

# THESIS

ROBERT COLLEGE GRADUATE SCHOOL  
BEBEK, ISTANBUL

PAGE

**FOR REFERENCE**

NOT TO BE TAKEN FROM THIS ROOM

SPEED CONTROL OF  
INDUCTION MOTORS  
BY CHANGING AIR-GAP FLUX

SPEED CONTROL OF THREE-PHASE INDUCTION MOTORS  
BY CHANGING AIR-GAP FLUX

Thesis Submitted in Partial Fulfillment  
of the Requirements for the Degree of  
M.S in Electrical Engineering

Submitted by:

Gönül Dinçer

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Electrical Engineering Dept .

Robert College

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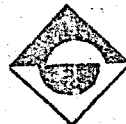
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experimental studies.

Respectfully submitted,

G.D.



124098

## A B S T R A C T

In this paper the speed control of three-phase induction motors by changing the air-gap flux is studied with D.C or Single-phase A.C voltage applied to the auxiliary stator winding.

In parts A and B the theoretical concept underlying the experimental results had been explained for both D.C and single-phase A.C control cases.

In parts C,D and E experimental studies and results are discussed.

D.C voltage applied to the auxiliary stator winding with six or twelve poles proved to be more convenient for the speed control.

The theoretical and experimental studies indicates the possibility of a smooth speed control method for three-phase induction motors within certain limits.

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## A. SPEED CONTROL OF THREE-PHASE INDUCTION MOTORS BY USING D.C IN THE AUXILIARY STATOR WINDING:

### I. Introduction:

The use of direct current in the stator of an induction motor was described as early as 1910 by Hellmund and as late as 1953 by La Pierre. After 1953 few other writers worked on the same subject, as it will be seen from the references. The economy of this method of braking as well as much empirical data has been presented by Harrel and Hough. A usefull graphical method for obtaining the speed torque curves for a wound-rotor induction motor during braking has been published by Harrison.

The application of direct current to two terminals of a 3-phase induction motor, auxiliary stator winding sets up stationary magnetic poles around the stator periphery. If the rotor is turning, the rotor bars cut the D.C flux and a voltage is produced in the rotor. The resulting rotor current acting with the flux produces a retarding torque on the rotor and the motor is braked, if we assume that only D.C is applied to the stator. Rotor reduces its speed if at the same time the normal three-phase voltage is applied to the main stator winding. The current flowing in the rotor is alternating at a frequency depending upon the speed.

A simple graphical construction is developed for predicting the dynamic braking torques of slipring and squirrel cage induction motors, taking full account of saturation and of the rotor leakage reactance. It has been shown by test results that the rotor-circuit reactance may often be neglected where great accuracy is not required.

## LIST OF PRINCIPAL SYMBOLS

- $V_1$  - Stator supply voltage perphase, volts.
- $V_2$  - Rotor E.M.F., perphase, at angular frequency
- $W_s$  - volts
- $I_1$  - Stator current per phase, amp
- $I_2$  - Rotor current perphase, amp
- $I_m$  - Magnetizing current perphase, amp
- $I_D$  - Direct stator excitation current, amp
- $V_D$  - Direct stator voltage, volts
- $X_1, X_2, X_m$  - Stator leakage reactance, rotor leakage reactance and magnetizing reactance, respectively expressed in ohms perphase at angular frequency, fs.
- $R_1$  - Stator resistance perphase, ohm
- $R_2$  - Rotor circuit resistance perphase, ohm
- $R$  - Equivalent rotor circuit resistance perphase, ohm
- $W_s$  - Unit angular frequency or synchronous speed, electrical rad/sec
- $W_1$  - Stator supply angular frequency, rad/sec
- $W_2$  - Rotor speed, electrical rad/ sec
- $N$  - Rotor speed, RPM
- $\phi$  - Air-gap flux, due to D.C excitation
- $\phi_2$  - Phase angle between E.M.F and current
- $T$  - Torque in synchronous watts per phase at synchronous speed  $W_s$  due to D.C. excitation.

II. Theory

The theory for auxiliary stator, D.C speed control of induction motor, is in fact similar to that system of electric braking called "Dynamic braking". The only difference is that, for the case of speed control or reduction we have to superimpose the "Dynamic Braking " torque and actual three-phase induction motor torque.

When the stator windings are excited by direct current, machine becomes an alternator. The alternating voltages generated in the rotor windings or bars cause currents to flow in the rotor circuit, the power dissipated in the rotor circuit resistance constituting the braking power. The resistance may be that of the rotor winding only; necessarily so in the squirrel-cage or may include external load resistances in a slip ring motor.

In view of the operating conditions, with speed ranging from zero to synchronous speed, and loads nearly always resistive, the methods of analysis used for normal alternators working are not convenient. The method described here, which is based on the equivalent circuit used for induction motors, is simple and straight forward, and takes full account of saturation and of the rotor bakage reactance.

Equivalent circuit:

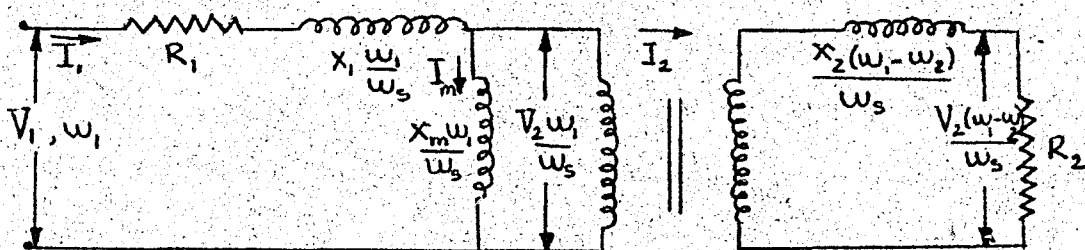


Fig. A-1

Fig. A-1 : General equivalent Circuit of Induction motor.

Stator frequency -  $\omega_1$

Rotor frequency -  $(\omega_1 - \omega_2)$

The equivalent circuit of one phase of a polyphase induction motor is shown in Fig-1, in a rather more general form than is usually adopted. The quantities  $V_2$ ,  $X_1$ ,  $X_2$  and  $X_m$  are all based on the unit angular frequency  $\omega_s$ , while the stator supply is at angular frequency  $\omega_1$ , the rotor speed is  $\omega_2$  and the rotor currents are of angular frequency  $(\omega_1 - \omega_2)$ .

For simplicity, the stator/rotor turn ratio is taken as unity, that is all quantities are referred either to stator or rotor, whichever is the more convenient. Fig-1, shows the actual values of the E.M.F and the reactances for the assumed conditions, and Fig-2 shows the corresponding vector diagram.

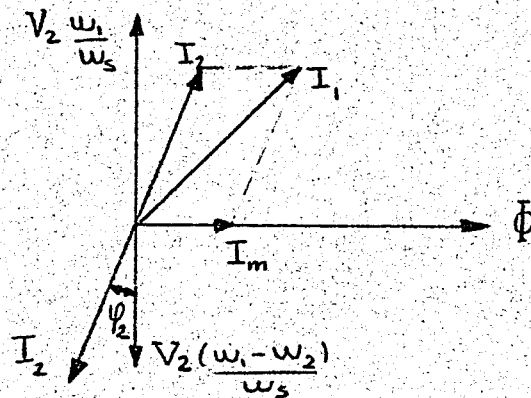


Fig. A-2 : Vector diagram for the equivalent circuit of the induction motor.

The stator iron loss is neglected so that  $I_m$  is in phase with  $\Phi$ , and in quadrature with  $V_2$ . This is fully justified when the stator current is direct, that is when  $\omega_1 - \omega_2 = 0$ .

The torque exerted on the rotor is proportional to  $\phi I_2 \cos Q_2$ , and since  $\phi$  can be taken as proportional to  $V_2$ , the torque expressed in synchronous watts perphase, based on the synchronous speed  $\omega_s$ , is given by :

$$T = V_2 I_2 \cos Q_2$$

The equivalent static circuit is then derived by dividing the rotor resistance, reactance and EMF by  $\frac{(\omega_1 - \omega_2)}{\omega_s}$ , so

that the actual rotor circuit is replaced by one with the same flux linkage but with angular frequency  $\omega_s$ . The stator circuit is also modified similarly dividing its voltage, resistance and reactance by  $\omega_1/\omega_s$ . This leaves the currents and phase angles unchanged. The ideal 1:1 ratio transformer may then be eliminated to give the resultant circuit shown in Fig-3, in which the angular frequency throughout is  $\omega_s$ .

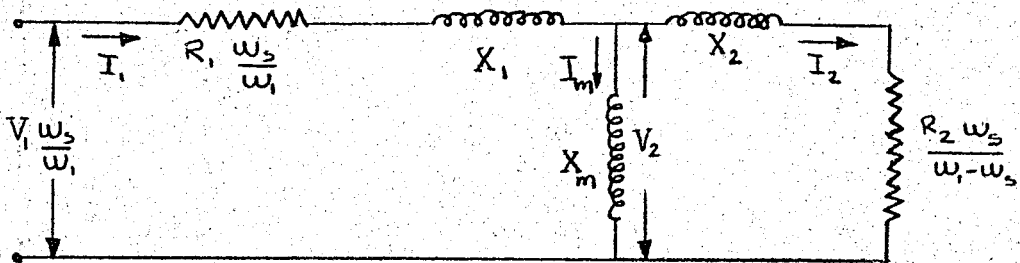


Fig A - 3 : General equivalent Circuit  
Frequency =  $\omega_s$

The torque per phase,  $T$ , expressed in synchronous watts referred to the synchronous speed  $\omega_s$  is now given by the power dissipated in the rotor resistance:

$$R_2 W_s / (W_1 - W_2) \quad \text{so that,} \quad T = I_2^2 R_2 W_s / (W_1 - W_2)$$

The circuit shown in Fig-3 is valid for any values of  $W_1$  and  $W_2$ . Under dynamic braking conditions and in our case, stator current is direct,  $W_1 = 0$ , and the equivalent stator applied voltage and resistance become infinite. This corresponds to the conditions assumed in the application of Thevenin's theorem for a constant current. When  $W_1 = 0$  the effective rotor resistance becomes

$$-R_2 \frac{W_s}{W_2} \quad \text{and} \quad T = -I_2^2 R_2 W_s / W_2 \quad \text{which, being negative}$$

represents a braking torque. In the usual dynamic braking equivalent circuit, the negative sign of the equivalent rotor resistance is omitted, which is quite in order, so long as it is recognized that the torque is opposite in direction to the rotation.

The stator current  $I_1$  is the alternating current of angular frequency  $W_s$ , which is equivalent to the actual direct stator excitation current  $I$ .

If all quantities are to be referred to the rotor, which is usually more convenient for a slipping motor, the values  $I_1$  must be multiplied by the transformation ratio of the machine which is 1 in this case.

By specifying the  $I$  and  $I_1$ , the stator impedance and voltage need not be shown and Fig-3 can be simplified to Fig-4:

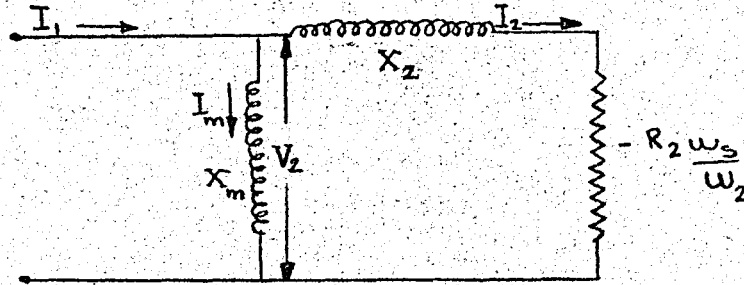


Fig A-4 : D.C control equivalent circuit.

Frequency =  $\omega_s$

For unsaturated conditions  $X_m$  is constant, and the maximum torque is found to occur when:

(1)  $R = (X_2 + X_m)$  (Eq. 5 of appendix II)

(2)  $T_{max} = \frac{I_1^2 X_m^2}{2(X_2 + X_m)}$  (Eq. 6 of appendix II)

The above equations show that the maximum torque for a given value of  $I_1$ , or of  $I_p$  is independent of  $R$ , but that the speed,  $\omega_2$ , at which the maximum torque occurs varies directly as the rotor-circuit resistance  $R_2$ , since  $R = R_2 \frac{\omega_s}{\omega_2}$ . This relation

ship between resistance and speed is analogous to that between resistance and slip in normal induction motor operation. Thus if the relationship between the torque and  $R$  for any particular value of  $I_1$  is derived as shown in appendix II. The torque/speed curves for a given value of  $R_2$  may be obtained from it and also the torque/resistance curves for given speeds. The result of the derivations done in Appendix III are:

$$(3) \quad W_2 = W_s \frac{R_2}{m} \tan \theta \quad (\text{Eq. 12 of Appendix III})$$

$$(4) \quad T = \frac{KmI_1^2}{2} \sin 2\theta \quad (\text{Eq. 13 of Appendix III})$$

In these equations:

$m$  = slope of the magnetization curve

$\theta$  = angle of  $I_1$

$$K = \frac{7.04 \times 3}{W_s}$$

To confirm the accuracy of the graphical construction described in Appendix III, for predicting the dynamic-braking torques of induction motors, a number of tests were carried out by D. Harrison on a 15 H.P., 3 phase, 400 volt, 50 cycle, 6 pole squirrel cage motor over the speed range 0 - 1000 rpm.

The results obtained together with those predicted are shown in Figures 9-10.

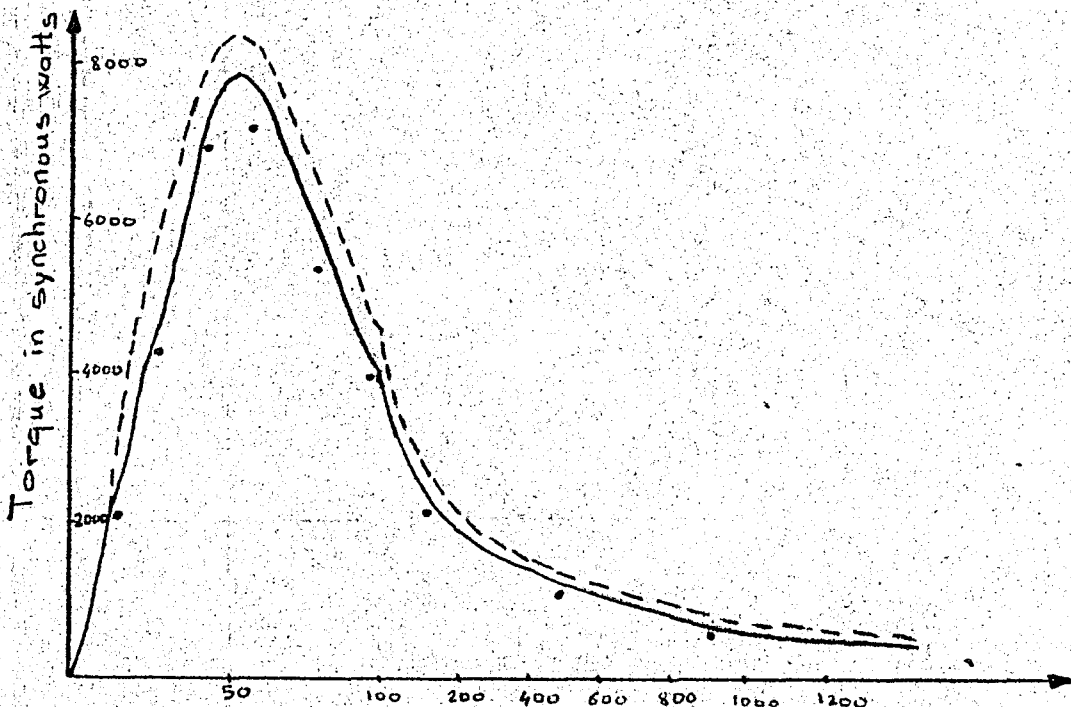


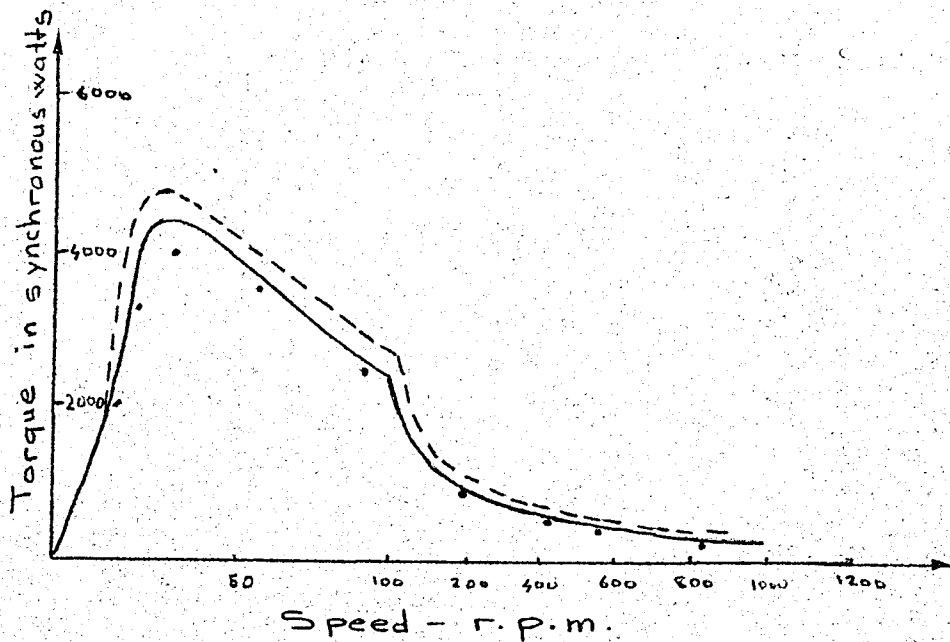
Fig- A-5 : Speed r. p. m.

Fig- A-5: Torque-speed curve characteristic of Squirrel Cage.

$I_{DC} = 14$  amp

$W_s = 800$  rpm

- \_\_\_\_\_ Predicted allowing for  $X_2$
- Predicted neglecting  $X_2$
- ..... Experimental



Speed rpm

Fig- A-6 : Torque speed characteristic of

$I_{D.C} - 10$  Amp

It should be noted that these curves show the torque-speed characteristics, when only the auxiliary winding is fed by DIRECT CURRENT AND ROTOR IS TURNING. When we, at the same time, apply the three-phase voltage to the main windings, the actual speed-torque curve will be the superimposition of the two speed-torque curves, one is the ordinary three-phase induction motor curve, the other is the curve due to the auxiliary winding, this will be shown later.

If we look at the figures 5-6, it is seen that the predicted torques allowing for  $X_2$  are more accurate than those obtained if  $X_2$  is neglected, particularly around the peak torque.

The torque-speed curves shown in figures 5-6 are the braking or the negative torques which will reduce the speed of the rotor of an induction motor. To see the overall effect, that is, the net effect of the auxiliary winding D.C voltage on the actual three-phase induction motor operation, we have to draw the induction motor torque-speed curve due to the main 3-phase A.C voltage and subtract from it for each speed value the negative torque effect due to the auxiliary D.C winding.

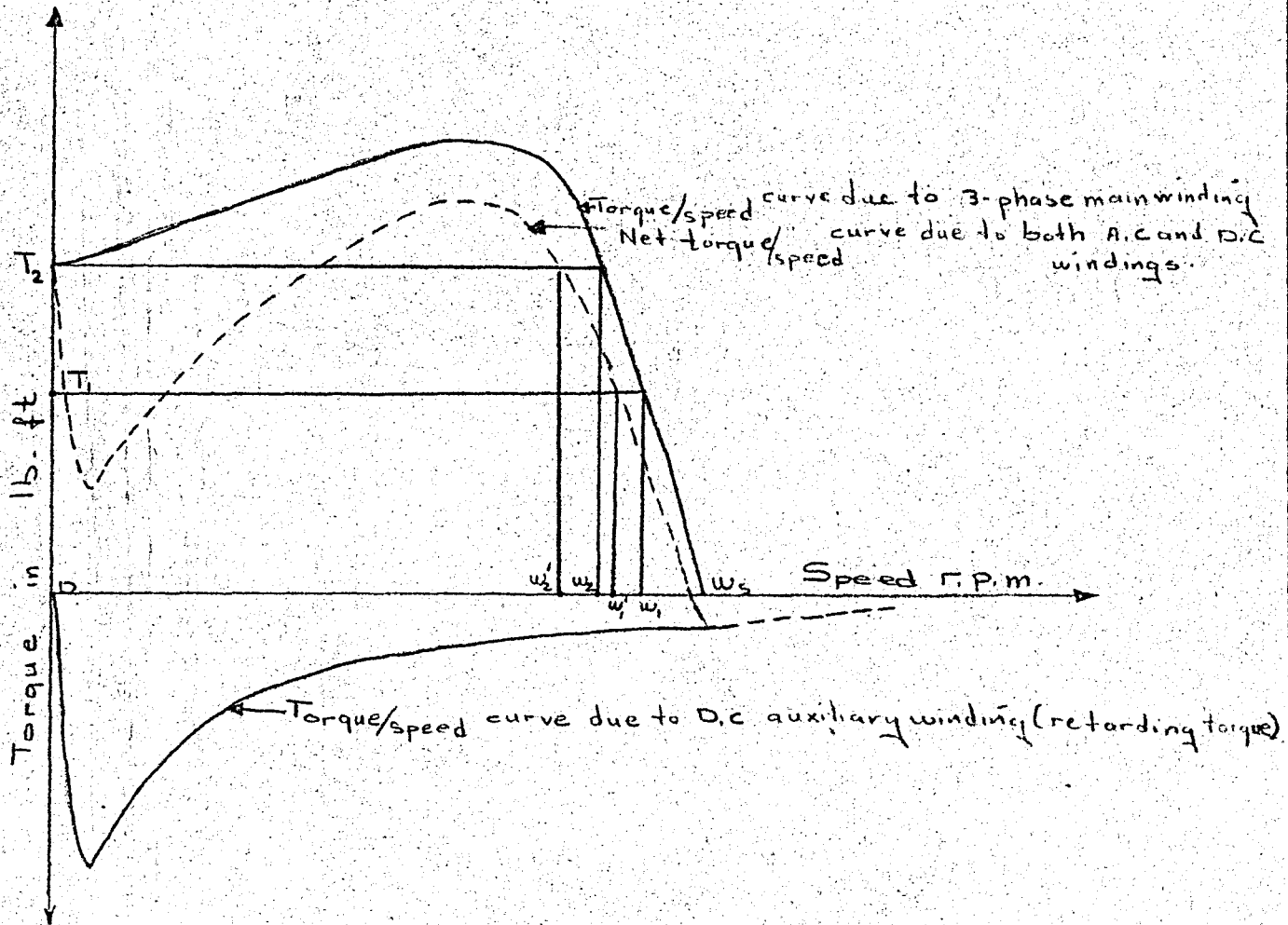


Fig. A-7 : Net Torque/Speed curve for speed control by means of D.C

If we examine the Figure 7, we see that for a given torque  $T_1$ , the speed corresponding to the actual three-phase characteristics is  $W_1$ , and the speed corresponding to the same torque on the composite characteristic for speed control is,  $W_1^1$ . But we should note that this difference in speed is not very large, so the effect of the auxiliary winding with D.C impressed to it is not very large. On the other hand, for larger values of torque, let's say,  $T_2$ , the change in speed that can be obtained by this method is larger:  $W_2^k - W_2 > W_1^1 - W_1$ . The other important point to be noted is the fact that the use of D.C in the second stator winding decreases the maximum torque that can be obtained.

The effect of larger D.C voltage to be applied to the auxiliary winding is to increase the speed change. This fact is illustrated by Fig. 8:

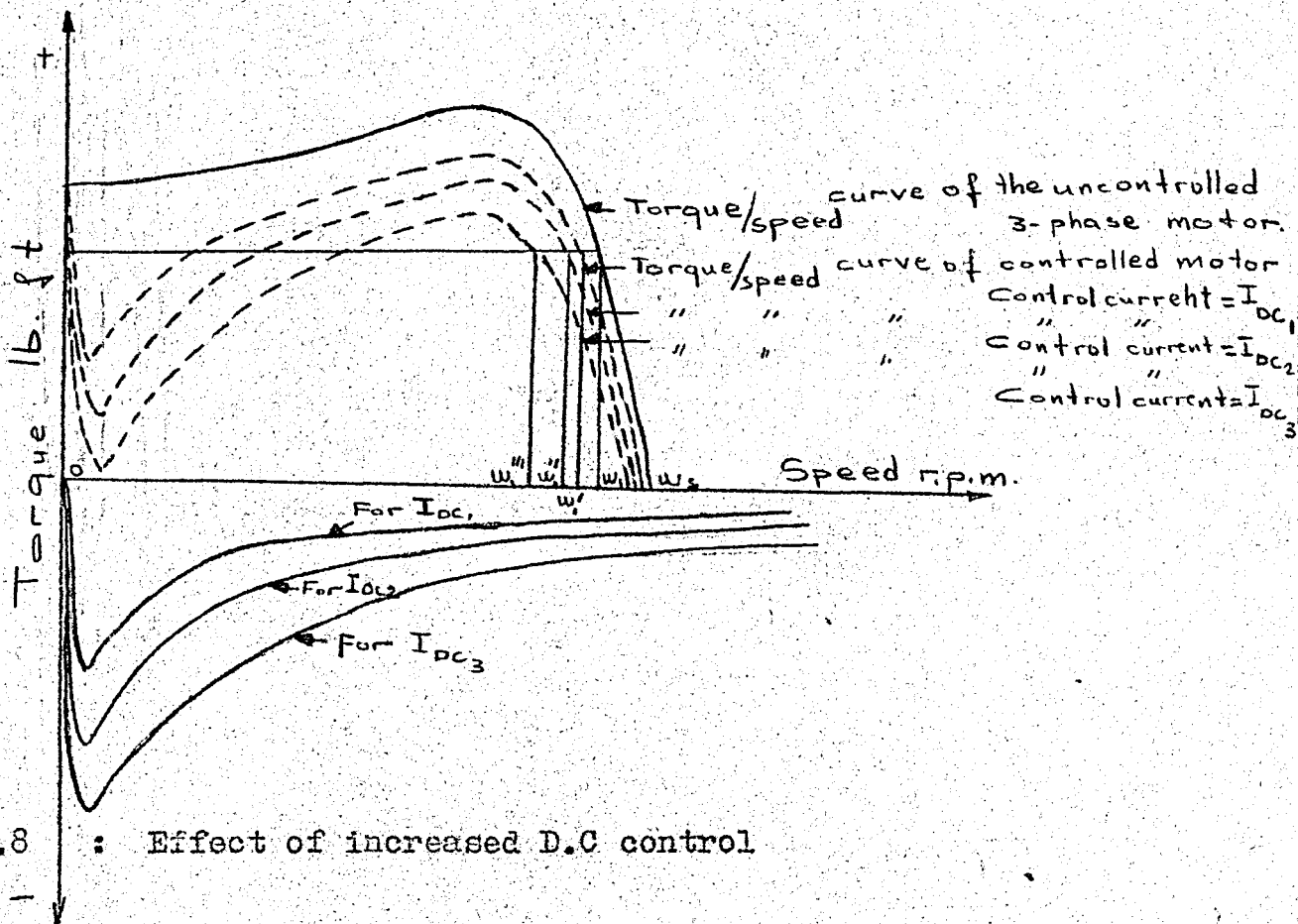


Fig. 8 : Effect of increased D.C control

As it is seen from Figure 8, the effect of increasing the control, D.C current is to decrease the speed more for the same torque  $T_1$  :

$$W_1^{111} - W_1 > W_1^{11} - W_1 > W_1^1 - W_1$$

Thus we can say that, by increasing the D.C control voltage or current, we can increase the effect of speed control; but it should never be forgotten that, the increase in the control voltage, will always decrease the maximum torque.

The analysis carried on is for the auxiliary winding wound FOR THE SAME NUMBER OF POLES AS THE MAIN STATOR WINDING. A question may arise as to the effect of the auxiliary stator winding with different number of poles than the actual three-phase stator winding. To answer this question we have to turn back to equation 3, where

$$W_2 = W_s \frac{R_s}{m} \tan \theta \quad \text{which gives the speed due to the braking}$$

torque. In this equation the only term which can change by the number of poles of the auxiliary winding is  $m$ , the slope of the magnetization curve. With greater number of poles for the auxiliary winding, the flux and the saturation will be such that the slope of the magnetization curve will be larger. In equation 3, the other terms being unrelated to the auxiliary winding and its magnetization their effect will be the same for a given D.C control excitation value. The only changing term, slope of the magnetization curve ( $m$ ), being increased in the denominator the rotor speed to the retarding effect of the braking torque will be smaller. This shows that we can obtain more powerful speed control with auxiliary winding poles larger than the main winding number of poles for a given D.C control voltage and power loss.

The experiments carried out in Robert College Laboratories with four, six and twelve poles for the auxiliary winding, and good results had been obtained especially with 6 and 12 poles connections, as shown in parts D-III; (A-1) , and E-I of this theisis.

## B. SPEED CONTROL OF THREE-PHASE INDUCTION MOTORS BY APPLYING SINGLE-PHASE A.C TO THE AUXILIARY STATOR WINDING:

### I. Introduction:

After studying the effect of D.C applied to the auxiliary winding during the normal three-phase operation, it will be wise to analyse also the effect of single-phase A.C applied to the same auxiliary winding instead of D.C. The experiments carried out show that it is again possible to get a speed change; however, the change is not very big.

### II. Theory:

The actual three-phase stator winding of the induction motor will provide a torque, at a given speed, determined by the torque/speed characteristics of the motor. And the rotor will turn in the same direction as the rotating three-phase magnetomotive force of the air-gap. When we apply single-phase A.C to the auxiliary winding wound for the same number of poles as the actual field structure, there will be a new field developed, alternating (pulsating) but stationary in space with respect to the stator. The effect of this new field on the torque/speed curve of the machine will be analysed. Considering this pulsating, magnetomotive force as composed of two rotating fields, each rotating in opposite directions with synchronous speed, as used in single-phase induction motor theory. This principle can be applied to every harmonic of the pulsating rectangular wave produced by the single-phase winding. The detailed analysis is given in Appendix IV at the end of the theory part. Figure 8 of Appendix IV gives the general shape of the torque/

speed curve of a single-phase A.C flux, due to main, third and fifth harmonics.

Since the flux due to the auxiliary winding is stationary in space with respect to the stator, but rotor is turning in the same direction as the rotating field of the actual three-phase winding, the effect of the pulsating field due to single-phase auxiliary winding is as if it is turning in the opposite direction with respect to the rotor or to the main rotating field. Thus we can reverse the curve shown above to see its subtractive effect on the main field torque.

The superposition of the auxiliary single-phase winding torque/speed curve on the 3-phase main field torque/speed curve will give us the actual effective torque/speed characteristic of the motor

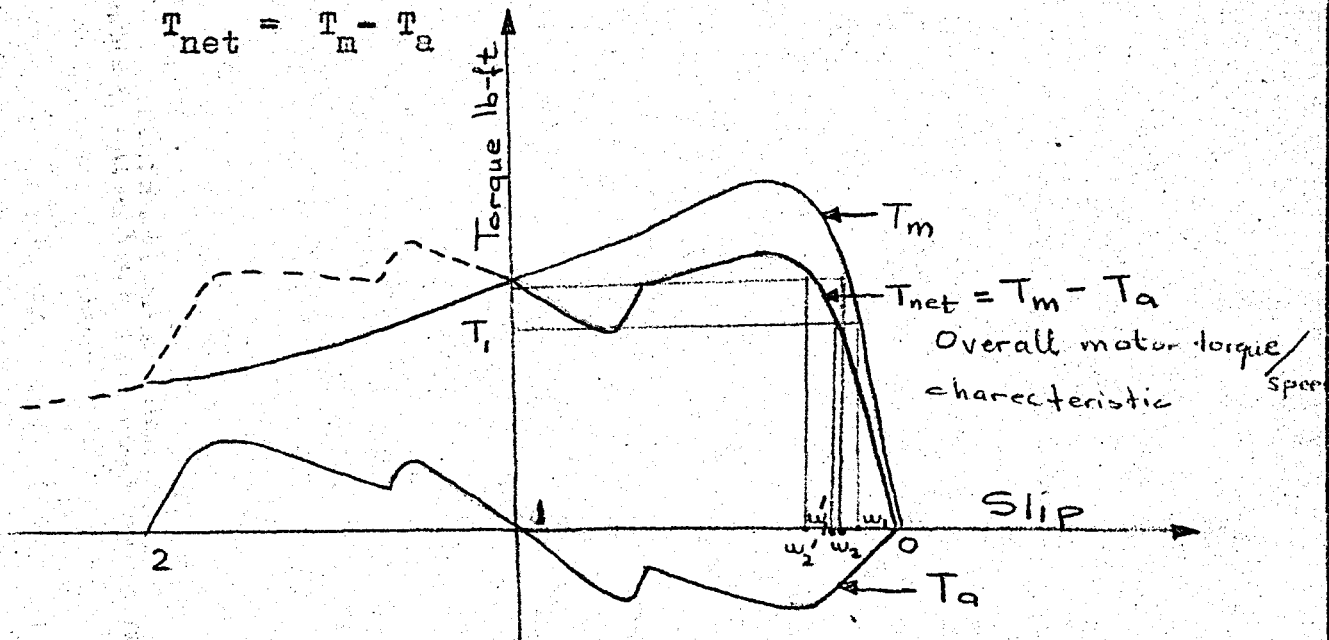


Fig. B-1 : Overall torque/speed characteristic for A.C single-phase auxiliary winding speed control.

By For an induction motor stable working region is the small slips. Keeping in mind this fact and examining Figure 9, it

can be seen that for a given torque  $T$ , the corresponding stable speed from the uncontrolled three-phase motor characteristic,  $T_m$ , is  $w_1$  and for the same torque corresponding stable speed from the controlled torque/speed characteristic,  $T_{net}$ , is  $w_1^1$ , where  $w_1^1 < w_1$ . But it should be noted that the change in speed of the motor obtained by this method is quite small. For larger torques than  $T_1$ , let's say for  $T_2$ , the speed change is larger:

$$w_2^1 - w_2 < w_1^1 - w_1 .$$

The other important point to be noted is the fact that the controlled motor maximum torque is smaller than the uncontrolled one, due to the retarding effect of the auxiliary winding torque,  $T_a$ . The speed control by the method mentioned above is in one direction only, that is, we can only decrease the speed, not increase it.

C. EXPERIMENTAL STUDIES

To study the control of the induct on motor with A.C and D.C applied auxiliary stator winding, an auxiliary winding is added to the stator of a 3-phase 2 H.P squirrel cage motor.

I. Description of the apparatus Used

1- Experimented Machine:

a) The original motor is:

3-phase, 2 H.P, 4 pole, squirrel cage, 50 cycle  
190 volt, Y connected one with following specifications

Speed : 1425 rpm

Cost  $\phi$  0.8

KW : 1.5

Rated Current : 6.2

Motor No. : 2444 -217

Type : D 20/4

FABRIQUE PARALA AEG BERLIN POUR BOURLA FRERES AND CO.

Stator winding of this machine is changed to give space for the auxiliary control winding as explained in section b.

The three-phase distributed stator field winding is given by Figure C-1.

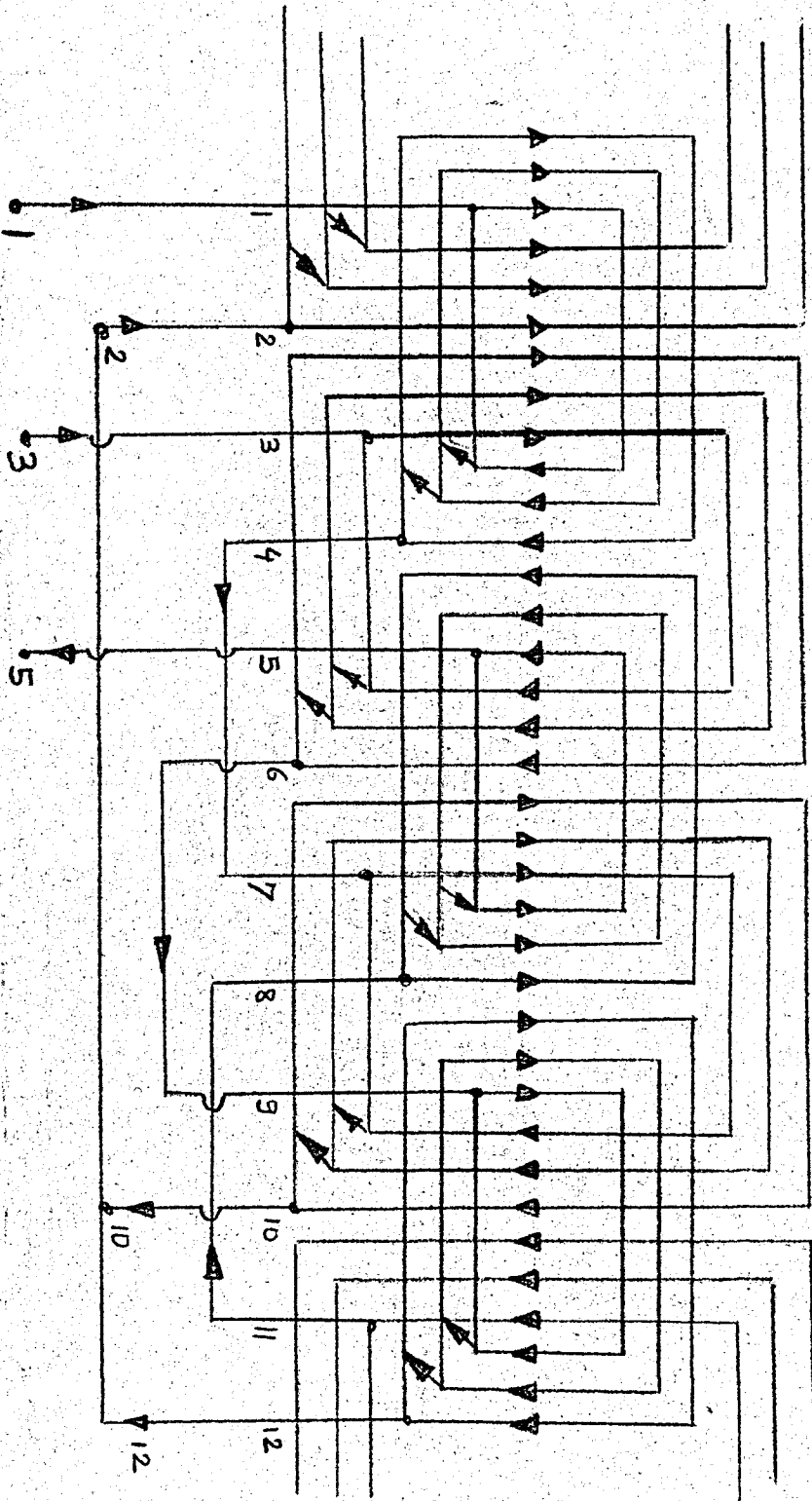


Fig. C-1 Main stator field winding of the 3-phase Y-connected induction motor.

b) An auxiliary winding was wound in the same form as the 3-phase distributed stator winding, but the coil ends are left free and numbered according to the winding diagram 10. The wire for auxiliary winding is copper wire of 1mm diameter, each coil has 52 turns and covers half of the depth of the slots, the other half being occupied by the main 3-phase field coils of 26 turns and of 1.78 mm diameter copper wire. The wiring diagram for auxiliary winding with coil ends numbered, given under Fig. C-3.

A.C Instruments:

a) Wattmeter

Range: 0 - watts

Ampers: 5      10      20

Volts :125      250      375

Voltmeter Resistance: 50 /v

Ampers : 5    10    20 amps

Resistances: 0,105      0,027      0,007

b) A.C voltmeter

Weston Electric Instrument Corp.

Newark, N.3, U.S.A

Model: 433 No.124762

Range: 0 - 300 v

0 - 150 v

25 - 125 cps

Resistances: 22594 for 300 v

11294 for 150 v

c) A.C Voltmeter

Range: 0 - 300 v

0 - 150 v

25 - 125 cps



0 - 15 v

0 - 3 v

Resistance: 1000  $\Omega/v$

b) D.C Voltmeter:

Model : 931 No. 38500 voltmeter

Range : 0 - 150 v

0 - 15 v

0 - 3 v

Resistance: 1000  $\Omega/v$

c) D.C Ammeter:

Model : 931 No. 36080

Ranges: 931 39573

Ranges: 0 - 30 Amp

0 - 15 amp

0 - 3 amp

2- Sources:

a) 3-phase a-c network supply

190 / 110 v. 50 cps.

b) Direct Current Generator:

120/160 v. 30 Amp. 1410 rpm.

Self Excitation:

Reference No. ZA 355 B

Manufactured by: The English Electric Company Ltd.  
London.

c) Autotransformer - 3-phase

Variac 380/190 volt

General Radio Company

Cambridge, Mass., U.S.A

- d) Autotransformer - Single-phase  
 Variac 220/110  
 General Radio Company  
 Cambridge, Mass., U.S.A.

3- Rheostats:

- a) 1 amp- 900  $\Omega$  No. 734213  
 b) 1,5 amp 340  $\Omega$  No. 714955  
 c) 2,0 amp 220  $\Omega$  No. 734218  
 d) 5. amp

4- Speed Measurement Devices:

a) Strobotac:

115 v. 50 cps 0,5 amp  
 Speed Range : High : 2500 -14500 rpm  
                   Low : 600 -3700 rpm  
 Type No. : 631 - BLS3  
 Serial No. : 25899  
 General Radio Co.

Cambridge, Mass., U.S.A

b) Tachometer : Jaquet

Speed Ranges:           60 - 240 rpm  
                               200 - 800 rpm  
                               600 -2400 rpm  
                               2000 -8000 rpm  
                               6000 -24000 rpm

No. K 507731

Made in Switzerland .

## II. Description of the Experiments Made:

The experiments for the speed control were performed on the three phase 190 c., 50 cps., 4 poles, distributed winding, Y connected squirrel cage 2 H.P induction motor. For the experimental study of the new speed control method a separate winding should be introduced on the three-phase stator winding. To manage this, the original winding of the motor unwound and a new winding of two parts was inserted, such that the first half of the slots are occupied by the three-phase field structure with the same winding type as the original stator winding but with less turns for each coil.

The second, or auxiliary winding occupied the other half of the slot depths. The auxiliary winding have the same type of winding as the three-phase one, but with thinner wire and more turns. The ends of the coils are left free and numbered according to the diagram shown in figure C-1 so that by various connections we can get several different pole structures for the auxiliary winding, and their corresponding effects on the speed changes were observed. Experiments were carried on by four, six and twelve poles for auxiliary structure.

The experiments for the speed control of induction motor by changing air-gap flux with the help of the auxiliary winding could be made only for no load. The change made in the original field structure to decrease its number of turns, and the opposing effect of the auxiliary field to the three-phase winding caused a larger current flow in the conductors even at no load. Thus it was not possible to load the rotor shaft with loads appreciably greater than the friction load.

### No Load Test:

The No-load tests can be classified into two main groups :

- 1- Experiments with D.C supplied to the auxillary winding.
- 2- Experiments with single-phase A.C supplied to the auxillary winding.

1- No load tests with D.C supplied auxillary winding:

The circuit corresponding to the experiments is given by Figure C-2.

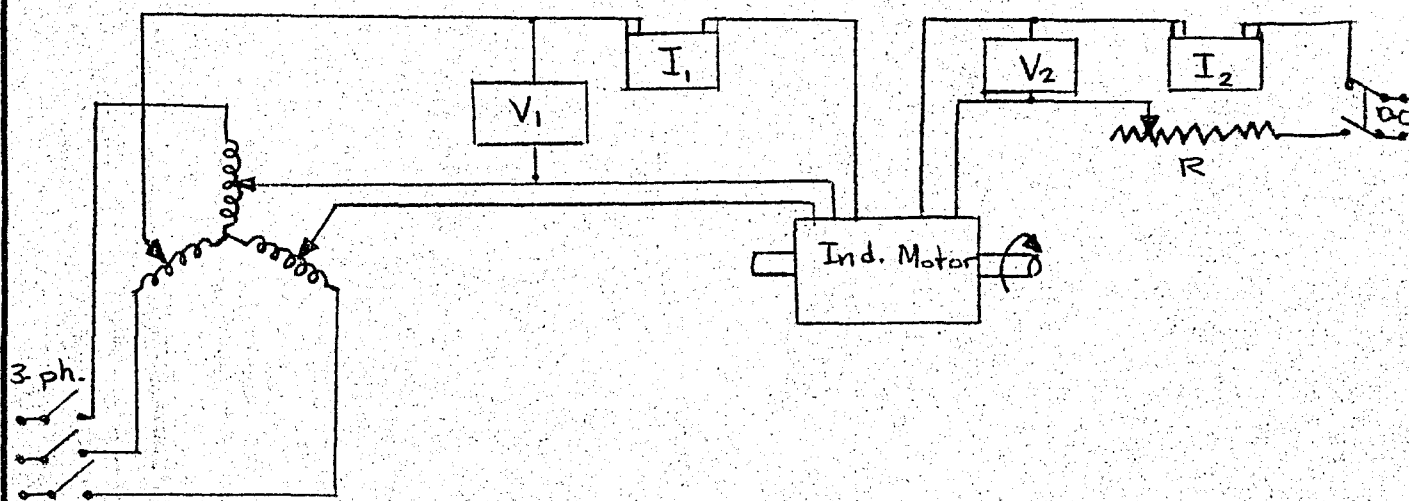


Figure C-2 : The circuit used for the speed control experiments, with D.C supplied auxiliary.

The no load experiments with D.C supply can be divided into four groups, such that:

- a- Auxiliary stator winding, connected for four poles, series connection.
- b- Auxiliary stator winding connected for four poles, parallel connection.
- c- Auxiliary stator winding connected for six poles.
- d- Auxiliary stator winding connected for twelve poles.

In each of these experiments, tests were made with different 3-phase supply voltages for the main field structure and D.C Excitation voltage current,  $V_2$  and  $I_2$  were changed in steps ,

so that we could observe the corresponding speed changes for each one.

The various supply voltages have been obtained by using three-phase autotransformer "variac" and also the excitation winding was supplied through a rheostat by the D.C generator. Excitation current  $I_2$  can be adjusted by means of rheostats,

The readings of supply voltage, supply current, excitation voltage, excitation current and speed of the rotor shaft had been taken and corresponding curves are drawn.

The experiments A,B,C, D will be described one by one:

a) Four pole series connected. D.C supplied auxiliary winding:

The necessary coil end connections to create series four pole is done by connecting ends 3 and 4; 6 and 7; 8 and 11; 9 and 10; 12 and 2, and D.C is applied across the ends 1 and 5. The corresponding winding diagram is given by Fig. C-3 . Different supply voltages applied and for each supply voltages under different excitation values speeds were read. The result is plotted in Fig. E-I-a.

b) Four pole parallel connected. D.C supplied auxiliary winding:

The necessary coil end connections is done by connecting ends 4 and 7; 6 and 9; 8 and 11. D.C is supplied across the points where ends 1-2-5 and 12-10-5 are connected. The corresponding winding diagram is given by Fig. C-4. (The result of experiments is plotted in Fig. E-I-b)

c) Six pole connected and D.C supplied auxiliary winding.

The necessary coil end connections are: 2 and 7; 4 and 9;

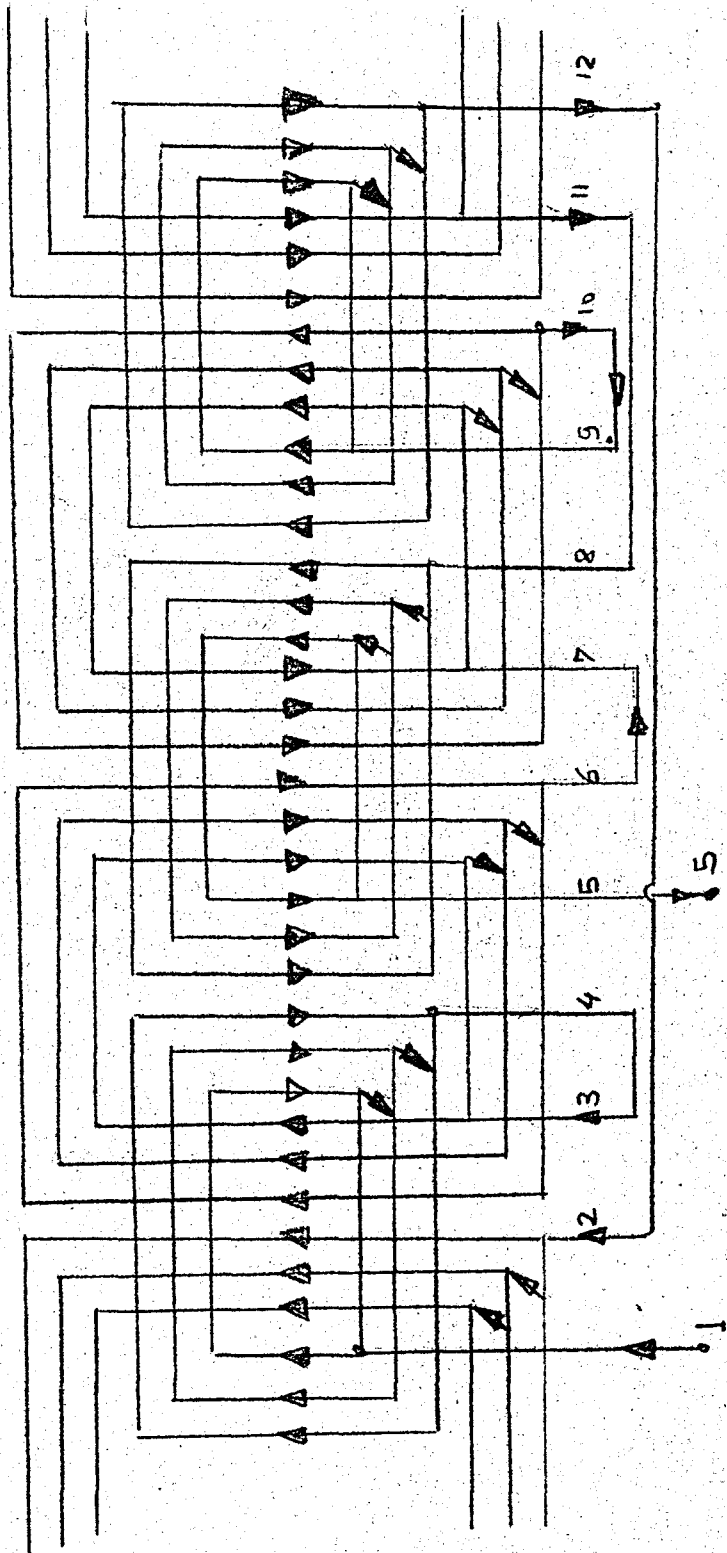


Fig. C-3 Auxiliary stator winding, 4 poles, series connected.

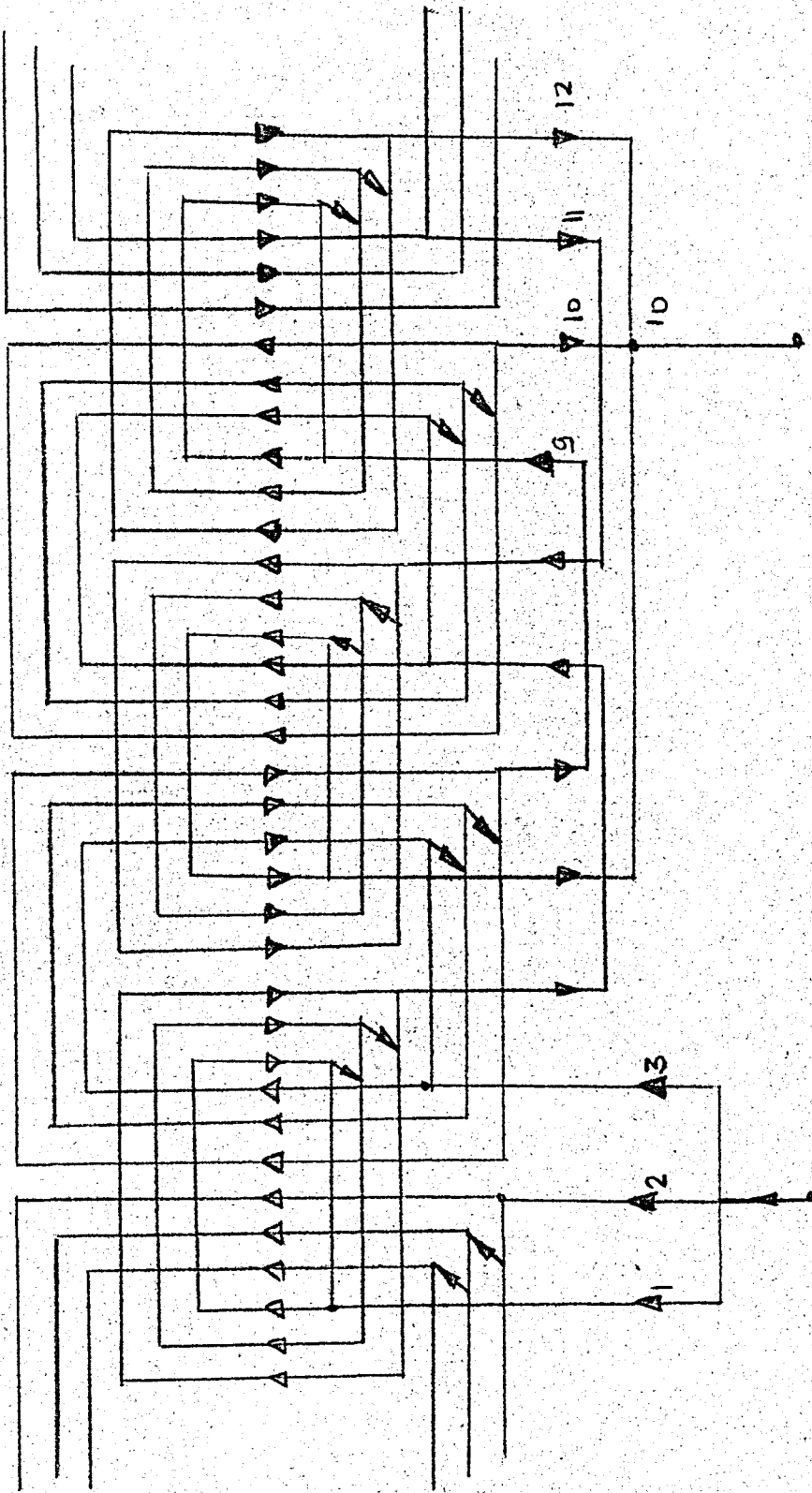


Fig. C-4 Auxiliary stator winding, 4 poles,  
Parallel connected.



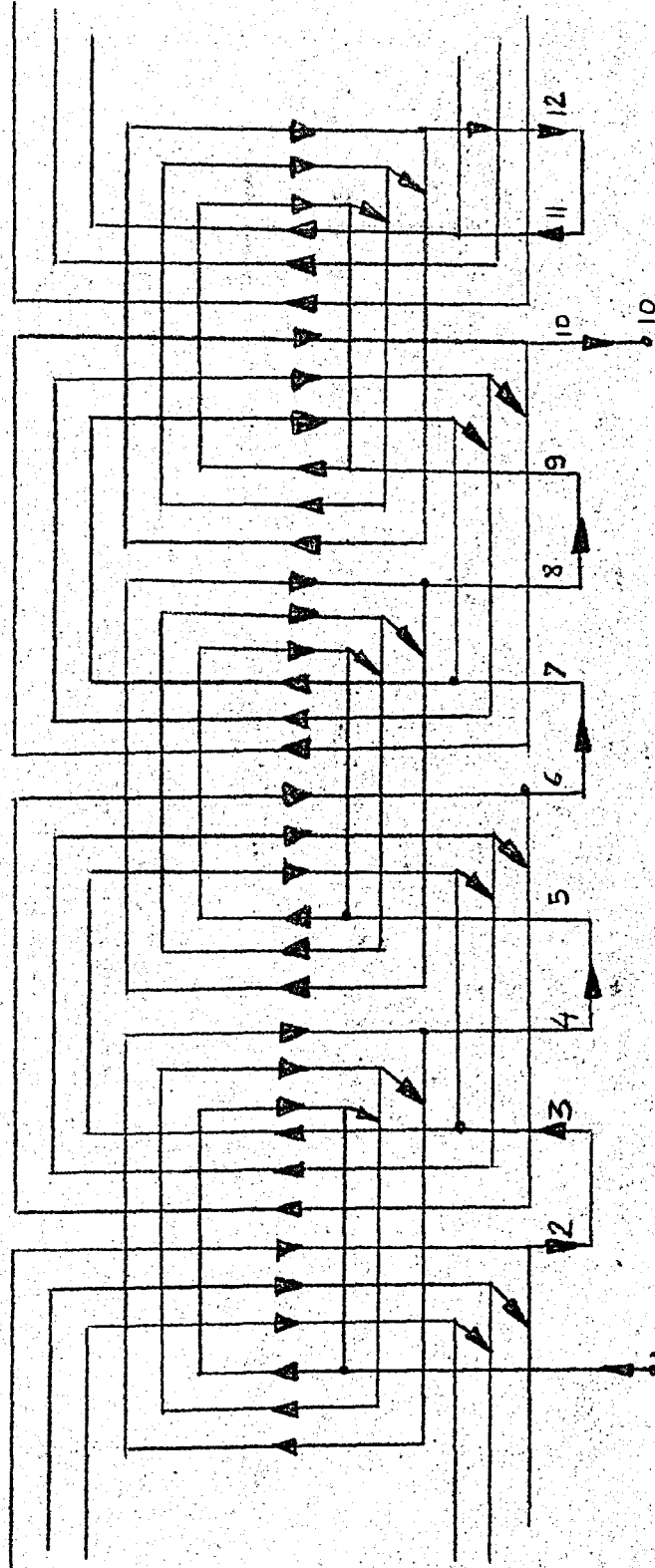


Fig. C-6 Auxiliary stator winding, 12 poles series connected.

6 and 8; 5 and 11; 10 and 12; D.C. is supplied across 1 and 3. The corresponding winding diagram is given by Fig. C-5. (Result of experiments is plotted in Fig. E-I-c.)

d) Twelve pole connected and D.C. supplied auxiliary winding.

The necessary coil end connections are: 2 and 3; 4 and 5; 6 and 7; 8 and 9; 11 and 12; D.C. is supplied across ends 1 and 10. The corresponding winding diagram is given by Fig. C-6. (Result of experiments is given in Fig. E-I-d)

2- No load test with A.C. supplied auxiliary:

The circuit corresponding to the experiments is given by Fig. C-7.

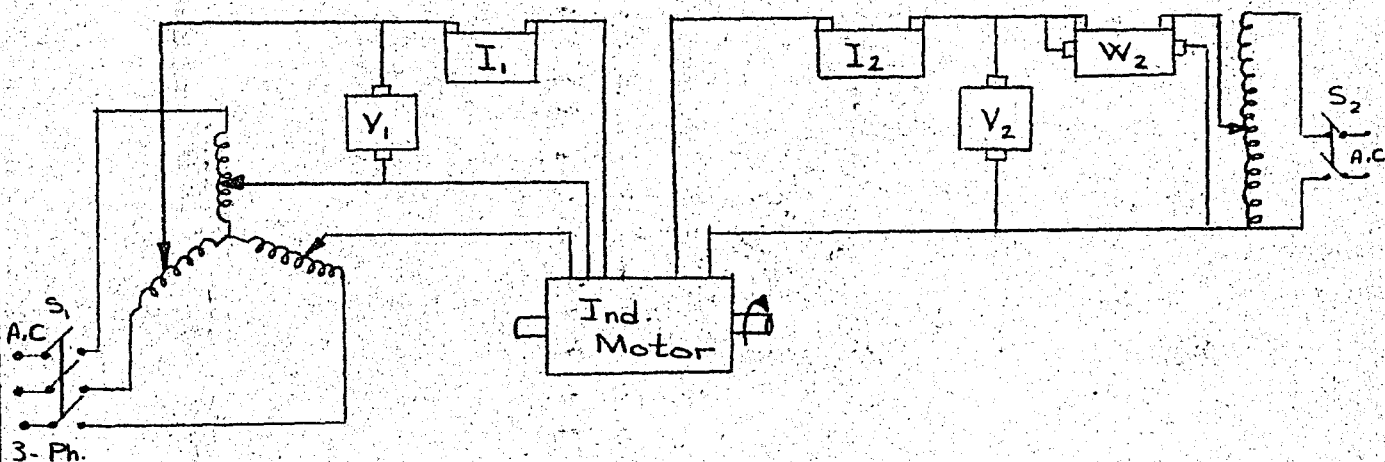


Figure C-7 : The circuit diagram for the speed control experiments with A.C. supplied auxiliary.

No load experiments with A.C. single phase supply can be divided into four groups also, as mentioned in connection with the D.C. experiments. The winding connections for four, six and twelve poles are the same as given by Figures 13,14,15,16.

### III. Comments on Experiments and Calculations:

The experiments to study the speed control by changing the air-gap flux, is done by introducing a new stator auxiliary winding.

The induction motor at hand, is designed for one speed only, so it is not suitable for all kinds of specific studies. Since in the laboratory it was not possible to find a motor more suitable for our purposes, the experiments are carried on with the motor described before, by unwinding it and again winding for half of the three-phase field structure turns, to reserve half of the slot depths for control winding.

The limited space for the stator windings changed the actual capacity and characteristics of the motor, especially its current and load-carrying capacities are decreased. Thus we could not load the motor with loads greater than friction load and so although at the beginning of the experiment studies with different loads had been planned, the experiments had to be carried out on no load.

The control current could not be increased beyond 4 amperes, due to the excess heat in the control winding. As the control current increased, as it will be seen from the readings taken, the current in the main field structure increased beyond the current carrying capacity of the wires, insulation of the main winding. The value of the maximum three-phase voltage to be applied was limited to 80-90 volts.

Small changes in the main voltage had occurred during the

experiments to avoid error, many readings are taken and the average is found to be the final value in plots.

Nevertheless, every effort had been made to perform satisfactory experimental study with the possibilities at hand and to some extent satisfactory results had been obtained, as it is seen from the speed curves.

The no load test was performed at different three-phase supply voltages up to 80-90 volts and for different pole numbers for the auxiliary winding. Curves of speed versus control current had been drawn for each value of three-phase supply voltage and this is repeated for different pole connections. The curves of speed versus control current A.C or D.C was plotted to study the effect of the control method.

### A- Control Current and Speed Relations:

#### 1. D.C control current and speed relations.

##### a) For four pole series connected auxiliary winding:

The control current versus speed curve shows a smooth change of speed with increased control current.

A greater speed change can be obtained, as the D.C control current is increased for a given supply voltage  $V$ . But for high control currents greater than 3 amps, the rate of speed change is smaller, due to D.C saturation, than for small control currents.

Per cent speed drops at  $I_2$ : 4 amp

$$\text{For } V_1 : 55 \text{ volt. Per cent speed drop} = \frac{(1485-1444)}{1485} \times 100 = 2.6\%$$

$$\text{For } V_1 : 50 \text{ volt. Per cent speed drop} = \frac{(1476 - 1432)}{1476} = 3\%$$

$$V_1 : 35 \text{ volt. Per cent speed drop} = \frac{(1450-1400)}{1450} = 3.45\%$$

From the comparison of the per cent speed drops it is easily seen that the speed change is greater with smaller 3-phase supply voltages for the same control current. This indicates the possibility that with loaded induction motor, greater speed changes can be obtained for the same control current. But to employ this speed control method with two stator windings, to loaded conditions, a special motor should be designed, with greater iron part so that the normal three-phase induction motor stator winding will have enough space, as well as the auxiliary.

b) Four pole parallel connected auxiliary:

From the speed versus control current curves it is seen that for a given supply voltage, speed change is greater for larger values of control current.

Per cent speed drops at  $I_2 = 4$  amp.

$$\text{For } V_1: 65 \text{ volt} \quad : \text{ Per cent speed drop} = \frac{(1486-1451) 100}{1486}$$

$$= 2,36 \%$$

$$\text{For } V_1: 50 \text{ volt. Per cent speed drop} = \frac{(1480-1438)100}{1480} = 2,9\%$$

$$V_1: 35 \text{ volt. per cent speed drop: } \frac{(1462-1412)100}{1462} = 3.42\%$$

For four poles parallel connected auxiliary winding, 3 parallel paths are obtained for the D.C control current to flow. Thus we can more than 4 amperes to the auxiliary winding mains without the danger of excessive heating, since actual current flowing in the auxiliary conductors is  $1/3$  of the applied current,  $I_2$ . Experiments are carried out up to 7 amperes for  $I_2$ . Thus:

Per cent speed drop at  $I_2$  : 6 amp.

For  $V_1$  : 65 volts. Per cent speed drop =  $\frac{(1486-1423)100}{1486} = 4,25\%$

$V_1$  : 50 volts. " " " " =  $\frac{(1480-1410)100}{1480} = 4,75\%$

$V_1$  : 35 volts. " " " " =  $\frac{(1462-1372)100}{1462} = 6,2\%$

Per cent speed drop at  $I_2$  : 7 amp.

For  $V_1$  : 65 volt. Per cent speed drop =  $\frac{(1486-1410)100}{1486} = 5,1\%$

$V_1$  : 50 volt. " " " " =  $\frac{(1480-1400)100}{1400} = 5,72\%$

$V_1$  : 35 volt. " " " " =  $\frac{(1462-1370)}{1462} = 6,3\%$

c) Six pole series connected auxiliary:

With six pole series connected auxiliary winding much greater speed changes were obtained for the same or less controlled current,  $I_2$ , than with four poles. This may be due to the more effective change in the magnetization of the iron part due to large number of poles. This point is explained more clearly in Part A-II.

Per cent speed drop at  $I_2$  : 3,5 amps.

For  $V_1$  : 65 volts. Per cent speed drop =  $\frac{(1470-1426)100}{1470} = 3\%$

$V_1$  : 50 volt. " " " " =  $\frac{(1467-1419)100}{1467} = 3,28\%$

$V_1$  : 45 volt. " " " " =  $\frac{(1463-1391)}{1463} = 4,95\%$

$V_1$  : 35 volt. " " " " =  $\frac{(1450-1280)}{1450} = 11,07\%$

Per cent speed drop at  $I_2 = 4,0$  amp.

For  $V_1$ : 65 volts. Per cent speed drop =  $\frac{(1470-1418)100}{1470} = 3,54\%$

$V_1$ : 50 volts. Per cent speed drop =  $\frac{(1467-1406)100}{1467} = 4,15\%$

$V_1$ : 45 volt. " " " " =  $\frac{(1463-1375)100}{1467} = 6\%$

$V_1$ : 35 volt. " " " " = 100% Motor is braked completely.

As it is seen from the calculations with 6 pole connection better results can be obtained, even it is possible to stop the motor with complete braking. This shows that for speed control by changing air-gap flux, better results are obtained if the auxiliary winding has greater number of poles than the main stator winding.

d) Twelve Pole connected auxillary:

After the good results taken with 6 poles, a trial was made with 12 poles. Result is as good as, and even somewhat better than, 6 poles.

Per cent speed drops at  $I_2 = 3,5$  amp.

For  $V_1$ : 65 volt. Per cent speed drop =  $\frac{(1486-1442)100}{1486} = 2,96\%$

$V_1$ : 50 volt. " " " " =  $\frac{(1480-1434)100}{1480} = 3,1\%$

$V_1$ : 45 volt. " " " " =  $\frac{(1470-1406)100}{1470} = 4,35\%$

$V_1$ : 35 volt. " " " " =  $\frac{(1439-1337)100}{1439} = 7,1\%$

Per cent speed drops at  $I_2 : 4$  amp.

For $V_1$ : 65 volt.	Per cent speed drop:	$\frac{(1486-1434)100}{1486}$	= 3,5%
$V_1$ : 50 volt.	" " " "	$\frac{(1480-1404)100}{1480}$	= 5,15%
$V_1$ : 45 volt.	" " " "	$\frac{(1470-1378)100}{1470}$	= 6,25%
$V_1$ : 35 volt.	" " " "	$\frac{(1439-1267)100}{1439}$	= 12%

Per cent speed drops for four different kinds of auxiliary connections had been calculated. The comparison of the results shows that for D.C speed control the best results are obtained by six and 12 pole connections for the auxiliary and the maximum speed change is 11-12% except for 100% corresponding to braking. Four-pole parallel connection gives better results in controlling the speed than the series one. It is not possible to obtain six and 12-pole parallel connection for the auxiliary due to the nature of the winding.

As a conclusion for D.C speed control, auxiliary field should be wound for more poles than the main stator field winding. Thus the change produced by the auxiliary field on the air-gap flux is more effective and strong. For large control currents, large speed changes can be obtained but there is a limit to the increase of the control D.C current, that is, care should be taken for excessive saturation.

2. A.C single-phase control current and speed relations:

A) Four-pole series connected auxiliary:

Speed decreases as the a.c control current is

increased for a given 3-phase supply voltage. Due to the transformer action a voltage is induced in the auxiliary winding, so we get a current, although we have had applied no A.C control voltage. The curves start from some other value of current corresponding to the transformer action voltage not from zero current.

Per cent speed drop at  $I_2$ : 2,8 amp.

For  $V_1$  : 25 volt. Per cent speed drop:  $\frac{(1394-1372)100}{1394} = 1,57\%$

$V_1$  : 20 volt. " " " " :  $\frac{(1310-1290)100}{1310} = 1,6\%$

Motor runs noisier than normal, probably due to opposing field flux of the auxiliary winding. An unbalance of the rotor shaft was observed. High currents had been obtained when even 20 or 30 volts 3-phase supply was applied; therefore, the experimental values are for smaller supply voltages than the D.C cases. But as we have concluded previously with D.C control the effect of control is increased with smaller supplied voltages, that is, per cent speed drops are higher for small supply voltages for a fixed value of control current.

Keeping in mind this fact, it is possible to say that the speed drops obtained by A.C control, 1,57% and 1,6% for 25 and 20 volts supplies is unimportant as compared to the values obtained with D.C control and for the same number of pole connections.

b) Four-pole parallel connected auxiliary:

Per cent speed drops at  $I_2 = 4$  amp.

For  $V_1$ :30 volt. Per cent speed drop:  $\frac{(1422-1406)100}{1422} = 0,85\%$

$V_1$ :20 volt. Per cent speed drop:  $\frac{(1359-1342)100}{1359} = 1,25\%$

These per cent speed drops obtained with four-pole parallel connected auxiliary are much lower than the ones obtained with D.C control voltages, that is, 2,36% , 2,9%, 3,42%, as given in C-III(A-1)

Per cent speed drops for  $I_2$ : 7 amp.

For  $V_1$ : 30 volt. Per cent speed drop:  $\frac{(1422-1402)100}{1422} = 1,31\%$

$V_1$ : 20 volt. Per cent speed drop:  $\frac{(1359-1317)100}{1359} = 3,1\%$

These drops are also much smaller compared to values obtained with D.C control current, that is: 5,1%, 5,72% and 6,3% as calculated before .

c) Six-pole connected auxiliary:

Per cent speed drop at  $I_2 = 2,5$  amp.

$V_1$ : 35 volt . Per cent speed drop:  $\frac{(1450-1433)100}{1450} = 1,17\%$

$V_1$ : 23 volt. Per cent speed drop:  $\frac{(1396-1364)100}{1396} = 1,63\%$

These values for per cent drops for  $I_2 = 2,5$  when compared with the D.C control per cent drops for the same control current, with the same supply voltage and six-pole connection, that is:

For  $V_1$ : 35 volt. per cent speed drop for  $I_2 = 2,5$  amp. with D.C:

$\frac{(1450-1376)100}{1450} = 5,1\%$ , which is much greater than the value obtained 1,17% with A.C control.

d) Twelve-pole connected Auxiliary:

Per cent speed drop at  $I_2$ : 3,5 amp.

$$V_1: 81 \text{ volt. per cent speed drop: } \frac{(1450-1427)100}{1450} = 1,59\%$$

$$V_1: 25 \text{ volt. " " " " : } \frac{(1435-1394)100}{1435} = 2,85\%$$

When we compare the per cent drop for  $V_1$ :25 volt  $I_2$ :3,5 amp which is 5,65% as calculated above, it is obvious that D.C control is much more effective . As a general conclusion, from control current versus speed curve analysis, we can say that D.C control is much more preferable, keeping in mind the comparisons made for each type of connection under the same conditions.

### B- Power and Speed Relations:

Till now the possibility and degree of speed control with different connections and voltages had been analyzed . The other very important point in relation with the feasibility of a speed control method is the energy requirements to obtain a given per cent speed drop. The power requirement analysis will also be made for each case and then will be compared with each other.

#### 1. D.C. Speed Control:

The power required for speed control is found by  $P_{D.C} = I_{D.C} V_{D.C}$  at given values of  $I_{D.C}$  and  $V_{D.C}$ .

##### a- Four-pole series connected auxiliary:

For  $V_1$ : 65 volt, 2,6% speed drop

Power Required:

$V_1$ : 50 volt, 3% " "

60 x 4 = 240 watts.

$V_1$ : 35 volt, 3,45% " "

For a control current of  $I_2 = 4$  amp.

$$R_s = \frac{60}{4} = 15$$

b) Four-pole parallel connected auxiliary:

For  $V_1$ : 65 volt, 2,36% speed drop

Power Required:

$V_1$ : 50 volt, 2,9% " "

$20 \times 4 = 80$  watts

$V_1$ : 35 volt, 3,42% " "

For a control current of  $I_2 = 4$  amp.

$$R_p = \frac{20}{4} = 5$$

$$R_s = 3 R_p$$

So, D.C Power requirement is 3 times less with parallel connection due to the smaller effective resistance obtained.

c) Six-pole connected auxiliary:

For per cent speed drops of 3,54%

4,15%

6%

100%

control power required is:

$54 \times 4 = 216$  watts.

corresponding to  $I_2 = 4$  amp.

For per cent speed drops of 3%

3,28%

4,95%

11,07%

control power required is:

$45 \times 3,5 = 157,5$  watts.

corresponding to  $I_2 = 3,5$  amp.

d- Twelve-pole connected auxiliary:

For per cent speed drops of: 3,5%

5,15%

6,25%

12%

D.C control power required is

$55 \times 4 = 220$  watts

corresponding to  $I_2 = 4$  amp.

For per cent speed drops of : 2,96%

3,1%

4,35%

7,1%

D.C control power required:

$45 \times 3,5 = 157,5$  watts.

These results show that, the most feasible speed control can be obtained by using 6 or 12 poles with 150-200 watts of D.C power requirement. With 4-pole parallel connection the power requirement is less, 80 watts, but the maximum obtainable speed change is not more than 3,42%.

2- A.C Speed Control:

a- Four-pole series connected auxiliary:

For per cent speed drops of 1,57%

1,6%

A.C power requirement is:

$45 \times 5 = 225$  watts

corresponding to  $I_2 = 2,8$  amp.

b- Four-pole parallel connected auxiliary:

For percent speed drops of 0,85%

1,25% A.C power requirement is:  
17 x 5 = 85 watts

corresponding to  $I_2 = 4$  amp.

For per cent speed drops of 1,31%

3,1 % A.C power requirement is:  
50 x 5 = 250 watts

corresponding to  $I_2 = 7$  amp.

c- Six-pole connected auxiliary:

For percent speed drops of : 1,17%

1,63% A.C power requirement is:  
50 x 5 = 250 watts

corresponding to  $I_2 : 2.5$  amp.

d- Twelve-pole connected auxiliary:

For per cent speed drops of : 1,59%

2,85% A.C power requirement is:  
60-70 watts

corresponding to  $I_2 = 35$  amp.

The maximum per cent speed drop that can be obtained by A.C control is 3,1% with a power requirement of 250 watts, by means of four-pole parallel connection, whereas 2,85% speed drop can be obtained for 60-70 watts with twelve-pole connection. This shows that 12-pole connection is preferable both in A.C and D.C controls, but much better results could be obtained with D.C, with less power requirements.

#### D. SUMMARY OF RESULTS AND CONCLUSIONS

The following results can be extracted from the experimental data obtained for the speed control of induction motors by changing air-gap flux.

1. Speed control of a three-phase induction motor is possible by changing air-gap flux. This is done by introducing an auxiliary winding on the main stator field winding. The experiments and results are for the no-load operation, but theoretical considerations show that the same or better control could be possible with loaded conditions, but a motor of special construction is needed with larger iron parts to carry two stator windings.

2. The degree of speed control that can be obtained is not very large as explained in <sup>the</sup> theoretical part. The best result that is obtained with the motor and experimental conditions at hand is not more than 12% with a control power consumption of 200-220 watts.

3. The speed control is more effective for voltages less than the rated one. Motors with larger iron parts can be more suitable for this kind of speed control but economical considerations that is, higher cost of the motor, be taken into consideration for the feasibility of the method.

4. Speed control, by the application of D.C, is much more effective than by the application of A.C.

The maximum speed change that was obtained in the experiments carried out with the special motor used in this study

is 12%. It is 4 times greater than the maximum speed change obtained with A.C, 3%. The energy consumption for D.C control is less than the A.C control method. But the availability of D.C power for the control purposes is an important problem to be solved.

5. The speed control experiments were carried on for four different types of connections of the auxiliary winding. Auxiliary winding being a distributed lap winding by various possible connections 4-pole series, 4-pole parallel, six-pole and 12-pole could be obtained. And the experimental results as it had been pointed out in the theory part indicates that one could obtain much better results with larger number of poles for auxiliary winding. The main field structure having 4 poles, with six or twelve poles connected auxiliary, we obtained 3 or 4 times better speed drops than with four-pole connected auxiliary.

6. Experiments done by Hasan Çelik, Grad.E.E.'63, Robert College, show that with concentrated auxiliary windings, good results could also be obtained.

7. The speed control method studied in this paper is an unidirectional one, that is, we can only decrease the speed and can not increase it. We cannot work with speeds above the synchronous speed.

8. Due to the additional fluxes of the auxiliary winding, there exists an unbalance which makes the motor run with a greater noise and vibration than the normal 3-phase operation. The efficiency of operation as well as the maximum torque is lowered as it is pointed out in the theory part.

9. Experiments for speed control can be done by applying 3-phase A.C to the auxiliary winding with proper connection of the auxiliary coils. Due to time limitation, only few trials had been

done and the following results had been taken.

60 volt, 3-phase voltage applied to the main stator field for normal operation speed is 1446 rpm with 28 ampere flowing. In addition to this, if we apply to the auxiliary stator winding 75 volts, 3-phase A.C. voltage, with 2,5 ampere flowing through the auxiliary, we obtain a speed of 1370 rpm. which corresponds to a speed drop of  $\frac{(1446 - 1370)}{1446} 100 : 5.2 \%$ .

This shows the possibility of speed control by means of second three-phase stator field structure. But care should be given to the fact that the two rotating fluxes should be rotating in opposite directions so that they will have a subtractive overall effect, which in turn will slow down the rotor. The same basic idea had been employed to the method of stopping the 3-phase induction motors called "plugging".

It is advisable to make speed control tests employing 3-phase A.C. voltage to the auxiliary with a better designed motor, so that excessive currents will not flow.

10. The experiments carried out in this study are with a squirrel cage motor. The same method of speed control can be applied to a three-phase slipring motor. In that case auxiliary windings can be put on the rotor, and interesting results may be obtained.

11. If economical methods of designing 3-phase induction motors suitable for two stator windings with greater slot depths or additional slots could be found, the speed control of induction motors by changing air-gap flux may be economically feasible, as is also theoretically, within possible limits.

12. The speed control method studied in this paper has

some advantages when it is economically feasible. It is possible to obtain a smooth control of speed, that is, it is possible to obtain a large number of different speeds with the same motor, just by changing the control current, whereas in pole or frequency changing methods of speed control only two different speeds can be obtained with one machine.

13. A servo system can be employed for the control operation with D.C path to be connected to the auxiliary winding.

E. TABLES OF EXPERIMENTAL STUDIES AND GRAPHS

I. D.C speed Control

A) Four-poles series connected auxiliary:

$V_1$ or $V_{A.C}$ (volts)	$I_1$ or $I_{A.C}$ (ampers)	$V_2$ or $V_{D.C}$ (volts)	$I_2$ or $I_{D.c_2}$ (ampers)	Speed (rpm)
50 v.	4 A.	0	0	1476
50 v.	4,8 A.	17 v.	1,35A.	1454
50 v.	5,7 A.	30 v.	2,50A.	1440
50 v.	6,4 A.	36 v.	3,00A,	1438
50 v.	7,4 A.	43 v.	3,50A.	1434
50 v.	8.0 A.	50 v.	4.00A.	1432

$V_1$ or $V_{AC}$ (volts)	$I_1$ or $I_{AC}$ (ampers)	$V_2$ or $V_{DC_2}$ (volts)	$I_2$ or $I_{DC_2}$ (ampers)	Speed (rpm)
65 v.	5 A.	0	0 A.	1485
65 v.	6,1 A.	17 v.	1,35 A.	1468
65 v.	6,8 A.	30 v.	2,50 A.	1456
65 v.	7,0 A.	36 v.	3,00 A,	1452
65 v.	7,3 A.	43 v.	3,50 A.	1447
65 v.	7,5 A.	50 v.	4,00 A.	1444

$V_1$ or $V_{AC1}$ (volts)	$I_1$ or $I_{AC1}$ ( Amperes <sup>1</sup> )	$V_2$ or $V_{DC2}$ (volts) <sup>2</sup>	$I_2$ or $I_{DC2}$ (ampers)	Speed (rpm )
35 v.	3 A.	0v.	0 A.	1450
35 v.	4,1 A.	17 v.	1,35 A.	1430
35 v.	4,7 A.	30 v.	2,5 A.	1420
35 v.	5,1 A.	36 v.	3,0 A.	1414
35 v.	5,5 A.	43 v.	3,5 A.	1405
35 v.	6,8 A.	50 v.	4,0 A.	1400

$V_1$ or $V_{AC1}$ ( volts )	$I_1$ or $I_{AC1}$ (ampers)	$V_2$ or $V_{AC2}$ (volts )	$I_2$ or $I_{DC2}$ (ampers)	Speed (rpm)
65 v.	5 A.	0 v.	0 A.	1486
65 v.	5,6 A.	8,5 v.	2 A.	1476
65 v.	5,9 A.	14,0 v.	3 A.	1465
65 v.	6,1 A.	20,0 v.	4 A.	1451
65 v.	6,3 A.	21,0 v.	5 A.	1436
65 v.	6,6 A.	29,5 v.	6 A.	1423
65 v.	7,2 A.	36,0 v.	7 A.	1410

$V_1$ or $V_{AC1}$ (volts)	$I_1$ or $I_{AC1}$ (ampers)	$V_2$ or $V_{DC2}$ (volts)	$I_2$ or $I_{DC2}$ (amps)	Speed (rpm)
50 v.	4.0 A.	0 v.	0 A.	1480
50 v.	4,7 A.	10.0 v.	2,0 A.	1459
50 v.	5,2 A.	14.0 v.	3,0 A.	1453
50 v.	5,7 A.	20 v.	4,0 A.	1438
50 v.	5,9 A.	25 v.	5,0 a.	1423
50 v.	6,3 A.	33 v.	6,5 A.	1410
50 v.	6,9 A.	35 v.	7.0 A.	1400

$V_1$ or $V_{AC1}$ (volts)	$I_1$ or $I_{AC1}$ (amps)	$V_2$ or $V_{DC2}$ (volts)	$I_2$ or $I_{DC2}$ (amps)	Speed (rpm)
35 v.	3,5	0	0	1462
35 v.	3,9	8.0 v.	2.0 A.	1438
35	4,1	12,5	3.0	1425
35	4,8	20,0	4,0	1412
35	5,1	25,0	5,0	1408
35	5,6	30	6,0	1372
35	6,2	35	7,0	1370

6) Six-pole connected auxiliary:

$V_1$ or $V_{AC_1}$ (volts)	$I_1$ or $I_{AC_1}$ (amps)	$V_2$ or $V_{DC_2}$ (volts)	$I_2$ or $I_{DC_2}$ (amps)	Speed (rpm)
65	5,0	0	0	1470
65	5,9	20	1,8	1466
65	6,2	33	2,5	1460
65	6,7	40	3,0	1437
65	7,4	47	3,5	1426
65	8,1	55	4,0	1418

$V_1$ or $V_{AC_1}$ (volts)	$I_1$ or $I_{AC_1}$ (amps)	$V_2$ or $V_{DC_2}$ (volts)	$I_2$ or $I_{DC_2}$ (amps)	Speed (rpm)
50	4,0	0	0	1467
50	4,7	20	1,8	1455
50	5,0	25	2,0	1450
50	5,5	32	2,5	1439
50	5,8	39	3,0	1435
50	6,7	45	3,5	1419
50	7,8	54	4,0	1406

$V_1$ or $V_{AC_1}$ (volts)	$I_1$ or $I_{AC_1}$ (amps)	$V_2$ or $V_{DC_2}$ (volts)	$I_2$ or $I_{DC_2}$ (amps)	Speed (rpm)
45	3,5	0	0	1463
45	4,4	20	1,8	1450
45	4,9	25	2,0	1445
45	5,7	32	2,5	1435
45	6,2	39	3,0	1406
45	7,1	45	3,5	1391
45	8	54	4,0	1375

$V_1$ or $V_{AC_1}$ (volts)	$I_1$ or $I_{AC_1}$ (amps)	$V_2$ or $V_{DC_2}$ (volts)	$I_2$ or $I_{DC_2}$ (amps)	Speed (rpm)
35	3,0	0	0	1450
35	4,3	20	1,8	1423
35	4,6	25	2,0	1390
35	5,5	32	2,5	1376
35	6,1	37	3,0	1364
35	7,0	44	3,5	1280
	inct. cases continuously unstable	54	4,0	Brake unstable

a) Twelve-poles connected auxiliary:

$V_1$ or $V_{AC1}$ (volts)	$I_1$ or $I_{AC1}$ (amps)	$V_2$ or $V_{AC2}$ (volts)	$I_2$ or $I_{AC2}$ (amps)	Speed (rpm)
65	5,0	0	0	1485
65	5,4	20	1,5	1479
65	5,5	25	2,0	1468
65	5,75	33	2,5	1452
65	6,25	40	3,0	1446
65	6,8	47	3,5	1442
65	7,5	54	4,0	1434

$V_1$ or $V_{AC1}$ (volts)	$I_1$ or $I_{AC1}$ (amps)	$V_2$ or $V_{AC2}$ (volts)	$I_2$ or $I_{AC2}$ (amps)	Speed (rpm)
50	4,00	0	0	1480
50	4,5	20	1,5	1467
50	4,75	25	2,0	1453
50	5,5	33	2,5	1448
50	6,0	40	3,0	1439
50	6,5	47	3,5	1434
50	7,1	54	4,0	1404
50	8,0	65	4,5	--

## II. AcC Single- Phase Speed Control:

a- Four-poles, series connected auxiliary:

$V_1$ (volts) 3-phase	$I_2$ (amps) 3-phase	$V_2$ (volts) single phase	$I_2$ (amps) single phase	$W_2$ (watts) single phase	Speed rpm
25:(	6,00	5	1,53	0	1394
25	6,25	15	1,70	6 x 5	1393
25	6,5	35	1,85	11 x 5	1391
25	6,75	52	2,06	17 x 5	1388
25	7,00	67	2,25	24 x 5	1384
25	7,5	90	2,58	36 x 5	1380
25	7,75	98	2,72	40 x 5	1373
25	8,00	107	2,88	45 x 5	1370

$V_1$ (volts) 3-ph.	$I_1$ (amps) 3-ph,	$V_2$ (volts) S. ph.	$I_2$ (amps) S. ph.	$W_2$ (watt) s. ph.	Speed rpm
:20	5,75	2-3	1,2	0	1310
20	5,80	33	1,6	10 x 5	1308
20	6,4	65	2,06	22 x 5	1300
20	7,25	91,5	2,58	36 x 5	1296
20	7,75	109	2,8	43,5x 5	1290

$V_1$ or $V_{AC_1}$ (volts)	$I_1$ or $I_{AC_1}$ (amps)	$V_2$ or $V_{AC_2}$ (volts)	$I_2$ or $I_{AC_2}$ (amps)	Speed (rpm)
45	3,5	0	0	1470
45	4,1	20	1,5	1461
45	4,45	25	2,0	1452
45	5,2	33	2,5	1446
45	5,8	40	3,0	1425
45	6,5	47	3,5	1406
45	7,5	54	4,0	1378
45	8,0	64	4,5	--

$V_1$ or $V_{AC_1}$ (volts)	$I_1$ or $I_{AC_1}$ (amps)	$V_2$ or $V_{AC_2}$ (volts)	$I_2$ or $I_{AC_2}$ (amps)	Speed (rpm)
35	3,0	0	0	1439
35	3,7	20	1,5	1427
35	4,25	25	2,0	1423
35	5,00	30	2,5	1413
35	5,80	40	3,0	1379
35	6,75	47	3,5	1337
35	9,0	54	4,00	1267

b) Four-poles, parallel connected auxiliary:

$V_1$ (volts) 3-ph.	$I_1$ (volts) 3-ph.	$V_2$ (volts) s.ph.	$I_2$ (amps) s.ph.	$W_2$ (watt) s.ph.	Speed rpm
20	4,30	25	2,06	7 x 5	1359
20	4,4	30	2,58	9 x 5	1351
20	4,5	37	3,5	14, x 5	1348
20	4,75	49	4,5	21 x 5	1337
20	4,80	56	5,5	32, x 5	1326
20	4,85	65	6,5	43 x 5	1321
20	5,2	72	7,25	53 x 5	1315

$V_1$ (volts) 3-ph.	$I_1$ (volts) 3-ph.	$V_2$ (volts) s.ph.	$I_2$ (amps) s.ph.	$W_2$ (watts) s.ph.	Speed rpm
30	4,9	15	0,4	0,5 x 5	1422
30	5,2	35	2,5	10 x 5	1410
30	5,3	44	3,5	16, x 5	1407
30	5,35	53	4,5	25 x 5	1406
30	5,40	63	5,5	35, x 5	1405
30	5,5	72	6,5	46 x 5	1404
30	5,7	78	7,25	59 x 5	1400

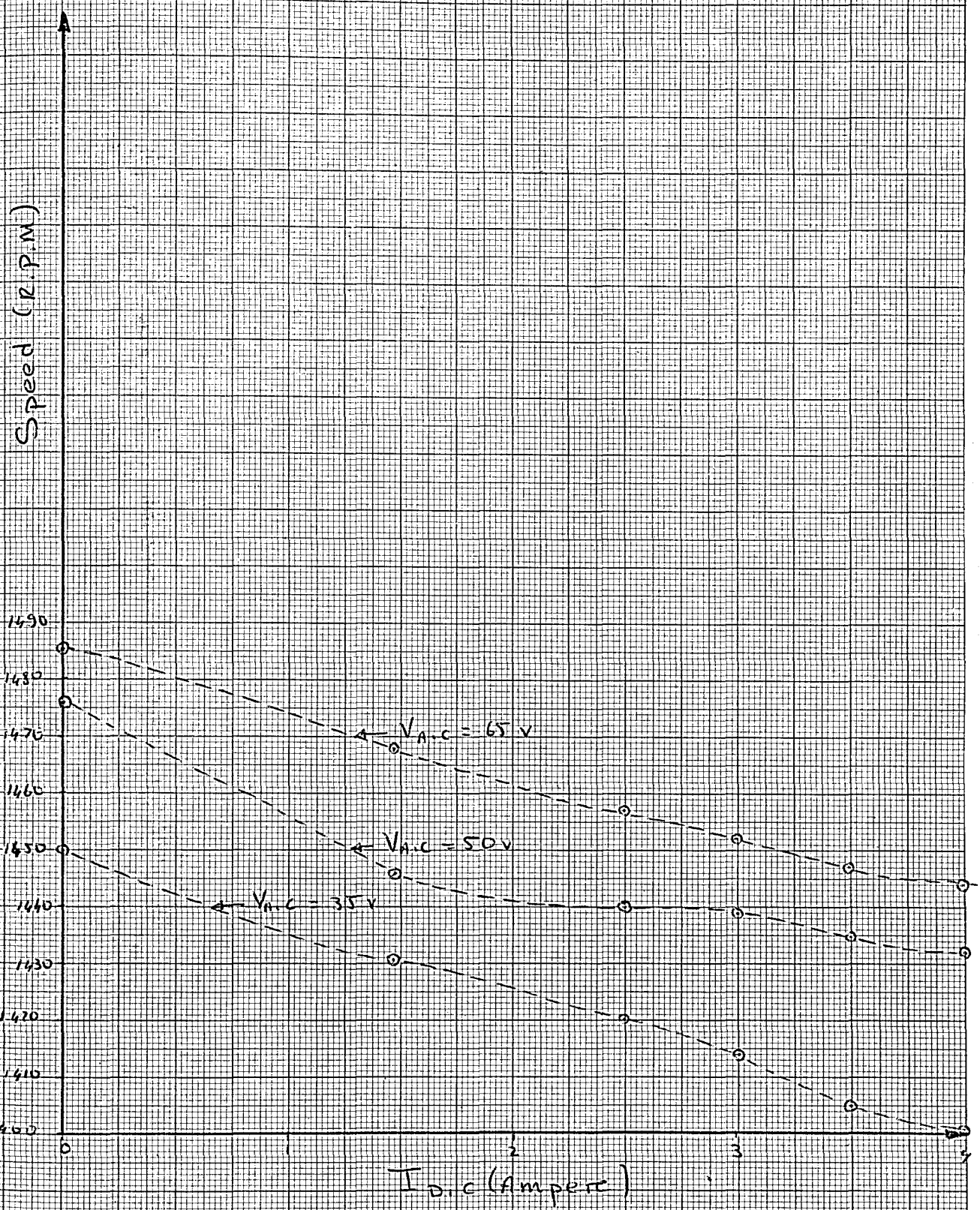
c) Six-poles connected auxiliary:

$V_1$ (volts) 3-ph.	$I_1$ (amps) 3-ph.	$V_2$ (volts) s-ph.	$I_2$ (amps) s-ph.	$W_2$ (watts)	Speed rpm
23	3,5	40	1,25,	12x5	1450
23	3,65	63	1,4	15x5	1396
23	3,70	77	1,6	21,5x5	1389
23	3,80	87	1,8,	27 x5	1381
23	3,90	96	2,0	31 x5	1375
23	4,00	106	2,2	36,5x5	1370
23	4,1	110	2,5	50 x5	1364

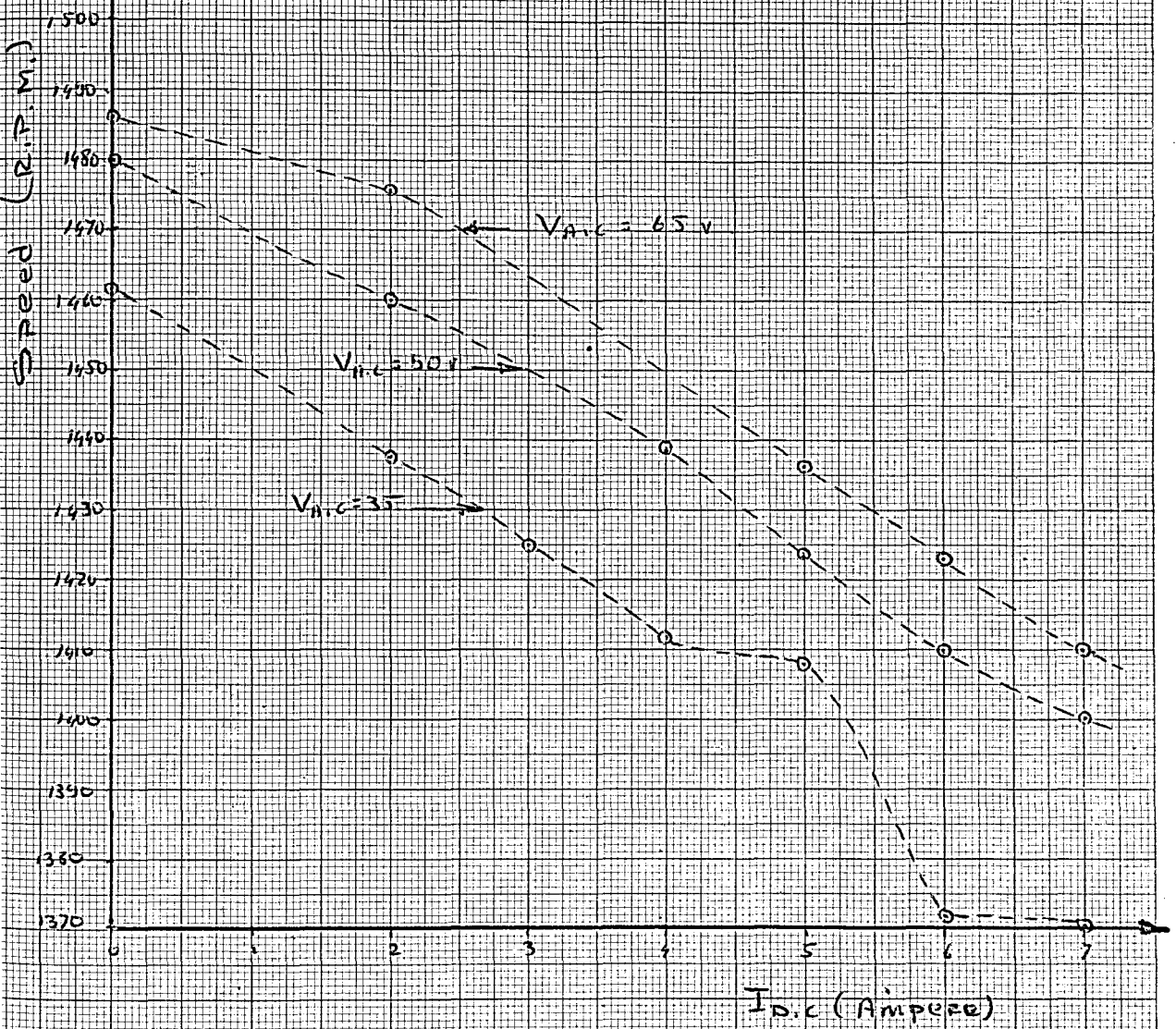
$V_1$ (volts) 3-ph.	$I_1$ (amps) 3-ph.	$V_2$ (volts) s-ph.	$I_2$ (amps) s-ph.	$W_2$ (watts)	Speed rpm
39	4,70	54	1,90	12x5	1450
39	4,75	96	2,00	32x5	1448
39	4,80	126	2,4	51x5	1438
39	5,00	140	2,75	60x5	1425

d). Twelve-poles connected auxiliary:

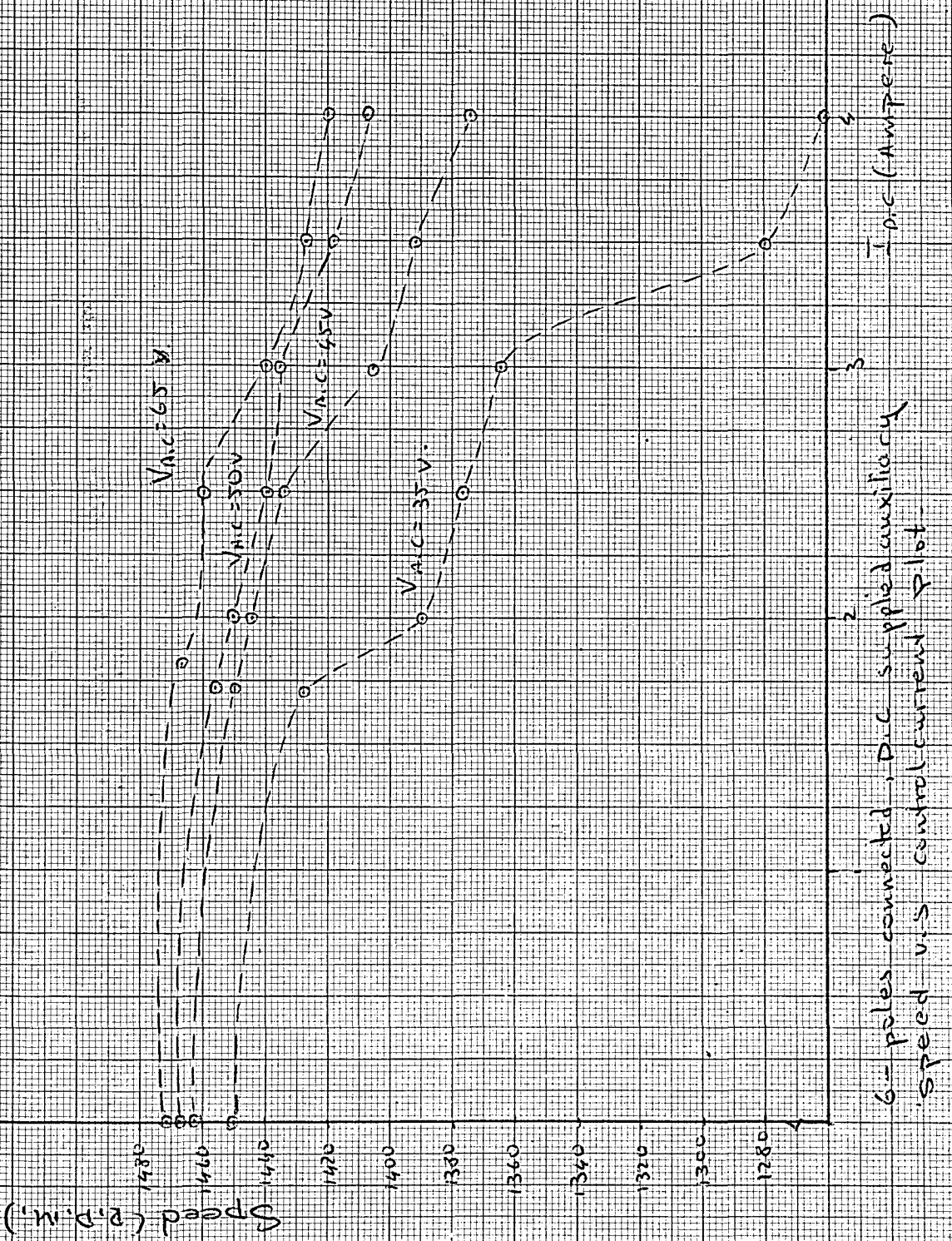
$V_1$ (volts) 3-ph.	$I_1$ (amps) 3-ph.	$V_2$ (volts) s-ph.	$I_2$ (amps) s-ph.	$W_2$ (watts)	Speed rpm
95	5	0	2,75	0	1461
95	5,1	30	3,0	0,5x5	1460
95	5,2	65	3,5	1x5	1446
95	5,5	90	3,95	7,5x5	1442
81	4,5	0	2,45	0	1450
81	4,70	49	3,0	0,5x20	1446
81	5,2	84	3,5	2 x 20	1427
81	5,25	91	3,7	3 x 20	1417
32	4	0	1,6	0	1435
32	4,5	28	2,0	0,5x5	1432
32	4,75	36	2,25	1 x 5	1430
32	4,8	45	2,5	2 x 5	1429
32	4,9	73	3,0	6 x 5	1427
32	5,00	87	3,25	11x5	1417
32	5,1	94	3,40	13x5	1403
20	3,6	0	1,25	0	1327
20	4,45	46	2,00	4 x 5	1261
20	4,75	68	2,5	8 x 5	unstable

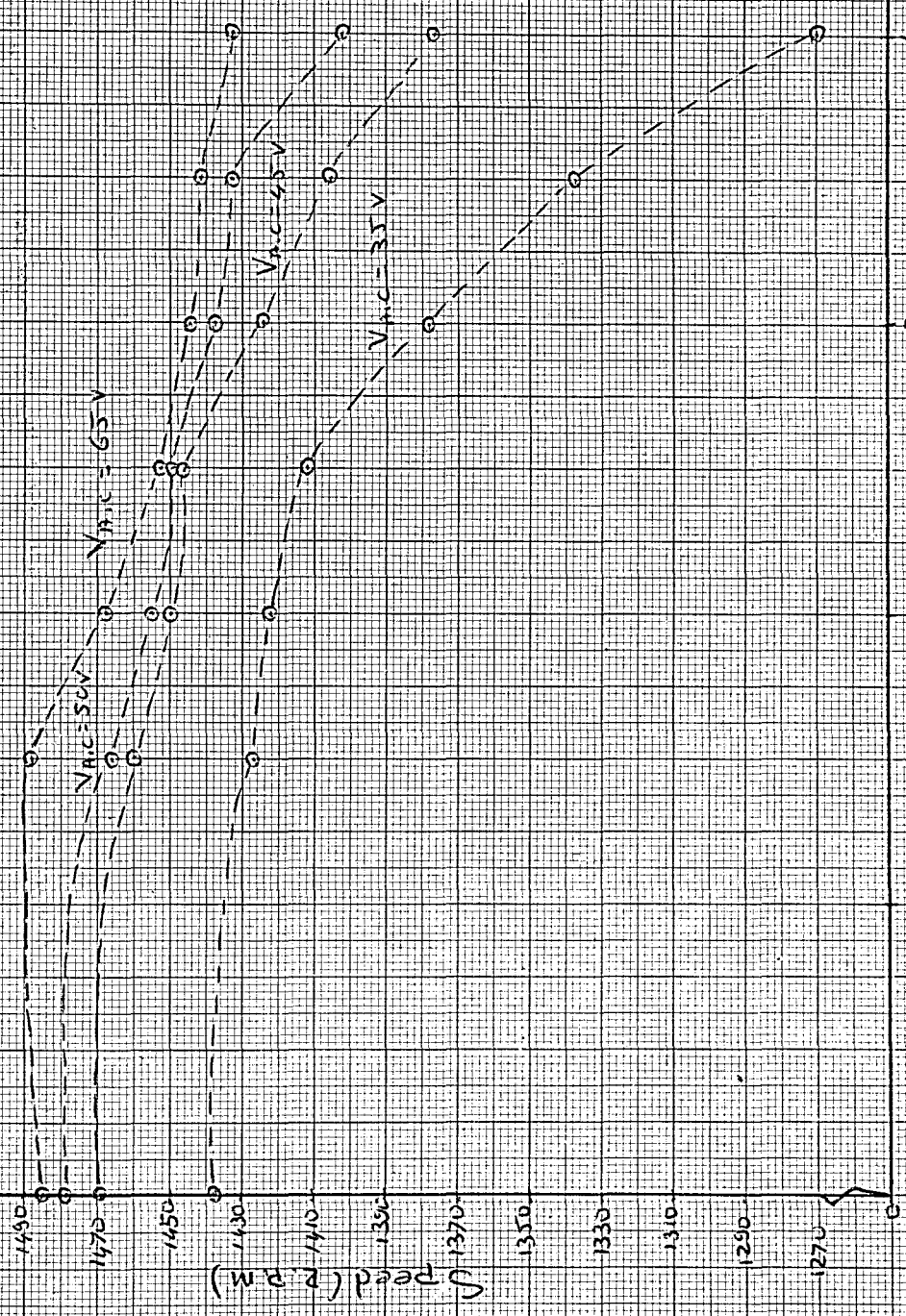


4-poles series connected, D.C supplied auxiliary speed v.s control current plot.



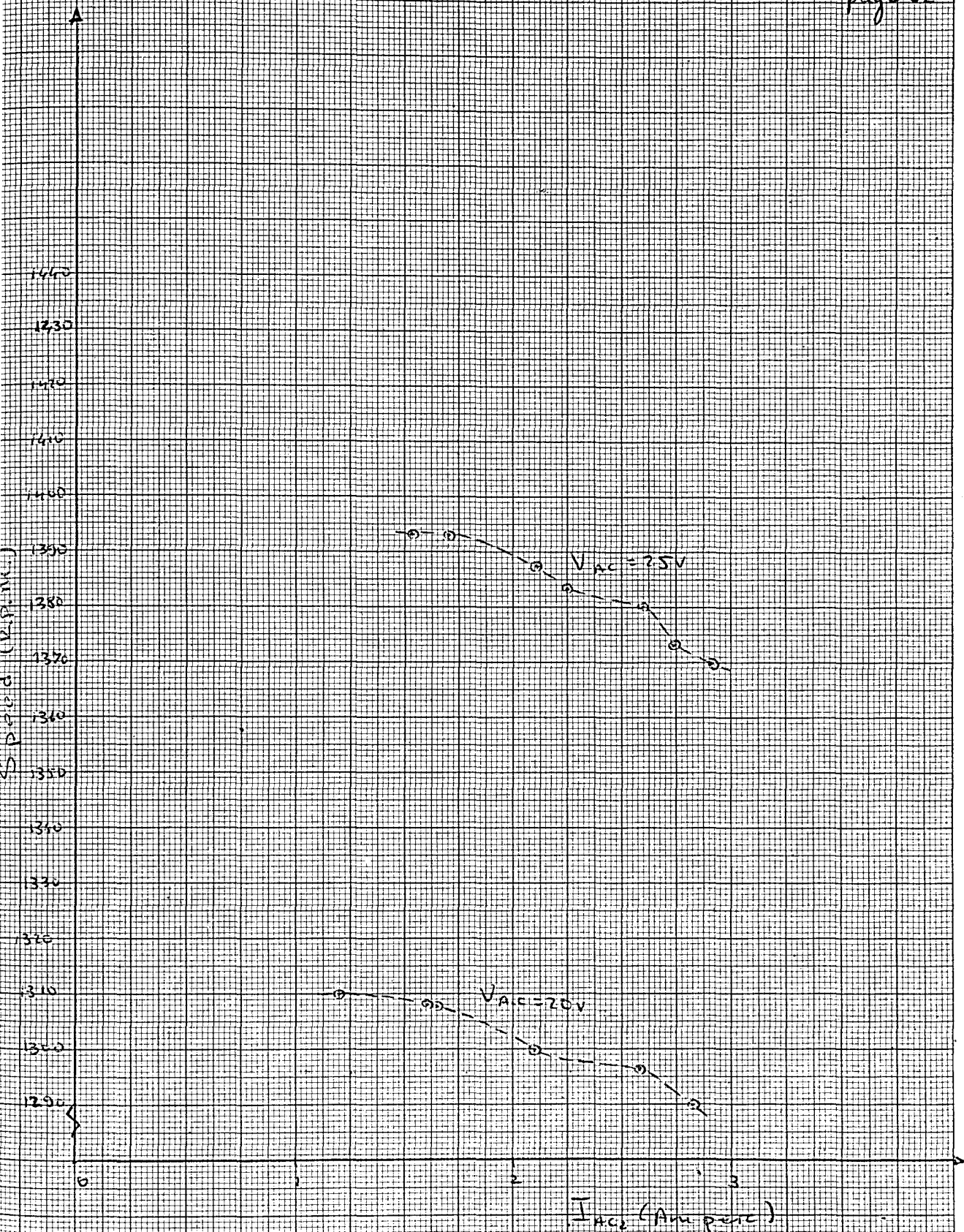
4-poles, parallel connected. D.C supplied auxiliary, speed vs control current curve.



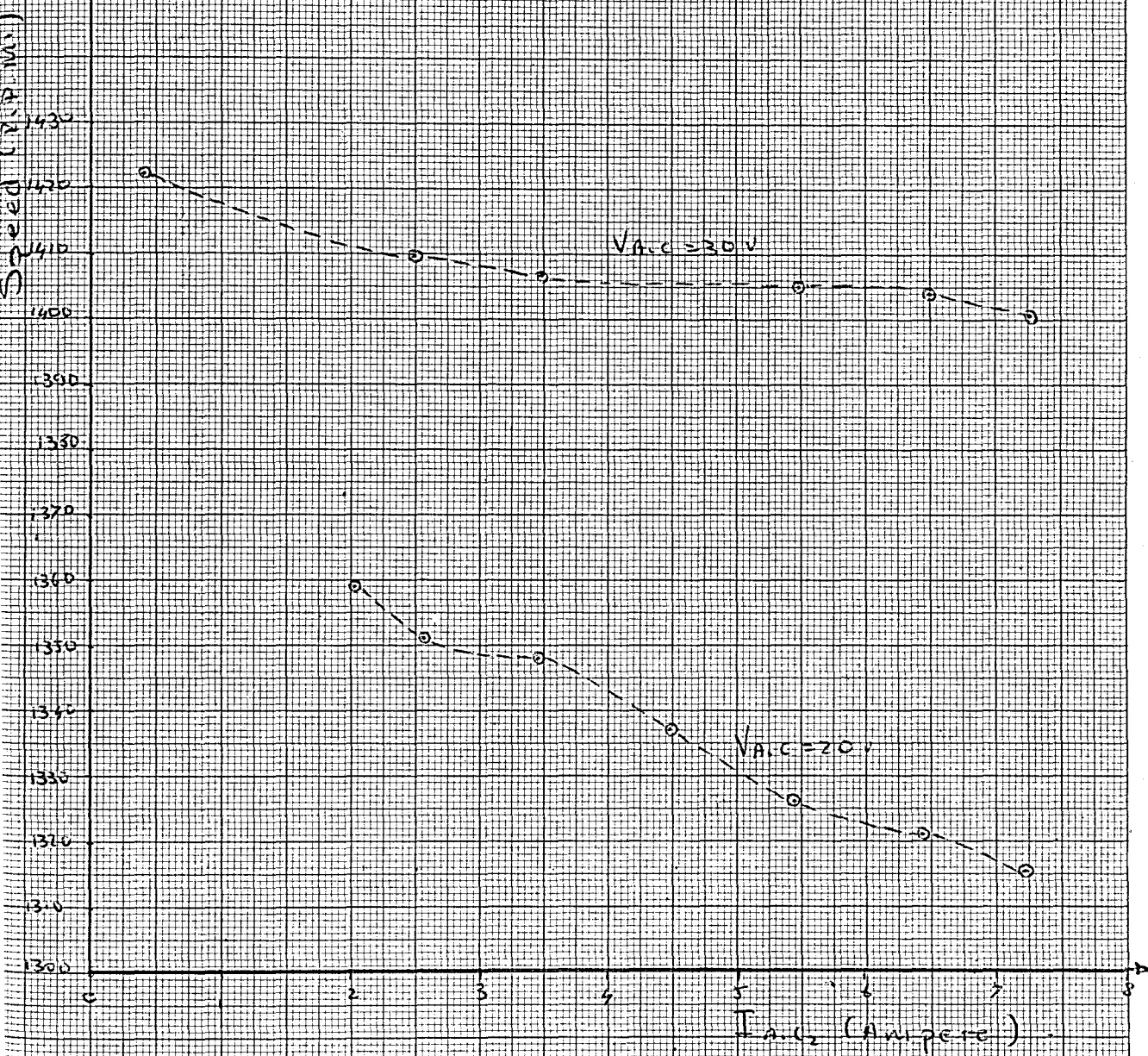


12-poles, D.C. supplied auxiliary, speed N's control current plot.

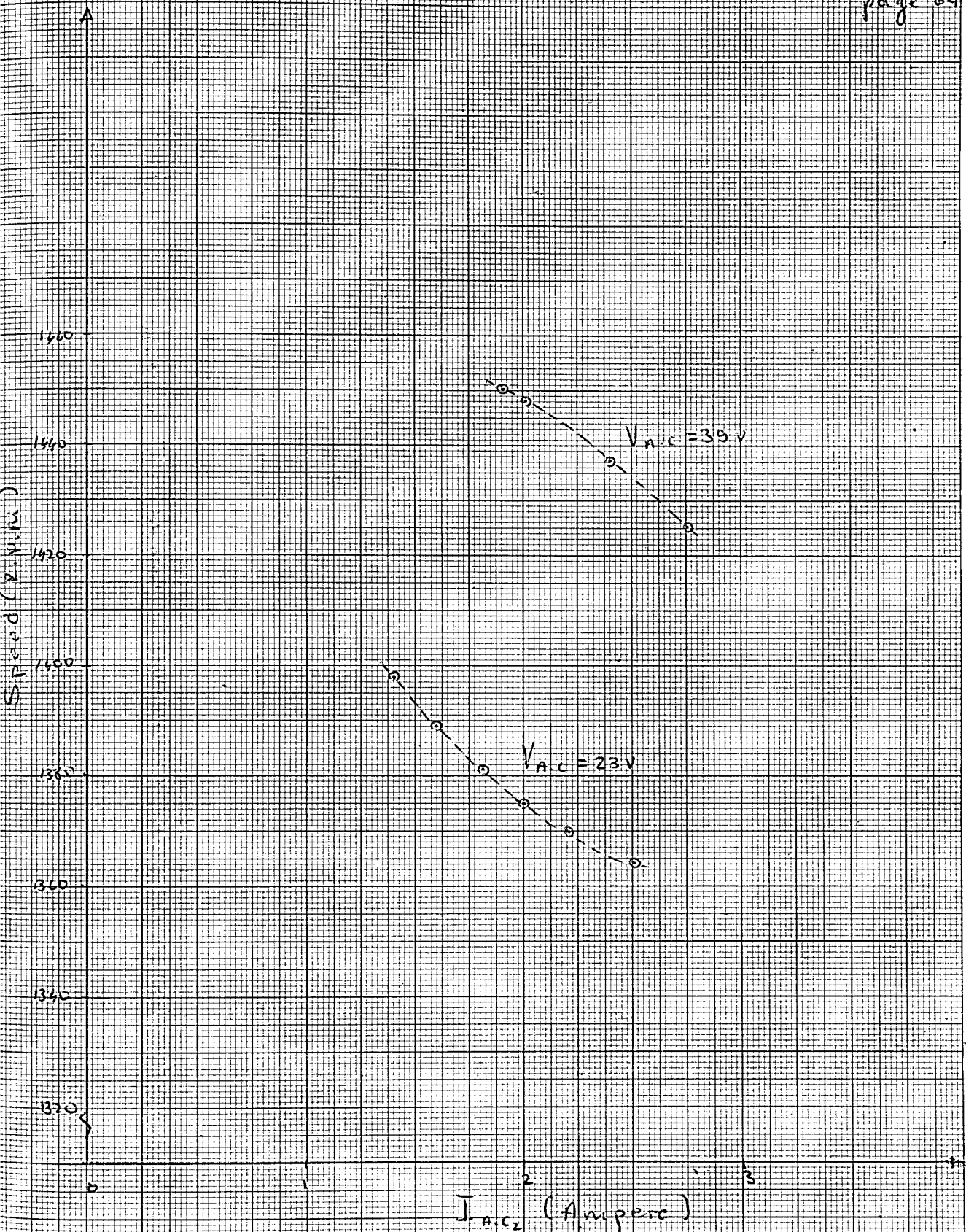
$I_{oc}$  (Ampere)



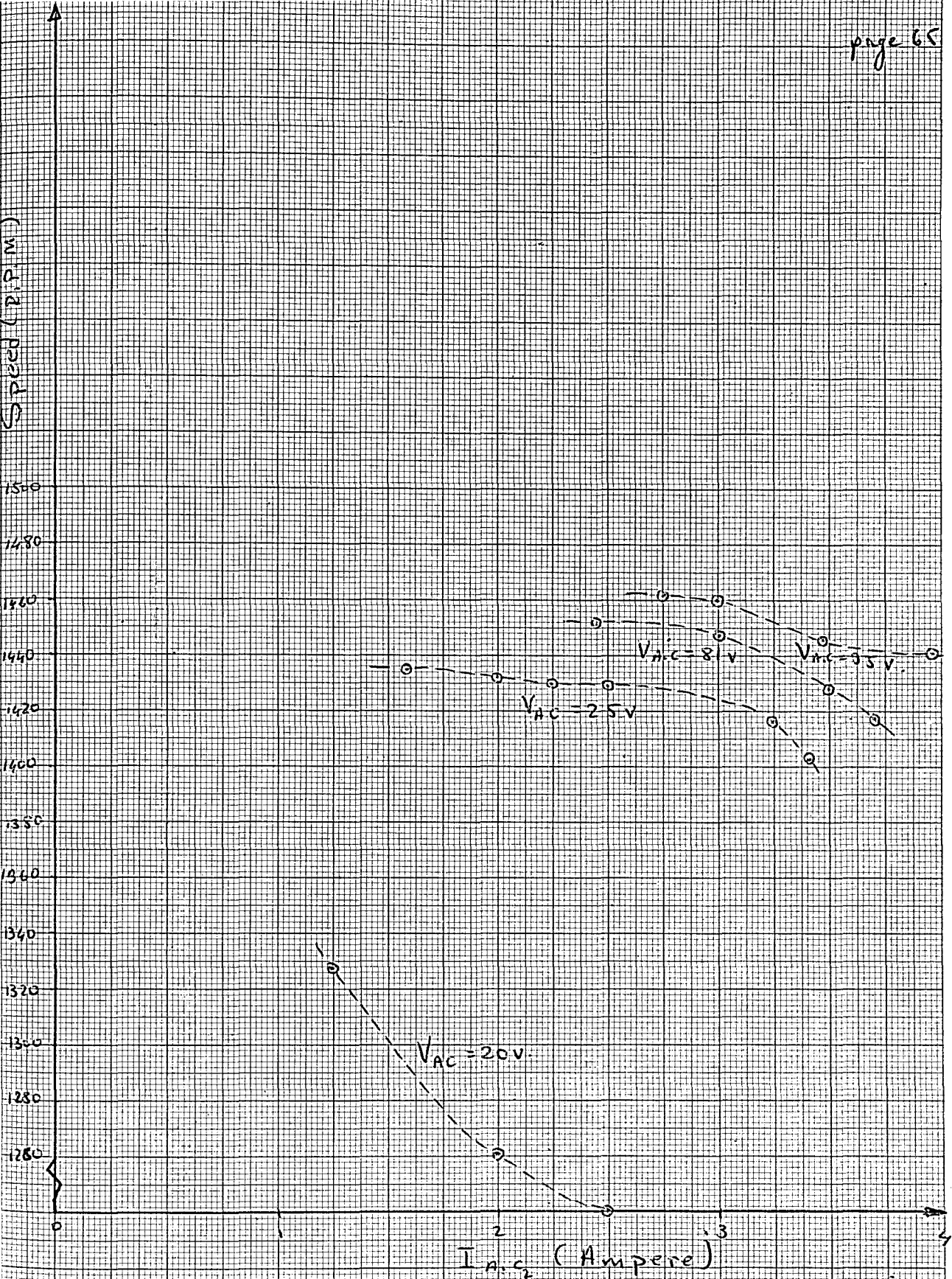
4-poles series connected, a.c single phase applied auxiliary, speed vs control current plot.



4-poles parallel connected, A.C single-phase applied auxiliary speed v.s Control current plot.



6-poles connected, A.C single phase applied auxiliary, speed vs control current plot.



12 - poles connected, A-C single-phase applied auxiliary, speed v.s A.C control current plot.

F. APPENDICES

APPENDIX -I-

Relation between direct excitation current,  $I_D$ , and the equivalent R.M.S alternating current,  $I_1$ .

The relationship between the direct stator excitation current,  $I_D$ , and the equivalent r.m.s alternating current,  $I_1$ , depends on the stator connection used. The commonly used schemes of connection are shown in Figures F- I, a-b for star connected machines and F-I, c-d for delta connected machines. These all require the same total power to produce the same effective excitation M.M.F.

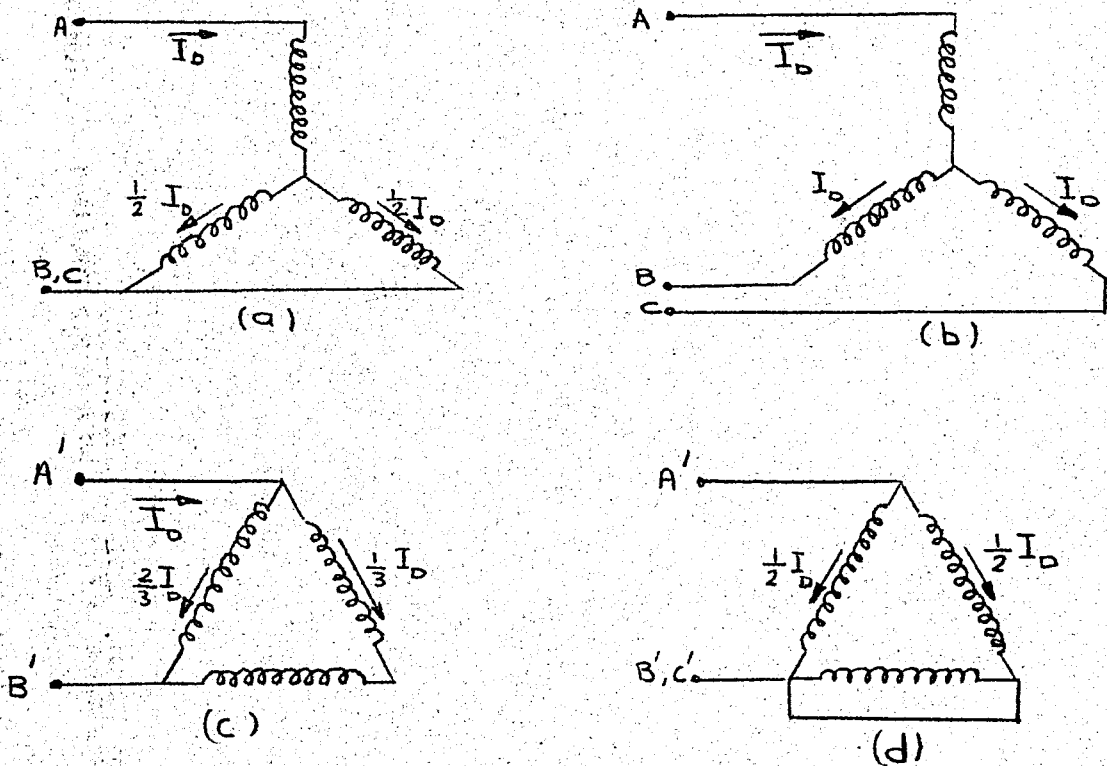


Fig. F-1 : Stator connections:

Scheme (a) gives more uniform heating than scheme (b), but the former requires a 3-pole braking contactor, whereas the latter only requires a 2-pole one.

Being simpler, (d) is more preferable for star connection and (c) for delta connection. In connection (a) the direct current flowing in the windings corresponds to the equivalent alternating

currents "frozen" at the instant when the current in phase A has its peak value. Therefore,  $I_D \approx \sqrt{2} I_1$ ,  $I_1$  being the r.m.s current. Thus,  $I_1$ , referred to the stator is  $I_1 \approx \frac{I_D}{\sqrt{2}}$ . For connection (b), the conditions correspond to the instant when the alternating current phase C is zero and that in A is  $\sqrt{3}/2$  times the peak value. Therefore

$$I_D \approx \frac{\sqrt{3} I_1}{\sqrt{2}} \quad \text{or} \quad I_1 \approx \frac{\sqrt{2} I_D}{\sqrt{3}}$$

Similar arguments are used to find the relationship for connections (c) and (d). In the former, if  $I_1$  is the r.m.s phase current

$$2I_{D/3} = \sqrt{2} I_1 \quad \text{or} \quad I_1 = \sqrt{2} I_{D/3} \quad \text{in the latter,}$$

$$I_1 = \frac{I_D}{\sqrt{6}}$$

APPENDIX -II-

TORQUE EQUATION:

By specifying the  $I_D$  and  $I_1$ , the stator impedance and voltage need not be shown and Fig. A-I can be further simplified to Fig.F-2.

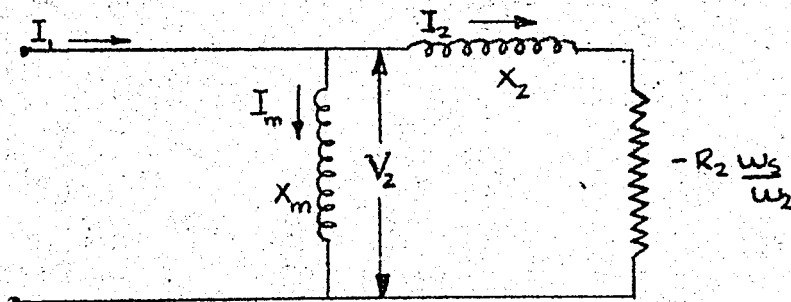


Fig. F-2, D.C control equivalent circuit.

Frequency:  $\omega_s$

Fig. F-2 : D.C control equivalent circuit.

Frequency :  $\omega_s$

To simplify the expressions the equivalent rotor resistance

$R_2 \frac{\omega_s}{\omega_2}$  will be denoted by  $R$ . Vector quantities are shown in

heavy lines. From the equivalent circuit diagram (Fig.F-2)

$$(1) \quad \vec{I}_1 = \vec{I}_m + \vec{I}_2$$

$$(2) \quad \vec{I}_m = \frac{\vec{V}_2}{jX_m}$$

$$(3) \quad \vec{I}_2 = \frac{\vec{V}_2}{R + jX_2}$$

From equations (2) and (3)

$$\vec{I}_m = \frac{\vec{I}_2 (R + jX_2)}{jX_m} \quad \text{substituting in equation (1) gives}$$

$$\vec{I}_1 = \frac{\vec{I}_2 (R + jX_2)}{jX_m} + \vec{I}_2 = \frac{\vec{I}_2 [R + j(X_2 + X_m)]}{jX_m}$$

Therefore, 
$$I_2 = \frac{I_1 J X_m}{R + j(X_2 + X_m)}$$
 and,

$$I_2^2 = \frac{I_1^2 X_m^2}{R^2 + (X_2 + X_m)^2}$$

(4) The net torque,  $T = I_2^2 R = \frac{I_1^2 X_m^2 R}{R^2 + (X_2 + X_m)^2}$

For unsaturated conditions  $X_m$  is constant, and the maximum Value of  $T$  is found from eq.(4) to occur when:

(5)  $R = (X_2 + X_m)$

(6)  $T_{max} = \frac{I_1^2 X_m^2}{2(X_2 + X_m)}$

$(X_2 + X_m)$  is the synchronous reactance at the angular frequency  $\omega_s$

Equation (4) can be used to find the torque even under saturated conditions, if the value of  $X_m$ , taken is the adjusted value as used in alternator calculations. However, this usually is a tedious process of trial and error, the graphical constructions given in Appendix-III- being much less laborious.

Even in the presence of saturation, when  $X_m$  cannot be taken as constant, it remains true that for a given value of  $I$ , the torque is a function of  $R$  only, since for a particular value of  $R$ , the quantities  $I_2, I_m$  and  $V_2$ , the degree of saturation and the value of  $X_m$  are all fixed.

APPENDIX-III-

GRAPHICAL CONSTRUCTION:

The curve OC shown in Fig. F-3 is the magnetization characteristic of the induction motor, that is, the relation between the air-gap F.M.F per phase,  $V_2$ , and the magnetizing current per phase,  $I_m$ , for the unit of synchronous speed  $\omega_s$ .

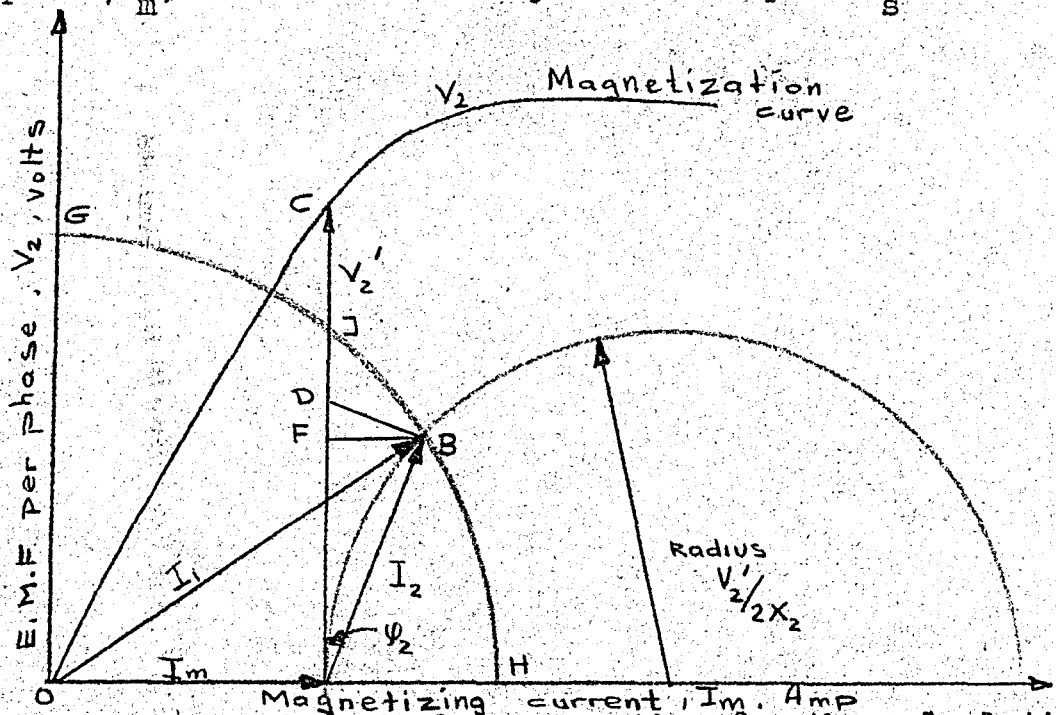


Fig. F-3 : Graphical construction for the calculation of DcC braking characteristic:

This curve may be measured as described in the next section. Also shown in Fig.-6 is the vector diagram OAB, for the circuit in Fig-5, showing the currents  $I_1$ ,  $I_m$ ,  $I_2$  for the particular E.M.F.

$V_2^1$  : AC. This vector diagram corresponds to equation(1). For a given direct stator excitation  $I_D$ ,  $I_1$  is also fixed in magnitude, so that the point B lies on the quadrant of circle GEH, radius  $I_1$ . Since the rotor circuit of Fig.-5 consists of a fixed

reactor in series with a variable resistor. The extremity of the current vector  $I_2$  lies on the circle AB, of diameter  $V_2^1 / X_2$  or radius  $V_2^1 / 2X_2$ . Hence, the point B must be the intersection of the two circles, and for the particular value of E.M.F chosen ( $V_2^1 : AC$ ). The vector diagram must be as shown above.

The phase angle between  $V_2^1$  and  $I_2 : \phi_2 : \text{angle CAB}$ , and if BF is drawn perpendicular to AC, the torque per phase in synchronous watt is given by:

$$(7) \quad T = V_2^1 I_2 \cos \phi_2 = AC \times AF$$

(AC is measured in volts and Af in amperes)

Also, if DB is drawn perpendicular to AB to cut AC at D,

AD :  $I_2 / \cos \phi_2$  and since

$$I_2 = \frac{V_2^1}{\sqrt{R^2 + X_2^2}} \quad \text{and,}$$

$$\cos \phi_2 = \frac{R}{\sqrt{R^2 + X_2^2}}$$

It follows that if AD is measured in amperes,  $AD = \frac{V_2^1}{R}$  and

therefore,

$$(10) \quad R = \frac{V_2^1}{AD}$$

By repeating this construction for a number of values of  $V_2^1$ , the relationship between T and R may be determined from which the torque/speed curves for given values of  $R_2$ , or the torque/resistance curves for given speeds may be derived as explained above. This procedure can be carried out for the values of  $I_1$  corresponding to any value of  $I_D$ .

If  $X_2$  is very small, the circle AB becomes very large, the angle  $\phi_2$  becomes very small, and the points D, F and B all tend to coincide J. In many cases it may be sufficiently accurate to assume that  $X_2$  is negligible, so that the torque is given by:

(8)  $T \approx A J$  (in amp)  $\times AC$  (in volts) and the equivalent rotor resistance by:

$$(9) \quad R \approx \frac{AC(\text{in volts})}{A \text{ (in amperes)}}$$

It should be noted that  $X_2$  is the leakage reactance of the rotor alone, and not the total leakage reactance which is used in the usual approximate equivalent circuit of the induction motor. For the case where  $X_2$  is negligible we can draw the new equivalent circuit and graphical construction.

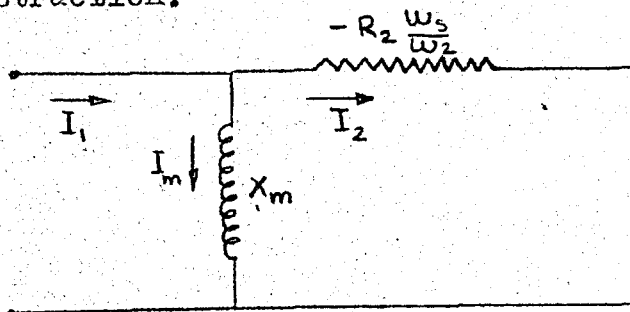


Fig. F-4 : Simplified braking circuit instator terms neglecting  $X_2$ .

We can draw another figure showing the magnetization curve and also the vector diagram of exciting and rotor currents and the loci of  $I_1$ .

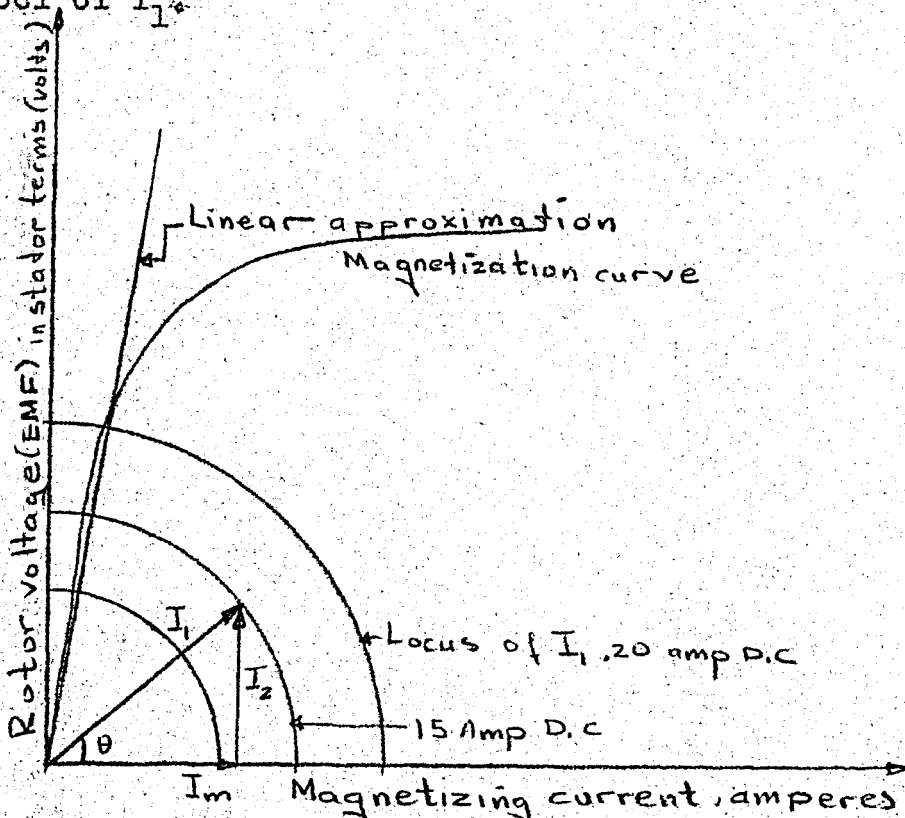


Fig. F-5: Magnetizing curve for an induction motor and linear approximation for the curve. The phasor diagram for the currents is superimposed.

For every value of  $I_2$  assumed, the corresponding value of  $I_m$  may be found, which in turn yields the proper value of rotor induced voltage from the saturation yields curve. The power per phase developed by the motor is  $E I_2$  synchronous watts. Also,

$$(10) \quad E I_2 : I_2^2 R_2 \frac{W_s}{W_2} \quad \text{so,}$$

$$(11) \quad \frac{W_2}{W_s} : \frac{I_2 R_2}{E}$$

The developed torque can be computed from the synchronous watts and the speed from the value of slip given in equation (11). In the regional operation where the magnetization curve is linear, that is,  $X_m$  is constant the speed and torque may be represented by:

$$(12) \quad W_2 = W_s \frac{R_2}{s m} \tan \theta$$

$$(13) \quad T = \frac{K I_1^2}{2} \sin 2\theta \quad \text{which are derived in the}$$

following way:

The power developed during braking is  $X : I_2 E$  synchronous watts per phase, but from Fig. F-5,  $I_2 = I_1 \sin \theta$ , if the magnetizing curve may be represented by a straight line.

$$E = m I_m + b \quad \text{where } m = \frac{E}{I_m}, \quad b = 0$$

Also from Fig. F-5, it may be seen that:

$$I_m = I_1 \cos \theta$$

Therefore,  $P = I_1 \sin \theta (m I_1 \cos \theta)$  synchronous watts per phase

$$P = \frac{m I_1^2}{2} \sin 2\theta \quad \text{or}$$

$$T = \frac{3 \times 7,04}{W_s} \frac{m I_1^2}{2} \sin 2\theta \quad \text{lbft or}$$

$$(13) \quad T = K \frac{I_1^2}{2} \sin 2\theta \quad \text{where } K = \frac{7,04 \times 3}{W_s}$$

If we put  $I_2 = I_1 \sin \theta$  and  $E = m I_1 \cos \theta$  in eq (11), we get:

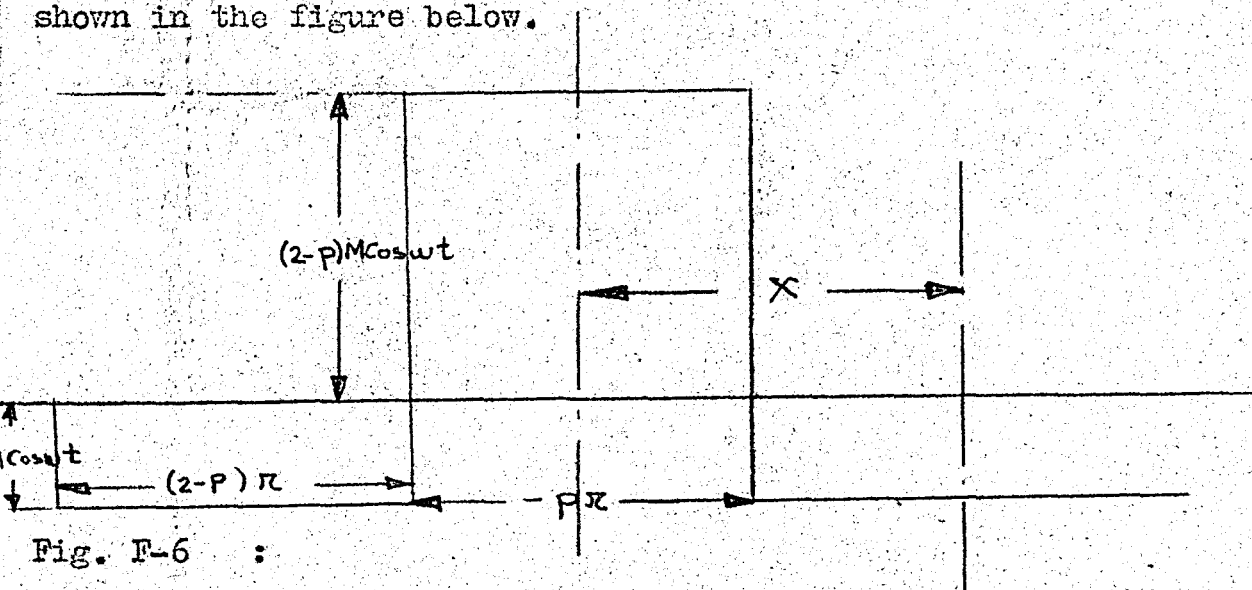
$$\frac{W_2}{W_s} = \frac{I_2 R_2}{E} = \frac{R_2 (I_1 \sin \theta)}{m I_1 \cos \theta} = \frac{R_2}{m} \tan \theta$$

using (12), 
$$W_2 = W_s \frac{R_2}{m} \tan \theta .$$

## APPENDIX - IV-

### ANALYSIS OF A.C SINGLE PHASE, AIR-GAP M.M.F AND TORQUE

Let us consider a single stator coil with a pitch of  $180/p$  degrees, or  $P$  times full pitch. An A.C flowing in this coil will produce pulsating magnetic flux across the air-gap over the coil width of  $p\pi$ , which returns over the remaining space of  $(2-p)\pi$ . Neglecting the reluctance of the iron on both sides of the gap, and considering the coil magnetomotive force (M.M.F) to be concentrated at slot openings of negligible width, the flux wave will consist of two dissimilar rectangles of equal area as shown in the figure below.



Since the same total flux crosses the gap in each direction, the flux density inside the coil is  $(2-p)B \cos \omega t$ , and that outside the coil is  $pB \cos \omega t$ , where  $B \cos \omega t$  is the flux density that would be produced if the coil were full pitch ( $p=1$ ).

The total flux is, therefore,  $p(2-p)$  times the flux that would be produced by the full pitch coil. By well-known Fourier method of analysis, the air-gap M.M.F distribution of such a shape may be represented as the sum of a series of sine waves. Thus M.M.F or  $m$  at any distance  $X$  from the center coil is:

$$(1) \quad m = a \cos X + b \cos 2X + c \cos 3X + \dots + n \cos kX + \dots$$

where,

$$(2) \quad m = -pM \cos wt \quad \text{from } X = -\pi \quad \text{to } X = \frac{p\pi}{2}$$

$$(3) \quad m = (2-p)M \cos wt \quad \text{from } X = \frac{p\pi}{2} \quad \text{to } X = \pi$$

$$(4) \quad m = -pM \cos wt \quad \text{from } X = \frac{p\pi}{2} \quad \text{to } X = \pi$$

where  $M \cos wt$  is the ampere turns of the single coil.

Multiplying equation (1) by  $\cos kX$  and integrating from

$X = -\pi$  to  $X = \pi$

$$(5) \quad \int_{-\pi}^{\pi} m \cos kX dx = n \int_{-\pi}^{\pi} \cos^2 kX dx = n\pi$$

All terms on the right side of equation (1) vanish except the term containing  $\cos kX$ , because

$$\int_{-\pi}^{\pi} \cos nX \cos kX dx = 0 \quad \text{unless } n=k$$

Multiplying equations (2), (3), (4) by  $\cos kX$  and integrating:

$$(6) \quad \int_{-\pi}^{\pi} m \cos kX dx = \left[ \frac{-pm \cos wt \sin kx}{k} \right]_{-\pi}^{-p\pi/2} + \left[ \frac{(2-p)M \cos wt \sin kx}{k} \right]_{-p\pi/2}^{p\pi/2} - \left[ \frac{pm \cos wt \sin kx}{k} \right]_{p\pi/2}^{\pi}$$

$$= \frac{4M \cos wt}{k} \sin kp \frac{\pi}{2}$$

Equating equations (5) to (6)

$$(7) \quad n = \frac{4 M \cos wt}{\pi k} \sin k \frac{p\pi}{2}$$

Substituting equation (7) in equation (1)

$$(8) \quad m = \frac{4 M \cos wt}{\pi} \left( \sin \frac{p\pi}{2} \cos X + \frac{1}{2} \sin 2 \frac{p\pi}{2} \cos 2X + \frac{1}{3} \sin \frac{3p\pi}{2} \cos 3X + \dots + \frac{1}{k} \sin k \frac{p\pi}{2} \cos kX + \dots \right)$$

Considering the  $K^{\text{th}}$  harmonic of equation (8) by itself we have an M.M.F wave with  $K$  times the fundamental number of poles sinusoidally distributed around the periphery, and alternating sinusoidally in time at a frequency,  $f$ , cycles per second.

At any point on the periphery, the M.M.F. is:

$$M_k = \frac{4M}{\pi k} \sin k \frac{p\pi}{2} \cos k \pi \cos 2 \pi ft$$

But this is identically equal to:

$$(9) \quad M_k = \frac{2M}{\pi k} \sin k \frac{p\pi}{2} \left[ \cos (kx - 2\pi ft) + \cos(kx + 2\pi ft) \right]$$

That is, the stationary, alternating field is at every point and at every instant indistinguishable from the sum of two equal constant magnitude fields revolving in opposite directions at the same speed of  $2f$  pole pitches per second. Since  $k^{\text{th}}$  harmonic has  $k$  times as many poles as the fundamental, or  $N/k$  revolutions per minute if  $N$  is the synchronous speed of the fundamental.

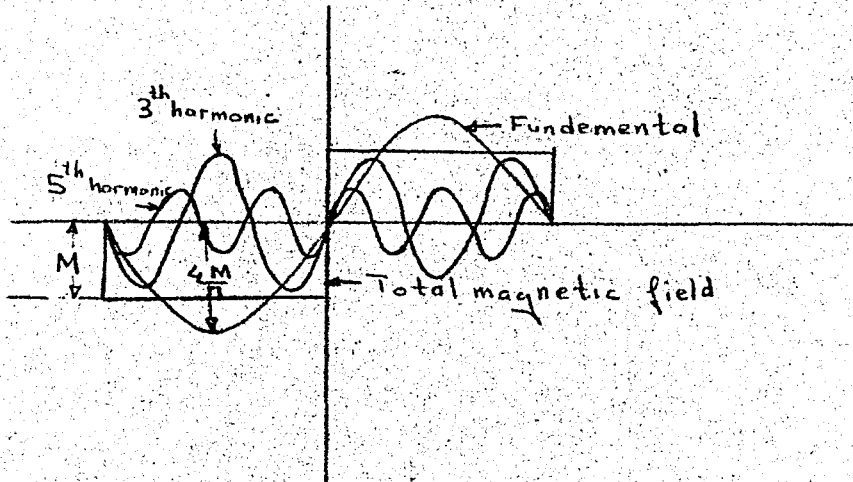


Fig. F-7 : Components of a rectangular pulsating M.M.F.

From equation (9) at  $x=0$  ,  $\frac{2\pi}{k}$  ,  $\frac{4\pi}{k}$  ,  $\frac{6\pi}{k}$  .....

representing the center of a coil at the time  $t=0$   $\frac{1}{f}$  ,  $\frac{2}{f}$  ,  $\frac{3}{f}$  ,  $\frac{4}{f}$  ..

the forward and backward revolving fields are coincident, giving their resultant its crest value of  $\frac{4M}{rk} \sin \frac{pk\pi}{2}$  . At the same time

$t : 0$  but at the point  $x = \frac{\pi}{2k}$  and  $x = \frac{3\pi}{2k}$  ,  $\frac{5\pi}{2k}$  ,  $\frac{7\pi}{2k}$  .....

the two fields are opposite, making their resultant equal to zero. This is shown down in the Figure-3.

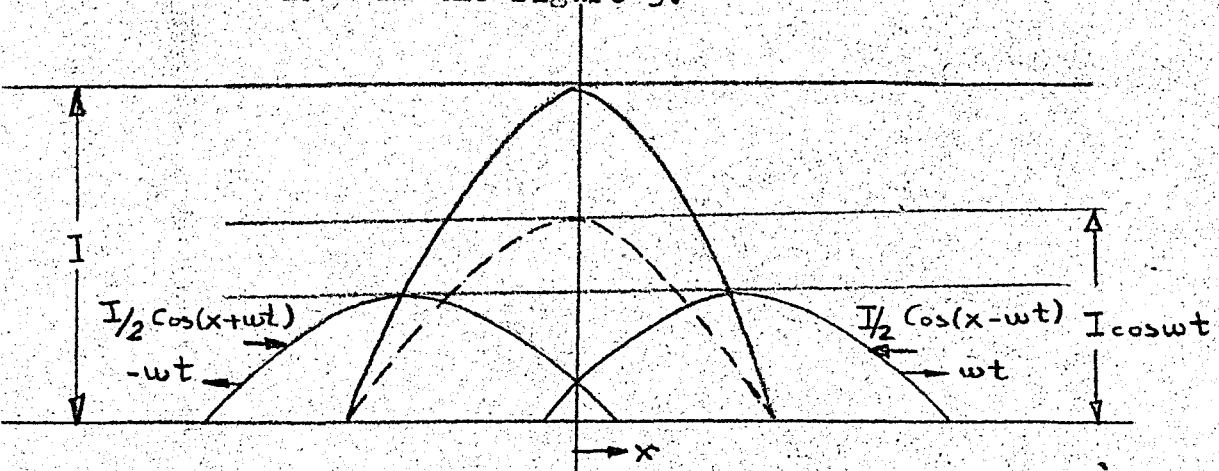


Fig. F-8 : Figure showing the two oppositely rotating fields, due to

one pulsating A.C. field.

If every coil in the winding is identical each one will produce a similar pair of oppositely revolving M.M.F waves for each harmonic.

Each phase belt composed of  $n$  consecutive coils carrying the same current, produces a  $k^{\text{th}}$  harmonic field equal to the vector sum of the  $n$  identical fields spaced consecutively one slot pitch apart. This resultant is found by vector addition of the chords of a circular arc. The corresponding distribution factor or ratio of the resultant to the numerical sum of the fields is given by equation 10.

$$(10) \quad K_{dm} = \frac{\text{Sin } \frac{mR}{2g}}{n \text{Sin } \frac{mR}{2ng}} \quad \text{for the } m^{\text{th}} \text{ harmonic.}$$

The mmf of a single phase belt of  $n$  coils is found by combining equations 8,9,10.

$$(11) \quad m = \frac{2Mn}{\pi} \left[ K_p K_d \text{Cos}(x-wt) + \frac{1}{2} K_{d_2} \text{Cos}(2x-wt) \right. \\ + \dots + \frac{1}{k} K_{pk} K_{dk} \text{Cos}(kx-wt) + \dots + \frac{2Mn}{r} \frac{k}{p} \frac{k}{d} \text{Cos} \\ (x+wt) + \frac{1}{2} K_{p_2} K_{d_2} \text{Cos}(2x+wt) + \dots + \frac{1}{k} K_{pk} K_{dk} \text{Cos}(kx+wt) \\ + \dots + \frac{2Mn}{r} K_p k \text{Cos}(x+wt) + \frac{1}{2} K_{p_2} K_{d_2} \text{Cos}(2x+wt) \\ \left. + \frac{1}{k} K_{pk} K_{dk} \text{Cos}(kx+wt) \dots \right]$$

The first bracketed term in equation (11) represents all the forward revolving fields, and the second one the backward

revolving fields. The value of  $K_{pm} = \sin pm \frac{\pi}{2}$ .

In a symmetrical winding, coils under consequent poles are  $180^\circ$  electrical apart, and carry currents  $180^\circ$  apart in time phase, that is, opposite in time phase. The resultant M.M.F of the complete phase winding expressed in ampere turns per pole, will be obtained by adding to equation (11) a similar series, except that the sign of each term is reversed and the angle  $(x + r)$  is substituted for  $X$  throughout and dividing the sum by two, that is:

$$(12) \quad m = \frac{1}{2} ( m \underline{x} - m \underline{x+r} )$$

Performing this operation on equation (11), we will note that all the odd harmonic terms add directly, since

$$- \cos [ ( 2k + 1 ) ( x + r ) ] = \cos ( 2k + 1 ) X;$$

and all the even harmonic terms cancel out, since :

$$\cos [ 2k ( x + r ) ] = \cos 2k x. \text{ Therefore, the M.M.F}$$

wave of phase A, whose current is  $\sqrt{2} I \cos wt$  is;

$$(13) \quad M_n = \frac{2Mn}{\pi} \left[ K_p K_d \cos ( X - wt ) + \frac{1}{3} k_{p3} K_{d3} \cos ( 3x - wt ) \right. \\ \left. + \frac{1}{5} K_{p5} K_{d5} \cos ( 5X - wt ) + \dots \right] + \frac{2Mn}{\pi} \left[ K_p K_d \cos ( x + wt ) \right. \\ \left. + \frac{1}{3} K_{p3} K_{d3} \cos ( 3x + wt ) + \frac{1}{5} K_{p5} K_{d5} \cos ( 5X + wt ) \dots \dots \right]$$

Where;  $M_n$  = peak ampere turns per coil :  $2 I$  times turns per coil, and,  $n$  = coils per pole per phase.

The  $\cos ( kX - wt )$  terms are forward revolving waves, and the  $\cos ( kX + wt )$  terms are backward revolving ones.

After having analysed the harmonic components of an alternating flux due to the single-phase auxiliary winding of the motor, we can study the effect of these components on the operational characteristic of the actual motor. As we have shown in the above analysis, an alternating flux can be replaced by two rotating M.M.F's travelling in opposite directions and each having an amplitude equal to half of that of the alternating M.M.F. To these two rotating flux which travels in the same direction of rotation as the rotor is called the forward rotating flux, while the other one travelling in the opposite direction to the rotor is called the backward rotating flux. So on the rotor there will be electromotive forces (EMF's) induced by two rotating fluxes for each harmonic and the influence of each flux on the rotor should be considered separately.

The effect of each of the two rotating fluxes on the rotor is the same as that of the single rotating flux on the rotor of the polyphase motor.

If the rotor has a speed of  $n$  rpm, according to the definition of the slip,  $s$ ; the slip of the rotor with respect to the forward rotating main harmonic flux is:

$$(14) \quad S_f = \frac{n_s - n}{n_s} = 1 - \frac{n}{n_s}$$

Since the backward main harmonic flux rotates opposite to the rotor, the slip of the rotor with respect to this backward rotating flux is:

$$(15) \quad S_b = \frac{n_s - (-n)}{n_s} = 1 + \frac{n}{n_s} = 2 - s$$

In order to make clear the influence of the two rotating fluxes on the rotor, it will be assumed that  $n < n_s$ . Then with respect to the forward rotating flux, and according to equation (14)  $S_f$  is positive

and smaller than unity. And  $S_b$  is greater than one and positive. Figure 4. shows the torque and mechanical power at the shaft as a function of slip for motor, generator and brake operation.

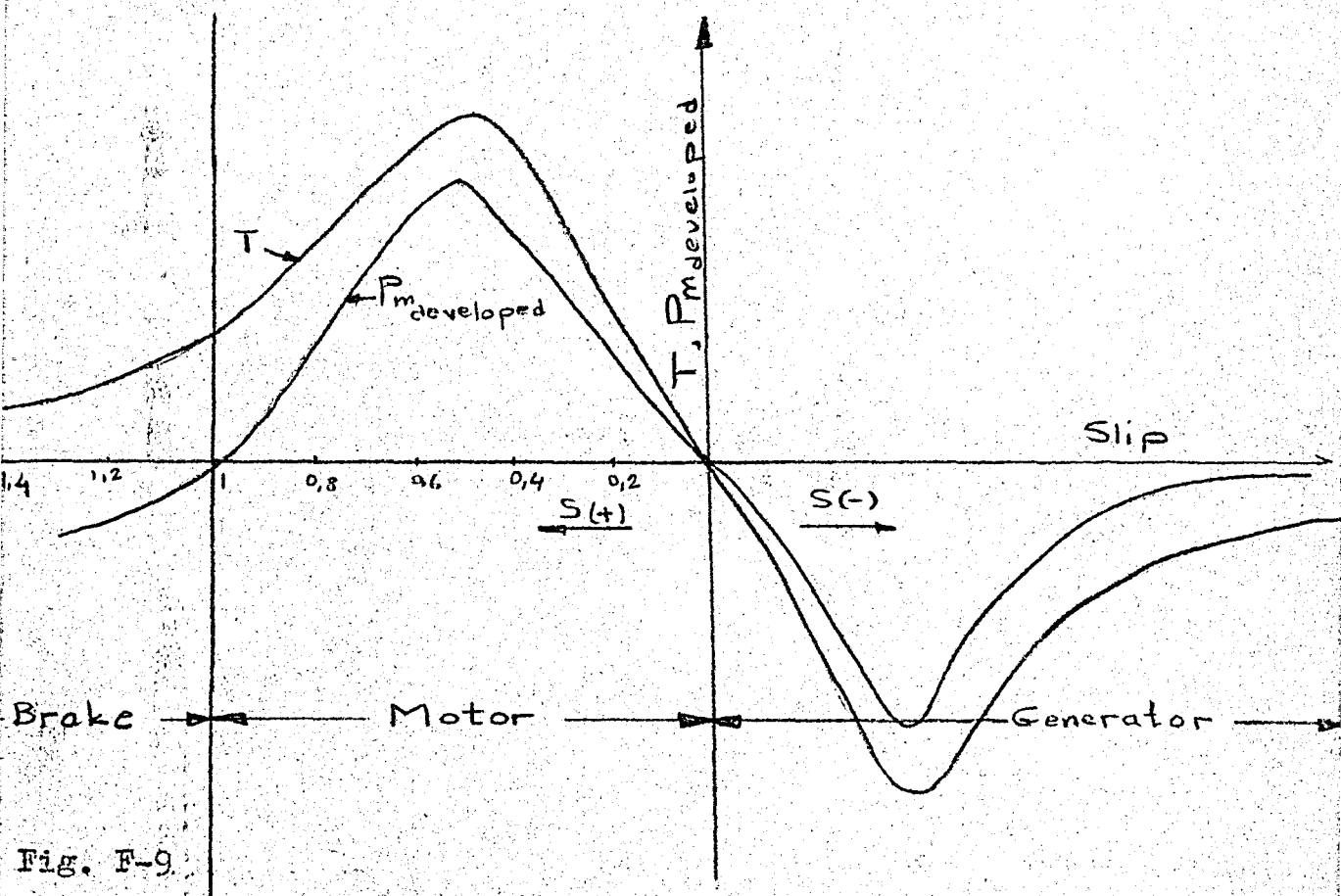


Fig. F-9.

Considering Figure F-9, it is seen that the rotor, under the influence of the forward rotating flux, operates as a motor. With S positive and smaller than 1, the slip of the rotor with respect to the backward flux is, according to equation (15), positive and larger than one. Again considering the figure drawn above, it is obvious that the rotor, under the influence of the backward rotating flux, operates in the region of brake operation. Thus the two rotating fluxes have an opposite influence upon the rotor.

The relation for the developed torque, derived for the poly phase induction motor:

$$T_{dev} = \frac{7.04}{n_s} M_1 I_2^{12} \frac{r_2}{s} \text{ lb ft} \quad \text{can be applied to}$$

each of the two rotating fluxes. Thus the torque developed by the rotor under the influence of the forward rotating main harmonic flux is:

$$(16) \quad T_{fm} = \frac{7,04 I_{2fm}^2 r_2}{n_s s} \quad \text{lb ft,}$$

and the torque developed by the rotor under the influence of the backward rotating main harmonic flux:

$$(17) \quad T_{bm} = \frac{7,04 I_{2bm}^2 r_2}{n_s (2-s)} \quad \text{lb-ft}$$

$I_{2fm}$  and  $I_{2bm}$  are the currents produced in the rotor by the forward and backward main harmonic fluxes respectively. The resultant developed torque due to main harmonic flux is the sum of  $T_{fm}$  and  $T_{bm}$ .

$$(18) \quad T_{rm} = T_{fm} + T_{bm}$$

Fig. P-10 shows both torques and their resultant for slips between zero and +2. The resultant torque is zero at standstill, in accordance with the fact that single-phase alternating flux is not able to start an induction motor.

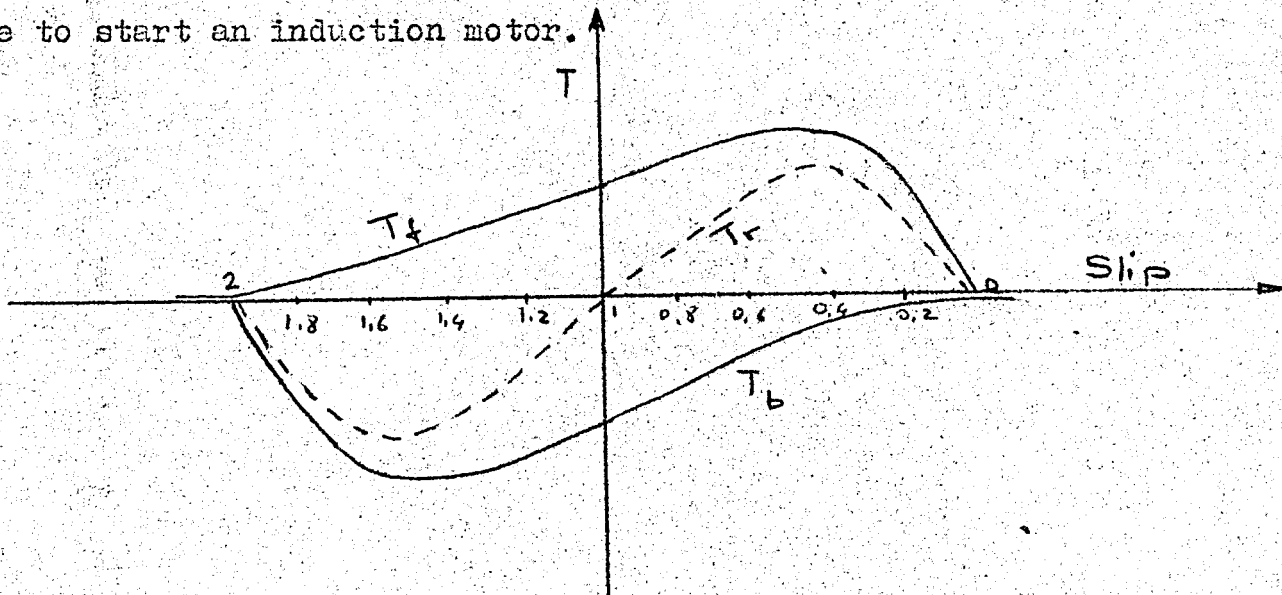


Fig. P-10

At stand still  $S=1$ ,  $2-S=1$  so both fluxes have equal and opposite influence on the rotor thus  $T_r=0$ . However, at any other slip  $|T_{fm}| \neq |T_{bm}|$  and there exists a driving torque  $T_{rm}$ . We can apply the same analysis for the other couples of harmonic fluxes. Since we are left with odd harmonics only from equation (13), it is seen that most effective harmonics are 3<sup>rd</sup>, 5<sup>th</sup>, 7<sup>th</sup>, since the amplitude of the  $K^{th}$  harmonic rotating flux is  $\frac{1}{K}$  of the main harmonic flux amplitude and the speed of rotation of the  $K^{th}$  harmonic flux is:

$$n_{sk} = \frac{120 f}{pK} = \frac{n_{sm}}{K}$$

thus:

$$(19) \quad S_{fk} = \frac{\frac{n_{sk} - n}{n_{sk}}}{\frac{n_{sk} - n}{n_{sk}}} = \frac{\frac{n_{sm}}{k} + n}{\frac{n_{sm}}{k}} = \frac{n_{sm} + kn}{n_{sm}} = 1 + k \frac{n}{n_{sm}}$$

$$(20) \quad S_{bk} = \frac{\frac{n_{sk} - (-n)}{n_{sk}}}{\frac{n_{sk} - (-n)}{n_{sk}}} = \frac{\frac{n_{sm}}{k} + n}{\frac{n_{sm}}{k}} = \frac{n_{sm} + nk}{n_{sm}} = 1 + k \frac{n}{n_{sm}}$$

For  $n \ll n_{sm}$  and  $k : 3, 5, 7, 9, \dots, k$ , from equation (19),  $S_{fk}$

is positive but less than one, for all important harmonics so rotor under the influence of forward rotating  $k^{th}$  harmonic flux, operates as in motor operation region. From equation (20),  $S_{bk}$  is positive and greater than one so the rotor under the influence of the backward rotating  $k^{th}$  harmonic flux operates in the region of brake operations.

For  $n < n_{sm}$  or considering only the first two harmonics for  $n \geq \frac{n_{sm}}{5} \gg n_{sk}$ . From equation (19)  $S_{fk}$  is negative, or zero.

Negative slip means according to Fig. F-9 generator region operation for the rotor, due to the forward flux of the harmonics. Zero slip corresponds to zero torque effect on the rotor, which is due to the forward rotating harmonic flux.

From equation (20)  $S_{bk}$  is positive and greater than one always so rotor under the influence of the backward rotating harmonic flux, always operates in the brake region.

$$S_{fk} = 0 \quad \text{when} \quad 1 = k \frac{n}{n_{sm}} \quad \frac{n}{n_{sm}} = \frac{1}{k} \quad \text{that is at}$$

$$S = 1 - \frac{1}{k}, \quad \text{when} \quad \frac{n}{n_{sm}} = \frac{1}{k} \quad S_{bk} = 1$$

$$S_{bk} = 0 \quad \text{when} \quad \frac{n}{n_{sm}} = -\frac{1}{k} \quad \text{thus} \quad S = 1 + \frac{1}{k}$$

In the analysis that will follow we will consider only the first two harmonic fluxes of the auxiliary winding namely, 3<sup>rd</sup> and 5<sup>th</sup> harmonics, however, the small their effect is.

$$\text{For } k=3 \quad s = 1 - \frac{1}{3} = \frac{2}{3} \quad \text{thus } S_{f3} = 0$$

$$\text{and } S = 1 + \frac{1}{3} = \frac{4}{3} \quad \text{makes } S_{b3} = 0$$

$$\text{for } k=5 \quad S = 1 - \frac{1}{5} = \frac{4}{5} \quad \text{makes } S_{f5} = 0$$

$$S = 1 + \frac{1}{5} = \frac{6}{5} \quad \text{makes } S_{b5} = 0$$

Using the above-mentioned data, we can draw the torque/speed characteristics of the third and fifth harmonic fluxes, Figures F-11-12. In the same way we can draw the torque/speed characteristic for the main harmonic and the super imposition of these three characteristics will give us the net torque-speed

characteristics due to the auxiliary winding with A.C single phase supply applied. Figure F-13.

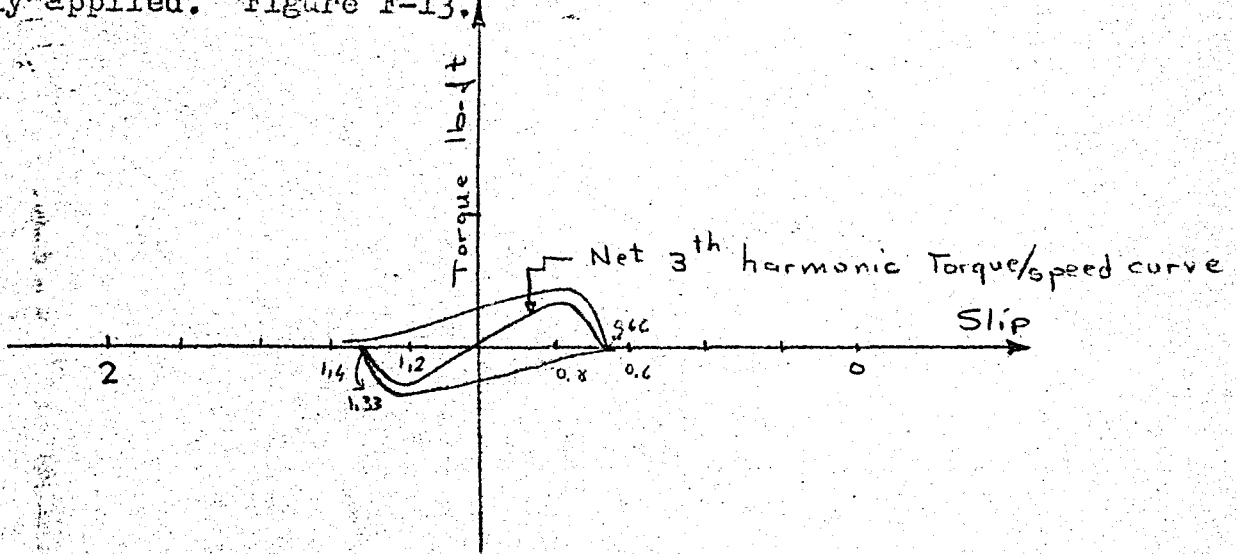


Fig. F-11 : Torque/speed characteristic of the third harmonic auxiliary single-phase flux

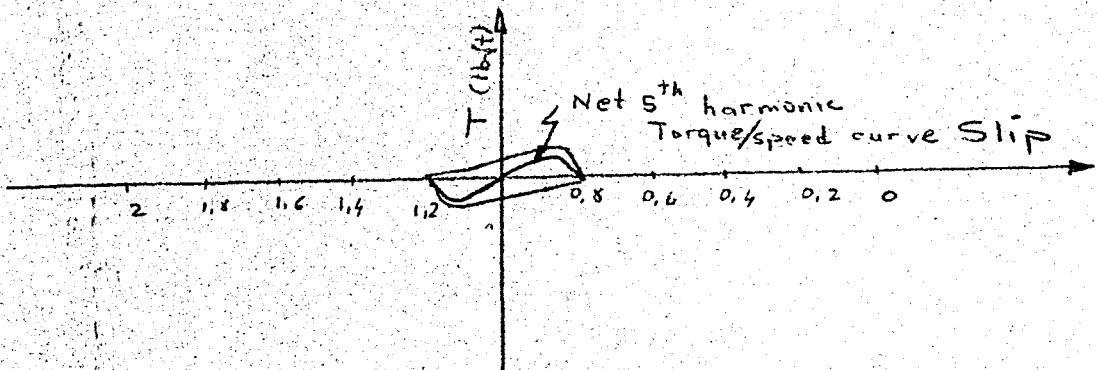


Fig. F-12 : Torque/speed characteristic of the fifth harmonic auxiliary single-phase flux.

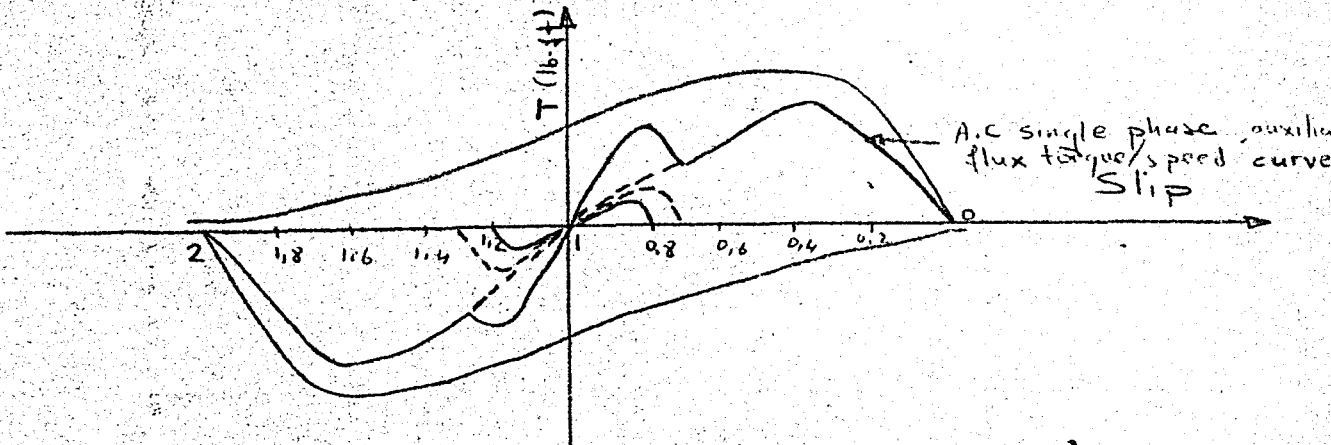


Fig. F-13 : Net torque speed curve due to main, third and fifth harmonics of single-phase A.C auxiliary flux.

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