THE EFFECTS OF USING CONTEXTUAL PROBLEMS AND STUDENT CENTERED TEACHING EPISODES ON 10[TH] GRADE STUDENTS' ACHIEVEMENT, AND VIEWS AND PERCEPTIONS IN THE SUBJECT OF TRIGONOMETRY

by

G. Ayça Eşlik

B.S., Secondary School Science and Education, Dokuz Eylül Universiy, 2003

Submitted to the Institute for Graduate Studies in
Science and Engineering in partial fulfillment of
the requirement for the degree of
Master of Science

Graduate Program in Secondary School Science and Mathematics Education
Boğaziçi University

2010

ACKNOWLEDGEMENTS

First of all, I would like to express my grateful thanks to my supervisor Assist. Prof. Dr Buket Yakmacı Güzel for her everlasting support, encouragement and concern during my research. One simply could not wish for a better supervisor. I would also like to express my sincere gratitude for her patience and valuable time she devoted to evaluate and read the drafts of this thesis. I attribute the level of my masters degree to her encouragement and effort.

Endless thanks must also go to Dr. Fatma Aslan Tutak for her valuable criticisms and for her kindly accepting to join to our meetings with my supervisor. She made important contributions to this study especially in the method part. I would like to express my special thanks for her smiling face, encouragement, support and suggestions.

I would like to thank the members of the supervisory committee, Assoc. Prof. Dr Ayşenur Yontar- Toğrol and Assist. Prof. Dr Hülya Kılıç for their knowledge and assistance.

I want to send my special thanks to assistants Ruhan Çirci and Füsun Şahin for their supports and interests. I would like to express my sincere gratitude to Ruhan Circi for her valuable contributions to this study especially in determining statistical tests we used.

I have also thanks to all of my friends who were with me during my study. Especially, I can not disregard the support of my friends Sümeyye Haliloğlu and Osman Kılıç who were helpful and supportive during my study and the encouragement and reinforcement of my friends Nuray Sönmez, Hatice Zengin, Cansu Şahin, Mübeccel Kahraman, Eren Aksoy, Ali Çakır and Tanju Tunalı. Also, I want to send my special thanks to my best friend Aslı Deniz, for encouraging me on my scientific way.

And, I have grateful thanks to my parents and my dear brother for their neverending patience, support, and love throughout all my life and my dear aunts who always encouraged me especially in though times.

I have furthermore thanks to Gülşen Pekcan, for her support, care and helpful advices to me.

ABSTRACT

THE EFFECTS OF USING CONTEXTUAL PROBLEMS AND STUDENT CENTERED TEACHING EPISODES ON 10TH GRADE STUDENTS' ACHIEVEMENT, AND VIEWS AND PERCEPTIONS IN THE SUBJECT OF TRIGONOMETRY

This study aimed to develop and measure the effectiveness of an instruction enriched with combination of contextual problems and student centered teaching episodes with regard to the extent of change in students' achievement in trigonometry, and their views and perceptions about trigonometry.

In order to analyze the effectiveness of the developed instruction on trigonometry achievement, and views and perceptions about trigonometry; a true experimental design was implemented in an Anatolian High School in Istanbul with the participation of 74 (Grade 10) students. 24 students received traditional instruction (Group A), 26 students received an instruction enriched with only contextual problems (Group B) and 24 students received an instruction enriched with combination of contextual problems and student centered teaching episodes (Group C). Data obtained (through Trigonometry Achievement Pretest, Trigonometry Achievement Posttest, Views and Perceptions about Trigonometry Prequestionnaire, Views and Perceptions about Trigonometry Postquestionnaire and interviews) from the sample were analyzed to examine the effect of instruction enriched with combination of contextual problems and student centered teaching episodes.

Kruscal Wallis Test was used in order to examine whether there is a significant difference between groups' trigonometry achievement after instructional sequences. The results of the analysis indicated that the group who received instruction enriched with combination of contextual problems and student centered

teaching episodes (Group C) performed significantly higher than the group who received traditional instruction (Group A), the group who received instruction enriched with contextual problems (Group B) performed significantly higher than the group who received traditional instruction (Group A), but there was no statistically significant difference between the group who received instruction enriched with combination of contextual problems and student centered teaching episodes (Group C) and the group who received instruction enriched with only contextual problems (Group B) in terms of trigonometry achievement of students. As a result, a positive effect of the instruction enriched with contextual problems on students' trigonometry achievement was found compared to traditional instruction. However, results implied that instruction enriched with combination of contextual problems and additional student centered teaching episodes may not have a significant positive effect in improving trigonometry achievement compared to the instruction enriched with only contextual problems.

Long term effects of instructional sequences on trigonometry achievement among groups were also explored in the study. The nonparametric test of Kruscall Wallis was used for the analysis of data and results indicated that there is significant difference in terms of retention test scores between Group A and Group B, and Group A and Group C. However, there is no significant difference between Group B and Group C in terms of retention test results. This result was parallel with the results of the immediate effects of instructional sequences on trigonometry achievement. As a further analysis, Related Samples Wilcoxon Signed Ranks Test was also carried out to compare retention test scores and posttest scores for each group. The results indicated that there is permanance of the trigonometry achievement for Group A and Group B, however there is no permanance of the trigonometry achievement in Group C.

Views and perceptions about trigonometry after instructions were also explored for each group. Descriptive analysis done on five questions asked before interventions and six questions asked after interventions implied that instruction enriched with combination of contextual problems and student centered teaching episodes has positive effects on students' views and perceptions about trigonometry

compared to the traditional instruction and instruction enriched with only contextual problems. The results also provided evidence on what students perceived to be most interesting and meaningful about the instructions.

ÖZET

BİR BAĞLAM İÇİNDE SUNULAN PROBLEMLERİN VE ÖĞRENCİ MERKEZLİ ÖĞRETİM UYGULAMALARININ KULLANIMININ 10. SINIF ÖĞRENCİLERİNİN TRİGONOMETRİ BAŞARISI VE TRİGONOMETRİ HAKKINDAKİ GÖRÜŞ VE ALGILARI ÜZERİNDEKİ ETKİLERİ

Bu çalışmada, bir bağlam içinde sunulan problemler ve öğrenci merkezli öğretim uygulamalarının kombine edilmesi ile zenginleştirilmiş bir öğretim uygulaması (dersler dizisi) geliştirilmesi ve bu uygulamanın trigonometri başarısı ve trigonometri hakkındaki görüş ve algıların değişimi ve etkililiğinin ölçülmesi amaçlanmıştır.

Geliştirilen öğretim uygulamasının trigonometri başarısı ve trigonometri hakkındaki görüş ve algılar üzerindeki etkililiğini analiz etmek için, tam deneysel araştırma deseni kullanılmış ve çalışma İstanbul'daki bir Anadolu Lisesi'nde 74 (10.sınıf) öğrencinin katılımıyla gerçekleştirilmiştir. 24 öğrenciye geleneksel bir öğretim, 26 öğrenciye sadece bağlamsal problemlerle zenginleştirilmiş bir öğretim ve 24 öğrenciye bağlamsal problemler ve öğrenci merkezli öğretim uygulamalarıyla kombine edilerek zenginleştirilmiş bir öğretim uygulanmıştır. Örneklemden elde edilen veriler (Trigonometri Başarı Ön Testi, Trigonometri Başarı Son Testi, Trigonometri Hakkında Görüşler ve Algılar Ön Anketi, Trigonometri Hakkında Görüşler ve Algılar Son Anketi, Röportajlar), bağlamsal problemler ve öğrenci merkezli öğretim uygulamalarının kombine edilmesi ile zenginleştirilmiş öğretimin etkisini araştırmak için analiz edilmiştir.

Ders uygulamalarından sonra grupların trigonometri başarısı arasında anlamlı bir fark olup olmadığını araştırmak için, Kruscal Wallis Testi kullanılmıştır. Analiz

sonuçları, bağlamsal problemler ve öğrenci merkezli öğretim uygulamalarının kombine edilmesi ile zenginleştirilmiş öğretim almış olan grubun (Grup C), geleneksel öğretim almış olan gruptan (Grup A) anlamlı derecede yüksek performans sergilediği, sadece bağlamsal problemler ile zenginleştirilmiş öğretim almış olan grubun (Grup B) geleneksel öğretim almış olan gruptan (Grup A) anlamlı derecede yüksek performans sergilediği, fakat bağlamsal problemler ve öğrenci merkezli öğretim uygulamalarının kombine edilmesiyle ile zenginleştirilmiş öğretim almış olan grup (Grup C) ile sadece bağlamsal problemler ile zenginleştirilmiş öğretim alan grubun trigonometri başarıları arasında istatistiksel olarak anlamlı bir fark olmadığı görülmüştür. Sonuç olarak, bağlamsal problemler ile zenginleştirilmiş öğretimin geleneksel öğretime kıyasla daha olumlu bir etkisi olduğu görülmüştür. Bununla birlikte sonuçlar, bağlamsal problemler ve ilave öğrenci merkezli öğretim uygulamalarının kombine edilmesi ile zenginleştirilmiş öğretimin sadece bağlamsal problemlerle zenginleştirilmiş öğretime kıyasla trigonometri başarısı üzerinde anlamlı bir olumlu etkisinin olmadığını göstermiştir.

Bu çalışmada, ders uygulamalarının grupların trigonometri başarısı üzerindeki uzun süreli etkisi de araştırılmıştır. Verilerin analizi için Kruscal Wallis Testi kullanılmış ve sonuçlar Grup A ile Grup B ve Grup A ile Grup C arasında kalıcılık testleri açısından anlamlı bir fark olduğunu göstermiştir. Bununla birlikte, Grup B ile Grup C arasında kalıcılık testleri açısından anlamlı bir fark bulunmamıştır. Bu sonuç ders uygulamalarının trigonometri başarısına anlık etkilerinin sonuçları ile paraleldir. İlave bir analiz olarak, her grubun kalıcılık testi puanları ve son test puanlarını karşılaştırmak için "İlişkili Örneklemler İçin Wilcoxon İşaretli Sıralar Testi" uygulanmıştır. Sonuçlar Grup A ve Grup B için trigonometri başarısının sürekliliğinin olduğunu, Grup C için ise olmadığını göstermiştir.

Her grup için trigonometri hakkında görüşler ve algılar da araştırılmıştır. Ders uygulamaları öncesinde sorulan beş soru ve sonrasında sorulan altı soru üzerinden yapılan betimsel analiz; bağlamsal problemler ve öğrenci merkezli öğretim uygulamalarının kombine edilmesi ile zenginleştirilmiş öğretimin, geleneksel ve sadece bağlamsal problemlerle zenginleştirilmiş öğretimlere kıyasla öğrencilerin trigonometri hakkındaki görüş ve algıları üzerine pozitif etkileri olduğunu göstermiştir. Sonuçlar ayrıca öğrencilerin öğretimlerde neyi en ilginç ve anlamlı bulduklarına dair kanıt sağlamıştır.

TABLE OF CONTENTS

LIST OF FIGURES

LIST OF TABLES

LIST OF SYMBOLS / ABBREVIATIONS

df	Degree of freedom
f	Frequency
N	Number
Adj. Sig.	Adjusted Significance
Asymp. Sig.	Asymptotic Significance
Sig.	Significance
Std. Deviation	Standard Deviation
Std. Error	Standard Error
Std. Test Statistic	Standard Test Statistic

1. INTRODUCTION

Mathematics is a difficult subject for most high school students. Students think of mathematics as a difficult subject and have difficulty in understanding abstract concepts such as angle measure and trigonometric ratios in a right triangle. Therefore, educators try to develop strategies intented to increase the efficiency of understanding and required skills. Making lessons relevant to real life and using instructional strategies might be the two of the attempts to improve mathematics learning.

Using mathematical symbols is the main characteristic of mathematics lessons. A board full of symbols is the first image that the students imagine when they think about mathematics lessons. Most students think that mathematics is a lesson which includes only numbers and equations. Nesin (as cited in Sertöz, 2003) says that mathematics is one of the lessons which is generally perceived as an abstract lesson, but in fact it is born from the 'concrete'. Every symbol in high school mathematics represents a concrete concept. For example; '3' does not exist in the environment, it is not palpable but there are 3 women, there are 3 pencils... '3' is not an abstract concept but it represents a concrete concept (as cited in Sertöz, 2003). For example, two same questions may be asked in two different ways as shown below:

i) $2x+y=10$

$x+2y=6$ $\quad$ $x=?$ $\quad y=?$

ii) Two bottles of milk and a bottle of coke cost 10TL, one bottle of milk and two bottles of cokes cost 6TL. So how much does each drink cost?

In the first question x is an abtract concept, but in the second question it is obvious that it represents a concrete concept, a bottle of milk. However, many students think that all mathematics subjects are abstract because of the traditional mathematics teaching strategies.Making lessons related to real life may be one of the attempts for letting students to set relationships between abstract and concrete concepts.

In recent years, the popularity of making the lessons relevant to real life contexts has been increased. In the last 15-20 years, serious and extended attention has been devoted to everyday mathematics (Arcavi, 1998). Researches point out the urgency of connecting school mathematics to the outside world (Gainsburg, 2007). Using real world situations in classrooms leaves the doors open for the students for decision making, experimental verification, conjecturing and even for construction of proofs (Güven, 2007). Realistic Mathematics Education (RME) is one of the popular theories which has progressed significantly since 1971.

There are stages of cognitive development which every child will experience. According to Piaget, the four stages are: sensorimotor - birth to 2 years; preoperational - 2 years to 7 years; concrete operational - 7 years to 11 years; and formal operational (abstract thinking) - 11 years and up. Although every child passes through the stages in exactly the same order, there is some variability in the ages at which children attain each stage (Evans, 1973). Piaget's research indicated that symbolic representations have meanings for students as old as 12 (Driscoll and Confrey, 1986). So, as it is obvious high school students aged 14-17 are abstract thinkers and because of the variability of ages described above, some of them may not become abstract thinkers yet. So, giving concrete problems related to real life contexts may be more effective especially for those students. Concrete daily life problems may be more influential for teaching higher level mathematical knowledge and skills. In this way, a greater number of students may be engaged in mathematics learning.

Problems that include real life contexts that students come across during their activities in their daily lives are rarely given to students especially in high school. On the other hand, these kind of concrete problems are essential in mathematics education for letting them to set relationships between concepts and understand why they are learning mathematics.

Content of the mathematics books may be one of the reasons of the difficulty students have in understanding abstract concepts. Many of the mathematics books are not written to help students to understand mathematics. Sertöz says: "I have come across a high school geometry book writing 'this formula has to be memorized'. However, there is nothing which should be memorized in mathematics. The writer wrote it, because he also didn't understand it." (Sertöz, 2003, p18). So, because of such books and wrong teaching techniques, students may not know where mathematics has been being used in life. They may think that mathematics is a lesson which must be memorized, because they do not know its real life application. However, relating the mathematics concepts with real life may be possible in several mathematics subjects. Adding contextual problems in lessons may help to take attention of the students, improve positive attitudes toward mathematics, provide them to understand the concepts better and finally increase their achievement.

The purpose of this study is to investigate,

i) the stand alone effect of using contextual problems

and

ii) the combined effect of using contextual problems and student centered teaching episodes

on 10^{th} grade high school students' achievement, and views and perceptions in the subject of trigonometry.

2. REVIEW OF LITERATURE

Students have difficulty to conceptualize the abstract concepts such as types of angles and trigonometric ratios in right triangles. There are not many researches on teaching and learning of trigonometry (Delice & Monaghan, 2005). However, many researchers emphasized the importance of connecting mathematics lessons to real life. The literature review will begin with a brief overview about importance of making real world connections in mathematics classrooms in order to provide a general framework. Research studies about trigonometry and student centered instruction will follow and it will continue with two of the popular theories; Situated Learning and Realistic Mathematics Education (RME), which has progressed significantly since 1971.

2.1. Importance of Making Real World Connections in Mathematics Classrooms

Many research studies pointed out the importance of connecting school mathematics to the outside world. The K-12 mathematics education community is virtually united on the importance of connecting classroom mathematics to the real world (Boaler, 1997). De Lange (as cited in Bishop *et al.,* 1996) says that real-world connections are expected to have many benefits, such as enhancing students' understanding of mathematical concepts and motivating mathematics learning. Real world connections help students apply mathematics knowledge to real problems particularly those arising in the workplace (National Research Council, 1998). Gainsburg (2008) says that the mathematics education literature locates a range of practices for real-world connections:

- simple analogies (e.g., relating negative numbers to subzero temperatures)
- classic "word problems" (e.g., "Two trains leave the same station…")
- the analysis of real data (e.g., finding the mean and median heights of classmates)
- discussions of mathematics in society (e.g., media misuses of statistics to sway public opinion)
- "hands-on" representations of mathematics concepts (e.g., models of regular solids, dice)
- mathematical modeling of real phenomena (e.g., writing a formula to express temperature as an approximate function of the day of the year).

Connecting school mathematics to the outside world is pointed out by many other research studies. Hoyles (as cited in Gainsburg, 2008) says that studies document the increasing importance of mathematical literacy in the modern workplace and the need for workers to understand the meaning of their calculations in the context of work. Yet many students fail to see the utility of mathematics.

Instead of learning skills in isolation, students learn mathematics in context where they can see how it is applied in real situations. In this way they come to recognize the importance of mathematics in their own lives. The connection between mathematics and the real world is a strong one. Along with learning fundamental mathematics skills, students learn to think logically, analyze data, make decisions and solve multifaceted problems that arise out of real life situations. Students use the skills they are learning in meaningful ways (Muschla and Muschla, 2006).

Teachers can make the problems more interesting by making it more concrete. For example; the classoom may be used in order to help to calculate the diagonal of prism, because the classroom is also a prism which has dimensions such as width, length and height. This helps students to visualize the problem and learn in meaningful ways (Polya, 1997). Gravemelger (cited in Hadi *et al.,* 2002) also says that teaching should begin with activities that the child has already been doing in the environment. Saxe (1991) suggests that the classroom practice should be constructed by using the characteristics of learning in everyday settings.

Civil (2002) describes 'everyday mathematics' as the "mathematics that occurs outside of the school (p.42). The author also describes 'academic mathematics' as the "mathematics as mathematicians do" such as exploring and representing mathematical objects and relations, formulating, summarizing ideas, and providing generalizations (p.43). Moschkovich (2002) provides an introduction to thinking about everyday and academic mathematical practices by discussing two proposals for classrom practices. The first proposal of Moschkovich would engage children in activities that parallel everyday mathematical practices and involve problems similar to those that adults might face at work or at home. The second proposal of the researcher would engage children in activites that parallel mathematicians' practices, thus involving students in exploring and representing mathematical objects and relations, formulating and testing conjectures, summarizing ideas, and providing generalizations (Moschkovich, 2002). Author also points out that these two approaches are neither incompatible nor contradictionary and proposes exploring the possibility of changing clasroom practices through a synthesis of these two proposals.

Arcavi (2002) draws on examples from first hand experiences as a curriculum developer and teacher educator in considering the pedagogical implications of bridging the gap between everyday and academic mathematics. He emphasizes that 'everydayness' varies a great deal depending on people and contexts. In addition to everyday experiences, he describes how contextualizing academic problems can increase the complexity and depth of mathematics that students experience. He points out a mathematics teaching innovation by proposing a combination of everyday mathematics and mathematicians' mathematics.

Masingila (2002) investigates children's views of mathematics and their use of mathematics in everyday activities. For most of the children, counting, measuring and designing were part of their activities. For those few who had a broader view of mathematics, locating, playing, and explaining were also described as part of their everyday mathematics activities (Masingila, 2002). Masingila's view is that we need a better understanding of how students perceive their use of mathematics outside the mathematics classroom so that we can build on that mathematical knowledge by extending and formalizing in the classroom.

Civil (2002) describes her experience in a teaching program that took children's everyday activities as starting points for exploring mathematics from mathematician's perspective. Her data show successful moments of children's engagements in mathematical activity that has characteristics which are parallel to both mathematicians' and everyday mathematical practices. By contrast, she describes how fewer and fewer students seemed engaged as the tasks became increasingly oriented to academic mathematics (Civil, 2002).

Moschkovich (2002) examined the mathematical activities in a middle-school mathematics classroom during an architectural design project. He examined student mathematical activities during assesments and compare mathematical activities by looking at three instances of assesment activities during this project, design reviews, final presentations, and classroom conversations. Overall, the design project shifted student work in the clasroom from traditional activities such as paper and pencil exercise to design and presentation activities.

Although the importance of connecting school mathematics to the real world is recognized, little is known about how and how often it actually happens in classrooms. American National Center for Education Statistics, the 1999 TIMSS Video Study found that in the average 8^{th} grade mathematics lesson, only 9–27% of the problems used a real-life situation in their set-up, in the six of the seven countries studied. The exception was the Netherlands, by 42%. In Daley and Valde's study about mathematics teaching in US district which was conducted in 2006, the vast majority of teachers made little or no effort to connect mathematics to students' lives (as cited in Gainsburg, 2008). Indeed, many teachers consider the real contexts of word problems irrelevant distractors. Pre-service teachers tend to exclude real-world knowledge and considerations when solving word problems and prefer nonrealistic approaches in others' solutions (Gainsburg, 2008).

These research above suggests that real world connections are infrequent and cursory in mathematics classrooms. The picture, however, is limited by the small number of studies, some of which examine only a narrow aspect of the phenomenon (e.g., word problems) or pre-service teachers (Gainsburg, 2008). Gainsburg's (2008) qualitative research has important results about reflecting teachers' views about making real life connections in secondary mathematics classrooms and the factors that influence teachers' use of connections.

Gainsburg (2008) surveyed 62 secondary mathematics teachers about their understanding and use of real-world connections, their purposes for making connections in teaching, and factors that support and constrain this practice. She also observed 5 teachers making real-world connections in their classrooms and conducted follow-up interviews. The results offer an initial portrayal of the use of real-world connections in secondary mathematics classes and raise critical issues for more targeted research, particularly in the area of teachers' beliefs about how to help different kinds of students learn mathematics. According to the qualitative data, secondary mathematics teachers count a wide range of practices as real-world connections. Gainsburg's study (Gainsburg, 2008, p.215) indicate important results about teachers' views:

- Teachers make connections frequently, but most are brief and many appear to require no action or thinking on the students' part.
- Standardized tests and externally mandated curricula sometimes suppress teachers' use of real-world connections, but most teachers do not blame them directly. These factors probably inhibit connections indirectly via the time pressure imposed by content laden curriculum schedules that are synchronized to external tests.
- Some teachers believe real world connections can prepare students for tests. Whether standardized tests inspire or inhibit connections seem to depend on the content of the test and the time duration of the connection.
- When evaluating a real-world task for classroom use, teachers primarily consider the importance of the mathematical topic and how real or familiar the context will be to students. The significance of the context, authenticity of the mathematical task, and potential of the task to teach about the context are far less relevant. These findings are consistent with the conclusion that teachers' main goal is to impart mathematical concepts and skills, and that developing students' ability and disposition to recognize applications and solve real problems is lower priority.
- Some teachers feel that students should master mathematical concepts and skills before connecting them to the real world.
- While some teachers value tasks that require critical thinking or promote literacy development, more fear that complex, ill-structured, or language-intensive tasks will overwhelm students.
- Teachers worry more about over-challenging students than under-challenging them.

Committee for Economic Development indicated that only 61% of American 12th graders agreed that mathematics is useful for solving problems in 2000, down from 73% in 1990 (as cited in Gainsburg, 2008). De Lange (as cited in Bishop et. al., 1996) stated that the concern is international: initiatives in the Netherlands, Portugal, Australia, the US, and Britain, among other countries have promoted application-oriented or context based curricula.

Brenner (2002) describes four teachers' implementation of an experimental unit aimed at providing children with opportunities to use their informal and everyday mathematical understanding and strategies as the foundations for algebraic thinking. She examined how teachers and students responded to the use of everyday mathematics in a pre algebra unit about choosing a pizza company for the school cafeteria. Four teachers and their junior high school students were videotaped repeatedly as they spent a month working on an experimental curriculum unit that emphasized realistic situations, open ended problem solving, and multiple representations as the means for introducing basic ideas about algebra. The students' use of everyday mathematics was significantly affected by how teachers introduced each lesson. Brenner's analysis of teachers' effectiveness in providing opportunities for their students to use everyday and informal knowledge leads to an interesting set of results that opens up a new series of questions concerning teachers' styles and students' learning. The analysis carefully considers the importance of context, describing wide variations among teachers in their implementation of the unit and the kinds of activities students engaged in (Brenner, 2002).

Altun and his colleagues' research (2004) about the way and frequency of using high school mathematics textbooks indicated that 65% of the teachers have considered that textbooks' way of presenting subjects is well. Other teachers stated that they are medium or insufficient. In addition to this, teachers stated that textbooks must involve practice and tests, examples in textbooks are stereotype, textbooks are understood only by themselves, ÖSYS's content has impaired high school mathematics education seriously (Altun et.al., 2004).

A high school teacher conducted a study about contextualizing calculus concepts in order to develop student understanding (Schwalbach, 2000). She provided students with concrete examples from their physics class to give them contextually rich environment. Students discovered connections between the physics concepts of position, velocity and acceleration and the calculus concepts of function, derivative and antiderivative. The qualitative study described several critical aspects of understanding: students' ability to explain concepts and procedures, to apply concepts in a physics context, and to explore their own learning. It included 32 seniors at a large, urban, comprehensive, religious school in a midwestern state. Samples of student work and reflections were collected by the researcher. The researcher kept a reflective journal. The study indicated that making connections between calculus and physics, namely contextualizing calculus concepts can yield deep understandings of semantic as well as procedural knowledge.

According to the synthesis of studies reviewed in literature, it is clear that the mathematics education community point out the importance of connecting school mathematics to the real world. Many research studies emphasize the importance of using real-world connections during the instructions. They mention that real world connections help students apply mathematics to real problems, motivate mathematics learning and increase their understanding of mathematical concepts.

2.2.Research Studies about Teaching of Trigonometry

Research studies on teaching and learning of trigonometry are very new and limited. As Delice and Monaghan (2005) states, there are not many research studies on teaching and learning of trigonometry. The review of literature about trigonometry instruction will be summarized in this part.

Delice and Monaghan (2005) examined tool use in senior high school trigonometry lessons in England and in Turkey in their comparative study. Besides tool use; curricula, assessment, classroom practices and aspects of student performance were also examined in the study. The study involved 55 English students, 65 Turkish students, 40 English and 65 Turkish teachers. It describes trigonometry in the two countries and student performance in several tests. According to the test results, Turkish students did better on symbolic tests and English students did better on 'real life' problem solving. This educational implications are drawn which include locating ethical questions in curricula change and possible ramifications.

Thompson (2008) conducted a research at a public high school from the theoretical framework of Realistic Mathematics Education. Three sections of precalculus started a unit of trigonometry by modeling the motion of an animated Ferris wheel in The Geometer's Sketchpad. The fourth section followed the design of the course textbook. Student reflections were collected periodically throughout the unit. In addition, interviews were conducted with students from the experimental and comparison classes. Students routinely commented concerning the benefits of the unit being linked to a real life and the specific benefits afforded for graphing trigonometric functions. The evidence showed the potential power of a contextual realistic problem solving scenario as the instructional starting point for a trigonometry unit of instruction. The Ferris wheel supported the concepts of amplitude, period, and the general behavior of the graph increasing or decreasing through various angles. The results showed that the students more than simply recognized these connections while looking at the Ferris wheel. Since the context was the starting point, the Ferris wheel supported students' understanding of graphing trigonometic functions and effectively using the unit circle by allowing them consider the animation as a model of trigonometric function.

Doğan (2001) examined the opinions of the high school students about the trigonometry subjects in Konya. The districts of Konya were divided into three groups as A, B and C according to their social and economical conditions. The multiple choice tests developed by the researcher were applied to the 1316 tenth grade students in these districts. The results showed that 21.89 % of the students like trigonometry subjects, 45.52 % of the students do not like trigonometry subjects, 30.93 % of the students said that they do not want to hear about trigonometry, 46.43 % of the students said that they do not want to study on trigonometry, trigonometry subjects are not necessary for their daily life and it is meaningless to learn trigonometry, 51.75 % of the students said that instructional materials were not used in the lessons, 65.35 % of the students think that their will to learn trigonometry were diminished since questions on trigonometry were not asked in university entrance examination and 65.96% of the students think that learning is easier when the levels of students are almost same. Besides these results, it is also observed that the students confuse the trigonometric conceptions and they have difficulty in solving trigonometric problems. They make mistakes in problems including trigonometric equations, identities, unit circle and trigonometric functions because of their misconceptions. Students are more successful at the questions with geometrical shapes which can be solved by using trigonometric formulas.

Quinlan conducted a lesson plan and reported his observations on the lesson which the pupils were involved in a practical activity designed to introduce the tangent ratio and demonstrate its utility in some real-life contexts (cited in Cavanagh, 2008). Quinlan defined some general principles for introducing new mathematical concepts and ideas. The author suggests an approach where concrete examples are considered before abstract definitions are taught. The author says that teachers might begin by allowing students to explore concrete examples of a concept before presenting its definition, and that the formal terminology and symbolism associated with the concept might be introduced much later, after students have developed a sound grasp of the basic ideas.

Tarhan (2007) examined the effect of trigonometry teaching by using the approach of constructivism on 10^{th} grade students' attitudes and success. The study group composed of 50 students chosen randomly from an Anatolian High School in Denizli. The data used in the research were collected through the 'Success Test' and the 'Attitude Scale' developed by the researcher. The prepared test was applied on both the experimental (n=25) and control group (n=25) students and t-test was applied on the obtained data. The results showed that there was statistically no significant difference between the attitude scores, pre and posttest scores of the students in the groups. Moreover, the experimental group students were observed to be curious and motivated, however the control group students were to be passive. As a result, although the statistical data does not show significant difference, researcher's observations show that the approach of constructivism is more effective than the traditional methods used.

Stephens (1997) discussed a trigonometry lesson in which students are asked to be energy conscious architects and design a window overhang that will block the midday summer sun but allow plenty of sunlight in the winter. Stephens states that architects can construct buildings by using trigonometry and emphasizes the importance of trigonometry for the training of energy conscious architects. Stephens asked a problem to the achitecture students which is about the design of a window overhang that controls the entry of sunlight. As a clue, he gave the angle of sunlight through window in the longest day of the summer and the shortest day of winter. Students in the classroom divided into groups. He observed that some groups had difficulty in recalling the trigonometric formulas and some groups had difficulty in applying the trigonometric formulas to the problem. However, he stated that students coped with these difficulties by group work and his guiding. He also observed that students developed positive perceptions towards trigonometry and trigonometry achievement increased when the subjects were narrated with stories and the groups were collaborated.

Örnek (2007) examined the effects of teaching trigonometry by using role playing (dramatization) on mathematics achievement, permanence of the knowledge learned, attitudes and anxiety towards mathematics. The study used a pretest-posttest design. The study composed of 69 eighth grade students from a primary school in İstanbul. The groups attending the research were applied a pretest, and mathematics attitude and anxiety scale before the study. Within the study the experimental group were taught by using dramatization method and the control group were taught by using traditional methods. After the study, pretest was used as posttest and applied to the groups. Also, mathematics attitude and anxiety scales were applied again. Eight weeks after the study, posttest was applied as permanence test in order to see the permanence of the knowledge learned. The findings showed that, there is a significant positive effect of using dramatization while teaching trigonometry to eighth grade students on mathematics achievement, permanence of the knowledge learned and attitudes towards mathematics.

Barnes (1999) emphasizes the importance of group work in the solution process of trigonometric problems. She wanted each group to write a story related with the problem and solve the problem like playing a game. She observed students' participation and interest towards trigonometry in this learning atmosphere. She wanted each group to shorten the expression:

$$2.\mathrm{Cos}(3t).[\mathrm{Sin}(2t).\mathrm{Cos}(t)+\mathrm{Cos}(2t).\sqrt{1-\cos^2(t)}\,]+1,\quad 0\le t\le 7^0$$

into an expression with maximum ten characters. She told the story of super hero Mcgyver's escape from prison by using the solution of this problem. She wanted students to write other stories about the problem, to think themselves instead of super hero Mcgyver and find the solution that will help them to escape from the prison. She placed students in a setting in which they use critical thinking skills, express their ideas verbally and have fun. This assignment required the students to use some trigonometric identities as well as some right triangle trigonometry. Barnes observed that every student joined to problem's solution process however they did not participate in the story writing process. She observed that only one student wrote a story, other students did not participate in this process. So, she said that she wanted every student to join to this process and write their own story. She observed that students were more active after this warning. She suggests that creative writing can help students overcome their fear of word problems while it might seem unusual to think of creative writing in a math class. Barnes described a number of successful math writing assignments in her study. She also stated that collaborative group work increased the trigonometry achievement and students' interest towards trigonometry.

Ağaç (2009) investigated the effect of the graphic calculating machine (GCM) utilization by 10^{th} grade class students in learning trigonometry on their academic achievements and problem solving skills. The implementation continued for nearly five weeks in a public school in Izmir. The study was conducted by using GCM in experimental group (n=18) and without using GCM in control group (n=20). In the research, both quantitative and qualitative data were collected. It is found out that there is no statistically significant difference between the academic achievements of the experimental and control group students who participated in the research. However, there is a significant difference among the problem solving skills of the students. Moreover, the results show that there is no significant difference according to the gender of the students who use GCM in learning trigonometry when their problem solving skills are concerned. According to the results, the benefits of the GCM utilization on the learning-teaching process are noteworthy. It provides permanence of the knowledge and rapid data transfer, helps in solving the problem by enabling cross-checks, ensures the materialization of the concepts in brains with its visuality, gives the opportunity for timesaving, facilitates learning and prevents the abstraction of interest and attention. It is stated that GCM utilization has no negative effects in general. It is noted that especially some difficulties were experienced at the first days of GCM utilization, however it is observed that opinions changed in time. The results demonstrate that trigonometric function drawing, sine and cosine theorems were learned in the best way by using GCM in trigonometry lessons. Most of the teachers and students said that their academic achievements would increase due to the GCM utilization in learning trigonometry.

Kendal and Stacey (1997) reported the research study which compared the two methods of teaching basic trigonometry named as Ratio Method and Unit Circle Method. They wanted to see which one promotes better understanding of the underlying concepts and mastery of skills in the subject of trigonometry. Both methods were outlined in more detail in the study. The study composed of 178 tenth grade students. 90 students in four classes were taught basic trigonometry by using the ratio method and the other 88 students in four classes were taught by using the unit circle method. Two tests were administered before and after the teaching. According to the results, students were better able to master the skills required and came a greater improvement in attitude to the subject when ratio method was used. The study showed that the teaching method, not the teacher, was the dominant effect. Students of all ability levels performed better with the ratio method, but the low ability students benefited the most.

Emlek (2007) examined the effect of computer aided teaching trigonometry by using dynamic modelling on the academic achievements of the vocational and general high schools' students. The study consisted 128 students of high school and 112 students of vocational school of higher education. The experimental and control groups were selected randomly. At the begining of the study, the pretest including 20 questions was applied to the groups. The instructional materials and dynamic models for the computer aided teaching trigonometry were designed and applied on the experimental group. The basic concepts of the trigonometry were taught by the traditional method on the control group during 3 weeks with 6 hours. Finally, the post test was applied to the groups. The data obtained from the pre and posttests were evaluated using descriptive statistical methods. Students' opinions were also examined by the material evaluation form. In general, the materials used in the experimental group were found interesting and inductive by the students. The academic achievement of the experimental group was found higher than the academic achievement of the control group.

Choi-Koh (2003) investigated the effect of a graphing calculator in trigonometry. The purpose of the research was to investigate the nature of student's mathematical thinking processes and behavior and describe the nature of difficulties the student underwent in trigonometry as the student conducted independent explorations within the interactive technology environment. Also, the research identified the connections among multiple representations, merits and shortcomings in using the graphing calculator, as a tool. In order to identify students' mathematical thinking processes and the nature of difficulties in learning trigonometry, a case study was conducted as the method for qualitative research. One 11^{th} grader, Sung Min who attended at a college preparatory high school in the suburb of Seoul, Korea, participated in the study. A take-based clinical interview procedure by qualitative research methodology was used to provide an opportunity to gain in-depth understanding of thinking processes in trigonometry, cognitive actions, and his interactions with the graphing calculator. The researcher found that the student moved hierarchically from operative stage, to constructive stage, then to applicable stage in thinking processes. Multiple representations Sung Min used were graphic representations, algebraic equations in correspondence to each graphic representation, diagrams and tables. Free movement between multiple representations with the relationships of correspondence facilitated Sung Min's visualization with dynamics and was considered as the best merit in learning mathematics.

Filiz and her colleagues (1999) designed an instructional material for eighth grade students in the subject of trigonometry. A scenario related with daily life was designed and presented with using computer programmes (MATHEMATICA and Powerpoint). Student centered and scenario based teaching strategies provided students to work in groups and collaborate. Open ended problems gave students the opportunity to discuss and express their ideas easily. Positive effects of the instruction were observed at the end of students' verbal comments. Filiz and her colleagues suggest that this study can have positive effects on students' attitudes towards trigonometry and can also be used by the educators. Students can feel as if they are role players during the lessons when games and scenarios are used as instructional methods. This innovative learning atmoshere may motivate students and let them participate in trigonometry lessons.

Weber (2005) investigated students' understanding of trigonometric functions in the context of two college trigonometry courses. 30 students in the first course were taught by a professor unaffiliated with the study in a lecture-based course, while the 40 students in the second course were taught using an experimental instruction paradigm based on Gray and Tall's notion of concept and current process-object theories of learning. In Tall et al's theory, a student can go through three stages to understand a concept. The student first learns how to apply an operation as a procedure, or as a step by step algorithm. At this stage, the procedure is highly mechanical and may be relatively meaningless to the student. In the second stage, if the student applies the procedure multiple times and is given the opportunity to reflect upon it, he or she may come to view the procedure as a process, or a meaningful method designed to accomplish a particular mathematical goal. In the last stage, the student who understands an operation as a process can begin to anticipate the results of this process without applying each of its steps. Via interviews and a paper-and-pencil test, Weber examined students' understanding of trigonometric functions for both classes. The results indicate that the students who were taught in the lecture-based course developed a very limited understanding of these functions. Students who received the experimental instruction developed a deep understanding of trigonometric functions.

Orhun (2004) investigated the students' mistakes and misconceptions in the subject of trigonometry. The research included totaly 77 tenth grade students from a high school and a science high school in Eskişehir. The research data were obtained from the exam which consists of 15 questions. According to the findings, students' mistakes are very systematical. It was seen that trigonometry is only understood as relations between the angle and the edges of a right triangle. For this reason, the students were successful on the questions concerned with angle and right triangle. The impression is that trigonometry is generally taught via teacher centered method, memorizing the ready knowledge and repeating them. So, students had difficulty to transfer the principles learned to new situations. The main reason of students' mistakes is due to teaching method. It was observed that measuring the angle in degree caused contradiction to general definitions and concepts in trigonometry. Orhun suggests that the measure of angle should be taught in radian. Trigonometric functions should be taught by using their graphs because students had difficulty in using these graphs. Orhun emphasizes that it is important to determine possible misunderstandings of students in order to improve teaching methods in trigonometry.

Research studies on teaching and learning of trigonometry were summarized in this part. Some of the studies examined the effects of trigonometry teaching by using different approaches on students' performance, attitudes and success. Some of them conducted lesson plans in the subject of trigonometry and reported the observations. Some of the studies examined students' opinions about trigonometry, understanding of trigonometric concepts and mistakes and misconceptions in the subject of trigonometry.

2.3. Student Centered Instruction

The concept of student centered learning has been credited as early as 1905 to Hayward and in 1956 to Dewey's work. The term student centered learning was also associated with the work of Piaget and more recently with Malcolm Knowles (O'Neill *et al.*, 2005). Rogers (1983) describes the shift in power from the expert teacher to the student learner, emerged by a need for a change in the traditional environment where students are passive, apathetic and bored. Simon highlighted that the concept of student centered education has been derived from the idea that the teacher should act as a guide (cited in O'Neill *et al.,* 2005).

Lea et al. (cited in O'Neill *et al.*, 2005) view that student centered learning involves increased responsibility on the part of the student. They summarize some of the literature on student centered learning to include the following principles:

- the reliance on active rather than passive learning,
- an emphasis on deep learning and understanding,
- increased responsibility and accountability on the part of the student,
- increased sense of autonomy in the learner,
- an interdependence between teacher and learner,
- mutual respect within the learner teacher relationship,
- and a reflexive approach to the teaching and learning process on the part of both teacher and learner.

In the traditional approach, the responsibility of communicating course material resides primarily with the instructor (Felder, 1996). In student centered instruction, some of this responsibility is shifted to the students. Learner-centered teachers are quides, facilitators, and designers of the learning experience (Weimer, 2002). Teacher's primary functions are lecturing, designing assignments and tests, and grading; in student centered instruction, the teacher still has these functions but also provides students with opportunities to learn independently and from one another and coaches them in the skills they need to do so effectively. In recent decades, the education literature has described a wide variety of student centered instructional methods and offered countless demonstrations that show student centered instruction leads to increased motivation to learn, greater retention of knowledge, deeper understanding, and more positive attitudes toward the subject being taught (Felder, 1996).

Student centered instruction is a broad approach that includes such techniques as substituting active learning experiences for lectures, holding students responsible for material, assigning open-ended problems and problems requiring critical or creative thinking, involving students in simulations and role-plays, assigning a variety of unconventional writing exercises, and using self-paced and/or cooperative learning (Felder, 1996).

Active learning is generally defined as any instructional method that engages students in the learning process (Bonwell & Eison, 1991). Active learning requires students to do meaningful learning activities and think about what they are doing (Prince, 2004). While this definition could include traditional activities such as homework, in practice active learning refers to activities that are introduced into the classroom. The primary elements of active learning are student activity and engagement in the learning process (Prince, 2004). Active learning is often contrasted to the traditional lecture where students passively receive information from the teacher.

Collaborative learning can refer to any instructional method in which students work together in small groups toward a common goal (Prince, 2004). As such, collaborative learning can be viewed as encompassing all group-based instructional methods, including cooperative learning (Millis & Cottell, 1998). In contrast, some authors distinguish between collaborative and cooperative learning as having distinct historical developments and different philosophical roots (Bruffee, 1995). The core element of collaborative learning is the emphasis on student interactions rather than on learning as a solitary activity (Bruffee, 1995). Cooperative learning can be defined as a structured form of group work where students pursue common goals while being assessed individually (Millis & Cottell, 1998). Johnson et. al. imply that the most common model of cooperative learning incorporates five specific tenets, which are individual accountability, mutual interdependence, face to face promotive interaction, appropriate practice of interpersonal skills, and regular self-assessment of team functioning (cited in Prince, 2004). While different cooperative learning models exist, the core element held in common is a focus on cooperative incentives rather than competition to promote learning (Prince, 2004).

Problem-based learning (PBL) is an instructional method where relevant problems are introduced at the beginning of the instruction and used to provide the context and motivation for the learning that follows (Prince, 2004). It is always active and usually (but not necessarily) collaborative or cooperative using the above definitions. Problem-based learning typically involves significant amounts of self-directed learning on the part of the students (Prince, 2004). Wilkerson and Gijselaers (1996) outline the following key principles in understanding and using PBL:

- Problems serve as a stimulus for learning,
- Problem-based learning is based on three important principles of learning:
  i) Learning is a constructive and not a receptive process,
  ii) Learning is associated with existing knowledge networks,
  iii) Knowing about knowing (metacognition) affects learning,

iv) Social and contextual factors influence learning, understanding how and when to use knowledge is as important as the knowledge itself,

- Problems reflect real world situations or professional practice,
- Small group work encourages student collaboration and independence,
- Students learn to share their ideas and share responsibility,
- Students learn to question their own assumptions about their reality,
- Conflicting views as part of discussion facilitate understanding,
- Problem-based learning may not be suitable for all types of learning and topic areas,
- Educators must have confidence in the students that they will use their time wisely and can be trusted to carry out the required tasks on time,
- Problems are encountered before all relevant knowledge has been acquired, not after it.

In the light of the literature review about student centered learning, student centered education emerged by a need for a change in the traditional environment where students are passive. Student centered learning is generally defined as any instructional method that engages students in the learning process. The primary elements of student centered learning are student activity and engagement in the learning process. Many research studies emphasize that the teacher should act as a guide in the classroom.

2.4. Situated Learning

A model named situated learning proposing that learning is social and comes largely from our experience of participating in daily life has emerged in the late 1980s and early 1990s. Situated cognition, or situated learning, has made a significant impact on education thinking since it was first proposed by Brown, Collin and Duguid (1989). They proposed that knowledge is situated, being in part a product of the activity, context, and culture in which it is developed and used. Work of some of the great educational thinkers (such as Vygotsky, Leontiev, and Dewey) has provided the research base for the theory (Herrington & Oliver, 1995). Resnick (1996) stated that situated learning is designed to bridge the gap between the theoretical learning in the formal instruction of the classroom and the real-life application of the knowledge in the work environment. Lave and Wenger (1991) emphasized the critical aspect of the situated learning model as the notion of the apprentice observing the 'community of practice'. The ideas of Lave and Wenger excited the interests of many of the thinkers and researchers at the time. Brown, Collins and Duguid were the first ones to use the ideas to produce a proposal for a model of instruction that has implications for classroom practice (Herrington & Oliver, 1995). The model arose out of observation of successful learning situations by the researchers. They set out to find examples of learning in any context or culture which were effective, and to analyse the key features of such models. They found examples of traditional school subjects, such as mathematics, reading, and writing, which were being taught in innovative and effective ways, and other areas of instruction such as snow skiing, where learning time had diminished from two years to two weeks as a result of instruction. An analysis of common features found in all the successful models was a set of six critical factors: apprenticeship, collaboration, reflection, coaching, multiple practice and articulation (McLellan, 1996).

In proposing their model of situated cognition, Brown, Collins and Duguid (as cited in Herrington & Oliver, 1995) argued that meaningful learning will only take place if it is integrated in the social and physical context within which it will be used. Formal learning is often quite distinct from authentic activity, or 'the ordinary practices' of the culture. Many of the activities undertaken by students are unrelated to the kind performed by practitioners in their everyday work (Herrington & Oliver, 1995). In order to achieve authenticity, they proposed the model of cognitive apprenticeship, a method designed to let students participate in authentic practices through activity and social interaction, and based on the successful and traditional apprenticeship model.

Lave and Wenger (1991) stated that social interaction is a critical component of situated learning, learners become involved in a community of practice which embodies certain beliefs and behaviors to be acquired. They also proposed that participation in a culture of practice can be observation from the boundary or 'legitimate peripheral participation'. As learning and involvement in the culture increase, the participant moves from the role of observer to fully functioning agent. Legitimate peripheral participation enables the learner to progressively piece together the culture of the group and what it means to be a member. Lave and Wenger (1991) emphasize the importance of legitimate peripheral participation:

> To be able to participate in a legitimately peripheral way entails that newcomers have broad access to arenas of mature practice (p.110).

Many of these authors emphasizing situated learning theory believe that useable knowledge is best gained in learning environments. They emphasize the importance of context and suggest learning is social and comes largely from our experience in daily life. Their model of situated learning proposed that learning involved a process of engagement in daily life. They argued that learning is a function of the activity, context and culture in which it occurs. This contrasts with most classroom learning activities which involve knowledge which is abstract and out of context.

According to the synthesis of studies about situated learning theory, the importance of the following two principles have been recognized and the instructional sequences were designed by making use of these principles:

- Knowledge needs to be presented in an authentic context, i.e., settings and applications that would normally involve that knowledge.
- Learning requires social interaction and collaboration in a community.

2.5. Realistic Mathematics Education (RME)

Realistic Mathematic Education (RME) is one of the popular theories which has progressed significantly since 1971. In 1971, Freudenthal founded the IOWO (Instituut Ontwikkeling Wiskundeonderwijs, Institute for Development of Mathematics Education) at Utrecht University, that after his death was renamed Freudenthal Institute, and it is called Freudenthal Institute for Science and Mathematics Education. The model of Realistic Mathematics Education (RME) has been developed by the teams at the Freudenthal Institute at the University of Utrecht (Zulkardi, 2004). Realistic Mathematics Education, or RME, is the Dutch answer to the world wide need to reform the teaching and learning of mathematics. The results of the Third International Mathematics and Science Study (TIMMS) show that students in the Netherlands gained high achievements in mathematics education (Moffet, 2008).

Amerom (cited in Mosvold, 2005) explains Hans Freudenthal's perspective as follows:

> According to Hans Freudenthal, mathematics is a human activity and it was invented from the natural environment. Freudenthal focused on the usefulness of mathematics in school. If mathematics education is intended for the majority of students, its main objective should be developing a mathematical attitude towards problems in the learner's every-day life. This can be achieved when mathematics is taught as an activity, a human activity, instead of transmitting mathematics as a pre-determined system constructed by others (p.59).

The events at the environment are mathematized (Altun,2000). A child can succeed to use mathematical language if the convenient environment is provided. Altun says that teaching mathematics should be environment centered. He also notes that every mathematics subject can be taught beginning with a life problem, this situation makes learning meaningful and also increases the motivation for learning. Gravemelger (cited in Hadi *et al.,* 2002) says that teaching should begin with activities that the child has already been doing in his/her environment .

This model is based on the principle that students realize meaning in schoolwork when they connect information with their own experience and RME demands that teachers establish a link between children's own world and the world of mathematical ideas. From a psychological perspective, this relates to Lave's theory of situated learning, which argues that knowledge needs to be presented in an authentic context, unlike most classroom learning activities which involve knowledge that is abstract or out of context (cited in Zulkardi, 2004). In RME, learning mathematics means doing mathematics, of which solving every day life problems (contextual problems) is an essential part. Other key principles are that student should be given the opportunity to reinvent mathematical concepts, and that the teaching-learning process be highly interactive (Fauzan, Slettenhaar, Plomp, 2002)

Streefland (as cited in Zulkardi, 2004) identified five educational principles of RME as follows:

1. The source of concept is the reality.
2. Pupils are given the opportunity to be constructors and actively contribute to the learning process by this involvement.

3. The learning process must be interactive so that in the course of constructing knowledge from real life situations, pupils discuss and collaborate when necessary.

4. Different lines of learning are entwined (e.g. fractions, ratio and proportion) so that both horizontal and vertical "mathematization" can take place.

5. The various tools used in the process of coming to understand mathematics, symbols, diagrams and visual models, should result from the need (p.9).

Van den Heuvel-Panhuizen (as cited in Moffet, 2008) expanded on these and outlined the key aspects of RME:

> It is realistic. The Dutch translation of 'to imagine' is 'zich REALISEren.' So the term 'realistic' refers to situations which can be imagined. In the RME approach, a problem presented to pupils can be based on a context from the real world but this is not always necessary; the fantasy world of fairy tales and even the formal world of mathematics can offer other suitable contexts, provided they are real in the pupil's mind (p.5).

The RME problems, set in real world contexts, involves "mathematization". Freudenthal (1991) stated that horizontal mathematization involves going from the world of life into the world of symbols, while vertical mathematization means moving within the world of symbols. But he adds that the difference between these two types is not always clear cut. Treffer (cited in Zulkardi, 2004) also defined horizontal and vertical mathematization. He says that horizontal mathematics is the students' discovery of mathematical tools that can help to organize and solve a problem located in a real-life situation. The following activities are examples of horizontal mathematics,

- identifying or describing the specific mathematics in a general context,
- schematizing, formulating and visualizing a problem in different ways,
- discovering relations,
- transfering a real world problem to a mathematical problem.

On the other hand, vertical mathematization is the process of reorganization within the mathematical system itself and refers to a greater use of abstract strategies. The following activities are examples of vertical mathematics,

- representing a relation in a formula,
- providing regularities,
- refining and adjusting models,
- using different models,
- formulating a mathematical model,

- generalizing.

RME and constructivism have some compatible characteristics. For instance, De Lange (as cited in Zulkardi, 2004) says that RME encourages "guided reinvention". Reinvention also exists in constructivism. Smith (as cited in Mosvold, 2005) explains the relationship between constructivism and reinvention:

> The constructivist stance is that mathematical understanding is not something that can be explained to children, nor is it a property of objects or other aspects of the physical world. Instead, children must "reinvent" mathematics, in situations analogous to those in which relevant aspects of mathematics were invented or discovered in the first place. They must construct mathematics for themselves, using the same mental tools and attitudes they employ to construct understanding of the language they hear around them (p.57).

De Lange (Zulkardi, 2004) emphasizes that RME stresses understanding processes, rather than learning algorithms. Students discover the mathematics for themselves, and so multiple solutions are encouraged and valued. Zulkardi (2004) says that RME requires a highly 'constructivist' approach to teaching, in which children are no longer seen as receivers of knowledge but the makers of it, and the role of the teacher is that of a facilitator. De Lange emphasizes the relationship between RME and constructivism; "RME is only applied to mathematics education while constructivism is used in many subjects" (as cited in Zulkardi, 2004, p.8).

As are the case to constructivism, the following conceptions are relevant to RME (Sutarto, 2002):

- Each learner brings his or her preconceptions to the educational experience. These preconceptions are highly influential on subsequent learning.
- Learners possess a diverse set of alternative conceptions about mathematical ideas that influence their future learning.
- Each learner actively constructs meaning. Learners acquire new knowledge by constructing it for themselves.
- Each learner is ready to share his or her personal meaning with others, and based on this negotiation process, reconceptualizes the initial knowledge structures. The construction of knowledge is a process of change that includes addition, creation, modification, refinement, restructuring, and rejection.
- Each learner takes responsibility for his or her learning. The new knowledge learners construct for themselves has its origin in a diverse set of experiences.
- Each learner is convinced that success in learning with understanding is possible. In other words, all students regardless of race, culture, and gender are capable of understanding and doing mathematics (p.36).

Gravenmeijer (cited in Zulkardi,2004) explains the role of the teacher in the classroom as a facilitator, an organizer, a guide, and an evaluator. The role of the teacher on the steps of the teaching-learning process based on the RME are:

- Giving the students a contextual problem that relate to the topic as the starting point.
- During interaction activity, giving the students a clue, guide the students individually or in a small group in case they need help.
- Stimulating the students to compare their solutions in a class discussion. The discussion refers to the interpretation of the situation sketched in the contextual problem and also focus on the adequacy and efficiency of various solution procedures.
- Letting the students find their own solution. It means the students are free to make discoveries at their own level, to build on their own experiental knowledge, and perform shortcuts at their own pace.
- Giving another problem in the same context (p.6).

According to the synthesis of studies reviewed in literature, and implications of Situated Learning Theory and Realistic Mathematics Education, two guiding principles emerged to design instructional sequences for this study:

- using contextual problems to make instruction relevant to real life,
- using student centered episodes for active construction of meaning via discussion, social interaction, collaboration, etc.

In light of the above studies, three instructional sequences were developed by the researcher. Two instructional sequences included these two principles above in varying degrees. Instructions were specified as ‘traditional’, ‘instruction enriched with only contextual problems’ and ‘instruction enriched with combination of contextual problems and student centered teaching episodes’. The effects of integrating contextual problems and student centered episodes into the instruction on students’ trigonometry achivement, and views and perceptions on trigonometry were examined in this study.

3. SIGNIFICANCE OF THE STUDY

According to National Council of Teachers of Mathematics (2000), "...school mathematics experiences at all levels should include opportunities to learn about mathematics by working on problems arising in contexts outside of mathematics. These connections can be other subject areas and disciplines as well as to students' daily lives..." (p.65)

There are so many studies conducted emphasizing the importance of making real life connections in mathematics lessons in the literature. Most of them offer a portrayal about the use of real world connections in mathematics lessons. However, there are not many studies offering an instructional sequence including contextual problems. As Delice and Monaghan (2005) states, there are not many research studies on teaching and learning of trigonometry. Instructional sequences including contextual problems were prepared after a very intense period of literature review. So, instructional sequence can be used by all the researchers even for different kinds of studies and by educators for educational purposes.

This study aimed to investigate the stand alone effect of using contextual problems and combined effect of using contextual problems and student centered teaching episodes in teaching one of the subjects of mathematics, trigonometry. It is expected that the results which were obtained from this study will have important implications for the educators.

4.STATEMENT OF THE PROBLEM

The present study questions the effects of using different instructional sequences which are developed by taking into account the main principles of Realistic Mathematics Education and Situated Learning in different forms on students' achievement in trigonometry, and their views and perceptions about trigonometry.

4.1. Variables

The independent variable of the research is type of instruction. Dependent variables of the study are the trigonometry achievement of students, and their views and perceptions about trigonometry. Trigonometry achievement of students was assessed via the Trigonometry Achievement Posttest developed by the researcher. Views and perceptions about trigonometry were measured with a set of questions prepared by the researcher. Detailed descriptions about measurement tools will be given in 'Instruments' section.

4.2. Research Questions

The research addresses the following questions about the immediate (just after the instruction) and longterm (sometime after instruction) effects of contextual problems and student centered episodes on achievement, and views and perceptions of 10^{th} grade students in the subject of trigonometry.

- *Research Question 1*: Is there any significant difference between trigonometry achievement scores of students in Groups A, B, and C as measured by Trigonometry Achievement Posttest which was administered immediately after the instructional sequences?

i) Is there any significant difference between trigonometry achievement scores of students who received traditional instruction (Group A) and those of who received an instruction enriched with only contextual problems (Group B) as measured by Trigonometry Achievement Posttest which was administered immediately after the instructional sequences?

ii) Is there any significant difference between trigonometry achievement scores of students who received traditional instruction (Group A) and those of who received an instruction enriched with combination of contextual problems and student centered episodes (Group C) as measured by Trigonometry Achievement Posttest which was administered immediately after the instructional sequences?

iii) Is there any significant difference between trignometry achievement scores of students who received an instruction enriched with only contextual problems (Group B) and those of who received an instruction enriched with combination of contextual problems and student centered episodes (Group C) as measured by Trigonometry Achievement Posttest which was administered immediately after the instructional sequences?

- *Research Question 2*: Is there any significant difference between trigonometry achievement scores of students in Groups A, B, and C as measured by Trigonometry Achievement Retention Test which was administered after 45 days from the instructional sequences?

i) Is there any significant difference between trigonometry achievement scores of students who received traditional instruction (Group A) and those of who received an instruction enriched with only contextual problems (Group B) as measured by Trigonometry Achievement Retention Test which was administered after 45 days from the instructional sequences?

ii) Is there any significant difference between trigonometry achievement scores of students who received traditional instruction (Group A) and those of who received an instruction enriched with combination of contextual problems and student centered episodes (Group C) as measured by Trigonometry Achievement Retention Test which was administered after 45 days from the instructional sequences?

iii) Is there any significant difference between trigonometry achievement scores of students who received an instruction enriched with only contextual problems (Group B) and those of who received an instruction enriched with combination of contextual problems and student centered episodes (Group C) as measured by Trigonometry Achievement Retention Test which was administered after 45 days from the instructional sequences?

- *Research Question 3:* Is there any change in terms of students' positive views and perceptions about trigonometry after instructional sequences in each group?

4.3. Statement of the Research Hypotheses

This study hypothesizes that there will be short term and long term differences in terms of trigonometry achievement, and views and perceptions among the groups. More specifically, it is hypothesized that:

1) There will be significant difference between trigonometry achievement scores of students in Groups A, B, and C as measured by Trigonometry Achievement Posttest which was administered immediately after the instructional sequences.

i) The group who received instruction enriched with only contextual problems (Group B) will perform significantly higher than the group who received traditional instruction (Group A) as measured by Trigonometry Achievement Posttest which was administered immediately after the instructional sequences.

ii) The group who received instruction enriched with combination of contextual problems and student centered episodes (Group C) will perform significantly higher than the group who received traditional instruction (Group A) as measured by

Trigonometry Achievement Posttest which was administered immediately after the instructional sequences.

iii) The group who received instruction enriched with combination of contextual problems and student centered teaching episodes (Group C) will perform significantly higher than the group who received instruction enriched with only contextual problems (Group B) as measured by Trigonometry Achievement Posttest which was administered immediately after the instructional sequences.

2) There will be significant difference between trigonometry achievement scores of students in Groups A, B, and C as measured by Trigonometry Achievement Retention Test which was administered after 45 days from the instructional sequences.

i) The group who received instruction enriched with only contextual problems (Group B) will perform significantly higher than the group who received traditional instruction (Group A) as measured by Trigonometry Achievement Retention Test which was administered after 45 days from the instructional sequences.

ii) The group who received instruction enriched with combination of contextual problems and student centered episodes (Group C) will perform significantly higher than the group who received traditional instruction (Group A) as measured by Trigonometry Achievement Retention Test which was administered after 45 days from the instructional sequences.

iii) The group who received instruction enriched with combination of contextual problems and student centered teaching episodes (Group C) will perform significantly higher than the group who received instruction enriched with only contextual problems (Group B) as measured by Trigonometry Achievement Retention Test which was administered after 45 days from the instructional sequences.

3) There will be more increase in terms of students' positive views and perceptions about trigonometry after instructional sequence in the group who received

instruction enriched with combination of contextual problems and student centered teaching episodes (Group C) compared to other groups.

5. METHOD

The present study compares the immediate and long-term effectiveness of the three types of instruction (specified as traditional, instruction enriched with only contextual problems and instruction enriched with combination of contextual problems and student centered teaching episodes) with regard to the extent of change in students' achievement in trigonometry, and their views and perceptions about trigonometry. Students in each group engaged in the lessons with different sequence of experiences. The instructional practices for the Group B and Group C differed from the instructional practices for the Group A in terms of using contextual problems. The instructional practices for the Group C differed from the instructional practices for the Group B in terms of using student centered teaching episodes. This study used a true experimental design with a pretest, a posttest and a retention test. The beginning of the trigonometry unit was chosen as the subject of instruction because it was more convenient to create contextual problems for this part of the unit. Besides achievement pre and posttests, the researcher asked five questions before the instructional sequences and six questions after the instructional sequences and wanted the students to write their anwers in order to understand their views and perceptions about trigonometry. Same six questions were asked verbally to six students from each group after one week from the posttests and these interviews were recorded by a tape recorder. The achievement posttest was aplied one more time to all groups after 45 days from the posttest as "retention test" to investigate long term effects of instructional sequences on students' achievement.

5.1. Subjects

Subjects in this study consisted of 74 tenth grade students from an Anatolian High School, in İstanbul. There are 5 tenth grade classes in this school. The students were assigned randomly to the classrooms by the school administration. Three of the classes consisted of students whose specialized area is Science & Mathematics. The study was conducted in Science & Mathematics classes because the students were assumed to have similar mathematics background. Since the sample size of each class was lower than 30, a nonparametric alternative of one-way ANOVA Test (Kruscal Wallis Test) was carried out to compare the average of the first, second and third examination scores for the previous semester for these three Science & Mathematics classes to determine whether there is a statistically significant difference between groups' mathematics background. It was found out that there is no statistically significant difference between the averages of the examination scores for the previous semester ($\chi^2(2)=0.772$, $p=0.680$) for these three classes. So, it is assumed that these three classes are similar in terms of their mathematics background (see in part 6.1). Kruscal Wallis Test was also carried out to compare Trigonometry Achievement Pretest scores of these three classes to determine whether there is a statistically significant difference between groups' trigonometry background. It was found out that there is no statistically significant difference ($\chi^2(2)=1.674$, $p=0.433$) between the groups' trigonometry background (see in part 6.1) also.

The number of the students in each classroom is 27, however there were 3 students who were absent either during the application of pretest or posttest in Group A, 1 student who was absent in Group B and 3 students who were absent in Group C. So, the number of students decreased to 24 in Group A, 26 in Group B and 24 in Group C. There were 12 female and 12 male students in Group A,12 female and 14 male students in Group B, 9 female and 15 male students in Group C. All the students (n=24) in one classroom (Group A) received traditional instruction, all the students (n=26) in the other classroom (Group B) received instruction enriched with only contextual problems and all the students (n=24) in the third classroom (Group C) received instruction enriched with combination of contextual problems and student centered teaching episodes. The type of instructional sequence to be applied was assigned randomly to each class.

5.2. Instructional Sequences

The instructional sequences were prepared and applied to three classes by the researcher. The researcher is also the teacher in the school but not the mathematics teacher of these three classes. However, the researcher taught during the instructions in three classes by obtaining permission of the mathematics teachers of the classes and school administration.

Traditional instruction is the typical learning environment for most of the students. In these environments, students may focus more on solving math problems without developing a deeper understanding (Martin, Rivale & Diller, 2007). Teachers generally assign problems and then solve them. There is very limited guidance, social interaction, collaboration and discussion in traditional instruction as described above. First group (Group A) received traditional instruction. Teacher presented the subject without making it relevant to real life and using contextual problems. The problems did not consist of daily life issues. Teacher showed the problems from the powerpoint slides and solved them herself on the board. The students were not encouraged to share their ideas during the problem solving process. There was almost no guidance, interaction and discussion during lessons. Instruction took three lesson hours. Each lesson hour was forty five minutes.

In the instruction of the second group (Group B), teacher presented the subject by giving emphasis to daily life connections of the subject of trigonometry, asked contextual problems related to daily life issues and then solved them herself on the board. The only difference of this instruction from the traditional instruction was the content and characteristics of the problems asked. Instruction took three lesson hours. Each lesson hour was forty five minutes.

In the instruction of the third group (Group C), teacher presented the subject by giving emphasis to daily life connections of the subject of trigonometry, asked contextual problems related to daily life issues and let the students solve the problems by themselves by using discussion, cooperation and interaction. Students participated in the activities and problem solving process during the instruction. Students actively constructed meaning in this process. Teacher gave the students clues and guided them individually or in small groups in case they need help. Teacher did not solve the problems directly, she let the students find the answer on their own or by help of a peer. Students were free to make conclusions, encouraged to construct their own knowledge. Instruction took four lesson hours (an extra hour more than others) because of the time needed for discussions in the class. The difference of this instruction from the instruction given to Group B was the structure of the instruction depending on student centered episodes which RME and Situated Learning perspective supports. The instruction was based on two main principles synthesized from the sayings of RME and Situated Learning theories which might be summarized as:

- using contextual problems to make instruction relevant to real life,
- using student centered episodes for active construction of meaning via discussion, social interaction, group work, etc.

Three different groups took three different instructional sequences. All of the problems asked during the instructions were shown to students by the researcher by using Powerpoint slides. Problems used in these three instructional sequences were developed by the researcher by using many resources (e.g. Akpınar, 2000; Altuntaş, 2009; Aydın, Asma, 2001 Brown, 1994; Çavdar, 2003; Derman, 2008; Gündüz, 2004; Kaplan, 2009; Merril, 1983; Öztürk, 2002). Contents of the instructional sequences in each group included:

- Type of angles (degree, radian and grad),
- Trigonometric ratios in a right triangle (sine, cosine, tangent, cotangent),
- Using trigonometric table,
- Relations between trigonometric ratios.

Beginning of the trigonometry unit was chosen as the subject of instruction because it was more convenient to create contextual problems for this part of the unit.

Objectives are already determined by the Turkish Ministry of National Education for 10^{th} grade curriculum (Talim ve Terbiye Kurulu Başkanlığı, 2009). Instructional sequences were based on the following objectives:

- To be able to explain the concepts of angle and arc.
- To be able to explain the types of angles and convert measures of angles.
- To be able to explain the trigonometric ratios in a right triangle (sine, cosine, tangent, cotangent).
- To be able to write the trigonometric ratios of special angles.
- To be able to find the trigonometric ratio of an angle by the help of trigonometric table.
- To be able to state the relations between trigonometic ratios.

Examples of the screens from the power-point slides used for each group can be seen in the following pages.

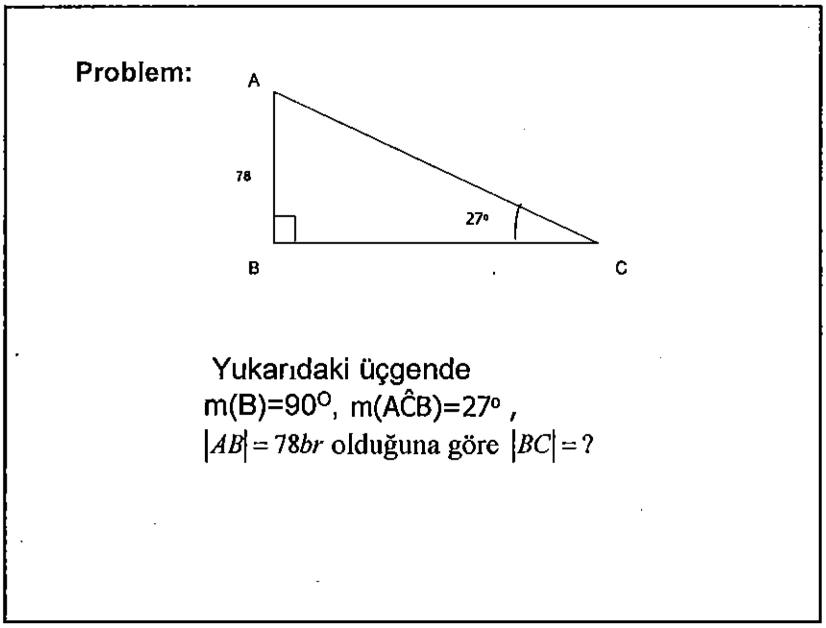


Figure 5.1. The screen representing the problem for Group A

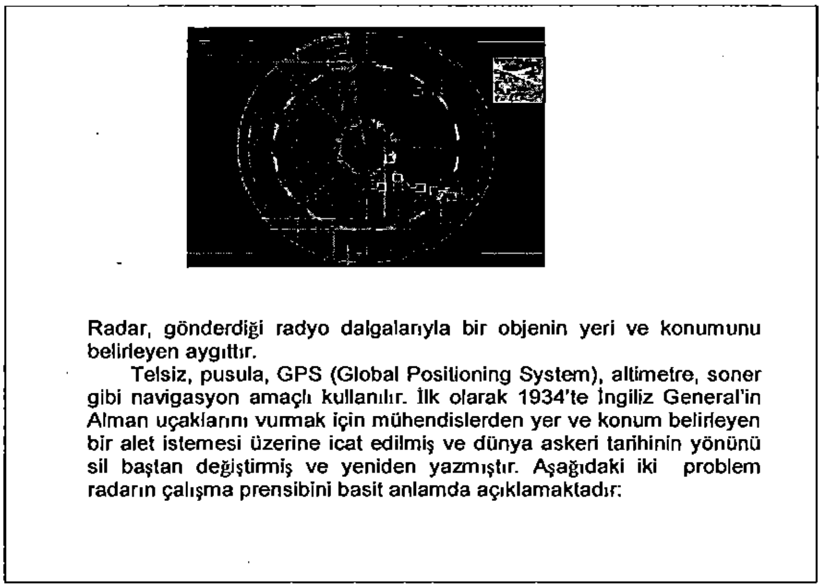


Figure 5.2. The additional screen for Group B and Group C.

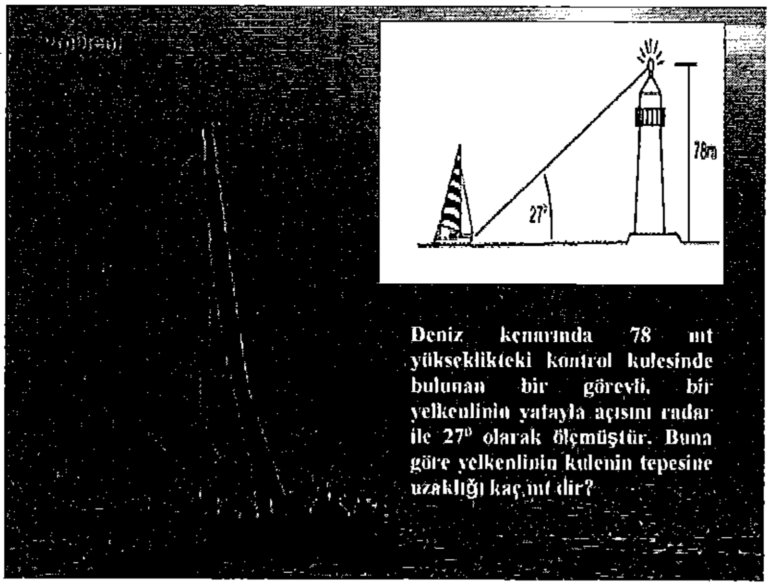


Figure 5.3. The screen representing the contextual problem for Group B and Group C

Figure 5.4. The additional screen for Group C.

As seen in the Figure 5.1, for Group A, the problem was not contextualized as it is for Group B and Group C (Figure 5.2, Figure 5.3, Figure 5.4). Teacher asked and solved the problem herself on the board for Group A. Students were not encouraged to share their ideas, discuss or participate in a group work.

In Group B, the same problem was contextualized. Information about radar systems, and navigation tools was given before the solution of the contextual problem. Teacher asked and solved the problem on the board. Students were not encouraged to say their ideas, discuss or collaborate. The only difference of this application from Group A was the content and characteristic of the problem.

In Group C, the problem was contextualized. Information about radar systems and navigation tools were given and discussed before the solution process of the contextual problem. Teacher did not solve the problem directly, she asked students' ideas about the solution process. She let the students solve the problem by themselves by using discussion, cooperation and interaction in small groups. Students participated in problem solving process during the instruction. She gave the students clues and guided them to use trigonometric table in case they need help.

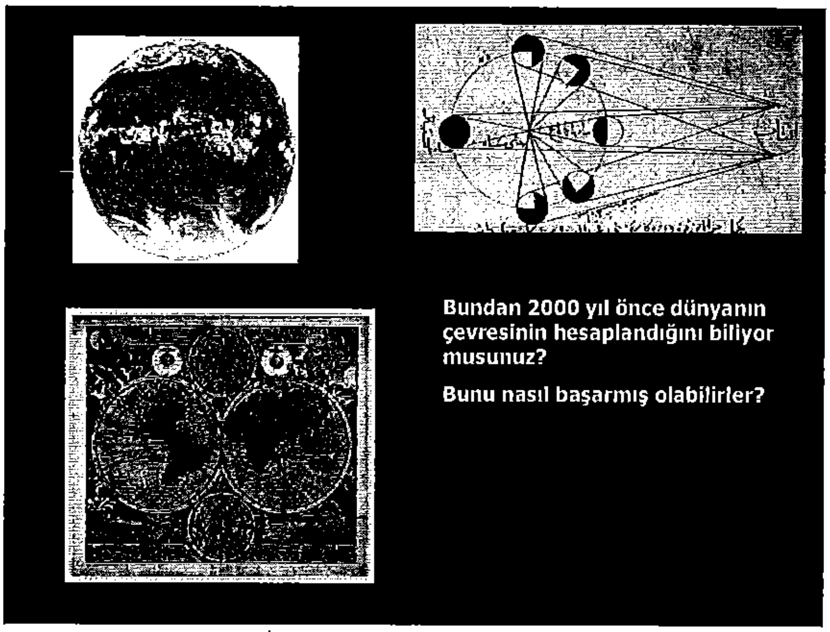


Figure 5.5. The screen representing the introduction of 'angle' in Group C.

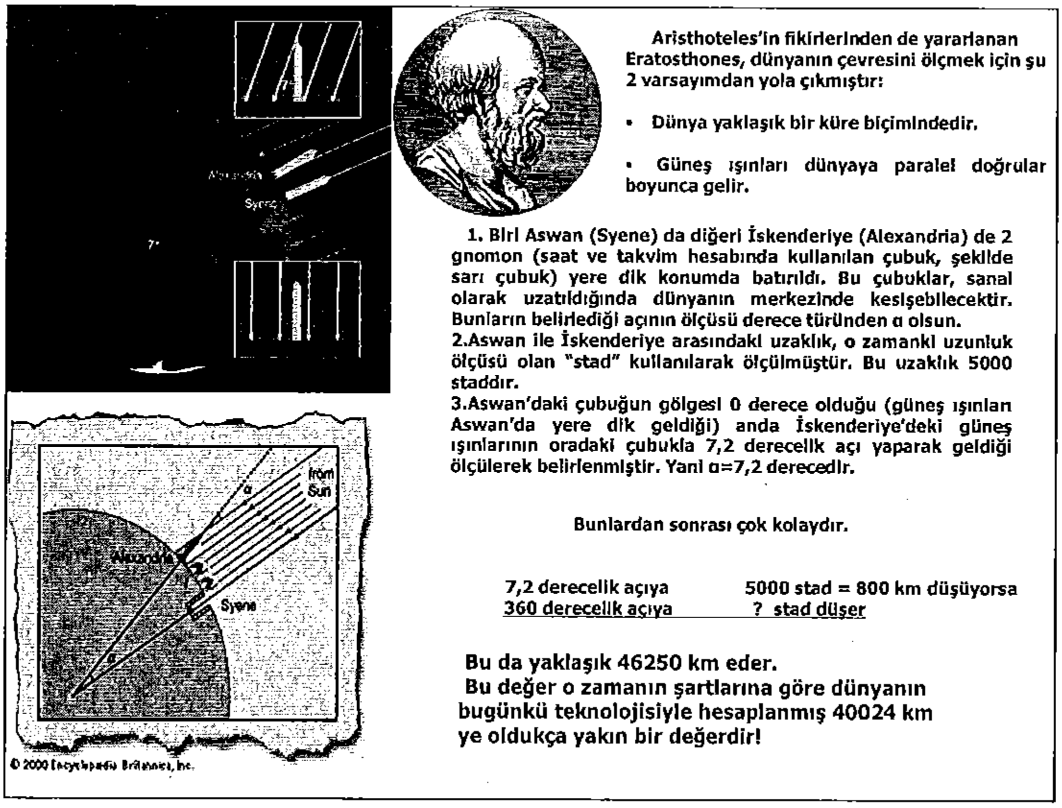


Figure 5.6. The screen representing the introduction of 'angle' in Group B and C.

The above information about the perimeter of the Earth was not given in Group A.

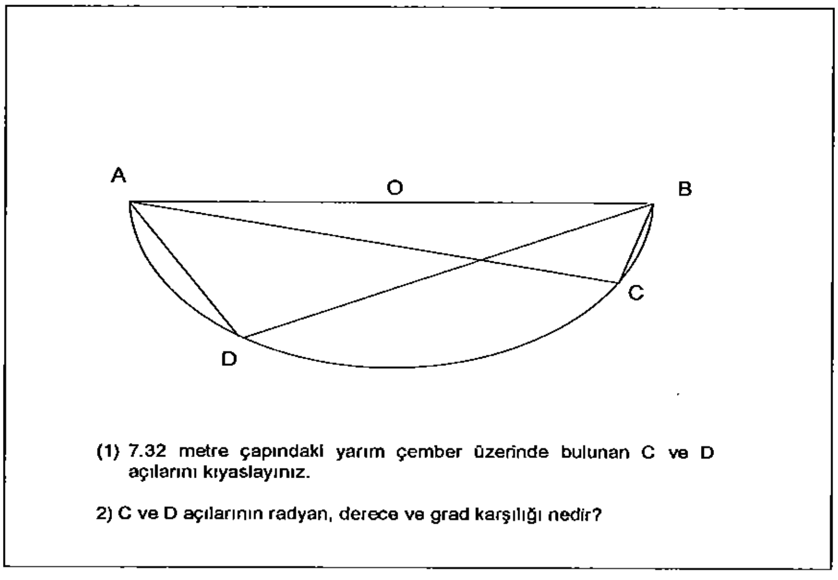


Figure 5.7. The screen representing the problem of 'angle' in Group A.

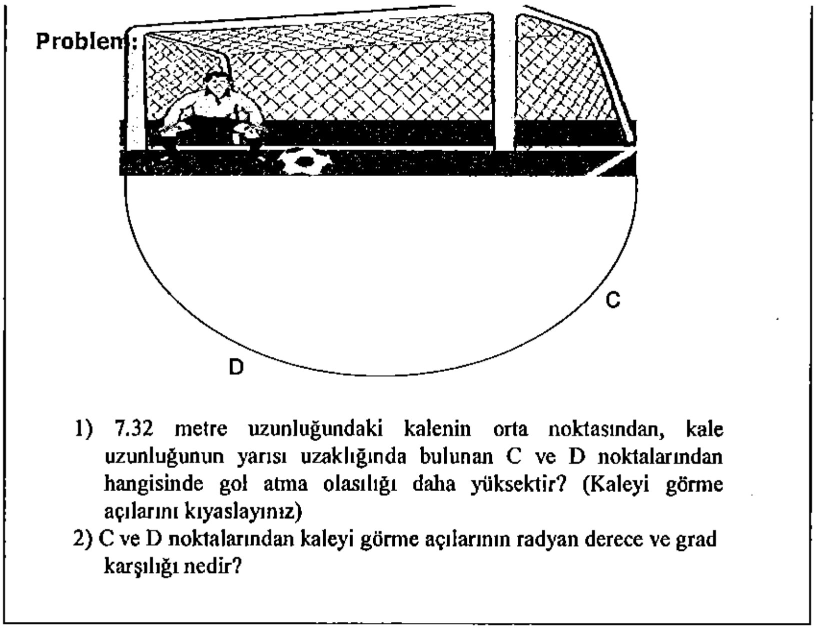


Figure 5.8. The screen representing the problem of 'angle' in Group B and Group C.

Worksheets, trigonometric table, protractor were used by the researcher in all groups during the instructions. However, compass was only used in Group B and Group C. The powerpoint slides and the flow of the instruction in three classes can be seen in Appendices E, F and G.

5.3. Instruments

5.3.1. Achievement Tests

Pre, post and retention trigonometry achievement tests were used to collect data on trigonometry achievement of students in this study. Tests were developed by the researcher and administered to three groups. Pre and post tests, each, included 10 parallel questions arranged to cover the objectives of subject determined by Ministry of Education (MEB). Four of the questions consisted of daily life connections and six of them did not. Questions of the pre and post tests were shown to the other five mathematics teachers in the school who teach 10th grade mathematics and two scholars from the university to check their content validity. It was important that the questions cover the objectives determined by MEB for 10th grade curriculum (Talim ve Terbiye Kurulu Başkanlığı, 2009) on the subject of this study. Pretest was piloted with a 10th grade class which was not a class that will take the treatment in order to test it. It was noticed that students had difficulty in understanding Question 3, Question 4 and Question 5. So, the wording of questions were modified and some drawings were added to the questions in order to ensure understanding. It was also noticed that it woud be better to provide more free space in the test paper, so the number of paper was increased to three. Posttest was also modified according to the results from pilot study.

The final forms of pre and post tests (see in APPENDIX C and D) consisted of questions about:

- type of angles (Question 1 and Question 2),
- trigonometric ratios in a right triangle (Question 3, Question 4, Question 5 and Question 6),
- using trigonometric table (Question 7 and Question 8),
- relations between trigonometric ratios (Question 9 and Question 10).

Rubrics were developed to score objectively the pre and post tests. The researcher gave 3 points for the answer of the question if the student used trigonometry during the solution process and the result is right, 2 points if the student used trigometry during the solution process and the result is partially true, 1 point if the student did not use trigonometry during the solution process and the result is right, 0 points if the student did not use trigonometry during the solution process and the result is wrong. The researcher also gave 0 points if the solution space in the paper is blank. Maximum score to be taken from the tests was 30 and minimum score was 0.

The posttest was aplied one more time to all groups after 45 days from the instructional sequences as "retention test" and were scored by the researcher. To check reliability of the posttest, Cronbach's Alpha and test-retest reliability coefficients were calculated. The internal consistency coefficient (Cronbach's Alpha) of posttest was found as 0.707 and test-retest correlation coefficient was found as 0.649. Another mathematics teacher also evaluated 12 posttest papers of Group A, 13 posttest papers of Group B and 12 posttest papers of Group C, totally 37 papers by using the rubric developed by the researcher, to check inter-scorer reliability. Posttest papers were selected randomly from each group's posttest papers. Inter-scorer reliability coefficient was found as 0.994.

5.3.2. Views and Perceptions about Trigonometry Questionnaires

The researcher asked five questions before instructional sequences and six questions after instructional sequences and wanted the students to write their anwers in order to understand their views and perceptions about trigonometry (See in Appendices A and B). The sixth question asked after the instructional sequences aimed to understand their perceptions about instructional sequences. Same six questions were asked by the researcher verbally to six students from each group, totally 18 students, after two weeks from the posttests and these interviews were recorded by a tape recorder. Six students in each group were chosen according to their Trigonometry Achievement Posttest scores. Two students were the students who took the maximum scores, the other two of them were the students who took minimum scores and the remaining two were the ones who took average scores. To conclude, 18 students (6 from Group A, 6 from Group B, 6 from Group C) were interviewed after the post test.

5.4. Design

The study aims to determine the relative effectiveness of three types of instruction with regard to the extent of change in students' achievement in trigonometry, and their views and perceptions about trigonometry. In this study, the students were already randomly assigned to the classrooms by the school administration. The type of instructional sequence to be applied was assigned randomly to three groups. To qualify as a true experimental design, at least random assignment must be involved (Gay, 1996). The pretest-posttest control group design involves at least two groups, both of which are formed by random assignment; both groups are administered a pretest of the dependent variable, one group receives a new, or unusual, treatment, and both groups are posttested. Posttest scores are compared to determine the effectiveness of the treatment. The pretest-posttest control group design may also be expanded to include any number of treatment groups (Gay, 1996). Therefore the design of this study is "true experimental design" with a pre and post test control group design with three groups. Table below summarizes the design of the study.

Table 5.1. Design of the Study

Group	PRE-MEASUREMENTS	INTERVENTION	POST-MEASUREMENTS	RETENTION MEASUREMENT
Group A (n=24)	Trigonometry Achievement Pretest + Views and Perceptions Prequestionnaire About Trigonometry (5 Questions)	Traditional Instruction	Trigonometry Achievement Posttest + Views and Perceptions Postquestionnaire About Trigonometry (6 Questions)	Trigonometry Achievement Retention Test
Group B (n=26)	Trigonometry Achievement Pretest + Views and Perceptions Prequestionnaire About Trigonometry (5 Questions)	Instruction Enriched with Only Contextual Problems	Trigonometry Achievement Posttest + Views and Perceptions Postquestionnaire About Trigonometry (6 Questions)	Trigonometry Achievement Retention Test
Group C (n=24)	Trigonometry Achievement Pretest + Views and Perceptions Prequestionnaire About Trigonometry (5 Questions)	Instruction Enriched with Combination of Contextual Problems and Student Centered Teaching Episodes	Trigonometry Achievement Posttest + Views and Perceptions Postquestionnaire About Trigonometry (6 Questions)	Trigonometry Achievement Retention Test

5.5. Procedure

Pre and post achievement tests were developed by the researcher to cover the main concepts of the trigonometry at hand. Principles of Realistic Mathematics Education and Situated Learning theories were taken into consideration during the development of lesson plans of instructional sequences for Group B and Group C. Both of the instructional sequences included contextualized problems. However, instructional sequence for Group C depended on student centered teaching episodes. Lesson plan of instructional sequence for Group A was prepared taken into consideration the aspects of traditional approach and did not include contextualized problems. Pretest was piloted with a 10^{th} grade class which was not a class that will take the treatment in order to test it. Then, some modifications were done in the pretest and posttest. All groups were administered 5 questions reflecting their views and perceptions about trigonometry, and Trigonometry Achievement Pretest including 10 questions. Pretest covered the main concepts about trigonometry. The completion of Trigonometry Achievement Pretest took 30 minutes, and Views and Perceptions about Trigonometry Prequestionnaire took 15 minutes. After one day from the pretests, groups received different instructions during three days. The type of instructional sequence to be applied was assigned randomly to three classes. 10-B class was assigned as Group A, 10-C class was assigned as Group B and 10-A class was assigned as Group C randomly. Group A received traditional instruction, Group B received instruction enriched with only contextual problems and Group C received instruction enriched with combination of contextual problems and student centered teaching episodes. Group A and B received three lessons, Group C received four lessons. Finally, all groups were administered 6 questions reflecting their views and perceptions about trigonometry and instructions and Trigonometry Achievement Posttest parallel to the pretest including 10 questions. Pretests were applied in one day (on monday), interventions were applied in three days (on tuesday, wednesday and thursday) and posttests were applied in one day (on friday) of the last week of April 2010 in all classes. After two weeks from the posttest, six questions (same with the ones in Views and Perceptions About Trigonometry Postquestionnaire) were asked by the researcher verbally to six students from each group and these interviews were recorded by tape recorder. 74 posttest papers were evaluated by the researcher with a rubric developed by the researcher. 37 posttest papers (12 from Group A, 13 from Group B and 12 from Group C) were also evaluated by another mathematics teacher. The posttest was re-administered as a retention test to all groups after 45 days from the posttest.

Prior to treatments, the three groups were compared in terms of students' previous semester mathematics exam scores' average (three exams taken), and their Trigonometry Achievement Pretest scores by using the nonparametric Kruscal Wallis Tests. Kruscal Wallis Test was also used to compare the Trigonometry Achievement Posttest scores for three groups in order to test the first hypothesis.

The nonparametric Kruscal Wallis Test was also carried out in order to see whether there is significant difference in terms of retention test results between the groups for testing the second hypothesis. As a further analysis, the nonparametric Related Samples Wilcoxon Signed Ranks Test was carried out to compare retention test scores and posttest scores for each group seperately.

In order to test the third hypothesis, analysis done on five questions asked before intervention and six questions asked after interventions were also reported. Questions had been prepared to reflect students' views and perceptions about trigonometry.

6. RESULTS

6.1. Data Analysis Prior to Instruction

Prior to treatments, the three groups were compared in terms of students' previous semester mathematics exam scores' average (three exams taken), and their Trigonometry Achievement Pretest scores. Throughout these analyses, the significance level was kept 0.05.

Students' previous semester math exam scores' average were compared using Kruscal Wallis Test which is the nonparametric alternative of one-way analysis of variance. ANOVA is a statistical data analysis technique that is used when the independent variable groups are more than two (Büyüköztürk, 2003). In ANOVA, we assume that distribution of each group should be normally distributed. In Kruskal-Wallis Test, we do not assume any assumption about the distribution. Kruscal Wallis Test is used to compare three or more independent groups of sampled data (Büyüköztürk, 2003; Kalaycı,2008). This test was preferred in data analyses, because the sample size for each group was lower than 30 and main assumptions of ANOVA were not satisfied. Kruscal Wallis Test makes no assumptions about the distribution of the data such as normality or equality of variance. This test uses the ranks of the data rather than their raw values to calculate the statistics. Tables show the test results.

Table 6.1. Average of the 1st, 2nd and 3rd Mathematics Examination Scores of the Previous Semester for Group A, B and C

Report			
Average of the 1., 2. and 3. Mathematics Examination Scores of the Previous Semester			
Group of the Student	Mean	N	Std. Deviation
A	47.90	24	18.14
B	43.86	26	16.97
C	45.54	24	15.72
Total	45.72	74	16.82

Table 6.2. Mean ranks of the 1st, 2nd and 3rd Mathematics Examination Scores of the Previous Semester for Group A, B and C

Ranks			
	Group of the Student	N	Mean Rank
Average of the 1., 2. and 3. Examination Scores of the Previous Semester	A	24	40.48
	B	26	35.19
	C	24	37.02
	Total	74	

Table 6.3. Results of Kruscal Wallis Test in terms of mean ranks of the 1st, 2nd and 3rd Mathematics Examination Scores of the Previous Semester

Test Statistics[a,b]	
	Mean of the 1., 2. and 3. Examination Scores of the Previous Semester
Chi-Square	0.772
df	2
Asymp. Sig.	0.680

a. Kruscal Wallis Test
b. Grouping Variable: Group of the Student

As seen from the table, there is no significant difference among the three groups in terms of their previous semester mathematics exam scores' average ($\chi^2(2)=0.772$, $p=0.680$). This means that these three classes are similar in terms of their mathematics background.

Kruscal Wallis Test was also carried out to compare Trigonometry Achievement Pretest scores of these three classes to determine whether there is a statistically significant difference between groups' trigonometry background. Tables below show the result of the test.

Table 6.4. Mean ranks of the total scores of Trigonometry Achievement Pretest for Group A, B and C

Ranks			
	Group of the Student	N	Mean Rank
Total Score of the Pretest	A	24	42.15
	B	26	35.31
	C	24	35.23
	Total	74	

Table 6.5. Results of Kruscal Wallis Test in terms of pretest scores between the groups

Test Statistics[a,b]	
	Total Score of the Pretest
Chi-Square	1.674
df	2
Asymp. Sig.	0.433
a. Kruscal Wallis Test	
b. Grouping Variable: Group of the Student	

It was found out that there is no statistically significant difference between the groups in terms of trigonometry background ($\chi^2(2)=1.674$, $p=0.433$).

6.2. Data Analysis Done on the Hypotheses

In this part, data analysis done on the hypotheses were reported. Throughout these analyses, the significance level was kept 0.05.

Table below shows descriptive statistics related to Trigonometry Achievement Pretest and Posttest scores of three groups.

Table 6.6. Descriptive statistics related to pretest and posttest scores of the three groups.

Groups	Trigonometry Achievement Test	N	Mean	Std. Deviation
Group A	Pretest	24	6.71	2.42
	Posttest	24	15.50	4.54
Group B	Pretest	26	6.27	4.26
	Posttest	26	20.19	4.40
Group C	Pretest	24	6.54	5.15
	Posttest	24	22.75	5.24

As can be seen in Table 6.6., there is an increase in means of posttest scores of students in all groups when compared to their pretest scores.

6.2.1. Hypothesis 1

- *Hypothesis 1:* There will be significant difference between trigonometry achievement scores of students in Group A, B, and C as measured by Trigonometry Achievement Posttest which was administered immediately after the instructional sequences.

i) The group who received instruction enriched with only contextual problems (Group B) will perform significantly higher than the group who received traditional instruction (Group A) as measured by Trigonometry Achievement Posttest which was administered immediately after the instructional sequences.

ii) The group who received instruction enriched with combination of contextual problems and student centered episodes (Group C) will perform significantly higher than the group who received traditional instruction (Group A) as measured by Trigonometry Achievement Posttest which was administered immediately after the instructonal sequences.

iii) The group who received instruction enriched with combination of contextual problems and student centered teaching episodes (Group C) will perform significantly higher than the group who received instruction enriched with only contextual problems (Group B) as measured by Trigonometry Achievement Posttest which was administered immediately after the instructional sequences.

In order to test the first hypothesis, the nonparametric Kruscal Wallis Test was carried out to compare the mean ranks of Trigonometry Achievement Posttest scores for three groups. With this test, mean ranks of students in terms of their posttest scores were compared. Kruscal Wallis was carried out because the assumptions of parametric one-way ANOVA were not satisfied. Tables below show the results of this analysis.

Table 6.7. Mean ranks of Trigonometry Achievement Posttest scores for Group A, B, and C

Ranks			
	Group of the Student	N	Mean Rank
Total Score of the Posttest	A	24	22.04
	B	26	39.35
	C	24	50.96
	Total	74	

Table 6.8. Results of Kruscal Wallis Test in terms of posttest scores between the groups.

Test Statistics[a,b]	
	Total Score of the Posttest
Chi-Square	22.073
df	2
Asymp. Sig.	0.000*

a. Kruscal Wallis Test
b. Grouping Variable: Group of the Student

It was found out that there is statistically significant difference between the mean ranks of students' Trigonometry Achievement Posttest scores in three groups (0.000<0.05). Pairwise comparisons were done in order to compare the groups in pair as it is stated in subhypotheses. Table below shows the pairwise comparisons of the groups.

Table 6.9. Pairwise comparisons for the posttest scores between groups

Groups	Test Statistics	Std. Error	Std. Test Statistics	Sig.	Adj. Sig.
B-A	17.304	6.076	2.848	0.004	0.013*
C-A	28.917	6.197	4.667	0.000	0.000*
C-B	11.612	6.076	1.911	0.056	0.168

According to the adjusted significance values (0.013<0.05 and 0.000<0.05), it was found that there is a significant difference between Group B and Group A, and Group C and Group A in terms of students' Trigonometry Achievement Posttest scores' mean ranks. However, there is no significant difference between Group B and Group C in terms of posttest scores' ranks (0.168>0.05).

The results indicated that two of the three subhypotheses were supported. That is to say the group who received instruction enriched with only contextual problems (Group B) performed significantly higher than the group received traditional instruction (Group A) in Trigonometry Achievement Posttest. Also, the group who received instruction enriched with combination of contextual problems and student centered episodes (Group C) performed significantly higher than the group who received traditional instruction (Group A) in Trigonometry Achievement Posttests, but there is no significant difference between the group who received instruction enriched with combination of contextual problems and student centered teaching episodes (Group C) and the group who received insruction enriched with only contextual problems (Group B) in terms of posttest scores of students as measured by Trigonometry Achievement Posttests. The results will be further discussed in the conclusion and discussion section (Section 7).

Furthermore, Kruscal Wallis Test was done on the ranks of the differences between Trigonometry Achievement Posttest scores and Trigonometry Achievement Pretest scores (defined as learning gain) of students among three groups. It was also found out that there is a statistically significant difference between the mean ranks of the differences between posttest scores and pretest scores of students' among three groups ($\chi^2(2)=26,060$, $p=0.000$). Pairwise comparisons were done in order to compare the groups in pair. The results indicated that the group who received instruction enriched with only contextual problems (Group B) showed more 'learning gain' than the group who received traditional instruction (Group A) (adj. sig. value 0.001<0.05). The group who received instruction enriched with combination of contextual problems and student centered episodes (Group C) showed more learning gain than the group who received traditional instruction (Group A) (adj. sig. value 0.000<0.05), but there is no significant difference between the groups' learning gains who received instruction enriched with combination of contextual problems and student centered teaching episodes (Group C) and the group who received instruction enriched with only contextual problems (Group B) (adj. sig. value 0.475>0.05). Results of the analysis done on the difference between Trigonometry Achievement Posttest and Trigonometry Achievement Pretest scores (defined as learning gain above) supported the results of the analysis done on the Trigonometry Achievement Posttest scores between the groups.

6.2.2. Hypothesis 2

- *Hypothesis 2:* There will be significant difference between trigonometry achievement scores of students in Groups A, B, and C as measured by Trigonometry Achievement Retention Test which was administered after 45 days from the instructional sequences.

i) The group who received instruction enriched with only contextual problems (Group B) will perform significantly higher than the group who received traditional instruction (Group A) as measured by Trigonometry Achievement Retention Test which was administered after 45 days from the instructional sequences.

ii) The group who received instruction enriched with combination of contextual problems and student centered teaching episodes (Group C) will perform significantly higher than the group who received traditional instruction (Group A) as measured by Trigonometry Achievement Retention Test which was administered after 45 days from the instructonal sequences.

iii) The group who received instruction enriched with combination of contextual problems and student centered teaching episodes (Group C) will perform significantly higher than the group who received instruction enriched with only contextual problems (Group B) as measured by Trigonometry Achievement Retention Test which was administered after 45 days from the instructional sequences.

Table below shows the means and standard deviations of Trigonometry Achievement Posttest and Trigonometry Achievement Retention Test scores of three groups.

Table 6.10. Descriptive statistics related to posttest and retention test scores of three groups

Groups	Test	N	Mean	Std. Deviation
Group A	Posttest Retention Test	24 24	15.50 14.25	4.54 4.68
Group B	Posttest Retention Test	26 26	20.19 19.50	4.40 4.53
Group C	Posttest Retention Test	24 24	22.75 18.50	5.24 5.31

There is a decrease in means of retention test scores of students when compared with their posttest scores. The nonparametric Kruscal Wallis Test was carried out in order to see whether there is significant difference in terms of retention test results between the groups. Tables below show the results of this analysis.

Table 6.11. Mean ranks of Trigonometry Achievement Retention Test scores for Group A, B, and C

Ranks			
	Group of the Student	N	Mean Rank
Rettotal	A	24	24.92
	B	26	45.65
	C	24	41.25
	Total	74	

Table 6.12. Results of Kruscal Wallis Test in terms of retention test scores between the groups

Test Statistics[a,b]	
	Rettotal
Chi-square	12.777
df	2
Asymp. Sig.	0.002*
a.Kruscal Wallis Test	
b. Grouping Variable: Group of the Student	

It was found out that there is statistically significant difference in terms of retention test scores between the groups ($\chi^2(2)=12.77$, $p=0.002$). Pairwise comparisons were done in order to compare the groups in pair. Table in the next page shows the pairwise comparisons of the groups.

Table 6.13. Pairwise comparisons for the retention test results between groups

Groups	Test Statistics	Std. Error	Std. Test Statistics	Sig.	Adj. Sig.
C-A	16.33	6.18	2.64	0.008	0.025*
B-A	20.73	6.06	3.41	0.001	0.002*
C-B	-4.40	6.06	-0.72	0.0468	1.000

According to the adjusted significance values, it was found out that there is significant difference between Group A and Group B, and Group A and Group C according to retention test scores. However, there is no significant difference between Group B and Group C according to retention test scores.

As a further analysis, Related Samples Wilcoxon Signed Ranks Test was carried out to compare retention test scores and posttest scores for each group seperately. Related Samples Wilcoxon Signed Ranks Test is a non-parametric statistical hypothesis test for the case of two related samples (Büyüköztürk, 2003; Kalaycı,2008). It can be used as a non-parametric alternative to the Related Samples t-test when the population cannot be assumed to be normally distributed. Like the Related Samples t-test, the Wilcoxon test involves comparisons of differences between measurements. In Related Samples Wilcoxon Signed Ranks Test, the null hypothesis is that the median of differences between observations is zero (Büyüköztürk, 2003). This is different from the null hypothesis of the t-test, which is that the mean of differences between observations is zero. The significance level was set as 0.05. Tables below indicates the results of this analysis for each group one by one.

Table 6.14. Related Samples Wilcoxon Signed Ranks Test for Group A

Null Hypothesis	Test	Significance	Decision
The median of differences between retention test scores and posttest scores equals 0.	Related Samples Wilcoxon Signed Ranks Test	0.153	Retain the null hypothesis

The above table shows that the null hypothesis which is the median of differences between retention test scores and posttest scores equals 0 could not be rejected for Group A.

Table 6.15. Ranks for Group A

		N	Mean Rank	Sum of Ranks
Posttest Scores-Retention Test Scores	Negative Ranks	9[a]	8.28	74.50
	Positive Ranks	12[b]	13.04	156.50
	Ties	3[c]		
	Total	24		

a. Posttest Score < Retention Test Score
b. Posttest Score > Retention Test Score
c. Posttest Score = Retention Test Score

Table 6.16. Test Statistics[b] for Group A

	Posttest Score- Retention Test Score
Z	-1.429[a]
Asymp. Sign. (2-tailed)	0.153

a. Based on negative ranks
b. Wilcoxon Signed Ranks Test

According to the significance values, it was found that there is no significant difference between retention test scores and posttest scores in Group A (Z=-1.429, p=0.153). This means that there is permanance of the trigonometry achievement for Group A.

Table 6.17. Related Samples Wilcoxon Signed Ranks Test for Group B

Null Hypothesis	Test	Significance	Decision
The median of differences between retention test scores and posttest scores equals 0.	Related Samples Wilcoxon Signed Ranks Test	0.548	Retain the null hypothesis

The above table shows that the null hypothesis which is the median of differences between retention test scores and posttest scores equals 0 could not be rejected for Group B.

Table 6.18. Ranks for Group B

		N	Mean Rank	Sum of Ranks
Posttest Scores- Retention Test Scores	Negative Ranks	8[a]	11.13	89.00
	Positive Ranks	12[b]	10.08	121.00
	Ties	6[c]		
	Total	26		

a. Posttest Score < Retention Test Score
b. Posttest Score > Retention Test Score
c. Posttest Score = Retention Test Score

Table 6.19. Test Statistics[b] for Group B

	Posttest Score- Retention Test Score
Z	-0.600[a]
Asymp. Sign. (2-tailed)	0.548

a. Based on negative ranks
b. Wilcoxon Signed Ranks Test

There is no significant difference between retention test scores and posttest scores in Group B (Z=-0.600, p=0.548). This means that there is permanance of the trigonometry achievement for Group B.

Table 6.20. Related Samples Wilcoxon Signed Ranks Test for Group C

Null Hypothesis	Test	Significance	Decision
The median of differences between retention test scores and posttest scores equals 0.	Related Samples Wilcoxon Signed Ranks Test	0.000*	Reject the null hypothesis

The above table shows that the null hypothesis which is the median of differences between retention test scores and posttest scores equals 0 was rejected for Group C.

Table 6.21. Ranks for Group C

		N	Mean Rank	Sum of Ranks
Posttest Scores-	Negative Ranks	1[a]	3.50	3.50
Retention Test Scores	Positive Ranks	18[b]	10.36	186.50
	Ties	5[c]		
	Total	24		

a. Posttest Score < Retention Test Score
b. Posttest Score > Retention Test Score
c. Posttest Score = Retention Test Score

Table 6.22. Test Statistics[b] for Group C

	Posttest Score- Retention Test Score
Z	-3.686[a]
Asymp. Sign. (2-tailed)	0.000*

a. Based on negative ranks
b. Wilcoxon Signed Ranks Test

There is a statistically significant difference between retention test scores and posttest scores in Group C ($Z=-3.686$, $p<0.001$). This means that there is no permanance of the trigonometry achievement in Group C.

To conclude, all these analysis show that there is permanance of the trigonometry achievement for Groups A and B, however there is no permanance of the trigonometry achievement in Group C. The results will be further discussed in the conclusion and discussion section (Section 7).

6.2.3. Hypothesis 3

- *Hypothesis 3:* There will be more increase in terms of students' positive views and perceptions about trigonometry after instructional sequence in the group who received instruction enriched with combination of contextual problems and student centered teaching episodes (Group C) compared to other groups.

Analysis done on five questions asked before intervention and six questions asked after interventions were reported in this part. Questions had been prepared to reflect students' views and perceptions about trigonometry.

Table 6.23. Percentage of the students' responses related to Question 1 and Question 2

	Percent of the students who gave mathematically sufficient answer in Group A (%)			Percent of the students who gave mathematically sufficient answer in Group B (%)			Percent of the students who gave mathematically sufficient answer in Group C (%)		
	Before intervention	After intervention	The direction of change	Before intervention	After inter vention	The direction of change	Before intervention	After intervention	The direction of change
1 a) What do you think when you hear the word "trigonometry"?	**67**	**80**	↑	**70**	**97**	↑	**63**	**92**	↑
1 b) Can you define trigonometry with your own words?	**21**	**63**	↑	**39**	**39**	=	**5**	**50**	↑
2) What are the daily life application areas of trigonometry? Can you give a few examples?	**21**	**42**	↑	**47**	**97**	↑	**30**	**100**	↑

When the percentages are considered as a whole there is an increase in the percentage of mathematically sufficient answers for all groups except Group B for the second part of the first question. The percent of the students who gave mathematically sufficient answer before and after intervention is same for Group B for the second part of the first question (39%). It is also obvious that Group B and Group C learned the daily life application areas of trigonometry (97% and 100%), however Group A's percentage is less (42%).

Mathematically sufficient answer for the first question includes words such as radian, grad, degree, sine, cosine, tangent, cotangent, ratios of the sides of a right triangle, relationship between angle and side, measurement by the help of triangle…etc.

For example a male student who took an average score from the posttest (student # 325) from Group C gave the mathematically sufficient answer to the first question. He also explained his answer in the interview: “Mathematics, sine, cosine, tanget, cotangent, ratios. Trigonometry is to be able to find the unknown distance by the help of trigonometric ratios”.

Mathematically sufficient answer for the second question includes words such as engineering, physics, construction sites, architecture, radar systems, submarine radars, GPS (Global Positioning Systems), navigation, mapping, aviation, marine, calculating Earth’s perimeter, calculating distance, calculating pitch angle in construction of buildings,..etc.

A male student who took a low score from the posttest (student # 113) from Group A gave mathematically insufficient answer to the second question in the interview: “I do not think it is used in daily life, I do not know.”

Another male student who took an average score from posttest (student # 112) from Group A gave mathematically sufficient answer to second question in the interview: “ I guess it may be used in engineering and construction. I think it mostly interests engineers and constructors.”

A female student who took an average score from the posttest (student # 214) from Group B gave the mathematically sufficient answer to the second question. She explained his answer in the interview: “Mapping, engineering, navigation, marine, radar systems, aviation, airplanes, calculating the position of a submarine, calculating the height of the flagpole”.

The researcher wondered how the student could remember so many things in spite of passing of two weeks and asked: “How could you remember so many things?”. She answered: “ Problems in the slides were nice and we saw them visually. I think that’s why I did not forget them.”

A male student who took a high score from the posttest (student # 320) from Group C also gave the mathematically sufficient answer to the second question in the interview: "Marine, radar systems, airplane radars, achitecture, measuring angles when constructing buildings. We learned where it is used during our lessons."

Table 6.24. Percentage of the students' responses related to Question 3

	Percent of the students in Group A (%)			Percent of the students in Group B (%)			Percent of the students in Group C (%)		
	Before intervention	After intervention	Direction of change	Before intervention	After intervention	Direction of change	Before intervention	After intervention	Direction of change
Trigonometry is not interesting when compared to other mathematics subjects.	25	17		23	19		13	17	
Trigonometry is a little interesting when compared to other mathematics subjects.	25	25		19	19		21	17	
Trigonometry is interesting when compared to other mathematics subjects.	33	21		23	30		50	13	
Trigonometry is very interesting when compared to other mathematics subjects.	**8**	**33**	↑	**19**	**38**	↑	**8**	**46**	↑

When the percentages before and after interventions are compared for each group there is an increase in the level of interest. Percent of the students who said that trigonometry is very interesting when compared to other mathematics subjects after interventions in each group is noteworthy. Group C (46%) has the highest percentage when compared with Group B (38%) and Group A (33%) in answering "it took my attention a lot".

One student who took a high score from the posttest (student # 104) from Group A explained his level of interest in the interview: " I think trigonometry is a difficult subject and difficult subjects attract my interest. Intervention was very simple, we had already known the things we learned, from 8th grade. It was good, it was like a reminder. It helped us to remember our previous knowledge about trigonometry. It seems simple but I think it will get more difficult in the next weeks. "

A student who took a high score from the posttest (student # 220) from Group B explained his level of interest in the interview: "Trigonometry is interesting because problems consist of visual components. It is amusing. Our three lessons were very interesting, related with daily life examples. Visual problems were very interesting, lessons were very different from monotone mathematics lessons. I think they were very efficient, I wish all the mathematics lessons were like them."

A student who took an average score from the posttest (student # 309) from Group C also explained his level of interest in the interview like that: "Trigonometry is more interesting to me when compared to other mathematics subjects because it is related with geometry. Visuality was also very interesting, I think I can solve trigonometry problems better compared to other mathematics subjects' problems."

Table 6.25. Percentage of the students' answers related to Question 4

	Percent of the students in Group A (%)			Percent of the students in Group B (%)			Percent of the students in Group C (%)		
	Before intervention	After intervention	Direction of change	Before intervention	After intervention	Direction of change	Before intervention	After intervention	Direction of change
Trigonometry is difficult when compared to other mathematics subjects.	54	46		50	62		75	54	
Trigonometry is not difficult when compared to other mathematics subjects.	**33**	**46**	↑	**35**	**35**	=	**21**	**42**	↑

There is an increase in the percent of the students who think trigonometry is not difficult when compared to other mathematics subjects after interventions in Group A (33% to 46%) and Group C (21% to 42%). However, the percent did not change in Group B (35%).

Table 6.26. Percentage of the students' responses related to Question 5

	Percent of the students in Group A (%)			Percent of the students in Group B (%)			Percent of the students in Group C (%)		
	Before intervention	After intervention	Direction of change	Before intervention	After intervention	Direction of change	Before intervention	After intervention	Direction of change
Trigonometry does not frighten me compared to other mathematics subjects.	**42**	**58**	↑	**35**	**31**	↓	**75**	**83**	↑
Trigonometry frightens me compared to other mathematics subjects.	54	29		58	54		25	17	

There is an increase in the percent of students who say that trigonometry does not frighten them compared to other mathematics subjects in all groups except Group B after intervention. Group C has the highest percent in this question (83%).

Table 6.27. Percentage of the students who used positive expressions related to Question 6

Percent of the students who used positive expressions in Group A	Percent of the students who used positive expressions in Group B	Percent of the students who used positive expressions in Group C
41.666	80.769	70.833

Question 6 was:

"In what degree did the intervention effect you to understand trigonometry?"

There were so many different answers for this question. Some were including negative expressions such as, "It did not affect me", "It did not make sense", " I did not understand why we did these lessons", "It was boring". However, some were including positive expressions such as, " I understood very well", "It helped me to understand better", "It affected me in good way", "Visual problems were interesting", "I understood the main idea of trigonometry", "It was amusing, I understood the lessons"...etc.

Group B has the highest percent of the students who used positive expressions (80.76%). However, Group A has the minimum percent of the students who used positive expressions (41.66%).

Students also answered this question in the interview. Here are the answers of the students from each group. A female student (student # 108) from Group A answered: "It was good for the beginning. Slides including types of angles were nice. It helped us to remember our previous knowledge about trigonometry. However, same things could be learned in shorter time."

A male student (student # 227) from Group B answered: "We solved many visual problems. Lighthouse problem and problem which we calculated the position of submarine were really very good. Playing with angles is amusing. Visuality made the lessons different and enjoyable. In our previous mathematics lessons, we firstly learned the formulas and we did not understand but in our lessons we firstly saw where and when trigonometry was born. It really made me surprised to see how people used trigonometry two thousand years ago when calculating Earth's perimeter. Seeing that mathematics has a verbal component besides numeric component made me delighted."

A student (student # 321) from Group C answered: "We saw the main, basic concepts of trigonometry. Converting angles, radian, degree, trigonometric ratios of the most used angles of 30^0, 45^0,60^0. We saw where they come from, how we find them. Because, I think memorizing is not important, learning is the most important thing. The lessons were different and enjoyable. I had been thinking that trigonometry was an ordinary mathematics subject but I saw that it is used in many things like calculating Earth's perimeter, radars, astronomy. I did not know that. It really made me interested. Astronomy already attracts my interest always. I said 'woow it is used in that area!'."

7. DISCUSSION AND CONCLUSION

This study aims to clarify the effects of three types of instruction defined as traditional, instruction enriched with only contextual problems and instruction enriched with combination of contextual problems and student centered teaching episodes in a true experimental design. Effects of instruction were measured by examining students' trigonometry achivement, and views and perceptions about trigonometry and how much they can recall what they have learned. 74 tenth grade students participated in this study and they were assigned into three groups. Students in all groups engaged in the lessons with different sequence of experiences. Group A received traditional instruction, Group B received instruction enriched with only contextual problems and Group C received instruction enriched with combination of contextual problems and student centered teaching episodes. Group A included 24, Group B included 26 and Group C included 24 students.

The design of the research was true experimental design with pretest, posttest and retention test. First, all three groups were administered 5 questions reflecting their views and perceptions about trigonometry and an achievement pretest including 10 questions. Pretest covered the main concepts about trigonometry. After one day from the pretest, groups received different instructions during three days. The type of instructional sequence to be applied was assigned randomly to three classes. Group A received traditional instruction, Group B received instruction enriched with only contextual problems and Group C received instruction enriched with combination of contextual problems and student centered teaching episodes. The instructional practices for the Group B and Group C differed from the instructional practices for the Group A in terms of the use of contextual problems. Group A and B received three lessons, Group C received four lessons. Finally, all groups were administered 6 questions reflecting their views and perceptions about trigonometry and an achievement posttest parallel to the pretest including 10 questions. Pretests were applied in one day, interventions were applied in three days and posttests were applied in one day of the same week in all groups. After two weeks from the posttest, six questions which were same with the ones asked before posttest were asked by the researcher verbally to six students from each group and these interviews were recorded by a tape recorder. The posttest was re-administered as a retention test to all groups after 45 days from the posttest.

First of all, whether all participants were similar in terms of their mathematics and trigonometry background was addressed. Three groups were compared using Kruscal Wallis Test in terms of students' previous semester mathematics exam scores' average, and their pretest scores on trigonometry subject. It was found that there was no significant difference among the three groups in terms of their previous semester mathematics exam scores' average and Trigonometry Achievement Pretest scores. These findings showed that these three classes are similar in terms of their background in mathematics and trigonometry. This was an important information for the group comparisons.

The study further questioned differences in trigonometry achievement between the three groups. To test Hypothesis 1, Kruscal Wallis Test was used in order to examine whether there is a significant difference between three groups' trigonometry achievement scores. The results of the analysis showed that there is a statistically significant difference between the mean ranks of students' posttest scores in three groups ($\chi^2(2)=22.073$, $p=0.000$). In other words, there is a significant difference in trigonometry achievement of the three groups. Pairwise comparisons were done in order to compare the groups in pair. According to the adjusted significance values ($0.013<0.05$ and $0.000<0.05$), it was found that there is significant difference between Group B and Group A, and Group C and Group A respectively in terms of trigonometry achievement. However, there is no significant difference between Group B and Group C in terms of trigonometry achievement ($0.168>0.05$). This means that, the group who received instruction enriched with combination of contextual problems and student centered episodes (Group C) performed significantly higher than the group who received traditional instruction (Group A); the group who received instruction enriched with only contextual problems (Group B) performed significantly higher than the group who received traditional instruction (Group A); but there is no significant difference between the group who received instruction enriched with combination of contextual problems and student centered teaching episodes (Group C) and the group who received instruction enriched with only contextual problems (Group B) in terms of trigonometry achievement of students. That is to say that the Hypothesis (1,i) and (1,ii) were supported but Hypothesis (1,iii) was not supported. Both Group B and Group C performed higher compared to Group A. Both of the instructions of

Group C and Group B included contextual problems. As a result, it may be considered as a positive effect of the instruction enriched with contextual problems on students' trigonometry achievement was found compared to traditional instruction. This result supports the statement of National Council of Teachers of Mathematics (2000), "...school mathematics experiences at all levels should include opportunities to learn about mathematics by working on problems arising in contexts outside of mathematics. These connections can be other subject areas and disciplines as well as to students' daily lives..." (p.65). Hypothesis (1,iii) was not supported. There is a difference between Group B and Group C in favour of Group C but this is not a significant difference. So, a positive effect of the instruction enriched with combination of contextual problems and student centered teaching episodes was not found different compared to the instruction enriched with only contextual problems. This implies that, instruction enriched with combination of contextual problems and additional student centered teaching episodes may not have an additional positive effect in improving trigonometry achievement compared to the instruction enriched with only contextual problems. This result supports the key principles of RME that states student should be given the opportunity to reinvent mathematical concepts, and that the teaching-learning process be highly interactive (Fauzan, Slettenhaar, Plomp, 2002). The result is also parallel with Felder's (1996) statement which suggests that student centered instruction leads to increased motivation to learn, greater retention of knowledge, deeper understanding, and more positive attitudes toward the subject being taught.

One possible reason behind this result may be that Group C students were not accustomed to take an instruction enriched with additional student centered episodes. This instruction type was not familiar to them. It was observed that a few students appeared to be less motivated and actively engaged to join the discussions. For example, one male student in Group C expressed his feelings by saying that it took a long time to discuss, it would be better to continue the lesson by solving the problems consecutively. These a few students might have answered the questions in the posttest in the same unwilling manner.

Another possible reason behind this result may be that the student centered episodes may not be used as they must be used because of the time limitation. Student centered instruction is a broad approach that includes such techniques as substituting active learning experiences for lectures, holding students responsible for material, assigning open-ended problems and problems requiring critical or creative thinking, involving students in simulations and role-plays, assigning a variety of unconventional writing exercises, and using self-paced and/or cooperative learning (Felder, 1996). Student centered instruction consists of too many techniques, however only a part of these techniques could be used during the instruction of Group C because of the time limitation of the syllabus. For example, students might be taken outside the classoom, to the garden. They could see the flagpole in the garden and participate in the discussions more actively. Both the teacher (researcher) and the students were not used to student centered teaching episodes combined with the contextual problems. Results might be different if the student centered episodes were used more effectively.

Another reason behind this result may be the students' curiosity about the results of the pretests, posttests and retention tests. They asked whether these tests would be evaluated and graded. The researcher did not give detailed information about this case, she only said that they will be evaluated. Almost all the students were seemed to be satisfied with this answer, however a few students from Group C said that he would answer the questions in posttest more seriously if the test would be graded and effect the average of his mathematics grade. This expression of him may have affected the other students' motivation of answering the posttest questions in Group C.

The researcher's role may be another reason behind this result. The researcher was not the real mathematics teacher of the classes, however she was the geometry teacher of the class defined as Group B. She had been giving two geometry lessons every week to the students in Group B for one year. So, students in Group B may be more familiar with the researcher's way of teaching compared to Group A and Group C. This might have caused inflated trigonometry achievement scores in Group B.

Long term effects of instructinal sequences on trigonometry achievement among groups were also explored. The posttest was readministered as a retention test to all groups after 45 days from the posttest. The nonparametric Kruscal Wallis Test was carried out in order to see whether there is significant difference in terms of retention test results between the groups. It was found out that there is statistically significant difference between the retention test scores between the groups ($\chi^2(2)=12.77$, $p=0.002$). Pairwise comparisons were done in order to compare the groups in pair. According to the adjusted significance values, it was found that there is significant difference between Group A and Group B, and

Group A and Group C in terms of retention test scores. However, there is no significant difference between Group B and Group C in terms of retention test results. This result is parallel with the results of the analysis done on hypothesis 1. It was also found that there is significant difference between Group B and Group A, and Group C and Group A in terms of posttest scores. However, there is no significant difference between Group B and Group C in terms of posttest scores. These findings on retention test scores are parallel with the findings on posttest scores. They support the results of the analysis done on Hypothesis 1.

As a further analysis, Related Samples Wilcoxon Signed Ranks Test (the nonparametric alternative of Related Sample t-test) was also carried out to compare retention test scores and posttest scores for each group seperately. It was observed that there is a general decrease in all groups. According to the significance values, it was found that there is no significant difference between retention test scores and posttest scores in Group A ($p=0.153$). Similarly, there is no significant difference between retention test scores and posttest scores in Group B ($p=0.548$). However, there is a statistically significant difference between retention test scores and posttest scores in Group C ($p=0.000$). The results indicate that there is permanance of the trigonometry achievement for Group A and Group B, however there is no permanance of the trigonometry achievement in Group C. Traditional instruction and instruction enriched with only contextual problems led greater retention of knowledge compared to the instruction enriched with the combination of contextual problems and student centered teaching episodes.

The main reason for this decrease may be the time passed, so students might have forgotten some concepts like trigonometric ratios, types of angles, or relations about trigonometry. It was also observed that students generally preferred to use geometric relations instead of trigonometric ratios in the retention test papers. For example, many students preferred to use geomety in 3^{rd}, 4^{th}, 5^{th} and 6^{th} questions in the retention test rather than using trigonometric relations. For instance, in the seventh question of the retention test, many students prefered to use the geometry knowledge that the length of the side at the opposite of 60^0 is $\sqrt{3}$ times of the adjacent side in a right triangle and wrote $20\sqrt{3}$ meter when they saw 20 meter as one side of a right triangle with an angle of 60^0. They preferred to use this simple and easy geometric information instead of using trigonometric relation of tangent 60^0. They omitted the fact that this geometric relation was a result of trigonometric relation, tangent 60^0. Geometry may be more practical and less tiring for some students. They got less point from questions which they solved by using other techniques instead of trigonometry although the answer was right. This may be one of the possible reasons they took lower scores in the retention tests.

A possible reason behind the significant decrease in Group C may be the way how students answered the retention test questions. It was observed that a few male students from Group C persistently asked the reason why they had to answer the same questions again although the researcher explained the reason by saying that it was important to see whether they forgot what they have learned. These students found it unnecessary and boring to answer the retention test questions. However, the researcher did not come across such behaviours in other groups. It was also observed that students in Group C might have preferred to use geometric relations instead of trigonometric ratios in the retention test papers compared to other groups. Some students preferred to use geomety in 3^{rd}, 4^{th}, 5^{th} and 6^{th} questions in the retention test instead of using trigonometric ratios. So, they got lower point from these questions. These students got 1 point for the question which they solved right but without using trigonometry in retention test papers of Group C. However, there were more students who answered these questions by using trigonometry in the posttest papers. So, they took 3 points from these questions because they used the right method, trigonometry, for solving these questions. They took 2 points when they tried to solve the question by using right trigonometric relation even though the result was wrong. This may be one of the possible reasons they took low scores in the retention tests.

Finally, views and perceptions about trigonometry after instructions were explored for each group. The researcher asked five questions before intervention and six questions after intervention to the students in all groups.

For the first question, when the percentages are considered generally there is an increase in the percentage of right answers for all groups for the first question asking "What are the things that comes to your mind when you think of trigonometry?" and " Can you define trigonometry with your own words?". The results indicated that students have learned the definition of the trigonometry in all types of instructions. It may be concerned as the effect of the type of instruction on learning the definition of trigonometry is not conclusive.

For the second question, the percentages showed that Group B and Group C learned the daily life application areas of trigonometry (97% and 100%), however Group A's percentage is less (42%). It is very clear that students in group C has learned the daily life application areas of trigonometry much more compared to the students in Group A and B. This can be considered as instruction enriched with combination of contextual problems and student centered teaching episodes has a positive effect on students learning the daily life application areas of trigonometry compared to traditional instruction and instruction enriched with only contextual problems.

For the third question, there is an increase in the level of interest when the percentages before and after interventions are compared for each group. Group C (46%) has the highest percentage when compared with Group B(38%) and Group A (33%) answering "it took my attention a lot". This can be considered as instruction enriched with combination of contextual problems and student centered teaching episodes has more positive effect on students level of interest in trigonometry compared to traditional instruction and instruction enriched with only contextual problems.

For the fourth question, the results showed that there is an increase in the percent of the students who think trigonometry is not difficult when compared to other mathematics subjects after interventions in Group A (33% to 46%) and Group C (21% to 42%). However, the percent did not change in Group B (35%). Group C showed the sharpest increase with a percentage of %21 compared to Group A with a percentage of %13 and Group B. This result may be related with students' level of understanding the trigonometric concepts during lessons. They may have thought that trigonometry is not difficult, on the contrary it is easy to understand, when they understood the lessons well. Hence, this result may be accepted as an indicator for the positive effect of using instruction enriched with the combination of contextual problems and student centered teaching episodes to the perceptions about trigonometry.

For the fifth question, there is an increase in the percent of students who say that trigonometry does not frighten them compared to other mathematics subjects in all groups except Group B after intervention. Group C has the highest percent in this question (83%). This result may also be related with the positive effect of using instruction enriched with the combination of contextual problems and student centered teaching episodes to students' perceptions about trigonometry.

Sixth question only existed in the questions asked after interventions. It was: "In what degree did the intervention effect you to understand trigonometry?". The answers given to this question were evaluated according to their content of including positive expressions such as " I understood very well", "It helped me to understand better", "It effected me in good way", "Visual problems were interesting", "I understood the main idea of trigonometry", "It was amusing, I understood the lessons"...etc. Group B has the highest percent of the students who used positive expressions (80.76%). However, Group A has the minimum percent of the students who used positive expressions (41.66%). Group C has notably high percent (70.83%). These results indicate that the instructions enriched with contextual problems have a positive effect on students perceptions on trigonometry lessons. The expressions in the interviews also supported this idea. Many students from Group B and Group C stated that lessons were interesting and understandable. For example, none of the students used expressions such as "Lessons were boring" in Group B and Group C. However there were some students saying "Lessons were boring" in Group A. This can be considered as the instructions enriched with contextual problems have a positive effect on students' perceptions on trigonometry lessons. However, the positive effect of using students centered teaching episodes was not so obvious. One possible reason behind this result may be that Group C students were not used to receive an instruction enriched with student centered episodes. This instruction type may be unfamiliar to them. Some students may have perceived the time given for the discussions as a waste of time.

To summarize the findings, the results showed that instruction enriched with only contextual problems caused positive effects on students' trigometry achievement, and views and perceptions about trigonometry, but results did not support the effectiveness of instruction enriched with combination of contextual problems and student centered teaching episodes on students' trigonometry achievement compared to the instruction enriched with contextual problems. However, it was found that instruction enriched with combination of contextual problems and student centered teaching episodes has positive effects on students' views and perceptions about trigonometry compared to the traditional instruction and instruction enriched with only contextual problems.

7.1. Limitations of the Study

First of all, it is not possible to generalize the results of this study to all 10^{th} grade students other than the students of this public high school in the study because the sample size of the study is small and sampling technique is not appropriate enough for drawing generalizations. The school was not selected by using randomization techniques. It was selected because the researcher was working as a teacher in this school, so it would be convenient to apply instructions, pre and post tests, and interviews. Furthermore, the numbers of the participants in all groups were below thirty and not equal. The number of students in Group A and Group C were equal (24), but the number of students in Group B was more than them (26). Sample sizes of each group decreased because of absent students during process. Group A decreased to 24 from 27, Group B decreased to 26 from 27, Group C decreased to 24 from 27.

The researcher's role may be stated as a limitation of the study. The researcher was not the real mathematics teacher of the classes, however she was the geometry teacher of the class defined as Group B. She had been giving two geometry lessons every week to the students in Group B for one year.

Another limitation seemed to be the time of application in Group B. Instruction times were generally sufficient in all groups. Instruction took three lesson hours in Group A and B. It took four lesson hours in Group C. Each lesson hour was forty five minutes. The subjects planned to teach were all taught during the instructions in all groups. However, the time need for the instruction in Group B may be a little longer. The researcher had to proceed a little faster at last 10 minutes of the third lesson in order to give all the problems. This may be another limitation about the study.

One of the limitations was the students' expectation about the feedback of the pretests, posttests and retention tests. They asked whether these tests would be evaluated and marked. The researcher did not give detailed information about this case, she only said that they will be evaluated. Nearly all the students were seemed to be satisfied with this answer, however one student from Group C said that he would answer the questions in posttest more seriously if the the test would be marked and affect the average of his mathematics grade. This expression of him may have affected the other students' motivation of answering the posttest questions in Group C.

To summarize the main limitations, it can be said that sample size of the study, sampling technique, researcher's role, time of implementation, students' expediency are the drawbacks of the study. However, although it has some limitations, it can contribute to our understanding on the effects of instruction enriched with combination of contextual problems and student centered teaching episodes on learning in trigonometry.

7.2. Recommendations for Further Research

This study investigated the effects of three types of instruction defined as traditional, instruction enriched with only contextual problems and instruction enriched with combination of contextual problems and student centered teaching episodes. The results showed that instruction enriched with only contextual problems has positive effects on students' trigonometry achievement, and views and perceptions about trigonometry, but results did not support the effectiveness of instruction enriched with combination of contextual problems and student centered teaching episodes on students' trigonometry achievement compared to the instruction enriched with only contextual problems. However, it was found that instruction enriched with combination of contextual problems and student centered teaching episodes has positive effects on students' views and perceptions about trigonometry compared to the traditional instruction and instruction enriched with only contextual problems. As mentioned before there are some limitations in this study and further research with better implementation can increase its effectiveness and provide necessary feedback to make revisions in the instructional sequences. Therefore, instructional sequences including contextual problems can be used as a model for developing similar sequences for different content areas and for different grade levels.

Instructional sequences including contextual problems were prepared after a very intense period of literature review. So, instructional sequence can be used by all the researchers even for different kinds of studies and by educators (teachers, curriculum developers) for educational purposes. It may be possible to measure the effectiveness of the instruction enriched with combination of contextual problems and student centered teaching episodes more accurately with a larger sample and better implementation.

Both the teacher and the students were not used to student centered teaching episodes combined with the contextual problems. Results might be different if the student centered episodes were used more effectively in a more convenient learning atmosphere.

In order to get more generalizable results the study should be carried out again with larger number of sample consisting of 10^{th} grade students. And teachers should also be integrated into the study such that they should participate in all the activities and interviews. Results may differ if students receive similar instructions from their mathematics teachers.

In order to increase students' motivation towards participating in lessons, answering the questions they should be given credits for their participation and their performances should be evaluated. For example, a homework for making a little research about the trigonometry in radar systems might be given and their products can be displayed in a poster, they can make their presentation in the classroom including their friends, teachers and maybe parents. Efforts of students can be rewarded by their mathematics teacher.

For further research, Turkish teachers' attitudes towards using instructions including contextual problems as developed in this study and their opinions about using contextual problems in the mathematics lessons can be studied. As it is seen in this study, students were not used to instruction enriched with contextual problems. The results of the study conducted by Gainsburg (2008) suggest that teachers make real life connections but most are brief and many appear to require no action or thinking on the students' part. He also states that some teachers feel that students should master mathematical concepts and skills before connecting them to the real world. Therefore, teachers attitudes towards using instructions including contextual problems can be studied.

This study was an attempt to investigate the effects of three types of instruction defined as traditional, instruction enriched with only contextual problems and instruction enriched with combination of contextual problems and student centered teaching episodes. However, teachers and researchers need to discover better ways in this issue in order to meet the desires and demands of students to provide high quality of education.

APPENDIX A: "VIEWS AND PERCEPTIONS ABOUT TRIGONOMETRY PREQUESTIONNAIRE"

SORULAR

Ad-Soyad:
Sınıf:

1.Trigonometri kelimesini duyduğunda aklına ne geliyor, trigonometriyi kendi cümlelerinle tanımlar mısın?

2.Trigonometrinin günlük hayattaki kullanım alanlarının neler olduğunu düşünüyorsun? Birkaç örnek verebilir misin?

3. Trigonometri ile diğer matematik konularını karşılaştıracak olursan, trigonometrinin ilgini çekme derecesini değerlendirir misin?

4. Trigonometri ile diğer matematik konularını karşılaştıracak olursan, trigonometrinin zorluk derecesini değerlendirir misin?

5. Trigonometri ile diğer matematik konularını karşılaştıracak olursan, trigonometri seni ne kadar korkutuyor?

CEVAPLAR

1.

APPENDIX B: "VIEWS AND PERCEPTIONS ABOUT TRIGONOMETRY POSTQUESTIONNAIRE"

SORULAR

Ad-Soyad:
Sınıf:

1.Trigonometri kelimesini duyduğunda aklına ne geliyor, trigonometriyi kendi cümlelerinle tanımlar mısın?

2.Trigonometrinin günlük hayattaki kullanım alanlarının neler olduğunu düşünüyorsun? Birkaç örnek verebilir misin?

3. Trigonometri ile diğer matematik konularını karşılaştıracak olursan, trigonometrinin ilgini çekme derecesini değerlendirir misin?

4. Trigonometri ile diğer matematik konularını karşılaştıracak olursan, trigonometrinin zorluk derecesini değerlendirir misin?

5. Trigonometri ile diğer matematik konularını karşılaştıracak olursan, trigonometri seni ne kadar korkutuyor?

6. Bu hafta gerçekleştirilen uygulamalar trigonometri konusunu anlamanı ne derece etkiledi?

CEVAPLAR

1.

APPENDIX C: TRIGONOMETRY ACHIEVEMENT PRETEST

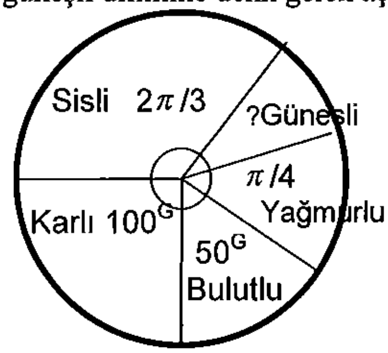
TRİGONOMETRİ SORULARI

Aşağıdaki soruları dikkatlice okuyunuz ve işlem basamaklarını detaylı ve anlaşılır biçimde göstererek çözümünüzü kağıt üzerindeki boş alanlara yapınız.

Ad-Soyad:

Sınıf-No:

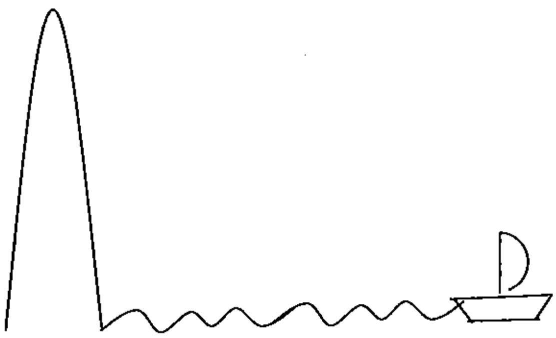
1. Aşağıdaki dairesel grafikte bir ilin bir aylık hava durumu gösterilmiştir. Buna göre güneşli dilimine denk gelen açının radyan karşılığı nedir?



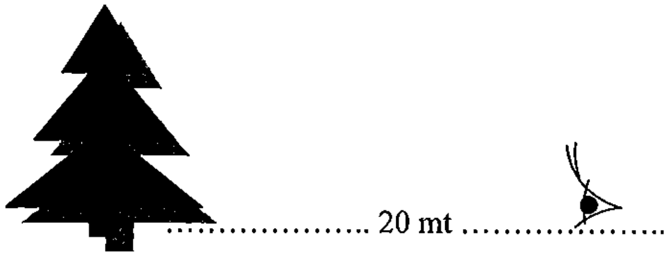
2. 150^0 lik açının radyan ve grad karşılığını bulunuz.

3. Yer çizgisiyle 60^o lik açı yapan bir yürüyen zeminin üzerindeki bir insan 1,5 metre yükseğe çıkmak için ne kadar yol alır?

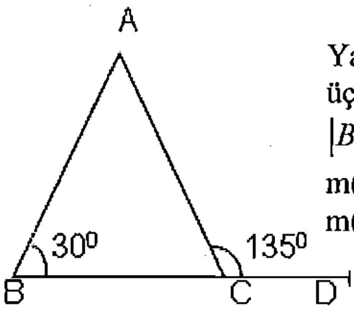
4. Deniz kıyısında bulunan 100 metre yüksekliğindeki bir tepeden denize bakınca düşeyle 30^o lik açı altında görülen bir yelkenlinin sahile olan uzaklığı kaç metredir?



5. 20 metre uzaklıktan bakıldığında tepe noktası yatayla 60° lik açı altında görülen bir ağacın boyu kaç metredir?



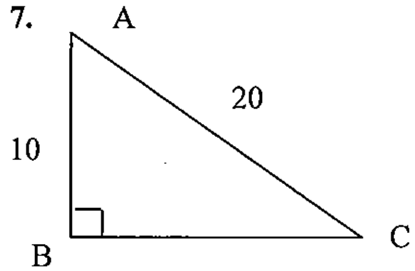
6.



Yandaki ABC üçgeninde
$|BC| = 10cm$
m(B)=30^{0},
m(AĈD)=135^{0}

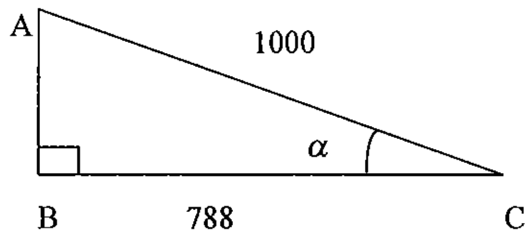
$|AB| = ?$
$|AC| = ?$

7.



Yandaki dik üçgende
m(B)= 90°
$|AB|$=10 cm,
$|BC|$=20cm
m(BĈA)=?

8.



Yukarıdaki üçgende $|AC| = 1000br$, $|BC| = 788br$ olduğuna göre $B\hat{C}A$ açısının ölçüsünü bulunuz.

9. Aşağıdaki soruyu bir dik üçgenden faydalanarak cevaplayınız.
tanx . cotx = ?

10. Aşağıdaki soruyu bir dik üçgenden faydalanarak cevaplayınız.
$\cos^2 x + \sin^2 x = ?$

APPENDIX D: TRIGONOMETRY ACHIEVEMENT POSTTEST

TRİGONOMETRİ SORULARI

Aşağıdaki soruları dikkatlice okuyunuz ve işlem basamaklarını detaylı ve anlaşılır biçimde göstererek çözümünüzü kağıt üzerindeki boş alanlara yapınız.

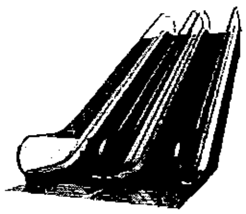
Ad-Soyad:

Sınıf-No:

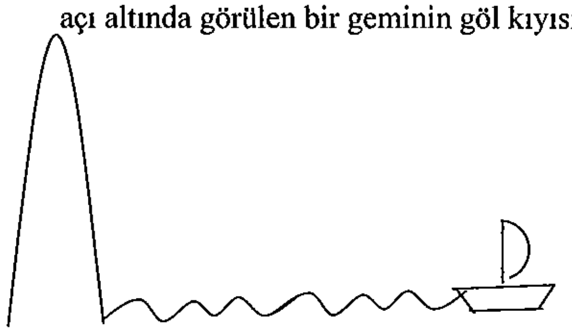
1. Aşağıdaki dairesel grafikte bir kişinin bir aylık harcama durumu gösterilmiştir. Buna göre gıdaya düşen dilime karşılık gelen açının radyan karşılığını bulunuz.

2. 135^0 lik açının radyan ve grad karşılığını bulunuz.

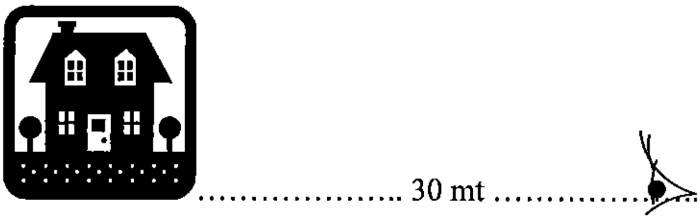
3. Yer çizgisiyle 60^o lik açı yapan bir yürüyen platformun üzerindeki bir yükün 3 metre yükseğe çıkması için ne kadar yol alması gerekir?



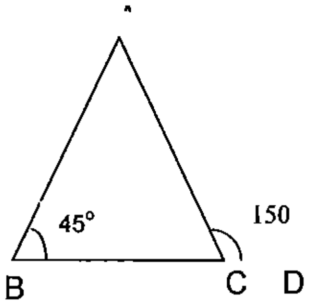
4. Göl kıyısında bulunan 60 metre yüksekliğindeki bir tepeden göle bakınca düşeyle 30^o lik açı altında görülen bir geminin göl kıyısına olan uzaklığı kaç metredir?



5. 30 metre uzaklıktan bakıldığında tepe noktası yatayla 60^{0} lik açı altında görülen bir binanın yüksekliği kaç metredir?



6.



Yandaki ABC üçgeninde

$|BC| = 5cm$

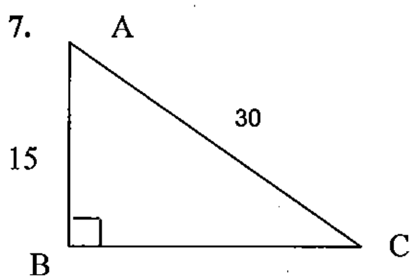
m(B)=45^{0},

m(AĈD)=150^{0}

$|AB| = ?$

$|AC| = ?$

7.



Yandaki dik üçgende

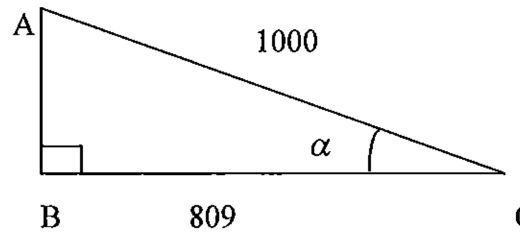
m(B)= 90^{0}

$|AB|$=15 cm,

$|AC|$=30cm

m(BĈA)=?

8.



Yukarıdaki üçgende $|AC| = 1000br$, $|BC| = 809br$ olduğuna göre $B\hat{C}A$ açısının ölçüsünü bulunuz.

9. Aşağıdaki soruyu bir dik üçgenden faydalanarak cevaplayınız.

$$\frac{1}{\cot x} \cdot \frac{1}{\tan x} = ?$$

10. Aşağıdaki soruyu bir dik üçgenden faydalanarak cevaplayınız.

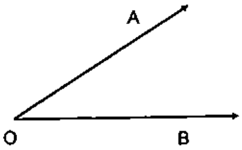
$$\frac{1}{\operatorname{cosec}^2 x} + \frac{1}{\sec^2 x} = ?$$

APPENDIX E: INSTRUCTIONAL SEQUENCE FOR GROUP A

TRİGONOMETRİ

Trigonometri terimi, Yunanca üçgen manasına gelen trigos ve ölçüm manasına gelen metron kelimelerinin birleşiminden meydana gelmiştir.

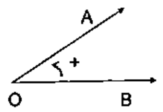
Yönlü Açılar



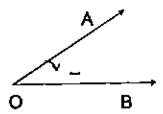
O başlangıç noktasına açılan köşesi [OA ve [OB ışınlarına da açının kenarları denir. Açıyı oluşturan 2 ışından birini başlangıç kenarı, diğerini de bitim kenarı olarak aldığımızda elde edilen açıya yönlü açı denir. Açılar adlandırılırken önce başlangıç kenarı, sonra da bitim kenarı yazılır.

Açının köşesi etrafında, başlangıç kenarından bitim kenarına 2 türlü gidilebilir.

Bunlardan birisi saatin dönme yönünün tersi, ikincisi ise saatin dönme yönünün aynısıdır. Saatin dönme yönünün tersi olan yöne pozitif, aynı olan yöne negatif yön denir.

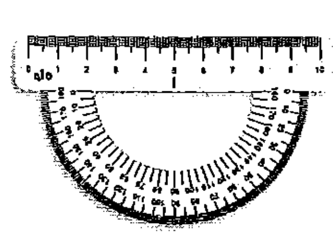


BÔA açısı pozitif yönlü bir açıdır.



A Ô B açısı negatif yönlü bir açıdır.

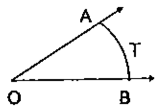
Negatif yönlü açıların ölçümü negatif sayılarla, pozitif yönlü açıların ölçümü pozitif sayılarla gösterilir.



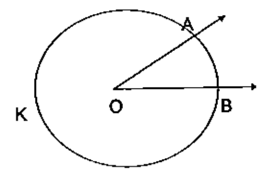
Açıların kolları arasındaki açıklığı sayısal olarak belirtme işlemine açı ölçümü denir ve bunun için açı ölçer kullanılır.

Yönlü Yaylar

Bir açının köşesini merkez kabul eden çemberin, açının iç bölgesiyle kesişen parçasına, o açının gördüğü yay denir. Yayın yönü açının yönüdür.

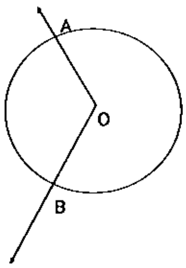


AB negatif yönlü yayı (veya ATB yayı)



AB pozitif yönlü yayı (veya AKB yayı)

örnek

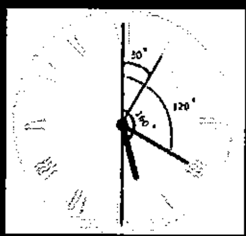


AB pozitif ve AB negatif yönlü yaylarını şekil üzerinde gösteriniz.

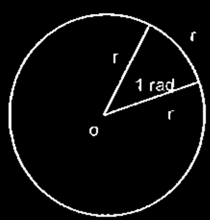
Açı ölçmek için 3 birim kullanılır:

Bir çember yayını gören köşesi çemberin merkezinde olan açıya merkez açı denir.

Bir çember yayı 360 eş parçaya bölündüğünde, bu eş parçalardan birini gören merkez açının ölçüsüne 1 derece denir ve 1 derece ile gösterilir. Tam açının ölçüsü 360 derecedir.



Bir çemberde yarıçap uzunluğuna eşit uzunluktaki yayı gören merkez açının ölçüsüne bir radyan denir ve 1R veya 1 rad ile gösterilir



Çember yayı 400 eş parçaya bölündüğünde bu eş parçalardan birini gören merkez açının ölçüsüne 1 grad denir ve 1^G ile gösterilir. Tam açının ölçüsü 400 graddır

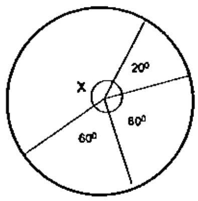
$360^0 = 2\Pi$ radyan= 400 grad

Sonuç $\frac{R}{\pi} = \frac{D}{180} = \frac{G}{200}$

Ödev: Tabloyu doldurunuz.

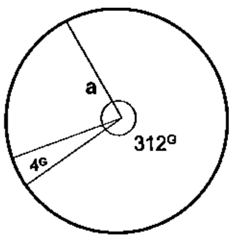
0°	30^{0}		60^{0}		120^{0}
		$\Pi/4$		$\Pi/2$	

Problem:

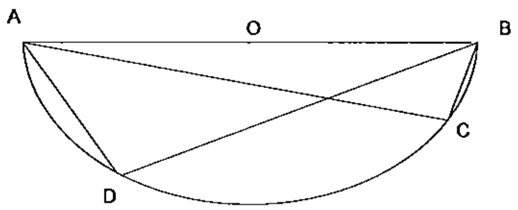


x açısının ölçüsünün radyan karşılığı nedir?

Problem



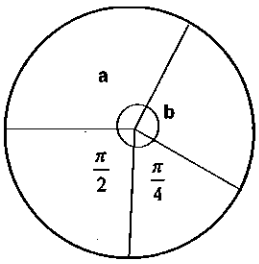
Yandaki şemada a açısına denk gelen dilimin açısının grad ve derece olarak değeri nedir?



(1) 7.32 metre çapındaki yarım çember üzerinde bulunan C ve D açılarını kıyaslayınız.

2) C ve D açılarının radyan, derece ve grad karşılığı nedir?

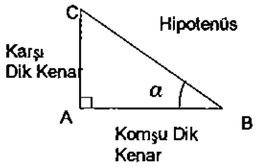
Problem:



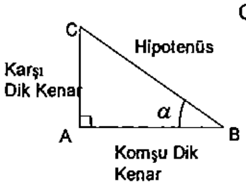
Yukarıdaki dairesel grafikte a ve b açılarının toplamı kaç radyandır?

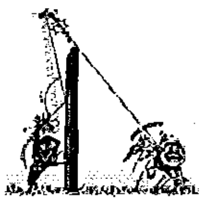
Şimdi bu dersimizde bildiğimiz açı ve mesafe bilgisini kullanarak bilmediğimiz uzunlukları hesaplamak için bize gerekli olan ve trigonometrinin temelini oluşturan kavramları, trigonometrik oranları öğreneceğiz.

Dik Üçgende Trigonometrik Oranlar



Trigonometrik Oranlar





Sinüsα = $\dfrac{\text{Karşı kenar}}{\text{Hipotenüs}}$

Cosinüsα = $\dfrac{\text{Komşu kenar}}{\text{Hipotenüs}}$

Cotan α = $\dfrac{\text{Komşu kenar}}{\text{Karşı kenar}}$

Tanα = $\dfrac{\text{Karşı kenar}}{\text{Komşu kenar}}$

Sec α = $\dfrac{1}{\cos\alpha}$

Cosecα = $\dfrac{1}{\sin\alpha}$

Trigonometrik Bağıntılar

1) $\tan\alpha \cdot \text{Cot}\,\alpha = 1$

2) $\sin^2\alpha + \cos^2\alpha = 1$

3) $\text{Tan}\,\alpha = \dfrac{1}{\cot\alpha}$

4) $\text{Cot}\,\alpha = \dfrac{1}{\tan\alpha}$

5) $\tan\alpha = \dfrac{\sin\alpha}{\cos\alpha}$

6) $\cot\alpha = \dfrac{\cos\alpha}{\sin\alpha}$

açıların trigonometrik oranları hesaplanabilir. Şimdi en çok kullanılan araçların trigonometrik oranlarının nasıl bulacağımızı öğrenelim. Diğer açılar için hesaplanış yöntemini daha sonra yardımıyla öğreneceğiz.

Eşkenar ve ikizenar üçgen yardımı ile ..

Sin 30 = ½

Cos 30 = √3/2

Tan 30 = 1/ √3 = √3 /3

Cotan 30 = √3

Sin 60 = √3/2

Cos 60 = ?
Tan 60 = ?
Cotan 60 = ?
Sin 45 = ?
Cos 45 = ?
Tan 45 = ?
Cotan 45 = ?

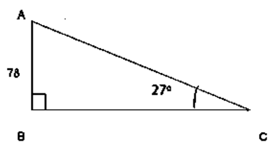
Açı	0°	30°	45°	60°
Sin				
Cos				
Tan				
cot				

Diğer tüm açılar için de trigonometrik oranlar hesaplanabilir. Bunu yaparken birim çemberden faydalanacağız. Bunu sonraki derslerimizde göreceğiz.

Tüm açıların trigonometrk oranları
yer alır.

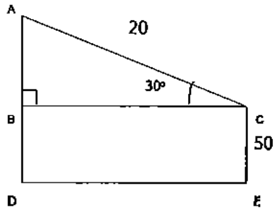
Şimdi bu öğrendiklerimizle ilgili başka problemler çözelim:

Problem:



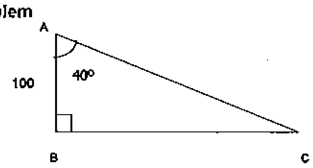
Yukarıdaki üçgende
m(B)=90°, m(AĈB)=27°, $|AB| = 78br$ olduğuna göre $|BC| = ?$

Problem:



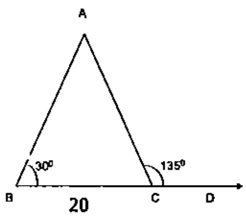
Yukarıdaki şekilde, BCED dikdörtgendir.
m(B)=90°,
$|AC| = 20br$
$|CE| = 50br$, m(AĈB)=30° olduğuna göre $|AD| = ?$

Problem



Yukarıdaki üçgende
m(B)=90°, m(BÂC)=40° $|AB| = 100br$ olduğuna göre $|AC| = ?$

Problem:

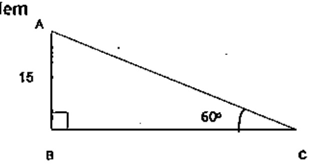


Yandaki ABC üçgeninde
IBCI=20 cm
m(B)=30°
m(AĈD)=135°
olduğuna göre
IABI=?

Problem:

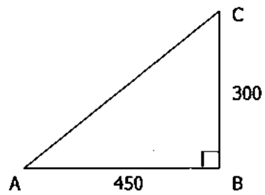
Boyut uzunlukları çok büyük olan bir üçgenin çevresini en kısa yoldan hesaplamak için neleri bilmek yeterlidir?

Problem



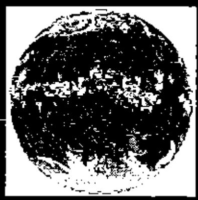
Yukarıdaki üçgende
m(B)=90°, m(BĈA)=60°, $|AB| = 15br$ **olduğuna göre** $|AC| = ?$

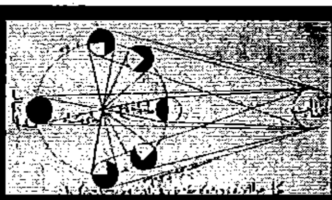
Problem:

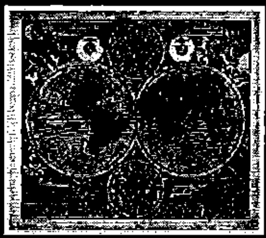


Yukarıdaki üçgende
m(B)=90°,
$|AB| = 450br$
$|BC| = 300br$ **olduğuna göre m(CÂB)=?**

APPENDIX F: INSTRUCTIONAL SEQUENCE FOR GROUP B

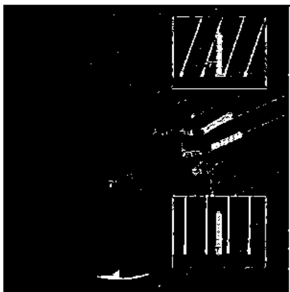


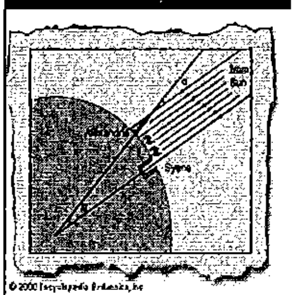




Bundan 2000 yıl önce insanlar dünyanın çevresini hesaplayabilmişlerdir.

Bunu yapan kişi asal sayılar konusundaki çalışmalarıyla tanından M.Ö 200 yıllarında yaşayan eski Yunan matematikçi Eratosthones (Eratosten) adlı bilim adamıydı (Brown, 1994).





Aristhoteles'in fikirlerinden de yararlanan Eratosthones, dünyanın çevresini ölçmek için şu 2 varsayımdan yola çıkmıştır:

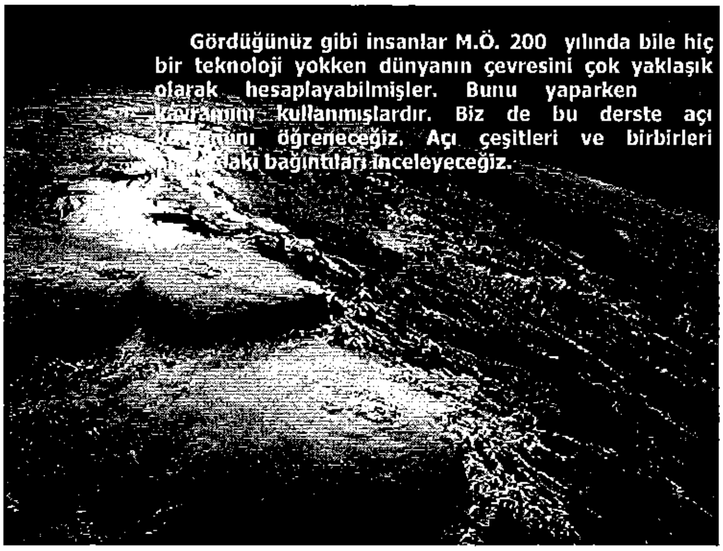
- Dünya yaklaşık bir küre biçimindedir.
- Güneş ışınları dünyaya paralel doğrular boyunca gelir.

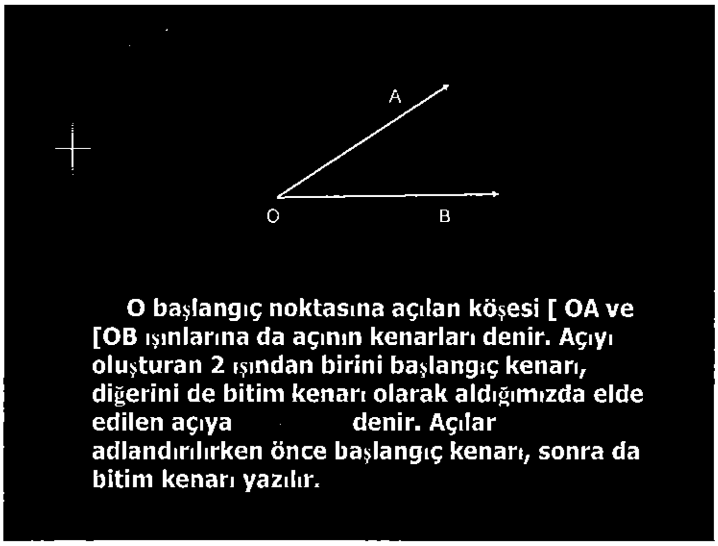
1. Biri Aswan (Syene) da diğeri İskenderiye (Alexandria) de 2 gnomon (saat ve takvim hesabında kullanılan çubuk, şekilde sarı çubuk) yere dik konumda batırıldı. Bu çubuklar, sanal olarak uzatıldığında dünyanın merkezinde kesişebilecektir. Bunların belirlediği açının ölçüsü derece türünden α olsun.
2. Aswan ile İskenderiye arasındaki uzaklık, o zamanki uzunluk ölçüsü olan "stad" kullanılarak ölçülmüştür. Bu uzaklık 5000 staddır.
3. Aswan'daki çubuğun gölgesi 0 derece olduğu (güneş ışınları Aswan'da yere dik geldiği) anda İskenderiye'deki güneş ışınlarının oradaki çubukla 7,2 derecelik açı yaparak geldiği ölçülerek belirlenmiştir. Yani α=7,2 derecedir.

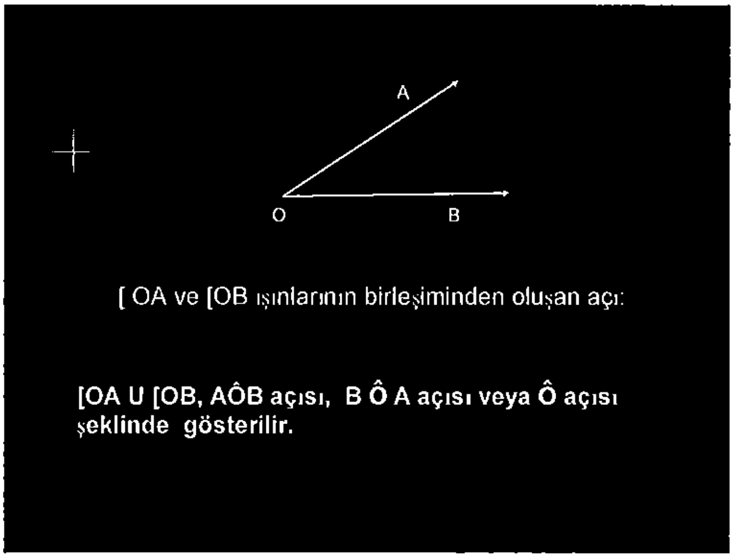
Bunlardan sonrası çok kolaydır.

7,2 derecelik açıya	5000 stad = 800 km düşüyorsa
360 derecelik açıya	? stad düşer

Bu da yaklaşık 46250 km eder.
Bu değer o zamanın şartlarına göre dünyanın bugünkü teknolojisiyle hesaplanmış 40024 km ye oldukça yakın bir değerdir!

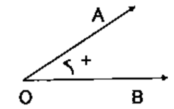

Gördüğünüz gibi insanlar M.Ö. 200 yılında bile hiç bir teknoloji yokken dünyanın çevresini çok yaklaşık
olarak hesaplayabilmişler. Bunu yaparken
kullanmışlardır. Biz de bu derste açı
öğreneceğiz. Açı çeşitleri ve birbirleri
bağıntıları inceleyeceğiz.


A
O
B
O başlangıç noktasına açılan köşesi [OA ve [OB ışınlarına da açının kenarları denir. Açıyı oluşturan 2 ışından birini başlangıç kenarı, diğerini de bitim kenarı olarak aldığımızda elde edilen açıya denir. Açılar adlandırılırken önce başlangıç kenarı, sonra da bitim kenarı yazılır.

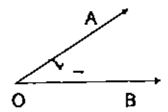

A
O
B
[OA ve [OB ışınlarının birleşiminden oluşan açı:
[OA U [OB, AÔB açısı, B Ô A açısı veya Ô açısı şeklinde gösterilir.

Açının köşesi etrafında, başlangıç kenarından bitim kenarına 2 türlü gidilebilir.

Bunlardan birisi saatin dönme yönünün tersi, ikincisi ise saatin dönme yönünün aynısıdır. Saatin dönme yönünün tersi olan yöne , aynı olan yöne yön denir.

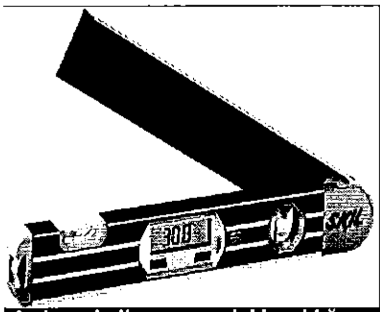


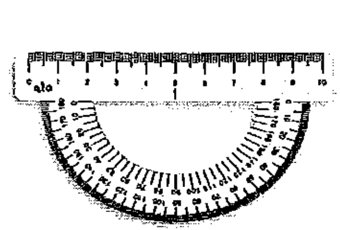
BÔA açısı pozitif yönlü bir açıdır.



A Ô B açısı negatif yönlü bir açıdır.

Negatif yönlü açıların ölçümü negatif sayılarla, pozitif yönlü açıların ölçümü pozitif sayılarla gösterilir.

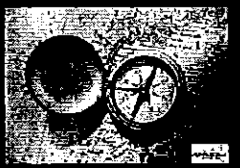




Açıların kolları arasındaki açıklığı sayısal olarak belirtme işlemine denir ve bunun için açı ölçer kullanılır.Daha ince hesap yapmak için kullanılan açıölçere ganyometre denir. İlk ganyometre 780'de Fransız minerolojist A. Carangeot'ca bulunmuş olup özellikle parlak ve yumuşak olmayan büyük kristallerin ölçümünde kullanılmıştı

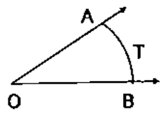
Pusula da yön bulmanın yanı sıra açı ölçümünde kullanılır. İtalyanca Bussola kelimesinden Türkçeye giren pusula MS. 300 lü yıllarda Çinliler tarafından bulunmuştur.

Çevremizde sıkça gördüğümüz arazi ölçüm aleti yapıların eğimini, yatayla yaptığı açıyı ölçmeye yarar.

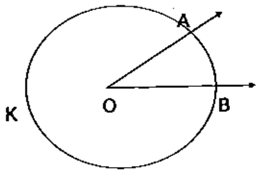


Yönlü Yaylar

Bir açının köşesini merkez kabul eden çemberin, açının iç bölgesiyle kesişen parçasına, o açının gördüğü yay denir. Yayın yönü açının yönüdür.

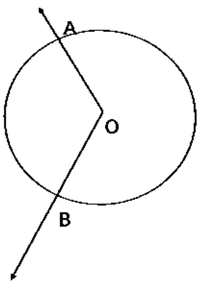


AB negatif yönlü yayı (veya ATB yayı)

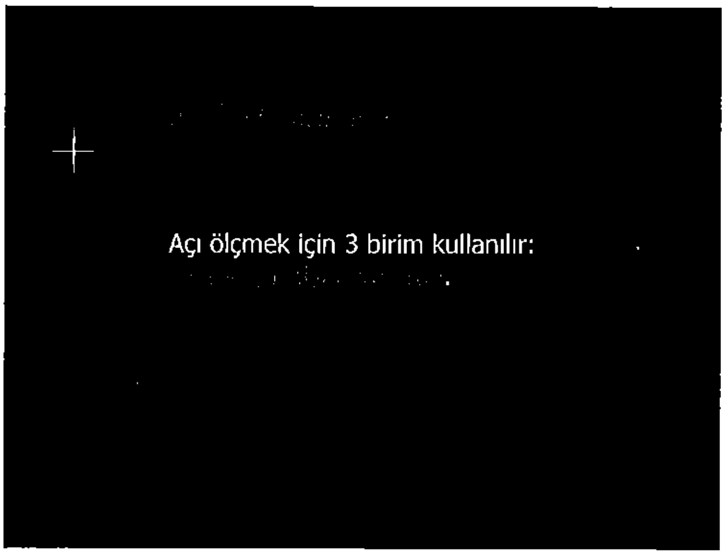


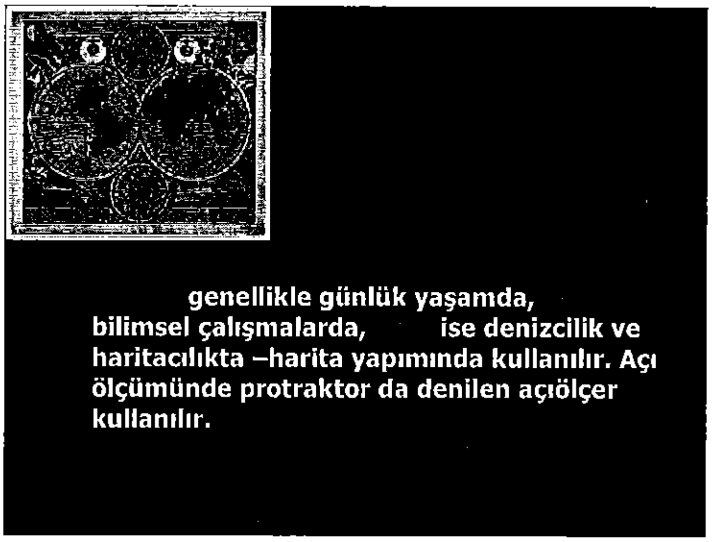
AB pozitif yönlü yayı (veya AKB yayı)

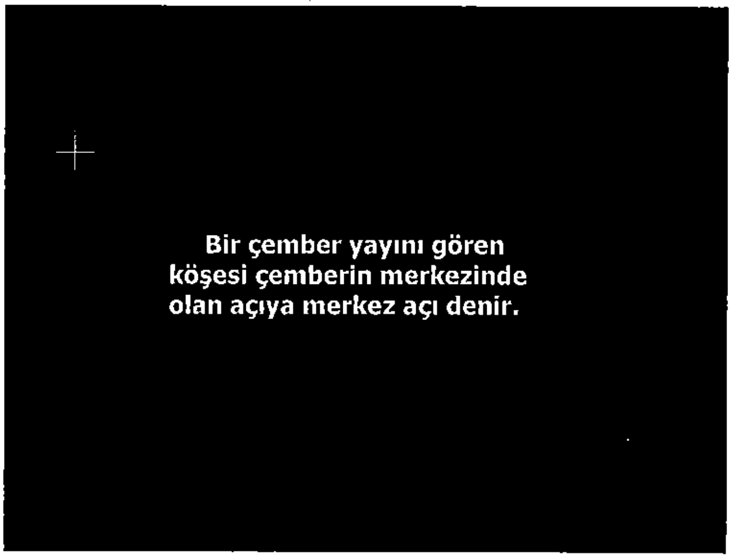
örnek



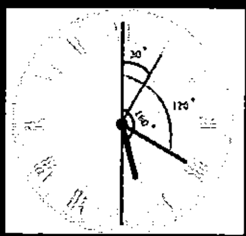
AB pozitif ve AB negatif yönlü yaylarını şekil üzerinde gösteriniz.


Açı ölçmek için 3 birim kullanılır:

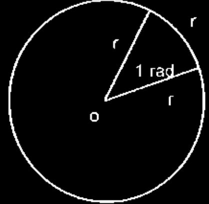

genellikle günlük yaşamda,
bilimsel çalışmalarda, ise denizcilik ve haritacılıkta –harita yapımında kullanılır. Açı ölçümünde protraktor da denilen açıölçer kullanılır.

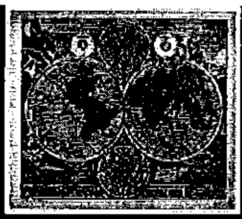

Bir çember yayını gören köşesi çemberin merkezinde olan açıya merkez açı denir.

Bir çember yayı 360 eş parçaya bölündüğünde, bu eş parçalardan birini gören merkez açının ölçüsüne 1 derece denir ve 1 derece ile gösterilir. Tam açının ölçüsü 360 derecedir.



Bir çemberde yarıçap uzunluğuna eşit uzunluktaki yayı gören merkez açının ölçüsüne bir radyan denir ve 1R veya 1 rad ile gösterilir





Haritacılık ve denizcilikte özellikle daha küçük açıların ölçümünde kullanılır.

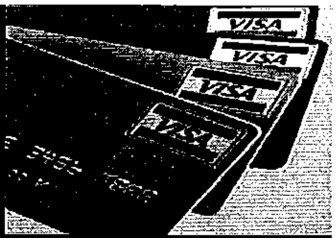
Çember yayı 400 eş parçaya bölündüğünde bu eş parçalardan birini gören merkez açının ölçüsüne 1 grad denir ve 1^G ile gösterilir. Tam açının ölçüsü 400 graddır.

$360^0 = 2\pi$ radyan $= 400$ grad

Sonuç $\frac{R}{\pi} = \frac{D}{180} = \frac{G}{200}$

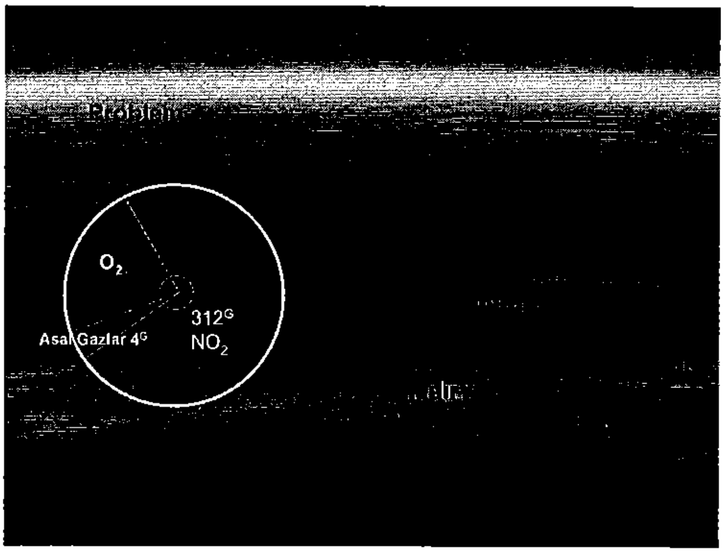
Ödev: Tabloyu doldurunuz.

0^0	30^0		60^0		120^0
		∏/4		]]/2	

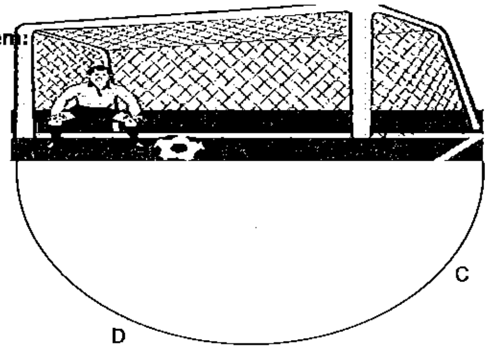


Hepimiz anne-baba veya kendi kredi kartımızda benzer şemayı görmüşüzdür. Yanda bir kredi kartına gelen harcamalar gösterilmiştir:

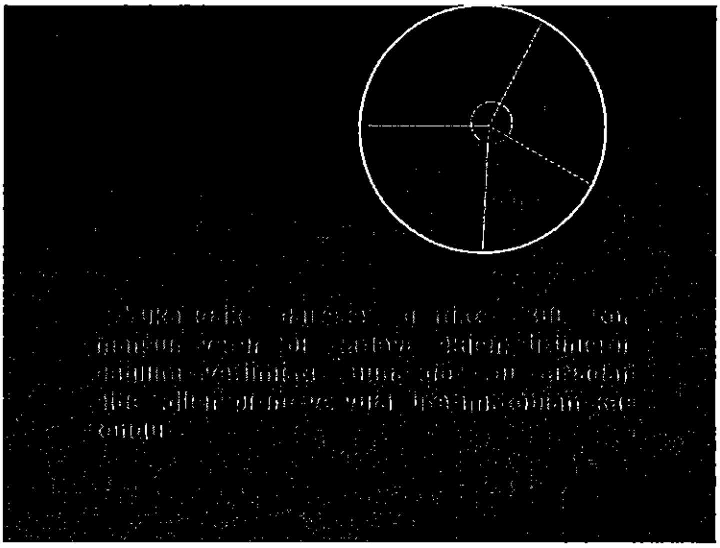
Buna göre gıdaya denk gelen açının ölçüsünün radyan karşılığı nedir?



Problem:



1) 7.32 metre uzunluğundaki kalenin orta noktasından, kale uzunluğunun yarısı uzaklığında bulunan C ve D noktalarından hangisinde gol atma olasılığı daha yüksektir? (Kaleyi görme açılarını kıyaslayınız)
2) C ve D noktalarından kaleyi görme açılarının radyan derece ve grad karşılığı nedir?



Açı kavramını ve açı çeşitlerini öğrendik. Bu kavram yardımıyla aşağıdaki problemlere cevap bulabiliriz:

1)Everest dağının yüksekliğini hesaplayabiliriz.

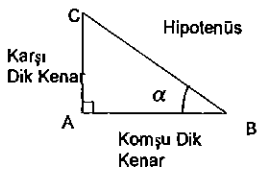
2) Bahçedeki bayrak direğinin boyunu hesaplayabiliriz.

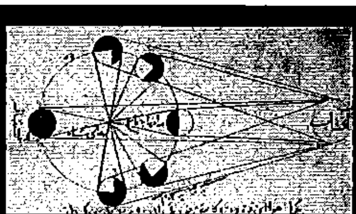
3) Bir gemi radarı denizaltının konumunu belirleyebilir.

Şimdi bu dersimizde

hesaplamak için bize gerekli olan ve trigonometrinin temelini oluşturan kavramları,
öğreneceğiz.

Dik Üçgende Trigonometrik Oranlar





Bir dik üçgende bir dar açı ve kenarlar arasında ilişki vardır.
ifade etmek için belli
tanımlayacağız. Bu oranlara yüzyıllar önce ilk olarak
yaparken (gezegenler arası uzaklık, yörünge hesabı) ihtiyaç duyulmuştur. Tarihi bilgilere göre; bu kavramları ilk kullananlar Müslümanlardı. Örneğin astronomik çalışmalarda geçen ceyb kelimesi daha sonradan Latince karşılığı kullanılarak sinüs olarak kaynaklara geçmiştir. Şimdi temel trigonometrik oranları tanımlayalım:

Trigonometrik Oranlar

Sinüs α = $\frac{\text{Karşı kenar}}{\text{Hipotenüs}}$

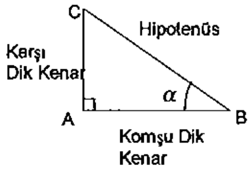
Cosinüs α = $\frac{\text{Komşu kenar}}{\text{Hipotenüs}}$

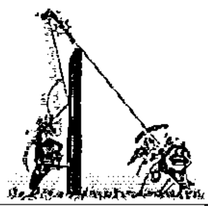
Cotan α = $\frac{\text{Komşu kenar}}{\text{Karşı kenar}}$

Tan α = $\frac{\text{Karşı kenar}}{\text{Komşu kenar}}$

Sec α = $\frac{1}{\cos\alpha}$

Cosec α = $\frac{1}{\sin\alpha}$





Trigonometrik Bağıntılar

1) $\tan\alpha \cdot \cot\alpha = 1$

2) $\sin^2\alpha + \cos^2\alpha = 1$

3) $\tan\alpha = \frac{1}{\cot\alpha}$

4) $\cot\alpha = \frac{1}{\tan\alpha}$

5) $\tan\alpha = \frac{\sin\alpha}{\cos\alpha}$

6) $\cot\alpha = \frac{\cos\alpha}{\sin\alpha}$

açıların trigonometrik oranları hesaplanabilir. Şimdi en çok kullanılan araçların trigonometrik oranlarının nasıl bulacağımızı öğrenelim. Diğer açılar için hesaplanış yöntemini daha sonra yardımıyla öğreneceğiz.

Eşkenar ve ikizenar üçgen yardımı ile ..

Sin 30 = ½

Cos 30 = √3/2

Tan 30 = 1/ √3 = √3 /3

Cotan 30 = √3

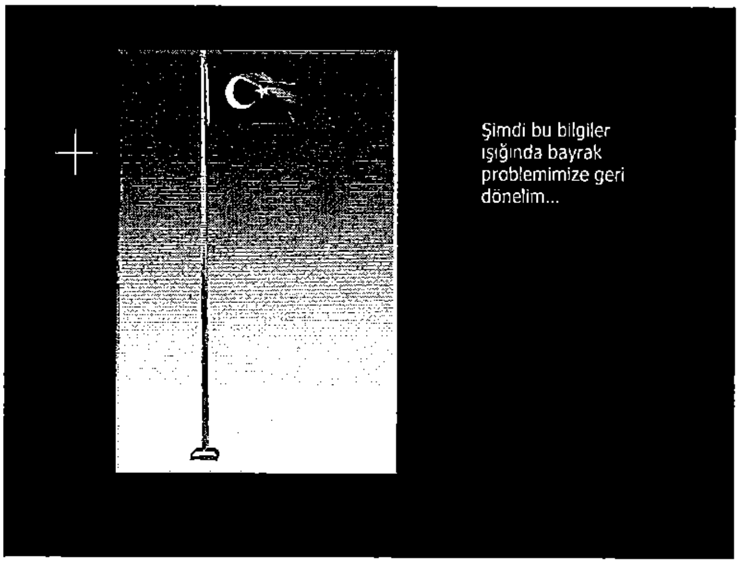
Sin 60 = √3/2

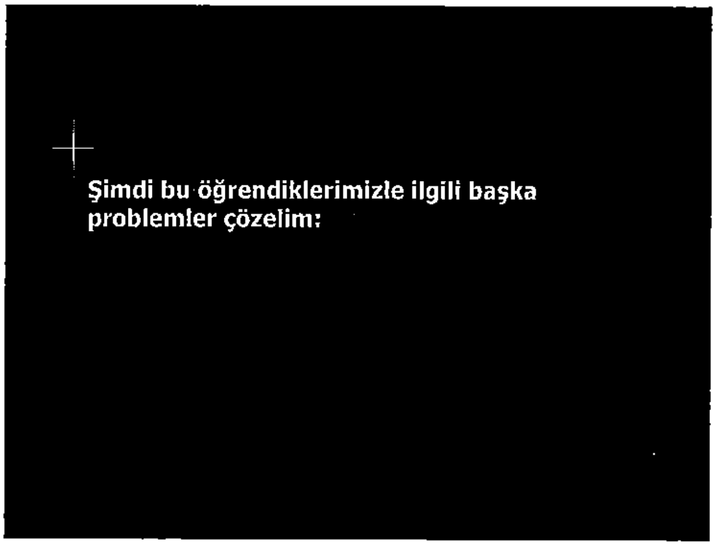
Cos 60 = ?
Tan 60 = ?
Cotan 60 = ?
Sin 45 = ?
Cos 45 = ?
Tan 45 = ?
Cotan 45 = ?

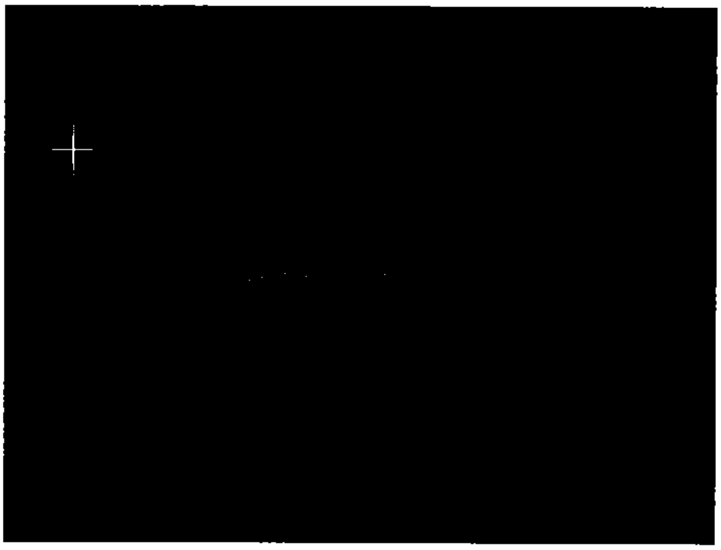
Açı	0°	30°	45°	60°
Sin				
Cos				
Tan				
cot				

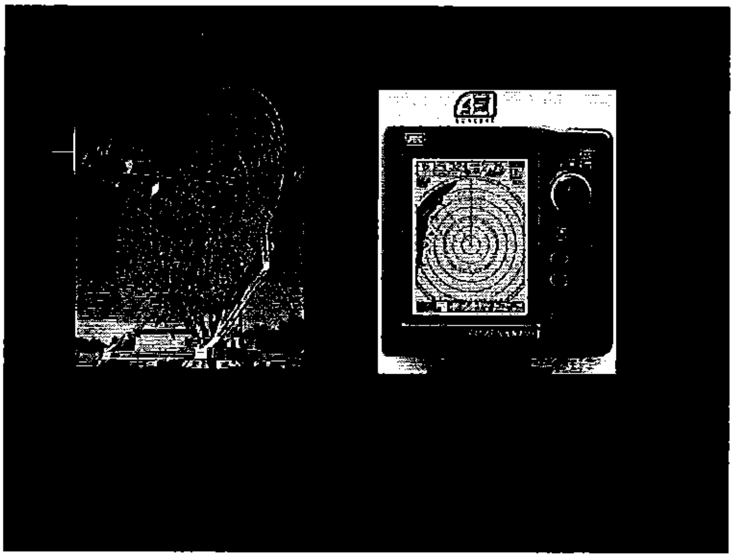
Diğer tüm açılar için de trigonometrik oranlar hesaplanabilir. Bunu yaparken birim çemberden faydalanacağız. Bunu sonraki derslerimizde göreceğiz.

Tüm açıların trigonometrk oranları
yer alır.


Şimdi bu bilgiler
ışığında bayrak
problemimize geri
dönelim...

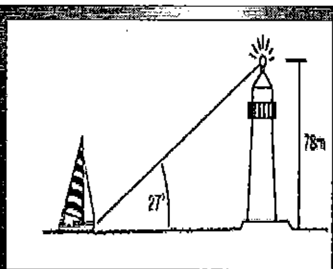

Şimdi bu öğrendiklerimizle ilgili başka
problemler çözelim:





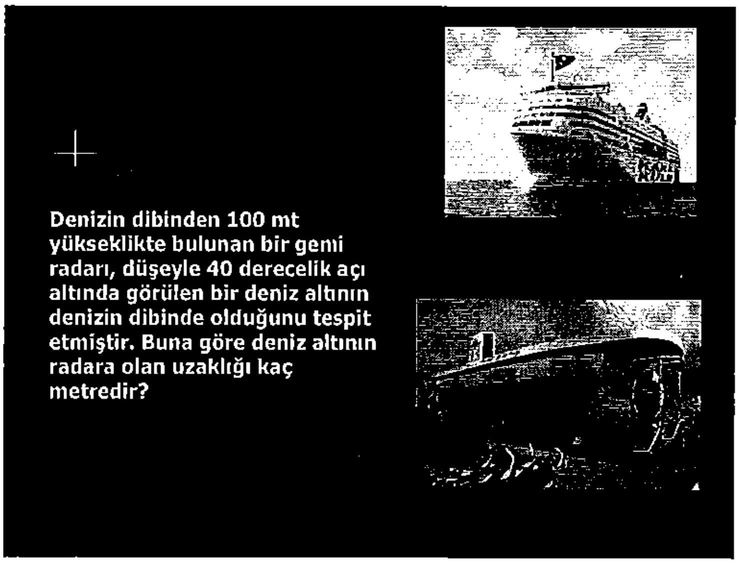
Radar, gönderdiği radyo dalgalarıyla bir objenin yeri ve konumunu belirleyen aygıttır.

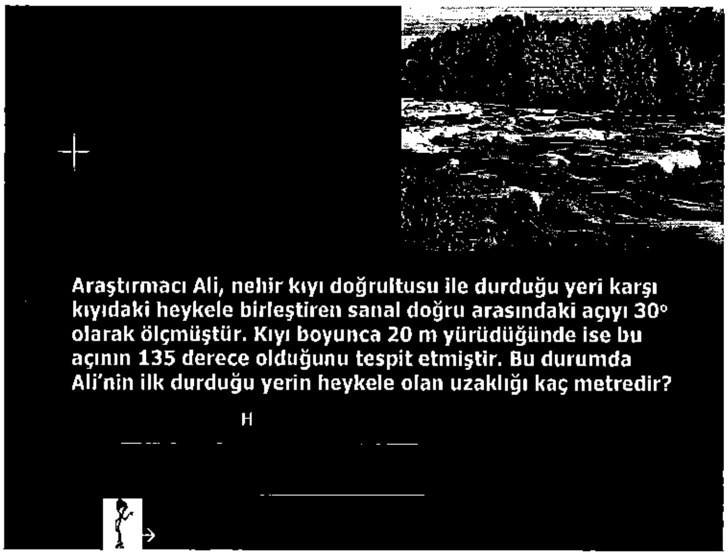
Telsiz, pusula, GPS (Global Positioning System), altimetre, soner gibi navigasyon amaçlı kullanılır. İlk olarak 1934'te İngiliz General'in Alman uçaklarını vurmak için mühendislerden yer ve konum belirleyen bir alet istemesi üzerine icat edilmiş ve dünya askeri tarihinin yönünü sil baştan değiştirmiş ve yeniden yazmıştır. Aşağıdaki iki problem radarın çalışma prensibini basit anlamda açıklamaktadır:



Deniz kenarında 78 mt yükseklikteki kontrol kulesinde bulunan bir görevli, bir yelkenlinin yatayla açısını radar ile 27° olarak ölçmüştür. Buna göre yelkenlinin kulenin tepesine uzaklığı kaç mt dir?

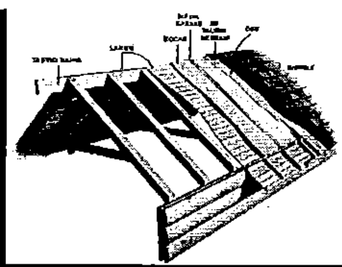

Problem:
50 metre yüksekliğindeki kontrol kulesinde
bir görevli, yere paralel uçan bir uçağın
uzaklığını radar ile 20 km olarak ölçmüştür.
(d=Vradyodalgası . t/2 formülü ile)
Görüş doğrusunun yatay ile yaptığı
olduğu bilindiğine göre uçağın yerden
nedir?


Denizin dibinden 100 mt yükseklikte bulunan bir gemi radarı, düşeyle 40 derecelik açı altında görülen bir deniz altının denizin dibinde olduğunu tespit etmiştir. Buna göre deniz altının radara olan uzaklığı kaç metredir?


Araştırmacı Ali, nehir kıyı doğrultusu ile durduğu yeri karşı kıyıdaki heykele birleştiren sanal doğru arasındaki açıyı 30° olarak ölçmüştür. Kıyı boyunca 20 m yürüdüğünde ise bu açının 135 derece olduğunu tespit etmiştir. Bu durumda Ali'nin ilk durduğu yerin heykele olan uzaklığı kaç metredir?
H

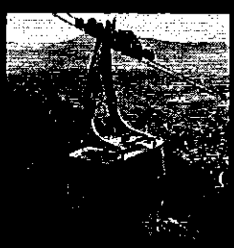
EMLAKÇI

Emlakçılık mesleğiyle uğraşan Ayşe, üçgen şeklinde bir arsanın krokisini çizmek ve müşteriler için dükkanında sergilemek istiyor. Arsanın boyutlar çok büyüktür. Ayşe'nin ölçme işlemini en kısa yoldan yapması için neleri bilmesi yeterlidir?

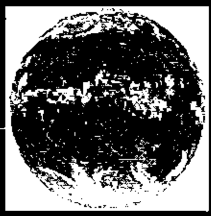


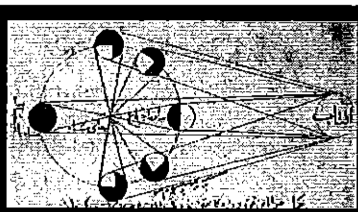
Bir çatının rüzgar ve basıncı kaldırabilmesi için 60 derecelik çatı eğimine sahip olması gerekir. Çatının yüksekliğinin 15 m olabilmesi için çatının eğimli bölümünün uzunluğunun kaç metre olması gerekir?

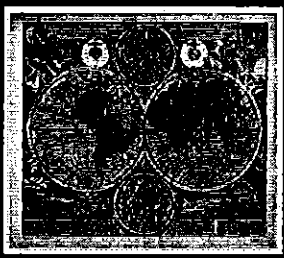
Teleferiklerin güvenli olması için teleferik halatının yer ile yaptığı açı 30 dereceyi geçmemelidir. 450 metre uzakta 300 metre yüksekliğe çıkması tasarlanan bir teleferik güvenli midir?



APPENDIX G: INSTRUCTIONAL SEQUENCE FOR GROUP C



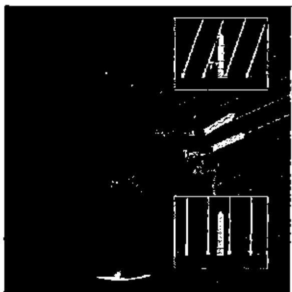


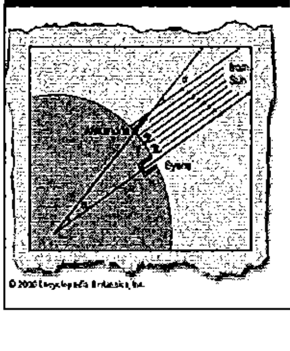


Bundan 2000 yıl önce dünyanın çevresinin hesaplandığını biliyor musunuz?

Bunu nasıl başarmış olabilirler?

Bunu yapan kişi asal sayılar konusundaki çalışmalarıyla tanından M.Ö 200 yıllarında yaşayan eski Yunan matematikçi Eratosthones (Eratosten) adlı bilim adamıydı (Brown, 1994).





Aristhoteles'in fikirlerinden de yararlanan Eratosthones, dünyanın çevresini ölçmek için şu 2 varsayımdan yola çıkmıştır:

- Dünya yaklaşık bir küre biçimindedir.
- Güneş ışınları dünyaya paralel doğrular boyunca gelir.

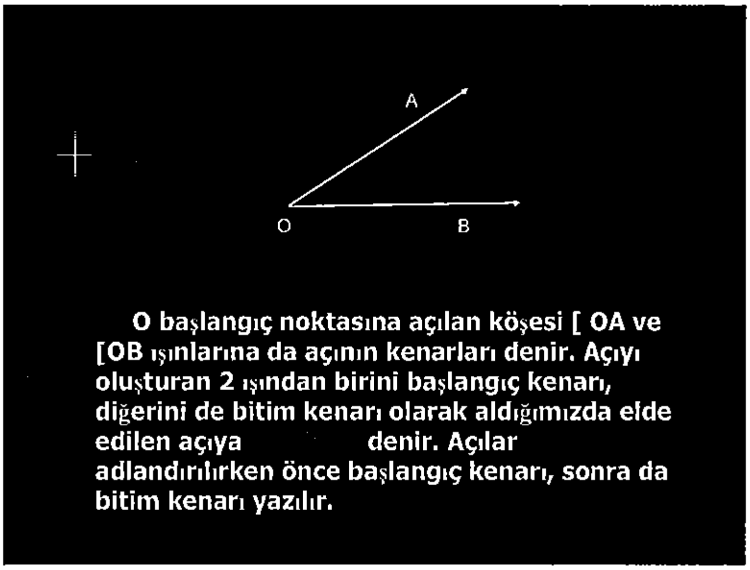
1. Biri Aswan (Syene) da diğeri İskenderiye (Alexandria) de 2 gnomon (saat ve takvim hesabında kullanılan çubuk, şekilde sarı çubuk) yere dik konumda batırıldı. Bu çubuklar, sanal olarak uzatıldığında dünyanın merkezinde kesişebilecektir. Bunların belirlediği açının ölçüsü derece türünden α olsun.
2. Aswan ile İskenderiye arasındaki uzaklık, o zamanki uzunluk ölçüsü olan "stad" kullanılarak ölçülmüştür. Bu uzaklık 5000 staddır.
3. Aswan'daki çubuğun gölgesi 0 derece olduğu (güneş ışınları Aswan'da yere dik geldiği) anda İskenderiye'deki güneş ışınlarının oradaki çubukla 7,2 derecelik açı yaparak geldiği ölçülerek belirlenmiştir. Yani α=7,2 derecedir.

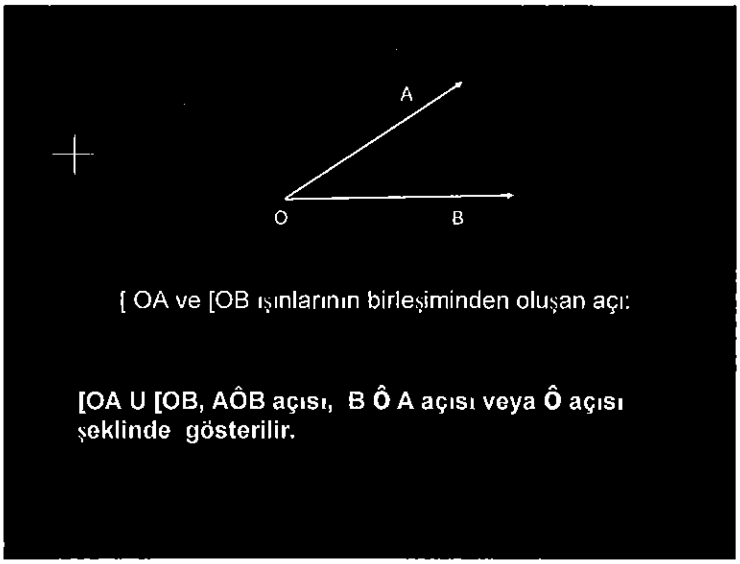
Bunlardan sonrası çok kolaydır.

7,2 derecelik açıya 5000 stad = 800 km düşüyorsa
360 derecelik açıya ? stad düşer

Bu da yaklaşık 46250 km eder.
Bu değer o zamanın şartlarına göre dünyanın bugünkü teknolojisiyle hesaplanmış 40024 km ye oldukça yakın bir değerdir!

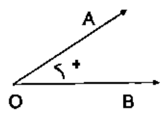

Gördüğünüz gibi insanlar M.Ö. 200 yılında bile hiç
bir teknoloji yokken dünyanın çevresini çok yaklaşık
olarak hesaplayabilmişler. Bunu yaparken
kullanmışlardır. Biz de bu derste açı
öğreneceğiz. Açı çeşitleri ve birbirleri
bağıntıları inceleyeceğiz.


A
O
B
O başlangıç noktasına açılan köşesi [OA ve
[OB ışınlarına da açının kenarları denir. Açıyı
oluşturan 2 ışından birini başlangıç kenarı,
diğerini de bitim kenarı olarak aldığımızda elde
edilen açıya denir. Açılar
adlandırılırken önce başlangıç kenarı, sonra da
bitim kenarı yazılır.

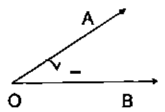

A
O
B
[OA ve [OB ışınlarının birleşiminden oluşan açı:
[OA U [OB, AÔB açısı, B Ô A açısı veya Ô açısı
şeklinde gösterilir.

Açının köşesi etrafında, başlangıç kenarından bitim kenarına 2 türlü gidilebilir.

Bunlardan birisi saatin dönme yönünün tersi, ikincisi ise saatin dönme yönünün aynısıdır. Saatin dönme yönünün tersi olan yöne , aynı olan yöne yön denir.

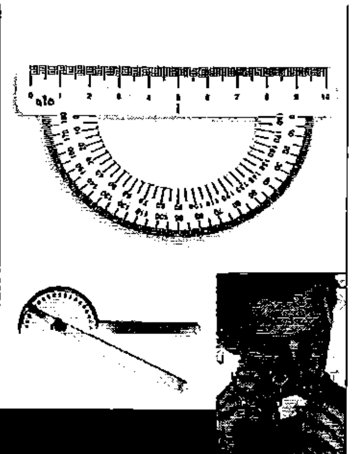


BÔA açısı pozitif yönlü bir açıdır.



A Ô B açısı negatif yönlü bir açıdır.

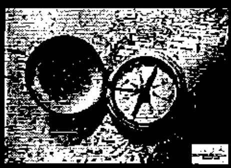
Negatif yönlü açıların ölçümü negatif sayılarla, pozitif yönlü açıların ölçümü pozitif sayılarla gösterilir.



Açıların kolları arasındaki açıklığı sayısal olarak belirtme işlemine denir ve bunun için açı ölçer kullanılır.Daha ince hesap yapmak için kullanılan açıölçere ganyometre denir. İlk ganyometre 780'de Fransız minerolojist A. Carangeot'ca bulunmuş olup özellikle parlak ve yumuşak olmayan büyük kristallerin ölçümünde kullanılmıştı

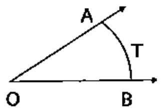
Pusula da yön bulmanın yanı sıra açı ölçümünde kullanılır. İtalyanca Bussola kelimesinden Türkçeye giren pusula MS. 300 lü yıllarda Çinliler tarafından bulunmuştur.

Çevremizde sıkça gördüğümüz arazi ölçüm aleti yapıların eğimini, yatayla yaptığı açıyı ölçmeye yarar.

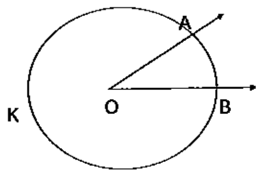


Yönlü Yaylar

Bir açının köşesini merkez kabul eden çemberin, açının iç bölgesiyle kesişen parçasına, o açının gördüğü yay denir. Yayın yönü açının yönüdür.

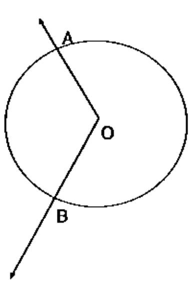


AB negatif yönlü yayı (veya ATB yayı)

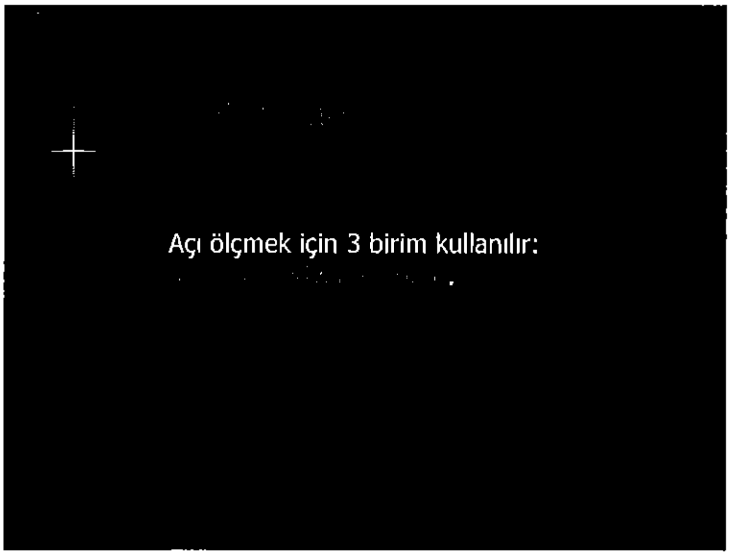


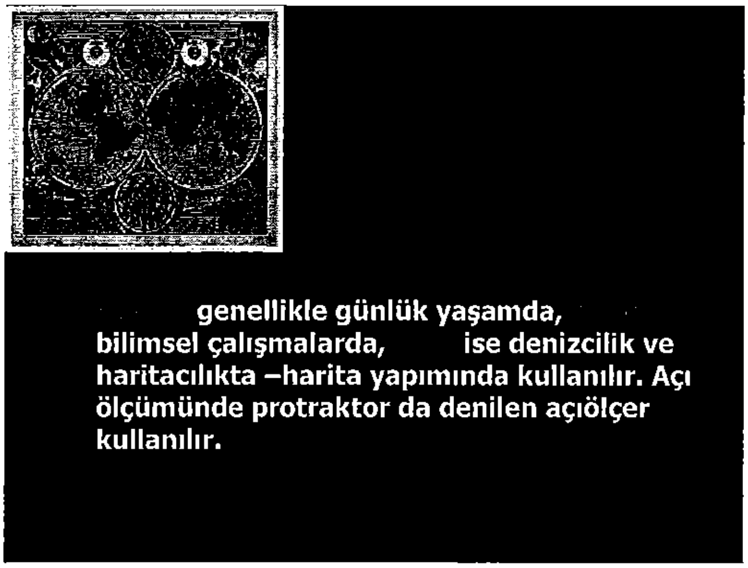
AB pozitif yönlü yayı (veya AKB yayı)

Örnek



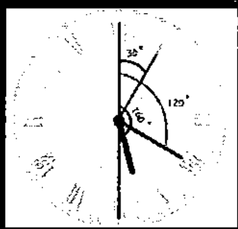
AB pozitif ve AB negatif yönlü yaylarını şekil üzerinde gösteriniz.


Açı ölçmek için 3 birim kullanılır:

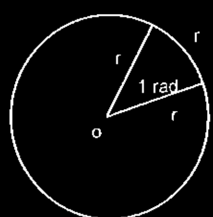

genellikle günlük yaşamda,
bilimsel çalışmalarda, ise denizcilik ve
haritacılıkta –harita yapımında kullanılır. Açı
ölçümünde protraktor da denilen açıölçer
kullanılır.

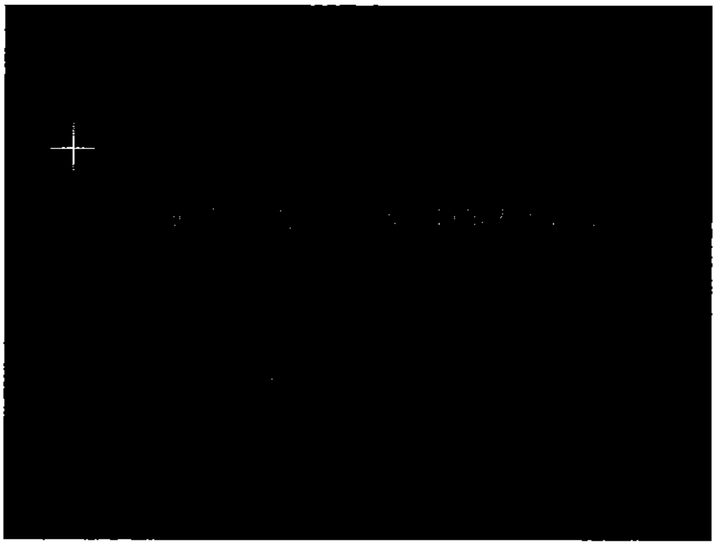

Bir çember yayını gören
köşesi çemberin merkezinde
olan açıya merkez açı denir.

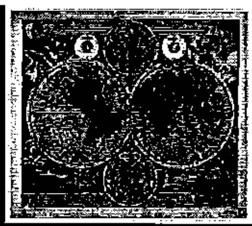
Bir çember yayı 360 eş parçaya bölündüğünde, bu eş parçalardan birini gören merkez açının ölçüsüne 1 derece denir ve 1 derece ile gösterilir. Tam açının ölçüsü 360 derecedir.



Bir çemberde yarıçap uzunluğuna eşit uzunluktaki yayı gören merkez açının ölçüsüne bir radyan denir ve 1R veya 1 rad ile gösterilir

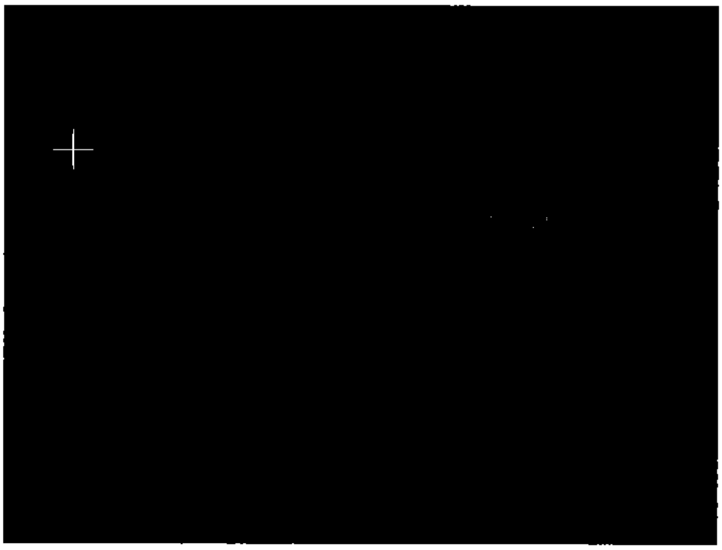


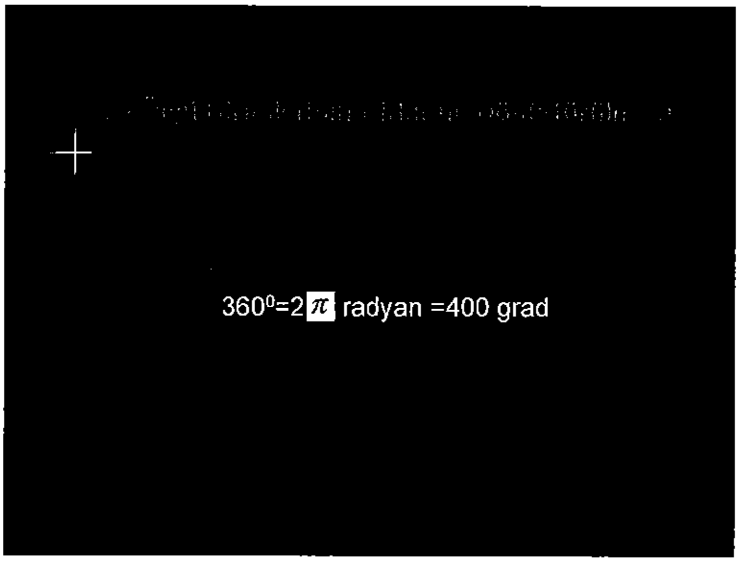


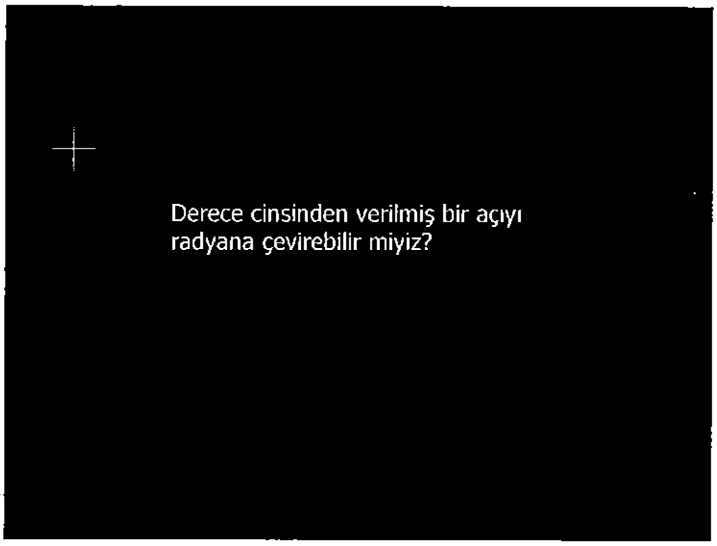


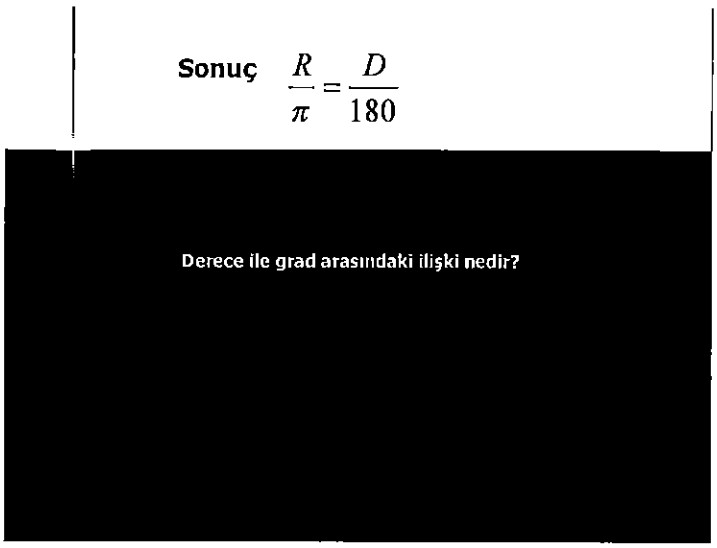
Haritacılık ve denizcilikte özellikle daha küçük açıların ölçümünde kullanılır.

Çember yayı 400 eş parçaya bölündüğünde bu eş parçalardan birini gören merkez açının ölçüsüne 1 grad denir ve 1^G ile gösterilir. Tam açının ölçüsü 400 graddır.




$360^0 = 2\pi$ radyan =400 grad


Derece cinsinden verilmiş bir açıyı
radyana çevirebilir miyiz?


Sonuç $\frac{R}{\pi}=\frac{D}{180}$
Derece ile grad arasındaki ilişki nedir?

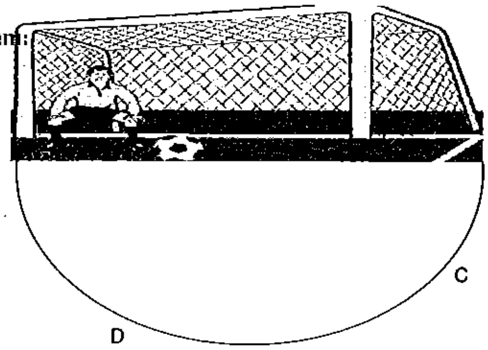
Ödev: Tabloyu doldurunuz.

0°	30°		60°		120°
		Π/4		Π/2	

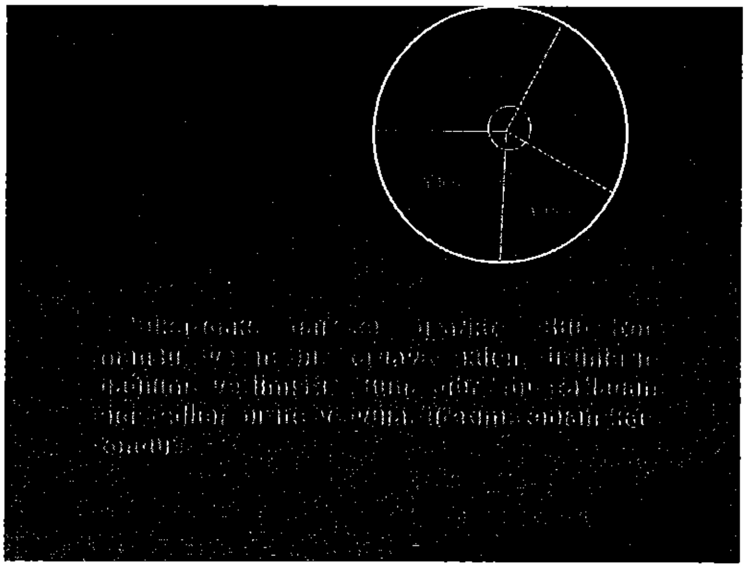
Hepimiz anne-baba veya kendi kredi kartımızda benzer şemayı görmüşüzdür. Yanda bir kredi kartına gelen harcamalar gösterilmiştir:

Buna göre gıdaya denk gelen açının ölçüsünün radyan karşılığı nedir?

Problem:



1) 7.32 metre uzunluğundaki kalenin orta noktasından, kale uzunluğunun yarısı uzaklığında bulunan C ve D noktalarından hangisinde gol atma olasılığı daha yüksektir? (Kaleyi görme açılarını kıyaslayınız)
2) C ve D noktalarından kaleyi görme açılarının radyan derece ve grad karşılığı nedir?



Açı kavramını ve açı çeşitlerini öğrendik. Şimdi acaba bu kavram yardımıyla aşağıdaki problemlere cevap bulabilir miyiz?

1)Everest dağının yüksekliğini hesaplayabilir miyim?

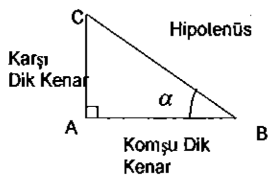
2) Bahçedeki bayrak direğinin boyunu hesaplayabilir miyim?

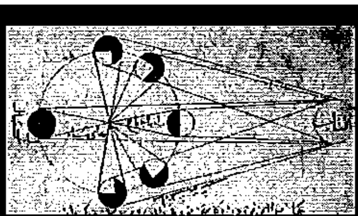
3) Bir gemi radarı denizaltının konumunu nasıl belirler?

Şimdi bu dersimizde

hesaplamak için bize gerekli olan ve trigonometrinin temelini oluşturan kavramları,
öğreneceğiz.

Dik Üçgende Trigonometrik Oranlar





Bir dik üçgende bir dar açı ve kenarlar arasında ilişki vardır.
ifade etmek için belli
tanımlayacağız. Bu oranlara yüzyıllar önce ilk olarak
yaparken (gezegenler arası uzaklık, yörünge hesabı) ihtiyaç duyulmuştur. Tarihi bilgilere göre; bu kavramları ilk kullananlar Müslümanlardı. Örneğin astronomik çalışmalarda geçen ceyb kelimesi daha sonradan Latince karşılığı kullanılarak sinüs olarak kaynaklara geçmiştir. Şimdi temel trigonometrik oranları tanımlayalım:

Trigonometrik Oranlar

Sinüsα = $\dfrac{\text{Karşı kenar}}{\text{Hipotenüs}}$

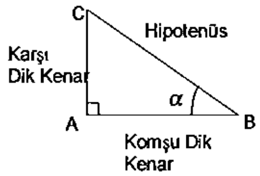
Cosinüsα = $\dfrac{\text{Komşu kenar}}{\text{Hipotenüs}}$

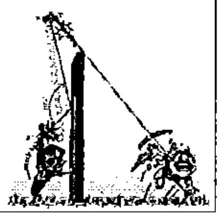
Cotanα = $\dfrac{\text{Komşu kenar}}{\text{Karşı kenar}}$

Tanα = $\dfrac{\text{Karşı kenar}}{\text{Komşu kenar}}$

Sec α = $\dfrac{1}{\cos a}$

Cosecα = $\dfrac{1}{\sin a}$





Trigonometrik Bağıntılar

1) tan α Cotα=1

2) $\sin^2\alpha + \cos^2\alpha = 1$

3)Tanα=$\dfrac{1}{\cot\alpha}$

4)Cotα = $\dfrac{1}{\tan\alpha}$

5)tanα=$\dfrac{\sin\alpha}{\cos\alpha}$

6)cotα= $\dfrac{\cos\alpha}{\sin\alpha}$

açıların trigonometrik oranları hesaplanabilir. Şimdi en çok kullanılan araçların trigonometrik oranlarının nasıl bulacağımızı öğrenelim2Diğer açılar için hesaplanış yöntemini daha sonra yardımıyla öğreneceğiz.

Eşkenar ve ikizenar üçgen yardımı ile ..

Sin 30 = ½

Cos 30 = √3/2

Tan 30 = 1/ √3 = √3 /3

Cotan 30 = √3

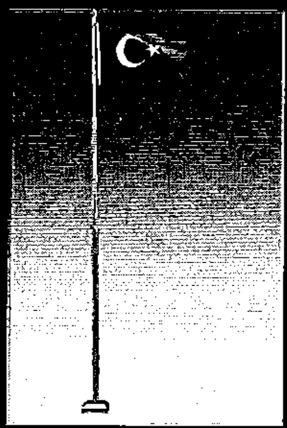
Sin 60 = √3/2

Cos 60 = ?
Tan 60 = ?
Cotan 60 = ?
Sin 45 = ?
Cos 45 = ?
Tan 45 = ?
Cotan 45 = ?

Açı	0°	30°	45°	60°
Sin				
Cos				
Tan				
cot				

Diğer tüm açılar için de trigonometrik oranlar hesaplanabilir. Bunu yaparken birim çemberden faydalanacağız. Bunu sonraki derslerimizde göreceğiz.

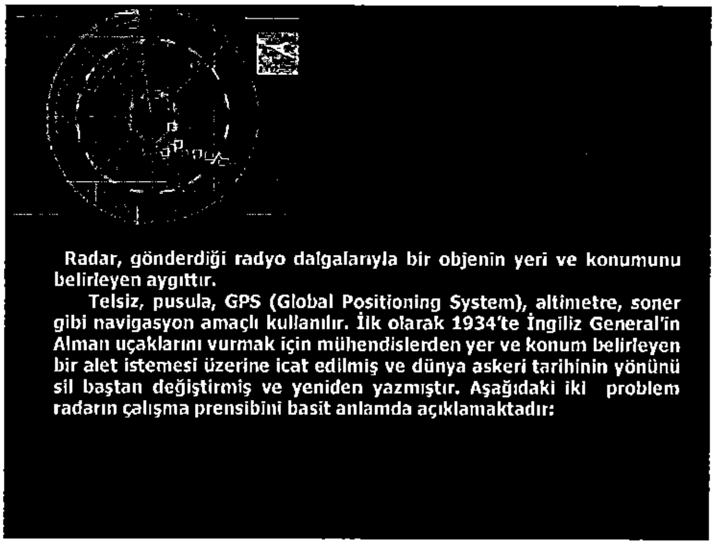
Tüm açıların trigonometrk oranları
yer alır.

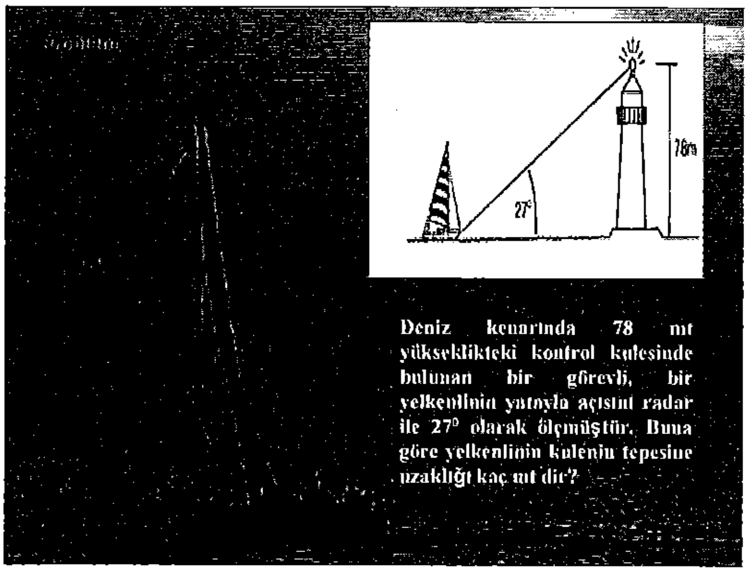


Şimdi bu bilgiler ışığında bayrak problemimize geri dönelim...

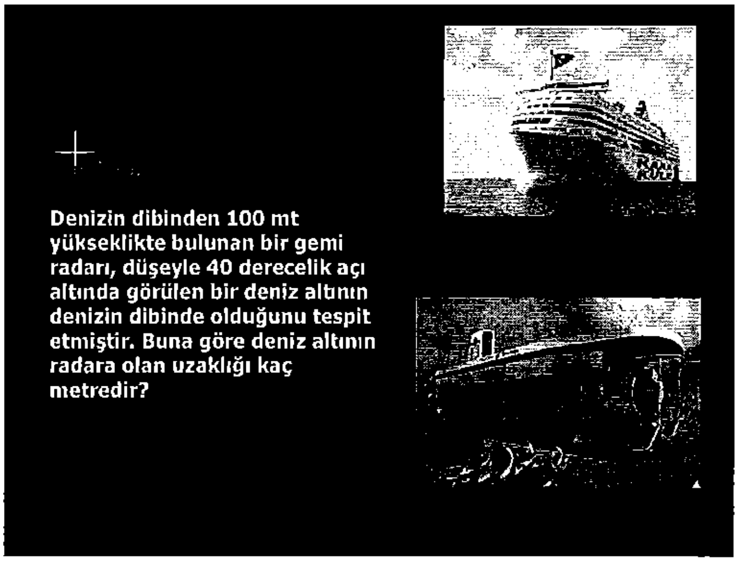
Şimdi bu öğrendiklerimizle ilgili başka problemler çözelim:

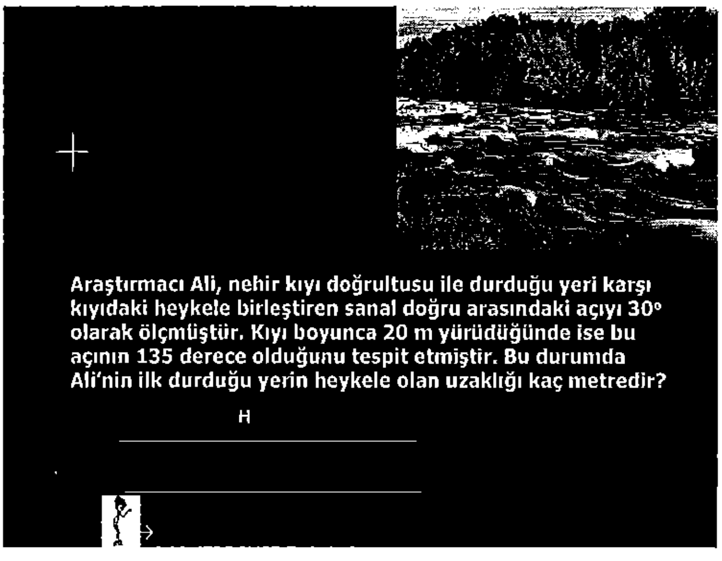

Bir radarın çalışma prensibini biliyor musunuz?
Radarı ilk kim ne zaman icat etmiş?


Radar, gönderdiği radyo dalgalarıyla bir objenin yeri ve konumunu belirleyen aygıttır.
Telsiz, pusula, GPS (Global Positioning System), altimetre, soner gibi navigasyon amaçlı kullanılır. İlk olarak 1934'te İngiliz General'in Alman uçaklarını vurmak için mühendislerden yer ve konum belirleyen bir alet istemesi üzerine icat edilmiş ve dünya askeri tarihinin yönünü sil baştan değiştirmiş ve yeniden yazmıştır. Aşağıdaki iki problem radarın çalışma prensibini basit anlamda açıklamaktadır:


78m
27°
Deniz kenarında 78 mt yükseklikteki kontrol kulesinde bulunan bir görevli, bir yelkenlinin yatayla açısını radar ile 27° olarak ölçmüştür. Buna göre yelkenlinin kulenin tepesine uzaklığı kaç mt dir?


Problem:
50 metre yüksekliğindeki kontrol kulesinde
bir görevli, yere paralel uçan bir uçağın
uzaklığını radar ile 20 km olarak
(d=Vradyodalgası . t/2 formülü ile)
Görüş doğrusunun yatay ile yaptığı
olduğu bilindiğine göre uçağın
nedir?


Denizin dibinden 100 mt yükseklikte bulunan bir gemi radarı, düşeyle 40 derecelik açı altında görülen bir deniz altının denizin dibinde olduğunu tespit etmiştir. Buna göre deniz altının radara olan uzaklığı kaç metredir?


Araştırmacı Ali, nehir kıyı doğrultusu ile durduğu yeri karşı kıyıdaki heykele birleştiren sanal doğru arasındaki açıyı 30° olarak ölçmüştür. Kıyı boyunca 20 m yürüdüğünde ise bu açının 135 derece olduğunu tespit etmiştir. Bu durumda Ali'nin ilk durduğu yerin heykele olan uzaklığı kaç metredir?
H

Emlakçılık mesleğiyle uğraşan Ayşe, üçgen şeklinde bir arsanın krokisini çizmek ve müşteriler için dükkanında sergilemek istiyor. Arsanın boyutlar çok büyüktür. Ayşe'nin ölçme işlemini en kısa yoldan yapması için neleri bilmesi yeterlidir?

Teleferiklerin güvenli olması için teleferik halatının yer ile yaptığı açı 30 dereceyi geçmemelidir. 450 metre uzakta 300 metre yüksekliğe çıkması tasarlanan bir teleferik güvenli midir?

REFERENCES

Ağaç, G., 2009, *Lise Öğrencilerinin Trigonometri Alanında Grafik Hesap Makinesi Kullanımının Akademik Başarıya ve Problem Çözme Becerisine Etkisi*, Yüksek Lisans Tezi, D.E.Ü. Eğitim Bilimleri Enstitüsü, İzmir.

Akpınar, A., 2000, *Lise 2 Matematik*, Oktay Yayıncılık.

Altun, M., 2000, *Matematik Öğretimi*, İstanbul, Alfa Yayınevi.

Altun, M., Ç. Arslan and Y. Yazgan, 2004, "Lise Matematik Ders Kitaplarının Kullanım Şekli Ve Sıklığı Üzerine", *Uludağ Üniversitesi Eğitim Fakültesi Dergisi*, pp.131.

Altuntaş, A., 2009, *Analitik Geometri Konu Özetli Soru Bankası*, Birey Yayıncılık.

Arcavi, A., 1998, "The Everyday and the Academic in Mathematics, Everyday and Academic Mathematics in the Classroom", *The National Council of Teachers of Mathematics, Reston*, p. 13, Virginia.

Aydın, N. and N. Asma, 2001, *Lise 2 Matematik Ders Kitabı*, Aydın Yayınları.

Barnes, J.A., 1999, "Creative writing in trigonometry", *The Mathematics Teacher*, Vol. 92, No: 6; pp. 498.

Bishop, A.J., K. Clements, C. Keitel, J. Kilpatrick and C. Laborde, 1996, *International handbook of mathematics education*, Boston: Kluwer Academic Publishers.

Boaler, J., 2005, *Connecting mathematical ideas: middle school video cases to support teaching and learning*, Portsmouth, NH : Heinemann.

Bonwell, C.C. and J. A. Eison, 1991, "*Active Learning: Creating Excitement in the Classroom,*" ASHEERIC Higher Education Report, George Washington University, Washington, DC.

Brenner, M.E, 2002, "Everyday Problem Solving and Curriculum Implementation: An Invitation to Try Pizza", *The National Council of Teachers of Mathematics*, Reston, Virginia.

Brown, J., A. Collins and P. Duguid, 1989, "Situated Cognition and the Culture of Learning", *Educational Researcher*; Volume:18, No:1, pp. 32-42.

Brown, R.G., 1994, *Advanced Mathematics*, Houghton Mifflin, Boston.

Bruffee, K., 1995, "Sharing Our Toys: Cooperative Learning Versus Collaborative Learning", *Change*, p. 12.

Büyüköztürk, Ş., 2003, *Veri Analizi El Kitabı*, Pagem Yayıncılık, Ankara.

Cavanagh, M., 2008, "Trigonometry from a Different Angle", *The Australian Mathematics Teacher,* Volume:60, No:1, pp.25-30.

Choi-Koh, S.S., 2003, "Effect of Graphing Calculator on a 10th Grade Student's Study of Trigonometry"., *The Journal of Educational Research,* Volume:96, No:6, pp.359-369.

Civil, M.,2002, "Everyday Mathematics, Mathematicians' Mathematics and School Mathematics: Can we bring them together?", The *National Council of Teachers of Mathematics*, Reston, Virginia.

Çavdar, A., 2003, *Lise Matematik 2*, Zambak Yayınları.

Delice, A. and M. Monaghan, 2005, "Tool Use in Trigonometry in Two Countries", *Fourth Congress of the European Society for Research in Mathematics Education,* p.989.

Derman, Z., 2008, *Matematik 10. Sınıf,* Ayrıntı Basımevi.

Doğan, A., 2001, *Genel Liselerde Okutulan Trigonometri Konularının Öğretiminde Öğrencilerin Yanılgıları, Yanlısları ve Trigonometri Konularına Karsı Öğrenci Tutumları Üzerine Bir Arastırma,* Doktora Tezi, Selçuk Üniversitesi Fen Bilimleri Enstitüsü, Konya.

Driscoll, M.J and J. Confrey, 1986, *Teaching mathematics : strategies that work, K-12,* Imprint Portsmouth, NH : Heinemann.

Emlek, B., 2007, *Dinamik Modelleme ile Bilgisayar Destekli Trigonometri Öğretimi,* Yüksek Lisans Tezi, Selçuk Üniversitesi Fen Bilimleri Enstitüsü, Konya.

Evans, R., 1973, "Jean Piaget: The Man and His Ideas", E. P. Dutton & Co., Inc., New York.

Fauzan A., D. Slettenhaar and TJ. Plomp, 2002, "Traditional mathematics education vs. realistic mathematics education: Hoping for changes", *Proceedings of the 3rd International Mathematics Education and Society Conference,* Copenhagen, Denmark: Center for Research in Learning Mathematics.

Felder, R.M., 1996, "Navigating The Bumpy Road to Student- Centered Instruction", *College Teaching,* Volume: 44, pp. 43-47.

Filiz, A., N. Özsoy and Z.F. Koçak, 1999, "Bilgisayar Destekli Trigonometri Öğretimi", www.ab.org.tr/ab05/tammetin/104.doc.

Freudenthal, H., 1991, *Revisiting mathematics education: China lectures,* Imprint Dordrecht ; Boston : Kluwer Academic Publishers.

Gainsburg, J., 2008, "Real world connections in secondary mathematics", *Teaching Mathematics and Its Applications,* Volume 26, No:2, pp.199-219.

Gay, L.R., 1996, *Educational Research, Merril,* New Jersey.

Gündüz, S., 2004, *Matematik Projeleri ve Sınıf Etkinlikleri,* Toroslu Kitaplığı.

Güven, B., 2007, "Using dynamic geometry software to convey real-world situations into the classroom: the experience of student mathematics teachers with a minimum network problem", *Teaching Mathematics and Its Applications*, Volume 27, No:1, pp.24.

Hadi., S., T. Plomp, and T. Suryanto, 2002, "Introducing Realistic Mathematics Education to Junior High School Mathematics Teachers in Indonesia", *2nd international conference on the teaching of mathematics*, New York, http://www.zentralblatt-math.org/matheduc/en/?q=au:Hadi%2C%20S%2A

Herrington, J. and R. Oliver, 1995, "Critical Characteristics of Situated Learning: Implications for the Instructional Design of Multimedia", *Learning with technology*, Parkville, Vic:University of Melbourne, pp. 235-262.

Kalaycı, Ş., 2008, *SPSS Uygulamalı Çok Değişkenli İstatistik Teknikleri*, Asil Yayın Dağıtım, Ankara.

Kaplan, E., 2009, *Ortaöğretim Matematik 10. sınıf Ders Kitabı*, Paşa Yayıncılık, Ankara.

Kendal, M. and K. Stacey, 1997, "Teaching Trigonometry", staff.edfac.unimelb.edu.au/~kayecs/publications/1997/KendalStacey-Trig.pdf

Lave, J. and E. Wenger, 1991, *Situated Learning: Legitimate peripheral participation,* Cambridge: University of Cambridge Press.

Martin, T., S. Rivale and K. Diller, 2007, "Comparing Challenge-Based and Traditional Instruction in Biomedical Engineering", International Conference on Engineering Education , pp.1-2.

Masingila, J.O., 2002, "Examining Students' Perceptions of Their Everyday Matematics Practice, Everyday and Mathematics in the Classroom", *The National Council of Teachers of Mathematics*, pp.30-31, Reston, Virginia.

McLellan H., 1996, *Situated learning perspectives*, Imprint Englewood Cliffs, N.J. : Educational Technology Publications

Merril, C., 1983, *Pre-Calculus Mathematics*, Merril Publishing Company, Ohio.

Millis, B. and P. Cottell, 1998, "Cooperative Learning for Higher Education Faculty", American Council on Education, ORYX Press.

Moffet, P., 2008, "Report on visits to the Freudenthal Institute, Utrecht University, the Netherlands and the Center for Mathematics Education at Manchester Metropolitan University", England, www.ucet.ac.uk/downloads/1532.pdf

Moschkovich, J.N, 2002, "An Introduction to Examining Everyday and Academic Mathematical Practices", *Everyday and Mathematics in the Classroom*, The National Council of Teachers of Mathematics, pp. 1-8, Reston, Virginia.

Mosvold, R., 2005, "Mathematics in everyday life:A study of beliefs and actions", *The Research Council of Norway* .p .57.

Moschkovich, J.N, 2002, "Bringing Together Workplace and Academic Mathematical Practices Duing Classroom Assesments", *Everyday and Mathematics in the Classroom*, The National Council of Teachers of Mathematics, pp.93-96, Reston, Virginia.

Muschla, J.A. and G.R. Muschla, 2006, *Hands-on Math Projects*, Jossey-Bass Wiley Imprint, San Fransisco.

National Council of Teachers of Mathematics, 2000, *Principles and Standarts for School Mathematics*, Reston, V.A.

National Research Council (NRC) ,1998, *High school mathematics at work: Essays and examples for the education of all students,* Washington: National Academy Press.

O'Neill, G., Moore, S. and B. McMullin, 2005, *Emerging Issues in the Practice of University Learning and Teaching*, Dublin, http://www.aishe.org/readings/2005-1/collection.pdf

Orhun, N., 2004, "Students' Mistakes and Misconceptions on Trigonometry", *Journal of Studies*, pp 208-211, http://math.unipa.it/~grim/AOrhun

Örnek, S., 2007, *Trigonometrik Kavramların Canlandırma Yöntemi İle Öğrenilmesinin Öğrencilerin Matematik Başarısına Etkisi*, Yüksek Lisans Tezi, Marmara Üniversitesi Eğitim Bilimleri Enstitüsü, İstanbul.

Öztürk, C., 2002, *Konu Anlatımlı-Çözümlü Geometri*, Kavram Dershaleri.

Polya, G., 1997, *Nasıl Çözmeli*, Sistem Yayıncılık, İstanbul.

Prince, M., 2004, "*Does Active Learning Work? A Review of the Research*", Journal of Engineering Education, Volume: 93, No: 3, pp. 223-231.

Resnick L., 1996, *Linking School and Work*, San Fransisco: Jossey-Bass Publishers.

Rogers, C.R., 1983, *Freedom to Learn, A View of What Education Might Become*, Merril Pub., Ohio.

Saxe, D.W., 1991, *Social studies in schools: a history of the early years*, Albany : State University of New York Pres.

Schwalbach, E.M., 2000, "Developing Student Understanding: Contextualizing Calculus Concepts", pp.90-93, *School Science and Mathematics*, Academic Research Library.

Sertöz, S., 2003, *Matematiğin Aydınlık Dünyası,* Tübitak Yayınları, pp.5, Ankara.

Stephens, G.P., 1997, "Trigonometry for the energy-conscious architect",*The Mathematics Teacher*, Vol: 90, No:7; pp. 564.

Sutarto, H., 2002, "Effective Teacher Professional Development for the Implementation of Realistic Mathematics Education in Indonesia", pp.36, Indonesia.

Talim ve Terbiye Kurulu Başkanlığı (TTKB) [Turkish Board of Educaton], 2009, Ortağretim 10. sınıf Matematik Dersi Programı, http://ttkb.meb.gov.tr/ogretmen/modules.php?name=downloads&d_op=viewdownload&cid=16

Tarhan, V.,2007, *Lise 2. Sınıfta Oluşturmacı Yaklaşımla Sunulan Trigonometri Öğretiminin Öğrencilerin Tutum ve Başarılarına Etkisi*, Yüksek Lisans Tezi, D.E.Ü. Eğitim Bilimleri Enstitüsü, İzmir.

Thompson, K.A., 2008, "Students' understanding of trigonometry enhanced through the use of a real world problem: Improving the instructional sequence", *Dissertation Abstracts International Section A: Humanities and Social Sciences*, Vol 68, No:9, pp. 3777.

Weber, K., 2005, "Students' Understanding of Trigonometric Functions", *Mathematics Education Research Journal* , Vol. 17, No. 3, pp. 91–112.

Weimer, M., 2002, *Learner Centered Teaching: Five Key Changes to Practice*, San Fransisco,http://www.google.com/books?hl=tr&lr=&id=X6IfawF1RNwC&oi=fnd&pg=PR11&dq=student+centered+teaching&ots=z1R06PEu3W&sig=8o3wBztVGJt1bjIXQ_OJCrLF944#v=onepage&q=student%20centered%20teaching&f=false

Wilkerson, L., and W.H. Gijselaers, 1996, *Bringing Problem-Based Learning to Higher Education: Theory and Practice. New Directions for Teaching and Learning*, San Francisco: Jossey-Bass Publishers.

Zulkardi, 2004, "How to Design Mathematics Lessons based on the Realistic Approach?", pp. 1-16, http://www.geocities.com/ratuilma/rme.html.