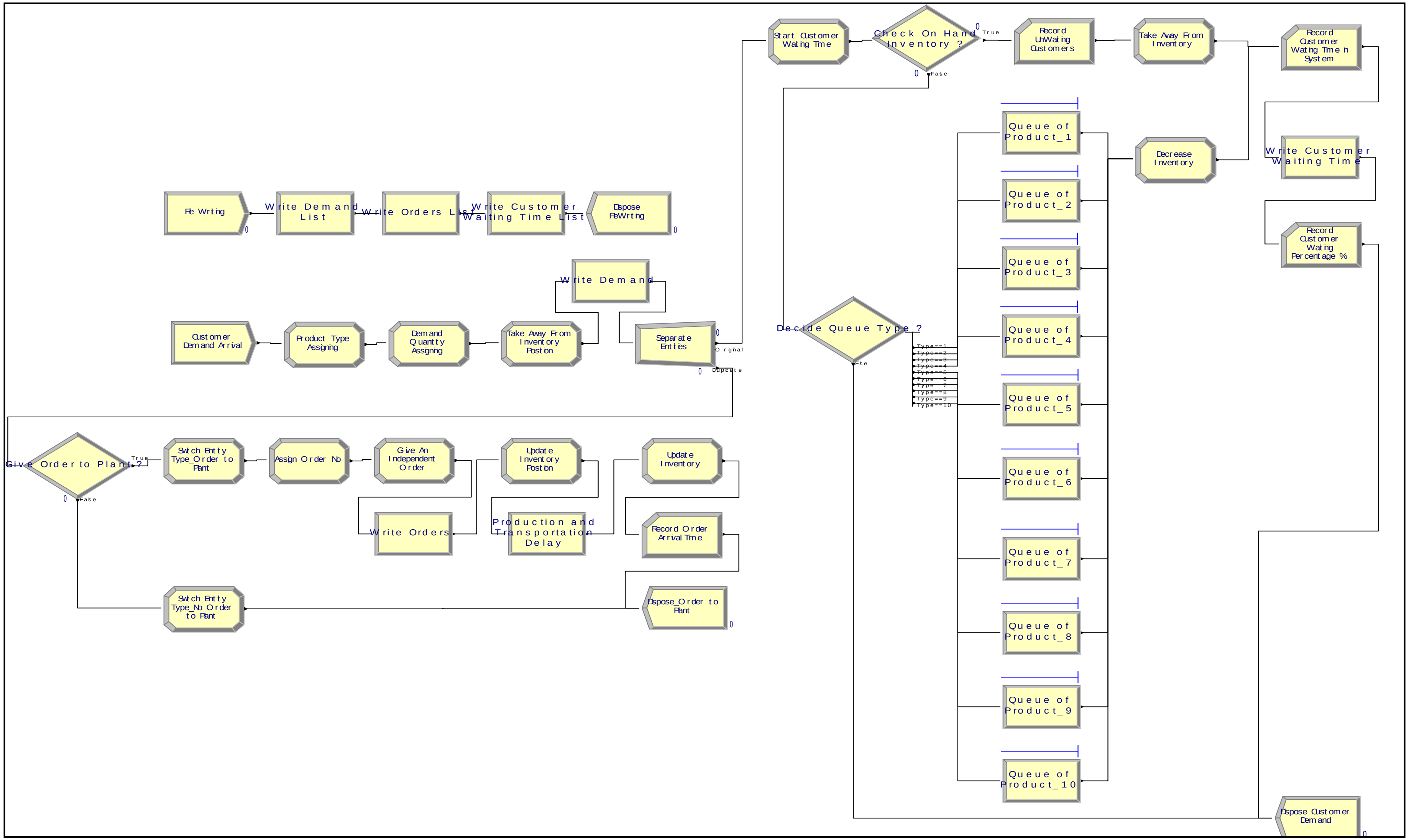


Figure 5.1 Independent control policy model

5.1.1. Customer Demand Generation Segment

The *Customer Demand Generation* segment generates multiple types of customers and their demands (see Figure 5.2). This segment models customer arrivals, demand assignment and decrements from inventory position.

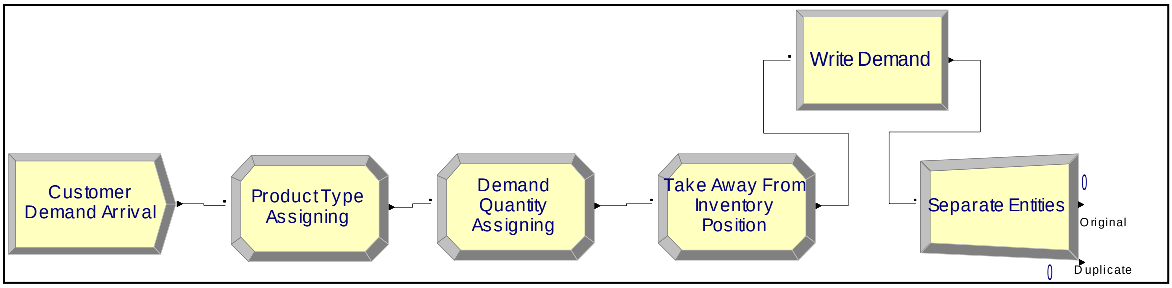


Figure 5.2. Customer demand generation segment

The create module called *Customer Demand Arrival* creates *Customer Demand* entities following Poisson process. For Poisson process, the probability distribution of the waiting time until the next occurrences is an exponential distribution. Waiting time until a next arrival comes called; *time between arrivals*. We show the calculation of the input value of *time between arrivals* in Table 5.1.

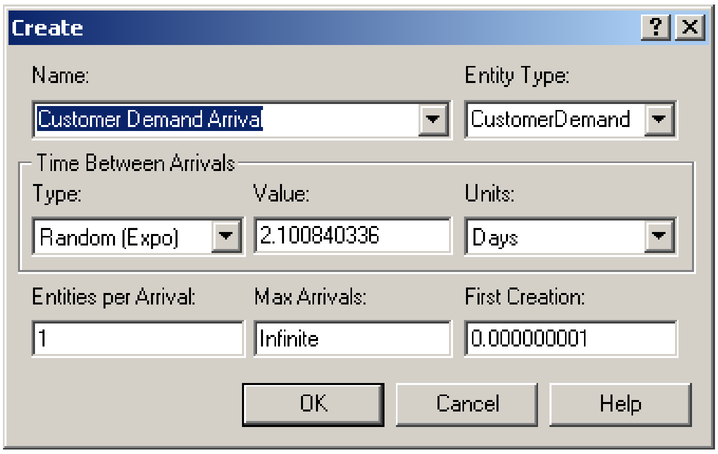
In Section 4, λ_i values are given. For simulation purposes; annual λ_i values are converted to daily value as shown in Equation 5.1.

$$\lambda_i\left(\frac{pcs}{day}\right) = \frac{\lambda_i\left(\frac{pcs}{year}\right)}{300\ days} \tag{5.1}$$

By dividing one over the summation of daily λ_i rate $\frac{1}{\sum \lambda_i} = \frac{1}{0{,}476} = 2.10084$, the time between arrivals is set to 2.10084 days (see Figure 5.3). There is no limitation for maximum arrival quantity. In conclusion, arena will generate a single entity for every customer arrival which follows Poisson Process.

Table 5.1. Calculation of time between arrivals

i Item	λ (Pieces / Year)	λ (Pieces / Day)	$1/\lambda$ Time Between Arrivals (Day)	$\frac{\lambda_i}{\sum\lambda_i}$	$\sum_{n=1}^{10}\frac{\lambda_i}{\sum\lambda_i}$
1	19	0.064	15.63	0.13445	0.1344538
2	15	0.050	20.00	0.10504	0.2394958
3	23	0.077	12.99	0.16176	0.4012605
4	39	0.130	7.69	0.27311	0.6743697
5	8	0.027	37.04	0.05672	0.7310924
6	8	0.027	37.04	0.05672	0.7878151
7	12	0.040	25.00	0.08403	0.8718487
8	5	0.017	58.82	0.03571	0.9075630
9	9	0.030	33.33	0.06303	0.9705882
10	4	0.014	71.43	0.02941	1
	$\boldsymbol{\sum\lambda_i =}$	**0.476**	**1/0.476 = 2.10084**		


Create

Name: Customer Demand Arrival

Entity Type: CustomerDemand

Time Between Arrivals

Type: Random (Expo)

Value: 2.100840336

Units: Days

Entities per Arrival: 1

Max Arrivals: Infinite

First Creation: 0.000000001

Figure 5.3. Dialog box of customer demand arrival module

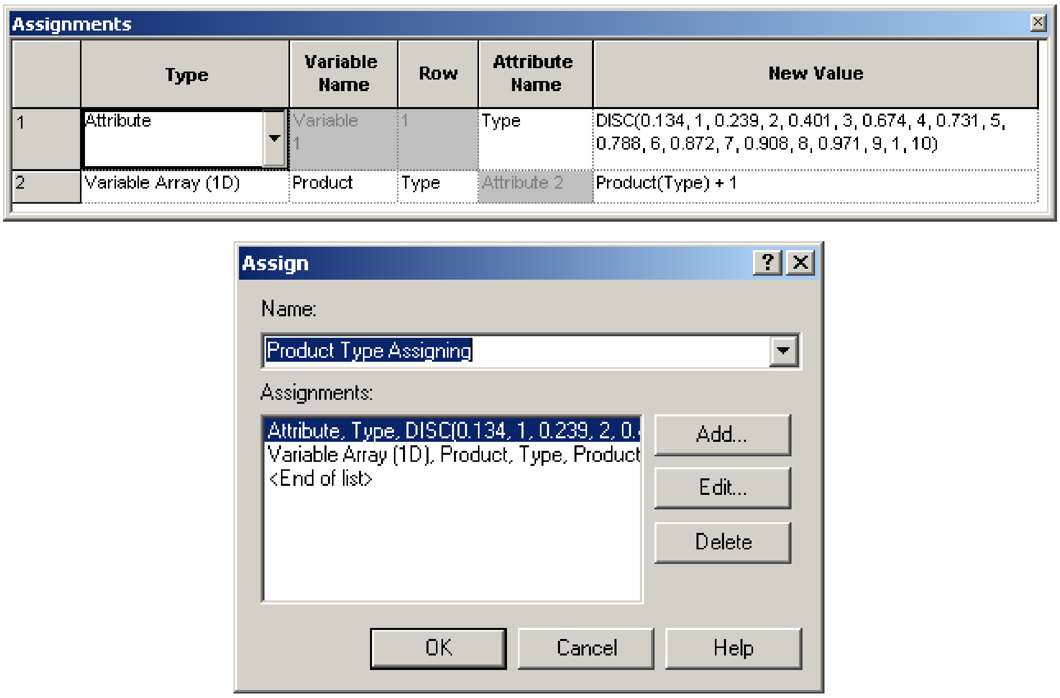
Following, *customer demand* entity enters an assign module named *Product Type Assigning* (see Figure 5.4). In this module an *attribute* is assigned called *Type* for the circulating *Customer Demand* entity and a special Arena function is set:

$$DISC(0.134, 1, 0.239, 2, 0.401, 3, 0.674, 4, 0.731, 5, 0.788, 6, 0.872, 7, 0.908, 8, 0.971, 9, 1, 10)$$

The *Disc* function recognizes the arriving entity generated in the previous create module, places it between the ranges given by $\Sigma(\lambda_i/\Sigma\lambda_i)$ values which shown in the Table 5.1 and assign to it the integer $i = 1,2, \dots 10$ as the *Type* attribute. Following, a one digit *Variable Array* is assigned called *Product* and selected the previous *Type* attribute as the row of this *Product* variable. A final assignment has done in Equation 5.2 as:

$$Product_{Type} = Product_{Type} + 1 \tag{5.2}$$

For each circulating Customer Demand entity labeled with $\mathrm{Product}_{\mathrm{Type}}$ variable, this assignment increment the value of the variable by one to calculate how many customers have arrived for each product type.



Assignments

	Type	Variable Name	Row	Attribute Name	New Value
1	Attribute	Variable 1	1	Type	DISC(0.134, 1, 0.239, 2, 0.401, 3, 0.674, 4, 0.731, 5, 0.788, 6, 0.872, 7, 0.908, 8, 0.971, 9, 1, 10)
2	Variable Array (1D)	Product	Type	Attribute 2	Product(Type) + 1

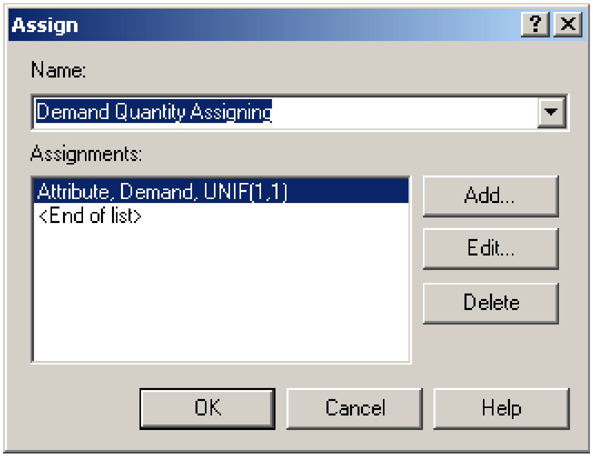
Assign

Name: Product Type Assigning

Assignments:
Attribute, Type, DISC(0.134, 1, 0.239, 2, 0.
Variable Array (1D), Product, Type, Product
<End of list>

Figure 5.4. Dialog box of product type assigning module

Following, *Customer Demand* entity enters another assign module called *Demand Quantity Assigning* shown in Figure 5.5. As underlined in previous sections, demand size is assumed as unite size. Therefore, this module samples a unit size demand and assigns it to the *Customer Demand* entity as an attribute called *Demand.* Unif (1,1) is used due to flexibility. By giving different integer values, demand size would easily being adapted to different requirements.



Name:

Demand Quantity Assigning

Assignments:

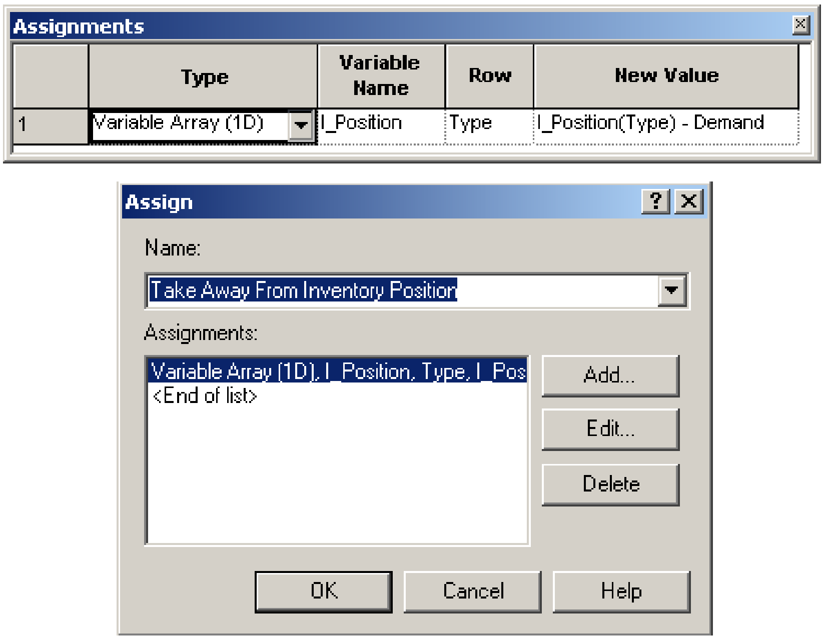
Attribute, Demand, UNIF(1,1)
<End of list>

Figure 5.5. Dialog box of demand quantity assigning module

The availability of sufficient stock will not be considered at this stage, keeping in mind that the product type information is stored in the *Type* attribute of the *Customer Demand* entity and this entity now enters an assign module called *Take Away From Inventory Position* where the corresponding inventory position type is decremented by the demand quantity (see Figure 5.6).

The Assignments expression is: $I\ Position_{Type} = I\ Position_{Type} - Demand$ where Type option in the row field indicates an assignment to a vector element. The inventory position level of each product is modeled by a vector called $\mathrm{I\ Position}_{\mathrm{Type}}$ which is indexed by product type. To update the inventory position level of the current product type, the corresponding inventory position level is simply decremented by the demand quantity.

Then, *Customer Demand* entity next enters a separate module called *Separate Entities*. The name of the original entity has remained same as *Customer Demand* and followed the way through to the decide module called *Check On Hand Inventory*, and the duplicated one then enters the decide module called *Give Order to Plant.*



Assignments

	Type	Variable Name	Row	New Value
1	Variable Array (1D)	I_Position	Type	I_Position(Type) - Demand

Assign

Name: Take Away From Inventory Position

Assignments:
Variable Array (1D), I_Position, Type, I_Pos
<End of list>

Figure 5.6. Dialog box of take away from inventory module

A method for changing the incoming entity into multiple outgoing entities is provided through the Separate module. This module has two methods for separating entities. First, an entity that has been temporarily batched in the Batch module may be split into its individual members by using the Split Existing Batch option. An incoming entity may be cloned into copies of the original entity by using the Duplicate Original option. (Altıok & Melamed, 2001) In our case, we have to choose the second option.

5.1.2. Replenishment Management Segment

The *Replenishment Management* segment is initiating orders to production plant for each product type as necessary and increases the inventories of related product types after replenishment arrivals. This segment also models the production & transportation delays. Figure 5.7 shows this segment.

As described in Section 5.1.1 the *Customer Demand* entity duplicated itself. Rather than the original entity, the duplicated one enters the decide module called "*Give Order to Plant?"* Decide module checks whether the reorder level is subsequently down-crossed or not (see Figure 5.8). Note that the logical expression in the value and if condition field compares two vector elements: the inventory position level *(I Position)* of the currently requested product type and the reorder level *(s small)* of the same product type. Two cases are then possible:

(i) Inventory position level of the product type is larger than the reorder level, or

(ii) Inventory position level of the product type is lower or equal to the reorder level and this triggers an order to the production plant.

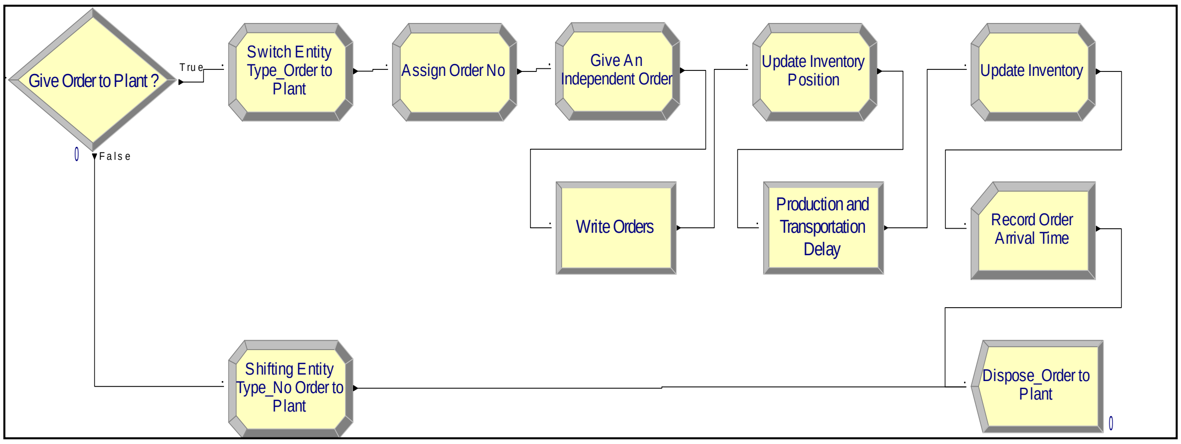
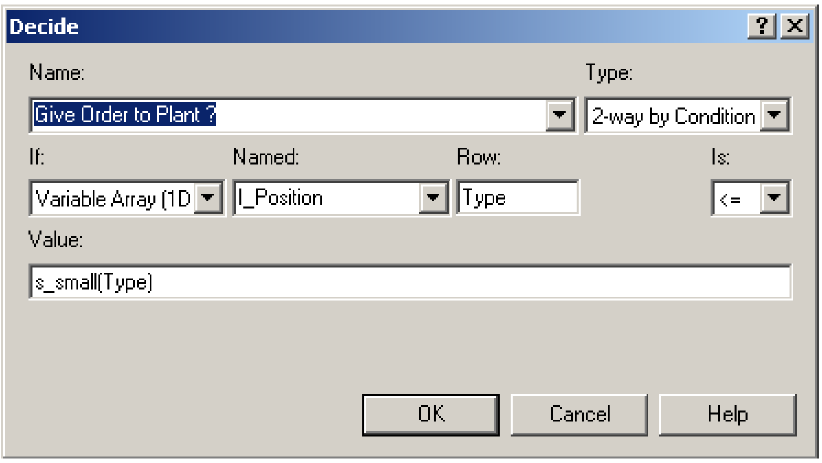


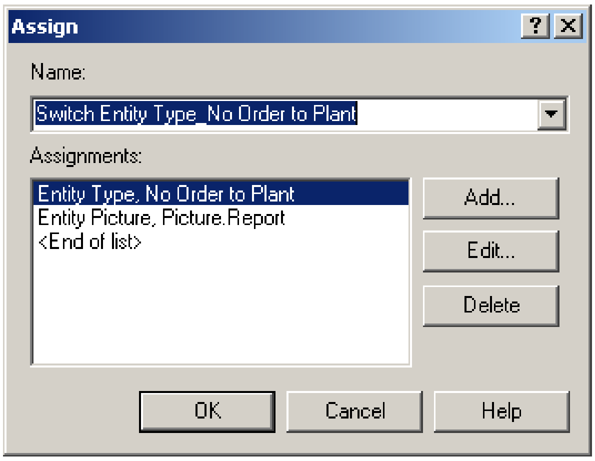
Figure 5.7. Replenishment management segment



Name: Give Order to Plant ?
Type: 2-way by Condition
If: Variable Array (1D
Named: I_Position
Row: Type
Is: <=
Value: s_small(Type)

Figure 5.8. Dialog box of give order to plant module

Consider the first case; if the position of the inventory is sufficient, then the duplicated entity proceeds to the assign module called "*Shifting Entity Type No Order to Plant*" to switch the *Entity* type as "*No Order to Plant*" (shown in Figure 5.9.). Following then, the renamed entity "*No Order to Plant*" enters the dispose module called "*Dispose Order to Plant*" and exits the circulation.

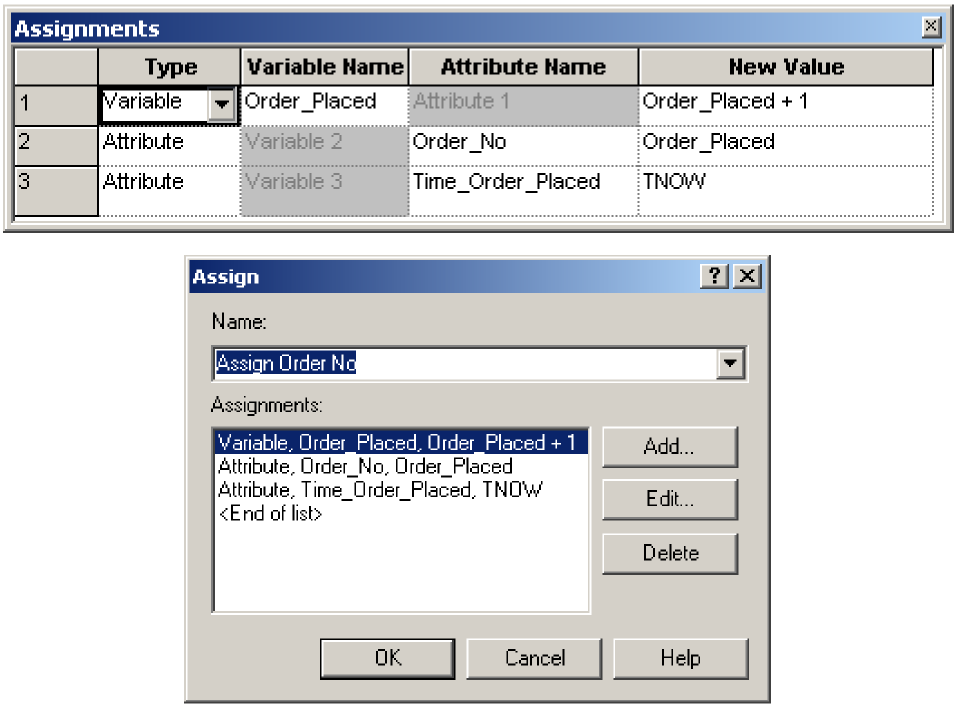


Name:
Switch Entity Type_No Order to Plant

Assignments:
Entity Type, No Order to Plant
Entity Picture, Picture.Report
<End of list>

Figure 5.9. Dialog box of "*switch entity type_no order to plant" module*

If we consider the second case, if the position of the inventory is equal or lower than the reorder level, like in the first case, the duplicated entity again proceeds to an assign module called "*Switch Entity Type Order to Plant*" to change the entity type as "*Order to plant*". Following then, the *Order to Plant* entity enters the assign module called "*Assign Order No*", shown in Figure 5.10.



	Type	Variable Name	Attribute Name	New Value
1	Variable	Order_Placed	Attribute 1	Order_Placed + 1
2	Attribute	Variable 2	Order_No	Order_Placed
3	Attribute	Variable 3	Time_Order_Placed	TNOW

Name:
Assign Order No

Assignments:
Variable, Order_Placed, Order_Placed + 1
Attribute, Order_No, Order_Placed
Attribute, Time_Order_Placed, TNOW
<End of list>

Figure 5.10. Dialog box of *"assign order no" module*

In the "*Assign Order No"* module for each *"Order to Plant"* entity it has assigned a different order number. To set an order number, this module first assigns a variable called $Order\ Placed$ which initially settled to zero and increments the value of the variable one by one every time an entity incomes to this module and counts the incoming entities. The expression is as shown:

$$Order\ Placed = Order\ Placed + 1 \tag{5.3}$$

Following that, the module assigns an *attribute* called $Order\ No$ and selects the previous variable $Order\ Placed$ as the new value of this attribute, as the expression shown in below. In addition, for each circulating *"Order to Plant"* entity with assigned $Order\ No$, the module assigns an *attribute* called $Time\ Order\ Placed$ to use as the beginning time of an order placement, for calculating the time difference of an order arrival .

$$Order\ No\ = Order\ Placed \tag{5.4}$$

We have to note here that; we use two different variables to keep track of inventory information. First is $I\ OnHand$, which tracks the on-hand inventory level and fluctuates between zero and S_{Big}. And the second is $I\ Position$, which tracks the inventory position level and fluctuates between s_{Small} and S_{Big}. Because the initial values of $I\ OnHand$ and $I\ Position$ are all equal to one for all product types at the beginning, we have to note it again that $I\ Position$ seems to fluctuate between 1 and S_{Big} but this should not considered. $I\ OnHand$ is used to satisfy a customer's demand, while the $I\ Position$ is used when triggering a replenishment order. Recall that whenever the inventory position down-crossess s_{Small}, a replenishment order of size $(S\ Big - I\ Position)$ is placed with the production plant, and the inventory position is immediately updated to S_{Big}.

Assign module called *"Give an Independent Order"* performs three assignments (see Figure 5.11). The first assignment expression is:

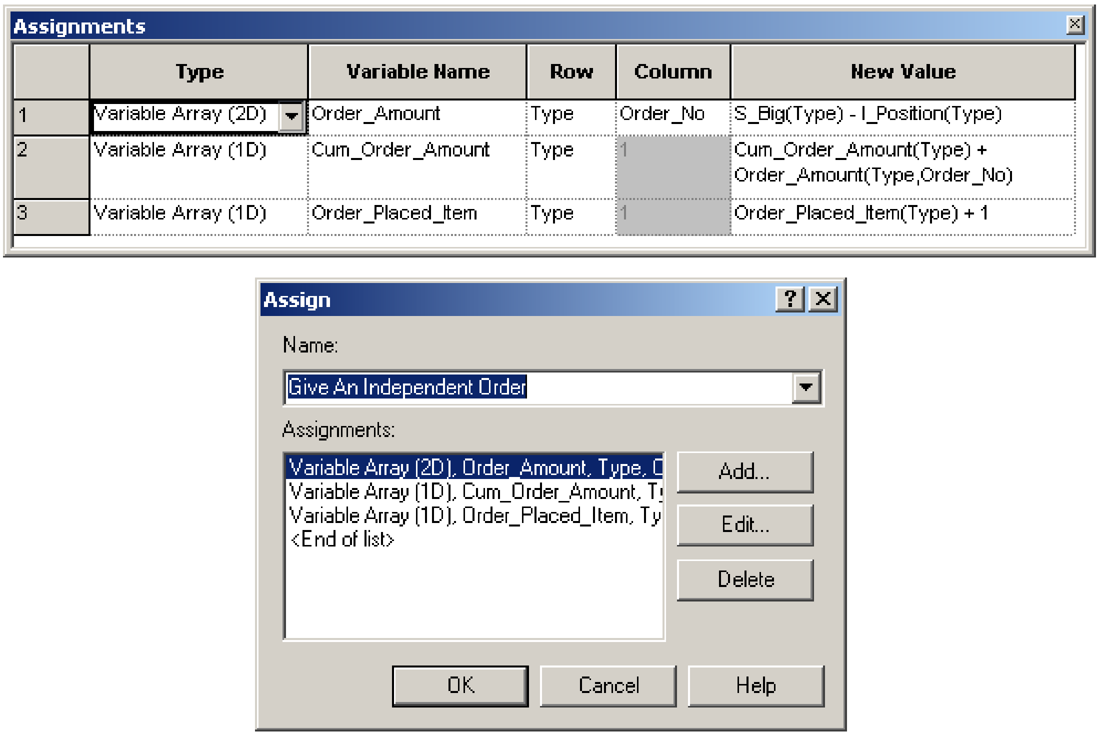
$$Order\ Amount_{Type\ ,Order\ \ No} = S_{Type}\ -\ I\ Position_{Type}$$

Where order amount of each product is modeled by a two-dimensional vector called *"Order Amount"* which is indexed by "*Type*" attribute of product type and "*Order No*" attribute. The second assignment calculates the cumulative order amount of the related product type as expressed below:

$$Cum\ Order\ Amount_{Type} = Cum\ Order\ Amount_{Type} + Order\ Amount_{Type\ ,Order\ \ No} \qquad (5.5)$$

Where the one-dimensional vector called *"Cum Order Amount"* stores and sum ups the every given order quantities for each product type. And the last assignment counts the replenishments for each product type as expressed below:

$$Order\ Placed\ Item_{Type} = Order\ Placed\ Item_{Type} + 1 \qquad (5.6)$$



	Type	Variable Name	Row	Column	New Value
1	Variable Array (2D)	Order_Amount	Type	Order_No	S_Big(Type) - I_Position(Type)
2	Variable Array (1D)	Cum_Order_Amount	Type	1	Cum_Order_Amount(Type) + Order_Amount(Type,Order_No)
3	Variable Array (1D)	Order_Placed_Item	Type	1	Order_Placed_Item(Type) + 1

Give An Independent Order

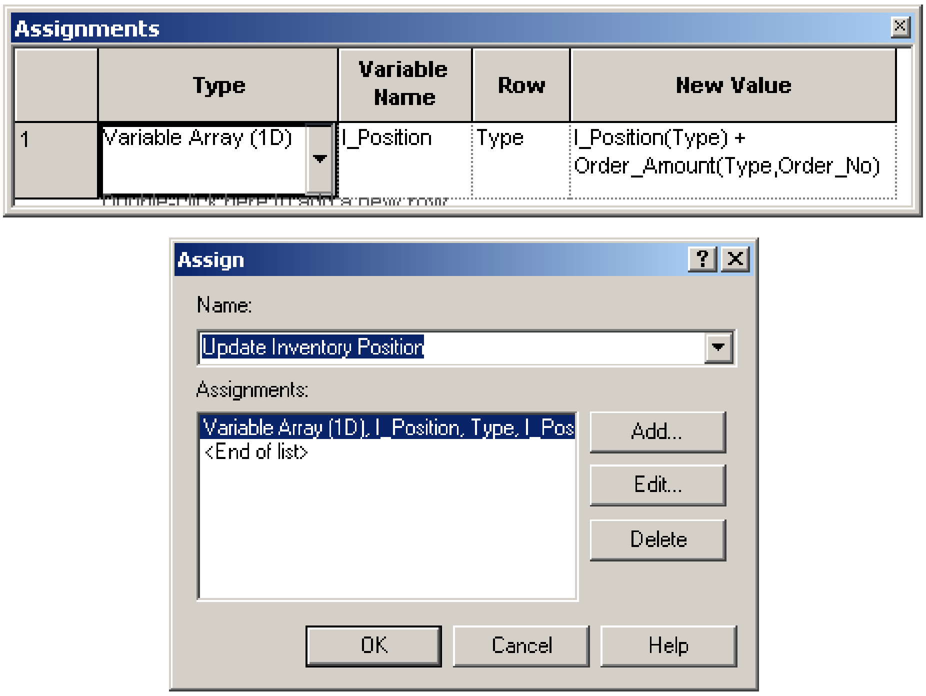
Variable Array (2D), Order_Amount, Type, O
Variable Array (1D), Cum_Order_Amount, Ty
Variable Array (1D), Order_Placed_Item, Ty
<End of list>

Figure 5.11. Dialog box of the *"give an independent order"* module

The *Order to Plant* entity next enters the assign module called *"Update Inventory Position"*. This module sets the new value of the inventory position for the related item which triggered the order in the previous *"Give Order to Plant"* decision module (see Figure 5.12). The assignment expression is given in Equation 5.7.

$$I\ Position_{Type} = I\ Position_{Type} + Order\ Amount_{Type\ ,Order\ \ No} \tag{5.7}$$

As previously described, inventory position of every product type is modeled by the one-dimensional vector called *"I Position"* and after an order triggered the *"Update Inventory Position"* module updates the inventory position by summing the order amount of the related product type with its inventory position.



Assignments

	Type	Variable Name	Row	New Value
1	Variable Array (1D)	I_Position	Type	I_Position(Type) + Order_Amount(Type,Order_No)

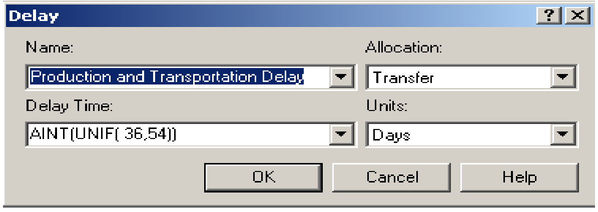
Assign

Name: Update Inventory Position

Assignments:
Variable Array (1D), I_Position, Type, I_Pos
<End of list>

Figure 5.12. Dialog box of *"update inventory position"* module

Next, the "*Order to Plant"* entity proceeds to a standard delay module called *"Production and Transportation Delay"* which models the production and transportation of the current product type. In here the delay module generates a product-independent processing time between 6 and 9 weeks. When processing of the current batch is done, that batch should be placed in the warehouse.



Delay

Name: Production and Transportation Delay

Allocation: Transfer

Delay Time: AINT(UNIF(36,54))

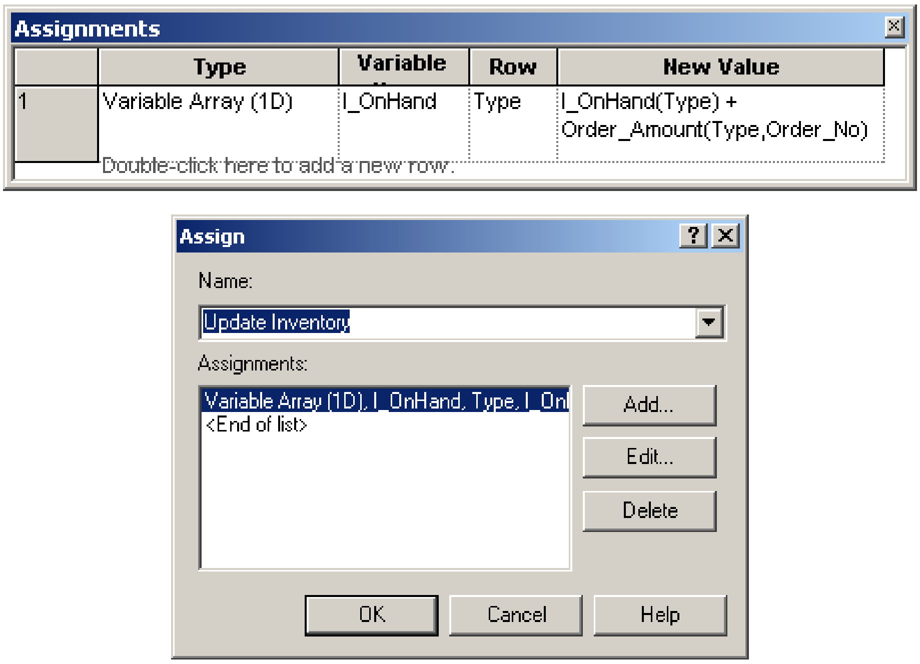
Units: Days

Figure 5.13. Dialog box of the *"production and transportation delay"* module

Following the delay module, the "*Order to Plant*" entity proceeds to the assign module, called *"Update Inventory"* which is shown in Figure 5.14. This module sets the new value of the on hand inventory of the related item after an order has arrived from the production plant. The assignment expression is:

$$I\ On\ Hand_{Type} = I\ On\ Hand_{Type} + Order\ Amount_{Type\ ,Order\ No} \qquad (5.8)$$

Like in the inventory position, the on hand inventory level of each product is modeled by a one-dimensional vector, called "*I On Hand*", which is indexed by product type. To update the inventory level of the current product type we simply incremented it by summing up with the received order amount of the related product type.



	Type	Variable	Row	New Value
1	Variable Array (1D)	I_OnHand	Type	I_OnHand(Type) + Order_Amount(Type,Order_No)

Double-click here to add a new row.

Name: Update Inventory

Assignments:
Variable Array (1D), I_OnHand, Type, I_OnH
<End of list>

Figure 5.14. Dialog box of the *"update inventory"* module

Finally, the *"Order to Plant"* entity has reached the end of its cycle and enters the dispose module called *"Dispose Order to Plant"* and exits the cycle.

5.1.3. Inventory Management Segment

Basically, the *Inventory Management* segment models the demand arrivals to the system and fulfillment of the regarding demand. As previously described, at the end of the

Separate Entities module, the *Customer Demand* entity duplicates itself. At that stage we skipped to describe the original entity and gave information about the duplicated one. Now, we'll turn our focusing to the original entity which named *Customer Demand.* Figure 5.15 shows the demand management segment of the Arena Model, which models the demand arrivals and fulfillments as explained before.

Following the *"Separate Entities"* module the *"Customer Demand"* entity needs to check whether or not there is sufficient inventory in the warehouse to satisfy the requested product quantity. To this end, all *"Customer Demand"* entities proceed to the decide module, called *"Check On Hand Inventory?"* whose dialog box is shown in Figure 5.16. The availability of sufficient product is expressed by the logical expression in the Value field as; $Demand\ \leq I\ On\ Hand_{Type}$. In here we again have to keep in mind that the product type information is stored in the "*Type"* attribute of the "*Customer Demand"* entity. Two options are then possible:

(i) If there is sufficient inventory in stock, then the demand of the current "Customer Demand" entity can be satisfied.

(ii) If not, then the demand of the "Customer Demand" entity could not be satisfied, waits in a queue of the related product type until a new replenishment arrives.

We will skip the second option and will continue the first option at this stage:

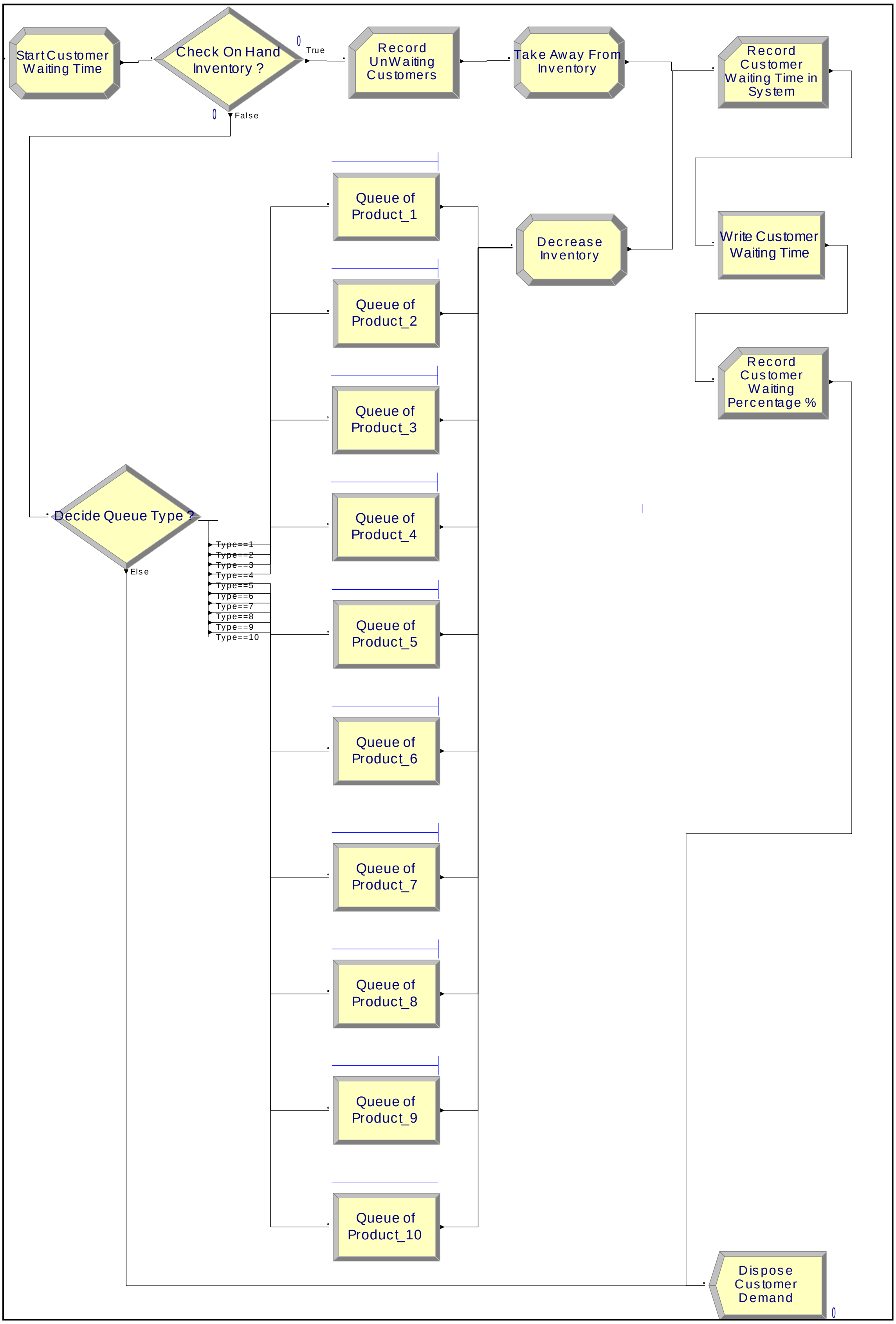
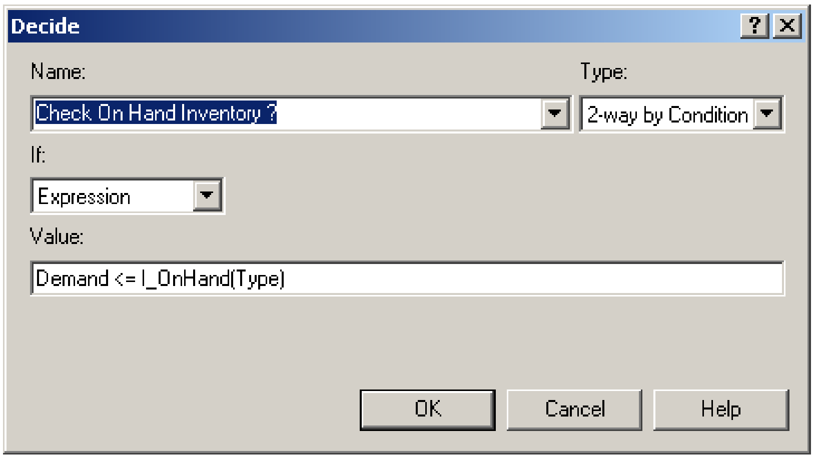


Figure 5.15. Inventory management segment



Check On Hand Inventory ?
2-way by Condition
Expression
Demand <= I_OnHand(Type)

Figure 5.16. Dialog box of "*check on hand inventory*" module

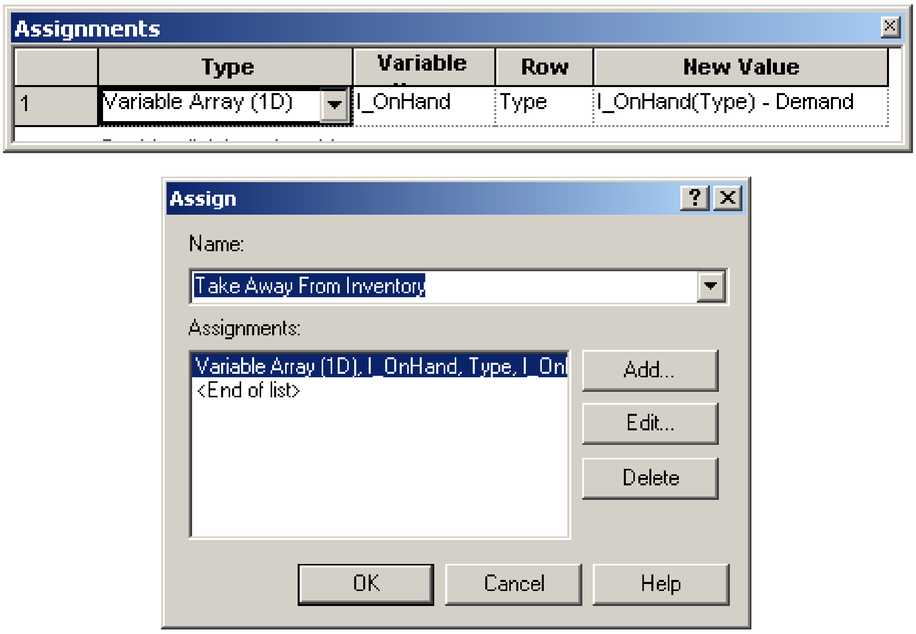
For the first option, the "Customer Demand" entity takes the true exit branch to the assign module, called "Take Away From Inventory", where the corresponding inventory of the product type is decremented by the demand quantity. Figure 5.17 displays the dialog box of this module. The Assignment expression is:

$$I\ On\ Hand_{Type} = I\ On\ Hand_{Type} - Demand \tag{5.9}$$

The on hand inventory level of each product type is modeled by a vector, called "I On Hand". The corresponding on hand inventory level is simply decremented by the demand size (unit size). Following this module the "Customer Demand" entity enters the dispose module "Dispose Customer Demand" and exits the cycle.

If we focus for the second decision option as the output of the "Give Order to Plant" module, the "Customer Demand" entity takes the false exit branch and enters the Decide module named "Decide Queue Type". In this module, whose dialog box is depicted in the Figure 5.18, the "Customer Demand" entity needs to check the queue type to wait in according to the related product type. Here we selected the *"N-way by Condition"* as the type of decision criterion. Then we expect to see ten different outcomes according to the ten different product types. As previously described in the *"Product Type Assigning"* module, an attribute named *"Type"* for each product has assigned for every *"Customer Demand"* entity. Then, the Decide module is deciding the queue type for the incoming entity by using an If-condition as seen in the Figure 5.19. For instance, if the value of a

"Type" attribute is equal to 4 then the entity will forwarded to the *Hold* module named *"Queue of Product 4"* and waits in the queue of product type 4. This is also valid for all other product types.



Assignments

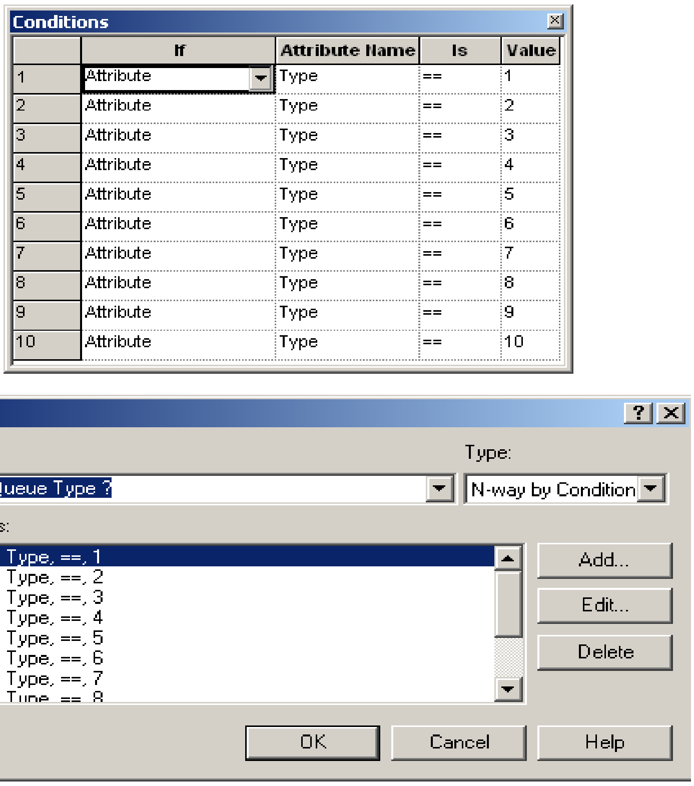
	Type	Variable	Row	New Value
1	Variable Array (1D)	I_OnHand	Type	I_OnHand(Type) - Demand

Assign

Name: Take Away From Inventory

Assignments:
Variable Array (1D), I_OnHand, Type, I_On
<End of list>

Figure 5.17. Dialog box of *"take away from inventory module"*



Conditions

	If	Attribute Name	Is	Value
1	Attribute	Type	==	1
2	Attribute	Type	==	2
3	Attribute	Type	==	3
4	Attribute	Type	==	4
5	Attribute	Type	==	5
6	Attribute	Type	==	6
7	Attribute	Type	==	7
8	Attribute	Type	==	8
9	Attribute	Type	==	9
10	Attribute	Type	==	10

Decide

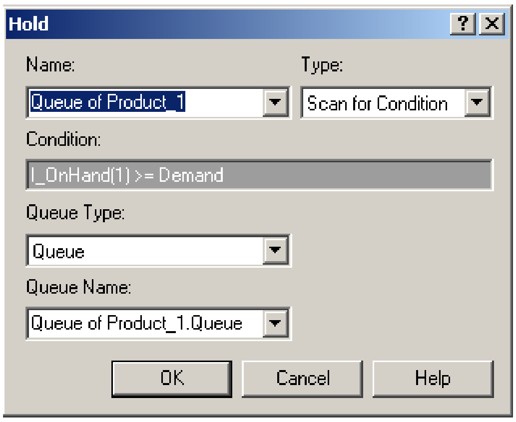
Name: Decide Queue Type ?

Type: N-way by Condition

Conditions:
Attribute, Type, ==, 1
Attribute, Type, ==, 2
Attribute, Type, ==, 3
Attribute, Type, ==, 4
Attribute, Type, ==, 5
Attribute, Type, ==, 6
Attribute, Type, ==, 7
Attribute, Type, ==, 8

Figure 5.18. Dialog box of the *"decide queue type"*

There are 10 different *Hold* modules, as shown in the Figure 5.15, and each of them is designed in the same way. For more focusing, we will describe the function of one of them in here. The *"Customer Demand"* entity enters the *Hold* module named *"Queue of Product_1"* which is depicted in Figure 5.19. The entity has also labeled with an attribute called *"Type"* and has a value of "1" which demonstrates the product type 1. For incoming entities the *Hold* module scans continuously to satisfy the condition expression, given in below, and whenever the condition is satisfied the module releases the incoming entities. The expression here is: $I\ OnHand_1 \geq Demand$ This means there is sufficient on hand inventory to meet the demand for product type 1. Unless the condition is not satisfied the module holds the "*Customer Demand"* entity and whenever the condition is satisfied the module releases the entity.



Name: Queue of Product_1
Type: Scan for Condition
Condition: I_OnHand(1) >= Demand
Queue Type: Queue
Queue Name: Queue of Product_1.Queue

Figure 5.19. Dialog box of the *"queue of product_1"* module

After a *"Customer Demand"* entity exits the waiting queue, it finally enters a *Assign* module called *"Decrease Inventory"*. This module functions the same as *"Take Away From Inventory"* module and the corresponding on hand inventory type is decremented by the demand quantity (unit size). The assignment expression of this module is given as:

$$I\ OnHand_{Type} = I\ OnHand_{Type} - Demand \tag{5.10}$$

Finally, the "Customer Demand" entity enters the Dispose module called "Dispose Customer Demand" to be removed from the Model.

5.2. Programming Coordinated Control Policy Model

This section will describe how the “*Coordinated Control Policy”* is modeled with Arena. First we will describe the model’s basic modules. Most of the main blocks of the model are same with the previous independent model so we will skip these blocks and focus on the different modules.

The Arena Model of the “***Coordinated Control Policy”*** is very similar with the *“Independent Control Policy”.* Like in the independent control, coordinated control model also has three main segments combined together. These are again; “*Customer Demand Generation”*, “*Replenishment Management*” and “*Inventory Management*” segments. Also the entity names are the same; *“Customer Demand”* and *“Order to Plant”* entities. And they are functioning by decrementing or incrementing inventory levels. Except of the *“Replenishment Management”* segment the other two segments, inventory management and customer demand generation, are completely same as *“Independent Control Policy”.* This similarity can also be understood by viewing the Figure 5.1 and Figure 5.20. Therefore we will skip to examine these similar segments.

The main difference of the coordinated control policy is modeled in the “*Replenishment Management”* Segment. Like in the independent control policy, this segment is initiating orders to production plant separately for each product type. But there are frequent occasions where the coordination of replenishment orders for selected groups of items can lead to significant savings in the costs of replenishment. And therefore, this segment combines the selected groups of items in the same replenishment. This segment also increases the inventory levels for each product type after the arrival of the either independent or coordinated orders to the warehouse. We next examine the model logic of the replenishment management segment of the coordinated policy in some detail.

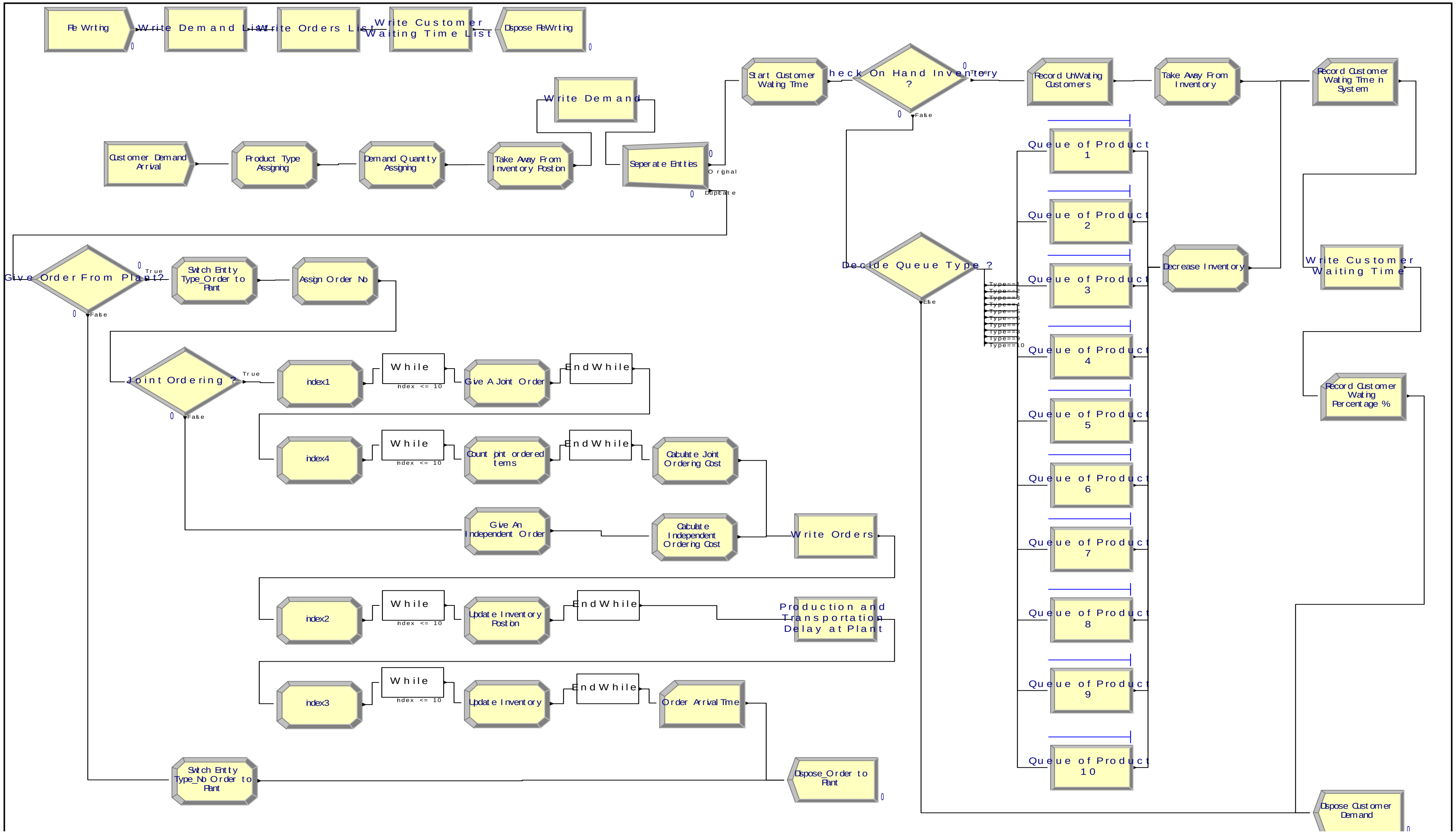


Figure 5.20. Arena model of the “*coordinated control policy*”

Before coming to describe the replenishment management segment it might be better to call the previous steps of the Arena model: As it was in the *"Independent Control Policy"* model, the "*Customer Demand Generation" s*egment generates different customer types. Arena proceeds this generation by entities which we called *"Customer Demand".* Following then Arena assigns a "*Type"* and "*Demand"* attributes to this *"Customer Demand"* entity and decrements the related item's inventory position in the *"Take Away From Inventory"* module. The *"Customer Demand"* entity next enters the "*Separate Entities"* module and we get two different entities at the end of this module. The name of the original entity remained same as *"Customer Demand"* and followed the way to the *"Check On Hand Inventory?"* module of the *"Inventory Management Segment".* The duplicated entity then enters the *"Replenishment Management"* segment.

5.2.1. Replenishment Management Segment

The model view of the replenishment management segment is depicted in Figure 5.21. Following the *"Separate Entities"* module the duplicated entity enters the decide module called *"Give Order to Plant?"*, again this decide module functions as the one described in the *"Independent Control Policy"* model in Section 5.3.2.2. To remember, the decision query of this decision module was; $IF\ (I\ Position_{Type}\ \leq s_{Type}\ )$. If the position of the inventory is sufficient, then the duplicated entity proceeds to the *Assign* module called "*Switch Entity Type-No Order to Plant"* to switch the *Entity* type as "*No Order to Plant"*. Then, "*No Order to Plant" entity* enters the Dispose module and exits the system. If we consider the second case, that the inventory position is equal or lower than the reorder level, then the duplicated entity again proceeds to an Assign module called "*Switch Entity Type-Order to Plant"* to change the *Entity* type as "*Order to plant"*. Following then, the "*Order to Plant"* entity enters the Assign module called "*Assign Order No"* to get a new order number.

This point forward, the *"Order to Plant" entity* proceeds to the most important module of this segment called *"Joint Ordering?"* to decide whether to give an independent or joint order to the production plant (see Figure 5.22). Before giving detailed information it would be beneficial to underline a key point: We know from the *"Give Order to Plant?"* decide module that, it is necessary for at least one product type, the inventory position of the product type should be equal or less than the reorder point to exceed the true branch of this "Give Order to Plant" module. Let assume in here that, inventory position of one of the product types is equal or less than the reorder point (s).Then, the "Order to Plant" entity enters the "Joint Ordering?" module.

In the *"Joint Ordering?"* module, inventory positions of all ten items are checked again by an if condition whether to find an item or items which of their inventory positions are between the related reorder points and can order levels. The if-expression value is given in Figure 5.22. We have to note that, the expression compares three vector elements; the inventory position level ($I\ Position$) of the currently requested product type, the reorder level ($s\ small$) of the same product type and finally the can order level($c\ CanOrder$). For giving a joint order, it is enough that an inventory position of one of the ten different products types is between reorder level and can order level as shown in below.

$$s\ Small_{Type} < I\ Position_{Type} \leq c\ CanOrder_{Type} \tag{5.11}$$

Two options are possible following inquiry of the decide module:

(i) There is at least one or more items which of their inventory position levels are between reorder and can order levels.
(ii) Except the item which triggered the order in the *"Order from Plant"* module, none of the other items have fallen under the can order level.

If the inquiry results with the second option and the *"Order to Plant"* entity exceeds the decide module from the false branch, then the entity enters an assign module called *"Give an Independent Order"*. This module functions like in the *"Independent Control Policy"* and therefore we will skip describing this module and focus on the second possible option. On the contrary, if the inquiry results with the first option then the "*Order to Plant*" entity proceeds to the assign module named "*Give a Joint Order*" and give a joint order to the plant.

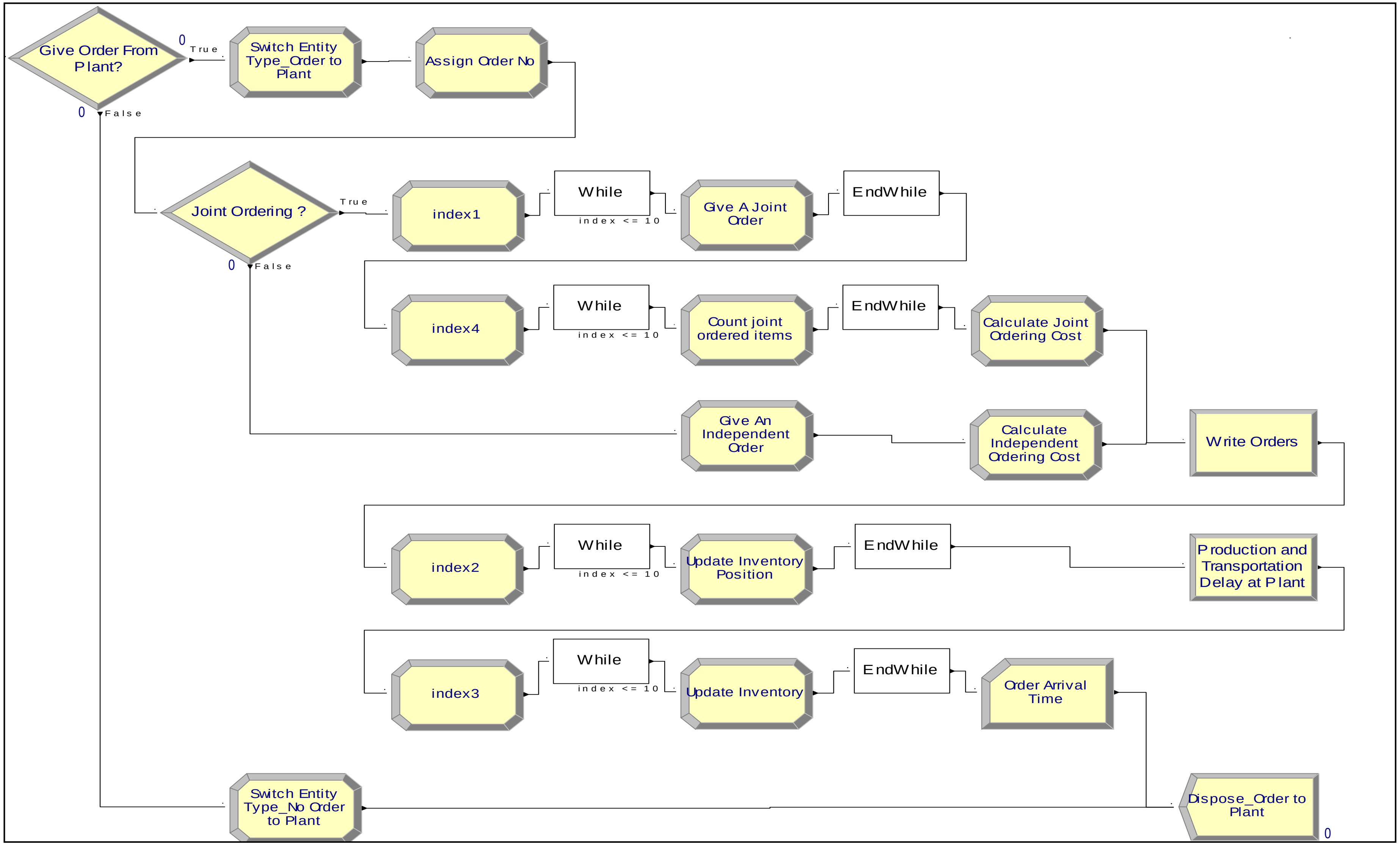
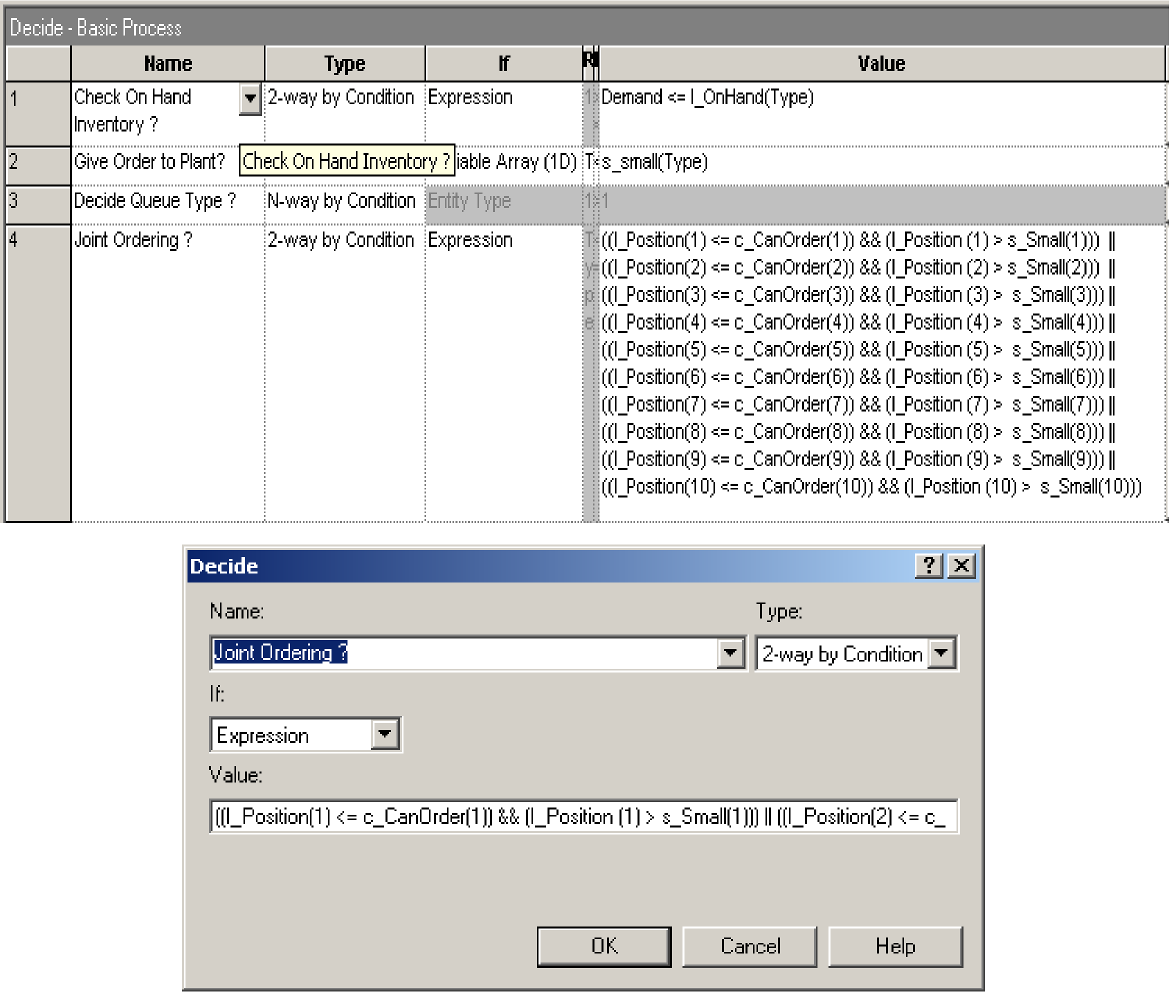


Figure 5.21. Replenishment management segment of coordinated control policy



Decide - Basic Process

	Name	Type	If	R	Value
1	Check On Hand Inventory ?	2-way by Condition	Expression	1	Demand <= I_OnHand(Type)
2	Give Order to Plant?	Check On Hand Inventory ?	iable Array (1D)	T	s_small(Type)
3	Decide Queue Type ?	N-way by Condition	Entity Type	1	1
4	Joint Ordering ?	2-way by Condition	Expression	T y p e	((I_Position(1) <= c_CanOrder(1)) && (I_Position (1) > s_Small(1))) || ((I_Position(2) <= c_CanOrder(2)) && (I_Position (2) > s_Small(2))) || ((I_Position(3) <= c_CanOrder(3)) && (I_Position (3) > s_Small(3))) || ((I_Position(4) <= c_CanOrder(4)) && (I_Position (4) > s_Small(4))) || ((I_Position(5) <= c_CanOrder(5)) && (I_Position (5) > s_Small(5))) || ((I_Position(6) <= c_CanOrder(6)) && (I_Position (6) > s_Small(6))) || ((I_Position(7) <= c_CanOrder(7)) && (I_Position (7) > s_Small(7))) || ((I_Position(8) <= c_CanOrder(8)) && (I_Position (8) > s_Small(8))) || ((I_Position(9) <= c_CanOrder(9)) && (I_Position (9) > s_Small(9))) || ((I_Position(10) <= c_CanOrder(10)) && (I_Position (10) > s_Small(10)))

Decide

Name: Joint Ordering ?

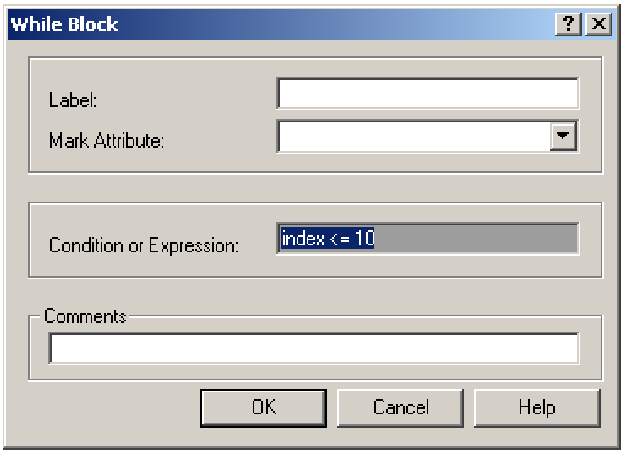
Type: 2-way by Condition

If: Expression

Value: ((I_Position(1) <= c_CanOrder(1)) && (I_Position (1) > s_Small(1))) || ((I_Position(2) <= c_

Figure 5.22. Dialog box of the *"joint ordering?"* module

Before giving descriptions about the "*Give a Joint Order*" module, it's beneficial to give brief information about the usage of variables and attributes again: Attributes stores information for each entity and it references the active entity only. But the usage of variables gives access to these attributes and other inactive entity related information. (Law & Kelton, 1991) Therefore, in the "*Give a Joint Order*" module Arena requires the usage of *"index"* variables instead of the "*Type*" attributes. We replaced an assign module called "*Index1*" just in front of the "*Give a Joint Order*" module and assigned a variable called *"index"* and settled its new value as 1. We also replaced "*While*" and "*End while*" blocks just in front and at the end of this *"Give a Joint Order"* module (shown in Figure 5.21). Therefore we examined this module with a while loop. This loop continue to process while the index number is less than or equal to ten as shown in Figure 5.23.



Condition or Expression: index <= 10

Figure 5.23. Dialog box of "*while*" block

Next, we will describe the "*Give a Joint Order*" module in more detail. When the "*Order to Plant*" entity enters this module four assignments are done by an order as shown in Figure 5.24. The first assignment expression is given below. Where order amount of each product is modeled by a two-dimensional vector called "Order Amount" which is indexed by "index" variable and "Order No" attribute. Like in the "Joint Ordering" module, we use a condition: $\mathrm{I\ Position}_{\mathrm{index}} \leq \mathrm{c}_{\mathrm{index}}$ to check all items to find which of them will be included in to the joint order. The order amounts for the included items satisfying this condition will be computed with the expression: $\mathrm{S}_{\mathrm{index}} - \mathrm{I\ Position}_{\mathrm{index}}$, and added to the Order Amount vector.

$$Order\ Amount_{index\ ,Order\ \ No} = (S_{index} - I\ Position_{index}) \times (I\ Position_{index} \leq c_{index}) \quad (5.12)$$

The second assignment calculates the cumulative order amount of the related product type as expressed below. We again have one-dimensional vector called *"Cum Order Amount"* as in the *"Independent Control"* policy. This vector stores and sums up the order quantities for each product type. Compared to the vector used in previous chapter, the only difference for this vector is the usage of *"index"* variable instead of *"Type"* attribute.

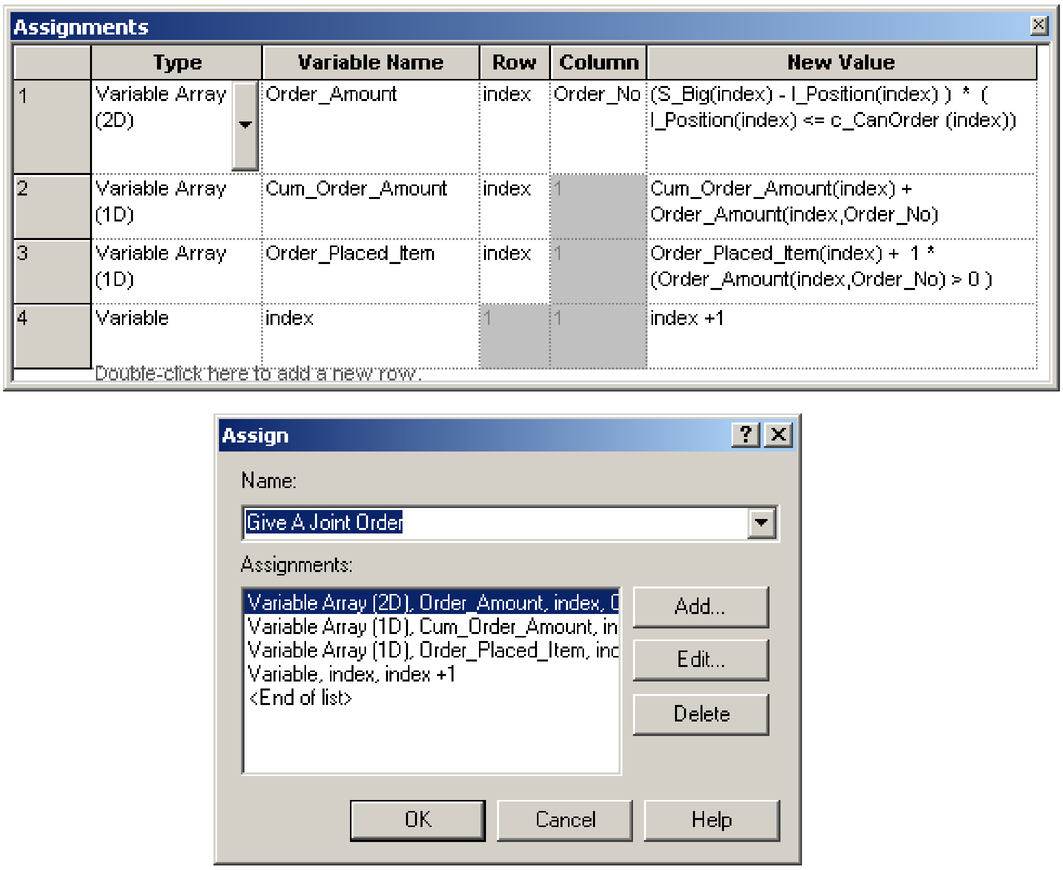
$$Cum\ Order\ Amount_{index} = Cum\ Order\ Amount_{index} + Order\ Amount_{index\ ,Order\ \ No} \quad (5.13)$$

And the third assignment counts the replenishments for each product type included in the order is expressed in Equation 5.14.

$$Order\ Placed\ Item_{index} = Order\ Placed\ Item_{index} + 1\ x\ (Order\ Amount_{index\ ,Order\ No} > 0) \qquad (5.14)$$

Finally the last assignment is incrementing the index variable by one as expressed below. As previously explained we assigned the *index* variable to one ($index = 1$) and used a while loop for the *"Give A Joint Order"* module. By the last assignment we described, the *"index"* variable will be incremented one by one for each product type in the product group and checked whether to find which items will be included in the joint replenishment. While the index value reaches ten then the while loop ends and the *"Customer Demand"* entity exceeds to the next module.

$$index = index + 1 \qquad (5.15)$$



Assignments

	Type	Variable Name	Row	Column	New Value
1	Variable Array (2D)	Order_Amount	index	Order_No	(S_Big(index) - I_Position(index)) * (I_Position(index) <= c_CanOrder (index))
2	Variable Array (1D)	Cum_Order_Amount	index	1	Cum_Order_Amount(index) + Order_Amount(index,Order_No)
3	Variable Array (1D)	Order_Placed_Item	index	1	Order_Placed_Item(index) + 1 * (Order_Amount(index,Order_No) > 0)
4	Variable	index	1	1	index +1

Double-click here to add a new row.

Assign

Name: Give A Joint Order

Assignments:

Variable Array (2D), Order_Amount, index, O
Variable Array (1D), Cum_Order_Amount, in
Variable Array (1D), Order_Placed_Item, inc
Variable, index, index +1
<End of list>

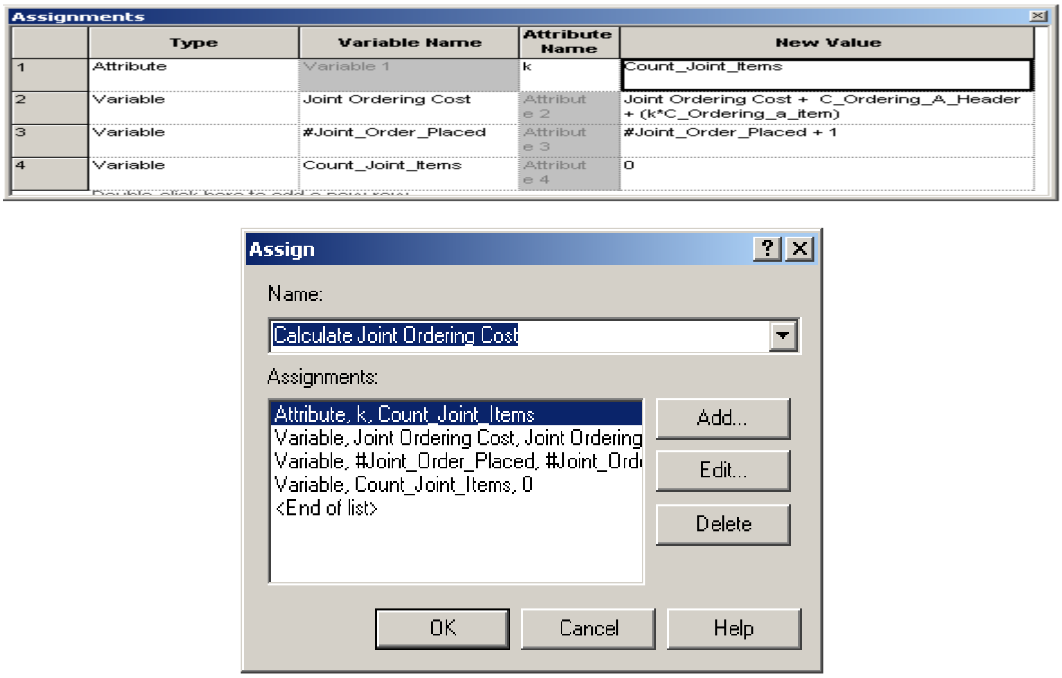
Figure 5.24. Dialog box of the *give joint order* module

Next, we will describe two different assign module called “*Count Joint Ordered Items”* and “*Calculate Joint Ordering Cost*” in short. We again used while loops for both modules and unless the index value reaches ten the loop does not stop cycling. The assignment expression for the “*Count Joint Ordered Items”* is given below. This assignment expression again uses a condition and increments the *Count Joint Items* variable one by one unless the inventory position of the indexed item is less than or equal to the can order level, so the module can select the items included to joint ordering to count.

$$Count\ Joint\ Items = Count\ Joint\ Items + 1 \times (I\ Position_{index} \ \le c_{index}) \tag{5.16}$$

The *“Calculate Joint Ordering Cost”* module uses four different assignments (see Figure 5.25). First it assigns an attribute, named “*k”* and set the result of the previous “*Count Joint Items”* variable as the new value of this attribute. The second assignment expression is given below. This assignment gives the total joint ordering cost at the end of the simulation application.

$$Joint\ Ordering\ Cost \ = Joint\ Ordering\ Cost + A + (k \times a) \tag{5.17}$$



	Type	Variable Name	Attribute Name	New Value
1	Attribute	Variable 1	k	Count_Joint_Items
2	Variable	Joint Ordering Cost	Attribut e 2	Joint Ordering Cost + C_Ordering_A_Header + (k*C_Ordering_a_item)
3	Variable	#Joint_Order_Placed	Attribut e 3	#Joint_Order_Placed + 1
4	Variable	Count_Joint_Items	Attribut e 4	0

Name:

Calculate Joint Ordering Cost

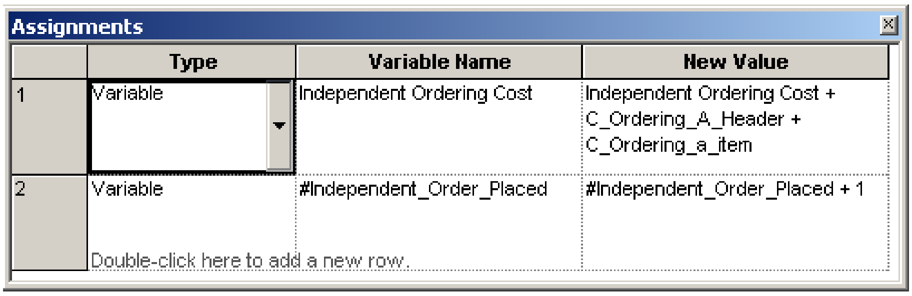
Assignments:

Attribute, k, Count_Joint_Items
Variable, Joint Ordering Cost, Joint Ordering
Variable, #Joint_Order_Placed, #Joint_Ord
Variable, Count_Joint_Items, 0
<End of list>

Figure 5.25. Dialog box of "*calculate joint ordering cost"* module

In the previous *"Joint Ordering?"* module, it is explained that there are two possible options following the decision inquiry and we skipped to describe the second option at that stage. Now if we look back to the second option, this time the *"Order to Plant"* entity enters another *Assign* module called "***Give An Independent Order****"* which process completely same with the one in the *"Independent Control"* Policy. Following, the entity enters anther *Assign* module called "***Calculate Independent Ordering Cost***" to compute the cost of the independent orderings and to count the number of independent orders (see Figure 5.26). The assignment expression to compute the independent ordering cost is given below. In the expression, again a variable called "*Independent Ordering Cost"* is used, every time an independent order placed, *A* (the header cost) and *a* (the cost per item involved in a replenishment) are added on the *"Independent Ordering Cost"* variable.

$$Independent\ Ordering\ Cost = Independent\ Ordering\ Cost + A + a \tag{5.18}$$



Assignments

	Type	Variable Name	New Value
1	Variable	Independent Ordering Cost	Independent Ordering Cost + C_Ordering_A_Header + C_Ordering_a_item
2	Variable	#Independent_Order_Placed	#Independent_Order_Placed + 1

Double-click here to add a new row.

Figure 5.26. Dialog box of "*calculate independent ordering cost"* module

We have to remind again that, after the *"Joint Ordering?" Decide* module the "*Order to Plant"* entity followed two different branches and proceeded different modules. At this point, these two different branches are again united and the *"Order to Plant"* entity enters the "***Update Inventory Position***" module. Very similar to the *"Independent Control"* policy, this module sets the new value of the inventory position for the related items included in either joint order or independent order. The assignment expression is given below. In the expression, the inventory position of every product type is again modeled by the one-dimensional vector called *"I Position"* and after an order triggered the *"Update Inventory Position"* module updates the inventory positions by summing the order amount of the related product type with its inventory position. Additionally, we again used a while loop to model this module similar to the one used in previous sections. And instead of using *"Type"* attribute, we indexed the vector by *"index"* variable.

$$I\ Position_{index} = I\ Position_{index} + Order\ Amount_{index\ ,Order\ \ No} \tag{5.19}$$

Next, the "*Order to Plant"* entity proceeds to a standard *Delay* module called ***"Production and Transportation Delay"*** which is same as with the same name in the section 5.3.2.2 and models the production and transportation of the current product type. In here the *Delay* module implements a product-independent processing time between 6 to 9 weeks. When processing of the current batch is done, that batch should be placed in the warehouse.

Following the *Delay* module, the "*Order to Plant"* entity proceeds to the *Assign* module, called ***"Update Inventory",*** which of the assignment expression is shown below. The on hand inventory level of each product is modeled by a one-dimensional vector, called "*I On Hand".* Different from the one in section 5.3.2.2, this vector is indexed by *"index"* variable because of the while loop we used. To update the inventory level of the current product type we simply incremented it by summing up with the received order amount of the related product type.

$$I\ OnHand_{index} = I\ OnHand_{index} + Order\ Amount_{index\ ,Order\ \ No} \tag{5.20}$$

Finally, the *"Order to Plant"* entity has reached the end of its cycle and enters the *Dispose* module called ***"Dispose Order to Plant"*** and exits the cycle.

5.3. Statistics Collection

Specifications of statistics collection is replaced in an Excel sheet for demonstration and illustrated in the Appendix G of the thesis. This statistics sheet includes statistical determinations for both *Independent* and *Coordinated Control Models.*

The spreadsheet includes *Time-Persistent* statistics of on-hand inventory levels of all items, as well as *Time-Persistent* statistics of inventory positions. The spreadsheet also includes *Output statistics* that count the amounts of total incoming demands for all items, as well as amounts of total given orders to production plant. Also, amount of joint and independent orderings are counted by *Output statistics.*

More important, the spreadsheet includes *output statistics* to estimate customer service levels in terms of fill rate, namely, the probability of orders that were satisfied from on-hand inventory, without experiencing backorders. Finally, the *Output* statistics estimate ordering cost, holding cost and total inventory costs.

5.4. Model Verifications and Validation

In general, it is always better to start with a moderate detailed model and then to develop a complex model because determination of errors and verification are extremely difficult when the large and complex model is run without any check or test during the model building stage. Therefore all models first should be constructed with a moderate detail level; then they should be refined step by step (Law & Kelton, Simulation Modeling And Analysis , 1991).

In this study, the last versions of *independent* and *coordinated models* are obtained as results of several iterations of improvements and evaluated in a long period. At the end of iteration simulations were run and results were checked many times for different extreme alternatives. First, an inventory control model with one product and zero lead time is modeled. Then, lost sales cost is added to the model for stockouts. After that, lead time is increased and backorders are allowed. In the model, instead of lost sales cost, backordering cost is charged. Following, product types are increased to five. Again first simulations were run with zero lead time and then the lead time was increased. The model was charging three types of costs, these were: holding cost, backordering cost and ordering cost. Then the model is redefined and allowed for joint replenishment. Finally product types are increased to ten. In final version of coordinated and independent models, backordering is allowed. But, instead of charging backordering cost, it is decided to measure the performance of the system with customer service levels. Then, some variables of interest like fill rate were added to the model. After each step, scenarios were run and checked whether they behave as expected or not.

Also, to check the logic of the scenarios, “trace” and extreme condition tests were used. Especially in every stage of the model improvement, extreme tests were done by changing the parameters of the lead time, control parameters and cost parameters. Observations of results of these extreme tests verify the model’s correctness. Moreover, animation was utilized to observe the values of some variables and expressions used in the scenarios.

Tracing (like debugging a computer program) is a very powerful technique. In a trace output, the state of the simulated system (i.e. all events, state variables, and certain statistics) is represented in all detail. Thus, trace outputs are compared with hand calculations to see if the model performs as intended. In Appendix H, one page of trace output is placed as a sample.

At the end, it can be concluded that the models pass the verification tests successfully. As explained previously, in real life the distributor company is not holding stock and we are not able to compare and validate the results of the study with the actual case. Hence, it is seen that it is unfortunately impossible to validate our model.

6. RESULTS OF THE STUDY

In this section, the results of simulation models are presented and their analyses are explained in detail. In Section 6.1 simulations are run with initial control parameters and results are given with tables and plot diagrams. During the search described in Section 6.1 it is seen that the solution algorithm gives insufficient results which unnecessarily high fill rates and high total inventory costs. Therefore in Section 6.2 simulations are run with new control parameters which are computed according to desired FDS which take values lower than 90 per cent. Again, results of these simulations are given with tables and plot diagrams. Finally, it is needed to find and adjust alternative scenarios to decrease the Total Inventory Costs in a sensible way. Therefore in Section 6.3 four different scenarios are developed to search the solution space to find better alternative solutions. Detailed results of the simulations are given in Appendix A, B, C and E respectively. New control parameters determined to use in the described scenarios are given in Appendix D of the thesis.

6.1. Results of the Simulations with Initial Control Parameters

A mathematical model to control the inventories was developed in Section 4. The solution algorithm of the heuristic to determine the sensible initial parameters are also determined and computed in Section 4. The computed control parameters are presented in Table 6.1 again. The control parameters (s, c, S) of the ten products are computed for three different desired levels of P_2.

Complete input data sets are given in Table 6.2. The starting conditions of inventory control variables, I_{OnHand} and $I_{Position}$ is set to one for each item in the group. Cost figures are also given in Table 6.2. *Independent* and *coordinated control models* are run in Arena Simulation Software, by using the control parameters in Table 6.1 and input data set in Table 6.2. The time horizon of each simulation run is 6,000 working days which is assumed to be equal to 20 years time. For each simulation run, 100 replications were done.

Table 6.1. Initial control parameters $(\boldsymbol{s}, \boldsymbol{c}, \boldsymbol{S})$ for 10 items in 3 different $\boldsymbol{P_2}$ level

i	P_2=0,90			P_2=0,95			P_2=0,99		
	s_i	c_i	S_i	s_i	c_i	S_i	s_i	c_i	S_i
1	3	7	11	4	8	12	6	10	14
2	2	6	10	3	7	11	5	9	13
3	3	9	17	4	10	18	6	12	20
4	4	14	29	6	16	31	9	19	34
5	1	4	8	2	5	9	3	6	10
6	1	5	9	2	6	10	3	7	11
7	1	6	13	2	7	14	4	9	16
8	1	3	6	1	3	6	2	4	7
9	1	5	10	2	6	11	3	7	12
10	1	3	5	1	3	5	2	4	6

Table 6.2. Input data set for 10 items

i Item	***λ*** Demand Rate **(Pcs./Yr.)**	***L*** Lead Time **(Days)**	***v*** Unit List Price **(€/Unit)**	***r*** Carrying Charge **(€/€/year)**	***v* x *r*** Unit Holding Cost **(€ / Unit /yr.)**	***A*** Header Cost **(€)**	***a*** Variable Cost **(€)**	$\boldsymbol{I_{Position}}$ Initial Variable	$\boldsymbol{I_{OnHand}}$ Initial Variable
1	19	U(36,54)	4,320	0.10	432	640	70	1	1
2	15	U(36,54)	3,422	0.10	342	640	70	1	1
3	23	U(36,54)	1,761	0.10	176	640	70	1	1
4	39	U(36,54)	913	0.10	91	640	70	1	1
5	8	U(36,54)	2,546	0.10	255	640	70	1	1
6	8	U(36,54)	1,971	0.10	197	640	70	1	1
7	12	U(36,54)	1,168	0.10	117	640	70	1	1
8	5	U(36,54)	2,761	0.10	276	640	70	1	1
9	9	U(36,54)	1,605	0.10	161	640	70	1	1
10	4	U(36,54)	3,205	0.10	321	640	70	1	1

Complete result tables of each simulation runs are given in Appendix A and Appendix B, respectively for *independent* and *coordinated policies*. These results are summarized below.

Simulation results for the i*ndependent policy* are summarized in Table 6.3. In this policy, joint replenishments are not allowed. For desired value of FDS at P_2 = 90 per cent, 95 per cent, and 99 per cent, simulation runs resulted in Fill Rates at 92.17 per cent, 96.05 per cent, and 98.71 per cent respectively and in Total Inventory Costs at € 20,728, € 22,511, and € 26,053 per year respectively. Fill Rates are higher than desired value of FDS at 90 per cent and 95 per cent. But it is slightly lower than desired customer service level at 99 per cent. Order arrival times are 7.5 weeks (each week has 6 working days in the model). This is constant for all the runs. Average customer waiting time falls down from 1.15 days at P_2 = 90 per cent to 0.2 days at P_2 = 99 per cent. Total costs rise from nearly 20,000 Euros up to 26,000 Euros. The rise is due to the increase in holding costs of the inventory. Since the number of replenishments is the same at all levels, ordering costs do not differ between different customer service levels.

Table 6.3. Summary of simulation results for independent control policy

Average Results for Different Values of P_2				
	Unit	P_2= 90%	P_2= 95%	P_2= 99%
Recorded Time Intervals				
Order Arrival Time (Lead Time)	days	44.56	44.56	44.56
Customer Waiting Time in System	days	1.15	0.53	0.21
Quantity of Replenishments Placed				
Independent Replenishment Orders	number	13.93	13.93	13.93
Joint Replenishment Orders	number	0	0	0
Total Replenishments	number	13.93	13.93	13.93
Realized Costs				
Cost of Independent Replenishments	€/yr.	9,889	9,889	9,889
Cost of Joint Replenishments	€/yr.	0	0	0
Total Ordering Cost	€/yr.	9,889	9,889	9,889
Total Holding Cost	€/yr.	10,839	12,622	16,164
Realized Total Costs for All Items	€/yr.	20,728	22,511	26,053
Realized Fill Rate				
Demand Satisfied Directly From Shelf	%	92.17	96.05	98.71

Simulation results of *coordinated policy* are summarized in Table 6.4. This model allows joint replenishment. For the same desired value of FDS, simulations resulted in Fill Rates at 96.43 per cent, 98.04 per cent, and 99.09 per cent respectively, and in Total

Inventory Costs at € 16,856, € 18,675, and € 22,239 respectively. Order arrival times are the same for all the runs again. Fill Rates are higher than desired customer service levels in all cases whereas the difference is very small at P_2 = 99 per cent. The customer waiting times fall from 0.51 days at P_2 = 90 per cent down to 0.17 days at P_2 = 99 per cent. Most of the replenishments are made jointly. In average 3.65 joint replenishment orders are made in comparison to 0.51 independent replenishment orders. Most of the inventory costs occur in holding costs, approximately 75 per cent. Total Ordering Costs are same for all desired service levels, but the total holding costs increase from around 13,000 Euros at P_2 = 90 per cent up to 18,000 Euros at P_2 = 99 per cent.

Table 6.4. Summary of simulation results for coordinated policy

Average Results for Different Values of P_2				
	Unit	P_2= 90%	P_2= 95%	P_2= 99%
Recorded Time Intervals				
Order Arrival Time (Lead Time)	Days	44.54	44.54	44.54
Customer Waiting Time in System	Days	0.51	0.29	0.17
Quantity of Replenishments Placed				
Independent Replenishment Orders	number	0.51	0.51	0.51
Joint Replenishment Orders	number	3.65	3.65	3.65
Total Replenishments	number	4.16	4.16	4.16
Realized Costs				
Cost of Independent Replenishments	€/yr.	364	364	364
Cost of Joint Replenishments	€/yr.	3,607	3,607	3,607
Total Ordering Cost	€/yr.	3,971	3,971	3,971
Total Holding Cost	€/yr.	12,885	14,704	18,268
Realized Total Costs for All Items	€/yr.	16,856	18,675	22,239
Realized Fill Rate				
Satisfied Directly From Shelf	%	96.43	98.04	99.09

In section 4.6.1.2, total expected cost is theoretically calculated as € 25,908 for P_2 = 99 per cent with *independent policy*. The simulation resulted in € 26,053 of Total Inventory Cost. The difference between theoretical and simulated calculation is very small. The difference is due to the nature of the simulation approach. This approximate equivalence verifies and proves once again that simulation models constructed in Section 5 are working accurately.

6.1.1. Advantages of the Coordinated Policy over the Independent Policy

Simulation results for the *independent* and *coordinated policies* are summarized in Table 6.3. and Table 6.4. Total Inventory Costs and direct demand satisfaction rates (Fill Rates) for both policies are plotted together in Figure 6.1. *Coordinated Control Policy* is better than the *Independent Control Policy* in all dimensions. Actual savings achieved by the *coordinated policy* are presented in detail in Table 6.5 i.e. with the desired level of P_2 = 90 per cent, *coordinated policy* can realize 18.68 per cent less Total Inventory Costs and reached Fill Rates is 4.63 per cent better than in the *independent* one.

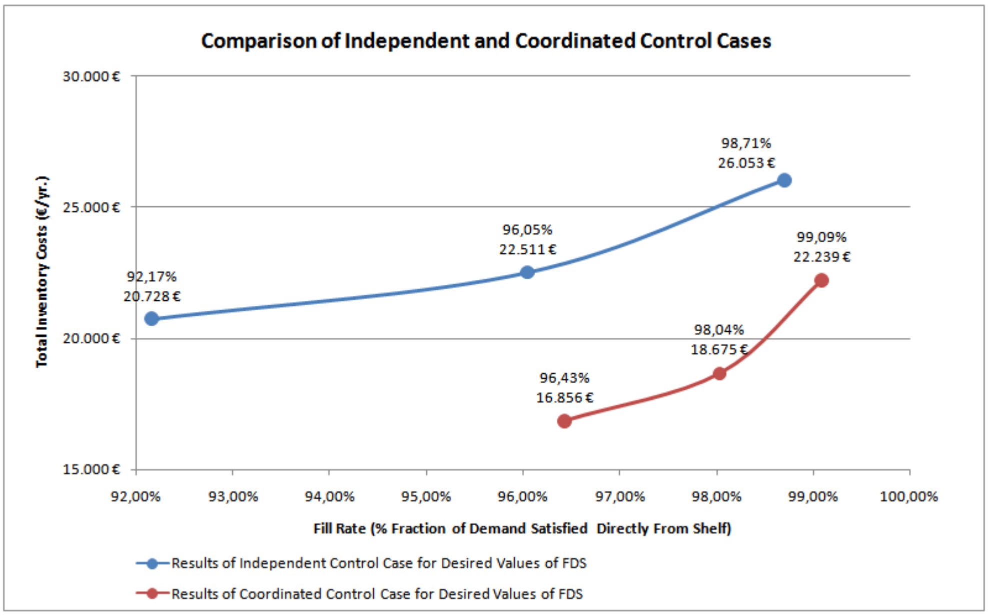


Figure 6.1. Comparison of independent and coordinated control policies

Additionally, some important points should be underlined such as; for P_2= 95 per cent, the *coordinated policy* results 4.16 replenishments averagely in a year with only € 3,971 of Total Ordering Cost while *independent policy* results 13.93 replenishments with cost of € 9,889 for all ordering operations. As seen from the numerical figures, *coordinated policy* has reduced the ordering costs to $\frac{1}{3}$ of the independent policy. Also, by reducing the number of replenishments to approximately 4 times in a year, *coordinated policy* has gained time to the company. In practice a replenishment means, lots of emailing, checking, operational stuffs and hours of work. In that means, *coordinated control policy* is clearly better than the independent one. Therefore, after this point of the study, we will only deal with the *coordinated control policy*.

Table 6.5. Cost savings and increase in fill rate with coordinated control policy

Desired FDS	Total Costs for Independent Control	Total Costs for Coordinated Control	% Cost savings of Coordinated Control Over Independent Control	Realized Fill Rate for Independent Control	Realized Fill Rate for Coordinated Control	% Increase in Fill Rate
P_2=90%	€ 20,728	€ 16,856	18.68%	92.17%	96.43%	4.63%
P_2=95%	€ 22,511	€ 18,675	17.04%	96.05%	98.04%	2.06%
P_2=99%	€ 26,053	€ 22,239	14.64%	98.71%	99.09%	0.38%

6.2. Effects of Desired Value of FDS to the Coordinated Control Policy

During the search described in Section 6.1 it is seen that the solution algorithm gives results with high costs. In other words, for P_2=90 per cent or 95 per cent of customer service level simulations resulted in 96.43 per cent or 98.04 per cent of satisfaction of customer service levels. Except of P_2=99 per cent level, algorithm is resulting the Fill Rate more than desired levels. In practice, those high levels of Fill Rates are unnecessary. As emphasized in the former sections of this study, since the distributor company holds very limited stocks on hand it is not possible to calculate numerically the current Fill Rate. But rather than the actual case, it can be assumed that the current Fill Rate is not more than 5 per cent. Therefore, 90 per cent levels of Fill Rate are extreme cases for the company, 75 per cent levels of Fill Rates seem more sensible.

In this section of the study effects of P_2 lower than 90 per cent levels will be searched. New control parameters are computed for the lower levels of desired FDS by using the methods described in Section 4.6, and results are given in Table 6.6. It should be noted that, there is no further need to investigate the values of P_2 lower than 75 per cent. Because, lower than this point, the algorithm computes all s_i values equal to one (results lower than one are rounded up to one by the *R* code). *Coordinated control model* has been run in Arena by using the new control parameters given with Table 6.6. and the same input data are given in Table 6.2. Again each simulation runs for 6,000 days with 100 replications.

Table 6.6. Control parameters for lower levels of $\boldsymbol{P_2}$

i	P_2=0.75			P_2=0.80			P_2=0.83			P_2=0.85			P_2=0.88		
	s_i	c_i	S_i	s_i	c_i	S_i	s_i	c_i	S_i	s_i	c_i	S_i	s_i	c_i	S_i
1	1	5	9	2	6	10	2	6	10	2	6	10	3	7	11
2	1	5	9	1	5	9	1	5	9	2	6	10	2	6	10
3	1	7	15	1	7	15	2	8	16	2	8	16	2	8	16
4	1	11	26	1	11	26	2	12	27	3	13	28	3	13	28
5	1	4	8	1	4	8	1	4	8	1	4	8	1	4	8
6	1	5	9	1	5	9	1	5	9	1	5	9	1	5	9
7	1	6	13	1	6	13	1	6	13	1	6	13	1	6	13
8	1	3	6	1	3	6	1	3	6	1	3	6	1	3	6
9	1	5	10	1	5	10	1	5	10	1	5	10	1	5	10
10	1	3	5	1	3	5	1	3	5	1	3	5	1	3	5

Simulation results are given in Appendix C and summarized below. Relation between the Fill Rate and Total Inventory Costs are plotted in Figure 6.2 to show the trade-off. Additionally, the previous results obtained in Section 6.1 are also combined in the table and chart. It is seen that holding costs are still very high compared to ordering costs and the algorithm continues to give pessimistic results with higher Fill Rates than it is expected. On the contrary; the gap between desired and realized customer service levels are increasing while P_2 decreases. Most probably, the reason for this contradiction is that, s_i parameters are remaining 1 for most items for the different levels of P_2. As a conclusion, those high values of Fill Rate results high values of Total Inventory Costs and can still be accepted as extreme for the distributor company. After this point of the study it is needed to find and alternative scenario to decrease the Total Inventory Costs in a sensible way.

Table 6.7. Summary of simulation results for coordinated control policy

Average Results for Different Values of P_2								
P_2	**75%**	**80%**	**83%**	**85%**	**88%**	**90%**	**95%**	**99%**
Recorded Time Intervals (days)								
Order Arrival Time (Lead Time)	44.54	44.54	44.54	44.54	44.54	44.54	44.54	44.54
Customer Waiting Time in the System	1.35	1.16	0.92	0.73	0.64	0.51	0.29	0.17
Realized Costs (€/yr.)								
Cost of Independent Replenishments	364	364	364	364	364	364	364	364
Cost of Joint Replenishments	3,607	3,607	3,607	3,607	3,607	3,607	3,607	3,607
Total Ordering Cost	3,971	3,971	3,971	3,971	3,971	3,971	3,971	3,971
Total Holding Cost	11,168	11,558	11,808	12,219	12,628	12,885	14,704	18,268
Realized Total Inventory Costs	15,139	15,529	15,779	16,190	16,599	16,856	18,675	22,239
Realized Fill Rate (%)								
Satisfied Directly From Shelf	92.00	92.94	94.08	95.06	95.66	96.43	98.04	99.09

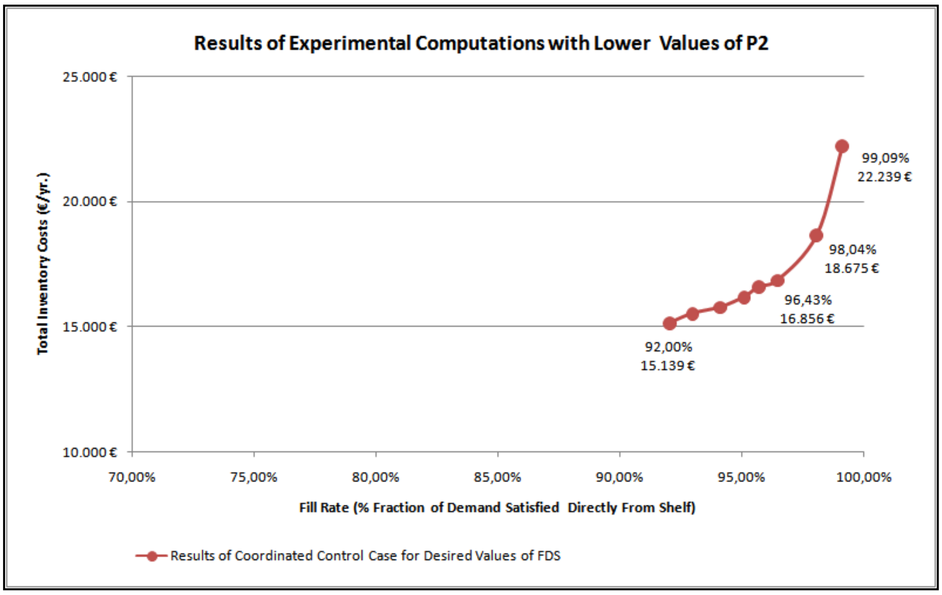


Figure 6.2. Total inventory costs vs. fill rate for different values of P_2

6.3. Performance of the Coordinated Control Policy under Changes

An interesting feature of the algorithm and inventory control model is their adaptability to changes in the values of the parameters. In a real life situation, the demand for certain items might decrease or increase. Also, change on the lead time may occur and new pricing practices instituted by the supplier will yield a change in the inventory carrying cost etc. (Silver, Pyke, & Peterson, Costs and Other Important Factors, 1998). Therefore, the algorithm and inventory control model has to be flexible to apply the changes needed. To expand the search of this thesis, the algorithm performance over the change in control parameters is examined in the following way.

If there is a need to change control parameters, it is not necessary to start calculating the parameters from the beginning. A scenario based search on the ex-near-optimal values of the algorithm would lead quickly to a new near-optimal solution.

The values of the current control parameters were determined using the heuristics (Silver E. A., 1974) which explained in Section 4 for different values of P_2. Those control parameter values are shown in Table 6.6.

New values of the control parameters are generated from the averages of the current values of S_i/λ_i . The averages of the current values are tabulated in Table 6.8. S_i/λ_i Is the ratio of upper limit of inventory holding to the yearly demand rate. The ratio shows what percentage of the yearly demand is hold in the inventory at maximum. As P_2 increases, the ratio of maximum inventory to yearly demand increases in order to satisfy the increasing desired customer service level.

The row averages of the Table 6.8 give averages of S_i/λ_i ratio for the same item at varying P_2 levels. For example S_i/λ_i ratio for the 1st item is 0.53. This means that, on average, the maximum inventory holding level of that item is at 53 per cent of its yearly demand rate for P_2 levels below 90 per cent.

Table 6.8. Averages of the current value of $\boldsymbol{S_i/\lambda_i}$ for $\boldsymbol{P_2}$ less than 90%

$\boldsymbol{i}$	$\boldsymbol{S_i/\lambda_i}$	**P_2=0.75**	**P_2=0.80**	**P_2=0.83**	**P_2=0.85**	**P_2=0.88**	$\boldsymbol{Avg\frac{S_i}{\lambda_i}}$
1	S_1/λ_1	0.47	0.53	0.53	0.53	0.58	0.53
2	S_2/λ_2	0.60	0.60	0.60	0.67	0.67	0.63
3	S_3/λ_3	0.65	0.65	0.70	0.70	0.70	0.68
4	S_4/λ_4	0.67	0.67	0.69	0.72	0.72	0.69
5	S_5/λ_5	1.00	1.00	1.00	1.00	1.00	1.00
6	S_6/λ_6	1.13	1.13	1.13	1.13	1.13	1.13
7	S_7/λ_7	1.08	1.08	1.08	1.08	1.08	1.08
8	S_8/λ_8	1.20	1.20	1.20	1.20	1.20	1.20
9	S_9/λ_9	1.11	1.11	1.11	1.11	1.11	1.11
10	S_{10}/λ_{10}	1.25	1.25	1.25	1.25	1.25	1.25
$K_{avg} = \frac{\left(\sum Avg\frac{S_i}{\lambda_i}\right)}{10} = 0.93$							

Overall average of the S_i/λ_i ratio is 0.93. This average value is denoted as K_{avg}. The row averages of S_i/λ_i ratio are denoted by $k_i^{K_{avg}}$ for each product i. Now, to determine new values of the control parameters (s, c, S) the following algorithm is used:

(i) The search domain of K is determined. Since the performance rates of the current scenarios are more than necessary, values for K are chosen mostly lower than K_{avg}. The chosen values for new K values are (0.5; 0.6; 0.7; 0.8; 0.9; 1.0)

(ii) To calculate the new values of S_i, it is needed to calculate new k_i^K values first: The k_i^K values are calculated from the current $k_i^{K_{avg}}$ values. The formula to determine the new k_i^K values is given in Equation 6.1.

$$k_i^K = k_i^{K_{avg}} \times \frac{K}{K_{avg}} \tag{6.1}$$

k_i^K Values are calculated for each of the search domain of K and for each product i. So there are 6 x 10 = 60 different k_i^K values. As an example: The formula to determine $k_3^{0.6}$ for $K = 0.6$ for the third item is calculated below. And all k_i^K values for each of the search domain K are computed and given in Table 6.9. Those values will be used to determine the new values of the control parameter S_i.

$$k_3^{0.6} = k_3^{K_{avg}} \times \frac{0,6}{0,93} = 0,68 \times \frac{0,6}{0,93} = 0,4387 \cong 0,44$$

(iii) New values of the control parameter $\mathrm{S_i}$ are calculated according the Equation 6.2. The $\mathrm{k_i^K}$ value corresponds to the $\mathrm{S_i/\lambda_i}$ ratio. So, new $\mathrm{S_i}$ values are found by multiplying $\mathrm{\lambda_i}$ with $\mathrm{k_i^K}$ values. It should be noted here that the result of this equation is rounded to the nearest integer value.

$$S_i^K = \lambda_i \times k_i^K \tag{6.2}$$

(iv) Now there remain two more control parameters: s and c. The new values of c are defined as a function of S (see Equation 6.3). f is a constant factor less than one. And 7 different values are chosen (0.2; 0.3; …; 0.8) for f to search in the new simulation runs.

$$c_i = f \times S_i \tag{6.3}$$

(v) All $\mathrm{s_i}$ values for ten different items are chosen as zero in the first scenario. So, the search space for all the control parameters is determined. All those values of $(\mathrm{s, c, S})$ control parameters are tabulated for Scenario-1 in the Table D.1 of Appendix D.

Four different scenarios are created to search for better results. Explanations given above are describing the algorithm of the first Scenario. Different from the first one, new scenarios are defined by only changing the s_i parameters. Other control parameters S_i and c_i will remain the same as in Scenario-1. As explained, s_i values for all items are set to zero in the first scenario. Opposite of the first one, in the second scenario, s_i values for all items are set to one. In the third scenario, except of the item-4, s_i is set to zero for all other items. Item-4 is selected because of its biggest annual demand rate $(\lambda_4 = 39)$ and s_4 is set to one. Finally, in the fourth scenario, except of the item-4 and item-3, s_i is set to zero for all other items. This time item-3 is selected with item-4 because of its second biggest annual demand rate ($\lambda_3 = 23$) and $s_3; s_4$ are set to one. Scenarios are briefly described below:

(i) Scenario-1: $s_i = 0$ for all items.
(ii) Scenario-2: $s_i = 1$ for all items.
(iii) Scenario-3: $s_i = 0$ for all items except; $s_4 = 1$
(iv) Scenario-4: $s_i = 0$ for all items except; $s_4 = s_3 = 1$

In light of the foregoing algorithm, (s, c, S) control parameters are determined respectively for Scenario-1, Scenario-2, Scenario-3 and Scenario-4 and given in Appendix D. *Coordinated Control Model* has been run in Arena for the defined scenarios. For each Scenario-42 different simulations are made. Each simulation has run for 6,000 days long with 100 replications. Simulation results of the defined scenarios are given in Appendix E.

Table 6.9. New k_i^K values

k_i^K	K=0.50	K=0.60	K=0.70	K=0.80	K=0.90	K=1.00
k_1^K	0.28	0.34	0.40	0.45	0.51	0.57
k_2^K	0.34	0.40	0.47	0.54	0.61	0.67
k_3^K	0.36	0.44	0.51	0.58	0.66	0.73
k_4^K	0.37	0.45	0.52	0.60	0.67	0.74
k_5^K	0.54	0.65	0.75	0.86	0.97	1.08
k_6^K	0.60	0.73	0.85	0.97	1.09	1.21
k_7^K	0.58	0.70	0.82	0.93	1.05	1.16
k_8^K	0.65	0.77	0.90	1.03	1.16	1.29
k_9^K	0.60	0.72	0.84	0.96	1.08	1.19
k_{10}^K	0.67	0.81	0.94	1.08	1.21	1.34

6.3.1. Evaluation of Simulation Results for the Defined Scenarios

Results of the Scenario-1 are given in Table E.1. As K decreases, customer service level decreases while this is not the case for Total Inventory Costs. It is possible that Total Inventory Costs is falling at the same time while Fill Rate increases.

Plot of the results of Scenario-1 is shown in Figure 6.3. Each curve represents seven different cases of costs vs. fill rate for different K values. It is better for the company to have high demand fill rate and low total inventory cost. Therefore, the optimum region lies in the south-east corner of the graph. But, since the marginal benefit of the demand fill rate is relatively small for values higher than 75 per cent, the company may probably choose to implement an inventory policy that is at the lower south region of the graph.

The lowest yearly Total Inventory Costs occurs at 11,354 Euros. The demand Fill Rate is 81.14 per cent which is better than the target of the company.

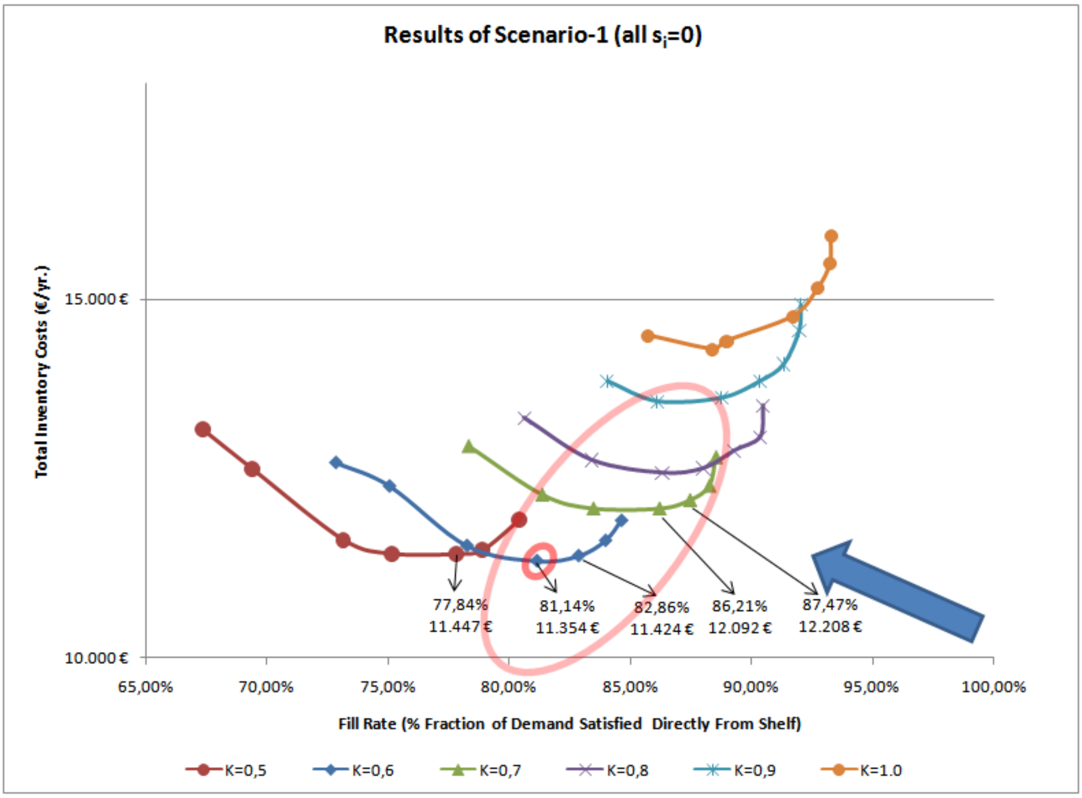


Figure 6.3. Results of scenario-1

The second scenario differs only in s_i parameters. The s_i values are chosen as one for all cases. The results of the simulation runs are plotted in Figure 6.4 and the lowest Total Inventory Cost results in yearly 13,603 Euros with demand Fill Rate 88.50 per cent.

Scenario-3 is a variant of Scenario-1. In Scenario-1 all the s_i values are taken as zero without any exception. In Scenario-3, all the s_i values except s_4 are taken as zero s_4 is set to one. The reason why s_4 is different is that, item 4 has the largest yearly demand rate. Therefore, keeping higher inventory level for item 4 seems a reasonable way to increase customer service level while keeping Total Inventory Costs limited. Table E.4 and Figure 6.5 show the performance results of the Scenario-3.

Since $s_4 = 1$ in Scenario-3 whereas it is equal to zero in Scenario-1, Scenario-3 results in slightly higher customer service levels than Scenario-1. This difference is probably due to two reasons. Firstly, the probability of a customer for item-4 to come up with no inventory of item-4 is slightly lower in Scenario-3. Secondly, the frequency of replenishments is slightly higher in Scenario-3, since s_4 is higher.

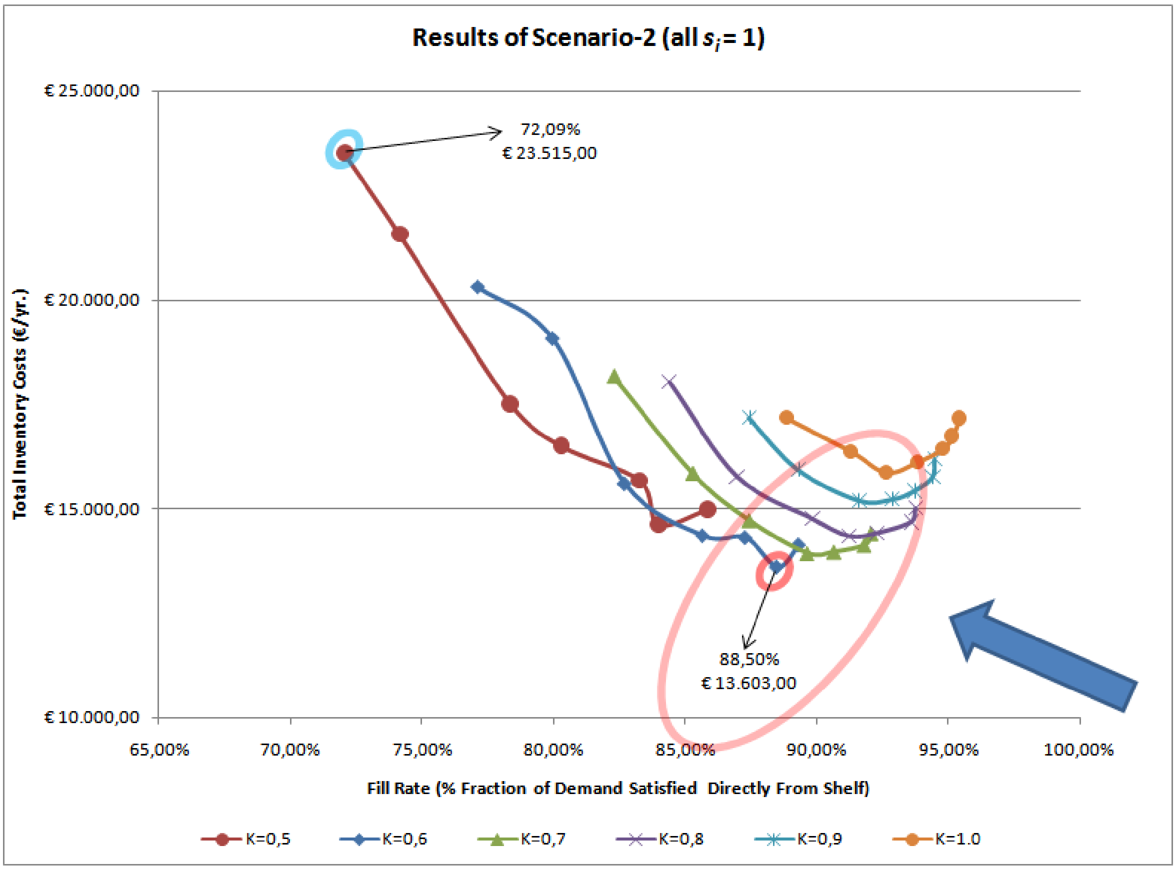


Figure 6.4. Results of scenario-2

The lowest yearly Total Inventory Costs in Scenario-3 occurs at 11,458 Euros with 83.16 per cent Fill Rate. The corresponding minimum in Scenario-1 is at 11,354 Euros with 81.14 per cent Fill Rate. So, the minimum of Scenario-3 improves customer service level for 2 points at a yearly cost of nearly 100 Euros.

Another interesting observation is that, the minima of yearly Total Inventory Costs of scenarios 1 and 3 occur with the same K value 0.6. Moreover f coefficients are very closed in both cases: f = 0.6 in Scenario-3 and f = 0.5 in Scenario-1.

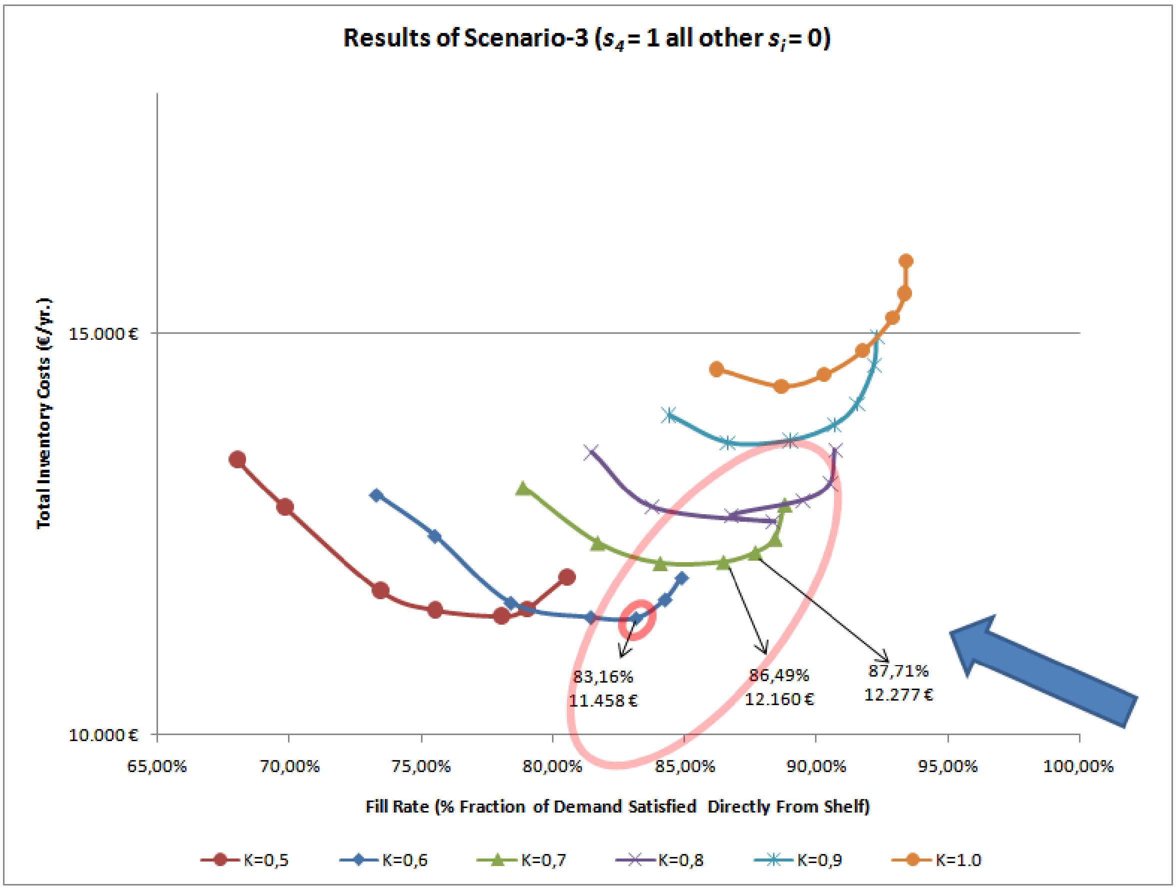


Figure 6.5. Results of scenario-3

Scenario-4 is another variant of Scenario-1. In Scenario-1 all the $s_i$values are taken as zero without any exception. In Scenario-3, all the $s_i$values except s_4 are taken as zero. In Scenario-4 all the s_i values except s_4 and s_3 are taken as zero. s_4 and s_3 are set to one. The reason why s_4 and s_3 are different is that, item 4 and 3 have the largest yearly demand rates among ten products. Therefore, keeping higher inventory level for these items may probably increase customer service level while keeping Total Inventory Costs limited. Table E.3 and Figure 6.5 show the performance results of the 42 cases in Scenario-4.

Since $s_4 = s_3 = 1$ in Scenario-4, whereas it is equal to zero in Scenario-1, Scenario-4 should result in slightly higher customer service levels than Scenario-1 as it is the case in Scenario-3.

The lowest yearly Total Inventory Costs in Scenario-4 occurs at 11,581 Euros with 83.48 per cent Fill Rate. The corresponding minima in Scenario-1 and 3 are at 11,354 Euros with 81.14 per cent Fill Rate and 11,458 Euros with 83.16 per cent Fill Rate respectively. So, the minimum of Scenario-4 improves customer service level for 2.3 points at a yearly cost of additional 230 Euros.

Again as it is in Scenario-3, it is interesting that the minima of yearly Total Inventory Costs of scenarios 1, 3, and 4 occur with the same K value 0.6. And f coefficients are very closed in all three cases again: f = 0.5 in Scenario-1, f = 0.6 in Scenario-3, and f = 0.6 in Scenario-4. Shapes of the curves are very similar in all three scenarios.

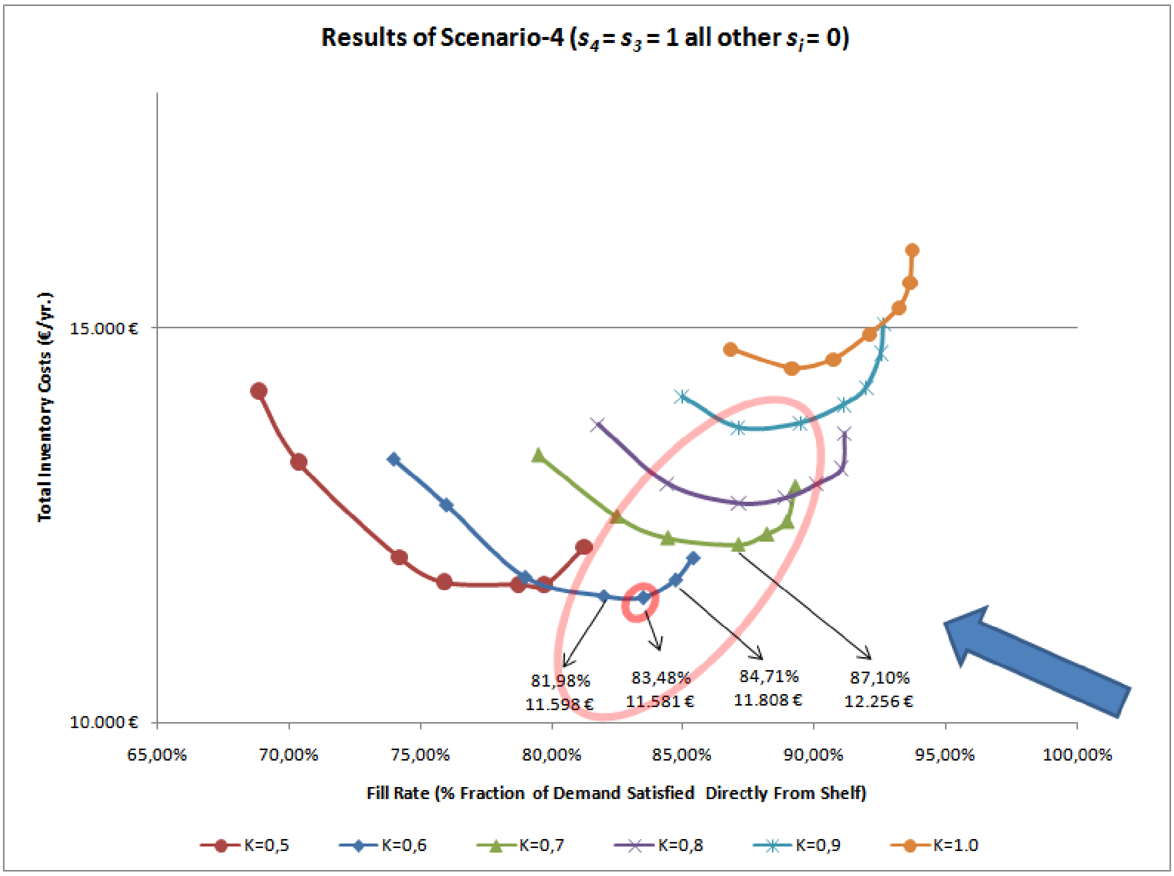


Figure 6.6. Results of scenario-4

Up to now, four different scenarios analyzed each with 42 different control parameter sets. Figure 6.7 shows the performance results of the 4 x 42 cases in four scenarios. Additionally, results obtained at the end of Section 6.2 are included in the figure with the legend called "*Results for Varying* P_2".

First observation from Figure 6.7 is that, there is a clear distinction between Scenario-2 and the rest. Performance data of Scenario-2 gathered around the north-east side of the graph. The remaining scenarios are very similar both in shape and in performance values. Scenario-4 has slightly higher Total Inventory Costs and demand Fill Rates than scenarios 1 and 3. But the difference between those three scenarios is not very distinctive. This difference is probably due to two reasons again as it is in the comparison between Scenario-3 and 1. Firstly, the probability of a customer for item 4 and 3 to come up with no inventory is slightly lower in Scenario-4. Secondly, the frequency of replenishments is slightly higher in Scenario-4, since $s_4$and s_3 are higher.

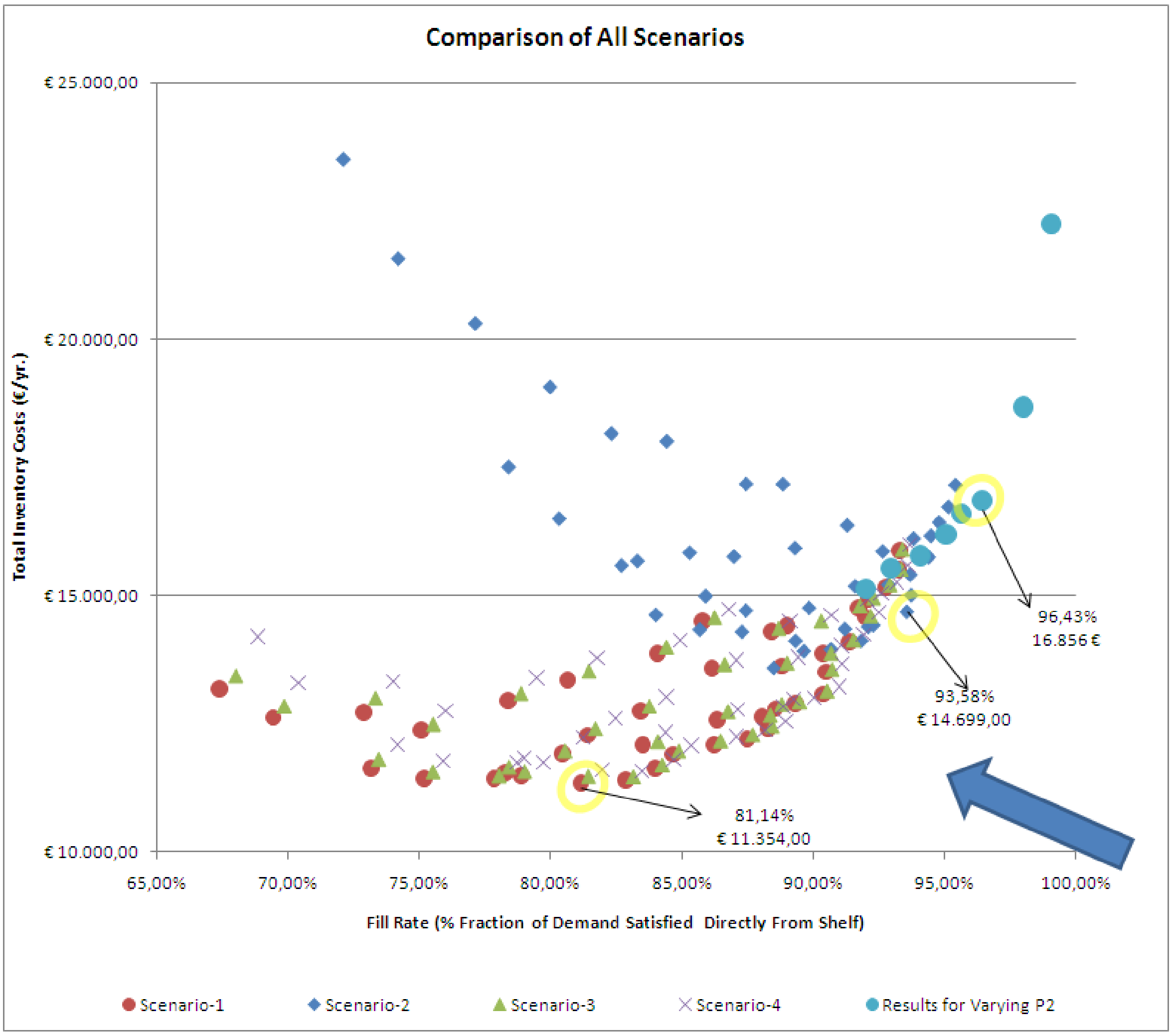


Figure 6.7. Comparison of all scenarios

Table 6.10 summarizes the minima and maxima of yearly Total Inventory Costs of all the scenarios. It is clear that Scenario-2 is at a distinct performance position than other scenarios. The minima of total costs for scenarios 1, 3, and 4 are very close to each other around 11,500 Euros while the customer service levels of scenarios 3 and 4 are about 2 points higher than that of the Scenario-1.

Interestingly, the minima in all scenarios occur at K = 0.6 and the maxima occur (except of Scenario-2) at K = 1.0. Additionally f values are very close in these scenarios. The minima in all scenarios occur in a range of 0.5-0.8 of f parameter. The maxima in all scenarios occur exactly at f = 0.8. The minimum of yearly Total Inventory Costs is at 11,354 Euros which occurs in Scenario-1. The maximum of inventory cost is at 23,515 Euros which occurs in Scenario-2.

Table 6.10. Minima and maxima of scenarios

Scenario #		***K***	***f***	**Average Total Holding Cost** €/yr.	**Average Total Ordering Cost** €/yr.	**Average Total Inventory Costs** €/yr.	**Average Fill Rate** %
1	Min	0.6	0.5	5,978	5,376	11,354	81.14
	Max	1.0	0.8	13,102	2,773	15,876	93.29
2	Min	0.6	0.7	6,697	6,906	13,603	88.50
	Max	0.5	0.2	4,475	19,040	23,515	72.09
3	Min	0.6	0.6	6,139	5,319	11,458	83.16
	Max	1.0	0.8	13,122	2,780	15,902	93.41
4	Min	0.6	0.6	6,166	5,415	11,581	83.48
	Max	1.0	0.8	13,191	2,808	15,999	93.70

7. CONCLUSIONS

The purpose of this research was to find a practical solution to a real life case which is the inventory management problem of a distributor company. The company faces performance and delivery problems. Selling products are very well known, high quality and problem solver industrial equipments which can be categorized as investment goods. In real life practice, it is not often seen that producers or distributors hold these types of goods in their stocks. The distributor company and its supplier are also not keeping stock on hand. Therefore, customers willing to buy these goods and giving order to the distributor have to wait until the lead time. But, although customers are willing to buy, because of the long delivery terms it is frequently seen that they are hesitating to give order. This situation leads to potential sales lost.

The solution needs to be a practical one such that business professionals can easily customize and use this method for their own companies. Therefore a reasonable and simple heuristics was used to help on deciding the levels of the inventory control parameters. In further, alternative scenarios were developed to improve the results obtained.

Our research shows that a certain parameter combination of joint order inventory control policy with yearly Total Inventory Costs around 11,350 Euros satisfies 81 per cent of orders directly from shelf with practically zero lead time to the customer. This is not the optimal solution, since the heuristics does not cover the whole search space, but it is reasonable to assume that the global optimum has a very marginal gain with respect to that solution.

In real life the distributor company, Leon Teknik, does not hold inventory practically. The ratio of direct satisfaction of orders from shelf is less than five per cent. In case, Total Inventory Costs which Leon Teknik currently pays consists only of inventory replenishment costs. The amount of replenishment costs is around 5,500 Euros for the 10 products covered in this research study. It can be assumed that the amount of replenishment costs is practically same in the real life. So, holding cost is the only element that makes the difference between the real case and suggested case. Therefore, the difference in Total Inventory Costs between the near optimum policy suggested by this study and current policy is around € 6.000 per year. For such a small amount of cost, the improvement in customer service level is huge: nearly 82 per cent of customer orders are delivered without any delay.

The yearly sales income of the company for the ten products is around € 285,000. The profit margin is approximately 30 per cent. So, the amount of yearly profits for the ten products is around € 85,000. Thus, the company needs to invest only seven per cent (6.000/85.000) of its yearly profit to increase the customer service level from five per cent to 85 per cent. This policy will put the company 5-9 weeks ahead of the competition in lead time to the customer.

In Table 3.3 of the thesis, fraction of sales success for the selected DEPA Pumps is given around 31.91 per cent. It can be assumed by this ratio that up to 70 per cent of Potential Sales are lost due to some reasons. Around 30 per cent of Potential Sales Lost of Leon Teknik is estimated to be caused by higher lead time in comparison to the competitors. Therefore, the improvement in customer service level from 5 to 85 per cent will increase the probability of sales success approximately to 55 per cent and annual sales income from € 285,000 to € 520,000.

The minimum expected amount of profits due to new policy is 85 thousand Euros per year. But, the actual amount will be probably much higher than that due to the increasing awareness of the potential customers for zero lead time. If it is accepted that the annual sales income will increase up to € 520,000, than there will be additional € 40,000 net profit. By only paying € 6,000 holding cost the distributor company seems to improve its gains € 40,000 more. This is six times more than the invested capital and can easily be accepted as a very good result besides the improvement in customer satisfaction.

This research develops an easy to use decision support tool that is able to present to the decision makers how much inventory carrying and replenishment costs are expected to actualize under different direct demand satisfaction rates that occur through various inventory control policies.

In conclusion, there are two major results gained at the end of this research study. These are:

- A sensible solution approach is developed for Leon Teknik's inventory management problem.
- An easy-practical decision taking tool is developed to use in inventory management area.

There are several ways to expand this research. One step is to customize this research for other products in the same industry or other industries. The idea in these applications will be the same. The simulation program in Arena requires only a few small changes for different products or industries. The number of queue elements in the simulation should be changed. The values for parameters will be customized for the new products. S, c, s parameters will change since they depend on several variables such as yearly demand rate, carrying cost, unit price. But, these customizations can be done very rapidly if the domain experts provide the required information. By applying this methodology in different industries and products, the business decision makers can easily plan their inventory management policies to increase the customer service level while holding the inventory costs below certain upper bounds.

Another further research opportunity occurs by adding the customer behavior variable into this research method. This research does not take into account how the customers will react to the change in customer service level. For example, how many of them turn to another supplier due to the additional lead time? How many new sales are expected to realize due to the improvement in customer service level?

In the same direction as customer behavior, it is also possible to add the feedback dynamics of the market into this study. Increasing the customer service level of a company will most probably attract many customers to the company. This movement will have several side effects which are difficult to predict in real life. For example, it is not unreasonable to expect that the company cannot satisfy increasing customer demand. Or it is very much expected that competitors will imitate the same inventory control policies maybe after some delay. So, the market conditions will most probably change after the new inventory control policy. Therefore a more comprehensive study should take the change in market dynamics into account as well.

APPENDIX A: SIMULATION RESULTS OF INDEPENDENT CONTROL POLICY WITH INITIAL CONTROL PARAMETERS

Table A.1. Simulation results for P_2= 90%

P_2= 90%	**Unit**	**Average**	**Half Width**	**Minimum Average**	**Maximum Average**
Recorded Time Intervals					
Order Arrival Time (Lead Time)	days	1.15	0.02	0.95	1.40
Customer Waiting Time in System	days	44.56	0.07	43.58	45.35
Quantity of Replenishments Placed					
Independent Replenishment Orders	number	13.93	0.05	13.15	14.60
Joint Replenishment Orders	number	0	0	0	0
Total Replenishments	number	13.93	0.05	13.15	14.60
Realized Costs					
Cost of Independent Replenishments	€/yr.	9,889	37.41	9,337	10,366
Cost of Joint Replenishments	€/yr.	0	0	0	0
Total Ordering Cost	€/yr.	9,889	37	9,337	10,366
Total Holding Cost	€/yr.	10,839	25.98	10,387	11,133
Realized Total Costs for All Items	€/yr.	20,728	33.7	20,370	21,280
Realized Fill Rate					
Satisfied Directly From Shelf	%	92.17	0.13	90.53	93.61
Fraction of Customer Demand Satisfaction					
Satisfied Less than in 1 week	%	94.17	0.11	92.97	95.31
Satisfied Less than in 2 week	%	95.92	0.08	94.85	96.85
Satisfied Less than in 3 week	%	97.36	0.06	96.67	97.99
Satisfied Less than in 4 week	%	98.44	0.05	97.73	99.04
Satisfied Less than in 5 week	%	99.21	0.04	98.37	99.69
Satisfied Less than in 6 week	%	99.66	0.03	99.15	99.93
Satisfied Less than in 7 week	%	99.89	0.01	99.68	100.00
Satisfied Less than in 8 week	%	99.98	0.01	99.86	100.00
Satisfied Less than in 9 week	%	100.00	0	100.00	100.00

Table A.2. Simulation results for P_2= 95%

P_2= 95%	**Unit**	**Average**	**Half Width**	**Minimum Average**	**Maximum Average**
Recorded Time Intervals					
Order Arrival Time (Lead Time)	days	0.53	0.01	0.36	0.71
Customer Waiting Time in System	days	44.56	0.07	43.58	45.35
Quantity of Replenishments Placed					
Independent Replenishment Orders	number	13.93	0.05	13.15	14.60
Joint Replenishment Orders	number	0	0	0	0
Total Replenishments	number	13.93	0.05	13.15	14.60
Realized Costs					
Cost of Independent Replenishments	€/yr.	9,889	37.41	9,337	10,366
Cost of Joint Replenishments	€/yr.	0	0	0	0
Total Ordering Cost	€/yr.	9,889	37	9,337	10,366
Total Holding Cost	€/yr.	12,622	26.67	12,158	12,919
Realized Total Costs for All Items	€/yr.	22,511	33.76	22,149	23,055
Realized Fill Rate					
Satisfied Directly From Shelf	%	96.05	0.09	94.84	97.15
Fraction of Customer Demand Satisfaction					
Satisfied Less than in 1 week	%	97.25	0.07	96.37	98.01
Satisfied Less than in 2 week	%	98.20	0.05	97.37	98.87
Satisfied Less than in 3 week	%	98.89	0.04	98.46	99.29
Satisfied Less than in 4 week	%	99.34	0.04	98.83	99.72
Satisfied Less than in 5 week	%	99.64	0.03	99.11	99.93
Satisfied Less than in 6 week	%	99.84	0.02	99.61	100.00
Satisfied Less than in 7 week	%	99.93	0.01	99.78	100.00
Satisfied Less than in 8 week	%	99.98	0	99.89	100.00
Satisfied Less than in 9 week	%	100.00	0	100.00	100.00

Table A.3. Simulation results for $\boldsymbol{P_2}$= 99%

P_2= 99%	**Unit**	**Average**	**Half Width**	**Minimum Average**	**Maximum Average**
Recorded Time Intervals					
Order Arrival Time (Lead Time)	days	0.21	0.01	0.11	0.42
Customer Waiting Time in System	days	44.56	0.07	43.58	45.35
Quantity of Replenishments Placed					
Independent Replenishment Orders	number	13.93	0.05	13.15	14.60
Joint Replenishment Orders	number	0	0	0	0
Total Replenishments	number	13.93	0.05	13.15	14.60
Realized Costs					
Cost of Independent Replenishments	€/yr.	9,889	37.41	9,337	10,366
Cost of Joint Replenishments	€/yr.	0	0	0	0
Total Ordering Cost	€/yr.	9,889	37	9,337	10,366
Total Holding Cost	€/yr.	16,164	27.29	15,698	16,461
Realized Total Costs for All Items	€/yr.	26,053	33.7	25,688	26,591
Realized Fill Rate					
Satisfied Directly From Shelf	%	98.71	0.05	97.96	99.19
Fraction of Customer Demand Satisfaction					
Satisfied Less than in 1 week	%	99.06	0.04	98.35	99.47
Satisfied Less than in 2 week	%	99.32	0.03	98.71	99.68
Satisfied Less than in 3 week	%	99.50	0.03	98.82	99.79
Satisfied Less than in 4 week	%	99.64	0.03	99.10	99.89
Satisfied Less than in 5 week	%	99.77	0.02	99.46	99.96
Satisfied Less than in 6 week	%	99.87	0.01	99.65	100.00
Satisfied Less than in 7 week	%	99.94	0.01	99.82	100.00
Satisfied Less than in 8 week	%	99.99	0	99.89	100.00
Satisfied Less than in 9 week	%	100.00	0	100.00	100.00

APPENDIX B: SIMULATION RESULTS OF COORDINATED CONTROL POLICY WITH INITIAL CONTROL PARAMETERS

Table B.1. Simulation results for P_2= 90%

P_2= 90%	**Unit**	**Average**	**Half Width**	**Minimum Average**	**Maximum Average**
Recorded Time Intervals					
Order Arrival Time (Lead Time)	days	44.54	0.12	43.09	46.41
Customer Waiting Time in System	days	0.51	0.02	0.34	0.84
Quantity of Replenishments Placed					
Independent Replenishment Orders	number	0.51	0.02	0.25	0.75
Joint Replenishment Orders	number	3.65	0.03	3.35	4.05
Total Replenishments	number	4.16	0.03	3.80	4.55
Realized Costs					
Cost of Independent Replenishments	€/yr.	364	14.65	178	533
Cost of Joint Replenishments	€/yr.	3,607	23.55	3,366	3,880
Total Ordering Cost	€/yr.	3,971	23.14	3,675	4,238
Total Holding Cost	€/yr.	12,885	28.79	12,507	13,237
Realized Total Costs for All Items	€/yr.	16,856	34.78	16,348	17,456
Realized Fill Rate					
Satisfied Directly From Shelf	%	96.43	0.09	95.21	97.52
Fraction of Customer Demand Satisfaction					
Satisfied Less than in 1 week	%	97.41	0.08	95.99	98.13
Satisfied Less than in 2 week	%	98.24	0.07	97.13	98.83
Satisfied Less than in 3 week	%	98.85	0.05	97.92	99.33
Satisfied Less than in 4 week	%	99.29	0.04	98.60	99.65
Satisfied Less than in 5 week	%	99.61	0.02	99.21	99.86
Satisfied Less than in 6 week	%	99.82	0.02	99.61	99.97
Satisfied Less than in 7 week	%	99.93	0.01	99.76	100.00
Satisfied Less than in 8 week	%	99.98	0	99.86	100.00
Satisfied Less than in 9 week	%	100.00	0	100.00	100.00

Table B.2. Simulation results for P_2= 95%

P_2= 95%	**Unit**	**Average**	**Half Width**	**Minimum Average**	**Maximum Average**
Recorded Time Intervals					
Order Arrival Time (Lead Time)	days	44.54	0.12	43.09	46.41
Customer Waiting Time in System	days	0.29	0.01	0.19	0.58
Quantity of Replenishments Placed					
Independent Replenishment Orders	number	0.51	0.02	0.25	0.75
Joint Replenishment Orders	number	3.65	0.03	3.35	4.05
Total Replenishments	number	364.23	14.65	177.50	532.50
Realized Costs					
Cost of Independent Replenishments	€/yr.	3,607	23.55	3,366	3,880
Cost of Joint Replenishments	€/yr.	4	0.03	4	5
Total Ordering Cost	€/yr.	3,971	23.14	3,675	4,238
Total Holding Cost	€/yr.	14,704	29.29	14,308	15,065
Realized Total Costs for All Items	€/yr.	18,675	35.05	18,149	19,280
Realized Fill Rate					
Satisfied Directly From Shelf	%	98.04	0.07	96.92	98.78
Fraction of Customer Demand Satisfaction					
Satisfied Less than in 1 week	%	98.59	0.06	97.31	99.15
Satisfied Less than in 2 week	%	99.05	0.05	98.17	99.39
Satisfied Less than in 3 week	%	99.35	0.04	98.57	99.73
Satisfied Less than in 4 week	%	99.57	0.03	98.96	99.93
Satisfied Less than in 5 week	%	99.74	0.02	99.36	99.97
Satisfied Less than in 6 week	%	99.87	0.01	99.68	100.00
Satisfied Less than in 7 week	%	99.94	0.01	99.82	100.00
Satisfied Less than in 8 week	%	99.99	0	99.90	100.00
Satisfied Less than in 9 week	%	100.00	0	100.00	100.00

Table B.3. Simulation results for $\boldsymbol{P_2}$= 99%

$\boldsymbol{P_2}$= 99%	Unit	Average	Half Width	Minimum Average	Maximum Average
Recorded Time Intervals					
Order Arrival Time (Lead Time)	days	44.54	0.12	43.09	46.41
Customer Waiting Time in System	days	0.17	0.01	0.09	0.40
Quantity of Replenishments Placed					
Independent Replenishment Orders	number	0.51	0.02	0.25	0.75
Joint Replenishment Orders	number	3.65	0.03	3.35	4.05
Total Replenishments	number	4.16	0.03	3.80	4.55
Realized Costs					
Cost of Independent Replenishments	€/yr.	364	14.65	178	533
Cost of Joint Replenishments	€/yr.	3,607	23.55	3,366	3,880
Total Ordering Cost	€/yr.	3,971	23.14	3,675	4,238
Total Holding Cost	€/yr.	18,268	29.9	17,859	18,636
Realized Total Costs for All Items	€/yr.	22,239	35.24	21,700	22,851
Realized Fill Rate					
Satisfied Directly From Shelf	%	99.09	0.05	98.32	99.51
Fraction of Customer Demand Satisfaction					
Satisfied Less than in 1 week	%	99.28	0.04	98.42	99.62
Satisfied Less than in 2 week	%	99.44	0.03	98.75	99.72
Satisfied Less than in 3 week	%	99.57	0.03	98.85	99.83
Satisfied Less than in 4 week	%	99.67	0.02	99.14	99.93
Satisfied Less than in 5 week	%	99.78	0.02	99.46	99.97
Satisfied Less than in 6 week	%	99.88	0.01	99.68	100.00
Satisfied Less than in 7 week	%	99.94	0.01	99.82	100.00
Satisfied Less than in 8 week	%	99.99	0	99.90	100.00
Satisfied Less than in 9 week	%	100.00	0	100.00	100.00

APPENDIX C: SIMULATION RESULTS SHOWING EFFECTS OF P_2 ON COORDINATED CONTROL POLICY

Table C.1. Simulation results for P_2 = 75%

SIMULATION OUTPUTS FOR THE GIVEN CUSTOMER SERVICE LEVEL			
P2=75%	**Unit**	**Average**	**Half Width**
Recorded Time Intervals			
Order Arrival Time (Lead Time)	days	44.54	0.12
Customer Waiting Time in System	days	1.35	0.03
Realized Costs			
Cost of Independent Replenishments	€/yr.	364	14.65
Cost of Joint Replenishments	€/yr.	3,607	23.55
Total Ordering Cost	€/yr.	3,971	23.14
Total Holding Cost	€/yr.	11,168	27.44
Realized Total Costs for All Items	€/yr.	**15,139**	**34.16**
Realized Fill Rate			
Satisfied Directly From Shelf	**%**	**92.00**	**0.14**

Table C.2. Simulation results for P_2 = 80%

SIMULATION OUTPUTS FOR THE GIVEN CUSTOMER SERVICE LEVEL			
P2=80%	**Unit**	**Average**	**Half Width**
Recorded Time Intervals			
Order Arrival Time (Lead Time)	days	44.54	0.12
Customer Waiting Time in System	days	1.16	0.03
Realized Costs			
Cost of Independent Replenishments	€/yr.	364	14.65
Cost of Joint Replenishments	€/yr.	3,607	23.55
Total Ordering Cost	€/yr.	3,971	23.14
Total Holding Cost	€/yr.	11,558	28.12
Realized Total Costs for All Items	€/yr.	**15,529**	**34.57**
Realized Fill Rate			
Satisfied Directly From Shelf	**%**	**92.94**	**0.13**

Table C.3. Simulation results for P_2 = 83%

SIMULATION OUTPUTS FOR THE GIVEN CUSTOMER SERVICE LEVEL			
P2=83%	**Unit**	**Average**	**Half Width**
Recorded Time Intervals			
Order Arrival Time (Lead Time)	days	44.54	0.12
Customer Waiting Time in System	days	0.92	0.02
Realized Costs			
Cost of Independent Replenishments	€/yr.	364	14.65
Cost of Joint Replenishments	€/yr.	3,607	23.55
Total Ordering Cost	€/yr.	3,971	23.14
Total Holding Cost	€/yr.	11,808	28.22
Realized Total Costs for All Items	€/yr.	**15,779**	**34.59**
Realized Fill Rate			
Satisfied Directly From Shelf	**%**	**94.08**	**0.12**

Table C.4. Simulation results for P_2 = 85%

SIMULATION OUTPUTS FOR THE GIVEN CUSTOMER SERVICE LEVEL			
P2=%85	**Unit**	**Average**	**Half Width**
Recorded Time Intervals			
Order Arrival Time (Lead Time)	days	44.54	0.12
Customer Waiting Time in System	days	0.73	0.02
Realized Costs			
Cost of Independent Replenishments	€/yr.	364	14.65
Cost of Joint Replenishments	€/yr.	3,607	23.55
Total Ordering Cost	€/yr.	3,971	23.14
Total Holding Cost	€/yr.	12,219	28.29
Realized Total Costs for All Items	€/yr.	**16,190**	**34.54**
Realized Fill Rate			
Satisfied Directly From Shelf	**%**	**95.06**	**0.11**

Table C.5. Simulation results for P_2 = 88%

SIMULATION OUTPUTS FOR THE GIVEN CUSTOMER SERVICE LEVEL			
P2=88%	**Unit**	**Average**	**Half Width**
Recorded Time Intervals			
Order Arrival Time (Lead Time)	days	44.54	0.12
Customer Waiting Time in System	days	0.64	0.02
Realized Costs			
Cost of Independent Replenishments	€/yr.	364	14.65
Cost of Joint Replenishments	€/yr.	3,607	23.55
Total Ordering Cost	€/yr.	3,971	23.14
Total Holding Cost	€/yr.	12,628	28.71
Realized Total Costs for All Items	€/yr.	**16,599**	**34.75**
Realized Fill Rate			
Satisfied Directly From Shelf	**%**	**95.66**	**0.11**

APPENDIX D: CONTROL PARAMETERS FOR DIFFERENT SCENARIOS

Table D.1. Control parameters determined for scenario-1

K	**i**	**s_i**	**S_i**		**c_i**						
			k_i^K	S_i^K	$f=0.2$	$f=0.3$	$f=0.4$	$f=0.5$	$f=0.6$	$f=0.7$	$f=0.8$
0.5	1	0	0.28	5	1	1	2	2	3	3	4
	2	0	0.34	5	1	1	2	2	3	3	4
	3	0	0.36	8	1	2	3	4	4	5	6
	4	0	0.37	15	3	4	6	7	9	10	12
	5	0	0.54	4	1	1	1	2	2	2	3
	6	0	0.60	5	1	1	2	2	3	3	4
	7	0	0.58	7	1	2	2	3	4	4	5
	8	0	0.65	3	1	1	1	1	1	2	2
	9	0	0.60	5	1	1	2	2	3	3	4
	10	0	0.67	3	1	1	1	1	1	2	2
0.6	1	0	0.34	6	1	1	2	3	3	4	4
	2	0	0.40	6	1	1	2	3	3	4	4
	3	0	0.44	10	2	3	4	5	6	7	8
	4	0	0.45	17	3	5	6	8	10	11	13
	5	0	0.65	5	1	1	2	2	3	3	4
	6	0	0.73	6	1	1	2	3	3	4	4
	7	0	0.70	8	1	2	3	4	4	5	6
	8	0	0.77	4	1	1	1	2	2	2	3
	9	0	0.72	6	1	1	2	3	3	4	4
	10	0	0.81	3	1	1	1	1	1	2	2

Table D.1. Control parameters determined for scenario-1 (Continue)

0.7	1	0	0.40	8	1	2	3	4	4	5	6
	2	0	0.47	7	1	2	2	3	4	4	5
	3	0	0.51	12	2	3	4	6	7	8	9
	4	0	0.52	20	4	6	8	10	12	14	16
	5	0	0.75	6	1	1	2	3	3	4	4
	6	0	0.85	7	1	2	2	3	4	4	5
	7	0	0.82	10	2	3	4	5	6	7	8
	8	0	0.90	5	1	1	2	2	3	3	4
	9	0	0.84	8	1	2	3	4	4	5	6
	10	0	0.94	4	1	1	1	2	2	2	3
0.8	1	0	0.45	9	1	2	3	4	5	6	7
	2	0	0.54	8	1	2	3	4	4	5	6
	3	0	0.58	13	2	3	5	6	7	9	10
	4	0	0.60	23	4	6	9	11	13	16	18
	5	0	0.86	7	1	2	2	3	4	4	5
	6	0	0.97	8	1	2	3	4	4	5	6
	7	0	0.93	11	2	3	4	5	6	7	8
	8	0	1.03	5	1	1	2	2	3	3	4
	9	0	0.96	9	1	2	3	4	5	6	7
	10	0	1.08	4	1	1	1	2	2	2	3
0.9	1	0	0.51	10	2	3	4	5	6	7	8
	2	0	0.61	9	1	2	3	4	5	6	7
	3	0	0.66	15	3	4	6	7	9	10	12
	4	0	0.67	26	5	7	10	13	15	18	20
	5	0	0.97	8	1	2	3	4	4	5	6
	6	0	1.09	9	1	2	3	4	5	6	7
	7	0	1.05	13	2	3	5	6	7	9	10
	8	0	1.16	6	1	1	2	3	3	4	4
	9	0	1.08	10	2	3	4	5	6	7	8
	10	0	1.21	5	1	1	2	2	3	3	4

Table D.1. Control parameters determined for scenario-1 (Continue)

1.0	1	0	0.57	11	2	3	4	5	6	7	8
	2	0	0.67	10	2	3	4	5	6	7	8
	3	0	0.73	17	3	5	6	8	10	11	13
	4	0	0.74	29	5	8	11	14	17	20	23
	5	0	1.08	9	1	2	3	4	5	6	7
	6	0	1.21	10	2	3	4	5	6	7	8
	7	0	1.16	14	2	4	5	7	8	9	11
	8	0	1.29	6	1	1	2	3	3	4	4
	9	0	1.19	11	2	3	4	5	6	7	8
	10	0	1.34	5	1	1	2	2	3	3	4

Table D.2. Control parameters determined for scenario-2

K	***i***	s_i	S_i		c_i						
			k_i^K	S_i^K	*f = 0.2*	*f = 0.3*	*f = 0.4*	*f = 0.5*	*f = 0.6*	*f = 0.7*	*f = 0.8*
0.5	1	1	0.28	5	1	1	2	2	3	3	4
	2	1	0.34	5	1	1	2	2	3	3	4
	3	1	0.36	8	1	2	3	4	4	5	6
	4	1	0.37	15	3	4	6	7	9	10	12
	5	1	0.54	4	1	1	1	2	2	2	3
	6	1	0.60	5	1	1	2	2	3	3	4
	7	1	0.58	7	1	2	2	3	4	4	5
	8	1	0.65	3	1	1	1	1	1	2	2
	9	1	0.60	5	1	1	2	2	3	3	4
	10	1	0.67	3	1	1	1	1	1	2	2
0.6	1	1	0.34	6	1	1	2	3	3	4	4
	2	1	0.40	6	1	1	2	3	3	4	4
	3	1	0.44	10	2	3	4	5	6	7	8
	4	1	0.45	17	3	5	6	8	10	11	13
	5	1	0.65	5	1	1	2	2	3	3	4
	6	1	0.73	6	1	1	2	3	3	4	4
	7	1	0.70	8	1	2	3	4	4	5	6
	8	1	0.77	4	1	1	1	2	2	2	3
	9	1	0.72	6	1	1	2	3	3	4	4
	10	1	0.81	3	1	1	1	1	1	2	2

Table D.2. Control parameters determined for scenario-2 (Continue)

0.7	1	1	0.40	8	1	2	3	4	4	5	6
	2	1	0.47	7	1	2	2	3	4	4	5
	3	1	0.51	12	2	3	4	6	7	8	9
	4	1	0.52	20	4	6	8	10	12	14	16
	5	1	0.75	6	1	1	2	3	3	4	4
	6	1	0.85	7	1	2	2	3	4	4	5
	7	1	0.82	10	2	3	4	5	6	7	8
	8	1	0.90	5	1	1	2	2	3	3	4
	9	1	0.84	8	1	2	3	4	4	5	6
	10	1	0.94	4	1	1	1	2	2	2	3
0.8	1	1	0.45	9	1	2	3	4	5	6	7
	2	1	0.54	8	1	2	3	4	4	5	6
	3	1	0.58	13	2	3	5	6	7	9	10
	4	1	0.60	23	4	6	9	11	13	16	18
	5	1	0.86	7	1	2	2	3	4	4	5
	6	1	0.97	8	1	2	3	4	4	5	6
	7	1	0.93	11	2	3	4	5	6	7	8
	8	1	1.03	5	1	1	2	2	3	3	4
	9	1	0.96	9	1	2	3	4	5	6	7
	10	1	1.08	4	1	1	1	2	2	2	3
0.9	1	1	0.51	10	2	3	4	5	6	7	8
	2	1	0.61	9	1	2	3	4	5	6	7
	3	1	0.66	15	3	4	6	7	9	10	12
	4	1	0.67	26	5	7	10	13	15	18	20
	5	1	0.97	8	1	2	3	4	4	5	6
	6	1	1.09	9	1	2	3	4	5	6	7
	7	1	1.05	13	2	3	5	6	7	9	10
	8	1	1.16	6	1	1	2	3	3	4	4
	9	1	1.08	10	2	3	4	5	6	7	8
	10	1	1.21	5	1	1	2	2	3	3	4

Table D.2. Control parameters determined for scenario-2 (Continue)

1.0	1	1	0.57	11	2	3	4	5	6	7	8
	2	1	0.67	10	2	3	4	5	6	7	8
	3	1	0.73	17	3	5	6	8	10	11	13
	4	1	0.74	29	5	8	11	14	17	20	23
	5	1	1.08	9	1	2	3	4	5	6	7
	6	1	1.21	10	2	3	4	5	6	7	8
	7	1	1.16	14	2	4	5	7	8	9	11
	8	1	1.29	6	1	1	2	3	3	4	4
	9	1	1.19	11	2	3	4	5	6	7	8
	10	1	1.34	5	1	1	2	2	3	3	4

Table D.3. Control parameters determined for scenario-3

K	***i***	***s_i***	***S_i***		***c_i***						
			k_i^K	S_i^K	***f* = 0.2**	***f* = 0.3**	***f* = 0.4**	***f* = 0.5**	***f* = 0.6**	***f* = 0.7**	***f* = 0.8**
0.5	1	0	0.28	5	1	1	2	2	3	3	4
	2	0	0.34	5	1	1	2	2	3	3	4
	3	0	0.36	8	1	2	3	4	4	5	6
	4	1	0.37	15	3	4	6	7	9	10	12
	5	0	0.54	4	1	1	1	2	2	2	3
	6	0	0.60	5	1	1	2	2	3	3	4
	7	0	0.58	7	1	2	2	3	4	4	5
	8	0	0.65	3	1	1	1	1	1	2	2
	9	0	0.60	5	1	1	2	2	3	3	4
	10	0	0.67	3	1	1	1	1	1	2	2
0.6	1	0	0.34	6	1	1	2	3	3	4	4
	2	0	0.40	6	1	1	2	3	3	4	4
	3	0	0.44	10	2	3	4	5	6	7	8
	4	1	0.45	17	3	5	6	8	10	11	13
	5	0	0.65	5	1	1	2	2	3	3	4
	6	0	0.73	6	1	1	2	3	3	4	4
	7	0	0.70	8	1	2	3	4	4	5	6
	8	0	0.77	4	1	1	1	2	2	2	3
	9	0	0.72	6	1	1	2	3	3	4	4
	10	0	0.81	3	1	1	1	1	1	2	2

Table D.3. Control parameters determined for scenario-3 (Continue)

0.7	1	0	0.40	8	1	2	3	4	4	5	6
	2	0	0.47	7	1	2	2	3	4	4	5
	3	0	0.51	12	2	3	4	6	7	8	9
	4	1	0.52	20	4	6	8	10	12	14	16
	5	0	0.75	6	1	1	2	3	3	4	4
	6	0	0.85	7	1	2	2	3	4	4	5
	7	0	0.82	10	2	3	4	5	6	7	8
	8	0	0.90	5	1	1	2	2	3	3	4
	9	0	0.84	8	1	2	3	4	4	5	6
	10	0	0.94	4	1	1	1	2	2	2	3
0.8	1	0	0.45	9	1	2	3	4	5	6	7
	2	0	0.54	8	1	2	3	4	4	5	6
	3	0	0.58	13	2	3	5	6	7	9	10
	4	1	0.60	23	4	6	9	11	13	16	18
	5	0	0.86	7	1	2	2	3	4	4	5
	6	0	0.97	8	1	2	3	4	4	5	6
	7	0	0.93	11	2	3	4	5	6	7	8
	8	0	1.03	5	1	1	2	2	3	3	4
	9	0	0.96	9	1	2	3	4	5	6	7
	10	0	1.08	4	1	1	1	2	2	2	3
0.9	1	0	0.51	10	2	3	4	5	6	7	8
	2	0	0.61	9	1	2	3	4	5	6	7
	3	0	0.66	15	3	4	6	7	9	10	12
	4	1	0.67	26	5	7	10	13	15	18	20
	5	0	0.97	8	1	2	3	4	4	5	6
	6	0	1.09	9	1	2	3	4	5	6	7
	7	0	1.05	13	2	3	5	6	7	9	10
	8	0	1.16	6	1	1	2	3	3	4	4
	9	0	1.08	10	2	3	4	5	6	7	8
	10	0	1.21	5	1	1	2	2	3	3	4

Table D.3. Control parameters determined for scenario-3 (Continue)

1.0	1	0	0.57	11	2	3	4	5	6	7	8
	2	0	0.67	10	2	3	4	5	6	7	8
	3	0	0.73	17	3	5	6	8	10	11	13
	4	1	0.74	29	5	8	11	14	17	20	23
	5	0	1.08	9	1	2	3	4	5	6	7
	6	0	1.21	10	2	3	4	5	6	7	8
	7	0	1.16	14	2	4	5	7	8	9	11
	8	0	1.29	6	1	1	2	3	3	4	4
	9	0	1.19	11	2	3	4	5	6	7	8
	10	0	1.34	5	1	1	2	2	3	3	4

Table D.4. Control parameters determined for scenario-4

K	***i***	s_i	S_i		c_i						
			k_i^K	S_i^K	*f = 0.2*	*f = 0.3*	*f = 0.4*	*f = 0.5*	*f = 0.6*	*f = 0.7*	*f = 0.8*
0.5	1	0	0.28	5	1	1	2	2	3	3	4
	2	0	0.34	5	1	1	2	2	3	3	4
	3	1	0.36	8	1	2	3	4	4	5	6
	4	1	0.37	15	3	4	6	7	9	10	12
	5	0	0.54	4	1	1	1	2	2	2	3
	6	0	0.60	5	1	1	2	2	3	3	4
	7	0	0.58	7	1	2	2	3	4	4	5
	8	0	0.65	3	1	1	1	1	1	2	2
	9	0	0.60	5	1	1	2	2	3	3	4
	10	0	0.67	3	1	1	1	1	1	2	2
0.6	1	0	0.34	6	1	1	2	3	3	4	4
	2	0	0.40	6	1	1	2	3	3	4	4
	3	1	0.44	10	2	3	4	5	6	7	8
	4	1	0.45	17	3	5	6	8	10	11	13
	5	0	0.65	5	1	1	2	2	3	3	4
	6	0	0.73	6	1	1	2	3	3	4	4
	7	0	0.70	8	1	2	3	4	4	5	6
	8	0	0.77	4	1	1	1	2	2	2	3
	9	0	0.72	6	1	1	2	3	3	4	4
	10	0	0.81	3	1	1	1	1	1	2	2

Table D.4. Control parameters determined for scenario-4 (Continue)

0.7	1	0	0.40	8	1	2	3	4	4	5	6
	2	0	0.47	7	1	2	2	3	4	4	5
	3	1	0.51	12	2	3	4	6	7	8	9
	4	1	0.52	20	4	6	8	10	12	14	16
	5	0	0.75	6	1	1	2	3	3	4	4
	6	0	0.85	7	1	2	2	3	4	4	5
	7	0	0.82	10	2	3	4	5	6	7	8
	8	0	0.90	5	1	1	2	2	3	3	4
	9	0	0.84	8	1	2	3	4	4	5	6
	10	0	0.94	4	1	1	1	2	2	2	3
0.8	1	0	0.45	9	1	2	3	4	5	6	7
	2	0	0.54	8	1	2	3	4	4	5	6
	3	1	0.58	13	2	3	5	6	7	9	10
	4	1	0.60	23	4	6	9	11	13	16	18
	5	0	0.86	7	1	2	2	3	4	4	5
	6	0	0.97	8	1	2	3	4	4	5	6
	7	0	0.93	11	2	3	4	5	6	7	8
	8	0	1.03	5	1	1	2	2	3	3	4
	9	0	0.96	9	1	2	3	4	5	6	7
	10	0	1.08	4	1	1	1	2	2	2	3
0.9	1	0	0.51	10	2	3	4	5	6	7	8
	2	0	0.61	9	1	2	3	4	5	6	7
	3	1	0.66	15	3	4	6	7	9	10	12
	4	1	0.67	26	5	7	10	13	15	18	20
	5	0	0.97	8	1	2	3	4	4	5	6
	6	0	1.09	9	1	2	3	4	5	6	7
	7	0	1.05	13	2	3	5	6	7	9	10
	8	0	1.16	6	1	1	2	3	3	4	4
	9	0	1.08	10	2	3	4	5	6	7	8
	10	0	1.21	5	1	1	2	2	3	3	4

Table D.4. Control parameters determined for scenario-4 (Continue)

1.0	1	0	0.57	11	2	3	4	5	6	7	8
	2	0	0.67	10	2	3	4	5	6	7	8
	3	1	0.73	17	3	5	6	8	10	11	13
	4	1	0.74	29	5	8	11	14	17	20	23
	5	0	1.08	9	1	2	3	4	5	6	7
	6	0	1.21	10	2	3	4	5	6	7	8
	7	0	1.16	14	2	4	5	7	8	9	11
	8	0	1.29	6	1	1	2	3	3	4	4
	9	0	1.19	11	2	3	4	5	6	7	8
	10	0	1.34	5	1	1	2	2	3	3	4

APPENDIX E: SIMULATION RESULTS FOR DIFFERENT SCENARIOS

Table E.1. Simulation results for scenario-1

K	*f*	**Average Total Holding Cost** €/yr.	**Average Total Ordering Cost** €/yr.	**Average Total Inventory Costs** €/yr.	**Average Fill Rate** %
0.5	0.2	4,119	9,056	13,184	67.37
0.5	0.3	4,194	8,433	12,628	69.43
0.5	0.4	4,433	7,210	11,643	73.16
0.5	0.5	4,578	6,874	11,452	75.17
0.5	0.6	4,856	6,591	11,447	77.84
0.5	0.7	5,043	6,458	11,501	78.88
0.5	0.8	5,274	6,653	11,928	80.43
0.6	0.2	5,156	7,563	12,720	72.86
0.6	0.3	5,232	7,160	12,393	75.07
0.6	0.4	5,584	5,979	11,564	78.26
0.6	0.5	5,978	5,376	11,354	81.14
0.6	0.6	6,116	5,307	11,424	82.86
0.6	0.7	6,441	5,194	11,636	83.97
0.6	0.8	6,621	5,293	11,915	84.63
0.7	0.2	6,726	6,225	12,951	78.35
0.7	0.3	7,060	5,222	12,283	81.38
0.7	0.4	7,406	4,686	12,093	83.49
0.7	0.5	7,946	4,146	12,092	86.21
0.7	0.6	8,207	4,000	12,208	87.47
0.7	0.7	8,420	3,986	12,406	88.27
0.7	0.8	8,732	4,066	12,799	88.54
0.8	0.2	7,650	5,698	13,348	80.64
0.8	0.3	8,003	4,764	12,768	83.40
0.8	0.4	8,485	4,103	12,588	86.33

Table E.1. Simulation results for scenario-1 (Continue)

0.8	0.5	8,962	3,688	12,651	88.04
0.8	0.6	8,332	3,561	12,893	89.29
0.8	0.7	9,603	3,480	13,084	90.37
0.8	0.8	9,939	3,578	13,517	90.49
0.9	0.2	9,130	4,736	13,866	84.06
0.9	0.3	9,504	4,078	13,582	86.13
0.9	0.4	10,211	3,420	13,632	88.79
0.9	0.5	10,715	3,148	13,863	90.37
0.9	0.6	11,107	2,993	14,101	91.36
0.9	0.7	11,563	3,006	14,569	92.00
0.9	0.8	11,835	3,087	14,922	92.07
1.0	0.2	10,267	4,227	14,494	85.76
1.0	0.3	10,741	3,563	14,304	88.41
1.0	0.4	11,312	3,106	14,419	88.99
1.0	0.5	11,927	2,836	14,763	91.71
1.0	0.6	12,447	2,705	15,153	92.75
1.0	0.7	12,803	2,701	15,505	93.26
1.0	0.8	13,102	2,773	15,876	93.29

Table E.2. Simulation results of scenario-2

K	*f*	**Average Total Holding Cost** €/yr.	**Average Total Ordering Cost** €/yr.	**Average Total Inventory Costs** €/yr.	**Average Fill Rate** **%**
0.5	0.2	4,475	19,040	23,515	72.09
0.5	0.3	4,551	17,031	21,582	74.18
0.5	0.4	4,847	12,672	17,519	78.39
0.5	0.5	4,989	11,524	16,513	80.32
0.5	0.6	5,344	10,344	15,688	83.30
0.5	0.7	5,529	9,109	14,638	84.01
0.5	0.8	5,896	9,109	15,005	85.90
0.6	0.2	5,538	14,780	20,318	77.12
0.6	0.3	5,657	13,424	19,081	79.97
0.6	0.4	6,007	9,594	15,601	82.70
0.6	0.5	6,453	7,900	14,353	85.68
0.6	0.6	6,630	7,679	14,309	87.29
0.6	0.7	6,697	6,906	13,603	88.50
0.6	0.8	7,224	6,906	14,130	89.33
0.7	0.2	7,183	10,988	18,171	82.31
0.7	0.3	7,512	8,338	15,850	85.30
0.7	0.4	7,878	6,844	14,722	87.44
0.7	0.5	8,413	5,519	13,932	89.65
0.7	0.6	8,747	5,217	13,964	90.68
0.7	0.7	9,045	5,084	14,129	91.83
0.7	0.8	9,400	5,006	14,406	92.10
0.8	0.2	8,147	9,874	18,021	84.42
0.8	0.3	8,484	7,290	15,774	86.98
0.8	0.4	8,981	5,793	14,774	89.85
0.8	0.5	9,472	4,893	14,365	91.22
0.8	0.6	9,862	4,575	14,437	92.30
0.8	0.7	10,269	4,430	14,699	93.58

Table E.2. Simulation results of scenario-2 (Continue)

0.8	0.8	10,648	4,374	15,022	93.75
0.9	0.2	9,656	7,533	17,189	87.45
0.9	0.3	10,013	5,927	15,940	89.31
0.9	0.4	10,691	4,505	15,196	91.61
0.9	0.5	11,208	4,016	15,224	92.88
0.9	0.6	11,685	3,737	15,422	93.71
0.9	0.7	12,130	3,633	15,763	94.41
0.9	0.8	12,489	3,691	16,180	94.49
1.0	0.2	10,758	6,427	17,185	88.86
1.0	0.3	11,248	5,134	16,382	91.31
1.0	0.4	11,838	4,038	15,876	92.66
1.0	0.5	12,468	3,658	16,126	93.84
1.0	0.6	13,063	3,376	16,439	94.81
1.0	0.7	13,454	3,288	16,742	95.17
1.0	0.8	13,836	3,332	17,168	95.43

Table E.3. Simulation results for scenario-3

K	***f***	**Average Total Holding Cost** €/yr.	**Average Total Ordering Cost** €/yr.	**Average Total Inventory Costs** €/yr.	**Average Fill Rate** %
0.5	0.2	4,114	9,317	13,431	68.02
0.5	0.3	4,200	8,637	12,837	69.86
0.5	0.4	4,453	7,348	11,801	73.46
0.5	0.5	4,581	6,977	11,558	75.53
0.5	0.6	4,862	6,617	11,479	78.05
0.5	0.7	5,055	6,510	11,565	79.02
0.5	0.8	5,281	6,684	11,965	80.56
0.6	0.2	5,155	7,841	12,996	73.34
0.6	0.3	5,245	7,240	12,485	75.54
0.6	0.4	5,590	6,058	11,648	78.42
0.6	0.5	6,002	5,467	11,469	81.45
0.6	0.6	6,139	5,319	11,458	83.16
0.6	0.7	6,467	5,222	11,689	84.26
0.6	0.8	6,646	5,317	11,963	84.89
0.7	0.2	6,747	6,340	13,087	78.88
0.7	0.3	7,073	5,326	12,399	81.72
0.7	0.4	7,431	4,715	12,146	84.10
0.7	0.5	7,957	4,203	12,160	86.49
0.7	0.6	8,241	4,036	12,277	87.71
0.7	0.7	8,448	4,003	12,451	88.45
0.7	0.8	8,773	4,101	12,874	88.82
0.8	0.2	7,689	5,834	13,523	81.47
0.8	0.3	8,044	4,797	12,841	83.77
0.8	0.4	8,517	4,145	12,662	88.38
0.8	0.5	9,009	3,726	12,735	86.77
0.8	0.6	9,347	3,576	12,923	89.49
0.8	0.7	9,629	3,505	13,134	90.54

Table E.3. Simulation results for scenario-3 (Continue)

0.8	0.8	9,965	3,585	13,550	90.73
0.9	0.2	9,162	4,831	13,993	84.42
0.9	0.3	9,550	4,103	13,653	86.64
0.9	0.4	10,226	3,454	13,680	89.02
0.9	0.5	10,721	3,150	13,871	90.69
0.9	0.6	11,113	3,014	14,127	91.55
0.9	0.7	11,581	3,021	14,602	92.20
0.9	0.8	11,855	3,097	14,952	92.31
1.0	0.2	10,301	4,269	14,570	86.26
1.0	0.3	10,760	3,599	14,359	88.70
1.0	0.4	11,359	3,140	14,499	90.32
1.0	0.5	11,928	2,867	14,795	91.79
1.0	0.6	12,476	2,730	15,206	92.93
1.0	0.7	12,809	2,701	15,510	93.37
1.0	0.8	13,122	2,780	15,902	93.41

Table E.4. Simulation results of scenario-4

K	***f***	**Average Total Holding Cost** €/yr.	**Average Total Ordering Cost** €/yr.	**Average Total Inventory Costs** €/yr.	**Average Fill Rate** %
0.5	0.2	4,169	10,036	14,205	68.86
0.5	0.3	4,221	9,085	13,306	70.39
0.5	0.4	4,494	7,605	12,099	74.20
0.5	0.5	4,610	7,174	11,784	75.93
0.5	0.6	4,899	6,858	11,757	78.74
0.5	0.7	5,094	6,655	11,749	79.72
0.5	0.8	5,841	6,387	12,228	81.25
0.6	0.2	5,196	8,139	13,335	74.01
0.6	0.3	5,276	7,480	12,756	76.01
0.6	0.4	5,628	6,211	11,839	78.99
0.6	0.5	6,034	5,564	11,598	81.98
0.6	0.6	6,166	5,415	11,581	83.48
0.6	0.7	6,508	5,300	11,808	84.71
0.6	0.8	6,679	5,403	12,082	85.37
0.7	0.2	6,799	6,599	13,398	79.49
0.7	0.3	7,127	5,490	12,617	82.48
0.7	0.4	7,469	4,873	12,342	84.41
0.7	0.5	8,000	4,256	12,256	87.10
0.7	0.6	8,277	4,113	12,390	88.18
0.7	0.7	8,496	4,056	12,552	88.96
0.7	0.8	8,845	4,147	12,992	89.27
0.8	0.2	7,710	6,073	13,783	81.78
0.8	0.3	8,083	4,946	13,029	84.42
0.8	0.4	8,550	4,232	12,782	87.13
0.8	0.5	9,041	3,808	12,849	88.88
0.8	0.6	9,390	3,632	13,022	90.06
0.8	0.7	9,668	3,557	13,225	91.02

Table E.4. Simulation results of scenario-4 (Continue)

0.8	0.8	10,027	3,644	13,671	91.12
0.9	0.2	9,202	4,936	14,138	84.95
0.9	0.3	9,592	4,153	13,745	87.08
0.9	0.4	10,289	3,511	13,800	89.44
0.9	0.5	10,803	3,231	14,034	91.10
0.9	0.6	11,187	3,060	14,247	91.94
0.9	0.7	11,637	3,053	14,690	92.52
0.9	0.8	11,907	3,143	15,050	92.62
1.0	0.2	10,358	4,381	14,739	86.81
1.0	0.3	10,812	3,688	14,500	89.13
1.0	0.4	11,414	3,202	14,616	90.70
1.0	0.5	12,013	2,918	14,931	92.08
1.0	0.6	12,524	2,747	15,271	93.22
1.0	0.7	12,869	2,725	15,594	93.62
1.0	0.8	13,191	2,808	15,999	93.70

APPENDIX F: SALES & DISTRIBUTION ORGANIZATION OF CRANE PFT

Table F. 1. Distributors & sales offices of Crane in EIMEA Region

***EIMEA* Countries**	**Product Segment**			
	Valves Group		**Pumps Group**	
	Responsible Operator's Name		**Responsible Operator's Name**	
Austria	SO	CPFT	SO	CPFT
	Crane PFT has its own local Sales-Office in this country. This office manages both Pumps & Valves Sales. Some of the South East European countries are also served from this office.			
Belgium	SO	CPFT	SO	CPFT
	Crane PFT has its own local Sales-Office in this country. This office manages both Pumps & Valves Sales. Also, there is a Distribution Center in this country.			
Czech Republic	None	-	DIST	Schmachtl CZ
	CRANE has only one distributors in the country. Schmachtl is responsible for Pump Group's sales & distribution operations in the country. Dusseldorf Office is servicing for Valve Group's sales & distribution operations.			
Denmark	DIST	Klinger	DIST	Granzow
	CRANE has two different distributors in the country. Klinger DK is responsible for Valve Group's sales & distribution operations and Granzow is for Pump Group.			
Finland	DIST	YTM Industrie	DIST	YTM Industrie
	CRANE's only distributor in the country is YTM-Industrie which is responsible for both Valve & Pump Group's sales & distribution operations.			
France	DIST	Eriks SARL	DIST	Mese SA
	CRANE has two distributors in the country. Eriks SARL is responsible for Valve Group's sales & distribution operations and Mese SA is for Pump Group.			
Great Britain	SO	CPFT	SO	CPFT
	Crane PFT has its own Central Sales-Office in this country. This office manages both Pumps & Valves Sales. This office also serves to some other countries. Also, there are three different Primary Locations in the country.			
North Ireland	SO	CPFT	SO	CPFT
	Crane PFT has its own local Sales-Office in this country. This office manages both Pumps & Valves Sales. Also, there is a Primary Location in the country.			
Hungary	DIST	Xomox	SO	CPFT
	Xomox is the distributor for Valve Group's sales & distribuion operations in the country. Xomox is also a contract manufacturer for Crane. For Pump Group, Crane PFT has its own local Sales Office in the country.			
Italy	SO	CPFT	SO	CPFT
	Crane PFT has its own local Sales-Office in this country. This office manages both Pumps & Valves Sales.			

Table F. 2. Distributors & sales offices of Crane in EIMEA Region (Continue)

Netherlands	DIST	Eriks B.V.	None	-
	CRANE has only one distributors in the country. Eriks B.V. is responsible for Valve Group's sales & distribution operations in the country. Dusseldorf Office is servicing for Pump Group's sales & distribution operations.			
Norway	DIST	GPA	DIST	GPA
	CRANE's only distributor in the country is GPA which is responsible for both Valve & Pump Group's sales & distribution operations.			
Poland	DIST	NPI	None	-
	CRANE has only one distributors in the country. NPI is responsible for Valve Group's sales & distribution operations in the country. Dusseldorf Office is servicing for Pump Group's sales & distribution operations.			
Portugal	DIST	Klinger & Saidi	DIST	Klinger & Saidi
	CRANE's only distributor in the country is Klinger&Saidi which is responsible for both Valve & Pump Group's sales & distribution operations.			
Russia & Province States	SO	CPFT	None	-
	Crane PFT has its own local Sales-Office in this country. This office manages only Valves Sales in this country and surrounding Natural-Gas-Markets.			
Slovakia	None	-	DIST	Schmachtl SK
	CRANE has only one distributors in the country. Schmachtl is responsible for Pump Group's sales & distribution operations in the country. Dusseldorf Office is servicing for Valve Group's sales & distribution operations.			
Spain	DIST	Saidi Central	DIST	Xomox
	CRANE has two different distributors in the country. Saidi Central is responsible for Valve Group's sales & distribution operations and Xomox is for Pump Group.			
Switzerland	DIST	Xomox	DIST	Adero AG
	CRANE has two different distributors in the country. Xomox is responsible for Valve Group's sales & distribution operations and Adero AG is for Pump Group.			
Sweden	DIST	Alnab	DIST	Lyme Kemiteknik
	CRANE has two different distributors in the country. Alnab is responsible for Valve Group's sales & distribution operations and Lyme Kemiteknik is for Pump Group.			
Turkey	DIST	Leon Teknik	DIST	Leon Teknik
	CRANE's only distributor in the country is Leon Teknik which is responsible for both Valve & Pump Group's sales & distribution operations.			
Germany	SO	CPFT	SO	CPFT
	EIMEA Region's Head Quarter is in Germany / Dusseldorf. Other *EIMEA* Countries which does not have sales organization or a distributor in a country are serviced from the Dusseldorf Head Quarter's sales teams. There are also three different *Primary Locations* located in the country.			

APPENDIX G: STATISTICS SPREADSHEET of ARENA MODELS

Table G. 1. Statistic Spreadsheet

NAME	TYPE	EXPRESSION	REPORT LABEL
Total Joint Ordering Cost	Output	Joint Ordering Cost/20	Total Joint Ordering Cost
Total Independent Ordering Cost	Output	Independent Ordering Cost/20	Total Independent Ordering Cost
Ordering Cost	Output	OVALUE(Total Joint Ordering Cost) + OVALUE(Total Independent Ordering Cost)	Ordering Cost
Holding Cost	Output	((C_Holding(1) * DAVG(I_OnHand_1)) + (C_Holding(2) * DAVG(I_OnHand_2)) + (C_Holding(3) * DAVG(I_OnHand_3)) + (C_Holding(4) * DAVG(I_OnHand_4)) + (C_Holding(5) * DAVG(I_OnHand_5)) + (C_Holding(6) * DAVG(I_OnHand_6)) + (C_Holding(7) * DAVG(I_OnHand_7)) + (C_Holding(8) * DAVG(I_OnHand_8)) + (C_Holding(9) * DAVG(I_OnHand_9)) + (C_Holding(10) * DAVG(I_OnHand_10)))	Holding Cost
Total Cost	Output	OVALUE(Cost_Holding_Total) +OVALUE(Cost_Ordering_Total)	Total Cost
Percent_Unwaited	Output	100* (NC(#_Unwaited)) / (NC(#_waited_lessthan_1wk) + NC(#_waited_lessthan_2wk) + NC(#_waited_lessthan_3wk) + NC(#_waited_lessthan_4wk) + NC(#_waited_lessthan_5wk) + NC(#_waited_lessthan_6wk) + NC(#_waited_lessthan_7wk) + NC(#_waited_lessthan_8wk) + NC(#_waited_lessthan_9wk) + NC(#_waited_lessthan_10wk) + NC(#_waited_lessthan_11wk) + NC(#_waited_lessthan_12wk))	Percent_Unwaited

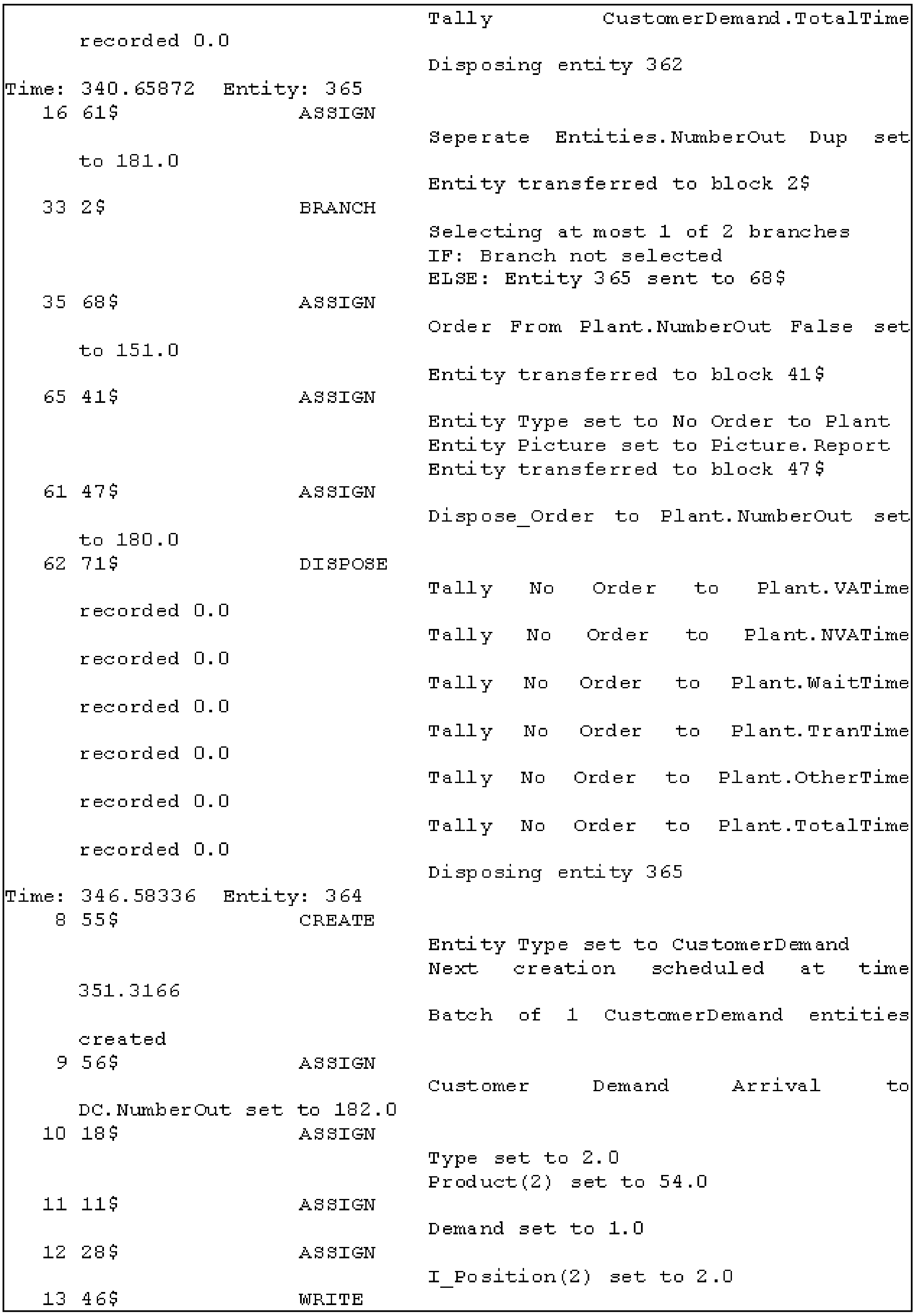
Table G. 1. Statistic Spreadsheet (Continue)

Total # of Joint Order Placed	Output	#Joint_Order_Placed/20	Total # of Joint Order Placed
Total# of Independent Order Placed	Output	#Independent_Order_Placed/20	Total# of Independent Order Placed
Total Ordering Number	Output	Order_Placed/20	Total Ordering Number
Incoming Demand for Product Type 1	Output	Product(1)/20	Incoming Demand for Product Type 1
Incoming Demand for Product Type 2	Output	Product(2)/20	Incoming Demand for Product Type 2
Incoming Demand for Product Type 3	Output	Product(3)/20	Incoming Demand for Product Type 3
Incoming Demand for Product Type 4	Output	Product(4)/20	Incoming Demand for Product Type 4
Incoming Demand for Product Type 5	Output	Product(5)/20	Incoming Demand for Product Type 5
Incoming Demand for Product Type 6	Output	Product(6)/20	Incoming Demand for Product Type 6
Incoming Demand for Product Type 7	Output	Product(7)/20	Incoming Demand for Product Type 7
Incoming Demand for Product Type 8	Output	Product(8)/20	Incoming Demand for Product Type 8
Incoming Demand for Product Type 9	Output	Product(9)/20	Incoming Demand for Product Type 9
Incoming Demand for Product Type 10	Output	Product(10)/20	Incoming Demand for Product Type 10
Cum Order Amount 1	Output	Cum_Order_Amount(1)/20	Cum Order Amount 1
Cum Order Amount 2	Output	Cum_Order_Amount(2)/20	Cum Order Amount 2
Cum Order Amount 3	Output	Cum_Order_Amount(3)/20	Cum Order Amount 3
Cum Order Amount 4	Output	Cum_Order_Amount(4)/20	Cum Order Amount 4
Cum Order Amount 5	Output	Cum_Order_Amount(5)/20	Cum Order Amount 5
Cum Order Amount 6	Output	Cum_Order_Amount(6)/20	Cum Order Amount 6
Cum Order Amount 7	Output	Cum_Order_Amount(7)/20	Cum Order Amount 7
Cum Order Amount 8	Output	Cum_Order_Amount(8)/20	Cum Order Amount 8
Cum Order Amount 9	Output	Cum_Order_Amount(9)/20	Cum Order Amount 9
Cum Order Amount 10	Output	Cum_Order_Amount(10)/20	Cum Order Amount 10

Table G. 1. Statistic Spreadsheet (Continue)

Inventory On Hand 1	Time-Persistent	I_OnHand(1)	Inventory On Hand 1
Inventory On Hand 2	Time-Persistent	I_OnHand(2)	Inventory On Hand 2
Inventory On Hand 3	Time-Persistent	I_OnHand(3)	Inventory On Hand 3
Inventory On Hand 4	Time-Persistent	I_OnHand(4)	Inventory On Hand 4
Inventory On Hand 5	Time-Persistent	I_OnHand(5)	Inventory On Hand 5
Inventory On Hand 6	Time-Persistent	I_OnHand(6)	Inventory On Hand 6
Inventory On Hand 7	Time-Persistent	I_OnHand(7)	Inventory On Hand 7
Inventory On Hand 8	Time-Persistent	I_OnHand(8)	Inventory On Hand 8
Inventory On Hand 9	Time-Persistent	I_OnHand(9)	Inventory On Hand 9
Inventory On Hand 10	Time-Persistent	I_OnHand(10)	Inventory On Hand 10
Inventory Position1	Time-Persistent	I_Position(1)	Inventory Position1
Inventory Position2	Time-Persistent	I_Position(2)	Inventory Position2
Inventory Position3	Time-Persistent	I_Position(3)	Inventory Position3
Inventory Position4	Time-Persistent	I_Position(4)	Inventory Position4
Inventory Position5	Time-Persistent	I_Position(5)	Inventory Position5
Inventory Position6	Time-Persistent	I_Position(6)	Inventory Position6
Inventory Position7	Time-Persistent	I_Position(7)	Inventory Position7
Inventory Position8	Time-Persistent	I_Position(8)	Inventory Position8
Inventory Position9	Time-Persistent	I_Position(9)	Inventory Position9
Inventory Position10	Time-Persistent	I_Position(10)	Inventory Position10

APPENDIX H: SAMPLE PAGE OF A TRACE OUTPUT



```
                                 Tally       CustomerDemand.TotalTime
      recorded 0.0
                                 Disposing entity 362

Time: 340.65872  Entity: 365
   16 61$             ASSIGN
                                 Seperate  Entities.NumberOut  Dup  set
      to 181.0
                                 Entity transferred to block 2$
   33 2$              BRANCH
                                 Selecting at most 1 of 2 branches
                                 IF: Branch not selected
                                 ELSE: Entity 365 sent to 68$
   35 68$             ASSIGN
                                 Order  From  Plant.NumberOut  False  set
      to 151.0
                                 Entity transferred to block 41$
   65 41$             ASSIGN
                                 Entity Type set to No Order to Plant
                                 Entity Picture set to Picture.Report
                                 Entity transferred to block 47$
   61 47$             ASSIGN
                                 Dispose_Order  to  Plant.NumberOut  set
      to 180.0
   62 71$             DISPOSE
                                 Tally   No   Order   to   Plant.VATime
      recorded 0.0
                                 Tally   No   Order   to   Plant.NVATime
      recorded 0.0
                                 Tally   No   Order   to   Plant.WaitTime
      recorded 0.0
                                 Tally   No   Order   to   Plant.TranTime
      recorded 0.0
                                 Tally   No   Order   to   Plant.OtherTime
      recorded 0.0
                                 Tally   No   Order   to   Plant.TotalTime
      recorded 0.0
                                 Disposing entity 365

Time: 346.58336  Entity: 364
    8 55$             CREATE
                                 Entity Type set to CustomerDemand
                                 Next   creation   scheduled   at   time
      351.3166
                                 Batch  of  1  CustomerDemand  entities
      created
    9 56$             ASSIGN
                                 Customer    Demand    Arrival    to
      DC.NumberOut set to 182.0
   10 18$             ASSIGN
                                 Type set to 2.0
                                 Product(2) set to 54.0
   11 11$             ASSIGN
                                 Demand set to 1.0
   12 28$             ASSIGN
                                 I_Position(2) set to 2.0
   13 46$             WRITE
```

Figure H.1. A sample trace output page

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