

RISK QUANTIFICATION FOR FOREIGN CURRENCY INVESTMENTS

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ABSTRACT

RISK QUANTIFICATION FOR FOREIGN CURRENCY INVESTMENTS

As the economies are globalized, more firms, investors and workers find their fortunes linked to the exchange rates. Therefore, management of exchange rate risk becomes more significant. As the measured risk can be managed better, the aim of this study is to quantify the foreign exchange risk. In this study, six different risk modeling methods - i.i.d Normal, i.i.d Student's t , Normal-GARCH, t -GARCH, unconditional EVT and GARCH filtered EVT - are used to quantify the daily risk of currency pairs. After the daily VaR of the currency pairs has been calculated, an extensive back testing study is performed using 4 different confidence levels to assess the performance of each risk model. The results of the back testing experiment show that the risk modeling methods that rely on the normality assumption, such as the i.i.d Normal and the Normal-GARCH model, result in significant underestimation of daily risk. The sophisticated methods, GARCH filtered EVT and t -GARCH models showed the best overall performance. Besides, it is observed that the performance of i.i.d Student's t is good especially for currency pairs that belong to the countries with strong economical base. The quality of the t -GARCH and GARCH filtered EVT models were observed in our study and it is recommended to use these models to quantify the FX risk. Also i.i.d Student's t model can be the method of choice if the simple implementation of the methods of choice is important.

ÖZET

DÖVİZ YATIRIMLARI İÇİN RİSK ÖLÇÜMÜ

Ekonomiler küreselleştikçe, daha çok şirket, yatırımcı veya çalışan kazancını döviz olarak sağlamaktadır. Döviz kurundaki değişimler kişilerin veya firmaların kazancını etkileyeceği için döviz kurundan kaynaklanan riskin yönetimi önem kazanmıştır. Ölçülebilen risk daha iyi bir şekilde yönetileceği için, bu çalışmadaki amacımız döviz kurundan kaynaklanan riskin ölçülmesidir. Çalışmamızda 9 farklı döviz çiftine ait günlük kur değerleri kullanılmıştır. Risk modellemesi için normal dağılım, t dağılımı, normal dağılımlı GARCH, t dağılımlı GARCH, eşik sınırın üzerinde kalan değerler yöntemi ve GARCH filtreli uç değer teorisi kullanılmıştır. Günlük risk ölçümü ise VaR modeli kullanılarak yapılmıştır. Günlük risk ölçümü tüm döviz çiftleri için ve her bir risk modelleme metodu kullanılarak yapıldıktan sonra, risk modelleme yöntemlerinin performanslarını değerlendirmek için 4 farklı güvenilirlik düzeyi kullanılarak geriye dönük testler uygulanmıştır. Gerçekleştirilen geriye dönük testlerin sonuçlarına göre normallik varsayımına dayanan metodlar, normal dağılım ve normal dağılımlı GARCH, riski düşük tahmin etmiştir. Gelişmiş yöntemlerden t dağılımlı GARCH ve GARCH filtreli uç değer teorisi, diğer risk modelleme yöntemleri ile karşılaştırıldığında en iyi risk modelleme performansına sahip yöntemlerdir. Ayrıca, t dağılımı da özellikle güçlü ekonomilere sahip ülkelerin döviz çiftlerinin risk modellemesinde iyi sonuçlar vermiştir. Çalışmamızın sonucunda öncelikle daha iyi bir risk modellemesi için gelişmiş yöntemlerin - t dağılımlı GARCH ve GARCH filtreli uç değer teorisi- kullanılması önerilmiştir. Fakat uygulama kolaylığını önemseyen kullanıcılar için riski diğer modellere göre daha yüksek tahmin eden t dağılımı önerilebilir.

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LIST OF SYMBOLS

F_t	Information set at time t
F_u	Excess distribution function
$I_{()}$	Indicator function
L	Loss function
l_θ	Log-likelihood function
N_u	Number of observations over the threshold
P	Price of an asset
r_t	Random return at time t
s	Scale parameter of Student's t model
u	Threshold value
V	Value of a portfolio
Y_i	Exceedance over the threshold value
z_t	Standardized residual at time t
α	Confidence level
α_i	ARCH parameters
β	Scale parameter of Generalized Pareto distribution
β_j	GARCH parameters
γ_i	A constant for EGARCH model
ε_t	The innovation of an asset return
θ	Parameter set
μ	Location parameter
ν	Degrees of freedom parameter
ξ	Shape parameter of GEV and GPD
$\rho(\cdot)$	Measure of risk over a chosen horizon
σ	Volatility
ω	Long term average value of ARCH and GARCH models

LIST OF ACRONYMS/ABBREVIATIONS

ACF	Autocorrelation Function
ARCH	Autoregressive Conditional Heteroskedastic
AUD	Australian Dollar
BIS	Bank of International Settlements
BMM	Block Maxima Model
BRL	Brazilian Real
CAD	Canadian Dollar
CHF	Swiss Franc
CVaR	Conditional Value at Risk
EGARCH	Exponential Generalized Autoregressive Conditional Heteroskedastic
EGP	Egyptian Pound
ES	Expected Shortfall
E.U.	European Union
EUR	Euro
EVT	Extreme Value Theory
FX	Foreign Exchange
G-30	Group of Thirty
GARCH	Generalized Autoregressive Conditional Heteroskedastic
GBP	British Pound
GEV	Generalized Extreme Value
GJR	Glosten-Jagannathan-Runkle
GPD	Generalized Pareto Distribution
IBM	International Business Machines
I.I.D	Independent and Identically Distributed
IMF	International Money Fund
JPY	Japanese Yen
MLE	Maximum Likelihood Estimation
MXP	Mexican Peso

NOK	Norwegian Krone
NZD	New Zealand Dollar
PACF	Partial Autocorrelation Function
POT	Peaks Over Threshold
Q-Q	Quantile-Quantile
RMB	Chinese Yuan Renminbi
RUB	Russian Ruble
SDR	Special Drawing Rights
SV	Stochastic Volatility
TGARCH	Threshold Generalized Autoregressive Conditional Heteroskedastic
THB	Thailand Baht
TRY	Turkish Lira
U.K.	United Kingdom
U.S.	United States
USA	United States of America
USD	United States Dollar
VaR	Value at Risk

1. INTRODUCTION

Financial risk can be defined as the prospect of financial outcomes (a loss or a gain) due to random changes in underlying risk factors. There are several types of risk in financial markets, one of which is the market risk. Market risk is emerged owing to the change in the value of an investment which is due to various movements in market factors. It can be classified into exchange rate risk, interest rate risk and so on, depending on whether the risk factor is an exchange rate or interest rate. Foreign exchange rate which is one of the most important policy tools for many transitional economies, can be viewed as security prices in the foreign exchange market. In the context of global integration, it is a common fact that the foreign exchange (FX) risk has become more and more significant. Regarding international investments, it should be emphasized that investors must not neglect this particular risk as it can affect or improve the value of investments significantly. Besides, exchange rate fluctuations have significant impacts on trade volume. Due to all these reasons, the risk of holding one currency against another is an important issue in financial markets.

The process, by which financial risk is identified, assessed, measured and managed in order to create economic value, is defined as financial risk management. During the last three decades, several crises have affected financial markets, such as the stock market crash of October 1987, the U.S. recession of 1992, the Asian financial crisis of 1997-1998, the Turkish economic crisis in 2001 and the global financial crisis in 2008. The common lesson learned from these financial disasters is that billions of dollars can be lost because of poor supervision and management of financial risk. Thus, the problem of managing risk has become a major concern [1–8].

Risk can never be entirely avoided so in order to balance risk against return, risk measurement is required. Hence, many risk measures, such as volatility, Value at Risk (VaR) or Expected Shortfall (ES) were suggested. The most commonly encountered risk measure is the Value at Risk. VaR attempts to answer the question, "Given a certain confidence level and a specified time horizon, what is the maximum potential

loss?”. While the concept of VaR is easy, its measurement is a challenging statistical problem. The main problem is related to the estimation of the distribution that adequately describes returns since it is practically impossible to accurately identify the distribution of financial returns. There are many models, parametric and nonparametric, used for modeling financial returns. The existence of various models stems from the two characteristics of financial data that are difficult to model, leptokurtosis and non-constant conditional volatility [8,9].

As economies become ”globalized”, more firms, investors and workers find their fortunes linked to the exchange rates and to their impact on trade and financial flows. This concentrates our attention on the FX market. The main problem is to quantify and manage the FX risk induced by the exchange rate fluctuations. At the macroeconomic level, these fluctuations can have significant impacts on trade volume. At the microeconomic level, exchange rate fluctuations can affect the value of firms that are involved in international business. In other words, poorly managed FX risk can have negative effects on investors and firms. Hence, the risk of holding one currency against another should be quantified in order to develop a beneficial investment strategy. The principle ”Risk that can be measured, can be managed better” explains the importance of risk quantification [2,10].

The aim of this study is to quantify the FX risk as good as possible. In order to achieve this goal, we find different risk modeling methods in the literature. In an effort to improve the performance of risk quantification, complicated risk models were suggested. Main risk modeling methods in the literature, that are used to quantify the VaR, are the i.i.d Normal, i.i.d Student’s t, Normal-GARCH, t-GARCH, unconditional EVT and GARCH filtered EVT. Though it is practically impossible to accurately identify the distribution of financial returns, we try to find the method that explains the data and quantifies the risk better than other selected models. These models are implemented to FX data, especially to the currencies that are significant for investors living in Turkey. By utilizing the statistical software R and its useful add-on packages, the implementation of these methods and the practical problems we have to solve while using them in practice will be further explained. An important objective is also the

evaluation of the performance of the different methods. In order to compare the quality of the risk estimates based on the different models, a back testing approach based on historical FX data will be used.

The thesis is organized as follows: in Chapter 2, a brief information is given about exchange rates and stylized facts of exchange rate data are presented. In Chapter 3, the basic concepts of risk measuring are introduced with the formal definitions of VaR and CVaR. In Chapter 4, the i.i.d log-return models and their parameter estimation and risk calculation steps are explained. In Chapter 5, different GARCH models are introduced. Besides, the parameter estimation and VaR calculation of the Normal-GARCH model are explained. GARCH filtered EVT method is also introduced in this chapter. In Chapter 6, the required data are introduced, VaR calculation and back testing are explained with simple examples. Moreover, we present the main back testing experiment and the coding of the required R functions. The performances of risk models are assessed at the end of Chapter 6. A small investment example is given in Chapter 7 and Chapter 8 contains the conclusion.

2. DESCRIPTIVE PROPERTIES OF EXCHANGE RATES

The foreign exchange (FX) market is the largest financial market in the world. In April 2010, the Bank for International Settlements (BIS) conducted a survey of trading volume around the world and found that the average amount of currency traded each business day was \$3,981 billion. In 2001, the trading volume of foreign exchange per day was \$1,239 billion. Thus, the amount of foreign exchange traded has recently grown tremendously. Even though there are over 150 currencies in the world, just two of them, the U.S. dollar (USD) and the euro (EUR), account for more than half of all trading volume in the foreign exchange market. USD and EUR are the three-letter currency codes for United States dollar and euro, respectively. Every currency has a three-letter currency code, such as CAD for the Canadian dollar. These codes have been developed by the International Organization for Standardization. In labeling currency pairs, market practice is to express the base currency first, as in “dollar–yen” or USD/JPY, which should be read as Japanese yen (JPY) per USD [11–13].

In Figure 2.1, the ten most traded currencies are shown. The total volume sums to 200%, not 100%, because there is a currency on the buy and sell side of every transaction. The dominance of the U.S. dollar (USD) in the FX market is clear, which reflects the market’s practice of trading minor currencies via a major currency which is called the vehicle currency. A trade from Mexican pesos (MXP) to Australian dollars (AUD), for instance, would typically involve two trades, one from MXP to USD and the second from USD to AUD. The Euro (EUR) is involved in 39% of trades, since it serves as the vehicle currency within the Euro zone. The next most actively traded currencies are the Japanese yen (JPY, 19%) and the U.K. pound (GBP, 13%). Together, these four currencies are known as “the majors”. On the 30th of October in 2015, the International Monetary Fund’s (IMF) Executive Board voted in favor of allowing China’s currency — the yuan (or renminbi) — to join the ranks of the world’s most elite currencies. On the 1st of October in 2016, the yuan will officially be added to the IMF’s Special Drawing Rights (SDR) basket of currencies, which previously included only the dollar, euro, yen, and pound. The SDR is an international reserve

asset, created by the IMF in 1969 to supplement its member countries' official reserves. Its value is currently based on a basket of four major currencies, and the basket will be expanded to include the Chinese Renminbi (RMB) as the fifth currency. SDRs can be exchanged for freely usable currencies. As of November 30, 2015, 204.1 billion SDRs had been created and allocated to members (equivalent to about \$285 billion). The next tier below the majors comprises the AUD (7.5%), the Swiss franc (CHF, 6.4%), and the Canadian dollar (CAD, 5.3%). A notable recent shift is the rising share of the so-called commodity currencies, specifically the Australian dollar (AUD), Canadian dollar (CAD), Norwegian krone (NOK), and the New Zealand dollar (NZD). These currencies' combined share rose from 7% in 1998 to 16.6% in 2010 [13–15].

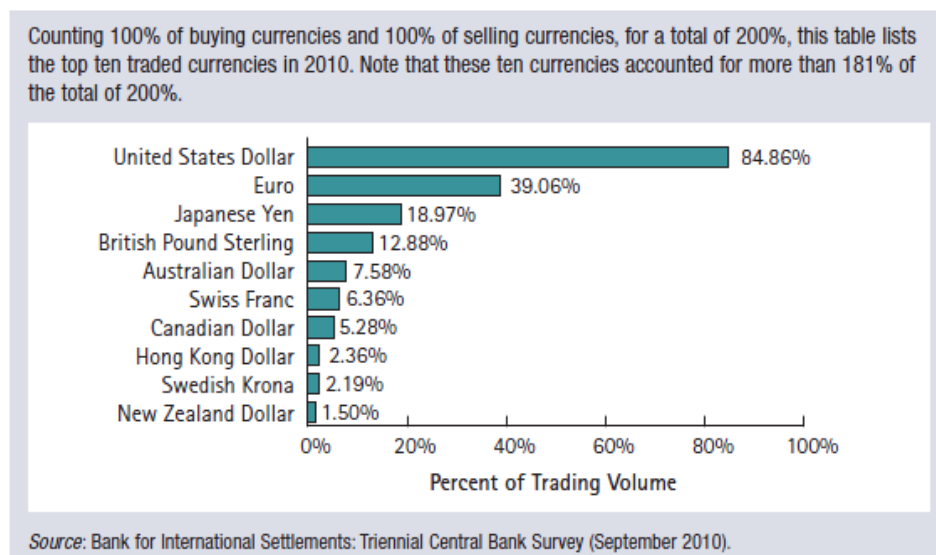


Figure 2.1. The most traded currencies.

Exchange rates are important to innumerable economic activities. Essentially, the increased importance being attached to exchange rates is a result of the internationalization of modern business, the continuing growth in world trade relative to national economies and the rapid pace of change in the technology of money transfer. Governments care about the prices of exports and imports and the domestic currency value of debt payments. Exchange rates affect the price of every country's imports and exports. As a result, changes in the exchange rate can impact trade flows. When a country's currency depreciates, its exports become less expensive to foreigners and im-

ports from overseas become more expensive to domestic consumers. Eventually, these changes in relative prices can cause the level of exports to rise and the level of imports to fall. Likewise, when a currency appreciates, the country's exports become more expensive to foreigners and imports become less expensive to domestic consumers. This can cause exports to fall and imports to rise. Besides, exchange rates determine the value of international investments. Hence, investors care about the effect of exchange rate fluctuations on their international portfolios. For instance, when a currency depreciates, the value of investments falls for overseas investors. Besides, if depreciation is expected in a currency, overseas investors may be reluctant to invest in assets or sell their assets in fear that their investments will become less valuable over time. Similarly, if appreciation is expected over time, assets in that country become more attractive to overseas investors. Moreover, interest rates, inflation and exchange rates are all highly correlated. Higher interest rates offer lenders in an economy a higher return relative to other countries. Therefore, higher interest rates attract foreign investment, increasing the demand for and value of the home country's currency. Conversely, lower interest rates tend to be unattractive for foreign investment and decrease the currency's relative value. Devaluation often invites recession and inflation and thus pushes the economy into an inflation-devaluation spiral, causing a serious setback in economic development. Therefore, central banks care about the value of their international reserves and open positions in foreign currency as well as about the impact of exchange rate fluctuations on their inflation objectives [16–20].

The level of foreign exchange risk, to which companies and individuals are subject, has increased significantly during the last few decades. In July 1944, representatives of 44 countries met in Bretton Woods, New Hampshire, where they reached an agreement to achieve relative currency stability by adopting a system of fixed exchange rates. The system revolved pegging foreign currencies to the U.S. dollar, which was in turn pegged to gold. This lasted until the early 1970s. The breakdown of the Bretton Woods Agreement in the early 1970s and the shift from fixed to flexible exchange rates led to a more volatile environment in which different currencies fluctuate, sometimes significantly, relative to one another [14, 21].

2.1. Stylized Facts of Exchange Rate Data

Market analysts attempt to explain a given market movement by relating it to an economic or political event or announcement. From this point of view, it could be considered that price series obtained from different assets and markets will exhibit different properties since assets are not necessarily influenced by the same events or information sets. However, it is seen that the properties of the Dollar/Yen exchange rate are similar to those of IBM shares or corn futures. The result of many empirical studies on financial time series indicates that this is the case if one examines the properties from a statistical point of view. The random variations of asset prices share some quite nontrivial statistical properties. Such properties, common across a wide range of instruments, markets and time periods are called stylized empirical facts [22].

A risk model that does not capture the characteristics of financial market data adequately, will also not be useful for deriving risk measures. Stylized facts have important implications for assessing whether the chosen risk model is appropriate or not. Thanks to the growth of computational power in the last two decades, the huge amount of empirical data produced by financial markets are now easily available in electronic format and can be massively studied. The systematic study of stylized facts has begun since the 1990s and by now, a number of stylized facts are widely acknowledged. In the next subsections, stylized facts of exchange rate data are explained in detail [23–25].

2.1.1. Heavy tails in the return distributions

The distribution of financial market returns is leptokurtic, namely, it has a narrower central part and fatter tails than the normal distribution. The fat tailed property results from the fact that financial data have sometimes very large positive or negative returns which are very unlikely to be observed if returns were normally distributed. In 1963, Mandelbrot pointed out the insufficiency of the normal distribution to model the marginal distribution of asset returns as the occurrence of extreme events is more likely compared to the normal distribution. Since then, the non-Gaussian character of the return distribution has been repeatedly observed in various market data [22, 24].

Financial risk modeling is usually concerned with obtaining the probability of important but infrequent events, such as market crashes or how much money we may expect to lose in a worst case scenario. For such applications, the main concern is on the far left tail of the return distribution. Hence, in the field of risk management, it is crucial to make the distribution assumption carefully for asset returns since an assumption of normal distribution for risk calculation may lead to a gross underestimation of risk, often with dire consequences. The presence of heavy tails has caused problems for many financial institutions. For instance, in the beginning of the 2007-2009 crisis, several banks lost large amounts of money on their quantitative trading funds, such as Goldman Sachs. Although the assumption of normality enables anyone

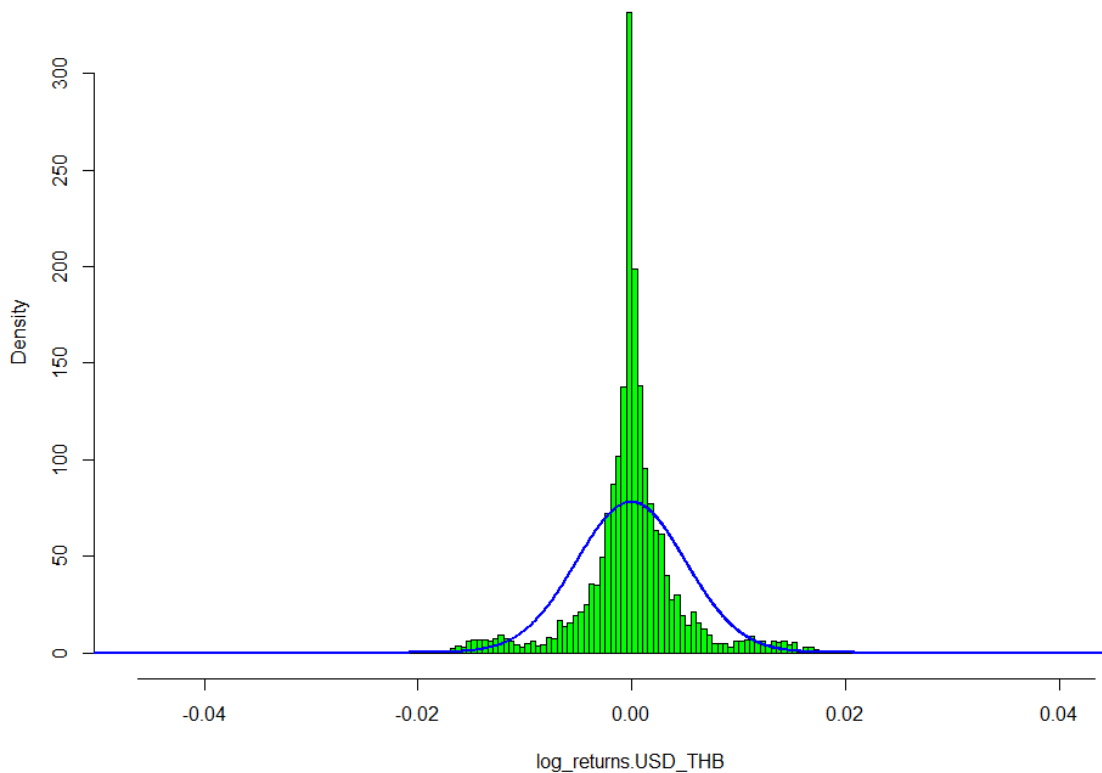


Figure 2.2. Fat tailed return distribution (USD/THB).

to drop off the huge amount of complexity in the equations, non-normality assumption is unfortunately what characterizes the real world. The heavy-tailed property of USD/THB exchange rate is shown in Figure 2.2. THB is the currency code for Baht and the Thai Baht is the currency of Thailand. Figure 2.2 shows the histogram of the USD/THB logarithmic returns overlain with a normal density line fitted to the mean

and standard deviation of the empirical returns. It is seen that USD/THB exchange rate has more relatively large and small outcomes than one would expect under the normal distribution, and conversely, fewer returns of intermediate magnitude [26, 27].

2.1.2. Absence of autocorrelation in returns

Autocorrelation, also known as serial correlation, measures the relationship between price changes in consecutive time periods such as hourly, daily or weekly. It is a measure of how much the return in any period depends upon the return over the previous time period. Positive and statistically significant serial correlation could be viewed as evidence of price momentum in markets, and would suggest that returns in a period are more likely to be positive (negative) if the prior period's returns were positive (negative). A serial correlation, that is negative and statistically significant, could be the evidence of price reversals and would be consistent with a market where positive returns are more likely to follow negative returns and vice versa. A serial correlation of zero would imply that price changes in consecutive time periods are uncorrelated with each other, and can thus be viewed as a rejection of the hypothesis that investors can learn about future price changes from past ones [28–30].

The absence of autocorrelation, which has been empirically observed quite consistently in the last 20 years, is an obvious stylized fact of financial time series [24]. The absence of significant linear correlations in price increments and asset returns has been widely documented [31, 32] and is often cited as support for the efficient market hypothesis. The efficient market hypothesis states that financial markets make efficient use of all available information, thereby instantly incorporating any hidden value of an asset into its price. If the efficient market hypothesis is strictly true, it is impossible to make profit without dealing with a risky investment. This implies that the market can be seen as an open system which continuously reacts to the economic situation of the world and self-organizes in order to quickly eliminate arbitrage opportunities. The absence of simple arbitrage in financial markets means that, given the price time series up to now, the sign of the next price variation is unpredictable. If returns were significantly autocorrelated, one could make better predictions of returns at least on average,

which could be used by traders to make profitable investment decisions. Figure 2.3 indicates the absence of autocorrelation in returns. It is seen that the autocorrelation at lag zero is always one. This is because a series is always perfectly correlated with itself [22, 24, 25, 29].

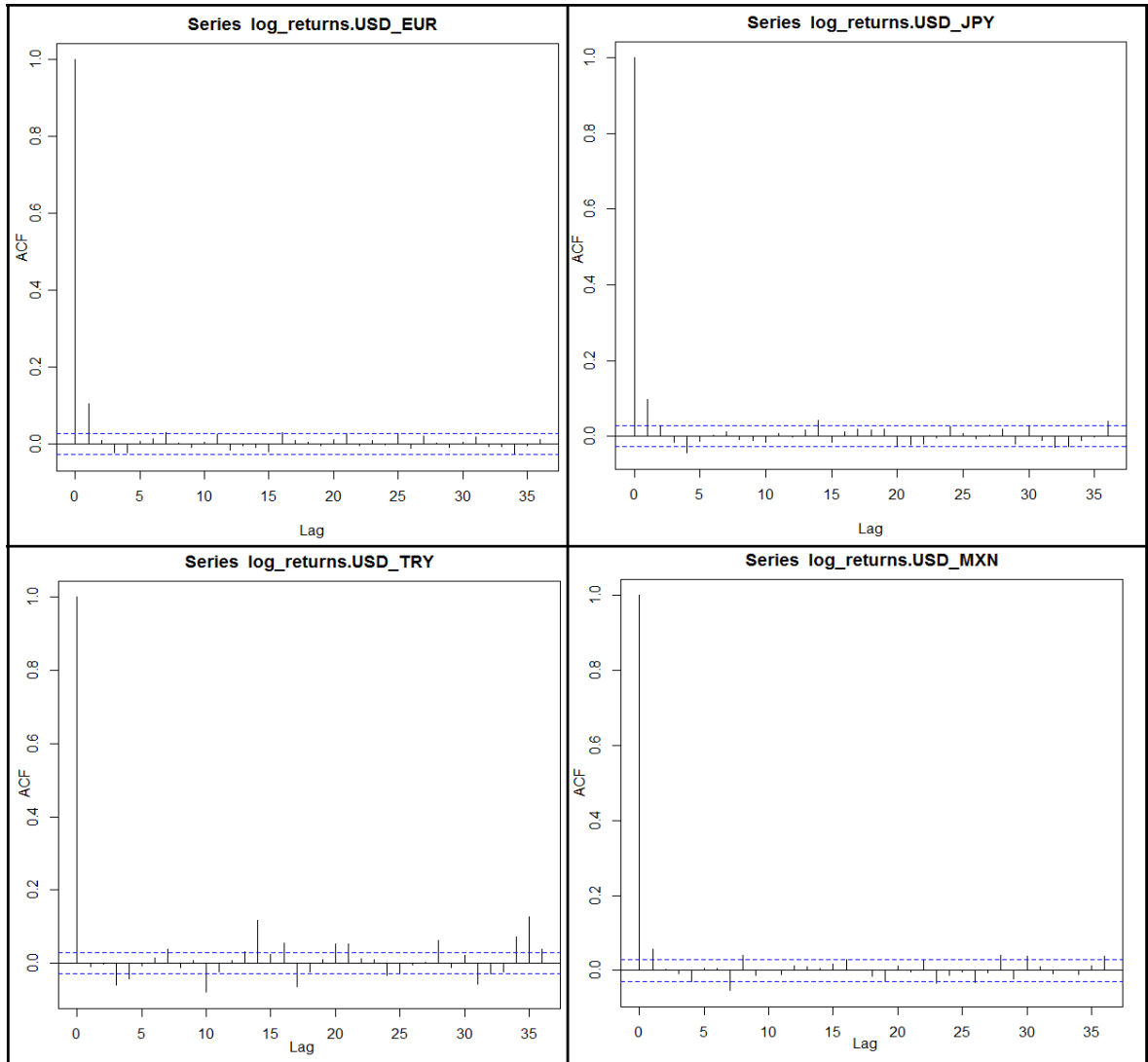


Figure 2.3. The absence of autocorrelation in exchange rate returns.

2.1.3. Volatility clustering

Linear models have some characteristics which might be serious limitations for return modeling. Some of the most important characteristics and limitations of the linear models are that the dynamics is constant for all values of the process and for

all time. For instance, the variance of any forecast is constant. However, empirical evidences have rejected the constant volatility assumption. Volatility is a measure of the amount of fluctuation in the asset price and is considered as a proxy for risk. As seen in Figure 2.4, calm and volatile periods are experienced and this is an evidence of time-varying volatility of returns [13, 33–35].

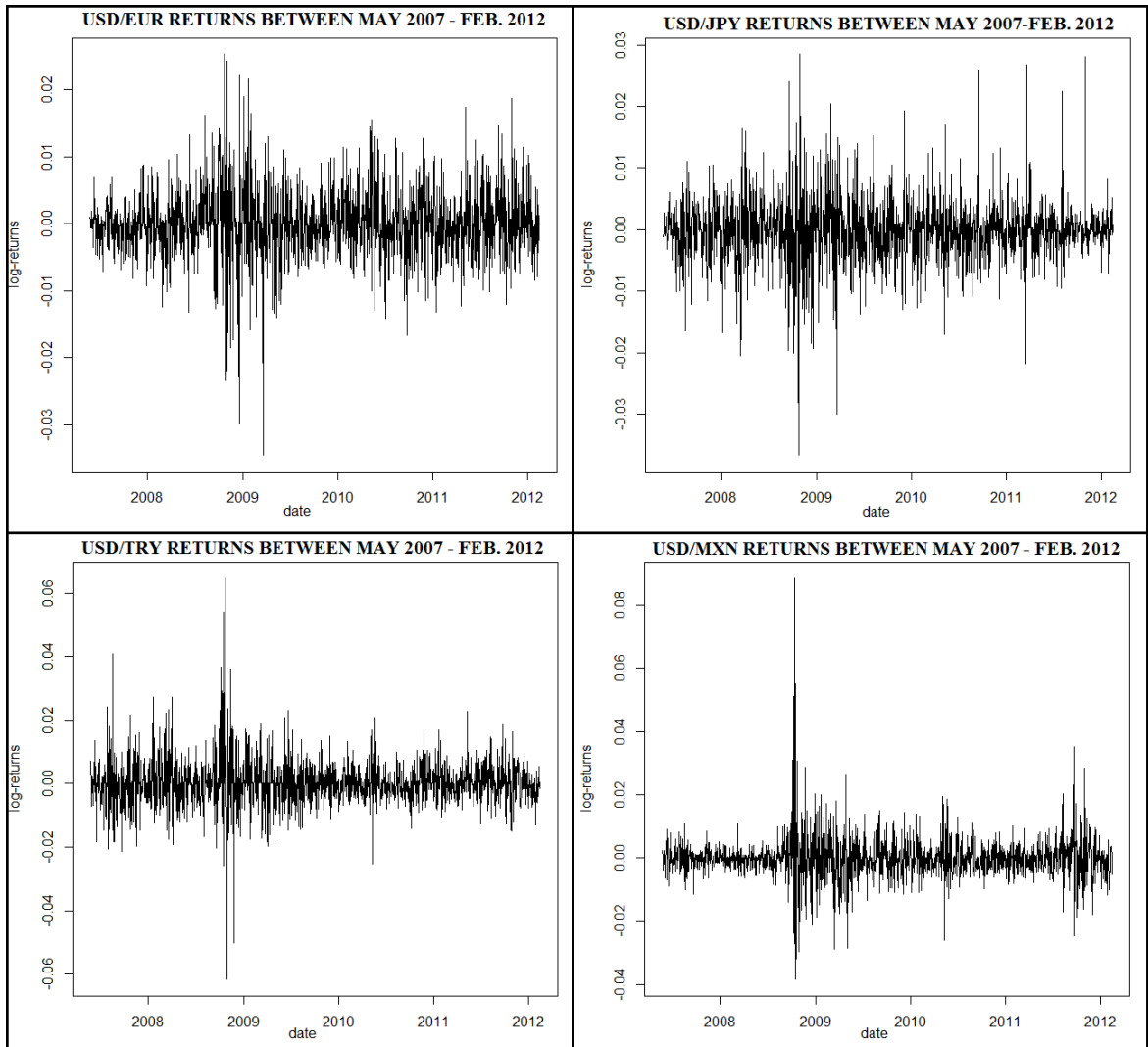


Figure 2.4. Exchange rate returns [USD/EUR (left top), USD/JPY (right top), USD/TRY (left bottom), USD/MXN (right bottom)].

Figure 2.4 also demonstrates that periods of high and low volatility have a strong tendency to persist over time, alternating each other in an unpredictable way. This phenomenon received the name of volatility clustering (volatility persistency). Volatil-

ity clustering behavior refers to the tendency of large changes in asset prices (either positive or negative) to be followed by large changes, and small changes to be followed by small changes. The clustering of large and small moves (of either sign) in the price process, which was one of the first documented features of the volatility process of asset prices, was reported by Mandelbrot (1963) and later confirmed by Fama (1965). Volatility clustering is recognized as a stylized property inherent in most financial time series in a number of different markets, including equity, currency and futures markets [13, 34, 36, 37].

The majority of problems in finance only have analytical solutions under simplifying assumptions, one of which is that returns are independent and identically distributed. The identically distributed assumption ensures that the mean and the variance of returns do not vary over time and conforms to a fixed probability assumption. The independence assumption ensures that speculative price changes are unrelated to each other at any point of time. However, the assumption of independent and identically distributed returns is not realistic. Time varying volatility of the series causes a violation of the assumption that variance is constant over time (identically distributed returns). Although the lack of autocorrelation in returns gives some empirical support for ‘random walk’ models of prices in which the returns are considered to be independent random variables, only this stylized fact is not enough to demonstrate independence of the increments. Independence implies that any nonlinear function of returns, such as the squared and absolute returns, will also have no autocorrelation. As shown in Figure 2.5, absolute and squared returns display a positive, significant autocorrelation, slowly decaying with the length of the lags. This is a quantitative signature of the phenomenon, volatility clustering. By Mandelbrot’s investigations in 1963, it was observed that security returns are governed by non-linear processes that allow successive price changes to be linked through the variance [13, 22, 34, 38–41].

Moreover, volatility clustering could also explain the appearance of fat tails. Extreme observations are drawn from periods with high volatility. This could cause the appearance of fat tails when combining periods of low and high volatility [4].

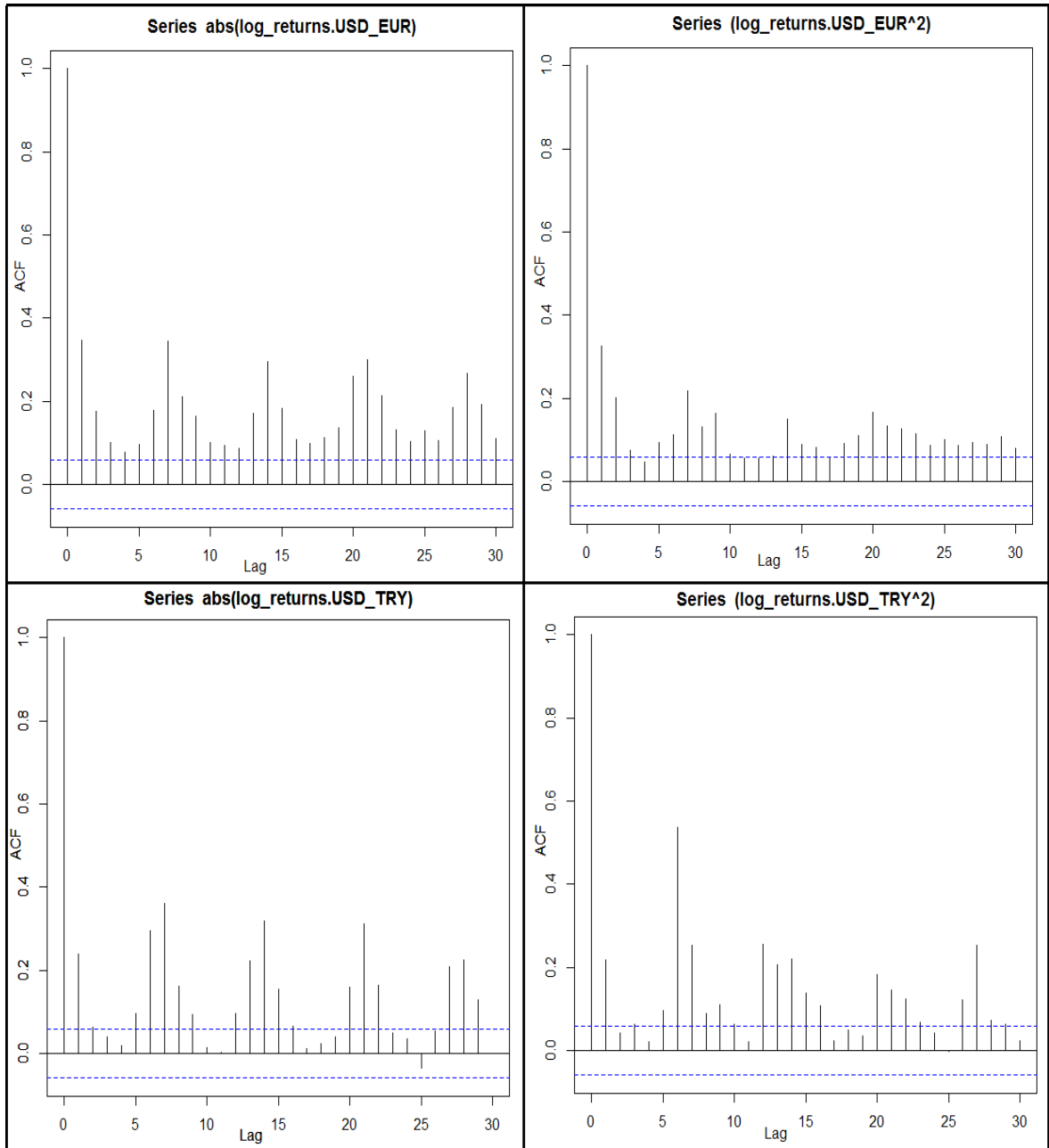


Figure 2.5. Serial correlation in absolute and squared returns [USD/EUR (top), USD/TRY (bottom)].

2.1.4. Asymmetry in the return distributions

A probability distribution may be symmetric or asymmetric around its mean. Asymmetry in distribution, is found in many important economic variables such as stock index returns and exchange rate changes. An appropriate measure to check for an asymmetric return distribution is called skewness and is the third standardized moment of the return distribution. A negative skewness measure indicates that the distribution is skewed to the left; that is, compared to the right tail, the left tail is elongated. A positive skewness indicates that the distribution is skewed to the right; that is, compared to the left tail, the right tail is elongated.

Regarding the stock market, it can be said that the distribution of returns is often negatively skewed reflecting the fact that the downturns of financial markets are often much steeper than the recoveries. Whereas, considering the FX market, there is no reason for exchange rate returns to have any particular sign associated with the skewness. This is owing to the fact that country A's positive skewness is country B's negative skewness for the currency pair B/A. Figure 2.6 shows the plot of TRY/USD (USD per TRY) and USD/TRY (TRY per USD) exchange rate returns from January 2000 to December 2015. The distribution of TRY/USD is negatively skewed which suggests that there exist some strong negative values, namely, depreciation of Turkish Lira against the American Dollar. As USD is a developed country's currency, it is expected that TRY stands to lose against USD. On the other hand, the distribution of USD/TRY is positively skewed [35, 37, 42–45].

2.1.5. Descriptive statistics of daily log-returns of the observed currencies

In this part of the study, the descriptive statistics of the chosen exchange rates are demonstrated. USD/EUR, TRY/USD and TRY/EUR are preferred as USD/EUR is significant for everyone on the World and TRY/USD and TRY/EUR are significant currencies for Turkey. Descriptive statistics of daily returns are reported in Table 2.1 in order to specify the distributional properties of daily returns of the exchange rate series.

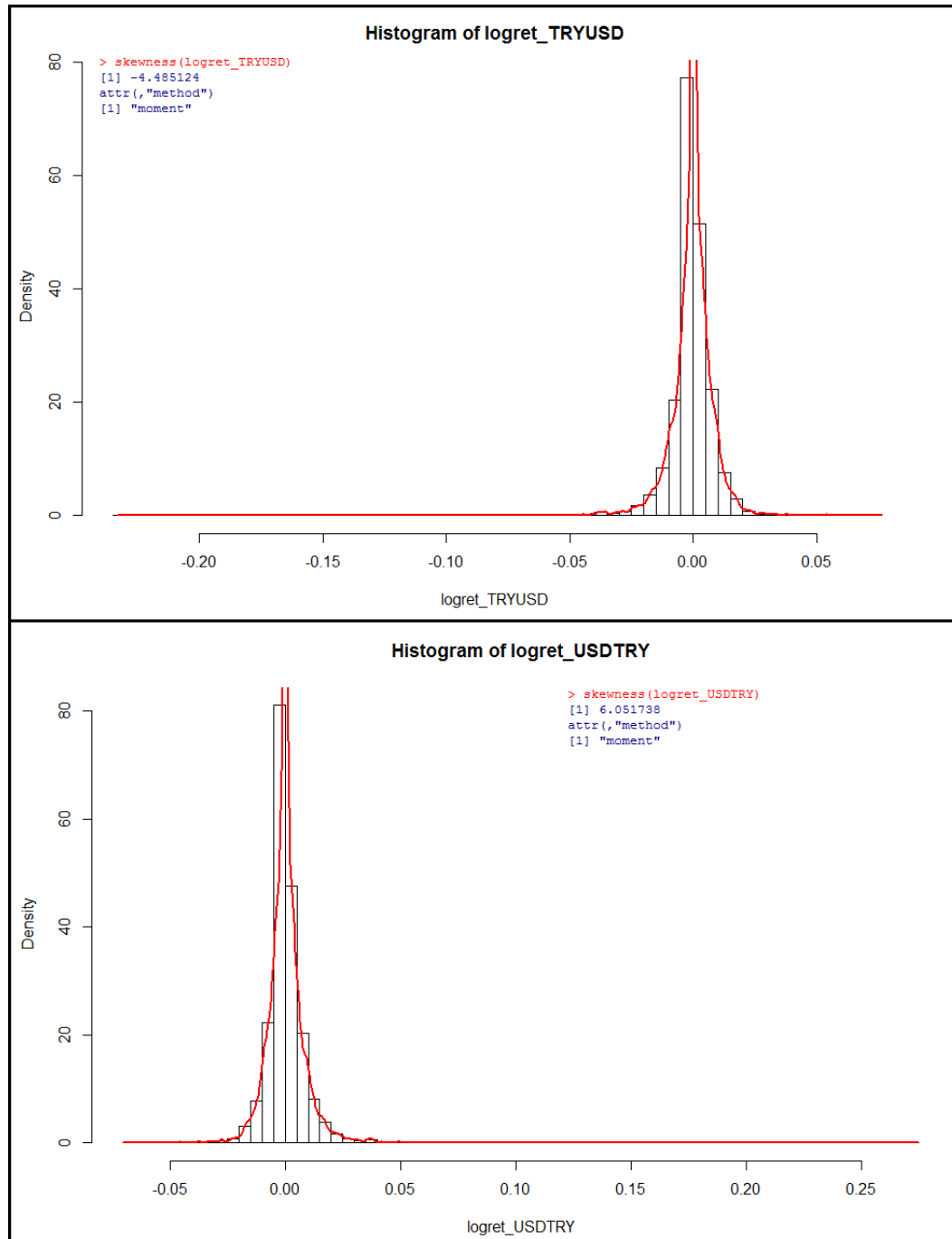


Figure 2.6. Density graphs of TRY/USD (negatively skewed) & USD/TRY (positively skewed).

Table 2.1. Descriptive statistics of daily log-returns for the period from 01/01/2000 to 31/12/2015.

Currency	Mean	Min.	Max.	Std.D.	Skew.	Kurt.
TRY/USD	0.000355	-0.2311	0.0740	0.00906	-4.485	103.85
TRY/EUR	-0.00037	-0.2369	0.1457	0.01003	-3.661	97.99
USD/EUR	-0.000015	-0.1596	0.1433	0.00716	-0.609	105.84

Considering the table, the mean values indicate that our assumption - the mean of logarithmic returns for each currency is zero - is realistic. Regarding Table 2.1, it can be said that skewness and excess kurtosis are clearly observed for the daily returns of all currencies which indicates the departure from normality. Likewise, tails of the log-return distributions are heavier than those of the normal distribution as it is highlighted explicitly in Figures 2.7 and 2.8. Figure 2.7 indicates that the return distribution of exchange rates tends to exhibit a sharper peak and heavier tails relative to the normal distribution. Namely, by using the Normal model, it is assumed that the probability of an extreme event is considerably lower than it is in fact and this results in an underestimation of risk on moving further into the tails. Besides, if the Normal model fits the data, the Q-Q plot should have a linear form. The more linear the plot, the more appropriate the model in terms of goodness of fit yet it is seen that the empirical distribution exhibits a curve to the top at the right end and to the bottom at the left end.

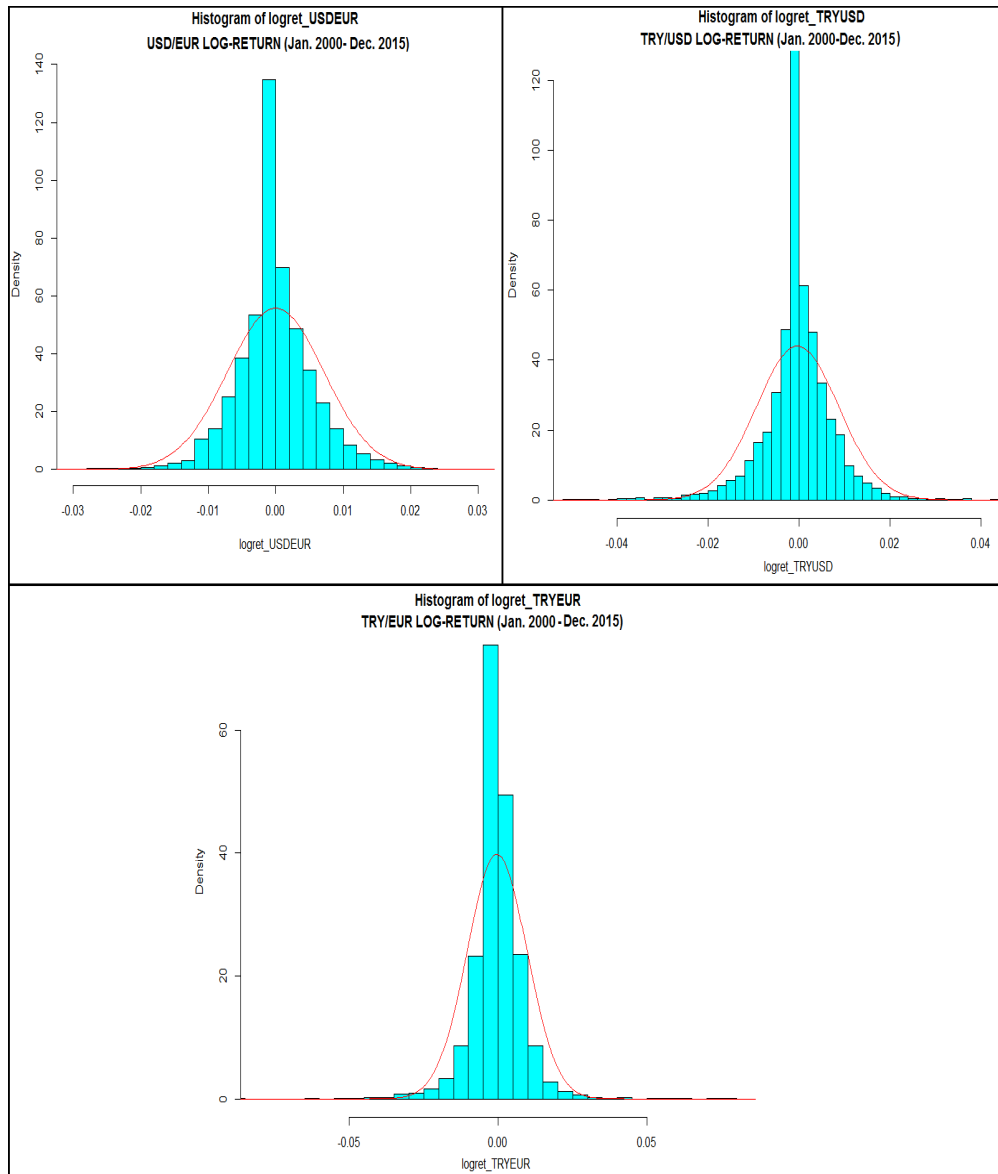


Figure 2.7. Histogram of log-returns (USD/EUR - TRY/USD - TRY/EUR).

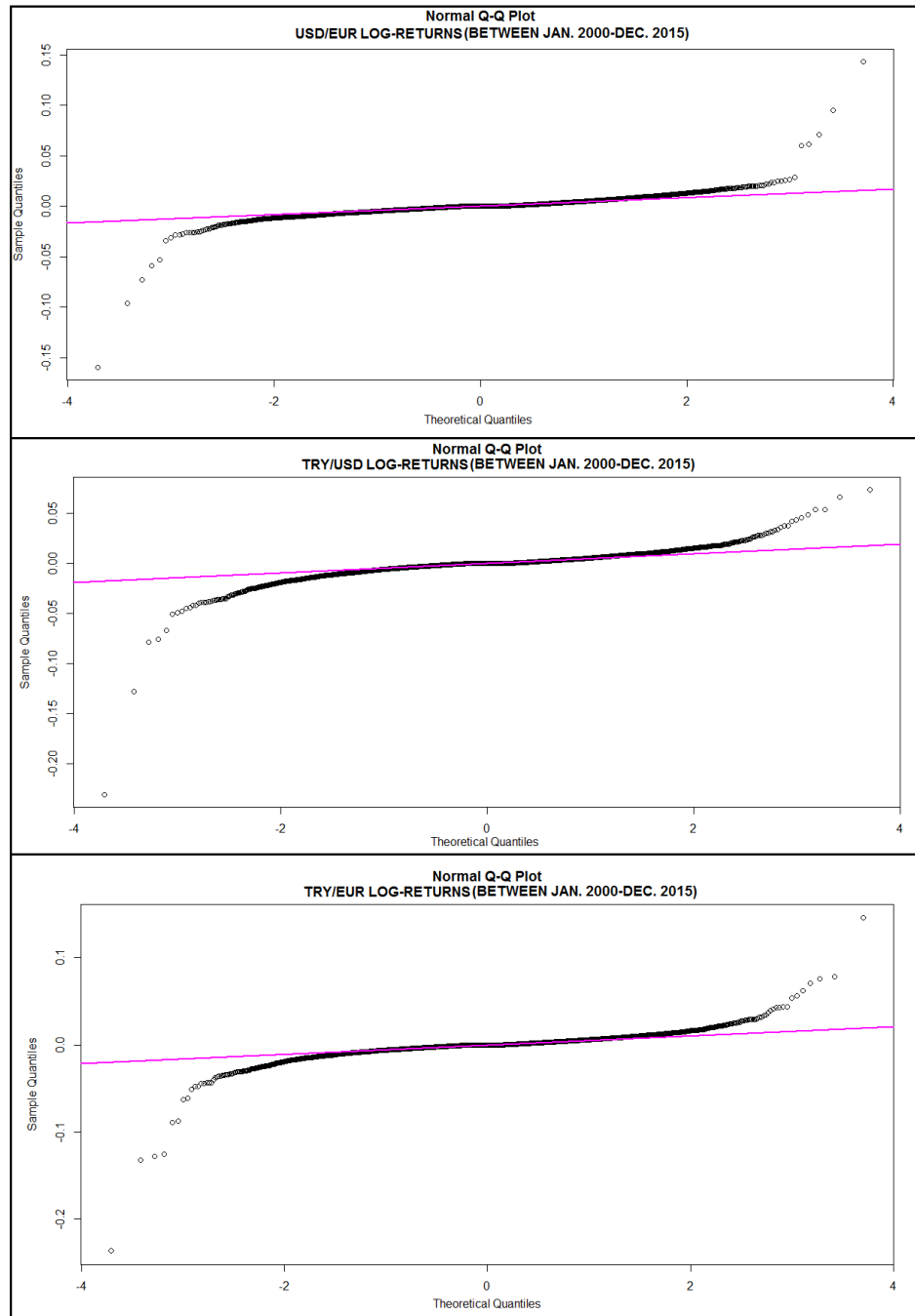


Figure 2.8. Q-Q plots against normal distribution for logarithmic returns of exchange rates ($\text{TRY/USD} - \text{TRY/EUR} - \text{USD/EUR}$).

3. RISK MEASURES

Although financial risk is generally defined as danger of loss, theory describes it as the dispersion of unexpected outcomes due to the movements in financial variables. Risk evokes notions of uncertainty, randomness and probability. The random outcomes might be good (i.e. there is a gain) or bad (i.e. we might suffer a financial loss) and in order to protect ourselves against “bad” events, it is preferred to focus on the risks associated with them [8,46]. The risk of financial assets can be quantified with different risk measures. One of the risk measures is the volatility - the standard deviation of returns. This is the easiest risk measure to quantify individual assets or portfolios’ risk. It is sufficient as a risk measure only when financial returns are normally distributed since all statistical properties of normal distribution are captured by the mean and variance. However, the normality assumption for returns is violated for most of the financial assets. Hence, the use of volatility as a risk measure can lead to misleading conclusions. There are more reliable risk measures such as Value at Risk and Expected Shortfall [4,5].

3.1. Value at Risk (VaR)

Value at Risk appeared as a risk measure in 1993, after its endorsement by the Group of Thirty (G-30) which is a private, nonprofit association, consisting of senior representatives of the private and public sector and of academia. However, the methodology behind VaR is not new. Value at Risk is a statistical measure of downside risk based on current positions. It is an estimate of the worst loss of a financial position over a target horizon such that there is a low, prespecified probability that the actual loss will be larger. When Value at Risk’s theoretical properties, issues in implementation and ease of backtesting are considered, the reason why it has been a widely accepted risk measure since 1990s becomes clear. The measure can be used by financial institutions to assess their risk or by a regular committee to set margin requirements. In either case, VaR is used to ensure that the financial institutions can still be in business after a catastrophic event [4,33,46–48].

Value at Risk is a quantile of the loss distribution. The potential loss is represented by a single number, its value at some confidence level which indicates the likelihood that an outcome- not worse than VaR- is obtained [8,49]. In order to define Value at Risk more precisely, V is assumed as the value of a given portfolio and V_0 as its initial value. $V - V_0$ represents loss whenever $V < V_0$. If $1 - \alpha$ is the desired confidence level (a defined percentage of potential scenarios where extremely adverse scenarios are excluded, i.e. 99%), then $VaR_{(1-\alpha)}$ is the amount such that [50]

$$P(Loss > VaR_{(1-\alpha)}) = \alpha \quad (3.1)$$

Figure 3.1 demonstrates how Value at Risk is defined. The plot on the left shows the entire density of profit and loss function and the plot on the right side zooms in on the left tail, where the shaded areas identify the 1% and 5% probabilities [26].

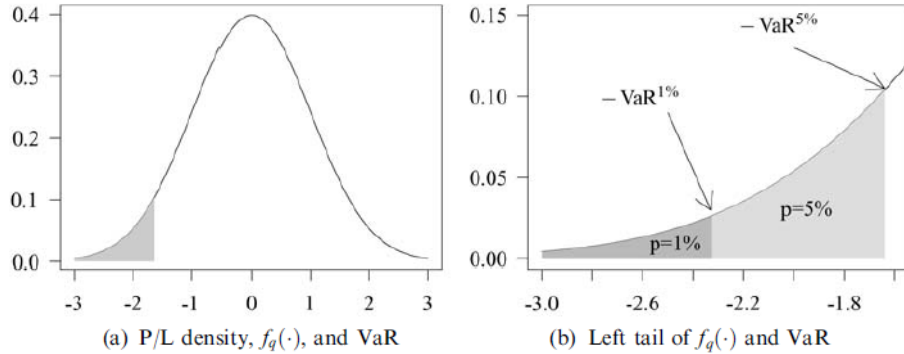


Figure 3.1. Demonstration of VaR.

Regarding the calculation of Value at Risk, it can be said that there are three main steps. First, it is necessary to specify the probability that losses exceed the VaR. The choice of probability level is mainly determined by how the user of risk management system wishes to interpret the VaR number yet the most common probability levels are between 1% and 5%. In this study, 5%, 1%, 0.5%, 0.1% are used as probability levels. Second step is the decision of holding period. One day is usually chosen as the holding period but this can be more or less depending on particular circumstances. The longer the holding period, the larger the VaR. In this study, the holding period is

chosen as one day. The final step is the identification of the probability distribution of returns. This is the most difficult and important aspect of risk modeling. In this study, the return distribution is estimated by using past observations and different statistical models in order to choose the one that best fits the return distribution [26].

Value at Risk has some drawbacks. Its estimates can be too imprecise because of the errors that arise from the model risk (i.e. the risk of errors arising from the models being based on incorrect assumptions) or implementation risk (i.e. the risk of errors arising from the way in which systems are implemented). For instance, if VaR is computed under the assumption that returns are light-tailed (i.e. normal) when in fact, they are heavy-tailed (i.e. Student's t distribution with ν degrees of freedom), the risk may be seriously underestimated for high confidence levels. Besides, VaR measures only percentiles of profit-loss distributions and disregards any loss beyond the value at risk level, namely, tail risk. Provided that a tail event occurs, it is expected to lose more than the VaR yet it gives no indication of how much this loss might be [8,47,51].

The notion of risk is hard to conceptualize without a clear idea of what is meant by a measure of risk. To clarify these issues, Artzner *et al.* (1999) postulated a set of axioms, the axioms of coherency. Assuming that the value of a portfolio is V and $\rho(\cdot)$ is a measure of risk over a chosen horizon, the risk measure is said to be coherent if it satisfies the following properties:

- Monotonicity: If $V_1 < V_2$, then $\rho(V_1) > \rho(V_2)$. If portfolio 1 has lower return than portfolio 2, then its risk is greater.
- Positive homogeneity: $\rho(hV_1) = h\rho(V_1)$. Increasing the size of the portfolio by h should scale its risk by the same factor.
- Translational invariance: $\rho(k + V_1) = \rho(V_1) - k$. Adding cash in the amount of k to a portfolio should reduce its risk by k .
- Subadditivity: $\rho(V_1 + V_2) \leq \rho(V_1) + \rho(V_2)$. Merging portfolios can not increase risk.

Artzner *et al.* (1999) showed that the quantile-based value at risk can not satisfy the subadditivity property so it is not coherent. This property is an important property of incremental risk. That is, the sum of the incremental risks of the positions in a portfolio equals the total risk of the portfolio. However, nonsubadditivity suggests the ridiculous conclusion that putting all the eggs into one basket might be a good risk management practice. To remedy these shortcomings inherent in VaR that are mentioned above, Artzner *et al.* (1999) have proposed the use of the Conditional Value at Risk as a risk measure [8, 46, 47, 52].

Although Value at Risk has some shortcomings, it has a number of significant attractions over traditional risk measures. The reason why VaR is useful is that it is able to collect a great deal of information into a single number. It enables risk managers to aggregate the risk of sub-positions into an overall measure of portfolio risk and it takes account of the ways in which different risk factors correlate with each other. It is expressed in the simplest and most easily understood unit of measure, namely, ‘lost money’. Besides, Value at Risk provides a common consistent measure of risk across different positions and risk factors. It is holistic since it takes account of all driving risk factors, whereas many traditional approaches either only consider risk factors one at a time or resort to simplifications to collapse multiple risk factors into one. Moreover, VaR is probabilistic and gives a risk manager useful information on the probabilities associated with specified loss amounts [8, 53].

3.2. Conditional Value at Risk (Expected Shortfall)

Although Value at Risk has become a standard risk measure for financial risk management due to its conceptual simplicity and ease of computation, it has been charged as having some conceptual problems. For instance, VaR uses percentiles of the profit-loss distribution and disregards any loss beyond VaR level. Besides, as explained in the previous section, it is not subadditive. In order to remedy these problems inherent in VaR, an alternative risk measure, Conditional Value at Risk (CVaR), is introduced by Artzner *et al.* (1999). CVaR, which is also referred as Expected Shortfall, provides an estimate of loss severity in the tail part of return

distribution. In fact, there is a close correspondence between CVaR and VaR. With the same confidence level, CVaR considers loss beyond the VaR level and VaR is a lower bound for CVaR [54–57].

CVaR is defined as the weighted average of losses exceeding VaR for discrete distributions. It is defined mathematically in Equation 3.2.

$$CVaR_{(1-\alpha)} = E[L|L \geq VaR_{(1-\alpha)}] \quad (3.2)$$

CVaR is defined as the conditional expected loss under the condition that it exceeds VaR for continuous distributions. The fact that an expectation is taken means that CVaR opposite to VaR considers the distribution of the far tail. A mathematical expectation for continuous distribution is defined by Equation 3.3.

$$E[X] = \int_{-\infty}^{\infty} xf(x)dx. \quad (3.3)$$

However, in the case of CVaR, the expectation is not taken from $-\infty$ to ∞ , but from $-\infty$ to $-VaR(1 - \alpha)$. The area under $f_q(\cdot)$ in the interval $[-\infty, -VaR(1 - \alpha)]$ is less than one, implying that $f_q(\cdot)$ is not a proper density function. By defining a new density, $f_{VaR}(\cdot)$, which is obtained by scaling $f_q(\cdot)$ up, this situation can be overcome as can be seen in Figure 3.2 [26].

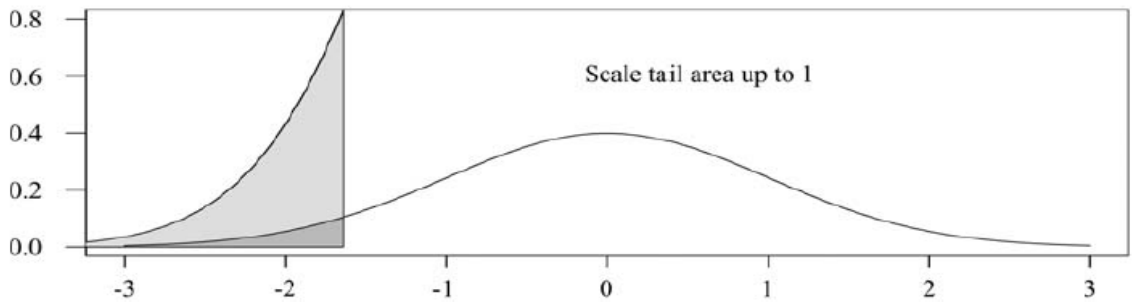


Figure 3.2. Graphical representation of the areas under the density functions $f_q(\cdot)$ and $f_{VaR}(\cdot)$.

The tail density, $f_{VaR}(\cdot)$, is shown in Equation 3.4.

$$1 = \int_{-\infty}^{-VaR(1-\alpha)} f_{VaR}(x)dx = \frac{1}{1-\alpha} \int_{-\infty}^{-VaR(1-\alpha)} f_q(x)dx. \quad (3.4)$$

CVaR is the negative expected value of losses over the tail density $f_{VaR}(\cdot)$ as shown in Equation 3.5 [26].

$$CVaR = - \int_{-\infty}^{-VaR(1-\alpha)} x f_{VaR}(x)dx. \quad (3.5)$$

The main drawback of VaR is that it is only a statement about one particular point (quantile) on the distribution of losses, and does not say anything about the magnitude of large losses when they occur. CVaR, as a risk measure, fills this gap since it is defined as the probability weighted average of the possible losses conditional on the loss being equal to or exceeding the specified VaR. In other words, it corresponds to the average of the $(1 - \alpha)$ largest outcomes of the loss distribution. CVaR attempts to answer the question, "If the $(1 - \alpha)\%$ worst scenario actually happens, how much can I expect to lose?" [58]. A loss distribution with VaR and CVaR is illustrated in Figure 3.3. As seen in the plot, CVaR takes into account the extremely large losses that may occur, with low probability in the tail of return distribution [59].

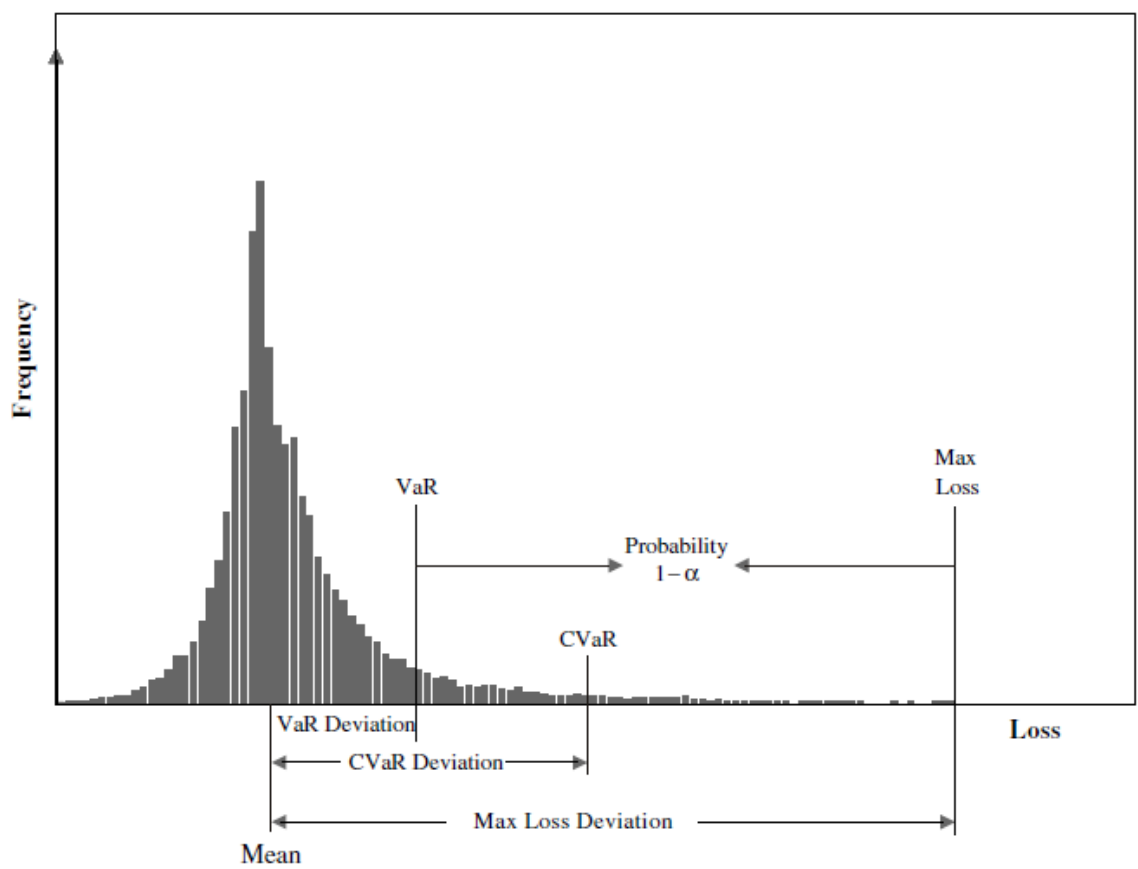


Figure 3.3. Graphical representation of VaR and CVaR.

4. INDEPENDENT AND IDENTICALLY DISTRIBUTED (I.I.D) LOG-RETURN MODELS

Modeling volatility, which is the standard deviation of change in the value of a financial instrument, is important since it is considered as a measure of risk and investors want a premium for investing in risky assets. However, volatility modeling is a difficult process because volatility is not directly observable - it is, unlike prices, a latent variable. Thus, it must be inferred by analyzing how much market prices move. The major challenge in implementing risk analysis is the specification of the probability distribution of the returns which is used in the parameter estimation process. This is because financial returns follow a complicated and ever-changing probability distribution function [26, 46, 60].

In this study, logarithmic returns are used, rather than simple returns, to estimate model parameters and the next day's value at risk. The primary reason why log-return is preferred in this empirical work is that it has some convenient statistical properties that simplify the analysis. Log-returns easily allow extensions into multiple periods, namely, they are additive over time. In contrast to simple returns, it is much easier to derive time series properties of sums than of products. As shown in Equation (4.1), multi-period log-return can be calculated by summing up intermediate logarithmic returns. In this equation, P_0 is the initial price of an asset and R_1 is the random return at time 1 [26, 33, 46, 60].

$$\begin{aligned}
 P_1 &= P_0(1 + R_1) \\
 P_2 &= P_0(1 + R_1)(1 + R_2) \\
 P_N &= P_0 \prod_{i=1}^n (1 + R_i) \\
 \sum_{i=1}^N \ln(1 + R_i) &= \ln(1 + R_1) + \ln(1 + R_2) + \dots + \ln(1 + R_N) = \ln(P_N) - \ln(P_0) \quad (4.1)
 \end{aligned}$$

Besides, using a logarithmic return may be more consistent for some series, such as exchange rates. Exchange rates can be defined in two different base currencies. Using $P(\$/GBP)$ as the dollar price of the British pound, the random variable of interest is $x = \ln(P_t/P_{t-1})$. Taking the viewpoint of a British investor, who measures asset values in pounds, the variable is $y = \ln((1/P_t)/(1/P_{t-1})) = -\ln(P_t/P_{t-1}) = -x$. Thus, if y is symmetrically distributed, x has the same distribution. Therefore, the two distributions are consistent with each other, which is not the case with discrete returns. Let's assume that a German investor wants to measure returns in euros. This can be derived from dollar-based data as shown in Equation (4.3). The euro-based return is equal to the difference between the dollar-based return on the pound and the dollar-based return on the euro. Thus, the euro-based distribution can be derived from the combination of two dollar-based distributions [46].

$$\ln(P(EUR/GBP)) = \ln(P(EUR/\$)) + \ln(P(\$/GBP)) \quad (4.2)$$

$$= \ln(P(\$/GBP)) - \ln(P(\$/EUR)) \quad (4.3)$$

Moreover, logarithmic return is often economically more meaningful than the simple return, since it ensures that the price of an asset can never become negative regardless of how negative the returns might be. On the other hand, with simple returns, a very low return or a high loss implies that the asset's value can become negative and a negative asset price does not make economic sense [8].

In this chapter, volatility estimation relies on the assumption that returns are independent and identically distributed. The identically distribution assumption ensures that the mean and the variance of returns do not vary over time. The independence assumption ensures that speculative price changes are unrelated to each other at any point of time. In this study, return models are divided into two essential groups, one of which is i.i.d logarithmic return models. Different return models will be explained thoroughly in the next sections.

4.1. Normal Model

The normal distribution which is usually the first choice when modeling asset returns, plays a special role in statistics as it is easy to handle. Besides, it is closed under addition, meaning that a weighted sum of jointly normal variables is itself normal. Empirically, the normal distribution provides a rough, first order approximation to the distribution of many random variables, such as rates of changes in currency prices, stock prices or bonds. All of these are characterized by many occurrences of small moves and fewer occurrences of large moves. Therefore, the bell-shaped normal distribution is preferred for many applications. Its graph, called the normal curve, can be seen in Figure 4.1. The highest point represents the most probable event (mean) and all possible occurrences are distributed around the mean. The normal distribution is characterized by its first two moments, the mean (μ) and the variance (σ^2). The first parameter, μ , represents the location and dispersion is represented by the variance, σ^2 . The normal distribution is symmetric around its mean [4, 46, 61].

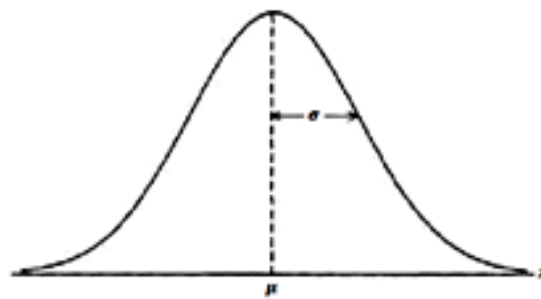


Figure 4.1. The Normal Curve.

Normality assumption of financial returns is widely used in theoretical finance. In the 1900s, Bachelier introduced mean and variance using the assumption of Normal distribution and the bell curve in his model. About half a century later, in 1952, the normality assumption was made by Harry Markowitz, who chose to use variance as a risk measure. In the beginning of 1970s, Black and Scholes (1973) and Merton (1973) derived the price and the hedging strategy of an option by assuming a Brownian motion for the price of the underlying asset implying a Gaussian distribution for returns. More

recently, with the changes in the banking and financial regulation on risks and capital, value-at-risk models, which were developed and implemented by financial institutions, also relied intensively on the Gaussian distribution [62].

Regarding the risk quantification, the first step is to define the risk factor. In this study, risk factor is the movement in exchange rates. In order to estimate next day's risk, logarithmic returns are used. Then, an appropriate model among distributions is selected. The normal model is chosen as the first alternative to estimate the volatility since it is easy to implement. In many practical cases, the methods developed using normal theory work quite well even when the real distribution is not normal. Besides, it is a good starting point to obtain some results (i.e. estimation of parameters by using exchange rate data).

4.1.1. Parameter estimation of the Normal model

Parameter estimation is the procedure of finding those parameter values that best fits to the data. In finance, it is hard to have data of the whole population. Provided that we are able to work with the population data, as we have taken all possible outcomes, no estimation will be required. However, in this study, we have to work with a sample taken from a population and parameter estimation is required as we have no prior knowledge of the magnitude of population parameters. Besides, this approach can also be used to check whether the parametric distribution considered is appropriate for the data [4, 63].

In this section, parameter estimation is done by using maximum likelihood estimation. The idea of this method is to estimate model parameters that best explain the data in the sense of yielding the largest likelihood. Let $x_1, \dots, x_n$ be an independent and identically distributed (i.i.d) sample with probability density function $f(x_i; \theta)$ where θ is a $(k \times 1)$ vector of parameters that characterize $f(x_i; \theta)$. The joint density of the sample is, due to the independence assumption, equal to the product of the marginal

densities.

$$f(x_1, \dots, x_n; \theta_1, \dots, \theta_k) = f(x_1; \theta_1, \dots, \theta_k) \dots f(x_n; \theta_1, \dots, \theta_k) = \prod_{i=1}^n f(x_i; \theta_1, \dots, \theta_k) \quad (4.4)$$

The joint density is an n dimensional function of the data $x_1, \dots, x_n$ given the parameter vector θ . The likelihood function is defined as the joint density treated as a function of the parameters $\theta_1, \theta_2, \dots, \theta_k$ [64].

$$L(\theta_1, \theta_2, \dots, \theta_k | x_1, \dots, x_n) = f(x_1, \dots, x_n; \theta_1, \theta_2, \dots, \theta_k) = \prod_{i=1}^n f(x_i; \theta_1, \theta_2, \dots, \theta_k) \quad (4.5)$$

Maximum likelihood estimation is the procedure of finding for a given sample the value of one or more parameters that maximize the likelihood function. In order to find the maximum of a function, it is necessary to take its derivative and solve for the parameter being estimated. For many applications, it is more convenient to work with the natural logarithm of the likelihood function, called the log-likelihood, than to work with the likelihood function. Because, the log-likelihood is a series of sums whereas the likelihood is a series of products and sums are easier to deal with than products [65,66].

In this thesis, one-period logarithmic returns are assumed to be normally distributed with zero mean since the mean return of daily data is usually very small. Besides, the assumption of mean return being zero is reasonable over short horizons such as a day, since the volatility of risk factors tends to dominate their mean over short horizons. The only parameter that should be estimated is the standard deviation (σ) for the Normal model. It is estimated by MLE using the return data in the sliding window $(r_{t-1}, r_{t-2}, \dots, r_{t-ndata})$. As the Normal model is a static method, the model parameters stay constant within the sample period. The only source of change in distribution parameters is the rolling window forecasting approach.

Initially, the logarithm of the likelihood function is obtained in order to find the maximum likelihood estimator of the parameter, $\hat{\sigma}$. Equation (4.6) shows the likelihood

function of Normal distribution [39].

$$f(x_1, \dots, x_n | \sigma) = \prod_{i=1}^n \frac{1}{\sigma \sqrt{2\pi}} \exp \left(-\frac{(x_i - \mu)^2}{2\sigma^2} \right) \quad (4.6)$$

The logarithm of the likelihood equation is shown in (4.7).

$$\ln f(x_1, \dots, x_n | \sigma) = -\frac{1}{2}n \ln 2\pi - n \ln \sigma - \frac{\sum_{i=1}^n (x_i - \mu)^2}{2\sigma^2} \quad (4.7)$$

To find the maximum likelihood estimator of the parameter, $\hat{\sigma}$, log-likelihood function should be differentiated with respect to σ , and then set to zero.

$$\frac{\partial \ln f}{\partial \sigma} = -\frac{n}{\sigma} + \frac{\sum_{i=1}^n (x_i - \mu)^2}{\sigma^3} = 0 \quad (4.8)$$

The maximum likelihood estimator, $\hat{\sigma}$ is shown by Equation (4.9) .

$$\hat{\sigma} = \sqrt{\frac{\sum_{i=1}^n (x_i - \hat{\mu})^2}{n}} \quad (4.9)$$

After the volatility estimation process, logarithmic returns can be simulated for the fitted Normal model. Lastly, simulated prices for the periods between 01.01.2008 and 31.12.2008 are obtained. In Figure 4.2, it is shown that the trend is almost the same. Blue line shows the simulated prices and the red line shows the real prices.

4.1.2. Risk quantification for the Normal model

In the previous section, the standard deviation is estimated using MLE. In this section, risk quantification is carried out by using the estimated parameter as input. Value at Risk associated with normally distributed logarithmic returns is shown in Equation (4.10). P is the current value of an asset and $qnorm(\alpha)$ is the α quantile of the standard normal distribution. As it is assumed that log-returns are normally

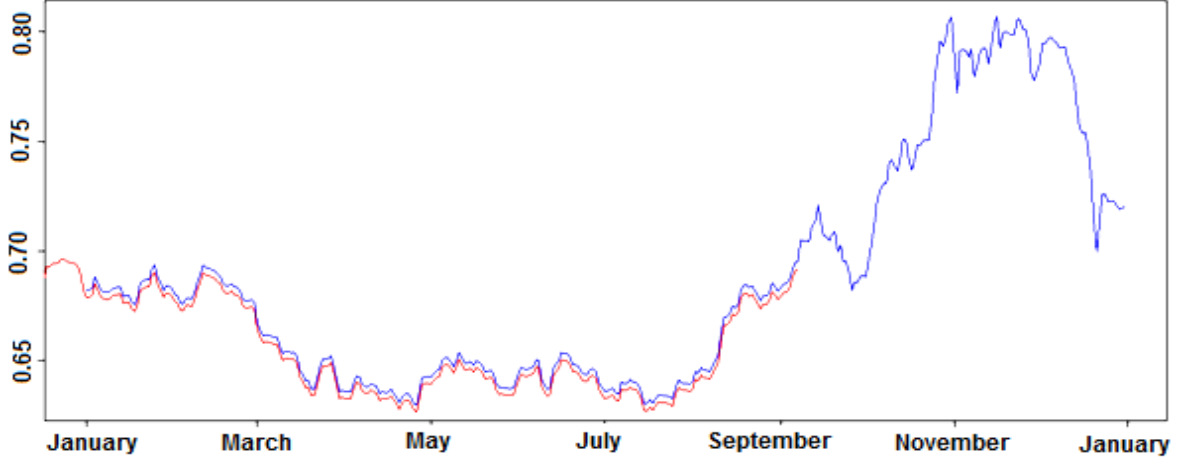


Figure 4.2. Time plot of simulated prices for the fitted Normal Model and real prices.

distributed with zero mean, μ is equal to zero.

$$VaR_{(1-\alpha)} = P(1 - \exp(\mu + \sigma \cdot qnorm(\alpha))) \quad (4.10)$$

Table 4.1 shows estimated volatilities and VaR values for the period between 11.10.2008 and 27.10.2008. Although the maximum return is on 23rd of October, there is a slight increase in the volatility and VaR values. This is owing to the fact that conditional volatility is constant for the Normal Model.

Regarding long histories of price/return data available for many financial instruments, it is easy to see that the assumption - logarithmic returns are normally distributed- is inadequate. The distribution of logarithmic returns is leptokurtic. More precisely, it has a higher peak and fatter tails than normal distribution. This phenomenon was first discovered by Mandelbrot in 1963. Since then, fat tails and excess kurtosis have been discovered in many markets including the foreign exchange market [4, 67]. Exchange rate data used in this thesis, are also fat-tailed and have a higher peak than the normal distribution. The histogram of daily logarithmic returns, seen in Figure 4.3, is obtained by using USD/AUD exchange rate data including the periods between 01.01.2000 and 19.03.2013. The density of the normal distribution with

Table 4.1. Estimated volatilities and VaRs for Normal Model.

Date	Price	log-return	vol. est.	VaR ($\alpha = 0.005$)	VaR ($\alpha = 0.001$)
2008-10-11	0.7385	0.0093872	0.00355783	0.009122487	0.01093429
2008-10-12	0.7459	0.0099704	0.00357516	0.009166720	0.01098726
2008-10-13	0.7453	-0.00080472	0.0035945	0.009216083	0.01104637
2008-10-14	0.7363	-0.0121492	0.003593541	0.009213640	0.01104344
2008-10-15	0.7318	-0.0061304	0.00362116	0.009284134	0.01112786
2008-10-16	0.7360	0.00572287	0.00362797	0.009301506	0.01114866
2008-10-17	0.7435	0.01013865	0.003633652	0.009316002	0.01116602
2008-10-18	0.7431	-0.000538141	0.003653415	0.009366433	0.01122641
2008-10-19	0.7452	0.00282201	0.003652671	0.009364534	0.01122414
2008-10-20	0.7452	0	0.00365012	0.009358012	0.01121633
2008-10-21	0.7453	0.000134183	0.003650108	0.009357995	0.01121631
2008-10-22	0.7548	0.01266599	0.003650116	0.009358014	0.01121633
2008-10-23	0.7741	0.02524825	0.003678599	0.009430694	0.01130336
2008-10-24	0.7800	0.00759286	0.003794820	0.009727191	0.01165838
2008-10-25	0.7901	0.0128656	0.003804533	0.009751966	0.01168805
2008-10-26	0.7872	-0.00367717	0.003828356	0.009812729	0.01176080
2008-10-27	0.7919	0.00595278	0.00382725	0.009809908	0.01175743

estimated σ^2 is also indicated in the plot. The leptokurtic pattern is apparent. However, when the time interval over which the returns are computed increases, such as monthly returns, leptokurtosis tends to vanish and the empirical distribution gets close to normal which can be seen in Figure 4.4.

Although the Normal distribution is the first choice due to its simplicity, there are some weaknesses of the model. Conditional volatility is constant for the Normal model and the constant volatility assumption is not realistic for financial data. The most serious problem with the Normal model is that its tails disappear faster than what is empirically observed in financial data. Namely, compared to the empirical return distribution, the number of rare events occurred in the Normal model is not enough. The empirical distribution of returns has more probability mass in the tails. As a consequence, the implication is that a risk measure, derived from the normal assumption, generally underestimates riskiness of a position in a financial instrument. The fatter the tails of the asset return distribution, the larger the discrepancy to the

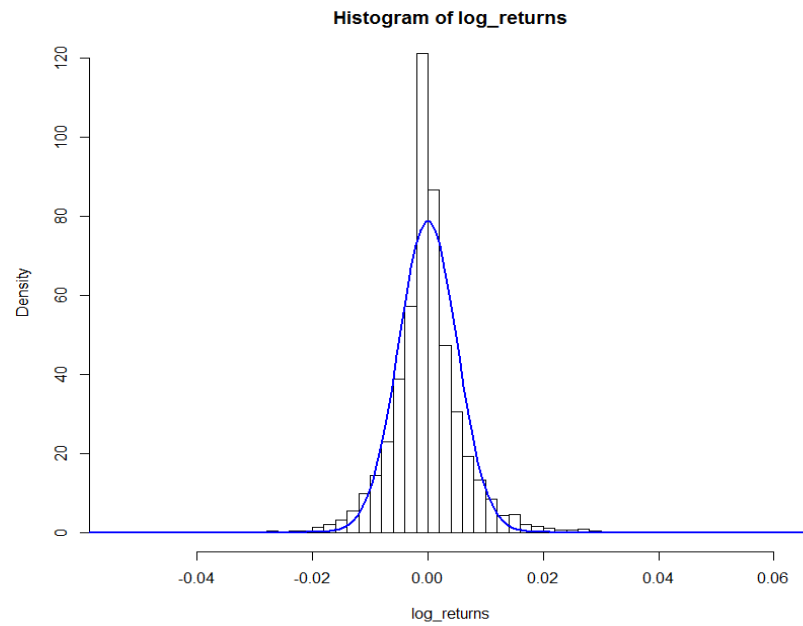


Figure 4.3. Histogram of daily log-returns of exchange rate data (USD/AUD) and the density of normal distribution.

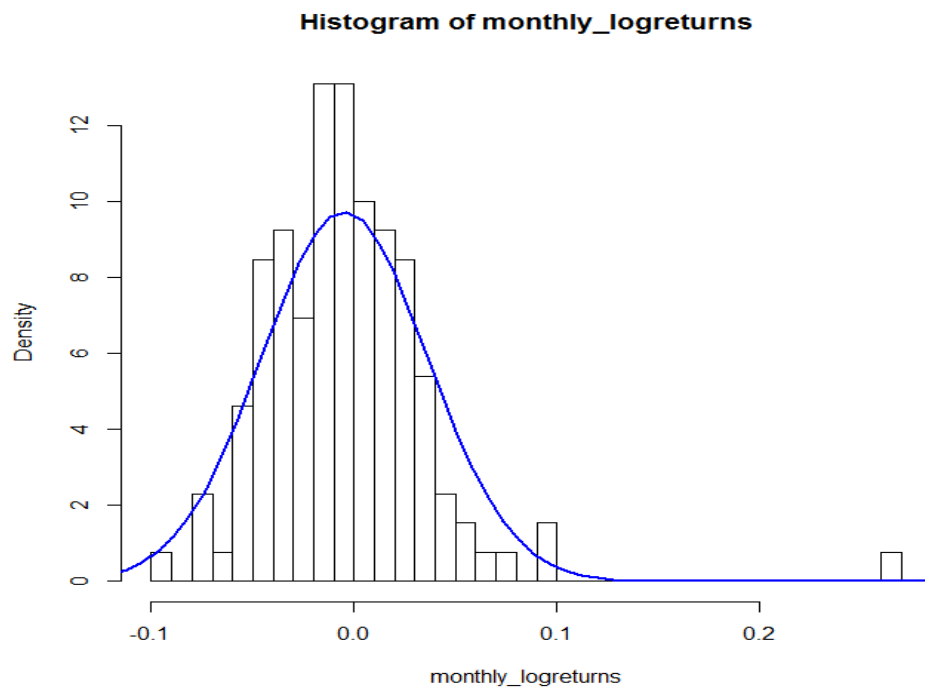


Figure 4.4. Histogram of monthly log-returns of exchange rate data (USD/AUD) and the density of normal distribution.

normal distribution and the larger the errors made in VaR estimation. Therefore, in the next section, an alternative model, Student's t distribution is used with the hope of decreasing the modeling error since this distribution is fatter tailed than the Normal distribution. It is expected that Student's t distribution fits the data better than the Normal model [4, 23, 68].

4.2. Student's t Model

Regarding the accuracy of value at risk estimates, it is important to make realistic assumptions concerning the actual form of the return distribution since VaR is estimated by using the probability distribution of returns. Although, for simplicity and convenience, asset returns are often assumed to be normally distributed, the return distribution on many assets exhibits fatter tails than Normal. Therefore, the normal assumption results in an underestimation of VaR. There is a need for simple methodologies to estimate VaR which can capture tail fatness apparent in return distributions. It seems natural to try a distribution that has fatter tails, namely, a distribution that generates more realizations in the tails than Normal distribution. Student's t distribution can also deal with the phenomenon, leptokurtosis [59, 69]. Consequently, Student's t distribution is used in this part of the thesis expecting better VaR estimates.

Student's t distribution is the most commonly used alternative of Normal distribution when attempting to address heavy tails of assets' return distribution. It is symmetric and bell-shaped like the Normal distribution. However, it is more peaked around the center and has fatter tails. As the number of degrees of freedom increases, Student's t distribution approaches the Standard Normal distribution which is the special case of Student's t distribution when the degrees of freedom tend to infinity. The plots can be seen in Figure 4.5.

Student's t distribution is characterized by three parameters; the location parameter (μ), the scale parameter ($\sigma > 0$) and the degrees of freedom parameter (ν). The degrees of freedom parameter, ν , controls tail thickness. It takes only positive values, and lower values signify heavier tails. In order to capture tail fatness correctly, it is

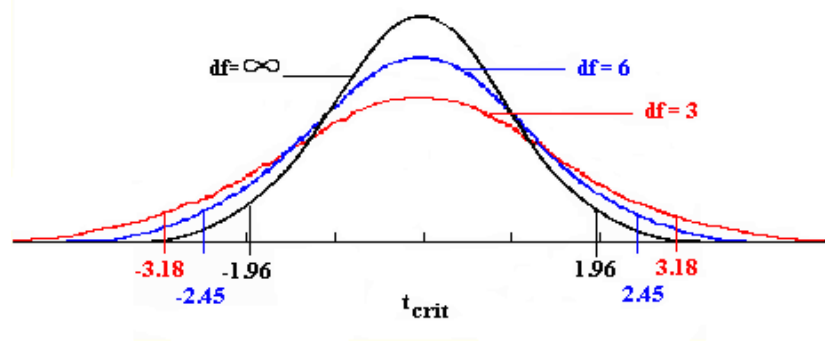


Figure 4.5. Plots of t distribution with different degrees of freedom values.

essential to estimate the degrees of freedom well [55, 59, 69, 70].

In this thesis, due to the fact that the mean return of daily data is usually very small and the volatility of risk factors tends to dominate their mean over short horizons, it is assumed that the mean of one-period logarithmic returns is zero. The density function of that distribution is described by Equation (4.11) where σ and ν are positive [55].

$$f(y|\nu, \sigma) = \frac{\Gamma\left(\frac{\nu+1}{2}\right)}{\sigma\Gamma\left(\frac{\nu}{2}\right)\sqrt{\nu\pi}} \left(1 + \frac{1}{\nu} \left(\frac{y}{\sigma}\right)^2\right)^{-\frac{(\nu+1)}{2}} \quad (4.11)$$

The variance of y is shown by Equation (4.12).

$$\text{var}(y) = \sigma^2 \frac{\nu}{\nu - 2} \quad (4.12)$$

4.2.1. Parameter estimation for Student's t model

Student's t distribution parameters, ν and σ , are estimated by MLE using the return data in the sliding window $(r_{t-1}, r_{t-2}, \dots, r_{t-ndata})$. In this model, returns are assumed to be i.i.d Student's t distributed. Therefore, the model parameters stay constant within the sample period. The only source of change in distribution parameters is the rolling window forecasting approach.

Unfortunately, in contrast to Normal distribution, analytical expressions are not available for maximum likelihood estimates of the parameters. Therefore, optimization has to be done numerically. The likelihood and log-likelihood functions are shown by Equation (4.13) and Equation (4.14) respectively. In this thesis, the statistical software, R, is used to optimize the log-likelihood function.

$$L(\nu, \mu, \sigma, y) = \left(\frac{1}{\sigma}\right)^n \left[\frac{\Gamma\left(\frac{\nu+1}{2}\right)}{\Gamma\left(\frac{\nu}{2}\right) \sqrt{\nu\pi}} \right]^n \sum_{i=1}^n \left[\left(1 + \frac{\left(\frac{y_i - \mu}{\sigma}\right)^2}{\nu}\right) \right]^{-\frac{(\nu+1)}{2}} \quad (4.13)$$

$$l(\nu, \mu, \sigma, y) = -n \ln(\sigma) + n \left[\ln\left(\Gamma\left(\frac{\nu+1}{2}\right)\right) - \frac{1}{2} \ln(\nu\pi) - \ln\left(\Gamma\left(\frac{\nu}{2}\right)\right) \right] - \left(\frac{\nu+1}{2}\right) \sum_{i=1}^n \ln\left(1 + \frac{\left(\frac{y_i - \mu}{\sigma}\right)^2}{\nu}\right) \quad (4.14)$$

4.2.2. Risk quantification for Student's t model

After parameters have been estimated, the next step is to calculate daily Value at Risk. VaR calculation formula for Student's t distribution is shown in Equation (4.15). σ is the scale parameter and $qt(\alpha, \nu)$ is the α -quantile of the t distribution with ν degrees of freedom.

$$VaR_{(1-\alpha)} = P(1 - \exp(\mu + \sigma \cdot qt(\alpha, \nu))) \quad (4.15)$$

In Table 4.2, logarithmic returns, estimated volatilities and VaR values are shown. On the 23rd of October in 2008, the maximum return is gained yet the risk is not increased significantly for the next day as time-varying volatility is not taken into account in Student's t model. Nevertheless, due to the fact that its tails are heavier than the Normal distribution, Student's t distribution estimates a higher VaR value

for the same day compared to the Normal model. (for $\alpha = 0.005$, $VaR_t = 0.01796$ and $VaR_{normal} = 0.00973$)

Table 4.2. Estimated volatilities and VaRs for Student's t model.

Date	Price	log-return	vol. est.	VaR ($\alpha = 0.005$)	VaR ($\alpha = 0.001$)
2008-10-11	0.7385	0.0093872	0.005256041	0.015989691	0.03147134
2008-10-12	0.7459	0.0099704	0.005325530	0.016118073	0.03179310
2008-10-13	0.7453	-0.00080472	0.005400956	0.016249282	0.03213065
2008-10-14	0.7363	-0.0121492	0.005504114	0.016351021	0.03249210
2008-10-15	0.7318	-0.0061304	0.005657561	0.016573970	0.03309837
2008-10-16	0.7360	0.00572287	0.005575094	0.016533813	0.03287072
2008-10-17	0.7435	0.01013865	0.005562707	0.016542418	0.03285530
2008-10-18	0.7431	-0.000538141	0.005659085	0.016697481	0.03325822
2008-10-19	0.7452	0.00282201	0.005786801	0.016806832	0.03366074
2008-10-20	0.7452	0	0.005763153	0.016762715	0.03355963
2008-10-21	0.7453	0.000134183	0.005762073	0.016760625	0.03355454
2008-10-22	0.7548	0.01266599	0.005766067	0.016767840	0.03357176
2008-10-23	0.7741	0.02524825	0.006186704	0.017167382	0.03484885
2008-10-24	0.7800	0.00759286	0.007380251	0.017959531	0.03742239
2008-10-25	0.7901	0.0128656	0.007506485	0.018060624	0.03770262
2008-10-26	0.7872	-0.00367717	0.008366621	0.018399938	0.03886060
2008-10-27	0.7919	0.00595278	0.008407128	0.018414131	0.03890754

Using the estimated parameters of t distribution, logarithmic returns and prices are simulated for the year 2008. Simulation is performed by using the data of 31.12.2005 and 31.12.2007. Figure 4.6 shows the real prices and simulated prices of Student's t Model. Blue line is used for simulated prices and red line is for real prices. It is seen that paths are similar.

Besides, in order to show the difference between the simulated prices of two fitted models, Normal model and Student's t model, Figure 4.7 is shown. In this plot, black line is used for Normal model and orange line is for Student's t model.

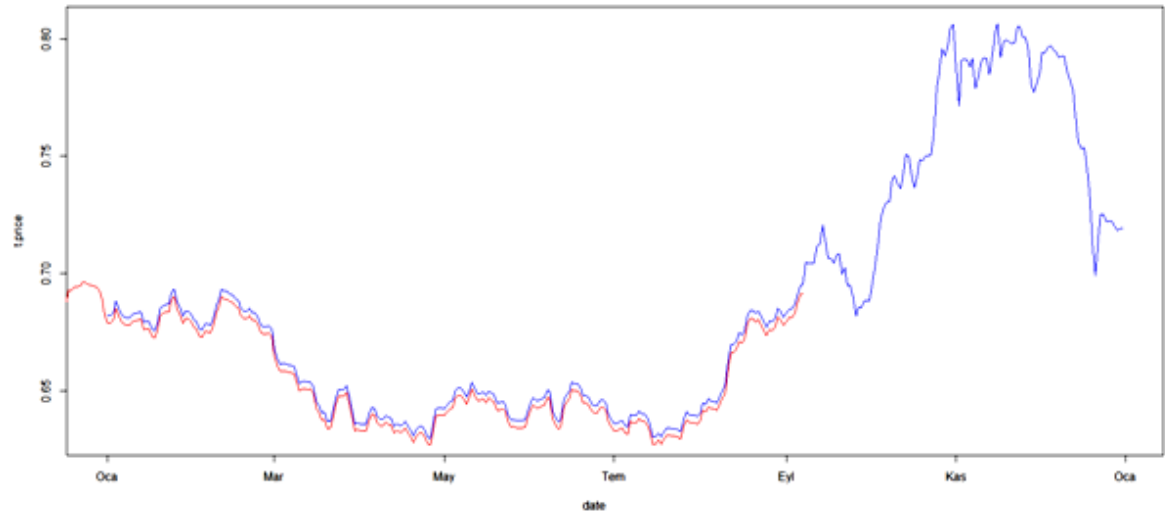


Figure 4.6. Time plot of simulated prices for fitted Student's t Model vs real prices.



Figure 4.7. Time plot of simulated prices for fitted Student's t Model and for fitted Normal Model.

Financial returns exhibit fat tails, namely, extreme outcomes occur more frequently than predicted by the Normal distribution. Owing to the fact that Student's t densities are more peaked around the center and have fatter tails unlike the Normal distribution, this model is chosen with the expectation of better return modeling. However, constant volatility assumption is also valid for Student's t distribution and this assumption is not realistic for financial data. There are fluctuations in volatility over time. This is one of the reasons why Student's t distribution is not an appropriate model for financial returns. Besides, similar to the Normal distribution, Student's t distribution is also symmetric yet time-varying volatility leads to asymmetric tails in the asset return distribution and usually big losses are more likely to occur than big returns. Thus, in the next section Extreme Value Theory is used as the third alternative model [26, 71–73].

4.3. Extreme Value Theory (EVT)

Extreme value theory is a field of statistics that studies the tail behavior of a distribution. The distinguishing feature of extreme value analysis is the objective to quantify the stochastic behavior of a process at unusually large or small levels. In particular, extreme value analysis usually requires estimation of the probability of events that are more extreme than any that have already been observed. Consequently, it promises to be more accurate for applications focusing on the extremes. Hence, extreme value theory is used as an alternative model with the hope of better modeling the return distribution since EVT allows separate analyses of the lower and upper tails of the distribution [26, 33, 74, 75].

The interest in accurately modeling the lower tail of the unknown distribution is deeply rooted in the essence of risk management, which aims to prevent one's own position from a huge unexpected loss. Hence, main concern in employing EVT techniques in finance is related to risk management. The fact that Value at Risk and Expected Shortfall only deal with extreme quantiles of the distribution, disregarding the center and the right tail, suggests the use of extreme value theory as an effective tool for providing reliable estimates for these risk measures. EVT only considers the tail of

the unknown distribution, neglecting its central part. Besides, EVT allows separate analysis of lower and upper tails of the distribution. This is important in many applications, since financial time series are sometimes asymmetric, namely, losses are more likely to occur than returns [60,67]. Extreme Value Theory was introduced to financial applications by Koedijk *et al.* (1990) and Jansen and de Vries (1991). EVT also has a long and successful history in areas such as climatology and hydrology where rare events are frequent, like extreme temperatures or floods [26,61].

Based on the generalized assumption of normality for financial returns, extreme events are expected to occur with an almost negligible probability since the normal density function has tails which assign very small probability to values far from the mean. Thus, for instance, with independent realizations that are observed once a day, a "4-sigma event" should not be expected to occur on average more often than once in 86 years but evidence shows that these events happen far more often [60,67]. Therefore, the Normal distribution is ill-suited for handling extremes. In addition, standard i.i.d modeling uses the entire distribution, where observations in the center of the distribution dominate the estimation process given the scarcity of extreme observations [8,26]. On the other hand, EVT focuses directly on the tails of the distribution and could, therefore, potentially perform better than other approaches in terms of predicting unexpected extreme changes [9].

The main aim of EVT is to draw inferences about rare events. It provides statistical tools to estimate the tails of the return distribution without requiring a prior assumption about the distribution. Such questions, "What are the extreme movements that can be expected in financial markets?", "Have we already seen the largest ones or are we going to experience even larger movements?" or "Are there theoretical processes that can model the type of fat tails that are observed in our empirical analysis?", can be answered within the framework of EVT [76]. The distribution of financial returns is heavy-tailed and asymmetric so neither Normal distribution nor Student's t distribution provide an adequate framework for analyzing extreme returns. Besides, as we care about more extreme probability levels, for instance 0.1 or 0.5 % VAR, more advanced techniques are important. Therefore, in this study, EVT is included as a method to

model extreme price movements.

EVT provides parametric models to capture the extreme tails of a distribution. Mainly there are two methods of applying EVT with their own strengths and weaknesses. In the next subsections, these methods are explained.

4.3.1. Block maxima method (BMM)

The first method of EVT, Block Maxima Method, is based on the extreme value distributions of the Gumbel, Frechet or Weibull distributions and known as the block maxima method (BMM). In the BMM, repeated observations of an n block maximum is needed, namely, the data should be divided into n blocks of size m . Afterwards, maximum value of each block is obtained. Selected values are considered as extreme events that constitute the block maxima. According to the theorem of Fisher and Tippett and Gnedenko, which will be explained below, distribution of the block maxima can be approximated for large enough n by a three-parameter Generalized Extreme Value distribution [60, 77, 78].

Consider X_t , $t = 1, 2, \dots, (nmx)$ is a sequence of i.i.d random variables over a time span of n periods with cumulative distribution function $F(x) = Pr(X_t \leq x)$. The time span is a calendar period such as a month, quarter, half a year or a year and the observed data within this period is of the daily frequency [23, 79]. Denote the n^{th} sample maxima in a sample of m losses by $M_n = \max(X_1, X_2, \dots, X_m)$ for $n \geq 2$. The cdf of M_n is given by Equation (4.16).

$$P(M_n \leq x) = P(X_1 \leq x, \dots, X_n \leq x) = \prod_{t=1}^n P(X_t \leq x) = F^n(x) \quad (4.16)$$

The Fisher and Tippett (1928) and Gnedenko (1943) theorem is the fundamental result in EVT stating that the maximum of a sample of properly normalized I.I.D random variables converges in distribution to one of three possible distributions - Frechet,

Gumbel or Weibull, regardless of the original distribution of the observed data [26].

Theorem 4.1 (Fisher and Tippett (1928); Gnedenko (1943)). *Let X_n be a sequence of i.i.d random variables. If normalizing constants $c_n > 0$ and $d_n \in \mathbb{R}$ exist and are defined as $d_n = n \cdot E(X_1)$ and $c_n = \sqrt{\text{Var}(X_1)}$, then the distribution of standardized maxima, M_n is shown in the equation below.*

$$\lim_{n \rightarrow \infty} P\left(\left(\frac{M_n - d_n}{c_n}\right) \leq x\right) = H(x) \quad (4.17)$$

The limiting distribution of the maxima $H(x)$ belongs to a Generalized Extreme Value (GEV) distribution.

$$H_\xi(x) = \begin{cases} \exp\left(-\left(1 + \xi \cdot \left(\frac{z - \mu}{\sigma}\right)\right)^{-1/\xi}\right), & \text{if } \xi \neq 0 \\ \exp\left(-\exp(-x)\right), & \text{if } \xi = 0 \end{cases} \quad (4.18)$$

where x satisfies the condition $1 + \xi \cdot x > 0$. $\xi = 1/\alpha$ is the shape parameter and α is the tail index.

GEV distribution is a generalized representation of the three distributions, Frechet, Gumbel and Weibull. This distribution is a three-parameter distribution where μ is the location and σ is the scale parameter of the limiting distribution and ξ is the shape parameter which gives an indication of the heaviness of the tail of the limiting distribution. If $\xi > 0$, the GEV becomes the Frechet distribution. This case is particularly useful for financial returns because they are heavy tailed. If $\xi = 0$, the GEV becomes the Gumbel distribution. This distribution has relatively light tails such as those obtained with normal distribution. If $\xi < 0$, the GEV becomes the Weibull distribution. This distribution has lighter tails than normal distribution and is not particularly useful for modeling financial returns since mostly the distribution of financial returns is heavy tailed. The standardized ($\mu = 0, \sigma = 1$) Frechet and Gumbel probability density functions are illustrated in Figure 4.8. It is seen that both distributions are skewed to the right, but the Frechet is more skewed and has noticeably larger right tail which means that the Frechet distribution has considerably higher probabilities of producing

large return values [8, 23, 26].

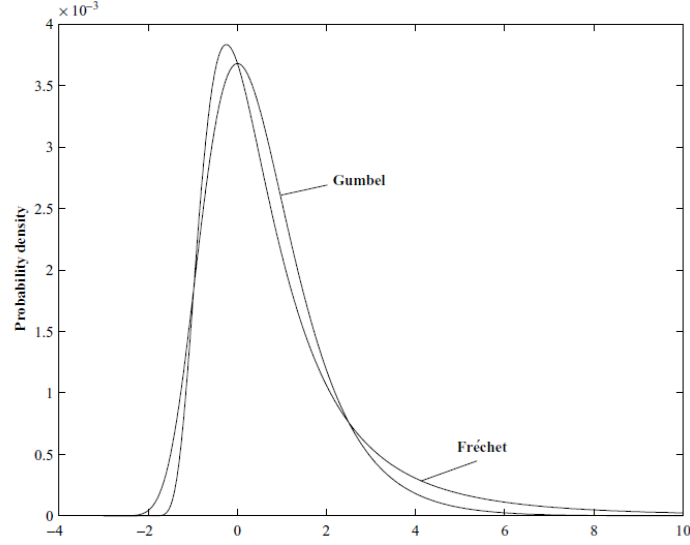


Figure 4.8. Standardized Frechet and Gumbel probability density functions.

4.3.2. Peaks over threshold method (POT)

Second method of EVT is the Peaks Over Threshold (POT) which models the behavior of extreme values above a high threshold. If X is a random i.i.d loss with distribution function $F(x)$ and u is a threshold value, the distribution of excess losses over the threshold value can be defined as shown in Equation 4.20. This gives the probability that a loss exceeds the threshold by at most x , given that it does exceed the threshold. If u is the VaR, then Equation 4.20 asks "what is the probability that we exceed VaR by a particular amount given that VaR is violated?" [8, 26].

$$F_u(x) = Pr(X - u \leq x | X > u) = \frac{F(x + u) - F(u)}{1 - F(u)} \quad (4.19)$$

The distribution of X can be any of the distributions such as normal, lognormal, t ... etc. and will usually be unknown. However as the threshold value gets large, the Pickands-Balkema-de Haan theorem, explained below, which is often called the second theorem in extreme value theory, states that the exceedances $(X - u)$ are distributed by the generalized Pareto distribution (GPD) [8, 23, 80].

Theorem 4.2 (Pickands, 1975; Balkema and de Haan, 1974). *For a large class of distributions F , the excess distribution function F_u can be approximated by GPD for a sufficiently large threshold value, u .*

$$G_{\xi,\beta}(x) = \begin{cases} 1 - \left(1 + \xi \cdot \left(\frac{x}{\beta}\right)\right)^{-1/\xi}, & \text{if } \xi \neq 0 \\ 1 - \exp\left(\frac{-x}{\beta}\right), & \text{if } \xi = 0 \end{cases} \quad (4.20)$$

where $\beta > 0$ and $x \geq 0$ when $\xi \geq 0$ and when $\xi < 0$, $0 \leq x \leq \frac{-\beta}{\xi}$.

GPD has two parameters which are a positive scale parameter (β) and a shape parameter (ξ) that can be positive, zero or negative. If the shape parameter is zero, the distribution becomes the Gumbel, if it is negative, the distribution becomes the Weibull and when it is positive, the distribution becomes the Frechet. Due to the fact that financial time series are heavy tailed, the case that $\xi > 0$ is usually considered in this thesis [8, 26]. The Pickands-Balkema-de Haan theorem tells that the distribution of excess losses always has the same form in the limit as the threshold value gets high, regardless of the distribution of the losses. In order to apply the generalized Pareto distribution, a reasonable threshold value (u) should be chosen to determine the number of observations (N_u) above the threshold value. However, in practice, a difficulty arises when an adequate threshold is selected. A sufficiently high threshold should be selected to apply the theorem reasonably closely. On the other hand, if the chosen threshold is too high, there is not enough excess-threshold observations to make reliable estimates. Hence, there is a trade-off between a biased estimate and a large estimation error [8, 23].

In this study, POT method is applied to financial data. One of the advantages of this method is that it uses data more efficiently. Besides, it is better adapted to the risk measurement of tail losses as it focuses on the distribution of exceedances over a threshold. Thus, POT method is more appropriate than BMM for financial applications. Two approaches are compared in Figure 4.9. On the left, the selected observations are the maxima of groups of three. On the right, observations are selected whenever they are

greater than u . The block maxima approach ignores extreme values x_9 , x_1 and x_8 because they appear in a block with already one extreme value. However, POT method includes these extreme values. Block Maxima procedure is rather wasteful of data and a relatively large sample is needed for accurate estimation yet sufficiently long data is often not available for a financial instrument. Besides, all observations that should be considered as extremes, are not included in the estimation of distribution parameters. Thus, available useful information is not fully taken into account. Moreover, given the stylized facts for return series, in particular volatility clustering, data points during tranquil periods could be selected as block maxima values, which they are not indeed.

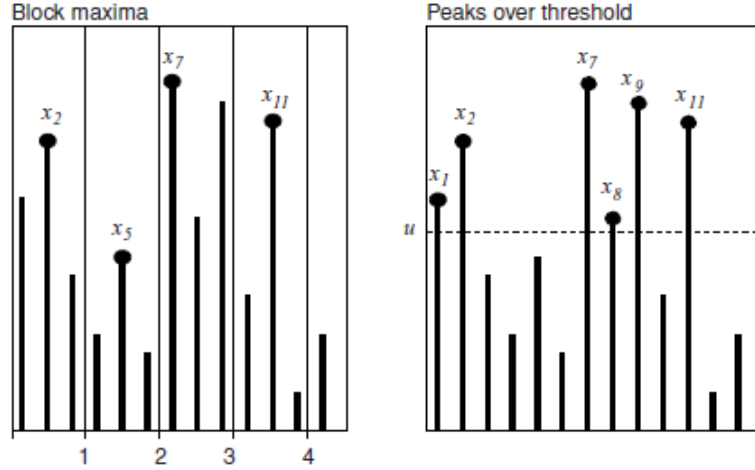


Figure 4.9. EVT approaches (BMM and POT).

4.3.3. Parameter estimation for Generalized Pareto distribution

Before the parameter estimation process, firstly a reasonable threshold value (u) is chosen and then the number of excess-threshold observations (N_u) is determined. The choice of threshold is the weak spot of POT theory. It is inevitably arbitrary and therefore judgemental. By trial, it is seen that the number of excess-threshold observations between 70 and 110 is adequate to estimate the parameters. Therefore, it is decided that 100 is used as the number of excess-threshold observations. After u and N_u are chosen, the next step is to estimate the parameters, ξ and β , by maximum likelihood estimation. However, there is no closed-form solutions for the parameters,

so the maximum likelihood approach requires the use of an appropriate numerical solution method which requires a suitable software. In this thesis, the software, *R*, and the package, *fExtremes*, are used for parameter estimation process. The likelihood function of (ξ, β) given x , is defined as $L(\xi, \beta; x) = \prod_{i=1}^n g_{\xi, \beta}(x_i)$ which is shown by Equation 4.21 [8, 81].

$$L(\xi, \beta; x) = \begin{cases} \frac{1}{\beta^{N_u}} \exp\left(-\frac{\sum_{i=1}^{N_u} x_i}{\beta}\right), & \text{if } \xi = 0 \\ \frac{1}{\beta^{N_u}} \prod_{i=1}^{N_u} \left(1 + \xi \frac{x_i}{\beta}\right)^{-\frac{1}{\xi}-1}, & \text{if } \xi > -\frac{\beta}{x_{(N_u)}} \text{ and } \xi \neq 0 \\ 0, & \text{if } \xi \leq -\frac{\beta}{x_{(N_u)}} \end{cases} \quad (4.21)$$

Maximum likelihood approach involves the numerical maximization of the log-likelihood function which is shown by Equation 4.22 [81].

$$l(\xi, \beta) = \begin{cases} -N_u \ln \beta - \left(1 + \frac{1}{\xi}\right) \sum_{i=1}^{N_u} \ln \left(1 + \xi \frac{x_i}{\beta}\right), & \text{if } \xi = 0 \\ -N_u \ln \beta - \left(\frac{1}{\beta}\right) \sum_{i=1}^{N_u} x_i & \text{if } \xi \neq 0 \end{cases} \quad (4.22)$$

4.3.4. Risk quantification for Generalized Pareto distribution

Let's assume that X_i ($i = 1, \dots, n$) are the random variables and u is the chosen threshold value. N_u is the number of exceedances and these are denoted as $Y_i = X_i - u$ for $i = 1, 2, \dots, N_u$. After the parameter estimation process, the next step is to calculate the daily Value at Risk. The Pickands-Balkema-de Haan theorem justifies that $F_u(y) = G_{\xi, \beta_{(u)}}(y)$ provided the threshold is sufficiently large. By setting $x=u+y$, an approximation of $F(x)$ for $x > u$ can be obtained from Equation 4.23 [9, 75].

$$F(y + u) = (1 - F(u)) \cdot G_{\xi, \beta_{(u)}} + F(u) \quad (4.23)$$

The function $F(u)$ can be estimated by Equation 4.24. This is an empirical estimate where n is the total number of observations and N_u is the number of observations above

the threshold [82].

$$F(u) = 1 - F(\bar{u}) = 1 - \left(\frac{1}{n} \sum_{i=1}^{N_u} 1_{(X_i > u)} \right) = \frac{n - N_u}{n} \quad (4.24)$$

$(n - N_u)/n$ is substituted for $F(u)$ and by plugging Equation 4.20 into Equation 4.23, $F(x)$ is obtained.

$$F(x) = 1 - \frac{N_u}{n} \left(1 + \xi \left(\frac{x - u}{\beta} \right) \right)^{-\frac{1}{\xi}} \quad (4.25)$$

In particular, we are concerned with the inverse of the Equation 4.25. Value at Risk is α -quantile of the distribution of returns. For a chosen probability α , Equation 4.26 shows VaR estimator for confidence level, α [75].

$$VaR_\alpha = u + \frac{\beta}{\xi} \left(\left(\frac{n}{N_u} (1 - \alpha) \right)^{-\xi} - 1 \right) \quad (4.26)$$

In this thesis, extreme value theory is chosen on the purpose of modeling the tail behavior of return distributions. It is considered that as VaR estimations are only related to the tails of a probability distribution, techniques from EVT may be particularly effective. However, applying EVT to the logarithmic returns is not appropriate as logarithmic returns are not independent and identically distributed. Besides, constant parameter assumption is not correct. The applied POT method for quantile estimation does not yield VaR estimates which reflect the current volatility background. Therefore, in order to overcome the drawback of EVT, GARCH model which is explained in the next chapter, is selected as another alternative [75, 82, 83].

5. GARCH MODEL

Economic crises, war or political disorders, that threaten human society, make financial markets react nervously. Hence, the volatility of logarithmic returns from financial market variables, such as exchange rates and equity prices, fluctuates over time while econometric models and conventional time series operate under an assumption of constant variance. Though earlier financial models assumed volatilities to be constant, it is widely recognized among both practitioners and academics that volatility varies over time. Time variation in the volatility of financial time series and volatility forecasting are important and immense effort has been expended in improving volatility models, since better volatility forecast can allow a more precise estimation of the value at risk, that gives rise to better risk management [78, 84–86].

Regarding the volatility forecasting, a common approach is to use the standard deviation of the returns of the last T days, such that T is the number of most recent observations used in order to calculate the standard deviation. This heuristic approach is based on the assumption that volatility changes only slightly over a certain period of time and can be treated as "almost" constant. However, constant volatility assumption is not realistic for financial return series. Given the rapid growth in financial markets over the past two decades, along with the explosive development of new and more complex financial instruments, an ever-growing need has emerged for accurate and efficient volatility forecasting to use in numerous practical applications of financial data. More sophisticated techniques have been developed for forecasting the volatility including ARCH, GARCH and stochastic volatility (SV) models which allow the volatility to vary over time in a specific way. In this thesis, only GARCH models are studied among the time-varying volatility models [86, 87].

In 1977, Friedman suggested that the unpredictability of inflation was a primary cause of business cycles and this unpredictability would affect the investors' behaviors. Thereafter, Robert Engle worked on a model that could assess the validity of the conjecture of Friedman. Time-varying volatility has been observed and documented in as

early as 1982 by Engle and was initially concerned with an economic phenomenon – time-varying and autoregressive variance of inflation. Engle has introduced Autoregressive Conditional Heteroscedastic (ARCH) model in 1982, owing to the requirement of a model that describes the variation of volatility with respect to time. For Autoregressive models, the output variable depends linearly on its previous values. The term “conditional heteroscedasticity” means that conditional variance is not constant and depends on the information in previous returns. Namely, the process has a short-time memory and this memory influences the behavior of the process [60, 77, 88].

Assume that the price on day t is P_t and daily logarithmic return on day t is r_t .

$$r_t = \ln P_t - \ln P_{t-1} \quad (5.1)$$

F_t is the information set or history at time t . ε_t is a random variable that has a mean and a variance conditional on the information set F_t . $E[\varepsilon_t|F_t] = 0$ and the conditional variance $\sigma_t^2 = E[\varepsilon_t^2|F_t]$ is a nontrivial positive-valued parametric function of F_t . The basic idea of ARCH models is that ε_t , the shock of an asset return, is serially uncorrelated, but dependent. Namely, the conditional mean is constant but the conditional variance is non-constant. The dependence of the conditional variance on the past causes the shock to be dependent and the independence of the conditional mean on the past is the reason that the shock is uncorrelated. The sequence, ε_t , may be observed directly, or it may be an error or innovation sequence of an econometric model. The following mean equation (5.2) and variance equation (5.4) define an ARCH model of order q . Z_t is a sequence of independent and identically distributed random variables with zero mean and unit variance [60, 68, 89].

$$r_t = \mu + \varepsilon_t \quad (5.2)$$

$$\varepsilon_t = Z_t \sigma_t \quad (5.3)$$

$$\sigma_t^2 = \omega + \sum_{i=1}^q \alpha_i \varepsilon_{t-i}^2 \quad (5.4)$$

$\omega > 0$, $\alpha_i \geq 0$ for $i = 1, \dots, q-1$ and $\alpha_q > 0$. The parameter restrictions of the variance equation form a necessary and sufficient condition for positivity of the conditional variance [60].

Even though the ARCH model is simple and some properties of asset returns can be modeled by ARCH such as the leptokurtic distribution of asset returns and volatility clustering, due to the large persistence in volatility, large orders (q) are often required to describe the volatility process of an asset return sufficiently. Moreover, the ARCH model is likely to overpredict the conditional volatility as the model responds slowly to the large shocks of the return series. Bollerslev introduced a useful extension of the ARCH model known as the Generalized ARCH (GARCH) model which overcomes certain weaknesses of the ARCH model. GARCH gained popularity in finance and among practitioners since it often gives a reasonable fit to real life data and some of the stylized facts of financial log returns can be explained well by this model [84, 88–90].

The Generalized ARCH (GARCH) model, which improves the original specification by addition of the lagged conditional variance, has been proposed by Tim Bollerslev in 1986. Bollerslev avoids the problem of large values of q in Engle's ARCH model by generalizing the ARCH(q) model in the Generalized Autoregressive Conditional Heteroscedastic (GARCH) model, in much the same way as an Autoregressive (AR) model extends to the Autoregressive Moving Average (ARMA) model. GARCH model asserts that the best predictor of the variance in the next period is a weighted average of the long-run average variance, the variance predicted for this period, and the new information in this period that is captured by the most recent squared residual. GARCH models typically fit as well as a high order ARCH yet remain parsimonious. Therefore, as fewer lags are necessary to capture the time varying variance, in empirical applications GARCH models, especially GARCH(1,1), is widely adopted. The (1,1) in parentheses is a standard notation in which the first number refers to how many autoregressive lags, or ARCH terms (q), appear in the variance equation, while the

second number refers to how many moving average lags are specified, which is often called the number of GARCH terms (p). Assuming that r_t is the logarithmic return, ε_t is the innovation and Z_t is a sequence of independent and identically distributed random variables with zero mean and unit variance. Mean equation (5.5) and variance equation (5.6) of GARCH model are seen below [88, 91–93].

$$r_t = \mu + \varepsilon_t \quad (5.5)$$

$$\sigma_t^2 = \omega + \sum_{i=1}^q \alpha_i \varepsilon_{t-i}^2 + \sum_{j=1}^p \beta_j \sigma_{t-j}^2 \quad (5.6)$$

$$\varepsilon_t = \sigma_t Z_t \quad (5.7)$$

The variance equation reduces to a pure ARCH (q) model if $p = 0$. ω is the long term average value. The parameters α_i and β_j are referred to as ARCH and GARCH parameters respectively. A sufficient condition for the conditional variance to be positive is $\omega > 0$, $\alpha_i \geq 0$ for $i = 1, \dots, q$ and $\beta_j \geq 0$ for $j = 1, \dots, p$. As the name of the model suggests, the variance specified above is conditional. The variance equation shows that large past squared returns ε_{t-i}^2 or large past conditional variances σ_{t-j}^2 give rise to large values of σ_t^2 . The unconditional variance of the GARCH model is obtained by applying the expectation operator to both sides of the variance equation as seen in Equation (5.9). $E[\sigma_t^2]$ is the unconditional (long-run) variance, the value to which the variance will tend to revert in the long run. The unconditional variance is shown in Equation (5.10) [60, 89].

$$E[\sigma_t^2] = E[\varepsilon_{t-i}^2] = E[\sigma_{t-j}^2] \quad (5.8)$$

$$E[\sigma_t^2] = \omega + \sum_{i=1}^q \alpha_i E[\varepsilon_{t-i}^2] + \sum_{j=1}^p \beta_j E[\sigma_{t-j}^2] \quad (5.9)$$

$$E[\sigma_t^2] = \sigma^2 = \frac{\omega}{1 - \left(\sum_{i=1}^q \alpha_i + \sum_{j=1}^p \beta_j \right)} \quad (5.10)$$

The GARCH(p,q) model builds on the ARCH(q) by including p lags of the conditional variance. The effect of the lagged variance is to produce a model which is actually an ARCH(∞) in disguise. Thus, GARCH(1,1) can be written as an ARCH(∞). By backward substituting, the ARCH(∞) representation is derived as seen in the equations below [93].

$$\begin{aligned} \sigma_t^2 &= \omega + \alpha_1 \varepsilon_{t-1}^2 + \beta_1 \sigma_{t-1}^2 \\ &= \omega + \alpha_1 \varepsilon_{t-1}^2 + \beta_1 (\omega + \alpha_1 \varepsilon_{t-2}^2 + \beta_1 \sigma_{t-2}^2) \\ &= \omega + \beta_1 \omega + \alpha_1 \varepsilon_{t-1}^2 + \beta_1 \alpha_1 \varepsilon_{t-2}^2 + \beta_1^2 \sigma_{t-2}^2 \\ &= \omega + \beta_1 \omega + \alpha_1 \varepsilon_{t-1}^2 + \beta_1 \alpha_1 \varepsilon_{t-2}^2 + \beta_1^2 (\omega + \alpha_1 \varepsilon_{t-3}^2 + \beta_1 \sigma_{t-3}^2) \\ &= \omega + \beta_1 \omega + \beta_1^2 \omega + \alpha_1 \varepsilon_{t-1}^2 + \beta_1 \alpha_1 \varepsilon_{t-2}^2 + \beta_1^2 \alpha_1 \varepsilon_{t-3}^2 + \beta_1^3 \sigma_{t-3}^2 \\ &= \sum_{i=1}^{\infty} \beta_1^i \omega + \sum_{i=1}^{\infty} \beta_1^i \alpha_1 \varepsilon_{t-i-1}^2 \end{aligned}$$

Sometimes models with more than one lag are needed to find good variance forecasts. However, many researchers in the literature, also Bollerslev et. al. (1992) and Zivot (2008), suggested that usually a GARCH(1,1) model with only three parameters in the conditional variance equation is adequate to obtain a good model fit for financial time series. In the recent literature that compares GARCH(1,1) with the higher-lag GARCH (p,q) or even other types of volatility models, the former is found to perform better in most cases. Pacelli (2012) finds that, when using GARCH(1,1), GARCH models can better forecast exchange rate dynamics. In the view of such information, GARCH(1,1) is chosen in this study to forecast the volatility of exchange rate time

series. Therefore, variance equations are given for GARCH(1,1) model rather than GARCH(p,q). In the next sections, various GARCH models are explained [91, 94, 95].

5.1. GARCH Model With Gaussian Innovations (Normal-GARCH)

The most common conditional distribution in the GARCH model is the normal distribution, that is, the error terms follow the normal distribution. In this thesis, GARCH(1,1) model with normal distributed innovations is chosen as the fourth alternative with the hope of better return modeling. This model is very strong in incorporating volatility persistence of returns. As the plots of exchange rate returns show the clustering patterns of volatility and time-varying volatility is not taken into account in the previous models, it is anticipated that GARCH(1,1) can better model daily exchange rate return series.

The mean and variance equations of GARCH(1,1) model is shown in the next equations. Conditional mean equation is estimated with only a constant, as ACF and PACF of exchange rates series do not indicate significant serial correlation. Normal-GARCH(1,1) models the conditional volatility as a function of past conditional volatility and return. The main assumption of Normal-GARCH(1,1) model is the conditional normality, namely, today's return is normally distributed conditional on the information of the previous day. Therefore, the innovations are i.i.d normal. As seen in Equation (5.13), the innovation (ε_t) is interpreted as the multiplication of conditional standard deviation and Gaussian noise. The Gaussian noise (Z_t) follows identical and independent normal distribution (i.i.d) with zero mean and unit variance [96].

$$r_t = \varepsilon_t \tag{5.11}$$

$$\sigma_t^2 = \omega + \alpha_1 \varepsilon_{t-1}^2 + \beta_1 \sigma_{t-1}^2 \tag{5.12}$$

$$\varepsilon_t = \sigma_t Z_t \quad (5.13)$$

$$Z_t \sim N(0, 1) \quad (5.14)$$

5.1.1. Parameter estimation of the Normal-GARCH model

Regarding the volatility forecasting, the first step is to estimate the model parameters. Due to the nonlinear relation between volatility and returns, the GARCH family of models is nonlinear. Although there are many techniques available for estimating nonlinear models, the most common method is maximum likelihood estimation (MLE) which is also used in this study. As θ is a vector of parameters, values of θ , that have high likelihood, correspond to models which give high probability to the observed data. The principle of maximum likelihood estimation is to adopt the model with greatest likelihood, since of all the models under consideration, this is the one that assigns highest probability to the observed data [4, 97].

The GARCH model in the equations (5.11) and (5.12) specifies that the distribution of r_t is normal with zero mean and conditional variance σ_t^2 . The conditional distribution of r_t is given in Equation (5.15).

$$f(r_t | r_{t-1}, r_{t-2}, \dots; \theta) = \frac{1}{\sqrt{2\pi\sigma_t^2}} \exp\left(-\frac{(r_t - \mu_t)^2}{2\sigma_t^2}\right) \quad (5.15)$$

The log-likelihood function for Normal GARCH(1,1) is shown in Equation (5.17).

$$l(\theta) = \sum_{t=1}^n \left(-\ln(\sqrt{2\pi}) - \frac{(r_t - \mu_t)^2}{2\sigma_t^2} - \frac{1}{2} \ln(\sigma_t^2) \right) \quad (5.16)$$

$$= -\frac{n}{2} \ln(2\pi) - \frac{1}{2} \sum_{t=1}^n \ln(\sigma_t^2) - \frac{1}{2} \sum_{t=1}^n \frac{(r_t - \mu_t)^2}{\sigma_t^2} \quad (5.17)$$

As the mean is assumed to be zero in our model, the log-likelihood simplifies as seen in Equation (5.18) [98]. Owing to the complicated likelihood function, the software R is used for numerical optimization.

$$l(\theta) = -\frac{n}{2} \ln(2\pi) - \frac{1}{2} \sum_{t=1}^n \ln(\sigma_t^2) - \frac{1}{2} \sum_{t=1}^n \frac{r_t^2}{\sigma_t^2} \quad (5.18)$$

5.1.2. Risk quantification of the Normal-GARCH model

After the parameters are estimated, the next step is to estimate the next day's volatility by using an iterative optimization algorithm. As the mean of the returns is assumed to be zero, only a starting value for σ_1^2 is needed. The unconditional variance, which is simply the sample variance of r_t , can be used as a starting value. When the volatility for the next day is forecasted, Equation (5.19) is used to quantify the risk. In the equation, μ is assumed to be zero. Equation (5.19) is the same equation used to calculate the VaR of normally distributed log-returns in Chapter 4 (Equation (4.10)).

$$VaR_{(1-\alpha)} = P(1 - \exp(\mu + \sigma \cdot qnorm(\alpha))) \quad (5.19)$$

5.2. Other GARCH Models

A variety of different specifications for the conditional variance equation of the GARCH model have been proposed. These extensions were motivated by the need of more effectively models for some particular features observed in financial data or by concerns regarding computational simplicity. In the following subsections, relevant variants of the GARCH model are described. These models are able to explain particular features of asset return series by introducing [99]:

- Conditional non-normality of the error process (ε_t) to better explain the leptokurtosis of the return series (by applying non-normal innovations).

- Asymmetric responses to negative and positive return innovations to model the asymmetry in the reaction of conditional volatility to the arrival of different news.

5.2.1. GARCH model with Student's t innovations (t-GARCH)

Although the GARCH model with Gaussian innovations has been extensively used, it is often the case that observed returns have fatter tails than would be implied by the GARCH model with Gaussian innovations. This suggests that the Normal-GARCH model could be improved by replacing the conditional normal distribution with a conditionally heavy-tailed distribution, where the parameter determining heaviness is to be estimated along with other model parameters. In 1987, Bollerslev suggested replacing the assumption of conditional normality of the error with that of conditional Student's t distribution. Although several distributional proposals have been made, the most common is the GARCH model with Student's t innovations [26, 60]. In this study, the t-GARCH model is also used as an alternative model in order to check whether the t-GARCH model better estimates VaR since t-GARCH takes into account the leptokurtosis of the residuals.

If a random variable (r_t) has a Student's t distribution with ν degrees of freedom and a scale parameter (s_t), then the probability density function of r_t is shown in Equation (5.20). The variance of r_t is given by Equation (5.21).

$$F(r_t) = \frac{\Gamma[(\nu + 1)/2]}{\Gamma[\nu/2] (\pi\nu)^{1/2}} \frac{s_t^{-1/2}}{\left[1 + \frac{r_t^2}{s_t\nu}\right]^{(\nu+1)/2}} \quad (5.20)$$

$$var(r_t) = \frac{s_t\nu}{\nu - 2}, \nu > 2 \quad (5.21)$$

The mean and variance equations of this model are also same as those (Equations (5.11) and (5.12) respectively) in the Normal-GARCH model. The difference is that

the innovation (ε_t) is interpreted as the multiplication of conditional standard deviation and the Student's t distributed noise, namely, $Z_t \sim t(\nu)$. The error term (ε) in this model follows a Student's t distribution with ν degrees of freedom which is an extra parameter that should be estimated. If $var_{t-1}(\varepsilon_t) = \sigma_t^2$, the scale parameter s_t should be chosen as seen in Equation (5.22) [60].

$$s_t = \frac{\sigma_t^2(\nu - 2)}{\nu} \quad (5.22)$$

5.2.2. Exponential GARCH (EGARCH)

In 1991, Nelson introduced the EGARCH model. EGARCH models the natural logarithm of the variance instead of the variance itself. Thus, the conditional variance is guaranteed to be positive at each point in time as the actual variance is obtained by exponentiation. However, in the GARCH model, parameter restrictions are required to ensure that the conditional variance is positive. Moreover, EGARCH captures asymmetric responses of the time-varying variance to shocks. This asymmetric relationship is called the leverage effect and describes how a negative shock causes volatility to rise more than a positive shock with the same magnitude. Even though GARCH models successfully capture the thick tails of return distribution and volatility clustering, it is not appropriate to capture this asymmetric behavior. In the GARCH model shocks of the same magnitude, positive ($\varepsilon_{t-1} > 0$) or negative ($\varepsilon_{t-1} < 0$), result in the same increase in volatility since it is only the absolute size of the news that matters, namely, ε_{t-1}^2 enters the equation [98–100].

An EGARCH(p,q) process is defined in the below equations. In the Equation (5.24), $\varepsilon_t = \sigma_t Z_t$ and $g(\varepsilon_t) = \theta \varepsilon_t + \gamma[|\varepsilon_t| - E(|\varepsilon_t|)]$ are the weighted innovations that model asymmetric effects between positive and negative asset returns and θ and γ are the constants. When ε_{t-i} is positive or there is "good news", the total effect of ε_{t-i} is $(1 + \gamma_i)|\varepsilon_{t-i}|$; in contrast when ε_{t-i} is negative or there is "bad news", the total effect

of ε_{t-i} is $(1 - \gamma_i)|\varepsilon_{t-i}|$ [60, 99].

$$r_t = \varepsilon_t \quad (5.23)$$

$$\ln(\sigma_t^2) = \omega + \sum_{i=1}^q \alpha_i \frac{|\varepsilon_{t-i}| + \gamma_i \varepsilon_{t-i}}{\sigma_{t-i}} + \sum_{j=1}^p \beta_j \ln(\sigma_{t-j}^2) \quad (5.24)$$

$$\varepsilon_t = \sigma_t Z_t \quad (5.25)$$

$$Z_t \sim N(0, 1) \quad (5.26)$$

5.2.3. Threshold GARCH (TGARCH)

Another GARCH variant that is capable of modeling leverage effects is the threshold GARCH (TGARCH) model. Zakoian (1991) and Glosten, Jagannathan and Runkle (1993) explicitly treat the asymmetric responses of time-varying variance in their extensions of the GARCH model. TGARCH model is also known as the GJR-GARCH model because the model was named after the authors who introduced it. A TGARCH(p,q) process is defined in the below equations where $I_{(\varepsilon_{t-i} < 0)}$ is an indicator function that takes the value 1 if $\varepsilon_{t-i} < 0$ and 0 otherwise. That is, depending on whether ε_{t-i} is above or below the threshold value of zero, ε_{t-i}^2 has different effects on the conditional variance σ_t^2 . If ε_{t-i} is positive, the total effects are given by $\alpha_i \varepsilon_{t-i}^2$, on the other hand, if ε_{t-i} is negative, the total effects are given by $(\alpha_i + \gamma_i) \varepsilon_{t-i}^2$. Hence, it would be expected that γ_i is positive for bad news to have larger impacts on the variance [60, 99, 101].

$$r_t = \varepsilon_t \quad (5.27)$$

$$\sigma_t^2 = \omega + \sum_{i=1}^q \alpha_i \varepsilon_{t-i}^2 + \sum_{i=1}^q \gamma_i I_{(\varepsilon_{t-i} < 0)} \varepsilon_{t-i}^2 + \sum_{j=1}^p \beta_j \sigma_{t-j}^2 \quad (5.28)$$

$$\varepsilon_t = \sigma_t Z_t \quad (5.29)$$

$$Z_t \sim N(0, 1) \quad (5.30)$$

5.3. GARCH Filtered EVT Model (Conditional EVT)

Financial risk management is intimately concerned with tail probabilities. Extreme probabilities are of particular interest since the ability to assess them accurately translates into the ability to manage extreme financial risks effectively, such as those associated with currency crises, stock market crashes, and large bond defaults. Unfortunately, traditional parametric, statistical and econometric methods, typically based on estimation of entire densities, are ill-suited to the assessment of extreme quantiles and event probabilities. These parametric methods implicitly strive to produce a good fit in regions where most of the data fall, potentially at the expense of good fit in the tails. On the other hand, as described in Chapter 4, the estimation method of EVT is tailored to the object of interest, the tail of the distribution, rather than the center of the distribution. Extreme value theory provides quantification of the stochastic behavior of a process at unusually large or small levels so it is a suitable method to capture the extreme tail risk. As Value at Risk and Expected Shortfall estimations are only related to the tails of a probability distribution, techniques from EVT may be particularly effective [102, 103].

The EVT procedures, described in Chapter 4, are all unconditional; namely, they are applied directly to the random variable of interest, which is daily logarithmic return for this study. One important assumption of EVT is that the tail events are independent and identically distributed (i.i.d). However, it is widely agreed that

high frequency financial asset returns are conditionally heteroscedastic and hence not independent and identically distributed. If EVT methods are directly applied to logarithmic returns, this assumption is likely to be violated due to the stochastic volatility and volatility persistence which suggests that one tail event is likely to be followed by another tail event. One way to overcome this violation of the i.i.d assumption is to filter the data with a volatility model and study the volatility filtered i.i.d residuals using EVT. This method, conditional EVT, was proposed by McNeil and Frey (2000). Unconditional EVT has the advantage that it focuses attention directly on the tail of the distribution. However, it does not recognize the fact that returns are non-i.i.d. The econometric models of volatility dynamics such as the GARCH process under different innovation distributions yield VaR estimates which reflect the current volatility background. However, the weakness of the GARCH modeling approach is that it focuses on modeling the whole conditional return distribution as time-varying, not only on the part of interest, the tail. The two methodologies are combined to come up with a robust model with much enhanced predictive abilities. Conditional EVT reflects two stylized facts exhibited by most financial return series, namely stochastic volatility and the fat-tailedness of conditional return distributions over short time horizons. Taking all these into account, in this study, conditional EVT method is used as the fifth alternative. Conditional EVT is a two-stage approach and these stages are summarized as follows [8, 82, 83, 102–105].

- A GARCH-type model is fitted to the return data using MLE. The standardized residuals of this model, which should turn out to be i.i.d, are extracted. The GARCH type model is then used to make one step ahead predictions of next period's location and scale parameters, μ_{t+1} and σ_{t+1} . In this study, the location parameter is assumed to be zero.
- EVT is applied to the residuals and then VaR estimates are derived by taking into account both, the dynamic (GARCH) structure and the residual process.

5.3.1. Parameter estimation of GARCH filtered EVT model

The first step of conditional EVT model is to fit a GARCH type model to the return data. In this study, initially GARCH(1,1) model is chosen to remove volatility clustering of the time series, which may be inconsistent with the i.i.d exceedances assumption given that financial return series typically have non-normal properties. In order to estimate the next day's volatility, GARCH(1,1) model parameters (ω, α, β) are estimated by MLE. The parameter estimation step is performed by using the statistical software R. After the parameter estimation step, the next day's volatility is obtained by using Equation (5.33). The software also gives the standardized residuals which are calculated by using Equation (5.31). n is the number of data to estimate the parameters. The location parameter, $\hat{\mu}$, is assumed to be zero in this study. If the fitted model is tenable, the standardized residuals should be i.i.d. Therefore, it would be useful to plot the correlograms for the standardized residuals and absolute standardized residuals.

$$\left(z_{(t-n+1)}, \dots, z_t \right) = \left(\frac{r_{(t-n+1)}}{\hat{\sigma}_{(t-n+1)}}, \dots, \frac{r_{(t)}}{\hat{\sigma}_{(t)}} \right) \quad (5.31)$$

$$r_t = \varepsilon_t \quad (5.32)$$

$$\hat{\sigma}_{t+1}^2 = \hat{\omega} + \hat{\alpha}\hat{\varepsilon}_t^2 + \hat{\beta}\hat{\sigma}_t^2 \quad (5.33)$$

After the standardized residuals are obtained, the next step is to use the peaks over threshold (POT) method for tail estimation. The first step in this modeling is to estimate the threshold for identifying the relevant tail region. There is an important bias-variance trade-off when deciding on the number of exceedances N_u . Increasing N_u (moving toward the center of the distribution) reduces variance as more data are used but increases bias. In this study, the threshold value u and the number of

excess-threshold N_u are chosen by trial. The choice of the threshold value and the parameter estimation process are carried out by using the standardized residuals of the GARCH(1,1) model. After u and N_u are chosen, the next step is to estimate the parameters, ξ and β , by maximum likelihood estimation. However, there exist no closed-form solutions for the parameters, so the maximum likelihood approach requires the use of an appropriate numerical optimization method which requires a suitable software. In this thesis, the software, *R*, and the package, *fExtremes*, are used for the parameter estimation process [82, 83].

5.3.2. Risk quantification of GARCH filtered EVT model

After specifying conditional EVT model completely by estimating the parameters, the next step is to obtain the quantile estimate by fitting the Generalized Pareto distribution to the excesses of the standardized residuals. Equation (5.34) is used to calculate the quantile estimate of the standardized residuals $VaR(Z)_\alpha$.

$$VaR(Z)_\alpha = u + \frac{\beta}{\xi} \left(\left(\frac{n}{N_u} (1 - \alpha) \right)^{-\xi} - 1 \right) \quad (5.34)$$

Lastly, the robust dynamic VaR estimates are calculated by using Equation (5.35).

$$VaR_\alpha = \hat{\sigma}_{(t+1)} \cdot VaR(Z)_\alpha \quad (5.35)$$

6. APPLICATION TO REAL DATA

In this chapter, initially, the required data we used for implementations, are introduced. In the second step, we show in a simple example how the daily VaR values of the exchange rate data were calculated using different risk models. Thirdly, how we implemented the back testing is explained by using a simple example. Lastly, the main back testing example, we performed in this study to assess the accuracy of the risk modeling methods, is presented with the necessary codings and the performances of the models are assessed.

6.1. Overview of The Data

Daily spot exchange rate data, obtained from Wiki Exchange Rates of Quandl's open sources, are used in this study. Various currency pairs, such as USD/TRY, EUR/TRY and USD/EUR, are analyzed for the period between January, 2000 and December, 2015 which includes 4801 trading days. Therefore, the sample consists of 4801 daily observations for each series. Daily logarithmic returns - the difference between logarithmic levels of prices in two successive days- are used as explained in Chapter 4. After transformation by Equation (6.1), 4800 daily log-returns are obtained.

$$r_t = \ln(P_t) - \ln(P_{t-1}), t = 1, \dots, 4800 \quad (6.1)$$

All results of the analyzed exchange rate series, such as plots and back testing results, can be seen in the Appendices. However, as USD/EUR, TRY/USD and TRY/EUR are significant currencies for Turkey, the plots and back testing results for them are shown throughout this chapter. The descriptive statistics of these currencies are demonstrated in the subsection 2.1.5. Besides, Figure 6.1 shows that the volatility of logarithmic returns is not constant over time. Historical volatility, calculated by Equation (6.2), is used to make this plot. The advantage of this measure is that it is easy to compute. The only critical part is the choice of the window length k , since if

the moving window is too long, the estimate will not be dynamic enough but if it is too short, the estimate will be too noisy (responds to new information more quickly). In order to plot Figure 6.1, k is chosen as 100 days.

$$\sigma_t^2 = \frac{1}{k} \sum_{i=1}^k r_{t-i}^2 \quad (6.2)$$

The visual inspection of logarithmic returns (Figure 6.2) reveals the presence of volatility clustering, implying that volatility changes through time and is prone to periods of clustering. Namely, large positive and negative changes in prices tend to be followed by large changes and small changes tend to be followed by small changes. Regarding Figure 6.1 and 6.2, highly volatile periods are observed for all currencies in the years 2008 and 2009 due to the global financial crisis. Moreover, highly volatile periods are observed for the currencies TRY/USD and TRY/EUR in 2001 owing to the Turkish economic crisis. On the other hand, there are tranquil periods in 2012 and 2013. Although normally distributed logarithmic returns and their constant variance through time are the two general assumptions made in the study of financial markets, these empirical evidences demonstrate that neither assumption holds in the context of exchange rates. In summary, log-return series do not conform to the normal distribution, they are highly leptokurtic in reference to the kurtosis values and considering the plots, constant volatility assumption is not realistic [98].

6.2. Quantifying Value at Risk Using Different Models

In this study, the main practical problem is to quantify the risk of a currency investment. As mentioned earlier, Value at Risk has emerged as the most prominent measure of downside market risk. Therefore, the first goal is to quantify the daily risk by using VaR measure and in order to accomplish this, different risk models are used. Each method has its strengths and weaknesses and certainly, it is not possible to find a model that captures all the stylized facts of the empirical return distribution and predicts the risk exactly. Nevertheless, as there were observed turbulent market

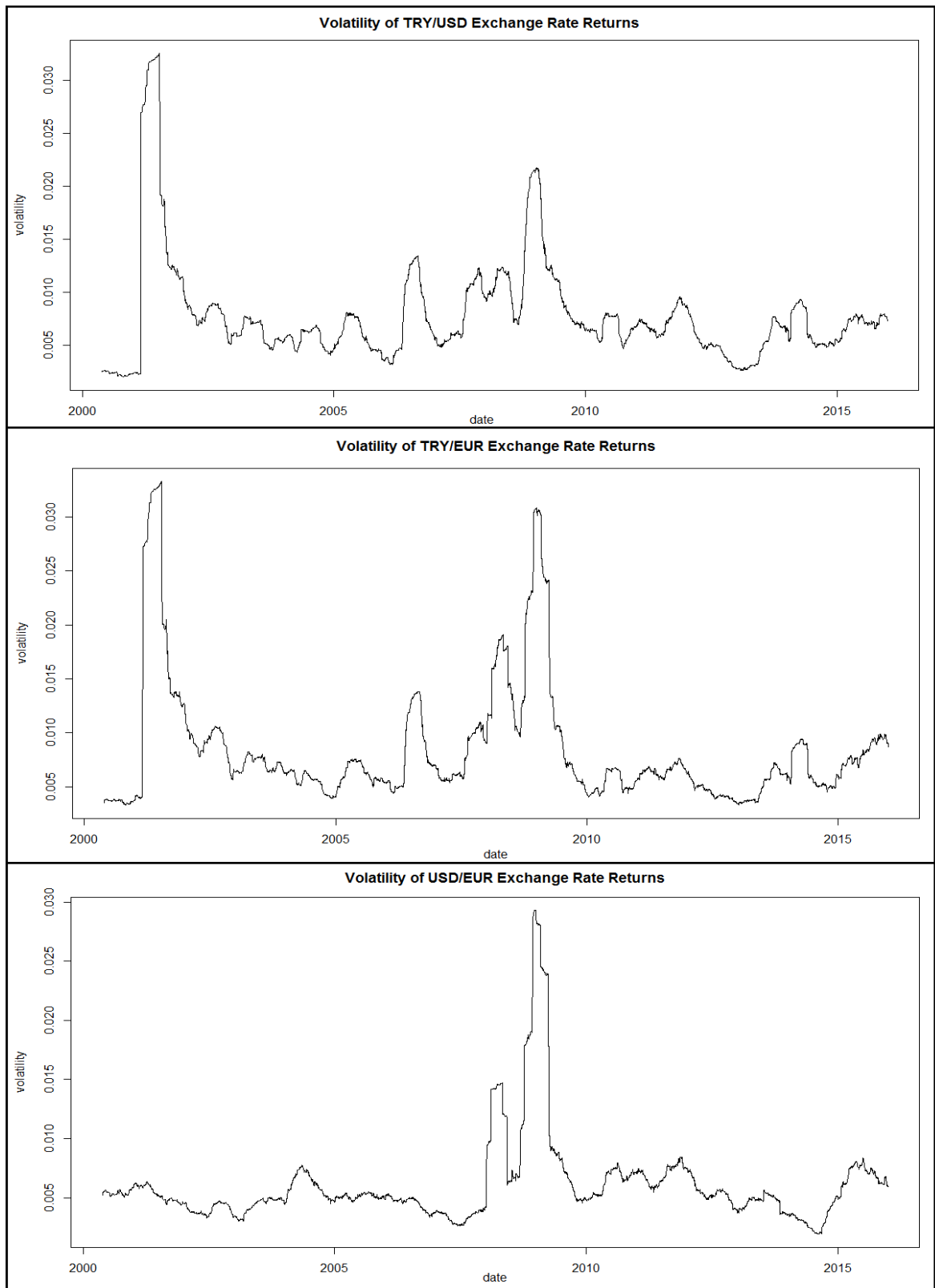


Figure 6.1. Historical volatility of logarithmic returns (TRY/USD - TRY/EUR - USD/EUR).

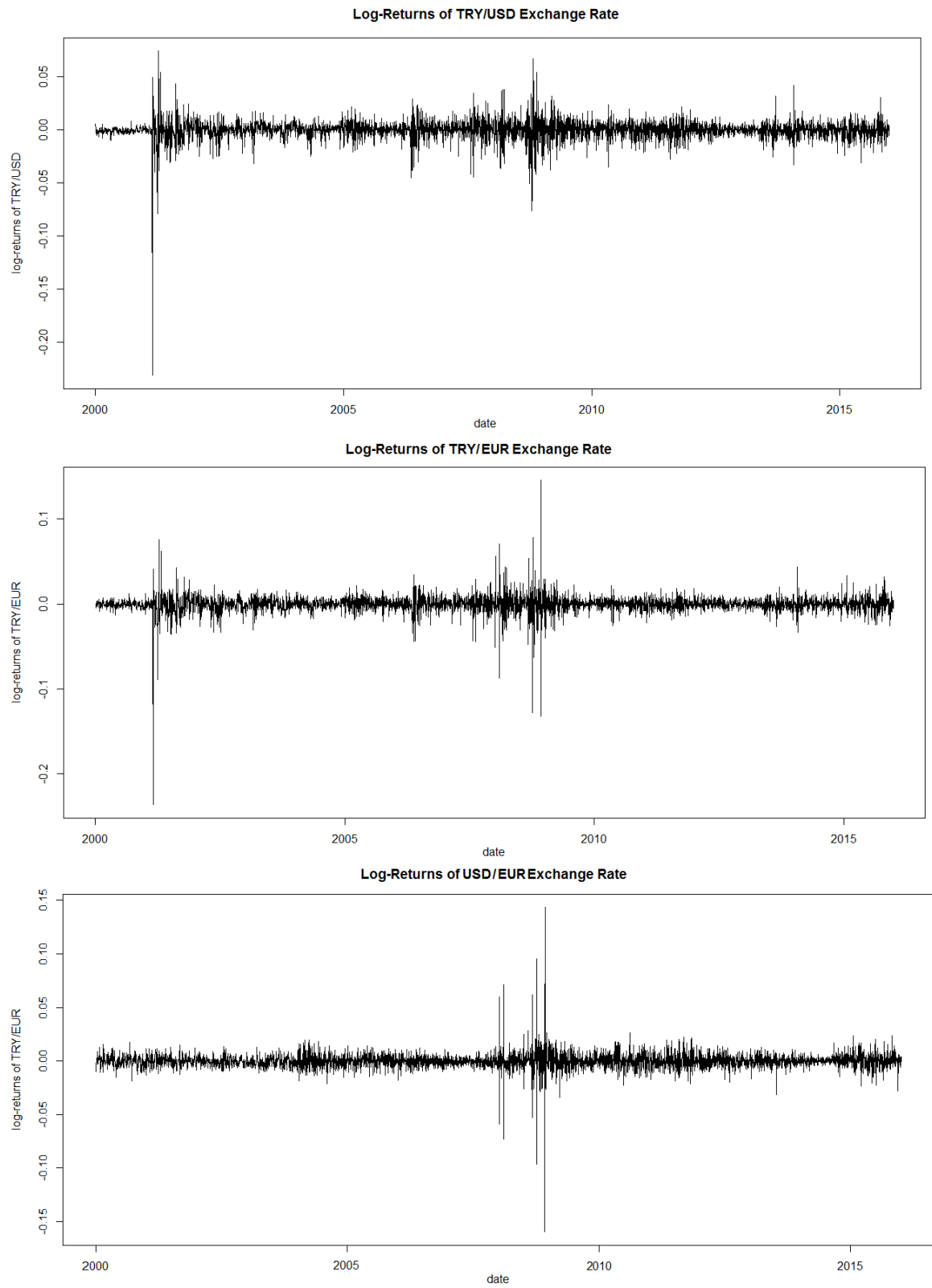


Figure 6.2. Logarithmic returns ($\text{TRY/USD} - \text{TRY/EUR} - \text{USD/EUR}$).

conditions often in the last two decades, especially the global financial crisis in 2008, it is critical to find a model that quantifies the extreme risk. In this study, the following models are used to calculate the one day ahead Value at Risk.

- i.i.d Normal model
- i.i.d Student's t model
- Normal-GARCH(1,1) model
- t-GARCH(1,1) model
- Unconditional EVT (POT) model
- GARCH filtered EVT (Conditional EVT) model

We start here presenting VaR calculations for USD/EUR currency for several days in the past. Initially, 20 days out of 10 years (from January 2006 to December 2015) were arbitrarily chosen. The second step is the coding process of the R functions which are used for VaR calculation of the Normal model and shown in Figure 6.3. As the Normal model is the simplest method for parameter estimation and VaR calculation, this code is shown in this part. The functions of other models also work in a similar way. The whole code can be seen in Appendix A.

As seen in Figure 6.3, there are three functions to calculate the daily VaR for the chosen day and for different confidence levels. The first function, *FXdata_matrix*, is used to take the FX data from the sources of Quandl. The second function is *Parest_Normal*. Taking the past data as input, this function estimates the parameters of the Normal model, which is only the standard deviation in our example, and gives the estimated parameters as output. The third function is *VaR_Calculation_Normal*. By taking the vector of model parameters, that are estimated in the previous step, and the confidence levels as input, this function calculates the daily VaRs for the chosen day and for different confidence levels. *DailyVaRFunc*(*code* = "CURRFX/TRYUSD", *ndata* = 730, *VaRdate* = "2006 - 02 - 15", *alpha* = 0.05, *ParestFunc*, *VaRFunc*) is the wrapper function. This wrapper function takes the Quandl code for the required currency, the number of data that is used for parameter estimation (*ndata*), the date

```

library(Quandl)

FXdata_matrix <- function(code, firstdate, lastdate){
#This function takes FX data from Quandl

fxdata <- Quandl(code,start_date=firstdate,end_date=lastdate)
fxdata
}

ParEst_Normal <- function(pastdata){
#This function calculates volatility
#pastdata...a vector of past data to estimate parameter (volatility)

vol <- 0
logret<-diff(log(pastdata)) #past log-returns
vol <- sd(logret) #current volatility
vol
}

VaR_Calculation_Normal<- function(estpar, alpha=0.05){
#This function calculates VaR for different alpha values
#estpar...estimated set of parameters in a vector
#alpha...confidence level

VaR <- 0
a <- length(alpha)
VaR[1:a]<- 1-exp(estpar[1]*qnorm(alpha)) #calculates value at risk for different alpha values
names(VaR) <- alpha
VaR
}

DailyVaRFunc <- function(code="CURRFX/TRYUSD", ndata=730, VaRdate="2006-02-15", alpha=0.05, ParestFunc=ParEst_Normal, VaRFunc=VaR_Calculation_Normal){
#This is the wrapper function to calculate the daily VaR for different values by using parameter estimation and VaR calculation functions
#ndata ...length of pastdata to estimate parameters
#VaRdate ...the day the VaR is calculated for
#lastdate ... last date of the foreign exchange rate data to estimate the parameters
#firstdate ... first date of the foreign exchange rate data to estimate the parameters
#alpha ... confidence level
#ParestFunc ... parameter estimation function for the chosen model
#VaRFunc ... VaR estimation function for the chosen model

lastdate <- as.Date(VaRdate)-1
firstdate <- as.Date(VaRdate)-1-ndata
pastdata <- rev(FXdata_matrix(code, firstdate, lastdate)[,2])
estpar <- ParestFunc(pastdata)
Valueatrisk <- VaRFunc(estpar, alpha)
Valueatrisk
}

alphav <-c(0.05, 0.01, 0.005, 0.001)
DailyVaRFunc(code="CURRFX/TRYUSD", ndata=730, VaRdate="2006-02-15", alpha=alphav, ParestFunc=ParEst_Normal, VaRFunc=VaR_Calculation_Normal)

```

Figure 6.3. The R code for VaR calculation.

of the day for which VaR is calculated (VaRdate), the confidence level (alpha), the parameter estimation function and VaR calculation function of the chosen model, namely, the Normal model for the code given in the figure. The wrapper function utilizes all three functions and the necessary inputs, calculates the daily VaR and returns the daily VaR values for different confidence levels. By using this code, the daily VaR results are obtained for the chosen 20 days and for different confidence levels ($\alpha = 0.005$ and $\alpha = 0.001$).

This naive example is performed not only to quantify the risk but also to examine whether the R code and the modeling methods work without problem. While estimating VaRs, it is seen that this code works well, namely there is no problem in the coding process. We have totally 6 risk modeling methods and five of these (the i.i.d Normal, i.i.d Student's t, Normal-GARCH, unconditional EVT and GARCH filtered EVT) calculated the VaRs for the 20 selected days without problem. However one method, the t-GARCH model, has a problem with the parameter estimation and therefore can not estimate the VaRs for two days out of the 20 days. This indicates that the numerical optimization necessary to calculate the MLE is sometimes problematic for the t-GARCH model. It is possible to use an alternative method such as the GARCH filtered EVT model for such situations. The VaR results of this naive example are shown in Figure 6.4. The table, showing the VaR values for $\alpha = 0.005$ and $\alpha = 0.001$, can be found in the Appendix. The plots of VaR measures for different confidence levels may give some idea about the pattern. Regarding the VaR estimates of the chosen risk models, the values for 14/03/2008 (one of the most riskiest day in the chosen 20 days) show that the static models (i.i.d Normal, i.i.d Student's t and EVT models) do not respond quickly to the changing volatility. This is different for the dynamic models (Normal-GARCH, t-GARCH and GARCH filtered EVT models). The i.i.d Normal, i.i.d Student's t and the unconditional EVT seem to underestimate the risk on the risky days (14/03/2008 and 23/11/2008). Although Normal-GARCH is a dynamic model, it also results in a comparatively small VaR for 14/03/2008. This may be due to the fact that Normal-GARCH does not take into account the heavy tails of the GARCH residuals. It is observed that appropriate increases in the VaR measures of the GARCH filtered EVT and t-GARCH models ensure the market safety in this

risky period (yellow and black lines between the periods 14/03/2008 and 23/11/2008). It is clearly observed that i.i.d Student's t model results in a comparatively high VaR for 09/04/2009 (the red line). This may occur due to the fact that the past data taken from a risky period (2007 and 2008) influence the estimation process negatively. On the other hand, the t -GARCH model can not estimate VaRs for 08/01/2007 and 13/06/2007 due to the problem of parameter estimation so there are no values to be shown in the plot for these days (the black line).

Although the interpretation of the VaR results is done by observing the plots, this is not a formal method to assess the quality of the methods. VaR models are only useful if they can predict risk reasonably well. That is why the application of these models should always be accompanied by validation. Model validation is the general process of checking whether a model is adequate [46]. In this study, it is important to find the method that is sufficient for risk modeling of the exchange rates. Therefore, back testing is used as a model validation method to make a general suggestion about which models are sufficient to use.

6.3. Back Testing

Comparing forecasting performance of competing models is one of the most important aspects of a forecasting exercise. In the previous section, Value at Risk estimates of different models were compared. However, any risk model should be checked for consistency with reality and it is not a robust validation method to evaluate the performance of the models by visual inspection of the VaR measures in a figure. Hence, back testing is chosen to verify the accuracy of the models. This process is the most important part of the study as the main issue is to find the most accurate model for quantifying the risk of exchange rates.

Back testing is a statistical procedure that compare systematically the VaR estimates with actual returns. For instance, if the confidence level used for calculating the daily VaR is 99%, an exception is expected to occur once in every 100 days on average. In order to implement back testing, initially VaR measures are calculated for every

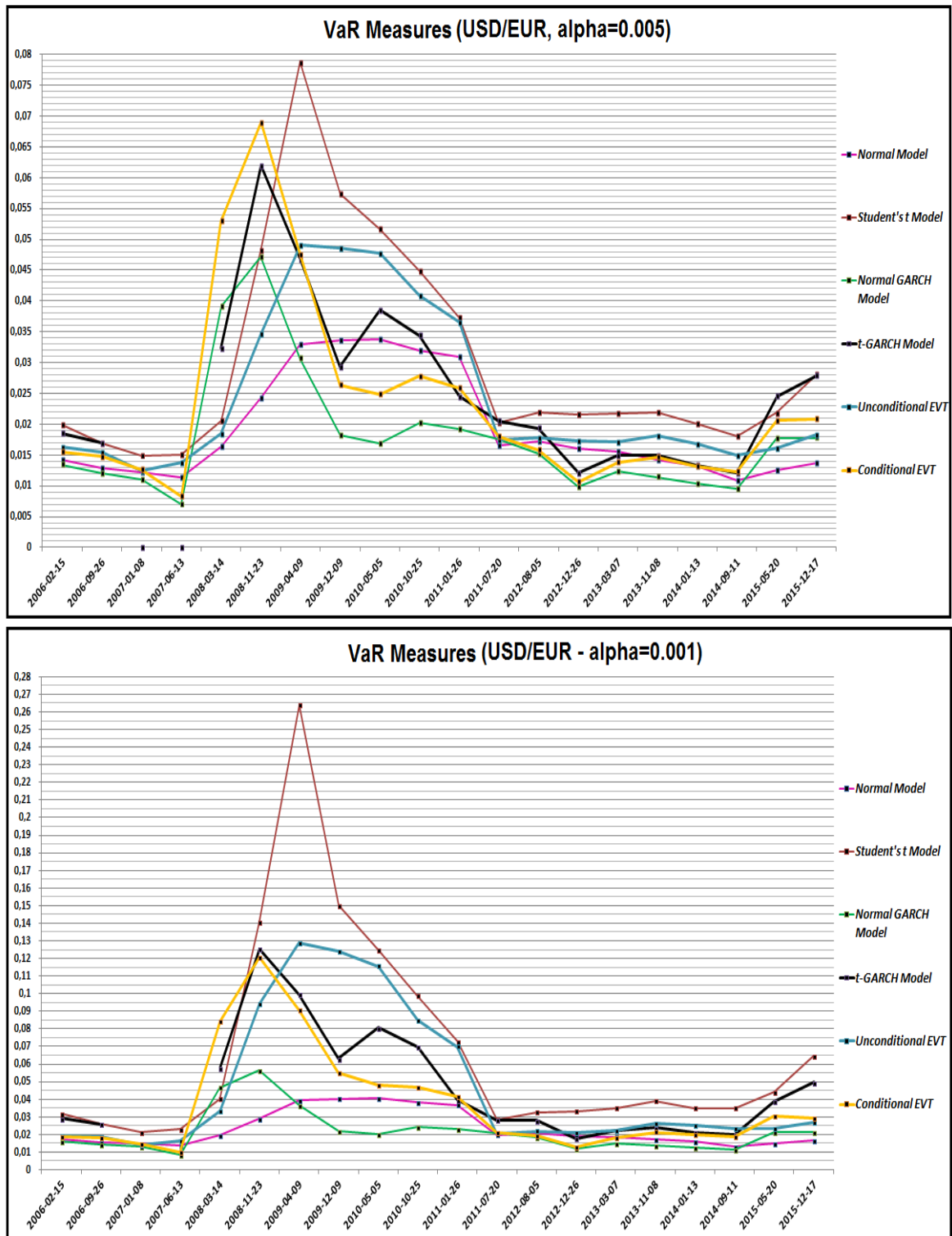


Figure 6.4. Comparison of VaR estimates for different models (USD/EUR).

day in the back testing period by using different risk modeling methods. Then, VaR of each day is compared to the return realized on that day. Whenever a loss exceeds the VaR measure, a violation has occurred and it is counted as a crossing (exceedance). This comparison is done for each day in the back testing period and crossings are counted. In the end, the total number of crossings is obtained. Back testing is useful in identifying the weaknesses of risk quantification models and providing ideas for improvement yet it is not informative about the causes of weaknesses. Models that do not perform well during back testing should have their assumptions and parameter estimates questioned [26, 105, 106].

In this study, the basic frequency (binomial) test, suggested by Kupiec (1995), is utilized. The idea of this method is to test whether the number of losses that exceed VaR is consistent with the confidence level. The information required to implement this test is the total number of observations (n), number of exceedances (x) and confidence level $(1 - \alpha)$ [8]. The sample size (n) is obtained by simply counting the number of observations, α is apparent from the confidence level, and x is obtained by counting the number of observations in which a loss exceeds the VaR. In order to apply back testing, initially, model parameters are estimated and then using the parameters, daily VaRs are calculated and lastly, the comparison of the calculated daily VaR measures with the daily losses is done iteratively. On each day, there either is or is not an exceedance. So the number of exceedances follows only an approximate binomial distribution as the past data used to estimate the daily risk, are almost the same. By utilizing this approximate binomial distribution, we can evaluate the performance of VaR modeling methods. In this part we apply one-sided binomial test. The null hypothesis is that the model is satisfactory against the alternative that the model is not satisfactory. By using the standard commands of R, we calculate the p-values for the number of crossings. $1 - pbinom((noofcrossing - 1), 312, \alpha)$ is used to calculate the p-value for the right tail and $pbinom(noofcrossing, 312, \alpha)$ is used for the left tail. If the calculated p-value is greater than the significance level (0.05), then we can not reject the null hypothesis that the model is satisfactory. However, if the p-value is less than the significance level, then we reject the null hypothesis and conclude that the model is not satisfactory. Too many observed exceedances imply that the model

underestimates risk whereas too few exceedances indicate an overestimation of the risk. The underestimation or overestimation of the risk causes the investor to give wrong decisions about his investment. For instance, the underestimation of the risk shows that the risk is higher than it is measured. Thus, if the investor makes the investment and the true risk materializes, he can face painful losses. On the other hand, if the true risk is overestimated, then most likely the investor does not make this risky investment and in case of actualization of the true risk, the investor may lose the chance to earn. Therefore, both underestimation or overestimation of the risk harms the investor. The decision of the back testing - whether the model underestimates or overestimates the risk or is accurate - is given depending on the probability values given in Table 6.1.

Table 6.1. Decision rule for back testing.

Decision Rule	Decision	Zone
$0.001 \geq pbinom(x, n, \alpha)$	The model overestimates risk.	RED
$0.05 \geq pbinom(x, n, \alpha)$	The model may overestimate risk.	YELLOW
$0.95 \geq pbinom(x, n, \alpha)$	The model is accurate.	GREEN
$0.999 \geq pbinom(x, n, \alpha)$	The model may underestimate risk.	YELLOW
$1 \geq pbinom(x, n, \alpha)$	The model underestimates risk.	RED

6.3.1. Implementation of R code for back testing

In this section, a back testing example is implemented in order to clarify how this method is applied, what is the output and how the results are interpreted. The exchange rate data were taken from Quandl and USD/EUR currency was chosen for this experiment. As in the next section the back testing code will be explained in detail, in this part the main functions of the code are shortly explained. An R code, that estimates the model parameters, calculates the VaR and performs back testing, was written. By using this code, the experiments are performed for each year from 2008 to 2015. In this experiment, after the daily VaR values are calculated in the back testing period, the actual log-returns are compared to the VaR estimates. If the actual loss is greater than the VaR, the code counts this as a crossing. The results of the experiment

are demonstrated in Figure 6.5. The yearly number of crossing values of each model for different confidence levels are the output of the back testing code. With the aim of testing the accuracy of the models, confidence intervals are built for each confidence level ($\alpha = 0.01$, $\alpha = 0.005$ and $\alpha = 0.001$). $1 - pbinom((noofcrossing - 1), 312, \alpha) \leq 0.05$ is used for the upper bound and $pbinom(noofcrossing, 312, \alpha) \leq 0.05$ is used for the lower bound. 312 is the number of days in the back testing period. The results for $\alpha = 0.05$ are presented in the Appendix C.

On the other hand, total back testing results of USD/EUR currency for all models and quantiles are summarized in Table 6.2. In this table, the total back testing results of 8 years are shown. The total number of observations are 2492 days and Target shows the expected number of crossings for the respected confidence levels. For instance, with the confidence level $\alpha = 0.01$, the target is 25 considering the total observations of 2492 days. We applied one-sided binomial test with 95% significance level to test the null hypothesis that VaR model is sufficient against the alternative hypothesis that the model is insufficient. The p-values are also shown in the table. If the p-value is above the desired significance level (5%), then the null hypothesis is not rejected and it is concluded that the VaR model is sufficient. However, if the p-value is below the significance level, we reject the null hypothesis and conclude that the model is not sufficient. Although p-values help to evaluate the performance of the models, we should not totally rest on these. The major drawback of this method is that it is dependent on the sample of actual returns used. We make a back testing example and used consecutive 312 days and assuming that 10 crossings are found. We may find a greater or fewer number of crossings if we use a different 312-day period. Besides, exchange rate series show the volatility clustering pattern and we use consecutive days so we do not have independence. This sample dependence makes it hard to exactly assess the accuracy of the risk models. The p-values given in Table 6.2 must be interpreted with caution.

It is misleading only to examine the total crossings in the whole back testing period (totally 8 years) as it is possible that the risk is clearly underestimated in one time period and overestimated in a different time interval. Therefore, based on

the independence assumption and using the binomial distribution, we calculate a 90% interval containing the number of likely crossings. Then, year by year, the observed number of crossings is compared with that interval. This assessment gives useful results since although the Normal-GARCH model gives sufficient results considering the total crossings in the whole back testing period, it is observed that for the riskiest year (2008), it gives too many crossings compared to the other models. Therefore, by considering the whole back testing period we first check the p-values if they are greater than the significance level ($\alpha = 0.05$) and then check whether the number of crossings in each year is within the expected bounds or not.

Table 6.2. Total number of crossings and p-values (USD/EUR).

	$\alpha = 0.05$	p-val.	$\alpha = 0.01$	p-val.	$\alpha = 0.005$	p-val.	$\alpha = 0.001$	p-val.
Target	125		25		13		3	
i.i.d Normal	91	0.00076	42	0.00105	31	$6.52 \cdot 10^{-6}$	19	$3.1 \cdot 10^{-12}$
i.i.d Student's t	132	0.2603	21	0.251	10	0.3003	4	0.1077
Normal-GARCH	78	$3.11 \cdot 10^{-6}$	30	0.1767	22	0.00463	12	$1.2 \cdot 10^{-5}$
t-GARCH	113	0.1536	19	0.1357	10	0.3003	6	0.0414
Unconditional EVT	138	0.1189	36	0.021	15	0.2707	9	0.0011
GARCH filt. EVT	113	0.1536	22	0.3221	14	0.3677	8	0.0041

Considering all the results presented in Table 6.2 and Figure 6.5, it can be said that i.i.d Normal and Normal-GARCH models clearly do not work well. Although i.i.d Student's t model does not take into account the time-varying volatility, the results show that it is sufficient especially for the extreme quantiles ($\alpha = 0.005$ and $\alpha = 0.001$). Moreover, regarding the t-GARCH, unconditional EVT and GARCH filtered EVT models, these models work well with the USD/EUR exchange rate data.

6.4. Main Back Testing Experiment

The goal of this study is to quantify the risk of the exchange rates by using different risk modeling methods, to check the validity of each model by back testing and to find the most accurate one in the chosen methods by interpreting the back testing results. In this section, the details of the experiments are explained and their

		USD/EUR exchange rate (alpha= 0.01)											
		i.i.d. Normal		i.i.d Student's t		Normal-GARCH		t-GARCH		Unconditional EVT		GARCH filt. EVT	
The Year of Back Testing	Interval for # of crossings	No of crossing	Decision	No of crossing	Decision	No of crossing	Decision	No of crossing	Decision	No of crossing	Decision	No of crossing	Decision
2015	[1,7]	16	Too many crossings	4	Accurate	5	Accurate	3	Accurate	11	Too many crossings	5	Accurate
2014		2	Accurate	1	Accurate	4	Accurate	2	Accurate	1	Accurate	2	Accurate
2013		2	Accurate	1	Accurate	5	Accurate	2	Accurate	2	Accurate	2	Accurate
2012		2	Accurate	1	Accurate	2	Accurate	2	Accurate	2	Accurate	2	Accurate
2011		4	Accurate	3	Accurate	5	Accurate	2	Accurate	4	Accurate	3	Accurate
2010		0	Too few crossings	0	Too few crossings	2	Accurate	0	Too few crossings	0	Accurate	1	Accurate
2009		1	Accurate	0	Too few crossings	1	Accurate	1	Accurate	1	Accurate	1	Accurate
2008		15	Too many crossings	11	Too many crossings	6	Accurate	7	Accurate	15	Too many crossings	6	Accurate
		USD/EUR exchange rate (alpha=0.005)											
		i.i.d. Normal		i.i.d Student's t		Normal-GARCH		t-GARCH		Unconditional EVT		GARCH filt. EVT	
The Year of Back Testing	Interval for # of crossings	No of crossing	Decision	No of crossing	Decision	No of crossing	Decision	No of crossing	Decision	No of crossing	Decision	No of crossing	Decision
2015	[0,4]	10	Too many crossings	2	Accurate	5	Too many crossings	1	Accurate	4	Accurate	3	Accurate
2014		1	Accurate	0	Accurate	2	Accurate	0	Accurate	0	Accurate	0	Accurate
2013		2	Accurate	1	Accurate	3	Accurate	2	Accurate	1	Accurate	2	Accurate
2012		2	Accurate	0	Accurate	2	Accurate	1	Accurate	1	Accurate	2	Accurate
2011		3	Accurate	1	Accurate	2	Accurate	0	Accurate	2	Accurate	1	Accurate
2010		0	Accurate	0	Accurate	1	Accurate	0	Accurate	0	Accurate	0	Accurate
2009		1	Accurate	0	Accurate	1	Accurate	0	Accurate	0	Accurate	0	Accurate
2008		12	Too many crossings	6	Too many crossings	6	Too many crossings	6	Too many crossings	7	Too many crossings	6	Too many
		USD/EUR exchange rate (alpha= 0.001)											
		i.i.d. Normal		i.i.d Student's t		Normal-GARCH		t-GARCH		Unconditional EVT		GARCH filt. EVT	
The Year of Back Testing	Interval for # of crossings	No of crossing	Decision	No of crossing	Decision	No of crossing	Decision	No of crossing	Decision	No of crossing	Decision	No of crossing	Decision
2015	[0,2]	8	Too many crossings	0	Accurate	1	Accurate	0	Accurate	2	Accurate	1	Accurate
2014		1	Accurate	0	Accurate	1	Accurate	0	Accurate	0	Accurate	0	Accurate
2013		1	Accurate	0	Accurate	2	Accurate	1	Accurate	1	Accurate	2	Accurate
2012		0	Accurate	0	Accurate	1	Accurate	0	Accurate	0	Accurate	1	Accurate
2011		1	Accurate	0	Accurate	0	Accurate	0	Accurate	1	Accurate	0	Accurate
2010		0	Accurate	0	Accurate	0	Accurate	0	Accurate	0	Accurate	0	Accurate
2009		0	Accurate	0	Accurate	1	Accurate	0	Accurate	0	Accurate	0	Accurate
2008		8	Too many crossings	4	Too many crossings	6	Too many crossings	5	Too many crossings	5	Too many crossings	4	Too many

Figure 6.5. Comparison of yearly back testing results (USD/EUR).

results will be given.

6.4.1. Experimental design

The statistical software, R, is used to perform the experiments. The R code, that is written for this study, consists of three parts, the parameter estimation part, the VaR calculation part and the back testing part. The first and second part of the code, parameter estimation and VaR calculation functions, were explained in section 6.2. If the Normal or Student's t approach is used for risk modeling, there is no need to use an extra package to estimate the parameters. The functions, *ParEst_Normal* and *ParEst_Student.t* use standard commands of R to estimate the model parameters. The parameter estimation functions, *ParEst_Garch*, *ParEst_Garch_Student.t*, *ParEst_EVT* and *ParEst_CONDEVT* need *fGarch* package for estimating the GARCH parameters and *fExtremes* package for estimating the parameters of EVT. We use our simple code *t.Loglike* while estimating the parameters of Student's t model. The execution time of *ParEst_Student.t* is much longer than the execution times of the other parameter estimation functions since we did not try to optimize the code with respect to execution time.

The second part includes the functions for VaR calculation. In order to calculate the VaR for EVT and GARCH filtered EVT models, the number of exceedances over the threshold value (*nth*) should also be given as input. The parameter estimation part contains six functions, each of which is used for the respective method. Unlike for the parameter estimation part, there are four functions in the VaR calculation part, *VaR_Calculation_Normal*, *VaR_Calculation_t*, *VaR_Calculation_EVT* and *VaR_Calculation_CONDEVT*. The Normal model and Normal-GARCH model use the same VaR calculation function, *VaR_Calculation_Normal* due to the normality assumption and Student's t and t-GARCH models use the same VaR calculation function due to the Student's t distributed assumption. The outputs of these functions are the calculated VaRs for different confidence levels.

The last part contains the main function, *Backtesting_daily*. This function works as a wrapper function which does not implement the actual functionality itself, but rather calls other functions with little computation. *Backtesting_daily* calls the parameter estimation and VaR calculation functions of the chosen model. Besides, the functions *countcrossing* and *crossing* are also called by the wrapper function to calculate the number of crossings and give the decision about the performance of the model for different confidence levels.

Backtesting_daily(*ndata* = 730, *code* = "CURRFX/USDEUR", *firstdate* = "2000 - 01 - 01", *lastdate* = "2015 - 12 - 31", *sdate* = "2008 - 01 - 01", *edate* = "2008 - 12 - 31", *alpha* = *alphav*, *ParEstFunc* = *ParEst_Normal*, *VaRCalcFunc* = *VaR_Calculation_Normal*) is the interface of the wrapper function. *Backtesting_daily* takes the past data for the chosen period from Quandl by using *FXdata_matrix* function. It is necessary to know Quandl's currency code to extract data from the data base. *Backtesting_daily* takes all the necessary inputs such as the number of data to use for parameter estimation, Quandl currency code to extract the past data, the start and end dates of the back testing period, the confidence level, the names of the parameter estimation and VaR calculation functions ... etc.

Initially, *Backtesting_daily* estimates the model parameters by using the past data. Although we hand over all the data in the wrapper function, while estimating the parameters, it is guaranteed that parameter estimation is performed by using just the necessary past data, not all data. The back testing function calculates many parameters in the back testing period and it is possible to use wrong data while estimating the model parameters. Therefore, it is crucial to hand over only the past data of the required period in order to obtain true parameter estimates and prevent wrong VaR calculations and inaccurate back testing results. After the parameter estimation process of the chosen model, the wrapper function calls the VaR calculation function and calculates the daily VaR. After obtaining all the VaR values in the back testing period, the real log-returns and the VaR measures are compared. If the loss is greater than the estimated VaR, *countcrossing* function counts this as a crossing. After the number of exceedances are obtained, *crossing* function is used to check the number of crossings

and evaluate the performance of the methods by using Table 6.1. The outputs of *Backtesting_daily* function are the number of crossings and the decision about the accuracy of the model for each confidence level, the daily real log-returns and daily VaR estimates for each confidence level. The R code, that is used to make experiments in this study, is given in Appendix A.

6.4.2. Assessment of the forecasting performances

Back testing experiments are performed for different currencies. As Euro and Dollar are significant for Turkey, in this section back testing results for only TRY/USD and TRY/EUR are presented. While assessing the performances, the results, seen in Table 6.2 and Figure 6.5, are also considered. The back testing period is from 01/01/2008 to 31/12/2015, a total of eight years. It is considered that, being a highly volatile year, 2008 should be in the back testing period to clearly observe the performances of the risk models. The total number of crossings in eight years and their p-values of TRY/USD and TRY/EUR are given in Table 6.3 and 6.4 respectively. Moreover, the yearly back testing results of TRY/USD and TRY/EUR are shown in Figures 6.6 and 6.7 respectively. All back testing results for the other currency pairs can be seen in Appendix D.

The risk model is useful for anybody provided that it gives satisfactory results for the volatile periods. Therefore, while making some comments about the models, the results of 2008 is used. Initially, considering all the results for the Normal model, it can be said that Normal model never works good. The tables, which contain the total number of crossings and p-values, indicate very low p-values that leads to rejecting the null hypothesis that the model is acceptable. Therefore, Normal model is not a satisfactory risk model for this study. Likewise, the results of the Normal-GARCH model shows that this model does not give good results either. In Table 6.3 and 6.4, low p-values are observed for Normal-GARCH. Thus, we reject the null hypothesis that the Normal-GARCH model is acceptable. Normal-GARCH model is not a satisfactory risk model for this study.

Table 6.3. Total number of crossings and p-values (TRY/USD).

	$\alpha = 0.05$	p-val.	$\alpha = 0.01$	p-val.	$\alpha = 0.005$	p-val.	$\alpha = 0.001$	p-val.
Target	125		25		13		3	
i.i.d Normal	147	0.0242	62	$2.29 \cdot 10^{-10}$	44	$2.63 \cdot 10^{-12}$	26	0
i.i.d Student's t	176	$4.66 \cdot 10^{-6}$	21	0.251	7	0.0708	0	0.08264
Normal-GARCH	162	$5.5 \cdot 10^{-4}$	72	$8.43 \cdot 10^{-15}$	54	0	28	0
t-GARCH	176	$4.66 \cdot 10^{-6}$	34	0.0472	14	0.3676	2	0.5458
Unconditional EVT	136	0.1582	27	0.3641	14	0.3676	4	0.2407
GARCH filt. EVT	124	0.5018	31	0.1319	17	0.1276	2	0.5458

		TRY/USD exchange rate (alpha= 0.01)											
		i.i.d. Normal		i.i.d Student's t		Normal-GARCH		t-GARCH		Unconditional EVT		GARCH filt. EVT	
The Year of Back Testing	Interval for # of crossings	No of crossing	Decision	No of crossing	Decision	No of crossing	Decision	No of crossing	Decision	No of crossing	Decision	No of crossing	Decision
2015	[1,7]	11	Too many crossings	4	Accurate	12	Too many crossings	6	Accurate	4	Accurate	6	Accurate
2014		7	Accurate	3	Accurate	10	Too many crossings	5	Accurate	4	Accurate	4	Accurate
2013		6	Accurate	3	Accurate	11	Too many crossings	7	Accurate	3	Accurate	6	Accurate
2012		0	Too few crossings	1	Accurate	3	Accurate	0	Too few crossings	0	Too few crossings	0	Too few crossings
2011		11	Too many crossings	3	Accurate	11	Too many crossings	6	Accurate	3	Accurate	3	Accurate
2010		1	Accurate	0	Too few crossings	8	Too many crossings	3	Accurate	0	Too few crossings	3	Accurate
2009		4	Accurate	0	Too few crossings	5	Accurate	1	Accurate	0	Too few crossings	2	Accurate
2008		22	Too many crossings	7	Accurate	12	Too many crossings	6	Accurate	13	Too many crossings	7	Accurate
		TRY/USD exchange rate (alpha= 0.005)											
		i.i.d. Normal		i.i.d Student's t		Normal-GARCH		t-GARCH		Unconditional EVT		GARCH filt. EVT	
The Year of Back Testing	Interval for # of crossings	No of crossing	Decision	No of crossing	Decision	No of crossing	Decision	No of crossing	Decision	No of crossing	Decision	No of crossing	Decision
2015	[0,4]	9	Too many crossings	1	Accurate	9	Too many crossings	2	Accurate	2	Accurate	3	Accurate
2014		6	Too many crossings	1	Accurate	6	Too many crossings	2	Accurate	2	Accurate	2	Accurate
2013		6	Too many crossings	1	Accurate	8	Too many crossings	4	Accurate	1	Accurate	4	Accurate
2012		0	Accurate	0	Accurate	0	Accurate	0	Accurate	0	Accurate	0	Accurate
2011		4	Accurate	1	Accurate	8	Too many crossings	2	Accurate	1	Accurate	1	Accurate
2010		1	Accurate	0	Accurate	7	Too many crossings	1	Accurate	0	Accurate	2	Accurate
2009		2	Accurate	0	Accurate	4	Accurate	0	Accurate	0	Accurate	2	Accurate
2008		16	Too many crossings	3	Accurate	12	Too many crossings	3	Accurate	8	Too many crossings	3	Accurate
		TRY/USD exchange rate (alpha= 0.001)											
		i.i.d. Normal		i.i.d Student's t		Normal-GARCH		t-GARCH		Unconditional EVT		GARCH filt. EVT	
The Year of Back Testing	Interval for # of crossings	No of crossing	Decision	No of crossing	Decision	No of crossing	Decision	No of crossing	Decision	No of crossing	Decision	No of crossing	Decision
2015	[0,2]	3	Too many crossings	0	Accurate	4	Too many crossings	1	Accurate	1	Accurate	1	Accurate
2014		3	Too many crossings	0	Accurate	3	Too many crossings	0	Accurate	1	Accurate	0	Accurate
2013		3	Too many crossings	0	Accurate	4	Too many crossings	1	Accurate	0	Accurate	1	Accurate
2012		0	Accurate	0	Accurate	0	Accurate	0	Accurate	0	Accurate	0	Accurate
2011		2	Accurate	0	Accurate	4	Too many crossings	0	Accurate	0	Accurate	0	Accurate
2010		0	Accurate	0	Accurate	3	Too many crossings	0	Accurate	0	Accurate	0	Accurate
2009		1	Accurate	0	Accurate	3	Too many crossings	0	Accurate	0	Accurate	0	Accurate
2008		14	Too many crossings	0	Accurate	7	Too many crossings	0	Accurate	2	Accurate	0	Accurate

Figure 6.6. Comparison of yearly back testing results (TRY/USD).

Table 6.4. Total number of crossings and p-values (TRY/EUR).

	$\alpha = 0.05$	p-val.	$\alpha = 0.01$	p-val.	$\alpha = 0.005$	p-val.	$\alpha = 0.001$	p-val.
Target	125		25		13		3	
i.i.d Normal	134	0.2053	56	$4.93 \cdot 10^{-8}$	43	$9.49 \cdot 10^{-12}$	23	$4.33 \cdot 10^{-15}$
i.i.d Student's t	163	$4.07 \cdot 10^{-4}$	26	0.4407	10	0.3003	1	0.2888
Normal-GARCH	143	0.0522	71	$2.51 \cdot 10^{-14}$	51	$2.2 \cdot 10^{-16}$	30	0
t-GARCH	137	0.1375	25	0.5206	7	0.0708	3	0.4542
Unconditional EVT	134	0.2053	31	0.1319	20	0.0294	6	0.0414
GARCH filt. EVT	144	0.0435	37	0.0135	18	0.0818	7	0.0139

		TRY/EUR exchange rate (alpha= 0.01)											
		i.i.d. Normal		i.i.d Student's t		Normal-GARCH		t-GARCH		Unconditional EVT		GARCH filt. EVT	
The Year of Back Testing	Interval for # of crossings	No of crossing	Decision	No of crossing	Decision	No of crossing	Decision	No of crossing	Decision	No of crossing	Decision	No of crossing	Decision
2015	[1,7]	14	Too many crossings	5	Accurate	14	Too many crossings	7	Accurate	6	Accurate	8	Too many
2014		8	Too many crossings	5	Accurate	11	Too many crossings	7	Accurate	5	Accurate	7	Accurate
2013		9	Too many crossings	6	Accurate	14	Too many crossings			6	Accurate	8	Too many
2012		0	Too few crossings	0	Too few crossings	1	Accurate	0	Too few crossings	0	Too few crossings	0	Too few crossings
2011		5	Accurate	1	Accurate	11	Too many crossings	2	Accurate	1	Accurate	1	Accurate
2010		0	Too few crossings	0	Too few crossings	7	Too many crossings	3	Accurate	0	Too few crossings	4	Accurate
2009		1	Accurate	0	Too few crossings	5	Accurate	1	Accurate	0	Too few crossings	1	Accurate
2008		19	Too many crossings	9	Too many crossings	8	Too many crossings	5	Accurate	13	Too many crossings	6	Accurate
		TRY/EUR exchange rate (alpha= 0.005)											
		i.i.d. Normal		i.i.d Student's t		Normal-GARCH		t-GARCH		Unconditional EVT		GARCH filt. EVT	
The Year of Back Testing	Interval for # of crossings	No of crossing	Decision	No of crossing	Decision	No of crossing	Decision	No of crossing	Decision	No of crossing	Decision	No of crossing	Decision
2015	[0,4]	11	Too many crossings	0	Accurate	9	Too many crossings	0	Accurate	5	Too many crossings	3	Accurate
2014		7	Too many crossings	2	Accurate	8	Too many crossings	2	Accurate	3	Accurate	2	Accurate
2013		6	Too many crossings	3	Accurate	10	Too many crossings			4	Accurate	6	Too many
2012		0	Accurate	0	Accurate	0	Accurate	0	Accurate	0	Accurate	0	Accurate
2011		2	Accurate	0	Accurate	8	Too many crossings	0	Accurate	0	Accurate	0	Accurate
2010		0	Accurate	0	Accurate	6	Too many crossings	0	Accurate	0	Accurate	1	Accurate
2009		1	Accurate	0	Accurate	4	Accurate	0	Accurate	0	Accurate	1	Accurate
2008		16	Too many crossings	5	Too many crossings	6	Too many crossings	5	Too many crossings	8	Too many crossings	5	Too many
		TRY/EUR exchange rate (alpha= 0.001)											
		i.i.d. Normal		i.i.d Student's t		Normal-GARCH		t-GARCH		Unconditional EVT		GARCH filt. EVT	
The Year of Back Testing	Interval for # of crossings	No of crossing	Decision	No of crossing	Decision	No of crossing	Decision	No of crossing	Decision	No of crossing	Decision	No of crossing	Decision
2015	[0,2]	5	Too many crossings	0	Accurate	5	Too many crossings	0	Accurate	0	Accurate	0	Accurate
2014		4	Too many crossings	1	Accurate	4	Too many crossings	1	Accurate	2	Accurate	1	Accurate
2013		4	Too many crossings	0	Accurate	5	Too many crossings			2	Accurate	4	Too many
2012		0	Accurate	0	Accurate	0	Accurate	0	Accurate	0	Accurate	0	Accurate
2011		1	Accurate	0	Accurate	4	Too many crossings	0	Accurate	0	Accurate	0	Accurate
2010		0	Accurate	0	Accurate	4	Too many crossings	0	Accurate	0	Accurate	0	Accurate
2009		0	Accurate	0	Accurate	2	Accurate	0	Accurate	0	Accurate	0	Accurate
2008		9	Too many crossings	0	Accurate	6	Too many crossings	2	Accurate	2	Accurate	2	Accurate

Figure 6.7. Comparison of yearly back testing results (TRY/EUR).

On the other hand, considering the results of Student's t model in the tables, mostly the p -values are greater than the significance level (0.05) so we can not reject the null hypothesis that the model is acceptable. Student's t model is also a satisfactory risk model for this study. The results for the t -GARCH model show that the model generally works good. p -values are high enough to conclude that we can not reject the null hypothesis. t -GARCH is a satisfactory risk model in this study. The drawback of this model is that the optimization for the MLE is problematic so we can not always get results by using this model yet if the model can estimate the parameters, it quantifies the risk satisfactorily. Regarding the results of EVT, it can be said that the method works good for the extreme quantiles ($\alpha = 0.005$ and $\alpha = 0.001$). However, for $\alpha = 0.01$ the performance of the model is not as good as t -GARCH model. The p -values in the tables show that the model is acceptable. Therefore, EVT is a satisfactory risk model in this study. Lastly, the results of GARCH filtered EVT are considered. High p -values indicate that the model is acceptable. GARCH filtered EVT is a satisfactory model for this study.

The risk model is useful for anybody provided that it gives satisfactory results for the volatile periods. Therefore, while making some comments about the models, the results of 2008 is used. Initially, considering all the results for the Normal model, it can be said that Normal model never works good. The tables, which contain the total number of crossings and p -values, indicate very low p -values that leads to rejecting the null hypothesis that the model is acceptable. Therefore, Normal model is not a satisfactory risk model for this study. Likewise, the results of the Normal-GARCH model shows that this model does not give good results either. In Table 6.3 and 6.4, low p -values are observed for Normal-GARCH. Thus, we reject the null hypothesis that the Normal-GARCH model is acceptable. Normal-GARCH model is not a satisfactory risk model for this study. On the other hand, considering the results of Student's t model in the tables, mostly the p -values are greater than the significance level (0.05) so we can not reject the null hypothesis that the model is acceptable. Student's t model is also a satisfactory risk model for this study. The results for the t -GARCH model show that the model generally works good. p -values are high enough to conclude that we can not reject the null hypothesis. t -GARCH is a satisfactory risk model in this

study. The drawback of this model is that the optimization for the MLE is problematic so we can not always get results by using this model yet if the model can estimate the parameters, it quantifies the risk satisfactorily. Regarding the results of EVT, it can be said that the method works good for the extreme quantiles ($\alpha = 0.005$ and $\alpha = 0.001$). However, for $\alpha = 0.01$ the performance of the model is not as good as t-GARCH model. The p-values in the tables show that the model is acceptable. Therefore, EVT is a satisfactory risk model in this study. Lastly, the results of GARCH filtered EVT are considered. High p-values indicate that the model is acceptable. GARCH filtered EVT is a satisfactory model for this study.

Moreover, regarding the figures that show yearly number of crossings, it can be said that in 2008, Normal model always underestimates risk (totally 12 observations) as the number of crossings is too high. Besides, the number of crossing values are also too high for the Normal-GARCH model which indicates underestimation. 12 observations are obtained for 2008 and in 8 cases the Normal-GARCH model underestimates the risk. On the other hand, considering the results of Student's t model, it can be said that Student's t model better models the risk than the Normal and Normal-GARCH models in the calm periods. However, in 2008, the performance of this model is not better than the Normal-GARCH model. This model, just as the Normal-GARCH, underestimates the risk in 8 observations in 2008. t-GARCH model, in 2008, underestimates the risk in 3 observations and in 9 cases, it gives good results. Unconditional EVT can not give satisfactory results as it underestimates the risk in 10 observations and works good in 2 observations. Taking into account the results of the risky years, it can be said that GARCH filtered EVT models the risk better than the other models. In 2008, the performances of t-GARCH and GARCH filtered EVT are alike. GARCH filtered EVT also underestimates the risk of 2008 in 3 cases.

6.4.3. Recommendation which model to use

In this study, six different risk models were used each of which has advantages and disadvantages. Obviously, the Normal model is the simplest model in this study yet it is observed that it is not a satisfactory risk model.

Regarding the risk quantification quality of the model, GARCH filtered EVT or t-GARCH models are suggested. Although GARCH filtered EVT or t-GARCH give good results, the implementation is difficult without using R together with the *fGarch* and *fExtremes* packages. Anybody who knows how to use R, can learn how to use these packages in a short time. Thus, considering the ease of package learning and usage, both of them are easy to use. Nevertheless, while using the package, we faced some problems and it is not easy to find the solution as we do not know what is happening in the back. If R or extra packages are not used in a study, the suggested method should give satisfactory results and should not be complicated. Therefore, i.i.d Student's t model may be the method of choice for users who do not use R as the parameter estimation for this model is easy and the results are satisfactory.

7. A SMALL INVESTMENT EXAMPLE

In this chapter, it is explained how this study can be used practically in the daily life by presenting a simple investment example. Assuming a worker who is willing to go to Barcelona with his family in his yearly vacation, makes the next year's holiday plan. According to his calculations, he needs €10,000 in July, 2017. The current exchange rate, EUR/TRY, is 3.30 and he owns 33,000 TL in his TL bank account. As EUR/TRY is highly volatile in these days, he should manage the risk as good as he can in order to earn more. He has two alternatives, one of which is to convert his TL into €10,000 and invest this money into a euro bank account which gives 1% annual interest. The second alternative is to convert half of his money into euro, €5,000, and invest it into a currency bank account which gives 1% annual interest and invest the other half, 16,500 TL, into a TL bank account which gives 10% annual interest. A third alternative is to put all 33,000 TL on a TL bank account. It is assumed that the bank accounts have the property that when you withdraw your money, you will not lose the interest you earn until that time, like the orange account in ING Bank. The first alternative is not risky and he owns the amount he needs without the risk of a loss. However, he only earns €100 interest if he chooses this alternative. With the aim of earning more interest, he decides to choose the second alternative. As EUR/TRY is highly volatile in these days, he does not want to take more risk by investing all of his money to the TL account so he guarantees half of the money he needs for the holiday and also earns €50 interest. If TL strengthens against euro, he will earn not only the annual interest but also the foreign exchange gain. However, if euro strengthens against TL and he does not convert his money to euro, he will earn the annual interest yet lose the amount due to the change in the exchange rate. If at the end of one year the foreign exchange loss is not more than 10%, he can bear this loss yet he needs extra money if the loss is more than 10%. Considering his situation, he can use the R code of this study and quantify the daily risk of his investment using the most recent exchange rate data. Hence, by taking the probable daily risk into consideration, he can decide whether to convert his money to euro or stay in the TL on the next day.

8. CONCLUSION

In this study, we considered to quantify the daily risk of 9 different currency pairs, TRY/USD, TRY/EUR, USD/EUR, TRY/AUD, TRY/GBP, TRY/RUB, USD/BRL, USD/EGP and USD/JPY. Value at Risk was used as the risk measure and risk quantification was done by implementing six different risk models to the exchange rate log-returns, the i.i.d Normal, i.i.d Student's t, Normal-GARCH, t-GARCH, unconditional EVT and GARCH filtered EVT. The risk quantification performances of these models were evaluated by an extensive back testing study. The statistical software, R, was used to write the code, composed of three main parts -estimation of the model parameters, VaR calculation and back testing- to implement all these methods to real data. All the results were obtained by using this code for the desired currency. The backtesting results of TRY/USD, TRY/EUR and USD/EUR, which are significant currencies for Turkey, are presented in Chapter 6 and Appendices B & C. Besides, additional experiments are performed for the currency pairs TRY/AUD, TRY/GBP, TRY/RUB, USD/BRL, USD/EGP and USD/JPY. The back testing results of these six currency pairs are presented in Appendix D.

Combining the results of Chapter 6 and Appendices C & D, it was observed that all the risk modeling methods can be implemented to exchange rate data by using R. Each model can give the one-day-ahead VaR results although some of the models overestimate or underestimate the daily risk in certain situations. Considering the currency pairs, the results are similar with the exception of USD/EGP. Thus, we do not need to recommend a different model for each of the currency pairs. Back testing of the one-day-ahead VaR estimations for several α values revealed that the risk modeling methods that rely on the normality assumption, whether it is the i.i.d Normal or Normal-GARCH model, result in significant underestimation of daily VaR. Although Normal-GARCH can model the time-varying volatility, the overall performance of this method is not any better than the i.i.d Normal. The results of this study show that modeling exchange rate returns with just a heavy-tailed distribution gives acceptable results; it turned out that the performance of i.i.d Student's t is better than that of

Normal-GARCH. As the quantile, at which VaR is estimated, becomes more extreme, the importance of heavy tail modeling increases. Back testing results indicate that heavy tailed distributions, i.i.d Student's t and t-GARCH, can model the VaR better at the quantiles $\alpha = 0.005$ and $\alpha = 0.001$ than $\alpha = 0.01$ and $\alpha = 0.05$. Besides, dynamic volatility modeling can also significantly improve the performance of the model in case of considering the heavy tails. Though the performance of the Normal-GARCH model is not satisfactory, the t-GARCH model gives good VaR estimates. However, in this study we performed 72 experiments using t-GARCH model and in 4 of them, the t-GARCH model does not give results due to the problems in the numerical optimization necessary to calculate the MLE of t-GARCH. This clearly limits the practical value of that model. As expected, it was observed that the most sophisticated methods - GARCH filtered EVT and t-GARCH - are best in quantifying the risk. However, the back testing results of USD/EGP, which is the riskiest currency pair in this study, showed that only GARCH filtered EVT is satisfactory to model the daily risk of this currency. Therefore, it can be concluded that GARCH filtered EVT showed the best overall performance.

Considering the practical implementation of the different methods, we observed that without using R and its packages, the implementation of the sophisticated methods, based on EVT and t-GARCH is difficult and requires a lot of coding. Therefore, the i.i.d Student's t model can be the method of choice for a user, who does not utilize R yet wants to get satisfactory risk estimates without too much coding. However, it should be emphasized that if the economic situations of two countries are different such as USA and Egypt, the distribution of the currency pair that belongs to these countries, are clearly non-symmetric so in such situations GARCH filtered EVT works good and i.i.d Student's t is not satisfactory. Nevertheless, if the currency pairs belong to the countries which have strong economies such as USA and the countries in the E.U., i.i.d Student's t works good. This is owing to the fact that the distribution of these currency pairs are close to symmetric.

Summarizing, it can be said that sophisticated methods, such as GARCH filtered EVT or t-GARCH, can be used to quantify the risk of exchange rates. The quality of

the risk estimates of these models was clearly underlined in our experiment. Nevertheless, if the use of R is not desired, the i.i.d Student's t model can be recommended.

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APPENDIX A: R CODES

```

FXdata_matrix <- function(code, firstdate, lastdate){
  #This function takes FX data from Quandl
  fxdata <- Quandl(code,start_date=firstdate,end_date=lastdate)
  fxdata }

#####
#THIS PART INCLUDES THE PARAMETER ESTIMATION FUNCTIONS
PEst_Garch_Normal <- function(pastdata){
  #This function calculates the parameters for Garch Model
  #pastdata...a vector of past data to estimate parameters
  logret<-diff(log(pastdata))
  prmtrs <- garchFit(~garch(1,1), data = logret, trace = FALSE)
  vol <- volatility(prmtrs, type = "sigma")
  res <- c(vol[length(vol)], prmtrs@fit$coef[2], prmtrs@fit$coef[3],
  prmtrs@fit$coef[4])
  names(res) <- c("volatility", "omega", "alpha", "beta")
  res }
PEst_Garch_t <- function(pastdata){
  #This function calculates the parameters for Garch Model
  #pastdata...a vector of past data to estimate parameters
  logret<-diff(log(pastdata))
  prmtrs <- garchFit(~garch(1,1), data=logret, cond.dist="std", trace=FALSE,
  include.shape=TRUE)

```

Figure A.1. R Codes for parameter estimation.

```

vol <- volatility(prmtrs, type="sigma")
res <- c(vol[length(vol)], prmtrs@fit$coef[2], prmtrs@fit$coef[3],
prmtrs@fit$coef[4],
prmtrs@fit$coef[5], prmtrs@fit$coef[1])
names(res) <- c("volatility", "omega", "alpha", "beta", "dof", "loc")
res }

ParEst_Normal <- function(pastdata){
#This function calculates volatility
#pastdata...a vector of past data to estimate parameter (volatility)
vol <- 0
logret<-diff(log(pastdata)) #past log-returns
vol <- sd(logret) #current volatility
vol }

ParEst_Garch <- function(pastdata){
#This function estimates the parameters (volatility and garch parameters) for
GARCH Model with Normal distributed logarithmic returns
#pastdata...a vector of past data to estimate parameters and calculate volatility
var <- 0
vol <- 0
logret<-diff(log(pastdata)) #vector of past log-returns from the historical data
omega <- PEst_Garch_Normal(pastdata)[2] #garch parameters
alph1 <- PEst_Garch_Normal(pastdata)[3] #garch parameters
beta <- PEst_Garch_Normal(pastdata)[4] #garch parameters
vol <- PEst_Garch_Normal(pastdata)[1] #conditional volatility
res <- c(vol, omega, alph1, beta)
names(res) <- c("volatility", "omega", "alpha", "beta")
res }

```

Figure A.1. R Codes for parameter estimation (cont.).

```

t.Loglike <- function (x, t.vector) {
  #loglikelihood function of Student's t distribution
  v = x[1] #degrees of freedom
  a = x[2] #location parameter
  b = x[3] #scale parameter
  f.sum = 0 #initialize
  for(i in 1:length(t.vector)){
    f.sum[i] = log((1/b) * dt(((t.vector[i] - a)/b), v))
  }
  f.sum.all = 0 - sum(f.sum) #in order to get a minimization for optim function
  return(f.sum.all)}

ParEst_Stud.t <- function(pastdata) {
  #This function estimates the parameters by maximum likelihood estimation
  method (scale and degrees of freedom)
  #pastdata...a vector of past data to estimate parameters
  vol <- 0
  logret<-diff(log(pastdata)) #vector of past log-returns from the historical data
  MLE_t <- optim(c(7,0,(sqrt(5/7)*sd(logret))), t.Loglike, t.vector = logret) #pa-
  rameter estimation by MLE method
  vol<- sqrt(MLE_t$par[3]) * sqrt(MLE_t$par[1]/(MLE_t$par[1] - 2))
  res <- c(vol, MLE_t$par[1], 0 , MLE_t$par[3]) #location parameter is assumed to
  be zero otherwise MLE_t$par[2] is used as location parameter
  names(res) <- c("volatility", "degrees of freedom", "location", "scale")
  res }

```

Figure A.1. R Codes for parameter estimation (cont.).

```

ParEst_Garch_Stud.t <- function(pastdata){
  #This function estimates the parameters (volatility and garch parameters) for
  GARCH Model with scaled Student's t distributed logarithmic returns
  #pastdata...a vector of past data to estimate parameters and calculate volatility
  logret<-diff(log(pastdata)) #vector of past log-returns from the historical data
  omega <- PEst_Garch_t(pastdata)[2] #garch parameters
  alph1 <- PEst_Garch_t(pastdata)[3] #garch parameters
  beta <- PEst_Garch_t(pastdata)[4] #garch parameters
  dof <- PEst_Garch_t(pastdata)[5] #degrees of freedom (shape parameter)
  loc <- 0 #location parameter is assumed to be zero otherwise use
  PEst_Garch_t(pastdata)[6] as location parameter
  vol <- PEst_Garch_t(pastdata)[1] #conditional volatility
  res <- c(vol, dof, loc,(vol * sqrt((dof - 2)/dof)))
  names(res) <- c("volatility", "dof", "loc. par.", "sca. par.")
  res }

ParEst_EVT <- function(pastdata, nth){
  #This function estimates the parameters (volatility, shape and scale parameters)
  for Extreme Value Theory
  #pastdata...a vector of past data to estimate parameters and calculate volatility
  #nth...threshold value
  prmtrs <- 0
  logret <- -diff(log(pastdata))
  prmtrs <- gpdFit(logret, u=sort(logret)[length(logret)-nth], type="mle")
  res      <-      c(prmtrs@fit$par.ests[1],          prmtrs@fit$par.ests[2],
  sort(logret)[length(logret) - nth], length(logret))
  names(res) <- c("xi", "beta", "threshold", "length of data")
  res }

```

Figure A.1. R Codes for parameter estimation (cont.).

```

ParEst_CONDEVT <- function(pastdata, nth){
  #This function estimates the parameters (volatility, shape and scale parameters)
  for Generalized Pareto Distribution
  #pastdata...a vector of past data to estimate parameters and calculate volatility
  #nth...threshold value

  logret <- 0
  stdres <- 0
  u <- 0
  vol <- 0

  logret<- diff(log(pastdata)) #vector of past log-returns from the historical data
  fit <- garchFit(~garch(1,1), data = logret, trace = FALSE)
  stdres <- sort(-residuals(fit, standardize=TRUE))
  u <- stdres[length(stdres)-nth]
  vol <- volatility(fit, type="sigma")
  prmtrs <- gpdFit(stdres, u, type="mle")
  res <- c(vol[length(vol)], u, prmtrs@fit$par.ests[1], prmtrs@fit$par.ests[2],
  length(stdres))
  names(res) <- c("volatility", "threshold", "xi", "beta", "length of data")
  res }

```

Figure A.1. R Codes for parameter estimation (cont.).

```

#THIS PART INCLUDES VAR CALCULATION FUNCTION FOR EACH
MODEL

qt_3p <-function(alpha, dof, loc, sca){
#This function returns the inverse cdf value of alpha in a scaled t distribution
with given dof and scale parameters
#dof... degrees of freedom parameter
#loc... location parameter
#sca... scale parameter
#alpha... confidence level

qt_3par <- -loc + (sca*qt(alpha, dof)) #location parameter is assumed to be zero
qt_3par
}

VaR_Calculation_t<- function(estpar, alpha=0.05){
#This function calculates VAR for different alpha values
#estpar...estimated set of parameters in a vector
#alpha...confidence level

VaR <- 0
a <- length(alpha)
VaR[1 : a] <- -1 - exp(qt_3p(alpha, estpar[2], estpar[3], estpar[4])) #calculates
value at risk for different alpha values
names(VaR) <- alpha
VaR
}

```

Figure A.2. R Codes for VaR calculation.

```

VaR_Calculation_Normal<- function(estpar, alpha=0.05){
#This function calculates VAR for different alpha values
#estpar...estimated set of parameters in a vector
#alpha...confidence level

VaR <- 0
a <- length(alpha)
VaR[1 : a] <- -1 - exp(estpar[1] * qnorm(alpha)) #calculates value at risk for
different alpha values
names(VaR) <- alpha
VaR
}

VaR_Calculation_EVT <- function(estpar, alpha=0.05, nth=100){
#This function calculates VAR for different alpha values
#estpar...estimated set of parameters in a vector
#alpha...confidence level
#nth...threshold value

VaR <- 0
a <- length(alpha)
VaR[1 : a] <- -estpar[3] + ((estpar[2]/estpar[1]) * (((estpar[4]/nth) * (alpha)) ^
(-estpar[1])) - 1)) #calculates value at risk for different alpha values
names(VaR) <- alpha
VaR
}

```

Figure A.2. R Codes for VaR calculation (cont.).

```

VaR_Calculation_CONDEVT <- function(estpar, alpha=0.05, nth=100){
  #This function calculates VAR for different alpha values
  #estpar...estimated set of parameters in a vector
  #alpha...confidence level
  #nth...threshold value
  quantile <- 0
  VaR <- 0
  a <- length(alpha)
  quantile[1 : a] < -estpar[2] + ((estpar[4]/estpar[3]) * (((estpar[5]/nth) *
(alpha)) ^ (-estpar[3])) - 1))
  VaR[1 : a] < -estpar[1] * quantile[1 : a]
  names(VaR) <- alpha
  VaR
}

```

Figure A.2. R Codes for VaR calculation (cont.).

```

# THIS PART INCLUDES THE DAILY BACK TESTING
countercrossing <- function(VaRres, backtper.data){
#This function counts the number of crossings (exceptions) in the backtesting
period for different alpha values
#VaRres ...matrix of VaR results for different alpha values
#backtper.data ...vector of foreign exchange rates in the backtesting period

numbercrossing <- 0
logret <- diff(log(backtper.data)) #log-returns in the backtesting period

d <- nrow(VaRres)
a <- ncol(VaRres)
noofcrossings <- matrix(nrow=d, ncol=a)

for(i in 1:d){
  noofcrossings[i,(1 : a)] <- ifelse(-VaRres[i,(1 : a)] > logret[i],1,0)
#checks if there is a crossing
  numbercrossing[(1:a)] <- colSums(noofcrossings) #counts number of crossings
  numbercrossing
}

```

Figure A.3. R Codes for back testing.

```

crossing <- function(d, numbercrossing, alpha){
# This function checks if the number of crossings is too large or too small
#d ... number of days in the backtesting period
#numbercrossing ... number of crossing for different alpha values
a <- length(alpha)
decision <- matrix(nrow=1, ncol=a)
for(i in 1:a) {
if(0.001 >= pbinom(numbercrossing[i], d, alpha[i])){
decision[1,i] <- ("number of crossings is clearly too small")
print(c("alpha value is", alpha[i]))
print(c(decision[1,i], "the model overestimates risk")) }
else if(0.05 >= pbinom(numbercrossing[i], d, alpha[i])){
decision[1,i] <- ("no of crossings may be too small")
print(c("alpha value is", alpha[i]))
print(c(decision[1,i], "the model may overestimate risk")) }
else if(0.95 >= pbinom(numbercrossing[i], d, alpha[i])){
decision[1,i] <- ("no of crossings is ok")
print(c("alpha value is", alpha[i]))
print(c(decision[1,i], "the model is accurate")) }
else if(0.999 >= pbinom(numbercrossing[i], d, alpha[i])){
decision[1,i] <- ("no of crossings may be too large")
print(c("alpha value is", alpha[i]))
print(c(decision[1,i], "the model may underestimate risk")) }
else {
decision[1,i] <- ("no of crossings is clearly too large")
print(c("alpha value is", alpha[i]))
print(c(decision[1,i], "the model underestimates risk")) }
} }

```

Figure A.3. R Codes for back testing (cont.).

```

Backtesting_daily <- function(ndata=730, code="CURRFX/USDEUR",
firstdate="2000-01-01", lastdate="2013-03-19", sdate="2008-01-01",
edate="2008-12-31", alpha=0.005, ParEstFunc=ParEst_Normal, VaRCalc-
Func=VaR_Calculation_Normal, ...){
#This function calculates all daily real log-returns and daily VaR and number
of crossings (exceptions) for different alpha values in the backtesting period for
Normal Model or GARCH Model with Normal Innovations
#data ...vector of daily foreign exchange rates
#ndata...length of pastdata to estimate parameters
#firstdate ... first date of the foreign exchange rate data
#lastdate ... last date of the foreign exchange rate data
#sdate ... start date of backtesting period
#edate ... end date of backtesting period
#alpha ... error probability
#ParEstFunc ... parameter estimation function for the chosen model
#VaRCalcFunc ... VaR estimation function for the chosen model

sdate <- adjust.next(sdate, cal=Calendar(name="Actual", week-
days=c("saturday", "sunday"),dib=365))
edate <- adjust.previous(edate, cal=Calendar(name="Actual", week-
days=c("saturday", "sunday"),dib=365))

FXdata <- FXdata_matrix(code, firstdate, lastdate)
workingdays <- rev(FXdata_matrix(code, firstdate, lastdate) [,1])
data <- rev(FXdata_matrix(code, firstdate, lastdate) [,2])
volat <- 0
no_edate <- 0
no_sdate <- 0

```

Figure A.3. R Codes for back testing (cont.).

```

pastdata <- 0
real_logret <- 0
a <- length(alpha)

for(m in 1:length(data)) {
  if(edate==workingdays[m]) {
    no_edate <- m
  } else {
  }}

for(k in 1:length(data)) {
  if(sdate==workingdays[k]) {
    no_sdate <- k
  } else {
  }}

d <- no_edate-no_sdate #number of days in the backtesting period
p <- length(data)-no_sdate #difference between the last date of the data and the
first date of the backtesting period
t <- length(data)-no_edate #difference between the last date of the data and the
last date of the backtesting period

if(t >= 0){
  numbercrossing <- -matrix(ncol = a, nrow = 1)
  VaRres <- -matrix(ncol = a, nrow = (d + 1))

```

Figure A.3. R Codes for back testing (cont.).

```

for(j in 1:(d+1)){
  real_logret[j] <- -log((data)[no_sdate + j - 1]) - log((data)[no_sdate - 2 + j])
  pastdata <- -data[(no_sdate - ndata - 2 + j) : (no_sdate - 2 + j)]
  estpar <- -matrix(0, nrow = (d + 1), ncol = length(ParEstFunc(pastdata, ...)))
  estpar[j,] <- ParEstFunc(pastdata,...)
  VaRres[j,(1:a)] <- VaRCalcFunc(estpar[j,], alpha,...)
}

numbercrossing <- countcrossing(VaRres, data[(no_sdate - 1) : (no_sdate - 1 +
d + 1)]) #counts number of crossings
crossing(((d + 1), numbercrossing, alpha) #checks if the model underestimates or
overestimates the risk or it is accurate
names(numbercrossing) <- alpha
rownames(VaRres) <- rownames(VaRres,do.NULL=FALSE,prefix="day.")
colnames(VaRres) <- alpha
list(crossing=numbercrossing, reallogret=real_logret, VAR=VaRres)
} else{
  print("edate should not be a date coming after the last date available in the data")
}
}

```

Figure A.3. R Codes for back testing (cont.).

APPENDIX B: VAR RESULTS OF THE EXAMPLE

DATE	Normal VaR (alpha=0,005)	Student's t VaR (alpha=0,005)	Normal- GARCH VaR (alpha=0,005)	t-GARCH VaR (alpha=0,005)	EVT VaR (alpha=0,005)	GARCH filt. EVT VaR (alpha=0,005)
2006-02-15	0,0142485	0,01981677	0,01342958	0,01851163	0,01623814	0,0155391
2006-09-26	0,01283921	0,01690771	0,01205614	0,01690749	0,01543803	0,01473223
2007-01-08	0,01226257	0,0149489	0,01101113	0	0,01239377	0,01269622
2007-06-13	0,01143108	0,01496563	0,007031929	0	0,01383513	0,008219207
2008-03-14	0,01641753	0,02063475	0,039132292	0,03235683	0,0185062	0,052984494
2008-11-23	0,02427631	0,04810568	0,047191844	0,06200003	0,03464636	0,06893871
2009-04-09	0,03291149	0,07870531	0,03064081	0,04670757	0,04915322	0,04743991
2009-12-09	0,03365526	0,05737451	0,0182143	0,02933533	0,04851132	0,02630663
2010-05-05	0,03386347	0,051743	0,0168218	0,03849938	0,04771251	0,0249019
2010-10-25	0,03189257	0,04476812	0,02025392	0,03444606	0,04081213	0,02776236
2011-01-26	0,03085428	0,03722906	0,01914238	0,02445566	0,03654814	0,02581357
2011-07-20	0,01659781	0,02021089	0,01756806	0,02056037	0,0174024	0,01799303
2012-08-05	0,01714616	0,02191385	0,01522463	0,0193527	0,01784359	0,01577999
2012-12-26	0,01597477	0,02149917	0,00986867	0,01210323	0,01727244	0,01065851
2013-03-07	0,01556728	0,02177413	0,012349237	0,01498624	0,01712135	0,0137489
2013-11-08	0,01428639	0,02191811	0,011439567	0,01488722	0,01806894	0,014646106
2014-01-13	0,01327927	0,02007102	0,010311198	0,01334522	0,01676312	0,01309709
2014-09-11	0,01083394	0,01811517	0,009471184	0,012069791	0,01493928	0,012362349
2015-05-20	0,01246343	0,02182271	0,017721108	0,024527239	0,01607457	0,020568538
2015-12-17	0,01374798	0,02816076	0,0177827	0,02787086	0,01833997	0,0208654

Figure B.1. Daily VaR results for arbitrarily chosen 20 days USD/EUR ($\alpha = 0.005$).

DATE	Normal VaR (alpha=0,001)	Student's t VaR (alpha=0,001)	Normal- GARCH VaR (alpha=0,001)	t-GARCH VaR (alpha=0,001)	EVT VaR (alpha=0,001)	GARCH filt. EVT VaR (alpha=0,001)
2006-02-15	0,01706956	0,03184265	0,01608983	0,0291307	0,01942767	0,01885491
2006-09-26	0,01538343	0,02588441	0,01444633	0,02588331	0,01879504	0,01802055
2007-01-08	0,01469338	0,02125899	0,01319553	0	0,01397621	0,01491574
2007-06-13	0,01369821	0,02287125	0,008430298	0	0,01660005	0,009844723
2008-03-14	0,01966374	0,04013915	0,046761768	0,05783922	0,03327474	0,084330215
2008-11-23	0,02905333	0,14027839	0,05634602	0,12554652	0,09441766	0,12061476
2009-04-09	0,03935313	0,26402482	0,0366465	0,09981753	0,12877603	0,09086254
2009-12-09	0,04023942	0,1499401	0,02181183	0,06313895	0,1239451	0,05482583
2010-05-05	0,0404875	0,1248689	0,02014712	0,08056205	0,1156644	0,04822473
2010-10-25	0,03813875	0,09888906	0,02424929	0,0698758	0,08499194	0,04707125
2011-01-26	0,036901	0,07277636	0,02292106	0,03966188	0,06993628	0,04158221
2011-07-20	0,01987931	0,02848884	0,02103932	0,02821336	0,02076509	0,02102028
2012-08-05	0,02053494	0,03247529	0,01823716	0,0280263	0,02191499	0,01908795
2012-12-26	0,01913429	0,03312023	0,01182778	0,01757411	0,02109924	0,01309091
2013-03-07	0,01864698	0,03497705	0,0147971	0,02226552	0,02204039	0,01816635
2013-11-08	0,01711489	0,03891768	0,01370837	0,0238821	0,02654891	0,02137448
2014-01-13	0,01590999	0,03503858	0,01235761	0,02108239	0,02492956	0,02013758
2014-09-11	0,01298342	0,03509475	0,011351843	0,0199771	0,0233078	0,018798291
2015-05-20	0,01493375	0,04397173	0,021222283	0,03895317	0,02356809	0,03036574
2015-12-17	0,01647078	0,06447227	0,02129591	0,04956505	0,02687298	0,02911047

Figure B.2. Daily VaR results for arbitrarily chosen 20 days USD/EUR ($\alpha = 0.001$).

APPENDIX C: DETAILED BACK TESTING RESULTS (USD/EUR, TRY/USD AND TRY/EUR)

[illegible]

Figure C.1. Back testing results of USD/EUR using each model.

Figure C.2. Back testing results of TRY/USD using each model.

[illegible]

Figure C.3. Back testing results of TRY/EUR using each model.

APPENDIX D: DETAILED BACK TESTING RESULTS (OTHER 6 CURRENCY PAIRS)

Considering the p-values of the back testing results of the currency pair TRY/AUD, it can be said that i.i.d Normal and Normal-GARCH models are not satisfactory. On the other hand, i.i.d Student's t, t-GARCH, unconditional EVT and GARCH filtered EVT works good for this currency pair as p-values are higher than the significance level ($\alpha = 0.05$) and number of crossings are inside the bounds. Hence, these 4 models are sufficient to model the daily risk of the currency pair TRY/AUD.

Table D.1. Total number of crossings and p-values (TRY/AUD).

	$\alpha = 0.05$	p-val.	$\alpha = 0.01$	p-val.	$\alpha = 0.005$	p-val.	$\alpha = 0.001$	p-val.
Target	125		25		13		3	
i.i.d Normal	84	$4.68 \cdot 10^{-5}$	28	0.2962	22	0.0091	14	$6.58 \cdot 10^{-8}$
i.i.d Student's t	141	0.0766	21	0.7515	8	0.1254	0	0.0823
Normal-GARCH	110	0.0927	40	0.0032	30	$1.72 \cdot 10^{-5}$	13	$2.3 \cdot 10^{-6}$
t-GARCH	166	$1.71 \cdot 10^{-4}$	23	0.3962	9	0.2023	0	0.0823
Unconditional EVT	125	0.5311	23	0.3962	14	0.3698	5	0.1082
GARCH filt. EVT	159	0.0014	32	0.0975	16	0.1918	5	0.1082

		TRY/AUD exchange rate (alpha=0.01)											
		i.i.d. Normal		i.i.d Student's t		Normal-GARCH		t-GARCH		Unconditional EVT		GARCH filt. EVT	
The Year of Back Testing	Interval for # of crossings	No of crossing	Decision	No of crossing	Decision	No of crossing	Decision	No of crossing	Decision	No of crossing	Decision	No of crossing	Decision
2015	[1,7]	11	Too many crossings	5	Accurate	10	Too many crossings	6	Accurate	6	Accurate	7	Accurate
2014		5	Accurate	4	Accurate	5	Accurate	3	Accurate	4	Accurate	4	Accurate
2013		6	Accurate	4	Accurate	4	Accurate	3	Accurate	4	Accurate	3	Accurate
2012		0	Too few crossings	0	Too few crossings	3	Accurate	0	Too few crossings	0	Too few crossings	2	Accurate
2011		6	Accurate	4	Accurate	10	Too many crossings	4	Accurate	4	Accurate	7	Accurate
2010		0	Too few crossings	0	Too few crossings	6	Accurate	1	Accurate	0	Too few crossings	3	Accurate
2009		0	Too few crossings	0	Too few crossings	2	Accurate	1	Accurate	0	Too few crossings	1	Accurate
2008		0	Too few crossings	4	Accurate	0	Too few crossings	5	Accurate	5	Accurate	5	Accurate
		TRY/AUD exchange rate (alpha= 0.005)											
		i.i.d. Normal		i.i.d Student's t		Normal-GARCH		t-GARCH		Unconditional EVT		GARCH filt. EVT	
The Year of Back Testing	Interval for # of crossings	No of crossing	Decision	No of crossing	Decision	No of crossing	Decision	No of crossing	Decision	No of crossing	Decision	No of crossing	Decision
2015	[0,4]	9	Too many crossings	1	Accurate	9	Too many crossings	1	Accurate	3	Accurate	3	Accurate
2014		4	Accurate	2	Accurate	4	Accurate	3	Accurate	3	Accurate	3	Accurate
2013		4	Accurate	1	Accurate	4	Accurate	1	Accurate	2	Accurate	1	Accurate
2012		0	Accurate	0	Accurate	2	Accurate	0	Accurate	0	Accurate	0	Accurate
2011		5	Too many crossings	3	Accurate	7	Too many crossings	2	Accurate	4	Accurate	4	Accurate
2010		0	Accurate	0	Accurate	3	Accurate	0	Accurate	0	Accurate	1	Accurate
2009		0	Accurate	0	Accurate	1	Accurate	0	Accurate	0	Accurate	1	Accurate
2008		0	Accurate	1	Accurate	0	Accurate	2	Accurate	2	Accurate	3	Accurate
		TRY/AUD exchange rate (alpha= 0.001)											
		i.i.d. Normal		i.i.d Student's t		Normal-GARCH		t-GARCH		Unconditional EVT		GARCH filt. EVT	
The Year of Back Testing	Interval for # of crossings	No of crossing	Decision	No of crossing	Decision	No of crossing	Decision	No of crossing	Decision	No of crossing	Decision	No of crossing	Decision
2015	[0,2]	5	Too many crossings	0	Accurate	4	Too many crossings	0	Accurate	0	Accurate	0	Accurate
2014		4	Too many crossings	0	Accurate	3	Too many crossings	0	Accurate	1	Accurate	3	Too many crossings
2013		2	Accurate	0	Accurate	1	Accurate	0	Accurate	1	Accurate	0	Accurate
2012		0	Accurate	0	Accurate	0	Accurate	0	Accurate	0	Accurate	0	Accurate
2011		3	Too many crossings	0	Accurate	3	Too many crossings	0	Accurate	3	Too many crossings	1	Accurate
2010		0	Accurate	0	Accurate	1	Accurate	0	Accurate	0	Accurate	1	Accurate
2009		0	Accurate	0	Accurate	1	Accurate	0	Accurate	0	Accurate	0	Accurate
2008		0	Accurate	0	Accurate	0	Accurate	0	Accurate	0	Accurate	0	Accurate

Figure D.1. Comparison of yearly back testing results (TRY/AUD).

[illegible]

Figure D.2. Back testing results of TRY/AUD using each model.

Considering the p-values of the back testing results of the currency pair TRY/GBP, it can be said that i.i.d Normal and Normal-GARCH models are not satisfactory. On the other hand, considering p-values and yearly number of crossings, i.i.d Student's t, t-GARCH, unconditional EVT and GARCH filtered EVT are satisfactory to model the daily risk of the currency pair TRY/GBP. However, the drawback of t-GARCH model is that the risk for 2012 and 2013 can not be estimated.

Table D.2. Total number of crossings and p-values (TRY/GBP).

	$\alpha = 0.05$	p-val.	$\alpha = 0.01$	p-val.	$\alpha = 0.005$	p-val.	$\alpha = 0.001$	p-val.
Target	125		25		13		3	
i.i.d Normal	135	0.1844	55	$1.18 \cdot 10^{-7}$	42	$3.47 \cdot 10^{-11}$	26	0
i.i.d Student's t	158	0.0018	24	0.477	6	0.0347	0	0.0824
Normal-GARCH	144	0.0448	57	$2.19 \cdot 10^{-8}$	47	$5.13 \cdot 10^{-14}$	21	$3.72 \cdot 10^{-13}$
t-GARCH	121	0.3875	18	0.0925	8	0.1257	0	0.0824
Unconditional EVT	130	0.3273	27	0.3663	16	0.1914	6	0.0416
GARCH filt. EVT	130	0.3273	29	0.2325	17	0.1286	3	0.455

		TRY/GBP exchange rate (alpha= 0.01)											
		i.i.d. Normal		i.i.d Student's t		Normal-GARCH		t-GARCH		Unconditional EVT		GARCH filt. EVT	
The Year of Back Testing	Interval for # of crossings	No of crossing	Decision	No of crossing	Decision	No of crossing	Decision	No of crossing	Decision	No of crossing	Decision	No of crossing	Decision
2015	[1,7]	9	Too many crossings	2	Accurate	10	Too many crossings	3	Accurate	2	Accurate	4	Accurate
2014		7	Accurate	4	Accurate	7	Accurate	4	Accurate	4	Accurate	4	Accurate
2013		14	Too many crossings	8	Too many crossings	13	Too many crossings			9	Too many crossings	9	Too many crossings
2012		1	Accurate	0	Too few crossings	2	Accurate			0	Too few crossings	1	Accurate
2011		2	Accurate	1	Accurate	6	Accurate	4	Accurate	1	Accurate	4	Accurate
2010		0	Too few crossings	0	Too few crossings	4	Accurate	1	Accurate	0	Too few crossings	1	Accurate
2009		4	Accurate	1	Accurate	5	Accurate	1	Accurate	2	Accurate	1	Accurate
2008		18	Too many crossings	8	Too many crossings	10	Too many crossings	5	Accurate	9	Too many crossings	5	Accurate
		TRY/GBP exchange rate (alpha= 0.005)											
		i.i.d. Normal		i.i.d Student's t		Normal-GARCH		t-GARCH		Unconditional EVT		GARCH filt. EVT	
The Year of Back Testing	Interval for # of crossings	No of crossing	Decision	No of crossing	Decision	No of crossing	Decision	No of crossing	Decision	No of crossing	Decision	No of crossing	Decision
2015	[0,4]	5	Too many crossings	1	Accurate	8	Too many crossings	1	Accurate	2	Accurate	3	Accurate
2014		7	Too many crossings	1	Accurate	4	Accurate	2	Accurate	3	Accurate	3	Accurate
2013		11	Too many crossings	2	Accurate	11	Too many crossings			5	Too many crossings	5	Too many crossings
2012		0	Accurate	0	Accurate	1	Accurate			0	Accurate	0	Accurate
2011		1	Accurate	0	Accurate	5	Too many crossings	2	Accurate	1	Accurate	2	Accurate
2010		0	Accurate	0	Accurate	4	Accurate	0	Accurate	0	Accurate	1	Accurate
2009		4	Accurate	0	Accurate	5	Too many crossings	0	Accurate	1	Accurate	0	Accurate
2008		14	Too many crossings	2	Accurate	9	Too many crossings	3	Accurate	4	Accurate	3	Accurate
		TRY/GBP exchange rate (alpha= 0.001)											
		i.i.d. Normal		i.i.d Student's t		Normal-GARCH		t-GARCH		Unconditional EVT		GARCH filt. EVT	
The Year of Back Testing	Interval for # of crossings	No of crossing	Decision	No of crossing	Decision	No of crossing	Decision	No of crossing	Decision	No of crossing	Decision	No of crossing	Decision
2015	[0,2]	2	Accurate	0	Accurate	3	Too many crossings	0	Accurate	1	Accurate	1	Accurate
2014		3	Too many crossings	0	Accurate	3	Too many crossings	0	Accurate	2	Accurate	0	Accurate
2013		8	Too many crossings	0	Accurate	5	Too many crossings			2	Accurate	2	Accurate
2012		0	Accurate	0	Accurate	0	Accurate			0	Accurate	0	Accurate
2011		1	Accurate	0	Accurate	2	Accurate	0	Accurate	0	Accurate	0	Accurate
2010		0	Accurate	0	Accurate	1	Accurate	0	Accurate	0	Accurate	0	Accurate
2009		3	Too many crossings	0	Accurate	2	Accurate	0	Accurate	0	Accurate	0	Accurate
2008		9	Too many crossings	0	Accurate	5	Too many crossings	0	Accurate	1	Accurate	0	Accurate

Figure D.3. Comparison of yearly back testing results (TRY/GBP).

Considering the p-values of the back testing results of the currency pair TRY/RUB, it can be said that i.i.d Normal and Normal-GARCH models are not satisfactory. Besides, the performance of unconditional EVT is not good for this currency pair, especially for $\alpha = 0.01$ and $\alpha = 0.005$. On the other hand, considering p-values and yearly number of crossings, i.i.d Student's t, t-GARCH and GARCH filtered EVT are satisfactory to model the daily risk of the currency pair TRY/RUB.

Table D.3. Total number of crossings and p-values (TRY/RUB).

	$\alpha = 0.05$	p-val.	$\alpha = 0.01$	p-val.	$\alpha = 0.005$	p-val.	$\alpha = 0.001$	p-val.
Target	125		25		13		3	
i.i.d Normal	117	0.2596	62	$2.29 \cdot 10^{-10}$	50	$7.77 \cdot 10^{-16}$	34	0
i.i.d Student's t	161	$7.4 \cdot 10^{-4}$	24	0.4794	10	0.3003	1	0.2888
Normal-GARCH	128	0.3899	48	$2.35 \cdot 10^{-5}$	29	$4.13 \cdot 10^{-5}$	17	$1.43 \cdot 10^{-9}$
t-GARCH	130	0.3222	20	0.8643	7	0.0708	1	0.2888
Unconditional EVT	149	0.0158	43	0.0006	19	0.0501	8	0.0041
GARCH filt. EVT	128	0.3899	19	0.1357	14	0.3677	4	0.2407

		TRY/RUB exchange rate (alpha=0.01)											
		i.i.d. Normal		i.i.d Student's t		Normal-GARCH		t-GARCH		Unconditional EVT		GARCH filt. EVT	
The Year of Back Testing	Interval for # of crossings	No of crossing	Decision	No of crossing	Decision	No of crossing	Decision	No of crossing	Decision	No of crossing	Decision	No of crossing	Decision
2015	[1,7]	13	Too many crossings	1	Accurate	5	Accurate	2	Accurate	9	Too many crossings	2	Accurate
2014		13	Too many crossings	8	Too many crossings	4	Accurate	2	Accurate	13	Too many crossings	2	Accurate
2013		6	Accurate	1	Accurate	9	Too many crossings	4	Accurate	2	Accurate	4	Accurate
2012		0	Too few crossings	0	Too few crossings	2	Accurate	1	Accurate	0	Too few crossings	1	Accurate
2011		2	Accurate	1	Accurate	8	Too many crossings	3	Accurate	1	Accurate	3	Accurate
2010		0	Too few crossings	0	Too few crossings	2	Accurate	0	Too few crossings	0	Too few crossings	0	Too few crossings
2009		8	Too many crossings	4	Accurate	8	Too many crossings	3	Accurate	6	Accurate	4	Accurate
2008		20	Too many crossings	9	Too many crossings	10	Too many crossings	5	Accurate	12	Too many crossings	3	Accurate
		TRY/RUB exchange rate (alpha=0.005)											
		i.i.d. Normal		i.i.d Student's t		Normal-GARCH		t-GARCH		Unconditional EVT		GARCH filt. EVT	
The Year of Back Testing	Interval for # of crossings	No of crossing	Decision	No of crossing	Decision	No of crossing	Decision	No of crossing	Decision	No of crossing	Decision	No of crossing	Decision
2015	[0,4]	11	Too many crossings	0	Accurate	3	Accurate	1	Accurate	2	Accurate	2	Accurate
2014		12	Too many crossings	6	Too many crossings	2	Accurate	1	Accurate	8	Too many crossings	2	Accurate
2013		3	Accurate	1	Accurate	5	Too many crossings	1	Accurate	1	Accurate	3	Accurate
2012		0	Accurate	0	Accurate	1	Accurate	0	Accurate	0	Accurate	1	Accurate
2011		2	Accurate	0	Accurate	3	Accurate	2	Accurate	0	Accurate	3	Accurate
2010		0	Accurate	0	Accurate	1	Accurate	0	Accurate	0	Accurate	0	Accurate
2009		8	Too many crossings	1	Accurate	6	Too many crossings	0	Accurate	3	Accurate	1	Accurate
2008		14	Too many crossings	2	Accurate	8	Too many crossings	2	Accurate	5	Too many crossings	2	Accurate
		TRY/RUB exchange rate (alpha=0.001)											
		i.i.d. Normal		i.i.d Student's t		Normal-GARCH		t-GARCH		Unconditional EVT		GARCH filt. EVT	
The Year of Back Testing	Interval for # of crossings	No of crossing	Decision	No of crossing	Decision	No of crossing	Decision	No of crossing	Decision	No of crossing	Decision	No of crossing	Decision
2015	[0,2]	5	Too many crossings	0	Accurate	2	Accurate	0	Accurate	0	Accurate	1	Accurate
2014		8	Too many crossings	1	Accurate	2	Accurate	0	Accurate	6	Too many crossings	1	Accurate
2013		1	Accurate	0	Accurate	3	Too many crossings	1	Accurate	1	Accurate	1	Accurate
2012		0	Accurate	0	Accurate	1	Accurate	0	Accurate	0	Accurate	0	Accurate
2011		1	Accurate	0	Accurate	3	Too many crossings	0	Accurate	0	Accurate	1	Accurate
2010		0	Accurate	0	Accurate	0	Accurate	0	Accurate	0	Accurate	0	Accurate
2009		7	Too many crossings	0	Accurate	3	Too many crossings	0	Accurate	0	Accurate	0	Accurate
2008		12	Too many crossings	0	Accurate	3	Too many crossings	0	Accurate	1	Accurate	0	Accurate

Figure D.5. Comparison of yearly back testing results (TRY/RUB).

[illegible]

Figure D.6. Back testing results of TRY/RUB using each model.

Considering the p-values of the back testing results of the currency pair USD/BRL, it can be said that i.i.d Normal and Normal-GARCH models are not satisfactory. On the other hand, considering p-values and yearly number of crossings, i.i.d Student's t, t-GARCH, unconditional EVT and GARCH filtered EVT are satisfactory to model the daily risk of the currency pair USD/BRL.

Table D.4. Total number of crossings and p-values (USD/BRL).

	$\alpha = 0.05$	p-val.	$\alpha = 0.01$	p-val.	$\alpha = 0.005$	p-val.	$\alpha = 0.001$	p-val.
Target	103		21		10		2	
i.i.d Normal	94	0.2017	37	$6.46 \cdot 10^{-4}$	25	$6.83 \cdot 10^{-5}$	12	$1.76 \cdot 10^{-6}$
i.i.d Student's t	126	0.0126	21	0.596	6	0.1128	2	0.6615
Normal-GARCH	88	0.0714	31	0.0183	24	$1.71 \cdot 10^{-4}$	12	$1.76 \cdot 10^{-6}$
t-GARCH	112	0.1883	23	0.3229	11	0.4520	2	0.6615
Unconditional EVT	125	0.01605	22	0.4040	10	0.5479	3	0.3385
GARCH filt. EVT	105	0.4259	24	0.2506	10	0.5479	3	0.3385

		USD/BRL exchange rate (alpha=0.01)											
		i.i.d. Normal		i.i.d Student's t		Normal-GARCH		t-GARCH		Unconditional EVT		GARCH filt. EVT	
The Year of Back Testing	Interval for # of crossings	No of crossing	Decision	No of crossing	Decision	No of crossing	Decision	No of crossing	Decision	No of crossing	Decision	No of crossing	Decision
2015	[0,5]	10	Too many crossings	8	Too many crossings	3	Accurate	1	Accurate	8	Too many crossings	2	Accurate
2014		4	Accurate	3	Accurate	5	Accurate	2	Accurate	3	Accurate	2	Accurate
2013		3	Accurate	2	Accurate	6	Too many crossings	4	Accurate	2	Accurate	5	Accurate
2012		2	Accurate	1	Accurate	3	Accurate	3	Accurate	1	Accurate	3	Accurate
2011		0	Accurate	0	Accurate	1	Accurate	1	Accurate	0	Accurate	1	Accurate
2010		6	Too many crossings	0	Accurate	5	Accurate	4	Accurate	0	Accurate	4	Accurate
2009		4	Accurate	0	Accurate	6	Too many crossings	6	Too many crossings	0	Accurate	5	Accurate
2008		8	Too many crossings	7	Too many crossings	2	Accurate	2	Accurate	8	Too many crossings	2	Accurate
		USD/BRL exchange rate (alpha= 0.005)											
		i.i.d. Normal		i.i.d Student's t		Normal-GARCH		t-GARCH		Unconditional EVT		GARCH filt. EVT	
The Year of Back Testing	Interval for # of crossings	No of crossing	Decision	No of crossing	Decision	No of crossing	Decision	No of crossing	Decision	No of crossing	Decision	No of crossing	Decision
2015	[0,3]	8	Too many crossings	2	Accurate	2	Accurate	1	Accurate	3	Accurate	1	Accurate
2014		3	Accurate	0	Accurate	2	Accurate	0	Accurate	0	Accurate	0	Accurate
2013		3	Accurate	0	Accurate	6	Too many crossings	1	Accurate	1	Accurate	1	Accurate
2012		2	Accurate	1	Accurate	3	Accurate	2	Accurate	1	Accurate	3	Accurate
2011		0	Accurate	0	Accurate	1	Accurate	0	Accurate	0	Accurate	0	Accurate
2010		1	Accurate	0	Accurate	4	Too many crossings	3	Accurate	0	Accurate	3	Accurate
2009		1	Accurate	0	Accurate	4	Too many crossings	3	Accurate	0	Accurate	0	Accurate
2008		7	Too many crossings	3	Accurate	2	Accurate	1	Accurate	5	Too many crossings	2	Accurate
		USD/BRL exchange rate (alpha= 0.001)											
		i.i.d. Normal		i.i.d Student's t		Normal-GARCH		t-GARCH		Unconditional EVT		GARCH filt. EVT	
The Year of Back Testing	Interval for # of crossings	No of crossing	Decision	No of crossing	Decision	No of crossing	Decision	No of crossing	Decision	No of crossing	Decision	No of crossing	Decision
2015	[0,1]	3	Too many crossings	1	Accurate	1	Accurate	1	Accurate	1	Accurate	1	Accurate
2014		0	Accurate	0	Accurate	0	Accurate	0	Accurate	0	Accurate	0	Accurate
2013		2	Too many crossings	0	Accurate	3	Too many crossings	0	Accurate	0	Accurate	0	Accurate
2012		1	Accurate	0	Accurate	3	Too many crossings	1	Accurate	1	Accurate	1	Accurate
2011		0	Accurate	0	Accurate	0	Accurate	0	Accurate	0	Accurate	0	Accurate
2010		0	Accurate	0	Accurate	3	Too many crossings	0	Accurate	0	Accurate	0	Accurate
2009		0	Accurate	0	Accurate	1	Accurate	0	Accurate	0	Accurate	0	Accurate
2008		6	Too many crossings	1	Accurate	2	Too many crossings	0	Accurate	1	Accurate	1	Accurate

Figure D.7. Comparison of yearly back testing results (USD/BRL).

Considering the p-values of the back testing results of the currency pair USD/EGP, it can be said that i.i.d Normal and Normal-GARCH models are not satisfactory. Besides, the performance of unconditional EVT is not good for this currency pair, especially for $\alpha = 0.005$. On the other hand, considering p-values and yearly number of crossings, only GARCH filtered EVT is satisfactory to model the daily risk of the currency pair USD/EGP.

Table D.5. Total number of crossings and p-values (USD/EGP).

	$\alpha = 0.05$	p-val.	$\alpha = 0.01$	p-val.	$\alpha = 0.005$	p-val.	$\alpha = 0.001$	p-val.
Target	125		25		13		3	
i.i.d Normal	119	0.317	57	$2.22 \cdot 10^{-8}$	36	$3.99 \cdot 10^{-8}$	19	$2.59 \cdot 10^{-11}$
i.i.d Student's t	113	0.1493	2	$4.38 \cdot 10^{-9}$	0	$3.68 \cdot 10^{-6}$	0	0.0823
Normal-GARCH	139	0.1055	74	$9.99 \cdot 10^{-16}$	60	0	39	0
t-GARCH	182	$4.69 \cdot 10^{-7}$	61	$6.22 \cdot 10^{-10}$	35	$1.18 \cdot 10^{-7}$	10	$2.71 \cdot 10^{-4}$
Unconditional EVT	138	0.1226	17	0.0605	5	0.0148	1	0.3385
GARCH filt. EVT	138	0.1226	31	0.1337	14	0.3698	3	0.7585

		USD/EGP exchange rate (alpha= 0.01)											
		i.i.d. Normal		i.i.d Student's t		Normal-GARCH		t-GARCH		Unconditional EVT		GARCH fit. EVT	
The Year of Back Testing	Interval for # of crossings	No of crossing	Decision	No of crossing	Decision	No of crossing	Decision	No of crossing	Decision	No of crossing	Decision	No of crossing	Decision
2015	[1,7]	4	Accurate	0	Too few crossings	5	Accurate	12	Too many crossings	2	Accurate	2	Accurate
2014		2	Accurate	0	Too few crossings	10	Too many crossings	20	Too many crossings	0	Too few crossings	3	Accurate
2013		3	Accurate	0	Too few crossings	5	Accurate	8	Too many crossings	0	Too few crossings	1	Accurate
2012		19	Too many crossings	0	Too few crossings	21	Too many crossings			7	Accurate	12	Too many crossings
2011		11	Too many crossings	0	Too few crossings	12	Too many crossings	4	Accurate	3	Accurate	4	Accurate
2010		7	Accurate	1	Accurate	9	Too many crossings	4	Accurate	3	Accurate	4	Accurate
2009		4	Accurate	0	Too few crossings	6	Accurate	5	Accurate	0	Too few crossings	3	Accurate
2008		7	Accurate	0	Too few crossings	6	Accurate	8	Too many crossings	2	Accurate	2	Accurate
		USD/EGP exchange rate (alpha= 0.005)											
		i.i.d. Normal		i.i.d Student's t		Normal-GARCH		t-GARCH		Unconditional EVT		GARCH fit. EVT	
The Year of Back Testing	Interval for # of crossings	No of crossing	Decision	No of crossing	Decision	No of crossing	Decision	No of crossing	Decision	No of crossing	Decision	No of crossing	Decision
2015	[0,4]	4	Accurate	0	Accurate	5	Too many crossings	10	Too many crossings	0	Accurate	1	Accurate
2014		2	Accurate	0	Accurate	9	Too many crossings	16	Too many crossings	0	Accurate	2	Accurate
2013		2	Accurate	0	Accurate	2	Accurate	0	Accurate	0	Accurate	1	Accurate
2012		11	Too many crossings	0	Accurate	16	Too many crossings			3	Accurate	6	Too many crossings
2011		8	Too many crossings	0	Accurate	10	Too many crossings	2	Accurate	1	Accurate	3	Accurate
2010		4	Accurate	0	Accurate	9	Too many crossings	2	Accurate	1	Accurate	1	Accurate
2009		2	Accurate	0	Accurate	4	Accurate	2	Accurate	0	Accurate	0	Accurate
2008		3	Accurate	0	Accurate	6	Too many crossings	3	Accurate	0	Accurate	0	Accurate
		USD/EGP exchange rate (alpha= 0.001)											
		i.i.d. Normal		i.i.d Student's t		Normal-GARCH		t-GARCH		Unconditional EVT		GARCH fit. EVT	
The Year of Back Testing	Interval for # of crossings	No of crossing	Decision	No of crossing	Decision	No of crossing	Decision	No of crossing	Decision	No of crossing	Decision	No of crossing	Decision
2015	[0,2]	4	Too many crossings	0	Accurate	4	Too many crossings	5	Too many crossings	0	Accurate	0	Accurate
2014		0	Accurate	0	Accurate	5	Too many crossings	5	Too many crossings	0	Accurate	2	Accurate
2013		0	Accurate	0	Accurate	1	Accurate	0	Accurate	0	Accurate	1	Accurate
2012		7	Too many crossings	0	Accurate	13	Too many crossings			0	Accurate	0	Accurate
2011		3	Too many crossings	0	Accurate	4	Too many crossings	0	Accurate	0	Accurate	0	Accurate
2010		2	Accurate	0	Accurate	6	Too many crossings	0	Accurate	1	Accurate	0	Accurate
2009		1	Accurate	0	Accurate	3	Too many crossings	0	Accurate	0	Accurate	0	Accurate
2008		2	Accurate	0	Accurate	3	Too many crossings	0	Accurate	0	Accurate	0	Accurate

Figure D.9. Comparison of yearly back testing results (USD/EGP).

Considering the p-values of the back testing results of the currency pair USD/JPY, it can be said that i.i.d Normal and Normal-GARCH models are not satisfactory. Besides, the performance of unconditional EVT and GARCH filtered EVT are not good for this currency pair and for $\alpha = 0.001$. On the other hand, considering p-values and yearly number of crossings, i.i.d Student's t and t-GARCH are satisfactory to model the daily risk of the currency pair, USD/JPY. As USA and Japan have strong economies, the distribution of this countries' currency pair is symmetric. Thus, i.i.d Student's t works good for this currency pair.

Table D.6. Total number of crossings and p-values (USD/JPY).

	$\alpha = 0.05$	p-val.	$\alpha = 0.01$	p-val.	$\alpha = 0.005$	p-val.	$\alpha = 0.001$	p-val.
Target	125		25		13		3	
i.i.d Normal	101	0.0148	40	0.0031	29	$4.13 \cdot 10^{-5}$	16	$9.89 \cdot 10^{-9}$
i.i.d Student's t	133	0.2319	16	0.0383	6	0.0350	2	0.5458
Normal-GARCH	108	0.0671	41	0.00183	27	0.00023	15	$6.45 \cdot 10^{-8}$
t-GARCH	129	0.3555	20	0.1884	8	0.1265	0	0.0826
Unconditional EVT	121	0.3927	22	0.3221	13	0.6323	7	0.0139
GARCH filt. EVT	122	0.4287	25	0.5593	15	0.2707	8	0.0041

		USD/JPY exchange rate (alpha= 0.01)											
		i.i.d. Normal		i.i.d Student's t		Normal-GARCH		t-GARCH		Unconditional EVT		GARCH filt. EVT	
The Year of Back Testing	Interval for # of crossings	No of crossing	Decision	No of crossing	Decision	No of crossing	Decision	No of crossing	Decision	No of crossing	Decision	No of crossing	Decision
2015	[1,7]	3	Accurate	1	Accurate	7	Accurate	4	Accurate	1	Accurate	5	Accurate
2014		1	Accurate	0	Too few crossings	4	Accurate	0	Too few crossings	0	Too few crossings	4	Accurate
2013		11	Too many crossings	3	Accurate	5	Accurate	3	Accurate	7	Accurate	3	Accurate
2012		1	Accurate	0	Too few crossings	1	Accurate	0	Too few crossings	1	Accurate	1	Accurate
2011		0	Too few crossings	1	Accurate	3	Accurate	3	Accurate	1	Accurate	2	Accurate
2010		1	Accurate	0	Too few crossings	4	Accurate	1	Accurate	0	Too few crossings	2	Accurate
2009		0	Too few crossings	0	Too few crossings	5	Accurate	2	Accurate	0	Too few crossings	1	Accurate
2008		23	Too many crossings	11	Too many crossings	12	Too many crossings	7	Accurate	12	Too many crossings	7	Accurate
		USD/JPY exchange rate (alpha= 0.005)											
		i.i.d. Normal		i.i.d Student's t		Normal-GARCH		t-GARCH		Unconditional EVT		GARCH filt. EVT	
The Year of Back Testing	Interval for # of crossings	No of crossing	Decision	No of crossing	Decision	No of crossing	Decision	No of crossing	Decision	No of crossing	Decision	No of crossing	Decision
2015	[0,4]	2	Accurate	0	Accurate	4	Accurate	2	Accurate	1	Accurate	3	Accurate
2014		0	Accurate	0	Accurate	1	Accurate	0	Accurate	0	Accurate	0	Accurate
2013		7	Too many crossings	0	Accurate	3	Accurate	0	Accurate	4	Accurate	3	Accurate
2012		1	Accurate	0	Accurate	1	Accurate	0	Accurate	0	Accurate	0	Accurate
2011		0	Accurate	0	Accurate	2	Accurate	0	Accurate	0	Accurate	1	Accurate
2010		0	Accurate	0	Accurate	2	Accurate	0	Accurate	0	Accurate	1	Accurate
2009		0	Accurate	0	Accurate	3	Accurate	0	Accurate	0	Accurate	1	Accurate
2008		19	Too many crossings	6	Too many crossings	11	Too many crossings	6	Too many crossings	8	Too many crossings	6	Too many crossings
		USD/JPY exchange rate (alpha= 0.001)											
		i.i.d. Normal		i.i.d Student's t		Normal-GARCH		t-GARCH		Unconditional EVT		GARCH filt. EVT	
The Year of Back Testing	Interval for # of crossings	No of crossing	Decision	No of crossing	Decision	No of crossing	Decision	No of crossing	Decision	No of crossing	Decision	No of crossing	Decision
2015	[0,2]	1	Accurate	0	Accurate	2	Accurate	0	Accurate	0	Accurate	2	Accurate
2014		0	Accurate	0	Accurate	0	Accurate	0	Accurate	0	Accurate	0	Accurate
2013		3	Too many crossings	0	Accurate	3	Too many crossings	0	Accurate	2	Accurate	0	Accurate
2012		0	Accurate	0	Accurate	0	Accurate	0	Accurate	0	Accurate	0	Accurate
2011		0	Accurate	0	Accurate	1	Accurate	0	Accurate	0	Accurate	1	Accurate
2010		0	Accurate	0	Accurate	1	Accurate	0	Accurate	0	Accurate	1	Accurate
2009		0	Accurate	0	Accurate	1	Accurate	0	Accurate	0	Accurate	0	Accurate
2008		12	Too many crossings	2	Accurate	7	Too many crossings	0	Accurate	5	Too many crossings	4	Too many crossings

Figure D.11. Comparison of yearly back testing results (USD/JPY).

[illegible]

Figure D.12. Back testing results of USD/JPY using each model