COMMODITY TAXATION WITH PRICE-DEPENDENT PREFERENCES

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COMMODITY TAXATION WITH PRICE-DEPENDENT PREFERENCES

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ABSTRACT

Commodity Taxation with Price Dependent Preferences

This study focuses on price-dependent preferences and their taxation. They have been subject to research in the field of economics since the beginning of the twentieth century. In general, the researchers classified these goods as either status-goods or quality-indicators. This thesis searches for other possible perceptions of price-dependent preferences and brings a new approach to them, showing that they may be necessities in the virtue of income elasticity.

Even though there are many pieces of research about the price-dependent preferences, the situation is not the same for their taxation. The second motivation of this thesis is prompted by this inadequacy. How would the optimal taxes behave when the utility functions were to include prices? The methodology is a constraint optimization with excise taxes in a static and deterministic world. In particular, I solved an optimal commodity tax problem of a representative agent who derives utility from two goods, and one of its prices enters the utility function. The first result is that price-dependency is not a synonym for luxury items, in fact, they may be necessities according to their income elasticities. In other words, a consumer may derive utility from the good's low price in addition to the good itself. Another important finding of this research supports that on the contrary to the classical approach, levying a higher tax on the more elastic good might yield an efficient outcome for some cases.

ÖZET

Fiyatlara Bağlı Tercihlerle Mal Vergilendirmesi

Bu çalışma fiyata bağlı tercihler ve bunların vergilendirilmesine odaklanmıştır. Bu tercihler 20. yüzyılın başından beri ekonomi alanında araştırmalara konu olmuşlardır. Genel olarak, araştırmacılar bu ürünleri statü malları veya kalite göstergeleri olarak sınıflandırdılar. Bu tez, fiyata bağlı tercihlerin diğer olası algılarını araştırır ve bunlara yeni bir yaklaşım getirir, gelir esnekliği vasıtasıyla ihtiyaç malları da olabileceğini gösterir.

Fiyata bağlı tercihlerle ilgili çok sayıda araştırma yapılsa da vergilendirilmesinde durum aynı değildir. Bu tezin ikinci motivasyonu bu yetersizlikten kaynaklanmaktadır. Fayda fonksiyonları fiyatları da içerdiğinde optimal vergiler nasıl davranırdı? Metodoloji, durağan ve deterministik bir dünyada tüketim vergileri ile bir kısıt optimizasyonudur. Özellikle, iki maldan fayda elde eden ve bu mallardan birinin fiyatı fayda fonksiyonuna giren temsili bir ajanın optimal bir tüketim vergisi problemini çözdüm. İlk sonuç, fiyat bağımlılığının lüks ürünlerle eş anlamlı olmadığı, hatta vergi sonrası fiyat esnekliklerine göre ihtiyaç malları olabileceğidir. Diğer bir deyişle, bir tüketici, malın kendisine ek olarak malın düşük fiyatından da fayda elde edebilir. Bu araştırmanın bir başka önemli bulgusu, klasik yaklaşımın tersine, daha esnek mala daha yüksek bir vergi uygulanmasının bazı durumlarda verimli bir sonuç verebileceğini desteklemektedir.

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CHAPTER 1

INTRODUCTION

Prices have a prominent influence on consumers' preferences. The traditional theory suggests that the relationship amongst the two is negative. Nevertheless, the relationship may be the other way around. The remarkable study of Veblen (1899) on the 'leisure class' discusses that prosperous households consume conspicuous goods. Households publicize their wealth and obtain greater social status by doing so. It can be said for such cases that they show the snob appeal. Alternatively, households may judge the quality by price, and higher prices may reflect superior quality. (Pollak, 1977,). In both cases, high prices imply these concepts. With this logic, low prices should mean inferior quality or low-status.

This study provides a new concept that is implied by low prices. Low prices should not always be associated with inferiority or low status. Low prices improve the budget so, it increases the quantity, therefore, the utility. Nevertheless, the only channel for low prices to affect the utility is not the budget. A consumer whose preferences are price-dependent can enjoy the low price of a product and the higher quantity due to low prices at the same time. In other words, the cheapness of a product can make a consumer happy. For instance, outlet shopping stores or online shopping platforms are some places where increasing happiness with the cheapness of goods is observable.

People buy goods according to their prices just because they are on sale or they are underpriced. Outlet shopping is a convenient real-life example of this consumer behavior because the prices in outlet stores are mostly less than the retail prices at the mall. These shoppers tend to be the lower-income households (Hausman and Leibtag, 2007). Outlet stores do not only consist of faulty products, in fact, they typically are a combination of current factory overruns, last season's styles, seconds, and samples. Although they include some faulty products or damaged goods, all of the products there are not necessarily inferiors. In the US, less than 15% of the products at the outlet stores were irregular and damaged merchandise. (Barnes, 1998).

In this study, the utility function depends on two goods and one of it is a regular consumption good whereas the other one is in a price-dependent form. The relationship between the price-dependent good's demand and its own price is negative so the good is neither Giffen nor Veblen. It is shown that the price-dependent one can be a normal consumption good, either a necessity or a luxury. If the consumer gets utility from the price of the good and the good is a necessity, then the consumer is likely to be enjoying the low price of the product. In real life, these goods can easily be found in outlet stores.

Even though the price-dependent preferences are subject to research since Veblen's conspicuous consumption theory mentioned above, there are still unanswered questions; how would the optimal tax behave when both price-dependent preferences and regular goods are together in the model? In the second quarter of the 1900s, it was proved by Ramsey that a uniform commodity tax system is not optimal (Ramsey, 1927). In general, taxing the elastic good less and the inelastic one more is the efficient solution to an optimal commodity tax problem. Does the inelastic good always have to be taxed more if preferences were to depend on prices? This paper is designed to conduct research on price-dependent preferences within a taxation context and seeks the answers to the aforementioned questions. To that end, a utility maximization problem is solved in a commodity tax framework, and the result tells the answer is no. An elastic good can be taxed more in some cases.

The methodology in this paper is simply a constraint optimization with a utility maximization problem of a representative agent and a dead-weight loss (DWL) minimization problem of the government. The utility function consists of two types of goods that the consumer derives utility from and of these two goods the government collects its target tax revenue. The utility maximization problem is solved with excise taxes and the utility function used here is the function introduced by Pollak (1977) where there are two price vectors that are observed in the function. One of them is called 'normal price' which gives utility to the consumer and the other one is 'market price' which enters the budget constraint. The normal prices are a function of current and past prices and they have two polar cases; either they may depend entirely on the past prices or current prices. The model constructed here uses the latter one since the problem is formed in a static environment. More details about the model can be found in Chapter 3.1.

The first critical assumption is that of perfectly elastic supply of the two goods to attain flat supply curves so that the producers are fully responsive to a change in the prices. Their extreme responsiveness causes consumers to bear the entire burden of the tax. The second one is that it is assumed there is a representative agent in the economy, which clears off the distributive concerns from the model. The third assumption is that the price-tags include only the final price of the product. Ultimately, they are the after-tax prices. The consumer's preference is shaped around the price and his information set on the prices contains only the after-tax prices. In other words, since his preference is based on how much he pays to acquire the product, the price influencing his taste is the after-tax price. As a result; his preference is taxed indirectly. On top of that; the buyer will be responding to the final prices on the price tag, rather than to the tax itself because only the tax-included price is visible to him. It is an intuitive assumption to eliminate a behavioral concern, tax salience. Consequently, the consumer is less salient to the taxes. The aforementioned assumptions will be valid throughout this paper.

One of the main findings of this study is that the after-tax price elasticity of the price-dependent good is not necessarily more than one. It is often misunderstood that when the price of a good is apparent in the utility function then it is a status good or a luxury good. In this paper, it is shown, using the income elasticities, that when a good's price is apparent in the utility function, it does not necessarily be one of the abovementioned goods. Being it a normal good is also the case under some circumstances which will be detailed in the subsequent section. On the other hand; the price-dependent good can be a luxury good as expected. According to the inverse elasticity rule, the luxuries are taxed less when the government has an efficiency concern. Another finding is that there exists at least one case that the elastic good is taxed more than the unit elastic one where the inverse elasticity rule contradicts this unconventional result.

A policy implication is that the tax basket may be reviewed according to the information above because there is a possibility of under-taxing some products. The government may raise the tax revenue by taxing more of these goods. Alternatively, the government may reduce the tax rate on some regular goods and increase the tax rate on the price-dependent goods. Eventually, the target revenue is collected but the burden is more on the consumers with price-dependent preferences. Consequently, taxing the price-dependent preferences may actually be welfare-improving.

Thereupon, altering the tax rates between two types of goods may be a channel for redistribution if equity is a concern of the government.

Behavioral aspects or implications for this type of consumer behavior may be a topic for further research which is excluded in this study. The anchoring effects may be measured in a dynamic setting of the normal prices since they will have a reference point in that setting. Consumers' salience to the tax over price-dependent preferences may be tested when price tags include both the ex-ante and the ex-post prices in the price tag and the problem is set in a dynamic world.

In the next section, the previous literature on price-dependent preferences and on commodity taxation will be scanned.

CHAPTER 2

LITERATURE REVIEW

The positive correlation between the prices and preferences is a part of the consumer theory since Veblen's reputed study 'The Theory of the Leisure Class' (Veblen, 1899). In his study, conspicuous consumption is observable when individuals demand more of a product as the price of it increases. In general, a higher price leads to a higher demand because it is either an indicator of quality or social status. Scitovszky (1944) argues that consumers perceive the price as an indicator of quality when they do not know all the products available to purchase. An increasing variety of consumer goods led the commodities in the consumer's budget to become complex. Consumers do not know all the set of available products so they are not expert-shoppers. Moreover; cheap is noticed as a synonym for inferior so this sensation makes the expensive a superior quality. Therefore; consumers judge quality by price. He concludes that this behavior is not irrational as long as the majority of the consumers are experts and they know what they buy.

Kalman (1968) formulates a utility function where prices enter the utility function and derive the Slutsky equation. His key interest is in the substitutability and complementarity of the goods. He derives a 'Generalized Slutsky Equation' and interprets it by imposing several qualitative restrictions.

Pollak (1977) provided a model for price-dependent preferences which is the benchmark model of this thesis. He introduced two price vectors that are normal prices and market prices. He distinguished the price vector which affects the consumer's preference and the prices that enter the budget constraint, and they coincide at the equilibrium. He also proved that price-dependent preferences are rationalizable.

Martin (1986) approached the prices being apparent in the utility function from a different point of view. He criticized Pollak for him to take the price as an indicator of quality in a deterministic environment. According to him, it is not rational for the consumer to use indirect quality indicators such as the price of the product in such an environment. He suggests that it is rational to form subjective expectations on the quality of the product according to prices only if the consumer is uncertain of the quality of the product. His statement implies that the expectations are price-dependent, rather than the preferences. Basmann, Molina, and Daniel (1987) discussed the relevance of the price-dependent preferences and used Fechner-Thurstone direct utility functions to study them. Their discussions do not contradict Pollak but, they allowed consumers to barter goods.

Bagwell and Bernheim (1996) explored factors that induce Veblen effects and investigated policy implications. Veblen effects arose from a desire to signal wealth so each individual's status depends upon perceptions of his wealth among social contacts in their model. Their findings suggest that luxury brands are sold at the consumer's preferred price, which is tax inclusive and does not vary with the tax rate. They found that tax on conspicuous goods may be welfare improving. They also found that excise taxes are non-distortionary.

Even though mass research on price-dependent preferences was done in the second half of the 20th century, they are still subject to current research. Balasko (2002) showed that the primary properties of competitive equilibria hold when preferences are price-dependent. The result is obtained in consideration of the qualitative comparative statics which remained true when preferences are price-dependent, by allowing resources to vary in the parameter space.

Ding, Ross, and Rao (2010) differ from the abovementioned researches with their behavioral approach to price-dependent preferences. They used price as an indicator of quality. On top of that; they used heterogeneous consumers, and established 5 different consumer types. They found strong empirical evidence on the existence of the 5 types.

Ng, (1987) introduced the 'diamond goods' and studied the taxation of them. Diamond goods are the goods that are valued for their values, not for their intrinsic consumption effects. He introduced taxes on the diamond goods, using the standard model of optimal taxation, and the main finding is that pure diamond good has an infinite tax in an optimal tax system. Put it differently, this study resulted that high taxes would not be distortionary. Notwithstanding; too high a tax induces tax evasion (including smuggling), especially if only a few countries are imposing high taxes. This paper is also related to optimal commodity taxation literature. To begin with, Ramsey showed that a uniform commodity tax system is not optimal. The main result was that the inelasticities are expensive. One of the most important papers in this literature is Optimal Taxation and Public Production II: Tax Rules. Diamond and Mirrlees (1971) solved a couple of problems to define optimum taxes in their study. They assumed constant returns to scale production function, and the result was that linear tax rates could be imposed on every product though the lump-sum taxes could not.

Sandmo (1975) solved an optimal commodity tax problem with one consumer and many goods. The result is that income and substitution effects arose because of the tax increase, and the income effects would be the same with the changes if the tax was a lump-sum. Indirect taxation is also studied by Deaton (1977) where optimal commodity taxation is not allowed to switch between a less distortionary income tax. There are many consumers, and they are all identical in preferences but differ in income. On the other hand, the government in this setting is not only raising the revenue, but the government's concern is also to maximize social welfare. Therefore; his problem was equity and efficiency trade-off since luxuries need to be taxed less to not distort efficiency. Additionally, he restricted the substitution between goods by using linear Engel curves. The main finding recommends that equity can be improved by subsidizing food, housing, and transportation.

Cremer and Gahvari (1993) worked with indirect taxes in the presence of tax evasion. They studied the effects of tax evasion on the link between commodity taxation and consumer welfare. They suggested that the traditional approach on the optimal tax theory should be revised intuitively, and they should reflect the impact of an audit probability while collecting tax revenues.

Using excise taxes has four motivations and they have been defined by Hines (2008). The first one is that they have significantly lower economic and political costs while generating remarkable government revenues on the contrary to income taxes. The second one is that the burden of the tax is on the consumer who benefits from the taxed good or service. The third one is that excise taxes can be used to control negative externalities. For instance, the substances that are polluting the environment can be taxed more by excise taxes. Lastly, the fourth motivation for excise taxes is that the consumption of harmful substances such as tobacco and alcohol can be controlled.

CHAPTER 3

THE MODEL

3.1 Before-Tax Analysis of the Model

This chapter starts with the introduction of the benchmark model, followed by the implementation of it to this study. The main reference to the model used here is the utility function which was first introduced by Pollak (1977) and that is;

$$U(q_1, q_2, \dots, q_n) = \sum_{k=1}^{n} a_k \log (q_k - b_k) \quad (3.1.1)$$

$$b_i = \hat{b}_i - \frac{\alpha_i}{P_i^M} A \prod (p_k^N)^{\alpha_k} \ \& \ \sum \alpha_k = 1 \quad (3.1.2)$$

where $a_i's$ denote the consumer's valuation of i^{th} good, and $\sum a_i = 1$. In other words; since this system is a linear expenditure system, they are the Cobb-Douglas weights of the corresponding goods. Moreover, q_i's are the quantities, P_i^M is the market price and P_i^N is the normal price of the i^{th}good where $i = 1, \dots, n$. Normal prices are the prices that influence consumers' tastes whereas market prices are the ones that enter the budget constraint. Prices influence consumers' tastes from two discrete perspectives in this model. Thus, they are separated notationally and conceptually one another to distinguish these roles that affect individuals' tastes Pollak (1977). He defines normal price function as a function of current and past prices. In reality, consumers who like Veblen goods do not consume a good just because of its current expensiveness. For example, if the previous prices of that good are remarkably lower than its current price then the households would not prefer to invest in that good to expose/exhibit their wealth. On the other hand; the consumers should buy the product to get utility from it so the market prices are the prices that the consumers pay for the product. Thus, the consumers are affected by the prices from two discrete channels; preferences and budget constraint. The normal prices are the ones that affect the preferences while the market prices influence the budget constraint. On the other hand, $\alpha_i's$ role is similar to the $a_i's$ but they show the consumer's valuation of the normal prices. The autonomous part of the model, denoted by $\hat{b}_i$, decreases the utility independent of the prices. It is likely to double-count on prices because the prices have two channels to influence the consumer's choice so, a neutralizing parameter might be needed. In a sense, it is a fixed cost to the consumer for having price-dependent preferences. Lastly, A can be interpreted as the degree of the price-dependency of the preferences.

Normal price specification yields two different types of demand function that are 'Normal Price Demand Function' and 'Market Price Demand Function'. The first one is a function of market prices, total expenditure, and normal prices while the latter one designates the normal prices as a function of the current and the past prices. At the point, where these two prices coincide, the market price demand functions imply a consumption pattern, and the normal price demand functions show this consumption pattern. (Pollak, 1977). If normal prices were to depend only on the past prices or both the current and the past prices, then the normal price demand function indicates the long-run or steady-state form. Other than these two exclusive cases; there is also a simultaneous specification. If the normal prices depend only on the current prices, then the distinction between these two prices is just an analytical one. In this case, the normal price demand functions indicate both long-run (or steady-state) and short-run demand functions (Pollak, 1977). This is the case that has been subject to the previous literature and to this study. That is why these two prices will be the same at the equilibrium.

People do not derive utility from only one type of good in reality, in fact, there are many types of goods that affect individuals' preferences. I aim to provide a model that theoretically simulates reality as simple as possible, hence, I constructed a model consistent with this point of view. The model establishes a utility function that depends on two goods. The first good is a regular normal consumption good, whereas the second good is a price-dependent one. I used this utility function with a few changes, which are reducing the price-dependent good to one good and leaving the other good in a price-independent form. Therefore; the utility function becomes of the following form as it is shown in Appendix A.

$$U(q_1, q_2) = a_1 \log(q_1) + a_2 \log(q_2 - b_2) \text{ where } b_2 = \hat{b}_2 - \frac{AP_2^N}{P_2^M} \quad (3.1.3)$$

The methodology used for this utility maximization problem is constraint optimization, therefore, the consumer with an income level of μ maximizes his utility function subject to the budget constraint by choosing how much to consume from each product. Therefore; the consumer's problem is:

$$\max_{q_1^*, q_2^*} U(q_1, q_2) \text{ subject to } P_1^M q_1 + P_2^M q_2 = \mu \quad (3.1.4)$$

The individual maximizes his utility function subject to his budget constraint. Since the methodology is constraint optimization, the Lagrangian becomes;

$$\mathcal{L} = a_1\, log(q_1) + a_2\, log\left(q_2 - \hat{b}_2 + \frac{AP_2^N}{P_2^M}\right) + \lambda(\mu - P_1^M q_1 - P_2^M q_2) \quad (3.1.5)$$

where λ denotes the Lagrangian multiplier. At the equilibrium, the corresponding normal price demand functions are obtained as the following two equations which were shown in Appendix A.

$$q_1^* = \frac{a_1}{P_1}\mu - a_1\hat{b}_2\frac{P_2}{P_1} + \frac{a_1}{P_1}AP_2 \quad (3.1.6)$$

$$q_2^* = \frac{a_2}{P_2}\mu + a_1\hat{b}_2 - a_1 A \quad (3.1.7)$$

The relationships between the demands and the parameters are presented in the vector below.

$$\begin{bmatrix} \frac{\partial q_1^*}{\partial a_1} & \frac{\partial q_2^*}{\partial a_1} \\ \frac{\partial q_1^*}{\partial a_2} & \frac{\partial q_2^*}{\partial a_2} \\ \frac{\partial q_1^*}{\partial (A-\hat{b}_2)} & \frac{\partial q_2^*}{\partial (A-\hat{b}_2)} \end{bmatrix} = \begin{bmatrix} \frac{\mu+P_2(A-\hat{b}_2)}{P_1} & (A-\hat{b}_2) \\ 0 & \frac{\mu}{P_2} \\ \frac{a_1 P_2}{P_1} & a_1 \end{bmatrix} \tag{3.1.8}$$

The first good's relationship with the valuation of the first good is affected by the consumer's income level, the good's own price, the second good's price, and the second good's type. The expression $(A-\hat{b}_2)$ determines the second good's type as will be shown in the subsequent section of this chapter. The demand for the first good is independent of the consumer's valuation of the good 2. The price-dependent good's relationship with the valuation of the first good is influenced by the type of the good 2. Its relationship with the consumer's valuation of this good depends on the ratio of the income and its own price. The final row tells the more luxurious the second good, the more is desired of both by the consumer.

Definition: The price effect on the demand is positive for Giffen goods. Every inferior good is not a Giffen good but every Giffen good is inferior (Spiegel, 1994).

The relationships between the demands and the prices are presented in the vector below.

$$\begin{bmatrix} \frac{\partial q_1^*}{\partial P_1} & \frac{\partial q_2^*}{\partial P_1} \\ \frac{\partial q_1^*}{\partial P_2} & \frac{\partial q_2^*}{\partial P_2} \end{bmatrix} = \begin{bmatrix} -\frac{a_1(\mu+P_2(A-\hat{b}_2))}{P_1^2} & 0 \\ \frac{a_1}{P_1}(A-\hat{b}_2) & -\frac{a_2\mu}{P_2^2} \end{bmatrix} \tag{3.1.9}$$

The price of the good 1 does not effect the demand for the good 2. The second good's demand is negatively related to its own price, yielding the result that this good is neither Giffen nor Veblen.

Proposition 1: The luxury degree or the necessity degree of the price-dependent good has an impact on the type of the price-independent good.

Proof: $P_2 > 0, \mu > 0$ and $a_1 > 0$ by construction. For sufficiently large value of $\hat{b}_2$, $(A - \hat{b}_2) < 0$ then $\frac{\partial q_1^*}{\partial P_1} > 0$, and $\eta_1^{\mu} < 0$ then the good 1 is Giffen □.

Proposition 2: The type of the price-dependent good determines whether the two goods are substitutes or complements.

Proof: $P_1 > 0$, and $a_1 > 0$ by construction. If the good 2 is a luxury, then $(A - \hat{b}_2) > 0$, then $\frac{\partial q_1^*}{\partial P_2} > 0$ so the goods are substitutes.

If the good 2 is a necessity, then $(A - \hat{b}_2) < 0$, then $\frac{\partial q_1^*}{\partial P_2} < 0$ so the goods are complements □.

The subsequent part contains the after-tax analysis of this model.

3.2 After-Tax Analysis of the Model

When the government decides to put excise taxes on both goods and the price tag shows only the tax-included price, then after-tax prices are the market prices and they enter the budget constraint. The commodity taxes can be injected into this model by replacing market prices with after-tax prices. The normal prices depend only on the current prices in this thesis as mentioned above. Therefore; the current price of the second good is its after-tax price now and so the normal prices. As an indirect result, the consumer's preference will be taxed. However; taxing the consumer's preference does not change the parameters of the consumer's preference. The optimal demands change because of the taxes, but not as an opposition to taxes. As Chetty, Looney, and Kroft (2009) proposed that "tax salience" is stronger if both ex-ante and the ex-post prices are visible to the consumers. When a consumer sees the commodity taxes in the posted prices, then he becomes more salient and the demand is affected more. The consumer will be less salient to taxes since the tax-exclusive price is not visible to him and the model's environment is not dynamic. In other words; the consumer does not have any reference point to compare the prices. Under these specifications, the equilibrium will occur where the normal price overlaps with the market price and the representative agent's problem is;

$$\max_{q_1^*, q_2^*} U(q_1, q_2) \text{ subject to } z_1 q_1 + z_2 q_2 = \mu \tag{3.2.1}$$

The equilibrium tax-included demand functions are now the following equations that were shown in Appendix B.

$$q_1^* = \frac{a_1}{z_1}\mu - a_1 \hat{b}_2 \frac{z_2}{z_1} + \frac{a_1}{z_1} A z_2 \tag{3.2.2}$$

$$q_2^* = \frac{a_2}{z_2}\mu + a_1 \hat{b}_2 - a_1 A \quad \text{where } z_i = p_i^M + t_i \text{ for } i = 1,2 \tag{3.2.3}$$

Furthermore; horizontal supply curves are assumed for both goods and the before-tax price of the first good is normalized to 1. The after-tax price demand elasticities and the income elasticities of the goods are derived as follows, which are shown in Appendix C.

$$\eta_1 = -1 \tag{3.2.4}$$

$$\eta_2 = -\frac{a_2\mu}{a_2\mu + a_1\hat{b}_2 z_2 - a_1 A z_2} \tag{3.2.5}$$

$$\eta_1^\mu = \frac{a_1\mu}{a_1\mu - a_1\hat{b}_2 z_2 + a_1 A z_2} \tag{3.2.6}$$

$$\eta_2^\mu = \frac{a_2\mu}{a_2\mu + a_1\hat{b}_2 z_2 - a_1 A z_2} \tag{3.2.7}$$

$$a_2\mu + a_1\hat{b}_2 z_2 - a_1 A z_2 > 0 \tag{3.2.8}$$

The first good is a normal good with unit elasticity where the demand elasticity of the second good depends on the parameters. The parameters on the nominator in the 3.2.7 are positive by definition so the income elasticity is positive if the denominator is positive too. The equation 3.2.8 restricts that the denominator is strictly greater than zero which ensures the normality of the second good. Whenever the reverse holds then the second good is inferior.

Proposition 3: A price-dependent preference does not always imply a luxury-good.

Proof: Suppose $a_2\mu + a_1\hat{b}_2 z_2 - a_1 A z_2 > 0$ so the second good is a normal good.

Claim 1: The second good is not an inferior good.

Proof: Let $a_2\mu + a_1\hat{b}_2 z_2 - a_1 A z_2 < 0$ so the second good is an inferior good. Recall that $a_2 > 0$ and $\mu > 0$, then $\eta_2 > 0$ by 3.2.5. Hence, it is a Giffen good.

If the second good is a Giffen good then $\frac{\partial q_2^*}{\partial z_2} > 0$.

But $a_2\mu + a_1\hat{b}_2 z_2 - a_1 A z_2 < 0$, $a_2 > 0$ and $\mu > 0 \Longrightarrow \frac{\partial q_2^*}{\partial z_2} = -\frac{a_2\mu}{z_2^2} < 0$.

Thus, the second good is not a Giffen good so the result contradicts with the result of the initial assumption.

Proof of Proposition 3-Continued: $a_2\mu + a_1\hat{b}_2 z_2 - a_1 A z_2 > 0 \Longrightarrow \eta_2^{\mu} > 0$. The following equation characterizes the type of the second good.

$$a_2\mu \gtreqless a_2\mu + a_1\hat{b}_2 z_2 - a_1 A z_2 \Longrightarrow A \gtreqless \hat{b}_2 \Longrightarrow \eta_2^{\mu} \gtreqless 1 \quad (3.2.9)$$

All of the three are possible because there is no upper-bound or lower-bound for the parameters A and $\hat{b}_2$. So, the ratio of the numerator to the denominator of the income elasticity of the good 2 can be separated into three cases as below.

$$\text{if } A = \hat{b}_2 \text{ then } \eta_2^{\mu} = 1 \quad (3.2.10)$$

$$\text{if } A > \hat{b}_2 \text{ then } \eta_2^{\mu} > 1 \quad (3.2.11)$$

$$\text{if } A < \hat{b}_2 \text{ then } \eta_2^{\mu} < 1 \quad (3.2.12)$$

Equation 3.2.12 proves the existence of the non-luxury items, the necessities, in the presence of price-dependent preferences □.

The first interpretation of the result is that the 3.2.10 tells if the degree of the price-dependency is equal to the fixed cost of having price-dependent preferences, then this model is trivial. The additional part, which was defined as b_i in original model, in the utility function for the second good disappears in the model because $b_2 = \hat{b}_2 - A = 0$ at the equilibrium. The second 3.2.11 tells if the degree exceeds the fixed cost then this good behaves like a luxury good. In other words, if the consumer's valuation of the price is more than the fixed cost of it, then this consumer derives utility from a status-good or he may judge the quality by price. The following equation, 3.2.12, tells the reverse of the 3.2.11. If the fixed cost of having such a preference is greater than the degree part, then this consumer still derives utility from its price but now utility generating part is not coming from the expensiveness of the product but from the cheapness of it. Outlet shopping or online shopping can be counted as real-life examples of this situation. People enjoy the low prices in the outlet stores or on the online shopping platforms and the goods there with low prices are not inferiors.

The following part introduces the government and contains the optimal tax analysis of this model.

3.3 Optimal Commodity Taxes for the Model

The above analysis concentrates on the consumer side of the model. The next step is the introduction of the government. The government has a target revenue $\bar{R}$ and collects it via commodity taxation. So, the government imposes excise tax on both of the two and raises the revenue $R_i = t_i q_i$ from the i^{th} good where $i = 1,2$. The government also wants to minimize the DWL, caused by the taxes, subject to the revenue constraint by choosing the tax rates. The government's minimization problem is;

$$\min_{t_1,t_2} \sum_{i=1}^{2} DWL_i \text{ subject to } \sum_{i=1}^{2} R_i \geq \bar{R} \qquad (3.3.1)$$

$$DWL_i = \left|\frac{\eta_i^s \eta_i^d}{\eta_i^s - \eta_i^d} \frac{t_i^2}{2} \frac{q_i}{p_i}\right| \text{ for } i = 1,2 \tag{3.3.2}$$

Since the methodology is, once again, constraint optimization so the Lagrangian becomes

$$\mathcal{L} = DWL_1 + DWL_2 + \theta(\bar{R} - R_1 - R_2) \tag{3.3.3}$$

where θ is the shadow price of the government's budget constraint. Intuitively, the tax rate should be positive and it requires $\theta > 0$ The equation D.1.6 of Appendix D shows this result.

Rearranging and solving for the first-order necessary conditions, visit Appendix D, of the problem in 3.3.3, the following two equations characterizes the optimal tax rates:

$$t_1^* = -1 + \sqrt{1 + 2\theta} \tag{3.3.4}$$

$$\frac{t_2^2 + 2p_2 t_2 + 2\theta p_2^2}{(p_2 + t_2)^2} = \frac{2p_2 t_1 a_1}{(1+t_1) a_2 \mu} \left(-\hat{b}_2 + A\right)\left(\theta - \frac{1}{2} t_1\right) + \frac{2\theta a_1 p_2}{a_2 \mu} \left(-\hat{b}_2 + A\right) \tag{3.3.5}$$

The first one is the optimal tax rate of the first good and it is independent of the tax rate of the second good even though its demand depends on it. The second equation yields the optimal tax for the second product but it is left as the arranged version of its first-order necessary condition because of interpretative reasons. These parameters will be interpreted in a parameter space in the following chapter.

If the number of price-dependent goods included in the utility function was more than one then the consumer's valuation to the normal prices, i.e. α_i, would be apparent in the model. In that case, the normal price effects on the demands, elasticities, and the tax rates would be more observable. I would expect the optimal tax rates to increase with $\alpha_i's$. Furthermore; if the utility function were to include only the price-dependent goods then the cross-price elasticities would be symmetric.

3.4 Comparative Statics

The comparative statics analysis around the type determinant of this good is given in the vector below.

$$\begin{bmatrix} \frac{\partial FOC_1}{\partial(A-\hat{b}_2)} \\ \frac{\partial FOC_2}{\partial(A-\hat{b}_2)} \\ \frac{\partial FOC_\theta}{\partial(A-\hat{b}_2)} \end{bmatrix} = \begin{bmatrix} \frac{a_1 z_2}{z_1}\frac{2t_1 z_1 + t_1^2 + 2\theta}{2z_1} \\ -\frac{1}{2}\frac{t_1^2 a_1}{z_1} - \frac{\theta t_1 a_1}{z_1} + \theta a_1 \\ -\frac{a_1 z_2 t_1}{z_1} - a_1 t_2 \end{bmatrix} = \begin{bmatrix} \frac{a_1 z_2(3\theta^2 + 4\theta)}{2(1+\theta)^2} \\ -\frac{3\theta}{2}\left(\frac{\theta a_1}{1+\theta}\right) + \theta a_1 \\ -\frac{a_1 z_2 \theta}{(1+\theta)} - a_1 t_2 \end{bmatrix} = \begin{bmatrix} + \\ \pm \\ - \end{bmatrix}$$

The first-order conditions give the equations that yield the minimum tax rates such that the government can minimize the cost of taxation to the consumer. Hence, the rows in the column vector above tell how these equations, yielding the minimum points of the tax rates, change with respect to the type of the price-dependent good $(A-\hat{b}_2)$.

The sign of the first entry is positive since all the parameters are greater than zero. It means that the minimum point for the first good's tax rate rises as the second good becomes more luxurious. The second row means that the equation that characterizes the minimum tax rate of the price-dependent good can be affected by its own type in two different ways. If $\theta > 2$ then the sign of the second entry is negative. The interpretation is that if the second good's luxuriousness increases then the tangent equation shifts down, so the minimum point it results with shifts down. As a result; as price-dependency increases, the tax collection ability of the government on that particular price-dependent good increases. The third entry means that if the consumer is more price-dependent then the equation yielding the minimum cost of relaxing the government's budget constraint by one unit shifts down.

The subsequent chapter introduces a vector in which the result is consistent with the comparative statics analysis above.

CHAPTER 4

A CASE AND ITS DISCUSSION

Since the equation 3.3.5 is not an intuition-friendly one, the optimal tax rate of the second product is exemplified from a parameter space. If the price-dependent good is a complement to leisure then it would be optimal to tax it more. Taxing leisure's complement is more efficient (Corlett and Hague, 1953). The case down below supports the Corlett-Hague rule.

Proposition 4: There exists at least a case for the price-dependent preferences where it is optimal to levy a higher tax on the more elastic good.

Proof: Pick a point from the parameter space of this model. The vector, $\langle (A - \hat{b}_2); \frac{a_1}{a_2}; p_2; \theta; \mu \rangle = \langle 1, \frac{1}{2}, 3, \frac{3}{2}, 4 \rangle \Rightarrow \langle \eta_1; \eta_2 \rangle = \langle |-1|; |-2.66| \rangle$ and $\langle t_1; t_2 \rangle = \langle 1;\ 2 \rangle$ at the equilibrium according to the 3.2.4, 3.2.5, 3.3.4 and 3.3.5. So, $\eta_1 < \eta_2$ but $t_1 < t_2$. One example proves the existence of this case □.

The interpretation of the parameters is as follows: $(A - \hat{b}_2) = 1$ means that the second good is a luxury good. The consumer's valuation of the second product is twice of the first one $\frac{a_1}{a_2} = \frac{1}{2}$. The pre-tax price of the luxury good exceeds the pre-tax price of the numeriare $p_2 = 3 > p_1 = 1$ which was a necessity with a unit elastic demand. Furthermore; it can be interpreted as the consumer's valuation of the second good doubles the first one while the pre-tax price of the second good triples the first good. The cost of relaxing the government's revenue constraint for one unit is one and a half units, denoted by $\theta = \frac{3}{2}$, and $\mu = 4$ means the agent's income is four. Under these parameters the corresponding tax rates are $t_1 = 1$ and $t_2 \approx 2$ so $t_1 < t_2$.

Usually, utility increases with the prices, so, with the taxes, in the presence of price-dependent preferences. However; the optimal tax rates are not infinite on the contrary to the diamond goods case of Ng (1987). Likewise, Bagwell and Bernheim (1996) did not end up with infinite tax rates for the taxation of Veblen goods and the factor that prevents infinite tax rates to be optimum is the increasing utility with the quantity. Their explanation of this difference is valid for this study and it comes from the difference in the setting of the utility. In Ng's setting, the consumer derives utility from the price of the good rather than the amount. In other words, what he paid for the diamond goods generates utility to the consumer rather than how many of the diamond goods he has. That is why the optimal tax rates are infinite for diamond goods. In the other study, the consumer's preference on the prices, which includes the taxes, determines the price that the luxury brands sold. "Thus, as long as the tax per unit does not exceed the difference between the consumer's preferred price and marginal cost, and as long as the tax does not fall on budget brands, an excise tax on conspicuous goods amounts to a non-distortionary tax on pure profits." (Bagwell and Bernheim, 1996, p.351).

The total effects of the commodity taxes on the goods are respectively -1,5675 and -0,532 as obtained in Appendix E. It should not be forgotten that this utility function is not quasi-linear so that the changes in the optimal demands due to the taxes are not decontaminated from the income effect. To separate the effects, the substitution and the income effects are calculated for this point. The income effects are negative since the tax increased the goods' prices so that the agent's purchasing power is declined. The relative price ratio increased from $\frac{p_1}{p_2} = \frac{1}{3} = 0.33$ to $\frac{z_1}{z_2} = \frac{2}{5} = 0.4$. The substitution effect for the first good is -0.377 and is 0.9964 for the second one. The positive substitution effect for the second good means that the consumer continues to consume the second good despite the tax since it is relatively less expensive.

The Inverse Elasticity Rule suggests that taxing the inelastic good more is more efficient. However; the finding above is quite the opposite. The deriving factor of this result may be that Ramsey Rule assumes cross-price elasticities are zero. On the contrary to this assumption, they are not here. As a matter of fact; there is an asymmetry between the cross-price elasticities. As can be found in 3.1.9 that the price of the first good has no influence on the price of the price-dependent good while the price-dependent good's price has an impact on the regular good. The anomaly observed here is not a new concept in the literature. It has been first introduced by Blattberg and Wisniewski as asymmetric price effect. The asymmetric price effect tells the discounts on the products with higher quality or higher prices affect the products with lower quality or lower priced ones. The magnitude of this effect is larger than the magnitude of the effect of the reverse direction (Blattberg and Wisniewski, 1989). In addition to the asymmetric price effects, Sethuraman, Srinivasan and Kim (1999) studied the neighborhood price effects and they explained the term as "The neighborhood price effect states that brands that are closer to each other in price have larger cross-price effects than brands that are priced farther apart." (p.38).

This result should not be interpreted as the government should levy the tax more heavily on the elastic good whenever a price-dependent preference is a case. In fact, this finding suggests that there is at least one case in which taxing the inelastic good more brings a less efficient outcome. What if there are many elastic goods that can be taxed more? In the light of the real data, a new basket may be formed for these goods and the government shall make its amends on the tax rates of them.

CHAPTER 5

CONCLUSION

This paper is designed to conduct research on price-dependent preferences within a taxation context. The benchmark model was the utility function that was introduced by Pollak (1977). I made modifications to the reference model which are reducing the overall number of consumption goods to two and restricting the consumer to have only one price-dependent good in his utility function. Once I rebuilt the utility function, the first stage was solving the utility maximization problem. The solution of the problem yielded the optimal demands and I searched for the relation between the optimal demands and the parameters, also the relationship between the regular consumption good and the price-dependent one. The price-dependent one could influence the type of the regular consumption good by turning it out to be a Giffen good. Furthermore; the type of the price-dependent good determined whether these two goods are substitutes or complements.

The first main finding is that the price-dependent good is not necessarily a luxury good or status good as it was expected according to the previous literature. According to elasticity interpretations, a price-dependent good may be either elastic or inelastic good depending on the difference between the consumer's degree of the price-dependency of the preferences and the fixed cost of having price-dependent preferences. This result can be interpreted as the consumer likes a necessity, and if that necessity product is even cheap then this consumer is happier. For example, if a consumer purchases a basic t-shirt from a budget brand then this consumer receives utility from this purchase. In addition to buying the t-shirt itself, if the t-shirt is cheap, then this consumer is even happier from this acquisition.

After obtaining the condition for the elasticity interpretations of the good, I introduced the government. The government imposed per unit consumption taxes on both goods and minimized the cost of the commodity tax to society. The first-order necessary conditions of this problem yield the optimal taxes for both products. The optimal tax rate of the regular good is obtained whereas the second commodity's optimal tax rate was left out as an equation, fully depending on the parameters. Since leaving the tax rate on the left-hand side would yield the equation but the interpretative power of the equation was low, I chose a parameter vector from the parameter space and obtained the corresponding tax rates of the two goods for a very specific scenario. The result was interesting because it showed that the inverse elasticity rule may fail when prices enter the utility function. In other words; the inelastic good may be taxed less and this would yield an efficient solution.

The deriving factor presumably was that the cross-price elasticities are assumed to be zero between the two goods while finding the optimal tax rates. Besides not being equal to zero, they had asymmetric cross-price elasticities here.

In broad terms, the methodology was a constraint optimization with commodity taxation and exemplification. I imposed heterogeneity in the consumption space; I allowed the consumer to have both types of preferences in his utility function in a static and deterministic environment. This problem had no distributive concerns because the problem here was a representative agent problem.

The second main finding of this research supports that levying a lower tax on elastic goods might yield an inefficient outcome, rather than the expected efficient one. Thus, revising the tax basket seems to be a sensible idea. A policy that alters the tax rates between the regular consumption goods and the price-dependent goods may be a welfare improving one. The change should either raise the tax rate of the price-dependent good, reduce the regular consumption goods, or do both.

In further studies, this study can be extended by setting the problem in a dynamic environment in the presence of price-dependent preferences. In addition to that, behavioral aspects of having price-dependent preferences, enjoying the low prices, anchoring effects and tax salience should be tested.

APPENDIX A

DERIVING THE BEFORE-TAX DEMANDS

The utility function established by Pollak is:

$$U(q_1, q_2, \dots, q_n) = \sum_{k=1}^{n} a_k log\,(q_k - b_k)$$

$$b_i = \hat{b}_i - \frac{\alpha_i}{P_i^M} A \prod (p_k^N)^{\alpha_k} \;\&\; \sum \alpha_k = 1$$

If the commodity space has two goods then n = 2 case for the utility function is;

$$U(q_1, q_2) = a_1 \log(q_1 - b_1) + a_2 \log(q_2 - b_2)$$

$$U(q_1, q_2) = a_1 \log\left(q_1 - \hat{b}_1 + \frac{\alpha_1}{P_1^M} A (p_1^N)^{\alpha_1} (p_2^N)^{\alpha_2}\right)$$

$$+ a_2 \log\left(q_2 - \hat{b}_2 + \frac{\alpha_2}{P_2^M} A (p_1^N)^{\alpha_1} (p_2^N)^{\alpha_2}\right)$$

$b_1 = 0, p_1^M = 1$ and $p_1^N = 0$

$$U(q_1, q_2) = a_1\, log(q_1) + a_2\, log(q_2 - b_2)$$

$b_2 = \hat{b}_2 - \frac{\alpha_2}{P_2^M} A (p_1^N)^{\alpha_1} (p_2^N)^{\alpha_2}$ & $\sum_{k=1}^{2} \alpha_k = 1$ and $\alpha_1 = 0$ then $\alpha_2 = 1$

So $b_2 = \hat{b}_2 - \frac{1}{P_2^M} A P_2^N$

The Lagrangian and the corresponding demands in this setting are;

$$\mathcal{L} = a_1\, log(q_1) + a_2\, log\left(q_2 - \hat{b}_2 + \frac{1}{p_2} A P_2^N\right) + \lambda(\mu - p_1 q_1 - p_2 q_2)$$

First-Order Necessary Conditions;

A.1 q_1 ↲ $\frac{a_1}{q_1} - \lambda p_1 = 0$

A.2 q_2 ↲ $\frac{a_2}{q_2 - \hat{b}_2 + \frac{1}{p_2} A P_2^N} - \lambda p_2 = 0$

A.3 λ ↲ $\mu - p_1 q_1 - p_2 q_2 = 0$

Combining A.1 & A.2 yields;

A.4 $\frac{a_1}{q_1}\frac{q_2 - \hat{b}_2 + A}{a_2} = \frac{1}{p_2}$

A.5 $q_1 = a_1 p_2 \frac{q_2 - \hat{b}_2 + A}{a_2}$

Combining A.3 & A.4;

A.6 $\mu - a_1 p_2 \frac{q_2 - \hat{b}_2 + A}{a_2} - p_2 q_2 = 0$

A.7 $q_1^* = \frac{a_1}{p_1} \mu - a_1 \hat{b}_2 \frac{p_2}{p_1} + \frac{a_1}{p_1} A p_2$

Combining A.4 & A.5;

A.8 $q_2^* = \frac{a_2}{p_2} \mu + a_1 \hat{b}_2 - a_1 A$

APPENDIX B

DERIVING THE AFTER-TAX DEMANDS

$$\mathcal{L} = a_1 \, log(q_1) + a_2 \, log\left(q_2 - \hat{b}_2 + \frac{1}{z_2} A P_2^N\right) + \lambda(\mu - z_1 q_1 - z_2 q_2)$$

First-Order Necessary Conditions;

B.1 q_1 ↲ $\frac{a_1}{q_1} - \lambda z_1 = 0$

B.2 q_2 ↲ $\frac{a_2}{q_2 - \hat{b}_2 + \frac{1}{z_2} A P_2^N} - \lambda z_2 = 0$

B.3 λ ↲ $\mu - z_1 q_1 - z_2 q_2 = 0$

Combining B.1 & B.2 yields;

B.4 $\frac{a_1}{q_1} \frac{q_2 - \hat{b}_2 + A}{a_2} = \frac{z_1}{z_2}$

B.5 $q_1 z_1 = a_1 z_2 \frac{q_2 - \hat{b}_2 + A}{a_2}$

Combining B.3 & B.4;

B.6 $\mu - a_1 z_2 \frac{q_2 - \hat{b}_2 + A}{a_2} - z_2 q_2 = 0$

B.7 $q_1^* = \frac{a_1}{z_1} \mu - a_1 \hat{b}_2 \frac{z_2}{z_1} + \frac{a_1}{z_1} A z_2$

Combining B.4 & B.5;

B.8 $q_2^* = \frac{a_2}{z_2} \mu + a_1 \hat{b}_2 - a_1 A$

APPENDIX C

DERIVING THE AFTER-TAX ELASTICITIES

Elasticities are obtained by the following equations;

$$\eta_i = \frac{\partial q_i^*}{\partial z_i}\frac{z_i}{q_i^*}$$

C.1 $\eta_1 = -\frac{(a_1\,\mu - a_1\hat{b}_2 z_2 + a_1 A z_2)}{z_1^2}\,\frac{z_1}{\frac{(a_1\,\mu - a_1\hat{b}_2 z_2 + a_1 A z_2)}{z_1}} = -1$

C.2 $\eta_2 = -\frac{a_2\mu}{z_2^2}\,\frac{z_2}{\frac{(a_2\,\mu + a_1\hat{b}_2 z_2 - a_1 A z_2)}{z_2}} = -\frac{a_2\mu}{a_2\mu + a_1\hat{b}_2 z_2 - a_1 A z_2}$

$$\eta_i^{\mu} = \frac{\partial q_i^*}{\partial \mu}\frac{\mu}{q_i^*}$$

C.3 $\eta_1^{\mu} = \frac{a_1}{z_1}\,\frac{\mu}{\frac{(a_1\mu - a_1\hat{b}_2 z_2 + a_1 A z_2)}{z_1}} = \frac{a_1\mu}{a_1\mu - a_1\hat{b}_2 z_2 + a_1 A z_2}$

C.4 $\eta_2^{\mu} = \frac{a_2}{z_2}\,\frac{\mu}{\frac{(a_2\,\mu + a_1\hat{b}_2 z_2 - a_1 A z_2)}{z_2}} = \frac{a_2\mu}{a_2\mu + a_1\hat{b}_2 z_2 - a_1 A z_2}$

APPENDIX D

DERIVING THE OPTIMAL-TAX RATES

The optimal tax rates obtained from the government's problem stated in 3.3.1 and formulated in 3.3.3 is solved below.

$$DWL_1 = \left|\frac{\eta_1^s \eta_1^d}{\eta_1^s - \eta_1^d} \frac{t_1^2}{2} \frac{q_1}{p_1}\right| \text{ and } DWL_2 = \left|\frac{\eta_2^s \eta_2^d}{\eta_2^s - \eta_2^d} \frac{t_2^2}{2} \frac{q_2}{p_2}\right|$$

Perfectly elastic supply was assumed for both goods, pre-tax price of the first good was normalized to 1 and C.1 and C.2 yields;

$$DWL_1 = -\frac{1}{2}\eta_1^d t_1^2 q_1 = -\frac{1}{2} t_1^2 q_1$$

$$DWL_2 = -\frac{1}{2}\eta_2^d t_2^2 \frac{q_2}{p_2}$$

The revenues are

$$R_1 = t_1 q_1$$

$$R_2 = t_2 q_2$$

Thus; the explicit version of the Lagrangian is;

$$\mathcal{L} = -\frac{1}{2} t_1^2 q_1 - \frac{1}{2}\eta_2^d t_2^2 \frac{q_2}{p_2}$$
$$+ \theta \left[\bar{R} - t_1 \left(\frac{a_1}{1+t_1}\mu - a_1 \hat{b}_2 \frac{(p_2+t_2)}{1+t_1} + \frac{a_1}{1+t_1} A (p_2 + t_2)\right) - t_2 \left(\frac{a_2}{p_2+t_2}\mu + a_1 \hat{b}_2 - a_1 A\right)\right]$$

First-Order Necessary Conditions;

D.1 t_1 ↲ $-t_1\left(\frac{a_1\mu - a_1\hat{b}_2 z_2 + a_1 A z_2}{z_1}\right) + \frac{1}{2} t_1^2 \left(\frac{a_1\mu - a_1\hat{b}_2 z_2 + a_1 A z_2}{z_1^2}\right) + \theta\left(\frac{a_1\mu - a_1\hat{b}_2 z_2 + a_1 A z_2}{z_1^2}\right) = 0$

D.1.1 $-t_1 + \frac{1}{2} t_1^2 \frac{1}{z_1} + \theta \frac{1}{z_1} = 0$

D.1.2 $t_1^2 + 2t_1 - 2\theta = 0$

D.1.3 $\Delta = 4 + 8\theta \geq 0$

D.1.4 $\theta \geq -\frac{1}{2}$

D.1.5 $t_1 = \frac{-2 \pm \sqrt{4(1+\theta)}}{2}$

D.1.6 $t_1 = -1 \pm \sqrt{(1+\theta)}$

Positive tax rate;

D1.6.1 $t_1 = -1 + \sqrt{(1+\theta)} \geq 0$

D.1.6.1.1 $\sqrt{(1+\theta)} \geq 1$

D.1.6.1.2 $\theta \geq 0$

D.1.6.2 $t_1 = -1 - \sqrt{(1+\theta)} \geq 0$

D.1.6.2.1 $-\sqrt{(1+\theta)} \geq 1$

D.1.6.2.2 $\sqrt{(1+\theta)} \leq -1$ contradicton

D.2 t_2 ↲ $-\frac{1}{2} t_1^2 a_1 \left(\frac{-\hat{b}_2 + A}{1+t_1}\right) - \frac{1}{2} \frac{a_2 \mu}{p_2} \frac{(t_2^2 + 2p_2 t_2)}{(p_2+t_2)^2} + \theta \left[-t_1 a_1 \frac{(-\hat{b}_2 + A)}{1+t_1} + a_1(-\hat{b}_2 + A) - \frac{a_2 \mu p_2}{(p_2+t_2)^2}\right] = 0$

D.2.1 $t_1 a_1 \frac{(-\hat{b}_2 + A)}{1+t_1}\left(\theta - \frac{1}{2} t_1\right) - \frac{a_2 \mu}{(p_2+t_2)^2}\left(\frac{t_2^2 + 2p_2 t_2}{2p_2} + \theta p_2\right) + \theta a_1(-\hat{b}_2 + A) = 0$

D.2.2 $\frac{t_2^2 + 2p_2 t_2 + 2\theta p_2^2}{(p_2+t_2)^2} = \frac{2p_2 t_1 a_1}{(1+t_1) a_2 \mu} (-\hat{b}_2 + A)\left(\theta - \frac{1}{2} t_1\right) + \frac{2\theta a_1 p_2}{a_2 \mu} (-\hat{b}_2 + A)$

APPENDIX E

A SPECIAL CASE

$$\langle (A - \hat{b}_2); \frac{a_1}{a_2}; p_2; \theta; \mu \rangle = \langle 1, \frac{1}{2}, 3, \frac{3}{2}, 4 \rangle$$

$$\frac{t_2^2 + 6t_2 + 27}{t_2^2 + 6t_2 + 9} = \frac{3}{2}$$

yielding $t_2 \approx 2$. So;

$$\langle t_1; t_2 \rangle = \langle 1;\ 2 \rangle$$

$$q_1 = 2.31\ , q_2 = 0.55\ \ pre - tax\ demands$$

$$q_1' = 0.7425, q_2' = 0.198\ post - tax\ demands$$

$$TE_1 = 0.7425 - 2.31 = -1.5675$$

$$TE_2 = 0.198 - 0.55 = -0.352$$

$$U(2.31; 0.55) = 0.5655$$

From B.5 $\frac{4}{5} q_1 = q_2$

$$0.5655 = 0.33\ log q_1 + 0.66\ log\ (\frac{4}{5} q_1)$$

$$q_1'' = 1.933$$

$$q_2'' = 1.5464$$

$$SE_1 = 1.933 - 2.31 = -0.377$$

$$TE_1 = SE_1 + IE_1 \rightarrow IE_1 = -1.5675 + 0.377 = -1.1905$$

$$SE_2 = 1.5464 - 0.55 = 0.9964$$

$$TE_2 = SE_2 + IE_2 \rightarrow IE_2 = -0.352 - 0.9964 = -1.3484$$

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