STOCHASTIC TRANSMISSION POLICIES FOR ENERGY HARVESTING WIRELESS NODES

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ABSTRACT

STOCHASTIC TRANSMISSION POLICIES FOR ENERGY HARVESTING WIRELESS NODES

We consider a single-link wireless communication system where the transmitter has a finite-capacity rechargeable battery that is receiving energy packets from an energy harvesting device (EHD) at certain time instants. The transmitter may be a sensor node, which is responsible for sensing certain inputs from the environment and send them to the base over an additive white Gaussian noise (AWGN) or a fading channel. We pick the system throughput until a transmission deadline as the utility function, since higher throughput yields a higher service quality in general. A random energy leakage is assumed during the energy transfer from the EHD to the energy buffer of the transmitter. For cases where the energy leakage can be described by statistical distribution models, we formulate the parametric average throughput maximization problem with probabilistic energy causality and battery capacity constraints. We present problem feasibility conditions which are affected by the introduced threshold parameters. Furthermore, we propose an iterative algorithm to solve $\mathbf{P}$ and develop parameter dependent *stochastic transmission policies* (STPs) to manage the transmission power over a finite transmission interval. Simulation results indicate that STP improves the collective throughput of numerous single link energy harvesting nodes, operating over both the AWGN and fading channels compared to other leakage unaware power allocation techniques.

ÖZET

ENERJİ HARMANLAYABİLEN KABLOSUZ DÜĞÜMLER İÇİN OLASILIKSAL İLETİM POLİÇELERİ

Bu tezde, sınırlı ve yeniden şarj edilebilir enerji belleğine ve bir enerji harmanlama aygıtına (EHA) sahip bir vericiyle donatılmış, tek linkli kablosuz haberleşme sistemi ele alınmıştır. Buradaki verici, çevresel ölçüm bilgilerini bir alıcıya kablosuz olarak gönderebilen bir algılayıcı düğüm olarak kabul edilebilir. Alıcı ile verici arasında toplanır beyaz Gauss gürültülü (TBGB) kanal ve sönümlü kanal varsayılmıştır. Haberleşme sisteminin yararlık fonksiyonu, sistemde vericiden alıcıya aktarılan toplam bilgi miktarı olarak kabul edilmiştir. EHA ile vericinin enerji deposu arasındaki enerji aktarımının belli anlarda gelen paketler halinde olduğu ve her paketten rassal bir miktar enerjinin kaybolduğu varsayılmıştır. Bu enerji kaybının istatistiksel dağılımlarla modellenebildiği durumlar için, eşik parametreli enerji nedenselliği ve enerji belleği kısıtlarına bağlı bir "olasılıksal data transfer hızını eniyileme problemi" biçimlendiriyoruz. Ayrıca, bu eşik parametrelerinin problem olurluk şartlarını nasıl etkilediğini ortaya koyuyoruz. Bahsettiğimiz eniyileme probleminin çözümüyle birlikte, probleme özel bir tekrarlı algoritma sunuyoruz. Son olarak modellediğimiz haberleşme sistemi için geliştirdiğimiz olasılıksal iletim poliçelerini (OİP) sunuyoruz. Benzetim sonuçlarımızın da önerdiği üzere, OİP'nin, çok fazla vericinin hem TBGB kanal hem de sönümlü kanal üzerinden, mühlet kısıtıyla yollanan veri miktarını, rassal enerji kaybını gözardı eden iletim poliçelerine göre daha iyi sonuç verdiğini gösteriyoruz.

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E_i	Energy packet size, released from the EHD by the epoch i
E_{MAX}	Energy storage capacity
$\mathbf{E}$	Set of incoming energy packets
h_i	Fading level during epoch i
$\mathbf{h}$	Set of all fading levels
L_i	Length of the epoch i
L	The Lagrangian
$\mathbf{L}$	Set of epoch durations
N	Number of transmission epochs
n_{ub}	Upperbound for the constant power transmission
$p(t)$	Transmission power function
p_i	Transmission power during the epoch i
$\mathbf{p}$	Transmission power set
$\mathbb{R}^+$	Set of all positive real numbers
$\Re_{EH}$	Rate region of the energy harvesting communication system
$\Re_{AWGN}$	Average rate region of the AWGN channel
$r(t)$	Transmission rate
T	Transmission deadline
$T(\mathbf{p})$	Throughput function
t_k	kth subinterval length
$\mathbf{t}$	Set of energy arrival instants

β	Region of all feasible power functions
Γ_ℓ	Total energy leakage upto the epoch ℓ
ϵ_1	Threshold probability for energy causality violation
ϵ_2	Threshold probability for battery overflow
η	Energy storage efficiency ratio
Φ_i	Energy leakage at the epoch i

LIST OF ACRONYMS/ABBREVIATIONS

AWGN	Additive White Gaussian Noise
CDF	Cumulative Distribution Function
EH	Energy Harvesting
EHCS	Energy Harvesting Communication System
EHD	Energy Harvesting Device
i.i.d.	independent and identically distributed
iCDF	Inverse Cumulative Distribution Function
FC	Feasibility Condition
KKT	Karush Kuhn Tucker
LN	Lognormal Random Variable
PDF	Probability Density Function
Pr	Probability
SNR	Signal-to-Noise Ratio
STP	Stochastic Transmission Policy

1. INTRODUCTION

1.1. History

The first successful attempt on wireless communication occured in 1880 when Alexander Graham Bell managed to send a speech signal to a receiver approximately 200 meters away, [4]. The device was named “Photophone”, since Bell used a beam of light as the carrier. The photophone did not gain commercial success, but it surely was the first building block of wireless communications, which has been a rich research field since then.

In this thesis, we focus on power management techniques for wireless communication with energy harvesting transmitters. The type of devices we consider here are called “nodes”, that are small in size, wireless and also cordless. These devices are used for various purposes, but mainly for sensing the environment and sending information to a base or another node by wireless communication. For example sensor nodes distributed in rain-forests to measure moisture and temperature. They are often equipped with nonrechargeable batteries, therefore their batteries have to be changed periodically. But usually it is too costly to find each node and replace the battery, therefore such nodes are for one use only.

With the use of rechargeable batteries, and by applying correct techniques to make the node recharge its battery by itself, the average lifetime of a single node can be increased significantly. There is a way for the node to recharge its battery and it is called energy harvesting. Now, the nodes can be equipped with devices which can convert a certain type of mechanical disturbance in its environment to chemically storable energy in the battery. Such nodes we call “Energy Harvesting Nodes”, the whole including the receiver and the channel as “Energy Harvesting Communication System”. This technology enables the node to last as long as the battery can be recharged repeatedly. Therefore the cost of each node can be reduced significantly compared to the others that carry nonrechargeable batteries.

To get the most out of such a communication system, we have to make sure that the scarce energy is used as efficiently as possible. For most of the cases the system throughput is a good measure for system utility. Therefore we aim to find the power policy that gives the best throughput in the energy harvesting framework.

In the following, we state our motivation for the specific throughput focused deadline constrained power allocation problem under random energy leakage from the harvester and/or the storage. Next, we present the related literature to build a better understanding of our problem. After that, we formulate the probabilistic average throughput maximization problem over fading and AWGN channels and propose threshold dependent *stochastic transmission policies* that solve the presented problem. Also, problem feasibility is analyzed for further remarks. We present simulation results for the proposed policies and finally the thesis is concluded with discussions on the proposed solution.

1.2. Motivation

Energy sufficiency is a huge constraint on sensor nodes ever since wireless communication became possible, [5]. Therefore efficient energy usage in wireless nodes has become a critical issue as information access depends more and more on wireless networks fed by non-rechargeable energy sources. In particular, sensor nodes are equipments that gather ambient information and transmit them to other nodes in the network. One problem with the sensor nodes is their short lifespans because of the small energy density of traditional batteries. In this context, energy flow to the node can be increased by using rechargeable batteries tied to energy harvesting devices. As a result, sensor nodes can last longer by equipping them with batteries that transforms environmental disturbances to storable chemical energy. This is a solution to nodes' energy shortage problem, however efficient usage of the harvested energy remains as an issue. Overcoming this problem is more challenging for the nodes operating under a random energy leakage from their energy harvesting devices. This is why current power scheduling techniques have to be adapted for these new generation sensor nodes. For such a scenario, we develop stochastic transmission policies for maximizing average throughput of a single-link energy harvesting node over a finite transmission interval.

Energy harvesting communication systems embody randomness in energy storage. This is the main difference between them and traditional communication systems. Energy may not be fully random; solar power or wind power can be predicted beforehand for a certain instant in a day, but even in that case prediction error features a random behaviour. So any power policy has to consider the randomness in available energy.

Some power policies can be designed way before the intended communication interval starts. Such policies are usually based on hard assumptions on incoming energy packets and disregard the random behaviour of energy harvesting. Therefore they are optimum for a communication model that does not quite fit the reality. However, these policies and their optimum performances give us insight on achievable performance of an energy harvesting communication system.

Additionally, prediction of harvested energy at a certain instant can be made less errorprone when the information on previous harvested energy packets is used. Power policies based on such observations are called "online power policies". Online power scheduling algorithms deal well with the uncertainties in arriving energy packet sizes and instances. These techniques improve the communication rate and quality significantly, however, they require a serious amount of processing power for this optimization process. Therefore offline power management policies may be preferred when the energy is a huge constraint on a communication system, such as wireless networks.

To the best of our knowledge, previously proposed offline power management policies for energy harvesting transmitters lack the flexibility to adapt to the unpredictable loss in the arriving energy packets from the harvester to the transmitter. Such policies push the communication system with lossy harvesters to outage. Hence there is a need for power policies for energy harvesting nodes with random energy leakage from harvested energy packets.

In the context of energy loss, we consider harvested energy leakages due to imperfections in energy storage as in [6]. Notice that, as pointed out in [7] and [8], the EHD is often implemented by a supercapacitor that is subject to energy losses caused by the parasitic resistance and switching actions of the integrated circuits. This leakage behavior can be approximated with a deterministic function using a curve fitting method in the long term as done in [8] especially for solar EHD. On the other hand, as in the case of piezoelectric EHDs operating with random vibrations such as those presented in [9, 10], the variations take place in short time intervals and a deterministic leakage model is not always possible. As a result the energy losses may not exactly be known to the EHD or the transmitter as a function of time, just like almost every other electronic device is exposed to uncertain leakage, [11]. For this reason, we consider random energy leakage models for the unknown loss during the transfer of energy packets from EHD to the battery. With the prior knowledge of energy predictions, the fading pattern and stochastic properties of leakage, we formulate the probabilistic average throughput maximization problem for fading channel and then for the AWGN channel as a special case. We formulate theoretical conditions on the feasibility of the probability thresholds, which define the feasible region of the problem. We also propose a practical algorithm that produces the proposed stochastic transmission policy within the feasible search domain.

Next, we present the related research on energy harvesting communication systems on single link/broadcast settings, with/without fading, equipped with a perfect/inefficient energy storages.

1.3. Related Work

In this section, we will review the latest research that provides solutions to increase the efficiency of energy harvesting sensor nodes.

Before diving into power allocation regimes, we think it is essential to understand the capacity of energy harvesting communication systems. After all, our work relies on an AWGN rate function assumption, therefore we have to be sure if the system can ever reach the specific rate function. Energy harvesting nodes are analysed in an information theoretic framework in [1]. In this work, the scalar capacity of the Additive White Gaussian Channel with an energy harvesting transmitter is derived. The transmitter is capable of adjusting the communication rate by changing the transmission power. Also, the function that relates the transmission power to the rate (power-rate function) is assumed to be monotonically increasing and concave in power. This means the capacity theorem and all the research in this thesis holds for cases when increasing the transmission power results in higher system throughput with decreasing rate.

The transmitter here has an infinite energy storage. Therefore it can hold any amount of harvested energy without additional cost. In fact, cost for storing harvested energy is assumed to be zero for any energy level, i.e. the storage is also assumed to be completely efficient. Also, the harvested energy is used for only transmission.

The transmitter has an infinite data buffer to send over the AWGN channel to the receiver. In other words, as long as there is energy in the transmitter's battery, there is available data to be transmitted, which increases the rate and consequently the throughput.

At the beginning of each channel use, an energy packet of a random size is available for data transmission. As an energy packet arrives at the transmitter's storage, the transmitter has to decide on the amount of energy to be used for the current channel use. The set of such decisions is called “transmission policy”. The packet can be fully consumed or a fraction of it can be saved for later channel uses. This communication setting is generally assumed to be close to the real applications and therefore employed by most of the works in the literature. We will refer to this setting as the “Energy Harvesting Communication System”. It is visualized in Figure 1.1.

Under such a scenario, the channel capacity is shown to be equal to the AWGN capacity constrained by the expected harvesting rate. So for a transmitter expecting an average of P Joules per second, the capacity is equal to the ergodic capacity of AWGN channel, which is equal to $C = \frac{1}{2}\log(1 + P)$. Furthermore it is shown that two

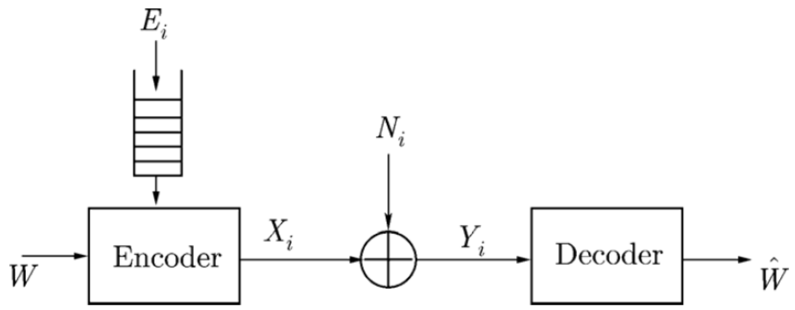

Figure 1.1. Energy harvesting communication system with stochastic energy arrivals [1].

schemes are shown to achieve the capacity, namely, best effort scheme and save and transmit scheme. This work is one of the main building blocks for communicating with energy harvesting transmitters.

Relying on the preceding work, we assume that the transmission rate is $\frac{1}{2}\log(1+P)$ which is valid for unitary noise variance. By this expression, we happen to assume that the transmission occurs at the capacity, however our work is valid for other increasing and concave rate functions.

A considerable amount of research has been conducted on throughput maximization for wireless sensor nodes and networks with offline knowledge of energy arrival packet sizes and instances. A power management algorithm is proposed in [12] with a nested two loop structure that considers *energy neutrality* while ensuring reliable data reception on the receiver side. In [2], a throughput maximizing algorithm is derived for a communication system consisting of a single energy harvesting transmitter and a receiver. Furthermore, the throughput maximization problem over the static AWGN channel is shown to be convex and the dual of the transmission duration minimization problem with a single data packet available at the beginning of the transmission. Similar identifications are done over a fading channel in [13]. Another transmission policy is proposed in [14], aiming to minimize the transmission duration when the data packets are allowed to arrive at the transmitter during the transmission. Throughput optimal policies for other channels are proposed in [15] and [16]. These policies are considered "leakage-unaware" since they do not assume any sort of energy loss.

A different line of research on EH transmission takes battery imperfections and storage losses into account, which are assumed to be perfectly predictable by the decision mechanism prior to the transmission. For instance, in [3], an energy harvesting transmitter with an inefficient energy storage device is studied and short-term throughput-optimal power allocation algorithms are derived over single-link and broadcast AWGN channels. The proposed *double threshold power policy* is shown to be optimal as a certain fraction of the withdrawn energy is lost in the employed system model. A similar transmission policy is shown to be optimal in [17] where the transmitter operates over a multiple access channel. A throughput optimal broadcast scheme is presented in [18] with a linear rate leakage model.

With the preceding quick overview, we will present the details of an energy harvesting communication system in the following chapters. In Chapter 2, the throughput analysis of an EHCS with a perfect energy storage will be given. Later in Chapter 3, an EHCS system with an imperfect storage will be analyzed. The main contribution of this thesis will be presented in Chapter 4, where the leakage is assumed to be unpredictable unlike the assumptions from Chapter 3.

2. TRANSMISSION OVER A SINGLE LINK STATIC CHANNEL

2.1. System Model

In this section, we present the optimum communication strategy for an energy harvesting communication system operating over a single link Gaussian channel as shown in Figure 2.1. The communication system is equipped with an energy harvesting device, feeding the finite rechargeable energy buffer of the transmitter. The transmitter is supplied with a large amount of information by its infinite data buffer. The order of magnitude of the harvested energy is assumed to be much lower than the required amount of energy to send the available information in the data buffer. Therefore, the storage capacity of data buffer is indicated as infinite.

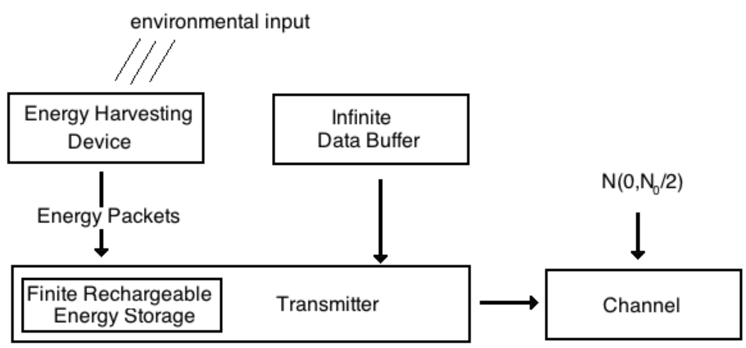

Figure 2.1. Communicating over AWGN channel with an energy harvesting transmitter.

The transmitter's energy storage is assumed to be finite with E_{MAX}. Energy is not lost during storing or withdrawing operations, i.e. the battery is perfectly efficient. Also power usage obeys the following “energy causality” and “battery capacity” constraints.

$$0 \leq E_{BAT} \leq E_{MAX},$$
$$\sum_{k=0}^{n-1} E_k - \int_0^{t'} p(t)dt \leq E_{MAX}.$$

Energy causality constraints imply that energy has to be available by the harvester before the transmitter can consume it. Such power functions are labeled as “feasible power functions”. The energy harvester may not be able to transfer harvested energy continuously, in fact, the harvesters may only be able to transfer the usable energy once the harvested energy reaches a certain amount, [19]. Therefore the energy harvesting process is modeled as packetized instantaneous energy arrivals at the transmitter. So, it is a discrete time process. In this specific system model, the nth harvest arrives at the instant $s_n \in \mathbb{R}^+$ with the size of $E_n \in \mathbb{R}^+$. Without loss of generality, the very first harvest is assumed to arrive at the instant s_0 with the size of E_0.

Although randomness is involved in harvested energy amounts, in this static channel model the arrival and size information of the harvested energy is available noncausally to the transmitter at the beginning of transmission. This may sound unreasonable at first, however certain cases exist where harvested energy follow a structured routine periodically, for example in the case of solar energy, [20].

2.2. Short-Term Throughput Maximization Problem

A transmission policy can be designed to achieve various objectives. One reasonable attempt is to maximize the total transmitted bits throughout a finite transmission interval. The power policy achieves such an objective when it solves the so-called “short term throughput maximization problem”, [2]. So the aim is maximizing total bits departed in $[0;T]$ over feasible power functions $p(t)$, T being the duration of transmission:

$$\textbf{P:} \quad \max_{p(t)} \int_0^T r(p(t))dt, \qquad \text{s.t. } p(t) \in \beta, \tag{2.1}$$

where

$$\beta = [\, p(t) \geq 0 \mid 0 \leq \sum_{k=0}^{n-1} E_k - \int_0^{t'} p(t)dt \leq E_{MAX},$$
$$\forall t',\; n \geq 0 \; s.t. \; s_{n-1} \leq t' \leq s_n \,] \tag{2.2}$$
$$= [\, p(t) \geq 0 \mid \sum_{k=0}^{n} E_k - E_{MAX} \leq \int_0^{s_n} p(t)dt \leq \sum_{k=0}^{n-1} E_k,\; n \geq 1 \,] \tag{2.3}$$

Note that, β is the region which includes all the feasible power functions that obeys the energy causality and battery capacity constraints.

2.3. Optimality Conditions

The stated short term throughput maximization problem has a concave objective function to be maximized that is subject to affine constraints. Hence it is a convex optimization problem and can be solved numerically by an appropriate software. However, there are certain characteristics of the optimum power policy that solves and are useful for the proposed problem in this thesis. These properties are essential for the optimum power set and therefore called as optimality conditions, summarized in the four lemmas below.

Lemma 2.1. *If total amount of energy comsumption is given in $[t_1; t_2]$; a constant power transmission is throughput optimal. If the consumed energy in the interval $[t_1; t_2]$ is given and equal to E, then the optimal power allocation for that interval is:*

$$p(t) = \frac{E}{t_2 - t_1} \tag{2.4}$$

Lemma 2.2. *Optimal power function does not allow the battery to overflow at energy arrival instances, since any lost energy could have been used to increase the throughput in the preceding transmission epoch. Therefore any $p(t)$ yielding a battery overflow is strictly suboptimal.*

Lemma 2.3. *In the optimal power allocation, the transmission power does not change unless the battery is either full or completely empty.*

Lemma 2.4. *For optimal power allocation, the change in the power level p_n at s_n cannot be positive unless the battery is depleted. Similarly, the change in the power level cannot be negative unless the battery is full.*

The collective result of the lemmas above can be summarized in Corollary 2.1.

Corollary 2.1. *In the optimal power allocation, transmission power is a piecewise linear function which is constant during the whole transmission and changes its rate only at energy arrival instants. The power function decreases at those instants when the battery is full and increases only at the instants when the battery is depleted. Also, the optimal power allocation expends all harvested energy by the end of the transmission, [13].*

2.4. Throughput Maximizing Policy

Previously, we introduced the fundamental properties of the optimum power allocation that maximizes the throughput of a single link energy harvesting communication system. In this section, we present a low complexity iterative algorithm, [2], that finds the optimal power function for given energy arrival information.

We denote the energy arrival instants with s_n and the instants at which transmission power changes as i_n. The energy packet arriving at $s_{n_{max}}$ cannot be used, since this is the end of transmission and therefore it is a dummy harvesting instant. The transmission power function is

$$p(t) = \begin{cases} p_n, & \text{if } i_{n-1} < t < i_n \\ 0, & \text{if } s_{n_{max}} < t. \end{cases} \tag{2.5}$$

2.4.1. Recursive Solution

Once the transmission for $[0, t_1]$ is optimized, the rest can be considered as a seperate throughput maximization problem, shifted by t_1. The shifted optimization problem can be solved with the exactly same algorithm following the update of the problem parameters.

The algorithm uses "comparative power levels" to reach the optimum solution. The power set $p_0[i]$ depletes the battery and $p_{max}[i]$ results in a full battery by the ith arrival instant. In other words, power levels that would result in an empty battery are shown as: $\{p_0[1], p_0[2], ...\}$ and in a full battery are shown as $\{p_{max}[1], p_{max}[2], ...\}$. The following comparative power level sets are defined, also depicted in Figure 2.2:

$$\mathbf{P} = \mathbf{P}[1], \mathbf{P}[2], ... \tag{2.6}$$

$$\mathbf{P}[n] = \{p_{max}[n], p_0[n]\} = \{p \mid p_{max} \leq p \leq p_0[n]\} \tag{2.7}$$

$$\mathbf{P}[n_{max}] = \{p_0[n_{max}]\}, \qquad s_{n_{max}} = T \tag{2.8}$$

Then, for example, we have the battery depleting power levels as:

$$p_0[n] = \frac{\sum_0^{n-1} E_k}{s_n}. \tag{2.9}$$

The corollary from the previous section suggest that the optimum constant power

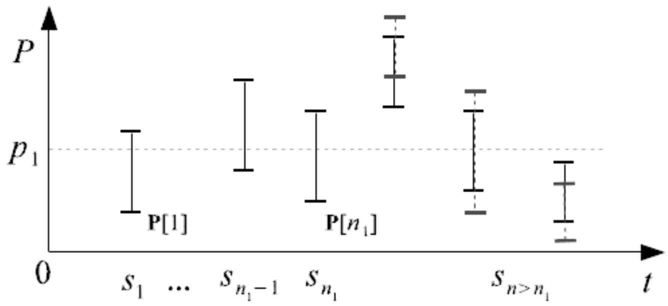

Figure 2.2. Comparative power level sets for the recursive algorithm [2].

transmission curve cannot be above the battery depleting curve or below the levels that result in a full battery. Therefore, the algorithm looks for the feasible power function that starts from $t = 0$ and lasts until the nth energy harvest, while its power level is contained in the range $\mathbf{P}[k]$ for $k = 1, 2, ..., n$. This search yields an upperbound n_{ub} on the length of the first constant power transmission:

$$n_{ub} = \max\{n | \bigcap_{k=1}^{n} \mathbf{P}[k] \neq \emptyset, \quad n = 1, 2, ..., n_{max}\} \tag{2.10}$$

In the optimal scenario, the transmission power stays constant until the n_{ub}th state. For instance, n_{ub} becomes n_1 for the power level sets given in Figure 2.2.

2.4.2. Determining the Power Level

Once the upperbound is obtained, the constant power transmission is specified according to the upcoming power level sets. Now, consider the case given in the Figure 2.2. The next level set to the upperbound stays above the previous one. This means when the constant power transmission remains fixed after the upperbound, the battery will face an overflow. Therefore, the upcoming transmission should consume more than the previous constant power, i.e. the transmission power should increase. We also know that according to the optimal policy, the increase in the transmission power follows the battery depletion at the energy arrival instant. So the algorithm concludes that the battery should be depleted at s_{n_1} for this specific example. So the constant power equals to $p_0[n_1]$. Similarly, if the upcoming power level set stays below, then the constant power transmission until the upperbound becomes $p_{\max}[n_1]$.

2.4.3. Throughput Maximizing Algorithm

The throughput optimizing recursive algorithm finds the first constant power transmission starting at $t = 0$ until the upperbound as explained above. At the last step, the problem parameters are updated and the algorithm is repeated until the transmission deadline is reached.

Step 1: Find n_{ub}. If $n_{ub} = n_{max}$, transmit with

$$p = \frac{\sum_{k=0}^{n_{max}-1} E_k}{T} \tag{2.11}$$

Step 2: Determine whether the next power interval $\mathbf{P}[n_{ub}+1]$ falls below or above $\bigcap_{k=0}^{n_{ub}} \mathbf{P}[k]$.

Step 3: If $\mathbf{P}[n_{ub}+1] > \bigcap_{k=0}^{n_{ub}} \mathbf{P}[k]$, transmit with:

$$i_1 = s_{n_1},\ p_1 = p_0[n_1] \tag{2.12}$$

where

$$n_1 = \max\{n | p_0[n] \in \mathbf{P}[n_{ub}+1] > \bigcap_{k=0}^{n_{ub}} \mathbf{P}[k]\} \tag{2.13}$$

If $\mathbf{P}[n_{ub}+1] < \bigcap_{k=0}^{n_{ub}} \mathbf{P}[k]$, transmit with:

$$i_1 = s_{n_1},\ p_1 = p_{max}[n_1] \tag{2.14}$$

where

$$n_1 = \max\{n | p_0[n] \in \mathbf{P}[n_{ub}+1] > \bigcap_{k=0}^{n_{ub}} \mathbf{P}[k]\} \tag{2.15}$$

Step 4: Parameter update:

$$E_0^{'} = \sum_{k=0}^{n_1} E_k - i_1 p_1 \tag{2.16}$$

$$s_n^{'} = s_{n+n_1} - i_1 \tag{2.17}$$

$$E_n^{'} = E_{n+n_1};\ n = 1, 2, ..., n_{max} \tag{2.18}$$

$$T^{'} = T - i_1 \tag{2.19}$$

$$n_{max}^{'} = n_{max} - n_1 \tag{2.20}$$

In this section, the throughput maximization problem is solved for an energy harvesting communication system, operating over a single-link AWGN channel. The proposed problem consists of a convex objective function and affine energy causality and battery capacity constraints. The throughput maximizing power allocation can be found via a convex optimization solver or the proposed low-complexity algorithm.

The optimum power allocation has notable characteristics and provides insights for the main problem considered in this thesis. First of all, in the optimum transmission scenario there is no silent interval. Secondly, the total harvested energy is completely consumed by the end of the transmission. Furthermore, the transmission approaches its optimum when the transmission power is small enough for higher throughput in the long run also large enough to prevent battery overflows at energy arrival instants. This result is illustrated by the tunnel analogy for transmission power.

3. INEFFICIENT ENERGY STORAGE SYSTEM MODEL

In this section, we review the energy harvesting communication system with an inefficient energy storage. Specifically, the throughput optimal power allocations are considered here. This work is important as a background material for this thesis, since we develop the inefficient storage problem to the randomly leaking battery problem.

The transmission with a single link energy harvesting transmitter, Figure 3.1, is optimized in [3]. The transmitter is able to control the transmission power according to a power scheme. Transmission power is denoted by p and $r(p)$ is the instantaneous transmission rate for the single link. This instantaneous utility function is non-decreasing, continuous and concave in p. For the broadcast setting (two receiver case), the transmission rates of both users are weighted into one single transmission rate.

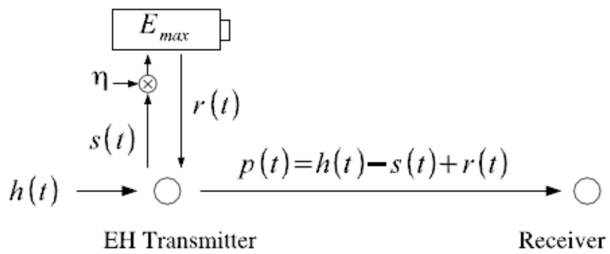

Figure 3.1. Energy harvesting communication system with inefficient energy storage [3].

The utility of the system at the time instant t is the integral of the instantaneous utility from the beginning of the transmission until t. The system utility for the single link becomes the total bit transmitted until that time instant. For the broadcast setting, system utility is any weighted sum of the number of bits delivered to all of the receivers.

The node harvests energy with a nonnegative rate, $h(t)$. The harvested energy can be consumed without being stored in the battery first. The unused energy is stored in the battery with an energy storage rate $s(t)$. If the power scheme requires more than the harvested energy, energy may be withdrawn from the battery with the withdrawing rate $u(t)$. When withdrawing energy from the battery, a certain fraction of the stored energy is lost [12]. This inefficiency is denoted by $0 \leq \eta \leq 1$ and corresponds to the fraction of energy that can be withdrawn from the battery per unit stored energy. The stored power, withdrawn power and harvested power obey $s(t) \leq h(t)$ and $s(t), u(t) \geq 0$.

3.1. Optimal Offline Power Policy for a Single Link

In this section, we review the optimal transmission schemes for the presented energy havesting communication system model. Here, the utility corresponds to the total transmitted bits until the transmission deadline, T. The convex average utility maximization problem becomes

$$
\begin{aligned}
\max_{s(t),u(t)} \quad & \frac{1}{T}\int_0^T r\Big(h(t) - s(t) + u(t)\Big)dt \\
\text{s.t.} \quad & 0 \leq \int_0^t \eta s(\tau) - u(\tau) d\tau \leq E_{max}, \\
& h(t) \geq s(t) \\
& s(t) \geq 0 \\
& u(t) \geq 0, \qquad\qquad 0 \leq t \leq T,
\end{aligned}
$$

with the Lagrangian

$$
\begin{aligned}
L = &\int_0^T r\Big(h(t) - s(t) + u(t)\Big)dt + \int_0^T \lambda(t)\int_0^t (\eta s(\tau) - u(\tau))d\tau dt \\
&- \int_0^T \beta(t)\Big(\int_0^t (\eta s(\tau) - u(\tau))d\tau - E_{max}\Big)dt \\
&+ \int_0^T \mu(t)\Big(h(t) - s(t)\Big)dt + \int_0^T \sigma(t)s(t)dt + \int_0^T \nu(t)u(t)dt
\end{aligned}
$$

The KKT conditions:

$$
\begin{aligned}
r'\Big(h(t) - s(t) + u(t)\Big) - \eta\int_t^T \lambda(\tau)d\tau + \mu(t) - \sigma(t) = 0 \\
-r'\Big(h(t) - s(t) + u(t)\Big) + \int_t^T \lambda(\tau)d\tau - \nu(t) = 0, \qquad 0 \le t \le T
\end{aligned}
\tag{3.1}
$$

The complementary slackness conditions:

$$
\begin{aligned}
\lambda(t)\Big(\int_0^t \eta s(\tau) - u(\tau)d\tau\Big) &= 0 \\
\beta(t)\Big(\int_0^t (\eta s(\tau) - u(\tau))d\tau - E_{max}\Big) &= 0 \\
\mu(t)(h(t) - s(t)) &= 0 \\
\sigma(t)s(t) &= 0 \\
\nu(t)u(t) &= 0, \qquad 0 \le t \le T
\end{aligned}
\tag{3.2}
$$

The idea is to test the presented KKT conditions for five mutually exclusive transmitter modes. The result of this analysis is as follows: The optimal policy obeys two thresholds such that:

$$r'(p_s(t)) = \eta\left(\int_t^T \lambda(\tau)d\tau - \int_t^T \beta(\tau)d\tau\right)$$
$$r'(p_u(t)) = \int_t^T \lambda(\tau)d\tau - \int_t^T \beta(\tau)d\tau \tag{3.3}$$

The optimal double threshold policy exists with three modes:

Mode 1: If $h(t) \geq p_s$ with $p(t) = p_s$, the battery is charged with $h(t) - p_s$.

Mode 2: If $h(t) \leq p_u$ with $p(t) = p_u$, energy is withdrawn from the battery with the rate of $p_u - h(t)$.

Mode 3: The battery is used for neither energy withdrawal nor storage. All of the harvested energy is used for transmission with $p_u \leq p(t) = h(t) \leq p_s$.

Additionally, energy withdrawal and storing do not occur simultaneously in any of the modes.

Using Equation 3.3, we see that:

$$\frac{r'(p_s)}{r'(p_u)} = \eta \tag{3.4}$$

The relation in Equation 3.4 hold for both offline and online policies.

Thresholds can only increase when the battery is empty, can only decrease when the battery is full and stay constant otherwise. Therefore thresholds are functions of time, $p_s(t)$ and $p_u(t)$. Note that the battery goes through battery events when it runs out of energy or become full. The terminating condition also depends on the battery:

$$E_{bat}(T) = 0 \tag{3.5}$$

Now all left is to find the optimum threshold pair $\Big(p_u(t), p_s(t)\Big)$ $\forall t$. The optimum threshold pair has certain properties. First, $p_u(t)$ yields an empty battery at time t_1 such that $t_1 \leq T$. Conversely, $p_s(t)$ yields a full battery at time $\hat{t}_1$ such that $\hat{t}_1 \leq T$. The optimal $p_u(t)$ is the lowest threshold that when applied past t_1 either yields a full battery at some t_2 such that $t_1 < t_2 \leq T$ or does not yield a battery event until the end of the transmission. Secondly, the optimal $p_s(t)$ is the highest threshold that when applied past $\hat{t}_1$ yields an empty battery at some $\hat{t}_2$ such that $\hat{t}_1 < \hat{t}_2 \leq T$.

With the above mentioned conditions, the optimum threshold pair can be found by a search algorithm. First, t_1 and $\hat{t}_1$ are found. This operation gives two threshold pairs, using Equation 3.4. Then the one which satisfies the aforementioned conditions becomes the optimum threshold.

3.2. Optimal Offline Power Policy for a Broadcast Channel

In the two-receiver setting, the aim is to find an average rate region $\Re_{EH} = (r_{1,avg}, r_{2,avg})$. $\Re_{EH}$ is the union of average rate pairs that can be achieved under energy causality and battery capacity constraints. Furthermore, $\Re_{EH}$ is convex under those constraints.

When the transmitter allocates $p(t)$ for transmission, it can achieve any rate pair in the achivable rate region $\Re(p(t))$. For the static AWGN white Gaussian channel, this rate region is proven to be convex for a fixed p and concave in p:

$$\Re_{AWGN}(p) = \left\{ (r_1, r_2) \middle| r_1 \leq \frac{1}{2}\log_2\left(1 + \frac{\alpha p}{\sigma_1^2}\right), r_2 \leq \frac{1}{2}\log_2\left(1 + \frac{(1-\alpha)p}{\alpha p + \sigma_2^2}\right), 0 \leq \alpha \leq 1 \right\} \tag{3.6}$$

Average sum-rate maximization problem:

$$\begin{aligned}
\max \quad & \frac{1}{T}\int_0^T r_\alpha^{BC}\Big(h(t)-s(t)+u(t)\Big)dt \\
\text{s.t.} \quad & 0 \le \int_0^t \eta s(\tau)-u(\tau)d\tau \le E_{max}, \\
& h(t) \ge s(t) \\
& s(t) \ge 0 \\
& u(t) \ge 0, \qquad\qquad 0 \le t \le T
\end{aligned} \tag{3.7}$$

where r_α^{BC} is the maximum achievable weighted sum-rate which is non-decreasing, continuous and concave in p for any coefficient $\alpha \ge 0$. Similar to the single-link setting, optimal broadcast channel power policy has a double threshold structure. The individual rates in this broadcast setting are the ones that maximize the weighted sum-rate with a fixed α and a given instantaneous power.

Since the sum-rate maximization problem in Equation 3.7 is reduced to one average rate function, the procedure that finds the optimum threshold pairs is identical to the single-link setting.

3.3. Optimal Online Transmission Policy

When the energy arrival information is not known noncausally, the throughput maximization problem can be modelled as a Markov decision process and the corresponding optimal power allocation can be found by dynamic programming, [21], [22]. First, a value function, proportional to system throughput, is formed under various assumptions. After that this value function is maximized recursively.

An example policy is given in [13]. Here, at any time t during $[0, T]$, the state of the value function is specified by four parameters: Energy stored in the battery, $E_{bat}(t)$, causal harvested energy information, $h^t = h(\tau)$, $0 \le \tau \le t$ and the time to the deadline, $T-t$. Based on these states, the transmit power is determined through the function $\phi(E_{bat}(t), h^t, T-t)$:

$$V(E_{bat}, h^t, T-t) = \max_{\phi} E\Big[\int_t^T r(\phi(E_{bat}, h^t, T-t))d\tau\Big] \tag{3.8}$$

Assuming a Markovian energy arrival process and sufficiently large or infinite T, this value function can be expressed as a simple recursive relation using the Bellman equation, [23]:

$$V(E_{bat}(t), h(t)) = \max_{\phi} r(\phi(E_{bat}(t), h(t)))\delta + \beta E\Big[V\big(E_{bat}(t+\delta), h(t+\delta)\big)\Big] \tag{3.9}$$

Note that E_{bat} changes linearly with ϕ. When $\phi(E,h) < h(t)$, it changes with the slope $-\eta$ and when $\phi(E,h) \geq h(t)$ with the slope -1 (none of the incoming energy is stored in the battery).

4. EHCS WITH A RANDOMLY LEAKING STORAGE

In this chapter, we consider a similar energy harvesting communication system as introduced in previous chapters, only with a leaking energy harvesting device. This leakage problem resembles the inefficient storage problem, however it is an entirely new problem. For instance, we use random variables to model unknown energy leakage whereas in other works the battery's inefficiency is deterministic and apparent to the transmitter. Further notes on our motivation for the considered leakage problem can be found in Chapter 1.

This chapter is organized as follows. First we introduce the energy harvesting communication system model exposed to random energy leakage. Then we formulate the probabilistic throughput maximization problem for the proposed system with energy leakage. Next, we state the problem feasibility conditions and propose *stochastic transmission policies* with the knowledge of leakage distribution. The STP performance is depicted in relation with the other policies and its results are discussed comparatively.

4.1. The Node Model

Traditional power scheduling techniques employ the assumption that a certain amount of energy is available to the node for its whole lifetime [24]. In our model, however, the node is exposed to the energy arrival process in Figure 4.2, whose certain characteristics can be used to derive a manipulated schedule to enhance the average throughput of the sensor node. Furthermore, when a number of nodes is deployed at a certain location, energy harvesting rate can be predicted out of the physical routine of the environment. Therefore Tx is aware of the sizes of packets that are caught by EHD, $\boldsymbol{E}$, and their arrival instants, $\boldsymbol{t}$, before the transmission starts.

Consider the communication system shown in Figure 4.1 with an energy harvesting device (EHD) and a transmitter (Tx). Tx is equipped with a rechargeable battery, B, and it is able to tune the transmission power to change its transmission rate by

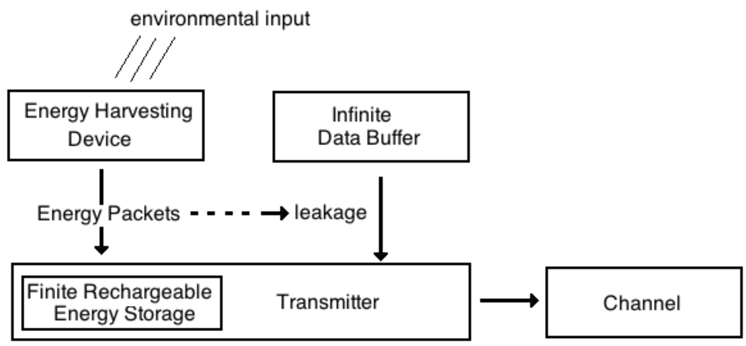

Figure 4.1. Energy harvesting communication system with energy leakage.

withdrawing energy from B. EHD converts a certain type of environmental disturbance into storable chemical energy. At the time instant t_i, EHD releases the energy packet E_i to B. We assume that during the transmission interval $[0, T]$, there are $N+1$ energy arrivals, $\mathbf{E} = [E_0, E_1, ..., E_N]$, at time instants $\mathbf{t} = [t_0, t_1, ..., t_N]$. The finite characteristic of B results in the complete loss of the received energy that overflows the storage capacity, E_{MAX}. Therefore each E_i is assumed to be less than or equal to E_{MAX}. Various imperfections rooting from the EHD and B, as pointed out in Chapter 1, can be modelled as a random loss, Φ_i, in each arriving energy packet. Hence B receives $\tilde{E_i} = E_i - \Phi_i$ at t_i. We denote the durations between any two energy packet arrival at B as $\boldsymbol{L} = [L_1, L_2, ..., L_N]$. The total random leakage from the battery until t_l is another random process denoted by $\Gamma_l = \sum_{i=0}^{l} \Phi_i$, which will be useful during the optimization process.

In the rest of this thesis, we use regular letters for scalars and bold letters for deterministic vectors unless noted otherwise. For a random variable X, $f_X(x)$ and $F_X(x)$ denote the probability density function (PDF) and cumulative distribution function (CDF) of X respectively, for which $Pr(X \leq x) = \int_{-\infty}^{x} f_X(q)dq = F_X(x)$ holds. The inverse CDF (iCDF) of X, $G_X(\tau) = x$, exists since CDF is both injective and surjective, and is defined as $F_X(x) = \tau$. Furthermore, the PDF of the random variable

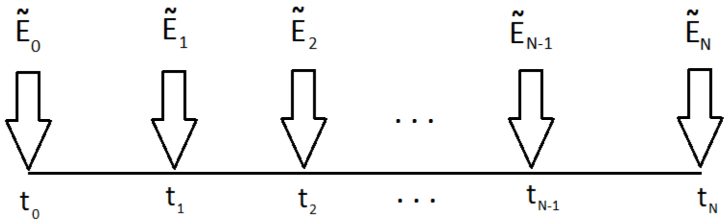

Figure 4.2. Energy packets arriving at Tx during [0, T].

$Y = \sum_{i=0}^{l} X_i$ is given by

$$f_Y(y) = f_{X_0}(x_0) * f_{X_1}(x_1) * f_{X_2}(x_2) * ... * f_{X_l}(x_l) \tag{4.1}$$

where X_i's are independent and identically distributed (i.i.d.) random variables and $(*)$ denotes the convolution operation.

4.2. Energy Leakage Model

We model the energy leakage as negative random uncertainties in energy arrivals which is an accurate approximation for the cases when the leakage can be characterized by probability distributions. We formulate the throughput maximization problem with three different candidate distributions, although it is worth to note that the problem can be stated for energy leakage from any distribution. The probability distributions that model individual and total leakage parameters are stated in the following sections.

4.2.1. Lognormal Leakage Model

Employing the lognormal leakage model, we assume that each Φ_i belongs to a zero-mean, unit-variance lognormal distribution, $\Phi_i \sim LN(0, 1)$ with the PDF

$$\begin{aligned} f_{\Phi_i}(\phi_i) &= \tfrac{1}{\phi_i\sqrt{2\pi}} \mathrm{e}^{-\frac{(\ln \phi_i)^2}{2}} \\ \phi_i > 0 & \quad \forall i = 0...N. \end{aligned} \tag{4.2}$$

We can approximate $\Gamma_l = \sum_{i=0}^{l} \Phi_i$ with another lognormal random variable, $\Gamma_l \sim LN(\mu_l, \sigma_l^2)$ by the Wilkinson method given in [25] as

$$\sigma_l^2 = \ln(\mathrm{e} + l) - \ln(l + 1) \tag{4.3}$$

$$\mu_l^2 = \ln(l + 1) + 0.5(1 - \sigma_l^2) \qquad \forall l = 1 \dots N. \tag{4.4}$$

Then the PDF for Γ_l is given by

$$\begin{aligned} f_{\Gamma_l}(\gamma_l) &= \tfrac{1}{\gamma_l \sigma_l \sqrt{2\pi}} \mathrm{e}^{-\frac{(\ln \gamma_l - \mu_l)^2}{2\sigma_l^2}} \\ \gamma_l > 0 & \qquad \forall l = 1...N. \end{aligned} \tag{4.5}$$

4.2.2. Uniform Leakage Model

Assuming $\Phi_i \sim U(0, 1)$, the PDF of the energy leakage becomes

$$f_{\Phi_i}(\phi_i) = \left\{ \begin{array}{ll} 1, & 0 \leq \phi_i \leq 1 \\ 0, & \text{otherwise} \end{array} \right\}, \quad \forall i = 0...N. \tag{4.6}$$

Then Γ_l is an Irwin-Hall random variable of order $l + 1$, [26] with the following PDF:

$$f_{\Gamma_l}(\gamma_l) = \left\{ \begin{array}{ll} \frac{1}{2l} \sum_{k=0}^{l+1} (-1)^k \binom{n}{k} (\gamma_l - k)^l \mathrm{sgn}(\gamma_l - k), & 0 \leq \gamma_i \leq l + 1 \\ 0, & \text{otherwise} \end{array} \right\}, \quad \forall l = 1...N. \tag{4.7}$$

4.2.3. Exponential Leakage Model

For exponential Φ_i's with the PDF , $f_{\Phi_i}(\phi_i) = \mathrm{e}^{-\phi_i}$ for $\phi_i \geq 0$, Γ_l is an Erlang random variable with the shape parameter $l + 1$ and rate 1, [27]:

$$f_{\Gamma_l}(\gamma_l) = \gamma_l^l \cdot \frac{\mathrm{e}^{-\phi_l}}{l!}, \quad \forall l = 1...N. \tag{4.8}$$

Considering random energy leakage from three candidate distributions, we seek the transmission power set, $\boldsymbol{p} = [p_1, p_2, \ldots, p_N]$, over $\boldsymbol{L}$, that maximizes the average throughput of the energy harvesting communication system.

4.3. Problem Formulation and Throughput Maximization

4.3.1. Static AWGN Channel

When Tx operates over a static channel exposed to AWGN, the transmission rate function can be modeled by a nonnegative and concave function, $r(t)$, which depends solely on the instantaneous Signal-to-Noise Ratio (SNR) of the transmitter. Although $SNR \sim p(t)$, [28], but not $SNR = p(t)$, we can assume the latter, which does not disturb the general system model. We assume no outage, that is any information package is transferred reliably to the receiver side independent of the SNR. Under these assumptions the transmission rate can be well modeled by

$$r(t) = \frac{1}{2}\log(1 + p(t)). \tag{4.9}$$

Based on the findings in [13], the optimum power allocation scheme has certain characteristics: First, the optimum power does not change until the battery is full, or completely depleted. Also the changes in the transmission power cannot be positive unless the battery is completely empty and cannot be negative unless the battery is full. As a collective result of these two remarks, the optimum transmission power is constant when there is no incoming energy packet, decreases only if the battery is full right after a new packet arrives and increases only if the battery is depleted just before a new packet arrives.

We can visualize the constraints and the resulting optimum power set with the use of the "energy tunnel" in Figure 4.3a.

Energy causality constraints force the transmitter to consume less energy than the

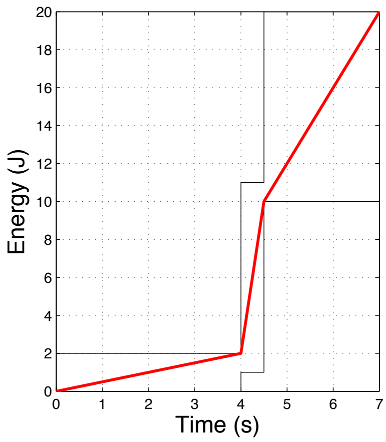

Figure 4.3. Optimum solution of (2.1) applied to an efficient EH transmitter.

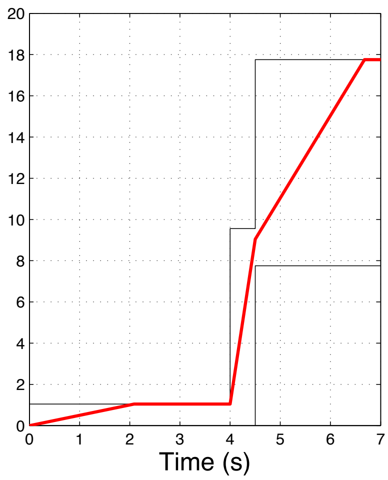

Figure 4.4. Optimum solution of (2.1) applied to an EH transmitter with random leakage.

total amount that B holds at any instance. Therefore energy causality constraints are visualized as the upperbound for the tunnel. Battery capacity constraints correspond to the lowerbound of the tunnel, forcing the transmitter to consume much enough to prevent battery overflow at energy arrival instances. Any curve that stays in the tunnel and connects the zero energy level at t_0 with the highest energy level at t_N gives a feasible transmission curve. Out of all these feasible power policies, only one of them depicted in Figure 4.3 is optimum.

When there is unpredictable energy leakage, however, tunnel upperbound is pushed downwards in practice, causing silent intervals (no transmission) which indicates suboptimality (Figure 4.4). Following the results in [13], any power policy that reduces the length of these silent intervals while consuming same power, results in a higher throughput in average. With these remarks, we form the objective function as

$$\max_{p_i \geq 0} \sum_{i=1}^{N} \frac{L_i}{2} \log(1 + p_i). \tag{4.10}$$

Under random energy leakage, it is reasonable to expect that the energy causality and battery capacity constraints of the communication system are violated with a certain probability. In particular, we force each energy causality constraint to be violated with a smaller probability than ϵ_1 and each battery capacity constraint to be violated with a smaller probability than ϵ_2:

$$\textbf{C1:}\ Pr\Big(\sum_{i=0}^{l-1}(E_i - \phi_i) \leq \sum_{i=1}^{l} L_i p_i\Big) \leq \epsilon_1 \tag{4.11}$$

$$\textbf{C2:}\ Pr\Big(\sum_{i=1}^{l} L_i p_i \leq \sum_{i=0}^{l}(E_i - \phi_i) - E_{MAX}\Big) \leq \epsilon_2 \qquad \forall l = 1...N. \tag{4.12}$$

We refer to $[\epsilon_1, \epsilon_2]$ as the threshold pair such that $0 \leq \epsilon_i \leq 1\ \forall i$, which are to be determined throughout the offline decision process. This problem statement can be considered in a stochastic optimization framework. We can transform the $Pr(\cdot) \leq \epsilon$ expressions first to equalities with CDFs as described in Section 4.1, then transform them to linear inequalities by using the iCDF relationships. The optimization problem finally becomes

$$\begin{aligned}
&\textbf{P1:} \quad \max \sum_{i=1}^{N} \frac{L_i}{2} \log(1+p_i) \qquad (4.13)\\
\textbf{s.t.} \quad &\textbf{C1:} \quad \sum_{i=1}^{l} L_i p_i - \sum_{i=0}^{l-1} E_i + G_{\Gamma_{l-1}}(1-\epsilon_1) \leq 0\\
&\textbf{C2:} \quad -\sum_{i=1}^{l} L_i p_i + \sum_{i=0}^{l} E_i - E_{MAX} - G_{\Gamma_l}(\epsilon_2) \leq 0 \qquad \forall l = 1...N.\\
&\textbf{C3:} \quad p_i \geq 0 \qquad \forall i = 1...N.
\end{aligned}$$

The problem in (4.13) is not convex in this form when p_i's, ϵ_1 and ϵ_2 are taken as optimization parameters. However **P1** is convex when ϵ_1 and ϵ_2 are taken as constants, since the constraints become affine and the objective function to be maximized are concave.

4.3.2. Fading Wireless Channel

For a transmitter operating over a block fading wireless channel, the transmission horizon can be represented by a pool (Figure 4.5) with semipermeable walls (energy causality constraints), bumps and holes at the bottom of the pool (fading levels). This pool analogy is used in [29] as well. Water level (transmission power) in each section depends on energy packets (water) arriving at the transmitter. In [13], a directional water-filling algorithm is proposed which gives the water equilibrium that corresponds to the optimum power allocation for this transmitter with the prediction set $[E_0, E_1, E_2]$ under no battery leakage (Figure 4.5).

When the optimum power allocation for the given prediction set is applied to a transmitter with unpredictable energy leakage, however, silent transmission intervals appear as if there are suboptimality walls in the pool (Figure 4.6). Again, we aim to find a new power allocation strategy that reduces the length of these silent intervals that will result in a higher throughput in average.

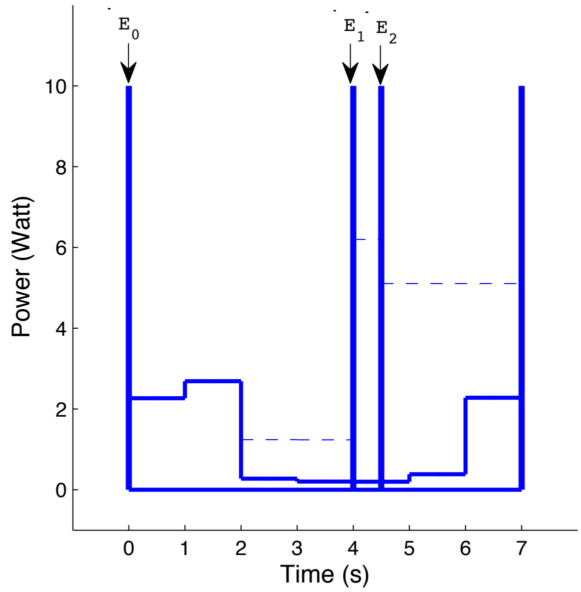

Figure 4.5. Optimum power allocation over the fading channel disregarding leakage.

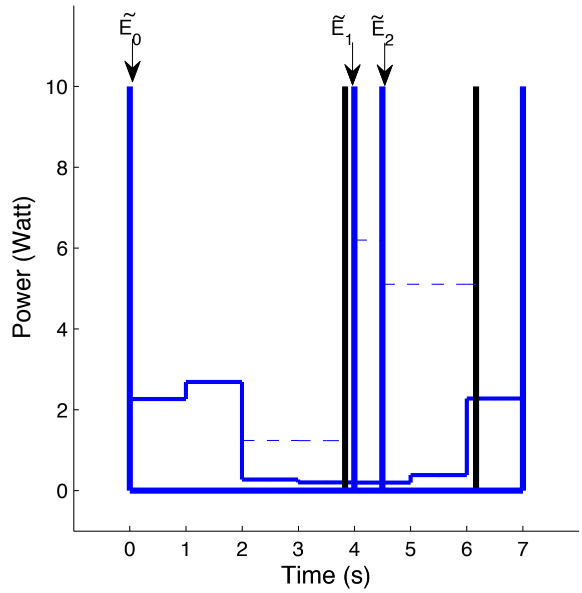

Figure 4.6. Optimum power allocation applied to an energy harvesting transmitter with random energy leakage.

When Tx is operating over a block fading wireless channel, the system throughput can be described by $\int_0^T \frac{1}{2}\log(1+h(t)p(t))dt$. Assuming perfect prior knowledge of the fading states and using the constant nature of optimum transmission power between energy arrivals, system throughput is given by $\sum_{i=0}^{M} \frac{L_i}{2}\log(1+h_i p_i)$. This throughput expression is similar to the one given in [13]. Applying the violation probabilities for battery capacity and energy causality constraints, the optimization problem becomes

$$
\begin{aligned}
\mathbf{P2:}\ \ & \max \sum_{i=1}^{M} \frac{L_i}{2}\log(1+h_i p_i) \qquad (4.14)\\
& \sum_{i=1}^{l} L_i p_i - \sum_{i=0}^{l-1} E_i + G_{\Gamma_{l-1}}(1-\epsilon_1) \leq 0\\
& -\sum_{i=1}^{l} L_i p_i + \sum_{i=0}^{l} E_i - E_{MAX} - G_{\Gamma_l}(\epsilon_2) \leq 0 \qquad \forall l = 1...M.\\
& p_i \geq 0 \qquad \forall i = 1...N.
\end{aligned}
$$

The problem in (4.14) is not convex in this form when p_i's, ϵ_1 and ϵ_2 are taken as optimization parameters. However **P2** is convex when ϵ_1 and ϵ_2 are taken as constants, since the constraints become affine and the objective function to be maximized are concave.

Hereby, please note that **P1** is just a special case of **P2** with $h_i = 1$, for all i. Therefore, from now on we focus on finding $\mathbf{p_2^*}$, the power allocation that solves **P2**. Our strategy is to employ a 3-step-procedure: First step, we find the feasible threshold pairs for a given prediction set and leakage distribution. Secondly, we form a group of power sets by solving **P2** with all feasible threshold pairs. Finally, we find one power set that maximizes the average throughput over many $\tilde{\mathbf{E}}$ realizations.

4.3.3. Feasibility Conditions

In this subsection, we analyze the feasibility of the introduced threshold pairs. Since both ϵ_1 and ϵ_2 can take values between 0 and 1, there will be infeasible pairs for any prediction set. We clarify this with the following lemma.

Lemma 4.1. *There is only one* $\mathbf{p_2^*}$ *that solves* ***P2****, if* $[\epsilon_1, \epsilon_2]$ *satisfies the following feasibility conditions:*

$$\textbf{\textit{FC-1:}}\ G_{\Gamma_{\ell-1}}(1-\epsilon_1) - G_{\Gamma_\ell}(\epsilon_2) \le E_{MAX} - E_\ell, \tag{4.15}$$

$$\textbf{\textit{FC-2:}}\ G_{\Gamma_{\ell-1}}(1-\epsilon_1) \le \sum_{i=0}^{\ell-1} E_i, \quad \forall \ell = 1, \ldots, N. \tag{4.16}$$

Proof. First, we state that **P2** has one and only one solution if it is convex and has at least one $\mathbf{p_2}$ in its feasible region. The convexity of **P2** is guaranteed by its affine constraints and concave objective function that is to be maximized. The feasible region of **P2**, $\mathscr{F} = \mathscr{F}_1 \cap \mathscr{F}_2$ formulated by **C1**, **C2** and **C3**, depends on $[\epsilon_1, \epsilon_2]$ and is given by:

$$\mathscr{F}_1 = \Big\{ p_\ell \ge 0,\, \forall \ell \,\Big|\, \sum_{i=1}^{\ell} L_i p_i \ge \sum_{i=0}^{\ell} E_i - E_{MAX} - G_{\Gamma_\ell}(\epsilon_2) \Big\},$$
$$\mathscr{F}_2 = \Big\{ p_\ell \ge 0,\, \forall \ell \,\Big|\, \sum_{i=1}^{\ell} L_i p_i \le \sum_{i=0}^{\ell-1} E_i - G_{\Gamma_{\ell-1}}(1-\epsilon_1) \Big\}.$$

In order for $\mathscr{F}$ to be nonempty, at least one $\mathbf{p_2}$ has to exist for the given $[\epsilon_1, \epsilon_2]$ pair. Therefore the upperbound in $\mathscr{F}$ always has to be nonnegative and higher than the lowerbound in $\mathscr{F}$. With this in mind, it can be shown via simple mathematics that $\mathscr{F}$ leads to the feasibility conditions **FC-1** and **FC-2**. □

We make a couple of observations on the feasibility conditions. First of all, not every threshold pair is feasible. Secondly, feasibility of a certain threshold pair depends on the distribution parameters of the leakage model, such as the type of the distribution, first and second moments etc. Among all feasible threshold pairs, there is one that gives the best average throughput for a certain prediction set, which can be found by Monte Carlo Analysis. After finding the most convenient threshold pair, the solution to **P2** becomes the STP for the energy harvesting transmitter under random energy leakage.

Note that, the problem formulation assumes that the transmission power changes only at energy arrival instances. However when unpredictable leakage takes place, like any other policy, STP results in silent intervals. This indicates that our policy is not optimum, which is an unreachable state since it requires the perfectly accurate energy leakage information. Despite of that, STP arbitrarily approaches the optimum policy as simulation results indicate.

Lemma 4.1 suggests that there are infeasible threshold pairs and we have to find the feasible ones to solve **P1** and **P2**. Indeed, we can make use of the feasibility conditions to find every feasible threshold pair. When feasibility analysis is complete, the transmission policy $\mathbf{p_{ij}}$ can be obtained by solving the corresponding **P1** or **P2**. Then $\mathbf{p_{ij}}$ is the transmission policy with the outage probability upperbound $\epsilon_1 = i$ and overflow probability upperbound $\epsilon_2 = j$.

Next, we present the average throughput maximizing algorithm that outputs the proposed stochastic transmission policy for the energy harvesting communication system with random energy leakage.

Algorithm 1 illustrates the decision process for transmission with the energy harvesting transmitter exposed to random energy leakage. This algorithm takes the battery capacity, the energy and fading pattern as the input sets. First of all, the feasible threshold pairs are found by using the idea given in the Lemma 4.1 and also by **FC-1** and **FC-2**. Then for each feasible pair, the offline throughput maximization problem under energy leakage is solved. Therefore each threshold pair results in a certain power allocation.

Get E_{MAX}, $\mathbf{E}$, $\mathbf{h}$ and $\mathbf{t}$

for all $[\epsilon_1, \epsilon_2]$ such that $0 < \epsilon_1, \epsilon_2 < 1$ **do**

  Step 1: Feasibility Analysis:

  for $\ell = 1$ **to** N **do**

    if $G_{\Gamma_{\ell-1}}(1-\epsilon_1) - G_{\Gamma_\ell}(\epsilon_2) \leq E_{MAX} - E_\ell$ **and** $G_{\Gamma_{\ell-1}}(1-\epsilon_1) \leq \sum_{i=0}^{\ell-1} E_i$ **then**

      $[\epsilon_1, \epsilon_2]$ is feasible

    else

      $[\epsilon_1, \epsilon_2]$ not feasible

      Exit the for loop

    end if

  end for

  Step 2: Throughput Analysis:

  if $[\epsilon_1, \epsilon_2]$ is feasible **then**

    p_{ij} = solve **P2**

  end if

end for

$\mathbf{p_2^*} = \arg\max_{i,j}(T(p_{ij}))$

Figure 4.7. Algorithm 1.

When resulting power allocations are applied to the energy harvesting node, the system throughput will depend on the amount of each leaking energy packet. Since in practice, the energy packets are exposed to random leakage, each power allocation does not have a strict throughput but an average throughput. At the end, the STP becomes the power allocation with the highest average throughput. So the transmission policy $\mathbf{p}^*$ is the STP, which gives the best average throughput, $T(\cdot)$, confirmed by Monte-Carlo simulations in the next section.

5. SIMULATION RESULTS

5.1. First Remarks

In this chapter, we present the stochastic transmission policies for an energy harvesting wireless node exposed to random energy leakage from various distributions. Then we compare the throughput performance of STP to the transmission power policy that disregards random energy leakage. We also state how close STP brings the system throughput to performance upperbound.

We test the performance of each policy first over the wireless static AWGN channel and then the fading channel. All our comparisons are in average, since the throughput depends on the total energy income of the transmitter, which is partially random.

The leakage disregarding policy assumes lossless energy transfer and usage within the node. Its application to a leaky energy harvesting node, therefore, consumes the stored energy faster than it should and results in outage. This inevitably results in poor throughput performance. STP, on the other hand, is a leakage-adapted power policy with shorter outage and better throughput.

Along with the leakage disregarding policy, we use the throughput upperbound as a performance indicator. The throughput upperbound is a utopic level, achieved when the transmitter has full knowledge over the leaking energy packets before the transmission starts. It is the performance bound of any energy harvesting wireless node operating under random energy leakage. This bound cannot be achieved in practice, since it requires the randomness to vanish, which conflicts with the system model. The throughput upperbound is the solution of the offline optimization problem in [2], after substituting $\mathbf{E}$, with the energy realization set, $\tilde{\mathbf{E}}$.

As pointed out in the previous chapter, the transmitter is fully aware of the energy arrival pattern (size and arrival instances of packets) and, if applicable, the fading pattern. Therefore the STP solution is an offline solution and gives us an insight on how far the system performance can reach when the energy pattern is not apparent to the transmitter.

5.2. Simulations

For convenience, we take harvested energy packets as $E = [2, 1, 6, 4, 8, 1, 10]$ Joules which becomes available to the transmitter at $t = [0, 2, 4, 5, 7, 11, 12]$ seconds. We assume that the transmitter has a storage capacity of $E_{MAX} = 10$ *Joules*. We test the throughput performance first over the AWGN channel then the fading channel. We aim to maximize the throughput over each channel for a $T = 12$ *s* long transmission.

For the AWGN channel, the noise power is assumed to be unity. This simplifies the problem formulation since changing the noise power would not effect the resulting power allocation.

First, using Lemma 4.1 and the given energy pattern, we find the feasible threshold pairs for each of the three distributions as given in Fig. 5.1.

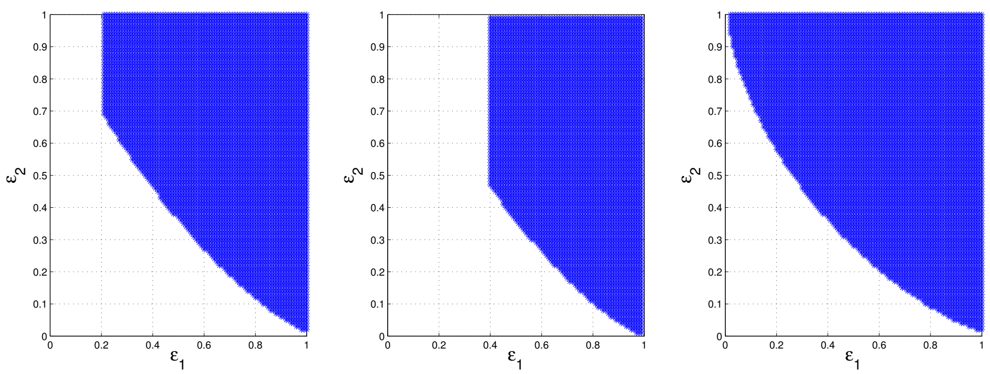

Figure 5.1. Feasible threshold pairs for exponential, lognormal and uniform leakage, respectively.

The feasible threshold pairs are valid for both channels since the energy patterns are the same.

Then, we solve the AWGN problem, **P1**, with the proposed algorithm for exponential, lognormal and uniform leakage. We show the resulting threshold pair-throughput relationship of all policies in Figures 5.2, 5.3 and 5.4 respectively.

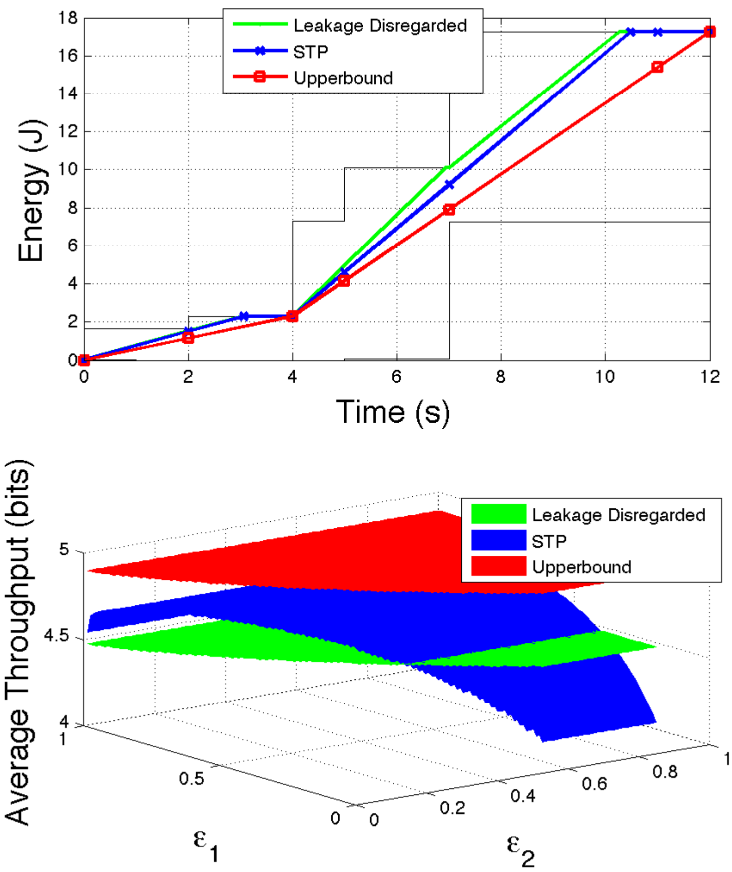

Figure 5.2. Average throughputs when leakage is exponential.

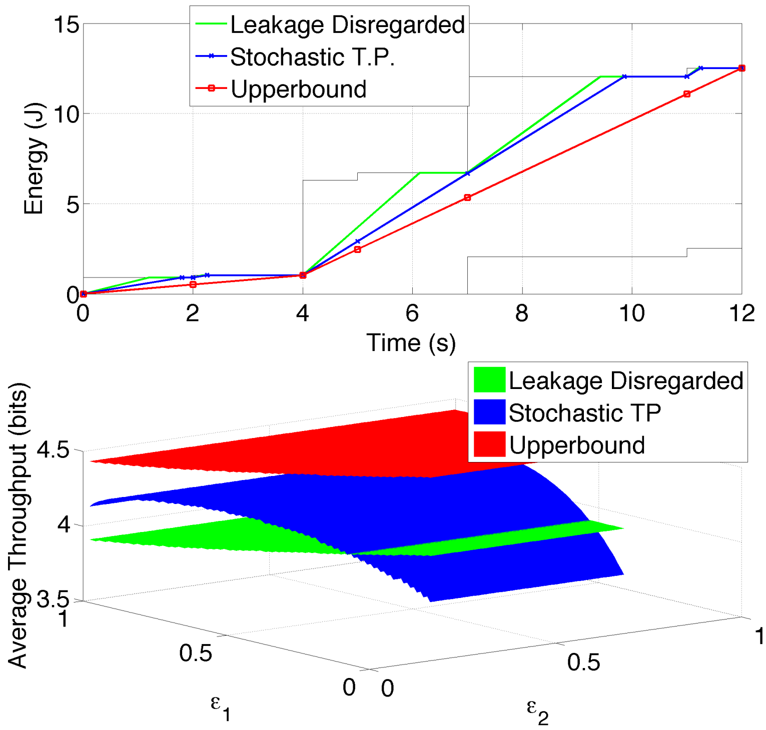

Figure 5.3. Average throughputs when leakage is lognormal.

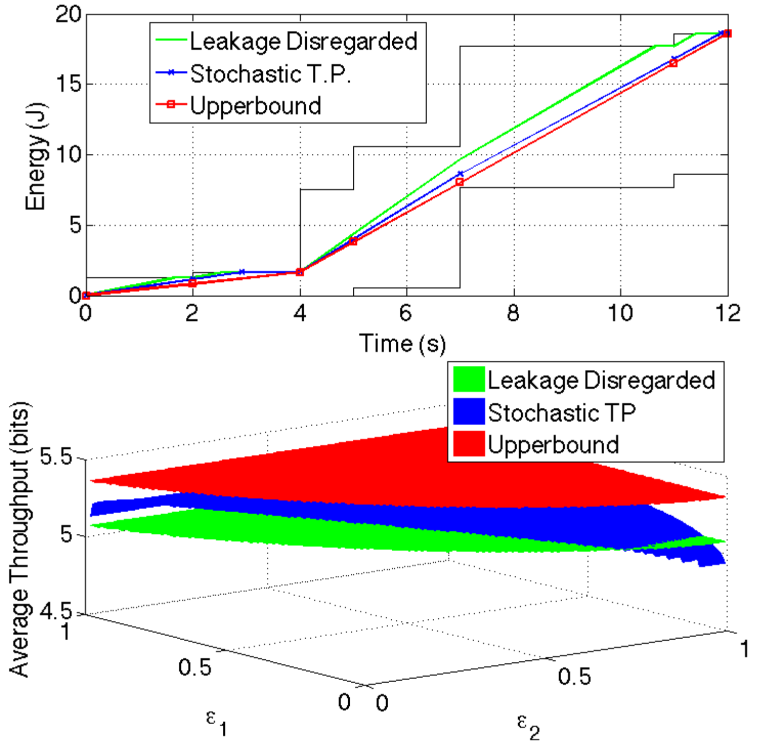

Figure 5.4. Average throughputs when leakage is uniform.

In Figures 5.2-3-4, the green curve is the average throughput resulting from the leakage disregarding policy over 1000 simulations. The red curve depicts the throughput upperbound. The blue curve represents the throughput performance of the stochastic power policies with different threshold pairs. For the given prediction set we find the leakage disregarding policy to be $\mathbf{p}_{\text{w/o}} = [0.75, 0.75, 2.67, 2.67, 2.2, 2.2]$ during $L = [2, 2, 1, 2, 4, 1]$ which is the same for any leakage distribution since it does not consider any sort of energy leakage. In case the leakage fits an exponential distribution, STP suggests to transmit with $p_S = [0.56, 0.56, 2, 2, 2, 2]$ Watts employing $[0.82, 0.31]$ as the threshold pair that results in 4.71 bits/Hz whereas the optimum upperbound is 4.89 bits/Hz. Under exponential leakage, STP improves the average throughput by around 5.2% compared to the leakage disregarding policy. When the leakage belongs to a lognormal distribution, the best threshold pair becomes $[\epsilon_1, \epsilon_2] = [0.87, 0.17]$. In this

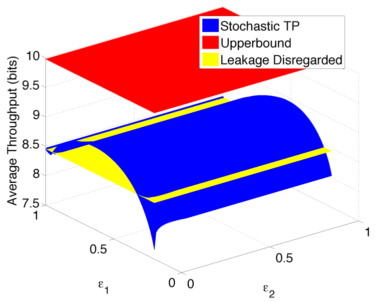

Figure 5.5. Average throughput performance of STP over fading channel for exponential leakage.

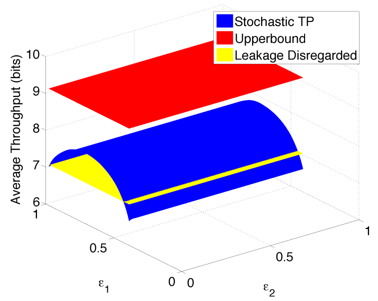

Figure 5.6. Average throughput performance of STP over fading channel for lognormal leakage.

case, STP is to transmit with $p_S = [0.5, 0.5, 1.88, 1.88, 1.88, 1.88]$ Watts. $\mathbf{p}_{\mathrm{w/o}}$ results in 3.91 bits/Hz in average whereas STP results in an average throughput of 4.21 bits/Hz with a 7.6% increase. When the leaking energy fits a uniform distribution, STP suggests to transmit with $p_S = [0.57, 0.57, 2.32, 2.32, 2.05, 2.05]$ and $[\epsilon_1, \epsilon_2] = [0.75, 0.13]$ that results in 5.27 bits/Hz in average whereas the optimum upperbound is about 5.36 bits/Hz. The leakage disregarding policy results in 5.07 bits/Hz in average, so our policy enhances the throughput by 3.8%.

Next, the STP is simulated over a fading channel. Here, the fading coefficients are produced randomly with a Rayleigh envelope, [30]. Over the fading channel, we assume that the channel state changes every second, therefore we have transmission epochs of one second. The results of a similar analysis over the fading wireless channel are shown in Figures 5.5-7. For all three leakage distributions, STP with the suitable

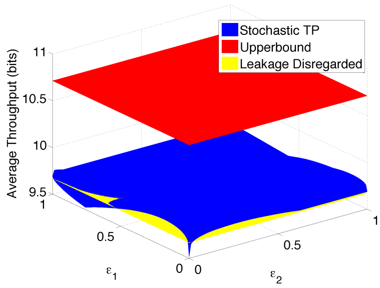

Figure 5.7. Average throughput performance of STP over fading channel for uniform leakage.

threshold pair brings the system throughput closer to the unreachable upperbound compared to the leakage disregarding policy.

The solution of **P2** gives the stochastic transmission policies for the fading problem with the threshold pair-throughput relationship in Figures 5.5-7. These figures demonstrate the average system throughput for a given threshold pair. The maximum throughput is achieved at *the best threshold pair* and STP is the power management policy with the best threshold pair.

Table 5.1. Throughput performance.

Channel	**AWGN**			**Fading**		
Leakage	Uni	Exp	Lgn	Uni	Exp	Lgn
STP	5.27	4.71	4.21	9.86	8.98	7.87
$\mathbf{p}_{\mathrm{w/o}}$	5.07	4.47	3.91	9.67	8.45	7.07
Improvement	3.87%	5.23%	7.39%	1.95%	6.08%	10.71%

Performance results for both policies over both channel conditions are summarized in Table 5.1. For instance, when energy leakage is an exponential random variable, STP can achieve upto 4.7 bits/Hz of throughput with a 5.2% improvement.

Simulation results indicate that the proposed stochastic transmission policy improves the throughput performance of the EH communication system compared to the power management policy disregarding energy leakage. The level of improvement is naturally better if energy leakage is large with a high probability compared to the case where energy leakage is small with a high probability. In our case, lognormal and exponential leakage correspond to the former and uniform leakage corresponds to the latter.

6. CONCLUSIONS AND FUTURE DIRECTIONS

In this thesis, we propose stochastic transmission policies for a single link energy harvesting wireless node with random leakage in noncausally known energy packets that are transferred from the energy harvester to the recharge able battery of the transmitter. Our method can be applied to systems with any leakage distribution. As shown in simulations, STP enhances the average throughput over AWGN channel under energy leakage compared to the leakage disregarding power management policies. Our proposed policy is confirmed to result in higher throughput in average, i.e. STP achieves better collective throughput-performance when it is applied to a high number of single-link nodes. The improved performance of STP relies on the required information on the distribution of the energy leakage. Once required information is obtained, then the throughput performance can be improved upto 7%.

There are many more trending problems to solve involving energy harvesting communication systems. For instance, in [31], transmission policies are studied for wireless sensor nodes when random energy harvesting is assumed at each node. Duty cycling algorithms are implemented on energy harvesting wireless sensor networks in [32]. For a point-to-point communication system with random energy harvesting, optimal policies are presented in [33]. In [34], the energy harvesting communication system is analyzed for the case when the transmitter talks to the receiver over a relay. In that system model, the source talks only to the relay, so there is no direct link to the receiver. The case with a direct link from the transmitter to the receiver on a 3-node setting, is studied in [35]. Additional to these efforts, it is also worthwhile to look into the energy diversity opportunities via cooperative transmission strategies over multiple access channels, [36]. In fact, while this thesis was on progress, we started investigating the throughput maximization and the deadline minimization problems for delay constrained and non-delay constrained transmission as a part of the TUBITAK project 113E556, with Prof. Onur Kaya, and we will continue working on these problems during my doctoral studies at Bogazici University. We expect that the findings in this thesis will shed light on the mentioned future research.

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