

AN APPLICATION OF INVENTORY POOLING / DISTRIBUTION MODELS IN
GLASS INDUSTRY

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ABSTRACT

AN APPLICATION OF INVENTORY POOLING / DISTRIBUTION MODELS IN GLASS INDUSTRY

Supply chain is a concept in which industrial engineering applications are very important in decision making processes by requiring systematic perspective, scientific approach and correct methodology. Supply chain concept covers a wide range of processes, from raw material procurement from suppliers to product delivery to customers. Inventory management, which is one of the subheadings of the supply chain, has both operational and financial consequences. While raw material inventory levels are kept high by the management in order to reduce the risk of being stockout, this reduces the net working capital due to the decrease in the inventory turnover rate of the companies. In this thesis, the benefits and costs arising from the coordination between the factories of a company operating in different locations in case of inventory pooling and selection of appropriate suppliers are evaluated. For this purpose, data set obtained from the company operating in the glass sector is used to provide input to the established mathematical model. The model achieves optimum results that are consistent with the supplier's capacity constraint by minimizing fixed costs, distribution costs, holding costs and purchasing costs while making supplier selection and warehouse location assignments. The nonlinear model, which emerges because the safety stock level calculation contains square root expression, is linearized by using piecewise linear function. The model results are compared under the scenarios of single warehouse use, the optimum number of warehouse use determined by the model and the use of all warehouses. On the other hand, since varying sales prices and profitability rates of suppliers create inequality while comparing scenarios, these factors are eliminated from the model and what-if study is performed. As a result, the decrease in inventory levels reduces the inventory holding cost and fixed cost, thus provides a cost advantage in spite of increasing in transportation cost by not using all available warehouses which are close to factories. This shows how important it is to ensure inter-location coordination in companies operating in multiple locations.

ÖZET

ENVANTER HAVUZLAMA / DAĞITIM MODELLERİNİN CAM SEKTÖRÜNDE BİR UYGULAMASI

Hammaddelerin tedarikçilerden satın alınmasından, ürünlerin müşterilere ulaştırılmasına kadar geçen her aşamada yer alan ve geniş bir kapsama sahip olan tedarik zinciri; karar süreçlerinde, sistemsel bakış açısı, bilimsel yaklaşım ve doğru metodoloji gerektirmesiyle endüstri mühendisliği uygulamalarının oldukça önem taşıdığı bir fonksiyon durumundadır. Tedarik zincirinin alt başlıklarından olan stok yönetimine dair yapılan çalışmalar hem operasyonel hem de finansal sonuçlar ortaya çıkartmaktadır. Hammadde envanter seviyeleri stoksuzluk riskine karşın yönetimler tarafından yüksek miktarda tutulmaya çalışılırken şirketlerin stok devir hızının düşmesi sebebiyle net işletme sermayesini azaltmaktadır. Bu tezde, bir şirketin farklı lokasyonlarda faaliyet gösteren fabrikaları arasında koordinasyon sağlanması ile envanter havuzlaması ve uygun tedarikçi seçimi yapılması durumlarında ortaya çıkacak fayda ve maliyet değerlendirilmektedir. Bu amaçla, cam sektöründe faaliyet gösteren bir şirketten alınan veriler matematiksel modele girdi sağlamak amacıyla kullanılmaktadır. Model, tedarikçi seçimi ve depo yeri seçimi atamalarını gerçekleştirirken sabit maliyetler ile dağıtım, elde bulundurma ve satın alma maliyetlerini minimize ederek tedarikçi kapasite kısıdına uygun olan optimum sonucu ortaya koymaktadır. Emniyet stok seviyesi hesaplamasının kareköklü ifade içermesi sebebiyle ortaya çıkan doğrusal olmayan model, parçalı doğrusal fonksiyon kullanılarak doğrusal hale getirilmektedir. Model sonuçları tek depo kullanımı, modelin belirleyeceği optimum sayıda depo kullanımı ve tüm depoların kullanımı senaryoları altında karşılaştırılmaktadır. Diğer yandan, tedarikçiye bağlı olarak değişen satış fiyatları ve karlılık oranları senaryolar karşılaştırılırken eşitsizlik yarattığı için model bu faktörlerden arındırılarak senaryo analizi yapılmaktadır. Sonuç olarak, optimum sayıda depo kullanılarak ortak stok tutulması durumunda taşıma maliyetleri artmasına karşın stok seviyelerinin dolayısıyla elde bulundurma maliyetlerinin ve sabit maliyetlerin düşmesi nedeniyle maliyet avantajı elde edildiği görülmektedir. Bu durum, çok lokasyonda üretim yapan şirketlerde fabrikalar arası koordinasyonun sağlanmasının ne kadar önemli olduğu sonucunu ortaya koymaktadır.

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LIST OF SYMBOLS

A	Set of candidate suppliers, $a=1,2,3$
a_i	Fixed cost of warehouse I
C_{ik}^r	Linear coefficient of segment R
d_{ij}	Shipment cost from warehouse I to factory J
F_{ik}^r	Linear coefficient of segment R
h_i	Holding cost per unit at warehouse I
I	Set of candidate warehouses, $i=1, 2, 3, \dots, 10$
J	Set of factories, $j=1, 2, 3, \dots, 10$
K	Set of raw materials, $k=1, 2, 3$
L_{ik}^r	Upper bound of segment R in the piecewise linearization of the safety stock
l_k	Lead time of raw material K
O_a	Supplier decision to use
P_{ak}	Capacity of supplier A for raw material K
R	Set of piecewise linearization segments
S_{aik}	Purchasing and shipment cost from supplier A to warehouse I for raw material K
U_{ik}^r	Piecewise segmentation variable
W_i	Decision to open a warehouse
X_{aik}	Quantity of raw material K shipped from supplier A to warehouse I
Y_{ijk}	Raw material allocation
Z_{α_k}	Standard normal deviate of desired service level for raw material K
α_k	Desired service level for raw material K
δ_{ijk}^r	Piecewise linear function variable
μ_{jk}	Mean demand of raw material K at factory J
σ_{jk}^2	Variance of demand of raw material K at factory J

1. INTRODUCTION

Companies focused on production in the 1900s, sales in the 1930s, marketing in the 1950s, logistics in the 1990s and supply chain management in the 2000s. In today's businesses, adapting to the developing and changing technology in parallel with the increasing customer expectations and achieving excellence in the entire value chain of the enterprise emerge as the most important differentiation points. Considering that it can be said that an enterprise consists of interconnected links. For example, supply chain which responds to the demands of sales departments, reduces the pressure on the production departments, and in short, ensures the unity of the whole chain plays an important role for a company to succeed. The supply chain takes on a task of coordination between departments and brings the simplicity and scientific approach to the business processes. Although each department in the enterprise tries to fulfill its duties perfectly, supply chain combines these departments around the same purpose. To concretize with examples;

- While production departments try to keep the raw materials to be used in production at the maximum level in order not to be stockout, supply chain focuses on the order points, optimum stock levels, determination of logistic costs, and inventory holding costs,
- Applying the right demand and production planning methods in order to increase the stock turnover rate,
- To take the necessary precautions with a proactive approach to ensure the flexible, fast and perfect structure of the logistics operations,
- Determination of appropriate purchasing methods by calculating the total cost of ownership in order to make the most profitable agreements,
- The use of intelligent integrated automation systems that come with Industry 4.0 to prevent employee-related errors and design systems that will avoid unnecessary waste to reach perfection in processes,
- Balance between customer satisfaction and total supply chain costs,
- Significant interpretation of big data in follow-up processes with developing information technologies.

With the impact of globalization, supply chains are becoming more complex with increasing speed, and competition between companies is concentrated on the supply chain. Since the effects of the supply chain can be seen in different areas such as sales, quality and production, companies define the supply chain as the focal point, which reduces costs and increases profitability (Chandak *et al.*, 2014). However, a disruption between the links of the chain due to the complex structure of the supply chain affects the entire system. For this reason, it is vital to manage the potential problems and their impacts at every stage of the supply chain by managing effective risk management (Bugert and Lasch, 2018).

When manufacturing companies are considered, it is difficult to manage whole process from supplying raw material to stocking product and reaching customer at the desired quality, at the desired place, despite the production lines equipped with advanced technologies. Just-in-Time Production System eliminates wastes and aims to produce enough quality to meet demand. When there is an error in the system, because there is not much stock in hand, it is necessary to intervene immediately. This implies that the control mechanism must be flawless and the system must be in continuous operation. Competitiveness, quality, efficiency and excellence in production have been increased thanks to Just-in-Time Production System. Many books have been written about the Just-in-Time Production System and have been adapted by many companies outside the automotive sector. Just in Time Production system has 7 basic waste categories which are time spent in a queue, overproduction, non-value-added activities, transportation, unnecessary movement, inventory, defects. By eliminating these wastes, effective utilization of working capital is ensured, cash flow is accelerated, inventories are reduced, and all non-value-added processes within the factory are eliminated with continuous improvement.

The difficulties in supply chain and inventory management have brought new management styles. Raw material inventories are now being managed with the Supplier Managed Inventory business model. In this model, suppliers and manufacturers integrate their information systems and the manufacturer leaves the raw material control completely to the supplier. Thus, the manufacturer aims to reduce the risk of being stockout and provide cost advantage. Suppliers, on the other hand, feel the bullwhip effect to a minimum, avoiding the cost of fluctuations in demand and information transfer. Naturally, it is a prerequisite that

the performance of suppliers should be closely monitored and their performance should reach high standards. In addition, suppliers must be reliable and make deliveries on time.

Inventory reduction is a concept that has recently gained popularity in the business. Reduced number of stock keeping unit, improved forecasting accuracy, improved inventory management software, coordination among all departments are among the most important tools on this path. Inventory pooling refers to the ability of a warehouse to serve multiple demand points that have their own demand distribution. The current literature evaluates that the inventory pooling decreases the cost of companies due to economies of scale. In addition, inventory pooling has important contributions to operational inventory management. For example, when there are multiple stochastic demands, combining multiple demands reduces total uncertainty (Eppen, 1979). In this way, the company can manage supply and demand balance more easily and effectively, which allows increased profitability. Inventory pooling is usually analyzed in two different contexts: the location pooling and the product pooling.

In this thesis, location pooling is considered. With the increase of information systems and e-commerce, the location pooling increased its popularity. Location pooling can be considered as combining requests from separate geographical regions. Additionally, inventory pooling allows a company to benefit from economies of scale thanks to consolidation of purchase orders. Inventory risk pooling is a phenomenon where changes in the demand for raw materials have decreased, with the collection of demands from more than one product. When used correctly, a company can avoid stockout and ensure lower stock levels without risk.

According to Das and Tyagi (1997), in sectors that require high investment expenditure, location pooling makes the supply chain a huge potential for savings and productivity gains. After all uncertainties surrounding demand are examined, these uncertainties are expressed in mathematical models. Thus, mathematical models which are compatible with real life conditions are introduced. From an academic point of view, such models are quite complex due to their integer and nonlinear structure. In order to obtain a reasonable solution and reach low error rates, linearization techniques should be applied on the models.

This thesis reveals the financial benefits that can be obtained thanks to inventory pooling, supplier selection and distribution by using a mathematical model based on a real life example. According to the results, 29% cost advantage can be achieved thanks to inventory pooling if the right supplier and distribution network are selected and all enterprises are allowed to communicate with each other.

Chapter 2 includes a literature survey on the concepts of inventory pooling, risk pooling and supply chain design system. Additionally, a brief review is made about the reasons to overstock. Chapter 3 presents a mathematical model for inventory pooling. The stages of the mathematical model in this thesis are as follows:

- A nonlinear model including the change in transportation costs, inventory holding costs, fixed costs and purchasing costs when a company holds raw material inventories in shared warehouses,
- Linearization of the obtained nonlinear model with a piecewise linear function,
- Selection of suppliers by adding capacity and sales price of candidate suppliers to the model,
- After all these have been put forward, scenario-based analysis will be carried out by comparing the scenarios giving the minimum total cost.

Chapter 4 provides information about the parameters used in the model. After parameter selection, the solutions of the scenarios and their comparison with each other are discussed. In Chapter 5, all results obtained during the thesis are summarized briefly, and the factors that are not discussed in the thesis but which will play an important role in the progress of the study are mentioned.

2. LITERATURE SURVEY

In this chapter, the importance of inventories for manufacturing companies and the factors that cause companies to maintain high stock levels are briefly examined in the light of the literature. There are many studies related to inventory in the literature since the warehouse related costs are one of the most important cost items of the supply chain and warehouse decisions are the result of many factors affecting each other. The studies reviewed in the literature are summarized according to the concepts such as inventory pooling, risk pooling and supply chain design system. Before the inventory pooling examined in the thesis, why the stock concept is important in the production sector is explained.

2.1. Definition of Stock

Hugos (2018) defines that all physical assets, raw materials, semi-finished products and finished goods stocked and used in an enterprise are evaluated in the stock concept. Everything stored within the entity has a tangible value and inventories are tracked by the monetary value and quantity of these assets. The stock level and stock turnover rate of an enterprise is also used as an indicator of how effectively an enterprise operates.

In Jacobs and Chase (2014), when assets are not used or the products produced are not sold or kept idle, they are called stocks. When the inventory is mentioned, the monetary value of all these assets in an enterprise comes to mind. In other words, inventory refers to the situation in which an entity invests in inventory and ties its money to inventory instead of keeping money in cash. Therefore, it is important to keep the inventory value at an optimum level.

Muller (2013) indicates it is more accurate to evaluate the resources used in production, such as equipment and labor, as capacity rather than to examine them within the stock concept because they directly affect the production capacity during production. Therefore, according to another definition, inventories are resources that are idle in the enterprise and have economic value.

2.2. Reasons for Stock Availability in Business

As long as a manufacturing company maintains its existence, it has to have raw materials, semi-finished products, and stocks in order to produce. The important thing is to manage the amount of stocks to be held accurately and efficiently. In the just in time production system, although the stocks are almost zero, many companies that do not have this production model have more stocks than necessary to prevent production disruption. Inventories in production companies are one of the most important and closely monitored cost items. Supply chain departments always try to reduce inventory, warehousing, depreciation and logistics costs. The most critical point is to reduce these costs with scientific approaches so as not to hinder customer satisfaction and stop the production. The reasons for keeping excessive stock can be explained under the headings of time, uncertainty and economic.

2.2.1. Time Factor

Berry *et al.* (1994) suggest that the time required for procurement of raw materials, conversion of raw materials into semi-finished and finished products, and delivery of products to customers should be examined under time factor. When looking at the details of the time factor, ordering the raw material, supplying the raw material, making the production planning and scheduling, performing the quality control procedures of the raw materials received, carrying out the production without quality problem, storing the product, making the product ready for shipment and delivering the product to the customer are seen. Considering that it can be said that although it is difficult because of many factors, customers' demands should always be met as soon as possible. For this reason, it is very important to minimize all these periods.

2.2.2. Uncertainty Factor

Muller (2013) states in all processes mentioned under the time factor, there are times that vary depending on people and machines. There is no doubt that these periods have their own standard deviations. As the standard deviation increases, the uncertainty factor increases

in direct proportion and the enterprises increase the amount of inventory held due to the uncertainty factor in order to use machinery, equipment and human resources effectively.

2.2.3. Economic Factor

Chopra and Sophi (2004) remark that companies are trying to increase their purchasing volume by consolidating their demands. The reason for this is to reduce the number of orders to be placed during the year and to save on order costs. On the other hand, economies of scale are utilized by increasing the amount purchased. With the increasing bargaining power against suppliers, raw material cost per unit can be reduced. Considering the fact that the raw material purchase costs of the companies operating in the glass sector are among the most important cost items taken into consideration even in the location selection, the smallest savings to be provided here are considered to have a significant effect for the financials. The point that should be noted here is that the cost reduction caused by the economies of scale should be higher than the inventory holding cost. Otherwise, there will be a negative impact on financials due to hidden costs.

2.3. Related Literature

The related literature under the headings of risk pooling, inventory pooling and supply chain design is summarized below. Since inventory management is one of the subheadings of the supply chain, the most important developments in the supply chain in the recent period are also presented under this heading.

Risks and uncertainties of companies are increasing due to the more complex structure of the supply chain in a globalized world. Therefore, it has become more important for companies to closely monitor the functioning and performance of the supply chain. Some developments increase the likelihood of existing risks. According to Norrman and Jansson (2004), current trends that increase the fragility of the supply chain are listed below:

- The emergence of Industry 4.0, the internet of things and big data concepts,
- Globalization of supply chains,

- Focus on more effective and efficient production of companies working with smaller margins,
- Technological advances in computer and communication,
- Increased centralization in company management,
- Increased use of outsourcing in production and R & D due to cost advantage,
- Reduction in the number of suppliers due to economic conditions,
- Increased interdependence of firms due to increased information flow,
- Increasing customer expectations and decreasing tolerances in markets with increasing number of producers,
- Variations in demand,
- Shortening of product life and more frequent introduction of new products,
- The spread of quality control practices and increased visibility,
- Shortening product life,
- Increased integration and interconnected processes between firms,
- Increased necessity of working within shorter time windows and delivery times,
- Increasing trade wars between countries.

Safety stock levels and average inventory levels decrease due to the centralized storage of products commonly used in multiple enterprises in regionalized warehouses. By managing this process in a controlled manner, no change in customer service levels occurs. In other words, a more homogeneous and plannable process is achieved by combining multiple demands that are not homogeneously distributed among the factories at a single point of demand.

Inventory management is one of the most challenging businesses. It is difficult to make modeling to estimate the amount of raw material stock to be held because there are many factors affecting the estimation. Customer demand for the product varies according to the complexity of the market and the preferences of end users. Risk pooling works best when used in common raw materials with different demand distributions. This is due to the high likelihood of balancing demand fluctuations of products with a completely different demand distribution due to keeping common stocks. This means that the risk pooling concept can be used in parts commonly used in every product group, such as accessories and fasteners. The

following approach should be followed when developing the risk pooling concept (Yang and Schrage, 2009):

- It is necessary to identify common parts used in multiple products.
- It is important that the actual and planned demand levels for common parts are constantly monitored.
- It is important that safety stock levels are set to exceed the estimated demand levels by a small amount, as actual demand levels may always contain margins of error.

The risk pooling has advantage depending on the standard deviation or coefficient of variation between different markets. The higher standard deviation / coefficient of variance ratio means the higher potential (Smith and Witter, 2004).

Different approaches and many combinations of these have been proposed in the literature to determine inventory levels and solve facility location selection problems. In many inventory problems, studies have been made to minimize all stock levels according to a certain level of customer service, but facility location selections and selection of customers' facilities allocations based on product groups have been used as pre-determined inputs in these problems. On the other hand, in general facility location selection problems included storage and transportation costs, while inventories were simplified and included in the models. Because of the complexity of modeling the problems, these problems were examined from different perspectives.

2.3.1. Inventory Pooling Models

Eppen (1979) is considered as the starting point for the introduction of the concept of inventory pooling into operational research literature. The author set out to explain the logic of wholesalers selling steel products in real life using distribution centers and explain inventory pooling in accordance with statistical foundations of economies of scale. A product with multiple demand points and demand points with linear penalties and inventory holding costs should be taken into account while opportunities for distribution centers are utilized. In the study, after carried out many assumptions, it is proved with mathematical

formulas that the benefits of inventory pool are maximized when variance between demand points increases and covariance between demand points decreases. When the covariance is zero, it is shown that the number of consolidated demand points is directly proportional to the square root. Unlike the assumption that demands are normally distributed, Federgruen and Zipkin (1984) also contribute to the study by taking into account exponential and gamma distributions. They approach the pooling problem from a much simpler perspective with their systematic approach.

Çömez Dolgan and Tanyeri (2015) suggest inventory pooling for different demand sources reduces optimum safety stock and cost. Variability between the demands of different demand sources has been addressed using firm-level data for 270 same sectoral mergers in the United States over 28 years. The inventory costs of the bidders and target firms started to decrease after the mergers. Through this study, real-life data on inventory economies in mergers and acquisitions have been handled and it is seen that firms with negative correlation between demands produce the most effective result.

Makaci *et al.* (2017) carried out qualitative studies in order to reveal the advantages, difficulties and risks of the pooled warehouse management which is becoming widespread today. Through interviews with companies, firms' perspectives on supplier-managed inventory, uncertainties in the pooled warehouse, and new key performance indicators were revealed.

In order to repair different fleets of machines in different locations, Sahba and Balcıoğlu (2011) are trying to find the effective one by comparing repair shop pooling and repair shops on-site. The difference of this model from other repair place models in the literature is that transportation times and costs are included as parameters in the model. To ensure this, the system is configured with a closed queuing network and a steady state distribution is provided. Repair shop pooling seems to produce beneficial results when transportation costs are reasonable. However, spare parts are not pooled in this model and each machine is considered to have its own spare part.

Das and Tyagi (1997) analyze the inventory pooling model from a broad perspective. By establishing an optimization model, the results of individual and combined effects of cost items on centralization are presented. The model is mainly used to decide the degree to which centralization is made by using the tradeoff between transportation and warehouse costs. Behaviors of the system were analyzed under different scenarios and the best degrees of centralization were revealed.

The cost allocation problem faced by multiple companies for pooling spare parts has been dealt with the game theory approach. One of the main focal point of Wong *et al.* (2007) is to achieve an acceptable cost allocation policy so that all participating companies can be encouraged to cooperate. This situation has been examined under scenarios where companies exhibit different behaviors such as cooperative and competitive.

A k-out-of-N system model with different standby modes is considered by Sleptchenko and Van Der Heijden (2016). Each component consists of several types of parts. In the event of a malfunction, the defected component is replaced by a spare part and repaired as soon as possible. Optimizing the component redundancy level with various spare parts stocks, the model was applied to a case study in Qatar to increase availability-cost ratio. According to the article, high redundancy levels appear to be beneficial when the parts are inexpensive and replacement times are high in case of failure. Otherwise, it is observed that low inventory levels are maintained.

Bittencourt *et al.* (2014) presents the problem of inventory location with two different mathematical models considering the decrease in safety stock levels by the use of centralized warehouses. In these formulations that address the problem from a two-stage and three-stage perspective, the three-stage perspective addresses the supply chain with a triple index flow variable that connects suppliers to customers. Models become linear and solved quickly thanks to segmented and quadratic linearization methods.

Guo *et al.* (2018) examine the location inventory problem in a closed supply chain network. Since the system is assumed to be closed, there are no external factors and products with quality problem return at the same time. With the mixed integer nonlinear program,

logistic flows between nodes and inventory quantities are determined. There is no routing part in the model. Since the problem involves the return of the products with quality problems in the closed system, it has the feature of being a rare subject in the literature. This problem also contributes to the sustainability of the supply chain. The model is solved with the heuristic algorithm.

Vidyarthi *et al.* (2007) evaluate the model of safety stock levels, which is the result of a joint product-inventory-distribution system. The inventory pooling and location selection problems minimize the transportation costs of the shipments from the warehouses to the distribution centers, the holding costs of the safety stock levels to be kept in the distribution centers and the total of fixed costs, and assigns products to the distribution centers. The model includes a nonlinear mixed integer programming model and the model is linearized using segmented-linear functions. Main problem is solved by optimizing the solution of the sub-problems thanks to Lagrangean relaxation due to the complexity of the model. The system provides a lower safety stock level than each vendor maintains its own safety stock level, thus reducing the cost of availability. As demand uncertainty increases, there is a considerable potential for savings in total cost. Supply chain planning decision can be evaluated at strategic, tactical and operational levels, and the results of the model are effective for all these layers.

Jorge (2008) recommends that inventory pooling should be examined under four separate headings which are the transfer of stocks to central locations (consolidation), order splitting, transshipment and standardization of product groups. The inventory cost of each warehouse should be considered as a function of the demand parameters and the correlation coefficient between the demands.

In the multi-stage production distribution system, Gebennini *et al.* (2009) realized an end-to-end modeling of the supply chain, unlike the classical approach that focuses only on inventory levels and costs of inventory pooling. Starting from the realization of product-factory assignments while creating the logistics networks of companies, it has provided a purpose function that includes and minimizes all costs such as production, storage, transportation. Based on the necessity of the whole supply chain to work in coordination, the

model provides strategic, tactical and operational decisions and the entire supply chain is optimized. This article provides an original model for the dynamic factory allocation problem while controlling rates such as customer service levels, production, line utilization, stock-turnover. The model was experimentally tested with an industrial application and optimization decisions were examined.

2.3.2. Transshipment Models

In transshipment models, inventories are kept in their existing warehouses rather than consolidated in a single warehouse and communication between enterprises is allowed. With this communication, each enterprise can see the stocks of other enterprises and it is allowed to withdraw products from the warehouse where the stock is in case of need. Although it is not allowed to use transshipment among the enterprises in the thesis, the literature on transshipment is also examined since it is presented as an alternative solution to pooling problems in the literature.

Needham and Evers (1998) report that the decisions of the logistics managers when making a transshipment decision were tested in a simulation environment using cost factors that affect the total cost. As a result of the simulation, it is seen that the cost item that affects the decision of transshipment is the cost of being stockout. The stockout cost also varies according to the penalty fees that depend on the sector in which the company operates. However, since transshipment has a significant impact on inventory availability, it is stated that managers generally prefer to do so.

In Nasr *et al.* (2012), a two-stage system in which a product from a source is distributed to plants in two different locations. The random distribution of the failures in the source and the stochastic failure times increase the possibility of the plants to be stockout. The model permits transshipment to allow for transshipment of stocks and even safety stocks between two plants with different holding costs. The nonlinear model determines the safety stock levels and transshipment quantities in both facilities and tries to minimize the total cost of ownership for both facilities. Delivery times are assumed to be zero during transshipment. The amount of raw material to be kept at the source where the products are supplied is not

included in the model as a decision variable and is assumed as a parameter. According to the results of the model, total cost shows a convex function characteristic. The function varies depending on the safety stock levels in both facilities and the quantity of product transshipped.

Wang and Yue (2015) offer an inventory pooling model for spare parts of critical systems serving multiple companies. It tries to show whether the companies operating in the same region can make savings by sharing storage cost, product replacement cost and downtime costs with each other or not. The model seeks the most appropriate solution under a predetermined cost structure based on the number of companies participating in the coalition and the threshold that determines whether an emergency order will be issued. The simulation test used to verify the developed model. Sensitivity analyses are also conducted to show effect of variables on the cost structure.

Evers (2001) studies the emergency transshipments of the product as an approach to solving stockout problems in multiple stock keeping facilities. If even the best forecasting methods are applied, variance in demand forecasts result from variance. In such cases, two different heuristic models have been developed in order to support the decisions of managers in managing companies. These models support the decision to make / not make transshipment while there is more stock in one enterprise and missing stock in the other. Taking into account the total related costs like transportation cost, order cost and stocking cost and holding cost are compared. Accordingly, the threshold level and the minimum order quantity for the transshipment decision are determined.

The high utilization of aircraft is one of the most important performance criteria for airline companies. For this reason, spare parts availability and spare parts replacement should be done in the case of failing aircraft. Since spare parts costs are high and aircraft breakdown times are randomly distributed, it is difficult to find the optimum number of spare parts to be stocked. The number of failures in a certain period showing the Poisson distribution is modeled in the simulation and costs of spare parts such as purchase, stocking and destocking are considered. Kilpi and Vepsalainen (2004) demonstrates that different airline companies' joint management of spare parts will create significant advantages. In this

way, companies will be able to take advantage of economies of scale and reduce their purchasing costs and manage their inventories with a lower number of stocks than keeping them separately.

2.3.3. Consumer Behavior Models

Anupindi and Bassok (1999) perform one of the first studies to analyze the interaction between a manufacturer and customers by adding consumer behavior to the inventory pooling model. According to this study, if end-users tend to do research instead of always buying the same product from the same market, that is if a product is present in more than one market, pooling can be harmful to retailer. Alfaro and Corbett (2003), on the other hand, suggest that, if inventory pooling is not performed optimally, companies will lose customers and hidden costs will occur. Accordingly, increasing the use of standardized products by stock keeping unit rationalization is a step that must be taken into account. It is emphasized that customer satisfaction should always be a priority target for retailers.

Chevalier and Goolsbee (2009) reveal that strategic consumer behavior should be taken into consideration in inventory pooling problems. If a company has a strategic partnership with its customers, customers who exhibit this behavior will be more moderate in their development projects. Because, in this way, customers' buying behaviors are aware that the cost advantages to be obtained will be reflected to them as product prices fall in the future.

The risk pooling problem was analyzed in a two-location inventory system of Li and Zhang (2012). In this study, the impact of inventory pooling on customer service levels is emphasized since it is known that inventory pooling is very effective in managing standard deviation due to uncertainty in demand. The effect of inventory pooling on type I and type II customer service levels is revealed. Study shows that inventory pooling always improves the no-stockout probability.

The contributions of this thesis to the literature can be summarized as follows:

- An original numerical study was carried out in the glass sector.
- The model is based on realistic data and a viable result is presented.
- Nonlinear model is converted to linear model by linear approximation.
- By combining supplier selection, inventory pooling and distribution network, the problem was evaluated from a broad perspective.

3. MODEL

In this chapter, after presenting briefly one of Turkey's most important glass manufacturing companies, the derivation of the resulting model used in the thesis is explained gradually. The objective function of nonlinear model consists of fixed cost, transportation cost and inventory holding cost which is linearized by applying piecewise linear function. Subsequently, the supplier selection decision is also included to make the model more complex but suitable for real-life. Based on the model introduced, the actual raw material consumption of the company in 2018 has been compared under various scenarios. In this way, the warehouse management decisions to be taken are based on mathematical basis.

3.1. Description of The Real Inventory / Distribution System

One of Turkey's most important holding companies has 3 different companies. These companies produce flat glass, glassware and glass packaging. All companies are managed separately and considered as independent businesses. Each factory also holds its own raw material inventory and communications between them is really low. Therefore, safety stock level increases and net working capital is used for inventories. Factory locations and candidate suppliers showed on Turkey map are as follows:

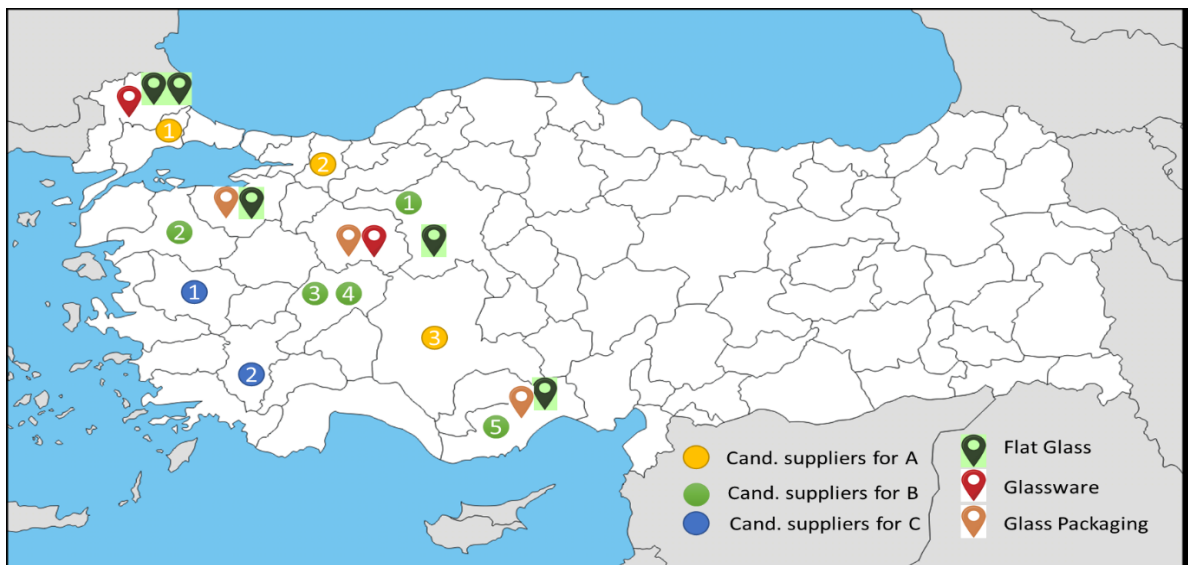


Figure 3.1. Factory locations and candidate suppliers

Glass sector is working with continuous production logic and the most important condition for production is not to be stockout. In this thesis, 10 factories and their 3 most commonly used raw materials are evaluated to keep their stocks together. It is aimed to examine the advantages and disadvantages of the factories if they keep common stock with each other. In the case of inventory pooling, the results will be evaluated by using cost-benefit analysis between decreasing inventory holding cost arising from the fact that demand is not distributed uniformly throughout the year and increasing raw material transportation costs to factories. Thesis is based on data-driven provided by company.

The assumptions used in the model are as follows:

- A new warehouse will not be opened and existing factory warehouses will be used.
- All raw material transportations will be carried out by a fully loaded vehicle.
- For the costs that will occur in the transportation between the warehouses, the matrix created depending on the distances is used.
- The demands of each factory are independent, uncorrelated and follow normal distribution.
- Demand is variable and lead time is constant while calculating safety stock. Lead time has been accepted as fixed since there are suppliers with sufficient capacity and they are in 2 days distance from each factory.
- The average demand and the standard deviations of demands using data set were calculated and used in the model.
- Since there are continuous production, the stockout causes line stops and high costs. It is aimed to meet the annual demand of each factory. Therefore, the service level is used to guarantee certain probability of stockout situation.
- Each warehouse can serve one or more raw materials of each factory.

The complexity of the problem comes from the complexity of the supplier-warehouse-factory network. There are 3 common raw materials. There are 10 candidate suppliers in total. There are 10 factories, so there are 10 candidate warehouses. Visualization of network can be seen below. Each combination of suppliers, warehouses and factories are allowed in this model to achieve the most cost-effective scenario.

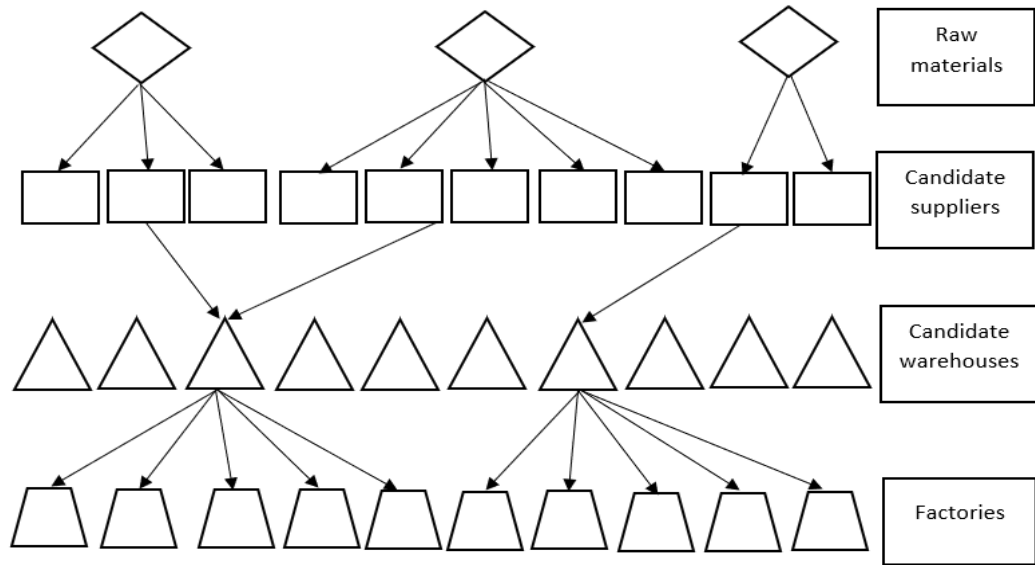


Figure 3.2. Network structure used in the model

3.2. Model Description

3.2.1. Inventory Pooling - Nonlinear Model

The problem consists of two stages. First aim is to find best locations for inventory pooling and second aim is to select suppliers and distribution networks giving the lowest total cost of ownership. Therefore, optimization model is created step by step. Firstly, each factory has its own warehouse. The model assumes that the warehouses owned by the factories will be used. On the other hand, three main raw materials, which are common to all glass production factories, are considered for the factories. It is the model of assigning each raw material of each factory to appropriate warehouses.

Assume that there are two factories and their own demands with $D_1: (\mu_1, \sigma_1^2)$ and $D_2: (\mu_2, \sigma_2^2)$. If we consolidate demands of two factories in a factory, new average and variance of D_{new} can be found below.

$$\rho = \text{correlation } (D_1; D_2) \text{ where } -1 < \rho < 1$$

$$\mu_{new} = \mu_1 + \mu_2$$

$$\sigma_{new}^2 = \sigma_1^2 + \sigma_2^2 + 2\rho\sigma_1\sigma_2$$

It is clear that $\sigma_{new} \leq \sigma_1 + \sigma_2$. Depending on ρ , analysis can be expanded:

- If D_1 and D_2 are positively correlated, $\rho > 0$, standard deviation of consolidated demand is less than sum of standard deviation of individual demands.
- If D_1 and D_2 are independent or negatively correlated, standard deviation of consolidated demand is much less.

That is, it can be said that location pooling decreases demand variability and expected inventory. On the other side, pooling increases distance between consolidated warehouse and factories. This create trade-off between inventory costs and transportation costs. The only way inventory pooling does not reduce safety stock is that all demands should be perfectly positively correlated ($\rho = 1$). In that case, pooled variance is equal to summation of each individual variances. Therefore, more negative correlation between demands means more variability reduction. In the thesis, since there is no correlation between the demands in the data sets provided by the company, it is accepted that $\rho = 0$.

In the current situation, each factory is working separately. There is no communication between factories regarding raw material stocks so safety stock level decisions are taken independently and this increases net working capital. Model assumes a centralized system and recalculates the safety stock levels in case factories communicate with each other and collect their common stocks by consolidating their demands in the same place. Inventory holding cost also varies depending on the location of the factories. To sum up, there are three main statements in the objective function. The aim is to minimize fixed costs, transportation costs between factories - warehouses and safety stock holding costs resulting from consolidation of inventories.

3.2.1.1. Indices

I : Set of candidate warehouses, $i=1, 2, 3, \dots, 10$

J : Set of factories, $j=1, 2, 3, \dots, 10$

K : Set of raw materials, $k=1, 2, 3, \dots, 10$

3.2.1.2. Parameters

a_i : Fixed cost of warehouse I

d_{ij} : Shipment cost from warehouse I to factory J

h_i : Holding cost per unit at warehouse I

μ_{jk} : Mean demand of raw material K at factory J

σ_{jk}^2 : Variance of demand of raw material K at factory J

α_k : Desired service level for raw material K

Z_{α_k} : Standart normal deviate of desired service level for raw material K

l_k : Lead time of raw material K

3.2.1.3. Decision Variables

$W_i = 1$ if a warehouse is opened at warehouse I ; 0 otherwise

$Y_{ijk} = 1$ if product K of factory J is assigned to warehouse I ; 0 otherwise

3.2.1.4. Model

$$\text{Minimize } \sum_{i \in I} a_i W_i + \sum_{i \in I} \sum_{j \in J} \sum_{k \in K} \mu_{jk} d_{ij} Y_{ijk} + \sum_{k \in K} \sum_{i \in I} h_i Z_{\alpha_k} \sqrt{l_k \sum_{j \in J} \sigma_{jk}^2 Y_{ijk}} \quad (1)$$

The derivation of $\sum_{k \in K} \sum_{i \in I} h_i Z_{\alpha_k} \sqrt{l_k \sum_{j \in J} \sigma_{jk}^2 Y_{ijk}}$ can be found below:

If we assume S_{ik} = Quantity ordered for not being stockout, we are able to define safety stock as expected surplus quantity remaining after the demand has been met, which is $E[(S_{ik} - \sum_{j \in J} D_{jk} Y_{ijk})^+]$. Therefore,

$$\Pr\{S_{ik} > \sum_{j \in J} D_{jk} Y_{ijk}\} = \alpha$$

$$\frac{S_{ik} - \sum_{j \in J} \mu_{jk} Y_{ijk}}{\sqrt{l_k \sum_{j \in J} \sigma_{jk}^2 Y_{ijk}}} = \Phi^{-1}(\alpha)$$

$$\begin{aligned}
S_{ik} &= \sum \mu_{jk} Y_{ijk} + Z_{\alpha_k} \sqrt{l_k \sum_{j \in J} \sigma_{jk}^2 Y_{ijk}} \\
\sum_{k \in K} \sum_{i \in I} h_i E \left[(S_{ik} - \sum_{j \in J} D_{jk} Y_{ijk})^+ \right] \\
&= \sum_{k \in K} \sum_{i \in I} h_i \left(\sum \mu_{jk} Y_{ijk} + Z_{\alpha_k} \sqrt{l_k \sum_{j \in J} \sigma_{jk}^2 Y_{ijk}} - \sum \mu_{jk} Y_{ijk} \right) \\
&= \sum_{k \in K} \sum_{i \in I} h_i Z_{\alpha_k} \sqrt{l_k \sum_{j \in J} \sigma_{jk}^2 Y_{ijk}}
\end{aligned}$$

Explanation of (1) can be found below:

- The fixed cost of operating warehouse i: $\sum_{i \in I} a_i W_i$
- Transportation cost from warehouse i to factory j for raw material k:
 $\sum_{i \in I} \sum_{j \in J} \sum_{k \in K} \mu_{jk} d_{ij} Y_{ijk}$
- Purchasing and shipment cost of raw material k from supplier a to warehouse I:
 $\sum_{a \in A} \sum_{i \in I} \sum_{k \in K} S_{aik} X_{aik}$

The constraints used in the model and their explanations are as follows:

subject to

$$\sum_{i \in I} W_i \geq 1 \tag{2}$$

$$\sum_{i \in I} Y_{ijk} = 1, \quad \forall j \in J, \forall k \in K \tag{3}$$

$$Y_{ijk} \leq W_i, \quad \forall i \in I, \forall j \in J, \forall k \in K \tag{4}$$

$$\sum_{j \in J} \sum_{k \in K} Y_{ijk} \geq W_i, \quad \forall i \in I \tag{5}$$

$$W_i \in \{0, 1\} \quad \forall i \in I \tag{6}$$

$$Y_{ijk} \in \{0, 1\} \quad \forall i \in I, \forall j \in J, \forall k \in K \tag{7}$$

- (2) - At least one warehouse should be opened.
- (3) - Common raw materials used in each factory must be assigned to only one warehouse.
- (4) - Only if one warehouse is opened, it will serve to factories for raw materials.
- (5) - If one warehouse is opened, it must serve at least one raw material of one factory.
- (6) and (7) - Decision variables are binary they will decide whether warehouse i will be used or not and raw material k of factory j will be assigned to warehouse i or not.

3.2.2. Linear Approximation

In the model, the safety stock level calculation is found to be nonlinear because it had a square rooted expression. It is hard to solve this model using classical optimization tools. Therefore, in order to linearize the model, new constraints have been added to the model by using the piecewise linearization method in the literature. According to Croxton *et al.* (2003), The below term can be used to replace nonlinear safety stock term which is

$Z_{\alpha_k} \sqrt{l_k \sum_{j \in J} \sigma_{jk}^2 Y_{ijk}}$ in this thesis. According to piecewise linearization method;

- Linear approximation is applied to the nonlinear function with a linear function.

$$F_{ik}^r + C_{ik}^r l_k \sum_{j \in J} \sigma_{jk}^2 Y_{ijk} \approx Z_{\alpha_k} \sqrt{l_k \sum_{j \in J} \sigma_{jk}^2 Y_{ijk}} \text{ where in range } [L_{ik}^{r-1}, L_{ik}^r]$$

- In order to make the function linear and continuous, the following equations are applied:

$$F_{ik}^{r+1} = F_{ik}^r + (C_{ik}^r - C_{ik}^{r+1}) L_{ik}^r$$

$$C_{ik}^1 > C_{ik}^2 > C_{ik}^3 > \dots \text{ and } \dots > F_{ik}^3 > F_{ik}^2 > F_{ik}^1$$

- $r = 1, 2, \dots, R$ is a set of points and indicates segments. In order to specify which segment is active, it is decided by

$$U_{ik}^r = 1 \text{ if segment } R \text{ and } [L_{ik}^{r-1}, L_{ik}^r] \text{ is active; } 0 \text{ otherwise.}$$

- Safety stock term, $Z_{\alpha_k} (\sum_{r \in R} F_{ik}^r + C_{ik}^r l_k \sum_{j \in J} \sigma_{jk}^2 Y_{ijk}) U_{ik}^r$ become nonlinear again because of multiplication of Y_{ijk} and U_{ik}^r . To correct expression, binary variable is defined:

$$\delta_{ijk}^r = 1 \text{ if } \mu_{ik}^r = 1 \text{ and } Y_{ijk} = 1; 0 \text{ otherwise}$$

- Finally, safety stock term $Z_{\alpha_k} (\sum_{r \in R} F_{ik}^r + C_{ik}^r l_k \sum_{j \in J} \sigma_{jk}^2 Y_{ijk}) U_{ik}^r$ is expressed by a set of linear expressions:

$$Z_{\alpha_k} \sum_{r \in R} (F_{ik}^r U_{ik}^r + C_{ik}^r l_k \sum_{j \in J} \sigma_{jk}^2 \delta_{ijk}^r) \quad \forall i \in I, \forall k \in K$$

$$U_{ik}^r + Y_{ijk} - 2\delta_{ijk}^r \geq 0 \quad \forall i \in I, \forall j \in J, \forall k \in K, \forall r \in R$$

$$U_{ik}^r + Y_{ijk} - \delta_{ijk}^r \leq 1 \quad \forall i \in I, \forall j \in J, \forall k \in K, \forall r \in R$$

The term expressed is adapted and the safety stock calculation included in the objective function has been rearranged accordingly. Piecewise linearization methods can be summarized that nonlinear function is broken into linearized segments equally using r segments. Depending on the number of segments, the limits are proportionally calculated and this is called linear segmentation because of linear relation to r . If one segment is active it should be between below terms in order to make function linearized:

$$L_{ik}^{r-1} U_{ik}^r \leq l_k \sum_{j \in J} \sigma_{jk}^2 Y_{ijk} \leq M(1 - U_{ik}^r) + L_{ik}^r U_{ik}^r \quad \forall i \in I, \forall k \in K, \forall r \in R$$

In this thesis, upper bound of segments r are calculated and required coefficients are selected properly. Thanks to linear segmentation, nonlinear models turns into linear models and linear optimization methods were used easily.

Graph summarizes what is done.

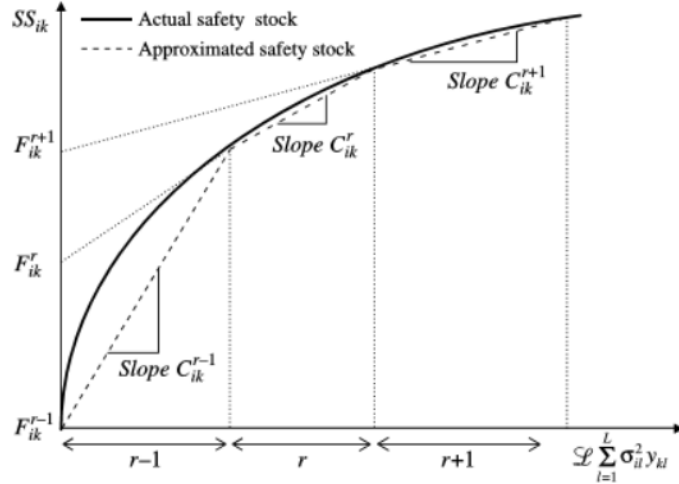


Figure 3.3. Piecewise linear function, Vidyarthi *et al.* (2007)

As a result, the resulting linear model is as follows:

3.2.2.1. Indices

R : Set of piecewise linearization segments

3.2.2.2. Parameters

L_{ik}^r : Upper bound of segment R in the piecewise linearization of the safety stock

F_{ik}^r : Linear coefficient of segment R

C_{ik}^r : Linear coefficient of segment R

3.2.2.3. Decision Variables

$\delta_{ijk}^r = 1$ if $\mu_{ik}^r = 1$ and $Y_{ijk} = 1$; 0 otherwise

$U_{ik}^r = 1$ if segment R is active; 0 otherwise

3.2.2.4. Model

The following part of the objective function must be replaced by the expression.

$$\sum_{k \in K} \sum_{i \in I} h_i Z_{\alpha_k} \sqrt{l_k \sum_{j \in J} \sigma_{jk}^2 Y_{ijk}} \approx Z_{\alpha_k} \sum_{k \in K} \sum_{i \in I} \sum_{r \in R} h_i \left(F_{ik}^r U_{ik}^r + C_{ik}^r l_k \sum_{j \in J} \sigma_{jk}^2 \delta_{ijk}^r \right) \quad (8)$$

In addition, the following constraints will be added the constraints.

$$L_{ik}^{r-1} U_{ik}^r \leq l_k \sum_{j \in J} \sigma_{jk}^2 Y_{ijk} \leq M(1 - U_{ik}^r) + L_{ik}^r U_{ik}^r \quad \forall_i \in I, \forall_k \in K, \forall_r \in R \quad (9)$$

$$\sum_{r \in R} U_{ik}^r = 1 \quad \forall_i \in I, \forall_k \in K \quad (10)$$

$$U_{ik}^r + Y_{ijk} - 2\delta_{ijk}^r \geq 0 \quad \forall_i \in I, \forall_j \in J, \forall_k \in K, \forall_r \in R \quad (11)$$

$$U_{ik}^r + Y_{ijk} - \delta_{ijk}^r \leq 1 \quad \forall_i \in I, \forall_j \in J, \forall_k \in K, \forall_r \in R \quad (12)$$

$$U_{ik}^r \in \{0, 1\} \quad \forall_i \in I, \forall_k \in K, \forall_r \in R \quad (13)$$

$$\delta_{ijk}^r \in \{0, 1\} \quad \forall_i \in I, \forall_j \in J, \forall_k \in K, \forall_r \in R \quad (14)$$

The explanation of expression added to objective function and constraints used to linearize in the model can be found below:

- (8) - Linearized safety stock holding cost:

$$Z_{\alpha k} \sum_{k \in K} \sum_{i \in I} \sum_{r \in R} h_i \left(F_{ik}^r U_{ik}^r + C_{ik}^r l_k \sum_{j \in J} \sigma_{jk}^2 \delta_{ijk}^r \right)$$

- (9) - If segment is active, it should be between lower and upper bounds.
- (10) - For each warehouse of each factory, only one segment should be active.
- (11) and (12) - To linearize $Y_{ijk} U_{ik}^r$, these constraints are added.
- (13) and (14) - Decision variables are binary.

3.2.3. Adding Supplier Selection to the Linear Model

In order to make the resulting model more suitable for the real life and to make the decision process end-to-end, the supplier selection and distribution decisions according to capacity and price has been added to the model. Each supplier has different capacity and when warehouses are selected by model, there may not be enough capacity of supplier in that region. Therefore, this situation should also be taken into account. Candidate suppliers and their costs are also important to meet the whole demands. In case supplier A is used, the model decides how much quantity will be ordered from supplier A. Ten candidate suppliers in total are described to the model.

Indices, parameters and decision variables added to the model at this stage are as follows:

3.2.3.1. Indices

A : Set of candidate suppliers

3.2.3.2. Parameters

S_{aik} : Purchasing and shipments cost from supplier A to warehouse I for raw material K

P_{ak} : Capacity of supplier A for raw material K

3.2.3.3. Decision Variables

X_{aik} = Quantity of raw material K shipped from supplier A to warehouse I

$O_a = 1$ supplier A is used; 0 otherwise

3.2.3.4. Model

The following is added to the objective function:

$$\sum_{a \in A} \sum_{i \in I} \sum_{k \in K} S_{aik} X_{aik}$$

Additional constraints:

$$\sum_{i \in I} \sum_{k \in K} X_{aik} \leq P_{ak} O_{ak} \quad \forall a \in A \quad (15)$$

$$\sum_{a \in A} X_{aik} = \sum_{j \in J} \mu_{jk} Y_{ijk} \quad \forall i \in I, \forall k \in K \quad (16)$$

$$O_a \in \{0, 1\} \quad O_a \in A \quad (17)$$

Addition to fixed costs, transportation costs and safety stock holding costs, new term added to objective function allows model to minimize purchasing cost of raw material including transportation freights from supplier to selected warehouses.

The constraints used in the model are as follows:

- (15) - Quantity shipped from supplier A should be less than its capacity.
- (16) - Ordered quantity for each raw material of each factory should meet total demand. Total demand calculated with average demand and its variance which depends on choice of warehouses to consolidate factories' demands. That is, model gives importance to suppliers' cost and capacity while selecting best warehouse location.
- (17) - Supplier selection decision is added to the model as a binary variable.

3.2.4. Resulting Model

According to Eppen (1979), if there is no correlation between demands average inventory increases proportionally to the square root of the number of locations in which inventory is held.

In decentralized system, the safety stock can be calculated by:

$$\sum_{k \in K} \sum_{i \in I} h_i Z_{\alpha_k} l_k \sum_{j \in J} \sigma_{jk} Y_{ijk}$$

If all inventories are kept in the one centralized warehouse, new formula of safety stock because of pooled variance is:

$$\sum_{k \in K} \sum_{i \in I} h_i Z_{\alpha_k} \sqrt{l_k \sum_{j \in J} \sigma_{jk}^2 Y_{ijk}}$$

Because of inventory pooling and pooled variance, there is lower demand variability which causes to lower safety stock levels for the protection against stockouts. Therefore, it is clear that transportation costs are expected to increase while safety stock holding costs are expected to decrease and a comparison will be made accordingly.

The resulting mixed integer linear programming model and all constraints are as follows:

3.2.4.1. Indices

I : Set of candidate warehouses, $i=1, 2, 3, \dots, 10$

J : Set of factories, $j=1, 2, 3, \dots, 10$

K : Set of raw materials, $k=1, 2, 3$

R : Set of piecewise linearization segments

A : Set of candidate suppliers, $a=1,2,3$

3.2.4.2. Parameters

a_i : Fixed cost of warehouse I

d_{ij} : Shipment cost from warehouse I to factory J

h_i : Holding cost per unit at warehouse I

μ_{jk} : Mean demand of raw material K at factory J

σ_{jk}^2 : Variance of demand of raw material K at factory J

α_k : Desired service level for raw material K

Z_{α_k} : Standart normal deviate of desired service level for raw material K

l_k : Lead time of raw material K

L_{ik}^r : Upper bound of segment R in the piecewise linearization of the safety stock

F_{ik}^r : Linear coefficient of segment R

C_{ik}^r : Linear coefficient of segment R

S_{aik} : Purchasing and shipments cost from supplier A to warehouse I for raw material K

P_{ak} : Capacity of supplier A for raw material K

3.2.4.3. Decision Variables

$W_i = 1$ if a warehouse is opened at warehouse I ; 0 otherwise

$Y_{ijk} = 1$ if raw material K of factory J is assigned to warehouse I ; 0 otherwise

$\delta_{ijk}^r = 1$ if $\mu_{ik}^r = 1$ and $Y_{ijk} = 1$; 0 otherwise

$U_{ik}^r = 1$ if segment R is active; 0 otherwise

X_{aik} = Quantity of raw material K shipped from supplier A to warehouse I

$O_a = 1$ supplier A is used; 0 otherwise

3.2.4.4. Model

Minimize

$$\sum_{i \in I} a_i W_i + \sum_{i \in I} \sum_{j \in J} \sum_{k \in K} \mu_{jk} d_{ij} Y_{ijk} + \sum_{a \in A} \sum_{i \in I} \sum_{k \in K} S_{aik} X_{aik} + Z_{\alpha k} \sum_{k \in K} \sum_{i \in I} \sum_{r \in R} h_i (F_{ik}^r U_{ik}^r + C_{ik}^r l_k \sum_{j \in J} \sigma_{jk}^2 \delta_{ijk}^r)$$

subject to

$$\sum_{i \in I} W_i \geq 1$$

$$\sum_{i \in I} Y_{ijk} = 1,$$

$$\forall j \in J, \forall k \in K$$

$$Y_{ijk} \leq W_i,$$

$$\forall i \in I, \forall j \in J, \forall k \in K$$

$$\sum_{j \in J} \sum_{k \in K} Y_{ijk} \geq W_i,$$

$$\forall i \in I$$

$$\sum_{i \in I} \sum_{k \in K} X_{aik} \leq P_{ak} O_{ak}$$

$$\forall a \in A$$

$$\sum_{a \in A} X_{aik} = \sum_{j \in J} \mu_{jk} Y_{ijk}$$

$$\forall i \in I, \forall k \in K$$

$$L_{ik}^{r-1} U_{ik}^r \leq l_k \sum_{j \in J} \sigma_{jk}^2 Y_{ijk} \leq M(1 - U_{ik}^r) + L_{ik}^r U_{ik}^r$$

$$\forall i \in I, \forall k \in K, \forall r \in R$$

$$\sum_{r \in R} U_{ik}^r = 1$$

$$\forall i \in I, \forall k \in K$$

$$U_{ik}^r + Y_{ijk} - 2\delta_{ijk}^r \geq 0$$

$$\forall i \in I, \forall j \in J, \forall k \in K, \forall r \in R$$

$$U_{ik}^r + Y_{ijk} - \delta_{ijk}^r \leq 1$$

$$\forall i \in I, \forall j \in J, \forall k \in K, \forall r \in R$$

$$W_i \in \{0, 1\}$$

$$\forall i \in I$$

$$Y_{ijk} \in \{0, 1\}$$

$$\forall i \in I, \forall j \in J, \forall k \in K$$

$$U_{ik}^r \in \{0, 1\}$$

$$\forall i \in I, \forall k \in K, \forall r \in R$$

$$\delta_{ijk}^r \in \{0, 1\}$$

$$\forall i \in I, \forall j \in J, \forall k \in K, \forall r \in R$$

$$O_a \in \{0, 1\}$$

$$O_a \in A$$

$$X_{aik} \geq 0$$

$$\forall a \in A, \forall i \in I, \forall k \in K$$

4. SCENARIO ANALYSIS AND NUMERICAL STUDY

The model is mixed-integer linear problem because there is both binary and integer variables. Objective function consists of linear terms. There are 3 raw materials, 10 suppliers, 10 factories, 10 potential warehouse locations and 2 segments in the problem. In the thesis, the nonlinear model that arises in the absence of linearization has 401 constraints. It is considered that nonlinear model can also be solved using nonlinear solvers without linearization because of the low number of constraints. However, since the thesis covers the modeling real-life example, it has been progressed by making linearization considering the possibility of a larger number of variables in the future. Actual data of glass manufacturer for 2018 were used as input in the model. The problem is modelled and solved using licensed GAMS 23.6 and CPLEX solver.

Three scenarios are created and they can be seen below. In each case, model is allowed to choose proper suppliers according to their capacity and cost.

- One warehouse is used for whole demand. (Hypothetical situation)
- Model is allowed to find and use optimal number of warehouses.
- All warehouses are used for whole demand. (Current situation)

4.1. Parameter Selection

In this thesis, parameter list and assumptions are as follows:

4.1.1. Transportation cost

- Each vehicle will be transported in full. Therefore, each vehicle can carry 23 tons.
- Transportation freights are directly proportional to road distance. 2 TL per kilometer will be paid.
- Matrix is created depending on distance and freight per ton are calculated.

$$\text{Transportation Freight} = \text{Distance (km)} \frac{\text{Cost (TL per km)}}{\text{Vehicle weight (ton)}}$$

Table 4.1. Transportation freights

Freight (TL per ton)	Glassware - Eskisehir	Glassware - Kirklareli	Flat Glass - Mersin	Flat Glass - Polatli	Flat Glass - Yenisehir	Flat Glass - Kirklareli	Glass Packaging - Mersin	Glass Packaging - Eskisehir	Glass Packaging - Yenisehir	Automotive - Kirklareli
Glassware - Eskisehir	1	45	59	14	12	45	59	2	12	45
Glassware - Kirklareli	45	1	100	58	32	2	100	45	32	2
Flat Glass - Mersin	59	100	1	44	70	100	2	59	70	100
Flat Glass - Polatli	14	58	44	1	26	58	44	14	26	58
Flat Glass - Yenisehir	12	32	70	26	1	33	69	12	2	33
Flat Glass - Kirklareli	45	2	100	58	33	1	100	45	32	2
Glass Packaging - Mersin	59	100	2	44	69	100	1	59	70	100
Glass Packaging - Eskisehir	2	45	59	14	12	45	59	1	12	45
Glass Packaging - Yenisehir	12	32	70	26	2	32	70	12	1	33
Automotive - Kirklareli	45	2	100	58	33	2	100	45	33	1

4.1.2. Inventory Holding Cost and Fixed Cost

Holding cost represents the principal costs of inventories. Inventories include costs related to rent, labor and utilities. After the raw materials are taken into the stocks, they are followed by using stock management systems. When the products are wanted to be moved from one point to the other during the stocking process within the factory, the products are handled. Logistic operation and handling costs are also added to the holding costs. On the other hand, raw materials in stock are insured against theft, fire and possible force majeure. In short, the holding cost is a figure that includes all these services.

The fixed cost is the cost incurred by the company due to having the warehouse regardless of the amount of movement realized within the warehouse. These costs must be paid even if the warehouse is never used. If they are fixed and not changing depending on quantities, rent, labor and utilities can be evaluated as fixed cost.

In the thesis, inventory holding and fixed cost figures were taken from factory accounting departments and can be seen below:

Table 4.2. Fixed and holding costs

Costs at warehouse i (TL)	Fixed cost	Holding cost per ton
Glassware - Eskisehir	470.000	40
Glassware - Kirklareli	650.000	43
Flat Glass - Mersin	520.000	38
Flat Glass - Polatli	680.000	40
Flat Glass - Yenisehir	580.000	52
Flat Glass - Kirklareli	700.000	57
Glass Packaging - Mersin	500.000	35
Glass Packaging - Eskişehir	430.000	47
Glass Packaging - Yenisehir	580.000	39
Automotive - Kirklareli	750.000	38

4.1.3. Mean and Variance

The model established in the thesis is evaluated by using real life numerical data. For this reason, the most common 3 materials used in 10 glass manufacturing plants were identified. Monthly average demand and standard deviations were calculated by summing the monthly consumption of all plants. These figures are used as input in the established model. Average demand is critical to see total consumption and variance of demand is critical in calculating the reorder point and safety stock level.

On the other hand, when the change of demands in the data set are analyzed, it is observed that the correlation coefficient varies between -0.1 and 0.1. Therefore, the model

was established assuming that the demands are uncorrelated with each other. The resulting figures of mean and variance are as follows:

Table 4.3. Mean and variance of demand for each raw material type

Factory / Product	Mean demand of raw material k at factory j			Variance of raw material k at factory j		
	A	B	C	A	B	C
Glassware - Eskisehir	12.200	3.200	890	38.440.000	608.400	184.041
Glassware - Kirklareli	14.600	2.820	1.450	14.440.000	1.210.000	160.801
Flat Glass - Mersin	10.050	3.980	620	25.000.000	1.742.400	94.864
Flat Glass - Polatli	5.400	1.700	150	10.758.400	1.000.000	7.396
Flat Glass - Yenisehir	8.500	3.500	670	5.760.000	902.500	25.600
Flat Glass - Kirklareli	6.500	3.200	700	8.410.000	184.900	126.736
Glass Packaging - Mersin	11.600	1.900	610	28.090.000	270.400	94.864
Glass Packaging - Eskişehir	5.400	2.200	2.230	2.250.000	792.100	336.400
Glass Packaging - Yenisehir	7.200	3.200	5.570	1.690.000	1.000.000	994.009
Automotive - Kirklareli	4.800	1.200	370	4.000.000	640.000	21.904

4.1.4. Safety Factor - Service Level

The glass sector has continuous production. A glass furnace operates continuously for 10-15 years after ignition. Stopping the production line causes very high costs. Therefore, inventory management is critical. In order not to be stockout, too much raw material stock is always kept. In this case, it increases inventory costs. Accordingly, the service level in the thesis was chosen as 99.29%. When the normal distribution table is examined, the value corresponding to the specified service level is taken as 2.45.

4.1.5. Supplier Capacities and Prices

When pricing methods of suppliers are taken into consideration, it is seen that pricing is made depending on product value and transportation cost. In this thesis, since real life model is modeled, the changing profit ratios of suppliers are also included while making supplier selection in the model. That is, in addition to the supplier-warehouse transportation which can be calculated depending on the distance, the profit ratios of the suppliers stand out as an element in the objective function. On the other hand, there is a capacity that each supplier can supply during the year.

Table 4.4. Prices and capacities of suppliers

Product	Supplier Capacity	Average Product Value (TL)*	Average cost of transporting supplier-warehouse locations (TL)	Capacity (Tons)	Average Demand (Tons)
A	Supplier 1	20	39	80.000	86.250
	Supplier 2	10	31	100.000	
	Supplier 3	15	42	90.000	
B	Supplier 1	88	36	15.000	26.900
	Supplier 2	103	40	30.000	
	Supplier 3	92	33	27.500	
	Supplier 4	82	33	20.000	
	Supplier 5	84	59	30.000	
C	Supplier 1	310	48	15.000	13.260
	Supplier 2	350	46	15.000	

*In the model, the prices of each supplier vary on the basis of the storage location, but the average prices are set here for example purposes.

From the table summarized above, it is seen that the main cost item is transportation costs in raw material A and the main cost item is raw material value in raw materials B and C. On the other hand, when supplier capacities and average demands are compared, it is seen that suppliers 1 for raw material A and suppliers 1 and 4 for raw material B do not have sufficient capacity to meet total demand.

4.2. Results

The results of the scenarios created are listed in this section. In each scenario, selected suppliers for each type of raw material, logistics network and the resulting costs are explained with the help of figures and tables.

4.2.1. Hypothetical Scenario - Single Warehouse

In the scenario where single warehouse is expected to be open, the model is solved by equalizing the constraint 1 below.

$$\sum_{i \in I} W_i = 1$$

In this scenario, all raw materials will be delivered to the single warehouse selected by the model and distribution to all factories will also take place from this warehouse. Selected suppliers and warehouse locations depending on capacities are as follows:

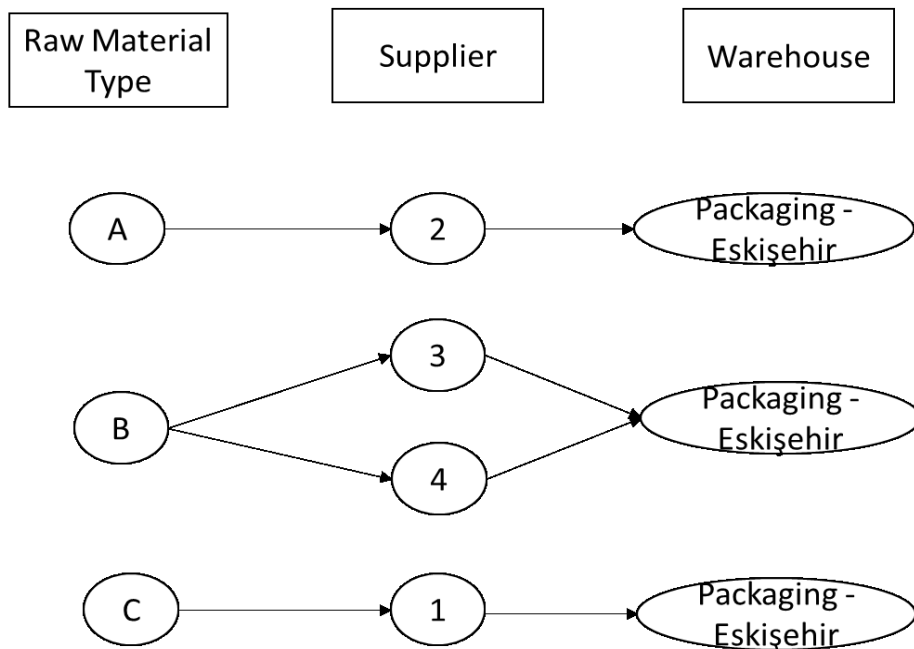


Figure 4.1. Suppliers selected and warehouse locations in single warehouse

Since the single warehouse will be selected, the model has chosen to use a warehouse in the Eskisehir region, which is the most optimal area and located geographically in the middle of all factories. When raw materials are taken into consideration, it appears that raw material A is supplied from supplier 2, raw material B from suppliers 3 and 4, and raw material C from supplier 1. Raw material B is supplied from 2 different suppliers because the capacity of one supplier is not sufficient to meet whole demands. On the other hand, when looked at the suppliers' locations, it is seen that the suppliers that are located closest to the warehouse are selected and thus cost minimization is achieved.

The selected suppliers, warehouse and distribution network are shown on the map in order to explain the logistics network more clearly.



Figure 4.2. Logistics network in a single warehouse scenario

Cost items are presented under four main headings: fixed cost, transportation cost, purchasing cost and holding cost. The results of this scenario are as follows:

Table 4.5. Costs incurred in a single warehouse scenario

Cost Type / Product (TL)	A	B	C	Total
Fixed Cost	143.333	143.333	143.333	430.000
Transportation Cost	2.710.965	776.424	262.691	3.750.080
Purchasing Cost	2.242.500	2.570.700	4.548.180	9.361.380
Holding Cost	606.783	148.813	73.671	829.267
Total	5.703.582	3.639.271	5.027.875	14.370.728

According to the result, the glass packaging Eskisehir factory holds all inventories instead of all factories and shipments to factories are done from this warehouse. For raw material A, both purchase and transportation costs are critical while purchase costs for raw materials B and C have priority. In addition, raw material C has a similar effect on the total cost although its quantity is less than A. This is because raw material C is more expensive than A.

4.2.2. Optimum Number of Warehouses Scenario

In the second scenario, the model was freed without any restriction on the number of warehouses to be used. The selected suppliers depending on raw material types are as follows:

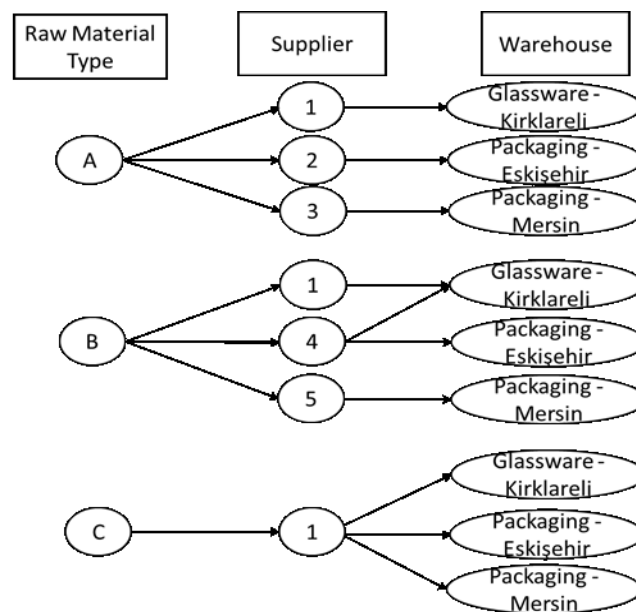


Figure 4.3. Suppliers selected and warehouse locations in optimum warehouse

While transportation costs were reduced with regional warehouses, an increase in holding cost was observed. The model has decided to use 3 regional warehouses in Kirklareli, Eskisehir and Mersin regions. Raw material A and B are supplied from 3 different suppliers while raw material C is supplied from a single supplier. Supplier 4 serves both Kirklareli and Eskisehir regions. The reason for this is that supplier 1, the main supplier of the Kirklareli factory, does not have sufficient capacity. Therefore, insufficient capacity is met by supplier 4.

Thanks to consolidated regional warehouses, factories located in Turkey are divided into 3 clusters. It is shown on the map as follows:

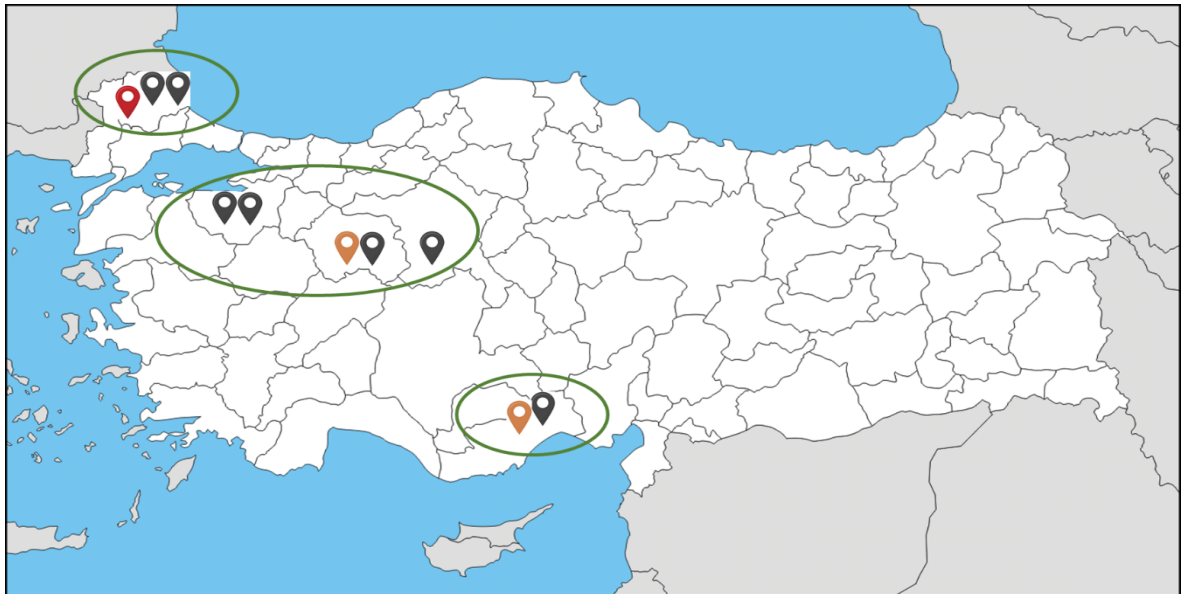


Figure 4.4. Logistics network in optimum warehouse scenario

The results depending on the costs are as follows:

Table 4.6. Costs incurred in optimum warehouse scenario

Cost Type / Product (TL)	A	B	C	Total
Fixed Cost	526.667	526.667	526.667	1.580.000
Transportation Cost	344.774	127.607	82.979	555.360
Purchasing Cost	2.735.800	2.266.260	4.651.530	9.653.590
Holding Cost	918.762	228.437	106.972	1.254.171
Total	4.526.002	3.148.971	5.368.147	13.043.120

Transportation costs are dramatically reduced by using regional warehouses. However, the purchase cost for the raw material C, which has 2 suppliers, increases significantly since it will be transported to 3 different regions. Although inventory holding cost and fixed costs increased, overall total cost decreased according to scenario used in single warehouse.

4.2.3. Current Situation

In the last scenario, the current situation of glass manufacturer is modeled. It is assumed that each of the 10 factories in the model stock their raw materials in their own warehouses. In this case, transportation costs will decrease and fixed and inventory holding costs are expected to increase.

The selected suppliers depending on raw material types are as follows:

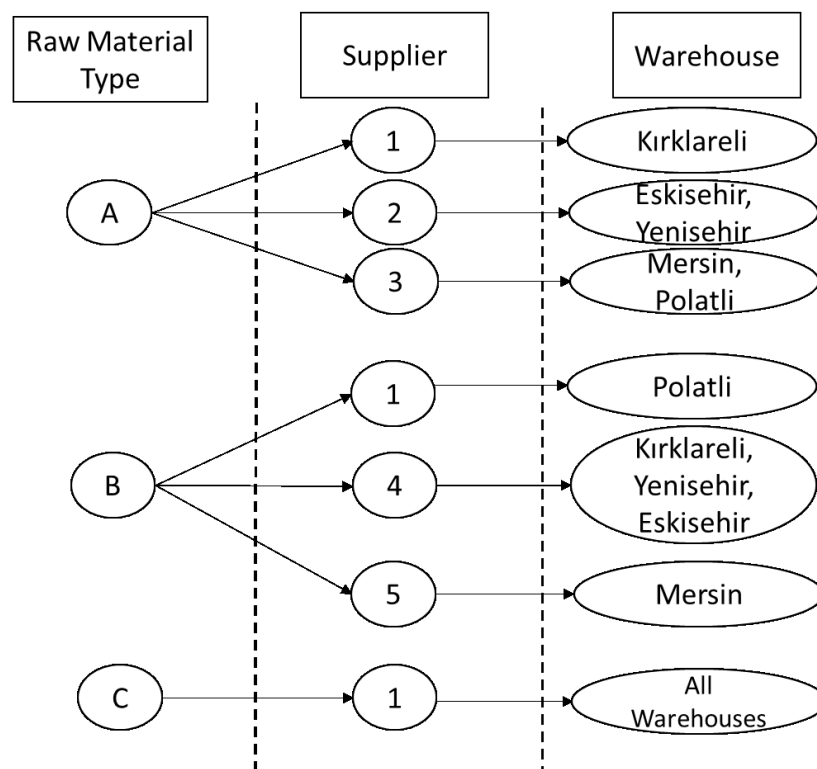


Figure 4.5. Suppliers selected and warehouse locations in current situation

Supplier choices are made according to supplier capacity availability and distance to the factory. Although supplier 1 for raw material B is located close to Polatli factory, supplier

1 is assigned to other factories because the cost of the whole system is minimized and raw material B is supplied to Polatli from supplier 4, which is further away than supplier 1.

Warehouse locations are shown on the map of Turkey below:



Figure 4.6. Logistics network in current situation

The results of the model are as follows:

Table 4.7. Costs incurred in current situation

Cost Type / Product (TL)	A	B	C	Total
Fixed Cost	1.953.333	1.953.333	1.953.333	5.860.000
Transportation Cost	86.250	26.900	13.260	126.410
Purchasing Cost	2.669.100	2.845.940	4.633.860	10.148.900
Holding Cost	1.539.859	407.590	176.081	2.123.531
Total	6.248.543	5.233.764	6.776.535	18.258.841

In the scenario where all warehouses are used, it is seen that transportation costs have reached the minimum point. Despite this decrease, it is seen that fixed cost and inventory holding costs have reached their maximum. When total costs are compared with other

scenarios, it is considered to be a more costly scenario than the other two scenarios. On the other hand, the fact that suppliers have their own profitability rates is one of the factors affecting the selection of suppliers even if they are far from the factories.

4.3. Comparison

While product pricing is made by suppliers, prices are determined according to the value of products and transportation cost. When the transportation costs are subtracted from the sales cost per ton, C is the most expensive raw material supplied. Additionally, raw material A is the most commonly used raw material. This information is considered important when comparing scenarios.

The results of the three scenarios presented by the model are shown below. There are 4 different graphs for each type of raw material. The column graph shows the total amounts according to the selected warehouses and cost items.

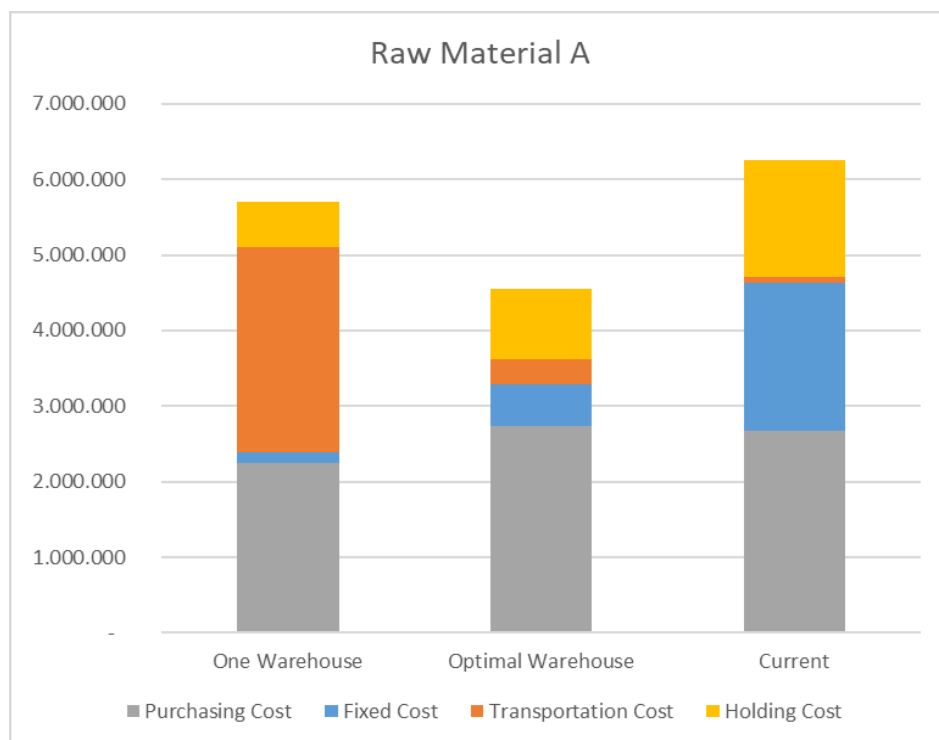


Figure 4.7. Change of costs on a scenario basis for raw material A

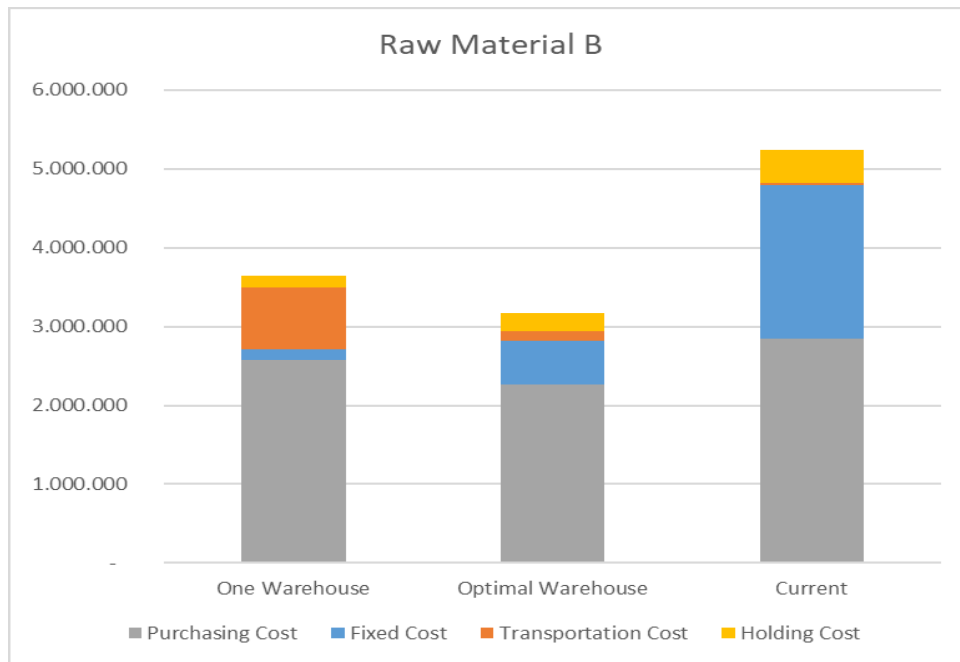


Figure 4.8. Change of costs on a scenario basis for raw material B

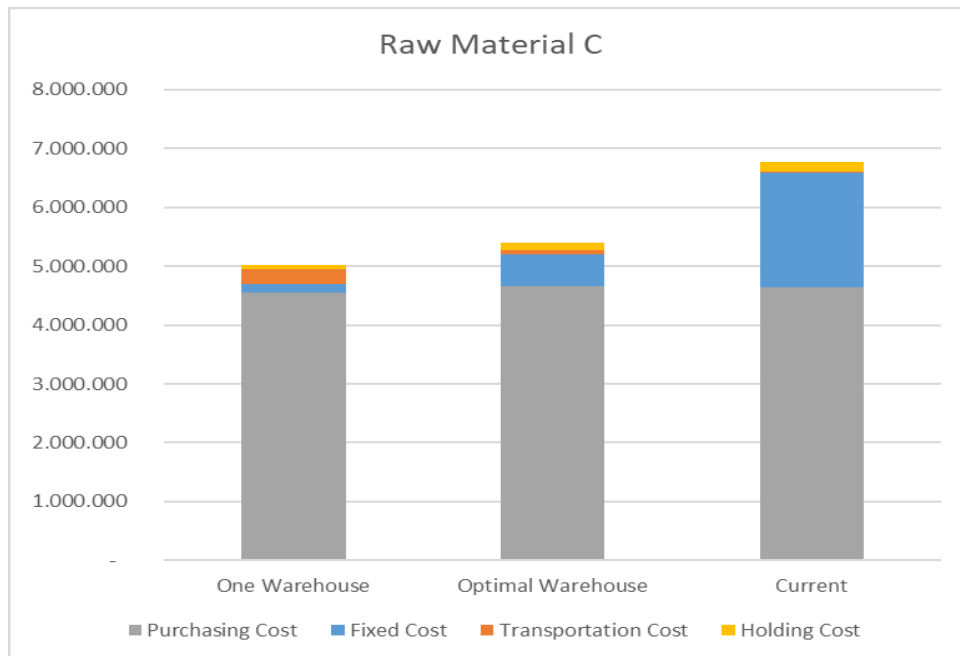


Figure 4.9. Change of costs on a scenario basis for raw material C

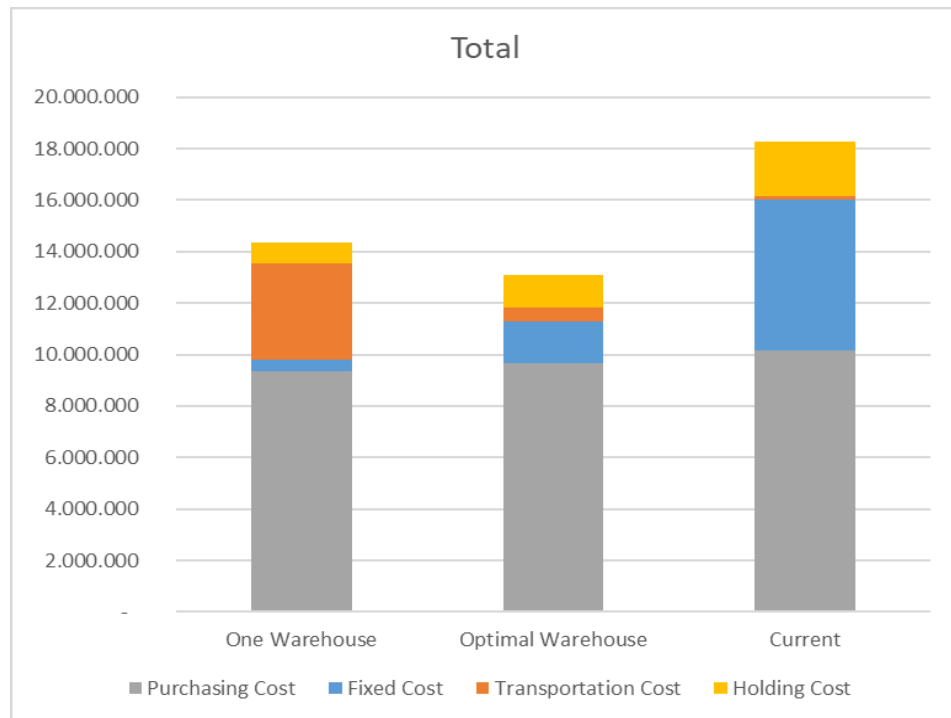


Figure 4.10. Change of costs on a scenario basis for all raw materials

When the column graph is examined, it is seen that the cost of purchasing raw materials is very important for all raw material types. This is similar to the fact that, in the glass sector, the decision of plant location selection is generally based on the distance to raw materials. Because the raw material consumption is very intense and each furnace produces 600 to 900 tons of glass per day continuously, closeness to the raw material is important.

It is observed that the most optimum result occurs in the scenario where 3 warehouses are used in Kirklareli, Eskisehir and Mersin Regions. In this case, fixed cost and holding cost emerge as an important cost item. However, by decreasing the number of warehouses from 10 to 3, these cost items are reduced and a more favorable cost structure can be obtained in spite of the increase in transportation cost. It is seen that cost improvement can be achieved by approximately 29% according to the current operation of the company thanks to optimum scenario.

In the case of a single warehouse use, the transportation cost becomes the most important item for the raw material A and also increases total cost. Although raw material A

is the least expensive raw material, it makes clear that transportation costs should be taken into consideration in the decisions to be made.

In order to further deepen the analyzes and to see the change of transportation and holding costs more closely in the selection of optimal storage numbers, the following line graph was created. In the line graph, how the cost items change according to the scenarios tested is revealed.

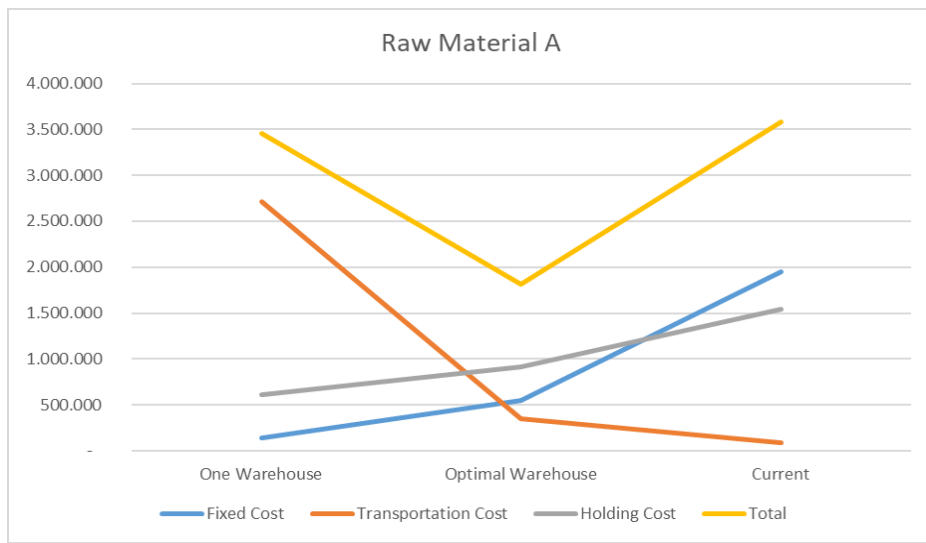


Figure 4.11. Change of costs other than procurement for raw material A

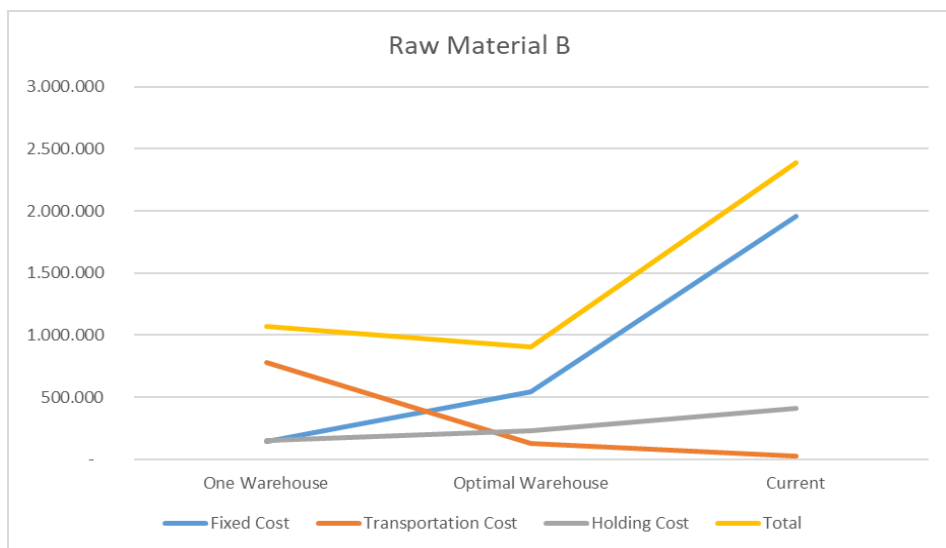


Figure 4.12. Change of costs other than procurement for raw material B

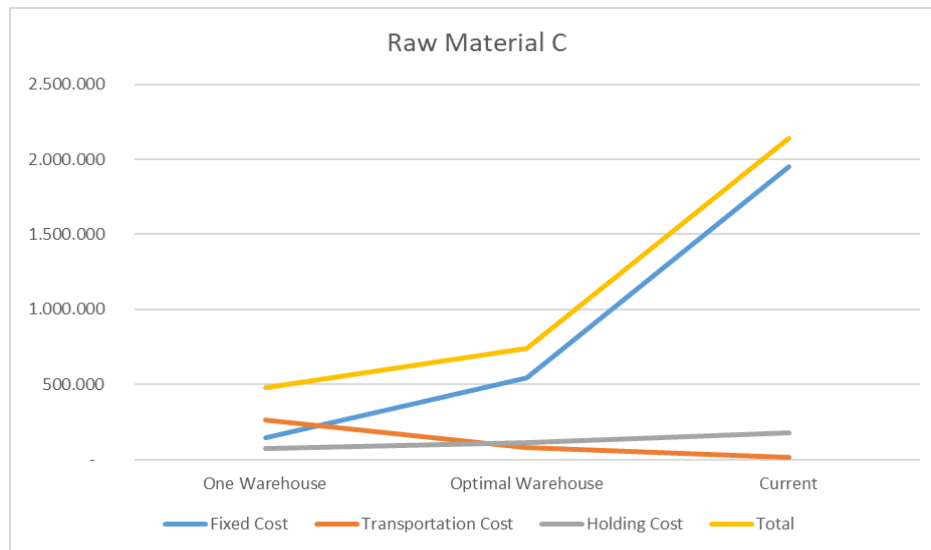


Figure 4.13. Change of costs other than procurement for raw material C

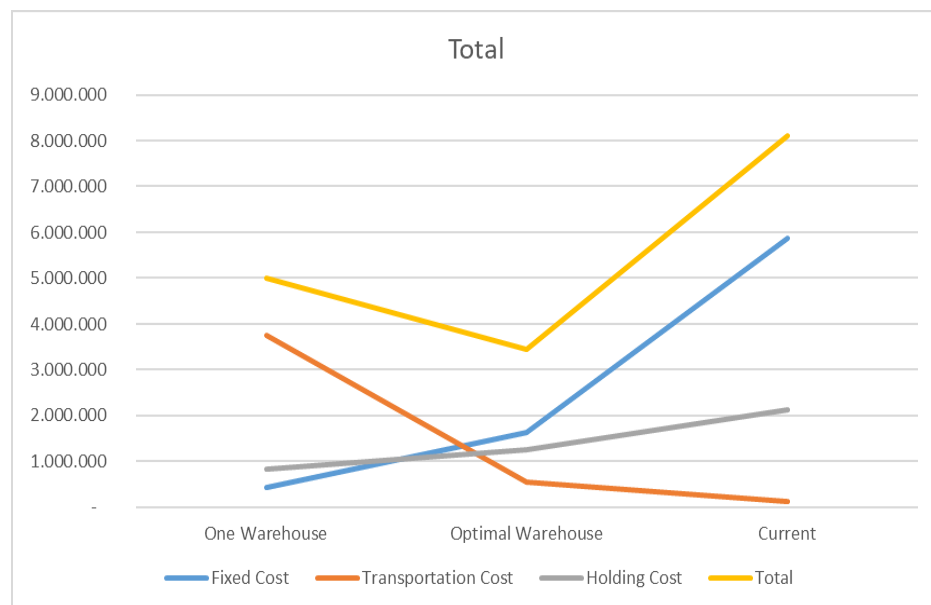


Figure 4.14. Change of costs other than procurement for all raw materials

It is seen that there is trade-off between transportation costs, holding costs and fixed costs depending on the number of warehouses used in all line graphs. As the number of warehouses decreases, transportation costs increase and holding and fixed costs decrease because standard deviations of demand are consolidated. Even when the raw material purchasing cost is excluded from the model, it is observed that the optimum scenario has the minimum cost for the remaining variables.

In the optimum scenario, transportation cost increases compared to the current situation while other cost items decrease. On the other hand, when the single warehouse scenario and the optimum warehouse scenario are compared, a significant increase in transportation cost is remarkable.

Table 4.8. Cost improvement on a scenario basis

Item / Cost (TL)	Current		Single		Optimum	Cost Improvement (%)
Purchasing Cost	10.148.900	▼	9.361.380	▲	9.653.590	-5%
Fixed Cost	5.860.000	▼	430.000	▲	1.580.000	-73%
Transportation Cost	126.410	▲	3.750.080	▼	555.360	339%
Holding Cost	2.123.531	▼	829.267	▲	1.254.171	-41%
Total	18.258.841	▼	14.370.728	▼	13.043.120	-29%

As a result, the cost improvement resulting from the hypothetical scenario to the optimal scenario is shown below in the waterfall graph. When a single warehouse is used, 21.3% cost improvement is achieved according to the current scenario. Besides, an additional 7.7% cost improvement is obtained with the optimum scenario.

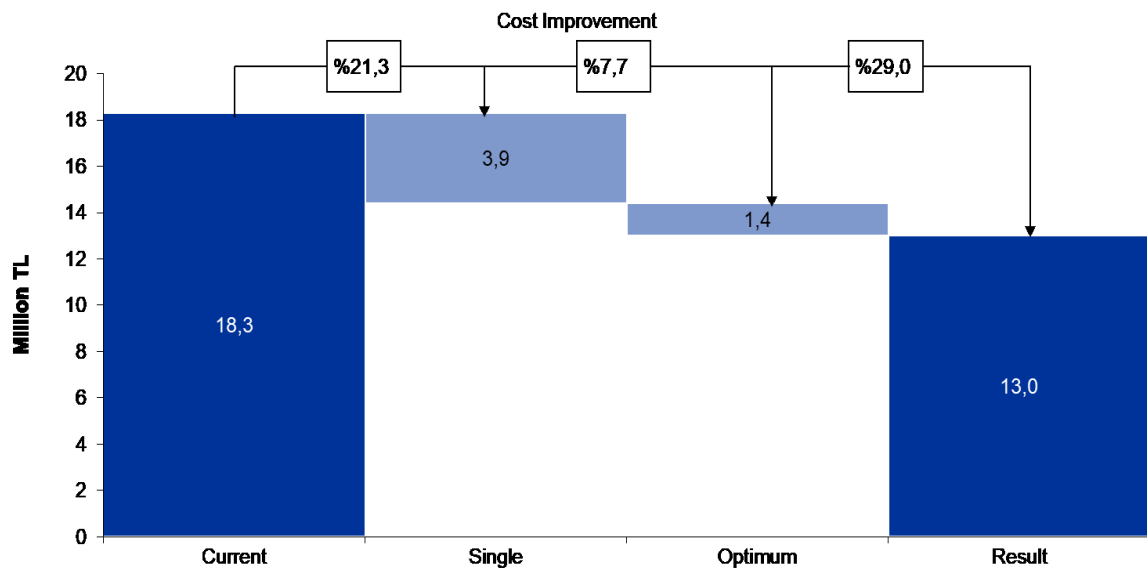


Figure 4.15. Cost improvement in waterfall chart

4.4. What-If Analysis

Since the cost of the supplier is of great importance in the model, an additional scenario has been made in order to see the effect on supplier selection by subtracting product cost and supplier profit from the supplier cost.

4.4.1. Supplier Cost without Profit and Product Value

In order to make what-if analyze after the optimal results were obtained, different profitability ratios of suppliers and product values were subtracted from raw material sales prices. That is, the objective function is modified in such a way that the supplier selections only consider transportation cost and available capacity. It is aimed to reveal which product types are affected by this change during the supplier selection stage. That is, the cost of suppliers' profitability and product value were eliminated and the best scenario where the optimum number of warehouses used is decided by the model is solved again to see changes in supplier selection.

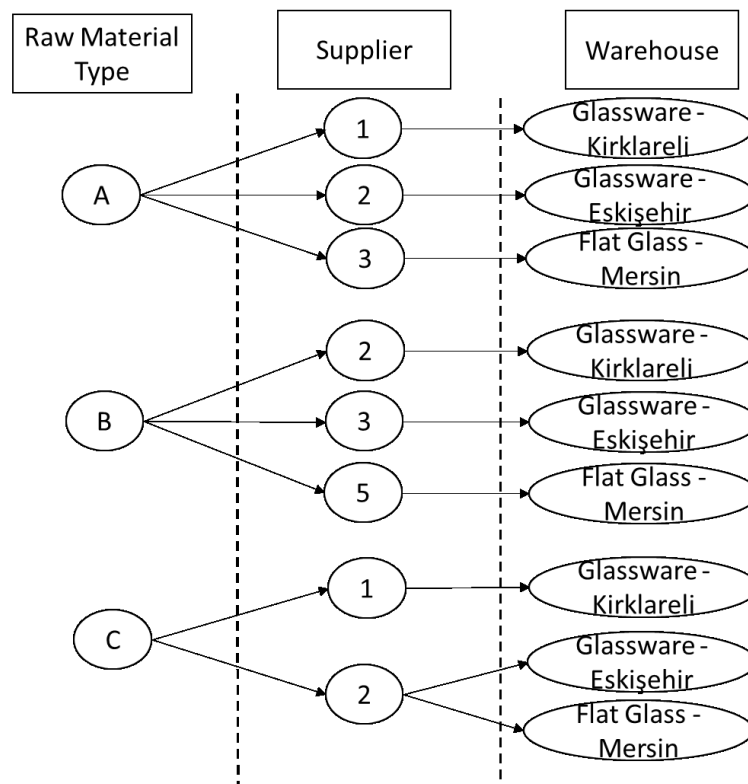


Figure 4.16. Suppliers selected and warehouse locations in what-if scenario

This shows that the profit rates and product values determined by suppliers are of great importance in supplier selection. In this scenario, regional warehouses have been established in Eskisehir, Kirklareli and Mersin regions again. However, while raw materials A with low product value used the same suppliers for the same regions, suppliers used for raw materials B and C with high product values changed according to the optimal scenario. This is an indication of how important raw material values are while choosing suppliers in the glass industry. Therefore, in the feasibility studies, due to the intensive raw material consumption activities, it shows us why the locations material are selected depending on supplier availability and procurement cost. If the cost models of suppliers can be analyzed well, it is considered to be useful in bargaining negotiations.

On the other hand, in this study, price changes of suppliers brought by quantity consolidation and economies of scale are neglected. However, this result shows that future studies on purchase prices and contract types proposed in the conclusion and extension chapter will produce significant results.

5. CONCLUSION AND EXTENSION

Taking everything into account, the supply chain, which collaborates with many different fields such as production, sales and quality, plays an important role in all departments that need to speak the same language in the enterprise. While the importance given to the supply chain in Turkey is increasing day by day, businesses are now positioning the supply chain as a concept that triggers the entire flow of the company. Every company wants to sell products to customers as soon as possible so as to generate revenue. Having a product in the inventory means that the company keeps all the resources and labor as a value-added product. However, the product is exposed to many risks such as loss of value and loss of quality when kept products in inventory. On the other hand, as inventory increases, warehouse management becomes more difficult. The fact that the number of stock-days is more than necessary indicates that there are problems in the production and sales balance of the enterprise. This can be seen as a sign that even the business has shown poor financial performance. When raw material inventories are analyzed, all raw materials that come to the factory are waiting to be converted into products. There are many methods that require an engineering perspective to determine the optimum raw material inventory levels, such as determining the safety stock level, the reorder point and the economic order quantity. All of these methods are critical in the supply chain in order to increase the sustainability of the supply and to obtain the most beneficial outputs for the enterprises.

Difficulties in adapting systems that look perfect in theory to real life should be considered as natural. Each company has its own internal dynamics and approaches like Just-in-Time Production and Supplier Managed Inventory should be considered as a strategy that aims to continuously improve, prevent waste, increase productivity, reduce inventories in production. In the years to come, enterprises that cannot perform a strong supply chain management are left behind in an increasingly competitive environment, while companies that make a difference will identify the hidden power behind their success through the supply chain. Because no business can survive by focusing on a single field, such as production, sales or marketing. The balance between all of departments is inevitable.

In this thesis, the data of a company operating in the glass industry in Turkey and making production in 10 different geographic locations is examined. Since each factory is managed by different teams, there is not enough communication between these teams. There is a total of 10 suppliers for 3 raw materials in the model and supplier selection was made in addition to the inventory pooling decision. Looking at the big picture, it is considered that there may be missed opportunities and inventory pooling can bring advantages. After the model was established, it was observed that the expression proposed for the calculation of the safety inventory level created nonlinearity in the model due to the square rooted expression. This expression has been made linear by applying piecewise linear function. It was aimed to examine the process from end to end by adding supplier selections to the model put forward for the inventory pooling.

Before applying inventory pooling to the real life, some actions should be taken. Although high service level is chosen in the model, there may be a risk of being stockout in factories caused by force majeure situations. An effective risk management mechanism should be established in order to prevent such situations. Alternative options for supplying the raw material in case of emergency should be available. Action lists, flow diagrams and role and responsibility matrices should be prepared systematically showing how to act in an emergency. All scenarios for prioritizing raw material demand must be studied in advance. On the other hand, if the operation of the shared warehouse is handed over to third-party logistics companies, the high penalty fees should be determined in the contract to prevent the stockout, so that the third-party logistics companies can take additional precautions in this regard.

The neglected but prospective research are is the purchasing cost. The increase in the amounts to be purchased gives the companies bargaining power due to the economies of scale because the demand consolidation enables the prices to decrease in the procurement negotiations. In the thesis, it is decided to work with 4 suppliers in single warehouse scenario and 7 suppliers were used in other scenarios depending on the supplier capacities. In this case, although it is predicted that additional benefit due to economies of scale can be provided by demand consolidation in a single warehouse scenario, this situation is not included in the thesis. It is considered that the unit purchasing cost will decrease due to the

demand consolidation. If unit purchasing cost can be written as a function of pooled demand and included to the objective function, the effect of pooled demand can be seen. It is known that if there is not capacity problem, suppliers tend to decrease unit purchasing cost to sell more raw materials when demand increases. That is, the inventory pooling creates additional cost advantages because selling price is sensitive to the amount of demand.

Contract types such as buy-back, revenue sharing, sales rebate, quantity-flexibility, which can be applied between the supplier and the company, can also be an important factor for the different solutions of the problem. On the other hand, due to supplier capacity constraint, the shared warehouses work with multiple suppliers. The benefits that may arise if the supplier invests in capacity increase by making a long-term agreement through the supplier development method can be examined.

As a result, scenarios for keeping all inventories in a single warehouse, keeping all inventories in an optimum number of warehouses to be found by the model, and keeping each factory's own inventories as in the current situation were studied and compared. As can be seen in the results, raw material costs and supplier capacities are critical for the factories operating in the glass sector. In each scenario, the most important cost item is purchase cost. On the other hand, due to inventory pooling, a trade-off arises between fixed costs, holding costs and transportation costs. The transportation cost decreases when the number of warehouses increases, but inventory holding costs and fixed costs increase. Contrary to the current situation, it was seen that the optimum result occurred in the scenario where 3 warehouses were used regionally and 29% cost improvement achieved. Therefore, it can be said that for a company operating in the glass industry with 10 different production locations, inventory pooling method is considered to have a very effective impact.

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