GENERATING SERVICIZING STRATEGIES FOR SUSTAINABLE SUPPLY CHAINS

by

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ABSTRACT

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As a result of technological developments, life-cycles of products have significantly decreased in industrialized countries resulting in a vast increase in waste and a decrease in natural resources. Consequently, various initiatives are taken by governing bodies to encourage supply systems to adopt environmentally sustainable practices. Many companies have reformed their business strategies to include product recovery activities in order to achieve sustainable development. Yet product recovery is not adequate to form a closed loop supply chain that minimizes the environmental impact in the long run. Hence supply systems have to incorporate *servicizing* based on providing functionality of products rather than product ownerships to achieve a truly closed-loop supply chain. Product recovery processes such as reuse with small repair, refurbishing and remanufacturing are very promising strategies in a sustainable world, due to their profitability and green properties. Yet in these systems uncertain timing, condition and quantity of returns complicate balancing returns with demands. In servicizing systems, uncertainties are eliminated by leasing products since manufacturers monitor and maintain the leased products and take back their products at the end of the leasing period. Servicizing systems have different structures for different industrial sectors. This fact affects all decisions in a servicizing strategy. Two most important decisions are the durability of the products and the decision of when to give up leasing and sell the product as a remanufactured good. In our study, we are trying to determine optimal durability and optimal number of times a product is leased with respect to total profit. To this end, we formulate a maximization function that characterizes the relation between design factors and leasing parameters.

ÖZET

SÜRDÜRÜLEBİLİR TEDARİK ZİNCİRLERİ İÇİN HİZMET SATIŞI STRATEJİLERİNİN OLUŞTURULMASI

Teknolojinin hızla ilerlemesiyle birlikte, birçok gelişmiş ülkede ürünler hızlı ve çok miktarda tüketilmeye başlamış ve bu durum atık miktarını artırırken, doğal kaynakların da hızla azalmasına sebep olmuştur. Tedarik sistemlerini çevresel olarak sürdürülebilir uygulamalara dönüştürmeleri için birçok şirket, ürün geri kazanım aktivitelerini işletme stratejilerine dahil etmiştir. Ancak uzun vadede, ürün geri kazanımı bir kapalı döngü tedarik zinciri oluşturmak için yeterli olmayacaktır. Bu nedenle, müşterilere ürünlerin satılmasının yerine, ürünün işlevselliğinin satılmasına dayalı bir hizmet satışı stratejisini, tedarik sistemlerine dahil etmek zorunlu hale gelmektedir. Çevresel katkılar ve karlılık göz önüne alındığında, ürünlerin yeniden kullanılması işlemleri sürdürülebilirlik açısından oldukça umut vericidir. Ancak bu sistemlerde geri dönen ürünlerin kalitesi, miktarı, geri dönme zamanları ve geri dönen ürünlerin talebi karşılaması konuları belirsizlik yaratmaktadır. Hizmet satışının esas alındığı sistemlerde bu belirsizlikler ortadan kalkmaktadır. Bunun sebebi, müşteri ve servis sağlayıcı arasında bir sözleşme ile, üreticinin ürünlerini takip etmesi, gerektiği takdirde bakım ve onarımını yapması ve ürünlerin kira anlaşmasının bitiminde üretici tarafından toplanması konularının kesinlik kazanmasıdır. Hizmet satışının sistemleri, birbirinden farklı endüstriyel sektörler için farklı yapılar sergilemektedir. Bu durum hizmet satışı stratejilerini doğrudan etkiler. Bu noktada, üretilen ürünün ömrü ve belli bir sayıda kiralama tekrarından sonra yeniden imal edilmiş ürün olarak ikinci el piyasasına sürülmesi konuları oldukça önem kazanmaktadır. Bu çalışmada, farklı sektörler için karlılık baz alınarak, ürünler için en uygun yaşam ömrü ve en uygun kiralama sayısı belirlenecektir. Bu amaç doğrultusunda, tasarım elemanları ve kiralama parametreleri arasındaki ilişkiyi nitelendirecek fonksiyonlar tanımlanmıştır.

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LIST OF SYMBOLS

c^d	Disposal cost of a product
$C(N)$	Design investment and manufacturing cost of a product with durability N
c_r	Fixed cost of remanufacturing for leased products
D	Market size
k	Index for lease periods
m	Power of variable design cost
n	Number of leasing times
n^*	Optimal number of leasing times
N	Useful life of a product
N^*	Optimal useful life of a product
P_0	Price of a new product
P_0^*	Optimal price of a new product
α	Percentage of a retained value of a product after one year
$(1-\alpha)$	Annual depreciation rate of a product
β	Ratio between the price of remanufactured products and new products

LIST OF ACRONYMS / ABBREVIATIONS

CLSC	Closed Loop Supply Chain
PSS	Product Service Systems

1. INTRODUCTION

With the development of technology, product lifecycles have significantly decreased in industrialized countries causing a significant increase in consumption of products. As a consequence, consumption of energy, and natural resources, and waste accumulation rates have reached critical levels triggering a worlwide effort in regulating waste and energy consumption. To this end, many countries have passed new laws and regulations supporting environmental conscious efforts. An increasing number of companies have started to change their business strategies to include product recovery activities in order to accomplish sustainable development. Recent efforts have shown that encouraging product recovery options are necessary yet not adequate in achieving sustainable development. The supply chain has to be redesigned to close the loop where return rates of end-of-life products are maximized and product life-cycles are increased without compromising technological improvements. To this end, a supply chain based on providing functions of products rather than product itself becomes important.

The management of all used and discarded products are summarized as the product recovery management. Brito [1] classifies product recovery into six groups which are reuse, refurbishing, remanufacturing, part recovery, recycling and incineration. In direct reuse, refurbishing, remanufacturing and part recovery, physical attributes of used products are retained while in the others, products identity and functionality are lost as the objective is material recovery. Retaining the physical attributes of the product saves much of the effort for producing a new one. Although, all of the activities defined by Brito [1] are geared towards minimizing waste, recycling activities are not as desirable as recovering the physical attributes in terms of carbon emissions and workload minimization. Therefore we consider that product recovery options comprise of direct reuse, refurbishing, remanufacturing and part recovery while recycling and incineration can be considered as material and energy recovery activities.

Product recovery presents a business model which makes production of materials both economically and environmentally attractive. Cost reductions, providing new business strategies, targeting new market segment and generating spare parts can be considered as benefits of product recovery activities from economic point of view. Raw material cost and processing cost are decreased by usage of returned products. Additionally, firms can target the second hand market by competing with products at lower prices. Moreover, remanufacturing strategies are being taken to bequeath a better environment for future generations. Large energy savings, cutting down on the amount of air pollution, and extending the lives of landfills are the consequences of reprocessing used products.

In order to undertake product recovery options, taking back used products has become a very important issue. Almost all challenges in those systems are caused from uncertainty of returned goods. Uncertainties are faced in timing and quantity of returns, balancing returns and demands, quality of returned products and complications rising from material matching [2]. Servicizing strategies have a potential in reducing these uncertainties. Servicizing is defined as selling functionality of products or selling services, instead of selling the product itself. A servicizing contract makes manufacturers service providers and enables them to closely monitor timing, quantity and quality of returned products. Furthermore, servicizing is an advantageous business model when considering revenue and environmental facts. First of all, servicizing systems have great potential to dramatically reduce industrial impacts on environment, because servicizing has a strong motivation for extending the product life cycle.

Traditionally, lots of firms in the industry have concentrated on forward supply chains based on manufacturing and selling products to the customers. Since customer is the last entity in the system, manufacturers are not responsible for any activities after selling the products. New laws and regulations, and the changing attitude of consumers towards preserving the environment drive manufacturer to take responsibility of their products at the end of their life. As the business environment becomes more competitive, firms have started to concentrate on collecting used products as well as the forward chain. The collection of used product, viz reverse supply chain, is the process of monitoring and taking back options for used products. Both forward and reverse chains form a closed loop supply chain (CLSC) which is defined as the design, control, and the operation of a system to maximize value creation over the entire life-cycle of a product with dynamic recovery of value from different types and volumes of returns over time [3]. Since product recovery activities such as used product acquisiton, product classification, remanufacturing, and repair are crucial for CLSC, facilitating a well controlled product recovery will provide a less problematic loop. According to Amezquita and Bras [4], the product and process design approach would be the most effective way to achieve facilitating and boosting product recovery.

Designing a product for remanufacturing includes design for used product collection, which is also known as the core collection in the literature, testing, dissasembly, reprocessing, reassembly, and reliability for multiple life cycles. Since, the servicizing business includes remanufacturing activities and aims at providing products to operate multiple life cycles, products can be more profitably servicized via proper design. Therefore, design is one of the most important issues that effects leasing parameters such as the durability of the product, the price of the leased product, duration of leasing period, and the number of times that the product is leased.

Fully closed loop chain structures such as servicizing are still not adequately efficient since recovering worn out products is not profitable. We design a supply chain model where the manufacturer either sell its products or leases them for multiple times. In addition to selling a product or leasing it during its entire life, a manufacturer can choose to lease the product multiple times and then sell it as a remanufactured good. The features such as durability, price, usage features etc. have different characteristics for different products and sectors. In this new supply chain structure, we are trying to compose a profit function that takes these characteristics into account. Using this function, we will be able to determine the relationship between design and leasing parameters. Also we will consider the optimal number of leasing a product before selling it as remanufactured. Furthermore, design for multiple life cycles are integrated and the effect of this design function is evaluated by means of durability and profitability. Since an extra emphasis on design represents extra cost in production, the price of the product becomes critical for profit maximization. We will consider the price and cost relation in a dynamic pricing setting.

The following chapters of the thesis are organized as follows. A detailed literature overview about product recovery, design issues in product recovery and servicizing are given in Chapter 2. In Chapter 3, the objectives of the thesis are described. After defining the main problem, model formulations are given in Chapter 4. In Chapter 5, numerical analysis for the proposed methods are illustrated. Finally the conclusions are drawn in the Chapter 6.

2. LITERATURE REVIEW

In our study, our problem has two main components which are servicizing systems and product recovery systems. In Section 2.1, literature on product recovery of returned products is discussed. The economical and environmental benefits are introduced as well as challenges and uncertainties highligted. Also supply chain structure of product recovery systems and the importance of design issue are explained. Literature on servicizing systems is discussed in Section 2.2. Firstly, the importance and basic features of servicizing systems are introduced. After mentioning about benefits and challenges of servicizing, used models and studies in the literature are discussed.

2.1. Product Recovery

In recent years, environmental issues such as consumer waste, shortage of natural resources and greenhouse effect have become the biggest problem that threaten the earth. The US Environmental Protection Agency (EPA) states that the amount of waste generated in the USA reached 196 million tons in 1990s from 88 million tons in 1960s [5]. This fact is also valid for other industrialized countries. As a result of continuous increase in consumer waste, many countries design legislations to force the manufacturing industry to adapt sustainable production methodologies.

For managing a sustainable production by the means of reducing waste and resource consumption, recycling was considered at first. However, recycling process is not the only way to achieve sustainability and the most of the time it is not the best solution [6]. Product type is the most important feature in deciding which end-of-life option has to be performed. Graedel and Allenby [7] proposed a priority list that can be used as a guide. The options from the most preferable to the least preferable are “reducing materials content, reuse components/refurbish assemblies, remanufacture, recycle materials, incinerate for energy (if safe), dispose of as waste." At this point, "product recovery management" has taken place in order to solve this problem. Thierry *et al.* [8], and Ferrer [9] consider product recovery management,which aims to recover the economic and ecological value as much as possible by managing all used and discarded products, components, and materials. A simplified illustration of the relationships between different product recovery options can be seen in Figure 2.1.

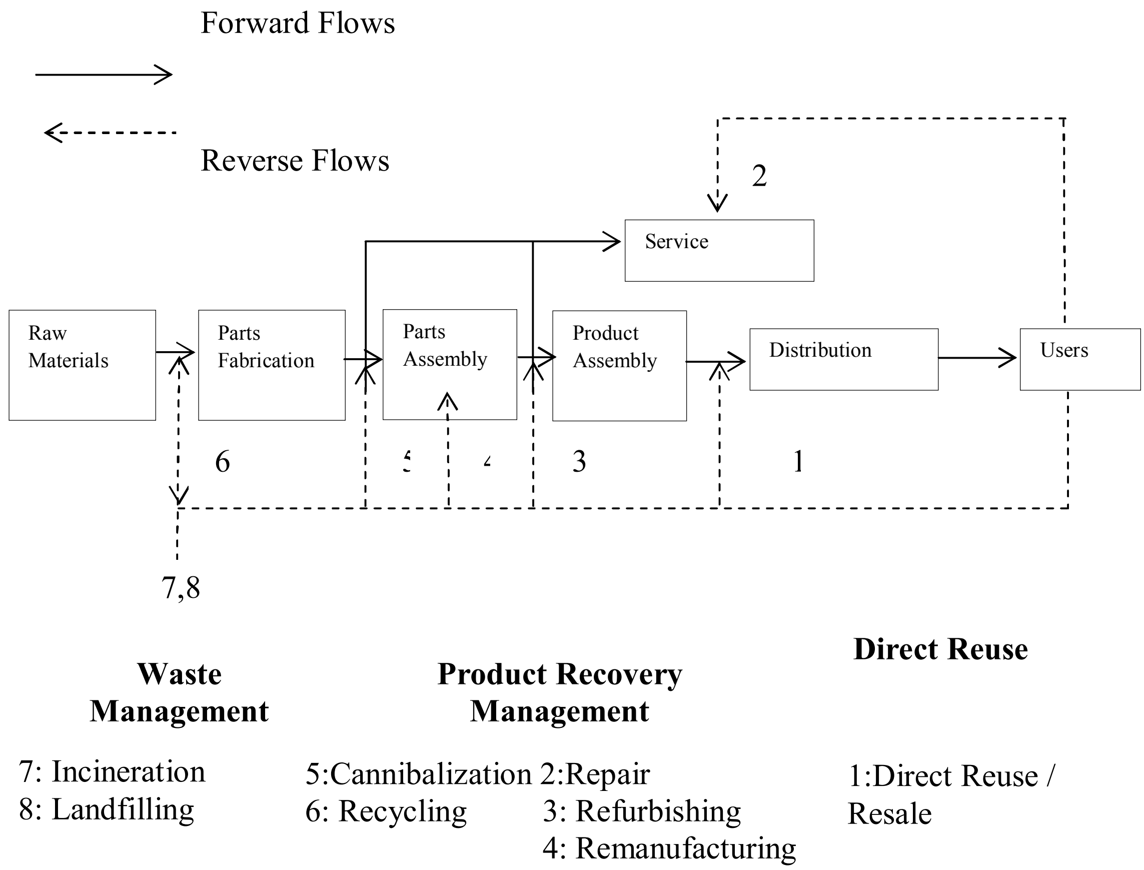

Figure 2.1. Product recovery in the closed-loop supply chain. [8]

The most preferrable product recovery activities which are direct reuse, refurbishing and remanufacturing are defined by various authors in the literature. According to Keoleian *et al.* [10] "Reuse is the additional use of a component, part or product after it has been removed from a clearly defined service cycle". Rodgers *et al.* [11] defines refurbishment "a product recovery process which a product is cleaned and repaired to return it to a 'like new' state". Sundin [12] defines remanufacturing as "an industrial process whereby products, referred to as cores, are restored to useful life. During this process, the core passes through a number of remanufacturing operations, for example, inspection, disassembly, part reprocessing, reassembly, and testing to ensure it meets the desired product standards."

The benefits of product recovery activities are both, economical and environmental. Cost reductions, providing new business strategies, targeting new market segment and generating spare parts can be considered as benefits of product recovery activities from an economic point of view [13]. There is a potential decrease in the cost of producing remanufactured products with respect to the new product cost. First of all, raw material cost is decreased by usage of returned products. Additionally, processing costs are reduced because of low reprocessing times than manufacturing a new product . Also, the low price of remanufactured products enables firms to target the second hand market and compete with products at lower prices. Therefore remanufacturing activities can be profitable. According to a case study by Toffel [14], Xerox Corporation saves millions of dollars within a year with remanufacturing activities.

The environmental benefits can be grouped as; energy conservation, raw material conservation, landfill space conservation and air pollution reduction. Reusing the product and reprocessing the waste materials decrease the raw material consumption. Energy consumptions and emissions while manufacturing new products can be decreased by extended life time of products. Seitz and Peattie [15] give an example from automotive part remanufacturing which reveals that the energy consumption of a new starter is more than eleven times of the energy required to remanufacture the product. Hilden *et al.* [16], compare manufacturing, remanufacturing and recycling in terms of usage of energy and raw material, emissions and the amount of waste. For a washing machine, energy used for manufacturing, remanufacturing and recycling are 1182 kWh, 16 kWh, and 20 kWh, respectively. Greenhouse gases emission rates are 214 kg CO_2 for manufacturing, 2.5 kg CO_2 for remanufacturing, and 7 kg CO_2 for recycling,. Also amounts of waste for manufacturing, remanufacturing, and recycling are 160, 1.1, and 13 kg, respectively. As a result we can say that remanufacturing is superior to the recycling when considering resource usage, environmental effects and waste management.

Product recovery system is generally very different in comparison to the traditional manufacturing system. Due to variability and uncertainty in processes, product recovery business is a complex business [2,15]. The high level uncertainty is mainly caused by two factors: the quantity and the quality of returned products, viz cores [17]. According to Krikke et al. [18], sources of the cores for remanufacturing can be categorized in four types which are end-of-life returns, end-of-use returns, the commercial returns and the reusable components. End-of-life returns are products that are taken back to the original equipment manufacturer at the end of their useful lives. End-of-use products are returned by customers after a limited period. This type is called end-of-use returns and they are usually sold again in the market as remanufactured goods. The commercial returns are retrieved in a short while after being sold according to warranty terms. Reusable components are the last type of sources of cores. Reusable components are not the actual products but they contain or carry the corresponding products such as plastic bottles.

Uncertainty of quality and quantity of cores also create variations regarding capacity requirements as well as the yield of the process. Guide [2] presents all seven characteristics of reprocessing as the uncertain timing and quantity of returns, the need to balance returns of used products with demand for remanufactured products, the need for disassembly of returned products, the uncertainty in materials recovered from returned items, the requirement for a reverse logistics network, the complication of material matching restrictions and, the problems of stochastic routings for materials for remanufacturing operations and highly variable processing times. These uncertainties make it hard for many remanufacturing companies to balance supply and demand. A survey conducted with 48 remanufacturing companies by Guide [2] showed that more than half of the companies had no control over the timing or the quantity of the returns. Therefore, remanufacturing business needs to develop new kinds of relationships between manufacturer and the customer which reduces the challenges faced while balancing supply and demand.

Product recovery operations are the integral parts of the closed-loop supply chain systems. The traditional supply chain includes activities from producer to customer, while the reverse chain includes flows from customer to producer. Then these flows are closed by product recovery operations [18]. The relationship between manufacturer and customer has an important role when managing closed loop supply chains. Some relationships depend on high level of collaboration and trust, while some of them have week linkage. Ostlin [13] has identified seven different type of structural relationships that can be helpful understanding customer relations management:

(i) Ownership-based : Manufacturer keeps ownership and customer buys function of the product such as rent or lease. Here, service contract provides high level of control for both sides of the contract.

(ii) Service contract : Manufacturer and customer are bound with a service contract which includes remanufacturing.

(iii) Direct-order : Remanufacturer take back the product from customer and after being remanufactured, customer gets the same product if product is still able to perform the same tasks.

(iv) Deposit-base : Customers act like a supplier to the remanufacturer by returning used product as an obligation.

(v) Credit-based : Customers that return used products receive credits for using as a discount when buying a remanufactured product again.

(vi) Buy-back : The remanufacturer buys the necessary used products from different entities like end user or a scrap yard.

(vii) Voluntary-based : The supplier or a customer can give the used products to the remanufacturer.

Among the relationships between manufacturer and the customers, because of high control level and the close relationship, the first two types of relationships seem to be the most preferrable for a successful closed loop supply chain. In our opinion, the service-based relationships seem to have the highest control of products when considering the complexity of recovery systems. In addition Thierry *et al.* [8] aggree with us by stating “companies that lease their products are generally in a more favourable position than companies that only sell products. Lease companies usually have more information on the quality and return of used products”.

Closed loop supply chain is defined as the design, control, and the operation of a system to maximize value creation over the entire life-cycle of a product with dynamic recovery of value from different types and volumes of returns over time [3]. Since product recovery activities such as used product acquisiton, product classification, remanufacturing, and repair are the crucial for a CLSC, facilitating product recovery will provide less problematic loop. According to Amezquita and Bras [4], the product and process design approach would be the most effective way to achieve facilitating and boosting product recovery. In the study of Sundin [19] , complexity of the product, fastening methods of the process, assembly and disassembly and part fragility are all referred as design related issues. Another study of Sundin [20] , and Barker and King [21] concluded that if the products are not designed to be reused or remanufactured, remanufacturing activities are not beneficial because of the high inspection, disassembly, reprocessing and testing costs.

Demirayak *et al.* [22] considered the effects of design for remanufacture on the profitability of the CLSC system. They observed that the increase in the profits meets the increase in the costs due to long production and design time. Profitability of the remanufacturing increases when decrease in the costs as a result of the good design is larger than the increase in the design and production costs. However, when the decrease in the remanufacturing cost can not compensate the increase in the production and design cost, the system profits decrease.

Zwolinski and Brissaud [23] propose a design approach which helps to develop products with properties considering both their remanufacturing and reuse quality and their profitability. Designers can use this approach to target a remanufacturing and reuse strategy according to the policy of the company. Also they highlighted external and internal criteria for product parameters. Their approach could be used for designing new products as well as redesign for current products to facilitate remanufacturing activities. Shu and Flowers [24] have developed a software program to determine the right fastening and joining methods. Bras and Hammond [25] developed design for remanufacturing metrics, applied to several product case studies, which indicate how well a product is designed for remanufacturing.

Umeda [6] discussed with designers and researchers from major Japanese industrial companies and determined key design elements for severeal types of product groups. According to the study, design elements can be classified into three groups. The first group is the traditional design elements which are well studied and already practiced in companies. These elements are design for disassembly/seperation, design for reliability, design for simplification/dematerialization and design for recycling. The second group is also traditional but becoming more important which are design for standardization / compatibility, and design for maintenance. The last group shows the design elements which has not been studied but are crucial for reverse manufacturing [6]. Here, design for upgrade, design for modularization, design for cleaning/inspection, design for reuse, and design for quality/longer life are considered as noteworthy to study.

2.2. Servicizing

Sustainable development can be reached by closing the supply chain loop. There are two main approaches to achieve this goal. The primary approach is to perform extensive product recovery to convert a traditional supply chain into a closed loop supply chain with remanufactured product sales [12]. An alternate way of closing the material flow is to focus on functional sales instead of physical product sales [11]. There are several terms for this fact in the literature and the practical life as, ‘product-to-service’’ , ‘‘servicizing’’ ‘‘functional sales / economy’’, ‘‘product service combinations’’, and ‘‘product service systems (PSS)’’ [26].

Makower [27] defined servicizing as “selling a service instead of product” and Fishbein *et al.* [28] defined servicizing as “selling the function of a product or the service it produces, rather than the product itself”. At the very basic level, these definitions are necessary yet not sufficient to define servicizing. A more comprehensive definition is given by White *et al.* [29], where manufacturers responsibility extends through the end of products’life cycle.

Servicizing is a new type of economical model that focuses on providing the functionality of the products. Basic features of this new business model are described in the study of Toffel [30]. Employing the servicizing paradigm, companies can sell the function of the product rather than the product itself, and maintain ownership of the product throughout its useful life. Under servicizing scenario, the manufacturer would be more inclined to manufacture a more durable product. This would result in fewer products manufactured, less resources employed, and less waste created [31]. Additionaly, customer pays a leasing price per leasing period and the manufacturer is responsible for maintaining and repairing its products at no additional cost to the customer. On the other hand, Blackburn *et al.* [32] showed that the time value of returned products varied widely across industries and product categories. Time sensitive products such as PCs lose value at a fast rate, while less time sensitive products like diesel engines preserve high percentage of its value. According to Fisher [33], functional sales are the best for products which are less time-sensitive and have low marginal values of time.

Evidence shows that servicizing is highly correlated with higher and more stable profits [34]. Suppliers can maintain sustainability while getting more profit by performing servicizing activities as this strategy extends the efficiency and value of their products [35]. Furthermore, servicizing systems have great potential to dramatically reduce industrial impacts on environment [28,29]. In the study of Stahel [36], it is mentioned that a servicizing system will help reduce the amount of material used and hence the energy consumption, because servicizing is introduced for extending the end-of-life of the product. Thus, selling the functionality of the product rather than the product itself entails an opportunity to decrease consumption of resources.

Rothenberg [35] made a cross-industry research that evaluates how three different companies implement strategies for more environmental friendly practices. According to this study, all three companies have attracted new customers with their new business strategy. Also, they are able to build closer customer relations. This relationship between customer and manufacturer provides manufacturer a stronger customer loyalty. Uchihira *et al*. [37] also conducted a research with 40 case studies. The results show that product-based servicizing contributes products to have better quality while raising the market share for the participant firms.

On the other hand, companies face some challenges worth mentioning, when implementing the servicizing model. Calthrop and Baveja [38] showed that only 21 percent of companies have achieved success with their service strategies. Rothenberg [31] discussed both advantages and challenges of moving a business to servicizing. These challenges are mostly caused by sales staff. It is hard to convert the traditional mindset from selling more products to leasing more products. In order to deal with this challenge, Rothenberg observed six strategies based on the interviews made with companies' managers and employees. These suggestions are "building on existing strengths, redefining the basis for profit in contractual agreements, communicating the new business model, changing incentives, acquiring new skills and highlighting environmental advantage" [31]. Again, Uchihira *et al*. [37] pointed out the challenges as: difficulties in designing a feasible service scheme in the organization of a manufacturing company, insufficient experience to manage service business, big process gaps between product businesses and service businesses.

Recently, some models considering CLSC with servicizing are proposed in the literature. Park *et al* [39] develop a generic model that optimizes service value from the perspective of the service delivery system by means of service quality and service cost. This study is conducted to maximize service value of a service organization such as a dental care facility by establishing a mathematical model. Wu and Ryan [40], present an integrated model that takes into account both the maintenance decisions and the inventory management decisions of a closed-loop supply chain in the context of a product–service system to minimize the total cost rate. Maintenance decisions are modeled as a Markovian model, while continuous review base stock policy is adopted for the inventory management of the closed-loop supply chain. They assigned an indicator for quality difference between preventively replaced products and failure replaced products, and they concluded that with higher quality differences, preventive replacements will be more often, and the stock level will be higher.

Demirayak *et al.* [22] compared traditional, sales with remanufacturing and servicizing supply chain structures by modeling G/G/1 queuing network and concluded that leasing is the most profitable of all when lease payments are high enough to compensate acquisition and process costs. According to this study, it is reasonable to incorporate servicizing systems with product recovery mechanisms. Additionally, this study shows that profitability starts to decrease after leasing a product number of times even for the highest lease payments which is equal to the new product price.

Literature research shows us there is no study about an integrated structure with an endogenous pricing mechanism that considers both design impact and optimal leasing periods. In our model, we have considered a hybrid supply chain system where products are sold to the market both as new or as remanufactured good or leased for a certain number of fixed periods. Our primary aim is to find an optimal value for product price, optimal durability and optimal number of leasing times before product loses its functionality. Design impact on durability and ease of remanufacturing are evaluated using a profit maximization model.

3. OBJECTIVES

The objective of this thesis is to investigate the potential benefits of functional sales over product ownership with respect to closed loop supply chains. To this end, a hybrid supply chain is designed where products can be either sold as new, or leased for multiple periods. Here, leased products can be remanufactured and sold to the second hand market after several leasing periods or can be leased throughout its useful life. Leased products are brought to the like new condition by product recovery after each leasing period, if they will be leased again, or they are brought to some acceptable quality level for the second hand market. Products that complete their total life are disposed of.

For this system, it is important that a product survives for long years and is remanufactured easily. Manufacturing such a product requires high investment costs for design and production efforts. Furthermore, this investment increases faster as the durability of the product is increased, resulting a high total product costs. Then, product prices increase the profitability of the system.Yet, higher product prices will decrease demand. Thus, the new product price determination is one of the most importans decisions, since, it reflects leased product price and remanufactured product price. Our aim is to measure this balance in a dynamic pricing setting.

To this end, we construct a profit function and investiate the results with respect to design and depreciation parameters. Also we evaluate the recovery effort for second hand market product at the same time. Here, we try to find optimal values for new product price, lifetime of a product and maximum number of times that a product can be leased and investigate the changes in profitability with respect to annual depreciation of products and remanufacturing efforts. Then, determining optimal product lifetime as a function of price and design investment becomes the primary objective of this study.

4. MODEL AND PROBLEM DEFINITION

4.1. Problem Definition

Traditional supply chains concentrate on manufacturing and selling products to customers. They are not concerned about the end of life issues of the products they produce. Government legislations and changing consumer behavior trigger the industry to collect, remanufacture, and sell used products. As the industry started to become more environmentally sensitive, firms have concentrated on collecting and reprocessing their own used products as well as the forward chain. Both forward and reverse chains form a closed loop supply chain (CLSC) which is defined as the design, control, and the operation of a system to maximize value creation over the entire life-cycle of a product with dynamic recovery of value from different types and volumes of returns over time. Since servicizing strategy is referred as one of the best ways of getting over challenges in product recovery, it is sensible to consider integrating sercivizing strategies in CLSC. In the literature, there are lots of studies on selling new products, selling remanufactured products or selling functionality of products vis. leasing. Yet there is no such study that integrates these three different supply chain structures.

In the study of Demirayak *et al.* [22], a supply chain model based on the multi-period leasing of products has been studied, where products are leased for a fixed period multiple times until the end of their useful lives. The study shows that leasing repetitively renders a concave profit function, indicating that profitability of leasing diminishes after a certain number of times a product is leased. In our study, we consider a model that measures the alternative decision of selling the product when it is more profitable.

In the proposed model, the manufacturer has different options at each decision epoch. Once a product is produced, it can either be sold directly in the new product market, or it can be leased a number of times for fixed periods. In the second case it is either leased until the end of the product's useful life or until it is sold in the second hand market as a remanufactured product. When a product is leased until the end of its useful life it is disposed of. The difference of our study from other studies about leasing is integrating the decision of selling used products to the second hand market as remanufactured goods.

Leasing price of a product is determined according to its depreciation rate. We define depreciation as an indicator which shows how the price of a product declines as it ages in the absence of inflation. Higher depreciation rates lead higher economic value loss, thus the lease price of a product which depreciates quickly will be higher than the lease price of a product which depreciates slowly. When leasing a product, we assume that there is no demand differentiation between new and remanufactured products as both product categories are capable of performing the same tasks, and only the functionality of the product is sold. Leased products are always returned to manufacturing facility after it completes its lease period, assuming that there is no acquisition cost for manufacturer, and the product replacements will be done instantaneously. Products which are decided to be leased in the next period go through a remanufacturing process which brings the products into as good as new shape. However, a product returning from a leasing cycle may be sold in the second hand market as a remanufactured product. In this case, we differentiate its price using a discount factor. This discount factor can also be considered as the level of remanufacturing effort. We assume that, as remanufacturing effort increases, price of remanufactured good in the second hand market will be increased too, since high effort leads to a higher quality product.

In our model, we use a dynamic pricing mechanism which decides for the new product price. We assume that market size is a constant and number of customers in the model is determined via the new product price by subtracting the fixed market size. Additionally, product’s useful life indicates the maximum number of times a product can be leased. In other words, the product is leased annually. We also assume that at the end of a lease term, each product is returned to undergo a remanufacturing process.

In previous sections we mentioned that proper design increases total profitability, since it decreases remanufacturing costs and extends the durability of a product. To this end, we assign a design investment cost and evaluate the design effect on optimal durability.

4.2. Model Formulation

In the model, we use a dynamic pricing strategy that determines flexible prices for a new product based on a current market size. Under such pricing strategy, manufacturer can choose optimal price for new products while reaching the number of demands which makes model the most effective by means of profitability. Here we assume that the market size is constant and symbolized as D, while new product price is shown as P_0. Demand for the single product is computed by (D - P_0). Since D is a constant value, as P_0 increases, demand for the product will be decreased. On the other hand, as P_0 increases, amount of money which can be gained from a single product will be increased as well. So model will decide optimal price of new product P_0^* that maximizes the profit. Here, we model the revenue from new product sales as:

$$(D - P_0)(P_0 - C(N)) \tag{4.1}$$

Here we defined $C(N)$ as the design and manufacturing cost of a product with durability N. We assume the durability of a product can be increased by design investment. Thus an increase in design cost indicates an improvement that leads to longer durability. Therefore, $C(N)$ is considered as an increasing function of N. We set N as a decision variable where N^* shows optimal durability of a product for the parameters of interest.

The second component of the model is the revenue obtained from leasing. Similar to new product sales, the number of customers is decided with (D - P_0). Price of the leased product is determined over the new product price and the depreciation rate of the product. Here, α is the percentage of retained value of a product that is still preserved after one year customer usage, then the price of the used product would be $P_0\alpha$. Customer is willing to pay $P_0 - P_0\alpha$ for one year usage, thus leased product price is computed by $P_0(1- \alpha)$, where (1- α) is the depreciation rate of the product. Here customer pays higher values for products which highly depreciate, and pays lower prices for products which depreciate slowly. A product which has durability N can be leased N times. Although, studies on servicizing showed that leasing a product during its entire life has a concave behavior where at some point profitability decreases. Therefore we need to define a decision variable which shows the number of times that a product is leased. Here n is the number of times that a product is leased where $0 \leq n \leq N$. The value of n is zero at the beginning of a manufacturing period. Manufacturer keeps track of the history of the products so that, after each return from leasing period, the value of n is incresed by 1. When n reaches to N, the product is no longer suitable for leasing, then it is disposed of. We find an optimal n^* value which shows the optimal point where leasing is still superior to selling the product to the second hand market as remanufactured good. Profit from leasing can be computed by using demand $(D - P_0)$, leasing price $P_0(1-\alpha)$, and number of times that product is leased n.

Expenses for leasing a product are accumulated from remanufacturing processes. Here we define two different remanufacturing costs. First remanufacturing cost is the variable cost per product which is the inverse function of $C(N)$. Since we assume, design investment facilitates the remanufacturing processes, it would be meaningful to use a decreasing function of N. In order to prevent drastic reductions, we took the m^{th} power of the variable remanufacturing cost where $0 < m < 1$. Here we need to multiply variable cost with n, since the product go through the recovery process n times. Another cost for remanufacturing is the c_r which represents fixed cost of remanufacturing. It includes the cost of other operations that design can not facilitate. In order not to ignore the age of the product, we multiply c_r with k which is an indicator of n. Leased product revenue is as follows:

$$P_0(1-\alpha)n(D-P_0) - \left(\sum_{k=1}^{n}\left(\frac{1}{C(N)}\right)^m + c_r k\right)(D-P_0) \tag{4.2}$$

Selling remanufactured products to the second hand market after leasing n periods is the another component of our model. The important issue here is to determine the price of the product in the second hand market. The second hand market price should be less than new product price so that customers will be willing to buy second hand product. In the problem definition section, we say that products which are sold in the second hand market have less quality than products which are leased. It means recovery effort for the second hand market leads to less costs. Here, we define β as a parameter which indicates both effort of recovery and the quality of remanufactured product. As the remanufacturing effort increases, price of remanufactured good in the second hand market will be increased too, since high effort leads to a higher quality product. Other costs are similar to leasing, except the cost is not repeated *n* times. Second hand market revenue is described in Equation 4.3.

$$(D-P_0)(P_0\beta-((\frac{1}{C(N)})^m+c_r n)\beta) \tag{4.3}$$

At some circumstances, leasing a product throughout its entire life would be the most profitable choice among the other structures. In such a scenario, $n^*=N$ and the manufacturing facility disposes of products at *N*. We set a constant disposal cost c^d for a single product. Therefore the disposal cost of entire system can be written as:

$$c^d(D-P_0) \tag{4.4}$$

Finally, we earlier stated that, designing and manufacturing a single product which has durability *N*, is *C(N)*. Thus the total cost of designing and manufacturing is:

$$(D-P_0)C(N) \tag{4.5}$$

When all components are combined a profit maximization model which includes new product sales revenue, leased products revenue, remanufactured product revenue,disposal cost, and the design cost is obtained as given in the Equation 4.6. Here the profit function is optimized with respect to which decides optimal values for *N,n* and P_0.

$$\max_{N,n,P_0} = \left\lfloor \frac{N-n}{N} \right\rfloor (D-P_0)(P_0 - C(N)) +$$
$$P_0(1-\alpha)n(D-P_0) - (\sum_{k=1}^{n} (\frac{1}{C(N)})^m + c_r k)(D-P_0) +$$
$$\left\lceil \frac{N-n}{N} \right\rceil \left\lceil \frac{n}{N} \right\rceil \left\{ (D-P_0)(P_0\beta - ((\frac{1}{C(N)})^m + c_r n)\beta) \right\}$$
$$-\left\lfloor \frac{n}{N} \right\rfloor c^d (D-P_0) - \left\lceil \frac{n}{N} \right\rceil (D-P_0)C(N) \qquad (4.6)$$

A concise description of all variables and parameters are given in Table 4.1.

Table 4.1. Definitions of decision variables and parameters.

Decision variables:
P_0*:* Sales price of a new (manufactured) product
N: Durability of a product (max.number of times a product can be leased)
n: Number of periods a product is leased where $0 \leq n \leq N$
Model Parameters:
α: Percentage of a retained value of a product after one year $0 \leq \alpha \leq 1$
β: Proportion of leasing efforts to remanufacturing efforts $0 \leq \beta \leq 1$
c_r*:* Fixed cost of recovery for leased products
$k\ c_r$*.:* Cost of recovery at k^{th} period where *k*=1,2,...*n*
c^d*:* Disposal cost of a product when *n*=*N*
D: Market size
Cost Functions:
C(N): Design cost of manufacturing a new product with durability *N*
$(\frac{1}{C(N)})^m$: Variable cost of remanufacturing for leased products with design investment *C(N)* where 0<*m*<1

The proposed model structure will be changed by the decision of number of leasing times. In other words, problem has three different cases when *n*=0, $1 \leq n <$ N, and *n*=N.

In the cases of n=0, the structure of the maximization problem would be:

$$\max_{N,P_0} = (D-P_0)(P_0-C(N)) \tag{4.7}$$

In the cases of $1 \leq n <$ N, the structure of the maximization problem would be:

$$\max_{\text{N,n,P}_0} = P_0(1-\alpha)n(D-P_0)-(\sum_{k=1}^{n}(\frac{1}{C(N)})^m + c_r k)(D-P_0)+$$
$$\left\{(D-P_0)(P_0\beta-((\frac{1}{C(N)})^m + c_r n)\beta)\right\}-(D-P_0)C(N) \tag{4.8}$$

In the cases of $n = N$, the structure of the maximization problem would be:

$$\max_{N,P_0} = P_0(1-\alpha)N(D-P_0)-(\sum_{k=1}^{N}(\frac{1}{C(N)})^m + c_r k)(D-P_0)$$
$$-c^d(D-P_0)-(D-P_0)C(N) \tag{4.9}$$

In our model, we evaluate different levels for N, so that the model can decide for an approximate value for N^*. In order to understand the behavior of the model, we can look at the derivatives of these three functions. Since we have three different structures, we will analyze them seperately, and approximately by assuming the given function is continuous and differentiable.

When $n = 0$,the structure of function will be $f(N,P_0)=(D-P_0)(P_0-C(N))$ where $N > 0, P_0 > 0,$ and $D > P_0$.

$$\frac{\partial f}{\partial N} = -(D-P_0)C'(N) \tag{4.10}$$

Our first assumption was $C(N)$ is an increasig function of N. Since the first derivative of an increasing function renders positive values, $\frac{\partial f}{\partial N} < 0$, for all values of $N > 0$. Therefore

$f(N, P_0) = (D - P_0)(P_0 - C(N))$ is a decreasing function with respect to *N*. The first derivative of *f* with respect to P_0 is given in Equation 4.11:

$$\frac{\partial f}{\partial P_0} = D - 2P_0 + C(N) \tag{4.11}$$

Function of $\frac{\partial f}{\partial P_0}$ has not a unique behavior like in $\frac{\partial f}{\partial N}$. This function has a local maximum which makes $D - 2P_0 + C(N) = 0$. We can clearly say that this function is an increasing function between $(0, \frac{D + C(N)}{2})$, since the P_0 values less than $\frac{D + C(N)}{2}$ leads $\frac{\partial f}{\partial P_0} > 0$, while decreasing in the interval $(\frac{D + C(N)}{2}, D)$ since the P_0 values more than $\frac{D + C(N)}{2}$ leads $\frac{\partial f}{\partial P_0} < 0$.

Since our model is a maximization model, we would like to have a concave function the function will be concave which means the second derivatives are negative. Following equations will give the second derivatives with respect to model's decision variables.

$$\frac{\partial^2 f}{\partial^2 N} = -(D - P_0)C''(N) \tag{4.12}$$

$$\frac{\partial^2 f}{\partial^2 P_0} = -2 \tag{4.13}$$

For the proposed model, we have to choose a function of *C(N)* where the second derivative of *C(N)* > 0. Then the proposed maximization model would be concave function.Therefore, the second derivative of *f* with respect to *N*, and the second derivative of *f* with respect to P_0 are negative, meaning that *f* is a concave function which has a local maximum point.

When $0 < n < N$, the structure of function will be :

$$f(N,n,P_0) = P_0(1-\alpha)n(D-P_0) - (\sum_{k=1}^{n}(\frac{1}{C(N)})^m + c_r k)(D-P_0)$$

$$+\left\{(D-P_0)(P_0\beta - ((\frac{1}{C(N)})^m + c_r n)\beta)\right\} - (D-P_0)C(N)$$

where $N > 0, P_0 > 0,\ D > P_0$, $0<\alpha<1$, $0<\beta<1$, $0<m<1$

In order to check whether this function is concave for the decision variables, we have to consider the partial second derivatives of the function. The second derivatives of the variables are:

$$\frac{\partial^2 f}{\partial^2 N} = -\beta m(1+m)(D-P_0)(\frac{1}{C(N)})^{2+m} C'(N)^2 - m(1+m)n(D-P_0)(\frac{1}{C(N)})^{2+m} C'(N)^2$$

$$-(D-P_0)C''(N) + \beta m(D-P_0)(\frac{1}{C(N)})^{1+m} C''(N) + mn(D-P_0)(\frac{1}{C(N)})^{1+m} C''(N) \quad (4.14)$$

$$\frac{\partial^2 f}{\partial^2 n} = -c_r(D-P_0) \quad (4.15)$$

$$\frac{\partial^2 f}{\partial^2 P_0} = -2\beta\text{-}2(1\text{-}\alpha)\text{n} \quad (4.16)$$

The second derivatives of f with respect to n, and P_0, are negative in any condition. But the second derivative of f with respect to N, needs a condition in order to be negative. Since we do not evaluate the hessian matrix for this function, we can only say that, f is a concave function for the single decision variable, thus we can not comment on the concavity of the model with respect to all decision variables.

When $n = N$, the structure of function will be:

$$f(N,P_0) = P_0(1-\alpha)N(D-P_0) - (\sum_{k=1}^{N}(\frac{1}{C(N)})^m + c_r k)(D-P_0)$$

$$-c^d(D-P_0) - (D-P_0)C(N)$$

where $N > 0, P_0 > 0,\ D > P_0$, $0<\alpha<1$, $0<m<1$

The second derivatives of the function are:

$$\frac{\partial^2 f}{\partial^2 N} = -(D - P_0)C''(N) - (D - P_0)(c_r - 2m(\frac{1}{C(N)})^{1+m} C'(N)$$

$$+Nm(1+m)(\frac{1}{C(N)})^{2+m} C'(N)^2 - Nm(\frac{1}{C(N)})^{1+m} C''(N)) \qquad (4.17)$$

$$\frac{\partial^2 f}{\partial^2 P_0} = -2N(1\text{-}\alpha) \qquad (4.18)$$

Since the second derivatives are negative, f is a concave function at $n=N$ with respect to decision variables. Similarly to the previous function, we do not evaluate the hessian matrix for this function, therefore, f is a concave function for the single decision variable. We can not comment on the concavity of the model with respect to all decision variables.

5. NUMERICAL ANALYSIS

In this chapter of the thesis, numerical experiments are performed in Mathematica 9.0.1 for the proposed model in order to find optimal values of decision variables and to see model behavior with respect to design investment *C(N)*, the percentage of preserved value α, and the remanufacturing factor β .

The important issue here is the structure of the design cost. At the model formulation section we have stated *C(N)*, design cost of manufacturing a new product with durability *N*, is an increasing function of *N*. Initially, we set *C(N)* as N^2. In the numerical analysis, we first evaluate the profit function for the percentage of preserved value α of a product using values 0.1 to 0.9 in 0.1 increments. Simultaneously, we evaluate values for remanufacturing effort β 0.1 to 0.9 in 0.1 increments. We set the market size *D* to 1000. We also fix recovery cost c_r, disposal cost c^d, and the value of *m* where $c_r = 50$, $c^d = 100$, and *m*=0.5 respectively. Using these parameters, we calculate total profit function by increasing *N* from 1 to 20, for the optimal P_0 and *n* value, exhaustively.

5.1. Optimal Values of Decision Variables

Depreciation rate (1-α) and remanufacturing effort β are two important parameters that affects the model outcomes so that all cases correspond different values of decision variables. Since there are 9 values of both α and β, we have 81 different cases. For simplicity, we want to show the results in 9x9 matrices for the optimal values of durability, number of leasing times and new product prices. The experiments are contucted for $N \in [1, 20]$

As a result of these experiments, optimal values of decision variables are determined with respect to total profitability. First of all, optimal number of leasing times n^* are compared with each other for all 20 cases of *N*. Since we have 81 different case according to α and β , this procedure is repated 81 times, and the optimal values n^*, and N^* are determined following an exhaustive search.Note that all optimal values are local optima since only a finite search is possible.

The experiment results summarized in Table 5.1 for the optimal values of N and n given as (N^*, n^*).

Table 5.1. Optimal durability N^* and leasing time n^* values for different α and β.

β / α	**0.1**	**0.2**	**0.3**	**0.4**	**0.5**	**0.6**	**0.7**	**0.8**	**0.9**
0.1	12,11	12,11	12,11	12,11	12,11	12,11	11,10	11,10	11,10
0.2	11,10	11,10	11,10	11,10	10,9	10,9	10,9	10,9	10,9
0.3	10,9	9,8	9,8	9,8	9,8	9,8	9,8	9,8	9,8
0.4	8,7	8,7	8,7	8,7	8,7	8,7	7,6	7,6	7,6
0.5	7,6	7,6	7,6	7,6	6,5	6,5	6,5	6,5	6,5
0.6	1,0	1,0	1,0	1,0	5,4	5,4	5,4	5,4	4,3
0.7	1,0	1,0	1,0	1,0	1,0	1,0	1,0	1,0	3,2
0.8	1,0	1,0	1,0	1,0	1,0	1,0	1,0	1,0	1,0
0.9	1,0	1,0	1,0	1,0	1,0	1,0	1,0	1,0	1,0

According to Table 5.1, percentage of preserved value α has a significant effect on optimal decisions of (N^*, n^*). We observed that products with small preserved value are more appropriate for multiple time leasing. Since this table is prepared considering maximum total profits among durability level $1<N<20$, for all α and β set , we can clearly say that, leasing is more profitable for quickly depreciated products. Here we observed that, a product is designed to survive 12 years and it should be leased 11 times for $\alpha = 0.1$ and $\beta < 0.7$. After 11th lease period, product will be sold as remanufactured good in second hand market. For other values of α, optimal values of n^* can be seen from the table.

Leasing a slowly depreciated product is not a logical option as it reduces profitability. In cases where α is higher than 0.6, the product should be sold as it is manufactured. This only changes when $\alpha = 0.6$, $\beta \geq 0.5$ and $\alpha = 0.7$, $\beta = 0.9$. Even though remanufacturing cost is high, the second hand market value will cover this loss and the effect of β on profitability will change the decision which is to lease a product a couple of times.

Another result from the Table 5.1 is optimal value of n is always less than one unit from the optimal value of N, meaningly the model tries to avoid disposal activies since it leads extra costs. It would be more profitable to remanufacture a used product rather than to perform disposal activities.

In Table 5.1 we see a significant jump on decisions from α=0.5 to α=0.6 for $\beta < 0.5$ and from α=0.6 to α=0.7 for $\beta \geq 0.5$. In order to understand this significant change, the increments are changed from 0.1 to 0.01 between α=0.5 and α=0.7. Table 5.2 will give the (N^*, n^*) values for this interval with respect to total profitability similarly in Table 5.1. All decisions are made by considering maximum profit.

In this analysis, change in N^*, n^* values can be clearly observed because of the small increments of α. Selling product as it is manufactured is a optimal decision for products which preserves its more than 0.55 value annually. This situation is valid for small values for β, since leasing is not as profitable as direct selling at first place. Here we can observe the effect of remanufacturing effort β. For larger values of β, model choses to lease product for couple of times and then to sell it in the second hand market. Yet effect of β is not significant with compared to α. In these experiments, depreciation rate seems to be the major factor on deciding leasing times. Products which still preserves half of their economic value after one year is not suitable for leasing.

Table 5.2. Optimal (N^*, n^*) values where $0.5 \le \alpha \le 0.7$ and for all values of β.

α \ β	**0.1**	**0.2**	**0.3**	**0.4**	**0.5**	**0.6**	**0.7**	**0.8**	**0.9**
0.50	7,6	7,6	7,6	7,6	6,5	6,5	6,5	6,5	6,5
0.51	7,6	7,6	7,6	6,5	6,5	6,5	6,5	6,5	6,5
0.52	7,6	7,6	6,5	6,5	6,5	6,5	6,5	6,5	6,5
0.53	7,6	6,5	6,5	6,5	6,5	6,5	6,5	6,5	6,5
0.54	6,5	6,5	6,5	6,5	6,5	6,5	6,5	5,4	5,4
0.55	6,5	6,5	6,5	6,5	6,5	6,5	5,4	5,4	5,4
0.56	1,0	6,5	6,5	6,5	6,5	5,4	5,4	5,4	5,4
0.57	1,0	1,0	6,5	6,5	5,4	5,4	5,4	5,4	5,4
0.58	1,0	1,0	1,0	5,4	5,4	5,4	5,4	5,4	5,4
0.59	1,0	1,0	1,0	5,4	5,4	5,4	5,4	5,4	5,4
0.60	1,0	1,0	1,0	1,0	5,4	5,4	5,4	5,4	4,3
0.61	1,0	1,0	1,0	1,0	5,4	5,4	5,4	4,3	4,3
0.62	1,0	1,0	1,0	1,0	1,0	5,4	4,3	4,3	4,3
0.63	1,0	1,0	1,0	1,0	1,0	1,0	4,3	4,3	4,3
0.64	1,0	1,0	1,0	1,0	1,0	1,0	4,3	4,3	4,3
0.65	1,0	1,0	1,0	1,0	1,0	1,0	4,3	4,3	4,3
0.66	1,0	1,0	1,0	1,0	1,0	1,0	1,0	4,3	4,3
0.67	1,0	1,0	1,0	1,0	1,0	1,0	1,0	4,3	3,2
0.68	1,0	1,0	1,0	1,0	1,0	1,0	1,0	3,2	3,2
0.69	1,0	1,0	1,0	1,0	1,0	1,0	1,0	1,0	3,2

In both Tables 5.1 and 5.2 we evaluated the changes in the first two decision vaiables which are N^* and n^* with respect to total profitability. Additionally, we defined price of a new product $P_{0,}$ as the last decision variable. In Table 5.3, we examine the optimal new product prices P_0^* that correspond to the results of Table 5.1 for all α and β values. P_0^* is an important decision variable as both leasing price and remanufactured product price is affected by P_0^*. Since α is the main factor of deciding leased product price and, β is the main factor of deciding remanufactured product price, P_0 is both related with these two parameters.

Table 5.3. Optimal new product price for different α and β values.

α \ β	**0.1**	**0.2**	**0.3**	**0.4**	**0.5**	**0.6**	**0.7**	**0.8**	**0.9**
0.1	675	676	677	678	679	680	666	667	668
0.2	680	681	682	683	667	668	669	669	670
0.3	687	669	670	670	670	671	671	672	672
0.4	674	674	674	674	675	675	652	652	652
0.5	682	681	680	679	652	651	650	649	648
0.6	501	501	501	501	649	647	645	643	608
0.7	501	501	501	501	501	501	501	501	583
0.8	501	501	501	501	501	501	501	501	501
0.9	501	501	501	501	501	501	501	501	501

Recall that the first row of Table 5.1 (N^*, n^*) values are (12,11) , and P_0^* takes values between 675 and 680 where β <0.7 , while (11,10) P_0^* takes values between 666 and 668 for $\beta \geq 0.7$. For the same α value, optimal decision of n^* has decreased 1 unit by the effect of β, and this alteration affects the P_0^*. Therefore, as n^* decreases P_0^* has decreased as well for a given α value. This trend can be seen also from other cases of α. On the other hand, in the cases where manufacturer decides to sell products once they are manufactured, decision of P_0^* value is fixed with a certain value. This value means product is sold at 501 unit of money and also 50% of the market is willing to buy the product. Since new product sales are not affected by neither retained percentage α, nor remanufacturing effort β, P_0^* has the same value for $N^* = 1$, and $n^* = 0$. As in Equation 4.11, which is $\frac{\partial f}{\partial P_0} = D - 2P_0 + C(N)$, P_0^*=(1000 + 1^2)/2.

The first three tables show optimal values for N, n, and P_0 when considering maximum profitability. Table 5.4 shows the total profit values and how it increases or decreases for different values of parameters. Model gives the maximum value when α=0.1 and β=0.9, namely with highest depreciation rate and highest remanufacturing effort. This result is meaningful, since leasing is not a profitable business model, when the product loses small amount of its economic value. Also with high β values, manufacturer can get more money when selling product to the second hand market. Depreciation rate (1- α) seems to be the most significant parameter which accelerates the total profit, while remanufacturing effort has a small contribution to profit when compared to depreciation rate. Additionally, Figure 5.1 will give the model behaviour according to the data in Table 5.4.

Table 5.4. Total profit corresponding N^*, n^*, P_0^* values as α and β changes.

α \ β	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9
0.1	1056250	1060320	1064410	1068510	1072640	1076780	1081540	1087090	1092650
0.2	827518	833286	839066	844857	852431	859661	866899	874146	881400
0.3	626247	634917	643820	652726	661634	67546	679460	688377	697297
0.4	455810	466368	476927	487485	498043	508602	519738	531983	544228
0.5	312971	325116	337267	349423	363283	377269	391260	405254	419252
0.6	249500	249500	249500	249500	258617	274380	290163	305963	323234
0.7	249500	249500	249500	249500	249500	249500	249500	249500	260292
0.8	249500	249500	249500	249500	249500	249500	249500	249500	249500
0.9	249500	249500	249500	249500	249500	249500	249500	249500	249500

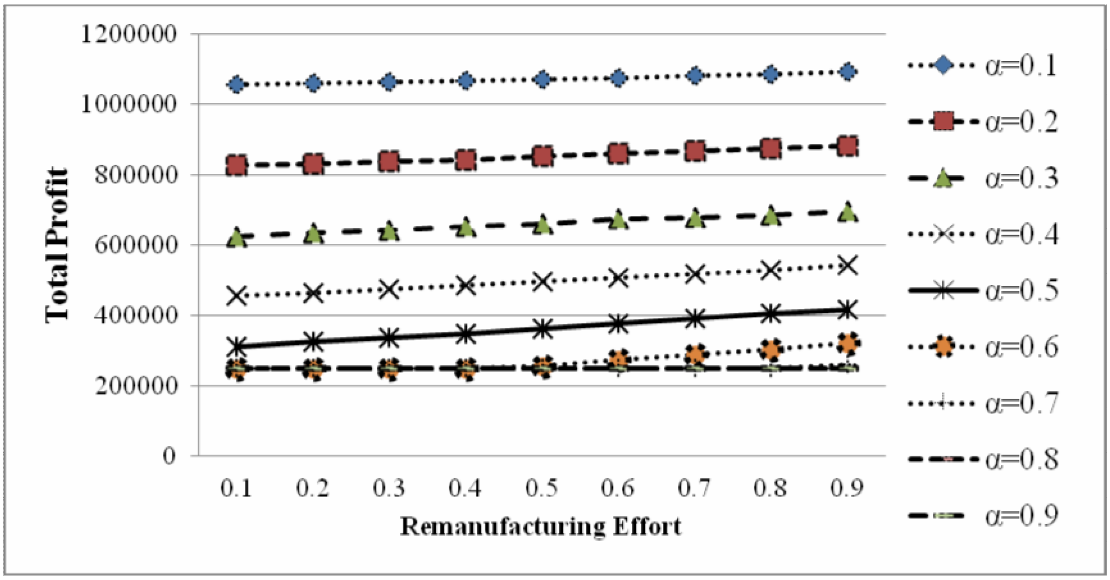

Figure 5.1. Change in total profit for corresponding N^*, n^*, P_0^* values as α and β changes.

5.2. Effect of Design Investment

This section focuses on the design issue which plays an important role in improving the efficiency of remanufacturing and leasing business. Design for remanufacture can cause high costs at the manufacturing phase, but through proper design, durable products can be produced and the remanufacturing cost can be decreased. Both product durability and componentwise design reduces remanufacturing cost consequently. We investigate whether the profitability is increased for the remanufacturing models when the remanufacturability of a product is improved by proper design. Here the important question is, investing on design always increases the profitability or is there a break point which profitability has started to decrease. To this end, we examine optimal values of leasing times n^* and P_0^* for all 20 cases of *N*. In order to see model behavior we have fixed α and β values at a certain value where α=0.1 and β=0.1 and give the result in Table 5.5. (Other cases can be seen from appendix).

In these experiments, *C(N)* has an increasing trend as *N* inreases, meaningly, products which have longer life cycles are produced with larger amount of costs. Table 5.5 shows that design investment has positive effect on number of leasing times until a certain point. It can be observed that, investing more on design does not effect the optimal number of leasing time after *n* reaches it optimal value. Here optimal value of number of leasing time is 11 for α=0.1 and β=0.1. Here the most profitable case occurs at *N*=12, and n^*=11, then profitability has started to decrease at $N > 13$. The changes in n^* is shown in Figure 5.2, and changes in total profit is shown in Figure 5.3 according to data in Table 5.5.

Table 5.5 demonstrates that until *N*=9, optimal number of leasing n^*=*N*. Here, model chooses to lease the product during its entire life and then to perform disposal activities in the manufacturing facility. This is because, investing less money on design leads larger remanufacturing costs and revenue from the second hand market does not cover the loss from disposal activities. Additionally, P_0^* has an increasing trend as design investment increases. In order to cover loss from initial design investment, the model assigns larger value for P_0^*. Figure 5.4 illustrates the change in P_0^* as design invesment increases.

Table 5.5. Optimal values for all cases of N where α=0.1 and β=0.1.

C(N)=N²	n^*	P_0^*	Total Profit
C(1)	0	501	249500
C(2)	2	570	331317
C(3)	3	575	485423
C(4)	4	585	617833
C(5)	5	597	729551
C(6)	6	609	821665
C(7)	7	623	895283
C(8)	8	636	951525
C(9)	9	650	991511
C(10)	9	646	1027020
C(11)	10	660	1048560
C(12)	11	675	1056250
C(13)	12	689	1051210
C(14)	12	690	1042870
C(15)	12	692	1033930
C(16)	12	693	1024420
C(17)	12	694	1014350
C(18)	12	696	1003710
C(19)	12	698	992527
C(20)	12	700	980804

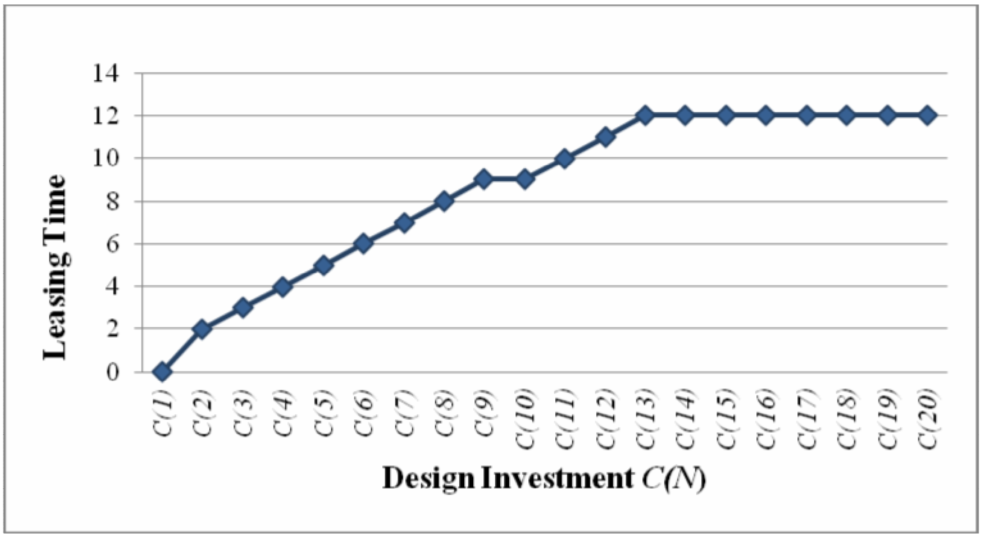

Figure 5.2. Change in n^* as design investment increases.

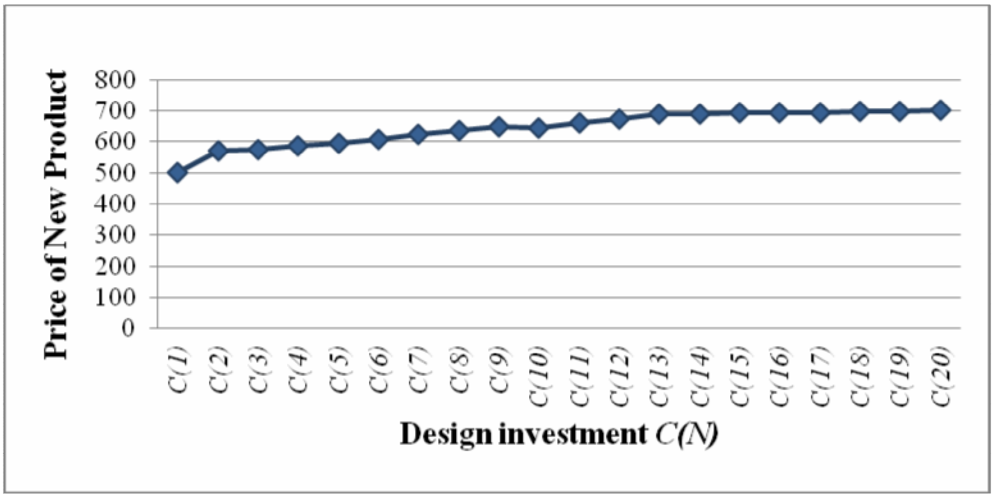

Figure 5.3. Change in P_0^* as design investment increases.

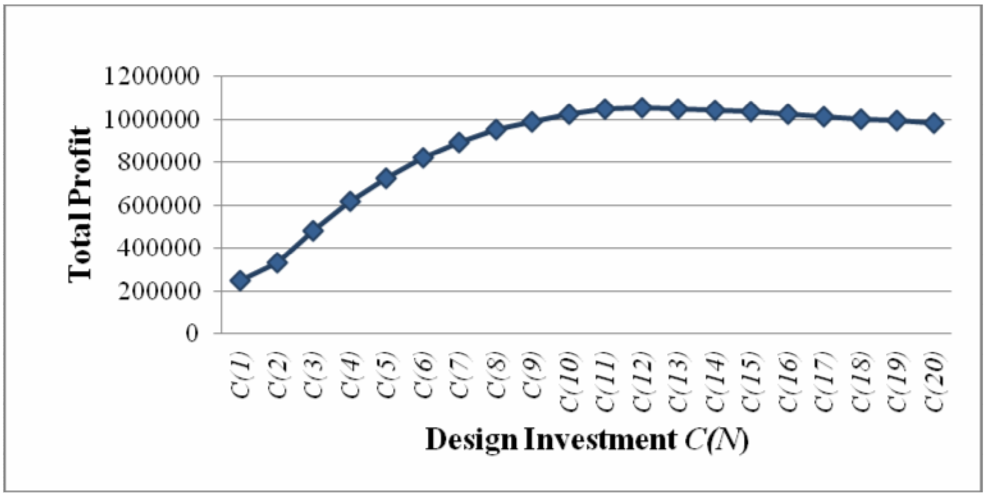

Figure 5.4. Change in total profit as design investment increases.

5.2.1. Design Investment and Profitability Analysis in the Traditional Supply Chain

Traditional supply chains are pure manufacturing systems where a product is manufactured and sold to the market as new good. In our model, the cases where n=0, is considered as a traditional supply chain. In this case, maximization problem will changed to $\max_{N,P_0} = (D - P_0)(P_0 - N^2)$. In such cases, the model is not affected by neither depreciation rate nor remanufacturing effort. Since α and β have no effect on the model, total profitability and product price are only affected by the design investment. It is expected that total profitability will decrease as N increases and optimal P_0 value increases in order to cover the loss caused from initial design cost.

Design for remanufacture is a tool which extends the life-cycle of a product as well as facilitates the remanufacturing activities such as disassembly, cleaning, refurbishment, reassembly and testing. When n=0, products are not intended to be collected back and to be recovered. In such case, it would be meaningless to invest large amount of money for design that eases the remanufacturing activities and extends products durability. We can observe this fact from Table 5.6 which show the results for the products that are sold once they are manufactured. Shu and Flowers [24] also stated that, it may be argued that adapting a product for disassembly, cleaning or reassembly is meaningless if the product or its parts are not intended to be reused. Here, most profitable case is investing the least money on design and selling product at 501 unit of money. Increasing design investment has no positive effect on both profitability and the number of customers. Since P_0 value increases to compensate the cost of design, number of customers in the system will be decreased.

According to data in Table 5.6, Figure 5.5, and Figure 5.6 demonstrates the change in the total profit and change in the P_0^*, respectively.

Table 5.6. Change in total profit and new product price when n=0.

C(N)=N²	P_0^*	Total profit
C(1)	501	249500
C(2)	502	247775
C(3)	505	245355
C(4)	508	241191
C(5)	512	237559
C(6)	518	232244
C(7)	524	226032
C(8)	532	218966
C(9)	540	211089
C(10)	550	202455
C(11)	560	193120
C(12)	572	183148
C(13)	584	172608
C(14)	598	161575
C(15)	612	150130
C(16)	628	138361
C(17)	644	126359
C(18)	662	114225
C(19)	680	102063
C(20)	700	89985

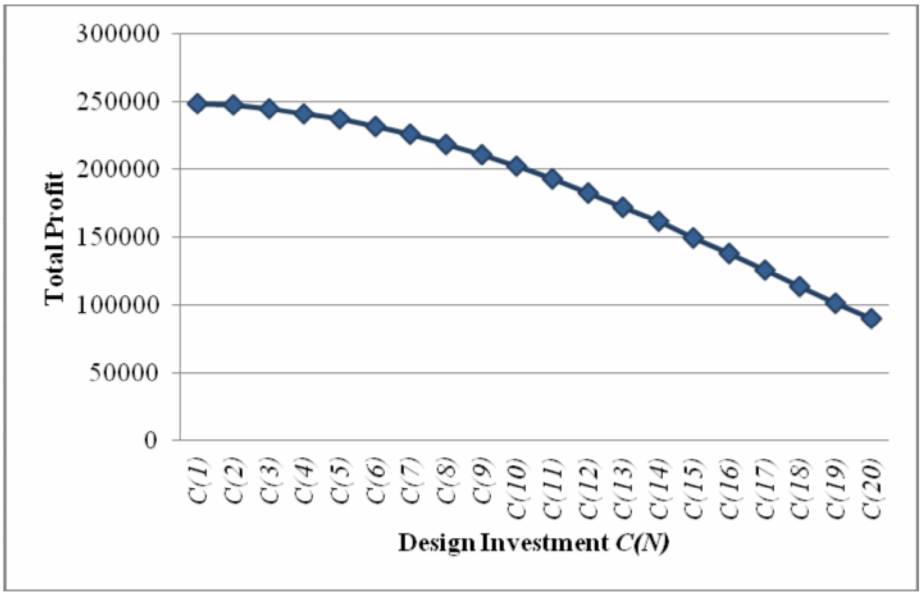

Figure 5.5. Change in total profit as *C(N)* increases at *n*=0.

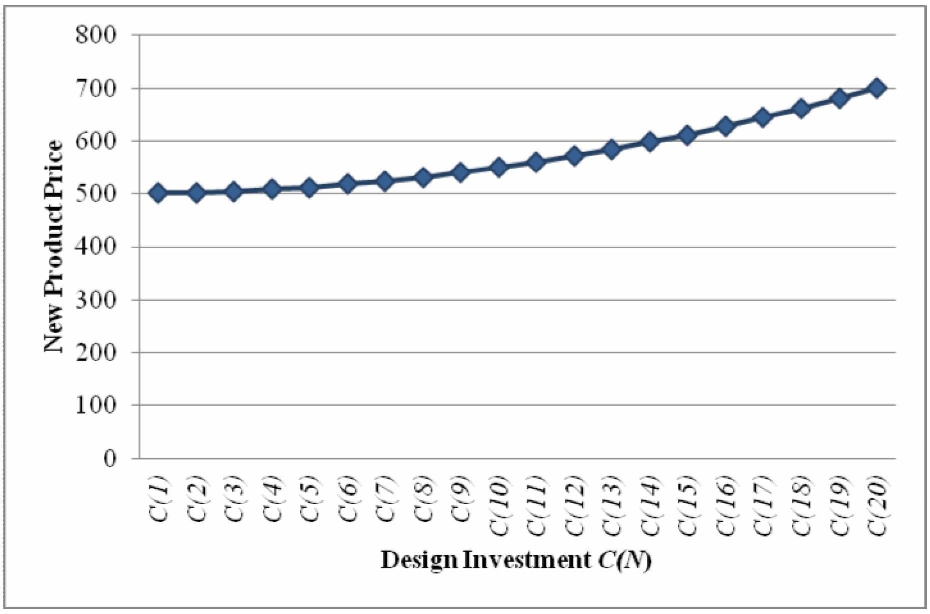

Figure 5.6. Change in new product price as *C(N)* increases at *n*=0.

5.2.2. Design Investment and Profitability Analysis in the Pure Leasing System

In pure leasing systems, products are leased for a leasing period, returned from customer and recovered in the manufacturing facility. This leasing and recovery actions are continued at the end of the products total life. In our model, $n=N$ cases are pure leasing systems. Since remanufacturing activities are the inseperable part of the servicizing business, products need to be designed for remanufacturing and thus servicizing. Design for remanufacturing is not only focus on facilitating demateralization, testing, and cleaning etc. it also focuses on the economical aspects and technological aspects of the product.

Profitability of the remanufactured product, and the value kept on products after their usage, and product's total life time are the economy related issues. In our experiments, we assume that product's total useful lifetime N is increased by proper design. In addition, preserved value of the product is presented as α. Therefore, in order to reach maximum profit via proper design , we have to consider the depreciation rate of the product. On the other hand, depreciation rate is also affected by tehcnological aspects of a product. Slowly depreciated products are less prone to response technological changes, while others depreciates faster as innovative technologies increases. In our experiments, model decisions are only affected by depreciation rate of the product and durability of the product. Total profit values when $n=N$ are shown in Table 5.7.

In Table 5.7 we see total profitability decreases as α increases for a fixed N, meaningly for a fixed design investment cost. On the other hand, we see the effect of design investment at different levels. For α=0.1, the design investment cost which gives maximum profit is $C(11)$, for α=0.2 is $C(10)$, for α=0.3 is $C(9)$ etc. Here we observe, products ,which still preserves high percentages of its value after one year usage, do not need to be designed for larger durability. Larger durability levels lead to higher design investment costs. Since quickly depreciated products are suitable for servicizing, their life time need to be extended. Here, products which loses only 10% of their economic value should not be leased during its life time.

In this section, we do not give the change in P_0, since we have observed in Section 5.1 and Section 5.1.1, new product price will be increased according to increase in design, in order to make profit.

Table 5.7. Change in total profit when n=N for all values of α.

	α=0.1	**α =0.2**	**α =0.3**	**α =0.4**	**α =0.5**	**α =0.6**	**α =0.7**	**α =0.8**	**α=0.9**
***N*=1**	155003	130815	106860	83253	60204	38130	18007	2761	0
***N*=2**	331317	282450	233907	185850	138570	92650	49450	13050	0
***N*=3**	485423	412372	339878	268219	197896	129911	66603	14988	0
***N*=4**	617833	521140	425393	331062	239000	150906	70749	10436	0
***N*=5**	729551	609883	491738	375877	263673	157866	64854	3831	0
***N*=6**	821665	679821	540307	404289	273864	153225	52161	343	0
***N*=7**	895283	732203	572528	417961	271567	139477	35993	0	0
***N*=8**	951525	768286	589836	418569	258795	119134	19699	0	0
***N*=9**	991511	789332	593673	407795	237565	94719	6644	0	0
***N*=10**	1016370	796606	585485	387325	209900	68762	199	0	0
***N*=11**	1027220	791373	566718	358846	177824	43791	0	0	0
***N*=12**	1025180	774899	538819	324046	143363	22339	0	0	0
***N*=13**	101140	748451	503235	284614	108544	6939	0	0	0
***N*=14**	986980	713295	461413	242238	75392	125	0	0	0
***N*=15**	953061	670698	414803	198609	45937	0	0	0	0
***N*=16**	910766	621929	364851	155415	22204	0	0	0	0
***N*=17**	861221	568253	313007	114347	6221	0	0	0	0
***N*=18**	805553	510938	260719	77093	17	0	0	0	0
***N*=19**	744887	451252	209434	45344	0	0	0	0	0
***N*=20**	680351	390461	160602	20789	0	0	0	0	0

Table 5.7 shows the effect of durability and percentage of the preserved value of a product on profitability. Figure 5.7, 5.8, and 5.9 shows the behaviour of profitability at α=0.1, α=0.4, and α=0.7, respectively according to data in Table 5.7.

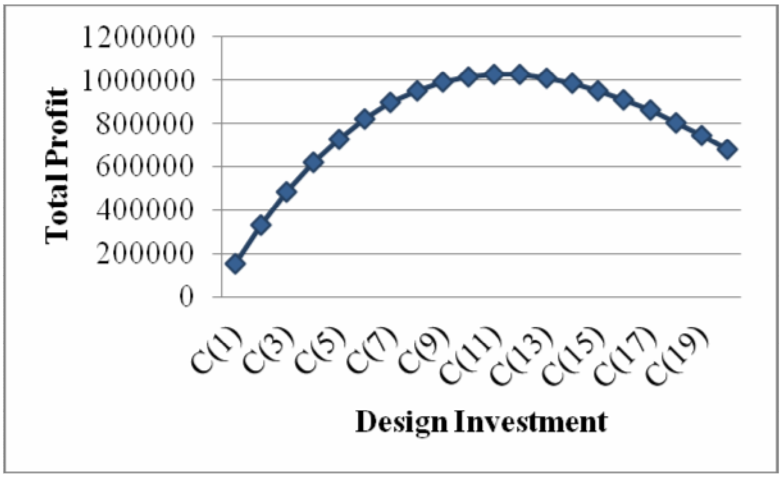

Figure 5.7. Change in total profit as *C(N)* increases when *α*:0.1.

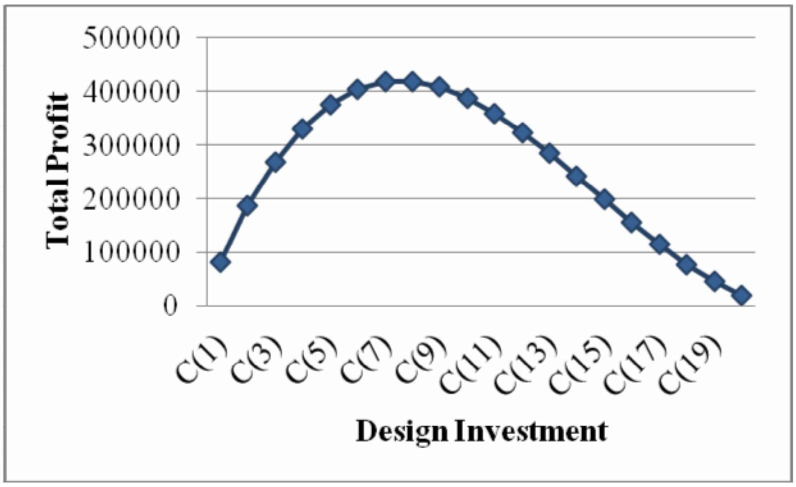

Figure 5.8. Change in total profit as *C(N)* increases when *α*:0.4.

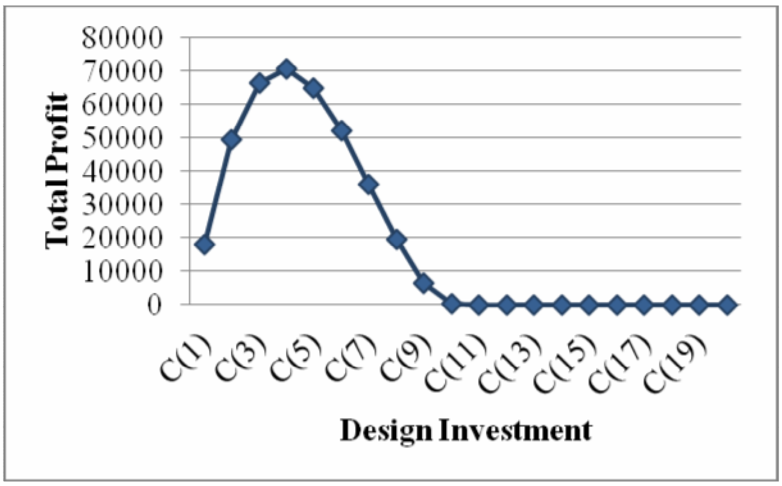

Figure 5.9. Change in total profit as *C(N)* increases when *α*:0.7.

5.3. Effect of Annual Depreciation Rate

In the previous sections, we evaluated optimal cases and the effect of design invetment. Almost in every table and figure we see the significant effect of depreciation rate on our model. In this section, we try to evaluate the effect of depreciation rate on the leasing or selling decisions. We examine the change in optimal durability, optimal number of leasing times, and price of new products as *α* changes.

. The economic value of a product after one year usage would be $P_0\alpha$, and the customer is willing to pay $P_0 - P_0\alpha$ amount of money. Therefore annual depreciation rate of a product is $(1-\alpha)$, and the leasing price would be $P_0(1-\alpha)$. According to this price determination low depreciation rates cause lower leased product price. Our expectation is leasing is a profitable and logical option at high depreciation rates. The results are rapport with our expectations.

5.3.1. Change in the Durability

Durability decision is affected by depreciation rate. According to table 5.1, we evaluate the behavior of the durability. In order to see only the effect of *α*, first we have fixed *β* value at *β*=0.1. Figure 5.10 will show the durability and the percentage of retained value relationship when *β*=0.1, and the figure 5.11 will show durability and the percentage of retained value relationship for all values of *β*.

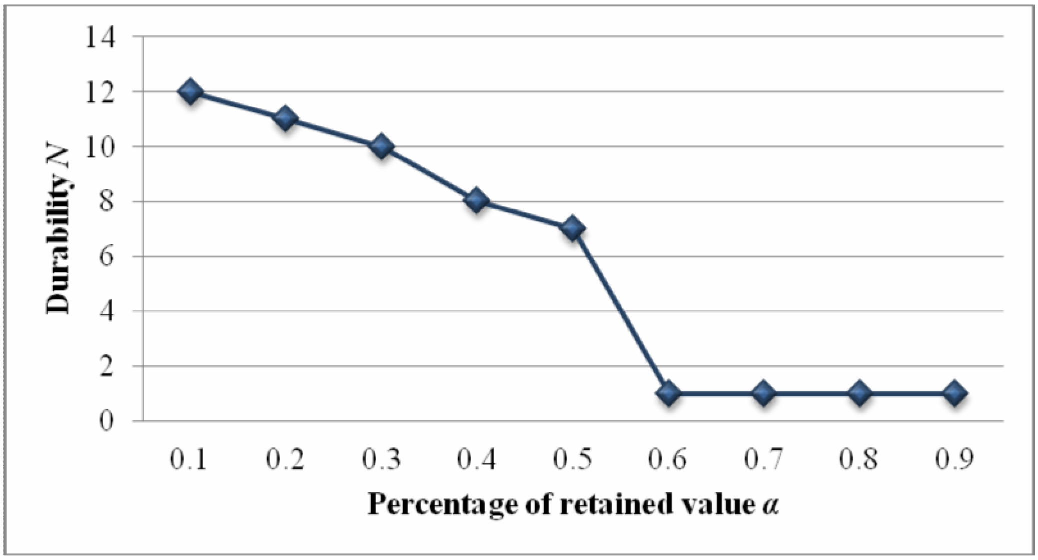

Figure 5.10. Change in durability N as α changes when β=0.10.

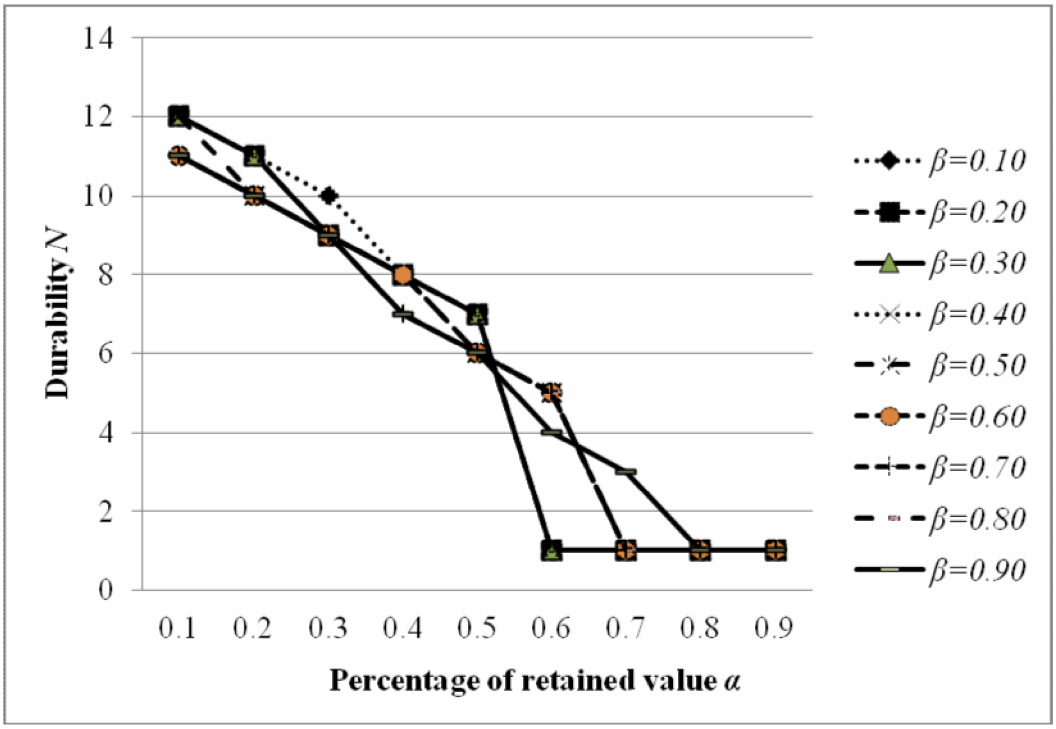

Figure 5.11. Change in durability N as α changes for all β values.

5.3.2. Change in the Number of Leasing Times

In this section, first we compare optimal number of leasing time for all values of α for a fixed β which is β =0.5. Optimal values is affected by both percentage of retained value α and remanufacturing effort β. In order to understand the exclusive effect of α on optimal number of leasing times , we compare different durability cases which are N=5, N=10, and N=15, for small, moderate and large values of durability, while fixing β value at 0.5. Figure 5.12 will show the change in the optimal number of leasing time according to data in Table 5.1. Then, Table 5.8 will show the results for N=5, N=10, and N=15. In addition Figures 5.13, 5.14, and 5.15 will give an insight for understanding the effect of depreciation rate on number of leasing times.

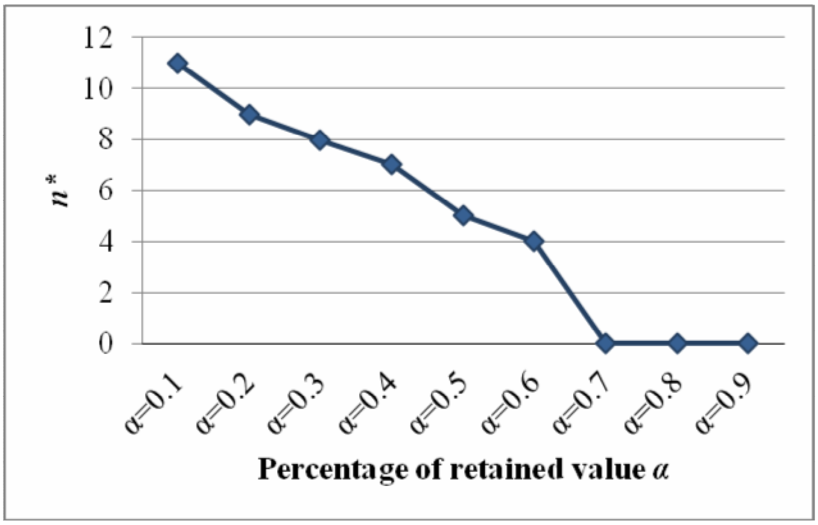

Figure 5.12. Change in optimal number of leasing times as α at β =0.5

In Table 5.8, for N=5, lower α values give higher profit rates with n=4 . It means leasing a product 4 times and then sell as remanufactured good is the best alternative for a product which has five year lifetime. Although with higher values of α, this decision changes as selling product immediately after manufacturing. Similarly in N=5, optimal leasing time has started to decrease for larger values of α when N=10, and N=15. With low depreciation values, leasing a product multiple times will no longer be a logical option.

Table 5.8. Total profit and P_0 values for N=5, N=10, and N=15 and β=0.5 for all α values.

	n^*	Total Profit	P_0^*
N=5			
α =0.1	4	735853	576
α =0.2	4	638437	584
α =0.3	4	541647	594
α =0.4	4	445743	608
α =0.5	4	351150	625
α =0.6	4	258617	649
α =0.7	0	237559	512
α =0.8	0	237559	512
α =0.9	0	237559	512
N=10			
α =0.1	9	1054880	649
α =0.2	9	852431	667
α =0.3	9	655947	689
α =0.4	7	486462	678
α =0.5	6	345479	685
α =0.6	4	233150	666
α =0.7	0	202455	550
α =0.8	0	202455	550
α =0.9	0	202455	550
N=15			
α =0.1	12	1045430	682
α =0.2	10	818163	689
α =0.3	9	617804	698
α =0.4	7	447166	691
α =0.5	6	307396	703
α =0.6	5	195879	720
α =0.7	0	150130	612
α =0.8	0	150130	612
α =0.9	0	150130	612

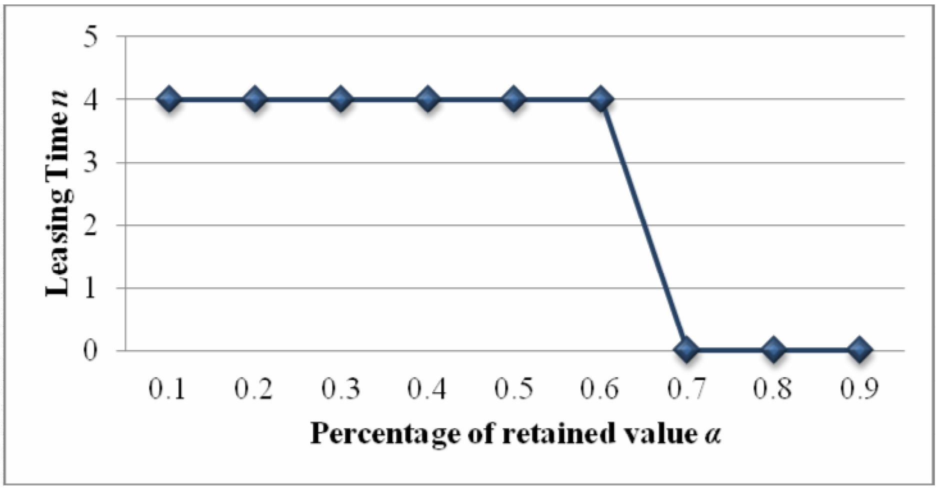

Figure 5.13. Change in *n* as *α* changes for products with *N*=5 years.

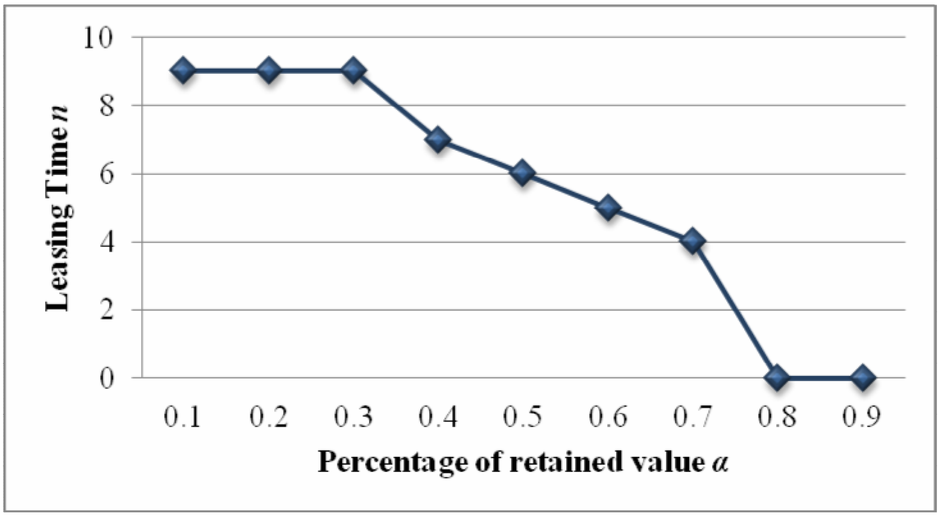

Figure 5.14. Change in *n* as *α* changes for products with *N*=10 years.

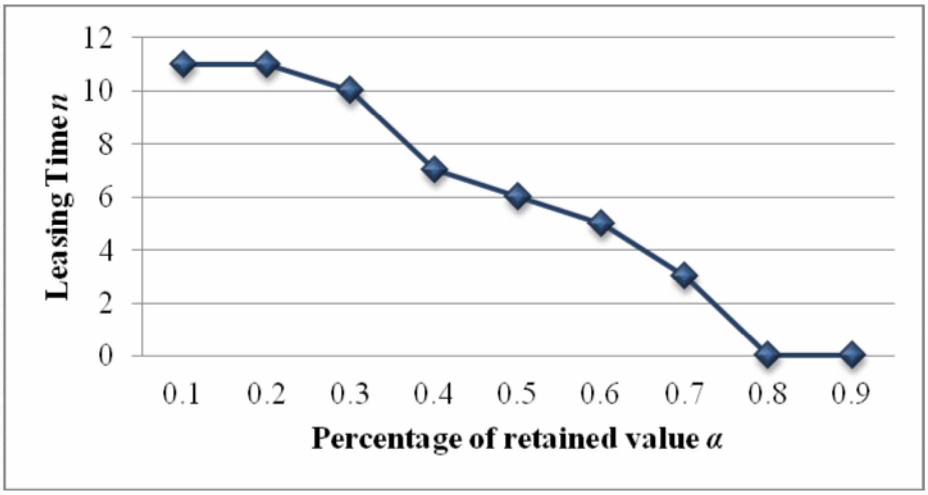

Figure 5.15. Change in *n* as *α* changes for products with *N*=15 years.

5.3.3. Change in the New Product Price

Optimality analysis in Section 5.1 shows that price of a new product is affected by preserved value percentage, remanufacturing effort, durability and the number of leasing times. It means this decision variable is affected by both parameters and the other decision variables. In this section we will only see the effect of preserved value α on the new product price. The effect of other parameters and decision variables will be examined in following sections.

In order to see the change in new product price as α changes, we have evaluated small, moderate and large durability cases which are N=5, N=10, and N=15 like in previous section. In order to get rid of the effect of β, we took the values of $n=N$. Here we have to remember that, this part is not focused on optimal cases, since there are too many factors that affects the value of P_0. Following table will show the corresponding data.

Table 5.9. P_0 values for different levels of α N=5, N=10,and N=15, for $n=N$.

	N=5	N=10	N=15
α	P_0	P_0	P_0
0.1	597.36	663.90	734.30
0.2	609.53	684.44	763.59
0.3	625.17	710.79	801.24
0.4	646.03	745.92	851.45
0.5	675.24	795.11	921.74
0.6	719.05	868.89	Inf
0.7	792.07	991.85	Inf
0.8	938.10	Inf	Inf
0.9	Inf	Inf	Inf

Table 5.9 includes infeasible P_0 values which represents that model can not make profit anymore for those α and n values. Following figures will give an insight for effect of α when N=5, N=10, and N=15 respectively.

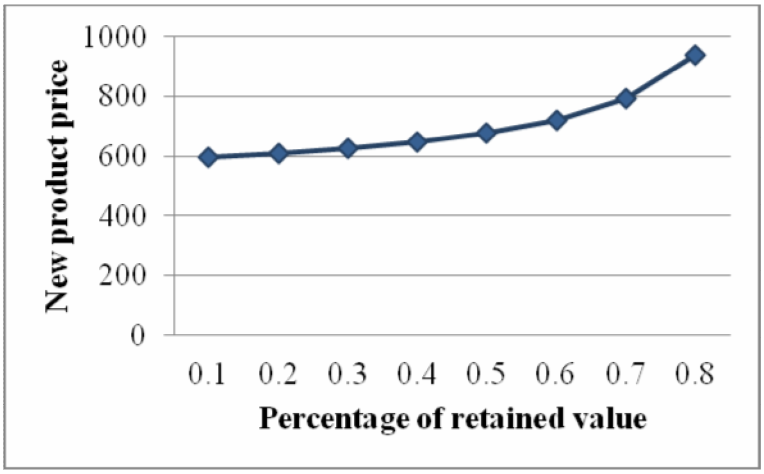

Figure 5.16. Change in P_0^* as α changes when N=5, and n=5.

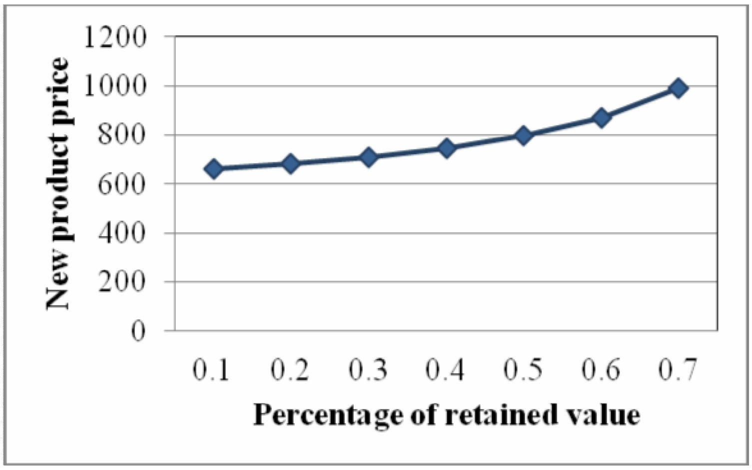

Figure 5.17. Change in P_0^* as α changes when N=10, and n=10.

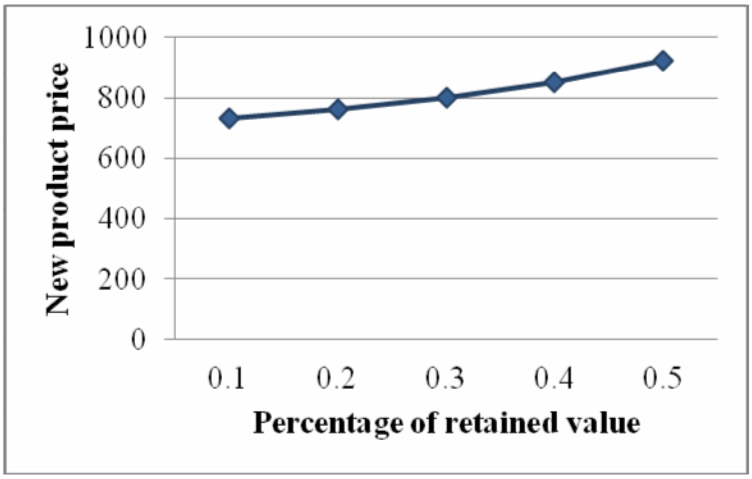

Figure 5.18. Change in P_0^* as α changes when N=15, and n=15.

5.3.4. Change in the Total Profit

Depreciation rate of the product is a parameter that determines the leasing price of the product. It is expected that total profit tends to decrease by decreasing depreciation because the lease payments is per product is decreased. On the other hand, depreciation rate of the product is one of the most significant factor that affects total profit since it affects optimal values of decision variables. In Table 5.5, we have examined that total profit is mostly affected by the number of durability and number of leasing times. The previous sections show that percentage of retained value is the key factor that affects *N* and *n*. Therefore depreciaton rate has both direct and indirect effect on total profit. In order to examine direct and indirect effect we perform experiments both fixed and optimal cases.

As a result of the experiments, as preserved value increases, total profit decreases. In order to understand model behavior, it is sufficient to analyze remanufacturing effort at a small, a moderate and a large level, namely β=0.1, β=0.5, and β=0.9 respectively. It can be observed from Table 5.10 that α has a adverse effect on total profit. Here we have to take into account that as α increases, optimal durability and leasing times are decreases. This table shows indirect effect of preserved value on total profit, since optimal *N* and *n* are not the same for all α values. Additionally, Figure 5.19 will give an insight for understanding the total profit behaviour at optimal cases.

Table 5.10. Total profit of optimal cases for all values of α and β=0.1, β=0.5, β=0.9.

Total Profit for Optimal Cases			
	β=0.1	β=0.5	β=0.9
α=0.1	1056250	1072640	1092650
α=0.2	827518	852431	881400
α=0.3	626247	661634	697297
α=0.4	455810	498043	544228
α=0.5	312971	363283	419252
α=0.6	249500	258617	323234
α=0.7	249500	249500	260292
α=0.8	249500	249500	249500
α=0.9	249500	249500	249500

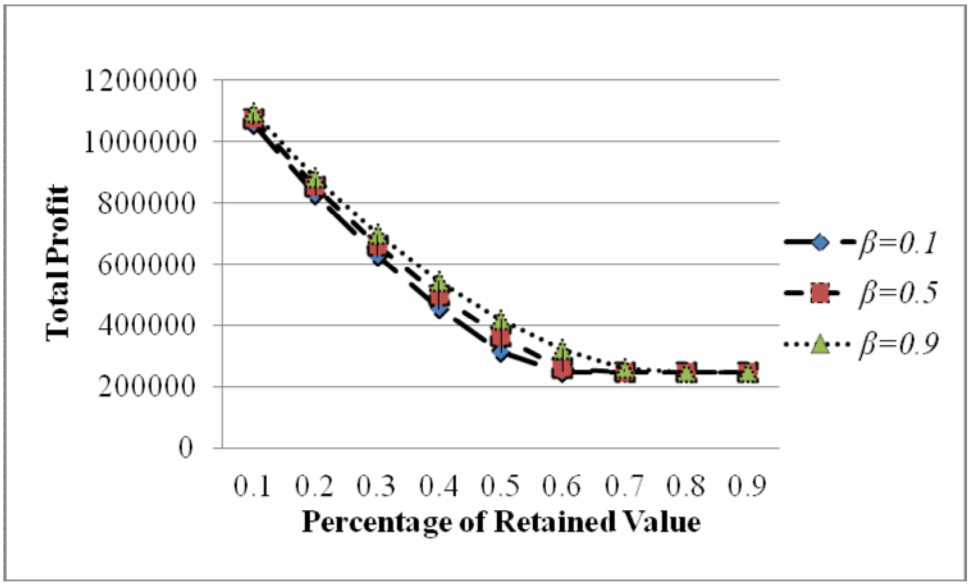

Figure 5.19. Change in the total profit as α changes for β=0.1, β=0.5, and β=0.9

Concentrating on just the optimal cases is not adequate for understanding depreciation effect. Therefore, we perform experiments for a certain values of N which are 5,10,15, and 20, and we took n=N in order to get rid of the effect of remanufacturing effort. Table 5.11 will give the results of these cases.

Table 5.11. Change in total profit as α changes for different levels of N,n.

Total Profit for all α values at certain *N,n* values				
	n=N=5	n=N=10	n=N=15	n=N=20
α=0.1	729551	1016370	953061	680351
α=0.2	609883	796606	670698	390461
α=0.3	491738	585485	414803	160602
α=0.4	375877	387325	198609	20789
α=0.5	263673	209900	45937	0
α=0.6	157866	68762	0	0
α=0.7	64854	199	0	0
α=0.8	3831	0	0	0
α=0.9	0	0	0	0

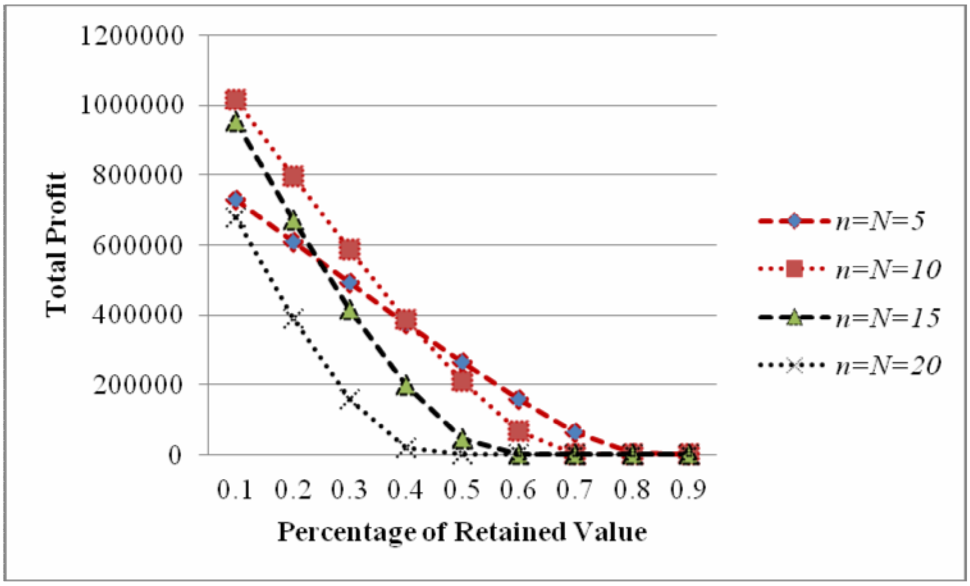

Figure 5.20. Change in the total profit as α changes for different levels of N,n.

5.4. Effect of Remanufacturing Effort

Remanufacturing is the operation of repairing used products and components, and put the product into a second life cycle, thus retaining the value of the extracted material, as well as some fraction of the original manufactured value. In our model, both leased and sold product in second hand market are recovered in a remanufacturing facility. We assume that products which are decided to be leased are remanufactured and brought back to “like-new” state, while products which are decided to be sold in second hand market are remanufactured to a state which can still function but with a lower quality. This quality discount is presented as β, which can be showed as:

β = Quality of remanufactured product / Quality of leased product.

Here, low quality remanufacturing leads low costs, and then manufacturer demands low prices for these remanufactured products. In the previous sections, we observed that percentage of retained value α, has a significant effect on profitability and decisions of the model, while remanufacturing effort β, is not effective as α. Still, examining the effect of β would be important, as it alters decisions after a certain level. In this part of the thesis, we would be able to understand the effect of β on durability, number of leasing times, new product price and the total profitability.

5.4.1. Change in Durability, and the Optimal Number of Leasing Times

In table 5.1 we observed that α is the main factor which decides N^*, and n^*, yet in some points β changes the decision where selling the remanufactured product to the price of $P_0\,\beta$ which has quality degree β is superior than the other options. For instance (N^*,n^*) is (12,11) for when α=0.1 until β=0.7, then the optimal decisions decrease in one unit. This is because, selling the product to the second hand market with 70% and more effort, would be more profitable when comparing other cases. Additionally, for high α, model chooses to sell product in first place unless it has high remanufacturing effort. In these experiments, we see the two sided effect of β on optimal decisions. High percentage of remanufacturing effort can decrease the optimal number of leasing times when it is superior to the leasing, and can increase the optimal number of leasing times when it is superior to selling the product in the first place. Figure 5.19 will show the behavior of the N^* by the change of β. Since n^* is less one unit than the N^*, we can also see how n^* changes by the change of β. The figure shows the change in N^*. β has a decreasing effect on number of leasing times for $0.1 \leq \alpha \leq 0.5$., while incrasing effect on n^* for α=0.6 and α=0.7. However, β has no effect on optimal decisions where α=0.8 and α=0.9. Even the highest percentage of remanufacturing effort is not sufficient to cover the loss value from α and the model chooses to sell the product as it is manufactured.

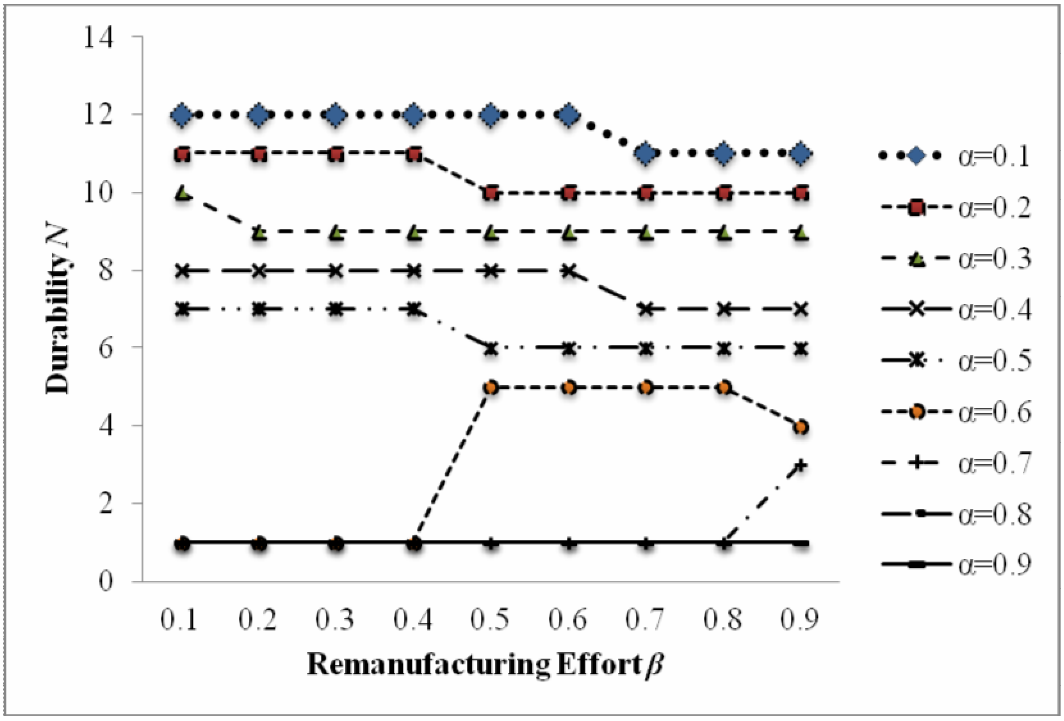

Figure 5.21. Change in the optimal N^*, as β changes for all values of α.

5.4.2. Change in Optimal Leasing Time, Total Profit and Optimal New Product Price for Fixed *N* Values

In Section 5.4.1. we examine the effect of β on optimal number of leasing times and optimal durability level. As optimal cases are affected by both depreciation rate and remanufacturing effort, making conclusions just evaluating the optimal cases is not sufficient in order to understand the single effect of β. To this end we make experiments for fixed small, moderate , and large *N* values as for *N*=5, *N*=10, and *N*=15 respectively. In addition, we fixed the value of α =0.5. Table 5.12 will give the optimal number of leasing times, total profit and the new product price for the parameters of interest.

Table 5.12 shows three different levels of durability, which are *N*=5, *N*=10, and *N*=15 respectively. As one can see, remanufacturing effort can not change the model behavior for small durability level. Product which has durability 5 years, should be leased 4 times, and then sold to the second hand market without considering the second hand market price and remanufacturing effort. For the moderate level of durability which is *N*=10 in our case, high values of remanufacturing effort alters the model behavior. Here the optimal number of leasing times is 6 for most cases, though this number decreases by β=0.8, and β=0.9. In such scenario, reducing the leasing number and selling product to the second hand market one period earlier provides more profit to the manufacturer. Similarly for *N*=15, optimal number of leasing times is 6 for most cases. This number changes to 7 for the smallest level of remanufacturing effort β, and changes to 5 for the largest level of remanufacturing effort β. Figure 5.22 shows the behavior of the model for number of leasing times for all values of β.

Another outcome of the table is that, as remanufacturing effort increases profitability of the model increases for all cases. Eventhough, increment of β also increases the cost of remanufacturing, the money which is gained from second hand market compensates the value from remanufacturing costs. Figure 5.23 shows the behavior of the model for total profit for all values of β.

Finally, new product price is also changed by the levels of β. All cases shows that as the quality of remanufactured product increases, price of new product is decreased. For the high values of β , the gained money from a singular product P_0 β leads higher values, therefore, the number of customers who is willing to buy second hand product will decrease. Figure 5.24 shows the behavior of the model for P_0^* for all values of β.

Table 5.12. Total profit values for N=5, N=10, N=15 and α =0.5 for all β values.

	n^*	Total Profit	P_0^*
N=5			
β=0.1	4	287483	630
β=0.2	4	303389	628
β=0.3	4	319303	627
β=0.4	4	335224	626
β=0.5	4	351150	625
β=0.6	4	367081	624
β=0.7	4	383017	623
β=0.8	4	398957	622
β=0.9	4	414901	621
N=10			
β=0.1	6	297070	690
β=0.2	6	309159	689
β=0.3	6	321257	687
β=0.4	6	333364	686
β=0.5	6	345479	685
β=0.6	6	357601	684
β=0.7	6	369730	683
β=0.8	5	383291	659
β=0.9	5	397237	658
N=15			
β=0.1	7	261216	730
β=0.2	6	271596	708
β=0.3	6	283511	706
β=0.4	6	295445	705
β=0.5	6	307396	703
β=0.6	6	319365	702
β=0.7	6	331348	700
β=0.8	6	343345	699
β=0.9	5	355734	676

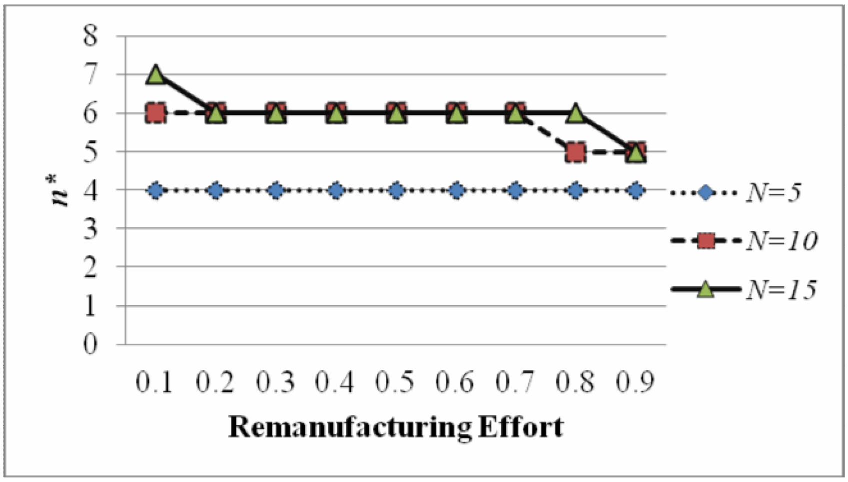

Figure 5.22. Change in n^* as β changes for fixed values of N.

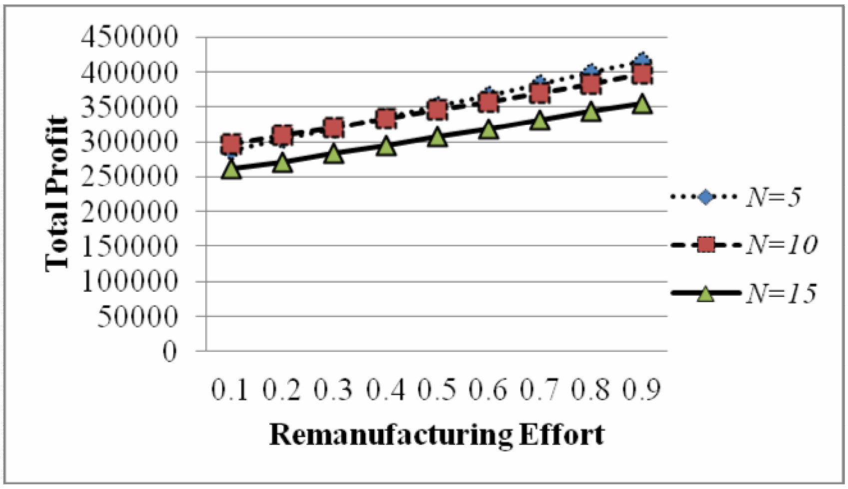

Figure 5.23. Change in total profit as β changes for fixed values of N.

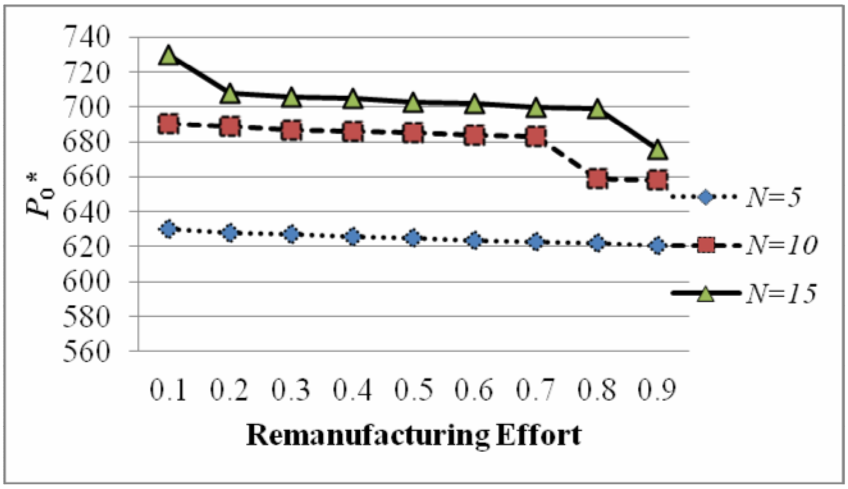

Figure 5.24. Change in P_0^* as β changes for fixed values of N.

5.5. Comparison of Disposal Cost and Salvage Value

In the previous experiments, one of the decisions that manufacturer can lease the product throughout its entire life, and then dispose the product with an associated cost. Since disposal activities requires materials and facility for proper disposal, therefore disposal activities come with a cost which we assume disposal cost c_d=100.

Most of the firms dispose its end of life products , while other firms chooses to sell end of life products to its salvage value. Salvage value of a product is the projected resale value of a product at the end of its useful life. Actually, salvage value is affected by both the depreciation rate of the product and the age of the product. Therefore the salvage value of a product can be determined by using (1-α), and N. Yet in our model, we assume that products are always brought back to like new condition. Thus we only use the depreciation rate of the product for estimating the salvage value. The wear off rate of a product is integrated into the cost structure, since older products require higher costs in order to recover them as good as new. Therefore, salvage value is estimated as a piece of property when the product is sold at new price value at first place regardless of the product's age.

Actually, salvage value is affected both by how the customer use the product and how long the product is used by costomer. In our assumption, customer uses the product without making extra harm, thus product can only loses its annual depreciation rate value. If annual depreciation rate of the product is small, meaningly the product is still in good operating condition, the salvage value can be relatively large. However, if the product loses lots of its economic value as a result of large depreciaitons, product's salvage value can be extremely low. Following equation will give an estimation for salvage value of the product.

$$\text{Salvage value: } P_0(1-DepreciationRate)$$

$$\text{Salvage value: } P_0(1-(1-\alpha)) = P_0\alpha$$

Here P_0 is the sale price of a new product, (1-α) is the annual depreciation rate of the product, and N is the age of the product at the end of its life as in previous sections.

Our mathematical model will be changed by this positive activity. Therefore new model would be:

$$\begin{aligned}
&\max_{\mathrm{N,n,P_0}} = \left\lfloor \frac{N-n}{N} \right\rfloor (D-P_0)(P_0 - C(N)) + \\
&P_0(1-\alpha)n(D-P_0) - (\sum_{k=1}^{n} (\frac{1}{C(N)})^m + c_r k)(D-P_0) + \\
&\left\lceil \frac{N-n}{N} \right\rceil \left\lceil \frac{n}{N} \right\rceil \left\{ (D-P_0)(P_0\beta - ((\frac{1}{C(N)})^m + c_r n)\beta) \right\} \\
&+ \left\lfloor \frac{n}{N} \right\rfloor P_0\alpha(D-P_0) - \left\lceil \frac{n}{N} \right\rceil (D-P_0)C(N)
\end{aligned}$$

Experiments are performed according to the new model and the optimal N^*, n^* results are represented in Table 5.13, and total profit values are demonstrated in Table 5.14. According to these tables, salvage value has a noteworthy effect on alteration in the optimal decisions. In the previous model, optimal N^* is more than one unit n^*, yet in the current model, optimal decisions are equal to each other in the most cases. It means that, manufacturer can get more money from selling product at the end of its useful life. This decision is only changes for the largest values of β. If the second hand market revenue is large enough to compensate the salvage value, then manufactuer chooses to sell product into the second hand market. Finally, the largest values of α, which are 0.8, and 0.9, manufacturer chooses to sell its product as the product is manufactured.

Table 5.13. Optimal N^{*},n^{*} values for the new model with salvage value.

N^{*},n^{*}									
β / α	**0.1**	**0.2**	**0.3**	**0.4**	**0.5**	**0.6**	**0.7**	**0.8**	**0.9**
0.1	11,11	11,11	11,11	11,11	11,11	11,11	11,11	11,10	11,10
0.2	10,10	10,10	10,10	10,10	10,10	10,10	10,10	10,9	10,9
0.3	9,9	9,9	9,9	9,9	9,9	9,9	9,9	9,9	9,8
0.4	7,7	7,7	7,7	7,7	7,7	7,7	7,7	7,7	7,6
0.5	6,6	6,6	6,6	6,6	6,6	6,6	6,6	6,6	6,5
0.6	4,4	4,4	4,4	4,4	4,4	4,4	4,4	4,4	4,3
0.7	3,3	3,3	3,3	3,3	3,3	3,3	3,3	3,3	3,2
0.8	1,0	1,0	1,0	1,0	1,0	1,0	1,0	1,0	1,0
0.9	1,0	1,0	1,0	1,0	1,0	1,0	1,0	1,0	1,0

Table 5.14. Total profit values for the new model with salvage value.

Total Profit									
β / α	**0.1**	**0.2**	**0.3**	**0.4**	**0.5**	**0.6**	**0.7**	**0.8**	**0.9**
0.1	1081720	1081720	1081720	1081720	1081720	1081720	1081720	1087090	1092650
0.2	872278	872278	872278	872278	872278	872278	872278	874146	881400
0.3	689957	689957	689957	689957	689957	689957	689957	689957	697297
0.4	539217	539217	539217	539217	539217	539217	539217	539217	544228
0.5	415840	415840	415840	415840	415840	415840	415840	415840	419252
0.6	321778	321778	321778	321778	321778	321778	321778	321778	323234
0.7	259881	259881	259881	259881	259881	259881	259881	259881	260292
0.8	249500	249500	249500	249500	249500	249500	249500	249500	249500
0.9	249500	249500	249500	249500	249500	249500	249500	249500	249500

5.6. Effect of Remanufacturing Cost on Model Behavior

In remanufacturing businesses, it is assumed that cost of remanufacturing c_r is less than the manufacturing cost, and it is generally considered that a remanufactured product can be produced and sold at a cost as a percentage of the price of a new product. Therefore, manufacturer can get sensible profit from remanufactured products.

In our model, we define two remanufacturing costs which are fixed and variable remanufacturing cost. Variable remanufacturing is $(\frac{1}{C(N)})^m$ and it is decreased by an increasing design investment. Other than the variable remanufacturing cost, we define c_r as a fixed remanufacturing cost which represents the processed that proper design can not facilitate. We assign a fixed value for c_r which is 50. But this value can not satisfy the rule that the remanufacturing cost should be lower than the manufacturing cost since, manufacturing cost *C(N)* has a dynamic behavior with respect to *N*. Since we assign N^2 for the function of *C(N)* , manufacturing cost is not larger than c_r where $N<8$. If remanufacturing cost of a product is lower than the production cost of a new product, then higher unit profit margin is obtained for the remanufactured product.

Lower remanufacturing costs compensate the increase in design investment, therefore, model will choose invest more money on design, thus the durability of a product will be increased. In addition, optimal number of leasing times will be increased, since reprocessing a leased product will be less costly.

In order to evaluate model sensitivity to remanufacturing cost we assign different values of c_r which are c_r=10, c_r=20, c_r=30, and c_r=40, and compare the results with c_r=50. In these experiments we run the model for all cases of α, where β is fixed at 0.1. Table 5.15 will give the optimal durability level and optimal number of leasing times as (N^*, n^*), and Table 5.16 will give the total profitability that corresponds to Table 5.15. Also Figure 5.24, and 5.25 are demonstrated the model behavior with respect to Table 5.15 and Table 5.16 respectively.

According to these experiments, smallest c_r value makes leasing business most profitable. For $c_r = 10$, the product is leased for its entire life for $0.1 \leq \alpha \leq 0.4$. For α=0.5 and α=0.6 manufacturer still produces the product with the lifetime of 20, yet it is leased for 19 times and then sold in the second hand market. The surprising outcome of these experiments, even with smallest remanufacturing cost, leasing is not an appropriate option for the products which are still preserve 80% and 90% of its value. Since lease payments are low in these cases, manufacturer chooses to sell the products as they are manufactured. For c_r =20, c_r =30, and c_r =40 model behavior changes as well. The decision of optimal number of leasing decreases as α and c_r increases.

As a result of these experiments, it is observed that the profitability of the model decreases as the cost of remanufacturing linearly increases. On the other hand, when α is high, selling the product becomes more profitable than leasing it number of times.We observed that as c_r inceases, optimal number of leasing times are decreased. This is an expected result since remanufacturing costs is crucial for leasing business as well as the percentage of retained value α. In addition, total profit is increased significantly by lower values of c_r. For example, the total profit is 1056250 unit of money for c_r =50, while the total profit is 3293440 unit of money for c_r =10.

In summary, we conclude that, the if cost of remanufacturing which is held for recovering the product at the end of each lease period is low enough, high profitability can be achieved by increasing the number of lease periods. Another important conclusion is that if the retained value of a product after one year customer usage is high, therefore lease payments will be lower and profitability decreases. Even with smallest value of remanufacturing cost, remanufacturing and leasing is not profitable.

Table 5.15. Optimal (N^*,n^*) values with respect to different values of c_r.

Optimal (N^*,n^*) values					
	c_r =10	**c_r =20**	**c_r =30**	**c_r =40**	**c_r =50**
α=0.1	20,20	20,20	19,18	15,14	12,11
α=0.2	20,20	20,19	17,16	13,12	11,1
α=0.3	20,20	20,19	15,14	12,11	10,9
α=0.4	20,20	19,18	13,12	10,9	8,7
α=0.5	20,19	16,15	11,10	8,7	7,6
α=0.6	20,19	13,12	9,8	7,6	1,0
α=0.7	17,16	10,9	1,0	1,0	1,0
α=0.8	1,0	1,0	1,0	1,0	1,0
α=0.9	1,0	1,0	1,0	1,0	1,0

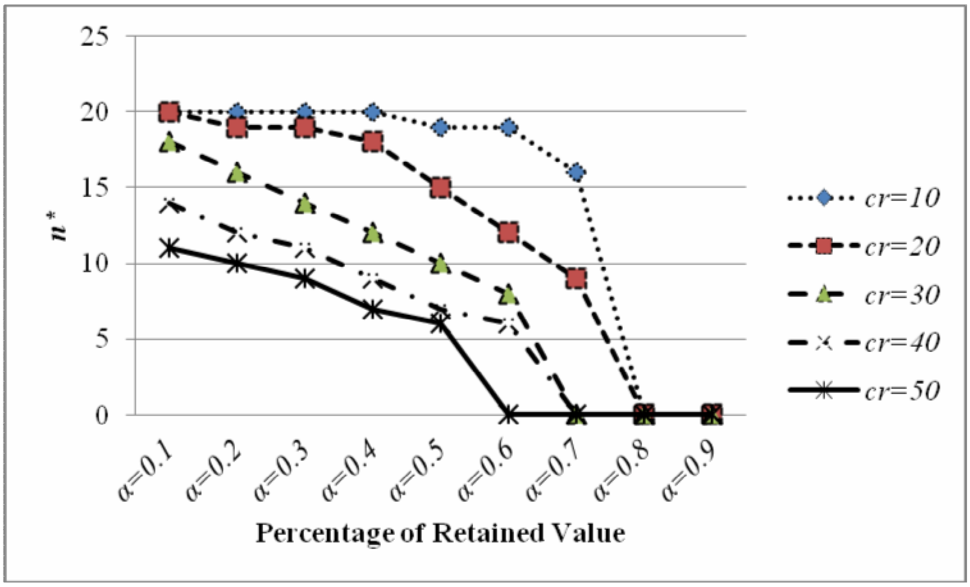

Figure 5.25. Optimal n^* values with respect to different values of c_r

Table 5.16. Total profit values with respect to different values of c_r.

Total Profit					
	c_r =10	c_r =20	c_r =30	c_r =40	c_r =50
α=0.1	3293440	2456420	1773750	1329440	1056250
α=0.2	2805190	1999110	1394270	1043170	827518
α=0.3	2320290	1565740	1061020	792011	626247
α=0.4	1840420	1150340	773988	577041	455810
α=0.5	1380160	793997	533174	395674	312971
α=0.6	939752	504963	388557	251535	249500
α=0.7	533608	283208	249500	249500	249500
α=0.8	249500	249500	249500	249500	249500
α=0.9	249500	249500	249500	249500	249500

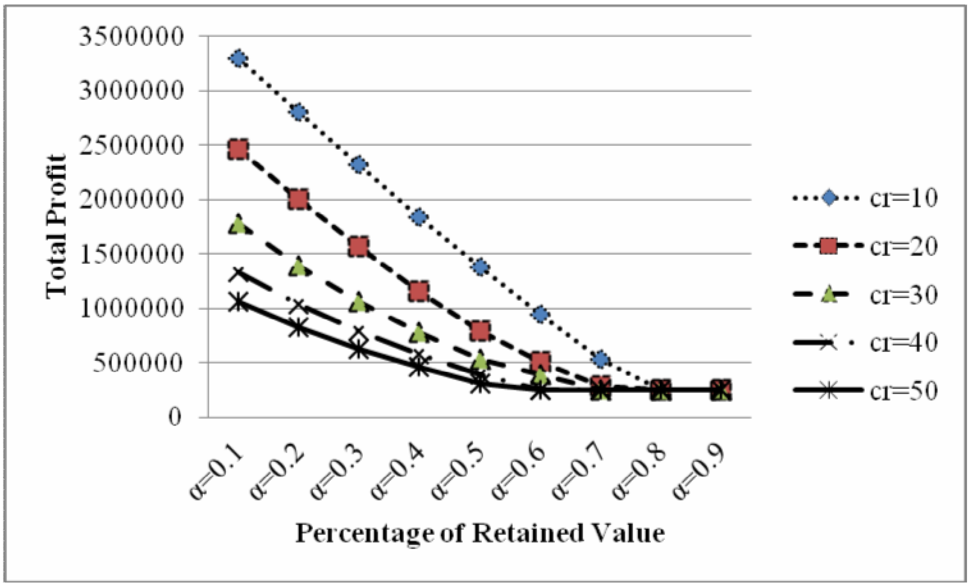

Figure 5.26. Total profit values with respect to different values of c_r.

6. CONCLUSIONS

In this thesis, three different supply chain structures are combined into a hybrid supply chain where products are sold or leased. At the beginning new products can either be sold to the customer or leased. The products which are sold to the new product market are the members of the first supply chain structure where a product is produced, sold to customer and used by customer during its useful life. The second structure includes a reverse supply chain where used products are collected from customers, and then recovered. The recovered product is sold to the second hand market after some periods. The last structure that appears in the model is a closed supply chain where returned products are recovered at the end of each lease period and leased during its entire life. Leased products are brought to the like new condition by product recovery, while the second hand market products are brought to the acceptable quality level. Products which complete their total life will be disposed and new product will be manufactured in this system.

We first evaluated the all cases of durability for all model parameters, and observed that the depreciation rate of a product and the remanufacturing effort are the two important parameters that affects the total profitability and the optimal values for the decisions variables. According to the results, leasing is a profitable business when the lease payments reach higher values. In our model lease payments are calculated by new product price P_0, and the annual depreciation rate of the product (1-α). Since quickly depreciated products lead to higher lease payments, leasing system is more suitable for the product which loses most of its value within a year. In the most profitable case, a product should be designed to survive 12 years and it should be leased 11 times for $(1-\alpha) = 0.9$. Leasing a slowly depreciated product is not a logical option as it reduces profitability. In cases where α is higher than 0.6, the product should be sold as it is manufactured. This only changes when $\alpha = 0.6$, $\beta \geq 0.5$ and $\alpha = 0.7$, $\beta = 0.9$. This implies that as remanufacturing effort increases, the profit from the second hand market will be increased, and the manufacturers want to sell their products into the second hand market after multiple leasing.

Design for remanufacturing is an important issue for leasing business since remanufacturing activities are cruial for leasing. Therefore design investments play a significant role by improving the efficiency of remanufacturing and leasing business. Proper design for remanufacture leads to higher costs in manufacturing, but it also extends the lifetime of a product and facilitates the remanufacturing activities. We investigate if the profitability is increased for the proposed model when the remanufacturability of a product is improved by proper design. Our experiments show that, a high design investment increases the durability of the product and also increases the number of leasing periods until a certain point. We observed that, investing more on design does not affect the optimal number of leasing time after *n* reaches it optimal value. In addition, design investment also affects the new product price, P_0^*, as it has an increasing trend with the increasing design investment. This is also an expected case since the model compensates the loss from initial design investment by increasing the price of a product.

Furthermore, we evaluate the impact of design in both scenarios which are traditional supply chains and pure leasing systems. Traditional supply chains are pure manufacturing systems where a product is manufactured and sold to the market as new good. Pure leasing systems are the fully closed loop supply chains where the product is leased throughout its entire life. In our model, the cases where n=0, is considered as a traditional supply chain, while the cases where $n=N$, is considered as a pure leasing systems. We observed that, adapting a product for product recovery activities is meaningless since the sold products are not intended to be used again. As design investment increases, profitability of the traditional supply chain model decreases. Here, the most profitable case is to invest the least amount of money on design and to sell products as soon as they are manufactured. For the pure leasing systems, design investment and the depreciation rate of the product are the most important components of the model. According to our experiments, products which still preserve a high percentages of their value after one year usage, do not need to be designed for larger durability. Since quickly depreciated products are suitable for servicizing, their life time need to be extended.

Also, we evaluate the effect of α and β partially in order to see the direct impact on proposed model. According to our experiments, α is the main factor which decides N^*,n^*, and P_0^*. Small values of α increses the profitability, durability and the number of lease periods. On the other hand, as β increses, profitability of the model is increased as well. Eventhough, large levels of β cause high costs for remanufacturing, the profit from the second hand market will cover this loss.

Another finding about the model is, for large durability levels, model avoids disposal activities. In order to understand this behavior, we recomputed the maximization model with using a salvage value. Here, salvage value is determined over the new product price and, the depreciation rate and the age of the product. Even with this positive value, model still avoids to lease products with large values of α.

In the last part of the thesis, we make a sensitivity analysis for the fixed remanufacturing costs. We observe that, leasing is a very profitable business strategy if remanufacturing cost is relatively low to the manufacturing cost. For c_r=10, model gives best value at higher durability levels and higher number of leasing times, even for the large values of α. Yet, for the largest values of α, model profitability decreases by leasing, therefore, selling in the first place would be the best option.

In conclusion, we see that the sales of function is a profitable model for highly depreciated products, and proper design makes the leasing business mor profitable. For design investment we only assign a single function that only effects the durability and remanufacturing cost. Future research can develop this design function by including attractiveness of the product, and ability to response innovative changes etc. We hope to have provided a starting point for the research on the effect of proper design in servicizing business with remanufacturing.

APPENDIX A: RESULT TABLES OF THE MAXIMIZATION MODEL

Table A.1. Optimal values for all levels of N where $0.1 \leq \alpha \leq 0.9$ and $0.1 \leq \beta \leq 0.9$.

α=0.1 β=0.1				α=0.1 β=0.2				α=0.1 β=0.3			
N	n*	Total Profit	P_0	N	n*	Total Profit	P_0	N	n*	Total Profit	P_0
1	0	249500	501	1	0	249500	501	1	0	249500	501
2	2	331317	570	2	2	331317	570	2	2	331317	570
3	3	485423	575	3	3	485423	575	3	3	485423	575
4	4	617833	585	4	4	617833	585	4	4	617833	585
5	5	729551	597	5	5	729551	597	5	5	729551	597
6	6	821665	609	6	6	821665	609	6	6	821665	609
7	7	895283	623	7	7	895283	623	7	7	895283	623
8	8	951525	636	8	8	951525	636	8	7	958353	618
9	9	991511	650	9	9	991511	650	9	8	1007590	633
10	9	1027020	646	10	9	1033970	647	10	9	1040920	647
11	10	1048560	660	11	10	1054020	661	11	10	1059490	662
12	11	1056250	675	12	11	1060320	675	12	11	1064410	676
13	12	1051210	689	13	12	1054000	689	13	12	1056810	691
14	12	1042870	690	14	12	1045680	691	14	12	1048510	692
15	12	1033930	692	15	12	1036780	692	15	12	1039640	693
16	12	1024420	693	16	12	1027300	694	16	12	1030190	695
17	12	1014350	694	17	12	1017250	695	17	12	1020180	696
18	12	1003710	696	18	12	1006650	697	18	12	1009610	698
19	12	992527	698	19	12	995499	699	19	12	998490	700
20	12	980804	700	20	12	983812	700	20	12	986838	701

Table A.1. Optimal values for all levels of N where 0.1≤ α ≤0.9 and 0.1≤ β ≤0.9 (cont.).

α=0.1 β=0.4				α=0.1 β=0.5				α=0.1 β=0.6			
N	n*	Total Profit	P_0	N	n*	Total Profit	P_0	N	n*	Total Profit	P_0
1	0	249500	501	1	0	249500	501	1	0	249500	501
2	2	331317	570	2	2	331317	570	2	1	333563	528
3	3	485423	575	3	3	485423	575	3	2	494951	545
4	4	617833	585	4	4	617833	585	4	3	633983	561
5	5	729551	597	5	4	735853	576	5	4	751789	576
6	6	821665	609	6	5	835553	591	6	5	849497	591
7	6	904133	605	7	6	916179	605	7	6	928233	606
8	7	968601	619	8	7	978859	620	8	7	989125	621
9	8	1016150	634	9	8	1247200	635	9	8	1033300	636
10	9	1047900	648	10	9	1054880	649	10	9	1061880	650
11	10	1064980	663	11	10	1070480	664	11	10	1076000	665
12	11	1068510	677	12	11	1072640	678	12	11	1076780	679
13	11	1060500	679	13	11	1064650	680	13	11	1068820	680
14	11	1051880	680	14	11	1056050	681	14	11	1060240	682
15	11	1042650	681	15	11	1046850	682	15	11	1051070	683
16	12	1033100	696	16	11	1037060	684	16	11	1041300	685
17	12	1023120	697	17	11	1026680	685	17	11	1030950	686
18	12	1012580	699	18	11	1015720	687	18	11	1020020	688
19	12	1001500	700	19	12	1004530	701	19	11	1008530	690
20	12	989881	702	20	12	992941	703	20	11	996494	691

α=0.1 β=0.7				α=0.1 β=0.8				α=0.1 β=0.9			
N	n*	Total Profit	P_0	N	n*	Total Profit	P_0	N	n*	Total Profit	P_0
1	0	249500	501	1	0	249500	501	1	0	249500	501
2	1	356100	528	2	1	378638	528	2	1	401176	527
3	2	515184	546	3	2	535417	546	3	2	555651	546
4	3	652018	562	4	3	670053	562	4	3	688089	562
5	4	767729	577	5	4	783671	577	5	4	799615	578
6	5	863445	592	6	5	877397	593	6	5	891353	593
7	6	940293	607	7	6	952358	608	7	6	964430	608
8	7	999399	622	8	7	1009680	622	8	7	1019970	623
9	8	1041890	636	9	8	1050490	637	9	8	1059100	638
10	9	1068890	651	10	9	1075920	652	10	9	1082960	653
11	10	1081540	666	11	10	1087090	666	11	10	1092650	667
12	11	1080950	680	12	11	1085120	681	12	11	1089320	682
13	11	1073000	681	13	11	1077200	682	13	11	1081420	683
14	11	1064450	683	14	11	1068680	683	14	11	1072920	684
15	11	1055300	684	15	11	1059550	685	15	11	1063810	686
16	11	1045560	685	16	11	1049830	686	16	11	1054120	687
17	11	1035240	687	17	11	1039540	688	17	11	1043850	689
18	11	1024340	689	18	11	1028670	689	18	11	1033010	690
19	11	1012880	690	19	11	1017240	691	19	11	1021610	692
20	11	1000870	692	20	11	1005260	693	20	11	1009660	694

Table A.1. Optimal values for all levels of N where $0.1 \leq \alpha \leq 0.9$ and $0.1 \leq \beta \leq 0.9$ (cont.).

α=0.2 β=0.1				α=0.2 β=0.2				α=0.2 β=0.3			
N	n*	Total Profit	P_0	N	n*	Total Profit	P_0	N	n*	Total Profit	P_0
1	0	249500	501	1	0	249500	501	1	0	249500	501
2	2	282450	579	2	2	282450	579	2	2	282450	579
3	3	412372	585	3	3	412372	585	3	3	412372	585
4	4	521140	596	4	4	521140	596	4	4	521140	596
5	5	609883	609	5	5	609883	609	5	5	609883	609
6	6	679821	623	6	6	679821	623	6	5	687180	600
7	7	732203	638	7	6	737269	616	7	6	749400	616
8	7	773680	631	8	7	784052	632	8	7	794431	633
9	8	806076	647	9	8	814804	648	9	8	823541	649
10	9	823600	664	10	9	830793	664	10	9	837997	665
11	10	827518	680	11	10	833286	681	11	10	839066	682
12	10	820210	681	12	10	825998	682	12	10	831796	683
13	10	812296	683	13	10	818105	684	13	10	823924	684
14	10	803788	684	14	10	809618	685	14	10	815458	686
15	11	794770	701	15	10	800548	687	15	10	806410	688
16	11	785548	702	16	10	790906	689	16	10	796791	690
17	11	775788	704	17	10	780703	691	17	10	786612	692
18	11	765501	706	18	11	770101	707	18	10	775886	694
19	11	754698	708	19	11	759326	709	19	10	764624	696
20	11	743393	710	20	11	748049	711	20	10	752842	698

α=0.2 β=0.4				α=0.2 β=0.5				α=0.2 β=0.6			
N	n*	Total Profit	P_0	N	n*	Total Profit	P_0	N	n*	Total Profit	P_0
1	0	249500	501	1	0	249500	501	1	0	249500	501
2	2	282450	579	2	1	286113	530	2	1	308649	530
3	3	412372	585	3	2	425175	550	3	2	445410	550
4	3	524147	567	4	3	542193	567	4	3	560239	567
5	4	622469	584	5	4	638437	584	5	4	654405	585
6	5	701176	600	6	5	715175	601	6	5	729176	601
7	6	761536	617	7	6	773676	617	7	6	785819	618
8	7	804816	633	8	7	815207	634	8	7	825603	635
9	8	832285	650	9	8	841036	650	9	8	849794	651
10	9	845209	666	10	9	852431	667	10	9	859661	668
11	10	844857	682	11	10	850659	683	11	10	856471	684
12	10	837606	684	12	10	843426	684	12	10	849255	685
13	10	829753	685	13	10	835592	686	13	10	841441	687
14	10	821308	687	14	10	827168	688	14	10	833036	688
15	10	812282	689	15	10	818163	689	15	10	824052	690
16	10	802686	690	16	10	808588	691	16	10	814499	692
17	10	792530	692	17	10	798455	693	17	10	804388	694
18	10	781827	694	18	10	787775	695	18	10	793731	696
19	10	770590	697	19	10	776562	697	19	10	782540	698
20	10	758832	699	20	10	764828	700	20	10	770829	700

Table A.1. Optimal values for all levels of N where $0.1 \leq \alpha \leq 0.9$ and $0.1 \leq \beta \leq 0.9$ (cont.).

α=0.2 β=0.7				α=0.2 β=0.8				α=0.2 β=0.9			
N	n^*	Total Profit	P_0	N	n^*	Total Profit	P_0	N	n^*	Total Profit	P_0
1	0	249500	501	1	0	249500	501	1	0	249500	501
2	1	331186	530	2	1	353722	529	2	1	376259	529
3	2	465645	550	3	2	485880	550	3	2	506115	550
4	3	578286	568	4	3	596333	568	4	3	614380	568
5	4	670375	585	5	4	686346	585	5	4	702318	586
6	5	743179	602	6	5	757185	602	6	5	771193	603
7	6	797966	619	7	6	810117	619	7	6	822271	620
8	7	836004	635	8	7	846410	636	8	7	856821	636
9	8	858559	652	9	8	867330	652	9	8	876108	653
10	9	866899	668	10	9	874146	669	10	9	881400	670
11	10	862293	685	11	10	868124	685	11	9	874519	671
12	10	855095	686	12	10	860943	687	12	9	867005	672
13	10	847298	687	13	10	853165	688	13	10	859040	689
14	10	838913	689	14	10	844799	690	14	10	850693	690
15	10	829949	691	15	10	835855	691	15	10	841768	692
16	10	820417	692	16	10	826343	693	16	10	832276	694
17	10	810328	694	17	10	816275	695	17	10	822229	696
18	10	799693	696	18	10	805662	697	18	10	811637	698
19	10	788525	698	19	10	794516	699	19	10	800512	700
20	10	77683	701	20	10	782850	701	20	10	788868	702

α=0.3 β=0.1				α=0.3 β=0.2				α=0.3 β=0.3			
N	n^*	Total Profit	P_0	N	n^*	Total Profit	P_0	N	n^*	Total Profit	P_0
1	0	249500	501	1	0	249500	501	1	0	249500	501
2	0	247755	502	2	0	247755	502	2	0	247755	502
3	3	339878	597	3	3	339878	597	3	3	339878	597
4	4	425393	610	4	4	425393	610	4	3	432619	575
5	5	491738	625	5	4	493680	594	5	4	509669	594
6	6	540307	641	6	5	553823	613	6	5	567865	613
7	6	584233	631	7	6	596444	631	7	6	608656	632
8	7	612496	650	8	7	622992	650	8	7	633492	650
9	8	626018	668	9	8	634917	669	9	8	643820	669
10	9	626247	687	10	9	633666	687	10	9	641089	688
11	9	619724	688	11	9	627155	689	11	9	634590	689
12	9	612611	690	12	9	620055	691	12	9	627502	691
13	9	604921	692	13	9	612378	693	13	9	619837	693
14	9	596666	694	14	9	604135	695	14	9	611608	695
15	9	587858	696	15	9	595341	697	15	9	602827	697
16	9	578513	699	16	9	586009	699	16	9	593507	700
17	9	568645	701	17	9	576154	702	17	9	583664	702
18	9	558270	704	18	9	565790	704	18	9	573312	705
19	9	547405	707	19	9	554935	707	19	9	562467	708
20	9	536066	710	20	9	543606	710	20	9	551148	711

Table A.1. Optimal values for all levels of N where $0.1 \leq \alpha \leq 0.9$ and $0.1 \leq \beta \leq 0.9$ (cont.).

α=0.3 β=0.4				α=0.3 β=0.5				α=0.3 β=0.6			
N	n^*	Total Profit	P_0	N	n^*	Total Profit	P_0	N	n^*	Total Profit	P_0
1	0	249500	501	1	0	249500	501	1	0	249500	501
2	0	247755	502	2	1	261217	533	2	1	283749	532
3	2	355496	555	3	2	375729	555	3	2	395961	555
4	3	450670	575	4	3	468722	575	4	3	486774	575
5	4	525658	594	5	4	541647	594	5	4	557636	595
6	5	581908	613	6	5	595952	614	6	5	609996	614
7	6	620870	632	7	6	633085	632	7	6	645302	633
8	7	643993	651	8	7	654497	651	8	7	665002	652
9	8	652726	670	9	8	661634	670	9	8	670546	671
10	9	648516	688	10	9	655947	689	10	8	664347	672
11	9	642028	690	11	9	649470	690	11	8	657519	674
12	9	634952	692	12	9	642406	692	12	8	650074	676
13	9	627300	694	13	9	634766	694	13	9	642234	694
14	9	619083	696	14	9	626561	696	14	9	634041	696
15	9	610314	698	15	9	617804	698	15	9	625295	698
16	9	601007	700	16	9	608508	700	16	9	616011	701
17	9	591175	702	17	9	598688	703	17	9	606202	703
18	9	580834	705	18	9	584188	690	18	9	595883	706
19	9	570000	708	19	9	577534	708	19	9	585068	708
20	9	558689	711	20	9	566232	711	20	9	573775	711

α=0.3 β=0.7				α=0.3 β=0.8				α=0.3 β=0.9			
N	n^*	Total Profit	P_0	N	n^*	Total Profit	P_0	N	n^*	Total Profit	P_0
1	0	249500	501	1	0	249500	501	1	0	249500	501
2	1	306283	532	2	1	328817	531	2	1	351352	531
3	2	416194	554	3	2	436427	554	3	2	456660	554
4	3	504826	575	4	3	522878	575	4	3	540930	575
5	4	573626	595	5	4	589616	595	5	4	605605	595
6	5	624041	614	6	5	638086	614	6	5	652132	615
7	6	657519	633	7	6	669738	634	7	6	681958	634
8	7	642341	664	8	7	686019	653	8	7	696530	653
9	8	679460	671	9	8	688377	672	9	8	697297	672
10	8	673270	673	10	8	682196	673	10	8	691123	673
11	8	666451	674	11	8	675386	675	11	8	684322	675
12	8	659015	676	12	8	667959	676	12	8	676904	677
13	8	650974	678	13	8	659927	678	13	8	668880	679
14	8	630599	664	14	8	651302	680	14	8	660264	681
15	8	633129	682	15	8	642099	683	15	8	651069	683
16	9	623516	701	16	8	632330	685	16	8	641308	685
17	9	613718	703	17	8	622011	688	17	8	630996	688
18	9	603408	706	18	8	611157	690	18	8	620148	691
19	9	592603	709	19	9	600139	709	19	8	608781	693
20	9	581318	711	20	9	588862	712	20	8	596912	696

Table A.1. Optimal values for all levels of N where $0.1 \leq \alpha \leq 0.9$ and $0.1 \leq \beta \leq 0.9$ (cont.).

α=0.4 β=0.1				α=0.4 β=0.2				α=0.4 β=0.3			
N	$n*$	Total Profit	P_0	N	$n*$	Total Profit	P_0	N	$n*$	Total Profit	P_0
1	0	249500	501	1	0	249500	501	1	0	249500	501
2	0	247755	502	2	0	247755	502	2	0	247755	502
3	3	268219	613	3	3	268219	613	3	2	285973	563
4	4	331062	628	4	3	341530	586	4	3	359569	586
5	4	381804	609	5	4	397788	608	5	4	413772	608
6	5	422167	630	6	5	436220	630	6	5	450273	630
7	6	446272	652	7	6	458516	652	7	6	470761	652
8	7	455810	674	8	7	466368	674	8	7	476927	674
9	8	452470	696	9	8	461465	696	9	7	471446	676
10	8	446746	698	10	8	455742	698	10	7	465346	678
11	8	440452	700	11	8	449449	700	11	8	458447	700
12	8	433604	702	12	8	442601	702	12	8	451598	702
13	8	426216	705	13	8	435211	704	13	8	444207	704
14	8	418305	707	14	8	427297	707	14	8	436289	707
15	8	409887	710	15	8	418874	710	15	8	427861	710
16	8	400981	713	16	8	409960	713	16	8	418940	713
17	8	391606	717	17	8	400575	716	17	8	409545	716
18	8	381782	720	18	8	390738	720	18	8	399695	720
19	8	371531	724	19	8	380470	724	19	8	389412	723
20	8	360877	728	20	8	369795	728	20	8	378716	727

α=0.4 β=0.4				α=0.4 β=0.5				α=0.4 β=0.6			
N	$n*$	Total Profit	P_0	N	$n*$	Total Profit	P_0	N	$n*$	Total Profit	P_0
1	0	249500	501	1	0	249500	501	1	0	249500	501
2	0	247755	502	2	0	247755	502	2	1	258866	535
3	2	306192	562	3	2	326412	561	3	2	346634	561
4	3	377610	585	4	3	395651	585	4	3	413693	584
5	4	429758	608	5	4	445743	607	5	4	461729	607
6	5	464326	630	6	5	478379	630	6	5	492433	630
7	6	483005	652	7	6	495250	652	7	6	507494	652
8	7	487485	674	8	7	498043	674	8	7	508602	674
9	7	482004	676	9	7	492563	676	9	7	503122	676
10	7	475904	678	10	7	486462	678	10	7	497020	678
11	7	469197	680	11	7	479754	680	11	7	490311	680
12	7	461901	683	12	7	472454	682	12	7	483008	682
13	7	454030	685	13	7	464578	685	13	7	475127	685
14	7	445601	688	14	7	456143	688	14	7	466685	688
15	8	436848	710	15	7	447166	691	15	7	457699	691
16	8	427921	713	16	7	437666	694	16	7	448188	694
17	8	418516	716	17	7	427663	698	17	7	438170	697
18	8	408654	719	18	8	417615	719	18	7	427668	701
19	8	398355	723	19	8	407301	722	19	7	416702	705
20	8	387641	726	20	8	396568	726	20	8	405498	725

Table A.1. Optimal values for all levels of N where $0.1 \leq \alpha \leq 0.9$ and $0.1 \leq \beta \leq 0.9$ (cont.).

α=0.4 β=0.7				α=0.4 β=0.8				α=0.4 β=0.9			
N	n^*	Total Profit	P_0	N	n^*	Total Profit	P_0	N	n^*	Total Profit	P_0
1	0	249500	501	1	0	249500	501	1	0	249500	501
2	1	281395	534	2	1	303925	534	2	1	326457	533
3	2	366858	560	3	2	387083	560	3	2	407308	559
4	3	431736	584	4	3	449779	584	4	3	467823	583
5	4	477716	607	5	4	493702	607	5	4	509690	606
6	5	506487	630	6	5	520541	629	6	5	534595	629
7	6	519738	652	7	6	531983	652	7	6	544228	652
8	7	519160	674	8	7	529719	674	8	7	540277	674
9	7	513681	676	9	7	524239	676	9	7	534798	676
10	7	507578	678	10	7	518137	678	10	7	528695	678
11	7	500867	680	11	7	511424	680	11	7	521981	680
12	7	493562	682	12	7	504116	682	12	7	514670	682
13	7	485677	685	13	7	496227	684	13	7	506777	684
14	7	477229	687	14	7	487772	687	14	7	498317	687
15	7	468234	690	15	7	478770	690	15	7	489306	690
16	7	458711	694	16	7	469236	693	16	7	479763	693
17	7	448680	697	17	7	459191	696	17	7	469705	696
18	7	438160	700	18	7	448654	700	18	7	459152	699
19	7	427173	704	19	7	437647	704	19	7	448125	703
20	7	415742	708	20	7	426192	708	20	7	436646	707

α=0.5 β=0.1				α=0.5 β=0.2				α=0.5 β=0.3			
N	n^*	Total Profit	P_0	N	n^*	Total Profit	P_0	N	n^*	Total Profit	P_0
1	0	249500	501	1	0	249500	501	1	0	249500	501
2	0	247755	502	2	0	247755	502	2	0	247755	502
3	0	245355	504	3	0	245355	504	3	0	245355	504
4	3	251213	603	4	3	269187	602	4	3	287171	600
5	4	287483	630	5	4	303389	628	5	4	319303	627
6	5	307393	656	6	5	321356	655	6	5	335326	653
7	6	312971	682	7	6	325116	681	7	6	337267	680
8	6	308263	684	8	6	320393	683	8	6	332530	682
9	6	302957	687	9	6	315068	686	9	6	327187	685
10	6	297070	690	10	6	309159	689	10	6	321257	687
11	6	290625	693	11	6	302686	692	11	6	314759	691
12	6	283642	697	12	6	295670	696	12	6	307712	694
13	7	276495	722	13	6	288134	699	13	6	300140	698
14	7	269075	726	14	6	280101	704	14	6	292064	702
15	7	261216	730	15	6	271596	708	15	6	283511	706
16	7	252941	734	16	7	263152	733	16	6	274506	711
17	7	244277	739	17	7	254433	737	17	6	265078	716
18	7	235252	744	18	7	245345	742	18	7	255462	740
19	7	225894	749	19	7	235915	747	19	7	245965	745
20	7	216236	754	20	7	226176	752	20	7	236148	750

Table A.1. Optimal values for all levels of N where $0.1 \leq \alpha \leq 0.9$ and $0.1 \leq \beta \leq 0.9$ (cont.).

α=0.5 β=0.4				α=0.5 β=0.5				α=0.5 β=0.6			
N	n^*	Total Profit	P_0	N	n^*	Total Profit	P_0	N	n^*	Total Profit	P_0
1	0	249500	501	1	0	249500	501	1	0	249500	501
2	0	247755	502	2	0	247755	502	2	0	247755	502
3	2	257086	571	3	2	277278	570	3	2	297476	568
4	3	305161	599	4	3	323158	598	4	3	341160	596
5	4	335224	626	5	4	351150	625	5	4	367081	624
6	5	349302	652	6	5	363283	652	6	5	377269	651
7	6	349423	679	7	6	361584	678	7	6	373750	677
8	6	344673	681	8	6	356822	680	8	6	368976	679
9	6	339314	684	9	6	351447	683	9	6	363587	682
10	6	333364	686	10	6	345479	685	10	6	357601	684
11	6	326841	689	11	6	338934	688	11	6	351034	687
12	6	319766	693	12	6	331832	692	12	6	343907	690
13	6	312160	696	13	6	324193	695	13	6	336238	694
14	6	304045	700	14	6	316040	699	14	6	328050	698
15	6	295445	705	15	6	307396	703	15	6	319365	702
16	6	286386	709	16	6	298287	708	16	6	310207	706
17	6	276896	714	17	6	288739	712	17	6	300604	711
18	6	267005	719	18	6	278781	717	18	6	290583	715
19	6	256743	725	19	6	268442	723	19	6	280172	721
20	7	246151	748	20	6	257755	728	20	6	269403	726

α=0.5 β=0.7				α=0.5 β=0.8				α=0.5 β=0.9			
N	n^*	Total Profit	P_0	N	n^*	Total Profit	P_0	N	n^*	Total Profit	P_0
1	0	249500	501	1	0	249500	501	1	0	249500	501
2	0	247755	502	2	1	279050	536	2	1	301577	535
3	2	317679	567	3	2	337884	566	3	2	358093	565
4	3	359166	595	4	3	377177	595	4	3	395190	594
5	4	383017	623	5	4	398957	622	5	4	414901	621
6	5	391260	650	6	5	405254	649	6	5	419252	648
7	5	386782	652	7	5	400768	651	7	5	414757	650
8	5	381626	654	8	5	395599	653	8	5	409577	652
9	6	375810	657	9	5	389768	656	9	5	403731	655
10	6	369730	683	10	5	383291	659	10	5	397237	658
11	6	363143	686	11	5	376188	662	11	5	390112	661
12	6	355991	689	12	5	368480	665	12	5	382378	664
13	6	348294	693	13	6	360360	692	13	5	374055	668
14	6	340072	696	14	6	352106	695	14	5	365165	672
15	6	331348	700	15	6	343345	699	15	5	355734	676
16	6	322146	704	16	6	334100	703	16	6	346069	702
17	6	312490	709	17	6	324395	707	17	6	336317	706
18	6	302409	714	18	6	314257	712	18	6	326125	710
19	6	291929	719	19	6	303713	717	19	6	315520	715
20	6	281083	724	20	6	292793	722	20	6	304531	720

Table A.1. Optimal values for all levels of N where $0.1 \leq \alpha \leq 0.9$ and $0.1 \leq \beta \leq 0.9$ (cont.).

α=0.6 β=0.1				α=0.6 β=0.2				α=0.6 β=0.3			
N	$n*$	Total Profit	P_0	N	$n*$	Total Profit	P_0	N	$n*$	Total Profit	P_0
1	0	249500	501	1	0	249500	501	1	0	249500	501
2	0	247755	502	2	0	247755	502	2	0	247755	502
3	0	245355	504	3	0	245355	504	3	0	245355	504
4	0	241941	508	4	0	241941	508	4	0	241941	508
5	0	237559	512	5	0	237559	512	5	0	237559	512
6	0	232244	518	6	0	232244	518	6	0	232244	518
7	0	226032	524	7	0	226032	524	7	0	226032	524
8	0	218966	532	8	0	218966	532	8	0	218966	532
9	0	211089	540	9	0	211089	540	9	0	211089	540
10	0	202455	550	10	0	202455	550	10	5	205314	701
11	0	193120	560	11	0	193120	560	11	5	199105	705
12	0	183148	572	12	0	183148	572	12	5	192409	710
13	5	158958	724	13	5	172067	720	13	5	185258	716
14	5	151626	731	14	5	164608	726	14	5	177684	722
15	5	143941	738	15	5	156779	733	15	5	169723	728
16	5	135946	745	16	5	148619	740	16	5	161413	735
17	6	127822	773	17	5	140172	747	17	5	152795	742
18	6	120035	780	18	5	131481	755	18	5	143913	749
19	6	112069	788	19	5	122596	763	19	5	134811	757
20	6	103968	796	20	6	114165	790	20	5	125538	766

α=0.6 β=0.4				α=0.6 β=0.5				α=0.6 β=0.6			
N	$n*$	Total Profit	P_0	N	$n*$	Total Profit	P_0	N	$n*$	Total Profit	P_0
1	0	249500	501	1	0	249500	501	1	0	249500	501
2	0	247755	502	2	0	247755	502	2	0	247755	502
3	0	245355	504	3	0	245355	504	3	2	248559	578
4	0	241941	508	4	3	251550	615	4	3	269449	613
5	4	242877	651	5	4	258617	649	5	4	274380	646
6	4	239121	654	6	4	254835	651	6	4	270574	649
7	4	234691	657	7	4	250372	654	7	4	266081	652
8	5	229773	690	8	4	245253	658	8	4	260924	655
9	5	224570	694	9	4	239503	662	9	4	255129	659
10	5	218817	698	10	4	233150	666	10	4	248721	663
11	5	212539	702	11	4	226222	671	11	4	241727	668
12	5	205764	707	12	5	219176	703	12	4	234177	673
13	5	198521	712	13	5	211848	708	13	4	226103	679
14	5	190841	718	14	5	204071	714	14	4	217540	685
15	5	182760	724	15	5	195879	720	15	5	209070	716
16	5	174313	730	16	5	187305	726	16	5	200379	722
17	5	165539	737	17	5	178387	732	17	5	191329	728
18	5	156480	744	18	5	169166	739	18	5	181958	735
19	5	147179	752	19	5	159683	747	19	5	172307	742
20	5	137684	760	20	5	149983	755	20	5	162418	750

Table A.1. Optimal values for all levels of N where $0.1 \leq \alpha \leq 0.9$ and $0.1 \leq \beta \leq 0.9$ (cont.).

α=0.6 β=0.7				α=0.6 β=0.8				α=0.6 β=0.9			
N	$n*$	Total Profit	P_0	N	$n*$	Total Profit	P_0	N	$n*$	Total Profit	P_0
1	0	249500	501	1	0	249500	501	1	0	249500	501
2	0	247755	502	2	1	254196	539	2	1	276715	538
3	2	268718	576	3	2	288887	575	3	2	309063	573
4	3	287364	611	4	3	305293	609	4	3	323234	607
5	4	290163	644	5	4	305963	642	5	4	321779	641
6	4	286336	647	6	4	302117	645	6	4	317915	643
7	4	281815	649	7	4	297571	647	7	4	313346	645
8	4	276624	653	8	4	292349	650	8	4	308095	648
9	4	270787	656	9	4	286473	654	9	4	302184	652
10	4	264328	660	10	4	279969	658	10	4	295637	656
11	4	257275	665	11	4	272860	662	11	4	288478	660
12	4	249654	670	12	4	265174	667	12	4	280733	664
13	4	241497	675	13	4	256942	672	13	4	272430	669
14	4	232837	681	14	4	248193	678	14	4	263601	675
15	4	223710	688	15	4	238964	684	15	4	254277	681
16	4	214154	694	16	4	229290	690	16	4	244494	687
17	5	204354	724	17	4	219209	697	17	4	234290	693
18	5	194845	731	18	4	208765	705	18	4	223704	700
19	5	185037	738	19	4	198000	712	19	4	212777	708
20	5	174973	745	20	5	187635	741	20	5	201556	716

α=0.7 β=0.1				α=0.7 β=0.2				α=0.7 β=0.3			
N	$n*$	Total Profit	P_0	N	$n*$	Total Profit	P_0	N	$n*$	Total Profit	P_0
1	0	249500	501	1	0	249500	501	1	0	249500	501
2	0	247755	502	2	0	247755	502	2	0	247755	502
3	0	245355	504	3	0	245355	504	3	0	245355	504
4	0	241941	508	4	0	241941	508	4	0	241941	508
5	0	237559	512	5	0	237559	512	5	0	237559	512
6	0	232244	518	6	0	232244	518	6	0	232244	518
7	0	226032	524	7	0	226032	524	7	0	226032	524
8	0	218966	532	8	0	218966	532	8	0	218966	532
9	0	211089	540	9	0	211089	540	9	0	211089	540
10	0	202455	550	10	0	202455	550	10	0	202455	550
11	0	193120	560	11	0	193120	560	11	0	193120	560
12	0	183148	572	12	0	183148	572	12	0	183148	572
13	0	172608	584	13	0	172608	584	13	0	172608	584
14	0	161575	598	14	0	161575	598	14	0	161575	598
15	0	150130	612	15	0	150130	612	15	0	150130	612
16	0	138361	628	16	0	138361	628	16	0	138361	628
17	0	126359	644	17	0	126359	644	17	0	126359	644
18	0	114225	662	18	0	114225	662	18	0	114225	662
19	0	102063	680	19	0	102063	680	19	0	102063	680
20	0	89985	700	20	0	89985	700	20	0	89985	700

Table A.1. Optimal values for all levels of N where $0.1 \leq \alpha \leq 0.9$ and $0.1 \leq \beta \leq 0.9$ (cont.).

α=0.7 β=0.4				α=0.7 β=0.5				α=0.7 β=0.6			
N	$n*$	Total Profit	P_0	N	$n*$	Total Profit	P_0	N	$n*$	Total Profit	P_0
1	0	249500	501	1	0	249500	501	1	0	249500	501
2	0	247755	502	2	0	247755	502	2	0	247755	502
3	0	245355	504	3	0	245355	504	3	0	245355	504
4	0	241941	508	4	0	241941	508	4	0	241941	508
5	0	237559	512	5	0	237559	512	5	0	237559	512
6	0	232244	518	6	0	232244	518	6	0	232244	518
7	0	226032	524	7	0	226032	524	7	0	226032	524
8	0	218966	532	8	0	218966	532	8	0	218966	532
9	0	211089	540	9	0	211089	540	9	0	211089	540
10	0	202455	550	10	0	202455	550	10	0	202455	550
11	0	193120	560	11	0	193120	560	11	0	193120	560
12	0	183148	572	12	0	183148	572	12	0	183148	572
13	0	172608	584	13	0	172608	584	13	0	172608	584
14	0	161575	598	14	0	161575	598	14	0	161575	598
15	0	150130	612	15	0	150130	612	15	0	150130	612
16	0	138361	628	16	0	138361	628	16	0	138361	628
17	0	126359	644	17	0	126359	644	17	0	126359	644
18	0	114225	662	18	0	114225	662	18	0	114225	662
19	0	102063	680	19	0	102063	680	19	0	102063	680
20	0	89985	700	20	0	89985	700	20	0	89985	700

α=0.7 β=0.7				α=0.7 β=0.8				α=0.7 β=0.9			
N	$n*$	Total Profit	P_0	N	$n*$	Total Profit	P_0	N	$n*$	Total Profit	P_0
1	0	249500	501	1	0	249500	501	1	0	249500	501
2	0	247755	502	2	0	247755	502	2	1	251877	541
3	0	245355	504	3	0	245355	504	3	2	260292	583
4	0	241941	508	4	0	241941	508	4	0	241941	508
5	0	237559	512	5	0	237559	512	5	2	253883	588
6	0	232244	518	6	0	232244	518	6	2	249431	592
7	0	226032	524	7	0	226032	524	7	2	244195	596
8	0	218966	532	8	0	218966	532	8	2	238208	601
9	0	211089	540	9	3	211224	647	9	2	231503	607
10	0	202455	550	10	3	204598	653	10	2	224116	613
11	0	193120	560	11	3	197393	659	11	2	216086	620
12	0	183148	572	12	3	189645	666	12	2	207455	628
13	0	172608	584	13	3	181397	673	13	3	198544	667
14	0	161575	598	14	3	172693	681	14	3	189687	675
15	0	150130	612	15	3	163581	689	15	3	180397	683
16	0	138361	628	16	3	154112	698	16	3	170722	692
17	3	128177	716	17	3	144341	708	17	3	160716	701
18	3	118467	727	18	3	134327	718	18	0	114225	662
19	3	108616	739	19	3	124132	729	19	3	139930	721
20	3	98695	751	20	3	113820	741	20	3	129271	732

Table A.1. Optimal values for all levels of N where $0.1 \leq \alpha \leq 0.9$ and $0.1 \leq \beta \leq 0.9$ (cont.).

α=0.8 β=0.1				α=0.8 β=0.2				α=0.8 β=0.3			
N	n^*	Total Profit	P_0	N	n^*	Total Profit	P_0	N	n^*	Total Profit	P_0
1	0	249500	501	1	0	249500	501	1	0	249500	501
2	0	247755	502	2	0	247755	502	2	0	247755	502
3	0	245355	504	3	0	245355	504	3	0	245355	504
4	0	241941	508	4	0	241941	508	4	0	241941	508
5	0	237559	512	5	0	237559	512	5	0	237559	512
6	0	232244	518	6	0	232244	518	6	0	232244	518
7	0	226032	524	7	0	226032	524	7	0	226032	524
8	0	218966	532	8	0	218966	532	8	0	218966	532
9	0	211089	540	9	0	211089	540	9	0	211089	540
10	0	202455	550	10	0	202455	550	10	0	202455	550
11	0	193120	560	11	0	193120	560	11	0	193120	560
12	0	183148	572	12	0	183148	572	12	0	183148	572
13	0	172608	584	13	0	172608	584	13	0	172608	584
14	0	161575	598	14	0	161575	598	14	0	161575	598
15	0	150130	612	15	0	150130	612	15	0	150130	612
16	0	138361	628	16	0	138361	628	16	0	138361	628
17	0	126359	644	17	0	126359	644	17	0	126359	644
18	0	114225	662	18	0	114225	662	18	0	114225	662
19	0	102063	680	19	0	102063	680	19	0	102063	680
20	0	89985	700	20	0	89985	700	20	0	89985	700

α=0.8 β=0.4				α=0.8 β=0.5				α=0.8 β=0.6			
N	n^*	Total Profit	P_0	N	n^*	Total Profit	P_0	N	n^*	Total Profit	P_0
1	0	249500	501	1	0	249500	501	1	0	249500	501
2	0	247755	502	2	0	247755	502	2	0	247755	502
3	0	245355	504	3	0	245355	504	3	0	245355	504
4	0	241941	508	4	0	241941	508	4	0	241941	508
5	0	237559	512	5	0	237559	512	5	0	237559	512
6	0	232244	518	6	0	232244	518	6	0	232244	518
7	0	226032	524	7	0	226032	524	7	0	226032	524
8	0	218966	532	8	0	218966	532	8	0	218966	532
9	0	211089	540	9	0	211089	540	9	0	211089	540
10	0	202455	550	10	0	202455	550	10	0	202455	550
11	0	193120	560	11	0	193120	560	11	0	193120	560
12	0	183148	572	12	0	183148	572	12	0	183148	572
13	0	172608	584	13	0	172608	584	13	0	172608	584
14	0	161575	598	14	0	161575	598	14	0	161575	598
15	0	150130	612	15	0	150130	612	15	0	150130	612
16	0	138361	628	16	0	138361	628	16	0	138361	628
17	0	126359	644	17	0	126359	644	17	0	126359	644
18	0	114225	662	18	0	114225	662	18	0	114225	662
19	0	102063	680	19	0	102063	680	19	0	102063	680
20	0	89985	700	20	0	89985	700	20	0	89985	700

Table A.1. Optimal values for all levels of *N* where 0.1≤ α ≤0.9 and 0.1≤ β ≤0.9 (cont.).

α=0.8 β=0.7				α=0.8 β=0.8				α=0.8 β=0.9			
N	*n**	Total Profit	P_0	*N*	*n**	Total Profit	P_0	*N*	*n**	Total Profit	P_0
1	0	249500	501	1	0	249500	501	1	0	249500	501
2	0	247755	502	2	0	247755	502	2	0	247755	502
3	0	245355	504	3	0	245355	504	3	0	245355	504
4	0	241941	508	4	0	241941	508	4	0	241941	508
5	0	237559	512	5	0	237559	512	5	0	237559	512
6	0	232244	518	6	0	232244	518	6	0	232244	518
7	0	226032	524	7	0	226032	524	7	0	226032	524
8	0	218966	532	8	0	218966	532	8	0	218966	532
9	0	211089	540	9	0	211089	540	9	0	211089	540
10	0	202455	550	10	0	202455	550	10	0	202455	550
11	0	193120	560	11	0	193120	560	11	0	193120	560
12	0	183148	572	12	0	183148	572	12	0	183148	572
13	0	172608	584	13	0	172608	584	13	0	172608	584
14	0	161575	598	14	0	161575	598	14	0	161575	598
15	0	150130	612	15	0	150130	612	15	0	150130	612
16	0	138361	628	16	0	138361	628	16	0	138361	628
17	0	126359	644	17	0	126359	644	17	0	126359	644
18	0	114225	662	18	0	114225	662	18	0	114225	662
19	0	102063	680	19	0	102063	680	19	0	102063	680
20	0	89985	700	20	0	89985	700	20	0	89985	700

α=0.9 β=0.1				α=0.9 β=0.2				α=0.9 β=0.3			
N	*n**	Total Profit	P_0	*N*	*n**	Total Profit	P_0	*N*	*n**	Total Profit	P_0
1	0	249500	501	1	0	249500	501	1	0	249500	501
2	0	247755	502	2	0	247755	502	2	0	247755	502
3	0	245355	504	3	0	245355	504	3	0	245355	504
4	0	241941	508	4	0	241941	508	4	0	241941	508
5	0	237559	512	5	0	237559	512	5	0	237559	512
6	0	232244	518	6	0	232244	518	6	0	232244	518
7	0	226032	524	7	0	226032	524	7	0	226032	524
8	0	218966	532	8	0	218966	532	8	0	218966	532
9	0	211089	540	9	0	211089	540	9	0	211089	540
10	0	202455	550	10	0	202455	550	10	0	202455	550
11	0	193120	560	11	0	193120	560	11	0	193120	560
12	0	183148	572	12	0	183148	572	12	0	183148	572
13	0	172608	584	13	0	172608	584	13	0	172608	584
14	0	161575	598	14	0	161575	598	14	0	161575	598
15	0	150130	612	15	0	150130	612	15	0	150130	612
16	0	138361	628	16	0	138361	628	16	0	138361	628
17	0	126359	644	17	0	126359	644	17	0	126359	644
18	0	114225	662	18	0	114225	662	18	0	114225	662
19	0	102063	680	19	0	102063	680	19	0	102063	680
20	0	89985	700	20	0	89985	700	20	0	89985	700

Table A.1. Optimal values for all levels of N where $0.1 \leq \alpha \leq 0.9$ and $0.1 \leq \beta \leq 0.9$ (cont.).

α=0.9 β=0.4				α=0.9 β=0.5				α=0.9 β=0.6			
N	n^*	Total Profit	P_0	N	n^*	Total Profit	P_0	N	n^*	Total Profit	P_0
1	0	249500	501	1	0	249500	501	1	0	249500	501
2	0	247755	502	2	0	247755	502	2	0	247755	502
3	0	245355	504	3	0	245355	504	3	0	245355	504
4	0	241941	508	4	0	241941	508	4	0	241941	508
5	0	237559	512	5	0	237559	512	5	0	237559	512
6	0	232244	518	6	0	232244	518	6	0	232244	518
7	0	226032	524	7	0	226032	524	7	0	226032	524
8	0	218966	532	8	0	218966	532	8	0	218966	532
9	0	211089	540	9	0	211089	540	9	0	211089	540
10	0	202455	550	10	0	202455	550	10	0	202455	550
11	0	193120	560	11	0	193120	560	11	0	193120	560
12	0	183148	572	12	0	183148	572	12	0	183148	572
13	0	172608	584	13	0	172608	584	13	0	172608	584
14	0	161575	598	14	0	161575	598	14	0	161575	598
15	0	150130	612	15	0	150130	612	15	0	150130	612
16	0	138361	628	16	0	138361	628	16	0	138361	628
17	0	126359	644	17	0	126359	644	17	0	126359	644
18	0	114225	662	18	0	114225	662	18	0	114225	662
19	0	102063	680	19	0	102063	680	19	0	102063	680
20	0	89985	700	20	0	89985	700	20	0	89985	700

α=0.9 β=0.7				α=0.9 β=0.8				α=0.9 β=0.9			
N	n^*	Total Profit	P_0	N	n^*	Total Profit	P_0	N	n^*	Total Profit	P_0
1	0	249500	501	1	0	249500	501	1	0	249500	501
2	0	247755	502	2	0	247755	502	2	0	247755	502
3	0	245355	504	3	0	245355	504	3	0	245355	504
4	0	241941	508	4	0	241941	508	4	0	241941	508
5	0	237559	512	5	0	237559	512	5	0	237559	512
6	0	232244	518	6	0	232244	518	6	0	232244	518
7	0	226032	524	7	0	226032	524	7	0	226032	524
8	0	218966	532	8	0	218966	532	8	0	218966	532
9	0	211089	540	9	0	211089	540	9	0	211089	540
10	0	202455	550	10	0	202455	550	10	0	202455	550
11	0	193120	560	11	0	193120	560	11	0	193120	560
12	0	183148	572	12	0	183148	572	12	0	183148	572
13	0	172608	584	13	0	172608	584	13	0	172608	584
14	0	161575	598	14	0	161575	598	14	0	161575	598
15	0	150130	612	15	0	150130	612	15	0	150130	612
16	0	138361	628	16	0	138361	628	16	0	138361	628
17	0	126359	644	17	0	126359	644	17	0	126359	644
18	0	114225	662	18	0	114225	662	18	0	114225	662
19	0	102063	680	19	0	102063	680	19	0	102063	680
20	0	89985	700	20	0	89985	700	20	0	89985	700

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