PROBABILISTIC EARTHQUAKE RESPONSE ANALYSIS OF SINGLE DEGREE OF FREEDOM STRUCTURES

by

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ABSTRACT

PROBABILISTIC EARTHQUAKE RESPONSE ANALYSIS OF SINGLE DEGREE OF FREEDOM STRUCTURES

For evaluation of seismic safety of structures, estimation of peak response values has an important role, since the main principle is comparing the capacity of the structure and the demand from exposed strong ground motion. However, the demand shall not be characterized with only the highest peak of the response. Especially, considering overall damage, performance of the structure is related with the amplitude of remaining peaks in the whole duration of the ground motion.

In this study, a probabilistic theory, based on order statistics, has been utilized to determine theoretical distribution functions for amplitudes of the local maximum values in the response of different SDOF systems, having five per cent damping ratio, assumed as a characteristic value for a wide range of structures.

In this determination procedure, a set of 317 pairs of horizontal earthquake acceleration records from PEER/NGA database has been used, of which the Mw vary between 4.5 and 8 with lowest usable frequency of 0.2 Hz. At first, the amplitudes of all the absolute peaks were calculated from the displacement response time histories of different SDOF systems for each ground motion. Secondly, the highest 50 of normalized peaks were chosen from both horizontal components of the records in order to analyze Probability Density Function (PDF) of the peaks at different oscillator periods. Also alignment of the results of the previous studies on the issue of probabilistic analysis of random functions or of the response of structures under the earthquake excitations which were created by using different methods with the results of this study was investigated.

At the end of this study, key parameters, which define the distribution of the peaks for each oscillator period, have been presented for use in the probabilistic estimations of displacement response values of SDOF systems, having a natural period of vibration vary between 0.1 sec and 5 sec. It can be said that the results of this study will contribute to the earthquake resistant design of structures.

ÖZET

TEK SERBESTLİK DERECELİ YAPILARIN OLASILIĞA DAYALI DEPREM TEPKİ ANALİZİ

Yapıların sismik güvenliğinin belirlenmesinde, dolayısıyla yapının kapasitesi ile maruz kalınan kuvvetli yer hareketi talebinin karşılaştırılmasında; en büyük davranış değerinin tahmin edilmesi başlıca hedeftir. Ancak, talebin yalnızca tek bir en büyük davranış değeri ile tanımlanması yeterli değildir. Özellikle, toplam hasar düşünüldüğünde, yapının performansı, yer hareketi süresince gerçekleşen diğer davranış değerleri (2., 3. ve sonraki en büyük değerler) ile de ilişkilidir.

Bu çalışmada, farklı tek serbestlik dereceli sistemlerin, yüzde beş sönüm oranı için (bu oran birçok bina türü yapılar için karakteristik değer olarak kabul edilmiştir) yerel en büyük davranış büyüklüklerinin teorik dağılım fonksiyonlarının belirlenmesinde, sıralı istatistiksel değerlere bağlı, olasılık teorisinden yararlanılmıştır.

Bu belirleme sürecinde, PEER/NGA veritabanından 317 çift kuvvetli yer hareketi ivme kaydı kullanılmıştır. En düşük frekans içeriği 0.2 Hz olan bu kayıtların, moment büyüklük değerleri 4.5 ile 8 arasında değişmektedir. İlk olarak, farklı tek serbestlik dereceli sistemlerin, yerdeğiştirme – zaman ilişkileri elde edilmiş, bütün uç noktaların mutlak değerleri (mutlak maksimum değerleri) her yer hareketi için belirlenmiştir. İkinci olarak, her salınım periyodunda ki en büyük davranış değerlerinin olasılık yoğunluk fonksiyonlarının oluşturulması için, kayıtların her iki yatay birleşeninden normalize edilmiş 50 uç davranış değeri belirlenmiştir. Ayrıca, bu çalışmada literaturde bugüne kadar rastgele sayı fonksiyonlarının ya da sistemlerin farklı metodlar kullanılarak oluşturulmuş yer hareketleri etkisindeki davranışlarının olasılığa dayalı analizleri üzerine yapılmış çalışmalardan çıkan sonuçlar ile bu çalışmadan çıkan sonuçların uyumu araştırılmıştır.

Çalışmanın sonunda, doğal titreşim periyodu 0.1 sn ile 5.0 sn arasında değişen tek serbestlik dereceli sistemlerin yerdeğiştirme davranış değerlerinin olasılığa dayalı analizlerinde kullanılması için, bu değerlerin dağılımını tanımlayan parametreler sunulmuştur. Bu çalışmadan elde edilen sonuçların, depreme dayanıklı yapı tasarımında katkı sağlayacağı söylenebilir.

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LIST OF SYMBOLS / ABBREVIATIONS

A	Rupture area
A_{11}, A_{12}, A_{21}, A_{22}	Recurrence coefficients
A^c	Complement of an event A
AI	Arias Intensity
a	Ground acceleration
a_0	Specified level of acceleration
a_{rms}	Root mean square value of f(t)
B_{11}, B_{12}, B_{21}, B_{22}	Recurrence coefficients
C1, C2, C3, C4	Integration constants
C_n	Amplitude related to the energy spectrum
c	Viscous damping coefficient
c_{cr}	Critical damping coefficient
Db	Bracketed duration
Ds	Significant duration
Du	Uniform duration
$\bar{D}$	Average amount of slip
E	Recurrence coefficients
E (ω)	Energy spectrum
Ei	Expected frequency
F	Recurrence coefficients
f	Natural cyclic frequency of vibration
f (t)	Random function of time
$f_D(t)$	Damping resisting force
$f_I(t)$	Inertial resisting force
$f_s(t)$	Structural resisting force
k	Lateral stiffness coefficient
M_D	Duration magnitude
M_L	Local magnitude

M_0	Seismic moment of an earthquake
M_S	Magnitude based on surface waves
M_W	Moment magnitude
m	Mass
m_b	Body wave magnitude
m_n	N-th moment of the energy spectrum
m_0	Zeroth moment of the energy spectrum
m_2	Second moment of the energy spectrum
m_4	Fourth moment of the energy spectrum
Oi	Observed frequency
P	Static loading
P (A)	Probability of an event A
P (A^c)	Probability of the opposite of an event A
P (t)	Dynamic loading
p (η)	Probability density function of the maxima of f(t)
s	Standard deviation
T	Natural vibration period of single degree of freedom system
t	Time
ti	Time station
tr	Total duration of accelerogram
U_{max}	Maximum spectral displacement value
u_h	Homogenous solution
u_p	Particular solution
u (t)	Relative displacement between the mass and ground
u (0)	Initial displacement of the mass
$u^g(t)$	Displacement of the ground
$u^t(t)$	Total (absolute) displacement of the mass
$\dot{u}(t)$	Relative velocity between the mass and ground
$\dot{u}(0)$	Initial velocity of the mass
$\dot{u}^t(t)$	Total (absolute) velocity of the mass
$\ddot{u}(t)$	Relative acceleration between the mass and ground

$\ddot{u}^t(t)$	Total (absolute) acceleration of the mass
X_0^2	Chi-Square test statistic value
β	Shape (Slope) parameter of the Weibull distribution
$\hat{\beta}$	Maximum likelihood estimator for the Weibull distribution' shape parameter
γ	Location (Shift) parameter of the Weibull distribution
Δt	Time interval
ε	Width of energy spectrum
η	Scale parameter of the Weibull distribution
$\hat{\eta}$	Maximum likelihood estimator for the Weibull distribution' scale parameter
λ	Rate parameter of the Exponential distribution
λ_1, λ_2	Roots of the quadratic formula
$\hat{\lambda}$	Maximum likelihood estimator for the Exponential distribution' rate parameter
μ	Mean
μ	Rupture strength of the material along the fault
$\hat{\mu}$	Maximum likelihood estimator for the mean
ξ	Damping ratio
σ	Scale (Rayleigh) parameter of the Rayleigh distribution
σ^2	Variance
$\hat{\sigma}$	Maximum likelihood estimator for the Rayleigh parameter
$\hat{\sigma}^2$	Maximum likelihood estimator for the variance
Φ_n	Random phase uniformly distributed between 0 and 2π
ω	Natural circular frequency of vibration
ω_D	Circular frequency of vibration for underdamped system
ω_E	Circular frequency of vibration for overdamped system
ω_n	Circular frequency

3D	Three dimensional
A-D	Anderson- Darling statistical test
CDF	Cumulative distribution function
DOF	Degree of freedom
EW	East-West component of record
K-S	Kolmogorov-Smirnov statistical test
MDOF	Multi-degree-of-freedom
MLE	Maximum Likelihood Estimation method
NGA	Next generation attenuation
NS	North-South component of record
PDF	Probability density function
PEER	Pacific Earthquake Engineering Research Center
PGA	Peak ground acceleration
PGD	Peak ground displacement
PGV	Peak ground velocity
SD	Spectral displacement
SDOF	Single degree of freedom

1. INTRODUCTION

As per the existing philosophy of earthquake-resistant design, the reductions are permitted in the linear design earthquake forces due to economical reasons and the structures are designed with sufficient ductility to withstand a few inelastic excursions as required by the ground motion without failure.

In earthquake engineering, ductility represents the potential of a structure to dissipate energy through inelastic deformations during the severe ground shaking. Most of the times, the design practices based on this philosophy require estimation of the maximum response value of structures using response spectra and ignore the rest of inelastic excursions, which differ in number, sequence and relative amplitudes, in their seismic responses.

However, to have knowledge of these repeated inelastic excursions should be helpful in understanding the number of times a certain response level may be exceeded, the progressing of damage in a structure, the probability of exceeding specified response levels, effective response durations and the relationship between the amplitudes of all the response peaks to the characteristics of structural system. Therefore, while the motivation for this thesis comes from the need to understand the seismic response of structures in detail, it becomes necessary to study the statistics of the ordered peaks in the response of structures to earthquake excitation.

Behavior of structures under earthquake ground motions are affected by several parameters that some of them are related to the characteristics of the structure under earthquake excitation and some are directly related to the characteristics of earthquake ground motion. Thus, below in separate subheadings, the widely used ground motion parameters that contribute to the seismic response of structures are discussed.

1.1. Overview of Strong Ground Motion Parameters

Ground motions induced by earthquakes are very complex and uncertain in nature. For this reason, various studies in earthquake engineering literature devoted to the identification of the ground motion parameters which will be helpful in describing the complex characteristics of earthquake ground motions in compact, quantitative form. These parameters can characterize the amplitude, frequency content and duration of strong ground motion.

Representation of earthquake ground motion characteristics with some parameters is desirable for quantitatively comparing different types of ground motions and for determining the relationship between the potential damage of structures and measured ground motions.

In this section, the most commonly used ground motion parameters which are essential to characterize and classify strong ground motions will be discussed briefly. However, prior to that below in separate subsections short descriptions of the various factors that affect the ground motion parameters most strongly is given.

The first influence is the magnitude of earthquake. Earthquake magnitude is a quantitative measure of the size of an earthquake. Earthquake magnitude is computed from measurements on seismographs. There are different magnitude scale definitions for classifying earthquakes (i.e., Richter Local Magnitude, M_L, Surface Wave Magnitude, M_S, Body Wave Magnitude, m_b, Moment Magnitude, M_W, and Duration Magnitude, M_D).

In this thesis, magnitude type of the selected records is moment magnitude since it is based on the seismic moment, which characterizes the overall deformation at the source.

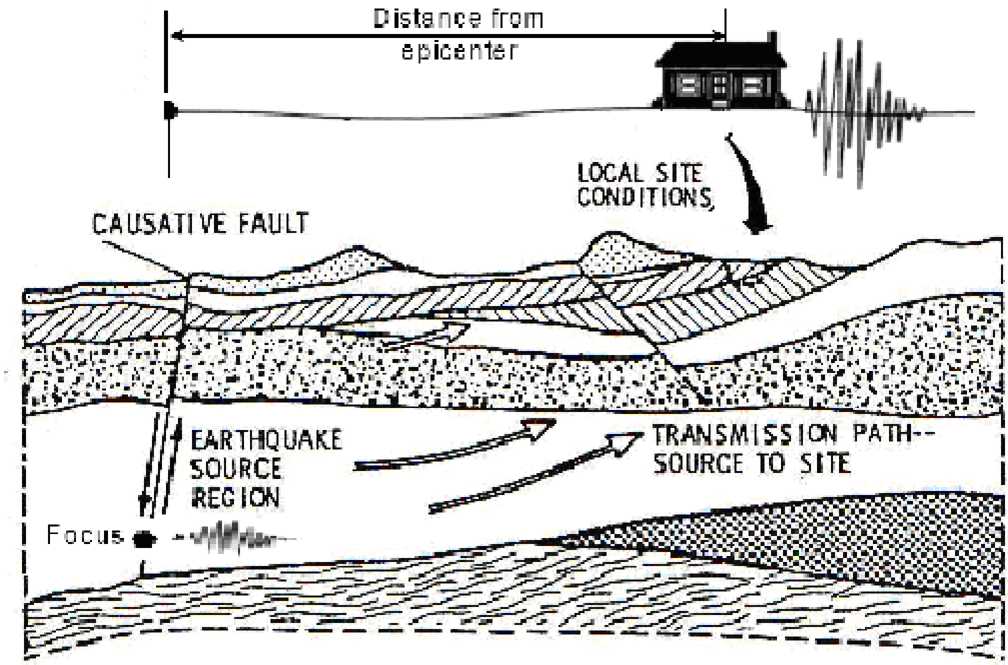

Figure 1.1. Schematic diagram of factors that affect ground motion (Werner, 1976)

The seismic moment of an earthquake is

$$M_0 = \mu A \bar{D} \tag{1.1}$$

Where μ is the rupture strength of the material along the fault, A is the rupture area and $\bar{D}$ is the average amount of slip.

The moment magnitude is given by

$$M_w = \frac{\log M_0}{1.5} - 10.7 \tag{1.2}$$

The second one is the distance between the source of an earthquake and a particular site. When the strength of tectonic plate in earth's lithosphere can no longer resist elastic strain that has generated because of slippage along the active fault, rupture begins along the fault and earthquake occurs. The point at which the initial rupture begins is the focus or hypocenter of the earthquake. The focus is located at some focal depth (or hypocentral depth) below the ground surface. The epicenter of the earthquake is the vertical projection of the hypocenter, or the focus onto the earth's surface.

The distance on the ground surface between an observer or site and the epicenter is known as the epicentral distance, and the distance between the observer and the focus is called the focal distance or hypocentral distance (Kramer, 1996).

The records in this thesis cover far field (at distances away from the fault or the source of energy release) and near field (at distances close to the fault).

The third one is the various geologic factors that affect the earthquake ground motions at a particular location. These factors are related to the earthquake source, wave propagation path, and /or local site conditions. The influences of these factors require careful consideration and will not be explained separately in this thesis.

After reviewing these factors, below in separate subtitles the most commonly used parameters that characterize the amplitude, frequency content and duration of ground motions are summarized.

1.1.1. Amplitude Parameters

The most commonly used amplitude parameters in time domain are peak acceleration, peak velocity and peak displacement.

Peak acceleration is closely related to the high frequency component of the ground motion but it alone does not provide any information about the duration or the frequency content and about the damage potential of the motion. When considering the amplitude of ground motion, peak ground acceleration (PGA) is the most widely used measure. PGA is the largest (absolute) value of the acceleration time history.

Peak velocity is another measure of ground motion amplitude that provides a good indication of the intermediate frequency component of the ground motion. As PGA, peak ground velocity (PGV), which corresponds to the maximum (absolute) value of the velocity time history, is used for characterization of ground motion amplitude. Peak velocity can not account for duration and frequency content of the motion like peak acceleration.

Peak displacement describes the amplitudes of the low frequency component of the ground motion more accurately and as in the case of peak acceleration and peak velocity, peak displacement provide no information on the frequency content and duration of ground motion. Ground motion can also be described by displacement time history and its peak value, peak ground displacement (PGD). PGD is another parameter that measures the amplitude of ground motion.

As mentioned above, the use of these parameters to characterize the ground motion has a number of limitations such as they can not account for duration and frequency content of the ground motion and also they are not sufficient enough to determine the relative strengths of different earthquake ground motions. Therefore, additional information such as frequency content or duration parameters is generally required to characterize the strong ground motion.

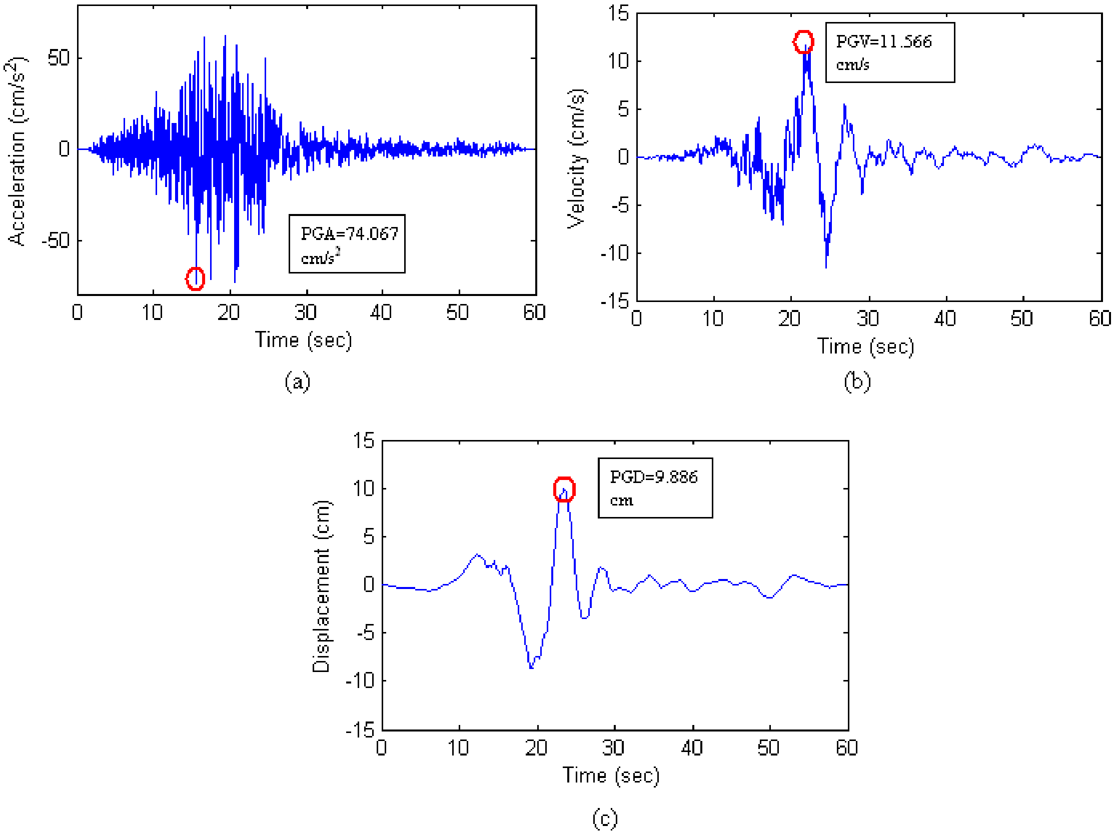

Figure 1.2. Time histories for the horizontal component of the 1999 Hector Mine earthquake recorded at Heart Bar State Park station: (a) ground acceleration; (b) ground velocity; (c) ground displacement

1.1.2. Frequency Content Parameters

Frequency content is one of the most important characteristics of strong ground motion which defines the distribution of ground motion amplitude through different frequencies. Earthquake shaking represents complex loading with components of motion that scatter a broad range of frequencies. Therefore, earthquake excitation should be characterized by taking the frequency content into consideration.

The frequency content of the ground motion is described by using different types of spectra such as Fourier spectra, Power spectra and Response spectra. In earthquake engineering practice, the response spectrum is extensively used for describing the frequency (or period) content of the ground motion and for analyzing the performance of structures in earthquake. Spectrum represents a single graph which summarizes the dynamic response of structures since many of them can be idealized as simple oscillators. The plotted response can be spectral accelerations, spectral velocities, and spectral displacements and so on.

The response spectrum represents maximum response (peak response) of a series of single degree of freedom systems (also known as simple oscillators) to a particular component of ground motion as a function of their natural vibration periods (or natural frequencies) and damping ratios.

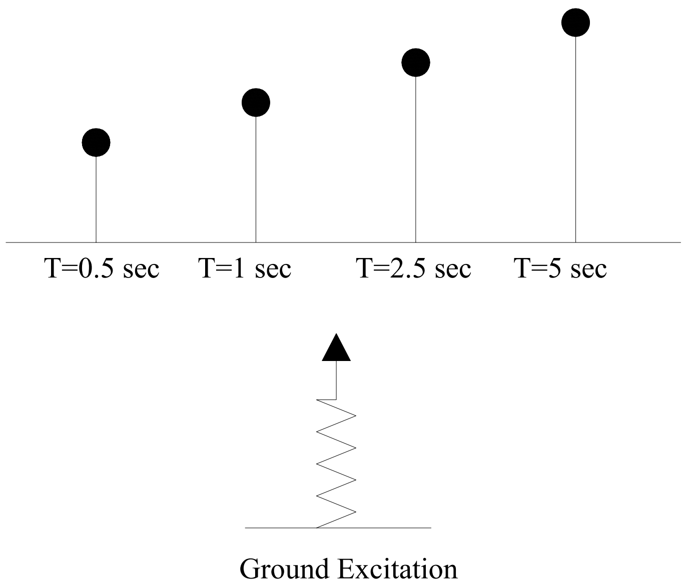

Figure 1.3. Linear SDOF systems with different vibration periods under the horizontal component of the 1999 Hector Mine earthquake at Heart Bar State Park station

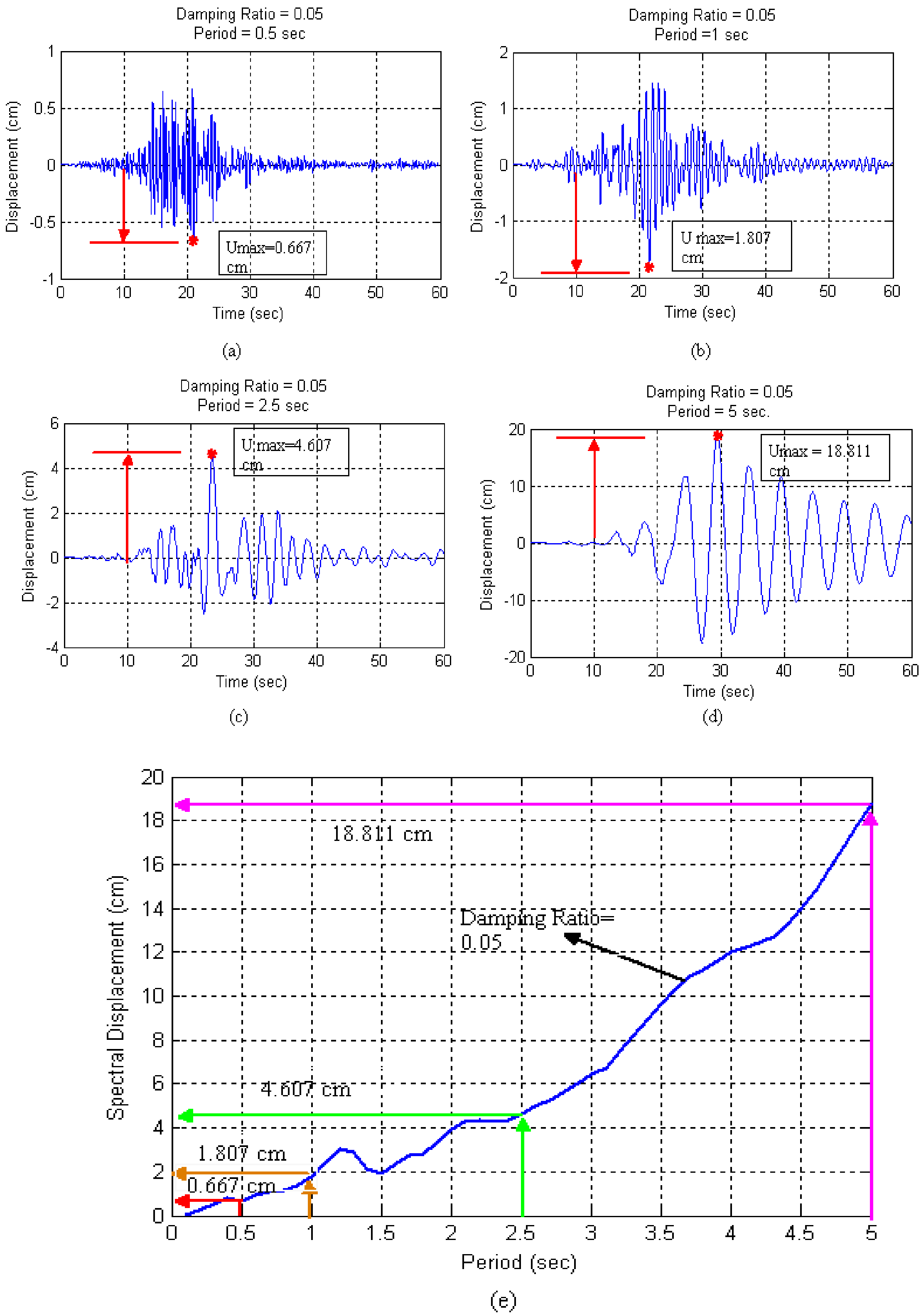

Figure 1.4. Displacement response of SDOF systems excited by the horizontal component of the 1999 Hector Mine earthquake recorded at Heart Bar State Park

station: (a) SDOF system with ξ = 5 % and Tn= 0.5 sec; (b) SDOF system with ξ=5% and Tn=1 sec; (c) SDOF system with ξ = 5 % and Tn= 2.5 sec; (d) SDOF system with ξ = 5 % and Tn= 5 sec; (e) Displacement response spectrum for ξ = 5 %

Response spectrum describes the actual ground motion characteristics indirectly because it essentially reveals the potential effects of the ground motions on structures. Produced spectral values are influenced by the amplitude, frequency content, and to a lesser extent duration of the ground motions. Response spectrum does not account for the duration of the ground motion that can have a strong influence on the damage sustained.

1.1.3. Duration

The duration of strong earthquake ground motion is another important parameter which can have a significant effect on the potential of the ground motion to produce damage.

In order to identify and measure the duration of strong motion, different approaches have been proposed by different researchers.

Strong motion duration definitions have been classified into four generic groups. The definitions of duration in the first three generic groups based on earthquake accelerograms and the definitions of duration in the fourth generic group based on the response of structures to earthquake loading (Bommer and Pereira, 1999).

- Bracketed Durations, Db: These are defined as the total time elapsed between the first and last excursions of a specified level of acceleration, a_0, as illustrated in Figure 1.5 (a).

- Uniform Durations, Du: These are also defined by a threshold level of acceleration, a_0, but rather than as the interval between the first and final peaks that exceed this level, the duration is defined as the sum of the time intervals during which the acceleration is greater than the threshold.

- Significant Durations, Ds: These are based on the accumulation of energy in the accelerogram represented by the integral of the square of the ground acceleration. These are defined as the time interval between the points at which five percent and 95 % of total energy has been recorded.

- Structural Response Durations: These definitions are based not on the characteristics of the record of ground motion, but on the characteristics of the response motion of a specified structure subjected to the ground motion.

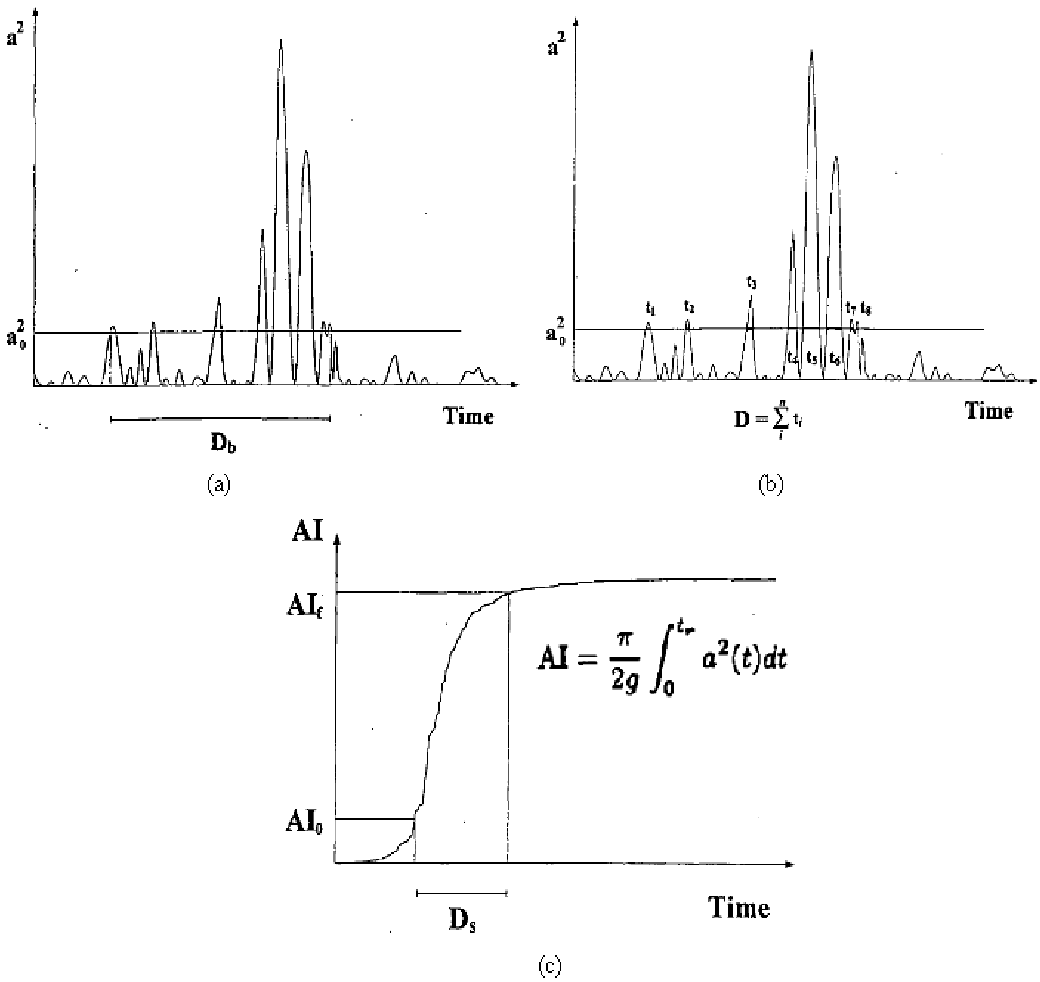

Figure 1.5. Generic definitions of strong ground motion duration: (a) Bracketed duration of an accelerogram; (b) Uniform duration of an accelerogram; (c) Significant duration of an accelerogram (Bommer and Pereira, 1999)

1.2. Literature Review

For earthquake engineering analysis and design, identification of earthquake ground motions with respect to their damage potential is the most important issue. Therefore, most of the studies in literature have been carried out to quantify the damage potential of ground motions by employing several parameters.

1.2.1. Strong Motion Duration

In the following subsections, some of the previous studies on the duration parameter, which usually correlates closely with the damage potential of ground motion, will be summarized.

Trifunac and Brady (1975) presented a simple definition of the duration of strong earthquake ground motion based on the mean-square integral of the motion. In this study, they defined significant duration as the time interval in which the 90 per cent of the total integral is attained.

They have also established correlations between Modified Mercalli intensity, earthquake magnitude, epicentral distance, and the type of recording site geology with the duration of strong motion acceleration, velocity and displacement by using 188 accelerograph records.

Bommer and Pereira (1999) reviewed and evaluated a large variety of approaches which have been proposed by different researchers to identifying and measuring the strong shaking phase of accelerograms. They classified strong motion duration definitions into four generic groups as stated above.

Based on the shortcomings that have been identified in all of the definitions reviewed, they have proposed a new definition for duration that is called 'effective duration'. This record-based duration definition is based on the significant duration concept.

In this study, they had also investigated the correlations between the new definition of duration and distance, magnitude and site conditions.

The work of Bommer and Pereira (1999) has been complemented by the study of Hancock and Bommer (2005). According to them, measures of effective number of cycles of motion are more likely to convey useful indications of the influence of the duration of shaking on the response of structures. They have reviewed, classified and compared different definitions of the effective number of cycles of the motion proposed in the literature and investigated the influence of the predictive variables, such as earthquake magnitude, site-to-source distance on the number of cycles.

On top of all this, they also explored correlations between cycle counts and different duration measures by using the databank of almost 500 strong-motion accelerogram and found that these correlations would be poor in the absence of additional parameters. However, as is the case with strong motion duration, there is no universally accepted approach to determining the effective number of cycles of motion, and the different methods that have been proposed can give widely varying results for a particular accelerogram, they recommended that the rainflow-counting definition can be employed to determine the number of cycles from accelerograms.

According to Bommer *et al.* (2006), duration of the shaking can be represented either by an interval of time during which the motion fulfils some specified criteria or else the number of effective cycles of motion. The main objective of this study was to improve the poor correlations between calculated durations and number of cycles, with the exception of uniform duration and number of cycles obtained using rainflow-counting, both using absolute rather than relative thresholds. In order to reduce the scatter in the correlation, they have proposed grouping the records according to a new measure of the period content of the motion. This measure is defined as the period interval between the first and last excursions of the five per cent damped acceleration spectrum at a level of 1.2 times the record PGA.

In the light of the foregoing discussions of the last two studies, Stafford and Bommer (2009) developed predictive equations for measures of the effective numbers of cycles of ground motions as functions of magnitude, distance, and site classification. Despite the fact that there are several different measures of the numbers of cycles in an accelerogram, rainflow-counting definition is adopted in this work due to the results of Hancock and Bommer' s (2005) study.

So far, all of the studies mentioned are mainly concentrated on measuring the duration of strong ground motion. However, when evaluating the seismic safety of a particular structural model, the definition of duration based on structural response would be the most appropriate one.

1.2.2. Structural Response Duration

In this section, a summary of the efforts of several researchers that contributed to the structural response duration is given

The structural response duration definition had been used by Sucuoğlu and Nurtuğ (1995a), in order to assess the sensitivity of seismic energy dissipation to ground motion and system characteristics. They pointed out that the energy dissipation is related to the response duration, which is dependent on, but different from, the strong motion duration. In this study, they have adapted the definition of the duration which had been presented by Trifunac and Brady (1975), and noted that the duration of strong motion may be substantially shorter than the total duration of a record, although for lightly damped structures, the duration of significant response of a structure can also be longer than the total duration of a recorded ground motion.

In the second study of Sucuoğlu and Nurtuğ (1995b); they had explored a relationship between the response duration of a linear SDOF system under earthquake excitation with the natural vibration period and viscous damping ratio of the system. The results of their study showed that

- Response duration increases with vibration period, but decreases with the amount of damping.
- For very stiff systems (short-period systems and systems with high damping), response duration approaches strong motion duration. However, response duration increases with vibration period almost linearly, with a similar slope and it approaches infinity for undamped systems which dissipate no energy.

Another study used the concept of structural response based duration was performed by Şafak (1998). In this study, a new method to incorporate duration in response spectra by introducing the concept of 3D Response Spectra has been presented. The principle of the method used for determining the duration is based on the identification of the distribution of the displacement response peaks of a single-degree-of-freedom (SDOF) oscillator.

The concept of 3D Response Spectra has been used in the following analysis to show the amplitudes of secondary peaks as well as the largest peaks of the displacement response of a series of single degree of freedom systems as a function of natural vibration period and the number of crossings.

Estimation of all the response peaks of a structure with known dynamic characteristics when it is subjected to a particular ground motion reveals very useful information for seismic design. Therefore, in the following subsections, a summary of previous studies carried out with the purpose of assessment of the statistics of the peaks in the response process is given.

The pioneering attempt to study the statistics of maxima was made by Rice (1944, 1945). In this study, the theory of probability distribution of a random function and its zeros and maxima were described.

The study of Cartwright and Higgins (1956) extended the work of Rice (1944, 1945) and presented results which find useful application in earthquake engineering. In this study, the distribution function, the expected values, and the most probable values of the peaks of a random function have been presented.

Many investigators have applied and developed the results of the above-mentioned studies.

Udwadia and Trifunac (1974) determined the response spectrum of a damped oscillator through the statistics of the maximum peak of the response.

The other work which was the generalization and extension of the theory of Cartwright and Higgins (1956) was carried out by Amini and Trifunac (1981, 1985). The objective of this study is to present some new results on the distribution of the first, second, third, … largest peak amplitudes in the response of structures to earthquake excitation based on the assumptions that the history of response is a stationary process in time and the local extrema of response time history are mutually independent.

A further study was made by Gupta and Trifunac (1988), to refine the theoretical distribution function for the amplitudes of all the local maxima in the random response function by following the order statistics approach. Even though the assumptions of stationarity and mutual independence of peaks had still been carried on, their theoretical results did not became poorer for higher-order peaks as in the study of Amini and Trifunac (1981, 1985) at which they did not use the order statistics to derive the distribution function for amplitudes of the higher-order peaks.

1.3. Objectives

The major objective of this study is to determine the statistical distribution of ordered peaks in linear earthquake response and thus give designer an opportunity to understand in great detail the response characteristics of structures to earthquake excitation.

Once the distribution of the peaks is known, then we can easily (1) describe the expected and the most probable values of the n-th order (first largest, second largest, third largest, etc.) peaks of response of linear, viscously damped SDOF system, (2) evaluate the relationship between all local response maxima, their number and amplitudes to the physical characteristics of the designed structural system, (3) understand the number of times certain response levels may be exceeded, (4) estimate the probabilities of exceeding or not exceeding a particular response peak amplitude, (5) understand the progressing of damage in a structure, and (6) determine the effective structural response duration.

In addition, this study is the last one in a series of studies carried out by many researchers mentioned in Chapter 1.2. Each one presented a probabilistic theory to determine the statistical distributions of peaks in a random process or in a structural response process. Although previous studies have applied different methods and made different assumptions in order to find the statistical distribution of the maxima, they have obtained the same results for the probability density function of the maxima. However, efforts towards the verifying these results were limited. Because, most of them have only used theoretical derivations to determine the distributions of maxima and/or synthetic (artificial) accelerograms have been considered to validate the obtained theoretical results. In this study, therefore, the secondary objective of the author is to justify the results of the previous studies by using large database of real ground motion records collected from all around the world.

2. INFORMATION ABOUT DYNAMIC ANALYSIS OF STRUCTURES

2.1. Dynamic Analysis of Structures

Loads are forces which act on a structure and then generate deformations, stresses and displacements in structures. For the assessment of behavior of the structure under external influences, or loads, they have to be defined because different types of loads create various influences on the behavior of structure on which it acts. In order to determine these effects, an analysis of structural system should be made. The methods of structural analysis based on the type of loading and type of structural system on which load acts.

2.1.1. Dynamic Loading

Earthquakes lead to dynamic loads and when evaluating structural response to dynamic loading there are two basically different approaches; deterministic and nondeterministic.

Deterministic analysis covers the structural response under the dynamic load whose time variation is exactly known in spite of its irregular and very vibrant character. However, in the nondeterministic analysis, time variation of the dynamic load can not be known exactly.

Throughout this study, the earthquake loads are assumed deterministic although they are nondeterministic in reality. Therefore, the used structural analysis method and the resultant response are also deterministic.

Before describing properties of the used analytical (dynamic) model, which represents the structural system, and mathematical model, which is composed of set of differential equations of motion that describe the analytical model, a small comparison will be made on the behalf of telling the difference between the static and dynamic analysis.

As it is well known, static load is an external force that is applied so slowly and does not change in magnitude or position with time as shown in below figure. All structures are subject to some static loading, e. g., their own weight.

Dynamic load may vary in magnitude, sense and direction during operation. Similarly, the structural response to a dynamic load is also time varying, or dynamic. Impact loads, waves, earthquake shocks, and high level wind gusts belong in this category.

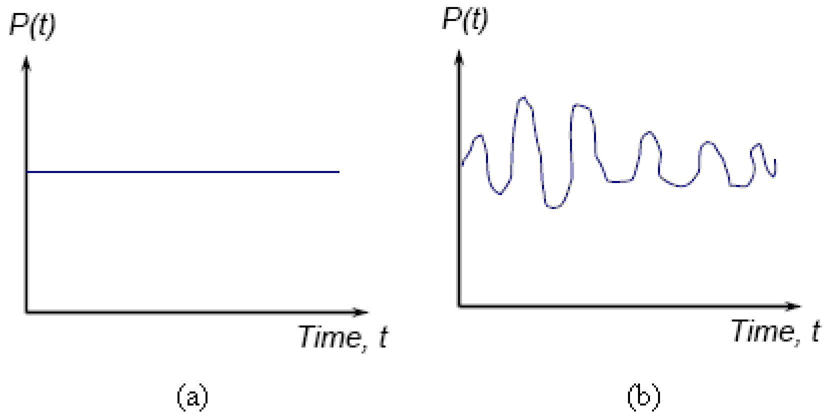

Figure 2.1. (a) Characteristics of static loads; (b) Characteristics of dynamic loads

Structural analysis for dynamic loads, P(t), is much harder than for static loads, P, because of the time varying nature of the dynamic load and structural response. However, in static problem due to the time independent nature of the load, it has a single solution. In addition to this, in a static problem, the response deformations only depends on the static load, it is therefore, force equilibrium equations are enough to solve the problem. Whereas, in a dynamic problem, resulting motion of the structure not only depends on applied dynamic load but also on the inertial forces which resist the accelerations producing them.

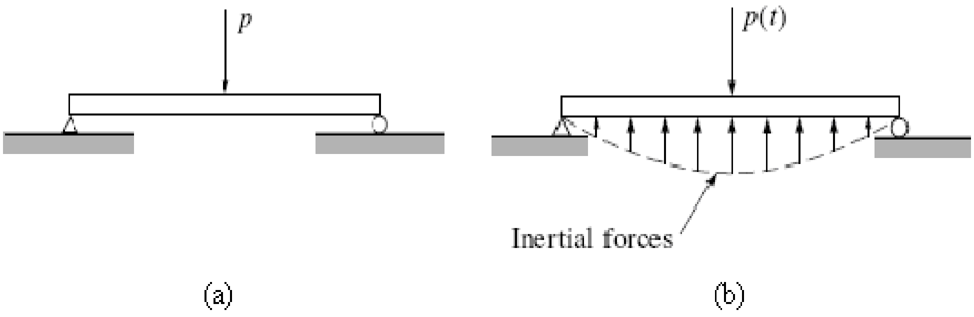

Figure 2.2. Difference between static and dynamic loading: (a) static loading (b) dynamic loading (Clough, 2003)

2.1.2. Analytical (Dynamic) Model

The most critical stage of structural analysis is to create the dynamic model that will simulate the behavior of the real structure. Structures respond to earthquake excitation as either simple or complex oscillators. When the simple oscillators are described with Single-degree-of-freedom (SDOF) systems, Multi-degree-of-freedom (MDOF) systems are used to represent the complex oscillators.

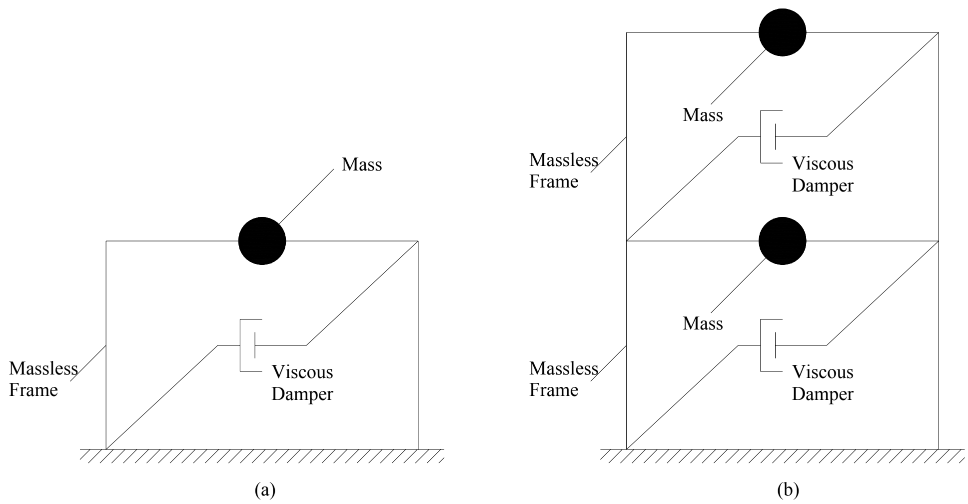

Figure 2.3. Idealization of structure: (a) single-degree-of-freedom system (b) multi degree- of-freedom system

A degree of freedom (DOF) represents an active translation or rotation component of the motion required to define the displaced positions of all the masses relative to their original position. For instance, if a structure deforms in three-dimensional space, then each mass will have a total of 6 DOF, namely three displacements and three rotations, while on the x-y plane, each mass will have 3 DOF, namely two displacements and one rotation.

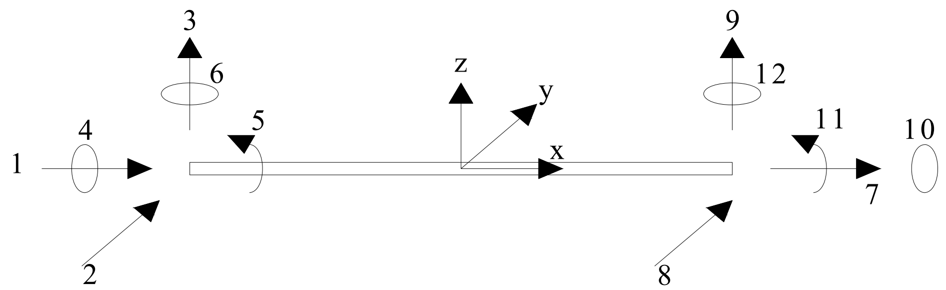

Figure 2.4. Three-dimensional 2 node frame

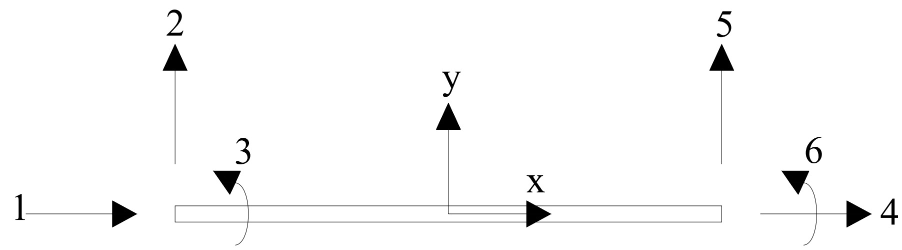

Figure 2.5. Two-dimensional 2 node frame

As showed above, the dynamic model which is an idealization of the structure is simplified by making some assumptions. The principal assumptions are;

- The mass of the structure is assumed to be lumped at the floor levels
- Diaphragms (Floors) are rigid
- The mass of the structure is assumed to displace only in one translational direction

2.1.3. Structural Analysis

The basic principles of structural analysis are to study and predict the behavior of structures. An analysis of structural system consists of determining the internal forces, stresses, deformations and the reactions caused by external loads.

For this study the simplest dynamic model, SDOF system is used as structural model. A SDOF system can be visualized as an idealized one story frame as shown in Figure (2.7) or a mass-spring-damper system as in Figure (2.6). If we consider the spring and damper to be massless, the mass to be rigid, and all motion to be in the direction of x-axis, we have a SDOF system (Chopra, 2001).

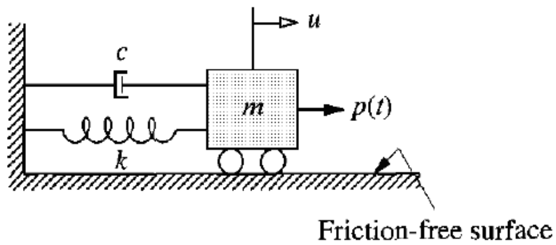

Figure 2.6. Mass-spring-damper system (Chopra, 2001)

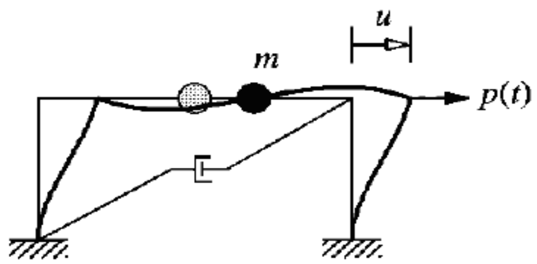

Figure 2.7. One story frame (Chopra, 2001)

A structure that is idealized as a SDOF system has only one degree of freedom (DOF) as the name suggests. In this idealized system, the mass of the actual structure is assumed to be concentrated at the roof level and it is constrained to move only in the direction of the excitation. An ideal system consists of three separate components; mass, stiffness and damping.

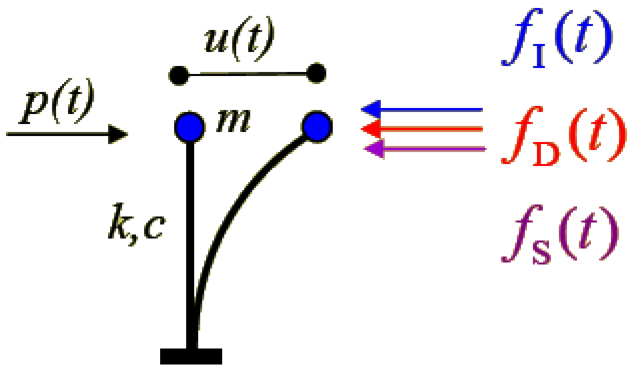

Figure 2.8. Components of idealized SDOF oscillator

Inertia force, f_I, which resists the acceleration of mass, is developed by the mass according to D'Alembert's principle. The inertia force associated with a mass is proportional to the acceleration of the mass and opposing it.

Another force which is the opposite direction to the force acting on the system is called damping force, f_D. Damping is the ability of the structural system to diminish the amplitude of vibration with time once the time-dependent applied force is removed. In damping, the energy of the vibrating system is dissipated by various mechanisms such as friction between the structure itself and nonstructural elements such as partition walls, internal friction due to the deformations in the structural material and so on (Chopra, 2001). Because of the complexity in defining each of these mechanisms, the actual damping in a SDOF structure can be idealized by a linear viscous damping mechanism (also known as a dashpot).

Damping force is related to the velocity with the constant proportionality referred to as the viscous damping coefficient, c. The damping coefficient can not be determined easily because it is not feasible to identify all the mechanisms that contribute to the vibrational energy dissipation in the actual structure. Therefore, some vibration experiments on the actual structure should have been done to determine the value of this coefficient.

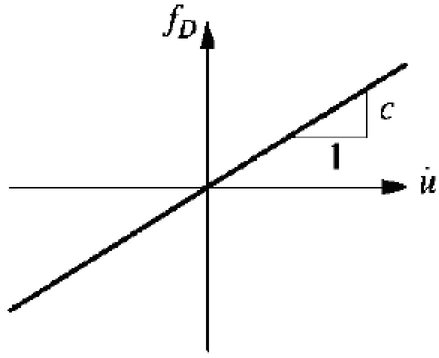

Figure 2.9. Damping force-velocity relation (Chopra, 2001)

$$f_D = c\dot{u} \tag{2.1}$$

The last component of the idealized system is lateral stiffness coefficient, k, which produces stiffness (or spring, or resistance) force. The resisting force is related to the resulting displacement, u. This force-displacement relation can be linear or nonlinear. At small deformations, this relation will be linear; hence it will be nonlinear at larger deformations.

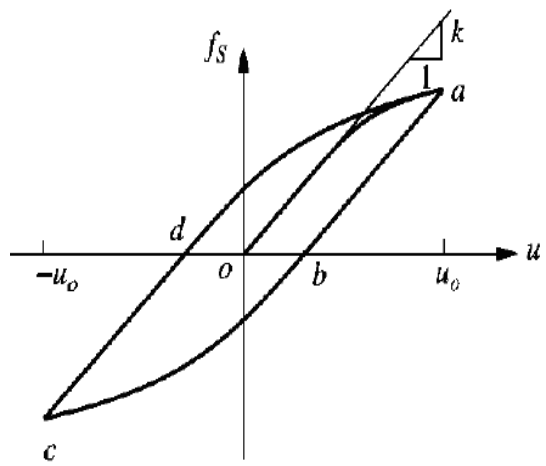

Figure 2.10. Nonlinear relationship between resistance force and displacement

(Chopra, 2001)

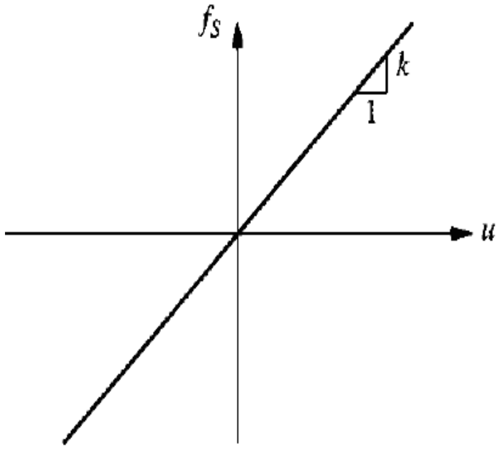

Figure 2.11. Linear relationship between resistance force and displacement

(Chopra, 2001)

As seen in Figure (2.11); in a linear relationship, loading and unloading curves are identical so that the structural resistance force is a single-valued function of u. However, in a nonlinear relation (Figure 2.10), loading and unloading curves differ from the initial loading branch so that the resistance force is not a single-valued function of u. A system is said to be elastic when the relationship between the resistance force and deformation is linear, however the system is said to be inelastic when the force-deformation relation is nonlinear.

Under strong earthquake motions, the structure will most likely display nonlinear behavior, which can be caused by material nonlinearity and / or geometric nonlinearity. Geometric nonlinearity is caused by the gravity loads acting on the deformed position of the structure. If the lateral displacements are small, this effect can be neglected. Material nonlinearity occurs when the stresses at certain critical regions in the structure exceed the elastic limit of the material (Naeim, 2001). However, in the dynamic analysis of engineering structures, it is generally assumed that the characteristics of the system, that is, its mass, stiffness and damping properties do not vary with time. It is further assumed that deformations of the structure are small and that the deforming material follows linear stress-strain relationship (Humar, 2002).

In the light of such information, in this thesis, we are interested in studying the dynamic response of linear SDOF systems. For a linearly elastic system, the relationship between the resistance force and resulting deformation can be given as

$$f_S = ku \tag{2.2}$$

The stiffness or resistance coefficient of the structure is calculated from the sizes of structural elements and dimensions of the structure.

Below in two separate subtitles; the equation of motion of the idealized system under earthquake excitation, whose pure components had already been discussed in the previous paragraphs, and the applied method for solving the equation of motion are given, consecutively.

2.1.4. Mathematical Model (Equation of Motion)

There is no applied external dynamic (or time-varying) force, p(t) when a structure subjected to the earthquake excitation because the system experiences earthquake-induced motion of the base.

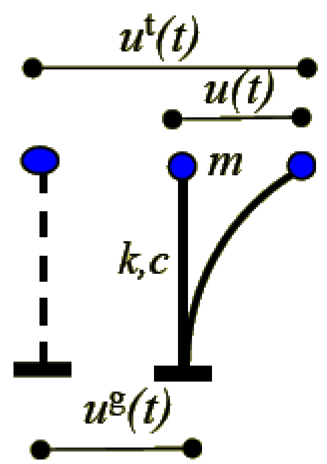

Figure 2.12. SDOF system subjected to earthquake excitation

As shown in Figure 2.12, the total (or absolute) displacement of the mass is denoted by $u^t(t)$ and the displacement of the ground is denoted by $u^g(t)$. Hence, the relative displacement between the mass and ground that is denoted by u(t) can be expressed as;

$$u^t(t) = u(t) + u^g(t) \tag{2.3}$$

Where;

$u^g(t) =$ Displacement of the base (or ground)

$u(t) =$ Displacement of the mass relative to the ground

The above relationship is valid for each instant of time. The forces acting on the mass at some instant of time are as shown in Figure 2.13.

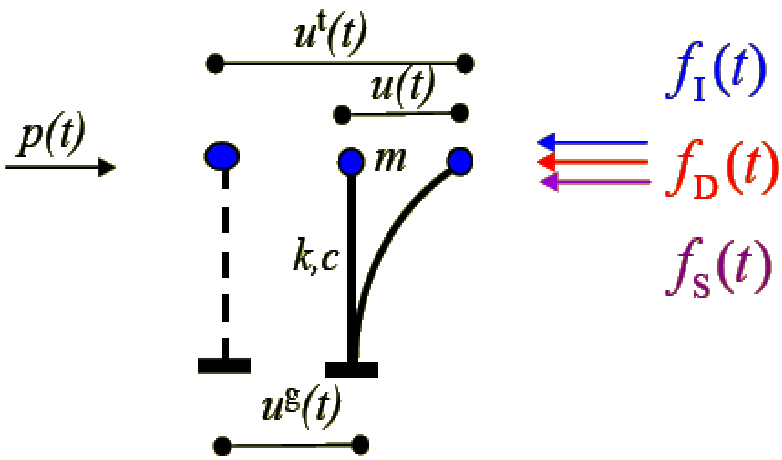

Figure 2.13. Forces acting on the mass at some instant of time

These forces include the external force, p(t) which equals to zero because no external dynamic forces are applied, the structural resisting force, $f_S(t)$, the damping resisting force, $f_D(t)$, and the inertial resisting force, $f_I(t)$. As stated above, the inertial resisting force is related to the total acceleration of the mass, therefore;

$$f_I(t) = m\ddot{u}^t(t) \tag{2.4}$$

Whereas the structural resisting and damping resisting forces are related to the relative motion, $u(t)$, between the mass and the base and for a linear system they can be given as

$$f_S(t) = ku(t) \tag{2.5}$$

$$f_D(t) = c\dot{u}(t) \tag{2.6}$$

Summing these forces, results in the following equation of dynamic equilibrium.

$$f_I(t) + f_D(t) + f_S(t) = p(t) = 0 \tag{2.7}$$

Substituting the physical parameters for $f_I(t)$, $f_D(t)$, and $f_S(t)$ in the above equation results in;

$$m\ddot{u}(t) + c\dot{u}(t) + ku(t) = -m\ddot{u}^g(t) \tag{2.8}$$

For earthquake engineering, the response quantities relative to the ground $u(t)$, $\dot{u}(t)$, $\ddot{u}(t)$ and the total (or absolute) responses $u^t(t)$, $\dot{u}^t(t)$, $\ddot{u}^t(t)$ have great importance. The term ‘response’ includes any response quantity, such as displacement, velocity or acceleration of the mass.

The state of the system under earthquake excitation is generally described by relative displacement, $u(t)$, relative velocity, $\dot{u}(t)$, and / or relative acceleration, $\ddot{u}(t)$, of the mass, where a dot represents differentiation with respect to time.

2.1.5. Free Vibration

The motion of linear SDOF systems under earthquake excitation is affected by the characteristics of earthquake excitation and also by the dynamic properties of the system.

The most important characteristic of ground motion that affects the structures is the period or frequency of the earthquake waves. The period or frequency of the waves indicates how the waves are slow and rolling or quick and unexpected. In general, just like sound waves, seismic waves vibrate at different frequencies

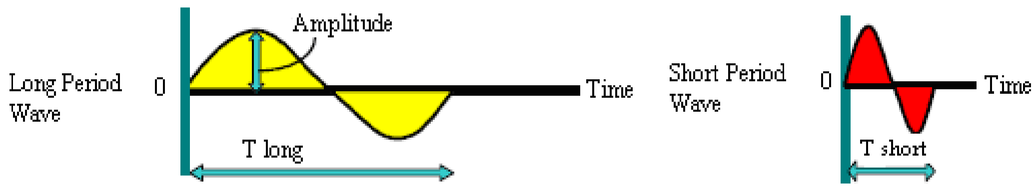

Figure 2.14. Strong earthquake ground motion which is transmitted by waves of different periods (Murty, 2003)

When the structure starts to vibrate under earthquake excitation, it will show the potential to 'wave' back and forth at its natural period (or fundamental period). In addition to the natural vibration period, the other important dynamic property of structures which plays a crucial role in determining the response of structures is called damping.

In this chapter, free vibration concept is discussed in order to explain these dynamic properties of a SDOF system.

If a system is distributed from its static equilibrium position with an initial input and then allowed to vibrate freely (without any external dynamic excitation), free vibration occurs.

The equation of motion of linear SDOF systems subjected to external force, p(t) is

$$m\ddot{u}(t)+c\dot{u}(t)+ku(t)=p(t) \tag{2.9}$$

When the system is allowed to vibrate freely, the external force, p (t), becomes zero. Then (2.9) becomes

$$m\ddot{u}(t)+c\dot{u}(t)+ku(t)=0 \tag{2.10}$$

This differential equation is a linear second order and homogenous (because all terms are of the same kind). For the solution of this equation, a function $u(t)$, such that a constant times its second derivative $\ddot{u}(t)$ plus another constant times $\dot{u}(t)$ plus a third constant times $u(t)$, is equal to zero can be obtained. The below exponential function has the property that its derivative is a constant multiple of itself.

$$u(t)=Ce^{\lambda t} \tag{2.11}$$

$$\dot{u}(t)=\lambda Ce^{\lambda t} \tag{2.12}$$

Furthermore,

$$\ddot{u}(t)=\lambda^2 Ce^{\lambda t} \tag{2.13}$$

Where, C is a constant.

Substituting these expressions into (2.10)

$$m\lambda^2 C e^{\lambda t}+c\lambda C e^{\lambda t}+k C e^{\lambda t}=0 \tag{2.14}$$

Dividing through by $Ce^{\lambda t}$ gives

$$m\lambda^2 + c\lambda + k = 0 \tag{2.15}$$

But, $Ce^{\lambda t}$ never equals to zero. Thus, $u(t) = Ce^{\lambda t}$ is a solution of (2.10) if λ is a root of the equation. The roots λ_1 and λ_2 are found by using the quadratic formula,

$$\lambda_1 = \frac{-b + \sqrt{(b^2 - 4ac)}}{2a} \tag{2.16}$$

$$\lambda_2 = \frac{-b - \sqrt{(b^2 - 4ac)}}{2a} \tag{2.17}$$

Where a, b, and c represent the m, c and k, relatively.

Thus;

$$\lambda_1 = \frac{-c + \sqrt{(c^2 - 4mk)}}{2m} \tag{2.18}$$

$$\lambda_2 = \frac{-c - \sqrt{(c^2 - 4mk)}}{2m} \tag{2.19}$$

$$\lambda = \frac{-c}{2m} \pm \sqrt{\left(\frac{c}{2m}\right)^2 - \frac{k}{m}} \tag{2.20}$$

The nature of the roots given by (2.20) depends on whether the quantity inside the square root is positive, exactly zero, or negative. Before discussing these three different cases, the below definitions should be given.

- $\omega^2 = \frac{k}{m}$

Where ω is the natural circular frequency of vibration; its units are [rad/sec]

- $T = \frac{2\pi}{\omega}$

Where;

T is the natural period of vibration of the system, in units of seconds. T is the time required to complete one cycle (one cycle= move back and forth) of free vibration for the undamped system. Natural period of vibration is unique for each structure and vary from 0.1 seconds to several seconds.

A rule of thumb is that the building period equals the number of stories divided by 10. Therefore, the fundamental period of the structure is affected by the height of the structure. In addition to height, the structural system, materials and geometric proportions will also affect the period (Arnold, 1982). Therefore, the taller the building means the longer the natural period it has.

- $f = \frac{\omega}{2\pi} = \frac{1}{T}$

Where, f is the natural cyclic frequency of vibration, in units of Hertz. f is also related to ω and inverse of T which means that a system generates 1/T cycles in a second or 1/T hertz. One hertz is one cycle per second.

If be considered, the terms, T, ω and f are defined with the same qualifier, natural, because all of them are natural properties of the system when it undergoes free vibratory motion. These natural vibration properties only depend on the mass and stiffness of the structure.

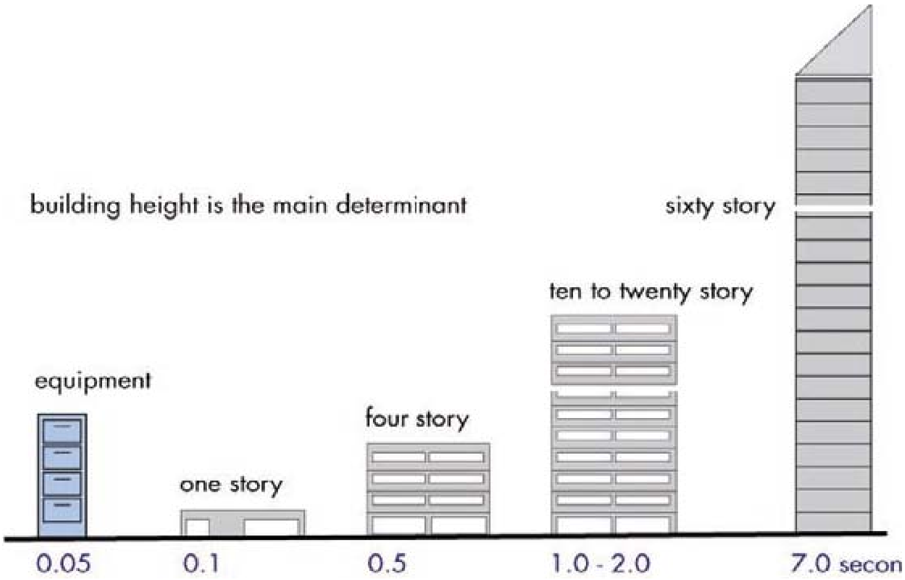

Figure 2.15. Comparative building periods, determined by height (Arnold, 1982)

- $c_{cr} = 2m\omega$

This particular value of c is called the critical damping coefficient. c_{cr} marks the boundary between oscillatory and nonoscillatory behavior. c_{cr} is the smallest value of c that restrains oscillation completely.

- $\xi = \frac{c}{c_{cr}} = \frac{c}{2m\omega}$

Where, ξ is the damping ratio or fraction of critical damping. This is the most generally used dimensionless measure of damping which is sometimes expressed in percentage terms. The term c, damping constant, in this equation is a measure of the dissipated energy in a cycle of vibration.

Damping is a dynamic property of the system which plays a crucial role in determining the response. Damping can be obtained by experiments because as mentioned before, there is no numerical method available for determining it.

As previously mentioned, damping limits the vibration of a structure by dissipating the energy of the earthquake ground shaking. Therefore, it is inversely proportional to the response of structure. The more damping the structure possesses, the smaller vibration amplitude the structure will have.

As can be seen, there is a relationship between the mass, stiffness and damping ratio. After these definitions, it will be helpful to write (2.10) and (2.20) in terms of the natural circular frequency and damping ratio as follows.

$$m\ddot{u}(t) + 2m\xi\omega\dot{u}(t) + m\omega^2 u(t) = 0 \tag{2.21}$$

Dividing through by m gives;

$$\ddot{u}(t) + 2\xi\omega\dot{u}(t) + \omega^2 u(t) = 0 \tag{2.22}$$

and

$$\lambda = \frac{-2m\xi\omega}{2m} \pm \sqrt{\left(\frac{2m\xi\omega}{2m}\right)^2 - \frac{\omega^2 m}{m}} \tag{2.23}$$

After making a simplification, the equation becomes;

$$\lambda = -\xi\omega \pm \sqrt{(\xi\omega)^2 - \omega^2} \tag{2.24}$$

$$\lambda = -\xi\omega \pm \omega\sqrt{\xi^2 - 1} \tag{2.25}$$

The quantity under the square root will affect the type of motion hence the form of the solution of (2.21)

First case corresponds to critical damping condition. In this case, the quantity inside the square root equals to zero. Thus $c=c_{cr}$ and also, $\xi=1$. As a result, the roots are equal to each other and both of them are real.

$$\lambda_1 = \lambda_2 = -\omega \tag{2.26}$$

A system that is critically damped returns to its equilibrium position without oscillating. This means that vibration is impossible.

Second case equals to overdamped condition. Different from the first case, inside the square root is positive in this case. Thus $c > c_{cr}$ and $\xi>1$. The roots are again real but they are different.

$$\lambda = -\xi\omega \pm \omega\sqrt{\xi^2 - 1} \tag{2.27}$$

Where;

$$\omega_E = \omega\sqrt{\xi^2 - 1} \tag{2.28}$$

Again the system under the condition of overdamped does not oscillate while returning to its static equilibrium position.

The last and the most common case correspond to underdamped condition. Most engineering structures fall into this category. In this case, the quantity inside the square root is negative which means that $c < c_{cr}$ also $\xi<1$.

$$\lambda = -\xi\omega \pm i\omega\sqrt{1-\xi^2} \tag{2.29}$$

Since,

$$i = \sqrt{-1} \tag{2.30}$$

and

$$\omega_D = \omega\sqrt{1-\xi^2} \tag{2.31}$$

An underdamped system oscillates about its equilibrium position under initial displacement and velocity conditions.

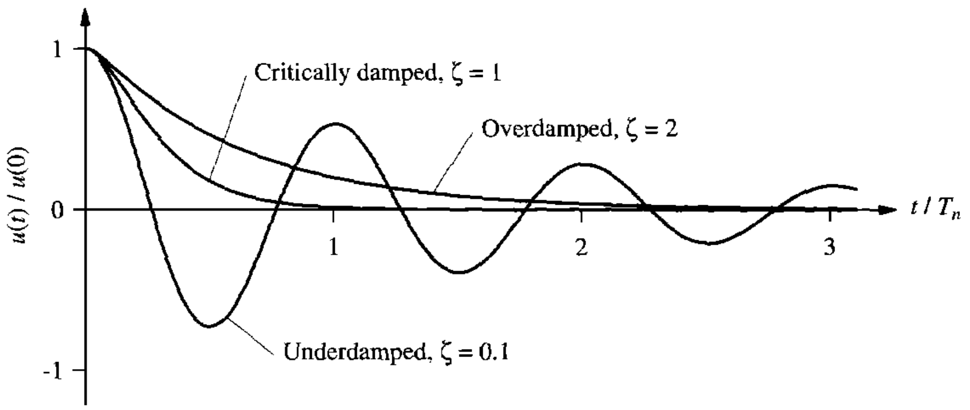

Figure 2.16. Free vibration of underdamped, critically damped, and overdamped systems (Chopra, 2001)

In this thesis, SDOF systems having five per cent (5 %) damping ratio, generally assumed as a characteristic value for concrete structures, are analyzed.

3. METHODOLOGY

3.1. Dynamic Response Analysis of SDOF Systems

Dynamic loads can be divided into two basic categories, periodic and nonperiodic. While harmonic and non-harmonic loads fall into periodic loading category, impulsive (short duration) and long duration (such as earthquake) loads fall into the category of nonperiodic loading.

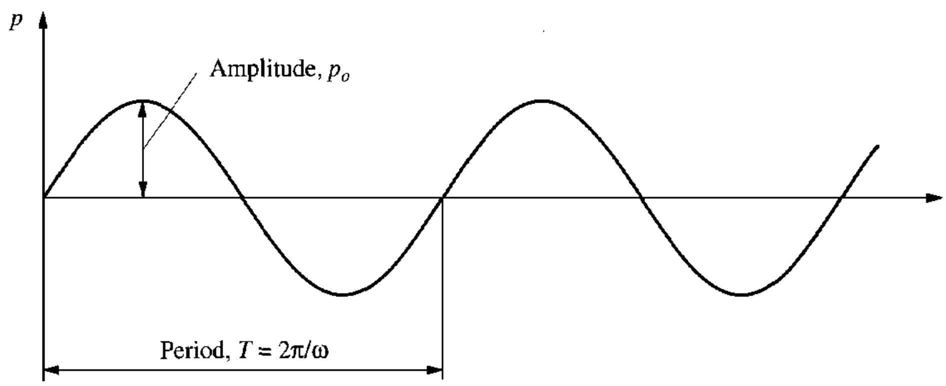

Figure 3.1. Harmonic Force (Chopra, 2001)

When a SDOF system is forced by dynamic excitation which can be described analytically such as harmonic, periodic as in the above (Figure 3.1), analytical solution of the equation of motion is possible. However, in situations the dynamic excitation is neither harmonic nor periodic and too complicated to be defined analytically and described only numerically such as earthquake ground motion, numerical methods are used for analyzing the response of a system.

Dynamic response analysis of SDOF systems to different types of dynamic loads, except the earthquake loads, is beyond the scope of this thesis, therefore; only a method used to solve the equation of motion of linear SDOF systems under ground accelerations will be stated in the following paragraphs.

Earthquake loads are very uncertain and complex. Earthquake acceleration varies with time in a highly irregular manner and it is not possible to have an easy mathematical representation (a function) to describe the time variation of this load. Therefore, analytical solution of the equation of motion is impossible. In this case numerical time-stepping methods are necessary for integration of differential equations. There are several numerical time stepping methods for solving different types of differential equations in literature but in this thesis, Piecewise linear exact method (by Aydınoğlu and Fahjan, 2003) is used.

3.2. Numerical Time-Stepping Method

3.2.1. Piecewise Linear Exact Method

Piecewise exact method is efficiently used for the response analysis of linear SDOF systems. This method is based on a simple recursive solution of equation of motion. The attractiveness of the method lies in the fact that it provides the exact solution when the loading time history is composed of piecewise linear segments, a condition that is perfectly satisfied for the earthquake excitation (Aydınoğlu and Fahjan, 2003).

For a linear system the equation of motion, governing the relative displacement or deformation u(t) when a system subjected to ground acceleration $\ddot{u}^g(t)$, to be solved numerically is

$$m\ddot{u}(t)+c\dot{u}(t)+ku(t)=-m\ddot{u}^g(t) \tag{3.1}$$

Where,

$$\ddot{u}^t(t)=\ddot{u}(t)+\ddot{u}^g(t) \tag{3.2}$$

Equation (3.1) can be written as by introducing the notations; linear viscous damping coefficient, c and stiffness coefficient, k, and dividing through by mass, m;

$$\ddot{u}(t) + 2\xi\omega\dot{u}(t) + \omega^2 u(t) = -\ddot{u}^g(t) \tag{3.3}$$

In order to solve this equation of motion, ''Piecewise linear exact method'' which expresses the ground acceleration as a linear function between the time stations $t = t_i$ and $t = t_{i+1}$ as

$$\ddot{u}^g(t) = \ddot{u}^g(t_i) + \left(\Delta\ddot{u}^g/\Delta t\right)\tau \tag{3.4}$$

Where,

$$\Delta\ddot{u}^g = \ddot{u}^g(t_{i+1}) - \ddot{u}^g(t_i) \tag{3.5}$$

$$\Delta t = t_{i+1} - t_i \tag{3.6}$$

$$\tau = t - t_i \tag{3.7}$$

Equation (3.5), (3.6) and (3.7) are used.

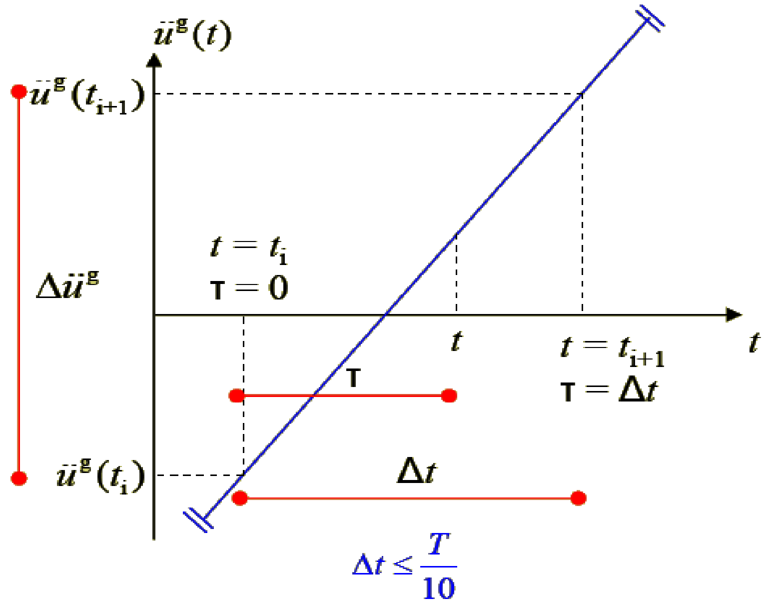

Figure 3.2. Piecewise linear representation of ground acceleration

The applied ground motion $\ddot{u}^g(t)$ is composed of a set of discrete values $\ddot{u}_i^g = \ddot{u}^g(t_i)$, for i=0 to N. $\Delta t = t_{i+1} - t_i$ represents the time interval. The shorter the time interval means the more reliable the results. According to Aydınoğlu and Fahjan (2003); the time step, Δt can be taken equal to the time step of the earthquake record for the response along the linear segments.

Therefore, in this analysis, Δt is determined separately for each strong ground motion record and it is the elapsed time between the two consecutive measurements of the earthquake excitation value.

This numerical solution method will help to determine the relative response quantities; $u(t)$, $\dot{u}(t)$, and $\ddot{u}(t)$, at all time instants. The known initial conditions, $u(0)$ and $\dot{u}(0)$ provide the necessary information to start the determination procedure of these response values.

Thus, Equation (3.3) can be written in the time span $t_i \le t \le t_{i+1}$, i.e. $0 \le \tau \le \Delta t$, as

$$\ddot{u}(\tau) + 2\xi\omega\dot{u}(\tau) + \omega^2 u(\tau) = -\ddot{u}^g(t_i) - \left(\Delta\ddot{u}^g / \Delta t\right)\tau \tag{3.8}$$

The solution of the response $u(\tau)$ over the time interval $0 \le \tau \le \Delta t$ in Equation (3.8) is the combination of a complementary solution and a particular solution.

$$u(\tau) = u_h(\tau) + u_p(\tau) \tag{3.9}$$

For a linear non-homogenous differential equation, the complete solution is the superposition of the particular solution and the complementary solution. The complementary solution is the general solution of the associated homogenous equation and the particular solution depends on the nature of the forcing function.

- The complementary solution corresponds to the response of the system that is allowed to vibrate freely. Therefore;

$$\ddot{u}(\tau) + 2\xi\omega\dot{u}(\tau) + \omega^2 u(\tau) = 0 \tag{3.10}$$

The solution of this equation gives the complementary solution and presented by Aydınoğlu and Fahjan (2003) as below.

$$u_h(\tau) = \left[C_1 \cos(w_D \tau) + C_2 \sin(w_D \tau)\right] e^{(-\xi\omega\tau)} \tag{3.11}$$

- The particular solution satisfies the right-hand side of the same differential equation. Therefore;

$$\ddot{u}(\tau) + 2\xi\omega\dot{u}(\tau) + \omega^2 u(\tau) = -\ddot{u}^g(t_i) - \left(\Delta\ddot{u}^g/\Delta t\right)\tau \tag{3.12}$$

The solution of this equation gives the particular solution and presented by Aydınoğlu and Fahjan (2003) as follows.

$$u_P(\tau) = C_3\tau + C_4 \tag{3.13}$$

Consequently the general solution combining complementary and particular solutions can be given

$$u(\tau) = \left[C_1\cos(w_D\tau) + C_2\sin(w_D\tau)\right]e^{(-\xi\omega\tau)} + C_3\tau + C_4 \tag{3.14}$$

Integration constants C_3 and C_4 are determined first from the particular solution and they are then substituted into general solution to obtain constants C_1 and C_2 by imposing displacement and velocity conditions at time station $t = t_i$ i.e as initial conditions at $\tau = 0$ (Aydınoğlu and Fahjan, 2003).

- Particular Solution;

$$u_P(\tau) = C_3\tau + C_4 \tag{3.15}$$

$$\dot{u}_P(\tau) = C_3 \tag{3.16}$$

$$\ddot{u}_P(\tau) = 0 \tag{3.17}$$

Then using Equation (3.15), (3.16) and (3.17); Equation (3.12) can be written as;

$$2\xi\omega C_3 + \omega^2(C_3\tau + C_4) = -\ddot{u}^g(t_i) - \left(\Delta\ddot{u}^g/\Delta t\right)\tau \tag{3.18}$$

$$C_3 = -\frac{1}{\omega^2}\left(\Delta\ddot{u}^g/\Delta t\right) \tag{3.19}$$

$$C_4 = -\frac{1}{\omega^2}\ddot{u}^g\left(t_i\right) + \frac{2\xi}{\omega^3}\left(\Delta\ddot{u}^g/\Delta t\right) \tag{3.20}$$

- Total Solution at $\tau = 0$ or $t = t_i$;

$$u\left(t_i\right) = C_1 + C_4 \tag{3.21}$$

$$\dot{u}\left(t_i\right) = \left(-\xi\omega C_1 + \omega_D C_2\right) + C_3 \tag{3.22}$$

$$C_1 = u\left(t_i\right) + \frac{1}{\omega^2}\ddot{u}^g\left(t_i\right) - \frac{2\xi}{\omega^3}\left(\Delta\ddot{u}^g/\Delta t\right) \tag{3.23}$$

$$C_2 = \frac{\xi\omega}{\omega_D}u\left(t_i\right) + \frac{1}{\omega_D}\dot{u}\left(t_i\right) + \frac{\xi}{\omega\omega_D}\ddot{u}^g\left(t_i\right) + \frac{1-2\xi^2}{\omega^2\omega_D}\left(\Delta\ddot{u}^g/\Delta t\right) \tag{3.24}$$

After substituting the integration constants C_1 to C_4, displacement and velocity expressions are evaluated for the time station $t = t_{i+1}$, i.e for $\tau = \Delta t$. Thus the following recursive relationships are obtained, through which the displacements and velocities at the time station $t = t_{i+1}$ are directly calculated in terms of those obtained at the previous time station, $t = t_i$.

Recurrence relations: $t = t_{i+1}$ or $\tau = \Delta t$

$$u\left(t_{i+1}\right) = A_{11}u\left(t_i\right) + A_{12}\dot{u}\left(t_i\right) + B_{11}\ddot{u}^g\left(t_i\right) + B_{12}\Delta\ddot{u}^g \tag{3.25}$$

$$\dot{u}\left(t_{i+1}\right) = A_{21}u\left(t_i\right) + A_{22}\dot{u}\left(t_i\right) + B_{21}\ddot{u}^g\left(t_i\right) + B_{22}\Delta\ddot{u}^g \tag{3.26}$$

$$\ddot{u}\left(t_{i+1}\right) = -\ddot{u}^{g}\left(t_{i+1}\right) - 2\xi\omega\dot{u}\left(t_{i+1}\right) - \omega^{2}u\left(t_{i+1}\right) \tag{3.27}$$

Recurrence coefficients:

$$A_{11} = E + \frac{\xi\omega}{\omega_D}F \tag{3.28}$$

$$A_{21} = -\omega^{2}A_{12} \tag{3.29}$$

$$A_{12} = \frac{F}{\omega_D} \tag{3.30}$$

$$A_{22} = E - \frac{\xi\omega}{\omega_D}F \tag{3.31}$$

$$B_{11} = \frac{A_{11} - 1}{\omega^{2}} \tag{3.32}$$

$$B_{21} = -A_{12} \tag{3.33}$$

$$B_{12} = \frac{A_{12} - 2\xi\omega B_{11} - \Delta t}{\omega^{2}\Delta t} \tag{3.34}$$

$$B_{22} = \frac{B_{11}}{\Delta t} \tag{3.35}$$

$$E = \cos\left(w_D\,\Delta t\right)\exp\left(-\xi\omega\,\Delta t\right) \tag{3.36}$$

$$F = \sin\left(w_D\,\Delta t\right)\exp\left(-\xi\omega\,\Delta t\right) \tag{3.37}$$

3.2.2. Displacement Response Time History Determination

In most cases the parameters related to structural displacement could reflect the structural damage during earthquakes more directly and consistently than structural strength (Guan *et al*., 2004). In addition to this; Decanini *et al*. (2003) has mentioned that an adequate damage control can be achieved if deformations are controlled and Chopra (2001) has presented that the relative displacement $u(t)$ associated with deformations of the structure is the most important since the internal forces in the structure are directly related to $u(t)$.

In the light of the foregoing instructions, in this thesis only the displacement response time histories of SDOF systems are taken into consideration. In order to determine the displacement response time histories of linear SDOF systems, piecewise linear exact method outlined above is used.

During the whole determination process in this thesis, Matlab, the fourth generation programming language, has been utilized.

3.3. Knowledge of Overall Structural Response

The conventionally used response spectrum technique for design of earthquake-resistant structures involve the estimation of only the largest peak amplitude of the response by ignoring much valuable information on overall structural response. However, the rest of the deformation time history, which includes the number, sequence and the relative amplitudes of the response peaks, provides very useful information that can not be extracted from standard response spectra.

For better understanding of the seismic response time history, it becomes necessary to have knowledge of the amplitudes of the first, second, third, … largest peaks of the response.

This information is invaluable for assessing the progressive damage in a structure, understanding the number of times a certain response level may be exceeded, considering the relationship between the amplitudes of all the response peaks to the characteristics of the structural system.

Therefore, in the following subsection, the applied method used for obtaining the amplitudes of the first largest, second largest, third largest, etc., peaks of the displacement response of each SDOF oscillator will be discussed.

3.3.1. Ordered Peak Statistics

After the displacement response time history of each SDOF oscillator is determined by the above-mentioned Piecewise linear exact method, order statistics approach, which is first applied by Gupta and Trifunac (1988) to refine the theoretical distribution function for the amplitudes of n-th-order peak in a total of N peaks of a random function, f(t), is followed. In their study, f(t) represents the response of a structure to an earthquake excitation.

The other attempts to propose theoretical distributions of peaks by using the order statistics approach were made by Basu *et al*. (1996) and Gupta and Trifunac (1998). According to the results of Gupta and Trifunac's (1998) work, which had been carried out to investigate the relative performance of approximate formulations owing to Amini and Trifunac (1981), and Gupta and Trifunac (1988) for probability distributions of ordered peaks, the formulation of Gupta and Trifunac (1988) describes the data well when comparing to the formulation of Amini and Trifunac (1981) in which order statistics approach was not used.

Order statistics approach is based on arranging the amplitudes of peaks in decreasing order. The use of order statistics has resulted in excellent improvement in theoretical predictions (Gupta and Trifunac, 1988; Basu *et al*., 1996 and Gupta and Trifunac, 1998).

Therefore, after the determination of the displacement response time history of each SDOF system, the absolute values of all peaks in the displacement response time histories of SDOF oscillators are arranged in decreasing order of amplitudes such as required by order statistics approach.

3.3.2. 3D Response Spectra Concept

As previously mentioned, conventionally used response spectrum represents the largest (absolute) peak amplitude of a response quantity as a function of the natural vibration period of the structural system. According to Şafak (1998) at any period, the response spectrum can be thought as the maximum displacement level that will be crossed only once with positive slope by the oscillator. Similarly, the maximum displacement levels that will be crossed with positive slopes only twice, three times, four times, etc., can be determined. These displacements corresponds to the amplitudes of second, third, fourth, and so on, absolute peaks of the displacement time history. The variation of the amplitude of each secondary peak with oscillator period gives the response spectra for the corresponding crossing level and the standard response spectra correspond to the first crossing level. Instead of plotting separate curves for each crossing level, they can be combined in a 3D plot by adding a third dimension to the standard response spectra representing the number of crossings. This result in a surface, which represents the peak displacements of the oscillator as a function of period and the number of crossings, called 3D Response Spectrum.

In this study, 3D Response Spectra surface have been used in order to show the spectral displacement values as a function of the oscillator's period and the peak number. This surface can be treated as an illustrative figure that will help a designer for evaluating

i. The relationship between the amplitudes and numbers of the maxima of the response to the characteristics of the structural model.
ii. The expected and most probable values of the first, second, third, … largest peaks of the response of linear, viscously damped SDOF systems.

3.4. Distribution of Ordered Peaks in Linear Earthquake Response

Defining the distribution functions for amplitudes of the ordered peaks in each SDOF oscillator's response assist in calculating all the other properties of the response such as effective response durations as in the work of Şafak (1998), the number of times particular amplitude of the response may be exceeded and probabilities of exceeding certain response levels.

There are many studies which have been carried out on the response of structures from stochastic viewpoint. In the following paragraphs, some of these studies' results on the theoretical distribution function for the amplitudes of response peaks will be presented.

The main purpose of presenting the below theoretical derivations is to investigate consistency between the results of previous studies and this thesis.

3.4.1. Statistical Distribution of Maxima of Random Function

Through generalizations and extensions of the work of Rice (1944, 1945) and Cartwright and Longuet Higgings (1956) on the theory of probability distributions of a random function which find useful applications in the field of earthquake engineering and strong motion seismology, a random function of time, f(t) may be represented by

$$f(t) = \sum_{n} C_n \cos\left(\omega_n t + \Phi_n\right) \tag{3.38}$$

Where, ω_n are the circular frequencies, Φ_n are the random phases uniformly distributed between 0 and 2π, t represents time and C_n are the amplitudes related to the energy spectrum, $E(\omega)$, of $f(t)$ by the following relation

$$\sum_{\omega n=\omega}^{\omega+d\omega} \frac{1}{2} C_n^2 = E(\omega)\, d\omega \qquad (3.39)$$

According to Gupta and Trifunac (1988) and Basu *et al*. (1996); f(t) may represent the response of a structure to an earthquake excitation.

Using previous definitions, Cartwright and Higgings (1956) derived the probability density function for the distribution of the maxima of f(t), in terms of the root-mean-square value of f(t), a_{rms}, and a parameter ε, in which represents a measure of the width of energy spectrum, $E(\omega)$ of f(t). These parameters are defined in terms of the zeroth, second and fourth moments of the energy spectrum as follows

$$a_{rms} = m_0^{1/2} \qquad (3.40)$$

and

$$\varepsilon = \left[\frac{m_0 m_4 - m_2^2}{m_0 m_4} \right]^{1/2} \qquad (3.41)$$

Where, in general, the n-th moment of the energy spectrum, m_n, given by

$$m_n = \int_0^{\infty} \omega^n E(\omega)\, d\omega \qquad n = (0,1,2,...) \qquad (3.42)$$

The probability density function of the maxima of f(t) normalized with respect to root-mean-square value $m_0^{1/2}$, is given by Cartwright and Higgings (1956) as

$$p(\eta) = \frac{1}{\sqrt{2\pi}} \left[\varepsilon\, e^{-\eta^2/2\varepsilon^2} + \left(1-\varepsilon^2\right)^{1/2} \eta\, e^{-\eta^2/2} \int_{-\infty}^{\eta(1-\varepsilon^2)^{\frac{1}{2}}/\varepsilon} e^{-x^2/2}\, dx \right] \qquad (3.43)$$

When $\varepsilon = 0$, the statistical distribution of the maxima tends to a Rayleigh distribution and when ε approaches its maximum value 1 the distribution of the maxima tends to a Gaussian distribution.

According to Şafak (1998) the displacements of a SDOF oscillator is a narrow-band response.

In the case of a narrow-band process, a band-width parameter, ε becomes zero and $0 < \varepsilon < 1$ for the wide-band process (Gupta, 1995).

Therefore, for a narrow-band response, $\varepsilon = 0$, the probability density function of the maxima approaches a Rayleigh distribution (Şafak, 1988, 1998; Gupta and Basu, 1996; and Cartwright and Higgings, 1956).

In addition to this, according to the study of Morikawa and Zerva (2007) in which they have discussed the stochastic characteristics of maximum response of a SDOF system which is excited by a non-stationary process; in a case where the damping factor is small namely $<< 1$, the local maxima of the response are random variables following a Rayleigh distribution.

In the current thesis, as stated before, the secondary objective is to prove the accuracy of the above statement because in the previous studies, different assumptions have been made to obtain the discussed result. In addition to this, most of them have used theoretical derivations in order to determine the distributions of the maxima and synthetic (artificial) accelerograms have been considered to validate the theoretical results.

Therefore, the results of this thesis will provide good opportunity to justify the results of the previous studies by using large database of ground motion records collected from around the world.

3.5. Probabilistic Earthquake Response Analysis

For better understanding of the forthcoming probabilistic theory application, the following concepts should be clarified on a preferential basis.

3.5.1. Probability Theory

Probability theory is the branch of mathematics which studies the likelihood of occurrence of random events for predicting the behavior of defined systems. The probability of an event can be expressed as a fraction, decimal, or percent with a value between 0 and 1.

In mathematics, a probability of an event A, P(A) is usually expressed as the ratio between the number of favorable outcomes and the total number of possible outcomes. If the event is certain to occur, the probability of an event equals to 1, and if the event is impossible, the probability of an event equals to 0. A probability between 0 and 1 means the event is possible. The opposite or complement of an event A is the event which is written as A^c . That is, the event of A will not occur and its probability, $P(A^c)$ is given by

$$P(A^c) = 1 - P(A) \tag{3.44}$$

3.5.2. Probability Density and Cumulative Distribution Functions

The probability density function is a fundamental concept in statistics. Probability density function (commonly abbreviated as PDF) of the underlying variable is a mathematical function that describes the relative likelihood for this variable to occur at a given point in the observation space. The function is usually denoted by the letter f. Specifying the function f gives a natural description of the distribution of data and serves to represent probability distribution in terms of integrals.

Probability distribution is a statistical function that describes all the possible values and likelihoods that a random variable can take within a given range. There are many different probability distributions that show up in various different applications. Each distribution has a predefined form of probability density functions and densities have the same name as their distribution functions. When using a probability distribution to model probability, the distribution used is selected to best model and fit the particular situation.

A probability distribution is a theoretical distribution that is expressed mathematically. These distributions were formulated by statisticians, mathematicians and engineers to mathematically model or represent certain behavior. Given a distribution function that describes the behavior of individual observations, you will derive distribution functions that describe the behavior of a wide variety of statistics (Dallal, 2000).

Each probability distribution is defined by certain parameters such as mean, variance which are estimated from analysis of data. These parameters are summary measures characterizing that distribution and having knowledge of them allows the distribution to be fully described.

Complete description of the probability distribution of the variable is given by probability density function and cumulative distribution function.

Before the definition of cumulative distribution function, properties of PDF can be given as follows.

i. Since the probabilities are nonnegative, the curve of f always be above the x-axis.

$$f(x) \geq 0$$

ii. The probability of X taking a value in the interval (a, b) is given by the corresponding area. Since the area between any curve and the x-axis is given by the integral of that curve with respect to x, we therefore have (Upton and Cook, 1996);

$$P(a < X < b) = \int_a^b f(x)\,dx$$

iii. The area bounded by the probability density function curve and the x-axis must be 1.

$$\int_0^\infty f(x)dx=1$$

iv. The probability that X takes on a specific value is 0. That is, X=a is a line, which has no area, so $P(X=a)$ is 0.

Cumulative distribution function, written as CDF for short is one of the most important statistical function in reliability like PDF and they are very closely related. The CDF represents the cumulative values of the PDF. If these functions are known, almost any other reliability measure of interest can be obtained or derived.

Cumulative distribution function is usually denoted by the letter F. The CDF indicates the probability that the outcome of X in a random trial will be less than or equal to any specified value of x. The CDF corresponds to the area under the PDF to the left of x (Mian, 2002).

$$F(x)=\int_0^x f(s)ds$$

Where, s is a dummy integration variable. Conversely;

$$f(x)=\frac{d\left(F(x)\right)}{dx}$$

3.6. Models for Histogram of the Peaks

Below in separate subtitles, properties of four distribution functions proposed as possible alternatives namely Gaussian (Normal), Weibull, Exponential and Rayleigh distributions for modeling the response displacement peaks' PDF are summarized.

3.6.1. Gaussian (Normal) Distribution

The Normal probability distribution is one of the most used models in applied probability theory. The Normal distribution is also known as the Gaussian distribution or the bell curve. The Normal probability model represents variables which arise as the sum of a number of random effects, no one of which dominates the others (Decanini et al., 2003).

Properties of Normal distribution can be given as;

i. The distribution is characterized by two parameters, the mean (μ or $\bar{x}$) and the standard deviation (s or σ) or variance (σ^2).
ii. The Normal curve is symmetrical about the mean, bell shaped.
iii. The total area under the curve is equal to 1.
iv. The mean, mode and median of the distribution are equal.
v. The mean is at the middle so it divides the area under the curve into two halves (i.e., fifty percent of all values lay either side of the mean value).
vi. Shifted to the right if the mean is increased and to the left if the mean is decreased (assuming constant variance) (Petrie and Sabin, 2005).
vii. Flattened as the variance is increased but becomes more peaked as the variance is decreased (for a fixed mean) (Petrie and Sabin, 2005).

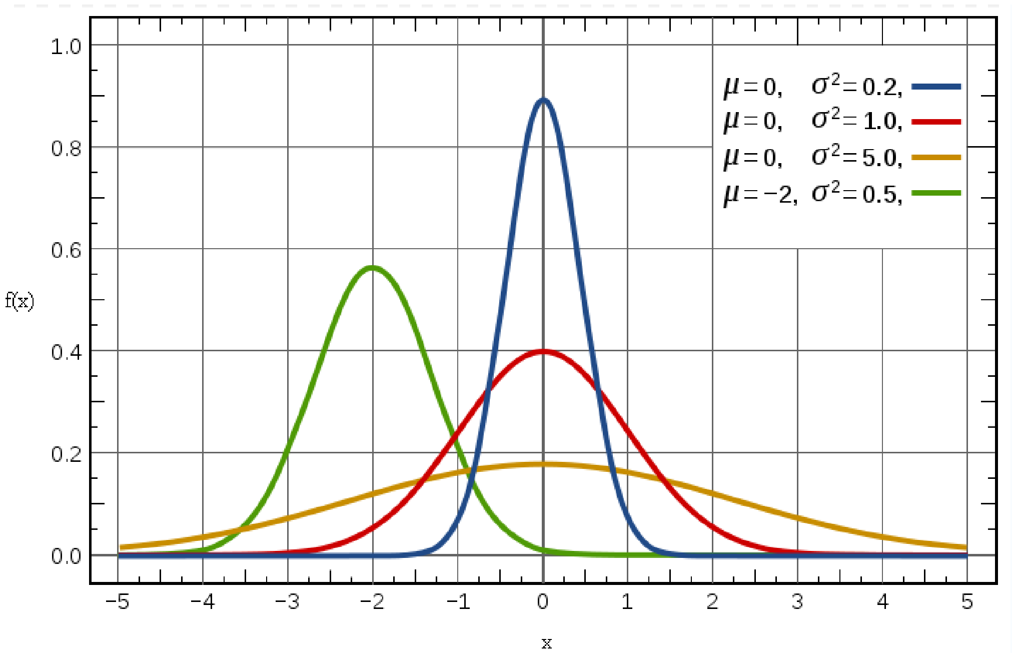

Figure 3.3. The probability density function of the Normal distribution with varying values of mean, μ and variance, σ^2 (Wikipedia)

The Normal probability density function is,

$$f(x)=\frac{1}{\sigma\sqrt{2\pi}}\exp\left[-\frac{1}{2}\left(\frac{(x-\mu)}{\sigma}\right)^2\right]; \quad x\in R \tag{3.45}$$

The mean of a Normal distribution, μ is given by

$$E(X)=\mu \tag{3.46}$$

and the variance (it describes how concentrated the distribution is around its mean) is designated commonly as, σ^2 ,

$$\text{Var}(X)=\sigma^2 \tag{3.47}$$

The standard deviation, σ, is the square root of the variance and the width of the density function.

3.6.2. Weibull Distribution

Another distribution which is the widely used in probability theory and statistics is the Weibull distribution. The Weibull distribution is a versatile distribution that can take on the characteristics of other types of distributions based on the value of shape parameter, β.

The two-parameter version of the Weibull distribution has the probability density function (where the location (shift) parameter, $\gamma=0$) as below,

$$f(x)=\frac{\beta}{\eta}\left(\frac{x}{\eta}\right)^{\beta-1}\ \exp\left[-\left(\frac{(x)}{\eta}\right)^{\beta}\right];\quad x\in[0;+\infty) \tag{3.48}$$

Where β and η denote the distribution parameters, shape (slope) and scale parameters, respectively. The Weibull distribution is defined for $x > 0$, and both distribution parameters are positive.

The mean and variance of a Weibull probability density function are given by

$$E\,(X)=\eta\,\Gamma\left(\frac{1}{\beta}+1\right) \tag{3.49}$$

$$\mathrm{Var}(X)=\eta^{2}\left[\Gamma\left(1+\frac{2}{\beta}\right)-\Gamma^{2}\left(1+\frac{1}{\beta}\right)^{2}\right] \tag{3.50}$$

Characteristics of the Weibull distribution can be given as:

Different values of the shape parameter, β, can have marked effects on the behavior of the distribution.

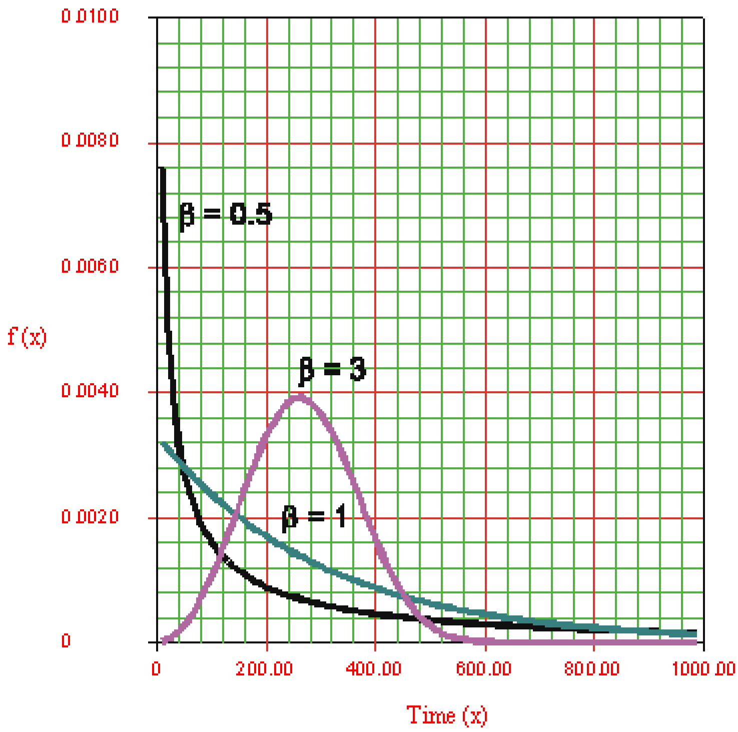

Figure 3.4. The effect of the Weibull shape parameter on the probability density function for 0<β<1, β=1, and β>1 (http://www.reliasoft.com/)

For instances, when, $\beta = 1$, this distribution reduces to the Exponential distribution and when $\beta = 2$, it mimics the Rayleigh distribution. In addition to these, it resembles the Normal distribution when $\beta = 3.5$.

In addition to shape parameter, a change in the scale parameter, η, has the effect on the Weibull distribution.

- If η is increased while β and γ are kept the same, the distribution gets stretched out to the right and its height decreases, while maintaining its shape and location.

- If η is decreased while β and γ are kept the same, the distribution gets pushed in towards the left (i.e. towards its beginning or towards 0 or γ), and its height increases.

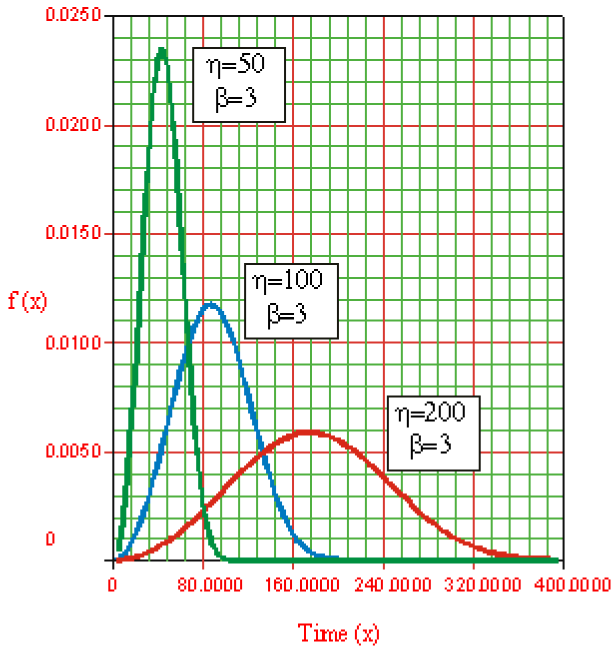

Figure 3.5. The effect of the Weibull scale parameter on the probability density function for a common β (http://www.reliasoft.com/)

As the name suggests, the location parameter, γ, locates the distribution along the horizontal axis.

- When γ = 0, the distribution starts at T = 0 or at the origin.
- If γ > 0, the distribution starts at the location γ to the right of the origin.
- If γ < 0, the distribution starts at the location γ to the left of the origin.

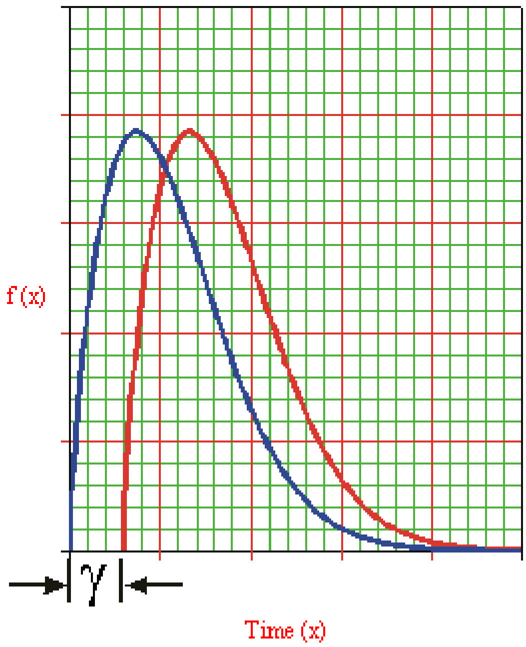

Figure 3.6. The effect of the Weibull location parameter on the probability density function (http://www.reliasoft.com/)

3.6.3. Exponential Distribution

The Exponential distribution is the other model that is commonly used in probability theory and statistics. The Exponential distribution is , in fact, a special case of the Weibull distribution as previously stated.

The theoretical one parameter Exponential distribution is described by the following probability density function,

$$f(x)= \lambda \exp\left(-\lambda x\right) = \frac{1}{m} \exp\left(-\frac{1}{m} x\right); \quad x \in [0;+\infty) \tag{3.51}$$

Where λ the parameter of the distribution often is called the rate parameter and m is a scale parameter of the distribution and it is the reciprocal of the rate parameter, λ (Wikipedia). The Exponential distribution has no shape parameter, as it has only one shape. It starts x = 0 at the level of f (x = 0) = λ and decreases thereafter exponentially and monotonically as X increases. This distribution requires the knowledge of only one parameter, λ, for its application.

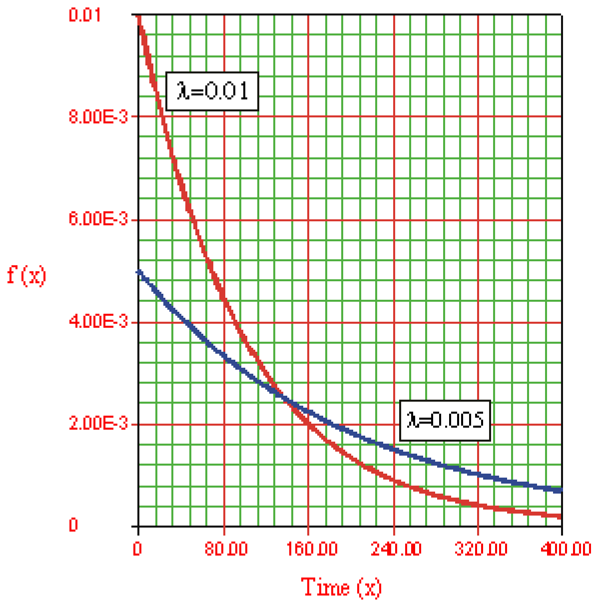

Figure 3.7. The effect of rate parameter on the Exponential probability density function (http://www.reliasoft.com/)

The mean or expected value of an exponentially distributed variable X with rate parameter λ is given by,

$$E(X) = \frac{1}{\lambda} \tag{3.52}$$

and variance of X is given by,

$$Var(X) = \frac{1}{\lambda^2} \tag{3.53}$$

3.6.4. Rayleigh Distribution

For a narrow band response, like a behavior of a SDOF oscillator when it is subjected to earthquake, the most widely accepted model that represents the distribution of the peaks is the Rayleigh distribution.

The Rayleigh probability density function is

$$f(x) = \frac{x}{\sigma^2} \exp\left(\frac{-x^2}{2\sigma^2}\right); \quad x \in [0;\infty) \tag{3.54}$$

Where, the only free parameter is σ (the so called scale parameter). As it is seen from the below figure, different values of the Rayleigh parameter can have marked effects on the behavior of the distribution.

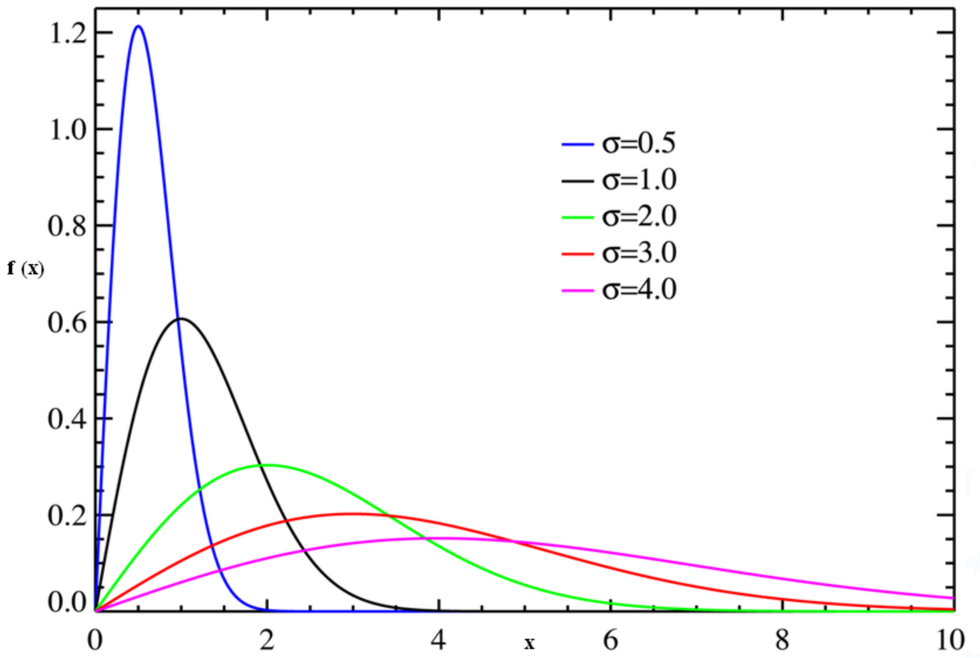

Figure 3.8. The probability density function of the Rayleigh distribution with varying values of the Rayleigh parameter (Wikipedia)

The mean and variance of a Rayleigh probability density function may be expressed as,

$$E\,(X) = \sigma\sqrt{\frac{\pi}{2}} \approx 1.25\sigma \tag{3.55}$$

$$Var\,(X) = \frac{4 - \pi}{2}\sigma^2 \approx 0.43\sigma^2 \tag{3.56}$$

4. APPLICATION AND RESULTS

4.1. Strong Motion Database

The strong ground motion records used to perform whole analysis in this thesis are obtained from the Next Generation Attenuation (NGA) database, which is an extended version of the Pacific Earthquake Engineering Research Center (PEER) strong motion database.

The overall NGA database includes 3551 multi-component recordings from 173 shallow crustal earthquakes. This web-based, searchable database of strong ground motion data is created by collecting the most important ground motion records from around the world, processing consistently that it is reliable in all regards and gathering related metadata such as earthquake magnitude, various site-to-source distance measures, style of faulting, local site conditions at the recording stations, and other relevant engineering parameters.

The database used for this thesis is a subset of these records covers 317 pairs of horizontal records from 51 earthquakes. When selecting the records to make up final dataset, any classification was not made according to their style of faulting, local site conditions, source-to-site distances and earthquake magnitude. However, a constraint of limiting high pass filter corner frequencies equal to 0.2 Hz for the two horizontal components of each record was created.

A list of selected earthquakes is tabulated in Table 4.1. The earthquakes in this table have moment magnitudes, Mw, ranging from 4.5 to 8.

In this database, all strong ground motion data have been processed. As it is well known, different agencies and different researchers use quite a few data processing techniques. The processing procedure of strong ground motion records in the PEER database is also different. Basically a reasonable digital filter (i.e., three types of filter namely Ormsby, Acausal Butterworth and Causal Butterworth) is chosen for filtering out the noise in ground motion signal and baseline correction of the records is performed by fitting a straight line or n-order polynomial to eliminate the drift in the integrated displacement ordinate. In order to get more information about the techniques used in processing the data, the entire dataset, a flatfile, along with documentation of each data column is available at PEER's website (http://peer.berkeley.edu/nga/).

In this thesis, the analysis is performed without applying any ground motion correction procedure, because an appropriate baseline correction procedure has already been applied and a suitable cut-off (or break or corner) frequency which must be compatible with the frequencies present in the signal of interest has already been determined for each component of the records in PEER database.

Table 4.1. Database of strong motion records used in probabilistic earthquake response analysis

Record Number	Earthquake Name	Year	Month & Day	Hour & Minute	Earthquake Magnitude	Corner frequency of the high-pass filter of component H1 (Hz)	Corner frequency of the high-pass filter of component H2 (Hz)	Number of Stations
1	Helena, Montana-01	1935	1031	1838	6	0.2	0.2	1
2	Imperial Valley-02	1940	0519	0437	6.95	0.2	0.2	1
3	Kern County	1952	0721	1153	7.36	0.2	0.2	1
4	Parkfield	1966	0628	0426	6.19	0.2	0.2	5
5	Borrego Mtn	1968	0409	0230	6.63	0.2	0.2	1
6	San Fernando	1971	0209	1400	6.61	0.2	0.2	8
7	Point Mugu	1973	0221	1445	5.65	0.2	0.2	1
8	Friuli, Italy-01	1976	0506	2000	6.5	0.2	0.2	2
9	Coyote Lake	1979	0806	1705	5.74	0.2	0.2	2
10	Imperial Valley-06	1979	1015	2316	6.53	0.2	0.2	4
11	Livermore-01	1980	124	1900	5.8	0.2	0.2	2
12	Mammoth Lakes-04	1980	0525	2035	5.7	0.2	0.2	1
13	Mammoth Lakes-06	1980	0527	1451	5.94	0.2	0.2	1
14	Victoria, Mexico	1980	609	328	6.33	0.2	0.2	4
15	Irpinia, Italy-01	1980	1123	1934	6.9	0.2	0.2	2
16	Taiwan SMART1(5)	1981	0129	0451	5.9	0.2	0.2	1
17	Coalinga-01	1983	0502	2342	6.36	0.2	0.2	40
18	Coalinga-02	1983	0509	0249	5.09	0.2	0.2	2
19	Coalinga-03	1983	611	309	5.38	0.2	0.2	1
20	Coalinga-04	1983	0709	0740	5.18	0.2	0.2	1
21	Taiwan SMART1(25)	1983	0921	1920	6.5	0.2	0.2	2
22	Borah Peak, ID-02	1983	1029	2329	5.1	0.2	0.2	1
23	Morgan Hill	1984	0424	2115	6.19	0.2	0.2	12
24	Taiwan SMART1(33)	1985	0612	1722	5.8	0.2	0.2	5
25	Taiwan SMART1(40)	1986	0520	0525	6.32	0.2	0.2	8
26	N. Palm Springs	1986	0708	0920	6.06	0.2	0.2	4
27	Chalfant Valley-03	1986	0721	1451	5.65	0.2	0.2	1
28	Kalamata, Greece-02	1986	0915	1141	5.4	0.2	0.2	1
29	San Salvador	1986	1010	1749	5.8	0.2	0.2	1
30	Whittier Narrows-01	1987	1001	1442	5.99	0.2	0.2	1
31	Superstition Hills-01	1987	1124	0514	6.22	0.2	0.2	1
32	Loma Prieta	1989	1018	0005	6.93	0.2	0.2	34
33	Georgia, USSR	1991	0615	0059	6.2	0.2	0.2	3
34	Big Bear-01	1992	0628	1506	6.46	0.2	0.2	5
35	Northridge-01	1994	0117	1231	6.69	0.2	0.2	26
36	Dinar, Turkey	1995	1001	1557	6.4	0.2	0.2	3
37	Chi-Chi, Taiwan	1999	0920		7.62	0.2	0.2	2
38	Sierra Madre	1991	0628		5.61	0.2	0.2	1
39	Northridge-06	1994	0320	2120	5.28	0.2	0.2	1
40	Hector Mine	1999	1016		7.13	0.2	0.2	1
41	Yountville	2000	0903		5	0.2	0.2	1
42	Big Bear-02	2001	0210		4.53	0.2	0.2	1
43	Mohawk Val, Portola	2001	0810		5.17	0.2	0.2	3
44	Anza-02	2001	1031		4.92	0.2	0.2	14
45	Gilroy	2002	0514		4.9	0.2	0.2	2
46	Big Bear City	2003	222		4.92	0.2	0.2	3
47	Chi-Chi, Taiwan-02	1999	0920	1757	5.9	0.2	0.2	2
48	Chi-Chi, Taiwan-03	1999	0920	1803	6.2	0.2	0.2	4
49	Chi-Chi, Taiwan-04	1999	0920	2146	6.2	0.2	0.2	16
50	Chi-Chi, Taiwan-05	1999	0922	0014	6.2	0.2	0.2	36
51	Chi-Chi, Taiwan-06	1999	0925	2352	6.3	0.2	0.2	41
								Total=317

4.2. Application Method

In an attempt to determine the statistical distribution of ordered peaks in linear earthquake response, these steps are followed in sequence.

- Step 1: Time History Response of Structural Systems

A preliminary step is to analyze the response of structural systems to earthquake motions. Piecewise linear exact method is used for determination of the displacement response time histories of linear SDOF systems over the whole range of natural vibration periods from very low, 0.1 sec, to very high, 5 sec, with 0.1 sec intervals. The viscous damping ratio, $\xi = 5$ %, is the same for all the above systems.

Below in Figure 4.1, a displacement response time history of a 2-second oscillator when subjected to the 2001, Anza earthquake recorded at Seven Oaks Dam Downstream Surf. station has been presented.

In this determination procedure, time history responses of structural systems have been evaluated for the two horizontal components namely: East-West and North-South components (EW and NS) of 317 accelerograms, which are recorded at different parts of the world during various earthquakes. All of these ground motions forming the database are presented in Table 4.1.

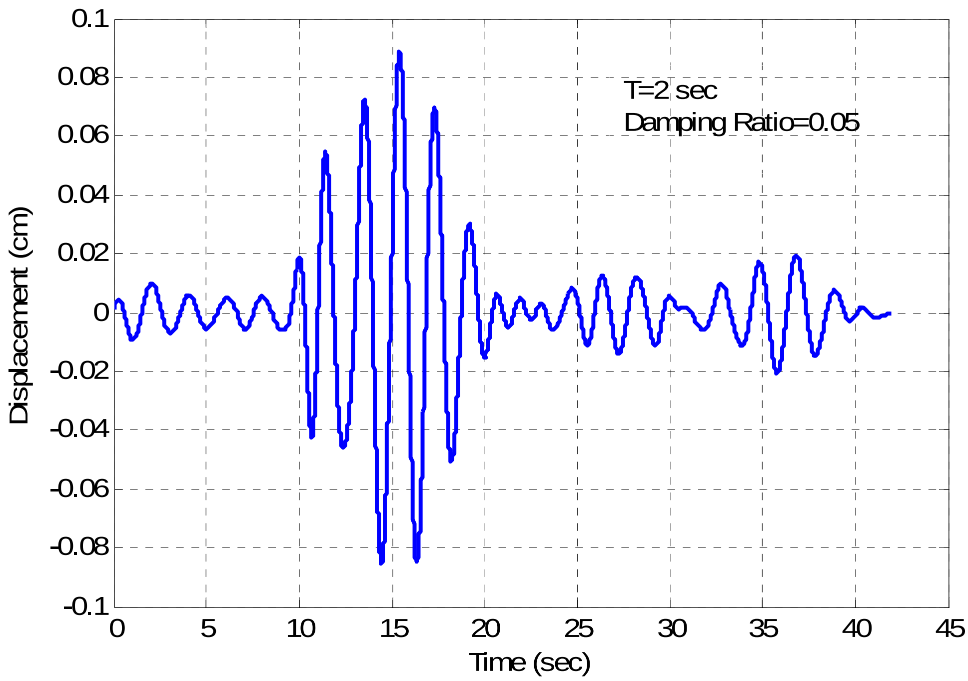

Figure 4.1. Displacement response time history of SDOF system

with ξ= 5% and T_n=2 sec.

- Step 2: Order Statistics Approach Application

In order to apply order statistics, firstly the values of local maxima of each displacement response time history have been determined. Then, all of these response peaks are arranged in decreasing order of amplitudes as illustrated in Figure 4.2 for all SDOF systems which are different in their natural vibration periods and are similar in their damping ratios.

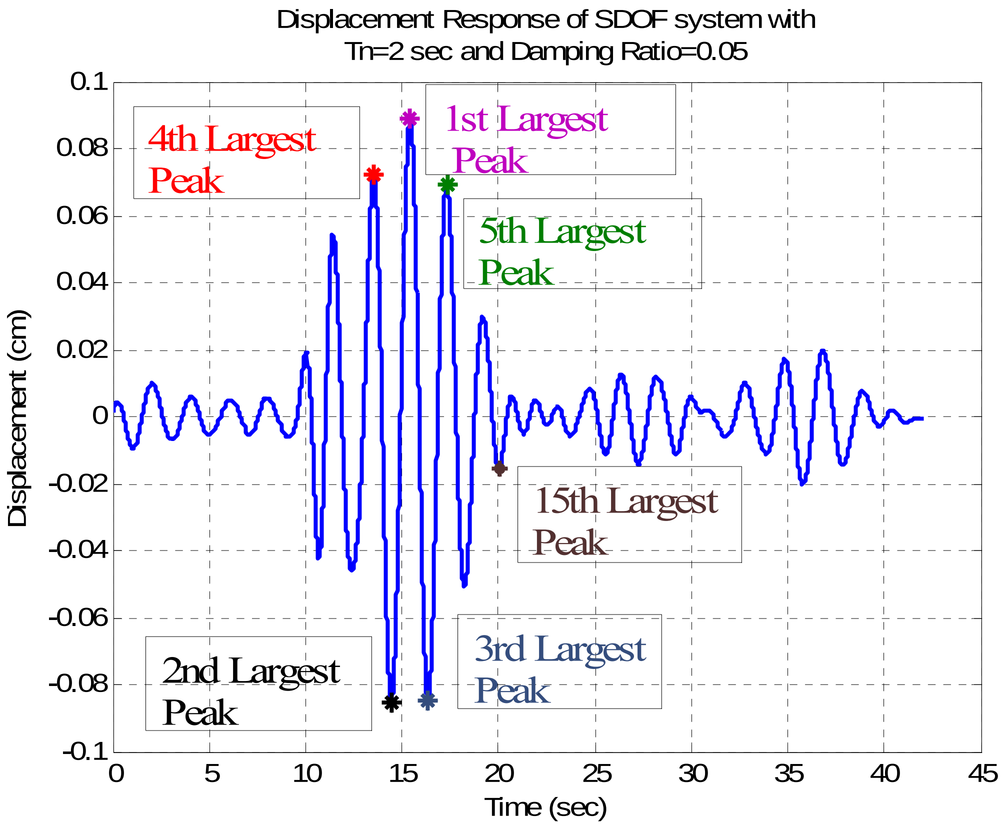

Figure 4.2. Typical example of displacement response time history of a 2-second oscillator with first, second, third order, etc., peaks

The above figure depicts the order of response displacement peaks of SDOF system with 5 % damping ratio and 2 second natural vibration period under 2001, Anza earthquake recorded at Seven Oaks Dam Downstream Surf. station (i.e., local maximum values of displacement response time history, maxima of displacement response time history).

- Step 3: 3D Response Spectra Calculation

In order to help a designer to consider the relationship between the amplitudes of response maxima to the physical characteristics of structural system and to present the results of the expected and most probable amplitudes of various orders of peaks, 3D Response Spectra plots have been constituted for the SDOF systems with five percent damping.

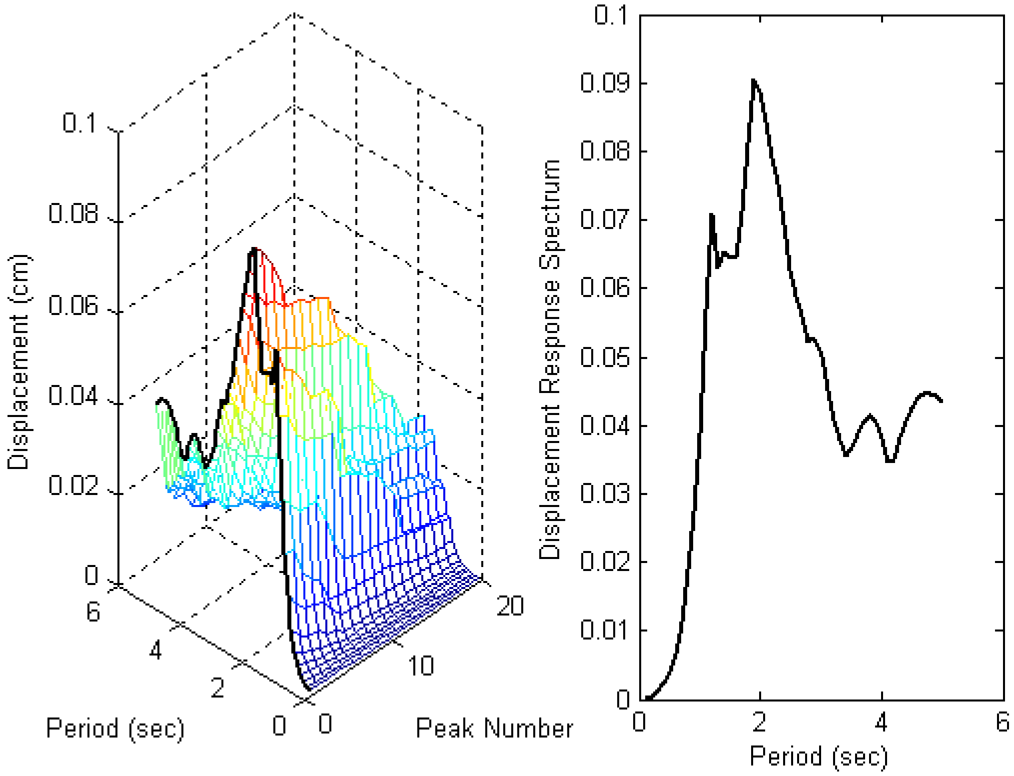

Figure 4.3. 3D Response Spectra and Standard Response Spectra of a series of linear SDOF systems under 2001, Anza earthquake

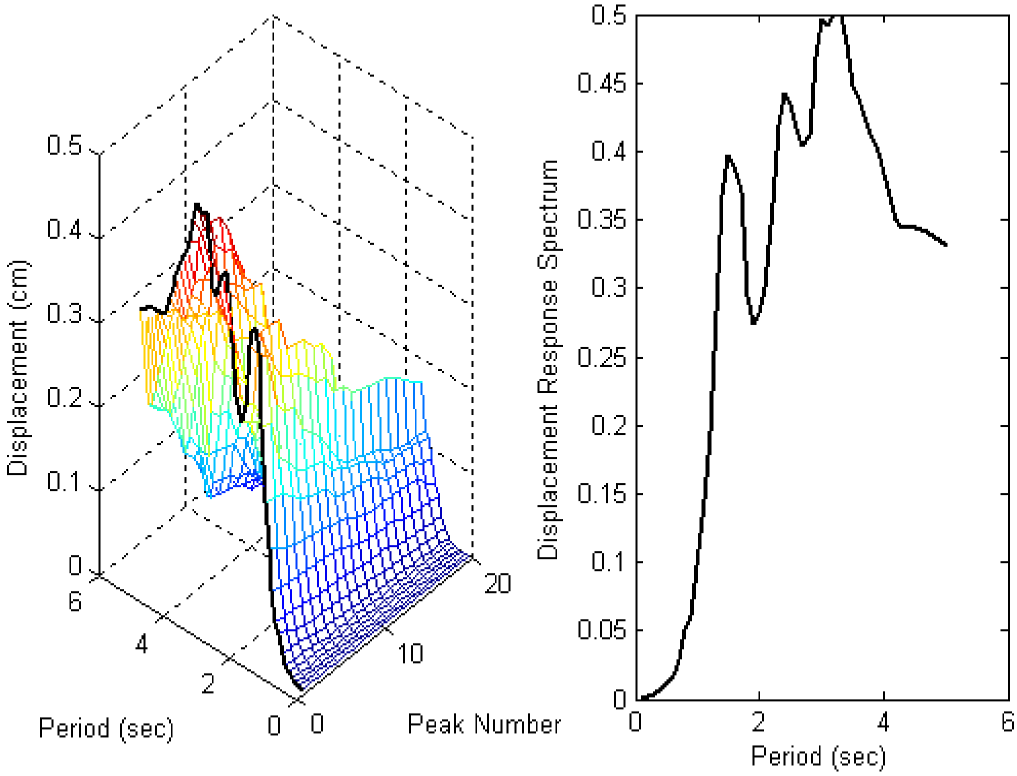

Figure 4.4. 3D Response Spectra and Standard Response Spectra of a series of linear SDOF systems under 1995, Dinar earthquake

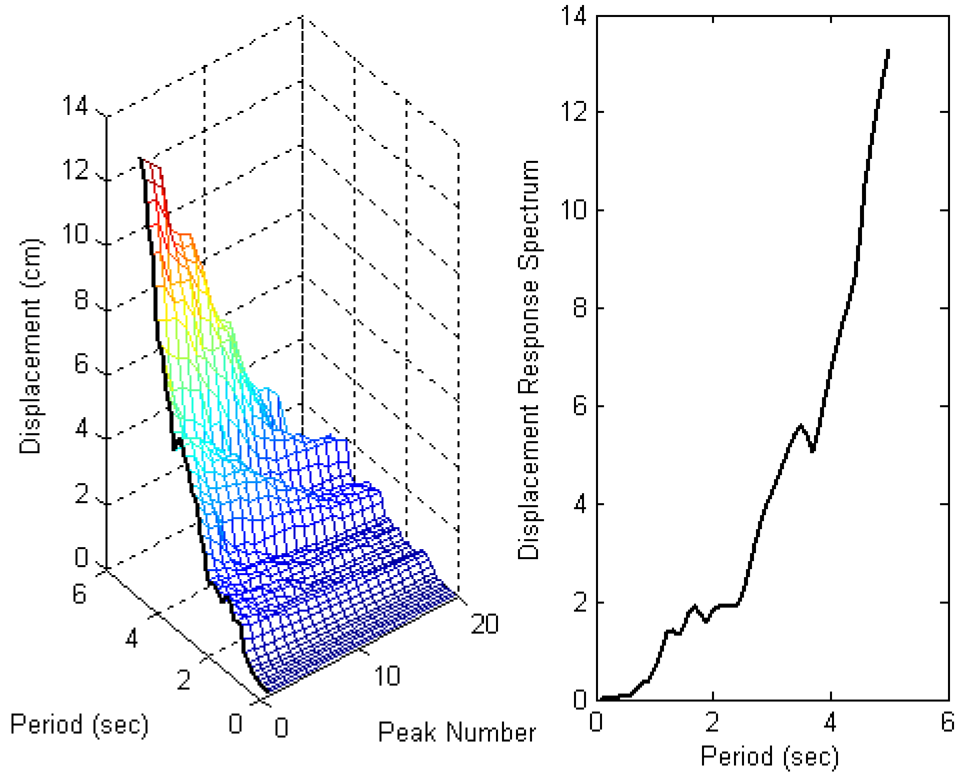

Figure 4.5. 3D Response Spectra and Standard Response Spectra of a series of linear SDOF systems under 1999, Hector Mine earthquake

In Figures (4.3), (4.4) and (4.5), the 3D displacement response spectra of linear SDOF systems with five per cent damping subjected to 2001 Anza, 1995 Dinar and 1999 Hector Mine earthquakes recorded at Seven Oaks Dam Downstream Surf., İzmir Trigger -2 , and Heart Bar State Park stations, respectively are given for the first 20 largest peaks.

In these Figures, thicker lines mark the standard response spectra. As it is seen, 3D Response Spectra give information about the amplitudes of the largest, second largest, third largest,….maxima of the response of different SDOF oscillators. This information is then be used to obtain the distribution of the peaks of the oscillator’s response.

- Step 4: Combination of EW and NS Components' Response Peak Amplitudes for Each Ground Motion

In this stage, the largest, second largest, third largest, …, fiftieth largest peak amplitudes, which are putted in order at the previous stage, are designated for each SDOF system (thus for each vibration period).

Then, for every SDOF system, obtained totally 100 peaks' amplitudes from the horizontal components of each earthquake record are combined together and the highest 50 of them are chosen as a result of sorting the amplitudes of gathered 100 peaks in descending order.

- Step 5: Normalization

The values of 50 highest ordered response peaks of each structural system are scaled to the maximum response peak value (that corresponds to the amplitude of the 1^{st} peak) of the oscillator in question. This normalization is needed in order to eliminate the influences of the factors mentioned in Section 1.1 such as earthquake magnitude, site-to–source distance, local site conditions, wave propagation path, etc., on the variations of ground motion parameters.

- Step 6: Mean Values of Ordered Peaks

In order to obtain earthquake independent and more general results, the normalized peak amplitudes of various orders of peaks are averaged for all considered SDOF systems.

This procedure can be explained simply as follows.

When a SDOF system subjected to the EW and NS components of a particular earthquake ground motion, the combined 50 largest normalized response peaks' amplitudes are taken into account for the forthcoming statistical evaluation. In this study, a ground motion database consisting of 317 earthquake excitation. Therefore, 1st largest peak, 2nd largest peak, 3rd largest peak, …, 50th largest peak of a specified SDOF oscillator will have 317 different normalized amplitudes.

By averaging these 317 different normalized values of each ordered response peak, the mean values of 50 normalized peak amplitudes of each oscillator response can be presented independently of earthquake characteristics.

After 50 largest averaged normalized response peaks' amplitudes have been obtained, the analysis based on probabilistic techniques is carried out to define the distribution of the peaks for different vibration periods in the following order.

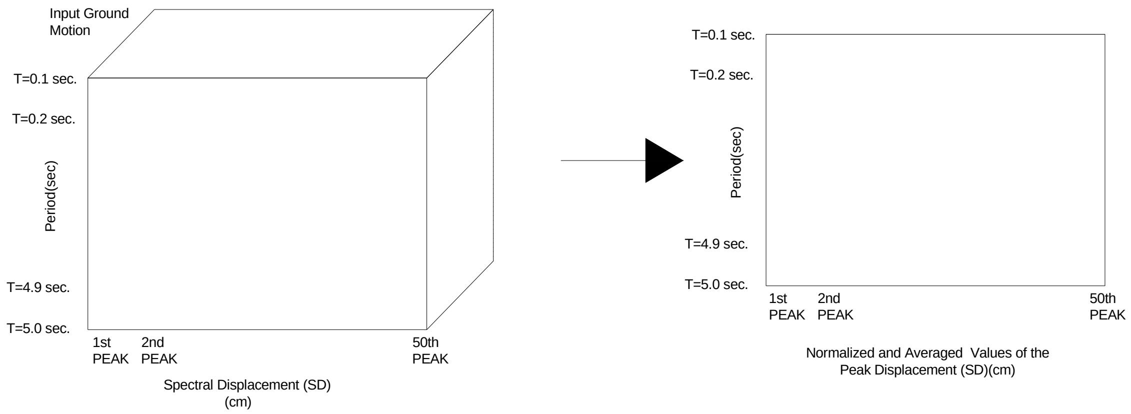

Figure 4.6. Illustrative figure which represents the applied procedure

- Step 7: Determination of the Probability Distribution of Peaks in Linear Earthquake Response

Using histogram will be necessary to group data, to portray it and to illustrate the distribution of a dataset. Therefore, histograms of the 50 normalized and averaged amplitudes of peaks at each period of vibration have been created.

Aside from using histograms to visualize the data, they can be applied to compare the fitted distributions to sample data and to select the best fitting model, or at least to reject the distributions that do not fit to data very well.

In order to fit a curve through the intersection of (x, y) points of the histogram, the histogram must be re-scaled so it is integral is one which is always true for a density function as previously stated. Only then, a curve (probability density function (PDF) or probability distribution curve) which provides an improved fit for overall statistical characterization of the peaks can be determined.

There are many probability distributions, however in this thesis, four distributions namely: Gaussian, Rayleigh, Exponential and Weibull are considered as candidates in order to observe the best model and fit which describes the distribution of peaks.

As for why these distributions were chosen follows that; According to Decanini et al. (2003), The Normal (Gaussian) is one of the most adopted probability model in civil engineering practice. In addition to this, the probability density function of the maxima of the structural response process can be represented by a Gaussian (Normal) distribution or a Rayleigh distribution of which the former is valid for a wide band response and the latter is valid in the case of a narrow band response (Şafak, 1988, 1998; Gupta and Basu, 1996; and Cartwright and Higgings, 1956). Moreover, we have also tested the other two well known distribution functions, the Exponential (a special case of the Weibull) and the Weibull to provide a unified description of the structural response peaks' probability distribution.

The comparison of these distributions was made for each SDOF system (i.e., for each vibration period). However, only the results of a 0.5 second oscillator and a 2 second oscillator were presented.

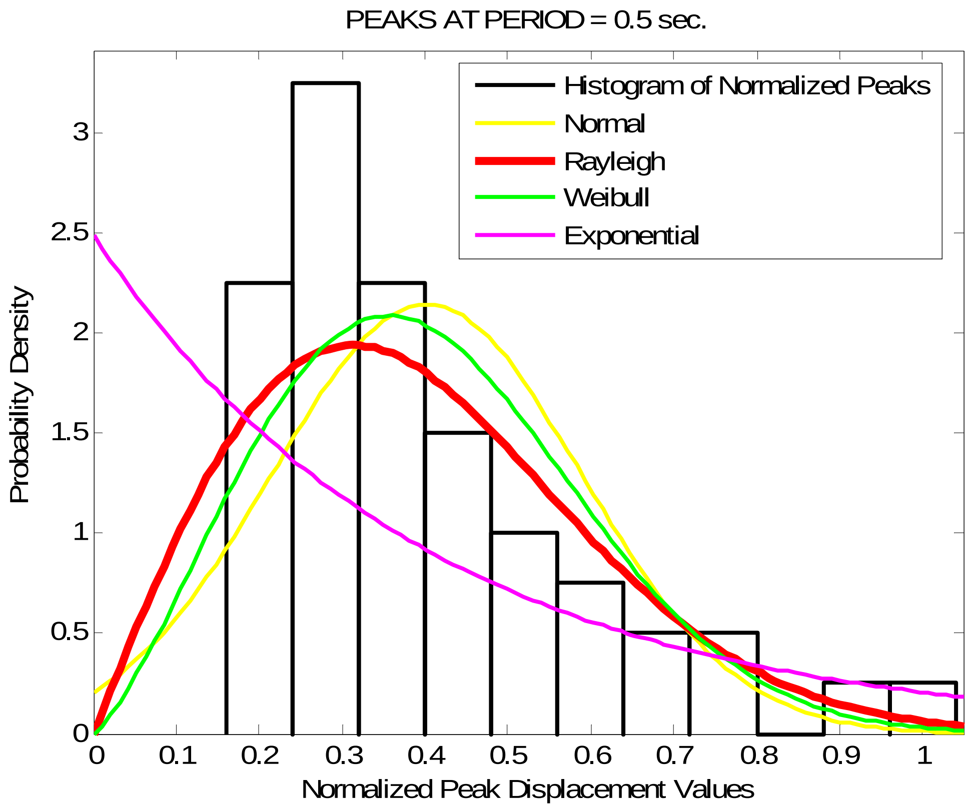

Figure 4.7. Probability density functions (probability distributions) for normalized displacement peaks of a 0.5 second oscillator

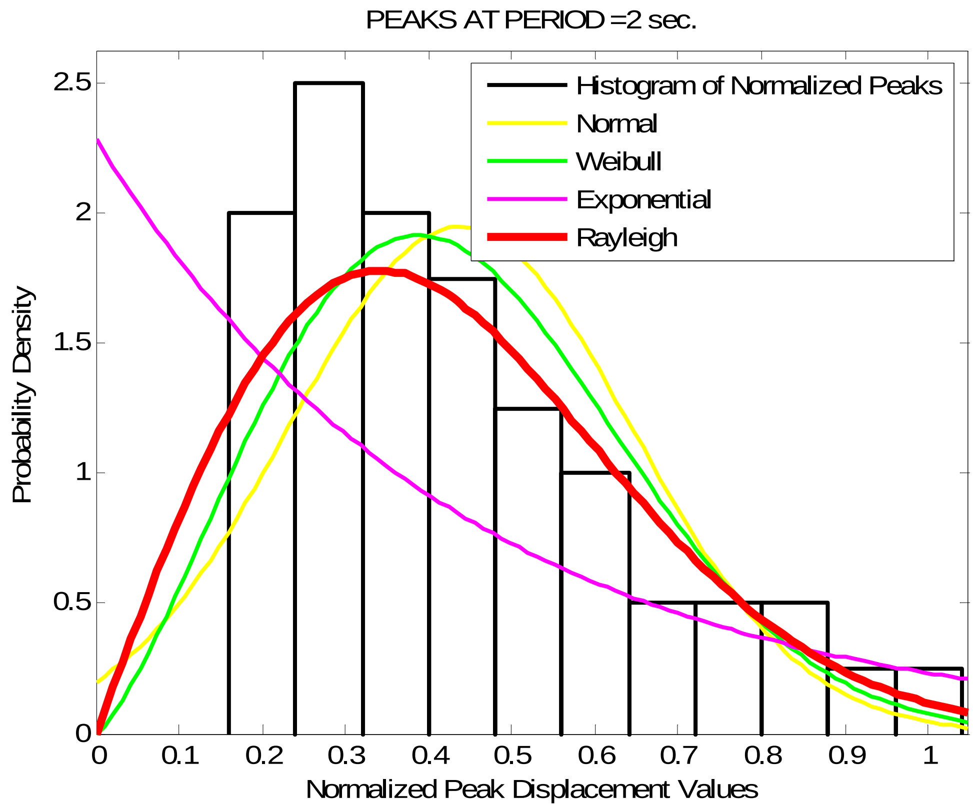

Figure 4.8. Probability density functions (probability distributions) for normalized displacement peaks of a 2 second oscillator

The parameters of these distributions have been estimated by using statistical Toolbox software (Matlab Toolboxes). In order to specify the distribution parameters, this toolbox have applied the Maximum Likelihood Estimation Method (MLE).

4.3. Maximum Likelihood Estimation Method

Maximum Likelihood Estimation (MLE) is one of the most popular statistical methods that are used to fit a statistical model to data and to provide estimates for the applied model's parameters. As the name suggests, the estimator will be the value of the parameter that maximizes the likelihood function. Maximum likelihood estimator can be used if the distribution of the data has been either known or assumed.

Maximum likelihood estimates for four types of distribution are given as follows.

4.3.1. MLE for Normal Distribution

The maximum likelihood estimator for the mean, $\hat{\mu}$ and variance, $\hat{\sigma}^2$ parameters are defined by the equation

$$\hat{\mu} = \bar{X} = \frac{1}{n}\sum_{i=1}^{n} X_i \tag{4.1}$$

$$\hat{\sigma}^2 = \frac{1}{n}\sum_{i=1}^{n}\left(X_i - \bar{X}\right)^2 \tag{4.2}$$

This method is based on generating a sample X1, …., Xn from a normal N(μ , σ^2) population.

4.3.2. MLE for Rayleigh Distribution

Given N independent and identically distributed Rayleigh random variables with parameter, σ, the maximum likelihood estimate of $\hat{\sigma}$ is

$$\hat{\sigma} = \sqrt{\frac{1}{2N} + \sum_{i=1}^{N}\left(X_i^{\,2}\right)} \tag{4.3}$$

4.3.3. MLE for Weibull Distribution

The maximum likelihood estimator for the Weibull distribution having two parameters, namely shape, $\hat{\beta}$ and scale, $\hat{\eta}$ parameters are defined by the below equations (Zaharim et al., 2009).

$$\hat{\eta} = \left[\left(\frac{1}{n} \right) \sum_{i=1}^{n} \left(X_i^{\hat{\beta}} \right) \right]^{1/\hat{\beta}} \tag{4.4}$$

$$\hat{\beta} = \frac{n}{\frac{1}{\hat{\eta}} \sum_{i=1}^{n} X_i^{\hat{\beta}} \log X_i - \sum_{i=1}^{n} \log X_i} \tag{4.5}$$

4.3.4. MLE for Exponential Distribution

The maximum likelihood estimate for the rate parameter, $\hat{\lambda}$, of the exponential distribution, a variable which is supposed to be distributed exponentially has been generated.

$$\hat{\lambda} = \frac{1}{\bar{X}} \tag{4.6}$$

Where, $\bar{X}$ is the sample mean.

$$\bar{X} = \frac{1}{n} \sum_{1}^{n} X_i \tag{4.7}$$

After the estimation of each distribution function's related parameters, goodness-of-fit test has been conducted in order to verify the assumption that the response peaks are distributed according to one of the previously defined theoretical probability density functions.

4.4. Goodness of Fit Test

To assess the underlying theoretical distribution of the data, evaluation of the fit plots only visually will not be sufficient. Therefore, goodness-of-fit tests should be performed in order to make a final assessment about the performance of the observed theoretical probability distributions and to see if the data came from some specified population.

Chi-squared, Anderson-Darling (A-D) and Kolmogorov-Smirnov (K-S) statistical tests are the most popular ones when establishing the goodness of fit between theoretical and observed distributions.

In this thesis, the Chi-square goodness of fit test is used to rank the fitted four distributions. The Chi-square test was adopted as it has the virtue that it can be used to assess model fit for most distributions (Meagher, 2003).

4.4.1. Chi-Square Test Application

The test procedure requires a random sample of size n from the population whose probability distribution is unknown. These n observations are arranged in a frequency histogram having k bins or class intervals. Let O_i be the observed frequency in the i^{th} class interval. From the hypothesized probability distribution, we compute the expected frequency in the i^{th} class interval, denoted E_i (Montgomery, 2003). The test statistic is;

$$X_0^2 = \sum_{i=1}^{k} \left(\frac{(O_i - E_i)^2}{E_i} \right) \tag{4.8}$$

Where X_0^2 is the test statistic value that reflects the departure of observed from expected. Intuitively if there is a big discrepancy between the observed frequencies and the expected frequencies, this value is large and if not, it is small.

The Chi-square goodness-of-fit test has been applied for determination of each oscillator's response peaks' distribution. However, the results of this test are given for the peaks of a 0.5 second oscillator in Table 4.2.

Table 4.2. Goodness of fit test results for the normalized and averaged displacement peaks of a 0.5 second oscillator

Class Interval	Observed Frequency (Oi)	Expected Frequency, (Ei)	Chi-Square Value	
0.2	0	6.95	6.95	Normal Distribution
0.2<x<=0.28	16	5.86	17.56	
0.28<x<=0.36	11	7.73	1.38	
0.36<x<=0.44	7	8.50	0.27	
0.44<x<=0.52	5	7.80	1.00	
0.52<x<=0.6	4	5.97	0.65	
0.6<x<=0.68	2	3.81	0.86	
0.68<x<=0.76	2	2.02	0.00	
0.76<x<=0.84	1	0.90	0.01	
0.84<x<=0.92	1	0.33	1.35	
0.92<x<=1	1	0.10	7.88	
Totals	50	50.0	37.91	
Class Interval	Observed Frequency (Oi)	Expected Frequency, (Ei)	Chi-Square Value	
0.2	0	9.25	9.25	Rayleigh Distribution
0.2<x<=0.28	16	7.26	10.51	
0.28<x<=0.36	11	7.71	1.40	
0.36<x<=0.44	7	7.19	0.01	
0.44<x<=0.52	5	6.03	0.18	
0.52<x<=0.6	4	4.61	0.08	
0.6<x<=0.68	2	3.53	0.47	
0.68<x<=0.76	2	2.09	0.00	
0.76<x<=0.84	1	1.25	0.05	
0.84<x<=0.92	1	0.70	0.13	
0.92<x<=1	1	0.36	1.14	
Totals	50	50.0	23.22	
Class Interval	Observed Frequency (Oi)	Expected Frequency, (Ei)	Chi-Square Value	
0.2	0	19.60	19.60	Exponential Distribution
0.2<x<=0.28	16	5.96	20.15	
0.28<x<=0.36	11	4.50	9.41	
0.36<x<=0.44	7	3.68	2.98	
0.44<x<=0.52	5	3.94	1.30	
0.52<x<=0.6	4	3.13	0.94	
0.6<x<=0.68	2	2.03	0.00	
0.68<x<=0.76	2	1.66	0.07	
0.76<x<=0.84	1	2.28	0.10	
0.84<x<=0.92	1	2.04	0.01	
0.92<x<=1	1	1.18	0.01	
Totals	50	50.0	54.56	
Class Interval	Observed Frequency (Oi)	Expected Frequency, (Ei)	Chi-Square Value	
0.2	0	6.93	6.93	Weibull Distribution
0.2<x<=0.28	16	6.93	11.86	
0.28<x<=0.36	11	8.15	0.99	
0.36<x<=0.44	7	8.12	0.15	
0.44<x<=0.52	5	7.01	0.58	
0.52<x<=0.6	4	5.31	0.32	
0.6<x<=0.68	2	3.55	0.67	
0.68<x<=0.76	2	2.09	0.00	
0.76<x<=0.84	1	1.09	0.01	
0.84<x<=0.92	1	0.51	0.48	
0.92<x<=1	1	0.31	3.06	
Totals	50	50.0	25.07	

It is obvious that the test statistics have minimum value for the Rayleigh distribution. For this reason, it has been concluded that the distribution that best represents the dataset is the Rayleigh distribution.

4.5. Results

Evaluation of the histogram plots of peaks visually and the goodness-of-fit test parameter, X_0^2, numerically shows that the probability density function (PDF) of the ordered peaks in linear earthquake displacement response of different SDOF oscillators can be represented by a Rayleigh distribution of the previously stated form.

Therefore, the result figures were only showed for the Rayleigh distribution.

As an example, Figure 4.9 illustrates the resulting probability density functions of the normalized and averaged amplitudes of the response peaks only for SDOF systems having vibration periods 0.1, 0.5, 1, 1.5, 2, 2.5, 3, 3.5, 4, 4.5, and 5 seconds. The histograms of the peaks, the matching Rayleigh distribution function and the fitted Rayleigh parameter are also shown in Figure 4.9.

In addition to PDF plots, cumulative distribution functions (CDF) over the considered variable are reported in Figure 4.10 for the same vibration periods to show the cumulative values of the PDF. These curves plot the probabilities of exceeding or not exceeding particular peak amplitude.

Such information is very useful for seismic design. Seismic design is primarily concerned with the balance between the potential of ground motion to cause damage on a particular structural system (demand) and the structure's ability to resist the seismic demand (capacity). According to Boore and Bommer (2005), when the seismic capacity of engineered structures can be assessed from experimentation, analytical modeling and field observations following earthquakes, and indeed from the interaction of these three channels of investigation, the destructive capacity of the ground motion (seismic demand) is represented by displacement response spectrum (Crowley et al., 2004).

However, the displacement demand imposed on structural systems shall not be quantified only with the largest spectral displacement amplitude of the response (conventionally used response spectrum technique). Because, the use of only response spectrum for design of earthquake- resistant structures clearly ignores much valuable information on overall structural response such as the overall duration of response and distribution of response maxima (Amini and Trifunac, 1981).

Therefore, especially considering cumulative damage, understanding all the peak amplitudes of the response and determining the statistical distribution functions of these peaks are necessary, because these repeated inelastic excursions in the duration of the ground motion have appreciable effects on the performance of structures.

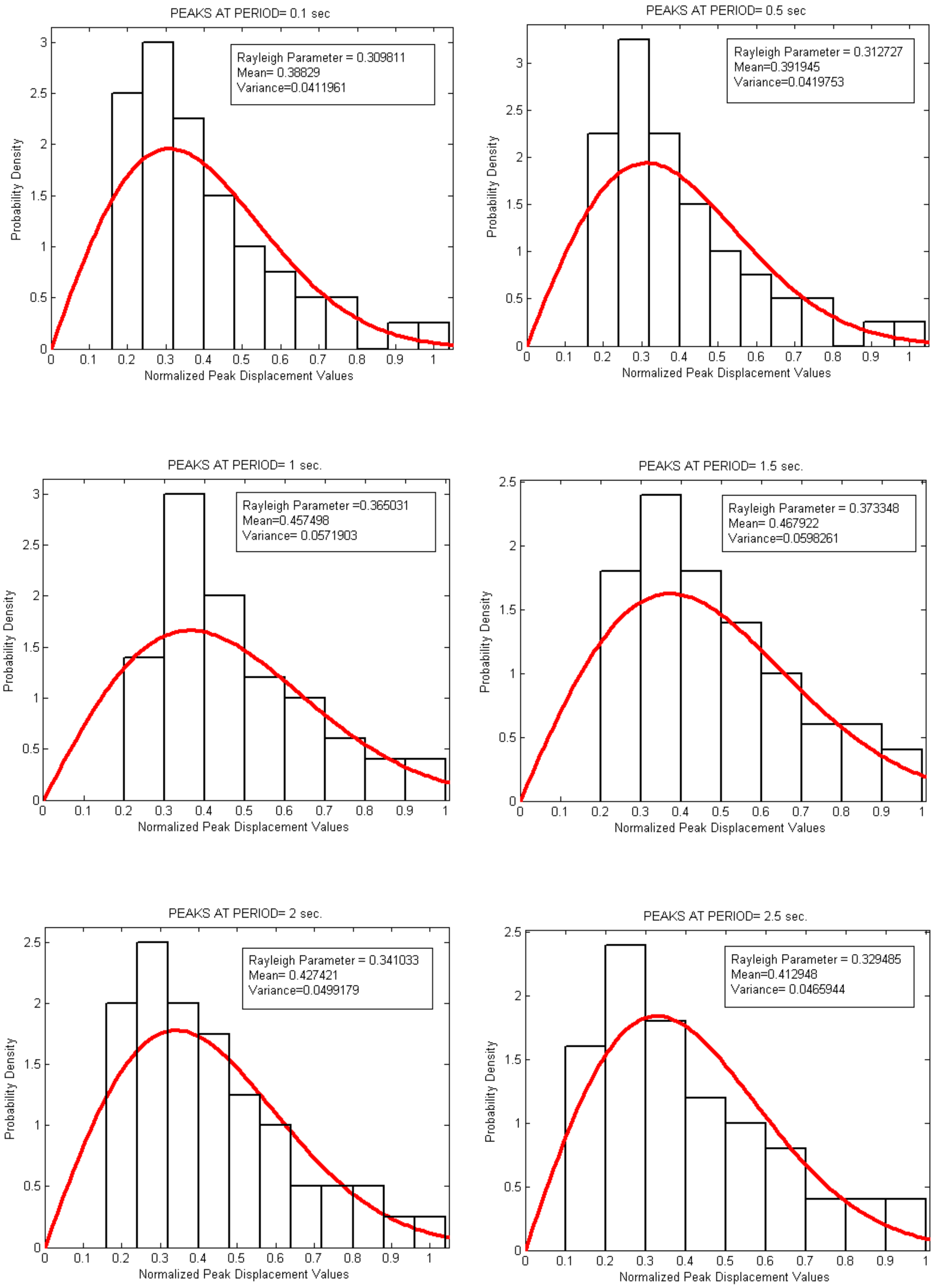

Figure 4.9. Distribution of the normalized and averaged displacement peaks of different linear SDOF oscillators with five percent (5%) damping

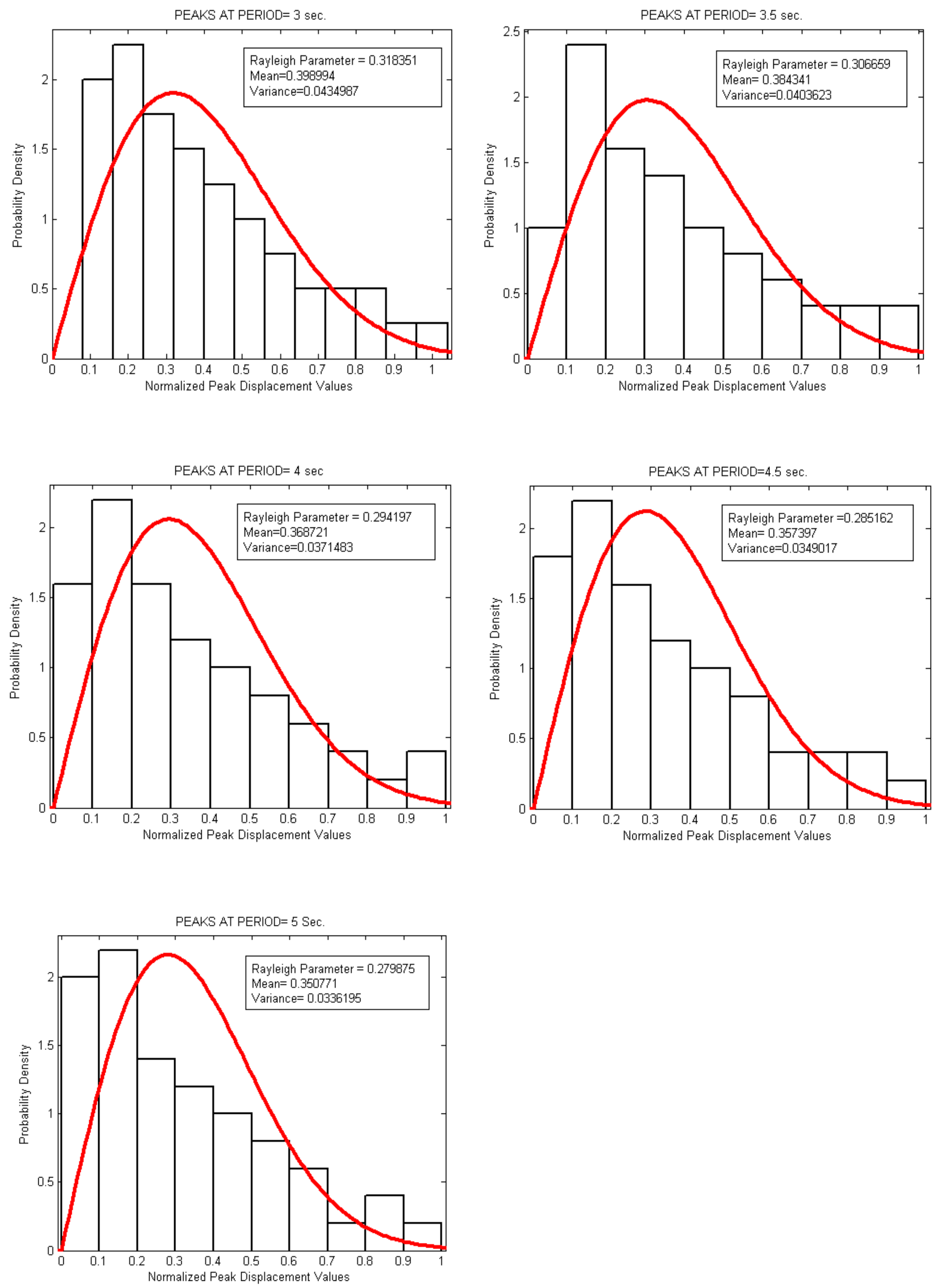

Figure 4.9. Distribution of the normalized and averaged displacement peaks of different linear SDOF oscillators with five percent (5%) damping (continue)

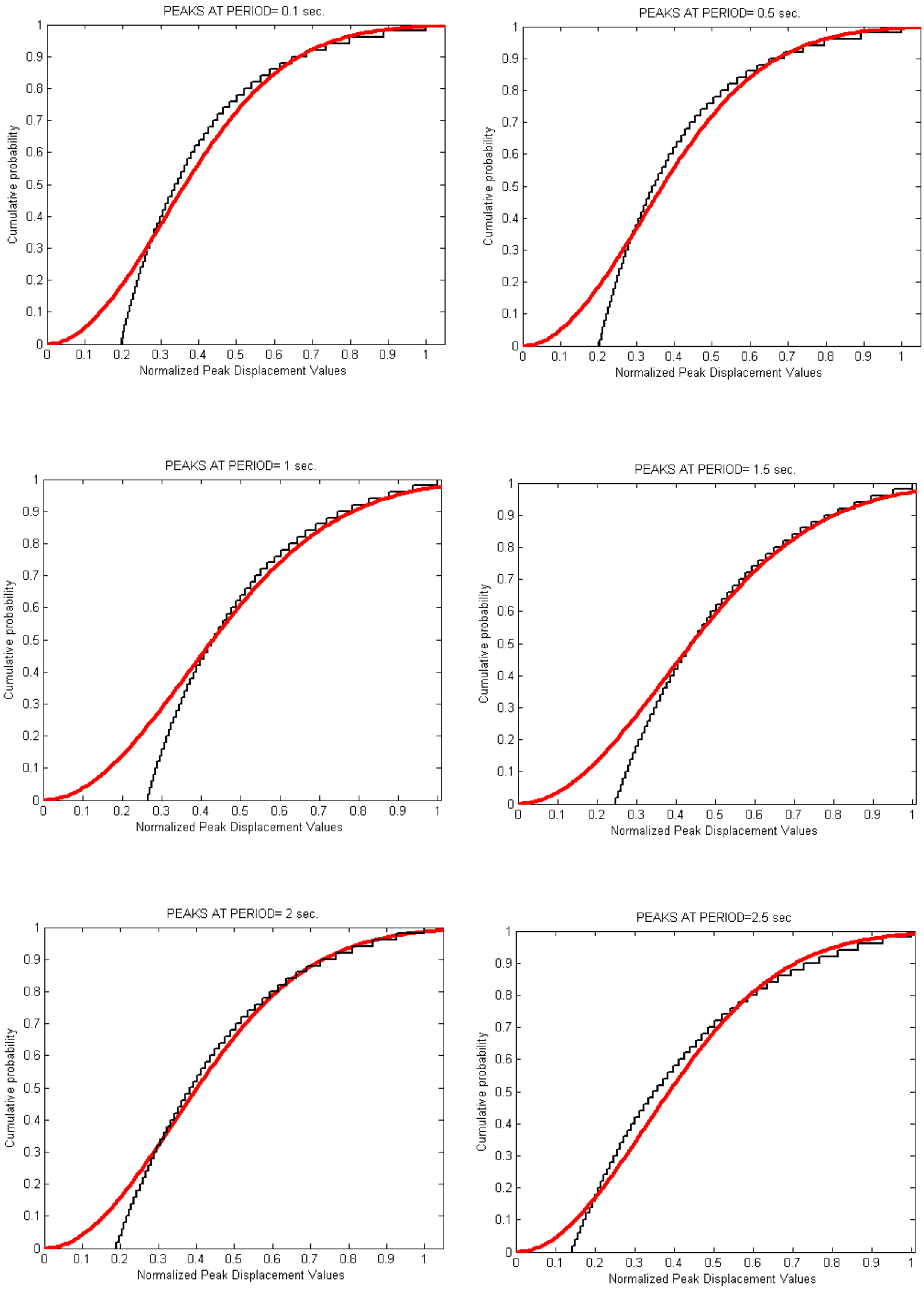

Figure 4.10. Cumulative distribution function (CDF) of the normalized and averaged displacement peaks of different linear SDOF oscillators with five percent (5%) damping

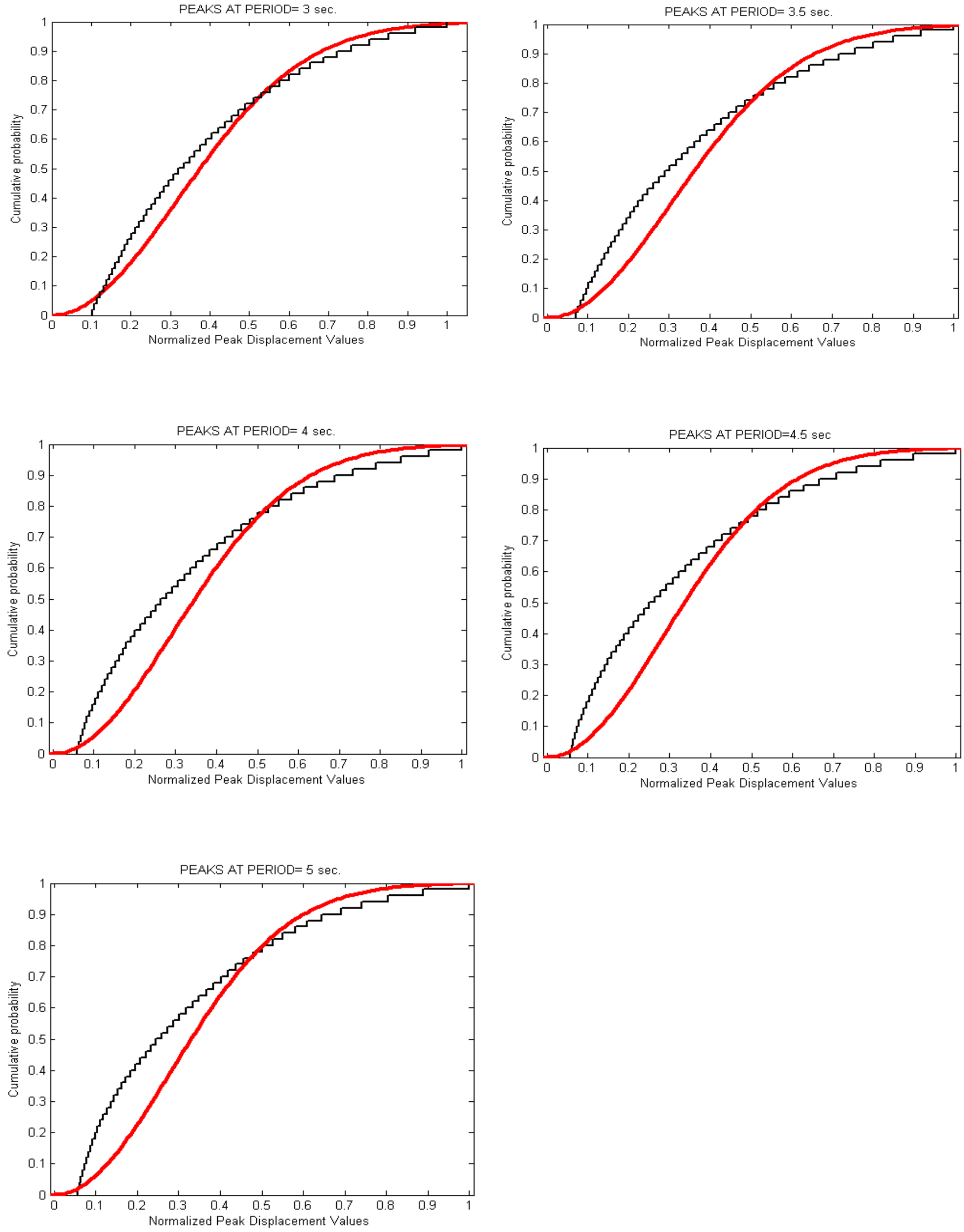

Figure 4.10. Cumulative distribution function (CDF) of the normalized and averaged displacement peaks of different linear SDOF oscillators with five percent (5%) damping (continue)

Table 4.3. Key parameter (Rayleigh parameter) which defines the distribution of the different oscillators' response peaks

Oscillator Period (sec)	Rayleigh Parameter	Mean	Variance
0.1	0.309811	0.38829	0.0411961
0.2	0.310082	0.38863	0.0412682
0.3	0.310619	0.389304	0.0414115
0.4	0.311535	0.390452	0.041656
0.5	0.312727	0.391945	0.0419753
0.6	0.314298	0.393914	0.0423981
0.7	0.320729	0.401975	0.0441511
0.8	0.327006	0.409841	0.0458959
0.9	0.347457	0.435473	0.0518163
1	0.365031	0.457498	0.0571903
1.1	0.371144	0.46516	0.0591218
1.2	0.370542	0.464406	0.0589303
1.3	0.370201	0.463978	0.0588217
1.4	0.373587	0.468222	0.0599028
1.5	0.373348	0.467922	0.0598261
1.6	0.365057	0.457531	0.0571985
1.7	0.356595	0.446926	0.0545776
1.8	0.352473	0.441759	0.0533231
1.9	0.34639	0.434135	0.0514984
2	0.341033	0.427421	0.0499179
2.1	0.34024	0.426427	0.0496859
2.2	0.338313	0.424013	0.0491249
2.3	0.33419	0.418845	0.0479348
2.4	0.332178	0.416324	0.0473594
2.5	0.329485	0.412948	0.0465944
2.6	0.325796	0.408324	0.0455569
2.7	0.322428	0.404104	0.04462
2.8	0.32108	0.402414	0.0442475
2.9	0.319832	0.40085	0.0439043
3	0.318351	0.398994	0.0434987
3.1	0.317008	0.39731	0.0431323
3.2	0.315483	0.395399	0.0427184
3.3	0.312732	0.391952	0.0419767
3.4	0.309117	0.387421	0.0410118
3.5	0.306659	0.384341	0.0403623
3.6	0.303794	0.380749	0.0396115
3.7	0.300933	0.377164	0.038869
3.8	0.2982	0.373738	0.0381661
3.9	0.296297	0.371353	0.0376806
4	0.294197	0.368721	0.0371483
4.1	0.292077	0.366064	0.0366149
4.2	0.289831	0.363249	0.0360539
4.3	0.288019	0.360978	0.0356045
4.4	0.28653	0.359112	0.0352373
4.5	0.285162	0.357397	0.0349017
4.6	0.283739	0.355614	0.0345541
4.7	0.282482	0.354039	0.0342488
4.8	0.28172	0.353084	0.0340642
4.9	0.280975	0.35215	0.0338843
5	0.279875	0.350771	0.0336195

Above in Table 4.3, calculated key parameters, which define the distribution of the displacement peaks for each oscillator vibration period with five percent damping, have been tabulated. By using these key parameters (Rayleigh parameter), all of the aforementioned properties of the response can be determined.

5. CONCLUSIONS

Response spectrum is of great significance to the seismic design. Although it is the important and widely used tool for description and characterization of strong ground motion amplitudes, it only represents the amplitude of the highest peak of the response over entire duration of ground shaking. Thus it clearly ignores the much valuable information on overall inelastic deformation time history such as the number, sequence and the relative amplitudes of the inelastic excursions.

However, these repeated inelastic excursions have appreciable effects on the structural response. Therefore, for better understanding of the response characteristics of structures to earthquake excitation, it is necessary to study the statistics of response maxima.

In this study, a probabilistic theory, based on order statistics approach has been applied to determine the theoretical distribution functions for the maxima of linear SDOF systems' response under earthquake excitation.

For this determination procedure, a subset of NGA data comprising 317 pairs of horizontal records with lowest usable frequency 0.2 Hz obtained during 51 earthquakes has been used. In addition, the response of SDOF systems over the whole range of vibration periods from very low, 0.1 sec, to very high, 5 sec, and with a damping value of five percent of critical is considered.

Evaluation of the histogram plots of the 50 largest normalized response peaks visually and the results of the goodness-of-fit test, Chi-Square statistical test, numerically have showed that the probability density function, PDF, of the ordered peaks in linear earthquake response of SDOF systems can be represented by a Rayleigh distribution.

Based on this result, key parameters, Rayleigh parameters, estimated by using Maximum Likelihood Estimation method have been presented for each SDOF system (for each vibration period) in order to define the distribution of the response maxima.

Once the statistical distribution of the maxima of the structural response is determined, then the following important information can be obtained on the basis of the presented key parameters.

i. Expected and most probable amplitudes of the n-th order (first largest, second largest, third largest, etc.) peaks of the response of linear, viscously damped SDOF system
ii. The probabilities of exceeding or not exceeding particular response peak amplitudes (by using the CDF plots)
iii. The number of times certain response levels may be exceeded
iv. The relationship between all local response maxima, their number and amplitudes to the physical characteristics of the designed structural system
v. The effective structural response duration
vi. The progressing of damage in a structure

Such kind of information can not be extracted from the conventionally used response spectra and it is very useful for seismic design as it gives better idea about the damage potential of an earthquake (demand).

Furthermore, this study gives opportunity to verify the results of previously conducted studies on this subject by using large database of real ground motion records collected from all around the world. This verification is required because in these studies, different assumptions have been made and/or different methods have been applied to determine the statistical distribution of the maxima besides using just theoretical derivations for determination of the distribution of response maxima and/or considering synthetic (artificial) accelerograms for confirmation of the obtained theoretical results.

The results reveal that the corresponding ordered peak distribution at each vibration period is in good agreement with the result obtained from the theoretical derivations which are used in the previously presented papers.

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