EXACT SEMANTICS FOR INDICATIVE CONDITIONALS

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EXACT SEMANTICS FOR INDICATIVE CONDITIONALS

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DECLARATION OF ORIGINALITY

I, Hüseyin Güngör, certify that

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ABSTRACT

Exact Semantics for Indicative Conditionals

This thesis extends Kit Fine's exact semantics and account of counterfactuals to non-habitual, non-predictive indicative conditionals. First, I provide Fine's exposition of exact semantics and his extension to counterfactuals. Later, I introduce a notion of context into the semantics and provide the truth-conditions for indicative conditionals by employing this notion of context. Afterwards, I turn to the logic of indicative conditionals under exact semantics and discuss the principles and inference rules which raise disagreements between variably strict and strict conditionals accounts. The account I provide shows its promise by validating a plausible combination of principles and strikes a balance between variably strict and strict conditional theories. I discuss certain principles in logic of indicative conditionals under exact semantics in detail and show how the present account validates the plausible combination of them. In the end, I draw comparisons between the viable theories for indicatives and the present one, and argue that the present account takes the advantage in several respects.

ÖZET

Bildirme Koşul Cümleleri için Kesinlik Semantiği

Bu tezde Kit Fine'ın kesinlik semantiğini ve bunun dilek koşul cümlelerine uygulanışını alışkanlık ve tahmin içermeyen bildirme koşul cümlelerine uygulayacağım. İlk önce Kit Fine'ın kesinlik semantiğinin ve onun dilek koşul cümlelerine uygulanışını özetleyeceğim. Ardından semantiğe formal bir bağlam kavramı tanımlayıp bu kavramı kullanarak bildirme koşul cümleleri için doğruluk koşullarını tanımlayacağım. Doğruluk koşullarını tanımladıktan sonra koşul cümlelerinin mantık kurallarına dönüp bizim semantiğimizin hangi çıkarım kurallarını destekleyip hangilerini desteklemeyeceğini tartışacağım. Bizim teorimizin diğerlerine nazaran güçlü yanları koşul cümlelerinin mantığında ortaya çıktığını göreceğiz. Bizim teorimizin şu zamana kadar ortaya atılmış katı ve monotonik olmayan katı koşul cümleleri teorilerinin desteklediği makul olmayan kuralları desteklemezken, onların avantajlarını ortaya çıkardıkları çıkarım kurallarını da destekleyerek literatürdeki teorilere kıyasla güçlü bir koşul cümlesi mantığı ortaya koyduğunu göstereceğim. Son olarak genel olarak teorimizin diğer teorilere nazaran nerede durduğunu tartışarak tezi sonlandıracağım.

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CHAPTER 1

INTRODUCTION

Kit Fine recently started fleshing out a version of truthmaker semantics called "exact semantics" (2012a; 2016; 2017). He thought it an alternative to possible world semantics to cater to its faltering aspects. As opposed to disjoint, complete and maximal nature of possible worlds *at which* statements are true or false, Fine takes (possibly) joint and partial fragments of worlds *which* make statements true or false, hence the name "truthmaker." Fine calls these fragments "states." The expectation is that this transition to a finer-grained framework will endow one with richer tools to engage natural language semantics with and, at the outset, it seems that this expectation is not let down, since Fine argues that exact semantics provides a potent array of tools for a variety of natural language phenomena such as partial content, subject matter, imperatives and scalar implicature (2017, §2). One of Fine's applications is interesting in particular—counterfactuals. In his 2012 papers, Fine argues that a contradiction can be drawn from Lewis' account for counterfactuals (1973) by employing additional plausible assumptions compatible with counterfactual reasoning and shows that exact semantics can positively resolve this contradiction (2012a; 2012b). Though Fine's account seems to require further elucidation and interpretation of the employed relations and tools which captures counterfactual reasoning, his groundwork is ripe for further development to capture conditional reasoning in general. I aim to provide the clarification and extend it to an additional application—indicative conditionals. Hence, the ultimate aim in this thesis is to extend exact semantics to provide the truth-conditions for indicative

conditionals and argue that the account can be a competitor to several other accounts on the market such as Stalnaker's (1968), Gillies' (2009; 2010) and Kratzer's (1986; 2012) in terms of its logic and other theoretical virtues.

The plan of the thesis is as follows: In Chapter 2, I provide an exposition of exact semantics in general, as Fine develops it (2017). In Chapter 3, I discuss how Fine extends his semantics to counterfactuals (2012a). Since his paper on counterfactuals precedes his general exposition of exact semantics, I take an anachronistic approach. However, this turns out to be more illuminating, since most of the ideas in 2012 paper becomes clearer, only when we view them as natural extensions of exact semantics. I keep the exposition as detailed as possible both in order to make the thesis self-contained and due to the framework being fairly new in the literature. In Chapter 4, I discuss how indicative and counterfactual conditionals are distinguished in general and present a way of capturing this distinction by introducing further tools into exact semantics. For this purpose, I develop a notion of context, which represents the preceding and mutually presupposed information in a conversational background. This allows us to capture the semantic distinction of indicatives from counterfactuals. By employing the notions, I have provided and in accordance with how I set up the semantics, I provide the truth-conditions for indicatives based on the truth-conditions Fine provides for counterfactuals. In Chapter 5, I turn to the logic of indicative conditionals under our account and show how certain principles are valid and other principles are invalid. Accordingly, I will comparatively show where the account stands with respect to other theories for indicatives in terms of which principles are validated and how certain problematic inferences which are valid under other theories are blocked under the current account. Lastly, I conclude by summarizing the features of the account

by discussing where it stands in general as a prospective account for indicative conditionals and noting the wanting aspects of the account, as it stands, such as inability to embed conditionals in the antecedent position, capture adverbs of quantification and habitual conditionals.

CHAPTER 2

TRUTHMAKER SEMANTICS

Exact semantics is a subdivision of truthmaker semantics. Truthmaker semantics is an umbrella term which is subdivided into various categories. First, there is the general clausal-objectual distinction, which is determined by the nature of truthmakers. For instance, Davidson (1967) takes the clausal approach. In Davidson's approach, truthmakers are not objects on the side of reality, but clauses or sentences which specifies the meaning of the statement or when the statement is true. The second major branch is the objectual approach which is mostly dominated by possible world semantics. However, possible world semantics was not the only game in town. There were several approaches which worked with smaller-than-world objects, which Fine calls in general "stately approach." Stately approach is mostly known and disseminated in the literature by Barwise & Perry's canonical work (1983). Fine's semantics also falls on the side of stately approaches. If we are to stick with Fine's classification (2017, p. 558), stately approach also consists of three subdivisions which are determined by the scope of their truthmakers. In an expanding order of their scope, these are called *exact, inexact* and *loose truthmakers*. Among these three approaches, Fine's approach falls within the exact approach. Exact approach seeks some complete and exhaustive relevance between the truthmaker and the statement made true by the truthmaker. With this briefest of the brief introduction to truthmaker semantics, I will start with Fine's exposition of exact truthmakers and show how other categories of truthmakers are recursively defined in terms of exact ones.

2.1 Exact semantics

Let us start with an analogy. In possible world semantics, a standard frame is an ordered pair (W, R) where W is the set of possible worlds and R is usually a binary relation on W, which determines how possible worlds are accessible from one another. A standard model for a propositional modal logic is an ordered triple (W, R, v) with v being the valuation function which assigns sentential letters and formulas of the relevant language to truth values in each world. Even though we call them "possible worlds," one does not need complete ontological commitment to possible worlds to the extent to which David Lewis committed (1986). For instance, Kripke treats possible worlds merely as indices or nodes of evaluation, which are purely mathematical objects, for formulas of a given language without interpreting possible worlds in a fully fleshed-out form (1963, p. 69). However, no matter which conception one adopts for possible worlds, it will be a common assumption or an implicitly accepted definition that possible worlds are maximal and consistent ("sets of propositions," "ways things could have been", "… disjoint entities" or what have you).[1] As opposed to a possible world frame, Fine adopts a frame $(S, \sqsubseteq)$ called "state space*"* where S is the set of all states and $\sqsubseteq$ is a parthood relation on S. States can be thought as fragments of possible worlds. Conceived in this way, they correspond to a wide-ranging family of truthmakers such as facts, events,

[1] "Maximal and consistent" is a common way to describe what a possible world is. Conjunction of maximal and consistent basically means that for every statement *P*, either *P* or ¬*P* will be true at a given world and never both. The former satisfies the condition of maximality and the latter—consistency. Even an axiomatic definition given to possible worlds meets these two criteria (see Zalta, 1993; 2014). I also want to note a general problem plaguing world and situation theory. There are several issues arising from each definition of worlds. For instance, taking them as maximal and consistent sets of propositions (see Adams, 1974; Kripke, 2017) leads to Kaplan's paradox (1995). My emphasis is on the conditions of maximality and consistency and *not* on what "maximal" and "consistent" adjectivize, i.e. sets, objects et cetera. If one wishes to remain clear of paradoxes or cardinality problems, she can take Zalta et al. (2014) construction of worlds, which are still maximal and consistent objects consisting of situations. They show that defining propositions and worlds in an object-theoretic framework renders their theory immune to paradoxes such as Russell's (1903, appendix B, §500) and Kaplan's (1995).

beliefs, emotions, attitudes et cetera.[2] By adopting a state space, the maximality requirement is dropped for the elements in our domain S. Instead, elements of the domain can be parts of each other.

In fact, this analogy is not completely accurate and we can be more precise. S is the set of all states which includes impossible states as well as possible states, whereas W is the set of possible worlds only. For instance, we have a state of its being the case P and $\neg P$ or a state of Fermat's Last Theorem being false in S, which are taken to be metaphysically impossible.[3] Therefore, a better analogy would be a frame (A, R) where A is the set of all worlds as opposed to merely possible ones. Of course, in such a model, there would be no logically valid formula, since we would have some impossible worlds where truth-functional tautologies would not hold. Moreover, we could not attach the necessity operator understood as truth in all worlds to any formula in such a model for the same reason, whereas the possibility operator would be attachable to every formula, contradictions included, since there will always be at least one world where even a truth-functional inconsistency or contradiction would hold. In order to regain validity, we need to put (A, W, R) where W is the set of possible worlds, which is a (proper) subset

[2] We take not only facts or situations, but also beliefs, emotions and other metaphysically contentious entities as states, since our concern is with any type of statement in the natural language. For a statement such as "Taylan believes that dogs are Martian agents," the state of Taylan's believing such-and-such will exist.

[3] Stalnaker and Lewis (1996; 1986 7n, 3) tentatively argued against the possible existence of impossible worlds on the grounds that it leads to a contradiction. However, the way they derive the contradiction assumes a bivalent background logic, which is exactly what is defined *not* to hold in impossible worlds or, at least, what is at stake at those worlds (see Priest et al. 2016). In other words, the purported existence of those worlds or meaningfulness of them exactly depends on the claim that this contradiction is derivable. So, it is in a sense self-defeating to attempt to prove the non-existence of impossible worlds by deriving a contradiction, because the existence of these worlds depends upon the derivability of this contradiction.

of A and define logical validity on W. For now, $(S, \sqsubseteq)$ is more similar to (A, R) than to (W, R).[4]

It is best to assume that the reader is not familiar with some version of situation or truthmaker semantics and show intuitively both how states can be parts of each other and how they make statements true in an example. For instance, take the statement "Da Vinci painted the *Mona Lisa*." This statement is made true by the state of Da Vinci having painted the *Mona Lisa*. However, we can imagine Da Vinci completing different tasks to paint the *Mona Lisa* and these correspond to different states as well. For instance, the state of Da Vinci having painted the *Mona Lisa* contains other states such as the state of Da Vinci having painted the right pillar of the *Mona Lisa*, the locks of hair et cetera. In a possible world where Da Vinci painted the *Mona Lisa* without painting the right pillar, the statement "Da Vinci painted the *Mona Lisa*" will be made true by a different composition of states, i.e. one which does not contain the state of Da Vinci having painted the right pillar. Furthermore, all Da Vinci did during the completion of the *Mona Lisa* was not to draw the *Mona Lisa*. Thus, looking down from above at the state of Da Vinci having painted the *Mona Lisa*, we will see that it is also part of a more comprehensive state such as Da Vinci having painted the *Mona Lisa*, taken a nap, read a tome, drunk water, et cetera.

Technically, parthood relation among states is effected by a partial order. $\sqsubseteq$ is a partial order on S, which allows the elements of the domain to be parts of each other. We will interpret "$s \sqsubseteq t$" as "s is a part of t." Being a partial order, $\sqsubseteq$ satisfies the following properties:

[4] For a general review of models for all worlds which include non-normal or impossible worlds, refer to Priest, 1992 and 2001.

Reflexivity: $\forall s \in S, s \sqsubseteq s$

Anti-symmetry: $\forall s, t \in S, s \sqsubseteq t \,\&\, t \sqsubseteq s$ implies $s = t$.

Transitivity: $\forall s, t, u \in S, s \sqsubseteq t \,\&\, t \sqsubseteq u$ implies $s \sqsubseteq u$.

Though we have shown how states can be parts of each other, we have not demonstrated how different states can be conjoined. To continue with the same example: in order to get to the *Mona Lisa* in its full beauty, we need to bring states of Da Vinci's painting the locks, the pillars, the paths together. This is what is called "fusion of states". Fusion is best conceived of as the union of states. From a theoretical perspective, this fusion is taken to be *the least upper bound* for some subset of states. This means that for any given subset T of S, s is an upper bound of T if $\forall t \in T$, $t \sqsubseteq s$ and s is a least upper bound if s is an upper bound and for all upper bounds s' of T, $s \sqsubseteq s'$. We will represent fusion of states $t_1, \ldots, t_n \in T \subseteq S$ as $t_1 \sqcup \ldots \sqcup t_n$. In order to secure that every subset of S has a least upper bound we impose the following condition:

Completeness: Every subset of S has a least upper bound.

This assures us of the fact that the fusion of any combination of states in the state space exists and relieves us of the requirement to define a separate operator $\sqcup$ to capture fusion, since any possible fusion will correspond to some least upper bound in the state space.

We have no distinction between possible and impossible states yet. S contains both possible and impossible states such as the possible state of one's talking and walking and the impossible state of one's talking and being silent at the same time. Fine modalizes the state space by putting $(S, S^{\Diamond}, \sqsubseteq)$ where $S^{\Diamond} \subseteq S$ is the set of possible states. What this achieves is the distinction of possible states from impossible states. Remember

the analogy above; we now have an analogue of (A, W, R) where W is the set of all possible worlds. If we wanted to define validity in a given model for our frame, we would do so with a modalized state space rather than a bare state space, because states in $S - S^{\Diamond}$, namely, impossible states, would stymie any formula from being logically valid. It seems that while providing the general framework, Fine wants to cast as wide a net as possible in order to give one the freedom of choice to tailor the framework to her specific needs.[5]

Modalizing the state space allows one to define two important notions, *compatibility* and *incompatibility*. If the fusion of two states $s \sqcup t$ is a possible state, i.e. $s \sqcup t \in S^{\Diamond}$, they will be compatible and if not, i.e. $s \sqcup t \notin S^{\Diamond}$, they will be incompatible. In order to block impossible states from being parts of possible states, Fine imposes the following restriction (Fine, 2017, p. 560):

Downward Closure under Part: $\forall s \in S^{\Diamond}, \forall t \in S$, if $t \sqsubseteq s$, then $t \in S^{\Diamond}$

This closure is downward in the sense that impossible states cannot be parts of possible states, but it is possible that possible states are parts of impossible states.[6]

For our purposes, Completeness is too loose. It gives us too many fusions and some of these fusions are incompatible and impossible states such as for any statement P, $P \wedge \neg P$-states. Now we have the notion of compatibility, we can impose a more rigid constraint on our possible state space than Completeness. In a sense, this is forced upon us, because we will have no business with impossible states for indicative conditionals. We rigidify Completeness by putting:

[5] For instance, if one were to engage counterpossibles, pinpointing which worlds are metaphysically impossible and which worlds are epistemically impossible would play a vital role.

[6] For instance, consider the states s, h and d for one's being happy, unhappy and sleepy respectively. $s \sqcup h \sqcup d$ is obviously impossible, but it contains $h \sqcup d$, which is possible.

Bounded Completeness: Every consistent subset of $S^{\Diamond}$ has a least upper bound. By a consistent subset, I mean that all of the states in the subset are compatible with each other. It is easy to see that the least upper bound for this subset is in $S^{\Diamond}$ as well. This along with Downward Closure prevents impossible states from having fusions in the possible state space. For instance, though both the state of a ball being smooth and the state of its being not smooth will be possible states separately, their fusion will be an impossible state and will have no fusion in $S^{\Diamond}$, though it will have a fusion in S. To reiterate, this choice is not forced on us and it is just what is proper for our purpose of providing a semantics for conditionals. If we wanted to deal with counterpossibles, we would be forced to include incompatible states and fusions thereof in our state space.

Within a possible state space, we can give a definition of what a possible world is by employing the notion of incompatibility. We briefly discussed above that no matter which position we adopt in terms of what possible worlds are, they share the property of being maximal and consistent. We can capture these properties within a possible state space with the following definition:

World State: w is a world state only if w is a possible state and any state $s \in S$ is either a part of w or incompatible with w. In symbols, w is a world state only if $w \in S^{\Diamond}$ and $\forall s \in S$, either $s \sqsubseteq w$ or $s \sqcup w \notin S^{\Diamond}$ (Fine, 2017, p. 561).

Since world states are possible, there will be no incompatible states which are parts of world states and we will not introduce impossible worlds into our possible state space. Moreover, they positively either exclude or include every other state (ibid). The former property preserves the consistency requirement of the definition of worlds and

the latter satisfies the maximality requirement. By this way, we can create a copy of the domain of possible worlds within the possible state space.

If we impose the restriction that every possible state be part of some world state, then we will make $(S, S^{\Diamond}, \sqsubseteq)$ a *world space* or W-space where world states do not leave any logical space of possible states uncharted. What this achieves in effect is that the logical space of possible states will exhaustively map onto the W-space and what this means, in turn, is that whenever a possible state verifies a statement, that statement will be verified by some world state. Of course, we could have assumed a state space or a possible state space without assuming a W-space. We are assuming a W-space specifically because we will be dealing with conditionals, as Fine does for his counterfactual account (2012a, p. 236, fn. 18). Fine does not elucidate why he assumes a W-space, but we can make clear why we are assuming a W-space. At this point we have not said anything about conditionals, but we can give an intuitive explanation without going into detail. While evaluating conditionals, we keep certain parts of the world as they are and consider the world *as if* the antecedent of the conditional holds there and evaluate whether the consequent is true under these circumstances. The truth of a conditional depends not only upon which truth-value its atomic constituents have, but also upon *under what conditions* the consequent can be true together with the antecedent. This means that whenever we evaluate a conditional, we assess some relation between the antecedent and consequent, that is, whether they can be true at the same world. Therefore, we always need to assess a conditional relative to a world state. This is the reason why we are assuming a W-space.

Let us now take a propositional language L consisting of countably many propositional variables p_1, p_2 ..., and closed under the Boolean operators of negation ¬, disjunction ∨ and conjunction ∧.[7] A modalized state space model on L will be a quadruple $(S, S^{\Diamond}, \sqsubseteq, |\bullet|)$ where $|\bullet|$ is a valuation function which maps each sentential variable of L to the ordered pair (V, F) of subsets of S where V is for exact verifiers and F for exact falsifiers. In symbols, s ||- p_i iff for some $s, s \in |p_i|^V$ where $|p_i|^F$ represents the set of exact verifiers for p_i and s -|| p_i iff for some s, $s \in |p_i|^F$ where $|p_i|^F$ represents the set of exact falsifiers. We require that no verifier of any statement be compatible with the falsifiers of that statement, i.e. for any $s \in |p_i|^V$ and $s' \in |p_i|^F$, $s \sqcup s' \notin S^{\Diamond}$, which is called Exclusivity (2017, p. 562). This precludes any statement from being both true and false. However, we do *not* impose what Fine calls Exhaustivity (2017, p. 562), which means that any statement is either true or false. Lack of Exhaustivity allows for truth-value gaps. This will turn the present account into a trivalent account, which lets indicative conditionals be true, false, or indeterminate. Up until now, we have been using the notions of verification and falsification without explication. Since these notions constitute the bedrock of the semantics at hand, we ought to put more flesh on their bones.

[7] It would be good to pause here and fix the language L's orthography. I shall use small-capped letters $p_1, \ldots, p_n$ to denote sentential variables given in L, capital letters P, Q, R, A, B, C to denote formulas which are formed out of the sentential variables, Boolean operators and conditional operators later to be defined into L. In the orthography of the proof theory, if a formula occurs at the left-hand side of a verification symbol, i.e. P ||- Q, the verification symbol will denote entailment or consequence instead of verification; otherwise, it will denote verification types as introduced in the thesis.

2.2 Verification

Verification is the basic notion of making a statement true. What verifies is a state and what is verified is a statement. For instance, the presence of some lantern on the table being alit verifies the statement "The lantern on the table is alit". Unlike possible world semantics which has one uniform notion of satisfaction or verification, our notion of verification comes in various forms and magnitudes:

Exact verification: A state s exactly verifies a statement A (symbolized as $s \Vdash A$) if s is *wholly relevant* to the truth of the statement A. Intuitively, it is plausible to assume that an exact verifier for a statement is a state which does not contain any *irrelevant* parts to the statement. For instance, the state of rain will be wholly relevant to the truth of "It is raining", whereas the state of rain and wind will be relevant more than to the truth of "It is raining."

Inexact verification: Inexact verification can be defined recursively on exact verification. A state s inexactly verifies a statement A, if some state t is a part of s and t exactly verifies A. In symbols,

$s \Vdash\!> A$, if there is a t such that $t \sqsubseteq s$, $t \Vdash A$. The presence of rain and wind will be an inexact verifier for the statement "It is raining," since the presence of rain is a part of it.

Loose verification: Loose verification is a modal notion. A state loosely verifies a statement *B* if it is incompatible with any exact falsifier of the statement. In symbols, $s \vDash B$ if $\forall t \in |B|^F$, $s \sqcup t \notin S^{\Diamond}$ where $|B|^F$ is the set of exact falsifiers for *B*. Any state will loosely verify any tautology. For instance, the state of my brother crying will loosely verify the statement "either it is sunny or not sunny," since nothing will falsify it. More importantly, by the definition of World State, if a world state verifies a statement,

it will always loosely verify it, since the world state will contain an exact verifier for the statement and, thus, it will be incompatible with its exact falsifier.

It is also important to note that every exact verifier for a statement will also be an inexact verifier for the same statement due to the reflexivity of $\sqsubseteq$ and every inexact verifier for the same statement will be a loose verifier for the same statement, because it will be incompatible with the exact falsifiers of the same statement. Therefore, if we have an exact verifier for a statement, then we will also have the same verifier as a loose verifier for the statement. However, the transition from loose verifiers to the exact verifiers will not be valid, as the reader can easily verify for herself.

There are three notions of consequence or entailment corresponding to three forms of verification (Fine, 2012a, pp. 235-236). If all the exact verifiers of a statement P are also the exact verifiers of Q, we will say Q is an exact consequence of P or P exactly entails Q, i.e. $P \Vdash Q$. The same goes for inexact and loose entailment. For instance, let p and q be exact verifiers for P and Q respectively. Then $P \wedge Q$ will not exactly verify Q, i.e. $P \wedge Q \nVdash Q$, since the exact verifier of $P \wedge Q$, which is, say, $p \sqcup q$ is not the exact verifier of Q, say, q, but it contains an exact verifier for Q. Thus, though $P \wedge Q \nVdash Q$, $P \wedge Q$ will inexactly entail Q, i.e. $P \wedge Q \Vdash\!\!> Q$.

The verification rules for atomic propositions, negation, conjunction and negations as Fine (2017, p. 562) provides them are as follows: For an arbitrary state s:[8]

(i)$^{+}$ $\quad s \Vdash A \quad$ iff $\quad s \in |A|^{+}$

(i)$^{-}$ $\quad s \dashv\Vdash A \quad$ iff $\quad s \in |A|^{-}$

[8] I use "-||" for exact falsification, which corresponds to valuation function assigning sentential variables to a set of falsifiers.

(ii)$^+$	s ||- $\neg A$	iff	s -|| A
(ii)$^-$	s -||$\neg A$	iff	s ||- A
(iii)$^+$	s ||- $A \wedge B$	iff	$\exists t, u(s = t \sqcup u$ & t ||- A & u ||- B)
(iii)$^-$	s -|| $A \wedge B$	iff	either s -|| A or s -|| B or s -|| $A \vee B$
(iv)$^+$	s ||- $A \vee B$	iff	either s ||- A or s ||- B or s ||- $A \wedge B$
(iv)$^-$	s -|| $A \vee B$	iff	$\exists t, u(s = t \sqcup u$ & t -|| A & u -|| B)[9]

Valuation function and verification rules provide the definition of propositionhood. In possible world semantics, we take a proposition to be the sets of possible worlds at which its corresponding statement is true, but do not specify the worlds at which it is false, because they will be the worlds at which it will not be true. For exact semantics, the idea is a bit more elaborate, because falsity cannot be defined upon complementation on the set of possible worlds:[10]

Proposition: There are two ways we can define propositionhood. The *unilateral* conception of propositionhood requires that propositions be the sets of their exact verifiers; symbolically, for a statement A, $|A|^V$ or $V_A = \{s \in S: s \Vdash A\}$. The *bilateral* conception requires that propositions be the ordered pair (V_A, F_A) where F_A is the exact falsifiers of A or $|A|^F$(Fine, 2017, p. 563). If we want to express A's falsity, then we can

[9] I have assumed an inclusive semantics from the outset, meaning exact verifiers for $A \wedge B$ are also exact verifiers for $A \vee B$ and falsifiers for $A \vee B$ are also falsifiers for $A \wedge B$. I am using the formulation, as it is presented in (Fine & Jago, Forthcoming)

[10] The reader might be unfamiliar with this definition of falsity in possible world semantics. Roughly, we do not need to define falsity for propositions in possible world semantics, because a proposition will be false at worlds where it is not true and "not true" is defined by complementation on the set of possible worlds. For instance, let (W, R, v) be a generic model for a propositional language K and A be a formula. If we denote truth in a world w as $v_w(A) = 1$ for some formula A and denote the truth of A in a set of some worlds $S = \{w_1, \dots, w_n\}$ as $v_S(A) = 1$, then for some set of possible worlds S, $v_S(A) = 0$ iff $v_{\bar{S}}(A) = 1$ where $\bar{S}$ represents the complement of S on W, $W - S$, which are the set of worlds where A is not true.

represent it as (F_A, V_A), that is, by changing the order of the elements in the tuple. I will assume a *unilateral* conception of propositionhood throughout to simplify the exposition at first, but since falsifiers will become significant in defining a notion of context and certain other places, I will make use of falsifiers as well. There will be no theoretical inconsistency hinging on this choice.

Before concluding the exposition, I want to address one issue concerning the exact verifiers for negated statements or exact falsifiers for normal, unnegated statements. Does exact semantics not suffer from the same problem as a generic truthmaker account does, that is, positing truthmakers for the negation of a proposition? Fine mentions two ways of defining falsification (2017, p. 562): the first is to define it in terms of exact verification, i.e. an exact falsifier of a statement *A* is the exact verifier for the statement $\neg A$. However, he realizes that it is not clear which sets of verifiers correspond to the negation of a proposition and, thus, takes exact falsification as another primitive notion similar to exact verification (ibid.). The general trouble with taking exact falsification as another primitive is that we seem to open a can of worms, that is, postulate truthmakers for the negations of propositions and it is far from clear what is on the side of the world which makes a statement such as "No one showed up at the meeting" true. However, Fine argues that we conflate the worries here:

> "Indeed, the general focus of the two projects is very different. If our aim is to understand the world, then our focus should be on the ultimate truthmakers, on what in the world ultimately makes something true, and the question of how the truthmakers make the statements of our language true is of no great concern. But if our aim is to understand language, then our focus should be on the immediate truthmakers, not the ultimate

> truthmakers, and the question of how they make the statements of the language true will be of greatest concern." (Fine, 2017, p. 557)

How convincing this distinction is up to the worry of the reader. If one is seeking a complete metaphysics + natural language semantics, which consists of a parsimonious metaphysics for truthmaking and complete truth-conditions for statements in the language, then this distinction is exactly what they would be arguing against. On the other hand, if one finds the division of labor in understanding the language and understanding the world justified, then they can rest content with positing exact falsifiers as another primitive notion in their semantics, since inflating or deflating the ontology is not one's concern here. Since this is a side point for the project at hand, I will accept the distinction and take falsifiers as another primitive notion in the framework.

Let us summarize where we are before extending the semantics to counterfactuals. First, we distinguished three different versions of verification—exact, inexact and loose. As long as we take exact verification as primitive, we showed that we can define inexact and loose verification recursively. Differences between these versions of verification will play an important role in determining the truth-conditions for conditionals. Second, we assumed a world space, W-space, which is a possible state space with a further constraint, since conditionals will be true or false relative to world states. We also assumed Exclusivity, but not Exhaustivity, since there will be certain indicatives which will be neither true, nor false. Now that we have taken stock, we will show how Fine introduces a counterfactual connective $>$ and its truth-conditions to his semantics.

CHAPTER 3

TRUTH-CONDITIONS FOR COUNTERFACTUALS

Upon this framework, Fine expands the language with a counterfactual connective $>$ in his 2012 paper (2012a). However, this is a non-trivial job and it gives rise to several complications that should be dealt with. I will follow Fine's steps, but provide elucidation and discussion where necessary. Fine's main motivation seems to be able to accommodate the failures of what is called "Substitution of Logical Equivalents" (henceforth, Substitution) in possible world semantics, which leads to difficulties in the logic of counterfactuals (2012b).[11]

We can again start from a more familiar ground and then make our way from there. In possible world semantics, a closeness relation is employed on worlds in order to evaluate counterfactuals. Roughly put, the idea is that a counterfactual $A > C$ will be true, if all closest worlds where A is true are also the worlds where C is true and closeness is determined by a ternary accessibility relation $\preccurlyeq$ on the domain W of possible worlds. $\preccurlyeq$ takes a centered world w and roughly compare two other worlds w', w'' relative to w and according to some interpretation metric defined on $\preccurlyeq$. Theoretically, this induces an ordering relative to every element in W. $w' \preccurlyeq_w w''$ is

[11] Two notes: 1) When I say "possible world semantics", I refer to David Lewis' and Robert Stalnaker's accounts, as Fine does (Lewis, 1973; Stalnaker, 1968) and not use it as an umbrella term, unless I explicitly make the distinction between several accounts which adopt some version of possible world semantics such as Gillies' (2007) or von Fintel's (2001). 2) Substitution is the principle which states that we can substitute logically equivalent statements with each other in antecedent and consequent positions. The main reasoning behind this is that, since logically equivalent statements are true in the same possible worlds, the truth-value of the statement does not change. We will discuss Substitution in Chapter 5 in detail.

interpreted as "w' is as close to w as w'' and represent it as $w' \preccurlyeq_w w''$.[12] Of course, this is purely a theoretical way of putting the relation without interpreting in terms of which aspect the worlds are close to each other. For instance, if we interpret closeness as matching up in terms of laws of nature, then we obtain Lewis' (1973) interpretation of the accessibility relation. Instead of working with a closeness relation, Fine introduces a transition relation:

Transition Relation: Fine's informal definition is as follows: "Let us use $t \rightarrow_w u$ to indicate that u is a possible outcome of imposing the change t on the world w"[13] (2012a, p. 237). The transition relation takes the *exact* verifiers of the antecedent, "imposes" it on the relevant world state and yields "a possible outcome." Whenever we see $t \rightarrow_w u$, we are supposed to interpret it as "u is a possible outcome of imposing *t* on *w*." What does "impose" mean, though? What I understand by "imposing" is analogous to Ramsey's now-ubiquitous idea in conditional reasoning:

> "If two people are arguing 'If p, will q?' and both are in doubt as to p, they are adding p hypothetically to their stock of knowledge and arguing on that basis about q." (Ramsey, 1931, p. 249)

Instead of taking a belief and adding it to our stock of knowledge, we add exact verifiers of the antecedent to the world state, make the necessary changes in the world to maintain consistency and take the outcome as this modified world. If we are evaluating a counterfactual, i.e. "If Oswald had not shot Kennedy, someone else would not have shot him," we impose the state of Oswald not having shot Kennedy on this world's history

[12] I am skipping various details about Lewis' framework, which is of no great importance for my purposes.
[13] This change talk should not mislead, though. Just as not every counterfactual has necessarily a false antecedent, not every transition relation induces a change on the world.

and keep the rest of the facts fixed. If investigations around the scene of crime yielded no information on the existence of other shooters, we obtain a possible outcome of no one else having shot Kennedy. This suggests that the counterfactual is true.

However, these are the types of cases where there is a determinate outcome arising out of the antecedent. I intend for "determinate" to mean that imposing the antecedent on the world does not yield outcomes which verify both the consequent and the negation of the consequent. However, not all transitions yield determinate outcomes. For instance, consider the counterfactual, "If Mehmet Emin had flipped a coin, then the coin would have landed heads." Suppose that Mehmet Emin always flips a fair coin and his tosses are stochastic events. Imposing the state of Mehmet Emin's having flipped a coin on the world of evaluation yields both the outcome of the coin having landed heads and tails. Since landing heads and landing tails are mutually exclusive and jointly exhaustive events, the statements exactly verified by these states are negations of each other. For now, we will accept that there are mutually contradictory outcomes which can be obtained by imposing a verifier on the world and leave it at that. I will not consider these types of cases for Fine's account of counterfactuals, since this would prolong the thesis unnecessarily. However, when we extend the semantics to capture indicative conditionals as well, we will turn back to this and provide our resolution within the trivalent framework we adopted by dropping Exhaustivity. Now that we have an intuitive grasp of the transition relation, we need to illuminate what a possible outcome of imposing a state amounts to.

Possible Outcome: Aside from stating that a possible outcome is naturally interpreted according to the type of counterfactual (future causal, backtracking et cetera) (Fine, 2012a, p. 237), Fine does not formally define what a possible outcome is. Without

specifying, we will define it in the most general way possible: a possible outcome of an antecedent is the way the world can be after we induce the exact verifier of the antecedent as a change on the world. If one wants his utterance to be read as backtracking, then the possible outcome will be yielded as a backtracking possible outcome. For instance, a backtracking counterfactual "If I had jumped out of the window, I would have died" will be false, because I would have jumped only if there is a safety net dragged across where I would have landed and the transition relation will return possible outcomes which guarantees the conditions for me to jump, namely, a safety net dragged. Therefore, in general imposing an exact verifier on a given world state will produce possible outcomes which can be taken as future causal outcomes, backtracking outcomes et cetera. One issue, which plagues possible outcomes, are underdetermination. We have mentioned a case of underdetermination by giving the example of flipping a coin. I will not resolve this issue of underdetermination here, but merely acknowledge that there may be possible outcomes of imposing an exact verifier for some antecedent, which verify mutually contradictory statements.

Another issue arises in terms of interpretation of possible outcomes. A possible outcome can also be taken to be an epistemic possible outcome, which reflects the salient possibilities attended by a speaker, depending on the information possessed by the speaker or it can be a metaphysical causal outcome, which reflects the possible ways the world can be, depending on the laws of nature or fundamental laws Fine (2012a, p. 237) naturally interprets the possible outcomes for counterfactuals as metaphysically causal outcomes, be that they are future or backtracking. We will see below that Fine's interpretation will not be suitable for our purposes. We will need further tweaking of

transition relation to capture epistemic outcomes. I will discuss the epistemic and metaphysical outcomes in §4 in more detail.

There is also a general worry about notions such as transition relation and possible outcome which might be thought to be endemic only to these notions in Fine's account: They are too vague. I believe that it would be a mistake to think that conditionals are usually non-vague and determinate and only when vague notions such as transition come into play, conditional semantics become blurry, indeterminate and hazy. In fact, this was exactly Mackie's criticism (Mackie, 1972, pp. 89-90) for Lewis' original conception of similarity relations and non-monotonic engagement with counterfactuals. However, I believe that Stalnaker's reaction to these objections marks an important point admissible for the metasemantics of conditionals in general (1984, p. 148).

Mackie criticizes Lewis' account, because Lewis' account fails to eliminate vagueness and indeterminacy in counterfactuals. Mackie's criterion of success for a theory of conditionals seems to be that a theory of conditionals must eliminate the vagueness of conditionals as much as possible and provide clear-cut and determinate truth-conditions for them. However, Stalnaker disagrees and believes that Mackie's criticism is misguided in two important ways. Stalnaker defends Lewis' account by emphasizing that Lewis' goal in the first place was not to eliminate the indeterminacy in counterfactuals; on the contrary, it was to recognize the wayward behavior of counterfactuals and give an equally flexible account to capture the inherently indeterminate behavior of counterfactuals.[14]

[14] Fine's example (1975, p. 452) can be given as exemplifying this indeterminacy. Consider the counterfactual "If Nixon had pressed the button, there would have been a nuclear holocaust." It is easy to

I believe that Stalnaker's defense can be taken a bit further. Stalnaker is right in claiming that Lewis' success lied in capturing something as indeterminate as counterfactuals in an equally indeterminate way and matching his theory to the indeterminacy of counterfactuals, while maintaining the connection between the theory and counterfactuals as lucid as possible. Of course, both Mackie's and anyone else's potential question is "What is the use of Lewis' theory then?" I believe that the use of a theory in general does not merely come from taming our use of a certain tool or concept, in this case, counterfactual utterances; it mostly comes from making the connection between counterfactuals and their formal modeling clear and precise or capturing the way counterfactuals are naturally used. If the use of counterfactuals is naturally indeterminate, then providing a theory which models the counterfactuals as used in an exact way would be an incorrect semantic theory for counterfactual reasoning. An analogy from physics may help us here: if the dynamics of a closed system is inherently probabilistic and indeterminate in certain aspects and allows for inherently probabilistic outcomes, then trying to construct a deterministic classical theory which allows for hidden classical variables or, in other words, does not allow for indeterminacy of measurement results will be an incorrect theory which will make inaccurate predictions, as Bell's thought experiment and Aspect's results show (Bell, 1964; Aspect et al, 1982).[15]

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imagine a scenario where this is true—simply a possible world where Nixon presses the button and the Armageddon occurs. However, it is also easy to imagine a scenario where this is false, since the likelihood of a nuclear war is smaller than the button malfunctioning, when we consider the comparative total similarity of the possible world up to some past time *t* where the button is pressed to the actual world where the nuclear holocaust did not happen.

[15] I am merely using this analogy to illustrate my point. I realize that Bell's results need not be as decisive as I make them sound. Those who are interested should refer to (Shimony, 2009).

The upshot seems to be that a successful theory of counterfactuals is *not* the one which eliminates the indeterminacy of counterfactuals, because that would require imposing unnatural rules on how people naturally use counterfactuals, but it is one which precisely models this indeterminacy. However, just because the nature of counterfactuals we construct a theory for is indeterminate does not mean that the relation between counterfactual utterances and our theory must be an indeterminate relation as well. To reiterate, Lewis' success was to model in a beautiful fashion the indeterminate way in which counterfactuals are uttered and used. I would like to show how Lewis' theory enhanced our understanding of counterfactuals and how Mackie's criticism misfires by employing another analogy from physics. We cannot claim that we are in no better position in our understanding the fluid dynamics with Navier-Stokes equations than without them, just because there are not always smooth solutions to these equations and we cannot predict precisely at which Reynolds number turbulent flows occur by way of these equations.[16] Equally, we cannot say that we are in no better position in understanding the semantics or uses of counterfactuals in natural language with Lewis' theory than without, because indeterminacy in counterfactuals persist. I believe that this argument equally applies for notions of transition and possible outcome. In our case, transition relation and possible outcome are tools to capture Ramsey's idea and the related indeterminacy in conditional reasoning.

This line of defense for semantics of conditionals also works against a general worry about notions such as closeness of possible worlds and transition. The objection runs that notions such as closeness and transition themselves hinge on some sort of counterfactual or conditional reasoning in general, since imposing a verifier on a world

[16] I am simplifying this process to the point of caricature just to make the analogy intelligible.

state may sound like a long-winded way of saying "If the antecedent were to obtain, then the consequent would obtain" (Fine, 2012a, p. 241). Fine considers this objection (2012a, p. 241), yet settles for a draw between his account and possible world semantics. We can provide a general defense for both Fine's account and possible world semantics along the lines sketched above. This objection rests on the same mistake as the objection against Lewis' not eliminating indeterminacy in counterfactuals. We are not seeking a long-concealed kernel within the use of conditionals upon the discovery of which we will open the gates of semantic heaven for conditionals. We are just trying to capture the already wayward use of conditionals in a formal framework which could also tell us about how we conceive the logic of conditionals. In this case, relying on another if-clause is not as problematic as it sounds, because it serves to illuminate what happens in other if-clauses.

After this long detour on the issue of indeterminacy in conditionals, we are ready to provide the truth-conditions of counterfactuals by employing the notions of transition and possible outcome:

TC: A counterfactual $A > C$ is true if any possible outcome of an *A*-state contains a *C*-state. Formally put, $A > C$ is true at w iff u inexactly verifies C whenever t exactly verifies A and u is a possible outcome of imposing t on w. Symbolically put, $w \vDash A > C$ iff $u \Vdash\!> C$ whenever $t \Vdash A$ and $t \rightarrow_w u$ (Fine, 2012a, pp. 236-237).

However, as we have seen from the verification of atomic sentences, certain statements may have more than one verifier. Take, for instance, $A \vee B > C$. Suppose a and b as exact verifiers for A and B respectively. Now a, b and $a \sqcup b$ are all exact verifiers for $A \vee B$. The question whether we should consider either one of these

verifiers or all of them to impose on a world state to obtain a possible outcome arises. According to Fine, we should consider all of them. He does this by bringing out two restrictions in order to define how we will determine which verifiers to send into the transition and how consequents are evaluated:

Universal Realizability of the Antecedent (URA): $A > C$ will be true only when it is true for *any* way in which the antecedent A might be true (Fine, 2012a, p. 236). For a counterfactual "If he had taken his pills or shot himself in the face, he would have gotten better" we will consider *not only* the exact verifier of "He took his pills" but also the exact verifier for "He shot himself in the face."

Universal Verifiability of the Consequent: $A > C$ will be true, given some way in which A is verified, only when C is verified under *any outcome* in which A is verified. Take the same example: for the counterfactual "If he had taken his pills or had shot himself in the face, he would have gotten better," the possible outcome of his taking his pills may make it true that he gets better, while shooting himself in the face will not. Thus, the possible outcome is not universally verified of the verifiers of the antecedent and the counterfactual is false.

These two rules will be crucial to the evaluation of conditionals in general. One may also realize that these restrictions make the semantics similar to that of a strict conditional. However, this should not immediately evoke the impression that the account will suffer from the same problems a generic strict conditional account does. The subtleties and idiosyncratic aspects of the account will emend most of the usual problems the strict conditional accounts inherit.[17]

[17] There is a long story behind the strict conditional account, which I will not tell here. For a detailed survey and analysis, refer to (Starr, Forthcoming).

Let us demonstrate how we carry out the evaluation of counterfactuals with the given account. Take the counterfactual "If Will struck the match or dipped it in water and struck it, the match would light up."[18] First, determine the exact verifiers of the antecedent taking URA into account: the state of striking the match in the room and the state of dipping the match in water and striking it. Impose these verifiers on the world state and check whether or not all of the possible outcomes of the verifiers for the antecedent *inexactly* verify the consequent. Imposing the state of striking the match yields an outcome which inexactly verifies the consequent. Consider the state of dipping the match in water and striking it: a possible outcome of this will inexactly verify the statement "the match will not light up", hence, will not inexactly verify the consequent "the match will light up". Since UVC is violated, the counterfactual will be false.

Note that transition relation is the main apparatus to govern counterfactual reasoning for Fine. If the account will validate any principles, then it will be through the modifications on the transition relation. Fine duly recognizes this and modifies the transition relation to validate certain plausible principles such as Modus Ponens:

Inclusion: If $t \rightarrow_w u$, then $t \sqsubseteq u$

Actuality: If $t \sqsubseteq w$, then $t \rightarrow_w u$ for some $u \sqsubseteq w$

Incorporation: If $t \rightarrow_w u$ and $u' \sqsubseteq u$, then $t \sqcup u' \rightarrow_w u$

Maximality: $t \rightarrow_w u$ only if u is a world state.

I will provide brief commentary on these rules. Inclusion makes sure that any possible outcome of a state imposed on a world state will include the state imposed. This settles the intuitive result that $A > A$ will always turn out to be true, since, when a state which

[18] I am assuming a lot here such as the context of utterance does not allow for insufficient oxygen in the vicinity and other relevant assumptions, which are harmless for the example at hand.

exactly verifies A imposed on a world state w, whatever the possible outcome is will turn out to have the verifier of A as a part, i.e. for any $t \rightarrow_w u,\ t \sqsubseteq u$ and $u \Vdash\!> A$ will hold. Actuality ensures that, if the imposed state is already a part of the world, then the possible outcome is also a part of that world. Actuality validates modus ponens. Incorporation ensures that for some outcome of an imposed state, the outcome is still the same outcome, when the fusion of any part of that outcome with the verifier of the antecedent is imposed on the world state. This principle makes sure that Weak Transitivity is valid.[19] For Fine, Maximality ensures the validity of classical weakening. For our purposes, it will also allow right-embedding of conditionals.[20]

This concludes my précis of Fine's counterfactual account. Taking stock, Fine's account seems to track intuitive truth-values of the counterfactuals by blocking Substitution better than a variably strict account such as Lewis'.[21] Lewis' account treat $A \vee (A \wedge B) > C$ as truth-functionally equivalent to $A > C$. That is, "If Will struck the match or dip the match in water and struck it, the match would light up" and "If Will struck the match, the match would light up" come out as equivalents truth-functionally, since $A \vee (A \wedge B)$ and A will be true at identical worlds. Even though Fine claims to raise his criticism against what he calls "possible world semantics," he has in mind only Lewis' account for counterfactuals, since there are other accounts for counterfactuals based on possible world semantics which may not be treated in the same way as Lewis' theory. When we survey the literature, we find at least two of them: Gillies (2007) and von Fintel (2001). Even though Gillies and von Fintel has not addressed the problems

[19] More on Transitivity at Chapter 5.3.
[20] For proofs of how these principles hold via the modification on the transition relation, see Fine, 2012a, pp. 239-240.
[21] There are other advantages of Fine's account such as validating Simplification of Disjunctive Antecedents (SDA). More on this at Chapter 5.

arising due to Substitution, one may imagine the possible resolutions they will provide based on their resolution of what is usually called *reverse Sobel sequences* or invalid inference patterns such as Antecedent Strengthening. I will deal with Gillies' possible response to Substitution in detail at Chapter 5.1. Now, I will proceed to extend the account to indicative conditionals.

CHAPTER 4

FROM COUNTERFACTUALS TO INDICATIVES

Since we have the truth-conditions for counterfactuals, our job is now to extend the account to indicatives. This means extending our language L with an indicative connective $\longrightarrow$. However, this is not easy, since we do not even have a distinction between counterfactuals and indicatives at hand. Even though counterfactual and indicative conditionals are both "if, then" utterances, we can come up with conditionals with same statements for their antecedents and consequents (modulo their tenses), but they may differ in truth-value and meaning. This is usually demonstrated with Adams' examples (1970, p. 90):

(1) If Oswald did not shoot Kennedy, then someone else shot him.

(2) If Oswald had not shot Kennedy, then someone else would have shot him.

In general, we can notice both a grammatical and a semantic difference between (1) and (2). I will focus on the semantic difference.[22] Though (1) and (2) share a syntactic similarity and similar statements for their antecedents and consequents, what they convey in meaning seem to be completely different. Intuitively put, if one wanted to put (1) in non-conditional terms, she would utter "Kennedy was shot" or "Kennedy died by being shooting," whereas she would put (2) in non-conditional terms as "There were back-up shooters."[23] What this tells us intuitively is that semantics of (1) and (2) differ

[22] For a more detailed analysis of the grammatical distinction, see Khoo, 2015.

[23] This is one of the strengths of Yablo's account for indicative conditionals (2016). It seems like, when we logically subtract the antecedent from the consequent in (1), we seem to obtain "Kennedy was shot," which is exactly what Yablo's account predicts. However, Yablo's approach does not encapsulate counterfactuals and does capture only a special form of indicative conditionals he calls "absolute conditionals" (2016, p. 4).

significantly. No matter what truth-conditions one ultimately provides for subjunctives and indicatives within any type of semantics, they should track these intuitions.

At the outset, one distinction between indicatives and subjunctives seems to be with the semantic presupposition of their antecedents. One may say that the antecedent of (1) seems to presuppose the possibility that Oswald did not shot Kennedy, whereas the antecedent of (2) seems to presuppose *the fact* that Oswald *actually* shot Kennedy or, in other words, the falsity of its antecedent. This distinction may seem to capture how we use subjunctives and indicatives. Yet, we can show that it is neither necessary, nor sufficient to draw the distinction. One of the reasons for this is that there are straightforward counterexamples where counterfactual[24] conditionals do not presuppose the falsity of their antecedents. Anderson provided such an example (1951; for further discussion, von Fintel, 2012): "If Jones had taken the arsenic, he would have shown just exactly those symptoms he actually shows." Anderson claims that a doctor would use this subjunctive as a premise for his argument that Jones actually took the arsenic. This means that Anderson's subjunctive is actually used as a premise to show the truth of the antecedent rather than the falsity thereof. One might also come up with other examples such as "If she were depressed, that would explain her silence" (example from Williamson, Forthcoming, Chapter 1). I might utter this to someone to let her affirm my suspicion that she is actually depressed. Examples of this kind present strong evidence for the thesis that the semantic presupposition of the antecedents of counterfactuals and

[24] One may sense a small inconsistency in the use; if the title "counterfactual" bothers the reader, she can substitute with its linguistic cousin "subjunctive."

antecedents is neither necessary, nor sufficient for one to capture the semantic distinction between counterfactuals and indicatives.[25]

If indicatives and counterfactuals cannot be individuated solely from their semantic presuppositions, we have to look elsewhere.[26] It is usually expressed that counterfactuals express a *metaphysically causal connection* between their antecedents and consequents, whereas indicatives express only an *epistemically causal* one (Khoo, 2015; Weatherson, 2001, p. 6; Zhao, 2015, pp. 496). It is generally argued that this distinction is marked by the tense shift in subjunctives or modal claims (Khoo, 2015, p. 2). For instance, (1) is dependent upon our information of the facts. So, something like the following sounds in the ballpark:

(1*) For all I know, Kennedy was shot and If Oswald did not shoot Kennedy, someone else did.

This leads to the following corollary to (1*):

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[25] There is another worry about drawing the distinction in this way. Forcing the constraint on counterfactuals to have false antecedents leads to more serious troubles such as the categorical invalidity of modus ponens for counterfactuals. Certain counterfactuals may not contain contrary-to-fact antecedents, meaning that they might have true antecedents in the world of evaluation. Indeed, denying this would be denying counterfactuals modus ponens (Some opted for this option; see, McGee, 1985). Due to this distinction, one might think that modus ponens is naturally invalid for counterfactuals. Let me present how one might doubt modus ponens in this context and put this doubt to rest. In a situation where one accepts "If Oswald had not shot Kennedy, someone else would not have," comes to learn that Oswald did not shoot Kennedy and retains his information that Kennedy is dead due to a shooting, one would *not* conclude that "No one else shot Kennedy." He would just realize that his counterfactual was false all along. In a sense, if $A > C$ is the counterfactual you assent to and you accept (in a slogan-like fashion) a counterfactual $A > C$ is true only if all closest A-worlds are C-worlds, then in the scenario above, you would be finding out that *the* closest A-world is an $A \wedge \neg C$-world, which forces your hand to reject your counterfactual $A > C$. Thus, the problem is not with modus ponens being invalid; the problem is with a false counterfactual you initially believed to be true. In another sense, counterfactuals are sensitive to actual data (the actual world is one of the possible worlds after all) and one would not drop his factual knowledge in order to accommodate her contradicting counterfactual. Instead, one would drop his counterfactual, if she learns that it contradicts her factual knowledge.

[26] I am skipping the discussion why a sequence of utterances such as "Oswald did not shoot Kennedy, #but if Oswald had not shot Kennedy, someone else would have shot him" still sounds infelicitous due to lack of space. The point is that it is a pragmatic infelicity, which is solely related to the order of utterance, but not to the truth-conditional content of the utterance. Conflating pragmatic and semantic infelicity is a cardinal sin.

(1**) It is not possible that Kennedy was not shot. (Epistemic)

(1**) expresses an epistemic impossibility along the lines of "according to the information I possess, there is no way Kennedy was not shot." Yet, it does not say anything along the lines of "according to the information I have, the world could not be such that Kennedy was not shot," which requires a metaphysical necessity to hold and, accordingly, projects a metaphysical modality.

On the other hand, consider the following:

(2) If Oswald had not shot Kennedy, someone else would have.

And the corollary:

(2*) It was not possible that Kennedy would not have been shot. (Metaphysical)

The metaphysical modality is individuated by the additional layer of past tense in the noun clause of (2*). (1**) says that it is incompatible with my knowledge of the facts that Kennedy was not shot, whereas (2*) says that the world could not be such that Kennedy would not be shot. (2*) projects a claim about how the world could be such-and-such as opposed to (1**), which projects what my epistemic status entails about the situation. The epistemic-metaphysical distinction carried by (1**) and (2*) are also carried by (1) and (2).[27]

Above we have provided counterexamples to the thesis that counterfactuals necessarily carry false antecedents. This led us to conclude that the semantic content of the antecedent is neither necessary, nor sufficient to draw the distinction between counterfactuals and indicatives. However, this does not automatically imply that both

[27] I am skipping possible fringe cases where subjunctives might have epistemic readings and indicatives might have metaphysical readings. For detailed discussion, see (Khoo, 2015, Chapter 2-3).

sides of the distinction (indicative antecedent's compatibility with context and subjunctive antecedent's incompatibility with context) were unsustainable as well. We have only provided counterexamples to the potential claim that every subjunctive conditional presupposes its antecedent's falsity. In fact, the semantic presupposition of antecedents provides us with a good clue about how indicative conditionals come to be felicitous. Consider the following:

(3) Oswald shot Kennedy, #but if Oswald did not shoot Kennedy, then someone else did.

Preceding information in (3) eliminates the possibility that Oswald did not shoot Kennedy, yet the antecedent of the indicative in (3) requires the epistemic possibility that he did not shoot him in order to be evaluable. Khoo (2016, p. 12) argues that truth-conditional evaluation of indicative conditionals should track our betting intuitions and, in this case, this argument provides us with strong evidence for the thesis that the antecedent of an indicative conditional must be an open possibility given the information presupposed for a speaker or conversational background. For instance, consider that I bet my friend on the following indicative conditional:

(4) If Mehmet Emin rolled a prime number, then he rolled an odd number.

Suppose that Mehmet Emin rolled the dice and informed me and my friend that the roll was not a prime. Now consider the following sequence:

(4*) Mehmet Emin did not roll a prime, #but if he rolled a prime, then he rolled an odd number.

After learning that he did not roll a prime, it is otiose for us to keep betting on the possibility that he rolled a prime and the bet would dissolve rather than determine a winner, if we learned that he did not roll a prime. The same intuition goes with the

evaluation of indicatives such as (3). If we came to learn that the antecedent is no longer a possibility, the truth-value of an indicative taking its antecedent as a live possibility would be indeterminate. If we have settled beforehand that the antecedent is not a live possibility, then the truth-value of an indicative goes indeterminate or what have you other than true or false.

The upshot of the above discussion is that there seem to be two ways in which indicatives individuate themselves from counterfactuals. First, they force their antecedents to be compatible with the preceding information and, second, they express an epistemic connection between their antecedents and consequents. Even though we may not accept the initial distinction between subjunctives and indicatives based on the semantic content of antecedents completely, we can sense a grain of truth for the indicative part. (3) might be cited as evidence for the thesis that indicative conditionals require their antecedents be compatible with the foregoing information. Therefore, we can partly accept the semantic-content distinction by dropping it for counterfactuals and by adopting the restriction that the antecedents of indicatives must be compatible with the foregoing information in the context. Fortunately, there is an exact correspondence to this distinction in the literature. Will Starr calls it Stalnaker's Distinction (2014, p. 1024):

Stalnaker's Distinction: An indicative conditional focuses solely on antecedent-worlds among the contextually live possibilities, which represent what's being taken for granted in the discourse. A subjunctive conditional focuses on antecedent-worlds that need not be among those possibilities, that is, they may be counterfactual from the perspective of the discourse. (for the original distinction: Stalnaker, 1975, pp. 144-145).

This is a plausible distinction, both because it captures the compatibility requirement between the preceding information in the conversational background and antecedent, and because it already alludes to the tools required to capture the epistemic connection. However, it poses two major troubles for us: 1) This is a distinction for possible world semantics, which cannot be taken as it is into exact semantics and 2) we have no defined notion of context. If we can define it, the notion of context will help us capture both the context-antecedent compatibility and the epistemic connection, since the notion of context will stand for the information presupposed in the conversational background. It will play the formal role of the phrase "for all the speakers at hand presuppose." For accounts such as Gillies' (2009) or Kratzer's (1986), context determines the base set of worlds through which the conditional is evaluated. For the present account, it will especially play a role in *verifiability*, which antecedents will be admissible and, *a fortiori*, how possible outcomes will turn out to be. With the above discussion in mind, let us attempt at a formal definition of context under exact semantics.

4.1 Defining context

I find it a bit funny that, though the notion of context has slipped into most of the discussions in semantics and pragmatics with the definition that context is the information taken for granted in the conversational background, the notion itself was not clear and precise enough to be taken for granted. Funny as it is, the notion of context is the key component in contemporary semantics and pragmatics discussions. For instance, dynamic strict accounts (i.e. von Fintel, 2001; Gillies, 2009; Starr, 2014) take the

semantic value of utterances as their context change potential (CCP), which lays the utmost importance in their semantics on the notion of context and operations on context.

One problem of defining context is that we have several candidates in the literature for the notion of context.[28] For our purposes, a modified version of what is usually called a "Stalnakerian context" will suffice (Stalnaker, 1999, pp.78-95). A Stalnakerian context is defined as a set consisting of the possible worlds which represent the salient possibilities ("live options recognized by the speaker" (1999, p. 85) relative to the information presupposed by the speakers. What this means is that speakers take for granted a given amount of information and this information rules out certain possible worlds from some given domain of possible worlds (1999, p. 86). In other words, all the worlds in the set bear the possibility to be actual and, as the context becomes better informed, the number of possible worlds as viable candidates in the set to be the actual one decreases. For instance, suppose we have a set of propositions, i.e. $C = \{P_1, P_2, P_3 \ldots\}$, which represents the information taken for granted by the speakers. Then the context is the set of possible worlds which remains after the given information rules out the possible worlds where the given information is false. Gillies takes a similar notion of context, which is a function from indices (say W) to sets of indices (say $\wp(W)$), i.e. $f: W \rightarrow \wp(W)$ (2009, p. 337), though he says that his notion of context need not be Stalnaker's (2009, p. 329, fn. 5). This function is supposed to return the worlds that are salient possibilities relative to the information presupposed. How does it do it? By removing the possible worlds which are incompatible with the given information. I shall formalize my own notion of context, though the idea operating in the background will be similar to Stalnaker's.

[28] For a survey, see (Stalnaker, 2014, Chapter 1-2).

Let us presume that all the information presupposed, the context set, is a set of propositions, i.e. $C = \{P_1, P_2, P_3, \dots, P_n\}$. Our context will be the set of verifiers for these propositions. We can define a set of sets of propositions as $\wp(\wp(S^{\Diamond}))$, since the power set of $S^{\Diamond}$ will contain only propositions as sets of exact verifiers and the power set of that set will contain the sets of propositions. A function $g: \wp(\wp(S^{\Diamond})) \to \wp(S^{\Diamond})$ defined as $C \mapsto \bigcup C$ will take us from a set of propositions to the set of the exact verifiers of the statements presupposed. If C is taken to be the set of propositions, i.e. $\{P_1, P_2, P_3, \dots, P_n\}$, then $\bigcup C$ will be the set of verifiers for these propositions, i.e. $\{p_1^1, \dots, p_n^1, \dots, p_1^n, \dots, p_n^n\}$ where $p_1^i, \dots, p_n^i \in |P_i|^V$. Of course, speakers may identify certain statements to be false in the conversation. If we ascertained that a certain statement is false or it is commonly shared information that the statement is false, we can project this onto our definition of context by allowing the context set to contain falsifiers for those statements by putting $\bar{p}_1^i, \dots, \bar{p}_n^i \in |P_i|^F$. Since a set of exact verifiers such as $\bigcup C$ will be a subset of $S^{\Diamond}$, there will be the least upper bound of $\bigcup C$ by Bounded Completeness, which will give us the context state of our chosen set of propositions. Call this *the context state* c.

There are two fundamental differences between our notion of context and that of Stalnaker's. Stalnakerian context is usually pretty huge consisting of all the possible worlds not ruled out by the information at hand. The huge size of the context is normal, given the function Stalnakerian context is assigned. It should represent all the epistemically open possibilities and, considering the fact that we usually have a limited amount of information which can shrink the context only to a limited degree, the context

representing the open possibilities remains to be pretty big.[29] Our notion, on the other hand, represents just the information presupposed and open possibilities are determined based on the compatibility between the verifiers of statements and the context state. We can say that a statement A is an open possibility only if its verifier is compatible with a given context state c. Then epistemically open possibilities in a Stalnakerian sense will be all the verifiers in the possible state space which are compatible with the context state, i.e. $\forall t, c \in S^{\Diamond}$ and c is a context state, if $c \sqcup t \in S^{\Diamond}$, then t is a salient possibility for a given context state. This will also mean that these open possibilities are situations which are candidates to be actual for a given world state w.

The second fundamental difference is that we represent falsities in the context, when the falsity of a statement is presupposed. In Stalnakerian notion of context, the function of an ascertained falsity is to eliminate the worlds at which this falsity is true from the context. In our conception, the presence of a falsifier for a given statement in the context state will be required, because it will guarantee the incompatibility of the context state with the verifiers of that statement. In case a confusion may arise, I feel the need to elucidate that I do not mean by the presupposition of a false statement the presuppositions which are false, yet presumed by the speakers to be true; I mean for a statement φ, "φ is false" is presupposed. This is a true presupposition, if φ is actually false and it will have a falsifier for φ in the context set. However, people can also

[29] This is one aspect by which the default notion of Stalnakerian context disappoints. It is too idealistic to suppose that speakers attend to all the possibilities which are not ruled out by their information. Most of these possibilities never become salient for the speakers, as they focus their attention on the subject matter at hand. Possibilities become salient, as they are brought to speakers' attention (sometimes even this is insufficient, see Schaffer and Knobe, 2012). I believe that our definition of context does not suffer from this, since salience of a possibility is defined to be the compatibility between the verifier of some statement and the context. Furthermore, compatibility is evaluated case-base rather than totally. The compatibility will reflect the salience of some possibility and, if the compatibility has never been considered, then the statement will remain to be an open possibility, but not a salient possibility. A full defense of this comparison would take us too far afield.

presuppose false statements on purpose or by error. Therefore, we impose no factivity constraint on our contexts. They can contain false presuppositions along with true presuppositions given a certain world state.[30] In summary, though Stalnakerian contexts and mine do the same job, they manage this job in the opposite directions. Stalnakerian contexts start with a limited amount of information and the size of the context is inversely proportionate to the amount of information presupposed by the speakers. We play the elimination game among possible worlds to ascertain the actual one relative to the information. In mine, we again start with a limited amount of information and the size of context is directly proportionate to the amount of information presupposed. As our context becomes better and better informed, the context itself tends to be closer to the actual world.

This also makes sense, when we think about the definition of world state in Chapter 2. Since the ultimate goal of a context is to determine which world is the actual one, the upper limit of our notion of context will be a world state where an omniscient speaker will be presupposing everything that is actually the case in that world. If the context state and world state of evaluation are equal, then the truth-values of the conditionals at hand will be determinately true or false for the given context and world states. Also, if the context state itself were a world state, then we would not need any other exact verifier in the context to consider in order to settle the truth-value of any

[30] Of course, the notion of truth will become gradated for indicative conditionals, since we dropped factivity constraint for context sets, which may render our account prone to Edgington's criticisms (1995, pp. 306-308). Dynamic strict accounts such as Gillies' (2009) associates the truth of an indicative with its acceptance conditions. I will not attempt at a critique of these accounts or reply to Edgington's objections in general, but merely say that the truth of an indicative conditional (*simpliciter*) will be established, only when given a context state, the truth of the antecedent and consequent is established in the world of evaluation. However, this does not mean that we have an anything-goes situation in terms of truth. The disagreements over the truth of indicatives can be objectively resolved by resorting to better and better informed contexts. Even though determinate truth-values seem to be at stake for the majority of indicatives, the objectivity is not.

conditional, since any state we wish to impose on a world state would already be included in it and the possible outcome would be the context itself. It is also too idealistic to suppose that our context state will be equivalent to a world state, because, if the context state is a world state, then an indicative $A \longrightarrow B$ will be trivialized in the sense that it will be just equivalent to the conjunction $A \wedge B$, all of whose conjuncts we know to be true.

Since we have a well-defined notion of context at hand, we can start employing it in other defined notions to extend the semantics to indicative conditionals. The notion of context will endow the account with an epistemic force. By this way, we will be able to account for the divergence of Adams' pair in terms of truth-conditions. In order to achieve this, first, we have to modify the transition relation in such a way that it takes context states into account. We can do this by redefining the transition relation:

Transition+Context: A possible outcome u of imposing an exact verifier t for the antecedent on a world state w will be yielded relative to a context state *c*. This will be represented by subscripting the arrow with the context state along with the world state, i.e. $t \rightarrow_{c,w} u$.

"Relative to context state" is not as lucid as we want and we must be more precise. In this account, "being relative to a context" means that possible outcomes are produced not only by the antecedent, but also by the information in the context. Hence, they will be compatible with the context, i.e. $c \sqcup u \in S^{\Diamond}$. However, it will not be mere compatibility. Possible outcomes will restrict the possible outcomes to those which contain all the states in the context state as parts and exclude those who do not contain the verifiers part of the context state. For instance, if one is not entertaining the

possibility of some statement or event in the context or, in other words, there are no verifiers or falsifiers relevant to that statement, then there will be no possible outcome yielded with respect to it. This restriction will reflect itself in the account by including or imposing context states onto the possible outcomes as well. Inclusion of context states can be captured as part of possible outcomes yielded relative to exact verifiers of the antecedent:

Context Inclusion: For all possible outcomes u of imposing an exact verifier of some antecedent t on a world state w relative to c, c is a part of u. In symbolic terms, $\forall u, c, w, t \in S^{\Diamond}$, if $t \rightarrow_{c,w} u$, then $c \sqsubseteq u$.

In a sense, Transition+Context works with a similar principle to that of modal bases in certain versions of strict conditional accounts (Gillies, 2010, p. 7; Kratzer, 2012, pp. 97-107). Modal base in those accounts determines the set of possible worlds via which the conditional is evaluated. Transition+Context restricts possible outcomes of imposing the verifier of the antecedent on a world state to those compatible with the context state. Transition+Context achieves two things in accordance with individuation conditions for indicatives from counterfactuals, as discussed above. First, it provides us with the framework upon which we can define the compatibility of the antecedent with the context. From the basic semantics, we already have the notion of compatibility defined on fusion of states. We can employ the same notion to define compatibility of the verifiers of the antecedent and context state. In particular, an utterance is semantically felicitous if and only if the exact verifiers of the uttered statement is compatible with the context state at hand, i.e. $c \sqcup t \in S^{\Diamond}$.

Second, it achieves, in part, projecting the epistemic states of speakers in a conversation. Since the context state represents the presupposed information in a conversation, possible outcomes will track this information as well. In turn, possible outcomes will include or exclude states in accordance with the information presupposed. If we couple this with our notion of context state which reflects falsities, unconsidered possibilities and presupposed truths in their respective forms, then we should see that our possible outcomes will be epistemically causal outcomes dependent upon these presuppositions in the context and antecedent. Hence, these two properties of Transition+Context satisfy two desiderata of individuating indicatives from counterfactuals.

We have no reason to force our context to be static throughout a sequence of utterances. If context is determined by what is taken for granted, then what is taken for granted must be shared via some communication. In an idealized form, if we assume that this communication is exchanged by declarative statements, then there will be propositions corresponding to these statements which will make their way into the context state. In a Ramseyan spirit, we will require that the context state be updated with the exact verifiers of the antecedent. This will mean that the context state will be fused with the exact verifiers of the antecedent. This yields:

Context Update: If for any indicative conditional $A \longrightarrow B$, context state c and state t such that $t \Vdash A$ and $t \rightarrow_{c,w} u$, c is also fused with t, i.e. $c \sqcup t$.

This provides us with a limited flexibility in terms of capturing the evolving nature of conversations. Suppose we have an indicative of the form $P \longrightarrow (Q \longrightarrow (R \longrightarrow T))$ and p, q, r, t are the verifiers for *P*, *Q*, *R*, *T* respectively. While evaluating

this indicative, successive transitions in obliging with the truth-conditions will look like as follows: First, $p \rightarrow_{c,w} u$; second $q \rightarrow_{c \sqcup p,w} u'$; third $r \rightarrow_{c \sqcup p \sqcup q,w} u''$ et cetera. Consider the example, "If Kripke is at the conference, then if Putnam is there, then if Lewis is there, then Stalnaker will be there." While evaluating the indicative, add the state of Kripke being there, evaluate whether "if Putnam is there, then if Lewis is there, then Stalnaker will be there" is plausible, do the same for Putnam being there and so on. What we are inquiring is whether Stalnaker will be there on the condition that Kripke, Putnam and Lewis is there. We cannot just forget that Kripke should be there for Stalnaker to be there, when we are assessing the plausibility of the indicative. Context Update lets us handily deal with McGee-type counterexamples to modus ponens (McGee, 1985).[31] We also require that all of the exact verifiers of the antecedent should be fused with the context state. For instance, if we have a conditional with a disjunctive antecedent $A \vee B$ and exact verifiers a and b respectively, then we will fuse a context state c with a, b and $a \sqcup b$, which will turn out to be just $c \sqcup a \sqcup b$.[32] With this minor unproblematic rule do we capture not only Ramsey's idea, but also Gillies' (2009).

Since we have laid out the basic machinery to be employed in the truth-conditions for indicatives, we can provide them at last. For counterfactuals, Fine's

[31] McGee's trick under our account culminates in how we apply the transition relation in the flux of a Modus Ponens argument. For the conclusion McGee wants to obtain, the indicative in the conclusion of the modus ponens argument needs to be evaluated through the transition $q \rightarrow_{c,w} u$, even though, while going through the modus ponens argument, we updated the context with the verifier of the embedded conditional premise. Therefore, it should be $q \rightarrow_{c \sqcup p} u$ rather than $q \rightarrow_{c,w} u$. This does not diminish the importance of McGee's result, though. I believe that McGee's result shows that any account for indicatives which do not take into account some notion of context is doomed to make incorrect predictions about McGee counterexamples. However, the detailed argument is left for another day.

[32] This operation is well-defined in our account due to how we formally defined our context. The context function will pick a set of propositions which includes $A \vee B$ in addition to its preceding propositions and map it to its union.

designated truth-conditions were bivalent. However, we provide a trivalent semantics for indicatives by dropping Exhaustivity. Truth-conditions are as follows:

$w \models_c A \longrightarrow B$ iff $u \models B$ whenever $t \Vdash A$, $c \sqcup t \in S^{\Diamond}$ and $t \rightarrow_{c,w} u$,

$w =|_c A \longrightarrow B$ iff $u =| B$ whenever $t \Vdash A$, $c \sqcup t \in S^{\Diamond}$ and $t \rightarrow_{c,w} u$,[33]

Indeterminate iff otherwise.

Though truth and falsity in our model is well-defined, we do not know what indeterminate corresponds to. The basic idea of "indeterminate" is that under the unilateral concept of propositionhood, $A \longrightarrow B$ has an empty set of verifiers $V_{A \rightarrow B} = \emptyset$. If we adopt a bilateral conception of propositionhood, then the falsifiers of the proposition will be empty as well, i.e. $V_{A \rightarrow B} = \emptyset$ and $F_{A \rightarrow B} = \emptyset$, i.e. $(V_{A \rightarrow B}, F_{A \rightarrow B}) = \emptyset$.[34] Note that $A \longrightarrow B$ will be indeterminate mostly due to the exact verifier of the antecedent and the context state being incompatible, since A will have an exact verifier so long as it is not a logical contradiction. If A is a logical contradiction, then both the outcome will not be a possible state, since imposing a contradiction on a world state will always yield an impossible state and A will not have a verifier at all let alone an exact one. However, the incompatibility of the exact verifier of the antecedent and the context

[33] This can also be read as $w \vDash_c A \longrightarrow \neg B$, since the possible outcome will loosely verify $\neg B$, when it loosely falsifies B, given that possible outcomes are world states by Maximality and the verification rule (ii)$^+$. Therefore, the negation of a true indicative, i.e. $\neg(A \longrightarrow B)$, will be $A \longrightarrow \neg B$. This blocks one implausible rule of inference which is valid for material conditional, namely, Material Negation: $\neg(A \longrightarrow B) \vDash A$. This inference rule is known to lead to cheap proofs for the existence of God. For instance:

It is not the case that if God exists, he is not my father.

#Therefore, God exists.

Since a material conditional $A \supset B$ is equivalent to $\neg A \vee B$, by negation of $\neg A \vee B$ and De Morgan, we obtain $A \wedge \neg B$, which lets us derive A by Simplification. This is a disappointing result and Material Negation should not reside in a plausible theory of indicatives.

[34] In bilateral conception, one can realize that if a proposition has an empty set of verifiers or falsifiers, then either there are no verifiers or falsifiers for that statement, which suggests that the statement is vacuous. This is a special case of vacuity in exact semantics. However, this vacuity does not lead to truth, because not even the empty state does not verify the indicative.

state will contingently arise. This incompatibility will not yield a true or false indicative. This, of course, intuitively squares with how we grok indicatives. Remember the discussion on rolling a dice at the start of §4:

(5) He rolled a non-prime, #but if he rolled a prime, he rolled an even.

(5) was counted as evidence that indicatives tend to obey the rule of semantic compatibility of their antecedents with the foregoing information as opposed to counterfactuals. Truth-conditions we have provided pay dividends to this distinction, since it assigns (5) an indeterminate truth-value due to the fusion of the exact verifier of the indicative and the context state being an incompatible state, i.e. $c \sqcup t \notin S^{\Diamond}$. This also means that the infelicity of (5) is semantic rather than pragmatic.[35]

This is one aspect of why we need the third truth-value for the truth-conditions of indicatives. Another reason can be seen by thinking in terms of determinate and indeterminate possible outcomes. Remember our discussion at transition relation subsection; we have left the discussion at recognizing that there are indeterminate outcomes of imposing certain verifiers. They emerge, when an exact verifier of the antecedent is imposed on a world state and two mutually contradictory outcomes are obtained. For instance, take the indicative "If he flipped a coin, it landed heads." Imposing the antecedent of one's flipping a coin (assuming flipping a coin is a stochastic event) will yield possible outcomes of both the coin landing heads and tails. In other words, the antecedent will not be sufficient to settle the consequent one way or another and leaves both possibilities open. We see that this issue is closely related to the truth-conditions we have provided above, since the scenario is such that for an exact verifier t,

[35] Whether infelicity *should* be pragmatic or semantic is an open discussion. Gillies takes the infelicity sometimes as semantic (2009, p. 346), whereas Veltman takes the infelicity as pragmatic (1987). For a detailed discussion, see (Starr, Forthcoming).

it is possible that for $t \rightarrow_{c,w} u$ and $t \rightarrow_{c,w} u'$, $u \vDash B$ and $u' \vDash \neg B$. This gives us for an indicative $A \longrightarrow B$ in a context state c, $w \models_c A \longrightarrow B$ and $w =|_c A \longrightarrow B$. What this means is that the indeterminate truth-value also captures scenarios where $A \longrightarrow B$ may turn out to be both true and false. It is easy to see that the cases where $u \vDash B$ and $u' \vDash \neg B$ are captured under "otherwise."[36]

The trivalent semantics captures an important aspect of how indicatives are understood in conversational settings. For instance, consider the following exchange:

Jack: If John flipped a fair coin, did the coin land heads?

Jill: #No.[37]

It sounds odd for Jill to say "No." to the question, unless she knows something definite about the outcome or the surrounding conditions such as the coin being biased towards tails. Assuming that John's flipping was a truly stochastic event, the following sounds more felicitous:

Jack: If John flipped a fair coin, did the coin land heads?

Jill: I don't know.

The felicity of the second exchange is supported by the third truth-value in our semantics. Jill would have replied "It either did or not did not" and that would have been true, since in any possible outcome, we obtain by imposing the antecedent above, Jill's reply would hold true, but with how the indicative stands as it does, the truth and falsity does not cut it. There is much more to say about the semantics of indicatives, but this

[36] A non-modalized indicative which has an indeterminate truth-value, i.e. "If he flipped a coin, it landed heads," may have a definite truth-value, when turned into an epistemically modalized indicative, i.e. "If he flipped a coin, it was possible that it landed heads." Epistemic modals lie beyond the scope of this thesis, but they can be captured with further modifications.

[37] Here I am assuming that the question is not asked in a betting environment. In a proper betting context, Jill might felicitously say "No" with the implication that she is betting on the possibility that the coin landed tails.

should suffice for a general understanding of our framework. I shall now proceed to discuss the logic of indicatives which comes with our semantics.

CHAPTER 5

LOGIC OF INDICATIVES UNDER EXACT SEMANTICS

If the present account were not to provide any advantages over several other theories of indicatives, then our project would be otiose. Fortunately, this is not the case and the account enjoys a pleasant logic of conditionals. Exact semantics offers a logic of indicatives which enjoys several advantages over other accounts. While showing how certain principles and inference rules hold under the present account, I will focus on two commonly accepted propositional theories of indicatives: Gillies' (2009; 2010) and Stalnaker's (1968).[38] Normally, one would present a problem with a given account at the outset and construct a theory to solve it. However, I thought it would be best, if we collect all the principles and problems under one section where we deal with them on a case-base. Some of the principles will pose threats to the core of the semantics for certain theories where they need employ pragmatics or some other patchwork outside the semantics; some of them will be downright semantically problematic. Kit Fine questioned the possible world semantics as a whole by posing problems with its fundamental assumptions such as the concept of proposition being the set of possible worlds at which the statement expressing the proposition is true et cetera (2012a, 2012b). Overall, if the charges stick, then these will witness as evidence that the present account should be preferred over them. Though the shift of semantics induces huge reverberations through all of the principles in the logic of conditionals, I will focus on three important principles. The importance of these principles comes from the fact that

[38] I take Stalnaker's account as the base account for any other variably strict account for indicatives. There are several versions of Stalnaker's account which other philosophers developed further (see Khoo, 2016; Kratzer, 1986).

several otherwise plausible accounts provide different answers to the validity or invalidity of these principles. Some say that the inference via such-and-such principle is valid, yet it is pragmatically unsound; some say that it is invalid, yet pragmatically sound.[39] The three principles I will focus on are as follows: 1) Substitution of Logical Equivalents; 2) Import/Export and 3) Antecedent Strengthening. After showing where they stand in the present account, I will discuss that the present account provides one of the most plausible solutions to the problems posed by these principles on the market and the main strength of the account comes from its robust logic of conditionals.

5.1 Substitution of logical equivalents

Since the works by Stalnaker (1968) and Lewis (1973), possible world semantics has shed light on one of the most elusive utterances in natural language—conditionals. Both probabilities of conditionals and their semantics have been dealt with in the framework of possible world semantics since their work. Even though several disagreeing accounts have been proposed in the framework of possible world semantics (see Stalnaker, 1975; Gillies, 2009; Kratzer, 1986), the disagreements among these accounts were generally on the issues of logic of conditionals[40] rather than on the issues of the framework itself. Fine's objection (2012a; 2012b; 2017, Chapter 2, Section 3) to Lewis' account of counterfactuals, on the contrary, was based on a problem with the framework of possible world semantics itself. Fine charged that possible world semantics validates the principle

[39] For a detailed discussion, see (Starr, Forthcoming).

[40] For instance, Stalnaker (1981) defends the law of Conditional Excluded Middle against Lewis' account (1973) or Gillies, for instance, motivates his account on pain of validating Import/Export and other forms of valid inferences which are invalidated by Stalnaker's variably strict analysis built upon selection functions (Gillies, 2009). Yet, all of these accounts are proposed within possible world semantics.

of Substitution, which leads to counterintuitive results in the truth-values of counterfactuals. First, the principle of Substitution:

> Substitution: $A \longrightarrow B \vDash A' \longrightarrow B$ or $A \longrightarrow B \vDash A \longrightarrow B'$ where A, B and A', B' are logical equivalents of each other.

Substitution is a principle deeply entrenched in the core of possible world semantics. It states that logical equivalents can be interchanged in antecedent or consequent positions, since logical equivalence in possible world semantics corresponds to *being true at the same worlds*. Though we will be concerned with a special application of Substitution, namely, the substitution of logical equivalents in antecedent position, the problem plagues possible world semantics in general. For instance, since the propositionhood expressed by a statement is defined to be the set of worlds at which that statement is true, necessary statements turn out to be the same set of worlds, namely, the set of all worlds, which implies that Brouwer's fixed-point theorem and $2 + 2 = 4$ express the same proposition.[41]

Fine brings out natural language counterexamples to Substitution in the context of conditionals. He, first, considers two counterfactuals with logically equivalent antecedents, but the examples are equally striking in indicative forms. I will engage with the indicative versions. Consider the following:

(6) If Sue took her pills, she got better.

[41] This is a well-known problem for possible world semantics. Especially, Perry considers this to be one of the reasons why he proposes his partial approach (1986, p. 86) and Zalta argues that necessarily equivalent propositions should be able to be distinct; otherwise, we would lose our ability to represent belief (1988, p. 57).

This is a plausible indicative which can be imagined to be true, given the right circumstances. However, consider (6'), which has the logical equivalent of (6) in the antecedent position:

(6') If Sue took her pills or her pills and the cyanide, she got better.

Logical form of (6) is $A \longrightarrow C$ and that of (6') is $A \vee (A \wedge B) \longrightarrow C$. It is easy to check A and $A \vee (A \wedge B)$ are logical equivalents.[42] Assume that (6) is true, given the circumstances. Would this entail (6')'s truth? It seems to me that it does not. Intuitively, someone would object as follows: "No! If she had taken her pills and the cyanide, she would die!" (6') makes us consider both horns of the disjunction and one of the scenarios seems to lead to the negation of the conclusion, which suggests that the inference from (6) to (6') is not valid.[43] If your intuitions track this objection as well, then you would agree that there is something wrong with the principle of Substitution in the context of logical equivalents as antecedents.

If Substitution causes problems for possible world semantics in general, then the problem should infect any theory of conditionals based on possible world semantics, since none of these accounts takes the definition of logical equivalence in possible world semantics differently from Lewis or Stalnaker.[44] Therefore, at this stage, one general

[42] If one has trouble in seeing this, just consider a truth table or set theoretic union and intersection for disjunction and conjunction respectively for any arbitrary sets of worlds for A and B.
[43] This objection itself relies on another principle Stalnaker or Lewis' account invalidates, namely, Simplification of Disjunctive Antecedents (SDA). If someone objects to this objection by holding that SDA is invalid, then we will see that it will be all the worse for her, since SDA is another principle for which variably strict accounts draw the short straw. More on SDA in Chapter 5.5.
[44] Of course, the jury is still out, as much of Fine's work on this remains to be either unresponded or unpublished. However, one point is clear that failures of Substitution in the case of conditionals cannot be explained away with recourse to the discrepancy between surface form vs. logical form, since it is exactly possible world semantics' premise to provide a more fine-grained conception of propositionhood than classical sentential calculus. The problem of Substitution seems to occur yet due to lack of a further degree of fine-grain.

motivation to propose and defend an account of conditionals in exact semantics might be that it blocks failures of Substitution.

For Stalnaker's indicative account (1968; 1975), Substitution is valid *simpliciter*. Let me briefly show how this is so. Stalnaker adopts the possible world semantics and takes the definition of a proposition as the set of possible worlds at which the statement is true. He further defines a selection function f which takes as its arguments the antecedent of an indicative and the world as it is and returns a set of the closest possible A-worlds. Thus, the truth-conditions for an indicative $A \longrightarrow B$ turns out to be:

$A \longrightarrow B$ is true in w iff B is true in $f(A, w)$ (Stalnaker, 1975, p. 143).

When we apply this to (6) and (6') respectively, we see that the function will return the same world, since the semantic value of the antecedents of (6) and (6') are the same. This will lead to an evaluation of the indicative through the same set of worlds, since $[\![A]\!]^{c,w} = [\![A \vee (A \wedge B)]\!]^{c,w}$ entails $f(A, w) = f(A \vee (A \wedge B), w)$.[45] Hence, Stalnaker's theory will validate Substitution and this means that Stalnaker's account predicts the truth-values of (6) and (6') incorrectly.

Stalnaker's account was a variably strict conditional theory. Let us consider a dynamic strict theory to see whether the puzzle persists. One of the most influential dynamic strict accounts in the literature is Gillies' (2009). For Gillies, the puzzle of Substitution might proceed in several ways. On his account, *if*s play two different roles. First, they determine the subordinate context represented by a modal base determined by speakers' information. We check whether the consequent is true in this subordinate context. Second, they restrict the modal base to the set of worlds where the antecedent holds to evaluate the consequent. Let E be a modal base function which takes as its input

[45] For further discussion, Warmbrōd, 1981.

a context set c and an index (generally, a world) w and outputs a set of worlds $E(c, w)$ which are compatible with the given information in *c*. *If*, first, determines a subordinate modal base with the antecedent of a given indicative, after it is determined by the salient possibilities, i.e. $E(c + A, w)$ for some antecedent A and $E(c + A, w)$ is to be just understood as $\lambda w. E(c, w) \cap [\![A]\!]^{c,w}$. This subordinate context should be understood as the shared information to which the information of the antecedent is added. Call this "the modal base determination function of *if*."

The second function of *if* is to restrict the modal base with A-worlds for some antecedent A. If all the A-worlds in the modal base are B-worlds for some consequent B, then the indicative will be true.[46] In symbols, $[\![A \longrightarrow B]\!]^{c,w} = 1$ iff $E(c + A, w) \subseteq [\![B]\!]^{c+A,w}$. Substitution *should* hold for Gillies, meaning that (6) must entail (6'), given that none of the functions *if* is assigned could prevent the equivalence of subordinate contexts from obtaining. Thus, nothing in his semantics[47] precludes Substitution from being invalid. This is so, because Gillies takes the notion of propositionhood as the set of worlds at which a given statement is true, as Stalnaker does, and if the expansion of the modal is effected merely by mechanically adding the given set of worlds to it, then technically $E(c + A, w)$ and $E(c + A \vee (A \wedge B), w)$ should turn out to be the same, since they denote the same propositions under possible world semantics. Therefore, Substitution would be valid for Gillies' account and his account would give an unsatisfactory response to the puzzle of Substitution.

However, Gillies may reply the same way as he does with Antecedent Strengthening (2009, p. 345). Gillies sides with Veltman (1987) on the thesis that

[46] I am skipping quite several details, but this picture does not distort Gillies' idea.
[47] As far as explicated in (2009; 2010; 2012).

indicatives imply that their antecedents are salient possibilities and that, when we add the implicatures of the antecedents to the entailments, the entailments must remain "happy" (2009, p. 345). (6') implies that both horns of the disjunction are salient possibilities and the entailment should not become unhappy, when we add either horn of the disjunction to the entailment. However, it is not clear how to achieve this. First, take the base entailment as:

If Sue took the pills, she got better;

_____

#So, if Sue took the pills or the pills and the cyanide, she got better.

If this were a happy entailment, then adding "Sue might have taken the pills and the cyanide" to the entailment must keep the entailment happy:

Sue might have taken the pills and the cyanide,

#If Sue took the pills, she got better

_____

So, if Sue took the pills or the pills and the cyanide, she got better.

But let alone keeping the entailment happy, adding the implicature made even the first premise (6) itself seem false. Some might take this to be a satisfactory resolution to the puzzle. I do not believe so. This resolution relies on an unnatural use of language. I believe that we treat salient possibilities, as they are brought to our attention. For instance, I would treat (6) to be true, since I take the antecedent to be a salient possibility, that is, compatible with my context state and imposing the antecedent on the world state would have me conclude that (6) is a plausible indicative and can be true. However, I would treat (6') to be false, because taking both horns of the disjunction as salient possibilities would have me conclude that there are possible outcomes where she

dies as opposed to getting better. Shrinking and expanding the domain of the indicative seems to never yield the circumstances where this intuition is satisfied, that is, taking (6) to be true, whereas (6') to be false. Therefore, I take the above approach to be an unsatisfactory solution to the puzzle.[48]

The present account is designed to be compliant with the intuition given above and provide an unequivocal solution to the puzzle of Substitution. The solution is simple: Substitution is invalid *simpliciter*. Rather than trying to accommodate the wayward cases of Substitution, it seems to be a better option to reject it in a principled way. Here is a way of doing so. Regardless of whether we take a unilateral or bilateral conception of propositionhood, A and $A \vee (A \wedge B)$ will differ in their semantic value under exact semantics, that is, the sets of their exact verifiers. For let a be the sole verifier of A and b for B. The semantic value of A will be the singleton $\{a\}$, whereas the semantic value of $A \vee (A \wedge B)$ will be $\{a, a \sqcup b\}$. Now consider the truth-conditions we have provided for indicatives with the principles such as URA and UVC. Since logical equivalents such as A and $A \vee (A \wedge B)$ have different set of exact verifiers and, by URA, we need to consider all of the verifiers of the antecedent, they will lead to different possible outcomes and one of these outcomes will falsify the consequent. This approach will make our account able to predict the truth value divergence of (6) and (6').

5.2 Import/export (I/E)

I/E is an intuitively plausible principle for indicatives. It reads as follows:

[48] It is possible that Gillies can also tackle the puzzle with a different approach. However, given the machinery he provides, I do not see how else he would. It is somewhat an open question for his theory, since, as far as I am aware, Gillies has never taken Substitution to be a problem.

Import/Export (I/E): for any given world w and context state c,

$w \vDash_c A \longrightarrow (B \longrightarrow C) \equiv A \wedge B \longrightarrow C.$

If we take Ramsey's observation as a key to understanding the truth-conditions of indicative conditionals, then there is no way that any account observing Ramsey's observation can risk invalidating Import/Export, since what I/E basically says is that there is no difference in adding the antecedents of the conditionals to our hypothetical stock of beliefs sequentially or all at once. In other words, when we check the truth and acceptability of indicatives, we assume their antecedents and evaluate their consequents, putting together both antecedents *seem* to yield equivalent propositions. For instance:

(7) "If Kripke is at the conference, then if Putnam is there, then there will be a causal-reference talk"

(8) "If Kripke is at the conference and Putnam is there, then there will be a causal-reference talk"

Despite its intuitive look, I/E has a checkered track record for conditionals due to the results it gives rise to. McGee countenances I/E against modus ponens for indicatives (McGee, 1985). Accordingly, McGee claimed that any account of subjunctives not supporting I/E would be a deficient account, because there is a plethora of evidence which supports I/E (McGee, 1985, p. 466). Against McGee, Katz charged that I/E is not valid for stronger-than-⊃ conditionals and modus ponens takes precedence (Katz, 1999, p. 415), but this cannot be right. Both Kratzer's and Gillies' theories validate I/E and both are stronger than material conditional ⊃ (Kratzer, 2012, p. 105; Gillies, 2009, p. 327). Gillies cites I/E as an independent reason why he provides his account (2009, p. 325). Kaufmann also argued against the validity of I/E by claiming that the probabilities

of $P \longrightarrow (Q \longrightarrow R)$ and $(P \wedge Q) \longrightarrow R$ may diverge under certain circumstances (Kaufmann, 2005, pp. 213-214), which may throw doubt on the principle. I will not deal with each of these arguments, but simply show that the principle holds for indicatives under exact semantics and provide a brief discussion why the I/E seems to be indefeasible.

Claim: $w \vDash_c A \longrightarrow (B \longrightarrow C) \equiv (A \wedge B) \longrightarrow C$ for any world state w and context state c.

Proof: Left-to-right. Assume for some arbitrary world state w and a context state c, $w \vDash_c A \rightarrow (B \rightarrow C)$, Show $w \vDash_c (A \wedge B) \longrightarrow C$. Suppose $a \sqcup b \Vdash A \wedge B$ and $a \sqcup b \rightarrow_{c,w} u$, show $u \vDash C$. Given $w \vDash A \longrightarrow (B \longrightarrow C)$, we have $u' \vDash B \longrightarrow C$ for $a \Vdash A$ and $a \rightarrow_{c,w} u'$. Since we have $u' \vDash B \longrightarrow C$, then we have $u'' \vDash C$ for $b \Vdash B$ and $b \rightarrow_{c \sqcup a,u'} u''$. By Inclusion, we have $a \sqsubseteq u'$ and since u is obtained by imposing b imposed on u', we also have $a \sqsubseteq u''$. Therefore, $a \sqcup b \rightarrow_{c \sqcup a,u'} u''$ is equivalent to $b \rightarrow_{c \sqcup a,u'} u''$. Now, observe that $a \sqcup b \rightarrow_{c \sqcup a,u'} u''$ entails $c \sqcup a \sqcup b \sqsubseteq u''$ and $a \sqcup b \rightarrow_{c,w} u$ entails $c \sqcup a \sqcup b \sqsubseteq u$ by Inclusion. But then $c \sqcup a \sqcup b \sqsubseteq u''$ and $c \sqcup a \sqcup b \sqsubseteq u$ entails $u = u''$, since these are the only determinants of possible outcomes by being imposed on any world state w and u and u'' are world states by Maximality. Since we have $u'' \vDash C$ from $u' \vDash B \longrightarrow C$ above, $u \vDash C$, as required. I am leaving out the right-to-left, since the idea is similar.

We have partly seen that I/E holds for the account, but one may demand a further reason for why I/E being valid is a good thing for a theory of conditionals. One may subscribe to other philosophers' reasons. I have mentioned above that Gillies (2009, p. 325) takes one of his primary reasons for providing a dynamic strict theory for

conditionals is that it validates I/E unlike variably strict accounts (Stalnaker's being the prime example (1968; 1975)). Another reason can be cited as the principle's resilience against counterexamples. Only Kaufmann (2005) has provided a counterexample to I/E in a probabilistic setting where he claims to show that the probabilities of $A \longrightarrow (B \longrightarrow C)$ and $A \wedge B \longrightarrow C$ diverge. However, the counterexample has a problem of ambiguity in that the antecedent and consequent seem like they should not have the same interpretation, though their semantic values are taken to be the same by Kaufmann. Besides Kaufmann's, there was no counterexample to I/E similar to counterexamples raised against Transitivity and Contraposition (Stalnaker, 1968) and, given the contentious nature of Kaufmann's example, I take it that the evidence favoring I/E outweighs the evidence disfavoring it.

I conclude, then, that it is an advantage for any account of conditionals to validate I/E and our account enjoys this advantage. As long as one has a principled way of rejecting the undesirable consequences of I/E (i.e., McGee, 1985), there seems to be no reason to reject I/E and any theory for conditionals which claims to observe Ramsey's idea should be in a position to validate I/E rather than invalidate it without bringing up a decisive counterexample to it.

5.3 Antecedent strengthening (AS)

AS might be one of the most troublesome principles, which haunts material and strict conditional theories of indicative conditionals, because it is the principle by which we violate our intuitive grasp of the truth-values of conditionals in the easiest fashion possible. Take Gillies' example:

(9) If there is sugar in the coffee, it tastes sweet,

and its strengthened version:

(9') If there is sugar and diesel oil in the coffee, it tastes sweet.

The truth-value diverge of this pair is obvious. Dynamic strict theories need to employ pragmatic implicatures or some other method extrinsic to their semantics to stymie AS from holding for their theories and, *a fortiori*, accounting for the truth-value divergence of (9) and (9'). Stalnaker's variably strict account (1968) does not validate AS, but it does not validate Import/Export, either, which is almost an introspective reasoning rule for conditionals (see, for instance, Gillies, 2009; 2010). I/E was one of the reasons why Gillies developed his own strict conditional account with shifty operators and his account validated it *simpliciter* (Gillies, 2009, p. 344). However, AS was still a problem, as it was a problem for Veltman's account (1987). The moral of the story for AS is that it seems with the current theories available to us that there is no way to prevent it from holding without going variably strict. However, going variably strict robs us of I/E. Thus, the natural conclusion is that a theory which validates I/E and invalidates AS is a better theory than one which validates both or does not validate either.

So far the present account reads like a strict conditional with context applying specific constraints on the possible outcomes and benefits from its strong aspects such as validating I/E. Reading like a strict conditional account, *the expectation* is that it also validates AS and falls short of providing a semantic solution to the problem of AS. However, this is not the case. The current account invalidates AS by way of the nature of the semantics it employs and without any recourse to pragmatics or implicatures. This virtue comes from the distinction between exact and inexact verification in addition to

general verification rules for conditionals. I will illustrate how this virtue is achieved. First, the definition of AS:

AS: $A \longrightarrow B \vDash A \wedge C \longrightarrow B$ for any C.

For material conditional, it is easy to show why AS holds. Suppose $A \longrightarrow B$ is true. Then $A \wedge C \longrightarrow B$ will be true either vacuously or having a true antecedent and a true consequent no matter which truth-value *C* has. For strict conditional, $[\![A \longrightarrow B]\!]^{c,w} = 1$ iff $\forall w' \in E(c,w) \cap [\![A]\!]^c$: $[\![B]\!]^{c,w'} = 1$, then for any *C*, $[\![A \wedge C \longrightarrow B]\!]^{c,w} = 1$ iff $\forall w' \in E(c,w) \cap [\![A]\!]^c \cap [\![C]\!]^c$: $[\![B]\!]^{c,w'} = 1$.[49] Notice that the set of worlds for $E(c,w) \cap [\![A]\!]^c \cap [\![C]\!]^c$ will still either be a subset of $[\![A]\!]^c$ or will be $\emptyset$ due to $[\![A]\!]^c \cap [\![C]\!]^c = \emptyset$. In either case, *B* will continue to be true, because the set of worlds *B* needs to be true at will not ever expand, but only shrink or go empty. Therefore, the same story goes for any version of the strict conditional account: either vacuously or non-vacuously true.

How does AS *not* hold for the present account? Since our verification is more fine-grained than the one in possible world semantics, we can employ the discrepancy between exact verification and inexact verification along with TCI to stymie AS. Fine, actually, hints at this opportunity, but does not drive the point home (2012a, p. 232; p. 239). Fine's account for counterfactuals validates what is called "Exact Strengthening (ES)." ES reads as $A \longrightarrow B \vDash A' \longrightarrow B$ where A' exactly entails A. Without much care, this can be read as the account *unconditionally* verifying AS, but that is not the case. I will show why this illusion might emerge and how it does not apply below.

[49] Why the notation change? $[\![\bullet]\!]^{c,w}$ is a semantic value function which assigns sentential atomics or propositional variables to truth-values at a given context *c* and world w. This notation is more prevalently used in possible world semantics, so I stuck to it.

For $A \longrightarrow B$, let $|A|^V = \{a\}$, meaning let a be an exact verifier for A and suppose we strengthen the antecedent to $A \wedge C \longrightarrow B$. Let us also suppose $|C|^V = \{c\}$. Then $|A \wedge C|^V = \{a \sqcup c\}$. In order to satisfy ES, we need to obtain $a \sqcup c \Vdash A$, since $A \wedge C$ must exactly imply A. However, $A \wedge C$ only *inexactly* implies A (i.e. $a \sqcup c \Vdash\!> A$), because $a \sqcup c$ contains a part which exactly verifies A, which violates ES[50] and there is no other rule which validates this transition from exact to inexact verifiers by strengthening. Thus, it is the case that $A \longrightarrow B \nvDash A \wedge C \longrightarrow B$, since ES does not validate this strengthening and there is *no* valid inference in the account such as Inexact Strengthening which would read $A \longrightarrow B \vDash A' \longrightarrow B$ where A' *inexactly* implies A. What this endows with the account is the semantic aspect to invalidate all of the counterexamples in the literature to show that AS is troublesome for natural language indicative conditionals without recourse to pragmatics.

Turn back to the Gillies' pair to see this situation clearly. The antecedent of (9') only inexactly implies the antecedent of (9), since the verifier for the antecedent of (9') contains a part for the antecedent of (9), which does not get a pass for ES and no other role permits this transition. What this endows the account with is the virtue of invalidating AS purely on semantic grounds rather than subscribing to implicatures or pragmatic grounds (unlike Gillies' (2009, pp. 344-346) or Veltman's (1987)).

There is another worry arising from the fact that our logic validates Transitivity, which is known to validate AS as a byproduct. There is a nuance here, which applies to all logics of conditionals in general. Even though it is not generally mentioned, there are

[50] This may not be as obvious as I make it sound. The explanation goes as follows: remember our definition for inexact verification: $s \Vdash\!> A$, if $\exists t, t \sqsubseteq s$ and $t \Vdash A$. Let $a \sqcup c$ be abbreviated as t. Now it follows that $a \sqsubseteq t$ and $a \Vdash A$. Notice that this precisely satisfies the definition of inexact verification.

two types of Transitivity, known as, Weak Transitivity (Starr, 2014, p. 1044) and Strong Transitivity. ST is usually what is taken as transitivity, which is:

Strong Transitivity: $A \longrightarrow B, B \longrightarrow C \ \vDash A \longrightarrow C$

Stalnaker (1968) brings his objection to Transitivity based on ST. His objection is raised via a counterexample and I believe that it can be explained away by sharp focus on context. Though his counterexample is not decisive to conclude that Transitivity in general is invalid for indicatives, ST gives us a bigger trouble than the counterexample Stalnaker provides does. This is because ST validates Antecedent Strengthening with some uncontroversial background logic. In order to see this, consider an arbitrary indicative $A \longrightarrow B$ as a premise. Since $A \wedge C \longrightarrow A$ is necessarily true by Inclusion, then by ST, we conclude $A \wedge C \longrightarrow B$, which is basically AS. Weak Transitivity prevents AS from holding as a byproduct. To show this, let us give the definition of WT:

Weak Transitivity: $A \longrightarrow B, A \wedge B \longrightarrow C \vDash A \longrightarrow C$.

Run the same argument. Take again as a premise $A \longrightarrow B$ and take $A \wedge C \longrightarrow A$ as a necessary truth, then you cannot conclude that $A \wedge C \longrightarrow B$, since you need to have $A \wedge C \wedge A \longrightarrow B$ as an extra premise to conclude $A \wedge C \longrightarrow B$.[51] Thus, this shows that the version of Transitivity valid under the present account does not imply AS.

The upshot of the discussion is that we seem to have identified a new way of explaining why the counterexamples to AS are legitimate counterexamples and why these inferences do not get a pass in the present account. When the antecedent is strengthened in those examples, their verification type changes, namely, from exact to inexact verification. Our framework supports Exact Strengthening, which forces the type

[51] To see it better, write it explicitly as $A \wedge C \longrightarrow A, A \longrightarrow B$ (as opposed to $A \wedge C \wedge A \longrightarrow B$) $\nvDash A \wedge C \longrightarrow B$. For his counterfactual account, Fine dubs his principle as Transitivity, but, in fact, he takes it as Weak Transitivity (2012a, p. 240).

of verification of the antecedents to remain the same type, when strengthened. This is sufficient to explain the invalidity of the inferences in the literature and keep those inferences away from our framework. Thus, we see that the problem is not with strengthening *per se*, since our account also validates a version of strengthening; it is with *unspecified* strengthening. Another advantage is that we do not need any extra pragmatic explanation for the infelicity of these inferences, which is a big advantage for our theory over the dynamic strict theories such as Gillies' (2009), Starr's (2014) or Willer's (2013).

5.5 General remarks on the logic of conditionals

We have shown above that the account validates I/E, which is an advantage over variably strict conditionals accounts and explains the counterexamples of AS usually raised against strict conditional accounts in a semantic and principled way (as opposed to, say, Gillies (2009) or Veltman (1987)). Though the present account validates its own strengthening called ES, ES does not admit any of the usual counterexamples of AS. Now, I want to show one more important result and make some general remarks on the logic of conditionals under our account. Afterwards, I will make a general map of where our account stands in terms of which principles it validates in general before concluding.

First, let us see where the account stands in terms of one of the sacrosanct principles for a compositional semantics of indicatives usually called Indicative Deduction (ID) or sometimes called Upper Bound, since it is equivalent to strict conditional (Gillies, 2009, p. 327):

ID: If $P \vDash Q$, then $\vDash P \longrightarrow Q$ where $\vDash$ is loose (classical) entailment.

We can easily check that our truth-conditions for $\longrightarrow$ validates ID. To see this, assume $P \vDash Q$ for arbitrary P and Q. This means that any falsifier for Q is incompatible with the verifiers of P. Then given an arbitrary world state w and an arbitrary context state c, no matter which exact verifier of P we impose on the world state, we cannot obtain any possible outcome which includes the falsifiers for Q as part.[52] Therefore, the possible outcomes for the verifiers of P will loosely verify Q, which satisfies the conditions for $\vDash P \longrightarrow Q$ for any arbitrary world state w and context state c.

A stronger result also follows from exact entailment and truth-conditions of indicatives. Call it *exact indicative deduction* (EID):

EID: If $P \Vdash Q$, then $\vDash P \longrightarrow Q$ where $\Vdash$ is exact and $\vDash$ is loose (classical) entailment.[53]

Proof follows smoothly from truth-conditions of indicatives and the principle of Inclusion we have given for the validity of $A \longrightarrow A$. Suppose $P \Vdash Q$. This means that every exact verifier of P is an exact verifier for Q. Then for any world state w and context state c, imposing any exact verifier of P on a world state will yield a possible outcome which will verify Q, since any imposed exact verifier of P will be a part of the possible outcome obtained via imposing the verifiers of P by Inclusion. But then any outcome obtained via imposing the verifiers of P will verify Q. Hence, it will satisfy our truth-conditions for $\longrightarrow$.

However, the other way around (if $\vDash P \longrightarrow Q$, then $P \Vdash Q$) does not hold, because all $\vDash P \longrightarrow Q$ tells us is that no matter which context and world state we

[52] If it did, then the possible outcome would become an impossible outcome.
[53] Since we have not given the exact truth-conditions for indicatives, there will not be exact verification for indicatives. This can be achieved, but it is non-trivial and not clear what problems it will cause. Fine provides the sketch for defining it (2017, p. 575, fn. 5).

consider, imposing the exact verifier of P yields a possible outcome which verifies Q, but this does not entail that every exact verifier of P is an exact verifier of Q, since some exact verifier of P may only be an inexact verifier for Q, when $\vDash P \longrightarrow Q$ holds. For a counterexample, suppose $\vDash P \longrightarrow Q$ and consider $P = R \wedge Q$, i.e. $\vDash R \wedge Q \longrightarrow Q$. Though $\vDash R \wedge Q \longrightarrow Q$ holds, $R \wedge Q \nVdash Q$, because $R \wedge Q$ will verify Q only inexactly, i.e. $R \wedge Q \Vdash\!> Q$.

So far, so good. Even though Modus Ponens is valid in our account by Actuality, we can separately show that $\longrightarrow$ is also bound below by the material conditional $\supset$.

Lower Bound: $P \longrightarrow Q \vDash P \supset Q$

To see this, assume $P \wedge \neg Q$ is true at some world state w and context state c, i.e. $w \vDash_c P \wedge \neg Q$. Show $w \vDash_c P \longrightarrow \neg Q$. That is, show $u =\!| Q$ for some $p \Vdash P$ and $p \rightarrow_{c,w} u$. Since we assumed for any context c, $w \vDash_c P \wedge \neg Q$, then there are some states $p, \bar{q}$ such that $p \sqsubseteq w$ and $\bar{q} \sqsubseteq w$ such that $p \Vdash P$ and $\bar{q} \text{ -}\!\Vdash Q$.[54] Since we have $p \sqsubseteq w$, by Actuality, we have $p \rightarrow_{c,w} u$ for $u \sqsubseteq w$. Now we have $p \Vdash P$ and $p \rightarrow_{c,w} u$, the question is whether $u \models Q$ or $u =\!| Q$? Remember that from our assumption we have $\bar{q} \text{ -}\!\Vdash Q$ for $\bar{q} \sqsubseteq w$. Since we have $u \sqsubseteq w$ from Actuality and u is a world state by Maximality, we have $u = w$. This entails that $\bar{q} \sqsubseteq u$ and, *a fortiori*, $u =\!| Q$, as required.

However, this brings out a more general question in the logic of indicatives: Since the account validates Modus Ponens, ID and Import/Export, how does it not collapse into material conditional, as Gibbard's and many others' collapse theorems show (Gibbard, 1981; McGee, 1985; Fitelson, 2013). Let me show briefly why the collapse proof does not follow for the present account. Collapse theorem basically says

[54] $\bar{q} \text{ -}\!\Vdash Q$ follows from $\bar{q} \nVdash \neg Q$ and the verification rule $(ii)^{+}$.

that if MP, ID and I/E is validated by a conditional connective $\longrightarrow$, then $\longrightarrow$ and $\supset$ will be equivalent, that is, they will assign the same truth-values, no matter whatever the antecedent and consequent are. We can show that $\longrightarrow$ and $\supset$ will assign different truth-values to same antecedents and consequents.

Suppose that we have our indicative connective $P \longrightarrow Q$ and the corresponding material conditional $P \supset Q$, that p and $\bar{p}$ are exact verifier and falsifier for P respectively and that $\bar{p} \sqsubseteq w$. $P \supset Q$ is true at w, since P has a falsemaker at w. Suppose also that imposing p on w yields a possible outcome u which falsifies Q. This entails that $P \longrightarrow Q$ is false by our truth-conditions, since imposing the exact verifier of P on w will yield a possible outcome which falsifies Q. Therefore, at w, $P \supset Q$ is true, whereas $P \longrightarrow Q$ is false. This shows that $\longrightarrow$ is not equivalent to $\supset$. This proof also shows an important aspect by which plausible theories for indicatives diverge from the material conditional. Consider the following utterance:

(10) If that animal is a frog, then it is a mammal.

Suppose that the animal in question is far away, but actually is a lizard. Then $\supset$ assigns truth to (10). However, in our account, imposing the verifier on the world of evaluation yields a possible outcome which cannot verify the consequent, so our present account assigns falsity to (10), which is the intuitive and correct result for the truth of the indicative. No matter whatever that animal is in the distance, I am inclined to say that this conditional is false and the present account predicts this intuitive result.

One more alluring aspect of the present account is that it validates Simplification of Disjunctive Antecedents (SDA).[55]

[55] SDA follows directly from what is called under the current account as Exact Strengthening of the Antecedent (Fine, 2012a, p. 239). I will not take long to discuss why SDA is an intuitive and plausible

SDA: $P \vee Q \longrightarrow R \vDash P(Q/P \wedge Q) \longrightarrow R$.[56]

Even though this rule is generally valid in the logic of conditionals, there is a famous counterexample to this inference rule by McKay and Inwagen (1977). Fine acknowledges this counterexample and defends the principle against the counterexample by subscribing to what he calls "Suppositional Accommodation" (2012a, p. 232). First, the counterexample:

(11) If Spain had fought for the Allies or the Axis in WWII, she would have fought with the Axis.

This sounds plausible and true, given Franco's inclinations during WWII. However, it would be a mistake to infer from (11) that:

(11*) #So, if Spain had fought for the Allies in WWII, she would have fought with the Axis.

But it should be inferable given SDA. We can easily find indicative analogues of this counterexample. However, we do not need to in order to see what would go wrong in the indicative analogue. If we admitted an indicative analogue of (11) true, then our context would be updated with the indicative due to Context Update as well. Since admitting an indicative analogue of (11) into the context would also entail admitting the presupposition that the falsity of the antecedent of (11*) into the context state, (11*) would go indeterminate, since it violates the semantic felicity rule we have set up above. Fine also gives an analogous explanation for the McKay-Inwagen counterexample, but since he has not set up the machinery that we did, he settles for saying that it violates Suppositional Accommodation. This can be taken to be a general rule for weakening

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inference rule. The attempts to make SDA work in possible world semantics can be cited as evidence for the plausibility for SDA (i.e., Willer, 2015).

[56] $P \wedge Q$ is included due to the assumption of inclusive disjunction at the outset.

SDA against the counterexamples: SDA is valid if and only if the antecedent of the conclusion is a salient possibility with regard to the context state at hand.

This concludes the general results and remarks I would like to make on the present account for indicatives. At the start of the thesis, I have promised that I will provide a general picture of the account, which includes which inferences are accepted in the logic. We can provide one which can be compared with others:[57]

- Present Account:
    - *Type of Infelicity*: Semantic (Chapter 4)
    - *Semantics*: Exact Semantics (Fine, 2017)
    - *Logic*: Identity (Fine, 2012a, p. 239), Weak Transitivity (Fine, 2012a, p. 240), Modus Ponens (Fine, 2012a, p. 239; Chapter 5.5), Exact Strengthening of the Antecedent (Fine, 2012a, p. 239), Inexact/Classical Weakening of the Consequent (Fine, 2012a, p. 239), Inclusive Disjunction (Fine, 2012a, p. 238), Simplification of Disjunctive Antecedents (Fine, 2012a, p. 239; Chapter 5.5), Conjunction (Fine, 2012a, p. 238), Import/Export (Chapter 5.2)

I believe that this list lacks the crucial implausible inference rules such as Material Negation, Antecedent Strengthening (especially, its projection Inexact Strengthening within the present account), Substitution of Logical Equivalents (in the antecedent form, as we have discussed it), False Antecedent et cetera. This list should suggest that the present account offers a pleasant logic of indicatives.

[57] I follow Starr's construction (Forthcoming, p.11).

CHAPTER 6

CONCLUSION

Now, let us take a look at what we have done, what we have achieved and what we have left wanting. Our general aim was to take exact semantics and counterfactual extension of exact semantics and extend it further to non-predictive and non-habitual indicative conditionals. This proved to be a heftier job than it sounded from the outset. We have distinguished indicatives from counterfactuals in a semantically salient way by utilizing the notion of context I have defined under exact semantics. The notion of context also turned out to be useful for defining the truth-conditions of indicatives. I have tried to show that the best candidate for truth-conditions is a trivalent semantics. This trivalent semantics enabled us to assign truth and falsity to certain indicatives, whereas leaving certain other ones as indeterminate. I have shown that this also fits our natural, everyday use of indicatives in conversational settings.

Afterwards, we have turned our attention to the logic of indicatives under the present account. I believe and I hope to have shown that the real strength of our account has shown through with the pleasant logic it provided for indicative conditionals. For instance, we have started from the ground with Fine's solution to the puzzle of Substitution of Logical Equivalents possible world semantics in general faces by applying the solution to the puzzle's indicative cousins. Furthermore, we have solved the puzzle of Antecedent Strengthening without recourse to any pragmatic implicatures, unlike Veltman (1987) or Gillies (2009). We have achieved this by showing that none of the valid inference patterns in the account allows inexact strengthening, which leads to the majority of the counterexamples provided in the literature against Antecedent

Strengthening. We have also shown that our account validates Import/Export, which is an essential inference pattern, if we care about preserving Ramsey's insight into conditionals. Aside from these, we have shown that the account validates the Holy Trinity of indicatives (Indicative Deduction, Lower Bound and Import/Export), yet does not fall prey to collapse theorems. This showed that our ⟶ said more than ⊃.

So much for the advantages. Since indicatives are a vast ocean of many threats and treasures, finding many treasures does not let us cut our profits and up-anchor. We have not defined a principled way of capturing epistemic modals in antecedent and consequent positions. Though I believe that this can be achieved, it is not obvious how to do so from the framework I have provided. Second, we have not provided a way of embedding conditionals in antecedent position, though we have provided a way of doing so for the consequent position.[58] This can also be captured, but we need to provide the truth-conditions for indicatives as exact verification rather than classical verification. Fine hints at a possible way of doing this (2017, p. 575, endnote 5), but we need to redefine the semantics altogether, if we want to capture this. For now, I accept the flaw and promise to emend it in future work. We have also provided no way of capturing probabilistic adverbs or adverbs of quantification. This can also be managed without making major changes in the semantic. Some of these aspects could not be captured merely due to lack of space rather than the inherent incapacity of the semantics.

Overall, it can be said that we have traversed some obstacles generally confronted in the literature of conditionals and we are a bit further ahead than where we started. Of course, many emendations and modifications could be attached to the

[58] The catch is, since the truth-conditions for indicatives are provided with loose or classic verification and the verification of consequent itself is provided with classical verification, we can recursively apply the definition to embed indicatives in the consequent position.

account to make it better and I have no claim whatsoever that I have provided the only or the best way of providing a semantics for indicative conditionals under Fine's semantics. However, I am hoping that the reader would agree that this is *one admissible way* of doing so.

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